JEAN-LOUIS L. ARCAND

University of Montreal Montreal, Quebec, Canada

ELISE S. BREZIS

Hebrew University of Jerusalem Jerusalem, Israel

Disequilibrium Dynamics during the Great Depression*

We test the flexibility of wages and prices in the U.S. before World War II using a simple two-market disequilibrium model. We test the model for four different tatônnement adjustment mechanisms and we find that the equilibrium restriction is strongly rejected in all cases. Hausman specification tests reject the equilibrium restriction but do not reject three of the disequilibrium specifications. Parameter estimates imply that the persistence of the Great Depression is not attributable to nominal rigidities but was caused by the system becoming dynamically neutral. We compute estimates of excess aggregate demand from 1892 to 1940 and find that a model in which adjustment obtains in prices in the goods market and in quantities in the labor market provides the best description of the data.

[I]t is an outstanding characteristic of the economic system in which we live that, whilst it is subject to severe fluctuations in respect of output and employment, it is not violently unstable. Indeed, it seems capable of remaining in a chronic condition of sub-normal activity for a considerable period without any marked tendency either towards recovery or towards complete collapse. (Keynes 1964, 249).

1. Introduction

The causes underlying the Great Depression of the 1930s still remain a subject of disagreement among economists. The stylized facts are clear: the magnitude of the collapse is illustrated by Table 1. Real GNP fell by 30% between 1929 and 1933. Consumption expenditures fell by 20%, and investment almost ceased. Accom-

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TABLE 1.

1937

1938

1939

1940

1240.6

1187.0

1279.3

1376.6

| Year | Real GNP | Consumption | Investment | M2 | CPI | Index |
|------|----------|-------------|------------|-----|-----|-------|
| 1928 | 1184.9 | 844.9 | 183.3 | 100 | 100 | 100 |
| 1929 | 1262.8 | 860.0 | 223.4 | 100 | 100 | 100 |
| 1930 | 1142.1 | 798.2 | 154.0 | 98 | 98 | 100 |
| 1931 | 1053.8 | 766.8 | 95.0 | 90 | 90 | 100 |
| 1932 | 908.2 | 695.2 | 32.0 | 76 | 80 | 100 |
| 1933 | 888.5 | 682.7 | 32.0 | 67 | 76 | 111 |
| 1934 | 956.4 | 707.0 | 52.9 | 73 | 78 | 122 |
| 1935 | 1040.2 | 750.5 | 96.7 | 84 | 80 | 122 |
| 1936 | 1182.2 | 825.6 | 127.5 | 94 | 82 | 122 |

159.9

96.4

133.5

179.1

Real

Indicies: 1929 = 100

84

82

82

82

Nominal

98

98

106

119

Real Wage

133

144

144

156

Billions of 1972 dollars

Real

854.5

834.8

879.8

919.0

panying this fall in real variables was a fall in the stock of money (M2) of 33%. Prices (CPI) fell by 24%.

The reasons for the severity of the Great Depression are not self-evident and the relations of causality linking the various variables lie at the core of the debate. Among all the variables which experienced sharp falls, which one can be pinned with the blame for being the impulse which led to the collapse? Monetarists choose to underline the money supply, while Keynesians push for consumption and investment. All schools of thought assume some sequence of exogenous negative shocks. The same question is asked about the ending of the Depression: was it the New Deal, World War II, or some other factor which led the U.S. economy out of the depths to which it had sunk?

Whatever one's view may be, it may be that it is impossible to isolate empirically those causal factors which led the economy into the Depression, and those which led it out. This is because whatever is assumed to be exogenous will invariably be found to be the "cause." As Temin has asserted:

The pursuit of exogenous variables whose movements can be said to have caused the Depression therefore must be judged unsuccessful at this time. The exogenous variables that have been proposed to fulfill such a role in econometric models either cannot be taken seriously as explanations of the Depression because their use is too arbitrary or they are better thought of as endogenous in a more completely specified model. . . . There are ways to view econometric models other than as guides to exogenous variables. These models all describe behavior over time, and there may be aspects of the dynamic behavior itself that will help us to understand the Depression (Temin 1976, 50, italics added).

Besides, it may not even be important to find the actual trigger. Many recessions begin and end, but none persisted as long as the Great Depression, and none has been as severe. Temin has underlined that "our interest lies in explaining why the Great Depression was different from other economic downturns" (Temin 1976, 62), which is why we try in this paper to analyze—not the exogenous factors which may or may not have contributed to the Depression—but instead, how the dynamics may have been different.

A view often taken by macroeconomists is that the long persistence of the Depression was caused by the sluggish adjustment of wages and prices. A corollary of this is the widely held view that wage and price flexibility are stabilizing. This last point is important because it highlights that the theoretical models underlying most explanations for the duration of the Depression are dynamically stable, meaning that the dynamics of the system tend to bring the economy back towards full employment. The sluggishness of wage and price adjustment ensures that the stable dynamics of the system do not bring the economy back to full employment instantaneously. The assumption of dynamic stability also creates the need for the sequence of exogenous negative shocks: whatever is propelling the economy down towards the abyss of 1933 must be powerful enough to overwhelm the equilibrating forces of the macrosystem.

One possible alternative is that the dynamics of the system may not have been stable, with the consequence that there was no tendency for the economy to converge back towards equilibrium following a negative shock. Instead, the economy would tend to remain in the depressed state into which it had been thrust. In order to investigate such a possibility, this paper sets out to estimate a simple AD-AS model where the "action" is not to be found in the chosen set of exogenous factors, and where the dynamics can be explicitly examined in order to determine whether some pathological condition lies behind the behavior of the U.S. economy in the interwar years.

The most appropriate method of investigation in this case is clearly a disequilibrium macro model. Previous empirical work on dynamic aspects of the Great Depression has chosen to operate within the edifice of equilibrium macroeconomics—"equilibrium" in the naïve sense that, either explicitly or implicitly, demand equals supply.² The problem with this assumption is that if the economy is not at a point where demand equals supply, then parameter estimates will be inconsistent.³ If there is any historical episode for which this edifice crumbles, if there is any period during which an

¹The notable exceptions to this view being Tobin (1975) and Delong and Summers (1986). An excellent survey of the labor market in the 1930s is provided by Baily (1983). Temin (1989) provides an alternative explanation based on the operation of the Gold Standard.

²An exception is the work of Smyth (1983) and of Holden and Peel (1986). Barro and Grossman (1971) provide an early example of the disequilibrium approach.

³This stems from the fact that assuming equilibrium constrains certain parameters to equal zero. If the assumption is false a standard omitted variables problem leads to inconsistency.

equilibrium weltanschauung is out of place, surely it must be the Great Depression. As Baily argues in his well-known analysis of the U.S. labor market during the 1930s: "explaining the decline in employment between 1929 and 1932 or 1933 within a model of competitive market-clearing is extremely hard" (Bailey 1983, 43). Tobin (1975), for his part, has underlined that Keynes's "equilibrium with involuntary unemployment" does not mean a static equilibrium but rather "the possibility of a protracted unemployment which the natural adjustment of a market economy remedy very slowly. . . . The phenomena he [Keynes] described are better regarded as disequilibrium dynamics."

Two advantages of a disequilibrium econometric approach are (i) that it provides one with consistent parameter estimates of the AD-AS system without imposing the doubtful null hypothesis that observations correspond to points of intersection of demand and supply and (ii) that it allows one to investigate the degree of rigidity present in the adjustment process. The adjustment process can take different forms. One possible assumption is that if at a given price, demand exceeds supply, then prices will rise. This is the Walrasian tatônnement mechanism. Marshallian dynamics correspond to a situation where quantities adjust to a discrepancy between "demand price" and "supply price." We have no a priori knowledge as to whether adjustment obtains in prices or in quantities; we will therefore allow for both Walrasian and Marshallian points of view.

This paper presents a simple disequilibrium macro-econometric model of the goods and labor markets. Our paper deals with the entire 1890–1940 period but pays particular attention to the dynamics of the interwar years. We focus on the following question: were the dynamics of the U.S. macro-system during the interwar years not dynamically stable? The framework of the paper is the

⁴For more details, see Samuelson (1983, 264), and Tobin (1975, 196). Samuelson points out that the term "Marshallian" is an historical error, since Marshall defined his adjustment in a manner similar to Walras. Note that the tatônnement adjustment mechanisms which we shall be using in this paper of necessity involve the variable on one axis being held constant while the other variable adjusts to bring about equilibrium. For this reason, it is impossible to specify a Bowden adjustment process in which both price and quantity adjust simultaneously. The reader will readily convince herself with a few lines of algebra that including both price and quantity adjustment simultaneously in the context of tatônnement will result in a single equation involving both changes in prices and in quantities—one will not be able to identify all parameters of interest. For a theoretical approach which might lead to simultaneous price/quantity adjustment, see Sondermann (1985).

following: in Part 2 we set up the basic disequilibrium framework for the goods and labor markets and sketch the four adjustment mechanisms we will consider. In Part 3 we present the estimation results, discuss the results of various statistical tests, and map out the dynamics our results yield for the interwar period. We also present a number of simulations based on our parameter estimates, as well as computed values for excess aggregate demand. We use these simulations to discuss and illustrate our argument regarding the dynamics of the U.S. macrosystem. Part 4 summarizes the principal empirical findings and offers some concluding remarks.

2. The Model

The model we construct and test revolves around the goods and labor markets. Aggregate supply is derived from labor demand and is therefore different from the standard specification which implicitly assumes a clearing labor market (see, for example, Dornbusch and Fischer 1987). The two markets may therefore be reduced to three structural equations: aggregate demand, aggregate supply, and labor supply.

