

# Crypto Refresher

Symmetric Cryptography, Asymmetric Cryptography, Hash Functions

Network Security AS 2020

*15 September 2020*

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# Other Lectures about Cryptography

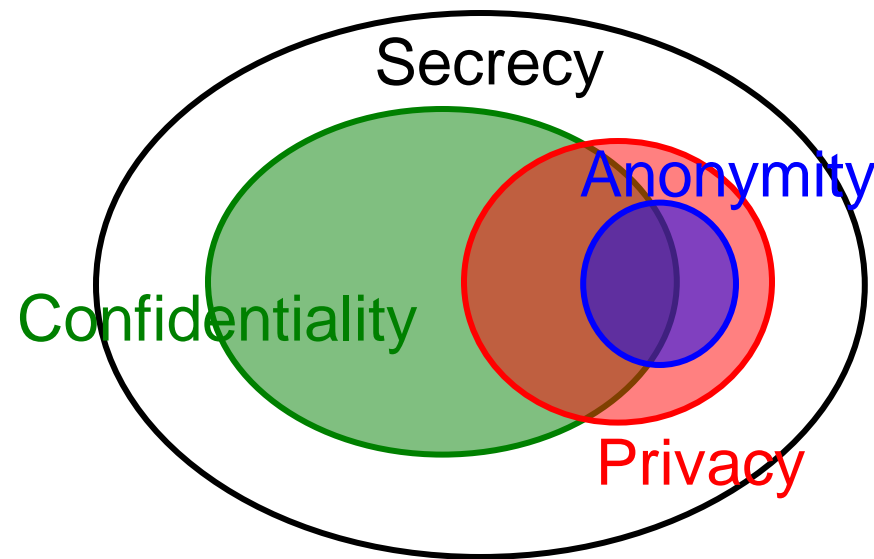
- Information Security (Prof. Dr. Srdjan Capkun, Prof. Dr. David Basin, Dr. Ralf Sasse)  
[https://infsec.ethz.ch/education/as15\\_ss19/ss2020/infsec.html](https://infsec.ethz.ch/education/as15_ss19/ss2020/infsec.html)
- Applied Cryptography (Prof. Dr. Kenny Paterson)  
<https://appliedcrypto.ethz.ch/education/lectures/appcry1.html>

# Secrecy, Confidentiality, Privacy, Anonymity

- Often considered synonymous, but are slightly different
- Secrecy
  - Keep data hidden from unintended receivers
  - “Alice and Bob use encrypted communication links to achieve secrecy”
- Confidentiality
  - Keep *someone else's* data secret
  - “Trent encrypts all user information to keep their client's information confidential in case of a file server compromise”
- Privacy
  - Keep *data about a person* secret
  - “To protect Alice's privacy, company XYZ did not disclose any personal information”

# Secrecy, Confidentiality, Privacy, Anonymity

- Anonymity
  - Keep *identity* of a protocol participant secret
  - “To hide her identity to the web server, Alice uses The Onion Router (TOR) to communicate”



# Integrity, Authentication

- Sometimes used interchangeably, but they have different connotations
- Data integrity
  - Ensure data is “correct” (i.e., correct syntax & unchanged)
  - Prevents unauthorized or improper changes
  - “Trent always verifies the integrity of his database after restoring a backup, to ensure that no incorrect records exist”
- Entity authentication or identification
  - Verify the identity of another protocol participant
  - “Alice authenticates Bob each time they establish a secure connection”
- Data authentication
  - Ensure that data originates from claimed sender
  - “For every message Bob sends, Alice authenticates it to ensure that it originates from Bob”

# Difference between Integrity and Authentication

- Integrity is often a property of *local or stored data*
  - For example, we want to ensure integrity for a database stored on disk, which emphasizes that we want to prevent unauthorized changes
  - Integrity emphasizes that data has not been changed
- Authentication used in network context, where entities communicate across a network
  - Two communicating hosts want to achieve data authentication to ensure data was not changed by network
  - Authentication emphasizes that data *was created by a specific sender*
  - Implies integrity, data unchanged in transit
  - Implies that identity of sender is verified

# Basic Cryptographic Primitives

- Symmetric (shared-key, same-key)
  - Block cipher (pseudo-random permutation PRP)
  - Stream cipher (pseudo-random generators PRG)
  - Message authentication code (MAC)
- Asymmetric (public-private key)
  - Diffie–Hellman key agreement
  - Public-key encryption
  - Digital signature
- Others (unkeyed symmetric)
  - One-way function
  - Cryptographic hash function

# Symmetric Cryptography



# Symmetric Encryption Primitives

- Encryption key = decryption key
- Encryption:  $E_K(\text{plaintext}) = \text{ciphertext}$
- Decryption:  $D_K(\text{ciphertext}) = \text{plaintext}$
- We write  $\{\text{plaintext}\}_K$  for  $E_K(\text{plaintext})$



# Stream Ciphers

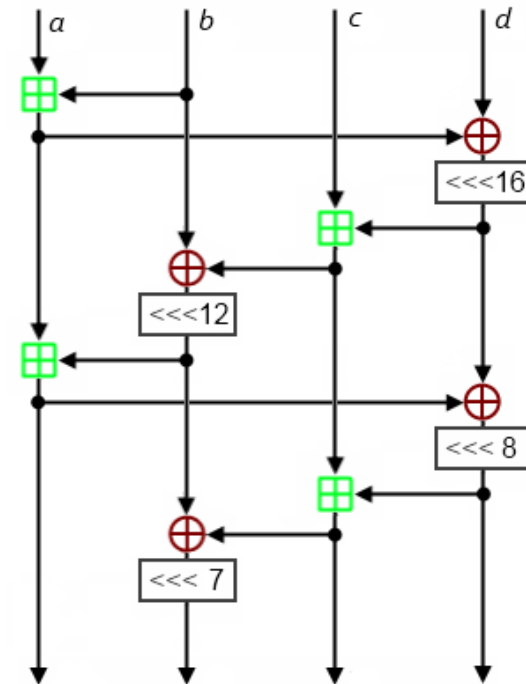
- One-time pad
  - Use unique random keystream for each message
  - Is this secure? **Yes.**
  - Is this secure if we re-use keystream? **No.**
- Stream ciphers use pseudo-random generator (PRG) to generate keystream from seed
  - Encryption: use shared key  $k$  and initialization vector  $IV$  for the seed  
ciphertext = plaintext  $\oplus$  PRG(  $k$ ,  $IV$  )
  - Send  $IV$ , ciphertext
- Example: AES in CTR mode

