MATH 351 – Assignment #1

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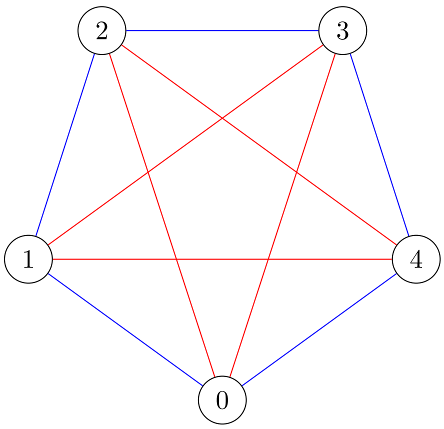
Section 03

20200829

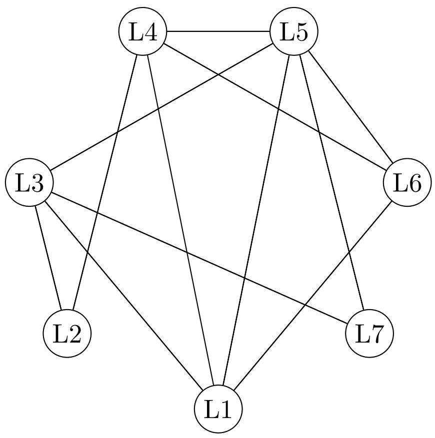
Part 2

Problem 1

False. This counter example shows the connections of G in blue, and the connections of the compliment in red. Both are connected.



Problem 2



Problem 3

I think that this comes down to how the removed vertices are picked. In the typical example of G-u and G-v, the points to pick are the vertices with the largest distance between them. If w is picked such that the distances between u, v, and w is maximized, I think the extention to theorem 1.10 holds.

Problem 4

Our presmise is that if G is a disconnected graph and !G is it's complement, !G will have a diameter less than or equal to 2. To prove that any two points in !G have a distance of 1 or 2, we will use a proof by contradiction.

1.For contradiction, assume that there exist two nodes, u and v, in !G whose distance is greater than 2. We will assume their distance is 3. For their distance to be 3, there must be two nodes in between them, x and y. So the edges between them are (u,x), (x,y), (y,v). Additionally, there may or may not be any connections between these 4 nodes assuming the connections do not shorten the u-v distance.

2. Between each of the four nodes (u,v,x,y) in !G, there must only be the connections ((u,x), (x,y), (y, v)) to maintain the distance of 3 between u and v.

3. This means that G must have the missing connections. The missing connections must include (y,u), (u,v), and (v,x). This means a walk between u and v still exists.

4. For all other nodes that are not the four:

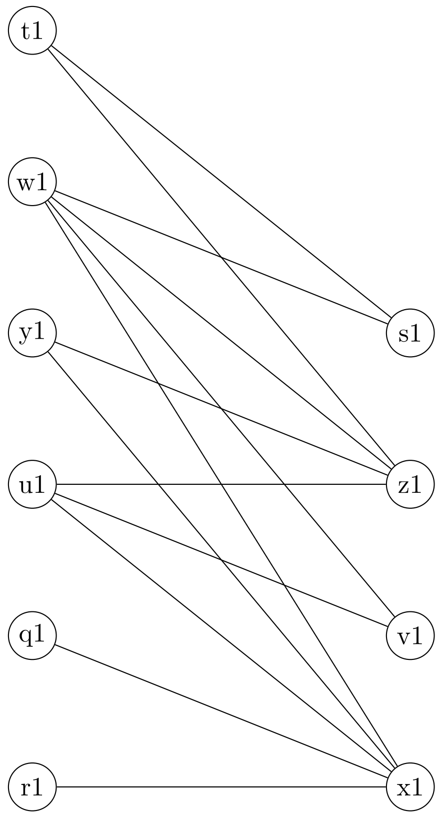
- If they had a distance of 1 to either u or v in !G, then they could have only had that connection to either u or v, and will be connected to the other, v or u respectivly.
- If they had a distance of greater than 1 to either u or v in !G, then they will be directly connected to that node in !G.

