Estimating Approximate Incentive Compatibility

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Tuomas Sandholm, and
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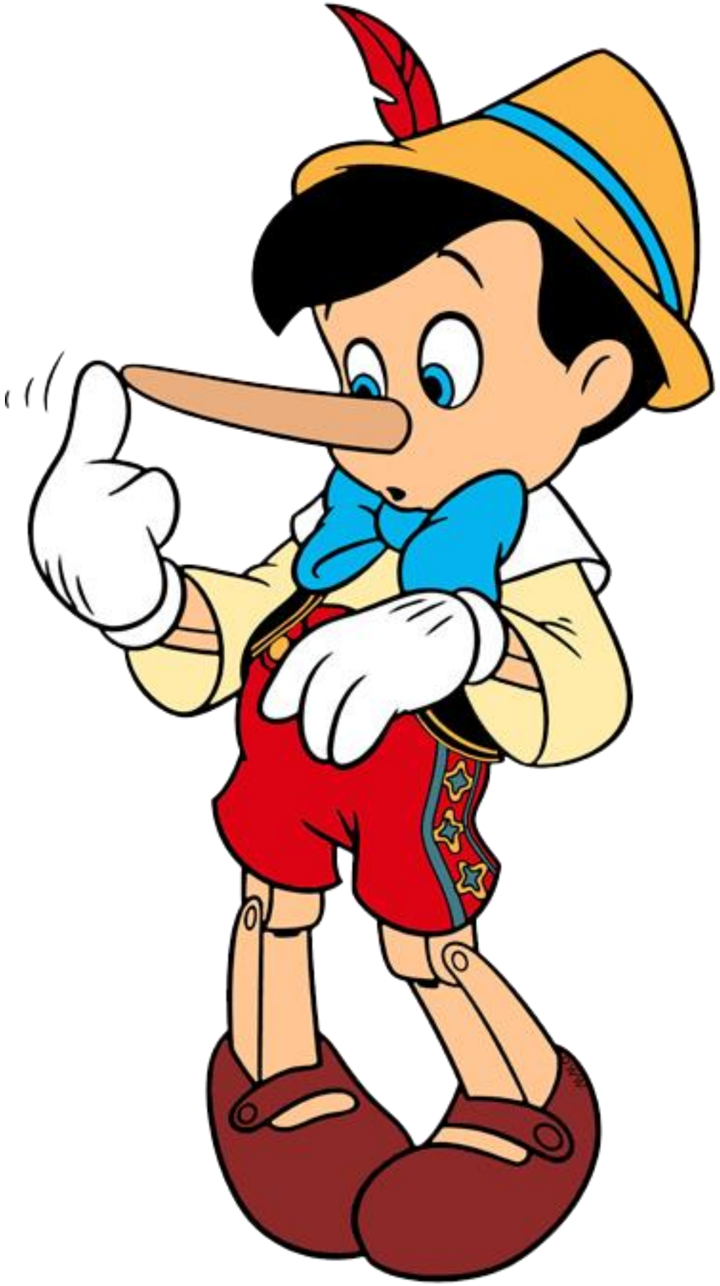
Incentive compatibility (IC)



