EXT 1: Vectors (Ext1), V1 Introduction to Vectors (Y12)

Vectors and Projectile Motion (Ext1)

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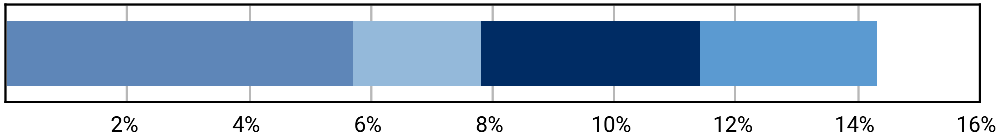
Exam Equivalent Time: 69 minutes *(based on HSC allocation of 1.5 minutes approx. per mark)*

V1 Vectors



Operations with Vectors
Vectors, Force and Velocity
Vectors and Geometry
Vectors and Projectile Motion

***Analytics based on the average contribution to past EXT1 exams since 2020.**

HISTORICAL CONTRIBUTION

- *Vectors* is an exciting new content inclusion into the Ext1 syllabus that has attracted an impressive 6 and 4 questions in the 2021 and 2020 Ext1 exams respectively.
- A supplementary database has been created with a focus on the new syllabus dot points, NESA sample exam questions and vector exam question history in other States.
- This topic has been split into four categories: *1-Operations with Vectors (5.7%), 2-Vectors, Force and Velocity (2.1%), 3-Vectors and Geometry (3.6%), and 4-Vectors and Projectile Motion (2.9%).*
- This analysis looks at *Vectors and Projectile Motion.*

HSC ANALYSIS - What to expect and common pitfalls

- *Vectors and Projectile motion (2.9%)* was examined for the first time in 2021 in a chunky 4-mark question that produced a 56% mean mark.
- Projectile motion in the new Ext1 course looks at the *parametric vector equations* of motion (functions of *t*).
- Historically, the most common concepts tested have been: range, maximum height and time of flight along with the more difficult angle of impact and range of initial angle. Each concept is addressed in the database.
- The vector magnitude and speed of motion relationship is an important "new" concept which is covered in *Ext1 V1 SM-Bank 8* and the challenging *Ext1 V1 SM-Bank 30*.
- We note that the sample NESA exam questions have also required students to convert these parametric equations into a Cartesian equation (*y* as a function of *x*), and *Ext1 V1 EQ-Bank 28* should be reviewed to cover this.

Questions

1. Vectors, EXT1 V1 EQ-Bank 28

A projectile is fired from a canon at ground level with initial velocity $\sqrt{300}$ ms^{-1} at an angle of 30° to the horizontal.

The equations of motion are $\dfrac{d^2x}{dt^2} = 0$ and $\dfrac{d^2y}{dt^2} = -10$.

i. Show that $x = 15t$. ***(1 mark)***

ii. Show that $y = 5\sqrt{3}t - 5t^2$. ***(2 marks)***

iii. Hence find the Cartesian equation for the trajectory of the projectile. ***(1 mark)***

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2. Vectors, EXT1 V1 2021 HSC 13b

When an object is projected from a point h metres above the origin with initial speed V m/s at an angle of $\theta°$ to the horizontal, its displacement vector, t seconds after projection, is

$$\underset{\sim}{r}(t) = (Vt\cos\theta)\underset{\sim}{i} + \left(-5t^2 + Vt\sin\theta + h\right)\underset{\sim}{j}. \quad \text{(Do NOT prove this.)}$$

A person, standing in an empty room which is 3 m high, throws a ball at the far wall of the room. The ball leaves their hand 1 m above the floor and 10 m from the far wall. The initial velocity of the ball is 12 m/s at an angle of 30° to the horizontal.