5. This means that G is a connected graph. This is a contradiction, proving that if !G has a diameter greater than 2, G must be connected.

Part 3

Problem 5

G1 is bipartite as it has no cycles of odd length.



G2 is not bipartite as it has the odd cycle x2,y2,z2,w2,r2,x2, with a length of 5.

Problem 6

For any graph (G) of order 5 or larger, if it is bipartite then all of the possible cycles in the graph are of even length We shall call the first 4 nodes in this cycle w, x, y, and z, but there may be more. For this to be the begining of a cycle in a bipartite graph w must be connected to x and not y, x must be connected to y and not z, y must be connected to z and not w, and z can connect to a further number of nodes, as long as that number is even, or it can connect to w, but not x. We also know that because this is a graph of order 5 or more, there is at least one other node, we'll call it node f. Due to it being a bipartite graph, we know that EITHER w or z is connected to it, not both. Because of that last point, we can determine one of two things.

If G contains f-w, we know that it does not contain f-z, and we know that it does not contain x-f, and due to the fact that x-y and y-z were in G, we know that G does not contain x-z. Because G does not contain those three edges, we know !G does, and those three edges form a triangle, a cycle of length 3, meaning !G is not bipartite.

If G contains f-z, we know that it does not contain f-w, and we know that it does not contain y-f, and due to the fact that x-y and y-z were in G, we know that G does not contain x-z. Because G does not contain those three edges, we know !G does, and those three edges form a triangle, a cycle of length 3, meaning !G is not bipartite.

So as long as G has an order greater than 5, a maximum of one between G and !G can be bipartite if either are.

Part 4

Problem 7

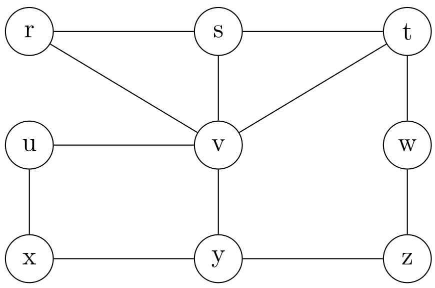


Figure 1: $G - X$

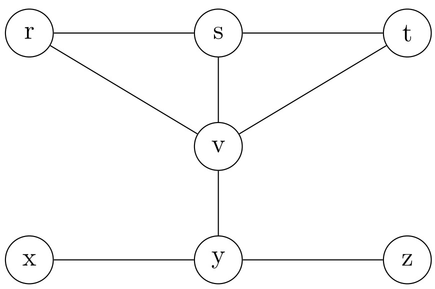


Figure 2: $G - U$

Problem 8

This is the only graph that satisfies the requirements that the distance between every two distinct vertices is odd. There are no other graphs that satisfy that requirement besides a complete graph where the distance between every two distinct vertices is exactly 1. In order to have any other graph that satisfies the requirement of having only odd distances between every vertex, there would have to be a distance of 3 between two vertices. To accomplish this, a walk would have to pass from the original vertex, through 2 other vertices to reach the destination, meaning that for the original and third vertex, the distance would be two, violating the rule.

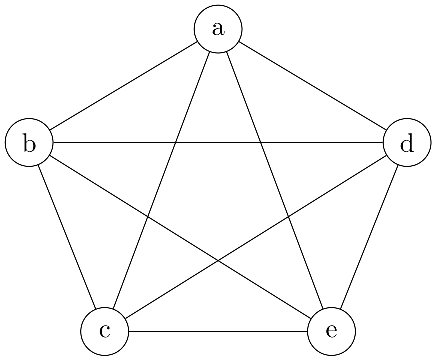


Figure 3: G

Problem 9

G belongs to the class of bipartite graphs.

