A Model for Software Development Effort and Cost Estimation

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Abstract—Several algorithmic models have been proposed to estimate software costs and other management parameters. Early prediction of completion time is absolutely essential for proper advance planning and aversion of the possible ruin of a project. Putnam's SLIM model offers a fairly reliable method that is used extensively to predict project completion times and manpower requirements as the project evolves. However, the nature of the Norden/Rayleigh curve used by Putnam, renders it unreliable during the initial phases of the project, especially in projects involving a fast manpower buildup, as is the case with most software projects. In this paper, we propose the use of a model that improves early prediction considerably over the Putnam model. An analytic proof of the model's improved performance is also demonstrated on simulated data.

Index Terms—Completion time, development time, early prediction, Gamma model, manpower, Norden/Rayleigh model.

1 Introduction

Software cost estimation is more of an art than a science. The proposed solutions are many, but usually too restrictive to apply across a wide range of projects. However, considerable analysis of empirical data collected over long periods of time does indicate that cost trends can be correlated with certain measurable parameters. This observation has spawned off active research in the field of process modeling, yielding a wide range of models that enable one to assess, control, and predict software costs on a real time basis. These models help to supplement, if not replace, or corroborate *expert judgment* to a great extent.

Software process models can be categorized on the basis of the paradigm employed in their conformation. Most mathematical models are definable in terms of an algorithm and can thus be termed *algorithmic models*. Boehm [1] classifies algorithmic models used for software cost estimation as follows:

- 1) *Linear models* or mathematical models that try and fit a simple line to the observed data;
- Multiplicative models that express effort as a product of constants with various cost drivers as their exponents:
- 3) *Analytic models* that usually express effort as a function that is neither linear nor multiplicative;
- 4) *Tabular models* that represent the relationship between cost drivers and development effort in a matrix form;
- 5) *Composite models* that use a combination of all or some of the aforementioned approaches.

Composite models have the advantage of being generic enough to represent a fairly large class of situations. Their

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Manuscript received 2 July 1996; revised 30 Apr. 1997. Recommended for acceptance by D. Perry. For information on obtaining reprints of this article, please send e-mail to: tse@computer.org, and reference IEEECS Log Number 101228. precise mathematical definition makes it easy to implement them on a computer. Two such composite models widely used in industry are the RCA PRICE S model and Putnam's SLIM Software lifecycle management) model. The CO-COMO model [1], proposed by Boehm, can also be considered a composite model, since it provides a combination of various functional forms made accessible to the user in a structured manner. Both the Putnam and the COCOMO models use the Rayleigh distribution as an approximation to the smoothed labor distribution curve. This is based on the observation by Norden [2], that the Rayleigh distribution provides a good approximation of the manpower curve for various hardware development processes. But Boehm points out that the slow manpower buildup and the long tail-off time, characteristic of the Rayleigh curve, is not in accordance with the labor curves of most "organicmode" software projects [1]. Software project generally have a faster buildup rate than hardware projects, and this is a deviation from the Rayleigh curve. To compensate for this the COCOMO model uses only the central portion of the Rayleigh curve to arrive at the labor estimating equation. An alternative model to the Rayleigh curve was proposed by Parr [3]. The Parr model uses a sech² curve to describe a work profile. However, a methodology to estimate the parameters constituting the Parr model is not well defined [4]. This, to a certain extent, accounts for the continued popularity of the Norden/Rayleigh model over the Parr model in industry.

The intrinsics of the RCA model are largely unpublished, which renders it unavailable for scrutiny and subsequent improvement. On the other hand, some information regarding the internal constitution of the SLIM [5] has been made available to academia. The Putnam model can be used in process analysis to assess the impact of a tightened schedule, and to predict long term costs as a project evolves. Given the total manpower available for a project, the Putnam model predicts probable completion times through straightforward curve-fitting techniques. The model is an extension to Norden's assumption, and pro-

vides an empirical solution to the software productivity problem. Fitting a mathematical model such as the one proposed by Norden, to data acquired while a project is in progress, enables one to predict the probable completion time of the project as well as its total manpower requirement.

The Putnam model provides a simple, computationally feasible way of predicting software costs. However, its capability in predicting development time and total manpower requirements at an early stage is not satisfactory [6]. It can be shown that this inability to predict certain parameters well ahead of time, is a natural mathematical limitation of the underlying Norden/Rayleigh equation in representing observed data. Early prediction is critical, and a model that can generate advance warning without resorting to excessive computation would be of considerable value as a substitute for the Rayleigh curve used by Putnam.

2 THE PUTNAM MODEL

There is empirical evidence to the effect that life-cycle patterns do follow the trend of a bell-shaped curve with a left skew. Norden [2] of IBM observed that the Rayleigh curve can be used as an approximate model for a range of hardware development projects that he surveyed at IBM. This approach was later extended by Putnam to apply to software projects. Putnam observed that the Rayleigh curve Fig. 1 was a close representation, not only at the project level, but also for software subsystem development. As many as 150 projects were studied by Norden [7] and subsequently by Putnam, and apparently both researchers observed the same tendency for the manpower curve to rise, peak, and then exponentially tail-off as a function of time.

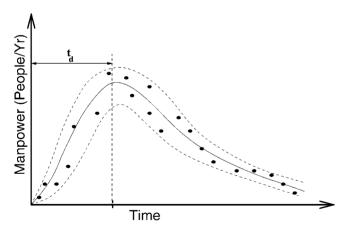


Fig. 1. Typical manpower profile.

The Rayleigh equation, it should be noted, is generally used to represent a Rayleigh probability distribution function. However, in this context, the equation does not carry its intended function of representing a distribution. It merely serves to generate a curve with the essential properties.

2.1 The Norden/Rayleigh Curve

The Norden/Rayleigh equation represents manpower, measured in people per unit time, as a function of time. It is usually expressed in manyear/year (MY/YR). Putnam points out that for a tolerance of ± 25 percent of the ex-

pected manpower value during the manpower buildup phase of the profile, the overall life-cycle can be represented as:

$$\dot{p} = 2Kate^{-at^2}MY / YR \tag{1}$$

Equation (1) expresses manpower as a function of time. The various parameters in the equation are defined as follows:

- \dot{p} represents the manpower in MY/YR;
- K represents the total area under the curve, and
- a is a constant such that $a = 1/(2t_d^2)$, where t_d is the time for manpower to peak.

