Light and Special Relativity

Module 7 Nature of Light

Light and Special Relativity

Inquiry question: How does the behaviour of light affect concepts of time, space and matter?

Students:

- analyse and evaluate the evidence confirming or denying Einstein's two postulates:
  - the speed of light in a vacuum is an absolute constant
  - all inertial frames of reference are equivalent (ACSPH131)
- investigate the evidence, from Einstein's thought experiments and subsequent experimental validation, for time dilation $t = \frac{t_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$ and length contraction $l = l_0\sqrt{\left(1-\frac{v^2}{c^2}\right)}$, and analyse quantitatively situations in which these are observed, for example:
  - observations of cosmic-origin muons at the Earth's surface
  - atomic clocks (Hafele–Keating experiment)
  - evidence from particle accelerators
  - evidence from cosmological studies

- describe the consequences and applications of relativistic momentum with reference to:
  - $p_v = \frac{m_0 v}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$
  - the limitation on the maximum velocity of a particle imposed by special relativity (ACSPH133)
- Use Einstein's mass–energy equivalence relationship $E = mc^2$ to calculate the energy released by processes in which mass is converted to energy, for example: (ACSPH134)
  - production of energy by the sun
  - particle–antiparticle interactions, eg positron–electron annihilation
  - combustion of conventional fuel

Einstein’s Postulates - Principle of Relativity

<u>Postulate - all inertial frames of reference are equivalent</u> (Principle of Relativity)

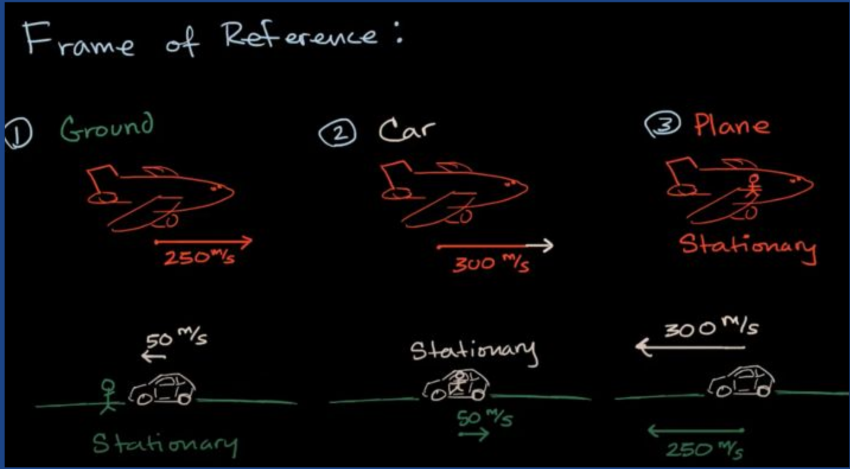
- Frame of reference (FOR) – a coordinate system (or fixed reference point) from which the motion of objects can be measured
- Inertial FOR – FOR where all forces are balanced; constant velocity or stationary
- All inertial (I) FOR are equal which means all measurements made from all IFOR are equally valid

Einstein’s Postulates - Principle of Relativity

- There is no single absolute FOR since every IFOR can be BOTH stationary or moving at a constant velocity relative to another IFOR.
    - E.g. a car travelling at 60 km/h East will be measured as travelling at 60 km/h East relative to a ‘stationary’ point along the road. However the ‘stationary’ point along the road will be measured as moving at 60 km/h West from the car. Furthermore, the car will be deemed stationary when its motion is measured from another vehicle also travelling at 60km/h East.
- Simply, all measurements of velocity are relative to another

Einstein's Postulates - Principle of Relativity

- There is no experiment that could be done inside an IFOR that could tell us if the FOR is moving or stationary
  - E.g. looking out a car window at night travelling at constant v it is impossible to tell if the car is moving or at rest (try this on a smooth road in an electric car, it feels weird because your mind knows you're moving but your body doesn't feel it)
- Videos:
- https://www.youtube.com/watch?v=bJMYoj4hHqU&t=3s
- https://www.youtube.com/watch?v=3yaZ7IkQPUQ



Einstein’s Postulates - Constancy of the Speed of Light

Postulate – the speed of light in a vacuum is an absolute constant

- James Maxwell has already shown through his equations that the speed of light is constant

