

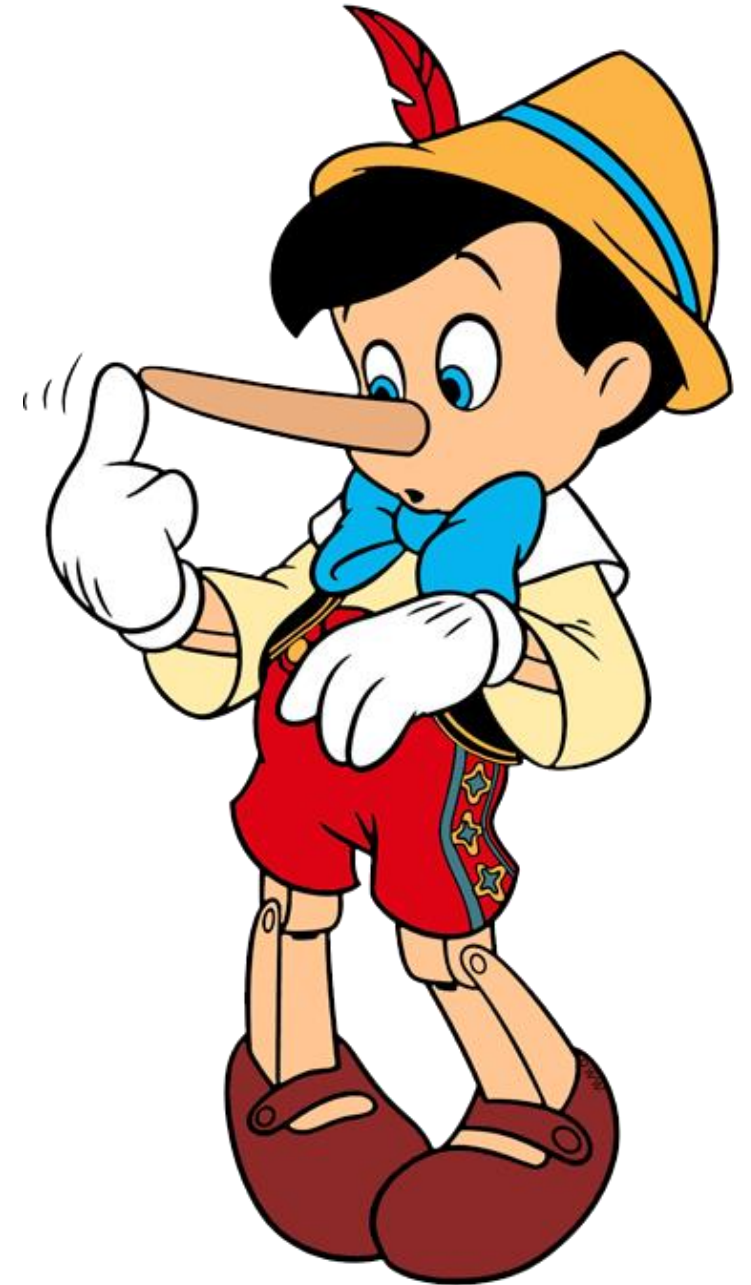
Estimating Approximate Incentive Compatibility

**Maria-Florina Balcan,
Tuomas Sandholm, and
Ellen Vitercik**

Incentive compatibility (IC)

