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# Syntax Analysis – Part V

(Finish LR(0) Parsing, Start on LR(1) Parsing)

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# From Last Time: Shift-Reduce Parsing

$S \rightarrow S + E \mid E$ $E \rightarrow \text{num} \mid (S)$
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derivation	stack	input stream	action
(1+2+(3+4))+5		(1+2+(3+4))+5	shift
(1+2+(3+4))+5	(	1+2+(3+4))+5	shift
(1+2+(3+4))+5	(1	+2+(3+4))+5	reduce $E \rightarrow \text{num}$
(E+2+(3+4))+5	(E	+2+(3+4))+5	reduce $S \rightarrow E$
(S+2+(3+4))+5	(S	+2+(3+4))+5	shift
(S+2+(3+4))+5	(S+	2+(3+4))+5	shift
(S+2+(3+4))+5	(S+2	+(3+4))+5	reduce $E \rightarrow \text{num}$
(S+E+(3+4))+5	(S+E	+(3+4))+5	reduce $S \rightarrow S+E$
(S+(3+4))+5	(S	+(3+4))+5	shift
(S+(3+4))+5	(S+	(3+4))+5	shift
(S+(3+4))+5	(S+(	3+4))+5	shift
(S+(3+4))+5	(S+(3	+4))+5	reduce $E \rightarrow \text{num}$
...			

# From Last Time: LR Parsing Table Example

We want to derive this in an algorithmic fashion

State	Input terminal					Non-terminals	
	(	)	id	,	\$	S	L
	1	s3	s2			g4	
	2	S→id	S→id	S→id	S→id		
	3	s3	s2			g7	g5
	4				accept		
	5	s6	s8				
	6	S→(L)	S→(L)	S→(L)	S→(L)		
	7	L→S	L→S	L→S	L→S		
	8	s3	s2			g9	
	9	L→L,S	L→L,S	L→L,S	L→L,S		

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# Start State and Closure

- **Start state**
  - Augment grammar with production:  $S' \rightarrow S \$$
  - Start state of DFA has empty stack:  $S' \rightarrow . S \$$
- **Closure of a parser state:**
  - Start with  $\text{Closure}(S) = S$
  - Then for each item in  $S$ :
    - $X \rightarrow \alpha . Y \beta$
    - Add items for all the productions  $Y \rightarrow \gamma$  to the closure of  $S$ :  $Y \rightarrow . \gamma$

# Closure

$$\begin{aligned} S &\rightarrow (L) \mid id \\ L &\rightarrow S \mid L, S \end{aligned}$$

DFA start state

$$S' \rightarrow \cdot S \$$$

closure

$$S' \rightarrow \cdot S \$$$
$$S \rightarrow \cdot (L)$$
$$S \rightarrow \cdot id$$

- Set of possible productions to be reduced next
- Closure of a parser state, S:
  - Start with  $\text{Closure}(S) = S$
  - Then for each item in S:
    - $X \rightarrow \alpha \cdot Y \beta$
    - Add items for all the productions  $Y \rightarrow \gamma$  to the closure of S:  $Y \rightarrow \cdot \gamma$