Problem 10

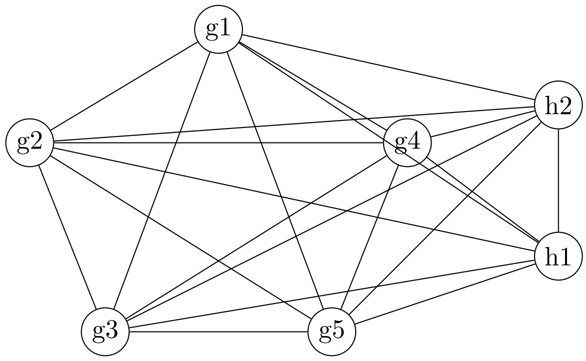


Figure 4: $K_5 + K_2$

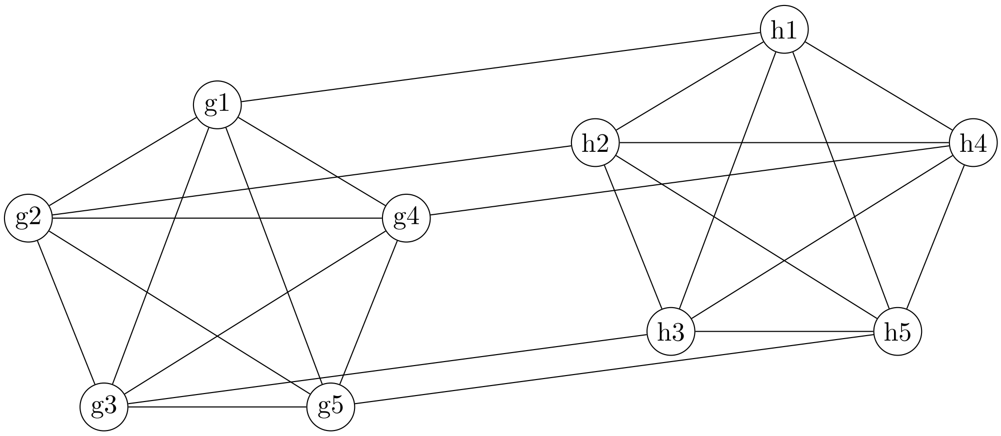


Figure 5: $K_5 \times K_2$

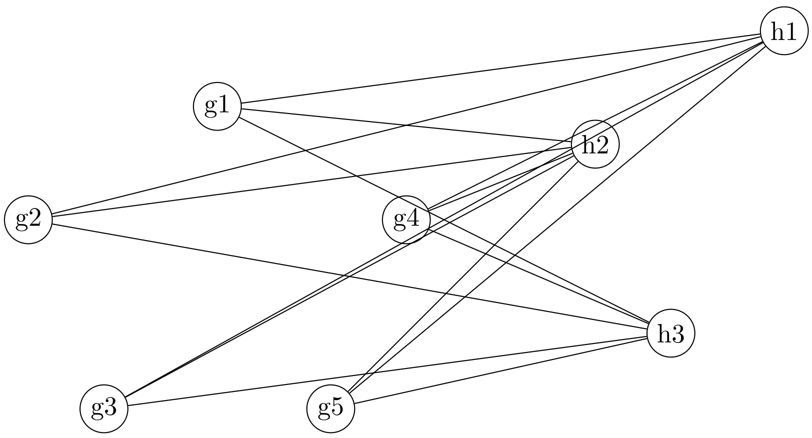


Figure 6: $!K_5 + !K_3$

