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SUPERVISOR'S USE ONLY

TOP SCHOLAR

NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Scholarship 2018
Calculus

9.30 a.m. Friday 9 November 2018
Time allowed: Three hours
Total marks: 40

There are five questions in this examination. Answer ALL FIVE questions.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Write ALL your answers in this booklet.

Make sure that you have Formulae and Tables Booklet S–CALCF.

Show ALL working. Start your answer to each question on a new page. Carefully number each question.

Answers developed using a CAS calculator require **ALL commands to be shown**. Correct answers only will not be sufficient.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION NUMBER

ASSESSOR'S USE ONLY

1a) ~~$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$~~

~~Let $P(\alpha-\beta)$, $P(\alpha)$ & $P(\beta)$ be points on the circle~~

Euler's theorem:

$e^{i\theta} = \cos\theta + i\sin\theta$

$e^{i(\alpha-\beta)} = \cos(\alpha-\beta+) + i\sin(\alpha-\beta)$

$\frac{e^{i(\alpha)}}{e^{i(\beta)}} = \frac{\cos\alpha + i\sin\alpha}{\cos\beta + i\sin\beta}$

$= \frac{(\cancel{\cos\beta}\ (\cos\alpha + i\sin\alpha)(\cos\beta - i\sin\beta)}{\cos\beta^2 + \sin^2\beta}$

$= \frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta + i(\sin\alpha\cos\beta - \cos\alpha\sin\beta)}{1}$

$\therefore e^{i(\alpha-\beta)} = \cos\alpha\cos\beta + \sin\alpha\sin\beta + i(\sin\alpha\cos\beta - \cos\alpha\sin\beta)$

$Re(e^{i\cancel{(\alpha-\beta)}}) = \cos\theta$

$\therefore Re(e^{i(\alpha-\beta)}) = \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

QUESTION NUMBER

ASSESSOR'S USE ONLY

1(3)) lengths OP & OA are fixed

$$\frac{\sin(\angle APQ)}{OP} = \frac{\sin(\angle OPA)}{OA}$$

let $|OP| = a$ $\quad |OA| = b$

$$\sin(\angle APQ) = \frac{a}{b}\sin(\angle OPA)$$

hence $\angle APQ$ is maximised, when $\angle OPA$ is maximised.

$\angle APQ$:
& $\angle OPG$

$\angle OPA$ is maximised for P being vertically above the midpoint of $\overline{OA}$

let $|OP| = a$ $\quad |OA| = b$

$$\frac{\sin(\angle APO)}{OP} = \frac{\sin(\angle PAO)}{OA}$$

$$\angle APO = \sin^{-1}\left(\frac{a}{b}\sin(\angle PAO)\right)$$

cosine rule : $\quad |OA| = b \quad |OP| = a, \quad |PA| = c$ variable

$$b^2 = a^2 + c^2 - 2ac\cos(\angle OPA)$$

$$\cos(\angle OPA) = \frac{a^2 + c^2 - b^2}{2ac}$$

$0° < \angle OPA < 90°$, is maximised for small $\cos(\angle OPA)$

$$y = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\frac{dy}{dc} = \frac{2c}{2ac} - \frac{a^2 + c^2 - b^2}{2ac^2}$$

$$0 = \frac{2c}{2ac} - \frac{a^2+c^2-b^2}{2ac^2} \quad \frac{a^2+c^2-b^2}{c} = 2c^2$$

$$a^2 + c^2 - b^2 = 2c^2$$

$$a^2 = b^2 + c^2$$ *Go to page 26

QUESTION NUMBER

ASSESSOR'S USE ONLY

1c)

$1 + \sin\theta + \cos\theta =$

$1 + \frac{1}{2}\sin(\frac{1}{2}\theta)\cos(\frac{1}{2}\theta) + 2\cos(\frac{1}{2}\theta)^2 - 1 =$

$\cancel{\frac{1}{2}\cos t} \quad \cos\frac{1}{2}\theta\left(\frac{1}{2}\sin\frac{1}{2}\theta + 2\cos(\frac{1}{2}\theta)\right) =$

$1 + \cos\theta = 2\cos(\frac{1}{2}\theta)^2$

$2\cos(\frac{1}{2}\theta)^2 + 2\cos(\frac{1}{2}\theta)^2 \cdot 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$\sin\theta + 1 =$

$\sin\theta + \sin(\frac{\pi}{2}) = 2\sin(\frac{1}{2}\theta + \frac{1}{4}\pi) \cdot \sin(\frac{1}{2}\theta - \frac{1}{4}\pi)$

$=$

$$\frac{\cos\theta + \cos^2\theta + \sin\theta - \sin^2\theta}{1 + \cos\theta + \sin\theta + \cos\theta\sin\theta} =$$