On integration, (1) yields the cumulative manpower at any instant t. The result of integration is a function that increases exponentially. As time elapses, manpower asymptotically approaches the value K.

$$p = K \left(1 - e^{-at^2} MY \right) \tag{2}$$

The cumulative curve is used extensively in the analysis of slow varying software metrics [8]. But it can smother useful information if high frequency variations are characteristic of the phenomenon being analyzed. The cumulative manpower curve is not of much use in practical applications [6]. This is due to the fact that the process of integration has the effect of removing fast variations from the data set, rendering the model considerably insensitive to variations. The slope variation is about the only information one can extract from a cumulative curve, as it would give some indication as to when the manpower buildup would saturate. But slope variations are directly available from the \dot{p} plot. Moreover, as Warburton [6] points out, management can easily misinterpret cumulative manpower curves owing to the fact that data smoothing introduced by the process of filtering the manpower curve, irreparably destroys precious time domain information.

2.2 Predicting t_d and K

The approach to solving (1) is fairly straightforward. The Putnam model can be fitted with partial manpower data when it becomes available. A Rayleigh curve that best fits the available data set is arrived at using a parameter transformation method. The values of K and t_d corresponding to this model will then provide an estimate of the total manpower requirement and the time for manpower to peak, respectively. Fitting a Rayleigh curve to a given data set through regular nonlinear regression is a complicated process, without the aid of specialized statistical tools [9]. Putnam circumvents this hurdle by transforming (1) with natural logarithms (Box and Pallesen [5]). This transformation removes the exponential factors, and (1) can then be rewritten as:

$$\ln\left(\frac{\dot{p}}{t}\right) = \ln\left(\frac{K}{t_d^2}\right) + \left(\frac{-1}{2t_d^2}\right)t^2 \tag{3}$$

Setting $\ln (\dot{p} / t)$ to y, and t^2 to x, (3) gets reduced to the simple linear form:

$$y = mx + c$$

where the slope m is the solution of the equation:

$$m = -1 / \left(2t_d^2\right) \tag{4}$$

and the y-intercept *c* is:

$$c = \ln(K / t_d^2) \tag{5}$$

As manpower data becomes available, it is transformed to $\ln (\dot{p} / t)$, and plotted against t^2 . The resulting graph is fitted with a straight line. Equations (4) and (5) can then be used to arrive at the predicted values of t_d and K once the slope and the intercept of the fitted line stabilize.

2.3 Limitations of the Norden/Rayleigh Equation

Though Putnam does not give any theoretical reasoning for the choice of the Rayleigh distribution as a representative model, it is obvious from simple visual inspection that the basic trend shown by most manpower profiles does resemble a Rayleigh curve. As pointed out earlier, linearizing the Norden/Rayleigh equation yields a function relating ln (\dot{p}/t) and t^2 . From (4), the time for development t_d , can be computed as $\sqrt{-1/(2m)}$. It should however be noted that, if the slope of the fitted line were to become zero or positive, t_d would make no practical sense. In other words, the linear regression estimate yields meaningful results only for negative, nonzero values of m, which is the slope of the fitted line. The manner in which $\ln(\dot{p}/t)$ varies with time, decides the validity of the computed value of t_d .

During the initial stages of the project, t values are naturally small by definition. However, there is a rapid initial buildup of manpower for small values of t. The cumulative effect of these two phenomena is that \dot{p} tends to increase faster than t, especially at the beginning of the project. This implies that the variation of \dot{p} /t with t is more than linear. The Putnam approach takes t^2 on the abscissa to fit a line to deemphasize the superlineal increase in \dot{p}/t . However, this does not reverse the increasing trend of the plotted parameter. This tendency will be illustrated shortly with actual data in Section 2.4. The resulting linearized model will have a non-negative slope, making t_d and K value predictions virtually impossible during the initial stages of the project. This is in concurrence with Warburton's observation [6] that the Putnam model prediction is reliable only after the manpower has peaked, by which time almost 40 percent of the project has been completed. Studies have also shown that medium scale projects [10], where rapid manpower buildup is fairly common, tend to deviate quite noticeably from the Putnam model.

2.4 A Real-Time Example

The data shown in Fig. 2, published by Warburton in his paper [6], depicts the manpower curve for a project involving a sonar and fire control system. The profile shown here is for the entire project, starting with design, up to the point of its delivery to the customer. The Putnam model offers a close fit for this particular data set. The data set shows a distinctive peak and tail-off that Norden and Putnam observed in their independent surveys of large scale projects [5].

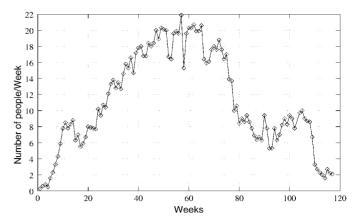


Fig. 2. Manpower profile for a project.

Prediction of K and t_d is a matter of iteratively fitting the Rayleigh curve to manpower data as and when it becomes available. The form of (3) is used to linearize the manpower equation. Linear regression is then applied on transformed data to arrive at the most representative model. Fig. 3 shows a plot of the transformed data, to which a straight line can be easily fitted. However, it is seen that the data has a tendency to first increase in magnitude, due to the fact that manpower increases at a considerable rate during the development phase of the project. As a result of this, early linear regression trials, with incomplete data, tend to yield lines with non-negative slopes, yielding a complex predicted value of t_d . The slope becomes negative only later into the project, after the peak of the Rayleigh curve has been attained.

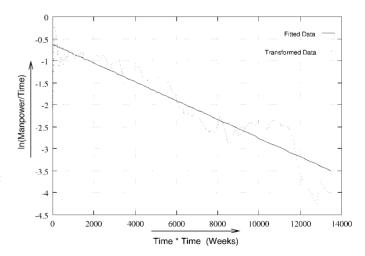


Fig. 3. Transformed profile for the project.

The predictive capability of the model is restricted to the range of data points for which the slope of the fitted line is negative. The manner in which the slope of the fitted line varies as data becomes available, is shown in Fig. 4. The predicted development time t_d , represented by (4), is a complex number up to the zero crossing point of the curve shown in the figure. And hence, it has no physical interpretation up to that point. It is observable from Fig. (4), that the slope becomes negative only after the 51st data point is

available. This corresponds to the completion of product development, which is about 40 percent of the project. A development project can be considered to be made up of several subcycles, as pointed out by Putnam. The different activities that comprise the project up to the completion of product development is shown in Fig. 5.

