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SCHOLARSHIP EXEMPLAR



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QUALIFY FOR THE FUTURE WORLD
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Tick this box if
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Scholarship 2020 Calculus

9.30 a.m. Monday 16 November 2020

Time allowed: Three hours

Total score: 40

ANSWER BOOKLET

There are five questions in this examination. Answer ALL FIVE questions.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Write ALL your answers in this booklet.

Make sure that you have Formulae and Tables Booklet S–CALCF.

Show ALL working. Start your answer to each question on a new page. Carefully number each question.

Answers developed using a CAS calculator require **ALL commands to be shown**. Correct answers only will not be sufficient.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Question	Score
ONE	
TWO	
THREE	
FOUR	
FIVE	
TOTAL	/40

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QUESTION
NUMBER

$$\text{Q1 a)} \lim_{x \rightarrow \infty} \frac{3x^2 + 2x - 4}{5x^2 + 8x - 1}$$

$$= \lim_{x \rightarrow \infty} \frac{3 + \frac{2}{x} - \frac{4}{x^2}}{5 + \frac{8}{x} - \frac{1}{x^2}}$$

$$= \frac{3}{5} \quad //$$

$$\text{Q1 b)} \int_0^a \frac{x^3}{\sqrt{a^4 + x^4}} dx$$

$$\text{let: } u = a^4 + x^4$$

$$du = 4x^3 dx$$

$$dx = \frac{1}{4x^3} du$$

$$\Rightarrow \int_0^a \frac{x^3}{\sqrt{u}} \cdot \frac{1}{4x^3} du$$

$$= \frac{1}{4} \int_0^a \frac{1}{\sqrt{u}} du$$

$$= \frac{1}{4} \int_0^a u^{-\frac{1}{2}} du$$

$$= \frac{1}{4} \left[2u^{\frac{1}{2}} \right]_0^a$$

$$= \frac{1}{4} \left[2\sqrt{a^4 + x^4} \right]_0^a$$

$$= \frac{1}{4} \left(2\sqrt{a^4 + a^4} - 2\sqrt{a^4 + 0} \right)$$

$$= \frac{1}{4} \left(2\sqrt{2a^4} - 2\sqrt{a^4} \right)$$

$$= \frac{1}{4} (2\sqrt{2} - 2) a^2$$

$$= \left(\frac{\sqrt{2}-1}{2}\right) a^2 \quad //$$

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Q1 c)(i) Form equation from roots:

$$(x-\alpha)(x-\beta)(x-\gamma)(x-\delta) = 0$$

$$\Rightarrow (x^2 - \beta x - \alpha x + \alpha\beta)(x-\gamma)(x-\delta) = 0$$

$$\Rightarrow (x^2 + (-\beta-\alpha)x + \alpha\beta)(x-\gamma)(x-\delta) = 0$$

$$\Rightarrow (x^3 - \gamma x^2 + (-\beta-\alpha)x^2 - \gamma(-\beta-\alpha)x + \alpha\beta x - \alpha\beta\gamma)(x-\delta) = 0$$

$$\Rightarrow (x^3 + (-\alpha-\beta-\gamma)x^2 + (\alpha\beta + \alpha\gamma + \beta\gamma)x - \alpha\beta\gamma)(x-\delta) = 0$$

$$\Rightarrow x^4 - \delta x^3 + (-\alpha-\beta-\gamma)x^3 - \delta(-\alpha-\beta-\gamma)x^2 + (\alpha\beta + \alpha\gamma + \beta\gamma)x^2$$

$$- \delta(\alpha\beta + \alpha\gamma + \beta\gamma)x - \alpha\beta\gamma x + \alpha\beta\gamma\delta = 0$$

$$\Rightarrow x^4 + (-\alpha-\beta-\gamma-\delta)x^3 + (\alpha\delta + \beta\delta + \gamma\delta + \alpha\beta + \alpha\gamma + \beta\gamma)x^2$$

$$+ (-\alpha\beta\delta - \alpha\gamma\delta - \beta\gamma\delta - \alpha\beta\gamma)x + \alpha\beta\gamma\delta = 0$$

$$a = 1$$

$$b = \cancel{\alpha + \beta + \gamma + \delta} - \alpha - \beta - \gamma - \delta$$

$$c = \alpha\delta + \beta\delta + \gamma\delta + \alpha\beta + \alpha\gamma + \beta\gamma$$

$$d = -\alpha\beta\delta - \alpha\gamma\delta - \beta\gamma\delta - \alpha\beta\gamma$$

$$\therefore -\frac{d}{a} = -\left(-\frac{\alpha+\beta+\gamma+\delta}{1}\right) = \alpha + \beta + \gamma + \delta$$

$$\therefore \frac{c}{a} = \beta\gamma + \gamma\alpha + \alpha\beta + \alpha\delta + \beta\delta + \gamma\delta$$

$$\therefore -\frac{d}{a} = -(-\alpha\beta\delta - \alpha\gamma\delta - \beta\gamma\delta - \alpha\beta\gamma) = \beta\gamma\delta + \gamma\alpha\delta + \alpha\beta\delta + \alpha\beta\gamma$$

$$\therefore \frac{c}{a} = \alpha\beta\gamma\delta \quad //$$

$$e = \alpha\beta\gamma\delta$$

$$\text{Q1(iii)} \quad x^4 - 8x^3 + 19x^2 + px + 2 = 0$$

~~Method~~ let roots be: $\alpha, \beta, \gamma, \delta$

$$\alpha + \beta = \gamma + \delta \quad \textcircled{1}$$

$$\alpha + \beta + \gamma + \delta = -\frac{-8}{1} = 8 \quad \textcircled{2}$$

$$\text{So: } 8 - \gamma - \delta = \gamma + \delta$$

$$\gamma + \delta = 4 \quad \text{and} \quad \alpha + \beta = 4$$

$$\beta\gamma\delta + \gamma\alpha\delta + \alpha\beta\delta + \alpha\beta\gamma = -\frac{p}{1} = -p$$

$$\beta\gamma + \gamma\alpha + \alpha\beta + \alpha\delta + \beta\delta + \gamma\delta = 19$$

$$\alpha\beta\gamma\delta = 2$$

$$\alpha(4-\alpha)(4-\gamma)\gamma = 2$$

$$\gamma(\beta + \alpha + \delta) + \alpha\beta + \alpha\delta + \beta\delta = 19$$

$$\gamma(8 - \gamma) + \alpha\beta + \alpha\delta + \beta\delta = 19$$

$$\gamma(\beta\delta + \alpha\delta + \beta\alpha) + \beta\alpha\delta = -p$$

~~$$\text{Method } \gamma(\beta\delta + \alpha\delta + \beta\alpha) = -p - \alpha\beta\delta$$~~

$$\alpha\beta\delta = \frac{3}{8}$$

$$\text{So } \gamma(\beta\delta + \alpha\delta + \beta\alpha) = -p - \frac{2}{8}$$

$$\gamma(8 - \gamma) + -\frac{p}{8} - \frac{2}{8} = 19 \quad \text{M}$$

$$\text{Q2a)} \quad x^4 + \frac{1}{x^4} = 7$$

$$(x^4 + \frac{1}{x^4})^2 = 49$$

$$x^8 + 2 + \frac{1}{x^8} = 49$$

$$x^8 + \frac{1}{x^8} = 47 \quad \text{M}$$

$$\text{Q2b)(i)} \quad f(x) = \frac{\cos x}{2+\sin x}$$

$$f'(x) = \frac{(2+\sin x)\sin x - \cos x \cdot \cos x}{(2+\sin x)^2}$$

$$= \frac{-2\sin x - \sin^2 x - \cos^2 x}{(2+\sin x)^2}$$

$$= \frac{-2\sin x - 1}{(2+\sin x)^2} \quad \text{as } \sin^2 x + \cos^2 x = 1$$

$f'(x) = 0$ for the turning points

$$-2\sin x - 1 = 0$$

$$2\sin x = -1$$

$$\sin x = -\frac{1}{2}$$

$$x_1 = \frac{7\pi}{6}, \quad x_2 = \frac{11\pi}{6} \quad \text{for solutions in the domain } 0 \leq x \leq 2\pi$$

$$\begin{aligned} y_1 &= \frac{\cos \frac{7\pi}{6}}{2+\sin \frac{7\pi}{6}} \\ &= \frac{-\frac{\sqrt{3}}{2}}{2+(-0.5)} \\ &= \frac{-\sqrt{3}}{\frac{5}{2}} \\ &= -\frac{2\sqrt{3}}{5} \end{aligned}$$

$$y_2 = \frac{\cos \frac{11\pi}{6}}{2+\sin \frac{11\pi}{6}}$$

$$= \frac{\frac{\sqrt{3}}{2}}{2+\frac{1}{2}}$$

$$= \frac{\sqrt{3}}{\frac{5}{2}}$$

$$= \frac{2\sqrt{3}}{5}$$

∴ Turning points at: $(\frac{7\pi}{6}, -\frac{\sqrt{3}}{2})$ (a local minima)
and $(\frac{11\pi}{6}, \frac{\sqrt{3}}{2})$ (a local maxima)

$$\begin{aligned} Q2b(ii) \quad f''(x) &= \frac{d}{dx} \frac{-2\sin x - 1}{(2 + \sin x)^2} \\ &= \frac{(2 + \sin x)^2 \cdot (-2\cos x) - (-2\sin x - 1) \cdot 2(2 + \sin x)\cos x}{(2 + \sin x)^4} \\ &= \frac{(2 + \sin x)^2 \cdot -2\cos x + 2\cos x(2\sin x + 1)(2 + \sin x)}{(2 + \sin x)^4} \end{aligned}$$

$f''(x) = 0$ for points of inflection.

$$\text{so } -2\cos x(4 + 4\sin x + \sin^2 x) + 2\cos x(4\sin x + 2\sin^2 x + 2 + \sin x) = 0$$

$$-8\cos x - 8\sin x \cos x - 2\sin^2 x \cos x + 10\sin x \cos x + 4\sin^2 x \cos x + 4\cos x = 0$$

$$2\sin^2 x \cos x - 4\sin x \cos x - 4\cos x = 0$$

$$2\cos x(\sin^2 x - 2\sin x - 2) = 0$$

so $\cos x = 0$ gives $x = 2n\pi \pm \frac{\pi}{2}$ where n is any integer for an inflection point

~~graphing calculator~~, it seems

$$f''(x) = \frac{2\cos x(\sin^2 x - 2\sin x - 2)}{(2 + \sin x)^4}$$

From $f''(x)$, note that $\sin^2 x - 2\sin x - 2$ is always negative so $f''(x) > 0$ for concave up for when $\cos x > 0$. Similarly, $f''(x) < 0$ for concave down when $\cos x < 0$.