Fundamental concept in mechanism design

Buyers maximize their utilities by bidding truthfully



Many real-world mechanisms are
not incentive compatible

Discriminatory auctions

Multi-unit variant of first-price auction

Not incentive compatible

Used to sell treasury bills since 1929

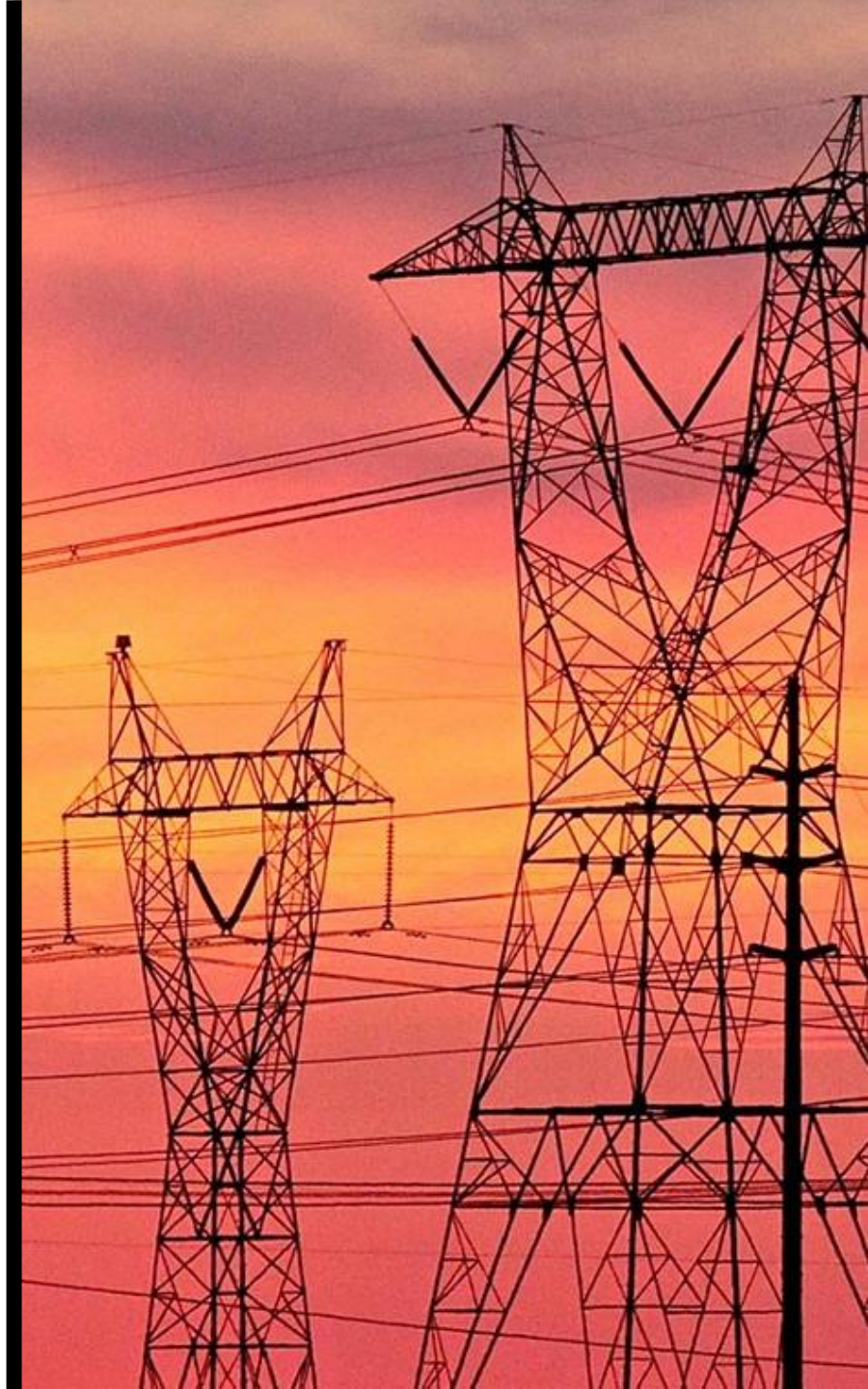


Discriminatory auctions

Multi-unit variant of first-price auction

Not incentive compatible

Used to sell treasury bills since 1929
and electricity in the UK



GSP auction

Used for sponsored search

Not incentive compatible

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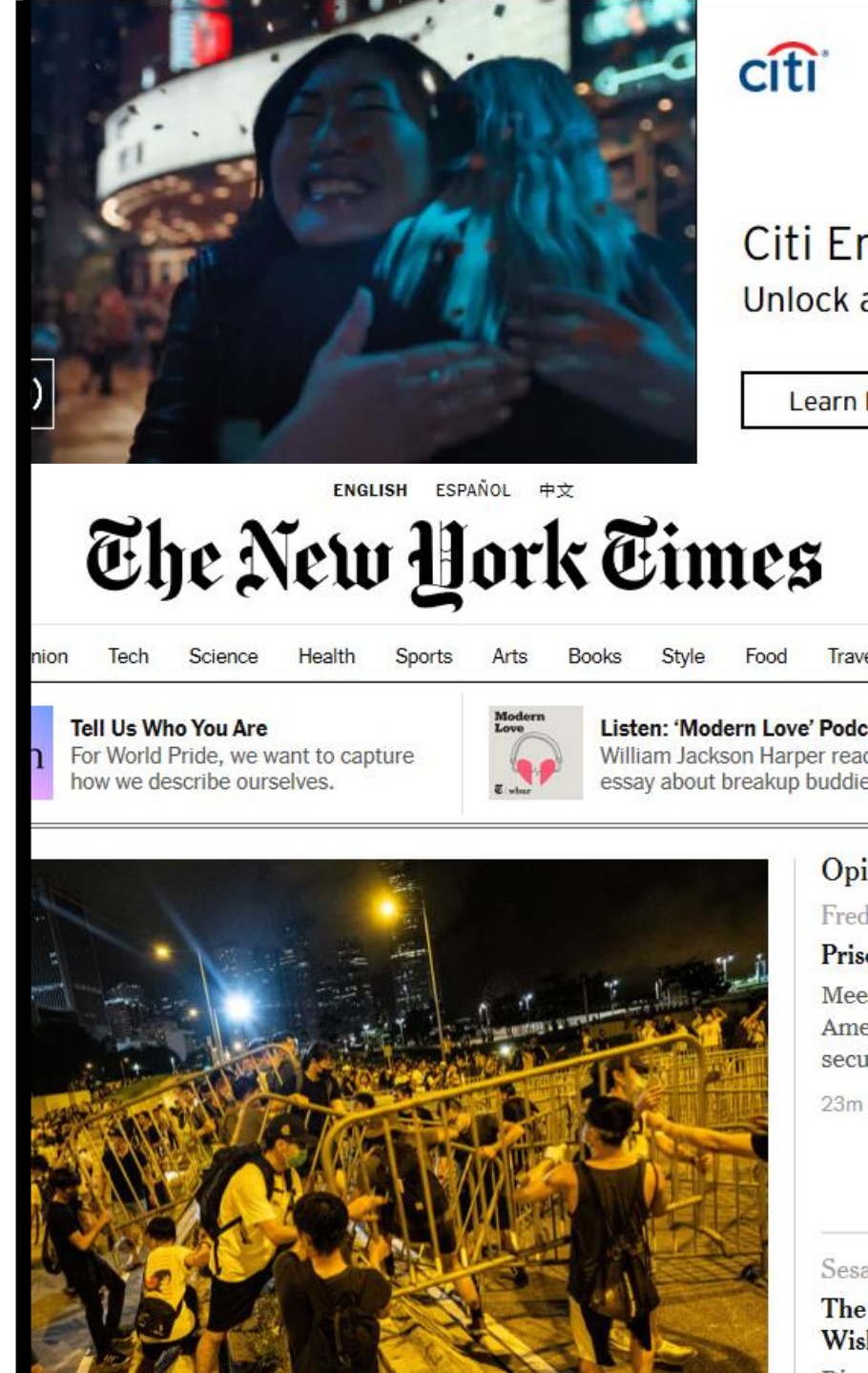


San Diego

First-price auction

Ad exchanges transitioning to FP auction

Not incentive compatible



The screenshot shows the top portion of The New York Times website. At the top right is a Citi advertisement featuring a woman hugging a child, with the Citi logo and text: "Citi En", "Unlock a", and a "Learn" button. Below the ad, the website header includes language options: "ENGLISH ESPAÑOL 中文". The main masthead reads "The New York Times". A navigation bar lists sections: "nion Tech Science Health Sports Arts Books Style Food Travel". Below this, there are two featured articles. The first is titled "Tell Us Who You Are" with the subtext "For World Pride, we want to capture how we describe ourselves." The second is a podcast preview titled "Listen: 'Modern Love' Podc" by William Jackson Harper, described as an "essay about breakup buddies". At the bottom of the screenshot is a large photograph of a crowd of people at night, some standing on a ledge, with city lights in the background. On the far right edge, a partial list of other articles is visible, including "Opin", "Fred", "Pris", "Mee", "Ame", "secu", "23m", "Sesa", "The", "Wis", and "Di".

