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SCHOLARSHIP EXEMPLAR

NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Pencil throughout

Scholarship 2015
Calculus

2.00 p.m. Tuesday 17 November 2015
Time allowed: Three hours
Total marks: 40

ANSWER BOOKLET

There are five questions in this examination. Answer ALL FIVE questions, choosing ONE option from part (b) of Question Four.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Write ALL your answers in this booklet.

Make sure that you have Formulae and Tables Booklet S–CALCF.

Show ALL working. Start your answer to each question on a new page. Carefully number each question.

Answers developed using a CAS calculator require **ALL commands to be shown**. Correct answers only will not be sufficient.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

The diagram for Question Four (b) Option 2 is on page 27 of this booklet.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

This examination consists of five questions.
Answer all FIVE questions, choosing ONE option from part (b) of Question Four.

QUESTION NUMBER

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$(x^{\frac{1}{2}})' =$

Question One.

(a) surface area $= \int_1^3 \left(2\pi\left(x^3 + \frac{1}{12x}\right)\sqrt{1+\left(3x^2 - \frac{1}{12x^2}\right)^2}\right) dx$ 0

$= \int_1^3 \left[\left(2\pi x^3 + \frac{\pi}{6x}\right) \cdot \sqrt{1 + 9x^4 - \frac{1}{2} + \frac{1}{144x^4}}\right] dx$

$= \int_1^3 \left[\left(2\pi x^3 + \frac{\pi}{6x}\right) \cdot \sqrt{\frac{1}{2} + 9x^4 + \frac{1}{144x^4}}\right] dx$

$= \int_1^3 \sqrt{\left(2\pi x^3 + \frac{\pi}{6x}\right)^2 \cdot \left(\frac{1}{2} + 9x^4 + \frac{1}{144x^4}\right)}\, dx$

$= \int_1^3 \sqrt{\left(4\pi^2 x^6 + \frac{\pi^2 x^2}{3} + \frac{\pi^2}{36x^2}\right)\left(\frac{1}{2} + 9x^4 + \frac{1}{144x^4}\right)}\, dx$

$= \int_1^3 \Big(2\pi^2 x^6 + \frac{\pi^2 x^2}{6} + \frac{\pi^2}{72x^2} + 36\pi^2 x^{10} + 3\pi^2 x^6 + \frac{\pi^2 x^2}{4}$
$+ \frac{\pi^2 x^2}{36} + \frac{\pi^2}{432x^2} + \frac{\pi^2}{5184x^6}\Big)^{\frac{1}{2}} dx$

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$= \int_1^3 \left[\frac{4\pi^2 x^2}{9} + \frac{7\pi^2}{432x^2} + 5\pi^2 x^6 + 36\pi^2 x^{10} + \frac{\pi^2}{5184x^6}\right] dx$

$= \left[\frac{4\pi^2 x^3}{27} - \frac{7\pi^2}{432x} + \frac{5\pi^2 x^7}{7} + \frac{36\pi^2}{11}x^{11} - \frac{\pi^2}{25920x^5}\right]_1^3$

$= 4\pi^2 - \frac{7\pi^2}{1296} + \frac{10935\pi^2}{7} + \frac{6377292\pi^2}{11} - \frac{\pi^2}{6298560}$

$- \frac{4\pi^2}{27} - \frac{7\pi^2}{432} + \frac{5\pi^2}{7} + \frac{36\pi^2}{11} - \frac{\pi^2}{25920}$

$=$

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(c) Initial amount of salt $= 200 \times 0.5 = 100$ kg.

~~0.8~~ x

0

QUESTION NUMBER

Question Two.

