Syntax Analysis – Part IV

(Bottom-Up Parsing)

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Grammars

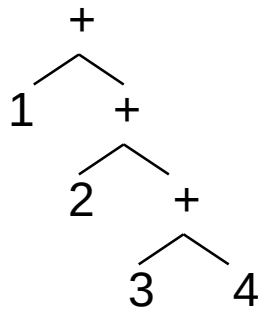
- **Have been using grammar for language "sums with parentheses" (1+2+(3+4))+5**
- **Started with simple, right-associative grammar**
  - S → E + S | E
  - E → num | (S)
- **Transformed it to an LL(1) by left factoring:**
  - S → ES'
  - S' → ε | +S
  - E → num (S)
- **What if we start with a left-associative grammar?**
  - S → S + E | E
  - E → num | (S)

Reminder: Left vs Right Associativity

Consider a simpler string on a simpler grammar: “1 + 2 + 3 + 4”

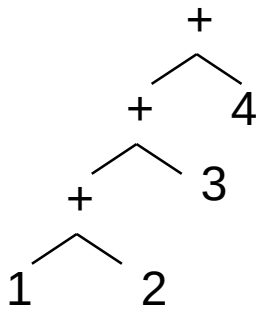
Right recursion : right associative

S → E + S
S → E
E → num



Left recursion : left associative

S → S + E
S → E
E → num



Left Recursion

S → S + E
S → E
E → num

"1 + 2 + 3 + 4"

derived string	lookahead	read/unread
S	1	1+2+3+4
S+E	1	1+2+3+4
S+E+E	1	1+2+3+4
S+E+E+E	1	1+2+3+4
E+E+E+E	1	1+2+3+4
1+E+E+E	2	1+2+3+4
1+2+E+E	3	1+2+3+4
1+2+3+E	4	1+2+3+4
1+2+3+4	$	1+2+3+4

Is this right? If not, what's the problem?

Left-Recursive Grammars

- **Left-recursive grammars don't work with top-down parsers: we don't know when to stop the recursion**
- **Left-recursive grammars are NOT LL(1)!**
  - $S \rightarrow S\alpha$
  - $S \rightarrow \beta$
- **In parse table**
  - Both productions will appear in the predictive table at row S in all the columns corresponding to FIRST(β)

Eliminate Left Recursion

- **Replace**
  - $X \rightarrow X\alpha 1 \mid \ldots \mid X\alpha m$
  - $X \rightarrow \beta 1 \mid \ldots \mid \beta n$
- **With**
  - $X \rightarrow \beta 1X' \mid \ldots \mid \beta nX'$
  - $X' \rightarrow \alpha 1X' \mid \ldots \mid \alpha mX' \mid \varepsilon$
- **See complete algorithm in Dragon book**