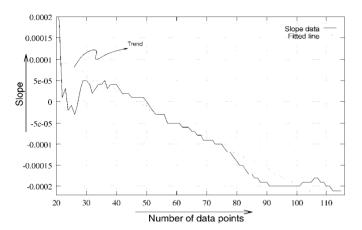


Fig. 4. Slope variation of fitted line vs. increasing record length.

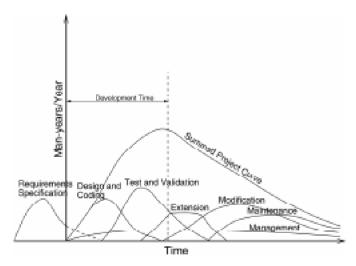


Fig. 5. The various activities that comprise a project.

Fig. 4 shows a plot of the slope variation, superimposed with a simple linear model. The linear model is fairly representative of the slope of the curve, more so after the slope variation monotonically decreases. Deviations such as the zero crossing between the 20th and the 30th data points can be attributed to the random deviations in the acquired data. In other words, the model provides a meaningful prediction for t_d and K, only beyond a small range close to t_d . It is evident that the predictive power of the model would be even more diminished for faster manpower buildup rates during the development stage.

A fast manpower buildup during the development phase would ultimately manifest itself as a bell-shaped profile that is excessively skewed to the left. In the linearized domain, represented by (3), this phenomenon translates to a positive deviation of the data from the fitted line. Early prediction failure occurs since the simple linear model fails to capture

the initial superlineal variation of the observations. There are basically two ways in which the initial partial data can be accommodated to yield interpretable regression results:

- Fit a curve with a higher degree to the data set, or
- Find an alternative transformation for the axes.

Fitting a higher degree polynomial to the transformed data would complicate the process of applying the inverse transformation to the data. The original Norden/Rayleigh equation would have to be drastically modified for this purpose. Furthermore, the model would be highly data dependent and not generic. In Section 3, we introduce a model that offers an alternative transformation of the parameter axes, in a manner that compensates for the limitations mentioned earlier. This is done without contributing to the overall complexity of the model.

3 THE GAMMA MODEL

An alternative transformation of the parameter axes will eventually yield a model that is different from the Rayleigh curve. But it can however, be easily mapped onto some generic model, such as the equation to the Gamma distribution. The standard Gamma distribution [11] used to model certain probability density functions, is described by the following equation:

$$Gamma(t|\alpha,\beta) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} t^{\alpha-1} e^{-t/\beta}$$
 (6)

where $0 \ge t \ge \infty$, and α , $\beta > 0$.

Through the appropriate choice of parameters α and β , one can generate functions such as the *exponential* (α = 1), and the *chi-squared*. The Rayleigh function is in fact, a special case of the exponential function. Even functions such as the *Weibull* and *Gumbel* can be derived from the exponential distribution, which has the *memoryless* property [11]. Thus, the Gamma distribution is ideal for modeling a variety of distributions owing to its very generic form. The choice of the parameters, α and β , decides the shape of the final curve generated by (6).

3.1 Alternative Axes Transformation

It is the choice of axes for the linearizing transformation on the Norden/Rayleigh equation that contributes to the excessive positive deviation of data points during the initial phase of the project. The linearization operation yields a semilog plot of the manpower deployment rate vs. the square of the time. By choosing the rate of change of the manpower deployment rate as the variable on the y axis, the rapid increase of manpower with time can be considerably suppressed or scaled down. The scaling is achieved through time compression, which involves dividing both the axes with time. In other words, a graphical representation of the rates of change is obtained. This translates to a semilog plot with t on the abscissa, and log (Manpower/Time 2) on the ordinate. A plot of the data (Fig. 2) under this new transformation, vs. time in weeks, is shown in Fig. 6. It is seen that the tendency for data points to lie above the final fitted line during the initial stage is considerably lesser, when compared to Fig. 3.

The negative slope of the final fitted line is predictable from the distribution of the data points at the very beginning of the project, implying possible early prediction. It can be shown that this trend is achieved without any loss of information with respect to the original model.

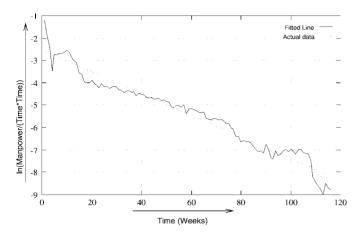


Fig. 6. Modified transformation for the project.

3.2 Equivalence of the Two Transformations

The equivalence of two transformations can be measured in terms of the information they capture from observed data. The difference between the actual observed data and the data predicted by a model, is termed the *residual* for that data set, corresponding to that model. The standard deviation and the mean evaluated over the residuals can be used to establish the *equivalence* of the two transformations. The difference in the observation and the predicted value can be thought of as a *prediction error*. The average of the prediction errors is termed the prediction *Bias* and its standard deviation is often used as a measure of the *Variation* in the predictions. The *root mean square prediction* error (RMSPE) is a measure of the closeness with which the model predicts the observation, and is given by:

$$RMSPE = \sqrt{Bias^2 + Variation^2} \tag{7}$$

Given

$$PE_{i} = (Actual_{i} - Predicted_{i})$$

$$Bias = \sum_{i=1}^{N} \frac{PE_{i}}{N}$$

and

$$Variation = \sqrt{\frac{\sum_{i=1}^{N}(PE_i - Bias)^2}{N-1}} \; .$$

The basic difference between the proposed transformation and (3) is that, both the abscissa and the ordinate are scaled down by *t*. The computed RMSPE, the *bias*, and the *Variation* are shown in Table 1.

TABLE 1
BIAS AND VARIATION FOR BOTH METHODS

Parameter	$\ln\left(\frac{\dot{p}}{t^2}\right)$ vs. t	$\ln\left(\frac{\dot{p}}{t}\right)$ vs. t^2
Bias Variation RMSPE	3.5987 ×10 ⁻¹⁶ 0.3624 0.3624	-1.5428×10^{-15} 0.3417 0.3417

It is observed that, for the data set chosen, the original transformation yields a slightly better fit, largely due to the bias term being negligible for this specific data set. However, the alternative transformation offers an improved bias value. It can safely be stated that there exists data sets for which the proposed transformation will yield a lower RMSPE than the original approach. In other words, as is intuitively evident, given a specific model, the goodness of fit is decided solely by the nature of the observed data.

3.2.1 Residual Plots for the Models

The degree to which a mathematical model is representative of the observed data set can be gauged by generating a plot of the residuals for the model. The *residue* is the amount by which each real data point deviates from the model. The residual plots can be used to indicate the actual information content in a given model, and are used here to show that the axes transformation introduced in Section 3.1 does not affect the information content of the Putnam model. This implies that the Gamma model has a certain level of equivalence with the Putnam model in terms of *information representativeness*, for the data set shown in Fig. 2.