Show that the ball will NOT hit the ceiling of the room but that it will hit the far wall without hitting the floor. ***(4 marks)***

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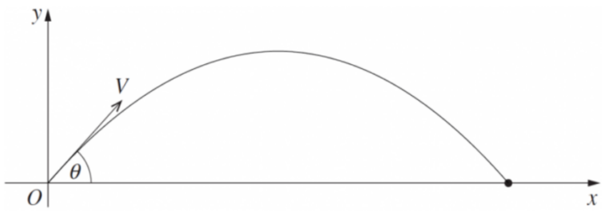
3. Vectors, EXT1 V1 SM-Bank 9

The diagram shows a projectile fired at an angle θ to the horizontal from the origin O with initial velocity $V\ \mathrm{ms}^{-1}$.



The position vector for the projectile is given by

$$\underset{\sim}{s}(t) = Vt\cos\theta\,\underset{\sim}{i} + \left(Vt\sin\theta - \frac{1}{2}gt^2\right)\underset{\sim}{j} \quad \text{(DO NOT prove this)}$$

where g is the acceleration due to gravity.

i. Show the horizontal range of the projectile is

$$\frac{V^2\sin 2\theta}{g}$$ *(2 marks)*

The projectile is fired so that $\theta = \frac{\pi}{3}$.

ii. State whether the projectile is travelling upwards or downwards when

$$t = \frac{2V}{\sqrt{3}g}$$ *(1 mark)*

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4. Vectors, EXT1 V1 SM-Bank 6

A cricketer hits a ball at time $t = 0$ seconds from an origin O at ground level across a level playing field.

The position vector $\underset{\sim}{s}(t)$, from O, of the ball after t seconds is given by

$$\underset{\sim}{s}(t) = 15t\underset{\sim}{i} + \left(15\sqrt{3}t - 4.9t^2\right)\underset{\sim}{j},$$

where, $\underset{\sim}{i}$ is a unit vector in the forward direction, $\underset{\sim}{j}$ is a unit vector vertically up and displacement components are measured in metres.

a. Find the initial velocity of the ball and the initial angle, in degrees, of its trajectory to the horizontal. ***(2 marks)***

b. Find the maximum height reached by the ball, giving your answer in metres, correct to two decimal places. ***(2 marks)***

c. Find the time of flight of the ball. Give your answer in seconds, correct to three decimal places. ***(1 mark)***

d. Find the range of the ball in metres, correct to one decimal place. ***(1 mark)***

e. A fielder, more than 40 m from O, catches the ball at a height of 2 m above the ground.
How far horizontally from O is the fielder when the ball is caught? Give your answer in metres, correct to one decimal place. ***(2 marks)***

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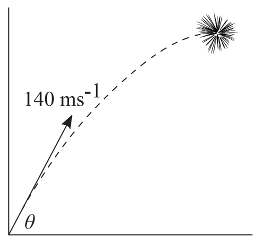
5. Vectors, EXT1 V1 SM-Bank 23

A fireworks rocket is fired from an origin O, with a velocity of 140 metres per second at an angle of θ to the horizontal plane.



The position vector $\underset{\sim}{s}(t)$, from O, of the rocket after t seconds is given by

$$\underset{\sim}{s} = 140t\cos\theta\underset{\sim}{i} + \left(140t\sin\theta - 4.9t^2\right)\underset{\sim}{j}$$

The rocket explodes when it reaches its maximum height.

i. Show the rocket explodes at a height of $1000\sin^2\theta$ metres. ***(2 marks)***

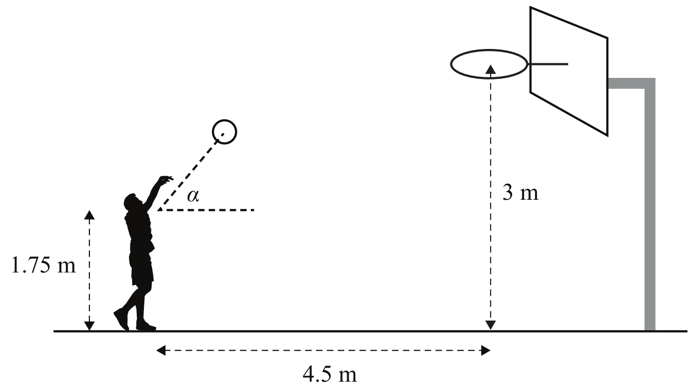
ii. Show the rocket explodes at a horizontal distance of $1000\sin 2\theta$ metres from O. ***(1 mark)***

iii. For best viewing, the rocket must explode at a horizontal distance of 500 m and 800 m from O, and at least 600 m above the ground.