Class Problem

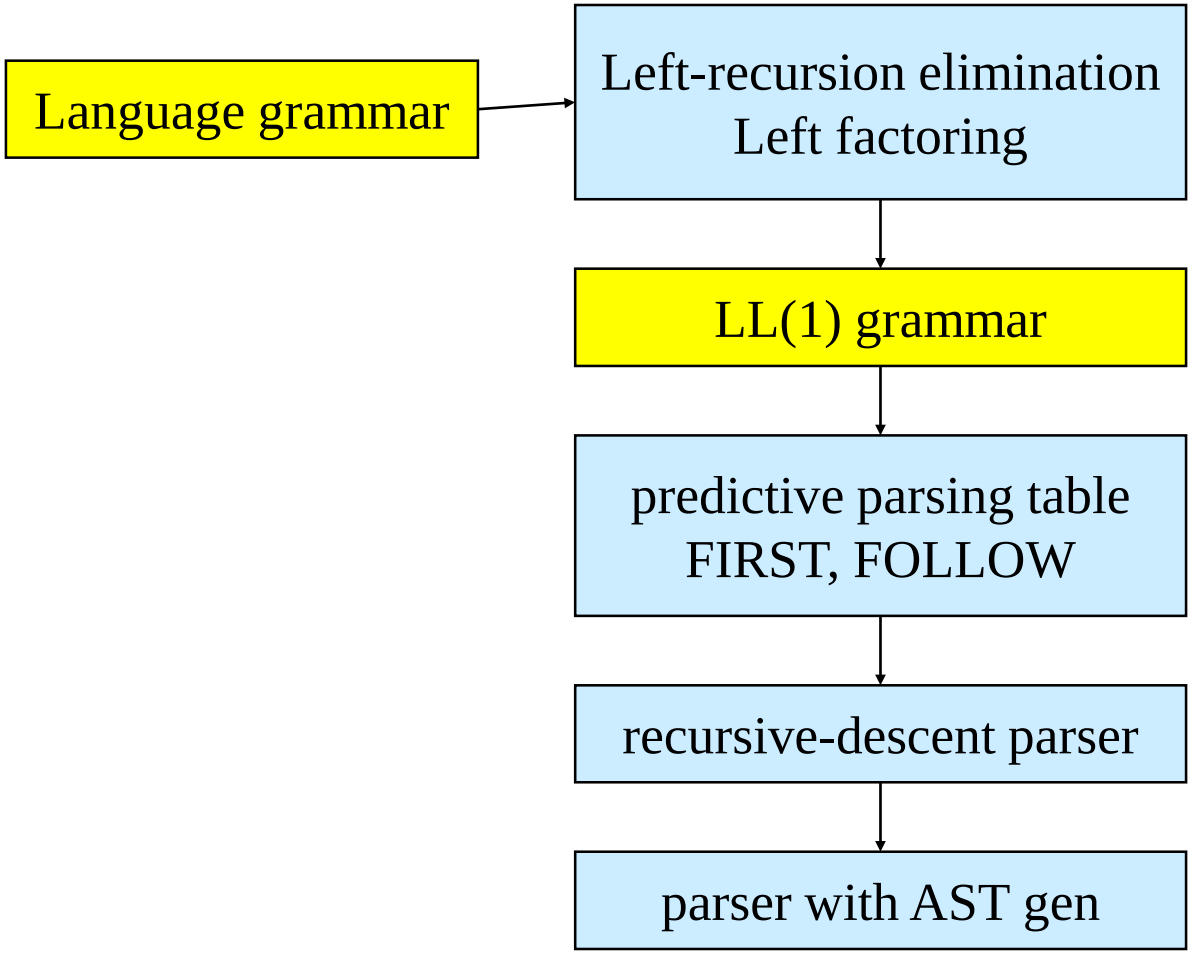
Transform the following grammar to eliminate left recursion:

E → E + T | T
T → T * F | F
F → (E) | num

Creating an LL(1) Grammar

- **Start with a left-recursive grammar**
  - S → S + E
  - S → E
  - and apply left-recursion elimination algorithm
    - S → ES'
    - S' → +ES' | ε
- **Start with a right-recursive grammar**
  - S → E + S
  - S → E
  - and apply left-factoring to eliminate common prefixes
    - S → ES'
    - S' → +S | ε

Top-Down Parsing Summary



New Topic: Bottom-Up Parsing

- **A more power parsing technology**
- **LR grammars – more expressive than LL**
  - Construct right-most derivation of program
  - Left-recursive grammars, virtually all programming languages are left-recursive
  - Easier to express syntax
- **Shift-reduce parsers**
  - Parsers for LR grammars
  - Automatic parser generators (yacc, bison)

Bottom-Up Parsing (2)

- **Right-most derivation – Backward**
  - Start with the tokens
  - End with the start symbol
  - Match substring on RHS of production, replace by LHS

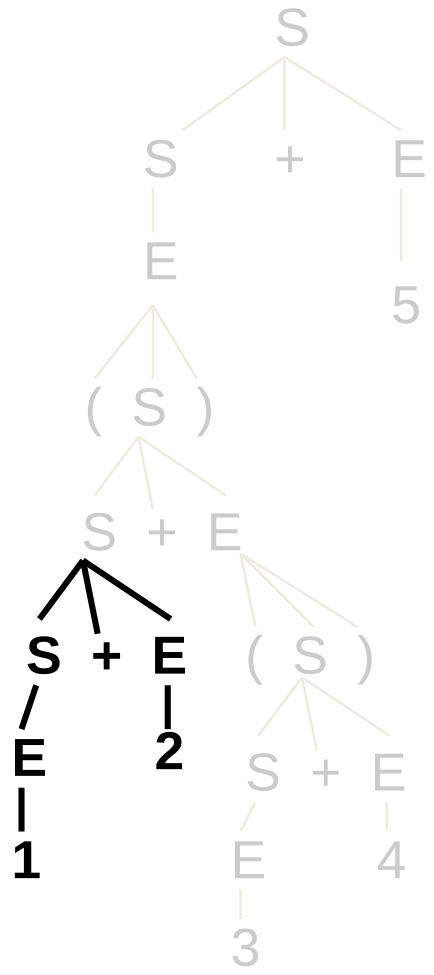
S → S + E | E
E → num | (S)