A residual plot of the deviations for the data set corresponding to the Putnam model is shown in Fig. 7. The residues for the plot of " $\ln\left(\frac{\dot{p}}{t}\right)$ " vs. " t^2 " were calculated as follows:

$$Residual_i = ln (Manpower_i/Time_i) - Fit_i$$

where Fit_i corresponds to the ith point on the fitted line.

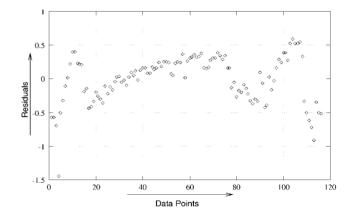


Fig. 7. Scatter plot for residuals from transformed data.

The residual plot should be ideally scattered in a random manner. In other words, it should not correlate with any observable function, or show any conspicuous trends. It is seen that the trend observed here is common to both the Putnam as well as the Gamma models and is a direct consequence of using a predefined function as a constraint to fit to a data set. A generic second order function may yield a more random scatter plot, but would not allow isolation of various management parameters such as K and t_d .

The scatter plot of the residuals for the alternative transformation introduced in Section 3.1 is shown in Fig. 8. The residuals for this new transformation, "ln $\left(\frac{\dot{p}}{t^2}\right)$ " vs. "t," are calculated as follows:

$$Residual_i = \ln(Manpower_i / Time_i^2) - Fit$$

The trend observed for the Putnam model is also seen in the scatter plot for the Gamma model implying that, for the data set under consideration, the information content lost by the Gamma model is not any more severe than that of the Putnam model.

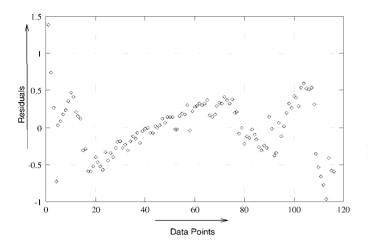


Fig. 8. Residual plot for proposed data transformation.

The trend shown by the residuals is due to the fact that a simple linear regression is being used to generate the model. A higher degree of randomness in the scatter plot is achievable if a quadratic or some other curvilinear model is used. But this cannot be achieved without increasing the complexity of the model several fold.

3.3 Ascertaining α and β : The Alternative Model

The Norden/Rayleigh equation no longer holds under the new transformation. The equation for the line fitted to the data set in Fig. 6 can be restated as following:

$$\ln\left(\frac{\dot{p}}{t^2}\right) = \gamma + \mu t \tag{8}$$

where μ is the slope, and γ the intercept of the line fitted to the data under consideration. Removing the log function on both sides gives us the following:

$$\dot{p} = e^{\gamma} t^2 e^{\mu t} \tag{9}$$

Equation (9) is a special case of the Gamma distribution curve, represented by (6). As previously mentioned, the equation to the Gamma distribution can model a wide range of functions that occur in nature. The Gamma equation can be reduced to the form of (9) with the following assignments:

$$\frac{1}{\Gamma(\alpha)\beta^{\alpha}} = e^{\gamma}$$

$$\alpha - 1 = 2$$
and $\frac{-1}{\beta} = \mu$ (10)

From (10), α = 3. Differentiating (6) with respect to t, and setting it to zero, we get the following:

$$\frac{df(t|\alpha,\beta)}{dt} = \frac{e^{-t/\beta}}{\Gamma(\alpha)\beta^{\alpha}} \left(\frac{-t^{\alpha-1}}{\beta} + (\alpha-1)t^{\alpha-2} \right)$$

$$0 = t^{\alpha-1}(-a/\beta) + (\alpha-1)t^{\alpha-2}$$

$$t = \beta(\alpha-1)$$
(11)

Equation (11) gives us the time at which the curve described by the Gamma equation peaks. In the Norden/Rayleigh curve, this occurs at the completion of product development (t_d). Thus, (11) can be restated, setting $\alpha = 3$, to arrive at a value for β as follows:

$$t = t_d = 2\beta$$

$$\beta = \frac{t_d}{2}$$
(12)

When integrated from zero to infinity with respect to t, the Gamma distribution described by (6) yields an area of unity. The model proposed by Putnam normalizes the area under the Norden/Rayleigh curve to K, the total manpower expended for the project. Doing likewise for (6), and setting α to 3, and β to $t_d/2$, the *Gamma model* can be represented as:

$$f(t) = \dot{p} = \frac{8K}{\Gamma(3)t_d^3} \left(t^2 e^{-2t/t_d}\right)$$

$$\dot{p} = \frac{4K}{t_d^3} t^2 e^{-2t/t_d}$$
(13)

Equation (13) is a special case of the Gamma distribution. It generates a curve that has a shape considerably similar to the Rayleigh curve. However, linearizing (13) does not lead to a situation yielding a complex solution for t_d , as will be demonstrated in Section 3.5. The transformation used, allows early prediction by forcing the ordinate variable to decay faster with time.

3.4 Intermodel Prediction Error

A comparison of (13) with the Norden/Rayleigh equation can now be done to quantify the difference between the two models. Fig. 9 shows the Gamma model superimposed over the Putnam model. Both models were computed for a time record of one hundred samples, and a *K* value of unity.

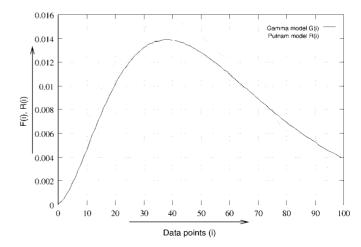


Fig. 9. The Rayleigh and the Gamma models superimposed.

The RMSPE measure is used here to quantify the behavioral differences between the two models. For this purpose, the Putnam model is taken as the reference model. The Gamma model is then used to predict the data values generated from the Putnam model. The prediction errors thus evaluated will give a measure of the difference between the two models.

The prediction bias of the Gamma model over the Putnam model is only 7.6081×10^{-04} . The prediction error has a standard deviation of 0.0013, which yields an RMSPE of only 0.0015. Though similar to the Putnam model, the Gamma model has distinct advantages. Unlike the Norden/Rayleigh equation, the shape of the Gamma distribution, in terms of its mean and variance, can be tuned to suit the class of the project being modeled. This can be achieved simply by adjusting the value of α or β . The gamma model is thus considerably more generic and *tunable* than the Putnam model. This makes it well suited for an estimation system that needs to be calibrated automatically, since it provides two independently tunable parameters.

