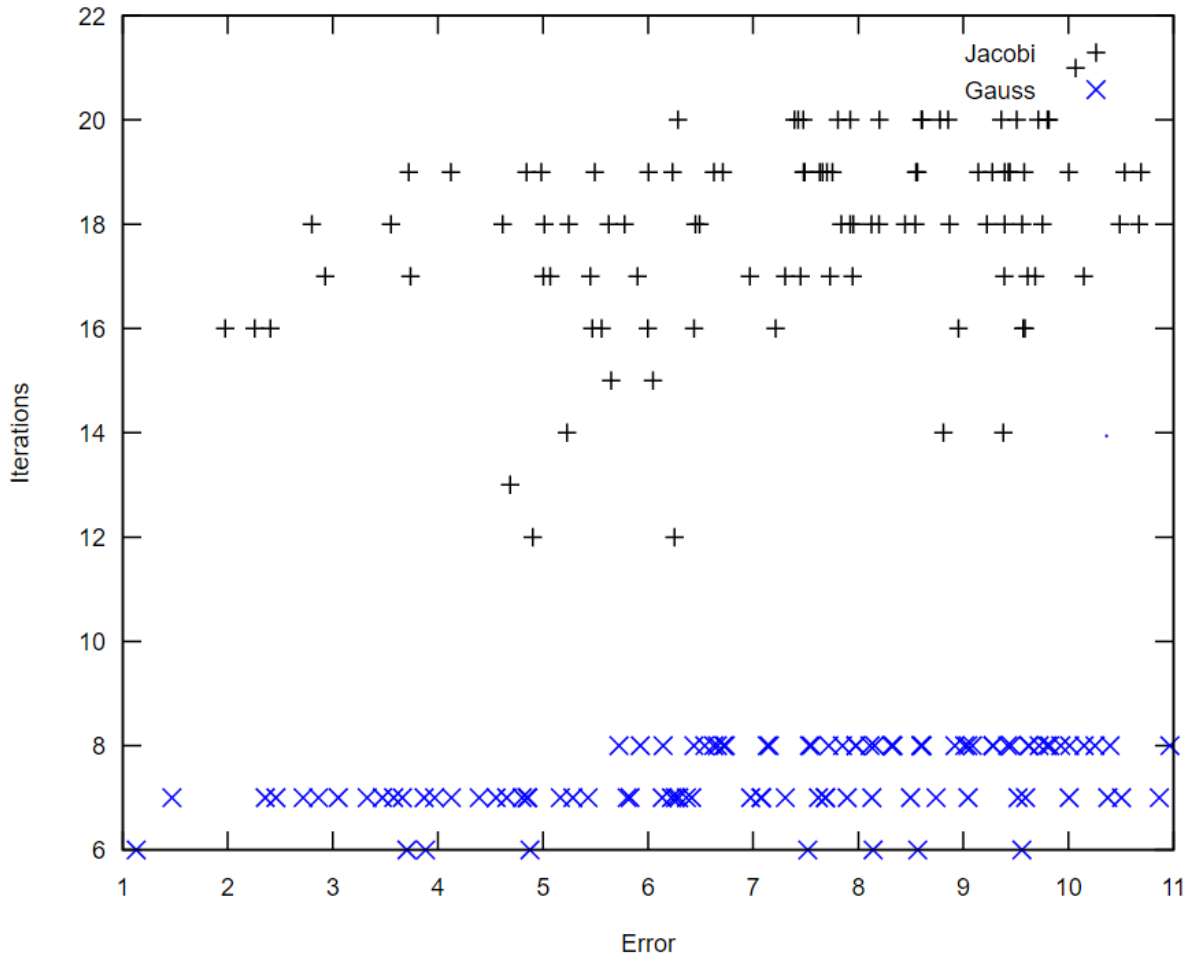
Iteration Convergence Analysis

Part 1

Both the Gauss-Seidel and Jacobi Iteration methods consistently converged on the correct solution resulting in a negligible approximate error for both of the methods: Jacobi had an approximate error of 1.17×10^{-6} and Gauss-Seidel had an approximate error of 4.71×10^{-7}, both values that are extremely close to 0. On average, Jacobi took 2.54 more iterations than Gauss-Seidel ($N_{jacobi}/N_{Gauss\text{-}seidel} = 2.54$). When plotting the error between the initial guess vector and the actual result versus the number of iterations for both methods, the following graph is generated.