(1+2+(3+4))+5 ← (E+2+(3+4))+5
← (S+2+(3+4))+5 ← (S+E+(3+4))+5
← (S+(3+4))+5 ← (S+(E+4))+5 ← (S+(S+4))+5
← (S+(S+E))+5 ← (S+(S))+5 ← (S+E)+5 ← (S)+5
← E+5 ← S+E ← S

Bottom-Up Parsing (3)

S → S + E | E
E → num | (S)

(1+2+(3+4))+5
← (E+2+(3+4))+5
← (S+2+(3+4))+5
← (S+E+(3+4))+5



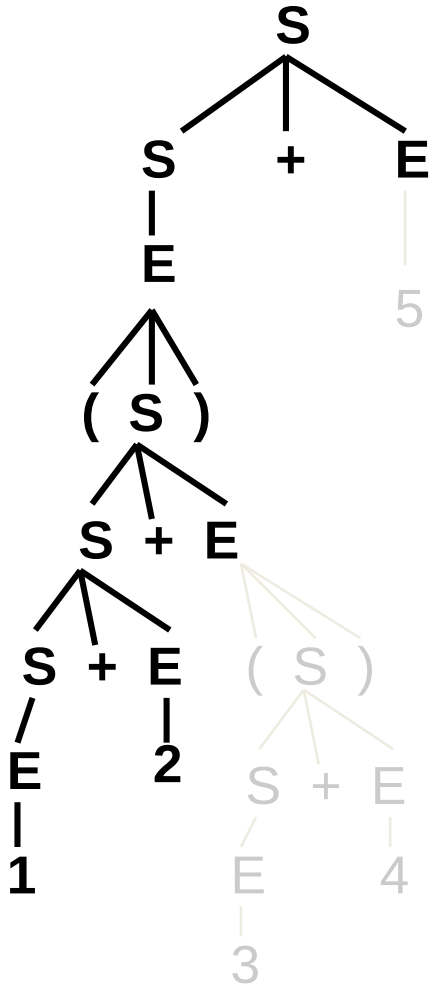
Advantage of bottom-up parsing: can postpone the selection of productions until more of the input is scanned

Top-Down Parsing

S → S + E | E
E → num | (S)

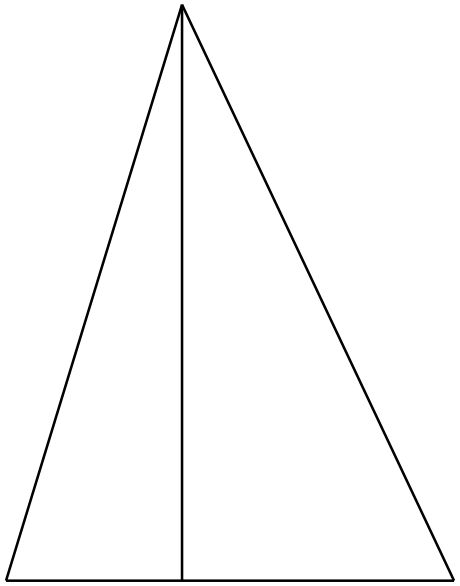
S → S+E → E+E → (S)+E → (S+E)+E
→ (S+E+E)+E → (E+E+E)+E
→ (1+E+E)+E → (1+2+E)+E ...

In left-most derivation, entire tree above token (2) has been expanded when encountered



Top-Down vs Bottom-Up

Bottom-up: Don't need to figure out as much of the parse tree for a given amount of input → More time to decide what rules to apply



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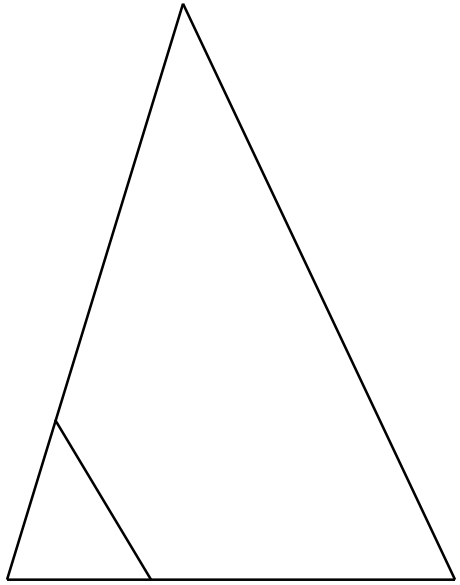
Top-down