3.5 Predicting t_d and K

The Gamma model can be fitted to the data previously introduced in Fig. 2. K and t_d can be evaluated by ascertaining the slope and the intercept of the line fitted to the transformed data. The mathematical relationships relating the two can be derived from the basic Gamma model. Linearizing the Gamma model by taking logarithms on both sides of (6), the manpower profile can be represented as:

$$\ln\left(\frac{\dot{p}}{t^{\alpha-1}}\right) = \ln\left(\frac{K}{\Gamma(\alpha)\beta^{\alpha}}\right) - \left(\frac{1}{\beta}\right)t$$

$$\gamma = \ln\left(\frac{K}{\Gamma(\alpha)\beta^{\alpha}}\right)$$
(14)

$$\mu = -\frac{1}{\beta} = \frac{-(\alpha - 1)}{t_d} \tag{15}$$

where μ is the slope of the line, and γ the intercept. In the Gamma model derived earlier, α has a value of 3. Substituting this value of α in (14) and (15), we obtain:

$$t_d = \frac{1 - \alpha}{\mu} = \frac{-2}{\mu} \tag{16}$$

$$K = \Gamma(\alpha)\beta^{\alpha}e^{\gamma} = \frac{t_d}{4}e^{\gamma}$$
 (17)

K and t_d predictions were carried out on the data shown in Fig. 2. A plot of the slope variation of the line fitted to the data vs. the number of data points available is shown in Fig. 10. There is definitely an upper bound on the rate of manpower buildup any environment can support. Thus, the deviations of data points from the ideal Rayleigh equation can never exceed a certain boundary due to real life limitations. Attempts have been made to empirically ascertain the bounds of this variation. In his book, Brooks [12] refers to estimations conducted by Vyssotsky of Bell Laboratories that indicate an upper limit on the manpower buildup rate. A buildup rate that is excessively high tends to complicate issues concerning information flow. The

lower bound on the buildup rate follows from the fact that marketing constraints often force product development to happen within a certain time frame. The Gamma model accommodates a wider range of deviations within these bounds than the Rayleigh curve. For μ to assume positive values, the rate of manpower buildup would have to be much higher than what one would encounter in a real development environment. Thus, even though an interpretable value of t_d becomes available only while μ remains negative (16) the possibility of this happening is now considerably lesser, since the Gamma curve offers more coverage over this super-Rayleigh manpower buildup range.

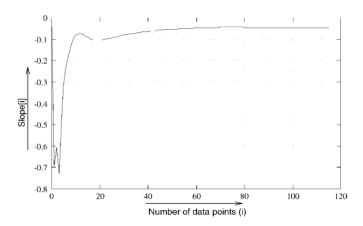


Fig. 10. Variation of slope of fitted line for the Gamma model.

It should also be noted that considerable information about the nature of the project is contained in the portion of the manpower profile that precedes the peak on the curve. The testing and maintenance phases are represented by a gradual exponential decay. Thus, the efficiency of the model is contingent on its ability to track the development phase of the product. The Gamma model tracks a wider range of positive variations during this phase of the project, yielding better performance.

A stable prediction is achieved only after the slope of the fitted line stabilizes. It can be seen from Fig. 10 that the slope converges on its final value much sooner than in the case of the Norden/Rayleigh equation (Fig. 4), thereby making early prediction of t_d and K possible.

3.6 Comparison of Predictive Capability

As mentioned earlier, the predictive capability of the model depends on how early in the project the model stabilizes. The predictions of t_d and K when both the models were run on real data (Fig. 2), are shown in Figs. 11 and 12 respectively. The parameters predicted by the two models are shown superimposed for ease of comparison.

It is readily observable from Fig. 11 that the final stable value of t_d is available from the Gamma model, by the time the 60th data point is fitted to it. The Putnam model, on the other hand, does not converge at this point. The predicted value for t_d , seems to be slightly lower for the Gamma model than for the Putnam model. However, a physically interpretable value of t_d , with a low margin of error, is available much in advance of the Gamma model. The t_d

value corresponding to the 52nd week of the project is 37.8562 for the Gamma model, and 214.5849 for the Putnam model, from Fig. 11. These readings correspond to an error of -8.1784 percent for the Gamma model, and 343.0106 percent for the Putnam model, expressed as a percentage of their final predictions of t_d .

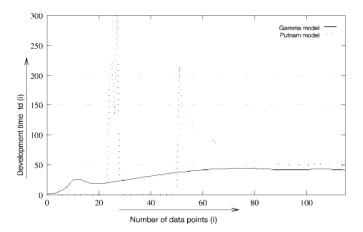


Fig. 11. Predicted t_d values for both models.

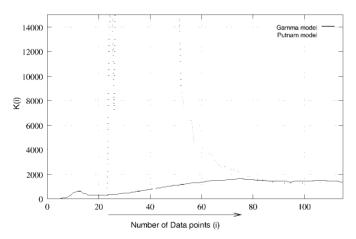


Fig. 12. Predicted *K* values for both models.

It is evident that the value of both K and t_d tend to fluctuate considerably in the case of the Putnam model. Though the final predicted value of K seems to be slightly higher for the Gamma model, a stable value for K is made available much earlier in the project by this model. Reliable K value predictions are dependent on the validity of the values predicted for t_d . For projects of long durations, extending to several years, the sampling period in time can be in terms of weeks, or even months. Early predictions of even a few points will thus translate to a fairly long period in time, owing to the coarse granularity of the measurements. Furthermore, the validity of the prediction is related to the stability of the model, as will be illustrated in Section 3.7.

3.7 Assuring Prediction Confidence

There is definitely a requirement for a criterion to indicate the confidence level of predictions. In other words, the point in time at which the model output is reliable should be clearly defined. The stability of the model, as it is fitted iteratively on evolving data, would serve this purpose considerably well, if it can be quantified. A reliable indicator of the stability of predictions, is offered by the differential of t_d values predicted by the model. The percentage differential of a running measure, is a gauge of the drift in an indicated measurement. The drift of a system, expressed as a percentage of the current reading, is generally used as a measure of system reliability in most navigation and control systems instrumentation. If one were to assume that the current prediction is true, then the percentage drift from that prediction is a measure of the system veracity. This is consistent with the philosophy of continuous skepticism employed in measurement technology [13]. A plot of this percentage differential, defined as $[t_d(i+1) - t_d(i)/t_d(i)]$, is shown in Fig. 13. Prediction confidence can then be defined in terms of a percentage tolerance limit, that offers upper and lower bounds on the percentage *drift* of predictions.

