Fifth Assignment in "Introduction to Numerical Analysis", Fall Sem. 2020

1) Finite Difference

Solve the Model Problem $\frac{dy}{dx} = x + y;\ y(0) = 1$ using Euler's Method (EM) with $h = 0.1$ and the 2nd order Runge-Kutta (RK2) with $\lambda = 2/3$, $h = 0.1$.

(a) Compare the solution with the exact solution $y = 2e^x - x - 1$ at x values between 0 and 1, with a step-size of 0.1 (i.e., x=0; x=0.1; x=0.2; … ;x=0.8; x=0.9; x=1.0).
(b) Compare between RK2 and EM.
(c) Count the number of times $f(x, y)$ was evaluated in both methods.
(d) How many time $f(x, y)$ is called with Euler's method with $h = 0.05$.
(e) Compare both methods for accuracy and efficiency: to be fair to both methods, adjust the value of h in EM so that both EM and RK2 use the same number of function evaluations. Now, compare the accuracy between the two methods.

2) Finite Element

Solve the Model Problem $\frac{d^2y}{dx^2} - \left(1 - \frac{x}{5}\right)y = x\ ;\ y(1) = 2\ ;\ y(3) = -1$ using the Rayleigh-Ritz method. Approximate $y(x)$ with $u(x) = \sum_{i=1}^{5} c_i\, N_i(x)$ where $N_i(x)$ are the triangular basis functions defined in class. Using the variational method find the coefficients C_1, C_2, C_3, C_4, C_5 (the first and last are given as the boundary conditions).

3) *Numerical differentiation*

(a) Determine as accurately as possible approximations for each missing entry in the following table. (hint: employ the formulas for numerical differentiation)

x	$f(x)$	$f'(x)$
-3.0	9.3678	
-2.8	8.2332	
-2.6	7.1803	
-2.4	6.2093	
-2.2	5.3203	
-2.0	4.5134	

(b) Given that $f(x) = e^{x/3} + x^2$ find the bound for the error in each case.

4) *Numerical integration*

(a) Use the Composite Trapezoidal and Composite Simpson's rules to approximate the following integral (the exact value is 19.9209…):

$$\int_{-2}^{2} x^3 e^x dx \qquad n = 4$$

(b) Determine the values of n and h required to approximate:

$$\int_{0}^{2} \frac{1}{x+4} dx \qquad within\ 10^{-5}$$

Good luck.