

WE HAVE IMPLEMENTED VALUES AS SPECIAL FORM NO SUPPORT FOR NESTED TUPLE

1.1 Stage 1: Rename bound variables

$((\text{lambda } (x1 \ y1) (\text{if } (> \ x1 \ y1) \ \#t \ \#f)) \ 8 \ 3) \Rightarrow ((\text{lambda } (x \ y) (\text{if } (> \ x \ y) \ \#t \ \#f)) \ 8 \ 3)$

Stage 2: Assign type variables for every sub expression

| Expression | Variable |
|---|------------|
| $((\text{lambda } (x \ y) (\text{if } (> \ x \ y) \ \#t \ \#f)) \ 8 \ 3)$ | T_0 |
| $(\text{lambda } (x \ y) (\text{if } (> \ x \ y) \ \#t \ \#f))$ | T_1 |
| $(\text{if } (> \ x \ y) \ \#t \ \#f)$ | T_2 |
| $(> \ x \ y)$ | T_3 |
| $>$ | $T_{>}$ |
| x | T_x |
| y | T_y |
| $\#t$ | $T_{\#t}$ |
| $\#f$ | $T_{\#f}$ |
| 8 | T_{num8} |
| 3 | T_{num3} |

Stage 3: Construct type equations. The equations for the sub-expressions are

| Expression | Equation |
|---|---|
| $((\text{lambda } (x \ y) (\text{if } (> \ x \ y) \ \#t \ \#f)) \ 8 \ 3)$ | $T_1 = [T_{num8} * T_{num3} \rightarrow T_0]$ |
| $(\text{lambda } (x \ y) (\text{if } (> \ x \ y) \ \#t \ \#f))$ | $T_1 = [T_x * T_y \rightarrow T_2]$ |
| $(\text{if } (> \ x \ y) \ \#t \ \#f)$ | $T_2 = T_{\#t} \text{ and } T_{\#t} = T_{\#f}$ |
| $(> \ x \ y)$ | $T_{>} = [T_x * T_y \rightarrow T_3]$ |
| $>$ | $T_{>} = [Number * Number \rightarrow Boolean]$ |
| $\#t$ | $T_{\#t} = Boolean$ |
| $\#f$ | $T_{\#f} = Boolean$ |
| 8 | $T_{num8} = Number$ |
| 3 | $T_{num3} = Number$ |

Stage 4: Solve the equations

| Equation | Substitution |
|--|--------------|
| 1. $T_1 = [T_{num8} * T_{num3} \rightarrow T_0]$ | $\{\}$ |
| 2. $T_1 = [T_x * T_y \rightarrow T_2]$ | |
| 3. $T_2 = T_{\#t}$ | |
| 4. $T_{\#t} = T_{\#f}$ | |
| 5. $T_{>} = [T_x * T_y \rightarrow T_3]$ | |
| 6. $T_{>} = [Number * Number \rightarrow Boolean]$ | |
| 7. $T_{\#t} = Boolean$ | |
| 8. $T_{\#f} = Boolean$ | |
| 9. $T_{num8} = Number$ | |
| 10. $T_{num3} = Number$ | |

Step1:

$$(T_1 = [T_{num8} * T_{num3} \rightarrow T_0]) \circ \text{Substitution} = T_1 = [T_{num8} * T_{num3} \rightarrow T_0])$$

$$\text{Substitution} = \text{Substitution} \circ T_1 = [T_{num8} * T_{num3} \rightarrow T_0]$$

| Equation | Substitution |
|--|--|
| 1. $T_1 = [T_x * T_y \rightarrow T_2]$ | $\{T_1 := [T_{num8} * T_{num3} \rightarrow T_0]\}$ |
| 2. $T_2 = T_{\#t}$ | |
| 3. $T_{\#t} = T_{\#f}$ | |
| 4. $T_{>} = [T_x * T_y \rightarrow T_3]$ | |
| 5. $T_{>} = [Number * Number \rightarrow Boolean]$ | |
| 6. $T_{\#t} = Boolean$ | |
| 7. $T_{\#f} = Boolean$ | |
| 8. $T_{num8} = Number$ | |
| 9. $T_{num3} = Number$ | |

Step2:

$$(T_1 = [T_x * T_y \rightarrow T_2]) \circ \text{Substitution} =$$

$$([T_{num8} * T_{num3} \rightarrow T_0] = [T_x * T_y \rightarrow T_2])$$

| Equation | Substitution |
|--|--|
| 1. $T_2 = T_{\#t}$ | $\{T_1 := [T_{num8} * T_{num3} \rightarrow T_0]\}$ |
| 2. $T_{\#t} = T_{\#f}$ | |
| 3. $T_{>} = [T_x * T_y \rightarrow T_3]$ | |
| 4. $T_{>} = [Number * Number \rightarrow Boolean]$ | |
| 5. $T_{\#t} = Boolean$ | |
| 6. $T_{\#f} = Boolean$ | |
| 7. $T_{num8} = Number$ | |

| | |
|------------------------|--|
| 8. $T_{num3} = Number$ | |
| 9. $T_{num8} = T_x$ | |
| 10. $T_{num3} = T_y$ | |
| 11. $T_0 = T_2$ | |