Combinatorial auctions

Nearly all fielded combinatorial auctions
(such as sourcing auctions)
aren't incentive compatible



**Why aren't real-world mechanisms
incentive compatible?**

Why not IC?

Expensive to compute true values

Rules are **easier** to explain

Bids used to tune **future** parameters

Might leak **private** values

Agents not **risk** neutral



Approximate incentive compatibility

Mechanism is γ -IC when for each bidder i :

If everyone except bidder i is truthful,

she can only increase utility by γ if she bids strategically

[Kothari, Parkes, and Suri, EC'03; Archer, Papadimitriou, Talwar, and Tardos, Internet Mathematics '04; Conitzer and Sandholm, IJCAI'07; Dekel, Fischer, and Procaccia, JCSS'10; Lubin and Parkes, Current Science '12; Mennle and Seuken, EC'14; Dütting, Fischer, Jirapinyo, Lai, Lubin, and Parkes TEAC'15; Azevedo and Budish, Review of Economic Studies '18; Feng, Narasimhan, and Parkes, AAMAS'18; Golowich, Narasimhan, and Parkes, IJCAI'18; Dütting, Feng, Narasimhan, Parkes, and Ravindranath, ICML'19]

Approximate incentive compatibility

Mechanism is **γ -IC** when for each bidder i :

If everyone except bidder i is truthful,

she can only increase utility by γ if she bids strategically

...in expectation over **others'** values

EX-INTERIM

(assume bidders independent)

...in expectation over **all** values

EX-ANTE

(no independence assumptions)

Approximate incentive compatibility

Literature on γ -IC assumes distribution is known in advance



Where does this knowledge come from?