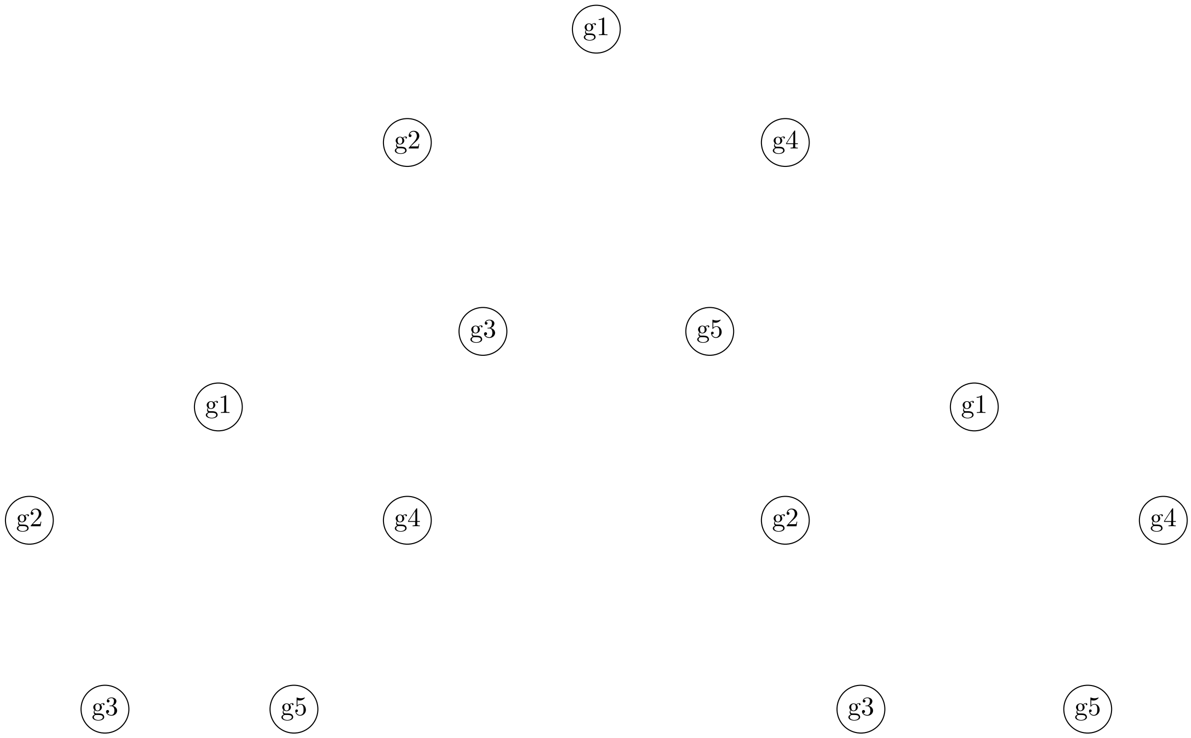


Figure 7: $!K_5 \times !K_3$

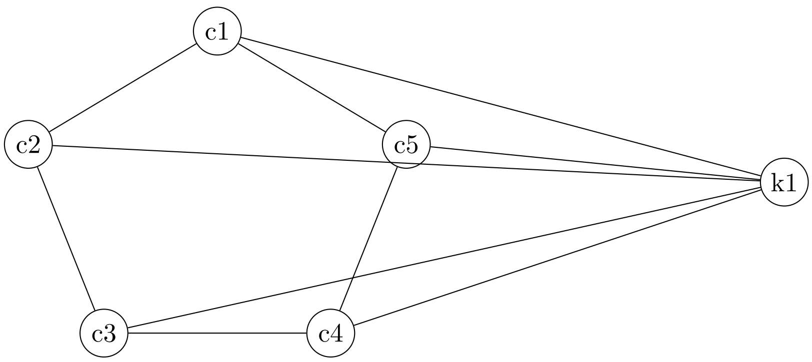


Figure 8: $C_5 + K_1$

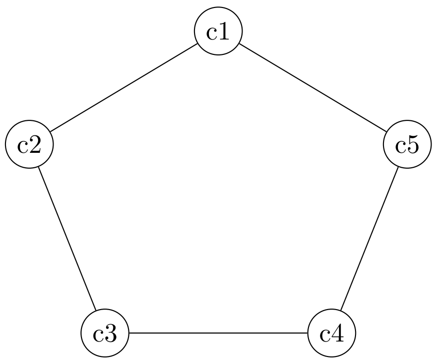


Figure 9: $C_5 \times K_1$