(a) $\begin{cases} 3^{4x+2y} - 3^{2x} \cdot 3^{y} = 6 \\ \log_{x+1}(y+3)(x+y+4) = \log_{x+1}(x+1)^3 \end{cases}$

$\Rightarrow \begin{cases} 3^{4x+2y} - 3^{2x+y} = 6 \\ (y+3)(x+y+4) = (x+1)^3 \end{cases}$

Since $3^2 - 3 = 6$

Therefore $\begin{cases} 4x+2y=2 \\ 2x+y=1 \end{cases} \Rightarrow y = 1-2x$

$xy + y^2 + 4y + 3x + 3y + 12 = x^3 + 3x^2 + \cancel{3x} + 1$

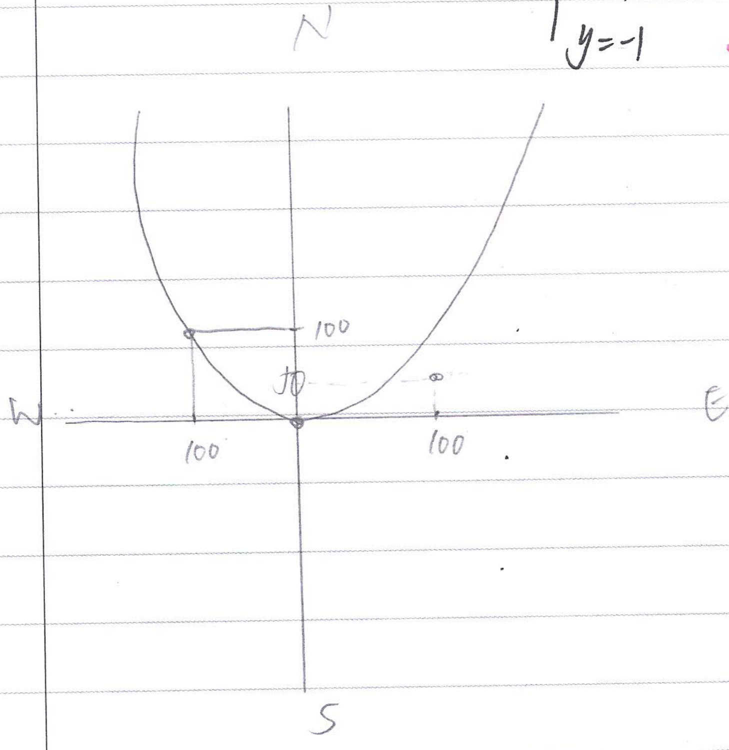
$x(1-2x) + (1-2x)^2 + 4(1-2x) + \cancel{3x} + 3(1-2x) \overset{+12}{=} x^3 + 3x^2 + \cancel{3x} + 1$

$x - 2x^2 + 1 - 4x + 4x^2 + 4 - 8x + 3 - 6x \overset{+12}{=} x^3 + 3x^2 + 1$

$x^3 + x^2 + 17x - 19 = 0$

$x = 1$ Therefore $y = 1 - 2 = -1$

$\begin{cases} x = 1 \\ y = -1 \end{cases}$



QUESTION NUMBER

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(c) $\int \frac{1}{S(N-S)}\, dS = \int k\, dt$

$(N-2S)\ln(SN-S^2) = kt+C$

when $t=0$ $S=2$

$C = (N-4)\ln(2N-4)$

$(N-2S)\ln(SN-S^2) = kt + (N-4)\ln(2N-4)$

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$z = \cos\theta + i\sin\theta \qquad z^6 = \cos^6\theta + i\sin 6\theta$

$z^6 + \frac{1}{z^6} = 2\cos 6\theta \Rightarrow \cos^6\theta + \cancel{i\sin 6\theta} + \frac{1}{z^6} = \cancel{2\cos 6\theta}$

$\cos^6\theta = -\frac{1}{z^6}$

$= -\frac{1}{\cos^6\theta + i\sin 6\theta}$

$\frac{1}{32}\left(\frac{1}{2}\cdot 2\cos 6\theta + 3\cdot 2\cos 4\theta + \frac{15}{2}\cdot 2\cos 2\theta + 10\right)$

$= \frac{1}{32}\left(\frac{1}{2}\cdot\left(z^6 + \frac{1}{z^6}\right) + 3\cdot\left(z^4 + \frac{1}{z^4}\right) + \frac{15}{2}\left(z^2 + \frac{1}{z^2}\right) + 10\right)$

$= \frac{1}{32}\left(\frac{1}{2}z^6 + \frac{1}{2z^6} + 3z^4 + \frac{3}{z^4} + \frac{15z^2}{2} + \frac{15}{2z^2} + 10\right)$