We relax this assumption:

Assume only **samples** from distribution over agents' types

[Likhodedov and Sandholm, AAAI'04, AAAI'05; Balcan, Blum, Hartline, and Mansour, FOCS'05, JCSS'08; Elkind, SODA'07; Cole and Roughgarden, STOC'14; Mohri and Medina, ICML'14; Huang Mansour, and Roughgarden, EC'15; Sandholm and Likhodedov, OR'15; Morgenstern and Roughgarden, NeurIPS'15, COLT'16; Roughgarden and Schrijvers, EC'16; Devanur, Huang, and Psomas, STOC'16; Balcan, Sandholm, and **Vitercik**, NeurIPS'16, EC'18; Alon, Babaioff, Gonczarowski, Mansour, Moran, and Yehudayoff, NeurIPS'17; Gonczarowski and Nisan, STOC'17; Cai and Daskalakis, FOCS'17; Syrgkanis, NeurIPS'17, Medina and Vassilvitskii, NeurIPS'17, ...]

Overriding goal:

Estimate IC approximation factor (γ) using samples

Our estimate (first try):

Maximum utility agent i can gain by misreporting her type, on average over samples $\{\mathbf{t}_{-i}^{(1)}, \dots, \mathbf{t}_{-i}^{(N)}\}$:

$$\max_{t_i, t_i' \in \mathbb{R}^D} \left\{ \frac{1}{N} \sum_{j=1}^N u \left(t_i, t_i', \mathbf{t}_{-i}^{(j)} \right) - u \left(t_i, t_i, \mathbf{t}_{-i}^{(j)} \right) \right\}$$

Utility from
strategic bid

Utility from
truthful bid

Overriding goal:

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Might not be
finite-time
procedure

Overriding goal:

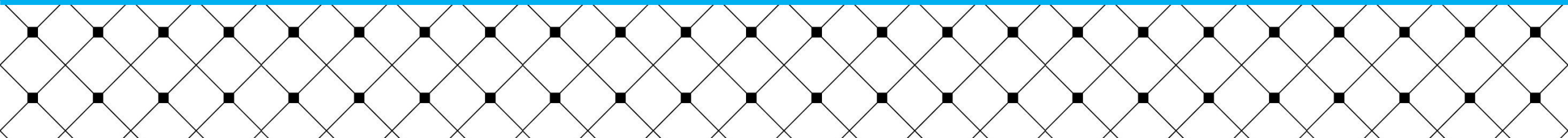
Estimate IC approximation factor (γ) using samples

Our estimate $\hat{\gamma}$:

Maximum utility agent i can gain by misreporting her type,
on average over samples $\{\mathbf{t}_{-i}^{(1)}, \dots, \mathbf{t}_{-i}^{(N)}\}$,

if true & reported types from **finite subset** F of type space

$$\hat{\gamma} = \max_{t_i, t_i' \in F} \left\{ \frac{1}{N} \sum_{j=1}^N u(t_i, t_i', \mathbf{t}_{-i}^{(j)}) - u(t_i, t_i, \mathbf{t}_{-i}^{(j)}) \right\}$$



Overriding goal:

Estimate IC approximation factor (γ) using samples

Our estimate $\hat{\gamma}$:

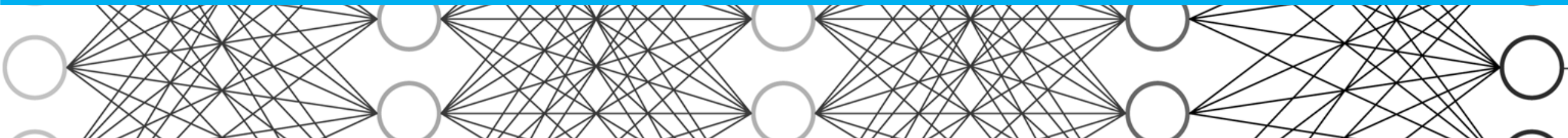
Maximum utility agent i can gain by misreporting her type,
on average over samples $\{t_{-i}^{(1)}, \dots, t_{-i}^{(N)}\}$,

if true & reported types from **finite subset** F of type space

Estimate used in mechanism design via deep learning:

Add constraint requiring this estimate be small

[Feng, Narasimhan, and Parkes, AAMAS'18; Golowich, Narasimhan, and Parkes, IJCAI'18;
Dütting, Feng, Narasimhan, Parkes, and Ravindranath, ICML'19]