# The Goto Operation

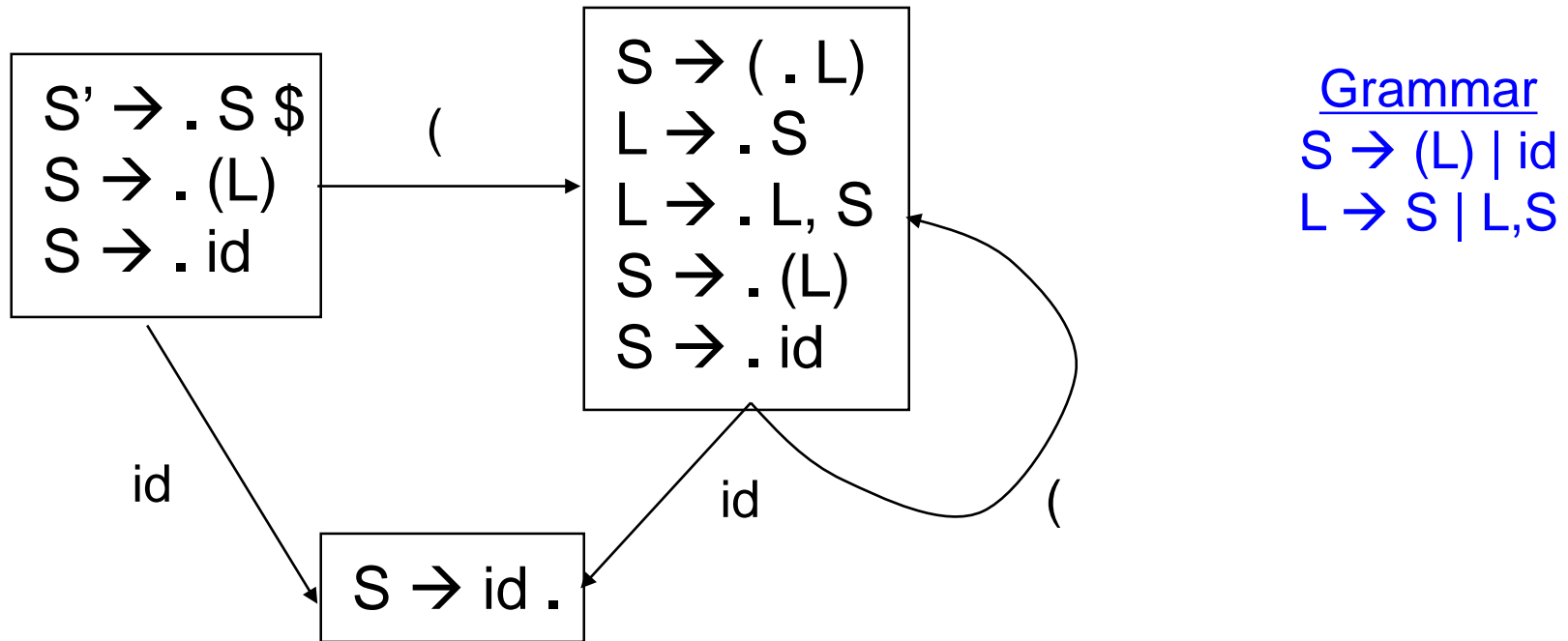
- **Goto operation** = describes transitions between parser states, which are sets of items
- **Algorithm:** for state  $S$  and a symbol  $Y$ 
  - If the item  $[X \rightarrow \alpha . Y \beta]$  is in  $I$ , then
  - $\text{Goto}(I, Y) = \text{Closure}([X \rightarrow \alpha Y . \beta])$