Fundamental concept in mechanism design

Buyers maximize their utilities by bidding truthfully

Many real-world mechanisms are **not** incentive compatible

Discriminatory auctions

Multi-unit variant of first-price auction

Not incentive compatible

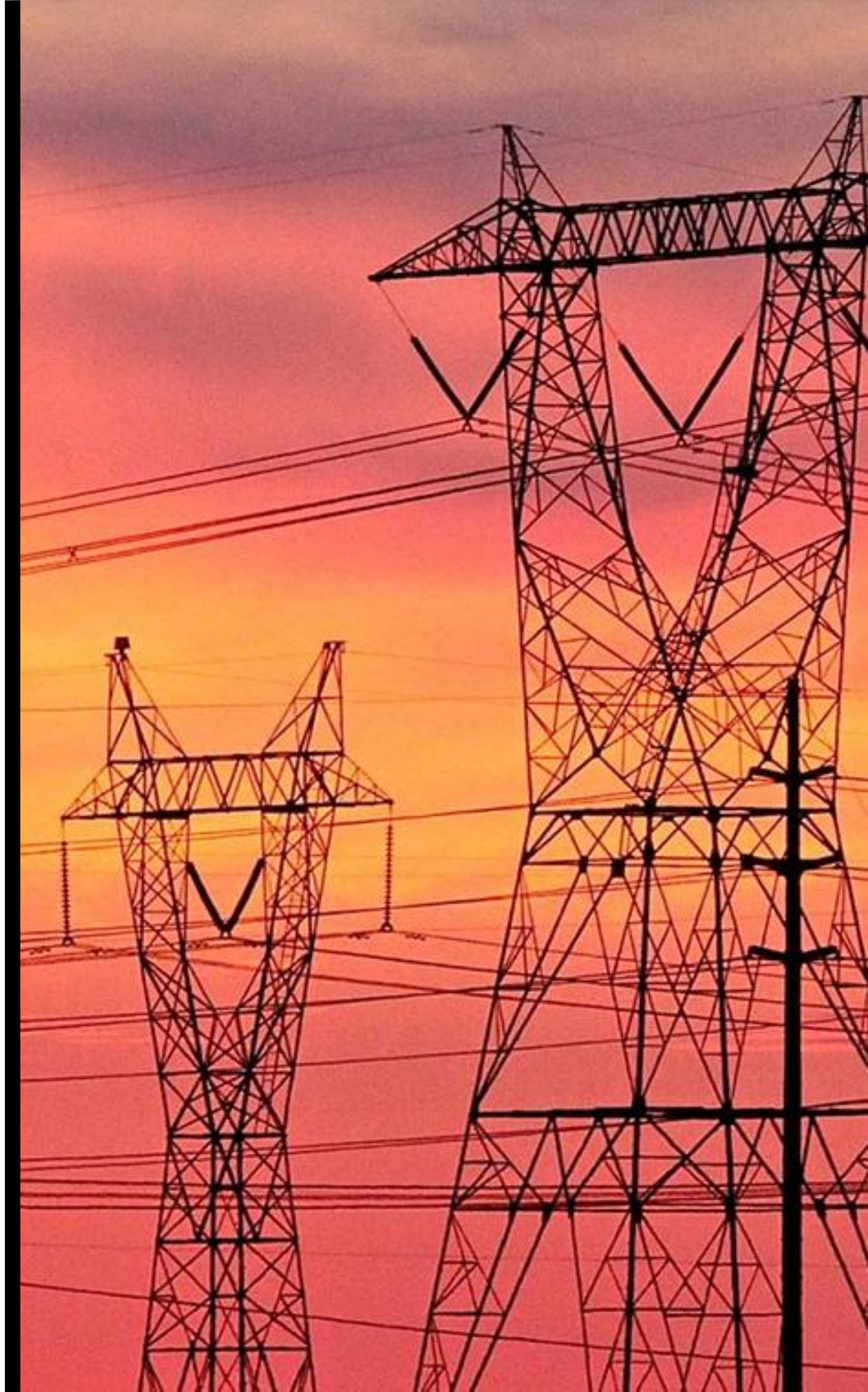
Used to sell treasury bills since 1929

Discriminatory auctions

Multi-unit variant of first-price auction

Not incentive compatible