Step3:

$$(T_2 = T_{\#t}) \circ \text{Substitution} = (T_2 = T_{\#t})$$

$$\text{Substitution} = \text{Substitution} \circ (T_2 = T_{\#t})$$

| Equation | Substitution |
|--|--|
| 1. $T_{\#t} = T_{\#f}$ | $\{T_1 := [T_{num8} * T_{num3} \rightarrow T_0]\}$ |
| 2. $T_{>} = [T_x * T_y \rightarrow T_3]$ | $\{T_2 := T_{\#t}\}$ |
| 3. $T_{>} = [Number * Number \rightarrow Boolean]$ | |
| 4. $T_{\#t} = Boolean$ | |
| 5. $T_{\#f} = Boolean$ | |
| 6. $T_{num8} = Number$ | |
| 7. $T_{num3} = Number$ | |
| 8. $T_{num8} = T_x$ | |
| 9. $T_{num3} = T_y$ | |
| 10. $T_0 = T_2$ | |

Step4:

$$(T_{\#t} = T_{\#f}) \circ \text{Substitution} = (T_{\#t} = T_{\#f})$$

$$\text{Substitution} = \text{Substitution} \circ (T_{\#t} = T_{\#f})$$

| Equation | Substitution |
|--|--|
| 1. $T_{>} = [T_x * T_y \rightarrow T_3]$ | $\{T_1 := [T_{num8} * T_{num3} \rightarrow T_0]\}$ |
| 2. $T_{>} = [Number * Number \rightarrow Boolean]$ | $\{T_2 := T_{\#t}\}$ |
| 3. $T_{\#t} = Boolean$ | $\{T_{\#t} := T_{\#f}\}$ |
| 4. $T_{\#f} = Boolean$ | |
| 5. $T_{num8} = Number$ | |
| 6. $T_{num3} = Number$ | |
| 7. $T_{num8} = T_x$ | |
| 8. $T_{num3} = T_y$ | |
| 9. $T_0 = T_2$ | |

Step 5:

$$T_{>} = [T_x * T_y \rightarrow T_3] \circ \text{Substitution} = (T_{>} = [T_x * T_y \rightarrow T_3]),$$

$$\text{Substitution} = \text{Substitution} \circ (T_{>} = [T_x * T_y \rightarrow T_3])$$

| Equation | Substitution |
|--|--|
| 1. $T_{>} = [Number * Number \rightarrow Boolean]$ | $\{T_1 := [T_{num8} * T_{num3} \rightarrow T_0]\}$ |
| 2. $T_{\#t} = Boolean$ | $\{T_2 := T_{\#t}\}$ |
| 3. $T_{\#f} = Boolean$ | $\{T_{\#t} := T_{\#f}\}$ |
| 4. $T_{num8} = Number$ | $\{T_{>} = [T_x * T_y \rightarrow T_3]\}$ |
| 5. $T_{num3} = Number$ | |
| 6. $T_{num8} = T_x$ | |
| 7. $T_{num3} = T_y$ | |
| 8. $T_0 = T_2$ | |

Step 6:

$$(T_{>} = [Number * Number \rightarrow Boolean]) \circ \text{Substitution} = T_{>}$$

$$= [T_x * T_y \rightarrow T_3] = [Number * Number \rightarrow Boolean]$$

| Equation | Substitution |
|------------------------|--|
| 1. $T_{\#t} = Boolean$ | $\{T_1 := [T_{num8} * T_{num3} \rightarrow T_0]\}$ |
| 2. $T_{\#f} = Boolean$ | $\{T_2 := T_{\#t}\}$ |
| 3. $T_{num8} = Number$ | $\{T_{\#t} := T_{\#f}\}$ |
| 4. $T_{num3} = Number$ | $\{T_{>} = [T_x * T_y \rightarrow T_3]\}$ |
| 5. $T_{num8} = T_x$ | |
| 6. $T_{num3} = T_y$ | |
| 7. $T_0 = T_2$ | |
| 8. $T_x = Number$ | |
| 9. $T_y = Number$ | |
| 10. $T_3 = Boolean$ | |