$\cancel{+\frac{1}{2}\sin 2\theta}$

$2\,\cancel{\cos\theta}\ \sin$

$$\frac{2(\cos\theta - \sin\theta)}{\cos^2\theta + \cos\theta + \sin^2\theta - \sin\theta} =$$

$$\frac{2(\cos\theta - \sin\theta)}{(\cos\theta + 1)\cos\theta + (\sin\theta + 1)\sin\theta} =$$

$$\frac{1}{\frac{(\cos\theta+1)\cos\theta + (\sin\theta+1)\sin\theta}{2(\cos\theta - \sin\theta)}} =$$

$$\frac{1}{\frac{\frac{\cos\theta}{2(\cos\theta - \sin\theta)}}{1+\cos\theta} + \frac{\frac{\sin\theta}{2(\cos\theta - \sin\theta)}}{1+\sin\theta}}$$

End of bklt

QUESTION NUMBER

ASSESSOR'S USE ONLY

2a)i) $x = -\frac{2}{3}$, $x = \frac{3}{5}$: denominator of first term

$x^2 - 5x + 3 = 0$: denominator of second term.

$x = \frac{5 \pm \sqrt{5^2 - 12}}{2} = \frac{5 \pm \sqrt{13}}{2}$

$x = \frac{5+\sqrt{13}}{2}$ $x = \frac{5-\sqrt{13}}{2}$

$x = -\frac{2}{3}$, $x = \frac{3}{5}$, $x = \frac{5+\sqrt{13}}{2}$, $x = \frac{5-\sqrt{13}}{2}$

~~$(3x+2)(5x-3) =$~~

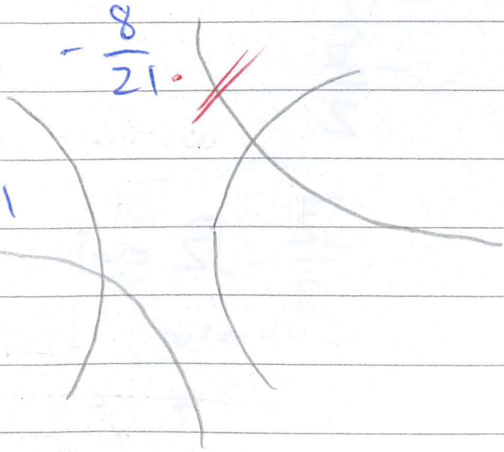
ii) f is not discontinous at $x = \frac{1}{2}$ so $\lim_{x \to \frac{1}{2}} f(x) = f\left(\frac{1}{2}\right)$

$$\lim_{x \to \frac{1}{2}} f(x) = \frac{\frac{1}{2} - 1}{\left(\frac{7}{2}\right) \cdot \left(-\frac{1}{2}\right)} - \frac{\frac{1}{2}}{\frac{3}{4}}$$

$$= \frac{\cancel{-\frac{1}{2}}}{\frac{7}{2}\cancel{\left(-\frac{1}{2}\right)}} - \frac{2}{3}$$