Used to sell treasury bills since 1929 and electricity in the UK



GSP auction

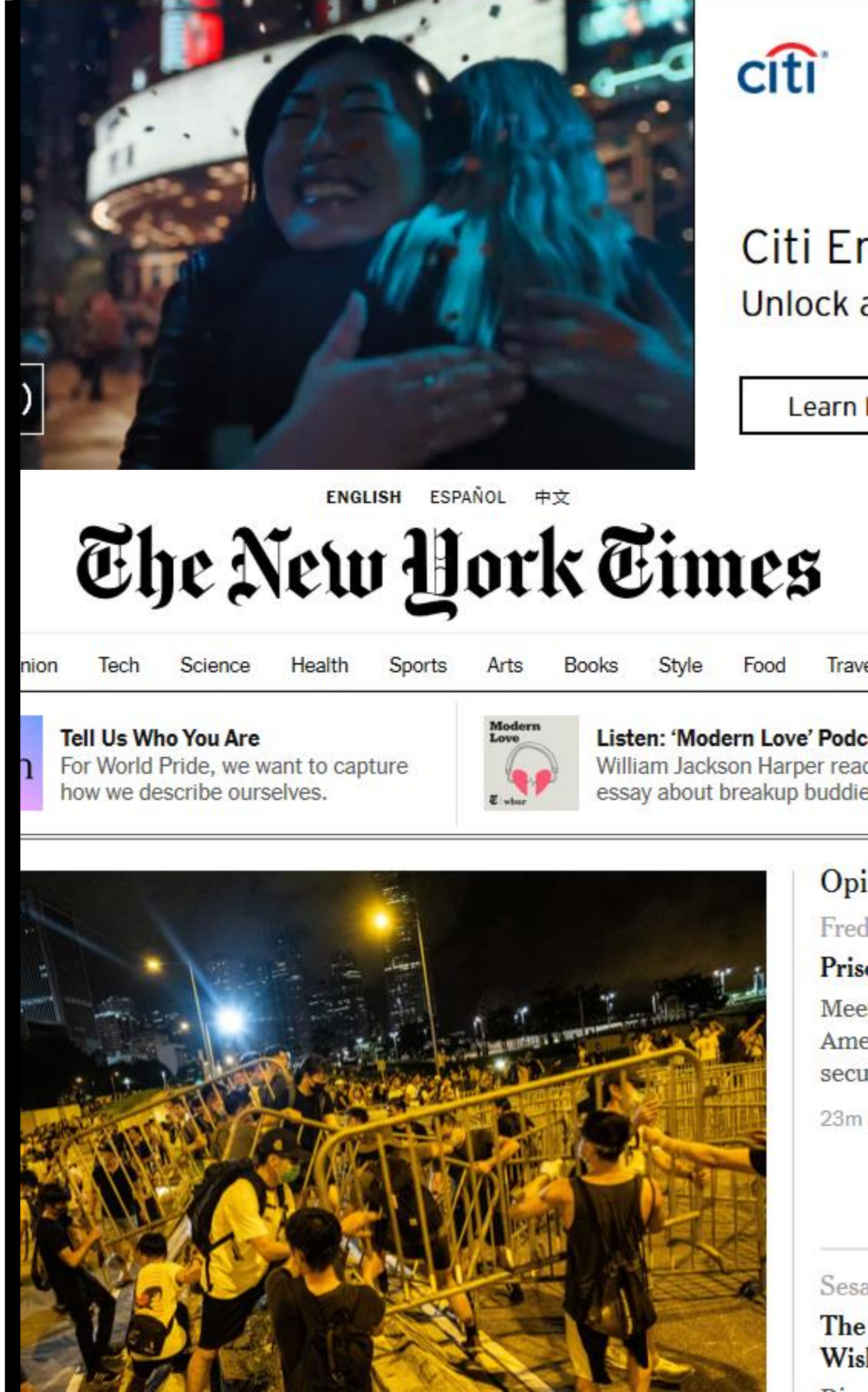
Used for sponsored search

Not incentive compatible

First-price auction

Ad exchanges transitioning to FP auction

Not incentive compatible



Combinatorial auctions

Nearly all fielded combinatorial auctions

(such as sourcing auctions)

aren't incentive compatible

Why aren't real-world mechanisms incentive compatible?

Why not IC?