~~This shows $f(x)$ is concave up on $(-\infty, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, \infty)$~~

This shows $f(x)$ alternates from concave up to concave down on intervals of 2π with the first concave up interval as $\frac{\pi}{2} < x < \frac{3\pi}{2}$, then concave down for $\frac{3\pi}{2} < x < \frac{5\pi}{2}$ and concave up for $\frac{5\pi}{2} < x < \frac{7\pi}{2}$ and so on...

Q2c(i) Because AC is the diameter of the circle, $\angle ABC = \angle ADC = 90^\circ$.

~~So $\angle AEC = 90^\circ$~~

$$\tan\left(\frac{\theta}{2}\right) = \frac{b}{a} \text{ follows from this}$$

$$\frac{a}{b} = \frac{\tan\left(\frac{\theta}{2}\right)}{\tan\left(\frac{\theta}{2}\right)}$$

$$= \frac{\frac{1}{2}\cos\left(\frac{\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)}$$

$$= \frac{\cos^2\left(\frac{\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}$$

$$= \frac{\frac{1}{2}\cos\theta + \frac{1}{2}}{\frac{1}{2}\sin\left(\frac{\theta}{2}\right)\cdot 2}$$

$$= \frac{\frac{1}{2}(\cos\theta + 1)}{\frac{1}{2}\sin\theta}$$

$$= \frac{\cos\theta + 1}{\sin\theta}$$

$$= \frac{1 + \cos\theta}{\sin\theta} \quad : \text{as required}$$

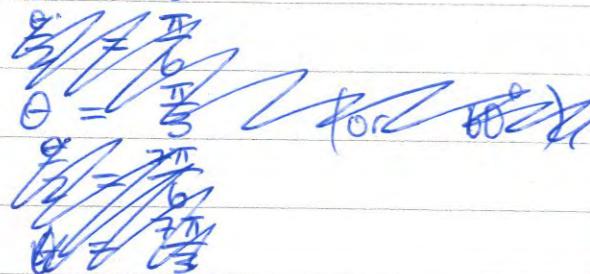
QUESTION NUMBER

$$2c)(ii) \tan\left(\frac{\theta}{2}\right) = \frac{b}{a}$$

$$b = \frac{a}{\sqrt{3}}$$

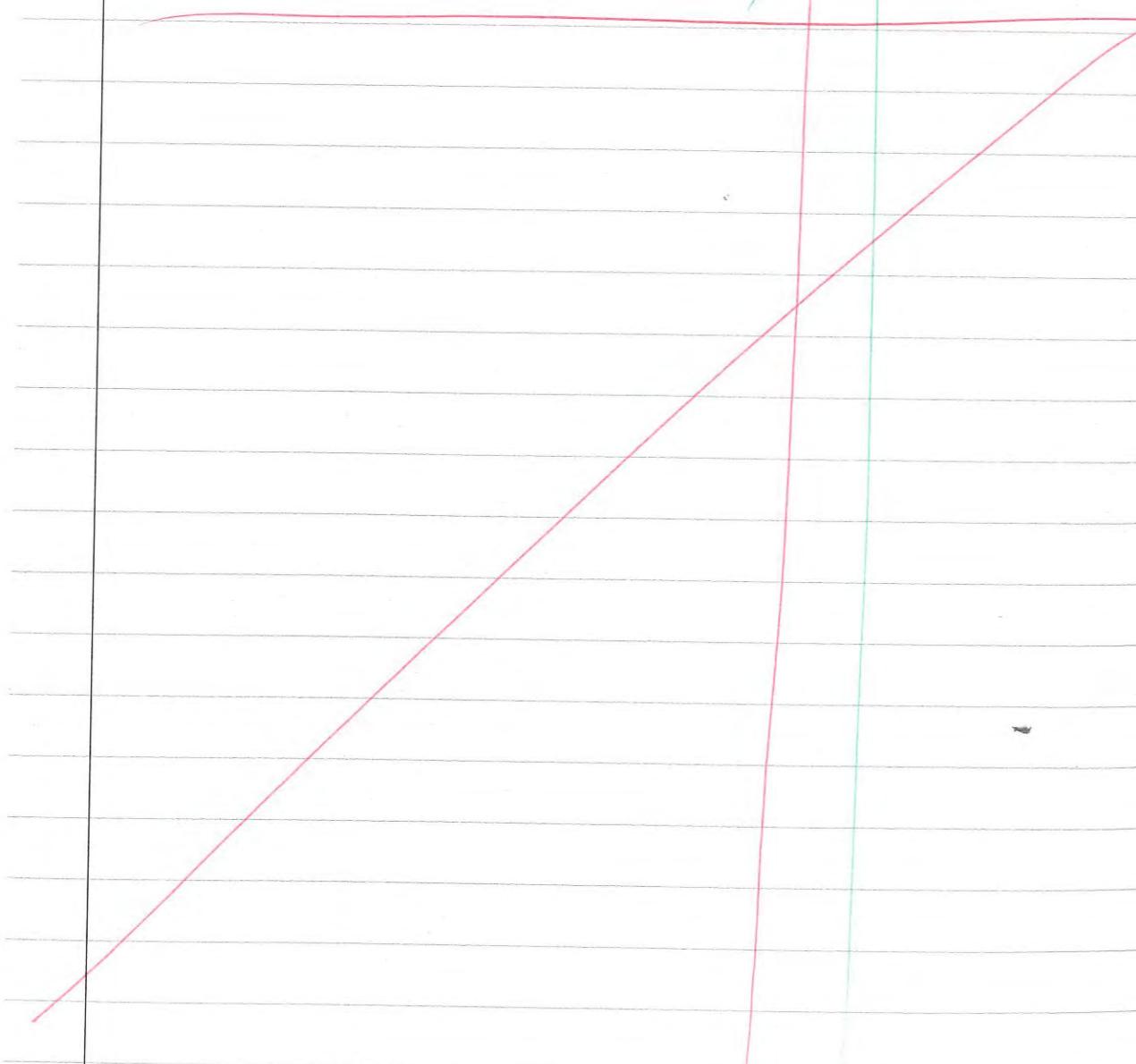
$$\frac{b}{a} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{3}}$$



$$\frac{\theta}{2} = \frac{\pi}{6}$$

$$\theta = \frac{\pi}{3} \text{ (or } 60^\circ)$$



Q2

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~~Find $f''(0)$ using x^2+ax+b~~

$f''(0)$ only exists if function is continuous at $x=0$
so $x^2+ax+b = e^x + \sin x$ and the derivatives
of both will be equal at $x=0$

$$y_1 = x^2 + ax + b$$

$$y_1' = 2x + a$$

$$y_1'' = 2$$

$$y_2 = e^x + \sin x$$

$$y_2' = e^x + \cos x$$

$$y_2'' = e^x - \sin x$$

$$y_1 = y_2 \text{ at } x=0$$

$$0 + a \cdot 0 + b = e^0 + \sin 0$$

$$b = 1$$

$$y_1' = y_2' \text{ at } x=0$$

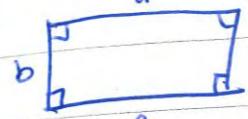
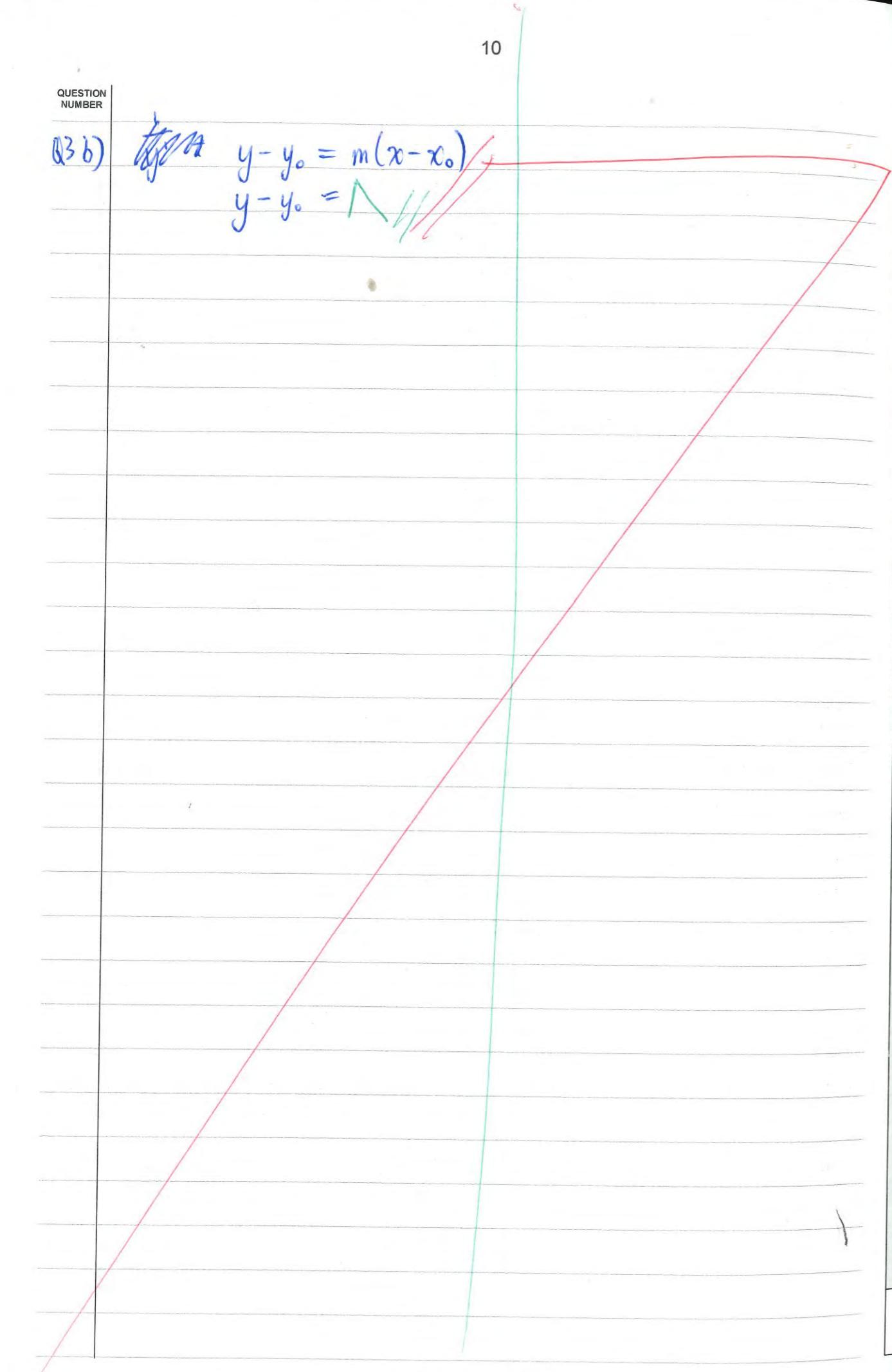
$$2 \cdot 0 + a = e^0 + \cos 0$$