Given a rough estimate of the time frame within which one operates, a certain amount of drift in the measured parameter can be tolerated. This is somewhat akin to saying that the cybernetics of a ship would drift off course by, say, 2° for every 100 nmi traversed. Given this specification, the margin of error involved in the ship reaching its final destination can be ascertained. Also, a high drift value implies that the ship is veering off course too rapidly for comfort. For the case in point, consider a tolerance limit of ± 1 percent, as shown in Fig. 13. It is seen that the Gamma model provides stable prediction within the specified confidence tolerance limits, much earlier than the Putnam model. A stable prediction is offered by the Gamma model by more than 20 data points over the Putnam model. Since the sampling period is a week, this translates to over five months of advance warning over the Putnam model, for this specific data set.

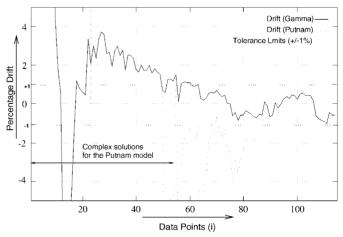


Fig. 13. Differential t_d values with tolerance limits.

The final fit on total data is shown in Fig. 14. It is seen that both models give a fairly close fit to the data sample under consideration. The Putnam model offers a slightly better fit in terms of the final RMSPE. However, the variation (standard deviation) of the residuals from either model is something solely incidental, and is totally data dependent. In other words, given a certain range within which

actual observed data can fall, it is possible to find a data set that is closer to either of the models in a least squares sense. The various prediction measures for both the models are shown in Table 2.

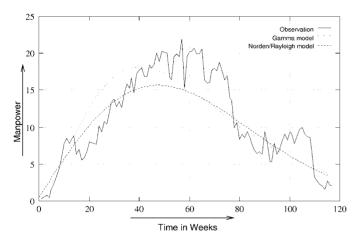


Fig. 14. Resultant fit for both models.

TABLE 2
PREDICTIVE CAPABILITY MEASURES

Parameter	Gamma model	Putnam model
Bias	0.2530	1.0185
Variation	3.0801	2.6937
RMSPE	3.0904	2.8799

The percentage by which the Gamma model prediction differs from the Putnam model, can be measured in terms of the *prediction error deviation* (PED).

$$PED = \frac{Gamma_{rmspe} - Putnam_{rmspe}}{Putnam_{rmspe}} \times 100$$
 (18)

The PED evaluated from Table 2 is only 7.3093 percent, and implies that the two models are considerably equivalent in terms of their RMSPE.

Table 3 gives the final results of the regression analysis. For the project which the data set represents, the actual product development time took 52 weeks [6]. The sudden spike around the 56th week could be attributed to ambient environmental conditions, and deviations from the assumption that manpower would peak at the completion of product development. The actual observed value of K was 1.3008×10^3 . The higher the fluctuations in the data under analysis, lesser the precision in the predicted value of t_d . Data smoothing techniques such as a five-point average can be used to smooth observational data to a certain degree. Smoothing of metrics data [14] highlights the underlying trend that might otherwise be obscured by excessive random noise. However, in some cases, filtering or smoothing might tend to make the model less sensitive.

TABLE 3 FINAL ESTIMATES OF K AND t_d

Model	t_d	K	
Gamma Putnam	41.228 48.4379	1.3812×10^{3} 1.2521×10^{3}	
1 utitalli	40.4379	1.2321 × 10	
Actual	52.0000	1.3008×10^{3}	

3.8 The Manpower Buildup Trend

The nature of manpower resource has undergone a tremendous change over the last decade or so. The medium of computer software offers the power to parallelize subtasks in a cost effective manner like never before. Unit level simulators, distributed cooperative work environments, and frameworks that accelerate productivity are available to the team leader to increase output. In addition to this new technology, the tendency of corporations to contract out jobs, combined with the availability of a large pool of highly trained contractors to draw from, enables a rapid buildup of the manpower deployed on a project. Contractors are generally experienced craftsmen who require very little or no training. The company that hires a contractor has no long term financial obligations toward him in terms of benefits or compensation beyond the terms of the binding contract. On the contrary, companies law [15] bestow upon full-time employees certain inalienable rights to insurance, compensation, and security from the employer.

The Software Contractors' guild defines a contractor's job as follows, on their web page "www.scguild.com":

A contractor's job is to come up to speed on your project, perform the task with little if any supervision, deliver a high quality product, **and leave**. You need not provide training or invitations to social events, and you need not expend efforts on elaborate team building. A contractor is an outside professional businessman who does a specific, usually highly specialized job and then goes away.

Regular employees, on the other hand cannot be employed and terminated on a short term basis, and hence encounter much more resistance from management as an option to increase productivity. Contractors can be deployed on a demand basis and can be terminated at the end of the manpower buildup phase. The net effect of this new degree of freedom reflects in a tendency for the manpower profile to buildup faster during the development phase of the project. This changes the manpower profile for software projects, making it considerably different from the conventional curves used to track hardware development. This buildup is of course, still within the constraints of Brooks's Law [16]. Managers attempt to speed up a project by employing more people. Brooks points out [12] that communication overheads and diversion of expertise for the training of new personnel may actually slow down the project, making it fall further behind schedule. The Gamma model is especially suited for situations where rapid manpower buildups and short development periods are inevitable, but still bounded by Brooks's constraints. On the other hand, the Putnam model, which is based on an extension of the hardware development solution, has limitations dealing with profiles of this nature. In spite of the potential dangers involved, manpower buildup is still an important parameter that is used by management to control productivity. Assigning newly hired people to activities such as documentation maintenance, running of test cases, and other support tasks that require minimal training can help reduce the diversion of actively deployed expertise [17]. Putnam observes [5] that some manpower curves can even be rectangular due to management intervention. Deviations thus introduced are attributed to factors such as impreciseness

and/or changes in requirements and communication losses between personnel. Such variations can be thought of as *noise* superimposed over the Rayleigh curve. In other words, a real world manpower profile can be considered to be the ideal Rayleigh curve combined with a random noise factor. In Section 3.9, a controlled amount of random noise of increasing intensity is added to the Rayleigh curve, to simulate a rapid buildup of manpower. The data thus generated is then used as an input to both the Gamma and the Putnam models. The performance of both the models are then rated under this non-ideal situation.