Step 7:

$$T_{\#t} = Boolean \circ \text{Substitution} = (T_{\#t} = Boolean)$$

$$\text{Substitution} = \text{Substitution} \circ (T_{\#t} = Boolean)$$

| Equation | Substitution |
|------------------------|--|
| 1. $T_{\#f} = Boolean$ | $\{T_1 := [T_{num8} * T_{num3} \rightarrow T_0]\}$ |
| 2. $T_{num8} = Number$ | $\{T_2 := Boolean\}$ |
| 3. $T_{num3} = Number$ | $\{T_{\#t} := T_{\#f}\}$ |
| 4. $T_{num8} = T_x$ | $\{T_{>} = [T_x * T_y \rightarrow T_3]\}$ |
| 5. $T_{num3} = T_y$ | $\{T_{\#t} = Boolean\}$ |
| 6. $T_0 = T_2$ | $\{T_{\#f} = Boolean\}$ |
| 7. $T_x = Number$ | |
| 8. $T_y = Number$ | |
| 9. $T_3 = Boolean$ | |

Step 8:

$$(T_{\#f} = Boolean \circ \text{Substitution} = (Boolean = Boolean))$$

| Equation | Substitution |
|------------------------|--|
| 1. $T_{num8} = Number$ | $\{T_1 := [T_{num8} * T_{num3} \rightarrow T_0]\}$ |
| 2. $T_{num3} = Number$ | $\{T_2 := Boolean\}$ |
| 3. $T_{num8} = T_x$ | $\{T_{\#t} := T_{\#f}\}$ |
| 4. $T_{num3} = T_y$ | $\{T_{>} = [T_x * T_y \rightarrow T_3]\}$ |
| 5. $T_0 = T_2$ | $\{T_{\#f} = Boolean\}$ |
| 6. $T_x = Number$ | $T_{\#t} = Boolean\}$ |
| 7. $T_y = Number$ | |
| 8. $T_3 = Boolean$ | |

Step 9:

$$(T_{num8} = Number) \circ \text{Substitution} = (T_{num8} = Number)$$

$$\text{Substitution} = \text{Substitution} \circ (T_{num8} = Number)$$

$$(T_{num3} = Number) \circ \text{Substitution} = (T_{num3} = Number)$$

$$\text{Substitution} = \text{Substitution} \circ (T_{num3} = Number)$$

| Equation | Substitution |
|---------------------|--|
| 1. $T_{num8} = T_x$ | $\{T_1 := [Number * Number \rightarrow T_0]\}$ |
| 2. $T_{num3} = T_y$ | $\{T_2 := Boolean\}$ |
| 3. $T_0 = T_2$ | $\{T_{\#t} := T_{\#f}\}$ |
| 4. $T_x = Number$ | $\{T_{>} = [T_x * T_y \rightarrow T_3]\}$ |
| 5. $T_y = Number$ | $\{T_{\#f} = Boolean\}$ |
| 6. $T_3 = Boolean$ | $\{T_{\#t} = Boolean\}$ |
| 7. | $\{T_{num8} = Number\}$ |
| 8. | $\{T_{num3} = Number\}$ |

Step 10:

$$(T_{num8} = T_x) \circ \text{Substitution} = (T_x = Number)$$

$$\text{Substitution} = \text{Substitution} \circ (T_x = Number)$$

$$(T_{num3} = T_y) \circ \text{Substitution} = (T_y = Number)$$

$$\text{Substitution} = \text{Substitution} \circ (T_y = Number)$$

| Equation | Substitution |
|--------------------|--|
| 1. $T_0 = T_2$ | $\{T_1 := [T_{num8} * T_{num3} \rightarrow T_0]\}$ |
| 2. $T_x = Number$ | $\{T_2 := Boolean\}$ |
| 3. $T_y = Number$ | $\{T_{\#t} := T_{\#f}\}$ |
| 4. $T_3 = Boolean$ | $\{T_{>} = [Number * Number \rightarrow T_3]\}$ |
| 5. | $\{T_{\#t} = Boolean\}$ |
| 6. | $\{T_{\#f} = Boolean\}$ |
| 7. | $\{T_{num8} = Number\}$ |
| 8. | $\{T_{num3} = Number\}$ |
| 9. | $\{T_x = Number\}$ |
| 10. | $\{T_y = Number\}$ |

Step 13:

$$(T_0 = T_2) \circ \text{Substitution} = (T_0 = \textbf{Boolean})$$

1.2 Are these typing statements true? Explain.

a. $\{f: [T_1 \rightarrow T_2], x: T_1\} \vdash (f\ x): T_2$

The statement is **true**.

$\Rightarrow f$ is function from T_1 to T_2 and $x \in T_1$ so $f(x) \in T_2$

b. $\{f: [T_1 \rightarrow T_2], g: [T_2 \rightarrow T_3], x: T_2\} \vdash (f\ g\ x): T_3$

The statement is **false**.