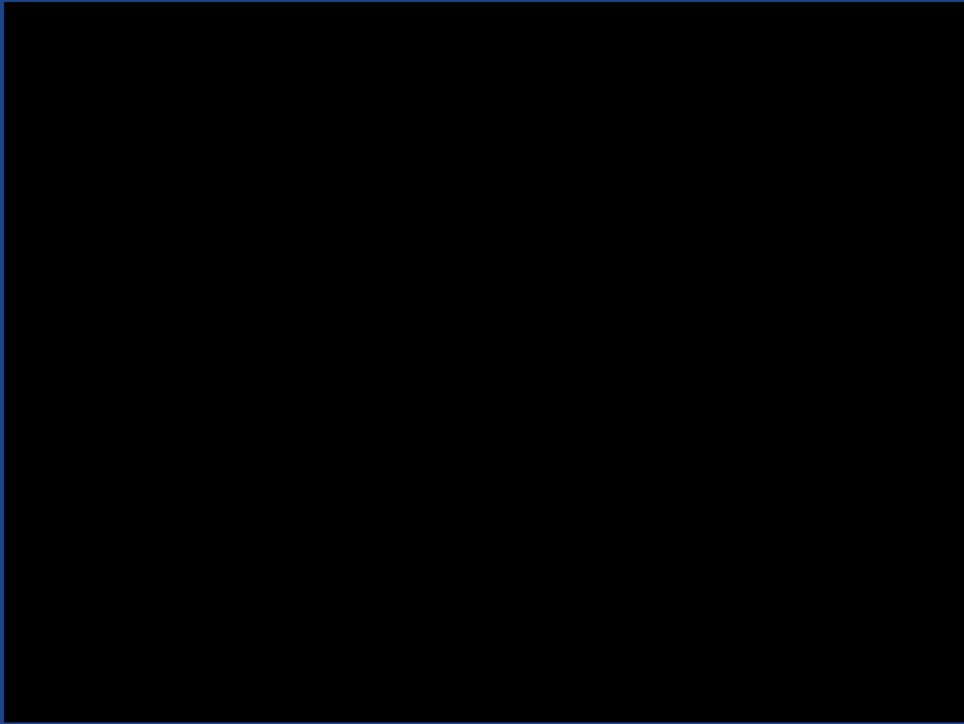
$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \text{ where } \varepsilon_0 \text{ and } \mu_0 \text{ are constants}$$

$$\therefore \text{ c is a CONSTANT!}$$

- The constancy of the speed of light provides evidence for the first postulate that all IFOR are equivalent.
    - If the speed of light is **absolute** (meaning *a value or principle which is regarded as universally valid or which may be viewed without relation to other things*) then for all IFOR to be equivalent the speed of light measured from any IFOR must be the same. (unlike the previous examples mentioned)

Einstein's Postulates - Constancy of the Speed of Light

- Thought experiment –
  - A person in a spacecraft moving at a constant velocity turns on their headlights and measured the speed of the light as *c*.
  - A second stationary person measures the speed of light from the headlight as also *c*
  - This is because the speed of light is constant in all frames of reference

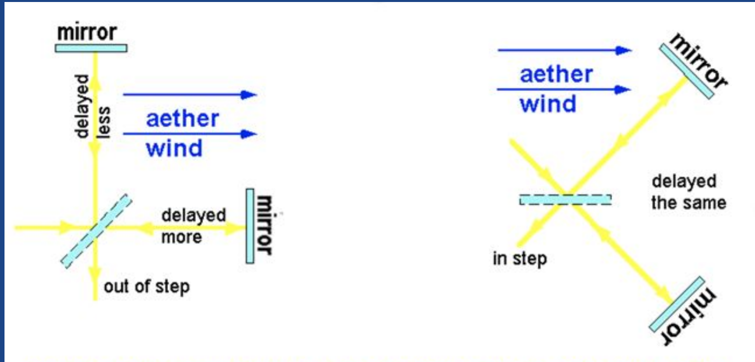


Einstein's Postulates - evidences for and against

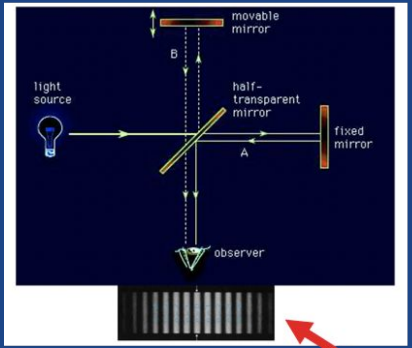
- Evidence confirming or denying Einstein's postulates
  - Einstein himself did not perform experiments
  - The aether is believed to be a medium through which EM waves are transmitted (because the idea of a a wave not requiring a medium was radical at the time)
  - The aether was thought to be fixed in space. All matter moves relative to the aether
  - It was considered an **absolute frame of reference** which is a **contradiction** to Einstein's postulate that all IFOR are equivalent and that there is no absolute FOR.
  - If the speed of light was measured as c in the aether then it would be different when measured from a FOR that is moving through the aether e.g. Earth
  - It also means that the speed of light will change if the direction which it travels relative to the aether is changed. This is the basis of the Michelson-Morley experiment
  - Video - https://www.youtube.com/watch?v=F2m_VZJM0Zc&list=PL5Tk152XIY6RovoIWX3tgOKpjVv9NHRNt