$S' \rightarrow . S \$$   
 $S \rightarrow . (L)$   
 $S \rightarrow . id$

$\text{Goto}(S, '(')$

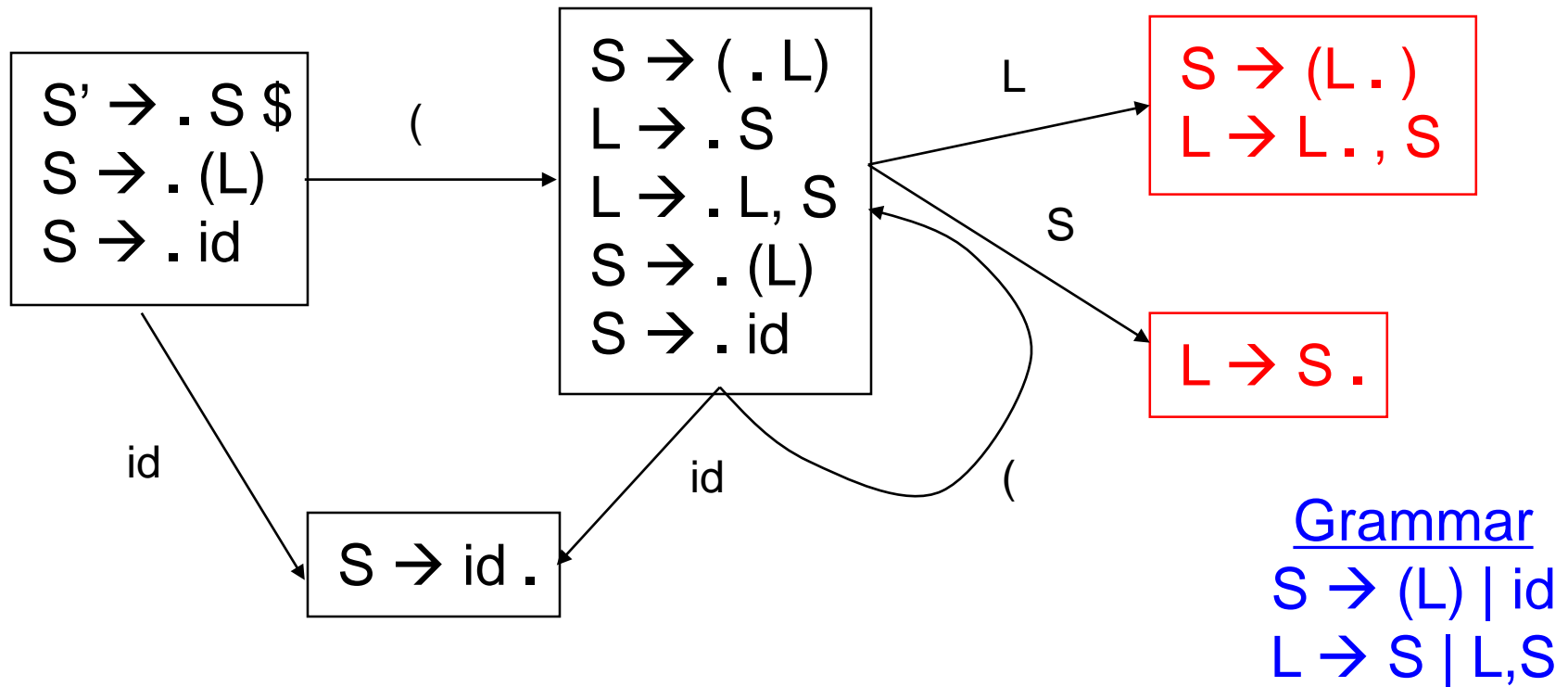
$\text{Closure}(\{ S \rightarrow ( . L) \})$

# Goto: Terminal Symbols



In new state, include all items that have appropriate input symbol just after dot, advance dot in those items and take closure