$$a = 1 + 1$$

$$a = 2$$

$$\text{So } a = 2 \text{ and } b = 1$$

Q3 b) ~~$y - y_0 = m(x - x_0)$~~
 $y - y_0 = \text{N}$



a is length
b is width
L is diagonal length

$$\frac{da}{dt} = 2$$

$$\frac{db}{dt} = 3$$

$$L = \sqrt{a^2 + b^2}$$

$$\frac{dL}{dt} = \frac{2a \frac{da}{dt} + 2b \frac{db}{dt}}{2\sqrt{a^2 + b^2}}$$

$$\text{at } a = 12$$

$$\text{at } b = 9$$

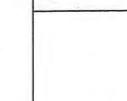
$$= \frac{2 \cdot 12 \cdot 2 + 2 \cdot 9 \cdot 3}{2\sqrt{12^2 + 9^2}}$$

$$= \frac{102}{2 \cdot 15}$$

$$= \frac{102}{30}$$

$$= \frac{17}{5}$$

Length increases at $\frac{17}{5}$ cm per second.



QUESTION NUMBER

Q3 d) let L be distance from some point (x_0, y_0) to $(1, 0)$

$$L = \sqrt{(x_0 - 1)^2 + (y_0)^2}$$

$$L = \sqrt{(x_0 - 1)^2 + y_0^2} \quad \because (x_0, y_0) \text{ is a point on } x=y^2$$

$$= \sqrt{x_0^2 - 2x_0 + 1 + x_0^2}$$

$$= \sqrt{x_0^2 - x_0 + 1}$$

$$L^2 = x_0^2 - x_0 + 1$$

$$2L \frac{dL}{dx_0} = 2x_0 - 1$$

$$\frac{dL}{dx_0} = 0 \quad \text{for minimum distance}$$

$$2x_0 - 1 = 0$$

$$x_0 = \frac{1}{2}$$

$$\text{now } y_0 = \pm \sqrt{\frac{1}{2}}$$

So points on $x=y^2$ shortest distance to $(1, 0)$

are $(\frac{1}{2}, \frac{\sqrt{2}}{2})$ and $(\frac{1}{2}, -\frac{\sqrt{2}}{2})$

$$2L \frac{dL}{dx_0} = 2x_0 - 1$$

$$2 \frac{dL}{dx_0^2} = 2$$

$$\frac{d^2L}{dx_0^2} = 1$$

Because $\frac{d^2L}{dx_0^2} = 1 > 0$, via the second derivative test, we conclude the found points ~~are~~ indeed have the minimum distance.

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QUESTION NUMBER

Q4a)

~~Add~~ let $p(x) = f(x) \cdot g(x)$

$$p'(x) = \lim_{h \rightarrow 0} \frac{p(x+h) - p(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$(f(x+h) - f(x))(g(x+h) - g(x))$$

$$= f(x+h)g(x+h) - g(x)f(x+h) - f(x)g(x+h) + f(x)g(x)$$

Prove this ↓

$$\frac{d f(x)}{dx} \cdot g(x) + f(x) \frac{d g(x)}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$\text{LHS} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} g(x) + f(x) \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{g(x)f(x+h) - f(x)g(x) + f(x)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x) + f(x)g(x+h) - 2f(x)g(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x) - g(x)f(x) + g(x+h)f(x) - g(x)f(x)}{h}$$

~~$$p'(x) = \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$~~

~~$$\lim_{h \rightarrow 0} \frac{(f(x+h) - f(x))g(x) + f(x)(g(x+h) - g(x))}{h}$$~~

Continued page 21 →

Q3

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QUESTION
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$$\begin{aligned}
 Q4b(i) \int e^{-x} \cos x \, dx &= e^{-x} \sin x - \int -e^{-x} \sin x \, dx \\
 &= e^{-x} \sin x + \int e^{-x} \sin x \, dx \\
 &\stackrel{\text{Rearrange}}{=} e^{-x} \sin x + e^{-x} \cdot -\cos x - \int -e^{-x} \cos x \, dx \\
 &= e^{-x} \sin x + e^{-x} \cos x - \int e^{-x} \cos x \, dx \\
 \Rightarrow 2 \int e^{-x} \cos x \, dx &= e^{-x} \sin x - e^{-x} \cos x \\
 \int e^{-x} \cos x \, dx &= \frac{1}{2} e^{-x} (\sin x - \cos x) \quad \text{mei}
 \end{aligned}$$

$$\frac{d}{dx} \frac{1}{2} (e^{-x} \sin x - e^{-x} \cos x)$$

$$\begin{aligned}
 &= \frac{1}{2} (e^{-x} \cos x - e^{-x} \sin x - e^{-x} \cos x + e^{-x} \sin x) \\
 &= \frac{1}{2} (2e^{-x} \cos x) \\
 &= e^{-x} \cos x
 \end{aligned}$$

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$$\begin{aligned}
 Q4b(ii) \text{ Area} &= \int_0^{2\pi} e^{-x} \cos(x) \, dx \\
 &= \left[\frac{1}{2} e^{-x} (\sin x - \cos x) \right]_0^{2\pi} \\
 &= \frac{1}{2} e^{-2\pi} (0 - 1) - \frac{1}{2} e^0 (0 - 1) \\
 &\stackrel{\text{cancel}}{=} -\frac{1}{2} e^{-2\pi} - \frac{1}{2} \cdot -1 \\
 &= -\frac{1}{2} e^{-2\pi} + \frac{1}{2} \\
 &= \frac{1 - e^{-2\pi}}{2} \quad \text{units}^2
 \end{aligned}$$

QUESTION
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$$\text{Q4c)} \quad xy + e^y = 2x + 1$$

$$y + x \frac{dy}{dx} + e^y \frac{dy}{dx} = 2$$

$$\frac{dy}{dx}(x + e^y) = 2 - y$$

$$\frac{dy}{dx} = \frac{2-y}{x+e^y}$$

$$\frac{d^2y}{dx^2} = \frac{(x+e^y) \cdot -\frac{dy}{dx} - (1+e^y \frac{dy}{dx})(2-y)}{(x+e^y)^2}$$

$$\left. \frac{dy}{dx} \right|_{x=0} = \frac{2-y}{e^y} = 2 \text{ for } y \text{ at } x=0$$

$$\left. \frac{d^2y}{dx^2} \right|_{x=0} = \frac{-e^y - \frac{2-y}{e^y} - (1+e^y \frac{2-y}{e^y})(2-y)}{e^{2y}}$$

$$\cancel{\left. \frac{d^2y}{dx^2} \right|_{x=0} = \frac{-e^y - (2-y) + 2(2-y) - y(2-y)}{e^{2y}}}$$

$$\text{at } x=0, \quad 0 \cdot y + e^y = 2 \cdot 0 + 1$$

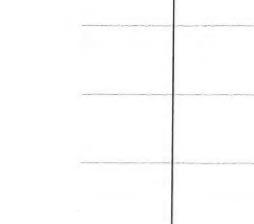
$$e^y = 1$$

$$y = 0$$

$$\left. \frac{d^2y}{dx^2} \right|_{x=0} = \frac{(0+e^0) \cdot -2 - (1+1 \cdot 2)(2-0)}{(0+e^0)^2}$$

$$= 1 \cdot -2 - (3)(2)$$

$$= \frac{1 \cdot -2 - (3)(2)}{1} = -8$$

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$$\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\text{RHS} = \frac{\cos \theta + i \sin \theta + \cos(-\theta) + i \sin(-\theta)}{2}$$

$$= \frac{\cos \theta + i \sin \theta + \cos \theta - i \sin \theta}{2}$$

$$= \frac{2 \cos \theta}{2}$$

$$= \cos \theta = \text{RHS}$$

Note: $\cos(-\theta) = \cos \theta$ since $\cos \theta$ is an even function

$\sin(-\theta) = -\sin \theta$ since $\sin \theta$ is odd function

$$\text{Q5a)(ii)} \quad \sin^3 \theta = \frac{3}{4} \sin \theta - \frac{1}{4} \sin(3\theta)$$