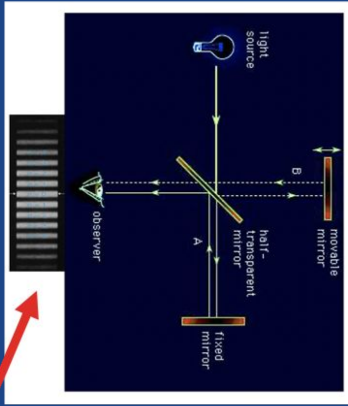
Einstein's Postulates - evidences for and against

- Evidence confirming or denying Einstein's postulates
  - Michelson-Morley experiment – aimed to measure the motion of the earth relative to the aether by detecting the 'aether wind' produced by the Earth's motion
  - The aether wind is kind of like when you move your hand through still water, causing a disturbance
  - They rationalised that light travelling with and against the aether wind will take longer to travel the same distance as light that travelled back and forth across the aether wind (see top diagram)
  - The light that was split by the half-silvered mirror will produce an interference pattern at the detector
  - When the apparatus is rotated there will be a change* in the interference pattern due to the change in the speed of light (see bottom diagram)
  - M & M weren't trying to measure the speed of light, but the **basis** of the experiment contradicts Einstein's postulate that the speed of light is constant because they are saying that the speed of light is variable relative to the aether



Rotating the apparatus would allow the direction of the aether wind to be deduced