The four disequilibrium models we present in this paper are all based on these three structural relationships. Where the models differ is in the dynamic process by which the macro-system is assumed to adjust. Because of its familiarity, we begin with a Walrasian model in which adjustment obtains through wages and prices: the PW model. At the other extreme, the economy may be "Marshallian" in that adjustment obtains through output and employment: this will constitute the YL model. We also consider two mixed models in which one market is Walrasian, the other Marshallian.⁵

Price and Wage Adjustment: The PW Model

Our initial specification is very similar to that of Smyth (1983) and clearly falls into the category of Quandt's Model C (Quandt 1988), which constitutes one of the standard methods of disequilibrium estimation.⁶ In the labor market, the model posits an explicit adjustment mechanism—sometimes referred to as a Bowden pro-

⁵Note that we do not provide explicit micro-foundations for these adjustment equations, which are simply tatônnement mechanisms familiar from general equilibrium theory.

⁶See Quandt (1988, ch. 2, especially p. 22) for the statement of the canonical form. Our original inspiration came from Rosen and Quandt (1986).

cess—in which nominal wages adjust to bring the labor market back towards a situation in which labor demand equals labor supply.

In this paper, we have chosen to keep our specification extremely simple so as to be able to interpret our results in the context of the dynamics of aggregate excess demand functions. An alternative to our simple aggregate relationships, particularly with regards to the labor market, would have been (i) a specification in the Muellbauer-Hajivassiliou tradition, which takes explicit account of aggregation issues, or (ii) a partial adjustment specification (à la Sarantis 1981 or Briguglio 1984), although the min approach we adopt is now the preferred form (see Quandt 1988, 109–32 for a good summary of the alternative specifications). A more complex error structure in the adjustment process is also easily handled in the context of this type of model (see, for example, Sneessens, 1981, 1983).⁷

The basic disequilibrium framework in the labor market is given by a labor demand function, a labor supply function, the short-side principle, and a pair of nominal wage adjustment equations:

$$L_{D,t} = \beta_0 + \beta_1(W_t - P_t) + \beta_2 Y_t + \epsilon_{D,t};$$
 (1)

$$L_{S,t} = \gamma_0 + \gamma_1(W_t - C_t) + \gamma_2 N_t + \epsilon_{S,t}; \qquad (2)$$

$$L_t = \min(L_{D,t}, L_{S,t});$$
 (3)

$$\dot{W}_t \approx W_t - W_{t-1} = \zeta_1 (L_{D,t} - L_{S,t})$$
 for $L_{D,t} < L_{S,t}$,

$$\dot{W}_t \approx W_t - W_{t-1} = \zeta_2(L_{D,t} - L_{S,t}) \quad \text{otherwise} , \qquad (4)$$

where $L_{D,t}$ is labor demand in period t; $L_{S,t}$ is labor supply; L_t is observed employment; W_t is the nominal wage rate; P_t is a measure of wholesale prices; C_t is a measure of consumer prices; Y_t is output; N_t is population; $\epsilon_{D,t}$ and $\epsilon_{S,t}$ are disturbance terms which are assumed to satisfy the usual assumptions. Note that Equation (1) represents the FOCs stemming from a CES aggregate production function, but that we do not impose the (non-linear) restriction implied by this specific functional form on the coefficients β_t .

 $^{^7\}mathrm{For}$ a different formulation without adjustment functions, see Artus, Laroque and Michel (1984).

⁸We consider the serial correlation issue below.

The basic problem to be solved in disequilibrium estimation is that the left-hand-side variables in Equations (1) and (2) are unobservable, and these equations must be somehow transformed into relationships among observable variables.

Note that $L_{D,t}-L_{S,t}=(\beta_1-\gamma_1)(W_t-W_t^*)$, where W_t^* is the equilibrium wage $(W_t^*$ such that $L_{D,t}=L_{S,t}$). Substituting into (4) yields

$$W_{t} - W_{t-1} = \zeta_{1}(\beta_{1} - \gamma_{1})(W_{t} - W_{t}^{*}), \qquad (5)$$

for downward adjustment and similarly for upward adjustment. Simple manipulations then yield

$$W_t - W_{t-1} = (1 - \mu_1)(W_t^* - W_{t-1})$$
 for $W_t < W_{t-1}$, $W_t - W_{t-1} = (1 - \mu_2)(W_t^* - W_{t-1})$ otherwise, where
$$1 - \mu_1 \equiv \frac{\zeta_1(\gamma_1 - \beta_1)}{1 + \zeta_1(\gamma_1 - \beta_1)}$$

and

$$1 - \mu_2 \equiv \frac{\zeta_2(\gamma_1 - \beta_1)}{1 + \zeta_2(\gamma_1 - \beta_1)},\tag{6}$$

or equivalently

$$\frac{1}{\zeta_1} \equiv \frac{\mu_1}{1-\mu_1} (\gamma_1 - \beta_1)$$

and

$$\frac{1}{\zeta_{o}} \equiv \frac{\mu_{2}}{1-\mu_{o}} \left(\gamma_{1}-\beta_{l}\right) \, . \label{eq:constraint}$$

Define

$$\Delta W_{t}^{-} = \begin{cases} \Delta W_{t} & \text{if } \Delta W_{t} < 0\\ 0 & \text{otherwise} \end{cases}$$

and

$$\Delta W_t^+ = \begin{cases} \Delta W_t & \text{if } \Delta W_t > 0 \\ 0 & \text{otherwise} \end{cases}.$$

Then Equations (3) and (4) allow one to write

$$L_t = L_{S,t} + \frac{1}{\zeta_1} \Delta W_t^-$$

and

$$L_t = L_{D,t} - \frac{1}{\zeta_2} \Delta W_t^+ .$$

Combining these last two expressions with Equations (1) and (2) yields the following equations in observable variables:

$$L_{t} = \beta_{0} + \beta_{1}(W_{t} - P_{t}) + \beta_{2}Y_{t} + \left[\frac{\beta_{1} - \gamma_{1}}{1 - \mu_{2}} \mu_{2}\right] \Delta W^{+} + \epsilon_{D,t}; \quad (7)$$

$$L_{t} = \gamma_{0} + \gamma_{1}(W_{t} - C_{t}) + \gamma_{2}N_{t} - \left[\frac{\beta_{1} - \gamma_{1}}{1 - \mu_{1}}\mu_{1}\right] \Delta W^{-} + \epsilon_{S,t}.$$
 (8)

As in Rosen and Quandt (1986), we will augment Equation (7) by Y_{t-1} and Equation (8) by $(W_{t-1} - C_{t-1})^9$

The goods market may be handled in largely the same manner. Aggregate demand is given by the usual solution to an IS-LM system in which the nominal interest rate enters the LM equation, while the real interest rate enters the IS equation. In order to keep things simple, we use the standard linear analytical form given in such texts as Dornbusch and Fischer (1987, 535):

$$Y_{D,t} = \delta_0 + \delta_1(M_t - P_t) + \delta_2(E_t P_{t+1} - P_t) + u_t ,$$

where M_t is the money supply, in logarithms, and $E_t P_{t+1}$ is the ex-

⁹The first modification is a consequence of costly adjustment; the second is a consequence of some simple expectational models. See Rosen and Quandt (1986) for a more detailed discussion.

pected price level, also expressed in logarithms. Aggregate demand is thus a function of expected inflation and the real money supply.

The usual aggregate supply specification used in equilibrium analysis cannot be used here because there may be binding constraints on one side of the labor market. Benassy has termed the usual aggregate supply equation "notional supply." (See Benassy 1982.) Aggregate supply in our model will differ from notional supply and we will call it—to use standard disequilibrium terminology—"effective aggregate supply." Realized employment, L_t , will constrain the representative firm's behavior. From Equation (7) one may therefore recover the effective aggregate supply equation:

$$Y_{S,t} = -\frac{\beta_0}{\beta_2} + \frac{1}{\beta_2} L_t - \frac{\beta_1}{\beta_2} (W_t - P_t) - \left[\frac{\mu_2}{1 - \mu_2} \right] \left[\frac{\beta_1 - \gamma_1}{\beta_2} \right] \Delta W_t^+ - \frac{\epsilon_{D,t}}{\beta_2}.$$
 (7')

Note that Equation (7) is not a conventional aggregate supply relationship in that L_t has not been substituted out on the R. H. S. ¹⁰ Our treatment of the supply-side will of necessity take on two forms because of our need to estimate the rigidity parameters as well as the "slopes" of aggregate demand and supply.

Suppose the aggregate production function is given by Y = f(L, t, ...), which takes the simple form $Y_t = \alpha_1 L_t + \alpha_2 t$, as in Dornbusch and Fischer (1987, 477). Then aggregate supply may be obtained by substituting (7) into the production function and solving for Y:

$$Y_{S,t} = \frac{\alpha_1 \beta_1}{1 - \alpha_1 \beta_2} (W_t - P_t) + \frac{\alpha_1}{1 - \alpha_1 \beta_2} \left[\frac{\beta_1 - \gamma_1}{1 - \mu_2} \mu_2 \Delta W_t^+ + \epsilon_{D,t} + \beta_0 \right] + \frac{\alpha_2}{1 - \alpha_1 \beta_2} t.$$
 (7")

Strictly speaking, we should substitute (7) into a CES aggregate

¹⁰Equation (7) allows us to estimate the rigidity parameters.