$= \frac{1}{32}\left(\frac{z^{12} + 1 + 6z^{10} + 6z^2 + 15z^8 + 15z^4 + 20z^6}{2z^6}\right)$

$= \frac{1}{32}$

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Question Three

(c)

$$\frac{2(\cos 4x+1)-4\cos^2 x+3(1-\cos^2 x)}{2[\cos(5\pi-2x)+1]-\frac{1}{2}[1-\cos(4x-4\pi]}$$

$$=\frac{2\cos 4x+2-4\cos x^2+3-3\cos x^2}{-2\cos 2x+2-\frac{1}{2}(1-\cos 4x)}$$

$$=\frac{2\cos 4x+5-7\cos x^2}{-2\cos 2x+\frac{3}{2}+\frac{1}{2}\cos 4x}$$

$$=\frac{2\cos 4x+5-7\cdot\frac{\cos 2x+1}{2}}{-2\cos 2x+\frac{3}{2}+\frac{1}{2}\cos 4x}$$

$$=\frac{2\cos 4x+5-\frac{7}{2}\cos 2x-\frac{7}{2}}{-2\cos 2x+\frac{1}{2}\cos 4x+\frac{3}{2}}$$

$$=\frac{2\cos 4x-\frac{7}{2}\cos 2x+\frac{3}{2}}{-2\cos 2x+\frac{1}{2}\cos 4x+\frac{3}{2}}$$

$$=\frac{2(2\cos^2 2x-1)-\frac{7}{2}\cos 2x+\frac{3}{2}}{-2\cos 2x+\frac{1}{2}(2\cos^2 2x-1)+\frac{3}{2}}$$

$$=\frac{4\cos^2 2x-2-\frac{7}{2}\cos 2x+\frac{3}{2}}{-2\cos 2x+\cos^2 2x+1}$$

$$=\frac{4\cos^2 2x-\frac{7}{2}\cos 2x-\frac{1}{2}}{-2\cos 2x+\cos^2 2x+1}$$

$$=\frac{8\cos^2 2x-7\cos 2x-1}{-4\cos 2x+2\cos^2 2x+2}$$

$$=\frac{(8\cos 2x+1)(\cos 2x-1)}{2(\cos 2x-1)^2}$$

$$=\frac{8\cos 2x+1}{2(\cos 2x+1)}$$

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QUESTION NUMBER

$$\boxed{\begin{aligned} x_1 + x_2 &= -\frac{b}{a} \\ x_1 \cdot x_2 &= \frac{c}{a} \end{aligned}}$$

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Question Four.

OR: (i) Assume the factory has x red pack, y white pack and z blue pack

$$\begin{cases} 6x+2y+2z \le 3x+4y+5z \\ 6x+2y+2z \le 500 \\ 3x+4y+5z \le 400 \\ 2x+4y+2z \le 300 \end{cases}$$

$10x+12y+8z \ge 1000$.

$3x-2y-3z \le 0$

$\frac{3x-3z}{2} \le y$

(ii) $x+y+z=100$

$z=100-x-y$

$3x-2y-3(100-x-y)\le 0$

$3x-2y-300+3x+3y\le 0$

① $6x+y\le 300$

$6x+2y+2(100-x-y)\le 500$

$6x+2y+200-2x-2y\le 500$

$4x \le 300$

② $x\le \frac{300}{4}$

$3x+4y+5(100-x-y)\le 400$

$3x+4y+500-5x-5y\le 400$

$-2x-y\le -100$.

③ $2x+y\ge 100$

$2x+4y+2(100-x-y)\le 300$

$2x+4y+200-2x-2y\le 300$

$2y\le 100$.

④ $y\le 50$

$10x+12y+8(100-x-y)\ge 1000$

$10x+12y+800-8x-8y\ge 1000$

$2x+4y\ge 200$

$x+2y\ge 100$

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Question Five.