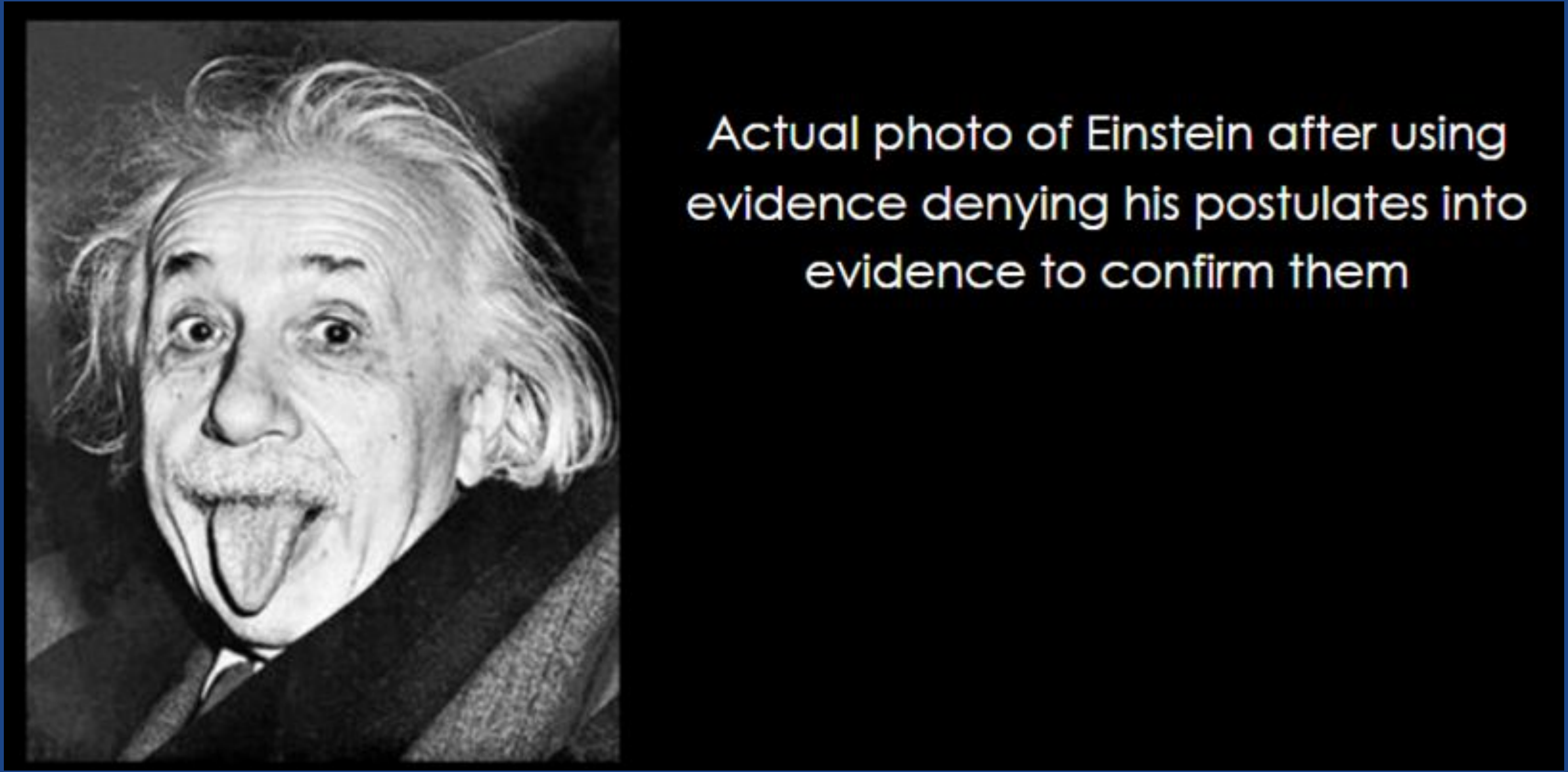


Different interference patterns

Einstein's Postulates - evidences for and against

- Evidence confirming or denying Einstein's postulates
  - The M-M experiment had a lot going for it: it was valid in that it was a fair test (only the orientation of the apparatus was changed); it was accurate and sensitive; it was reliable (repeated multiple times and at different positions of the Earth in its orbit and by different teams of scientists all yielding the same result…)
  - Michelson-Morley experiment showed no change interference pattern. It was a null result because it didn't support the hypothesis
  - The null result confirms Einstein's hypothesis in two ways
  - The aether wind was not detected hence there is no aether wind which means that there is **no absolute frame of reference**
  - All IFOR are indeed equivalent
  - That no ***change*** in interference pattern was detected supported the constancy of the speed of light
  - Furthermore, even if the aether existed (but is undetectable) it is still an IFOR and this again supports the idea that the speed of light is constant in all IFOR. So it doesn't matter if the aether exists or not the speed of light is still c. It's a win-win for Einstein!

Einstein’s Postulates - evidences for and against



Einstein’s Postulates - Other Evidence

- The relative motion of stars towards or away from the Earth results in changes in frequency (as detected in blue or red shift in star spectra) but does not affect the speed of light
    - as seen in rotational and translational velocity of stars

- The decay products of supernovae explosions can travel at 0.03c, yet the light reaches Earth from matter ejected towards the Earth and perpendicular to the line of sight from the Earth at the same time. Meaning, the velocity of the matter from which the EM radiation is released does not alter the velocity of the EM wave

Special Relativity

- Prior to Einstein's postulates time and space were viewed as absolute, that their measurement is the same regardless of FOR
- One of the consequences of the constancy of the speed of light is that time and space can no longer be viewed as absolute

Thought Experiments:

- Time dilation
- Length contraction

Time Dilation

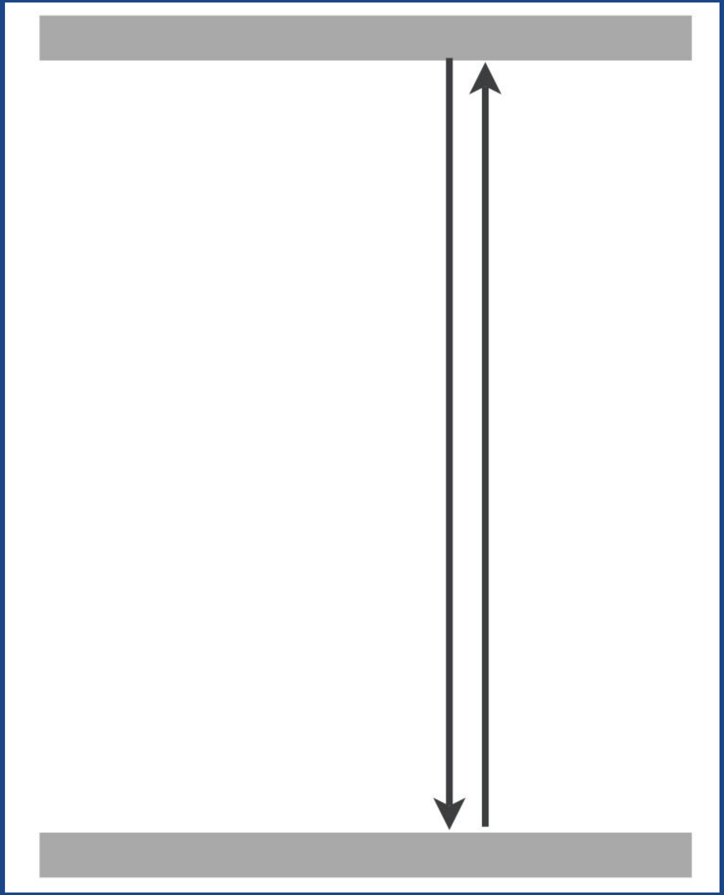
Time dilation derivation

Imagine a light clock which consists of two mirrors and

beam of light reflecting back and forth between the mirror

One "tick" is when the light goes from one mirror to the

other and back again

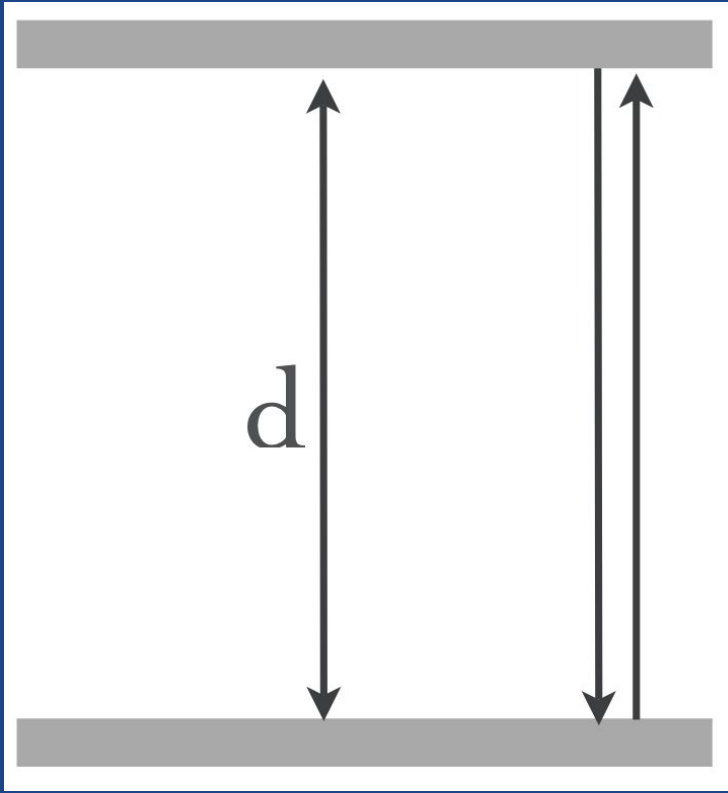


Time Dilation

Scenario 1

- You are in the same inertial frame as the light clock
- You are therefore measuring the proper time, denoted Δt_0
- The Mirrors are separated by distance "d"
- The light moves with a speed "c"
- The proper time for one tick is given by:

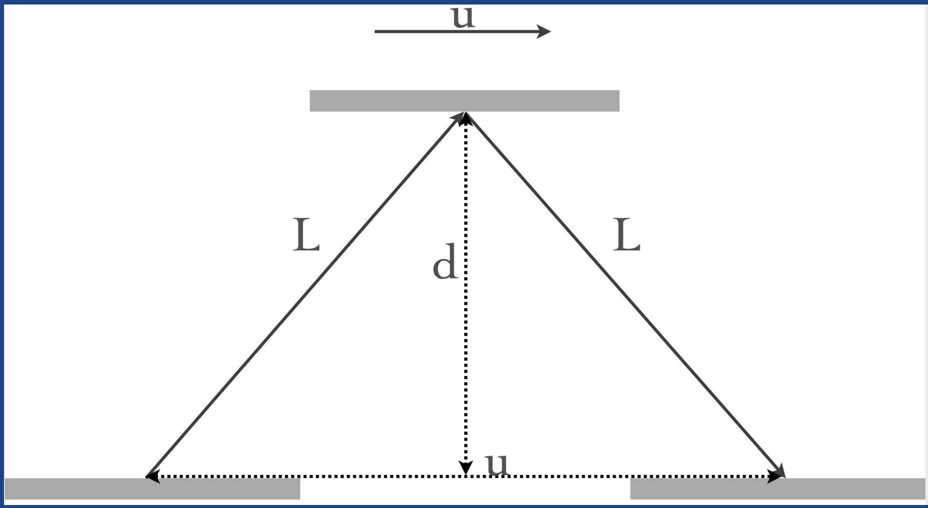
$$\Delta t_0 = \frac{2d}{c}$$



Time Dilation

Scenario 2

- You are in a different inertial frame to the light clock
- The light clock is moving with velocity u ms^{-1} in the x direction
- The path the light takes is now different as it has only a finite speed
- The time taken for one tick is denoted Δt, the observed time