Overriding goal:

Estimate IC approximation factor (γ) using samples

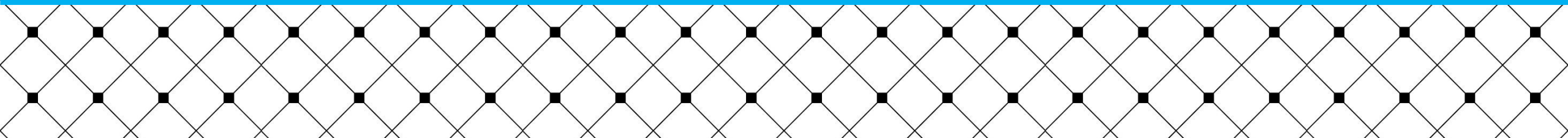
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if true & reported types from **finite subset** F of type space

Challenge:

Might miss pairs of true & reported types with large utility gains



Overriding goal:

Estimate IC approximation factor (γ) using samples

Our estimate $\hat{\gamma}$:

Maximum utility agent i can gain by misreporting her type,
on average over samples $\{t_{-i}^{(1)}, \dots, t_{-i}^{(N)}\}$,

if true & reported types from **finite subset** F of type space

- 1. Which finite subset?
- 2. $|\hat{\gamma} - \gamma| \leq ?$

Which finite subset?

1. Uniform grid



Easy to construct



Works if distribution is “nice”

2. Learning theoretic cover (standard ML theory techniques)



Can be hard to construct



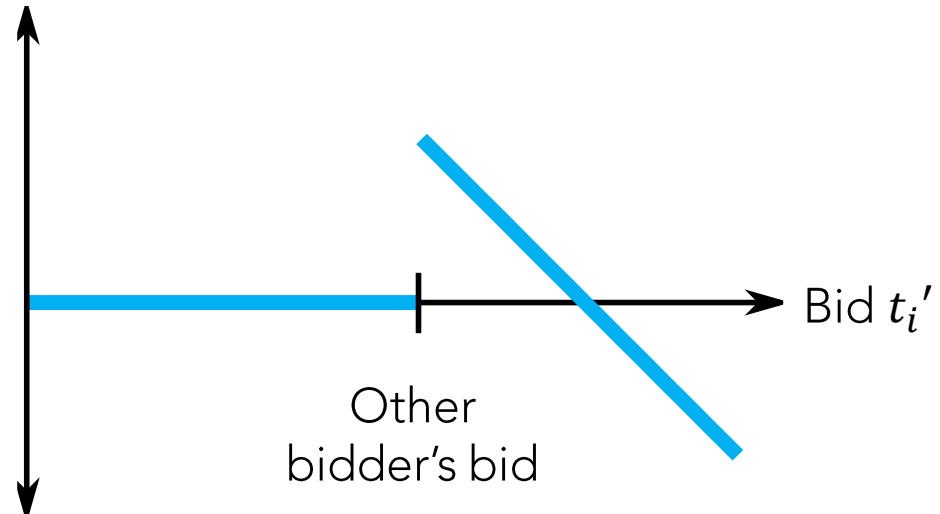
Always works

Uniform grid

Challenge:

Utility functions are volatile

First-price auction
utility $u(t_i, \cdot, \mathbf{t}_{-i})$



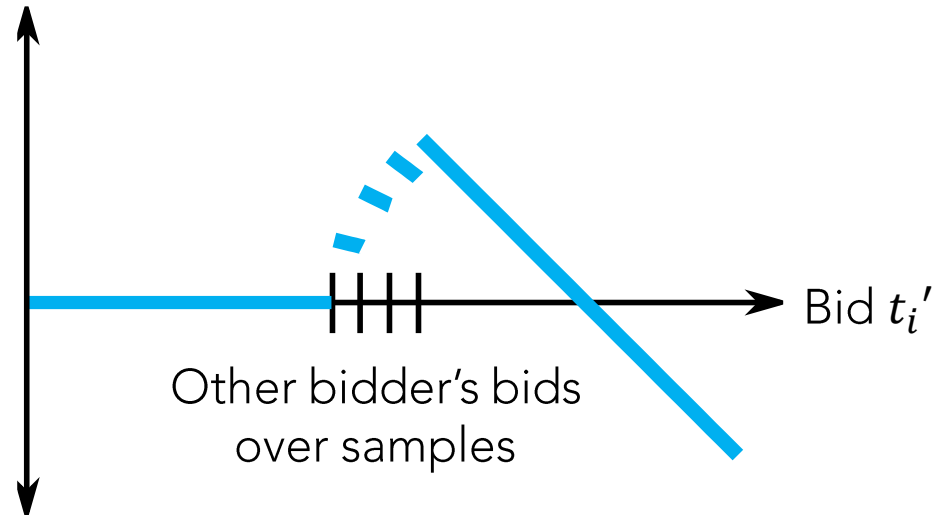
Uniform grid

Challenge:

Utility functions are volatile

First-price auction **average**

$$\text{utility } \frac{1}{N} \sum_{j=1}^N u(t_i, \cdot, \mathbf{t}_{-i}^{(j)})$$

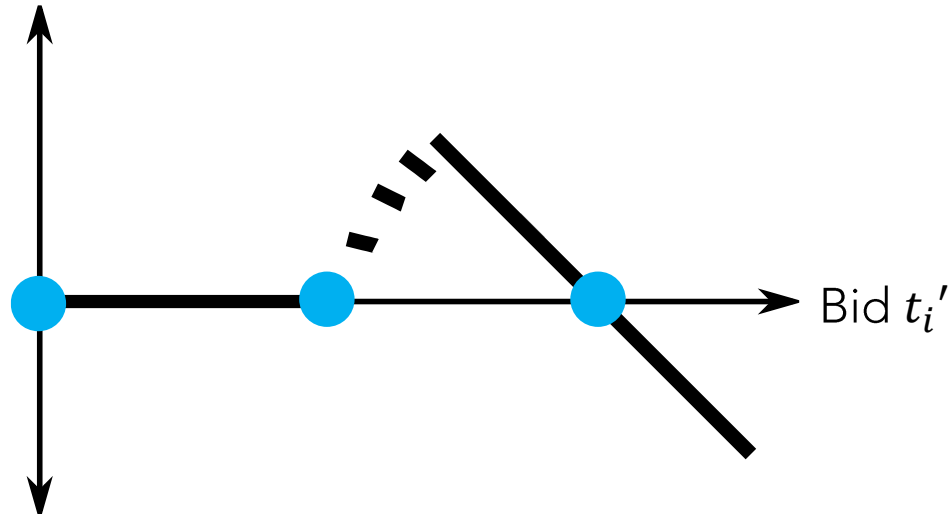


Uniform grid

Coarse discretization can lead to poor utility estimation

First-price auction **average**

$$\text{utility } \frac{1}{N} \sum_{j=1}^N u(t_i, \cdot, \mathbf{t}_{-i}^{(j)})$$



**When is the distribution “nice”
enough to use a grid?**

Dispersion

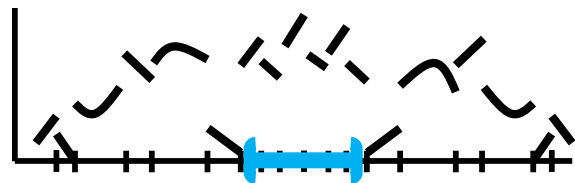
Functions u_1, \dots, u_N are (w, k) -dispersed if:

Every w -ball contains discontinuities of $\leq k$ functions

[Balcan, Dick, and [Vitercik](#), FOCS'18]

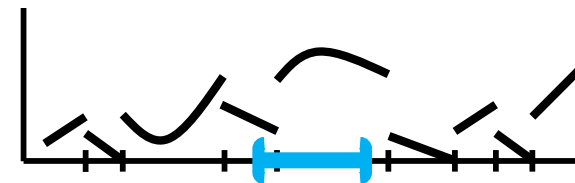
Plot $\frac{1}{N} \sum u_i$:

Not dispersed



Many discontinuities in interval

Dispersed



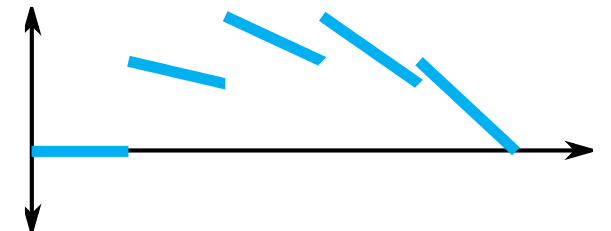
Few discontinuities in interval

Uniform grid: Guarantees

Our estimate $\hat{\gamma}$:

Maximum utility agent can gain by misreporting her type,
on average over samples,
if true & reported types from **finite subset** of type space

Theorem (informal): If utility functions induced by N samples are:
 (w, k) -dispersed and piecewise L -Lipschitz

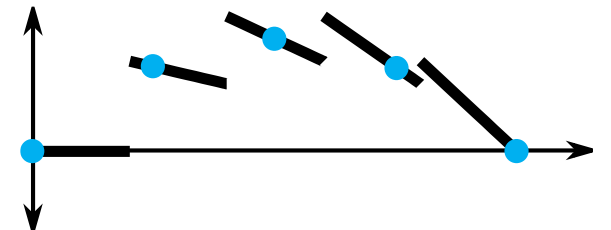


Uniform grid: Guarantees

Our estimate $\hat{\gamma}$:

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Theorem (informal): If utility functions induced by N samples are:
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 \Rightarrow Can use w -grid as finite subset



Uniform grid: Guarantees

Theorem (informal): If utility functions induced by N samples are:
(w, k)-dispersed and piecewise L -Lipschitz
 \Rightarrow Can use w -grid as finite subset

Estimation error: $|\hat{\gamma} - \gamma| = \tilde{O} \left(Lw + \frac{k}{N} + \sqrt{\frac{d}{N}} \right)$

d = standard ML measure of utility functions' **intrinsic complexity**

Uniform grid: Guarantees

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Proof idea:

- If snap types to grid, average utility only changes by $\leq Lw + \frac{k}{N}$

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Proof idea:

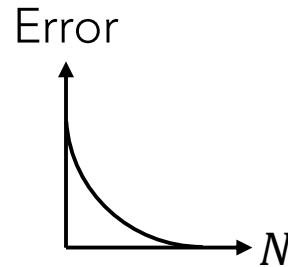
- If snap types to grid, average utility only changes by $\leq Lw + \frac{k}{N}$
- $\sqrt{\frac{d}{N}}$ additional error incurred from sampling

Uniform grid: Guarantees

Theorem (informal): If utility functions induced by N samples are:
(w, k)-dispersed and piecewise L -Lipschitz
 \Rightarrow Can use w -grid as finite subset

Estimation error: $|\hat{\gamma} - \gamma| = \tilde{O} \left(Lw + \frac{k}{N} + \sqrt{\frac{d}{N}} \right)$

When $w = O\left(\frac{1}{\sqrt{N}}\right), k = O(\sqrt{N})$:



We prove these (w, k) values hold when distribution is **nice**

Applications

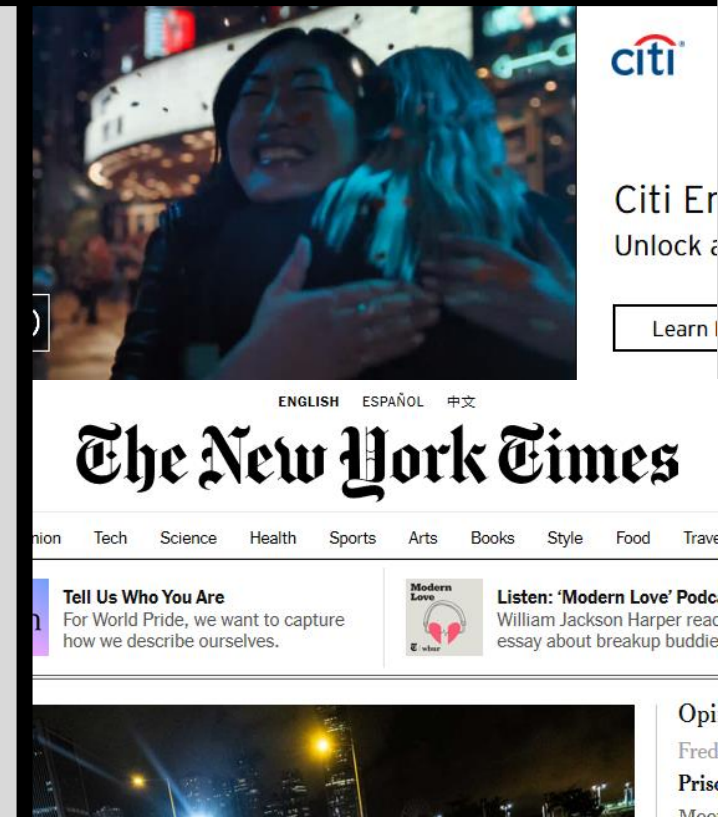
When does dispersion hold?

$[0, \kappa]$ = range of density functions defining agents' type distributions

First-price auction

$$\text{Error: } |\hat{\gamma} - \gamma| = \tilde{O}\left(\frac{(\#\text{bidders}) + \kappa^{-1}}{\sqrt{(\#\text{samples})}}\right)$$

Also analyze **combinatorial** first-price auctions



Applications

When does dispersion hold?

$[0, \kappa]$ = range of density functions defining agents' type distributions

Generalized second-price auction

$$\text{Error: } |\hat{\gamma} - \gamma| = \tilde{O} \left(\frac{(\#\text{bidders})^{3/2} + \kappa^{-1}}{\sqrt{(\#\text{samples})}} \right)$$

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Las Vegas

Applications

When does dispersion hold?