# Goto: Non-terminal Symbols



same algorithm for transitions on non-terminals

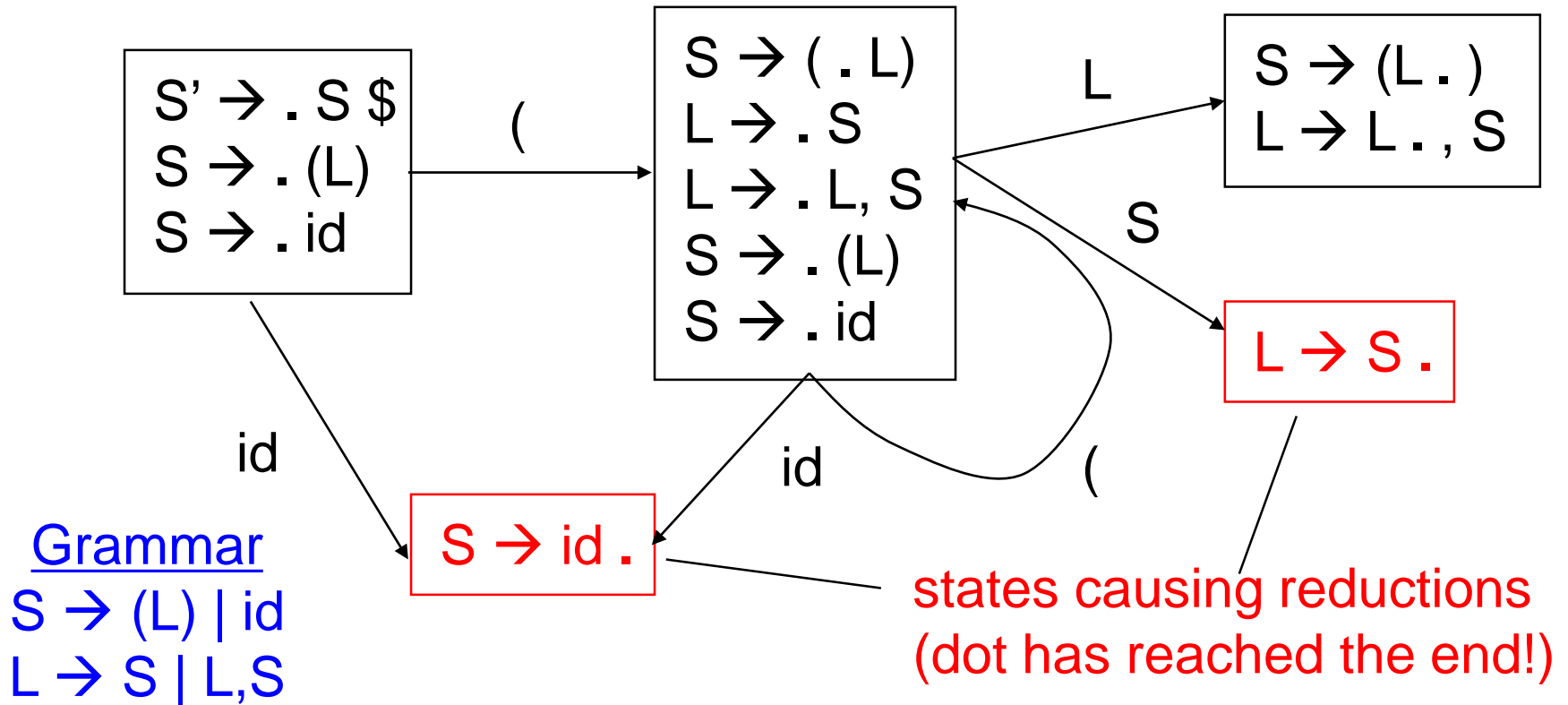


# Class Problem

$$\begin{array}{l} E' \rightarrow E \\ E \rightarrow E + T \mid T \\ T \rightarrow T * F \mid F \\ F \rightarrow (E) \mid \text{id} \end{array}$$

1. If  $I = \{ [E' \rightarrow \cdot E] \}$ , then  $\text{Closure}(I) = ??$
2. If  $I = \{ [E' \rightarrow E \cdot], [E \rightarrow E \cdot + T] \}$ , then  $\text{Goto}(I, +) = ??$

# Applying Reduce Actions



Pop RHS off stack, replace with LHS  $X$  ( $X \rightarrow \beta$ ),  
 then rerun DFA (e.g.,  $(x)$ )