# ChaCha Stream Cipher

- Dan Bernstein designed ChaCha in 2008 (adaptation of earlier Salsa stream cipher)
- Used in TLS 1.3 and Wireguard VPN
- Structure: 128-bit constant, a 256-bit key, a 64-bit counter (Position), and a 64-bit nonce, arranged as a  $4 \times 4$  matrix of 32-bit words
- Quarter round function illustrates cipher structure using addition, xor, and rotate
- High speed: around 4 cycles per byte

Initial state of ChaCha

| Cons | Cons | Cons  | Cons  |
|------|------|-------|-------|
| Key  | Key  | Key   | Key   |
| Key  | Key  | Key   | Key   |
| Pos  | Pos  | Nonce | Nonce |



# Stream Cipher Vulnerabilities

## ■ Keystream reuse attack

- Enormous security vulnerability if same keystream used to encrypt two different messages
- $c1 = p1 \oplus k$ ,  $c2 = p2 \oplus k$
- $c1 \oplus c2 = p1 \oplus p2$  (which is easy to analyze, because the unknown key is removed!)
- $c1 = p1 \oplus \text{PRG}(K, IV)$ , where  $IV$  = initialization vector, make sure  $IV$  is never used twice!

## ■ Ciphertext modification attack

- Alteration of ciphertext will alter corresponding values in plaintext after decryption
- Example, encrypt a single bit:  $c = p \oplus k$ , for  $p=1$ ,  $k=0$ , thus  $c=1$
- If attacker changes  $c$  to 0 during transmission, decrypted value is changed to 0!  $p = c \oplus k$ , if  $c=0$ ,  $k=0$ , then  $p=0$
- To defend, need to ensure authenticity of ciphertext

# Block Ciphers

- Block cipher is a pseudo-random permutation (PRP), each key defines a one-to-one mapping of input block to output block
  - Substitution cipher with large block size
- Encrypt each block separately
- Examples: DES, Rijndael (=AES)

# Advanced Encryption Standard: AES

- Officially adopted for US government work, but voluntarily adopted by private sector
- Winning cipher was Rijndael (pronounced Rhine-doll)
  - Belgian designers: Joan Daemen & Vincent Rijmen
- Adopted by NIST in November 2001
- {128, 192, 256}-bit key size
- High-speed cipher
  - Using native AESni instructions on Intel and AMD CPUs, 128-bit AES encryption requires only 30 clock cycles (~10 ns)
    - ~7x faster than loading a byte from DRAM!
    - Illustration: light travels 3 m in 10 ns

# Block-Cipher Modes of Operation

- Block cipher modes of operation

- ECB: Electronic code book
- CBC: Cipher block chaining
- CFB: Cipher feedback
- OFB: Output feedback
- CTR: Counter mode
- GCM: Galois Counter Mode:  
encryption and authentication in a single pass!

**Block cipher in a stream  
cipher mode**

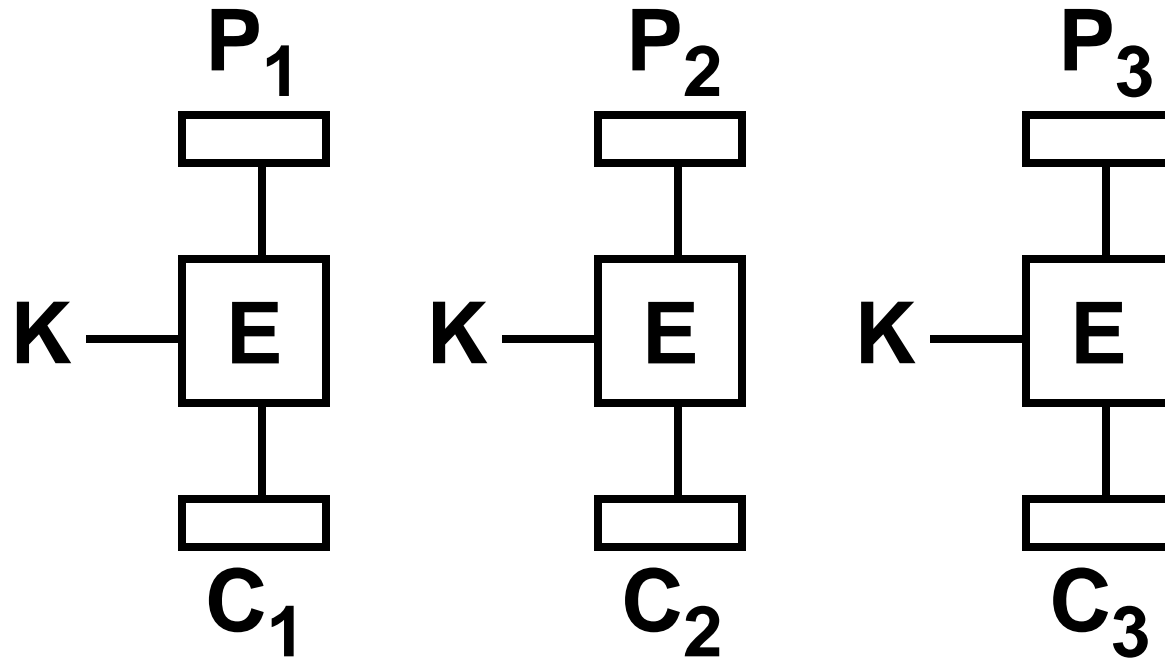
- Stream cipher: plaintext is XORed with keystream generated from secret key and initialization vector (IV)

# Electronic Code Book (ECB)

- Natural approach for encryption: given a message  $M$ , split  $M$  up into blocks of size  $b$  bits (where  $b$  = input size of block cipher)
- Ciphertext =  $\{M_1\}_K \{M_2\}_K \dots \{M_n\}_K$
- This approach is called Electronic Code Book mode (ECB mode)
- Advantages
  - Simple to compute
- Disadvantages
  - Same plaintext always corresponds to same ciphertext
  - Traffic analysis yields which ciphertext blocks are equal  $\rightarrow$  know which plaintext blocks are equal
  - Adversary may be able to guess part of plaintext, can decrypt parts of a message if same ciphertext block occurs
  - Adversary can replace blocks with other blocks