Expensive to compute true values

Rules are **easier** to explain

Bids used to tune **future** parameters

Might leak **private** values

Agents not **risk** neutral

Approximate incentive compatibility

Mechanism is **γ-IC** when for each bidder i:

If everyone except bidder i is truthful,

she can only increase utility by γ if she bids strategically

[Kothari, Parkes, and Suri, EC'03; Archer, Papadimitriou, Talwar, and Tardos, Internet Mathematics '04; Conitzer and Sandholm, IJCAI'07; Dekel, Fischer, and Procaccia, JCSS'10; Lubin and Parkes, Current Science '12; Mennle and Seuken, EC'14; Dütting, Fischer, Jirapinyo, Lai, Lubin, and Parkes TEAC'15; Azevedo and Budish, Review of Economic Studies '18; Feng, Narasimhan, and Parkes, AAMAS'18; Golowich, Narasimhan, and Parkes, IJCAI'18; Dütting, Feng, Narasimhan, Parkes, and Ravindranath, ICML'19]

Approximate incentive compatibility

Mechanism is **γ-IC** when for each bidder i:

If everyone except bidder i is truthful,

she can only increase utility by γ if she bids strategically

…in expectation over **others'** values	…in expectation over **all** values
EX-INTERIM	*EX-ANTE*
(assume bidders independent)	*(no independence assumptions)*

Approximate incentive compatibility

Literature on γ-IC assumes distribution is known in advance

Where does this knowledge come from?

We relax this assumption:

Assume only **samples** from distribution over agents' types

[Likhodedov and Sandholm, AAAI'04, AAAI'05; Balcan, Blum, Hartline, and Mansour, FOCS'05, JCSS'08; Elkind, SODA'07; Cole and Roughgarden, STOC'14; Mohri and Medina, ICML'14; Huang Mansour, and Roughgarden, EC'15; Sandholm and Likhodedov, OR'15; Morgenstern and Roughgarden, NeurIPS'15, COLT'16; Roughgarden and Schrijvers, EC'16; Devanur, Huang, and Psomas, STOC'16; Balcan, Sandholm, and Vitercik, NeurIPS'16, EC'18; Alon, Babaioff, Gonczarowski, Mansour, Moran, and Yehudayoff, NeurIPS'17; Gonczarowski and Nisan, STOC'17; Cai and Daskalakis, FOCS'17; Syrgkanis, NeurIPS'17, Medina and Vassilvitskii, NeurIPS'17, …]

Overriding goal:

Estimate IC approximation factor (γ) using samples

Our estimate (first try):

Maximum utility agent i can gain by misreporting her type,

on average over samples $\left\{\boldsymbol{t}_{-i}^{(1)}, \dots, \boldsymbol{t}_{-i}^{(N)}\right\}$:

$$\max_{t_i, t_i' \in \mathbb{R}^D} \left\{ \frac{1}{N} \sum_{j=1}^{N} \underbrace{u\left(t_i, t_i', \boldsymbol{t}_{-i}^{(j)}\right)}_{\text{Utility from strategic bid}} - \underbrace{u\left(t_i, t_i, \boldsymbol{t}_{-i}^{(j)}\right)}_{\text{Utility from truthful bid}} \right\}$$

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Might not be finite-time procedure

Overriding goal:

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if true & reported types from **finite subset** F of type space

$$\hat{\gamma} = \max_{t_i, t_i' \in F} \left\{ \frac{1}{N} \sum_{j=1}^{N} u\left(t_i, t_i', \boldsymbol{t}_{-i}^{(j)}\right) - u\left(t_i, t_i, \boldsymbol{t}_{-i}^{(j)}\right) \right\}$$

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Estimate used in mechanism design via deep learning:

Add constraint requiring this estimate be small

[Feng, Narasimhan, and Parkes, AAMAS'18; Golowich, Narasimhan, and Parkes, IJCAI'18; Dütting, Feng, Narasimhan, Parkes, and Ravindranath, ICML'19]

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Challenge:

Might miss pairs of true & reported types with large utility gains

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?

1. Which finite subset?
2. $|\hat{\gamma} - \gamma| \leq$ **?**

Which finite subset?