$$= \frac{2}{7} - \frac{2}{3}$$

$$= -\frac{8}{21}.$$



2b) $xy = \sqrt{2}$ $x^2 - y^2 = 1$

$y = \frac{\sqrt{2}}{x}$

$x^2 - \frac{2}{x^2} = 1$

$x^4 - 2 = x^2$

$x^4 - x^2 - 2 = 0$

$(x^2 - 2)(x^2 + 1) = 0$

$x^2 = 2$ $x^2 = -1$: no real solutions

QUESTION NUMBER

ASSESSOR'S USE ONLY

$x^2=2$ $\quad x=+\sqrt{2}$ $\quad x=-\sqrt{2}$

$y=1$ $\quad y=-1$

$xy=\sqrt{2}$

$y=\frac{\sqrt{2}}{x}$

$\frac{dy}{dx}=\frac{-\sqrt{2}}{x^2}$

$x^2-y^2=1$

$2x-2y\frac{dy}{dx}=0$

$\frac{dy}{dx}=\frac{-2x}{-2y}$

$=\frac{x}{y}$

$(\sqrt{2},\ 1)$

$xy=\sqrt{2}$

$\frac{-\sqrt{2}}{\sqrt{2}^2}=-\frac{\sqrt{2}}{2}$

$x^2-y^2=1$

$\frac{\sqrt{2}}{1}=$

$\sqrt{2}$

$-\frac{\sqrt{2}}{2}\cdot\sqrt{2}=$

-1

$\therefore$ Perpendicular tangents ~~(gradients~~ at intersection 1

$(-\sqrt{2},\ -1)$

$\frac{-\sqrt{2}}{(-\sqrt{2})^2}=-\frac{\sqrt{2}}{2}$

$\frac{(-\sqrt{2})}{-1}=$

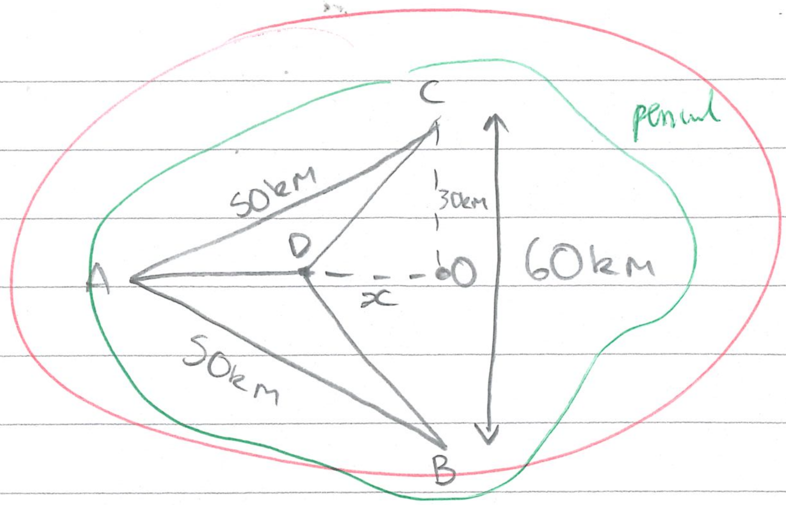
$\sqrt{2}$

$-\frac{\sqrt{2}}{2}\cdot\sqrt{2}=-1$

$\therefore$ Perpendicular tangents at intersection 2

QUESTION NUMBER | ASSESSOR'S USE ONLY

2c)



pencil

~~x lies alon~~

Let D be the midpoint of BC

x lies at a distance from O along OA.

$CD = BD = \sqrt{x^2+900}$

$AD = 40 - x$; $\sqrt{50^2 - 30^2}$:

Total length =

$L = 2\sqrt{x^2+900} + 40 - x$

$\frac{dL}{dx} = 2(2x) \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x^2+900}} + 0 - 1 = 0$

$\frac{2x}{\sqrt{x^2+900}} = 1$

$2x = \sqrt{x^2+900}$

$4x^2 = x^2 + 900$

$3x^2 = 900$

$x^2 = 300$ D is ...

$x = 10\sqrt{3}$: $10\sqrt{3}$ km from O, or $40 - 10\sqrt{3}$ km from A

$L = 2 \cdot \sqrt{1200} + 40 - 10\sqrt{3}$

$= 2 \cdot \sqrt{4}\sqrt{300}$

$= 2 \cdot 2 \cdot 10 \cdot \sqrt{3} + 40 - 10\sqrt{3}$

$= 40\sqrt{3} - 10\sqrt{3} + 40 = 30\sqrt{3} + 40$ km

QUESTION NUMBER

ASSESSOR'S USE ONLY

$$\frac{dL}{dx} = \frac{2x}{\sqrt{x^2+900}} - 1$$

$$\frac{d^2L}{dx^2} = \frac{2}{\sqrt{x^2+900}} - \frac{x \cdot 2x}{\sqrt{x^2+900}^3}$$

$$\frac{d^2L}{dx^2}(10\sqrt{3}) = \frac{2}{\sqrt{1200}} - \frac{2(10\sqrt{3})^2}{\sqrt{1200}^3}$$

$$= \cancel{0.0573163602S}$$ 0.0433012701 9

$\therefore \frac{d^2L}{dx^2}(10\sqrt{3}) > 0$

$\therefore x = 10\sqrt{3}$ is a Minimum

Q2

QUESTION NUMBER

ASSESSOR'S USE ONLY

3(a)(i) Let $u = 2x+5$ $\quad u(5)=15$, $u(0)=5$

$2x-5 = u-10$

$\cancel{\int_0^5 (u-10)\sqrt{u}}$ $\quad u=2x+5$

$\frac{du}{dx}=2$

$dx = \frac{1}{2}du$

$\int_{5}^{15} (u-10)\sqrt{u} \cdot \frac{1}{2}\, du =$

$\int_5^{15} \frac{1}{2}u^{\frac{3}{2}} - 5\sqrt{u} \cdot du =$

$\left[\frac{1}{5}u^{\frac{5}{2}} - \frac{10}{3}u^{\frac{3}{2}}\right]_5^{15} =$

$\frac{1}{5}15^2 \cdot \sqrt{15} - 50\sqrt{15}$

$-5\sqrt{5} + \frac{50}{3}\sqrt{5} =$

$\frac{35}{3}\sqrt{5} - 5\sqrt{15}$

ii) If we use a sub $v = 2x-5$ $\quad v(5)=5$, $v(0)=-5$

$dx = \frac{1}{2}dv$ and

we get

$\int_{-5}^{5} (u+10)\sqrt{u}\,\frac{1}{2}du =$

$\int_{-5}^{5} \frac{1}{2}u^{\frac{3}{2}} + 5u^{\frac{1}{2}} =$

$\left[\frac{1}{5}u^{\frac{5}{2}} + \frac{10}{3}u^{\frac{3}{2}}\right]_{-5}^{5}$

however when we substitute in $u=-5$, we get a complex number, for $u^{\frac{5}{2}}$ and $u^{\frac{3}{2}}$, so the result is undefined...