For what values of θ will this occur. ***(3 marks)***

6. Vectors, EXT1 V1 SM-Bank 7

A basketball player aims to throw a basketball through a ring, the centre of which is at a horizontal distance of 4.5 m from the point of release of the ball and 3 m above floor level. The ball is released at a height of 1.75 m above floor level, at an angle of projection α to the horizontal and at a speed of V ms^{-1}. Air resistance is assumed to be negligible.



The position vector of the centre of the ball at any time, t seconds, for $t \geq 0$, relative to the point of release is given by

$$\underset{\sim}{s}(t) = Vt\cos(\alpha)\underset{\sim}{i} + \left(Vt\sin(\alpha) - 4.9t^2\right)\underset{\sim}{j},$$

where $\underset{\sim}{i}$ is a unit vector in the horizontal direction of motion of the ball and $\underset{\sim}{j}$ is a unit vector vertically up. Displacement components are measured in metres.

For the player's first shot at goal, $V = 7$ ms^{-1} and $\alpha = 45^\circ$

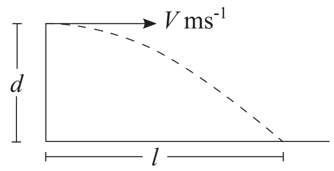
a. Find the time, in seconds, taken for the ball to reach its maximum height. Give your answer in the form $\dfrac{a\sqrt{b}}{c}$, where a, b and c are positive integers. ***(2 marks)***

b. Find the maximum height, in metres, above floor level, reached by the centre of the ball. ***(2 marks)***

c. Find the distance of the centre of the ball from the centre of the ring one second after release. Give your answer in metres, correct to two decimal places. ***(2 marks)***

7. Vectors, EXT1 V1 SM-Bank 8

A projectile is fired horizontally off a cliff at an initial speed of V metres per second.



The projectile strikes the water, l metres from the base of the cliff.

Let g be the acceleration due to gravity and assume air resistance is negligible.

i. Show the projectile hits the water when

$$t = \sqrt{\frac{2d}{g}}$$ *(2 marks)*

ii. If l equals twice the height of the cliff, at what angle does the projectile hit the water? *(2 marks)*

iii. Show that the speed at which the projectile hits the water is

$2\sqrt{dg}$ metres per second. *(1 mark)*

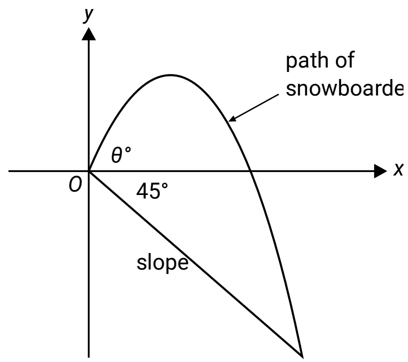
8. Vectors, EXT1 V1 2019 SPEC2-N 4

A snowboarder at the Winter Olympics leaves a ski jump at an angle of θ degrees to the horizontal, rises up in the air, performs various tricks and then lands at a distance down a straight slope that makes an angle of 45° to the horizontal, as shown below.

Let the origin O of a cartesian coordinate system be at the point where the snowboarder leaves the jump, with a unit vector in the positive x direction being represented by $\underset{\sim}{i}$ and a unit vector in the positive y direction being represented by $\underset{\sim}{j}$. Distances are measured in metres and time is measured in seconds.