¹¹Note also that having a single factor of production (L) is a modeling strategy also adopted in a different context by Artus, Laroque and Michel (1984).

production function given that Equation (1), labor demand, was derived as the FOC for a CES function. This, however, will lead to a non-linear expression which cannot be linearized in L and which will thus not be amenable to the simple adjustment dynamics of the Bowden process. Note that we did not impose the conditions on the parameters of (1) implied by the CES functional form, and one may thus choose to think of (1) and (7") as linear approximations to some constant returns aggregate production function. Given the high degree of aggregation of such macro-modeling, and the potential heterogeneity of production processes underlying aggregate supply, we would argue that the imposition of a specific aggregate functional form is not warranted.

The steps leading from unobservable latent variables to estimating equations in observable variables are then the same as for the labor market (whether we use [7'] or [7"]). Write the short-side principle and the price adjustment equations as

$$Y_t = \min \left(Y_{D,t}, Y_{S,t} \right); \tag{9}$$

$$P_t = P_{t-1} + \nu_1 (Y_{D,t} - Y_{S,t}) \qquad \text{for } Y_{D,t} < Y_{S,t},$$
 (10)

$$P_t = P_{t-1} + \nu_2(Y_{D,t} - Y_{S,t})$$
 otherwise,

which we re-parametrize, as in the case of the labor market, as

$$P_t - P_{t-1} = (1 - \eta_1)(P_t^* - P_{t-1})$$
 for $P_t < P_{t-1}$, (10')
 $P_t - P_{t-1} = (1 - \eta_2)(P_t^* - P_{t-1})$ otherwise.

For rigidity parameter estimation, we then obtain the following goods market estimating equations:

$$\begin{split} Y_{t} &= -\frac{\beta_{0}}{\beta_{2}} + \frac{1}{\beta_{2}} L_{t} - \frac{\beta_{1}}{\beta_{2}} (W_{t} - P_{t}) \\ &- \frac{\beta_{3}}{\beta_{2}} t - \left[\frac{\mu_{2}}{1 - \mu_{2}} \right] \left[\frac{\beta_{1} - \gamma_{1}}{\beta_{2}} \right] \Delta W_{t}^{+} \\ &+ \left[\frac{\beta_{1} + \beta_{2} (\delta_{1} + \delta_{2})}{\beta_{2}} \right] \left[\frac{\eta_{1}}{1 - \eta_{1}} \right] \Delta P_{t}^{+} + z_{t} ; \end{split} \tag{11}$$

Jean-Louis Arcand and Elise S. Brezis

$$Y_{t} = \delta_{0} + \delta_{1}(M_{t} - P_{t}) + \delta_{2}(P_{t+1} - P_{t})$$

$$-\frac{\eta_{2} \left[1 + \beta_{1} + \beta_{2}(\delta_{1} + \delta_{2})\right]}{1 - \eta_{2}} \Delta P_{t}^{+} + u_{t}.$$
(12)

For examining the slopes of aggregate demand and supply, we obtain

$$Y_{S,t} = \frac{\alpha_1 \beta_1}{1 - \alpha_1 \beta_2} (W_t - P_t) + \frac{\alpha_1}{1 - \alpha_1 \beta_2} \cdot \left[\frac{\beta_1 - \gamma_1}{1 - \mu_2} \mu_2 \Delta W_t^+ + \epsilon_{D,t} + \beta_0 \right] + \frac{\alpha_2}{1 - \alpha_1 \beta_2} t + \frac{\eta_1}{1 - \eta_1} \left[\frac{-\alpha_1 \beta_1}{1 - \alpha_1 \beta_2} + \delta_1 + \delta_2 \right] \Delta P_t^- ,$$
 (13)

and

$$Y_{t} = \delta_{0} + \delta_{1}(M_{t} - P_{t}) + \delta_{2}(P_{t+1} - P_{t})$$

$$-\frac{\eta_{2}}{1 - \eta_{2}} \left[\frac{-\alpha_{1}\beta_{1}}{1 - \alpha_{1}\beta_{2}} + \delta_{1} + \delta_{2} \right] \Delta P_{t}^{+} + u_{t} . \tag{14}$$

For both systems, the third equation in the system is (8), the labor supply equation. In order to endogenize consumer prices, we assume that

$$C_t = C_t(P_t) . (15)$$

Note that the system constituted by Equations (13), (14) and (8) does not allow one to estimate μ_2 and η_1 , but does give us an estimate of $\partial Y_{s,t}/\partial P_t$. The system constituted by (11), (12) and (8), on the other hand, gives us estimates for μ and η , but does not give us an estimate of $\partial Y_{s,t}/\partial P_t$. As will become clear in part 3, the slope of excess demand is the key to the dynamic stability issue.

One of the model's strengths is that—by going out on a "specification limb"—it provides an elegant and concise formulation of goods and labor market dynamics; namely, an explicit estimate of the stickiness of prices and nominal wages. Moreover, these esti-

mates of wage and price flexibility are defined in terms of the extent to which wages and prices adjust to their market-clearing values. This is in contrast to most discussions of flexibility, which confine their attention to measuring the magnitude of movements in wages and prices. We believe such approaches to be rather uninformative in that they fail to address the key issue which may be succinctly expressed as: flexibility with respect to what? A wage rate that displays a high variance over a period of time may not be evidence of flexibility at all if it consistently "misses" the equilibrium value. A wage rate that varies very little may not be evidence of inflexibility in that it may shadow the underlying equilibrium wage rate very closely.

Output and Employment Adjustment: The YL Model

So far the discussion has been Walrasian in that adjustment has been assumed to obtain through prices and wages. In a Marshallian approach, on the other hand, quantity supplied increases if "demand price" exceeds "supply price." In the labor market, the Marshallian adjustment equations are given by

$$W_t = \max(W_{D,t}, W_{S,t}); \tag{16}$$

$$\dot{L}_t \approx L_t - L_{t-1} = \phi_1(W_{D,t} - W_{S,t}) \qquad \text{for } W_{D,t} < W_{S,t} \,,$$

$$\dot{L}_t \approx L_t - L_{t-1} = \phi_2(W_{D,t} - W_{S,t}) \qquad \text{otherwise} , \qquad (17)$$

which may be re-parametrized as in the PW model as

$$L_t - L_{t-1} = (1 - \theta_1)(L_t^* - L_{t-1})$$
 for $L_t < L_{t-1}$, (17')

$$L_t - L_{t-1} = (1 - \theta_2)(L_t^* - L_{t-1})$$
 otherwise.

Marshallian adjustment in the goods market is described by

$$P_t = \max(P_{D,t}, P_{S,t});$$
 (18)

$$\dot{Y}_t \approx Y_t - Y_{t-1} = \lambda_1 (P_{D,t} - P_{S,t}) \quad \text{for } P_{D,t} < P_{S,t}$$

$$\dot{Y}_t \approx Y_t - Y_{t-1} = \lambda_2 (P_{D,t} - P_{S,t}) \qquad \text{otherwise} , \qquad (19)$$

which is re-parametrized as

$$Y_t - Y_{t-1} = (1 - \psi_1)(Y_t^* - Y_{t-1})$$
 for $Y_t < Y_{t-1}$ (19')
 $Y_t - Y_{t-1} = (1 - \psi_2)(Y_t^* - Y_{t-1})$ otherwise.

The estimating equations in terms of observable variables are obtained by following the same procedure as in the PW model.

Mixed Adjustment: The YW and PL Models

The PW and YL models are "pure" adjustment models in either prices or quantities. However, one can envisage worlds in which the labor market adjusts in wages and the goods market in quantities, and vice versa. These two models will be referred to as the YW and PL models. The adjustment equations for the YW model are given by (18) and (19) for the goods market, (3) and (4) for the labor market. For the PL model, the goods market adjusts according to (9) and (10), the labor market according to Equations (16) and (17).

Estimating Equations

Serial correlation in Quandt type-C models may be treated as a problem of lagged endogenous (latent) variables, and the systems presented below are modified to take this into account. ¹² The five systems we will be considering are summarized in Table 2.

The equilibrium specification is essentially a restriction on the parameters of any one of the four disequilibrium systems. For the PW model it is straightforward to show that the disequilibrium model converges to the equilibrium specification when $(\mu_1, \mu_2, \eta_1, \eta_2) \rightarrow (0, 0, 0, 0)$. A likelihood ratio test which compares

$$T\log\left[rac{\det\hat{\Omega}_e}{\det\hat{\Omega}_d}
ight]=W_p$$

with the critical value of $\chi^2(p)$ is thus the appropriate test (where Ω_e is the covariance matrix of the model constrained to the equilibrium specification, Ω_d is the covariance matrix under the unrestricted disequilibrium hypothesis, "hats" designate estimated val-

¹²See Quandt (1988, 132–8), who notes that this treatment does involve problems similar to those of the Cochrane-Orcutt transformation (p. 133).

¹³Quandt (1988, 83). This may also be shown for the three other models. For a general discussion of testing for equilibrium, see Quandt (1988, 80–87).