and this indicates the method's impossibility as the real integral should be real

QUESTION NUMBER

ASSESSOR'S USE ONLY

3b) Area beneath the equality diagonal =

$$100 \times 100 \times \frac{1}{2} = 5000$$

MEi

Area beneath the curve.

$$y = \int_0^{100} \frac{B-1}{B}x^2 + \frac{1}{B}x \, dx =$$

$$\left[\frac{B-1}{3B}x^3 + \frac{1}{2B}x^2 \right]_0^{100} =$$

$$\frac{B-1}{3B}(1000000) + \frac{1}{2B} \cdot 10000 - 0$$

$$\frac{5000 - \left(\frac{B-1}{3B}(1000000) + \frac{1}{2B}10000\right)}{5000} = \frac{20}{63}$$

$$\frac{200(B-1)}{3B} + \frac{1}{B} = \frac{20}{63}$$

$$200(B-1) + 3 = \frac{20}{21}B$$

$$\frac{4180}{21}B = 197$$

$$B = \frac{4137}{4180} \approx 0.9897129187$$

$$1 - \frac{200(B-1)}{3B} - \frac{1}{B} = \frac{20}{63}$$

$$\frac{200(B-1)}{3B} + \frac{1}{B} - 1 = -\frac{20}{63}$$

$$\frac{200B - 200}{3B} + \frac{1}{B} = \frac{43}{63}$$

$$200B - 200 + 3 = \frac{43}{21}B$$

$$\frac{4117}{21}B = 197$$

$$B = \frac{4137}{4117}$$

QUESTION NUMBER

ASSESSOR'S USE ONLY

$200B - 200r^3 = \frac{43}{\cancel{88}\,21}B$

$\cancel{\frac{1255\,7}{63}B = 197}$

$B = 0.9988373019$

$\cancel{B = 0.99}$

$\cancel{B = 0.988\,1331}$

$\frac{4157}{21}B = 197$

$B = \frac{4137}{4157}$

$B = 0.9951888381$

$\boxed{B = 0.995}$

QUESTION NUMBER

ASSESSOR'S USE ONLY

3c) For a height h, volume is

$$h \cdot \cancel{\tfrac{1}{2}} \frac{\pi D^2}{4} =$$

$$\frac{\pi D^2 h}{4} = V \qquad \frac{dV}{dh} = \frac{\pi D^2}{4}$$

For a height h, rate of change of volume =

$$v \cdot \frac{\pi d^2}{4} =$$

$$\sqrt{2gh}\,\frac{\pi d^2}{4} = \frac{dV}{dt}$$

$$\frac{dV}{dt} = \sqrt{2gh}\,\frac{\pi d^2}{4}$$

$$dV = \sqrt{2gh}\,\frac{\pi d^2}{4}\,dt$$

$$\cancel{\frac{dV}{dh}}\ \frac{\pi D^2}{4}\,dh = \sqrt{2gh}\,\frac{\pi d^2}{4}\,dt$$

$$\int \frac{1}{\sqrt{h}} \cdot \frac{\cancel{\pi} D^2}{\cancel{4}}\,dh = \int \frac{\cancel{\pi} d^2}{\cancel{4}}\,\cancel{dt} \cdot \sqrt{2g}\,dt$$

$$2\sqrt{h}\,D^2 = \sqrt{2g}\,d^2 t + C$$

at $t=0$, $h=h_1$

$$2\sqrt{h_1}\,D^2 = C$$

$$C = 2\sqrt{h_1}\,D_2 \quad \text{TE}$$

$$2\sqrt{h}\,D^2 = \sqrt{2g}\,d^2 t + 2\sqrt{h_1}\,D_2$$

$$t = \frac{2D_2(\sqrt{h} - \sqrt{h_1})}{\sqrt{2g}\,d^2} = \frac{D^2}{d^2}\sqrt{\frac{2}{g}}\left(\sqrt{h} - \sqrt{h_1}\right)$$

for $h = h_2$

$$t = \frac{D^2}{d^2} \times \sqrt{\frac{2}{g}}\left(\sqrt{h_2} - \sqrt{h_1}\right)$$