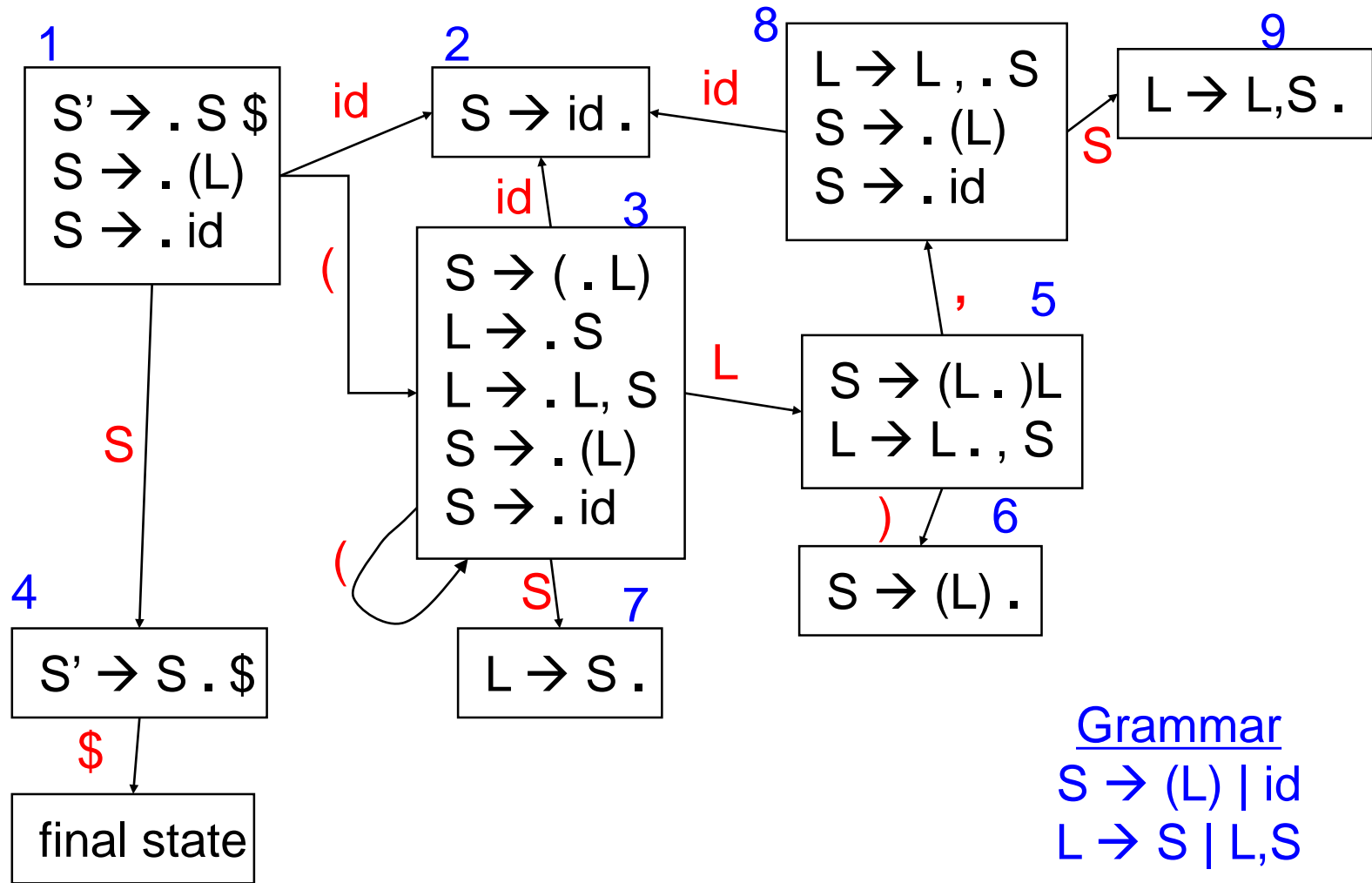
# Reductions

- On reducing  $X \rightarrow \beta$  with stack  $\alpha\beta$ 
  - Pop  $\beta$  off stack, revealing prefix  $\alpha$  and state
  - Take single step in DFA from top state
  - Push  $X$  onto stack with new DFA state

- Example

derivation	stack	input	action
$((a),b) \leftarrow$	1 ( 3 ( 3	a),b)	shift, goto 2
$((a),b) \leftarrow$	1 ( 3 ( 3 a 2	),b)	reduce $S \rightarrow id$
$((S),b) \leftarrow$	1 ( 3 ( 3 S 7	),b)	reduce $L \rightarrow S$

# Full DFA



Grammar  
 $S \rightarrow (L) \mid id$   
 $L \rightarrow S \mid L, S$

# Parsing Example ((a),b)

$S \rightarrow (L) \mid id$   
 $L \rightarrow S \mid L,S$

derivation	stack	input	action
((a),b) ←	1	((a),b)	shift, goto 3
((a),b) ←	1(3	(a),b)	shift, goto 3
((a),b) ←	1(3(3	a),b)	shift, goto 2
((a),b) ←	1(3(3a2	),b)	reduce $S \rightarrow id$
((S),b) ←	1(3(3S7	),b)	reduce $L \rightarrow S$
((L),b) ←	1(3(3L5	),b)	shift, goto 6
((L),b) ←	1(3(3L5)6	,b)	reduce $S \rightarrow (L)$
(S,b) ←	1(3S7	,b)	reduce $L \rightarrow S$
(L,b) ←	1(3L5	,b)	shift, goto 8
(L,b) ←	1(3L5,8	b)	shift, goto 9
(L,b) ←	1(3L5,8b2	)	reduce $S \rightarrow id$
(L,S) ←	1(3L8,S9	)	reduce $L \rightarrow L,S$
(L) ←	1(3L5	)	shift, goto 6
(L) ←	1(3L5)6		reduce $S \rightarrow (L)$
S ←	1S4	\$	done