Time dilation

- Applying Pythagoras' theorem the length L is given by: $L = \sqrt{d^2 + \left(\frac{u\Delta t}{2}\right)^2}$
- The value for the observed time is: $\Delta t = \frac{2L}{c}$
- Combine the two equations above $\Delta t = \frac{2}{c}\sqrt{d^2 + \left(\frac{u\Delta t}{2}\right)^2}$
- Rearrange the equation for proper time to make "d" the subject of the equation $d = \frac{c\Delta t_0}{2}$
- Combine the two equations above $\Delta t = \frac{2}{c}\sqrt{\left(\frac{c\Delta t_0}{2}\right)^2 + \left(\frac{u\Delta t}{2}\right)^2}$

Time Dilation

- This equation can now be solved to make the observed time the subject of the formula

(from previous slide) $$\Delta t = \frac{2}{c}\sqrt{\left(\frac{c\Delta t_0}{2}\right)^2 + \left(\frac{u\Delta t}{2}\right)^2}$$

- Square both sides and simplify

$$\Delta t^2 = \frac{4}{c^2}\left[\frac{c^2\Delta t_0^2}{4} + \frac{u^2\Delta t^2}{4}\right]$$

$$\Delta t^2 = \Delta t_0^2 + \frac{u^2\Delta t^2}{c^2}$$

- Collect like terms on the same side of the equation

$$\Delta t^2 - \frac{u^2\Delta t^2}{c^2} = \Delta t_0^2$$

- Factorise the left hand side

$$\Delta t^2\left(1 - \frac{u^2}{c^2}\right) = \Delta t_0^2$$

Time Dilation

- From previous slide $$\Delta t^2\left(1-\frac{u^2}{c^2}\right)=\Delta t_0^2$$
- Rearrange to make Δt the subject of the formula
$$\Delta t^2=\frac{\Delta t_0^2}{\left(1-\frac{u^2}{c^2}\right)}$$
- Take the square root of both sides $$\Delta t=\frac{\Delta t_0}{\sqrt{\left(1-\frac{u^2}{c^2}\right)}}$$

WORKED EXAMPLE (12.3)

A pilot in a rocket travelling with a velocity of $0.250c$ presses a button to flash a 'Hello' sign for 5.00 s at a space station as the rocket passes.

a For how long is the flash seen by an observer on the space station?

b Explain your reasoning.

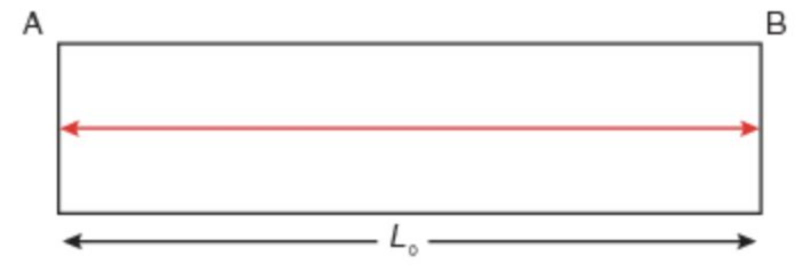
ANSWERS	LOGIC
a $t = \dfrac{t_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$	• Identify the relevant data from the question. • Write the equation connecting time in the two frames of reference.
$= \dfrac{5.00\ \text{s}}{\sqrt{1 - 0.25^2}}$	• Substitute the values with correct units.
$= \dfrac{5.00\ \text{s}}{\sqrt{0.9375}}$ $= 5.16\ \text{s}$	• Calculate the final answer with correct units and appropriate significant figures.
b The observer in the space station views the rocket travelling towards it at a velocity of $0.250c$. From this viewpoint, the clock on the rocket will be slow compared to the one in the space station. Any observer regards a clock that is moving relative to their frame of reference as running slow.	• Recognise that time dilation is important at this relative speed.