3.9 Simulated Instances of Rapid Manpower Buildup

Consider the ideal situation where the staffing profile follows the Rayleigh curve faithfully. The transformed plot of $\ln(\dot{p}/t)$ vs. t^2 is shown in Fig. 15. The slope of the line is what defines the time for the Putnam model to reach peak manpower. The Rayleigh curve that corresponds to the line shown, peaks around 40 percent of the duration of the project. A real world manpower profile would of course, deviate from this representation. A rapid manpower buildup during initial stages would make observed values lie above the fitted line during the development stage of the project. Let the abscissa of the plot be divided into five time zones as shown in Fig. 15. Now random values can be introduced in a controlled manner into these different zones to study the effects of non-idealities on the predictive capability of the Gamma and the Putnam models. The perturbations are samples drawn from a uniform distribution. The pseudocode used for the generation of the random sequences is shown:

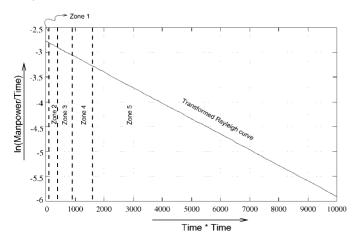


Fig. 15. Transformed manpower profile partitioned into zones.

The random numbers are weighted down arbitrarily. Zone 5 is perturbed with a weighted random number that is either added or subtracted from the transformed Rayleigh curve, based on the outcome of a "±" Bernoulli trial.

Let each zone be represented by $\{z_1, ..., z_5\}$. Table 4 shows the zones and the corresponding bounds within which the introduced random noise fluctuates. Each zone is arbitrarily chosen at intervals of 10 percent as shown in Table 4. An accelerated buildup is simulated by adding random noise of varying intensity to zones z_1, z_2, z_3 , and z_4 . Zone z_5 , on the other hand, is in the tail-off region and does not contribute considerably to predictive information. A random noise with a zero average is added to this zone to simulate actual field data. In cases where the manpower buildup is sluggish or sub-Rayleigh, both the Putnam model and the Gamma model tend to provide the same performance on predictive capability. Hence, only situations where a positive noise, resulting in a super–Rayleigh buildup in the first four time zones, are analyzed.

TABLE 4
Noise Boundary Values for Various Zones

Zone	Range of zone	Trial 1	Trial 2	Trial 3
z_1	<i>t</i> ≤ 10%	+0.3	+0.4	+0.6
Z_2	$10\% < t \le 20\%$	+0.4	+0.53	_0.8
Z_3	$20\% < t \le 30\%$	+0.5	+0.67	+1.0
Z_4	$30\% < t \le 40\%$	+0.6	+0.8	+1.2
z_5	40% < t	±0.3	±0.4	±0.6

The manpower profiles generated for different ranges of noise are shown in Fig. 16a. The development time predictions obtained from both models are shown in Fig. 16b. The random variations in the profiles are considerably more exaggerated than in real life, but it is more meaningful to investigate the response of the models under this worst case scenario. It is seen from Fig. 16b, that the Gamma model provides better accuracy, early prediction, and a low drift measure for the predicted development time t_d .

It serves well to draw a physical interpretation of the different terms that constitute the mathematical model at this stage. Certain terms that occur in both models are of considerable importance, since they can be correlated with the complexity of the product being developed. One such attribute is the *difficulty gradient* of the development process.

3.10 The Difficulty Gradient

Putnam observed in his study [5] of 100 different systems that the ratio K/t_d^2 , was found to have a correlation with the complexity of the product being developed. Hence, the ratio was interpreted as a quantitative measure of the difficulty of the project, termed the *Difficulty ratio*, *D*. The difficulty gradient, termed grad D, or the rate at which the difficulty ratio changes, is a parameter that characterizes a development process. The difficulty gradient, if defined, can be easily incorporated into the Gamma model.

An extremely rapid buildup cannot be sustained by most development environments. The rate of change of difficulty is thus an important parameter that denotes the capability of the development environment. The variation of D, which is grad D, has a magnitude of K/t_d^3 on the basis of the Putnam model. In a study of a group of U.S. Army Computer Sys-

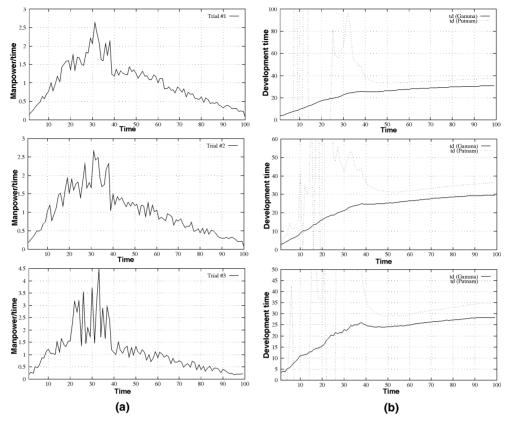


Fig. 16. (a) manpower profiles; (b) predicted values of t_d .

tems Command projects, Putnam [5] observed that the ratio K / t_d^3 remained a constant. It was also observed that the ratio, referred to as C in the study, could be used to categorize a group of systems of similar complexity. Based on this observation, different values of C could be arrived at empirically, indicating the skill level and other developmental attributes of the environment. The Gamma model can be restated to incorporate the difficulty gradient C, as follows:

$$\dot{p} = \frac{4K}{t_d^3} \left(t^2 e^{-2t/t_d} \right) = 4Ct^2 e^{-2t/t_d} \tag{19}$$

3.11 Mathematical Interpretation of the Putnam Model

The rate at which a fresh problem gets solved is dependent on the expertise of the development team, and their experience in solving problems of a similar nature. All teams tend to follow a "learning curve" when it comes to improving their skills on the job. The learning curve is naturally a function of the difficulty of the problem at hand, and therefore influences the development time of the product. This attribute corresponds to what is termed as the "skill function" of a development team for a certain problem.