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Bottom-up

Terminology: LL vs LR

- **LL(k)**
  - Left-to-right scan of input
  - Left-most derivation
  - k symbol lookahead
  - [Top-down or predictive] parsing or LL parser
  - Performs pre-order traversal of parse tree

- **LR(k)**
  - Left-to-right scan of input
  - Right-most derivation
  - k symbol lookahead
  - [Bottom-up or shift-reduce] parsing or LR parser
  - Performs post-order traversal of parse tree

Shift-Reduce Parsing

- **Parsing actions: A sequence of shift and reduce operations**
- **Parser state: A stack of terminals and non-terminals (grows to the right)**
- **Current derivation step = stack + input**

Derivation step	stack	Unconsumed input
(1+2+(3+4))+5 ←		(1+2+(3+4))+5
(E+2+(3+4))+5 ←	(E	+2+(3+4))+5
(S+2+(3+4))+5 ←	(S	+2+(3+4))+5
(S+E+(3+4))+5 ←	(S+E	+(3+4))+5
…		

Shift-Reduce Actions

- **Parsing is a sequence of shifts and reduces**
- **Shift: move look-ahead token to stack**

stack	input	action
(	1+2+(3+4))+5	shift 1
(1	+2+(3+4))+5	

- **Reduce: Replace symbols β from top of stack with non-terminal symbol X corresponding to the production: X→ β (e.g., pop β, push X)**

stack	input	action
(S+E	+(3+4))+5	reduce S → S+ E
(S	+(3+4))+5	

Shift-Reduce Parsing

S → S + E | E
E → num | (S)

derivation	stack	input stream	action
(1+2+(3+4))+5		(1+2+(3+4))+5	shift
(1+2+(3+4))+5	(	1+2+(3+4))+5	shift
(1+2+(3+4))+5	(1	+2+(3+4))+5	reduce E→ num
(E+2+(3+4))+5	(E	+2+(3+4))+5	reduce S→ E
(S+2+(3+4))+5	(S	+2+(3+4))+5	shift
(S+2+(3+4))+5	(S+	2+(3+4))+5	shift
(S+2+(3+4))+5	(S+2	+(3+4))+5	reduce E→ num
(S+E+(3+4))+5	(S+E	+(3+4))+5	reduce S → S+E
(S+(3+4))+5	(S	+(3+4))+5	shift
(S+(3+4))+5	(S+	(3+4))+5	shift
(S+(3+4))+5	(S+(	3+4))+5	shift
(S+(3+4))+5	(S+(3	+4))+5	reduce E→ num
…			

Potential Problems

- **How do we know which action to take: whether to shift or reduce, and which production to apply**
- **Issues**
  - Sometimes can reduce but should not
  - Sometimes can reduce in different ways

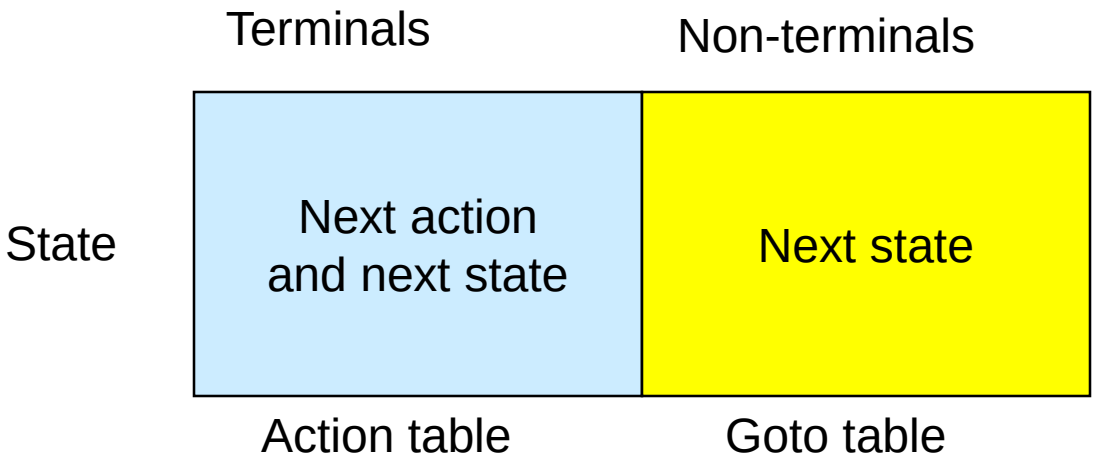
Action Selection Problem

- **Given stack β and look-ahead symbol b, should parser:**
  - Shift b onto the stack making it βb ?
  - Reduce X → γ assuming that the stack has the form β = αγ making it αX ?
- **If stack has the form αγ, should apply reduction X → γ (or shift) depending on stack prefix α ?**
  - α is different for different possible reductions since γ's have different lengths