The position vector of the snowboarder t seconds after leaving the jump is given by

$$\underset{\sim}{r}(t) = \left(6t - 0.01t^3\right)\underset{\sim}{i} + \left(6\sqrt{3}t - 4.9t^2 + 0.01t^3\right)\underset{\sim}{j},\ t \geq 0$$



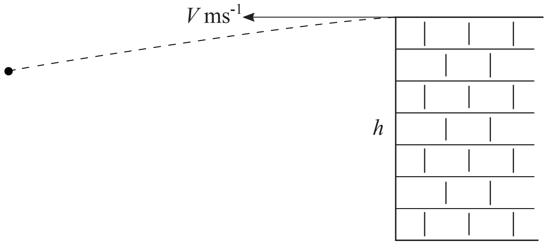
a. Find the angle θ°. *(2 marks)*

b. Find the speed, in metres per second, of the snowboarder when she leaves the jump at O. *(1 mark)*

c. Find the maximum height above O reached by the snow boarder. Give your answer in metres, correct to one decimal place. *(2 marks)*

d. Show that the time spent in the air by the snowboarder is $\dfrac{60\left(\sqrt{3}+1\right)}{49}$ seconds. *(3 marks)*

9. Vectors, EXT1 V1 SM-Bank 30



A canon ball is fired from a castle wall across a horizontal plane at V ms^{-1}.

Its position vector t seconds after it is fired from its origin is given by $\underset{\sim}{s}(t) = Vt\underset{\sim}{i} - \dfrac{1}{2}gt^2\underset{\sim}{j}$.

i. If the projectile hits the ground at a distance 8 times the height at which it was fired, show that it initial velocity is given by

$$V = 4\sqrt{2hg} \quad \textit{(2 marks)}$$

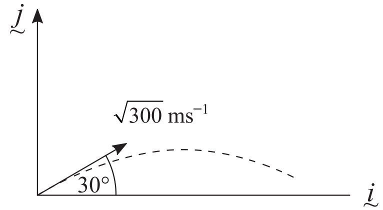
ii. Show that the total distance the canon ball travels can be expressed as

$$\int_0^{\sqrt{\frac{2h}{g}}} \sqrt{g\left(32h + gt^2\right)}\, dt \quad \textit{(2 marks)}$$

Worked Solutions

1. Vectors, EXT1 V1 EQ-Bank 28

i.



$$\dot{x} = \sqrt{300}\cos 30^\circ = 10\sqrt{3} \times \frac{\sqrt{3}}{2} = 15 \text{ ms}^{-1}$$

$$x = \int 15\, dt = 15t + C_1$$

$\text{When } t = 0,\ x = 0 \Rightarrow C_1 = 0$

$\therefore x = 15t$

ii. $\dot{y} = \displaystyle\int -10\, dt = -10t + C_1$

$\text{When } t = 0,\ \dot{y} = 10\sqrt{3}\sin 30^\circ = 5\sqrt{3} \Rightarrow C_1 = 5\sqrt{3}$

$\therefore \dot{y} = 5\sqrt{3} - 10t$

$$y = \int \dot{y}\, dt = 5\sqrt{3}t - 5t^2 + C_2$$

$\text{When } t = 0,\ y = 0 \Rightarrow C_2 = 0$

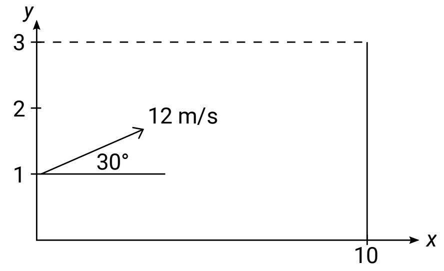
$\therefore y = 5\sqrt{3}t - 5t^2$

iii. $x = 15t \Rightarrow t = \dfrac{x}{15}$

$$y = 5\sqrt{3} \times \frac{x}{15} - 5\left(\frac{x}{15}\right)^2$$

$$\therefore y = \frac{\sqrt{3}}{3}x - \frac{x^2}{45}$$

2. Vectors, EXT1 V1 2021 HSC 13b



$r(t) = (Vt\cos\theta)\underset{\sim}{i} + \left(-5t^2 + Vt\sin\theta + h\right)\underset{\sim}{j}$