~~$$e^{i3\theta} = \cos(3\theta) + i \sin(3\theta)$$~~

$$e^{i3\theta} = (\cos \theta + i \sin \theta)^3$$

$$= \cos^3 \theta + 3i \cos^2 \theta \sin \theta - 3 \cos \theta \sin^2 \theta - i \sin^3 \theta$$

$$\text{so } \sin(3\theta) = 3 \cos^2 \theta \sin \theta - \sin^3 \theta \quad \text{by equating imaginary}$$

$$\sin(3\theta) = 3(1 - \sin^2 \theta) \sin \theta - \sin^3 \theta$$

$$\sin(3\theta) = 3 \sin \theta - 3 \sin^3 \theta - \sin^3 \theta$$

$$\sin(3\theta) = 3 \sin \theta - 4 \sin^3 \theta$$

$$4 \sin^3 \theta = 3 \sin \theta - \sin 3\theta$$

$$\sin^3 \theta = \frac{3}{4} \sin \theta - \frac{1}{4} \sin 3\theta \quad \text{as required} //$$

Q4
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QUESTION
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$$\text{Q5b)} \int \cos x \cdot e^x dx \quad \cos x = e^{ix} - i \sin x$$

$$= \int (e^{ix} - i \sin x) e^x dx$$

$$= \int e^x e^{ix} - i e^x \sin x dx$$

~~$\cancel{\int e^x e^{ix} dx}$~~

~~$\cancel{\int e^x e^{ix} dx}$~~

~~$\int e^x e^{ix} dx = \int e^x e^{ix} dx + \cancel{\int e^x \sin x dx}$~~

~~$\int e^x e^{ix} dx = \int e^x e^{ix} dx - \cancel{\int e^x (e^{ix} \cos x) dx}$~~

~~$\int e^x e^{ix} dx = \int e^x e^{ix} dx - \cancel{\int e^x e^{ix} \cos x dx}$~~

~~$\int e^x e^{ix} dx = \int e^{ix} e^x dx - \cancel{\int e^x dx}$~~

$$\int \cos x \cdot e^x dx = (\int e^{ix} e^x dx) - i(\int e^x \sin x dx - \int e^x \cos x dx)$$

$$\int \cos x \cdot e^x dx = (\int e^{ix} e^x dx) - i e^x \sin x + i \int e^x \cos x dx$$

$$\int \cos x \cdot e^x dx (1-i) = (\int e^{ix} e^x dx) - i e^x \sin x$$

$$\int \cos x \cdot e^x dx = \frac{1}{i+1} e^{(i+1)x} - i e^x \sin x$$

$$= \frac{e^{(i+1)x} - i e^x \sin x (i+1)}{i+1}$$

$$= \frac{e^{(i+1)x} + i e^x \sin x - i e^x \sin x}{(i+1)(1-i)}$$

$$= \frac{e^{(i+1)x} + e^x \sin x - i e^x \sin x}{2}$$

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QUESTION
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$$\text{Q5c)} z = x+iy$$

$$|z| = 1 \text{ so } \sqrt{x^2+y^2} = 1 \Rightarrow x^2+y^2 = 1$$

$$w = \frac{z+i}{i-z} = \frac{x+iy+i}{-x+iy+i}$$

$$\Rightarrow \frac{x+i(y+1)}{-x+i(-y-1)} = -\frac{x+i(y+1)}{x+iy-1}$$

$$= -\frac{(x+i(y+1))(x-i(y-1))}{(x+i(y-1))(x-i(y-1))}$$

$$= -\frac{(x^2 - ix(y-1) + iy(y+1) + (y+1)(y-1))}{x^2 + (y-1)^2}$$

$$= -\frac{(x^2 - ixy + ix + ixy + ix + y^2 - 1)}{x^2 + (y-1)^2}$$

$$= -\frac{(x^2 + 2ix + y^2 - 1)}{x^2 + (y-1)^2}$$

$$= -\frac{(2ix)}{x^2 + y^2 - 2y + 1}$$

$$= \frac{-2ix}{2-2y}$$

$$= \frac{-ix}{1-y}$$

$$= \frac{ix}{y+1}$$

Purely Imaginary NS

QUESTION NUMBER

Q8c

Q5d)

$$x^2 - yz = 1$$

$$y^2 - zx = 2$$

$$z^2 - xy = 3$$

$$x^2 + y^2 + z^2 - xy - xz - yz = 6$$

$$y^2 - zx = 2x^2 - 2yz$$

$$x^2 - yz = 1$$

$$x^4 - 2x^2yz + y^2z^2 = 1$$

$$x^4 - 2x^2(x^2 - 1) + (2+zx)(3+xy) = 1$$

$$x^4 - 2x^4 + 2x^2 + 6 + 2xy + 3xz + x^2yz = 1$$

~~$$x^4 - x^4 + 2x^2 + x^2(x^2 - 1) = -5 - 2xy - 3xz$$~~

~~$$-x^4 + 2x^2 + x^4 - x^2 = -5 - 2xy - 3xz$$~~

$$x^2 + 2xy + 3xz + 5 = 0$$

$$y^4 - 2y^2xz + z^2x^2 = 4$$

$$z^4 - 2z^2xy + x^2y^2 = 9$$

$$x^4 - 2x^2(x^2 - 1) + (x^2 - 1)^2 = 1$$

$$x^4 - 2x^4 + 2x^2 + x^4 - 2x^2 + 1 = 1$$

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Q4a)

$$\begin{aligned} \frac{d}{dx}(f(x) \cdot g(x)) &= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} \cdot \frac{g(x+h)f(x+h)}{g(x+h)f(x+h)} \\ &= \lim_{h \rightarrow 0} \frac{f^2(x+h)g^2(x+h) - f(x)g(x)g(x+h)f(x+h)}{h g(x+h) f(x+h)} \end{aligned}$$

$$\left(\frac{d}{dx} f(x) \right) g(x) + f(x) \cdot \frac{dg(x)}{dx}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} g(x) + \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} f(x)$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x) + g(x+h)f(x) - 2f(x)g(x)}{h}$$

~~$$\lim_{h \rightarrow 0} \frac{f(x+h)g(x) + g(x+h)f(x) - 2f(x)g(x)}{h}$$~~

$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h)g(x) + (g(x+h))^2f(x) - 2f(x)g(x)g(x+h)}{h g(x+h)}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h)}{h} \cdot \lim_{h \rightarrow 0} \frac{g(x)}{g(x+h)} - \lim_{h \rightarrow 0} \frac{(g(x+h))^2f(x) - 2f(x)g(x)g(x+h)}{h g(x+h)}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h)}{h} - \lim_{h \rightarrow 0} \frac{g(x+h)f(x) - 2f(x)g(x)}{h}$$