Length Contraction

Length contraction derivation

Scenario 1

- Imagine you are on a train with a light clock which consists of two mirrors and beam of light reflecting back and forth between the mirror



- One “tick” is when the light goes from one mirror to the other and back again (A to B, then B to A)

- The proper time for one tick is given by:

$$t_0 = \frac{2L_0}{c}$$

Length Contraction

Scenario 2

- Imagine you are in a reference frame of an observer with a speed of v relative to the clock
- We can measure the distance between the ends of the clock using the time for light to travel from one end to the other and back.
- From A to B

$L + vt_{AB} = ct_{AB}$

Where

L = the length of the clock as observed by the moving observer

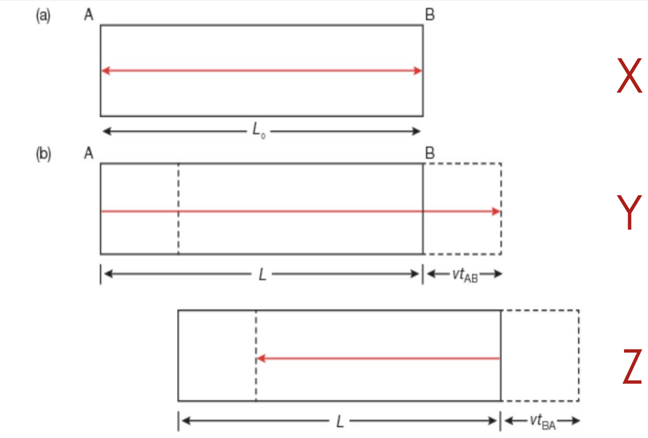
vt_{AB} = the distance the clock has moved in the time the light passes from A to B

ct_{AB} = the distance the light has travelled passing from A to B (the red line in Y that shows the total distance light has travelled going from A to B)

Make t_{AB} the subject of the equation:

$$t_{AB} = \frac{L}{c - v}$$

FIGURE 11.21 Light journeys in (a) a clock at rest and (b) a clock moving to the right at speed *v*.



Length Contraction

- From B to A (shown as Y):

  $L - vt_{AB} = ct_{BA}$

  where

  vt_{AB} = the distance the clock has moved in the time the light passes from B back to A

  ct_{BA} = the distance the light has travelled passing from B back to A (it's a shorter distance because the light clock has moved forward in the time that it takes for the light to be reflected off the mirror at B)

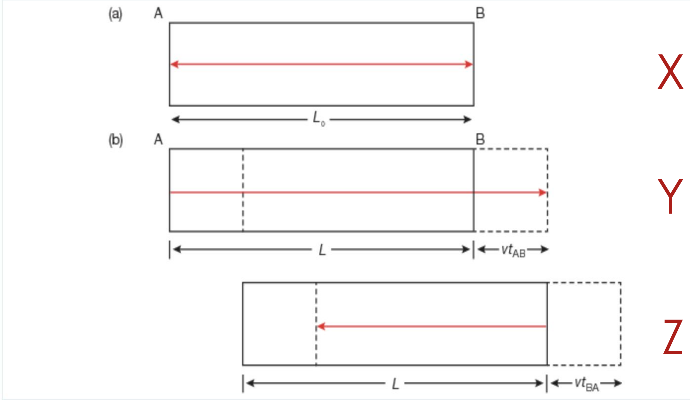
- Make t_{BA} the subject:

  $t_{BA} = \frac{L}{c+v}$

- As A moves to meet the light the time tBA is less than tAB. The total time is:

  $t = t_{AB} + t_{BA}$

FIGURE 11.21 Light journeys in (a) a clock at rest and (b) a clock moving to the right at speed v.



Length Contraction

- From previous slide:

  $t = t_{AB} + t_{BA}$

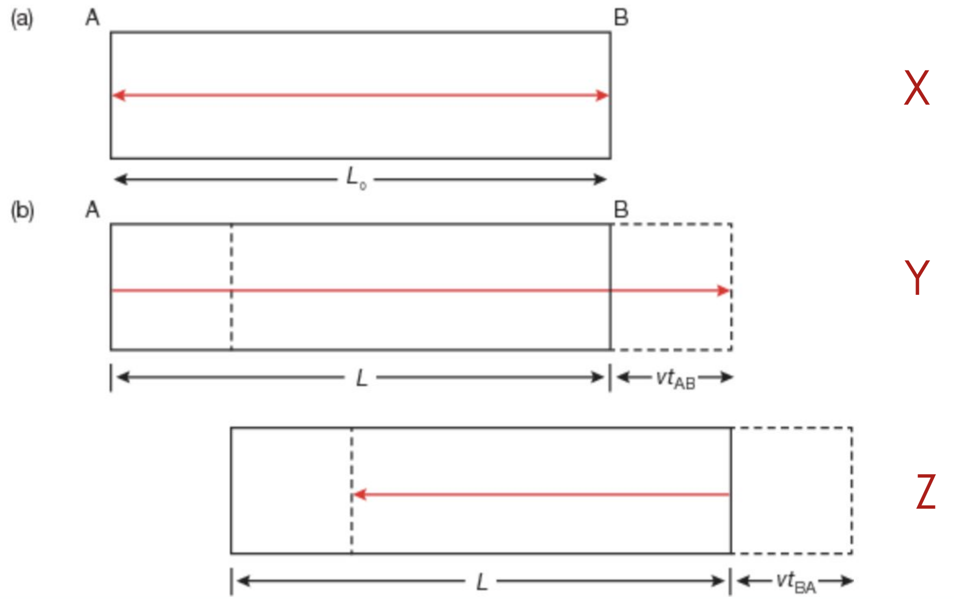
- Substitute expression for t_{AB} and t_{BA}