Q3

QUESTION NUMBER | ASSESSOR'S USE ONLY

4a) $\cancel{\log}\ \log_3(4-2x) + \log_3(x) \leq 2$

$\cancel{\dfrac{\log(4-2x)+\log(x)}{\log 3} \leq 2}$

$\cancel{\log((4-2x)x) \leq 2\log 3}$

$\cancel{4x - 2x^2 \leq 9}$

$2x^2 - 4x + 9 \geq 0$

$\log_3((4-2x)x) \leq 2$

$4x - 2x^2 \leq 3^2$

$4x - 2x^2 \geq 0$

$0 \leq 4x - 2x^2 \leq 9$

$4x - 2x^2 \geq 0$

$2x(2-x) \geq 0$

$x = 0 \quad x = 2$

$\cancel{4x - 2x^2 \geq 0}$

~~for~~

	0		2	
$4x-2x^2$	-ve	+ve		-ve

$4x - 2x^2 \geq 0$ for $0 < x < 2$

$4x - 2x^2 \leq 9$

$2x^2 - 4x + 9 \geq 0$

$2x^2 - 4x + 9 = 0$

$4^2 - 4 \times 2 \times 9 = -56 < 0$

$\therefore$ This has no real solutions

$2x^2 - 4x + 9$ is a polynomial so is continuous for all x.

$2(1)^2 - 4(1) + 9 = 7 > 0 \ \therefore$

$2x^2 - 4x + 9 > 0$ for all x

QUESTION NUMBER

ASSESSOR'S USE ONLY

so $(4x-2x^2) \leq 9$

for all x

$\therefore$

x is limited by

$4-2x>0$, $x>0$

$x<2$ $x>0$

$\therefore$

$0<x<2$

QUESTION NUMBER

ASSESSOR'S USE ONLY

4b) $\boxed{1}$ $x^2 - 16x + y^2 - 20y + 115 = 0$

$x^2 - 16x + 64 + y^2 - 20y + 100 - 64 - 100 + 115 = 0$

$(x-8)^2 + (y-10)^2 = 49$

∴ Circle ~~leng~~ radius 7, centre (8, 10)

$\boxed{2}$ $x^2 + 8x + y^2 - 10y + 5 = 0$

$(x^2 + 8x + 16) + (y^2 - 10y + 25) - 16 - 25 + 5 = 0$

$(x + 4)^2 + (y - 5)^2 = 36$

Circle radius 6, centre (-4, 5)

The distance between the centres d

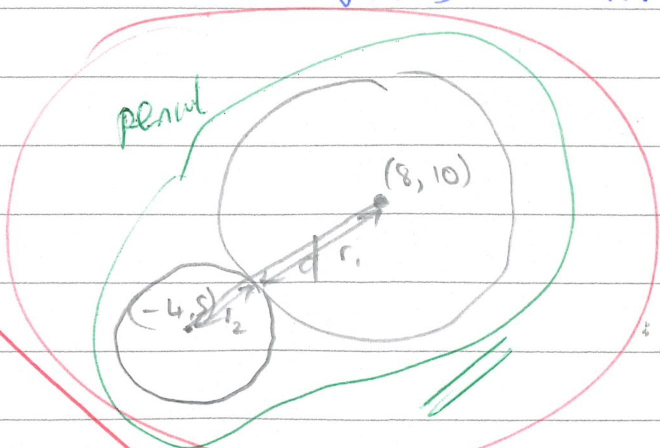
$d^2 = (8 - -4)^2 + (10 - 5)^2$

$= 169$

$d = 13$

$7 + 6 = 13$ ∴ They are tangential as the sum of the radii = the distance of the centres.

Co-ordinates of intersection ....



QUESTION NUMBER

ASSESSOR'S USE ONLY

Co-ords of intersection

Gradient of line between centres

$$\frac{10-5}{8--4} = \frac{5}{12} = \tan\theta$$

$$\sec^2\theta = 1+\left(\frac{5}{12}\right)^2 = \cancel{\frac{17}{144}}\ \frac{169}{144}$$