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# Building the Parsing Table

- **States in the table = states in the DFA**
- **For transition  $S \rightarrow S'$  on terminal  $C$ :**
  - $\text{Table}[S, C] += \text{Shift}(S')$
- **For transition  $S \rightarrow S'$  on non-terminal  $N$ :**
  - $\text{Table}[S, N] += \text{Goto}(S')$
- **If  $S$  is a reduction state  $X \rightarrow \beta$  then:**
  - $\text{Table}[S, *] += \text{Reduce}(X \rightarrow \beta)$

# Computed LR Parsing Table

	Input terminal					Non-terminals	
	(	)	id	,	\$	S	L
1	s3		s2			g4	
2	S→id	S→id	S→id	S→id	S→id		
3	s3		s2			g7	g5
4					accept		
5		s6		s8			
6	S→(L)	S→(L)	S→(L)	S→(L)	S→(L)		
7	L→S	L→S	L→S	L→S	L→S		
8	s3		s2			g9	
9	L→L,S	L→L,S	L→L,S	L→L,S	L→L,S		

blue = shift

red = reduce

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# LR(0) Summary

- **LR(0) parsing recipe:**
  - Start with LR(0) grammar
  - Compute LR(0) states and build DFA:
    - Use the closure operation to compute states
    - Use the goto operation to compute transitions
  - Build the LR(0) parsing table from the DFA
- **This can be done automatically**



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# Class Problem

Generate the DFA for the following grammar

$S \rightarrow E + S \mid E$

$E \rightarrow \text{num}$

# LR(0) Limitations

- An LR(0) machine only works if states with reduce actions have a single reduce action
  - Always reduce regardless of lookahead
- With a more complex grammar, construction gives states with shift/reduce or reduce/reduce conflicts
- Need to use lookahead to choose

OK

$L \rightarrow L, S.$

shift/reduce

$L \rightarrow L, S.$   
 $S \rightarrow S., L$

reduce/reduce

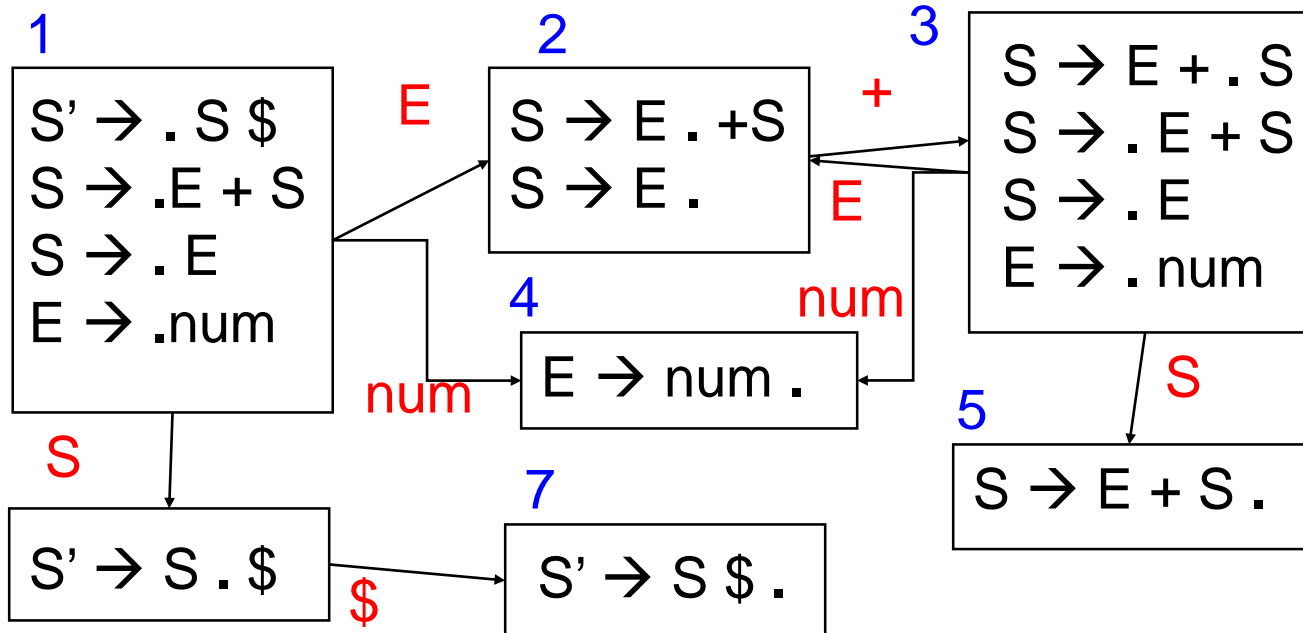
$L \rightarrow S, L.$   
 $L \rightarrow S.$