$$\boxed{\text{N/A}}$$

Q5

6

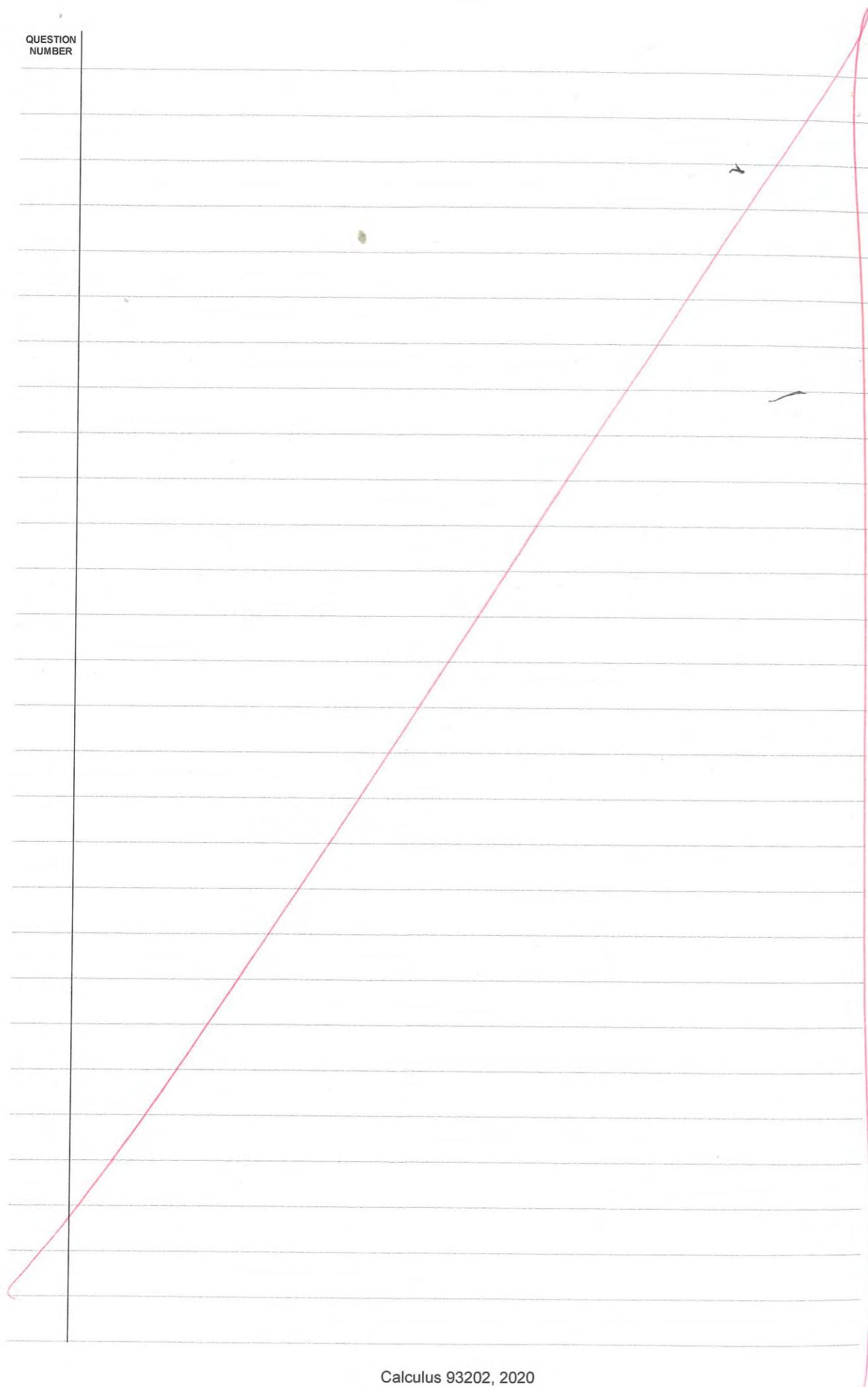
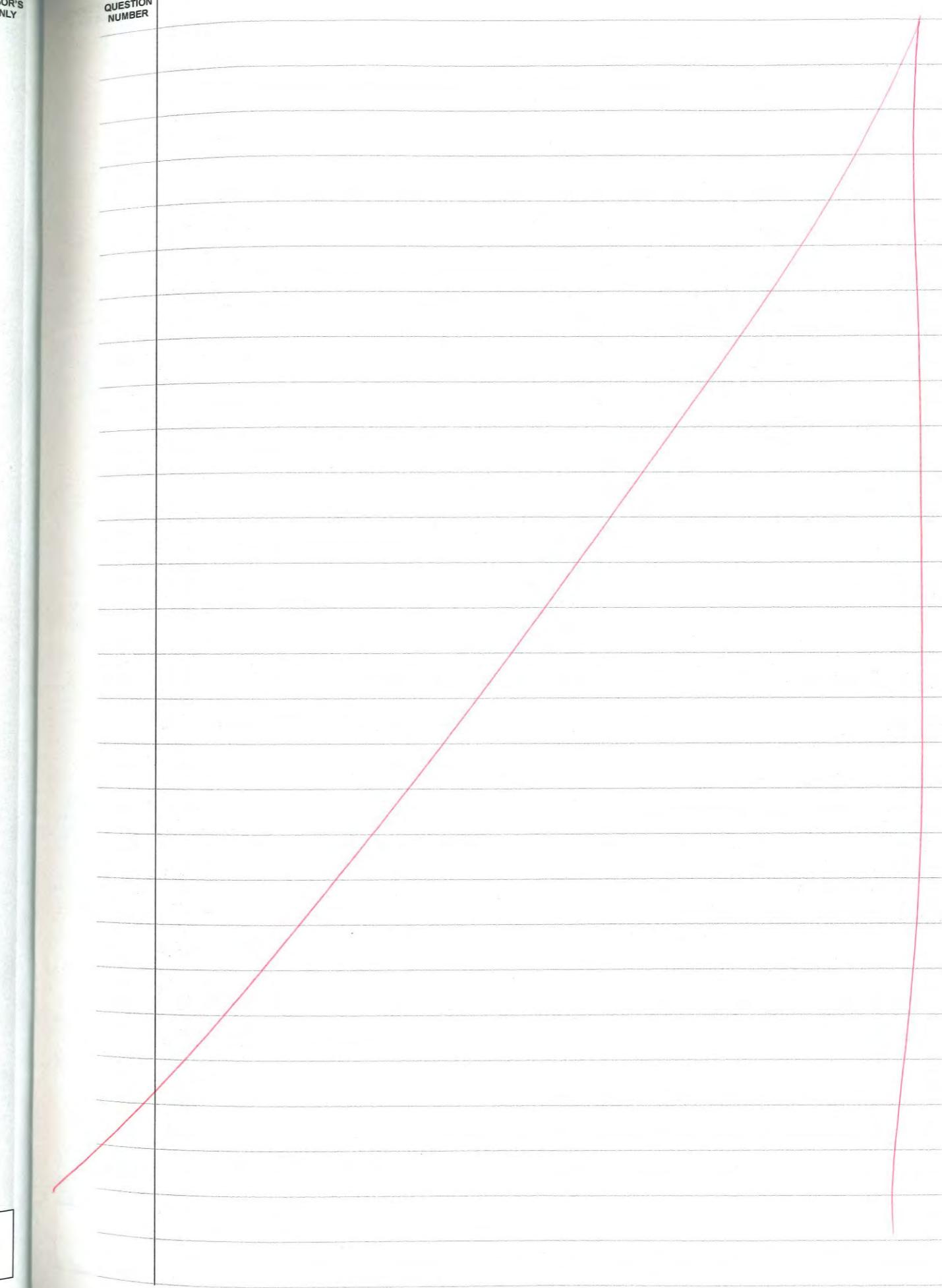
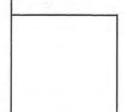
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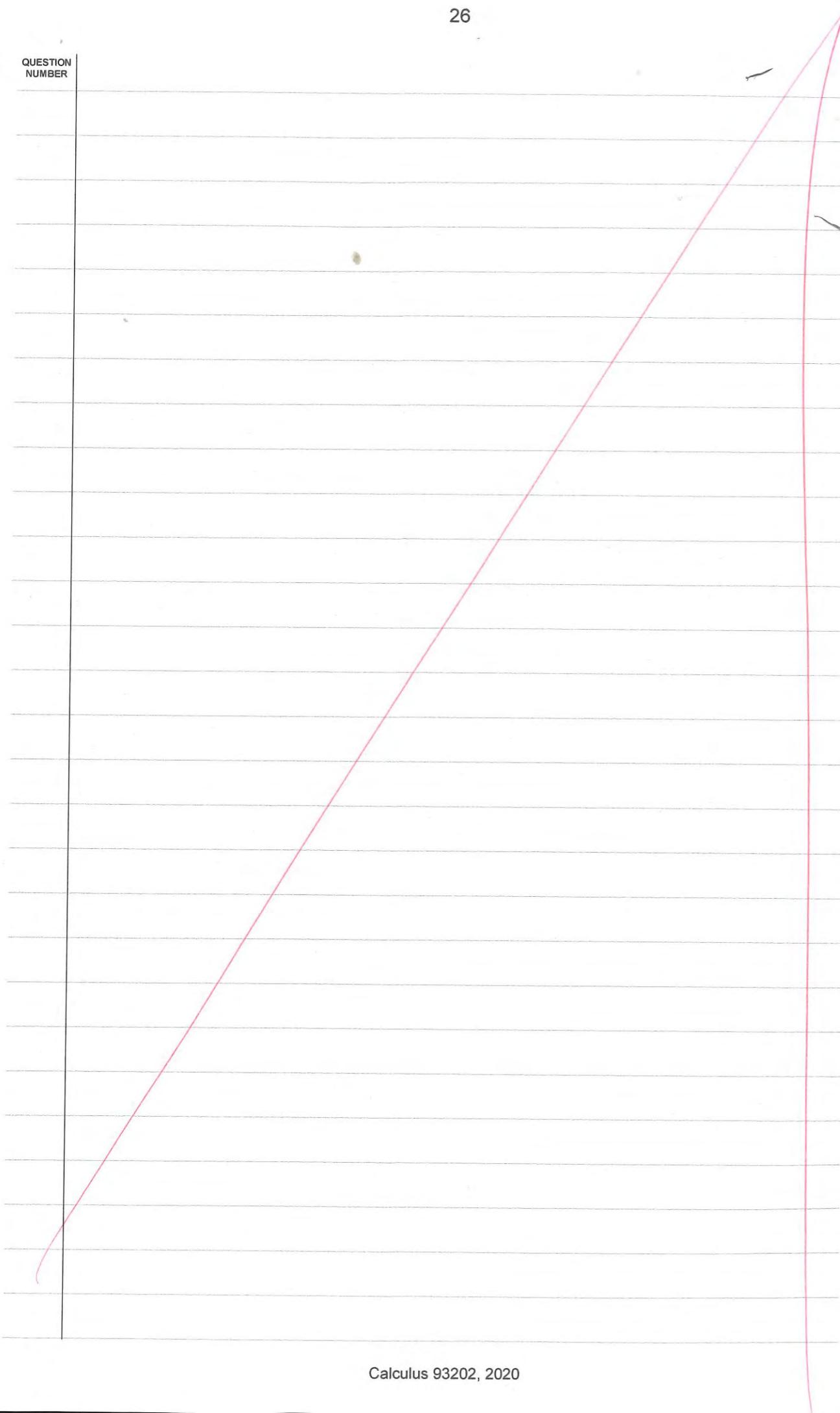
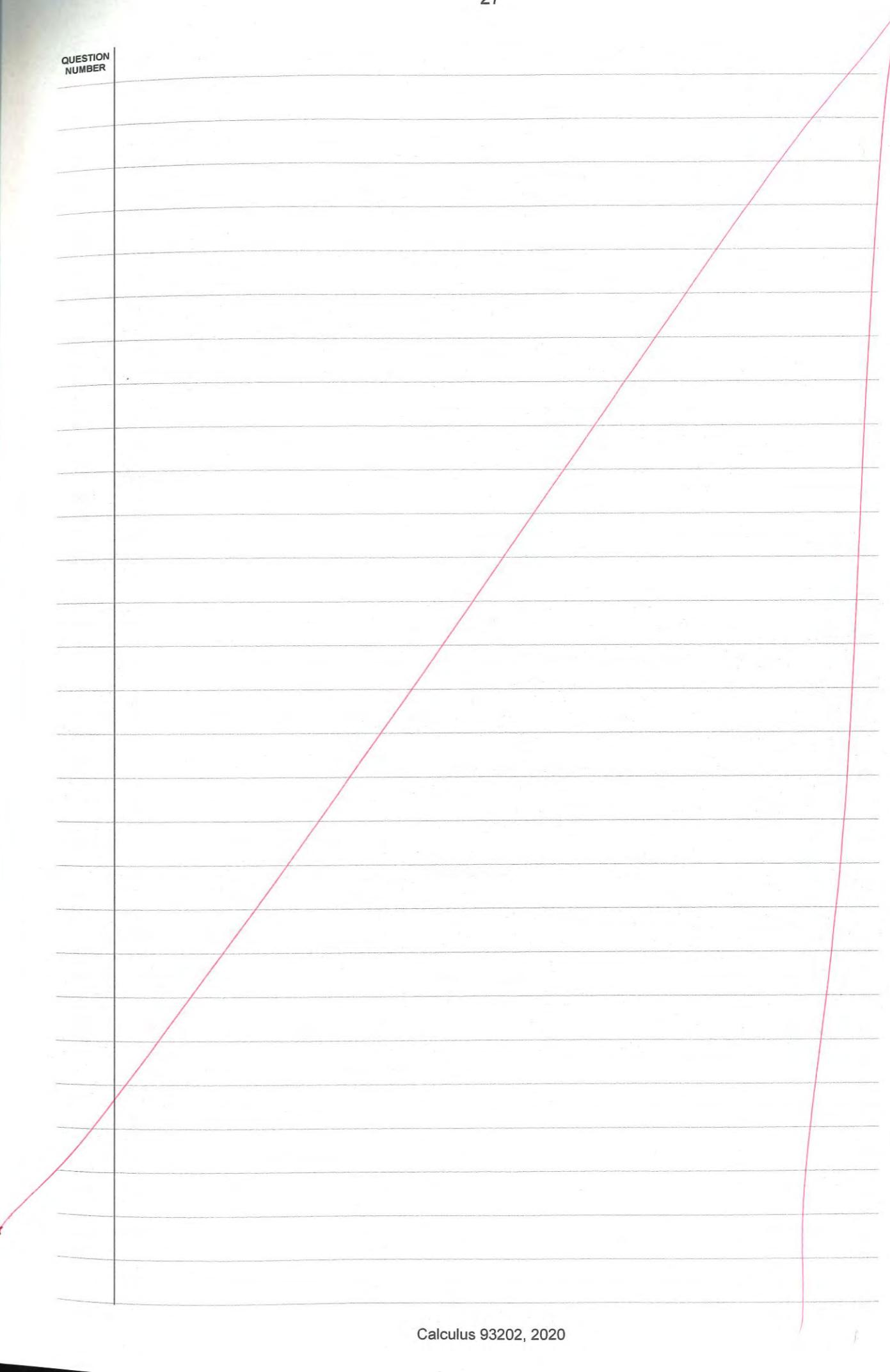
(Q4a)