# ECB Mode

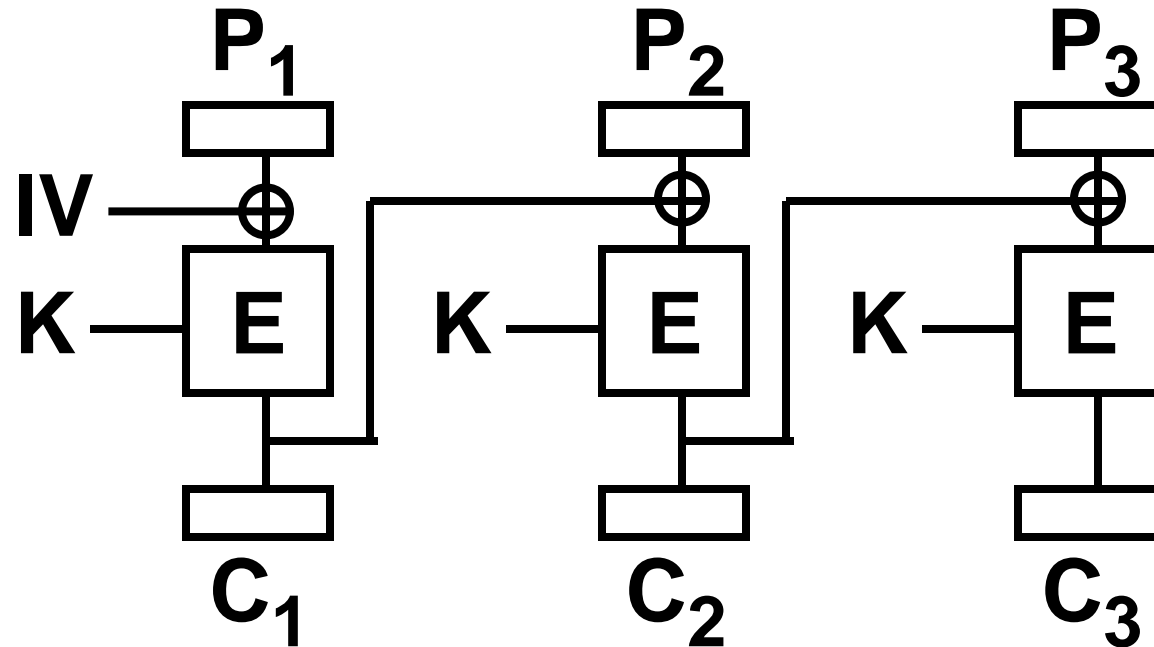


# Desired Properties

- Semantic security: if adversary had to guess value of a single plaintext bit, can guess at best at random
  - Even if we keep encrypting a 1-bit message with skewed distribution (e.g., a fire alarm message, which almost always carries the same plaintext bit), attacker cannot guess value of plaintext given ciphertext
- Adversary cannot cut-and-paste blocks of ciphertext, otherwise complete message garbled

# Cipher Block Chaining (CBC)

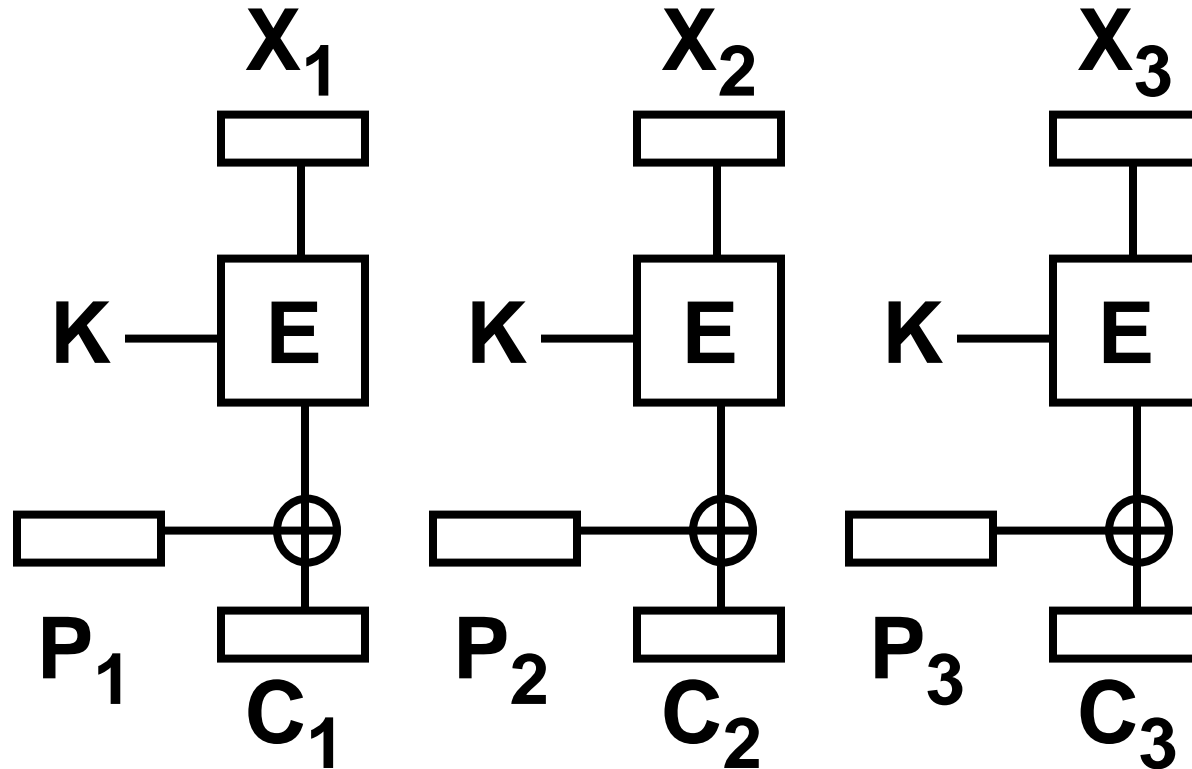
- $C_j = \{ P_j \oplus C_{j-1} \}_K$
- $C_0 = IV$  called initialization vector



- Advantages
  - Semantic security
- Disadvantages
  - Altered ciphertext only influences two blocks
  - Not secure for variable-sized messages!
  - However, self-synchronizing can be an advantage too!

