

Coq report 2

The part of Sets

First of all

Make a set

```
Section Sets.
```

Something else

```
Open Scope type_scope.  
Set Implicit Arguments.  
Implicit Arguments inl [A B].  
Implicit Arguments inr [A B].
```

- Using `Open Scope type_scope.` to set the scope of argument scopes.

Finite Sets

Inductive

Using `Inductive` to define the set what you want:

E.g. To define the empty set

```
Inductive empty_set : Set := .
```

To define the set of color only include two colors:

```
Inductive Color : Set :=  
| white: Color  
| black : Color.
```

Also can define the Boolean set:

```
Inductive bool : Set := true | false.
```

Products

Tips

- `Definition ident :type := define.` Defines an object that can selectively declare its type, but must have a definition.

E.g. before explaining that, we need define a Rank as a set:

Inductive Rank : Set :=

- | pawn : Rank
- | rook : Rank
- | knight : Rank
- | bishop : Rank
- | queen : Rank
- | king : Rank.

To define a new set $\text{Color} * \text{Rank}$ which called **Piece** . It's the set of pairs $(\text{Color} * \text{Rank})$.

In this case, the **blackKnight** is the set pairs $(\text{Color} * \text{Rank})$. And also the black and knight that belong to **Color** and **Rank** set.

Definition **Piece** : Set := Color * Rank.

Definition **blackKnight** : Piece := (black , knight).

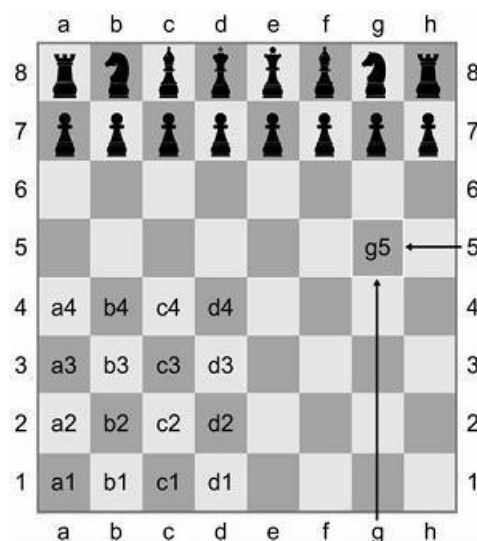
In this case, we also need to define the coordinates

Inductive XCoord : Set :=

- | xa : XCoord
- | xb : XCoord
- | xc : XCoord
- | xd : XCoord
- | xe : XCoord
- | xf : XCoord
- | xg : XCoord
- | xh : XCoord.

Inductive YCoord : Set :=

- | y1 : YCoord
- | y2 : YCoord
- | y3 : YCoord
- | y4 : YCoord
- | y5 : YCoord
- | y6 : YCoord
- | y7 : YCoord
- | y8 : YCoord.



Define some fuctions

Using this template to define the some functions:

```
Definition function(X: Type)(x: X) : nat->X :=  
  fun (k:nat) => x.
```

It needs a return value or a function.

In this case, we need to define some generic operations:

```
Definition fst(A B : Set)(p : A * B) : A :=  
  match p with  
  | (a , b) => a  
  end.
```

```
Definition snd(A B : Set)(p : A * B) : B :=  
  match p with  
  | (a , b) => b  
  end.
```

Extract the components

```
Eval compute in fst blackKnight.  
Eval compute in snd blackKnight.  
Eval compute in (fst blackKnight,snd blackKnight).
```

Destruct

```
destruct p as [a b].
```

All source code in that case

```
Section Sets.  
  
Open Scope type_scope.  
Set Implicit Arguments.  
Implicit Arguments inl [A B].  
Implicit Arguments inr [A B].  
  
Inductive Color : Set :=  
  | white: Color  
  | black : Color.  
  
Inductive Rank : Set :=  
  | pawn : Rank  
  | rook : Rank  
  | knight : Rank  
  | bishop : Rank  
  | queen : Rank  
  | king : Rank.  
  
Inductive XCoord : Set :=  
  | xa : XCoord
```

```

| xb : XCoord
| xc : XCoord
| xd : XCoord
| xe : XCoord
| xf : XCoord
| xg : XCoord
| xh : XCoord.

```

Inductive YCoord : **Set** :=

```

| y1 : YCoord
| y2 : YCoord
| y3 : YCoord
| y4 : YCoord
| y5 : YCoord
| y6 : YCoord
| y7 : YCoord
| y8 : YCoord.

```

Definition Piece : **Set** := Color * Rank.

Definition Coord : **Set** := XCoord * YCoord.

Definition blackKnight : Piece := (black , knight).

Definition e2 : Coord := (xe , y2).

Definition fst(A B : **Set**)(p : A * B) : A :=

```

  match p with
  | (a , b) => a
end.

```

Definition snd(A B : **Set**)(p : A * B) : B :=

```

  match p with
  | (a , b) => b
end.

```

Eval compute in fst blackKnight.

Eval compute in snd blackKnight.

Eval compute in (fst blackKnight,snd blackKnight).

Lemma surjective_pairing : **forall** A B : **Set**,

forall p : prod A B, (fst p , snd p) = p.

intros A B p.


destruct p as [a b].

simpl.

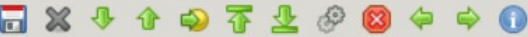
reflexivity.

Qed.

Run result screenshot

 CoqIde

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01.v

```

Definition Piece : Set := Color * Rank.
Definition Coord : Set := XCoord * YCoord.
Definition blackKnight : Piece := (black , kni
Definition e2 : Coord := (xe , y2).
Definition fst(A B : Set) (p : A * B) : A :=
  match p with
  | (a , b) => a
  end.

Definition snd(A B : Set) (p : A * B) : B :=
  match p with
  | (a , b) => b
  end.

Eval compute in fst blackKnight.
Eval compute in snd blackKnight.

Eval compute in (fst blackKnight, snd blackKnight).
Lemma surjective_pairing : forall A B : Set,
  forall p : prod A B, (fst p , snd p) = p.
intros A B p.
destruct p as [a b].
simpl.
reflexivity.
Qed.

```

```

1 subgoal
A, B : Set
a : A
b : B

(a, b) = (a, b) (1/1)

```

Messages Errors Jobs

Ready in Sets, proving surjective_pairing
 Line: 64 Char: 5 Coq is ready 0 / 0