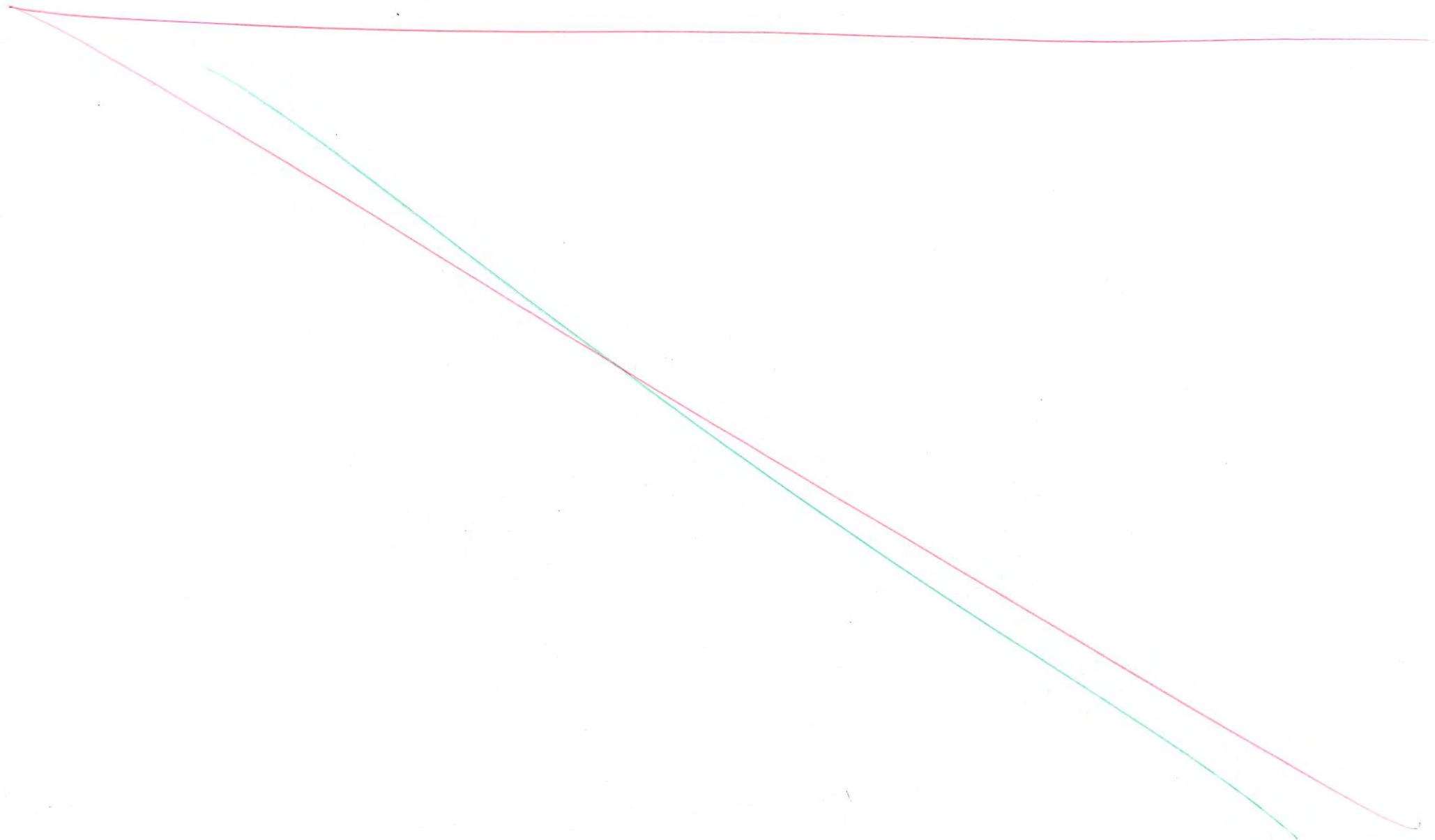
$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