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# A Non-LR(0) Grammar

- Grammar for addition of numbers
  - $S \rightarrow S + E \mid E$
  - $E \rightarrow \text{num}$
- Left-associative version is LR(0)
- Right-associative is **not LR(0)** as you saw with the previous class problem
  - $S \rightarrow E + S \mid E$
  - $E \rightarrow \text{num}$

# LR(0) Parsing Table



Grammar  
 $S \rightarrow E + S \mid E$   
 $E \rightarrow \text{num}$

Shift or  
reduce  
in state 2?

	num	+	\$	E	S
1	s4			g2	g6
2	$S \rightarrow E$	s3/ $S \rightarrow E$	$S \rightarrow E$		

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# Solve Conflict With Lookahead

- **3 popular techniques for employing lookahead of 1 symbol with bottom-up parsing**
  - SLR – Simple LR
  - LALR – LookAhead LR
  - LR(1)
- **Each as a different means of utilizing the lookahead**
  - Results in different processing capabilities

# SLR Parsing

- **SLR Parsing = Easy extension of LR(0)**
  - For each reduction  $X \rightarrow \beta$ , look at next symbol  $C$
  - Apply reduction only if  $C$  is in FOLLOW( $X$ )
- **SLR parsing table eliminates some conflicts**
  - Same as LR(0) table except reduction rows
  - Adds reductions  $X \rightarrow \beta$  only in the columns of symbols in FOLLOW( $X$ )

Example: FOLLOW( $S$ ) = { $\$$ }

Grammar

$S \rightarrow E + S \mid E$

$E \rightarrow \text{num}$

	num	+	\$	E	S
1	s4			g2	g6
2		s3			
			S→E		

# SLR Parsing Table

- Reductions do not fill entire rows as before
- Otherwise, same as LR(0)

Grammar

$S \rightarrow E + S \mid E$

$E \rightarrow \text{num}$

	num	+	\$	E	S
1	s4			g2	g6
2		s3	$S \rightarrow E$		
3	s4			g2	g5
4		$E \rightarrow \text{num}$	$E \rightarrow \text{num}$		
5			$S \rightarrow E + S$		
6			s7		
7			accept		

# Class Problem

Consider:

$S \rightarrow L = R$

$S \rightarrow R$

$L \rightarrow *R$

$L \rightarrow \text{ident}$

$R \rightarrow L$

Think of L as l-value, R as r-value, and  
\* as a pointer dereference

When you create the states in the SLR(1) DFA,  
2 of the states are the following:

$S \rightarrow L . = R$   
 $R \rightarrow L .$

$S \rightarrow R .$

Do you have any shift/reduce conflicts? (Not as easy as it looks)



# LR(1) Parsing

- Get as much as possible out of 1 lookahead symbol parsing table
- LR(1) grammar = recognizable by a shift/reduce parser with 1 lookahead
- LR(1) parsing uses similar concepts as LR(0)
  - Parser states = set of items
  - LR(1) item = LR(0) item + lookahead symbol possibly following production
    - LR(0) item:  $S \rightarrow . S + E$
    - LR(1) item:  $S \rightarrow . S + E \text{ , } \underline{+}$
    - Lookahead only has impact upon REDUCE operations, apply when lookahead = next input