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Equilibrium Restriction:  Y_{t} = Y_{t}(Y_{t-1}, M_{t}, M_{t-1}, P_{t+1}, P_{t}, P_{t-1}) . 
 Y_{t} = Y_{t}(Y_{t-1}, Y_{t-2}, W_{t}, W_{t-1}, P_{t}, P_{t-1}, t) . 
 L_{t} = L_{t}(L_{t-1}, W_{t}, W_{t-1}, W_{t-2}, C_{t}, C_{t-1}, C_{t-2}, N_{t}, N_{t-1}) . 
PW:  Y_{t} = Y_{t}(Y_{t-1}, M_{t}, M_{t-1}, P_{t+1}, P_{t}, P_{t-1}, \Delta P_{t}^{+}, \Delta P_{t-1}^{+}) . 
 Y_{t} = Y_{t}(Y_{t-1}, Y_{t-2}, W_{t}, W_{t-1}, P_{t}, P_{t-1}, \Delta W_{t}^{+}, \Delta W_{t-1}^{+}, \Delta P_{t}^{-}, \Delta P_{t-1}^{-}, t) . 
 L_{t} = L_{t}(L_{t-1}, W_{t}, W_{t-1}, W_{t-2}, C_{t}, C_{t-1}, C_{t-2}, N_{t}, N_{t-1}, \Delta W_{t}^{+}, \Delta W_{t-1}^{-}) . 
YL:  Y_{t} = Y_{t}(Y_{t-1}, M_{t}, M_{t-1}, P_{t+1}, P_{t}, P_{t-1}, \Delta Y_{t}^{-}, \Delta Y_{t-1}^{-}) . 
 Y_{t} = Y_{t}(Y_{t-1}, Y_{t-2}, W_{t}, W_{t-1}, P_{t}, P_{t-1}, \Delta L_{t}^{-}, \Delta L_{t-1}^{-}, \Delta Y_{t}^{+}, \Delta Y_{t-1}^{+}, t) . 
 L_{t} = L_{t}(L_{t-1}, W_{t}, W_{t-1}, W_{t-2}, C_{t}, C_{t-1}, C_{t-2}, N_{t}, N_{t-1}, \Delta L_{t}^{+}, \Delta L_{t-1}^{+}) . 
YW:  Y_{t} = Y_{t}(Y_{t-1}, M_{t}, M_{t-1}, P_{t+1}, P_{t}, P_{t-1}, \Delta Y_{t}^{-}, \Delta Y_{t-1}^{-}) .
```

PL:

 $Y_{t} = Y_{t}(Y_{t-1}, Y_{t-2}, W_{t}, W_{t-1}, P_{t}, P_{t-1}, \Delta W_{t}^{+}, \Delta W_{t-1}^{+}, \Delta Y_{t}^{+}, \Delta Y_{t-1}^{+}, t) .$ $L_{t} = L_{t}(L_{t-1}, W_{t}, W_{t-1}, W_{t-2}, C_{t}, C_{t-1}, C_{t-2}, N_{t}, N_{t-1}, \Delta W_{t}^{-}, \Delta W_{t-1}^{-}) .$

NOTE: These are the systems for which parameter estimates will be presented. In order to calculate the adjustment parameters, the second equation in each system is replaced by

```
PW: Y_{t} = Y_{t}(Y_{t-1}, Y_{t-2}, L_{t}, L_{t-1}, W_{t}, W_{t-1}, P_{t}, P_{t-1}, \Delta W_{t}^{+}, \Delta W_{t-1}^{+}, \Delta P_{t}^{-}, \Delta P_{t-1}^{-}, t);

YL: Y_{t} = Y_{t}(Y_{t-1}, Y_{t-2}, L_{t}, L_{t-1}, W_{t}, W_{t-1}, P_{t}, P_{t-1}, \Delta L_{t}^{-}, \Delta L_{t-1}^{-}, \Delta Y_{t}^{+}, \Delta Y_{t-1}^{+}, t);

YW: Y_{t} = Y_{t}(Y_{t-1}, Y_{t-2}, L_{t}, L_{t-1}, W_{t}, W_{t-1}, P_{t}, P_{t-1}, \Delta W_{t}^{+}, \Delta W_{t-1}^{+}, \Delta Y_{t}^{+}, \Delta Y_{t-1}^{+}, t);