$r'(t) = (V\cos\theta)\underset{\sim}{i} + (-10t + V\sin\theta)\underset{\sim}{j}$

Max height occurs when $\dot{y} = 0$:

$10t = V\sin\theta$

$$t = \frac{12 \times \sin 30^\circ}{10}$$

$= 0.6$ sec

Find y when $t = 0.6$:

$y = -5(0.6)^2 + 12 \times 0.6 \times \sin 30^\circ + 1$

$= 2.8 \text{ m} < 3 \text{ m}$

$\therefore$ Ball will not hit ceiling.

Find time of flight when $y = 0$:

$-5t^2 + 6t + 1 = 0$

$5t^2 - 6t - 1 = 0$

$$t = \frac{6 \pm \sqrt{(-6)^2 + 4 \cdot 5 \cdot 1}}{10}$$

$$= \frac{6 + \sqrt{56}}{10}$$

$= 1.348...$

Find x when $t = 1.348$:

$x = 12 \times 1.348 \times \cos 30$

$= 14.0 > 10$

$\therefore$ Ball will hit the wall on the full.

3. Vectors, EXT1 V1 SM-Bank 9

i. Time of flight when

$\underset{\sim}{j}$ component of $\underset{\sim}{v} = 0$

$$Vt\sin\theta - \frac{1}{2}gt^2 = 0$$

$$t\left(V\sin\theta - \frac{1}{2}gt\right) = 0$$

$$\frac{1}{2}gt = V\sin\theta$$

$$t = \frac{2V\sin\theta}{g}$$

Range $\Rightarrow$ $\underset{\sim}{i}$ component of $\underset{\sim}{s}$

when $t = \dfrac{2V\sin\theta}{g}$

$$\text{Range} = V\cdot\left(\frac{2V\sin\theta}{g}\right)\cdot\cos\theta$$

$$= \frac{V^2}{g}\cdot 2\sin\theta\cos\theta$$

$$= \frac{V^2\sin 2\theta}{g}$$

ii. Time of flight $= \dfrac{2V\sin\frac{\pi}{3}}{g} = \dfrac{\sqrt{3}V}{g}$

Since parabolic path is symmetrical,

$\Rightarrow$ Upwards if $t < \dfrac{\sqrt{3}V}{2g}$

$\Rightarrow$ Downwards if $t > \dfrac{\sqrt{3}V}{2g}$

$\therefore$ At $t = \dfrac{2V}{\sqrt{3}g}$, travelling downwards

as $\dfrac{2}{\sqrt{3}}\cdot\dfrac{V}{g} > \dfrac{\sqrt{3}}{2}\cdot\dfrac{V}{g}$

4. Vectors, EXT1 V1 SM-Bank 6

a. $\underset{\sim}{v}(t) = \underset{\sim}{\dot{s}}(t) = 15\underset{\sim}{i} + \left(15\sqrt{3} - 9.8t\right)\underset{\sim}{j}$

Initial velocity occurs when $t = 0$:

$\therefore \underset{\sim}{v}(0) = 15\underset{\sim}{i} + 15\sqrt{3}\underset{\sim}{j}$

Let $\theta =$ Initial trajectory,

$$\tan\theta = \frac{15\sqrt{3}}{15}$$

$$= \sqrt{3}$$

$$\therefore \theta = \frac{\pi}{3} \text{ or } 60^\circ$$

b. Max height $\Rightarrow$ $\underset{\sim}{j}$ component of $\underset{\sim}{v} = 0$.

$$15\sqrt{3} - 9.8t = 0$$

$$t = \frac{15\sqrt{3}}{9.8}$$

$$= 2.651...$$

Find max height when $t = 2.651...$

$\therefore$ Max height $= 15\sqrt{3}\times 2.651 - 4.9\times(2.651)^2$

≈ 34.44 m

c. Ball travels in symmetrical parabolic path.