# LR(1) States

- **LR(1) state = set of LR(1) items**
- **LR(1) item =  $(X \rightarrow \alpha . \beta , y)$** 
  - Meaning:  $\alpha$  already matched at top of the stack, next expect to see  $\beta y$
- **Shorthand notation**
  - $(X \rightarrow \alpha . \beta , \{x1, ..., xn\})$
  - means:
    - $(X \rightarrow \alpha . \beta , x1)$
    - ...
    - $(X \rightarrow \alpha . \beta , xn)$
- **Need to extend closure and goto operations**

$S \rightarrow S . + E$	$+, \$$
$S \rightarrow S + . E$	$num$

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# LR(1) Closure

- **LR(1) closure operation:**
  - Start with  $\text{Closure}(S) = S$
  - For each item in  $S$ :
    - $X \rightarrow \alpha . Y \beta, z$
    - and for each production  $Y \rightarrow \gamma$ , add the following item to the closure of  $S$ :  $Y \rightarrow . \gamma, \text{FIRST}(\beta z)$
  - Repeat until nothing changes
- **Similar to LR(0) closure, but also keeps track of lookahead symbol**

# LR(1) Start State

- Initial state: start with  $(S' \rightarrow . S , \$)$ , then apply closure operation
- Example: sum grammar

$S' \rightarrow S \$$   
 $S \rightarrow E + S \mid E$   
 $E \rightarrow \text{num}$

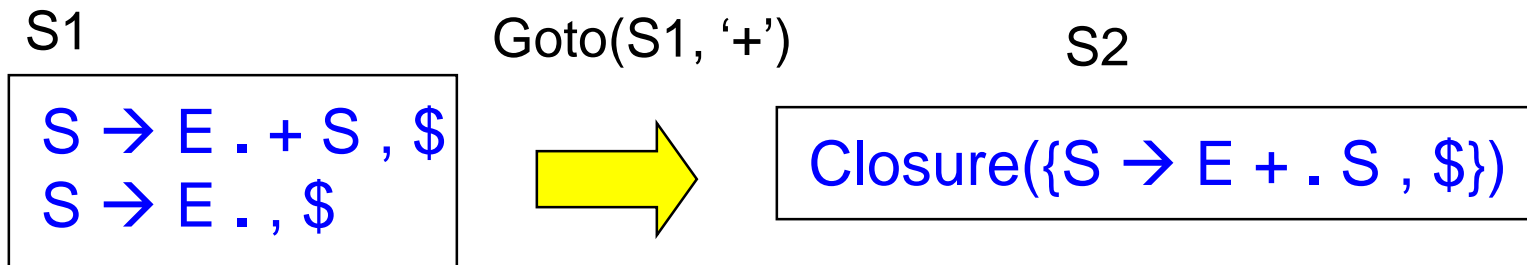
$S' \rightarrow . S , \$$

closure

$S' \rightarrow . S , \$$   
 $S \rightarrow . E + S , \$$   
 $S \rightarrow . E , \$$   
 $E \rightarrow . \text{num} , +, \$$

# LR(1) Goto Operation

- LR(1) goto operation = describes transitions between LR(1) states
- Algorithm: for a state  $S$  and a symbol  $Y$  (as before)
  - If the item  $[X \rightarrow \alpha . Y \beta]$  is in  $I$ , then
  - $\text{Goto}(I, Y) = \text{Closure}([X \rightarrow \alpha Y . \beta])$



Grammar:

$S' \rightarrow S\$$

$S \rightarrow E + S \mid E$

$E \rightarrow \text{num}$

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# Class Problem

1. Compute:  $\text{Closure}(I = \{S \rightarrow E + \cdot S, \$\})$
2. Compute:  $\text{Goto}(I, \text{num})$
3. Compute:  $\text{Goto}(I, E)$

$$\begin{aligned} S' &\rightarrow S \$ \\ S &\rightarrow E + S \mid E \\ E &\rightarrow \text{num} \end{aligned}$$