PL: Y_{t} = Y_{t}(Y_{t-1}, Y_{t-2}, L_{t}, L_{t-1}, W_{t}, W_{t-1}, P_{t}, P_{t-1}, \Delta L_{t-1}, \Delta L_{t-1}, \Delta P_{t}^{-}, \Delta P_{t-1}^{-}, t).
```

ues, and p is the difference in the number of parameters to be estimated in the two models, which here equals 8).

3. Estimation Results and the Dynamics of the Downward Spiral

1892-1940

The five specifications were estimated in 2SLS, 3SLS and I3SLS (Iterated-3SLS). Only the 3SLS results are presented. 14 Estimation results for the 1892-1940 sample period are shown in Table 3. Results presented in Table 4 indicate that the equilibrium restriction is strongly rejected in each of the four disequilibrium systems. On the other hand, all four disequilibrium specifications yielded estimates of rigidity parameters which were all "small," as indicated by the size of the coefficients on the ΔP , ΔW , ΔY and ΔL terms in the estimating equations (for example, the coefficient on ΔW^{+} in the aggregate supply equation for the PW model in Table 3 is an estimate of $-1/\zeta_1$. One can then easily compute estimates of μ_1). This absence of nominal rigidities is a general characteristic of our estimation results for both pre- and post-WWI periods. It implies that adjustment is rapid. Note that this is perfectly consistent with the finding that the equilibrium restriction is rejected because the test of the equilibrium restriction is essentially a test on the joint significance of the coefficients on the ΔP , ΔW , ΔY and ΔL terms. Moreover, these coefficients may be significantly different from zero (which will imply rejection of the equilibrium restriction, as an example, consider the coefficient on ΔW^- in Table 3 in the labor

¹⁴All variables except for the ΔW , ΔP , ΔL , ΔY variables and the time trend are expressed in logarithms. All variables are drawn from Long Term Economic Growth, 1973 (hereafter, LTEG), and Gordon 1986: L_t is manhours in non-farm employment (LTEG, series A70); W_t is the real wage index (LTEG, series B70) multiplied by CPI (LTEG, series B69); P_t is the GNP deflator (Gordon 1986); C_t is a general index of consumer prices (LTEG, series B69); Y_t is (real) gross national product (Gordon 1986); N_t is total population (LTEG, series A114); M_t is M2 (Gordon 1986). In the present version of the paper, we eschew more complex maximum likelihood estimation methods described in Quandt (1988) and we confine ourselves to simple two- and three-stage linear estimating procedures. Our estimates are not the most efficient possible because we fail to take into account the non-linearity of the endogenous parameters ΔW -, ΔW +, etc. They are, however, consistent. Moreover, 3SLS will be efficient in the class of linear estimators, as well as consistent under the null that the specification is correct. This will allow us to construct Hausman specification tests based on our linear estimators.

supply equation), while the implied adjustment parameters such as μ are not statistically different from zero. This is because the adjustment parameters are a combination of the coefficients on $\Delta P,$ $\Delta W,$ ΔY or ΔL and of the slope of the excess demand functions. We will have more to say on the implied dynamics of the system later on, when we consider the behavior of excess demand.

In order to ascertain the statistical robustness of our econometric results, we carried out Likelihood-Ratio-form Hausman (1978) specification tests on all five models. The results for the 1892-1940 sample period are reported in Table 4. In the Hausman test, the null hypothesis is that the system of equations is correctly specified. If this is true, then the 3SLS estimator will provide a consistent and efficient estimate of the parameters. However, if the null hypothesis is incorrect, then the 3SLS estimator will be inconsistent. On the other hand, the 2SLS estimator will provide a consistent estimate of the parameters if there is specification error, but in the event that the null-hypothesis is true will provide estimates which, though consistent, are not as efficient as those provided by 3SLS. These properties of the 3SLS versus 2SLS estimators allow one to construct a statistical test of the null hypothesis that the model is correctly specified. 15 The test results reported in Table 4 indicate that the equilibrium specification is very strongly rejected by the Hausman test, as is the PW model. On the other hand, the YL. YW and PL models are not rejected by the Hausman test for estimation over the entire sample period.

The equilibrium specification is rejected (i) by the test of the equilibrium restriction in each of the four disequilibrium models and (ii) by the Hausman test. We may therefore safely conclude that the equilibrium specification is strongly rejected by the data. Of course, implicit in any such structural estimation is the joint hypothesis that the theoretical model underlying the regression equations is also correctly specified. In other words, we may reject the equilibrium specification when a reduced-form IS/LM model is the basis of the (demand side of the) test, but not reject equilibrium when some other structural form is chosen. However, the fact that three of our disequilibrium specifications were not rejected by a Hausman test gives us some cause for cautious optimism regarding the robustness of our results.

¹⁵For more details on the Hausman specification test, see Hausman (1978) and his survey (Hausman 1983) in chapter 7 of volume I of the *Handbook of Econometrics*. The test is then constructed in the same manner as the likelihood ratio test for the equilibrium restriction, as given above.

Jean-Louis Arcand and Elise S. Brezis

Among the disequilibrium specifications, we would, on the basis of the Hausman specification tests, reject the purely Walrasian (PW) model. On the other hand, we have no firm statistical basis on which to base a choice among the three remaining disequilibrium models (PL, YL and YW), since they are not nested in any manner. We thus turn to more heuristic tools. Figure 1 plots a simulation resulting from the estimated equation for the PL model versus actual (log) GNP, over the entire sample period. Note that these are not aggregate demand and aggregate supply per se but aggregate demand and supply plus the adjustment terms in ΔP , ΔW ,

TABLE 3.

| Aggregate Demand 1 | Functions | | | | | |
|--------------------|-----------------|----------------|-----------------------|-----------------|-----------------|-----------------------|
| | | Eqbm | | | PW | |
| | 1890 -1940 | 1919 -1940 | 1919 -1940 Chow | 1890 -1940 | 1919 -1940 | 1919 -1940 Chow |
| Aggregate Demand | (1) | (2) | (3) | (4) | (5) | (6) |
| Y - 1 | 0.69 (4.74) | 0.58 (4.81) | 0.66 (5.04) | 0.61 (5.44) | 0.63 (5.93) | 0.66 (6.71) |
| M | 0.7 (2.96) | 0.73 (5.25) | 0.66 (4.27) | 0.67 (4.09) | 0.64 (3.93) | 0.53 (3.44) |
| P | -0.9 (1.3) | -0.48 (2.12) | -0.47 (1.47) | -0.23 (0.57) | 0.27 (0.71) | 0.48 (1.15) |
| P + 1 | 0.33 (1.08) | 0.1 (0.74) | 0.11 (0.54) | 0.13 (0.79) | -0.03 (0.26) | -0.03 (0.19) |
| M - 1 | -0.49 (1.82) | -0.45 (3.16) | -0.43 (2.59) | -0.4 (2.25) | -0.51 (3.3) | -0.43 (3.) |
| P - 1 | 0.37 (0.75) | 0.1 (0.61) | 0.13 (0.62) | -0.16 (0.48) | -0.36 (1.11) | -0.56 (1.65) |
| ΔP+ | | | | -0.01 (0.56) | -0.03 (1.87) | -0.04 (2.39) |
| $\Delta P+-1$ | | | | 0 (0.33) | -0.01 (0.69) | 0 (0.52) |
| ΔΥ- | | | | | | |
| ΔY1 | | | | | | |
| Constant | 1.58 (2.06) | 2.17 (3.39) | 1.78 (2.59) | 2.02 (3.46) | 1.99 (3.57) | 1.86 (3.6) |
| Rho | -0.15 | 0.22 | -0.01 | 0 | 0.01 | -0.05 |
| SEE R-Squared | 0.05 0.99 | 0.04 0.91 | 0.05 0.98 | 0.04 0.99 | 0.03 0.95 | 0.04 0.99 |

 ΔY or ΔL . Visually, the "fit" of this model is remarkably good, particularly when the simplicity of the underlying equations is considered. Unfortunately, the visual fit of all three models not rejected by the Hausman test proved to be quite good (see the additional figures presented in Arcand and Brezis 1992). As a result, these simulations do not allow us to (visually) "reject" any of the three remaining models.

Figure 2 plots the estimated differences between (log) aggregate demand and (log) aggregate supply using the parameter estimates of Table 3, excluding the ΔP and ΔW terms. That is, Figure

| | YL | | | YW | | | PL | |
|---------------|---------------|-----------------------|---------------|---------------|-----------------------|---------------|---------------|-----------------------|
| 1890 -1940 | 1919 -1940 | 1919 -1940 Chow | 1890 -1940 | 1919 -1940 | 1919 -1940 Chow | 1890 -1940 | 1919 -1940 | 1919 -1940 Chow |