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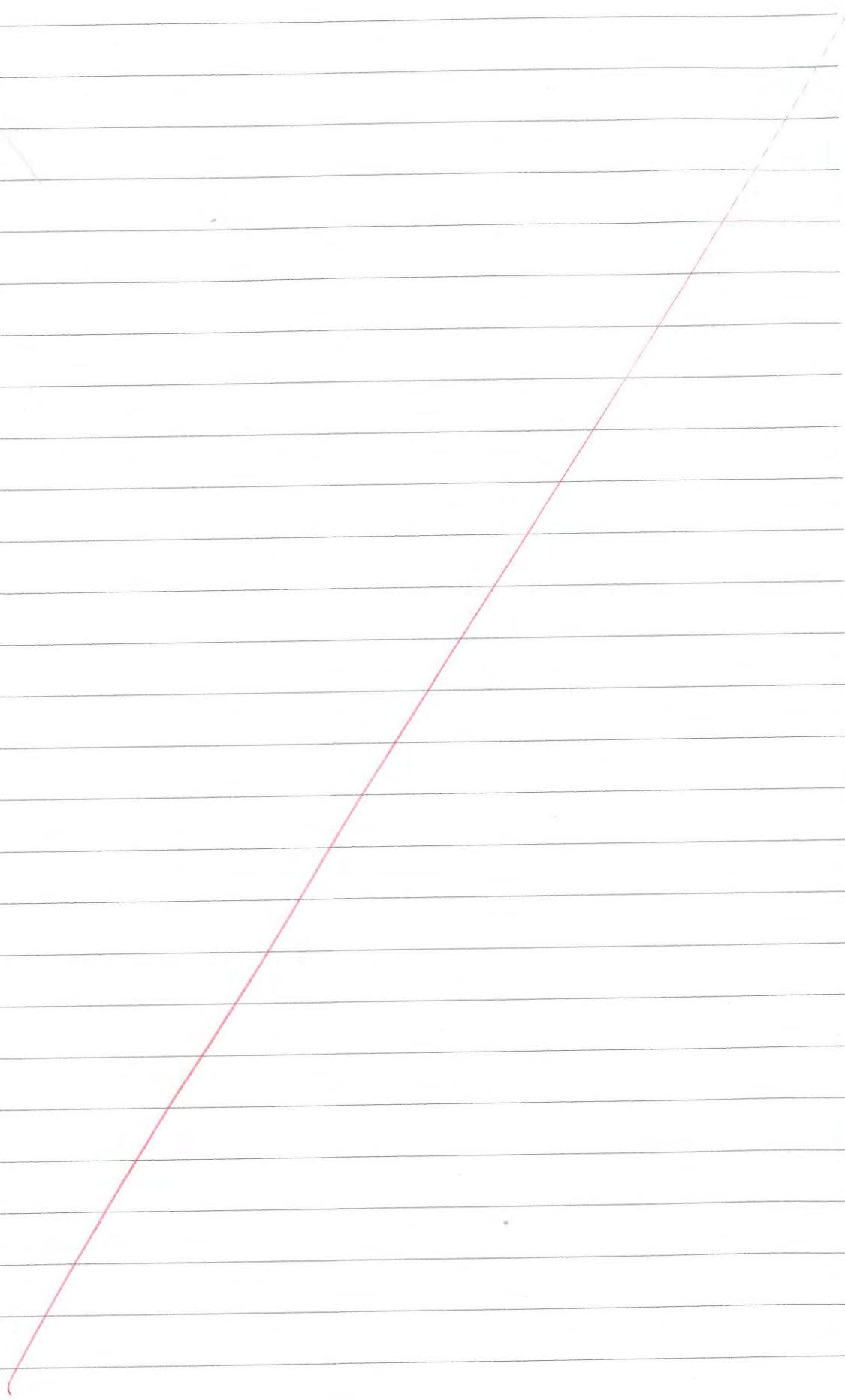
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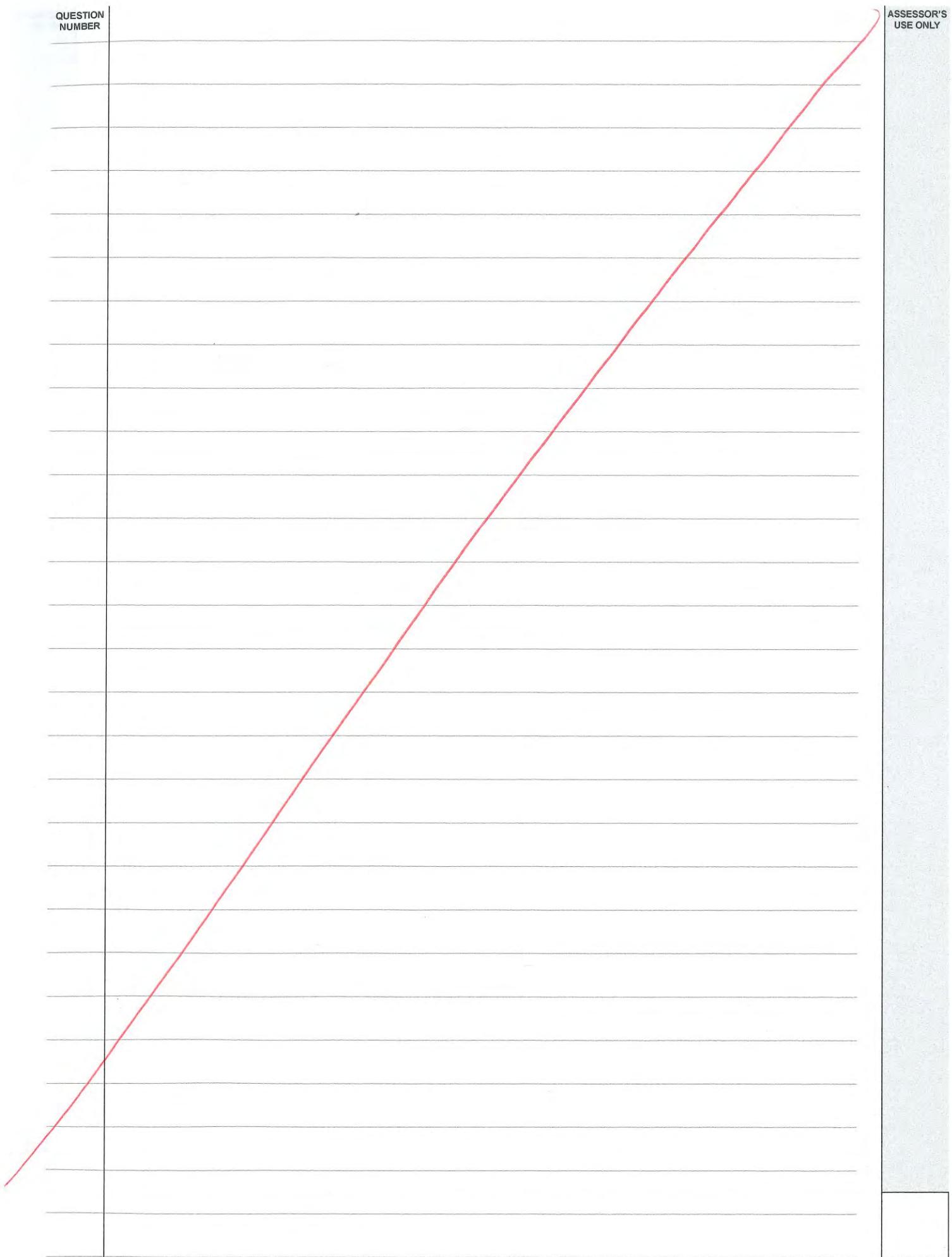
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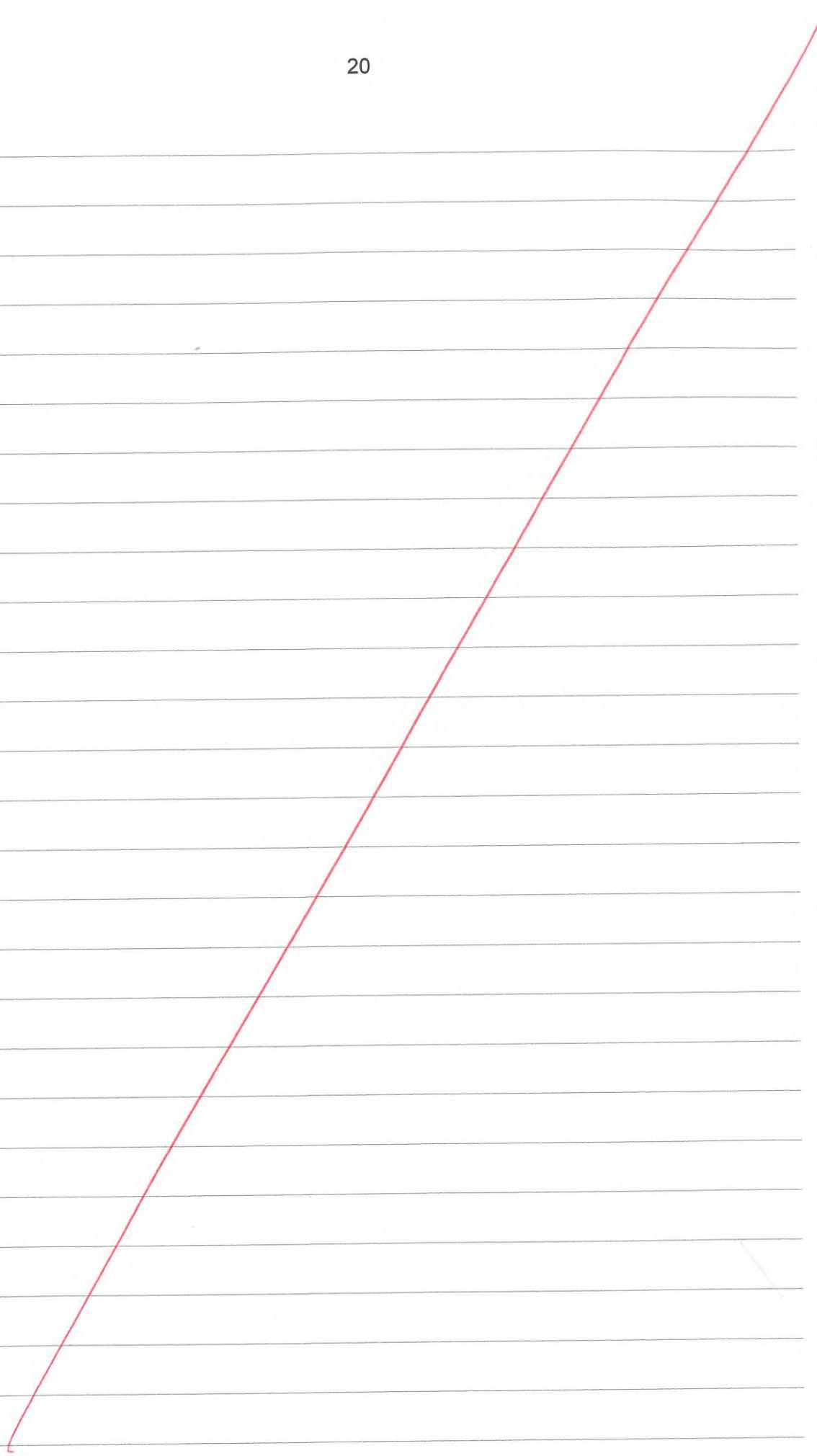