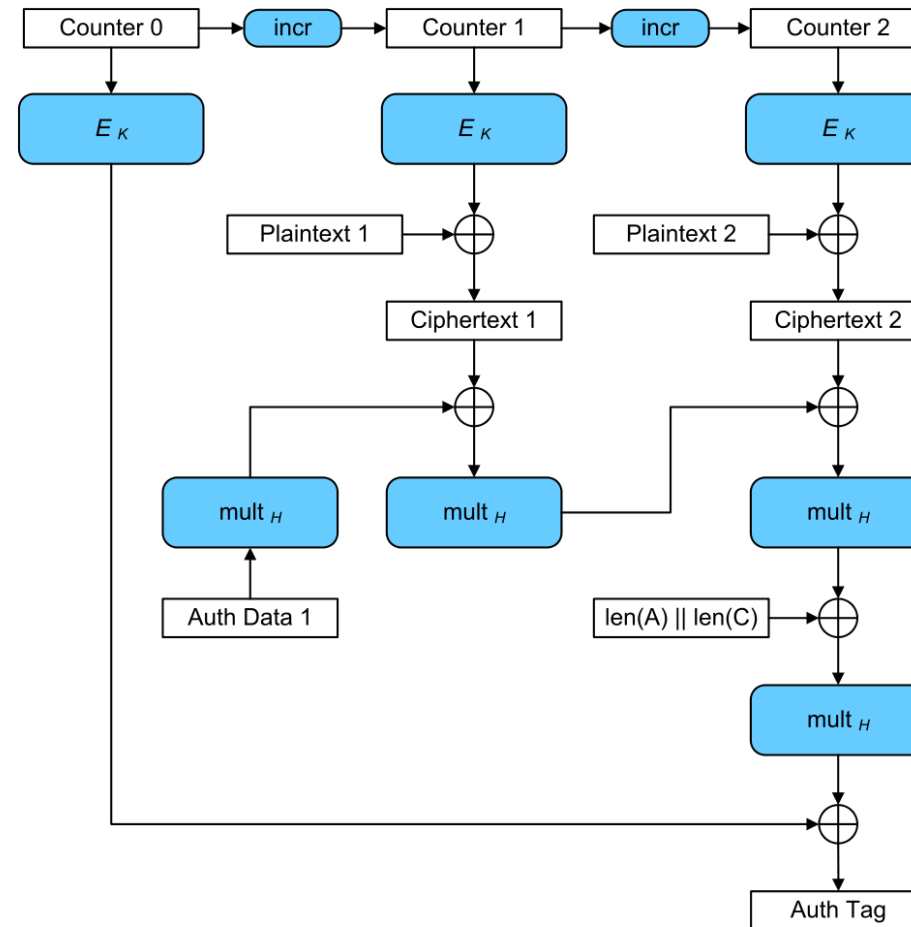
# Counter Mode (CTR)

- $X_1$  = IV called initialization vector
- $X_j = X_1 + i - 1$
- $C_j = \{ X_j \} K \oplus P_j$



- Advantages
  - Semantic security
- Disadvantages
  - Altered ciphertext only influences single block
  - Same vulnerabilities as any stream cipher

# Galois Counter Mode (GCM)

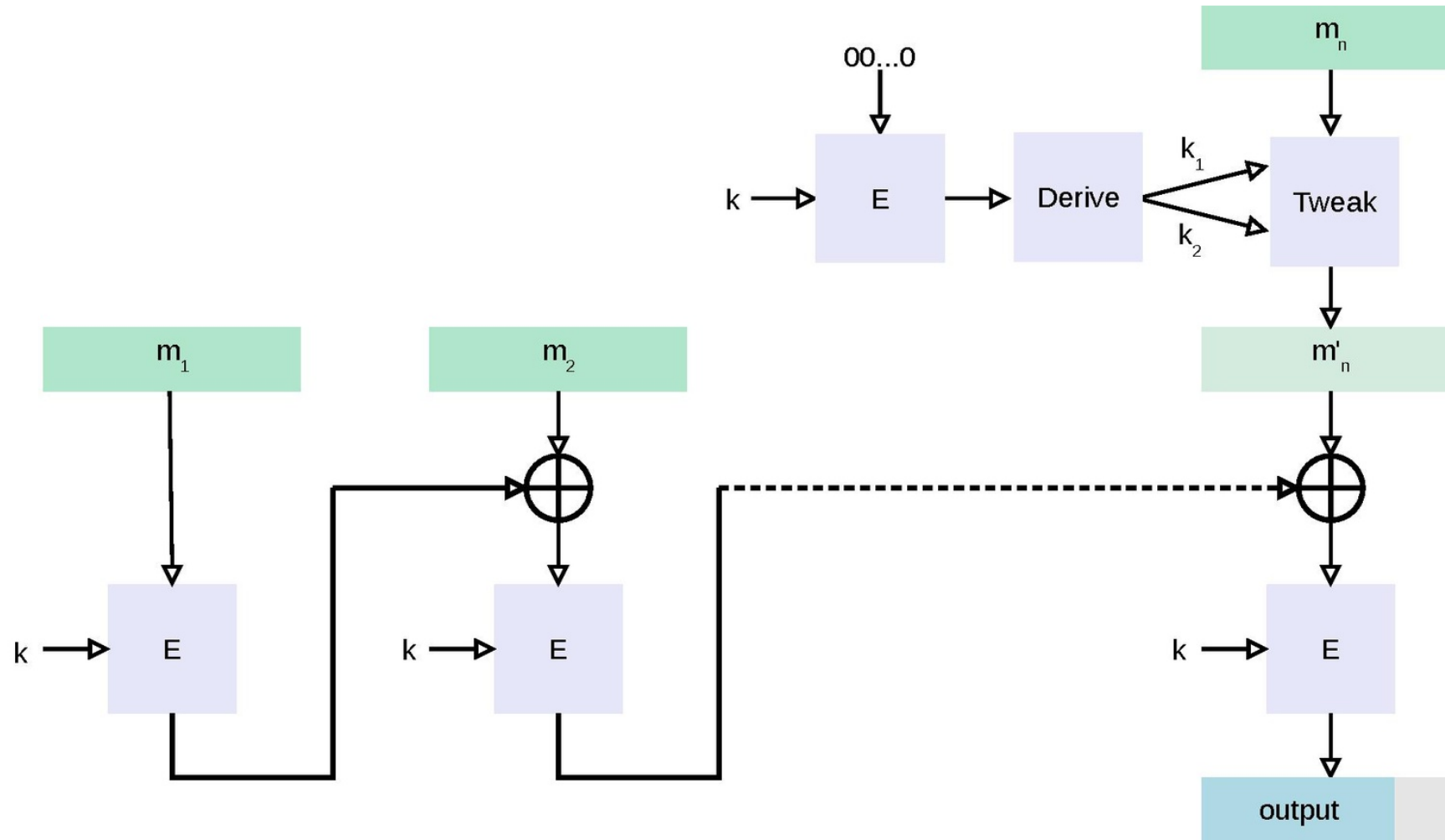


Graphic from [https://en.wikipedia.org/wiki/Galois/Counter\\_Mode](https://en.wikipedia.org/wiki/Galois/Counter_Mode)