A simple definition of the skill function may be used as the starting point of an alternative approach to deriving the Putnam model. There is, in fact, no rigorous mathematical derivation to prove the appropriateness of the Norden/Rayleigh equation. But Putnam has provided a mathematical interpretation of the model, on the basis of certain observations made by Norden [2]. Norden described the rate of accomplishment of a process as being proportional to two terms:

- 1) The level of *skill* deployable at instant *t*
- 2) The amount of work left to be done

The above observations can be interpreted mathematically as follows. If K is the total amount of work that needs to be done, p the amount of work accomplished at a given point in time, and s(t) the skill function deployable at instant t, then the rate of accomplishment \dot{p} is:

$$\dot{p} \propto s(t)$$
 $\dot{p} \propto K - p$

where (K - p) is a term that indicates the work that is left to be done. Its graphical interpretation is shown in Fig. 17. Now \dot{p} can be represented by a simple first-order differential equation as follows:

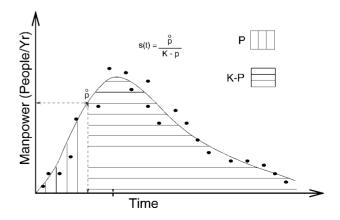


Fig. 17. Calculating skill s(t) in terms of K and p.

$$\dot{p} = \frac{dp}{dt} = s(t)(K - p) \tag{20}$$

The constant of proportionality in (20) is incorporated as a scaling factor into the skill function, for the sake of brevity.

The Putnam model uses the *linear learning law* of Norden. By approximating the *skill* function as being simply linear, and setting it to t/t_d^2 , the rate of doing work gets reduced to a Rayleigh curve. Separating variables and integrating (20) gives the following:

$$\int \frac{1}{K-p} dp = \int s(t)dt - C_i$$

$$p = K - e^{-\int s(t)dt + C_i}$$
(21)

However, at t = 0, the skill function s(t) = p(t) = 0. Thus, the constant of integration C_i reduces to $\ln K$, and (21) can be rewritten as:

$$p = K \left(1 - e^{-\int s(t)dt} \right) \tag{22}$$

Substituting this expression for p(t), in (20), the rate of doing work can be represented as:

$$\frac{dp}{dt} = s(t)Ke^{-\int s(t)dt}$$
 (23)

Setting $s(t) = t / t_d^2$, and $a = 1 / 2t_d^2$, (23) becomes:

$$\frac{dp}{dt} = \frac{t}{t_d^2} K e^{-t^2/2t_d^2}$$

$$\dot{p} = 2Kate^{-at^2}$$
(24)

Equation (24) is the Putnam model, which conversely implies that, fitting a Rayleigh curve to the manpower data forces one to assume that the skill function, or the learning curve is simply linear. However, as Parr points out [3], this assumption has no theoretical support. But then, the Parr model uses a method of decomposition of the process of problem solving, based again on a major assumption. The assumption is that "the change in size of a problem being solved is a linear function of the work completed" [3]. This again has no theoretical basis. The Gamma model is not arrived at from first principles, but the choice of the distribution does force an assumption on the nature of the *skill* function. The nature of the resulting skill function is analyzed in Section 3.12.

3.12 Mathematical Interpretation of the Skill Function

The skill function corresponding to the Gamma model can be arrived at, by integrating (13) to yield the following:

$$\int \dot{p}(t)dt = p(t)$$

$$= -Ke^{-2t/t_d} (1 + 2t/t_d + 2t^2/t_d^2) + C_i$$
(25)

The constant of integration C_i can be found by assuming that there is no manpower deployed at t = 0. Setting t = 0, and p(0) = 0, C_i evaluates to K. Solving for $e^{-2t/td}$ from (25), and eliminating this exponent factor from (13), we arrive at the following:

$$\dot{p} = \frac{4Kt^2t_d^2(1 - p / K)}{t_d^3(t_d^2 + 2t_dt + 2t^2)}$$

$$\dot{p} = \frac{4t^2}{t_d(t_d^2 + 2t_dt + 2t^2)}(K - p)$$

$$\Rightarrow \dot{p} = s(t)(K - p)$$

where

$$s(t) = \frac{4t^2}{t_d(t_d^2 + 2t_d t + 2t^2)}$$

$$\Rightarrow s(t) \propto \frac{1}{t_d}$$
(26)

and

$$s(t) \propto \left[\frac{4t^2}{t_d^2 + 2t_d t + 2t^2} = w(t) \right]$$
 (27)

The choice of the Gamma model as a representative model forces the skill function to be a curve rather than a line. Equation (27) implies that the pace of work is inversely proportional to the development time. In other words, the shorter the development time, the faster the rate at which resources are expended. This implication is consistent with observations. The function w(t) on the other hand, can be thought of as a weighting function, the nature of which can be investigated by plotting the curve for different values of t. Rearranging the terms of w(t) yields:

$$w(t) = \frac{1}{\frac{1}{2} + \frac{t_d}{2t} + \left(\frac{t_d}{2t}\right)^2}$$
 (28)

A plot of w(t), for one hundred data points with K=1, and $t_d=40$, is shown in Fig. 18. It is seen that the weighting function has the effect of providing a trend to the skill function. The skill level increases slowly at first, while the deployed manpower is still familiarizing itself with the problem at hand. It then increases at a faster rate at around half the development time, and then slows down again due to *marginal utility*. Though it does not make sense to attempt to draw a quantitative parallel between the weighting function w(t) and any observable phenomena, it may be thought of as an approximation of the nonlinearities associated with the development process.

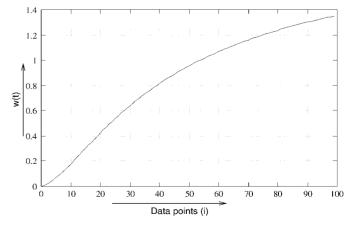


Fig. 18. Weighting function w(t).

However, in a qualitative sense, the weighting function seems to follow the nature of the "law of diminishing returns." Human skill cannot increase indefinitely with time, and hence is intuitively not linear, as assumed by Norden and Putnam. At some point in time, the additional amount of skill that can be achieved during a given interval of time, will naturally diminish. The function w(t) captures the tendency for the rate of work to saturate with passage of time. The combined effect of w(t) and t_d , is to provide an S-shaped curve that represents the overall rate at which work is done.

4 CONCLUDING REMARKS

The Gamma model, based on the equation of the Gamma distribution provides improved predictive capability over the Putnam model, especially in cases where the manpower buildup tends to be faster than that assumed in the Norden/Rayleigh Equation. The Putnam model is an extension of the Norden model which was primarily developed for hardware projects, and has generally a slower manpower buildup rate than software projects. The observational basis for the choice of different parameters hardly varies from the Putnam model to the model introduced in this article. As such, it forms an effective alternative to the Putnam model, offering advance prediction of management parameters, such as K and t_d . The software equation and the different indices proposed by Putnam can be incorporated into the model to make it *portable* across projects. Computationally, the Gamma model is no more complex than the Putnam model. Finally, the Gamma model can form a sound building block for an adaptive model, owing to its generic nature and easy tunability.

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