$$\cancel{\cos^2\theta = \frac{12}{17} \quad \sin^2\theta = \frac{5}{17}}$$

$$\cos^2\theta = \frac{144}{169} \qquad \sin^2\theta = \frac{25}{169}$$

$$\cos\theta = \frac{12}{13} \qquad \sin\theta = \frac{5}{13}$$

radius of second sphere is 6

so displacement of x & y coordinates from the centre of circle 2 is

$$(C_x + r\cos\theta,\ C_y + r\sin\theta) =$$

$$\left(-4 + 6\cdot\frac{12}{13},\ 5 + 6\cdot\frac{5}{13}\right) =$$

$$\left(\frac{20}{13},\ \frac{95}{13}\right)$$

QUESTION NUMBER

ASSESSOR'S USE ONLY

4c) $(x+1.5)^2 - 2.25 + (y+1.5)^2 - 2.25 = 8$

$(x+1.5)^2 + (y+1.5)^2 = 12.5$

$4x + 4y = 2 - xy$

$3x + 3y = \frac{3}{4}(2 - xy)$

$3x + 3y = \frac{3}{2} - \frac{3}{4}xy$

$x^2 + y^2 + \frac{3}{2} - \frac{3}{4}xy = 8$

$y(x+4) = 2 - 4x$

$y = \frac{2-4x}{x+4}$

$x^2 + \frac{(2-4x)^2}{(x+4)^2} + 3x + \frac{3(2-4x)}{x+4} = 8$

$x^2(x+4)^2 + (2-4x)^2 + (x+4)^2 3x + 3(2-4x)(x+4) = 8(x+4)^2$

$x^2(x^2+8x+16) + 16x^2 - 16x + 4 + 3x^3 + 24x^2 + 48x$

$+ 6x + 24 - 12x^2 - 48x = 8x^2 + 64x + 128$

$p(x)\ x^4 + 11x^3 + 36x^2 - 74x - 100 = 0$

~~$2 - 4x + 1.5x + 6 = 8 - 2.5x$~~

Trying integer ~~values~~ factors of 100

1 $p(1) = -126 \neq 0$

2 $\boxed{p(2) = 0}$

$\boxed{p(-1) = 0}$

$p(-2) = 120 \neq 0$

$x = 2$ $x = -1$

$(x-2)(x+1) = x^2 - x - 2$

QUESTION NUMBER | ASSESSOR'S USE ONLY

$$\begin{array}{r} x^2+12x+50 \\ x^2-x-2 \overline{\big)\, x^4+11x^3+36x^2-74x-100} \\ \underline{x^4-x^3-2x^2} \\ 12x^3+38x^2 \\ \underline{-\;12x^3-12x^2-24x} \\ 50x^2-50x-100 \\ \underline{-\;50x^2-50x-100} \end{array}$$

$x^2+12x+50=0$

$x=\dfrac{-12\pm\sqrt{12^2-50\times4}}{2}$

$=\dfrac{-12\pm\sqrt{-56}}{2}$

$=\dfrac{-12\pm2\sqrt{-14}}{2}$

$=-6\pm2\sqrt{-14}$

$=-6\pm2\sqrt{14}\,i$

$\boxed{x=2 \quad y=-1}$ $\boxed{x=-1 \quad y=2}$

$\boxed{x=-6+2\sqrt{14}i}$

$y=\dfrac{2+24-8\sqrt{14}i}{-2+2\sqrt{14}i}$

$=\dfrac{26-8\sqrt{14}i}{-2+2\sqrt{14}i}$

$=\dfrac{-52+16\sqrt{14}i+52\sqrt{14}i+224}{4+56}$

$=\dfrac{172+68\sqrt{14}i}{60}$

$\boxed{y=\dfrac{43+17\sqrt{14}i}{15}}$

$\boxed{x=-6-2\sqrt{14}i}$

$y=\dfrac{2+24+8\sqrt{14}i}{-2-2\sqrt{14}i}$

$=\dfrac{26+8\sqrt{14}i}{-2-2\sqrt{14}i}$

$=-52-16\sqrt{14}i-52\sqrt{14}i+224$

$\boxed{y=\dfrac{43-17\sqrt{14}i}{15}}$

Q4

QUESTION NUMBER		ASSESSOR'S USE ONLY
5a)	$z = x+iy \quad z^{-1} = (a+ib)^{-1} + (a+ic)^{-1}$	

$$\frac{1}{x+iy} = \frac{1}{a+ib} + \frac{\cancel{a+ic}\ 1}{a+ic}$$

$$\cancel{\frac{x-yi}{x^2+y^2} = \frac{a-bi}{a^2+b^2} + \frac{a-ci}{a^2+c^2}}$$

$$\cancel{\frac{x}{x^2+y^2} = \text{Real part}}$$

$$\frac{1}{x+iy} = \frac{a+ic+a+ib}{a^2+aic+aib-bc}$$

$$x+iy = \frac{a^2+aic+aib-bc}{2a+(b+c)i}$$

$$= \frac{(a^2+aic+aib-bc)(2a-(b+c)i)}{4a^2+(b+c)^2}$$

$$Re(x+iy) = \frac{2a^3 a-2abc+ac(b+c)+ab(b+c)}{4a^2+(b+c)^2}$$

QUESTION NUMBER

ASSESSOR'S USE ONLY

QUESTION NUMBER

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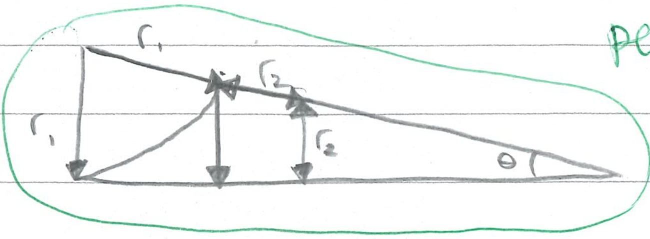
5b)i)