# Message Authentication Codes

- Message authentication codes (MAC) provide a “cryptographic checksum” for authentication, integrity
  - Write  $\text{MAC}(K, M)$ , or  $\text{MAC}_K(M)$  ( $K$  is a shared symmetric key)
  - Intuition: MAC for a specific message can only be calculated when knowing the key  $K$
- Use
  - A and B share symmetric key  $K_{AB}$
  - $A \rightarrow B: M, \text{MAC}(K_{AB}, M)$
- Hash-based MAC: HMAC
  - Example based on SHA256:
  - $\text{HMAC-SHA256}(K, M) = \text{SHA256}(K \oplus \text{opad} \parallel \text{SHA256}(K \oplus \text{ipad} \parallel M))$
  - $\text{ipad} = 3636\ldots36$ ,  $\text{opad} = 5C5C\ldots5C$
- Block-cipher based MAC: CMAC
  - Fixes length vulnerabilities of CBC-MAC

# Cipher-based Message Authentication Code (CMAC)



Graphic from [https://en.wikipedia.org/wiki/One-key\\_MAC](https://en.wikipedia.org/wiki/One-key_MAC)

# Asymmetric Cryptography



# Asymmetric Primitive: Diffie–Hellman

- Public values: large prime  $p$ , generator  $g$
- Alice has secret value  $a$ , Bob has secret  $b$
- $A \rightarrow B$ :  $g^a \pmod{p}$
- $B \rightarrow A$ :  $g^b \pmod{p}$
- Bob computes  $(g^a)^b = g^{ab} \pmod{p}$
- Alice computes  $(g^b)^a = g^{ab} \pmod{p}$
- Eve *cannot* compute  $g^{ab} \pmod{p}$

# Example

- $a=3, b=6, g=2, p=11$
- $A \rightarrow B: g^a \pmod{p} = 2^3 \pmod{11} = 8$
- $B \rightarrow A: g^b \pmod{p} = 2^6 \pmod{11}$   
 $= 64 \pmod{11} = 9$
- Bob computes  $(g^a)^b \pmod{p} = 8^6 \pmod{11}$   
 $= 262144 \pmod{11} = 3$
- Alice computes  $(g^b)^a \pmod{p} = 9^3 \pmod{11}$   
 $= 729 \pmod{11} = 3$

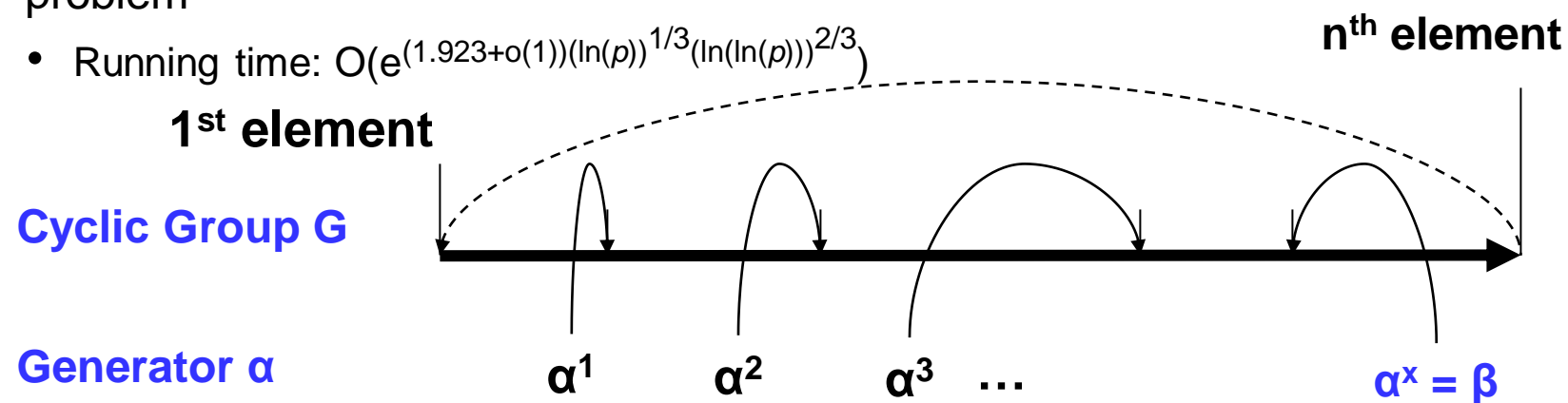
# Discrete Logarithm Problem

- Public values: large prime  $p$ , generator  $g$
- $g^a \bmod p = x$
- Discrete logarithm problem: given  $x$ ,  $g$ , and  $p$ , find  $a$
- Table  $g=2$ ,  $p=11$

|          |          |          |          |          |           |          |          |          |          |           |
|----------|----------|----------|----------|----------|-----------|----------|----------|----------|----------|-----------|
| <b>a</b> | <b>1</b> | <b>2</b> | <b>3</b> | <b>4</b> | <b>5</b>  | <b>6</b> | <b>7</b> | <b>8</b> | <b>9</b> | <b>10</b> |
| <b>x</b> | <b>2</b> | <b>4</b> | <b>8</b> | <b>5</b> | <b>10</b> | <b>9</b> | <b>7</b> | <b>3</b> | <b>6</b> | <b>1</b>  |

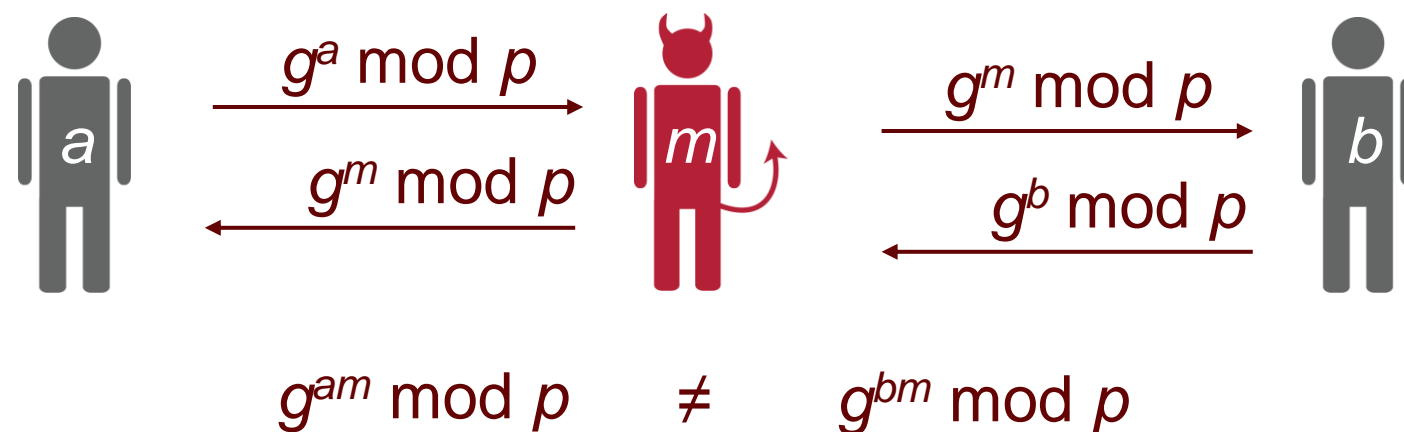
- Number field sieve is fastest algorithm known today to solve discrete logarithm problem