| (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) |
| 0.83 | 0.63 | 0.64 | 0.8 | 0.69 | 0.73 | 0.6 | 0.61 | 0.63 |
| (5.12) | (5.53) | (3.36) | (4.58) | (6.24) | (4.18) | (5.35) | (5.64) | (5.61) |
| 0.57 | 0.48 | 0.44 | 0.71 | 0.42 | 0.21 | 0.89 | 0.95 | 1.03 |
| (2.47) | (2.47) | (1.54) | (2.37) | (2.29) | (0.84) | (5.22) | (5.14) | (4.79) |
| -0.82 | -0.17 | -0.04 | -1.06 | -0.14 | 0.06 | -0.48 | -0.1 | -0.28 |
| (1.92) | (0.91) | (0.11) | (1.71) | (0.8) | (0.18) | (1.15) | (0.27) | (0.55) |
| 0.28 | -0.08 | -0.14 | 0.36 | -0.07 | -0.14 | 0.19 | -0.12 | -0.24 |
| (1.43) | (0.71) | (0.63) | (1.35) | (0.64) | (0.72) | (1.08) | (0.87) | (1.32 |
| -0.45 | -0.24 | -0.21 | -0.56 | -0.21 | -0.32 | -0.62 | -0.79 | -0.88 |
| (1.92) | (1.31) | (0.74) | (1.85) | (1.21) | (0.13) | (3.38) | (4.59) | (4.56 |
| 0.41 | 0.02 | -0.06 | 0.55 | 0.01 | -0.09 | 0.02 | 0.07 | 0.37 |
| (1.24) | (0.12) | (0.23) | (1.27) | (0.05) | (0.42) | (0.06) | (0.2) | (0.82) |
| | | | | | | -0.02 | -0.02 | 0 |
| | | | | | | (0.98) | (1.01) | (0.14) |
| | | | | | | 0.01 | -0.01 | -0.02 |
| | | | | | | (0.8) | (1.44) | (1.92) |
| 0.01 | 0 | 0 | 0 | 0 | 0.01 | | | |
| (2.48) | (3.44) | (2.32) | (1.46) | (4.12) | (3.72) | | | |
| 0 | 0 | 0 | 0 | 0 | 0 | | | |
| (0.41) | (1.05) | (0.67) | (0.17) | (0.56) | (0.46) | | | |
| 0.88 | 1.93 | 1.92 | 1.04 | 1.65 | 1.43 | 2.05 | 2.07 | 1.95 |
| (1.04) | (3.17) | (1.92) | (1.14) | (2.83) | (1.54) | (3.52) | (3.65) | (3.33) |
| -0.16 | -0.04 | -0.05 | -0.22 | -0.03 | -0.01 | -0.04 | -0.09 | 0.09 |
| 0.04 | 0.03 | 0.04 | 0.04 | 0.03 | 0.04 | 0.04 | 0.03 | 0.04 |
| 0.99 | 0.97 | 0.99 | 0.99 | 0.97 | 0.99 | 0.99 | 0.96 | 0.99 |

Jean-Louis Arcand and Elise S. Brezis

TABLE 3. cont'd

| | | Eqbm | | _ | PW | |
|------------------|--------|--------|--------|--------|--------|------------|
| | 1890 | 1919 | 1919 | 1890 | 1919 | 1919 |
| | -1940 | -1940 | -1940 | -1940 | -1940 | -1940 |
| | | | Chow | | | Chow |
| Aggregate Supply | (1) | (2) | (3) | (4) | (5) | (6) |
| Y - 1 | 0.78 | 0.72 | 0.82 | 0.68 | 0.7 | 0.73 |
| | (6.43) | (4.1) | (6.01) | (4.13) | (3.47) | (4.06) |
| Y-2 | -0.02 | -0.06 | -0.06 | 0.12 | -0.18 | -0.13 |
| | (0.18) | (0.41) | (0.44) | (0.61) | (0.94) | (0.74) |
| (W - P) | 0.06 | 0.36 | 0.51 | 0.47 | 0.72 | 1.01 |
| | (0.22) | (0.77) | (1.19) | (0.87) | (1.33) | (1.93) |
| (W - 1 - P - 1) | -0.2 | 0.3 | 0.05 | -0.54 | -0.07 | -0.55 |
| | (0.84) | (0.81) | (0.18) | (1.05) | (0.12) | (1.03) |
| ΔW + | | | | -0.01 | -0.01 | -0.02 |
| | | | | (0.51) | (1.91) | (2.66) |
| ∆W + −1 | | | | 0 | -0.01 | - 0 |
| | | | | (0.31) | (1.08) | (0.65) |
| ∆P- | | | | 0.03 | 0.02 | 0.03 |
| | | | | (2.06) | (1.94) | (2.83) |
| ∆P1 | | | | 0 | 0.01 | 0 |
| | | | | (0.12) | (0.72) | (0.38) |
| 7L- | | | | | | |
| ∆L1 | | | | | | |
| ΔY + | | | | | | |
| 1 ' | | | | | | |
| ∆Y+ −1 | | | | | | |
| : | 0.01 | -0.01 | -0.01 | 0.01 | -0.01 | -0.01 |
| | (1.68) | (0.74) | (0.8) | (1.0) | (1.04) | (0.83) |
| Constant | 1.09 | 2.09 | 1.53 | 0.9 | 2.99 | 2.51 |
| | (2.56) | (2.55) | (2.68) | (1.8) | (3.35) | (3.32) |
| Rho | 0.19 | 0.34 | 0.12 | 0.2 | 0.21 | 0.03 |
| SEE | 0.06 | 0.07 | 0.06 | 0.06 | 0.06 | 0.06 |
| R-Squared | 0.97 | 0.78 | 0.98 | 0.98 | 0.89 | 0.99 |

2 plots the realizations of ${\it unobservable}$ excess aggregate demand. This figure is quite revealing.

In considering Figure 2 it is important to keep in mind the location of zero on the vertical axis, since any point below this line corresponds to excess aggregate supply, and any point above to ex-

| | YL | | | YW | | | PL | |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1890 | 1919 | 1919 | 1890 | 1919 | 1919 | 1890 | 1919 | 1919 |
| -1940 | -1940 | -1940 | -1940 | -1940 | -1940 | -1940 | -1940 | -1940 |
| | | Chow | | | Chow | | | Chow |
| (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) |
| 0.73 | 1.07 | 1.06 | 1.04 | 1.29 | 1.26 | 0.58 | 1.06 | 0.88 |
| (4.54) | (3.17) | (3.43) | (6.1) | (4.45) | (5.16) | (3.66) | (4.06) | (3.12) |
| 0.21 | -0.11 | -0.09 | -0.12 | -0.45 | -0.39 | 0.38 | -0.1 | 0.05 |
| (1.25) | (0.32) | (0.31) | (0.67) | (1.27) | (1.31) | (2.26) | (0.44) | (0.19) |
| -0.05 | 0.53 | 0.47 | -0.28 | 0.14 | -0.02 | 0.44 | 0.13 | 0.47 |
| (0.29) | (2.17) | (1.81) | (1.35) | (0.41) | (0.08) | (1.49) | (0.34) | (1.0) |
| 0.01 | -0.08 | -0.06 | 0.14 | 0.18 | -0.22 | -0.35 | 0.24 | 0.16 |
| (0.09) | (0.34) | (0.27) | (0.65) | (0.55) | (0.79) | (1.43) | (0.67) | (0.45) |
| | | | 0 | 0 | 0 | | | |
| | | | (0.05) | (0.39) | (0.26) | | | |
| | | | 0 | 0.01 | -0.01 | | | |
| | | | (0.31) | (1.23) | (1.27) | | | |
| | | | | | | 0.01 | -0.01 | 0 |
| | | | | | | (0.88) | (1.08) | (0.22) |
| | | | | | | -0.01 | -0.01 | -0.01 |
| | | | | | | (0.77) | (1.57) | (1.68) |
| 0.01 | 0.01 | 0.01 | | | | 0.03 | 0.02 | 0.02 |
| (2.81) | (3.37) | (3.43) | | | | (4.22) | (4.48) | (3.38) |
| 0 | 0 | 0 | | | | 0.01 | 0 | 0.01 |
| (1.62) | (0.09) | (0.17) | | | | (1.31) | (0.7) | (0.93) |
| 0 | 0 | 0 | 0.01 | 0 | 0.01 | | | |
| (5.57) | (5.6) | (5.56) | (7.9) | (3.74) | (4.43) | | | |
| 0 | 0 | 0 | 0 | 0 | 0 | | | |
| (0.82) | (0.7) | (0.66) | (0.08) | (0.97) | (0.91) | | | |
| 0 | -0.01 | -0.01 | 0 | -0.01 | 0 | 0 | -0.01 | -0.01 |
| (0.4) | (1.9) | (1.88) | (0.85) | (0.77) | (0.53) | (0.04) | (0.5) | (1.09) |
| 0.25 | 0.51 | 0.47 | 0.33 | 1.02 | 0.83 | 0.23 | 0.4 | 0.69 |
| (1.21) | (1.53) | (1.66) | (1.24) | (1.38) | (1.29) | (0.57) | (0.63) | (1.05) |
| -0.07 | -0.05 | -0.16 | -0.08 | -0.21 | -0.06 | 0.01 | 0.09 | -0.06 |
| 0.02 | 0.21 | 0.02 | 0.03 | 0.03 | 0.03 | 0.05 | 0.04 | 0.04 |
| 1 | 0.98 | 1 | 0.99 | 0.96 | 1 | 0.98 | 0.94 | 0.99 |

cess aggregate demand. All three models not rejected by the Hausman test pick out the critical turning points in the time series of output. In particular, the depression following the end of World War I is predicted by all models, as is the Great Depression, the recovery of 1936–1937 and the depression of 1937.

TABLE 3. Cont'd.

| Labor Supply Fund | ctions | | | | | |
|-------------------|-----------------|-----------------|-----------------------|-----------------|-----------------|-----------------------|
| | | Eqbm | | | PW | |
| | 1890 -1940 | 1919 -1940 | 1919 -1940 Chow | 1890 -1940 | 1919 -1940 | 1919 -1940 Chow |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| L - 1 | 0.81 (9.17) | 0.74 (4.38) | 0.84 (8.76) | 0.81 (9.05) | 0.65 (4.01) | 0.82 (8.41) |
| (W - C) | 0.37 (1.31) | 0.22 (0.63) | 0.43 | 0.11 (0.4) | -0.26 (0.75) | 0.07 (0.29) |
| (W-1 - C-1) | -0.55 (1.93) | -0.23 (0.57) | -0.52 (1.82) | -0.35 (1.27) | -0.1 (0.26) | -0.3 (1.08) |
| (W-2 - C-2) | 0.15 (0.77) | 0.25 (0.87) | 0.12 (0.58) | 0.19 (1.04) | 0.41 (1.42) | 0.17 (0.79) |
| N | 4.34 (1.93) | 1.73 (0.28) | 5.08 (1.85) | 3.18 (1.52) | 5.15 (0.91) | 3.68 (1.47) |
| N - 1 | -4.1 (1.86) | -2.16 (0.37) | -4.94 (1.84) | -2.91 (1.42) | -5.1 (0.95) | -3.39 (1.39) |
| $\Delta W-$ | ` , | , , | , , | 0.03 (4.28) | 0.03 (3.56) | 0.03 (4.15) |
| $\Delta W1$ | | | | -0.01 (1.03) | -0.01 (0.48) | -0.01 (1.19) |
| ΔL+ | | | | (=::-, | (====) | (====7 |
| $\Delta L+-1$ | | | | | | |
| Constant | -2.08 (0.94) | 6.11 (1.07) | -0.98 (0.4) | -2.38 (1.04) | 0.7 (0.12) | -2.63 (1.05) |
| Rho | 0.19 | 0.32 | 0.14 | 0.13 | 0.15 | 0.13 |
| SEE R-Squared | 0.06 0.94 | 0.07 0.65 | 0.05 0.94 | 0.05 0.96 | 0.05 0.84 | 0.05 0.96 |

Quantitatively, however, some models do better than others. The YW model, for example, shows a fall in aggregate excess demand beginning in the early 1920s, but does not indicate any excess supply during the Great Depression: one would thus tend to reject it out of hand. Similarly, the YL model shows some excess aggregate supply during the Depression, but it is somewhat disturbing that its magnitude is dwarfed by the magnitude of excess supply following World War I. On the other hand, the PL model seems to perform quite well on both counts. The depression following the

| | YL | | | YW | | | PL | |
|---------------|---------------|-----------------------|---------------|---------------|-----------------------|---------------|---------------|-----------------------|
| 1890 -1940 | 1919 -1940 | 1919 -1940 Chow | 1890 -1940 | 1919 -1940 | 1919 -1940 Chow | 1890 -1940 | 1919 -1940 | 1919 -1940 Chow |
| (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) |
| 0.99 | 0.99 | 0.99 | 0.86 | 0.59 | 0.81 | 1.02 | 0.97 | 0.97 |
| (14.9) | (7.24) | (15.2) | (8.69) | (3.63) | (8.18) | (16.4) | (7.28) | (14.9) |
| -0.04 | 0.44 | 0.24 | 0.22 | -0.27 | 0.01 | -0.02 | 0.33 | 0.15 |
| (0.17) | (1.21) | (1.16) | (0.71) | (0.81) | (0.05) | (0.09) | (0.99) | (0.82) |