1. **Uniform grid**

Easy to construct

Works if distribution is “nice”

2. Learning theoretic cover (standard ML theory techniques)

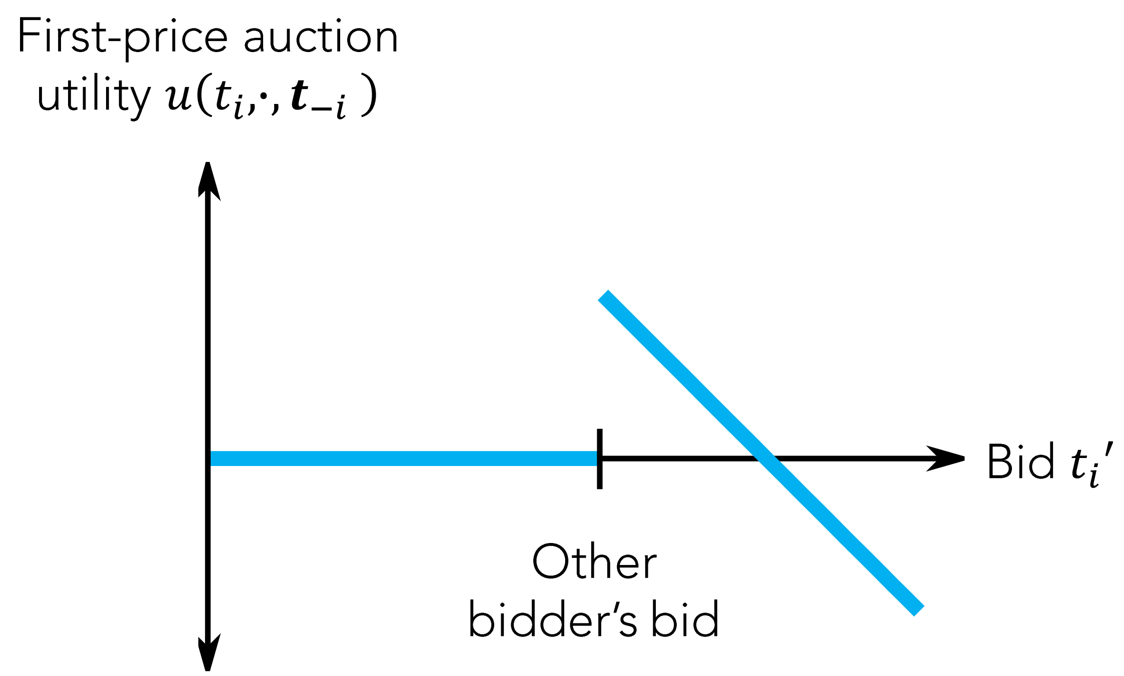
Can be hard to construct

Always works

Uniform grid

Challenge:

Utility functions are volatile



Uniform grid

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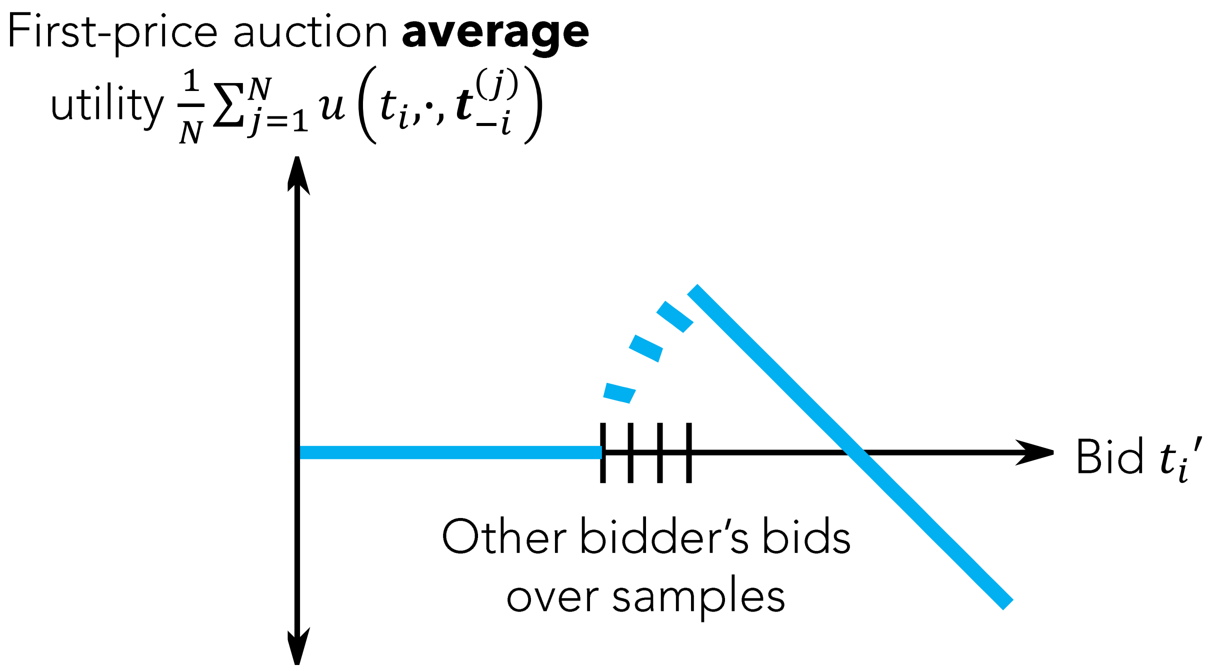
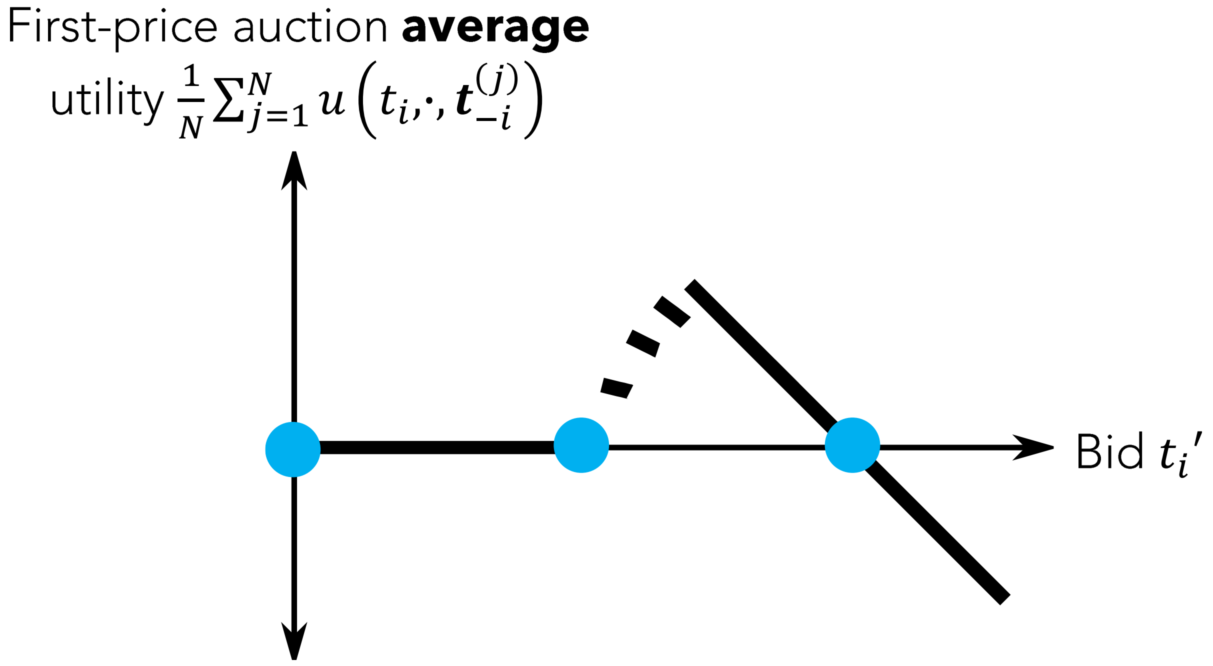


Uniform grid

Coarse discretization can lead to poor utility estimation



When is the distribution "nice" enough to use a grid?

