Exercise 3
Due: 24/12/2020

1. Given a graph $G = (V, E)$ with weights $w : E \to \mathbb{R}_+$, define for a spanning tree T the value $\lambda(T) = \max_{e \in T}\{w(e)\}$ (the maximal edge weight in T).
   (a) Prove that the MST of G has minimal value $\lambda(MST)$ among all spanning trees.
   (b) Devise a deterministic linear time algorithm for finding a spanning tree T with minimal $\lambda(T)$.
2. Show that there exists a linear time deterministic algorithm for MST in planar graphs.
3. Given a graph G with n vertices and m edges, show an $O(n + m)$ time algorithm that determines whether G has P_3 as an induced subgraph (recall P_3 is the path on 3 vertices).
4. Given a weighted directed graph $G = (V, E)$ with n vertices, and $0 < \epsilon < 1$, devise a randomized $O(n^{3-\epsilon} \log n)$ time algorithm, that computes a value $\hat{d}(u, v)$ for every $u, v \in V$, so that with high probability: For every pair u, v, if they have a shortest path that contains at least n^ϵ edges, then $\hat{d}(u, v) = d(u, v)$.
5. **Bonus** (3 points to course grade): Given a graph $G = (V, E)$ with $m = |E|$ edges, show a $O(m^{4/3})$ time algorithm to decide if G contains a C_4 (not necessarily induced).