- Running time:  $O(e^{(1.923+o(1))(\ln(p))^{1/3}(\ln(\ln(p)))^{2/3}})$



# Problem: Man-in-the-Middle Attack

- Public values: large prime  $p$ , generator  $g$
- Problem: in Man-in-the-Middle attack, Mallory impersonates Alice to Bob and Bob to Alice



# Asymmetric Primitive: RSA

- Invented by Rivest, Shamir, Adleman 1978
- Let  $p, q$  be large secret primes
- Pick  $e$ , compute  $d$  so  $ed \equiv 1 \pmod{\phi(pq)}$ 
  - Public key:  $N=pq, e$
  - Private key:  $p, q, d$
- Signature generation of message  $M$   
 $\Sigma = M^d \pmod{N}$
- Signature verification:  
 $\Sigma^e = M^{ed} = M^{1 + K\phi(pq)} = M \pmod{N}$

# Example

- $p=3, q=7, N=21, \phi(pq)=12, e=5, d=5$
- $\phi(pq) = (3-1)*(7-1) = 2*6 = 12$
- $e=5$ 
  - Want  $d$  such that  $ed \equiv 1 \pmod{\phi(pq)} = 1 \pmod{12}$
  - Pick  $d=5$
- $M=2$ 
  - Signature  $\Sigma = M^d \pmod{N} = 2^5 \pmod{21} = 11$
  - Signature verification:  
$$\Sigma^e = (M^d)^e \pmod{21} = 11^5 \pmod{21}$$
$$= 161051 \pmod{21} = 2$$
- Could also pick  $e=7, d=7...$

# Encrypted Key Exchange (EKE)

## DH Protocol

- A, B share password  $P$ , want to authenticate each other and establish a shared secret key
- $K = H(P)$ , A picks random  $a$ , B picks random  $b$
- 1:  $A \rightarrow B: \{g^a\}_K$
- $K' = H(g^{ab})$
- 2:  $B \rightarrow A: \{g^b\}_K, \{N_B\}_{K'}$
- 3:  $A \rightarrow B: \{N_A, N_B\}_{K'}$
- 4:  $B \rightarrow A: \{N_A\}_K$
- Dictionary attacks? (Enables verification by attacker if a given password was correct or not.)

# Difference between Authentication and Signature

- Authentication enables the receiver to verify origin, **but receiver cannot** convince a third party of origin
- Signature enables the receiver to verify origin, **and receiver can** convince third party of origin as well
- Signature also provides authentication



# Comparison Symmetric vs Asymmetric Crypto

## Symmetric crypto

- Need **shared** secret key
- 128 bit key for high security (year 2020)
- ~100,000,000 ops/s on 5GHz processor
- 10x speedup in HW

## Asymmetric crypto

- Need **authentic** public key  
→ public-key infrastructures (PKIs)
- 3072 bit key (RSA), 384 bit key (EC) for high security (year 2020)
- ~1000 signatures/s  
~10000 verify/s (RSA) on 5GHz processor
- Limited speedup in HW

# Hash Functions

# Cryptographic Hash Functions

- Maps arbitrary-length input into finite length output
- Properties of a *secure* (cryptographic) hash function
  - **One-way**: Given  $y = H(x)$ , cannot find  $x'$  s.t.  $H(x') = y$
  - **Weak collision resistance**: Given  $x$ , cannot find  $x' \neq x$  s.t.  $H(x) = H(x')$
  - **Strong collision resistance**: Cannot find  $x \neq x'$  s.t.  $H(x) = H(x')$
- Example: MD5, SHA-1, SHA-2, SHA-3, Poly1305
  - MD5 and SHA-1 have been broken to some extent

# Attack Complexity: One-Wayness

- Assume secure hash function with  $n$ -bit output
- One-wayness: given output  $y$ , how many operations does it take to find any  $x$ , such that  $H(x) = y$ ?
  - Assumption: best attack is random search
  - For each trial  $x$ , probability that output is  $y$  is  $2^{-n}$
  - $P[\text{find } x \text{ after } m \text{ trials}] = 1 - (1 - 2^{-n})^m$
  - Rule of thumb: find  $x$  after  $2^{n-1}$  trials on average

# Attack Complexity: *Weak* Collision Resistance

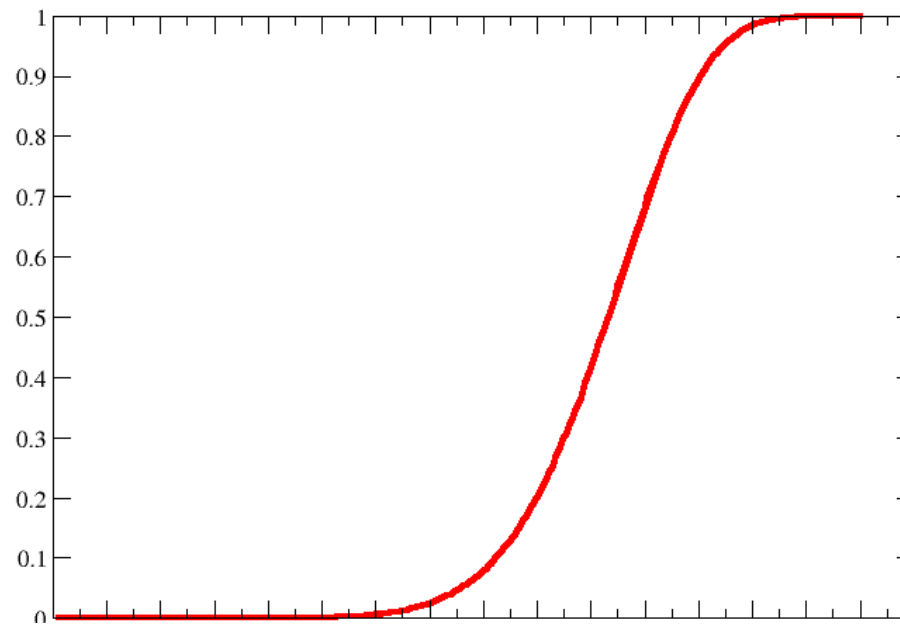
- Weak collision resistance (or second pre-image collision resistance): given input  $x$ , how many operations does it take to find another  $x' \neq x$ , s.t.  $H(x) = H(x')$ ?
  - Assumption: best attack is random search
  - For each trial  $x'$ , probability that output is equal is  $2^{-n}$
  - $P[\text{find } x \text{ after } m \text{ trials}] = 1 - (1 - 2^{-n})^m$
  - Rule of thumb: find  $x'$  after  $2^{n-1}$  trials on average