These results imply that both Gauss-Seidel and Jacobi are accurate iteration methods that given enough time should converge on a solution. As expected, GS consistently found converged on a solution in significantly less iterations and with greater accuracy. Both of these stats (number of iterations and approximate accuracy) imply that at each iteration step the GS method jumps higher in accuracy than does the Jacobi method. One drawback to using the GS method could be that each iteration is more complex and processor intensive than each of Jacobi's iterations which could lead to a greater time per iteration. This could mean that the Jacobi method is more suited for problems that require many iterations while the GS method would be more suited for problems that do not. Since we did not

measure time in seconds, but only in iterations, this is speculation based on knowledge of the algorithms.

0	-1/3	-1/9
-1/3	0	-1/3
-1/9	-1/3	0

$S^{-1}T$ for the Jacobi Method

Spectral radius: 0.53022

0	-1/3	-1/9
0	1/9	-8/27
0	0	1/9

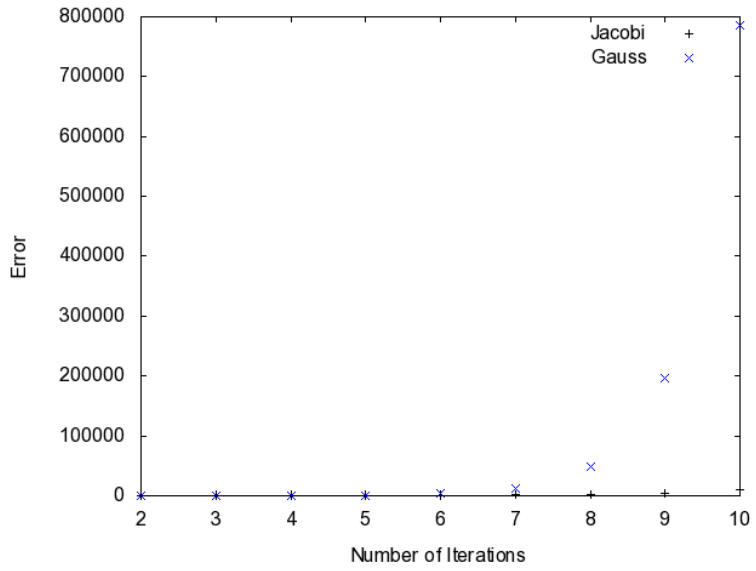
$S^{-1}T$ for the Gauss-Seidel Method

Spectral radius: 0.11111

As can be seen above, the Jacobi Method has a spectral radius of 0.53022 and the GS method has a spectral radius of 0.1111. For any iteration method, the convergence condition is that the spectral radius be under 1. The number of iterations is positively correlated with the magnitude of the spectral radius which is represented in our data. The spectral radius of the Jacobi Method is nearly 5 times that of the GS method and consistently took 2.5 times the amount of iterations. Three is also a clear correlation between the initial error and the amount of iterations required where a greater error seems more likely to require a larger amount of iterations. This correlation is displayed through the greater frequency in iteration as the error increases. Still, the data is noisy – some values for error have multiple points on the y-axis (iterations) which implies that initial error is not the only factor to determine the amount of iterations needed.

Part 2

The graph for the matrix used in part two of number of iterations vs. error is shown below



It seems here that every iteration increases the amount of error experienced by this algorithm, this begins to make more sense when looking at the spectral radius for the $S^{-1}T$ matrices.

0	-2
-2	0

$S^{-1}T$ for the Jacobi Method

Spectral radius: 2

0	-2
0	4

$S^{-1}T$ for the Gauss-Seidel Method

Spectral radius: 4

As stated earlier, a convergence condition is that the spectral radius be less than 1. Both of these spectral radius are above 1. Gauss-Seidel has an even higher spectral radius than Jacobi. These spectral radii seem to produce an exponential increase in error from one iteration to the next. It is also clear that a greater spectral radius produces much more error from one iteration to the next.

These two problems imply that Gauss-Seidel is able to receive a more extreme spectral radius. By this I mean a greater value for U where the spectral radius is either U or 1/U. This means that that it will either approach a correct answer in a lower number of iterations, or get to a massively incorrect answer in less iterations, but in the case where the spectral radius is > 1, Jacobi won't get a correct answer either.