$\therefore$ Total time of flight

$$= 2\times\frac{15\sqrt{3}}{9.8}$$

$$= \frac{15\sqrt{3}}{4.9}$$

≈ 5.302 s

d. Range $\Rightarrow$ $\underset{\sim}{i}$ component of $\underset{\sim}{s}(t)$ when $t = \dfrac{15\sqrt{3}}{4.9}$

$\therefore$ Range $= 15\times\dfrac{15\sqrt{3}}{4.9}$

$$= \frac{225\sqrt{3}}{4.9}$$

$\approx 79.5 \text{ m}$

e. Find t when height of ball $= 2$ m:

$$15\sqrt{3}t - 4.9t^2 = 2$$
$$4.9t^2 - 15\sqrt{3}t + 2 = 0$$

$$t = \frac{15\sqrt{3} \pm \sqrt{(15\sqrt{3})^2 - 4 \times 4.9 \times 2}}{2 \times 4.9}$$

$t \approx 0.078131 \quad \text{or} \quad t \approx 5.22406$

When $t = 0.0781$,

$x = 15 \times 0.0781 = 1.17 \text{ m} \ \ (\text{no solution} \to x < 40)$

When $t = 5.2241$,

$x = 15 \times 5.2241 = 78.4 \text{ m}$

$\therefore$ Ball is caught 78.4 m horizontally from O.

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5. Vectors, EXT1 V1 SM-Bank 23

i. $\underset{\sim}{s} = 140t\cos\theta\,\underset{\sim}{i} + \left(140t\sin\theta - 4.9t^2\right)\underset{\sim}{j}$

$\underset{\sim}{v} = 140\cos\theta\,\underset{\sim}{i} + (140\sin\theta - 9.8t)\underset{\sim}{j}$

Max height occurs when $\underset{\sim}{j}$ component of $\underset{\sim}{v} = 0$

$$0 = 140\sin\theta - 9.8t$$
$$t = \frac{140\sin\theta}{9.8}$$

Max height: $\underset{\sim}{j}$ component of $\underset{\sim}{s}$ when $t = \dfrac{140\sin\theta}{9.8}$

$$\begin{aligned}\text{Max height} &= 140\sin\theta \cdot \frac{140\sin\theta}{9.8} - \frac{4.9 \cdot 140^2 \sin^2\theta}{9.8^2} \\ &= 2000\sin^2\theta - 1000\sin^2\theta \\ &= 1000\sin^2\theta\end{aligned}$$

ii. Horizontal distance (d):

$\Rightarrow \underset{\sim}{i}$ component of $\underset{\sim}{s}$ when $t = \dfrac{140\sin\theta}{9.8}$

$$\begin{aligned}\therefore d &= 140\cos\theta \cdot \frac{140\sin\theta}{9.8} \\ &= \frac{140 \times 70 \times \sin 2\theta}{9.8} \\ &= 1000\sin 2\theta\end{aligned}$$

iii. Using part ii,

$$500 \leq 1000\sin 2\theta \leq 800$$
$$0.5 \leq \sin 2\theta \leq 0.8$$

In the 1st quadrant:

$$30^\circ \leq 2\theta \leq 53.13^\circ$$
$$15^\circ \leq \theta \leq 26.6^\circ$$

In the 2nd quadrant:

$$126.87^\circ \leq 2\theta \leq 150^\circ$$
$$63.4^\circ \leq \theta \leq 75^\circ$$

When $\theta = 26.6^\circ$:

$$\text{Max height} = 1000 \cdot \sin^2 26.6^\circ$$
$$= 200.5 \text{ metres } (< 600 \text{ m})$$

$\Rightarrow$ Highest max height for $15^\circ \leq \theta < 26.6^\circ$ does not satisfy.

When $\theta = 63.4^\circ$:

$$\text{Max height} = 1000 \cdot \sin^2 63.4^\circ$$
$$= 799.5 \text{ metres } (> 600 \text{ m})$$

$\Rightarrow$ Lowest max height for $63.4^\circ \leq \theta \leq 75^\circ$ satisfies.