$$t = \frac{L}{c - v} + \frac{L}{c + v}$$

$$= \frac{2Lc}{c^2 - v^2}$$

$$= \frac{2L}{c\,(1 - \frac{v^2}{c^2})}$$

FIGURE 11.21 Light journeys in (a) a clock at rest and (b) a clock moving to the right at speed *v*.



Length Contraction

According to the time dilation formula:

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Substituting this for our time in the moving clock gives:

$$\frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{2L}{c\left(1 - \frac{v^2}{c^2}\right)}$$

$$t_0 = \frac{2L}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

Substituting $t_0 = \frac{2L_0}{c}$ gives:

$$\frac{2L_0}{c} = \frac{2L}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Rightarrow L_0 = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}} \text{ or } L = \frac{L_0}{\gamma}$$

WORKED EXAMPLE (12.4)

1 An observer on the Moon notices a spacecraft travelling past at a speed of $2.08 \times 10^8 \text{ m s}^{-1}$. The spacecraft has a proper length of 120 m.

What length will the observer on the Moon measure?

ANSWERS	LOGIC
1 $l_0 = 120 \text{ m}, v_0 = 2.08 \times 10^8 \text{ms}^{-1}$	▪ Identify the relevant data from the question.
$l = l_0\sqrt{1 - \frac{v_0^{\ 2}}{c^2}}$	▪ Write the equation connecting length in the two frames of reference.
$l = 120 \text{ m} \times \sqrt{\left(1 - \frac{(2.08 \times 10^8 \text{ m s}^{-1})^2}{(3.00 \times 10^8 \text{ m s}^{-1})^2}\right)}$	▪ Substitute the values with correct units.
$l = 120 \text{ m} \times \sqrt{1 = 0.481}$ $= 120 \text{ m} \times 0.72$ $= 86.5 \text{ m}$	▪ Calculate the answer with correct units and appropriate significant figures.

2 A crewed mission is to be sent to a newly discovered exoplanet 8 light years away. It will travel at a velocity of $0.5c$ to get there.

a According to the mission crew:

i how far away from Earth is the exoplanet?

ii how long will the journey take?

b According to the mission command on Earth, how long will the journey take?

2 $l_0 = 8$ light years, $v_0 = 0.5\,c$	▪ Identify the relevant data from the question.
a i $l = l_0\sqrt{1-\frac{v_0^2}{c^2}}$	▪ Write the equation connecting length in the two frames of reference.
$l = 8 \text{ light years} \times \sqrt{1-\frac{(0.5c)^2}{c^2}}$ $= 8 \text{ light years} \times \sqrt{1-0.25}$	▪ Substitute the values with correct units.
$= 8 \times 0.866$ light years $= 6.9$ light years	▪ Calculate the answer with correct units and appropriate significant figures.
ii $t_{crew} = \frac{s}{v} = \frac{6.9 \text{ light years}}{0.5c} = 14$ years	▪ Substitute the values with correct units and calculate the answer with correct units and appropriate significant figures.
b $t_{Earth} = \frac{s}{v} = \frac{8 \text{ light years}}{0.5\,c} = 16$ years	▪ Substitute the values with correct units and calculate the answer with correct units and appropriate significant figures.

Special Relativity - evidence

- observations of cosmic-origin muons at the Earth's surface
  - https://www.youtube.com/watch?v=nxQ0kz_qWmc
- atomic clocks (Hafele–Keating experiment)
  - https://www.youtube.com/watch?v=LW4NsxCglp4
- evidence from particle accelerators
  - https://www.symmetrymagazine.org/article/inside-the-large-hadron-collider
  - https://web.stanford.edu/~rlbyer/PDF_AllPubs/2005/407.pdf

    (looks more into mass and energy than time dilation and length contraction)
- evidence from cosmological studies
  - https://iopscience.iop.org/article/10.1086/589568/pdf
  - https://phys.org/news/2018-03-strange-physics-jets-supermassive-black.html