pencil

$$\sin\theta = \frac{20}{120} = \frac{1}{6}$$

$$\theta = 30°$$

$$r_1 - r_1 \sin\theta = r_2 + r_2 \sin\theta$$

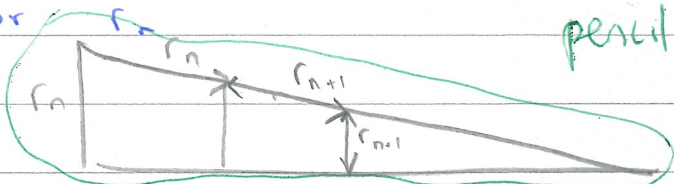
$$\frac{5}{6} r_1 = \frac{7}{6} r_2$$

$$r_2 = \frac{5}{7} \cdot 20$$

$$r_2 = \frac{100}{7} \text{ mm}$$

~~For~~

ii) For



pencil

$$r_n - r_n \cdot \frac{1}{6} = r_{n+1} + r_{n+1} \cdot \frac{1}{6}$$

$$\frac{5}{6} r_n = \frac{7}{6} r_{n+1}$$

$$r_{n+1} = \frac{5}{7} r_n \qquad r_n = \frac{5}{7}(r_{n-1})$$

$$r_n = \left(\frac{5}{7}\right)^{n-1} \cdot r_1$$

$$r_n = 20 \cdot \left(\frac{5}{7}\right)^{n-1}$$

iii)

$$V = \frac{4}{3}\pi r_n^3$$

$$= \frac{4}{3}\pi \cdot 8000 \left(\frac{125}{343}\right)^{n-1}$$

$$T_1 = \frac{32000\pi}{3} \qquad r = \left(\frac{125}{343}\right)$$

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$$S_{\infty} = \frac{32000}{\cancel{\pi}3} \cdot \pi \cdot \frac{1-\left(\frac{125}{343}\right)^{\infty}}{1-\frac{125}{343}}$$

$$\frac{32000 \cdot \pi}{3} \quad \frac{1}{\frac{218}{343}}$$

$$\frac{343 \times 32000 \times \pi}{3 \times 218} =$$

$$\frac{343 \times 16000}{3 \times 109} \times \pi =$$

$$\frac{5488000}{327} \pi =$$

$$\frac{5.488 \times 10^{6}}{327} \pi \text{ mm}^3$$

Q5

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1(c)

$\frac{(1-\sin\theta)\cos\theta}{(1-\sin\theta^2)} - \frac{\sin\theta(1-\cos\theta)}{1-\cos^2\theta} =$

$\cancel{1+\sin\theta}$

$\cancel{1+(\sin\theta\cos\theta)}$ -

$\frac{\cos\theta - \sin\theta\cos\theta}{\cos^2\theta} - \frac{\sin\theta - \sin\theta\cos\theta}{\sin^2\theta}$

$= \frac{2(\cos\theta - \sin\theta\cos\theta)}{2\cos^2\theta} - \frac{2(\sin\theta - \sin\theta\cos\theta)}{2\sin^2\theta}$

$= \frac{2(\cos\theta - \sin\theta\cos\theta)}{1+\cos 2\theta} - \frac{2(\sin\theta - \sin\theta\cos\theta)}{1-\cos 2\theta}$

$-2\cos 2\theta(\cos\theta - \sin\theta\cos\theta)$

$-2\cos 2\theta(\sin\theta - \sin\theta\cos\theta)$

$= \frac{2(\cos\theta - \sin\theta\cos\theta) - 2(\sin\theta - \sin\theta\cos\theta) \cancel{-\cos 2\theta}}{(1-\sin^2 2\theta)}$

=

$\frac{\cos\theta}{1+\sin\theta} - \frac{\sin\theta}{1+\cos\theta} =$

$1 + \sin\theta \quad 1 + \cos\theta$

$1 + \sin\theta + 2\cos\frac{1}{2}\theta^2 - 1 =$

$2\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta + 2(\cos\frac{1}{2}\theta)^2 =$