LR Parsing Engine

- **Basic mechanism**
  - Use a set of parser states
  - Use stack with alternating symbols and states
    - E.g., 1 (6 S 10 + 5 (blue = state numbers)
  - Use parsing table to:
    - Determine what action to apply (shift/reduce)
    - Determine next state
- **The parser actions can be precisely determined from the table**

LR Parsing Table



- **Algorithm: look at entry for current state S and input terminal C**
  - If Table[S,C] = s(S') then shift:
    - push(C), push(S')
  - If Table[S,C] = X→ α then reduce:
    - pop(2*|α|), S'= top(), push(X), push(Table[S',X])

LR Parsing Table Example

We want to derive this in an algorithmic fashion

State	Input terminal					Non-terminals	
	(	)	id	,	$	S	L
1	s3		s2			g4	
2	S→id	S→id	S→id	S→id	S→id		
3	s3		s2			g7	g5
4					accept		
5		s6		s8			
6	S→(L)	S→(L)	S→(L)	S→(L)	S→(L)		
7	L→S	L→S	L→S	L→S	L→S		
8	s3		s2			g9	
9	L→L,S	L→L,S	L→L,S	L→L,S	L→L,S		

LR(k) Grammars

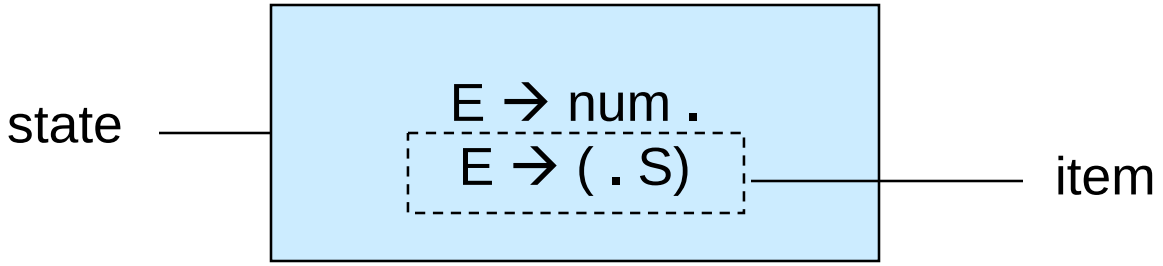
- **LR(k) = Left-to-right scanning, right-most derivation, k lookahead chars**
- **Main cases**
  - LR(0), LR(1)
  - Some variations SLR and LALR(1)
- **Parsers for LR(0) Grammars:**
  - Determine the actions without any lookahead
  - Will help us understand shift-reduce parsing

Building LR(0) Parsing Tables

- **To build the parsing table:**
  - Define states of the parser
  - Build a DFA to describe transitions between states
  - Use the DFA to build the parsing table
- **Each LR(0) state is a set of LR(0) items**
  - An LR(0) item: X → α **.** β where X → $\alpha\beta$ is a production in the grammar
  - The LR(0) items keep track of the progress on all of the possible upcoming productions
  - The item X → α **.** β abstracts the fact that the parser already matched the string α at the top of the stack

Example LR(0) State

- **An LR(0) item is a production from the language with a separator "." somewhere in the RHS of the production**



- **Sub-string before "." is already on the stack (beginnings of possible γ's to be reduced)**
- **Sub-string after ".": what we might see next**

Class Problem

For the production,

E → num | (S)

Two items are:

E → num .

E → (. S)

Are there any others? If so, what are they? If not, why?

LR(0) Grammar

- **Nested lists**
  - S → (L) | id
  - L → S | L,S
- **Examples**
  - (a,b,c)
  - ((a,b), (c,d), (e,f))
  - (a, (b,c,d), ((f,g)))

Parse tree for
(a, (b,c), d)