| 0.04 | -0.37 | -0.25 | -0.53 | -0.21 | -0.42 | 0.16 | -0.08 | -0.11 |
| (0.15) | (0.98) | (1.16) | (1.7) | (0.54) | (1.42) | (0.77) | (0.22) | (0.56) |
| -0.08 | 0.02 | -0.01 | 0.32 | 0.57 | 0.36 | -0.21 | -0.35 | -0.1 |
| (0.47) | (0.07) | (0.07) | (1.4) | (1.91) | (1.55) | (1.43) | (1.23) | (0.66) |
| -1.9 | 3.07 | 0.03 | 4.51 | 6.09 | 3.66 | -1.55 | 6.56 | 0.05 |
| (0.9) | (0.51) | (0.01) | (1.73) | (1.03) | (1.41) | (0.86) | (1.15) | (0.03) |
| 1.89 | -3.23 | -0.08 | -4.34 | -6.1 | -3.37 | 1.5 | -6.12 | -0.03 |
| (0.9) | (0.57) | (0.04) | (1.69) | (1.1) | (1.33) | (0.84) | (1.13) | (0.02) |
| | | | 0.05 | 0.03 | 0.03 | | | |
| | | | (4.9) | (3.52) | (3.98) | | | |
| | | | -0.01 | 0 | -0.01 | | | |
| | | | (1.65) | (0.24) | (0.66) | | | |
| 0.03 | 0.03 | 0.02 | | | | 0.03 | 0.03 | 0.03 |
| (9.0) | (4.41) | (8.52) | | | | (9.61) | (5.22) | (8.67) |
| 0 | -0.01 | 0 | | | | 0 | -0.01 | 0 |
| (0.56) | (0.86) | (0.03) | | | | (0.18) | (1.05) | (0.1) |
| 0.02 | 1.79 | 0.59 | -1.49 | 1.73 | -2.56 | 0.47 | -5.17 | -0.24 |
| (0.01) | (0.33) | (0.36) | (0.58) | (0.3) | (0.99) | (0.31) | (1.0) | (0.14) |
| 0.14 | 0.24 | 0.15 | 0.01 | 0.11 | 0.14 | 0.07 | 0.14 | 0.14 |
| 0.03 | 0.05 | 0.03 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.03 |
| 0.98 | 0.88 | 0.98 | 0.96 | 0.84 | 0.96 | 0.98 | 0.87 | 0.98 |

NOTE: t-statistics are in parentheses.

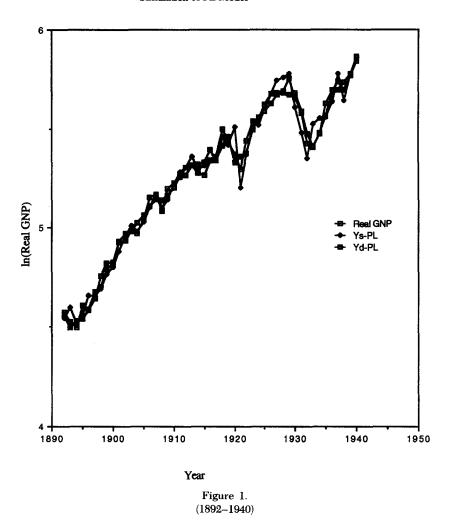
Great War is indicated by an important dose of excess aggregate supply, but so is the Great Depression, with excess supply building up during the mid-twenties, bottoming out in 1932–1933. There is then a spurt of excess demand in the mid-to-late thirties (the recovery of 1936–1937), followed by the excess supply of the 1937 recession, a situation which was corrected only by the advent of World War II (return to excess demand in 1940). Figure 3 plots estimated aggregate demand and aggregate supply, as well as realized GNP for the entire sample period, and confirms that GNP

TABLE 4. Tests Statistics for Equilibrium Restrictions and Hausman Specification Tests

| | E | Test Statistic for Equilibrium Restriction H | | | | Test Statistic for ausman Specification Test | | | |
|---------------------------|--------------|--|-------------|-------------|------|---|-----|------|-----|
| Sample Period | PW | YL | YW | PL | Eqbm | PW | YL | YW | PL |
| 1892–1940 (Table 3) | 39.05 (8) | 137.2 (8) | 68.6 (8) | 87.6 (8) | 75.9 | 73.0 | 1.6 | 0.9 | 1.7 |
| 1892–1940 (Table 3: Chow) | 61.3 (12) | 125.5 (12) | 59.8 | 70 (12) | 51.8 | 38.2 | 3.9 | 8.8 | 9.8 |
| 1919–1940 | 29.8 (8) | 66.5 (8) | 44 (8) | 84.4 (8) | 68.4 | 69.0 | 1.0 | 15.3 | 5.0 |

NOTE: Hausman tests are 2 SLS versus 3 SLS; the first line corresponds to the estimation results presented in Table 3, columns 1, 4, 7, 10 and 13; the second line to results presented in Table 3, columns 3, 6, 9, 12 and 15; numbers in parentheses for tests of equilibrium restriction are degrees of freedom.

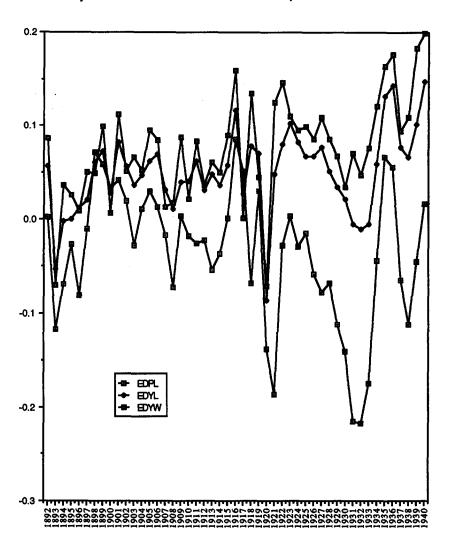
Simulation of PL Model



was constrained on the demand side during the downward spiral of the first years of the 1930s, and that aggregate demand bottomed out in 1933.

Thus, on the basis of (i) the statistical rejection of the equilibrium restriction, (ii) the results of the Hausman specification test (non-rejection), (iii) the "good fit" of the simulation runs, and (iv) the relative success of its predictions about excess aggregate demand and the manner in which those predictions square with our

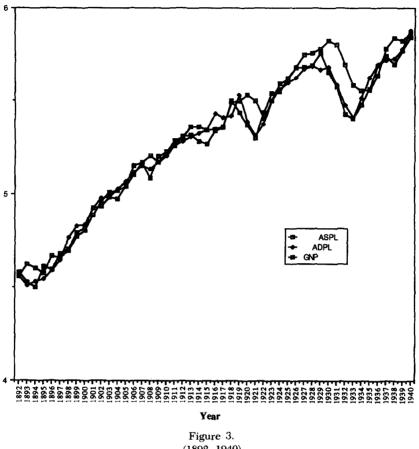
Comparison of Estimated Excess AD: PL, YL and YW Models



Year

Figure 2. (1892–1940)

Estimated AD and AS (PL Model) versus Realized GNP



(1892 - 1940)

general intuition about the events in question, one would tend to prefer the disequilibrium PL model (price adjustment in the goods market, quantity adjustment in the labor market).

Interwar Results

Table 3 (columns 2, 5, 8, 11, and 14) presents results from estimating the various specifications over the 1919-1940 sample period. Of greater interest are the results presented in Table 3, columns 3, 6, 9, 12, and 15, which give estimates of AS and AD for 1919-1940 for systems estimated over the entire 1892-1940 sample in which the coefficients of AS and AD are not constrained to be equal during the pre-WWI and interwar periods. ¹⁶ These estimates are obtained by applying a simple multiplicative dummy variable method to these two equations (as modified for instrumental variable estimation). ¹⁷ Again, the equilibrium restriction is strongly rejected in each of the four disequilibrium systems (Table 4). As was the case for the entire 1892–1940 period, no interwar rigidity parameter was found to be large, implying the absence of important nominal rigidities. Again, the equilibrium and PW models are rejected by Hausman tests (Table 4). The YL, YW and PL specifications are not rejected by the Hausman tests.

We carried out the same simple heuristic exercises as above, but this time with the interwar years. Here, our preference for the PL model was confirmed. As is shown in Figure 4, the model picks out 1932 and 1933 as the trough, while the recovery of 1935–1936 is clearly indicated, as is the recession of 1937. The other models (see Figures 15 and 16 in Arcand and Brezis 1992), on the other hand, all indicate severe excess aggregate supply during the Depression. But the YW model singles out 1933 and 1934 as a cyclical peak, while the YL model points to 1933 as a year near market clearing, clearly absurd when one quarter of the work force was out of work.

The Dynamics of the Downward Spiral: Stability

Consider the PW model. Equations (4) and (10) describe a tatônnement process. The key assumption of such a process is that price adjustment equals some smooth sign-preserving function of excess demand. This means of course that, to be economically

¹⁶Of course, such a constraint was implicitly imposed in those equations presented in Table 2.

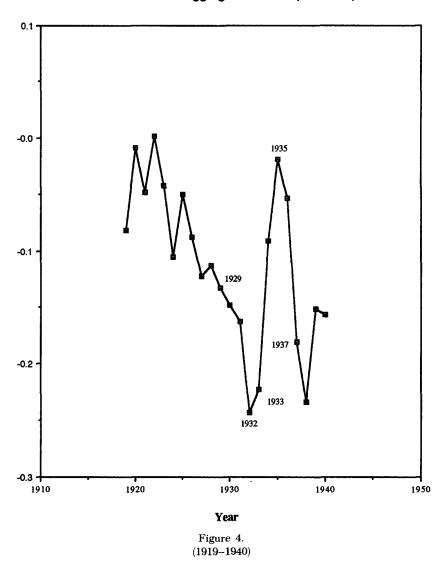
¹⁷This may equivalently be understood as a Chow test on the stability of the regression coefficients of the AD and AS equations between the 1892–1918 and 1919–1940 periods, modified so as to be applicable to instrumental variable estimation. This also yields the coefficient estimates of the pre-WWI period (not presented). The coefficients of the labor supply function are constrained to be the same during the two periods.

¹⁸It is also interesting to note that estimated excess aggregate demand under the PL model is strongly negatively correlated with the unemployment rate during the interwar years (which is what one would expect), while for the other two models the correlation is weak and positive.

¹⁹That is, if excess demand is given by $z(p) \equiv D(p) - S(p)$, where p is a k-dimensional vector of prices, a valid tatônnement adjustment rule is given by

$$\dot{p}_i = G_i(z_i(p)) \text{ for } i = 1, \ldots, k,$$

Estimated Excess Aggregate Demand (PL Model)



meaningful, $(\zeta_1, \zeta_2, \nu_1, \nu_2) > (0, 0, 0, 0)$. What restrictions does this impose on our estimated parameters $(\mu_1, \mu_2, \eta_1, \eta_2)$ and on the coefficients of the structural equations?

where G_i is some sign preserving function of excess demand. For an elementary treatment, see Varian (1984, 245), and the references cited therein.

Jean-Louis Arcand and Elise S. Brezis

Consider the labor market. The reparametrization given in Equation (6) yielded identities of the form

$$\frac{1}{\zeta} \equiv \frac{\mu}{1-\mu} \left[\frac{\partial L_{S,t}}{\partial W_t} - \frac{\partial L_{D,t}}{\partial W_t} \right].$$

Since ζ must be positive, the constraint is that

$$\frac{\mu}{1-\mu}$$

and

$$\left[\frac{\partial L_{\mathrm{S},t}}{\partial W_t} - \frac{\partial L_{D,t}}{\partial W_t}\right]$$

be of the same sign, and similarly for the goods market. Our estimates indicated that the transformed rigidity parameters, such as μ , were not statistically different from zero, so that we had no a priori restriction on the slope of estimated excess aggregate demand.

Defining EDY_t \equiv Y_{D,t} - Y_{S,t} and EDL_t \equiv L_{D,t} - L_{S,t}, the downward dynamics of the PW macro-system are given by

$$\begin{bmatrix} \dot{P}_t \\ \dot{W}_t \end{bmatrix} = \begin{bmatrix} v_1 \frac{\partial EDY_t}{\partial P_t} & 0 \\ 0 & \zeta_1 \frac{\partial EDL_t}{\partial W_t} \end{bmatrix} \begin{bmatrix} P_t - P_t^* \\ W_t - W_t^* \end{bmatrix},$$

and it is similar for the upward dynamics. Loosely speaking, the stability of the system depends upon the signs of the principal minors of the 2×2 matrix. For stability,