Dispersion

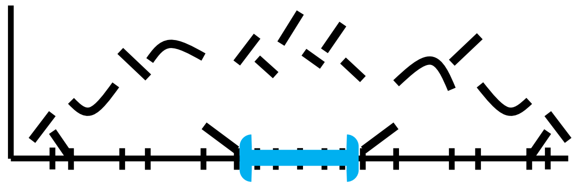
Functions $u_1, \ldots, u_N$ are $(\boldsymbol{w}, \boldsymbol{k})$-*dispersed* if:

Every w-ball contains discontinuities of $\leq k$ functions

[Balcan, Dick, and Vitercik, FOCS'18]

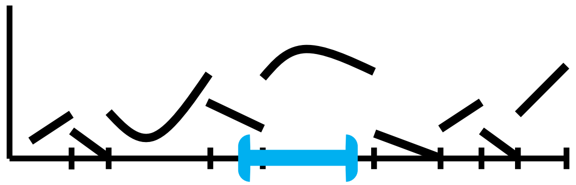
Plot $\frac{1}{N}\sum u_i$:

Not dispersed



Many discontinuities in interval

Dispersed



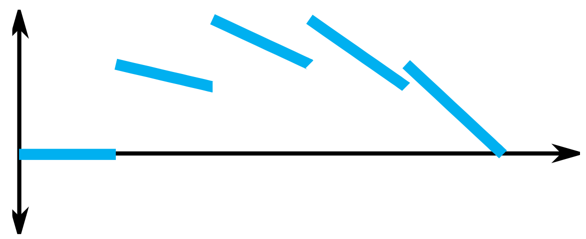
Few discontinuities in interval

Uniform grid: Guarantees

Our estimate $\hat{\gamma}$:

Maximum utility agent can gain by misreporting her type,
on average over samples,
if true & reported types from **finite subset** of type space

Theorem (informal)**:** If utility functions induced by N samples are:
(w, k)-dispersed and piecewise L-Lipschitz



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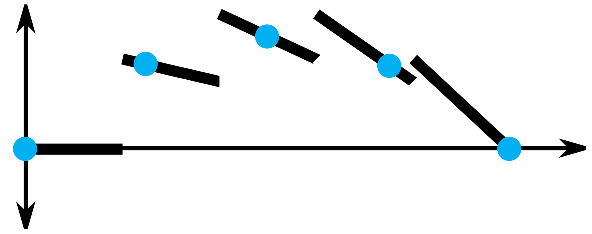
$\Rightarrow$ Can use w-grid as finite subset



Uniform grid: Guarantees

Theorem (informal)**:** If utility functions induced by N samples are:

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Estimation error: $|\hat{\gamma} - \gamma| = \tilde{O}\left(Lw + \frac{k}{N} + \sqrt{\frac{d}{N}}\right)$

d = standard ML measure of utility functions' **intrinsic complexity**

Uniform grid: Guarantees

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Proof idea:

- If snap types to grid, average utility only changes by $\leq \boldsymbol{Lw} + \boldsymbol{\frac{k}{N}}$

Uniform grid: Guarantees

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Proof idea:

- If snap types to grid, average utility only changes by $\leq Lw + \frac{k}{N}$
- $\sqrt{\frac{d}{N}}$ additional error incurred from sampling

Uniform grid: Guarantees

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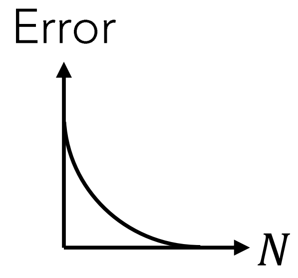
Estimation error: $|\hat{\gamma} - \gamma| = \tilde{O}\left(Lw + \frac{k}{N} + \sqrt{\frac{d}{N}}\right)$

When $w = O\left(\frac{1}{\sqrt{N}}\right), k = O(\sqrt{N})$:



We prove these (w, k) values hold when distribution is **nice**

Applications

When does dispersion hold?