(a) At (x_0, y_0)

gradient of tangent $= 2kx_0$

gradient of normal $= -\frac{1}{2kx_0}$

$y_0 = kx_0^2$

$-\frac{1}{2kx_0}(x - x_0) = y - y_0$

$-\frac{1}{2kx_0}(x - x_0) = y - kx_0^2$

$-\frac{1}{2kx_0} \cdot x + \frac{1}{2k} + kx_0^2 = y$

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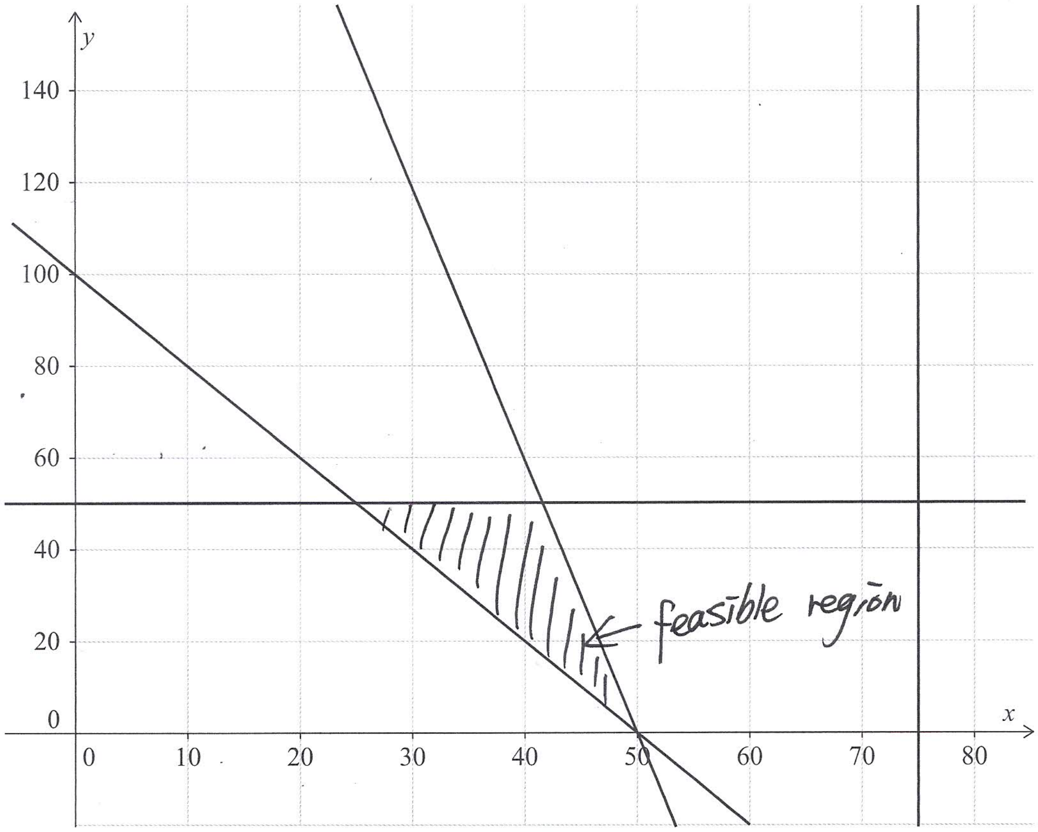
(b)

Diagram for Question Four (b).



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Annotated Exemplar for 93202 Calculus Scholarship		Total Score	20
Question	**Mark**	**Annotation**	
1	**1**	The candidate made little progress with this question, creating only the correct expression to be integrated in **1a**.	
2	**5**	The candidate has provided evidence in **2a** of competently solving simultaneous equations with logarithms and exponents. The candidate does not recognise that partial fractions were needed in **2c**.	
3	**4**	The candidate did not give an adequate proof or expansion in **3a**. The candidate has shown competence in **3c** in simplifying and expanding trigonometric expressions and sound use of trigonometric identities.	
4	**6**	In **4b Option 2**, the candidate has communicated clearly the equations which describe the linear programming context. The equations have been correctly simplified to be in two variables and the feasible region shaded.	
5	**4**	The candidate has made progress to obtain the equation of the normal to the curve at a general point in **5a**. No attempt was made at **5b** or **5c**.	