$$v_1 \frac{\partial EDY_t}{\partial P_t}$$

and

$$\zeta_1 \frac{\partial EDL_t}{\partial W_t}$$

| | PW | YL | YW | PL |
|------------------|--------|------|-------|--------|
| Slope of Excess | 0.75 | 6.04 | 0.435 | -0.9 |
| Demand for Labor | (0.94) | (0) | (0.9) | (0) |
| Slope of Excess | 1.4 | 22.8 | -33.3 | 0.18 |
| Demand for Goods | (2.04) | (0) | (0) | (0.27) |

TABLE 5. Slopes of Excess Demand Functions: 1919-1940

NOTE: t-statistics in parentheses

must both be negative. This should be intuitively clear: the partial derivative of excess demand with respect to own price is negative in the "usual" case (that is, downward-sloping demand and upward-sloping supply), and this case is also stable.²⁰

Table 5 presents estimates for the 1919–1940 period of (i) the partial derivative of excess aggregate demand with respect to price and (ii) the partial derivative of excess labor demand with respect to wage. ²¹ It is particularly noteworthy that the null hypothesis of stability is rejected for all four disequilibrium specifications for the interwar period. ²² That is, the econometric results imply that the estimated slope of the excess demand function is near zero, which implies that the PL system (among others) is essentially dynamically neutral during the interwar years.

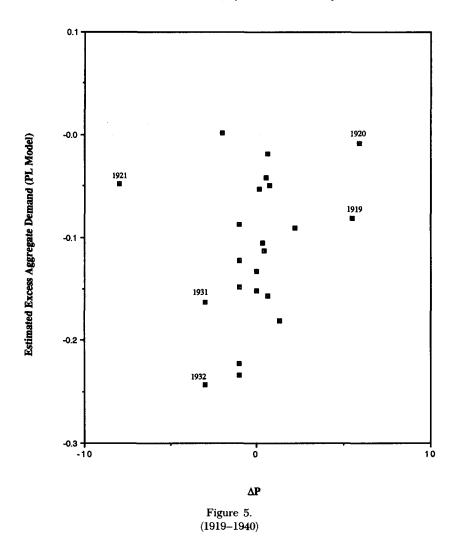
Figures 5 and 6 provide a visual illustration of these results. Consider Figure 5: on the horizontal axis, we plot the change in the price level; on the vertical axis, the estimated excess aggregate demand for the PL model estimated over the interwar period. If the Walrasian tatônnement mechanism which underlies all the estimation results is a fair proxy for reality, then the slope of the resulting skatter will indicate the stability properties of the macrosystem. The skatter plot of ΔP on estimated excess aggregate demand reveals that there are many observations where estimated excess aggregate demand is different from zero but where prices do

²⁰For a complete treatment of the "correspondence principle," see Samuelson (1983, ch. IX).

²¹Which corresponds to the systems presented in Table 5. Note that we did not reject the null hypothesis of stability for estimation over the entire sample period, not reported.

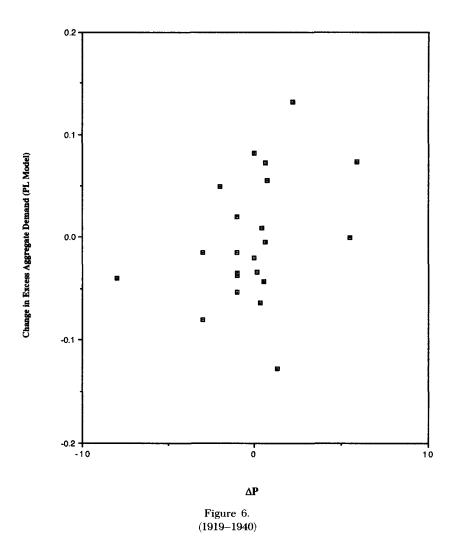
²²On the other hand, the null hypothesis of instability is also rejected for all specifications except the goods market in the PW model, but this model was rejected by a Hausman test.

An illustration of the Dynamics of the Macrosystem



not adjust. That is, there are many points clustered around the line $\Delta P=0$, but where estimated excess aggregate demand is different from zero. Thus, dynamic neutrality over certain values of excess aggregate demand is a distinct possibility. Figure 6 makes this even clearer. Here, we plot ΔP on the horizontal axis, versus the *change* in the excess aggregate demand on the vertical axis. Recall that if the system is dynamically stable, we will have $\Delta ED/\Delta P < 0$.

An Illustration of the Slope of Estimated Excess Aggregate Demand



Graphically, dynamic stability implies that the points should be arranged along a downward-sloping line passing through the origin. This is clearly not the case in Figure 6: there is certainly not any clear negative correlation between ΔP and ΔED , which would be the case if the system were dynamically stable. Of course, there is no clear positive correlation either, so the system is not dynamically

unstable. What these figures bring us back to is the initial quotation from Keynes: the U.S. macrosystem appears to be dynamically neutral during the interwar years.

4. Conclusions

Our empirical conclusions may be summarized in four points:

- (i) The equilibrium restriction is always strongly rejected in favor of one of its four disequilibrium counterparts. Though it is obviously not possible to generalize this finding to other models a priori, we conjecture that tests of the equilibrium restrictions in more complex models will yield much the same result. If anything, the fact that a very simple two-market equilibrium model does not survive a test for disequilibrium indicates that much of the equilibrium macro-econometric literature may be flawed.
- (ii) The equilibrium restriction and the PW model always fail Hausman specification tests. The YL, YW and PL models are not rejected by Hausman specification tests. Although a statistical test can never "accept" a null hypothesis, this robustness of the YL, YW and PL models is striking.
- (iii) There is very little wage, price, employment and output rigidity (depending upon the disequilibrium model estimated). This is so although the equilibrium specification is always rejected (both by tests of the equilibrium restriction and by Hausman tests).
- (iv) For the interwar period (1919–1940) the null hypothesis of stability is rejected for all four disequilibrium specifications. The PL model—the one we prefer because (i) it is not rejected by a Hausman test and (ii) it appears to square with our understanding of the sequence of events during the Depression—is essentially dynamically neutral. Thus, when the economy is hit by a negative shock, the lack of rigidity implies that output and prices will fall quickly. The lack of dynamical stability implies that once the economy reaches its new resting place, it will not tend to return back towards equilibrium. Rather, because of its dynamical neutrality, it will remain stuck at the new, lower level of output and employment.

Our view of the Great Depression—based on the dynamics uncovered by our econometric work—is that the United States economy was dynamically neutral during the interwar years. The economy may even have been slightly unstable—we need not belabor the point. The key is that it was not dynamically stable. It was not self-correcting. A sequence of negative shocks took the

economy out of equilibrium: there was nothing pulling it back. The length of the Depression was therefore caused by the lack of stability of the economy. What this means is that one no longer needs to identify an exogenous factor to explain why the economy stumbled so badly or to explain why the recovery obtained: the "cause" is endogenous to the dynamics of the system itself. To paraphrase Temin (1976), there appear to be aspects of the dynamic behavior itself that will help us to understand the Depression. We now need to better understand the underpinnings of those dynamics, both theoretically, and empirically, at a more disaggregated level.

Our findings lead us to reject, both statistically and conceptually, the equilibrium approach to the Great Depression. They also lead us to reject, again both statistically and conceptually, the notion that nominal rigidities were responsible either for the downward spiral or for the failure of the economy to return to full employment for so long.

Our main goal in this paper has been to find empirical regularities in the dynamics underlying the U.S. economy at a highly aggregated level. Our approach is in no way a substitute for a theoretical model which would explain why the dynamics were neutral. However, we do believe that our findings regarding dynamics indicate that the correct approach in understanding why the Depression took place should steer away from identifying the impulse and concentrate instead on understanding why the aggregate dynamics were perverse during the period.

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Appendix: Estimation Procedure

For those results presented in Tables 1, 2 and 3, the following variables are treated as endogenous for estimation purposes: Y_t , L_t , P_t , P_{t+1} , W_t , C_t , ΔP_t^- , ΔP_t^+ , ΔW_t^- , ΔW_t^+ , ΔY_t^- , ΔY_t^+ , ΔL_t^- , ΔL_t^+ . Aggregate demand is estimated subject to the constraint that the sum of the coefficients on M_t , M_{t-1} , P_{t+1} , P_t and P_{t-1} equal zero (if the serial correlation correction was not taken into account, the restriction would be that the sum of the coefficients on M_t , P_t and P_{t+1} equal zero). Aggregate supply is estimated subject to the constraint that the sum of the coefficients on W_t and P_t equal zero and that the sum of the coefficients on W_{t-1} and P_{t-1} equal zero. Labor supply is estimated subject to the constraint that the sum of the coefficients on W_t and C_t equal zero, the sum of the coefficients on W_{t-1} and C_{t-1} equal zero, and that the sum of the coefficients on W_{t-2} and C_{t-2} equal zero.