$[0, \kappa]$ = range of density functions defining agents' type distributions

First-price auction

Error: $|\hat{\gamma} - \gamma| = \tilde{O}\left(\frac{(\#\text{bidders}) + \kappa^{-1}}{\sqrt{(\#\text{samples})}}\right)$

Also analyze **combinatorial** first-price auctions

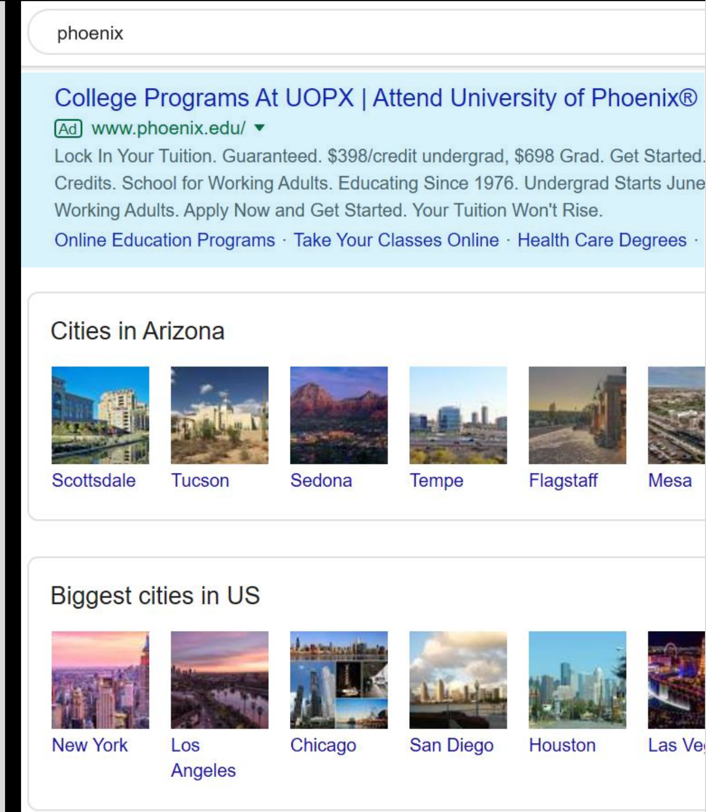
Applications

When does dispersion hold?

$[0, \kappa]$ = range of density functions defining agents' type distributions

Generalized second-price auction

Error: $|\hat{\gamma} - \gamma| = \tilde{O}\left(\frac{(\#\text{bidders})^{3/2} + \kappa^{-1}}{\sqrt{(\#\text{samples})}}\right)$



Applications

When does dispersion hold?

$[0, \kappa]$ = range of density functions defining agents' type distributions

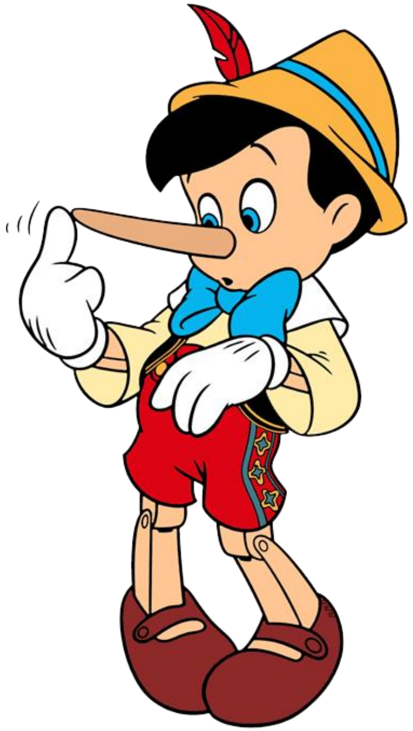
Discriminatory and uniform price auctions

Generalization of first-price auction to multi-unit settings

Error: $|\hat{\gamma} - \gamma| = \tilde{O}\left(\frac{(\#\text{bidders})(\#\text{units})^2 + \kappa^{-1}}{\sqrt{(\#\text{samples})}}\right)$

Conclusion

- Provide techniques for estimating how far mechanism is from IC
- Introduce empirical variant of approximate IC
- Bound estimate's error using *dispersion*
- Guarantees for:
  - First-price (combinatorial) auction
  - Generalized second-price auction
  - Discriminatory auction
  - Uniform price auction
  - Second-price auction under spiteful agents



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