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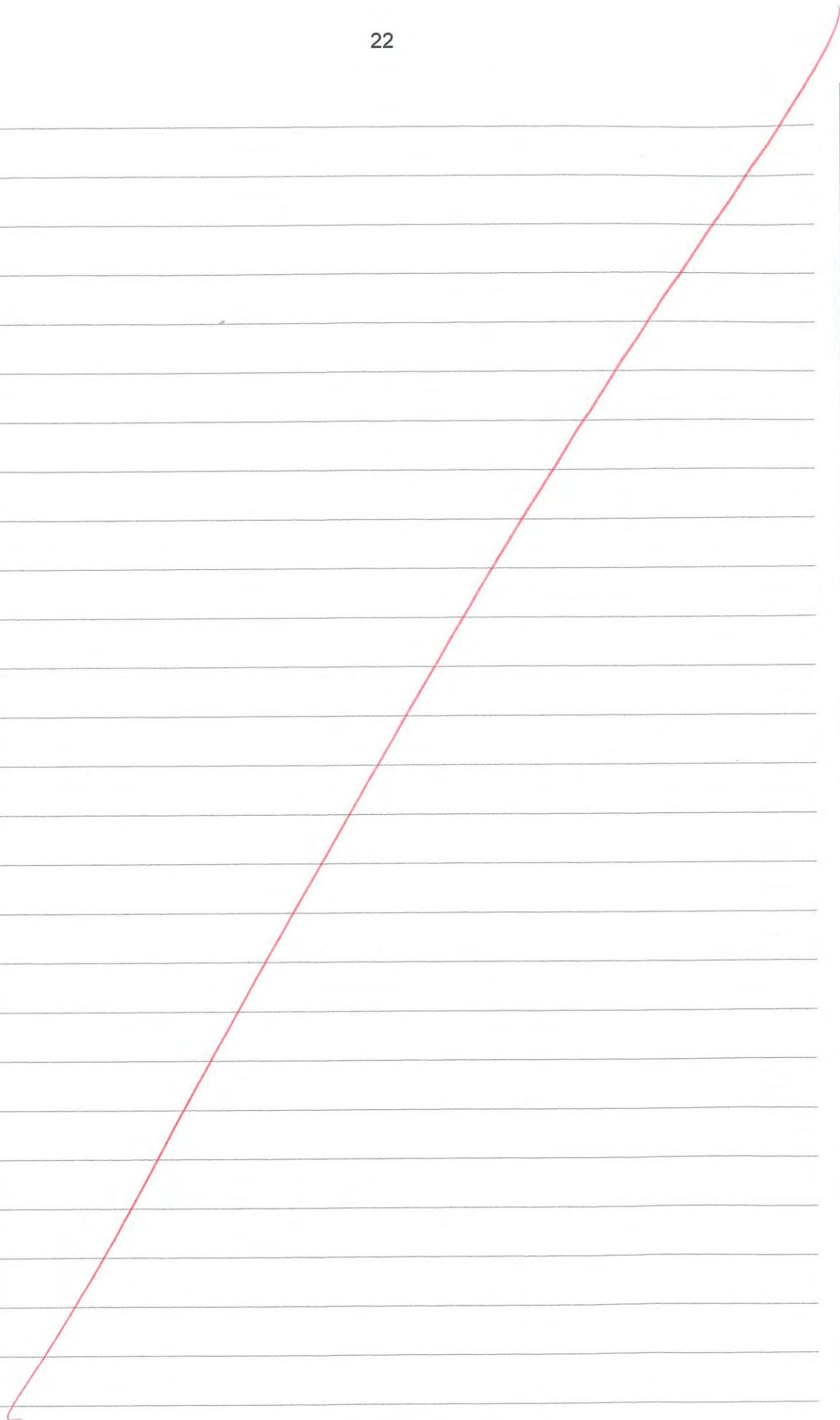
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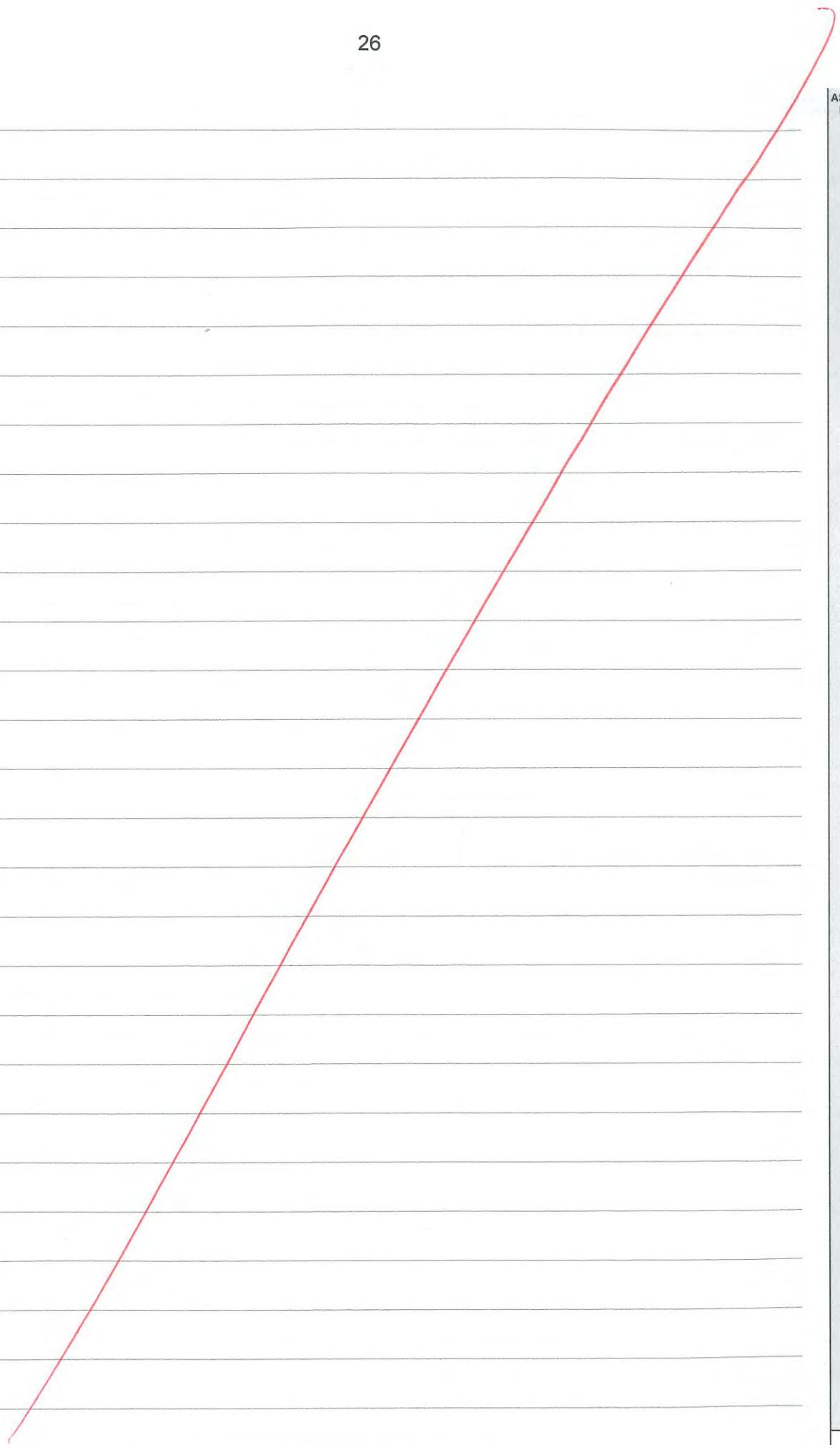
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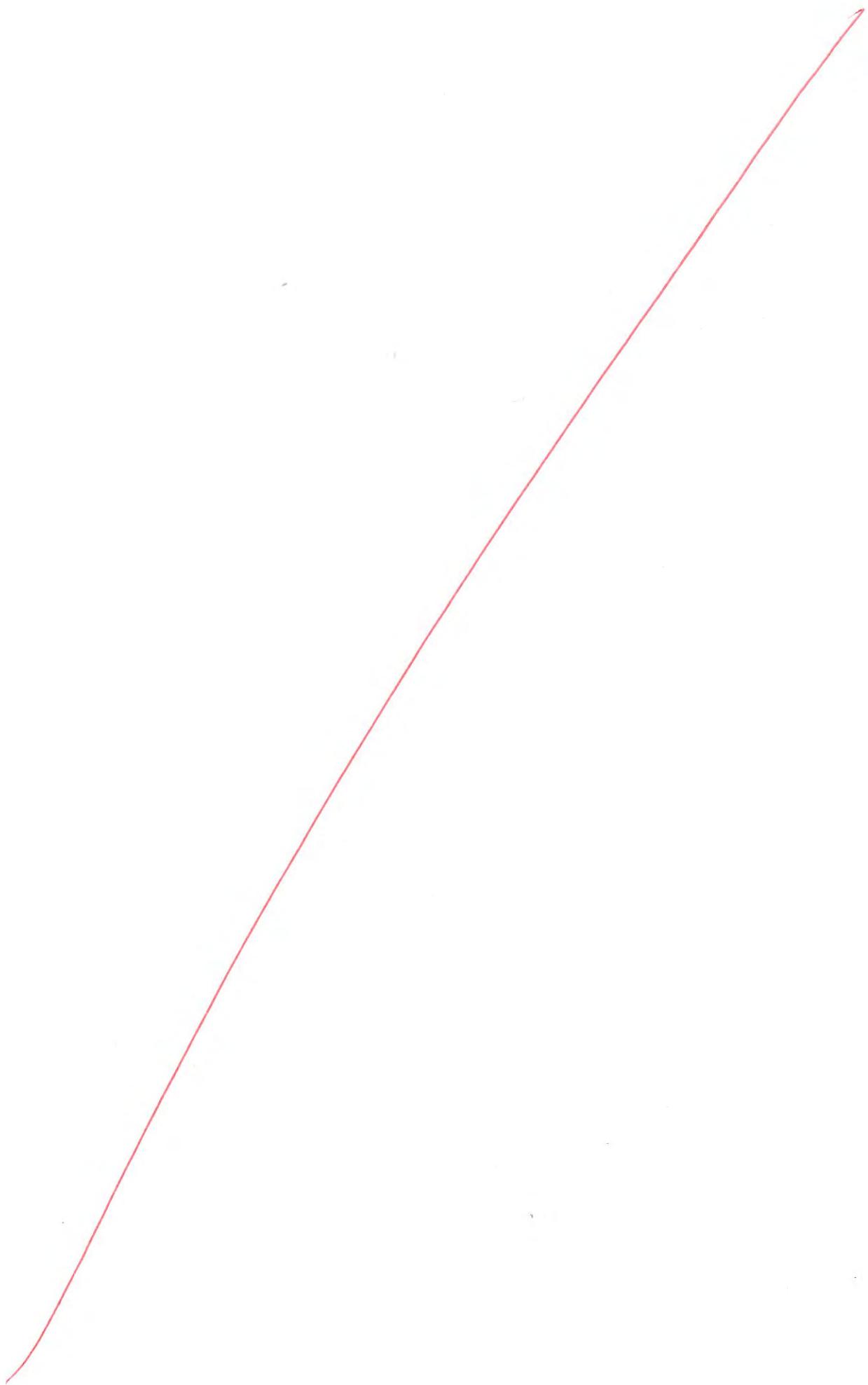
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Question	Mark	Annotation
1	6	The candidate successfully integrated the function by selecting a suitable substitution in 1b . Although the limits were left unchanged, it was treated as a minor error as they managed to obtain the correct answer by changing the integral back using original variable. The candidate also showed understanding of strategies in finding limit of a rational function in 1a .
2	7	The candidate accurately used ‘quotient rule’ while differentiating the function involving trigs in 2bi and subsequently solved the simple trig equation in the given domain, and gave exact x and y coordinates of the turning points as required. The candidate also successfully connected the ratio of b over a with the trig (cotangent) of the half angle in 2ci , and applied double angle formulae of ‘sine’ and ‘cosine’ to complete the proof.
3	6	The candidate could apply related rates of change rule correctly and concisely, therefore simplified the working process effectively in 3c . The candidate constructed a correct function in 3d , and successfully used differentiation to optimise it. Unfortunately, they made an error in finding the second derivative implicitly.
4	7	The candidate managed to correctly finding the second derivative by using implicit differentiation in 4c . They could apply ‘integrate by parts’ rule consistently in 4bi , and recognise the need for the rearrangement of the integral. Unfortunately, they failed to acknowledge the connection of the ‘area under the curve and the integral’ in 4bii .
5	6	The candidate worked with clear setting out and explained the steps taken in 5aii . The candidate correctly rationalised the complex number using conjugate, then successfully simplified it to an imaginary expression in 5b . They attempted 5d with little success.