$[0, \kappa]$ = range of density functions defining agents' type distributions

Discriminatory and uniform price auctions

Generalization of first-price auction to multi-unit settings

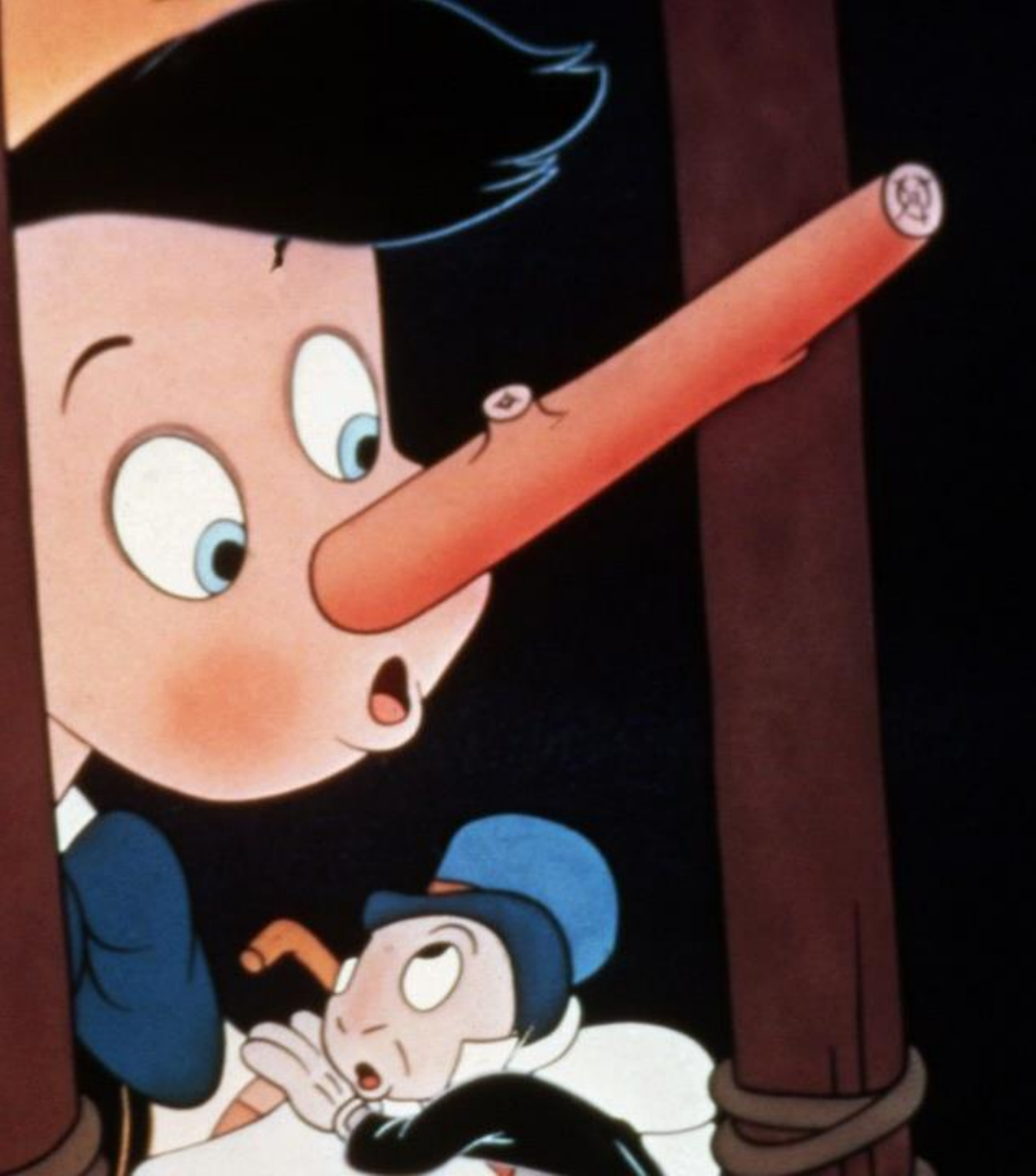
$$\text{Error: } |\hat{\gamma} - \gamma| = \tilde{O} \left(\frac{(\#\text{bidders})(\#\text{units})^2 + \kappa^{-1}}{\sqrt{(\#\text{samples})}} \right)$$



Conclusion

- Provide techniques for estimating how far mechanism is from IC
- Introduce empirical variant of approximate IC
- Bound estimate's error using *dispersion*
- Guarantees for:
 - First-price (combinatorial) auction
 - Generalized second-price auction
 - Discriminatory auction
 - Uniform price auction
 - Second-price auction under spiteful agents





Estimating Approximate Incentive Compatibility

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