$\Rightarrow f$ expected **one** argument of type T_1 but received **two** arguments.

c. $\{f: [T_2 \rightarrow T_1], g: [T_1 \rightarrow T_2], x: T_1\} \vdash (f\ (g\ x)): T_1$

The statement is **true**.

$\Rightarrow g$ is function from T_1 to T_2 , and $x \in T_1$.

In addition f is function from T_2 to T_1 as necessary.

d. $\{f: [T_2 \rightarrow \text{Number}], x: \text{Number}\} \vdash (f\ x\ x): \text{Number}$

The statement is **false**;

$\Rightarrow f$ expected **one** argument of type T_2 but received **two** arguments of type Number .

1.3 What is the type of the following primitive operators:

a. $\text{cons} = [T_1 * T_2 \rightarrow \text{Pair}(T_1, T_2)]$

b. $\text{car} = [\text{Pair}(T_1, T_2) \rightarrow T_1]$

c. $\text{cdr} = [\text{Pair}(T_1, T_2) \rightarrow T_2]$

1.4 Write the type of the following function: (Define f (lambda (x) (values x x x)))
(see question 2 for the definition of values).

$\Rightarrow [T_1 \rightarrow [T_1 * T_1 * T_1]]$

1.5 Write the MGU of the following expressions, or state that there is no such MGU.

| Substitution | MGU |
|--|---|
| $\{x: T_1\}, \{x: T_2\}$ | $\{T_1 = T_2\}$ |
| $\{x: \text{Number}\}, \{x: \text{Number}\}$ | $\{\}$ |
| $\{x: [T_1 * [T_1 \rightarrow T_2] \rightarrow \text{Number}]\}$ $\{x: [[T_3 \rightarrow \text{Number}] * [T_4 \rightarrow \text{Number}] \rightarrow \text{Number}]\}$ | $\{T_1 = T_4 = [T_3 \rightarrow \text{Number}],$ $T_2 = \text{Number}\}$ |
| $\{x: [T_1 \rightarrow T_1]\}$ $\{x: [T_1 \rightarrow [\text{Number} * \text{Number}]]\}$ | $\{T_1 = [\text{Number} * \text{Number}]\}$ |

2.3 Write the fully type-annotated version of this function:

- a. (define f (lambda (x) (values x (+ x 1))))

```
(lambda ((x : number)) : (number * number) (values x (+ x 1)))
```

- b. (define g (lambda (x) (values "x" x)))

```
(lambda ((x : T)) : (string * T) (values "x" x))
```

4.2 What are the benefits of the promise interface compared to the callback interface

- a. Dealing with errors is simpler with the use of "catch" mechanism instead of using different cases with callbacks.
- b. The code is more readable because of the ability to concat promises instead of doing so in a nested way when working with callbacks.

Part 3 code:

```
function* braid(gen1: Generator, gen2: Generator) {

    let iter1 = gen1.next();
    let iter2 = gen2.next();

    while(iter1.value!=undefined || iter2.value!=undefined){
        if(iter1.value!=undefined){
            yield iter1.value;
            iter1 = gen1.next();
        }
        if(iter2.value!=undefined){
            yield iter2.value;
            iter2 = gen2.next();
        }
    }
}

function* biased(gen1: Generator, gen2: Generator) {
    let iter1a = gen1.next();
    let iter1b = gen1.next();
    let iter2 = gen2.next();

    while(iter1a.value!=undefined || iter2.value!=undefined){
        if(iter1a.value!=undefined){
            yield iter1a.value;
            iter1a = gen1.next();
        }
        if(iter1b.value!=undefined){
            yield iter1b.value;
            iter1b = gen1.next();
        }
        if(iter2.value!=undefined){
            yield iter2.value;
            iter2 = gen2.next();
        }
    }
}
```

Part 4 code:

```
export const f = (x: number): Promise<number> => {
  return new Promise (
    (resolve, reject) => {
      if (x!= 0) resolve(1/x);
      else reject("CAN NOT DIVIDE BY ZERO!");
    });
}

export const g = (x: number): Promise<number> => {
  return new Promise (
    (resolve, reject) => {
      if(true) resolve(x*x);
      else reject("NOT POSSIBLE!");
    });
}

export const h = (x: number): Promise<number> => {
  return g(x)
    .then((c) => {return f(c)})
    .catch();
}

export const slower = <T1, T2>(arr: [Promise<T1>, Promise<T2>]): Promise<string> => {
  return new Promise(
    (resolve, reject) => {
      const f=(i:number)=>(value: T1 | T2)=>{
        done++;
        if(done===2){
          resolve(`${i} , '${value}'`)
        }
      }
      let done = 0;
      arr[0].then(f(0)).catch(() => reject("0 FAILED!"))
      arr[1].then(f(1)).catch(() => reject("1 FAILED!"))
    });
}
```