# Attack Complexity: *Strong* Collision Resistance

- Strong collision resistance: how many operations does it take to find  $x$  and  $x'$ , s.t.  $x' \neq x$  and  $H(x) = H(x')$ ?
  - Assumption: best attack is random search
  - Algorithm picks random  $x'$ , checks whether  $H(x')$  matches any other output value previously seen
  - $P[\text{find col after } m \text{ trials}] = 1 - (1 - 1/2^n)(1 - 2/2^n)(1 - 3/2^n) \dots (1 - (m+1)/2^n)$
  - Rule of thumb: find collision after  $2^{n/2}$  trials on average
    - $(1.17 \cdot 2^{n/2})$  to be a bit more precise)

# Birthday Paradox

- How many people need to be in a room to have a probability  $> 50\%$  that at least two people have the same birthday?
- Answer: approximately  $1.17 \cdot 365^{1/2} \sim 22.4$



# Lack of Collision Resistance

- How good is life without collision insurance?
- No real effect on most protocols
  - SSL, IPsec, SSH, etc. use MD5 in three ways
    - Key expansion
    - HMAC
    - Signatures
  - In most use cases, not affected by collisions
- What about PKI certificates?
  - Register certificate for [www.something.com](http://www.something.com) and use certificate for [www.bank.com](http://www.bank.com) if  $H(\text{Cert } \text{www.something.com}) = H(\text{Cert } \text{www.bank.com})$
  - Countermeasure?



# One-Way Hash Chains

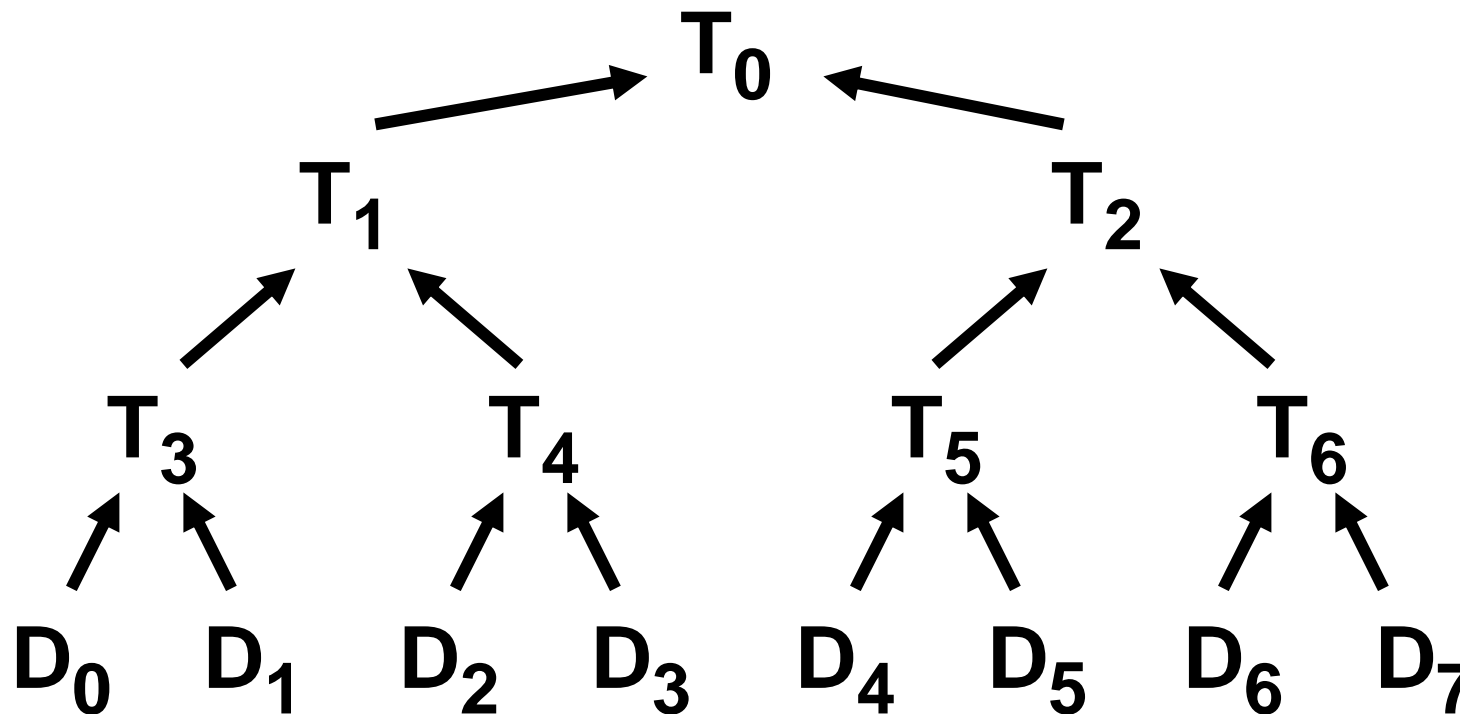
- Versatile cryptographic primitive
- Construction
  - Pick random  $r_N$  and public one-way function  $F$
  - $r_i = F(r_{i+1})$
  - Secret value:  $r_N$ , public value  $r_0$



- Properties
  - Use in reverse order of construction:  $r_0, r_1, \dots, r_N$
  - Infeasible to derive  $r_i$  from  $r_j$  ( $j < i$ )
  - Efficiently authenticate  $r_i$  using  $r_j$  ( $j < i$ ):  $r_j = F^{i-j}(r_i)$
  - Robust to missing values