$(\sin\frac{1}{2}\theta + \cos\frac{1}{2}\theta)\, 2\cos(\frac{1}{2}\theta)$

$2(\cos(\theta) - \sin\theta) =$

$2($

QUESTION NUMBER | ASSESSOR'S USE ONLY

$\sin\theta + \cos\theta = R\sin(\theta + \alpha)$

$R^2 = 1^2 + 1^2 = 2$ $\tan\alpha = 1$

$R = \sqrt{2}$ $\alpha = 45$

$1 + \sqrt{2}\sin\left(\theta + \frac{1}{4}\pi\right)$

$\cos\theta - \sin\theta =$

$\frac{2x}{1+y} - \frac{y}{1+x} = \frac{2(x-y)}{1+x+y}$

1c LHS. $(1+\cos\theta)(1+\sin\theta)(1+\cos\theta+\sin\theta)$

$\cos\theta(1+\sin\theta)(1+\sin\theta+\cos\theta) -$

$\sin\theta(1+\cos$

$(\cos\theta(1+\cos\theta) - \sin\theta(1+\sin\theta))(1+\sin\theta+\cos\theta)$

$= (\cos\theta + \cos^2\theta)(1+\sin\theta+\cos\theta)$

$\quad - (\sin\theta + \sin^2\theta)(1+\sin\theta+\cos\theta)$

$= \cos\theta + \cos^2\theta + \cos^2\theta\sin\theta + \cos^4\theta + \cos^3\theta + \cos\theta\sin\theta$

$- \sin\theta - \sin^2\theta - \sin\theta\cos\theta$

$= \cos\theta + \cos^2\theta + \sin\theta\cos\theta + \sin\theta\cos^2\theta + \cos^2\theta + \cos^3\theta$

$- \sin\theta - \sin^2\theta - \sin^2\theta - \sin^3\theta - \sin\theta\cos\theta - \sin^2\theta\cos\theta$

$= (2\cos^2\theta - 2\sin^2\theta) + \cos\theta + \cos^3\theta - \sin\theta - \sin^3(\theta)$

$+ \sin\theta\cos^2\theta - \sin^2\theta\cos\theta$

RHS. $(1+\cos\theta)(1+\sin\theta)(1+\cos\theta+\sin\theta)$

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$2(\cos\theta - \sin\theta)(1+\sin\theta)(1+\cos\theta) =$

$2(\cos\theta - \sin\theta)(1+\sin\theta + \cos\theta + \cos\theta\sin\theta) =$

$2\cos\theta + \cancel{2\cos\theta\sin\theta} + 2\cos^2\theta + 2\cos^2\theta\sin\theta$

$- 2\sin\theta - 2\sin^2\theta - \cancel{2\sin\theta\cos\theta} - 2\cos\theta\sin^2\theta$

$= (2\cos^2\theta - 2\sin^2\theta) + 2\cos\theta + 2\cos^2\theta\sin\theta$

$\quad - 2\sin\theta - 2\cos\theta\sin^2\theta$

$= (2\cos^2\theta - 2\sin^2\theta) + 2\cos\theta + \cos^2\theta\sin\theta$

$\quad + (1-\sin^2\theta)(\sin\theta)$

$\quad - 2\sin\theta - \cos\theta\sin^2\theta$

$\quad - (1-\cos^2\theta)\left(\frac{\cos\theta}{\sin\theta}\right)$

$= (2\cos^2\theta - 2\sin^2\theta) + 2\cos\theta + \cos^2\theta\sin\theta$

$\quad + \sin\theta - \sin^3\theta$

$\quad - 2\sin\theta - \cos\theta\sin^2\theta$

$\quad - \cos\theta + \cos^3\theta$

$= (2\cos^2\theta - 2\sin^2\theta) + \cos\theta + \cos^2\theta\sin\theta + \cos^3\theta$

$\quad - \sin\theta - \sin^3\theta - \cos\theta\sin^2\theta$

$= (2\cos^2\theta - 2\sin^2\theta) + \cos\theta + \sin\theta\cos^2\theta + \cos^3\theta$

$\quad - \sin\theta - \sin^3\theta - \sin^2\theta\cos\theta$

$=$ LHS $\therefore (1+\cos\theta)(1+\sin\theta)(1+\cos\theta+\sin\theta)$

$\therefore$ LHS $\cdot (1+\cos\theta)(1+\sin\theta)(1+\cos\theta+\sin\theta)$

$= $ RHS $\cdot (1+\cos\theta)(1+\sin\theta)(1+\cos\theta+\sin\theta)$

$\therefore$ LHS $=$ RHS

Q1

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1b	Page 26 continuing 1b	

~~Maxim~~ Min

For extremes of $\cos(\angle OPA)$,

$a^2 = b^2 + c^2$ ~~in~~

where $a = |OP|$

$b = |OA|$

$c = |PA|$

$\therefore$ ~~maximised~~ minimised for $\angle OAP$ = right angle

Maximum of $\angle APO$ occurs at P is vertically above A