$\therefore 63.4^\circ \leq \theta \leq 75^\circ$

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6. Vectors, EXT1 V1 SM-Bank 7

a.i. $\underset{\sim}{s}(t) = 7t\cos 45^\circ \underset{\sim}{i} + (7t\sin 45^\circ - 4.9t^2)\underset{\sim}{j}$

$$= \frac{7\sqrt{2}t}{2}\underset{\sim}{i} + \left(\frac{7\sqrt{2}}{2}t - 4.9t^2\right)\underset{\sim}{j}$$

Maximum height $\Rightarrow$ find t when $\underset{\sim}{j}$ component of $\underset{\sim}{V}(t) = 0$:

$$\underset{\sim}{V} = \frac{7\sqrt{2}}{2}\underset{\sim}{i} + \left(\frac{7\sqrt{2}}{2} - 9.8t\right)\underset{\sim}{j}$$

$$\frac{7\sqrt{2}}{2} - 9.8t = 0$$

$$t = \frac{5\sqrt{2}}{14} \text{ seconds}$$

a.ii. Max height $\Rightarrow$ Find $\underset{\sim}{j}$ component of $\underset{\sim}{s}(t)$ when $t = \dfrac{5\sqrt{2}}{14}$

$$\underset{\sim}{s}_{\underset{\sim}{j}}\left(\frac{5\sqrt{2}}{14}\right) = 7 \times \frac{5\sqrt{2}}{14} \times \frac{1}{\sqrt{2}} - 4.9 \times \left(\frac{5\sqrt{2}}{14}\right)^2$$

$$= 1.25 \text{ m}$$

$\therefore$ Height above floor $= 1.25 + 1.75 = 3$ m

a.iii. $\underset{\sim}{s}_{\text{ring}} = 4.5\underset{\sim}{i} + 1.25\underset{\sim}{j}$

After 1 second,

$$\underset{\sim}{s}_{\text{ball}} = \frac{7}{\sqrt{2}}\underset{\sim}{i} + \left(\frac{7\sqrt{2}}{2} - 4.9\right)\underset{\sim}{j}$$

$$\therefore d = \left|\underset{\sim}{s}_{\text{ring}} - \underset{\sim}{s}(1)\right|$$

$$= \sqrt{\left(4.5 - \frac{7}{\sqrt{2}}\right)^2 + \left(1.25 - \frac{7}{\sqrt{2}} + 4.9\right)^2}$$

$$\approx 1.28 \text{ m}$$

7. Vectors, EXT1 V1 SM-Bank 8

i. $\underset{\sim}{v} = V\cos\theta\underset{\sim}{i} + (V\sin\theta - gt)\underset{\sim}{j}$

$= V\cos 0\underset{\sim}{i} + (V\sin 0 - gt)\underset{\sim}{j}$

$= V\underset{\sim}{i} - gt\underset{\sim}{j}$

$$\underset{\sim}{s} = \int \underset{\sim}{v}\, dt$$

$$= Vt\underset{\sim}{i} - \frac{1}{2}gt^2\underset{\sim}{j} + c$$

When $t = 0$, $\underset{\sim}{s} = 0$, $c = 0$

Time of flight:

$\underset{\sim}{j}$ component of $\underset{\sim}{s} = -d$

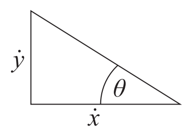
$$-\frac{1}{2}gt^2 = -d$$

$$t^2 = \frac{2d}{g}$$

$$t = \sqrt{\frac{2d}{g}}$$

ii. $l = 2d$ (given)

Projectile hits water at θ:



$\dot{y} = \underset{\sim}{j}$ component of $\underset{\sim}{v}$

$= -gt$

$$= -g \cdot \sqrt{\frac{2d}{g}}$$

$= -\sqrt{2dg}$

$\dot{x} = \underset{\sim}{i}$ component of $\underset{\sim}{v}$

$= V$

When $t = \sqrt{\frac{2d}{g}}$,

$\underset{\sim}{i}$ component of $\underset{\sim}{s} = 2d$

$$2d = V \cdot \sqrt{\frac{2d}{g}}$$

$$V = \frac{2d\sqrt{g}}{\sqrt{2d}} = \sqrt{2dg}$$

$$\tan\theta = \frac{|\dot{y}|}{|\dot{x}|} = \frac{\sqrt{2dg}}{\sqrt{2dg}} = 1$$

$\therefore \theta = 45^\circ$

iii. Speed $=$ magnitude of velocity

$$|\underset{\sim}{v}| = \sqrt{\left(\sqrt{2dg}\right)^2 + \left(\sqrt{2dg}\right)^2}$$

$= \sqrt{4dg}$

$= 2\sqrt{dg}$

8. Vectors, EXT1 V1 2019 SPEC2-N 4

a. $v(t) = \left(6 - 0.03t^2\right)\underset{\sim}{i} + \left(6\sqrt{3} - 9.8t + 0.03t^2\right)\underset{\sim}{j}$

When $t = 0$,

$v(t) = 6\underset{\sim}{i} + 6\sqrt{3}\underset{\sim}{j}$

$\tan\theta = \dfrac{6\sqrt{3}}{6} = \sqrt{3}$

$\therefore\ \theta = \tan^{-1}\sqrt{3}$

$= 60^\circ$

b. Speed $= |v(0)|$

$= \sqrt{6^2 + \left(6\sqrt{3}\right)^2}$

$= 12\ \text{ms}^{-1}$

c. Max height when $\underset{\sim}{j}$ component of $v(t) = 0$

Solve: $6\sqrt{3} - 9.8t + 0.03t^2 = 0$

$\Rightarrow t =$ 1.064 seconds

Max height $= 6\sqrt{3}(1.064) - 4.9(1.064)^2 + 0.01(1.064)^3$

≈ 5.5 m

d. Time of Flight $\Rightarrow$ Solve for t when $y = -x$:

$$6\sqrt{3}t - 4.9t^2 + 0.01t^3 = -\left(6t - 0.01t^3\right)$$

$$\left(6 + 6\sqrt{3}\right)t - 4.9t^2 = 0$$

$$t\left(6 + 6\sqrt{3} - 4.9t\right) = 0$$

$$4.9t = 6 + 6\sqrt{3}$$

$$t = \frac{6 + 6\sqrt{3}}{4.9}$$

$$= \frac{60\left(\sqrt{3} + 1\right)}{49}\ \text{seconds}$$

9. Vectors, EXT1 V1 SM-Bank 30

i. Time of flight $\Rightarrow$ find t when $y = -h$

$$-\frac{1}{2}gt^2 = -h$$

$$t^2 = \frac{2h}{g}$$

$$t = \sqrt{\frac{2h}{g}},\quad t > 0$$

Since the canon ball impacts when $x = 8h$:

$$Vt = 8h$$

$$V\sqrt{\frac{2h}{g}} = 8h$$

$$V = \frac{8\sqrt{hg}}{\sqrt{2}}$$

$$= 4\sqrt{2hg}$$

ii. $\underset{\sim}{v} = 4\sqrt{2hg}\underset{\sim}{i} - gt\underset{\sim}{j}$

$$|\underset{\sim}{v}| = \sqrt{\left(4\sqrt{2hg}\right)^2 + (-gt)^2}$$

$$= \sqrt{16 \times 2hg + g^2t^2}$$

$$= \sqrt{g\left(32h + gt^2\right)}$$

Distance $= \displaystyle\int_0^{\sqrt{\frac{2h}{g}}} |\underset{\sim}{v}|\, dt$

$$= \int_0^{\sqrt{\frac{2h}{g}}} \sqrt{g\left(32h + gt^2\right)}\, dt$$