

**Assignment 4 - Linear Algebra**

**Question 1.**

Show that:

$$\|x - y\| \geq |||x|| - \|y||; \quad x, y \in R^n$$

**Question 2.**

Show that:

$$\|AB\| \leq \|A\|\|B\|; \quad A, B \in R^n \times R^n$$

**Question 3.**

For the linear system  $Ax = b$  given by  $\begin{pmatrix} 9.7 & 6.6 \\ 4.1 & 2.8 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 9.7 \\ 4.1 \end{pmatrix}$  estimate the  $\text{cond}A$ .

**Question 4.**

Solve the linear system: 
$$\begin{cases} 4x_1 - x_2 + x_3 = 7 \\ 4x_1 - 8x_2 + x_3 = -21 \\ -2x_1 + x_2 + 5x_3 = 15 \end{cases}$$

using:

- **Jacobi**
- **Gauss Seidel**

Starting in both from the initial guess:  $x^0 = (1, 2, 2)$

**Question 5.**

The values  $x_1 = x_2 = 1.000$  are the solutions to: 
$$\begin{cases} 1.133x_1 + 5.281x_2 = 6.414 \\ 24.14x_1 - 1.210x_2 = 22.93 \end{cases}$$

- Use four-digit arithmetic and **Gaussian Elimination** without pivoting to find a computed approximate solution to the system.
- Same as above but use partial pivoting.

**Question 6.**

Use the power method with 9 iterations to locate an eigenvalue and eigenvector for the

matrix:  $\begin{pmatrix} 5 & -1 & 7 \\ -1 & -1 & 1 \\ 7 & 1 & 5 \end{pmatrix}$ . Check with Python/MATLAB.

**Good luck.**