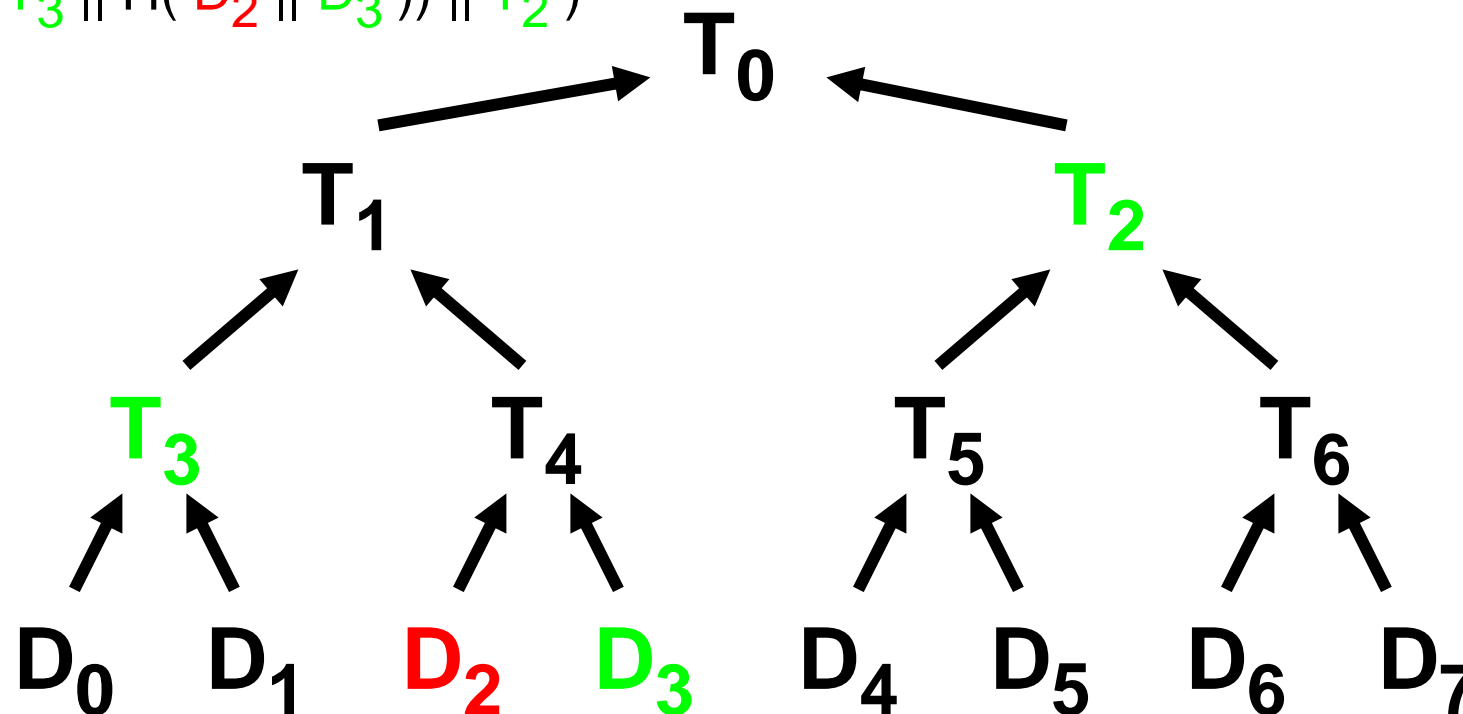
# Merkle Hash Trees

- Authenticate a sequence of data values  $D_0, D_1, \dots, D_N$
- Construct binary tree over data values



# Merkle Hash Trees




- Verifier knows  $T_0$
- How can verifier authenticate leaf  $D_i$  ?
- Solution: recompute  $T_0$  using  $D_i$
- Example authenticate  $D_2$  , send  $D_3 T_3 T_2$
- Verify  $T_0 = H( H( T_3 \parallel H( D_2 \parallel D_3 ) ) \parallel T_2 )$



# Selecting Cryptographic Key Lengths

## ■ Useful resources

- <https://www.keylength.com>
- [https://en.wikipedia.org/wiki/Key\\_size](https://en.wikipedia.org/wiki/Key_size)

| Method  | Date                    | Symmetric | Factoring<br>Modulus | Discrete Logarithm<br>Key Group | Elliptic Curve | Hash |
|---|-------------------------|-----------|----------------------|---------------------------------|----------------|------|
| [1] Lenstra / Verheul  | 2037                    | 99        | 2986 2464            | 175 2986                        | 186            | 197  |
| [2] Lenstra Updated    | 2032                    | 90        | 2278 3034            | 179 2278                        | 179            | 179  |
| [3] ECRYPT II   | 2031 - 2040             | 128       | 3248                 | 256 3248                        | 256            | 256  |
| [4] NIST  | 2016 - 2030<br>& beyond | 128       | 3072                 | 256 3072                        | 256            | 256  |
| [5] ANSSI   | > 2030                  | 128       | 3072                 | 200 3072                        | 256            | 256  |
| [6] IAD-NSA   | -                       | 256       | 3072                 | - -                             | 384            | 384  |
| [7] RFC3766          | -                       | 123       | 2986                 | 246 2986                        | 231            | -    |
| [8] BSI   | > 2022                  | 128       | 3000                 | 250 3000                        | 250            | 256  |

# Powers of Two Conversion & Useful Units

- $2^n = 10^m$   
 $m \sim (n/10) * 3$   
 $n \sim (m/3) * 10$
- Fast conversion trick:
  - $2^{10} \sim 10^3$  ,  $2^{20} \sim 10^6$  ,  $2^{30} \sim 10^9$
  - $2^0 = 1$  ,  $2^1 = 2$  ,  $2^2 = 4$  ,  $2^3 = 8$  ,  $2^4 = 16$  ,  $2^5 = 32$  ,  $2^6 = 64$  , 128 , 256 , 512
- Seconds per day  $\sim 2^{16}$  , seconds per year  $\sim 2^{25}$
- Schneier's Applied Cryptography, p18
  - Probability to get hit by lightning per day ( $10^{-10}$  ,  $2^{-33}$  )
  - Number of atoms on earth ( $10^{51}$  ,  $2^{170}$  )
  - Number of atoms in the universe ( $10^{77}$  ,  $2^{265}$  )
  - Time until next ice age (14,000,  $2^{14}$  years)
  - Duration until sun goes nova ( $10^9$  ,  $2^{30}$  years)
  - Age of the Universe ( $10^{10}$  ,  $2^{33}$  years)