Weakly Supervised Bayesian Estimation of Seizure Likelihood

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Outline

Background

Figure 1: 18th century iconograph of an epilepsy seizure. Etched by J. Duplessi-Bertaux. Credit: Wellcome Library, London. Licensed under the Creative Commons Attribution 4.0 International license.

Background

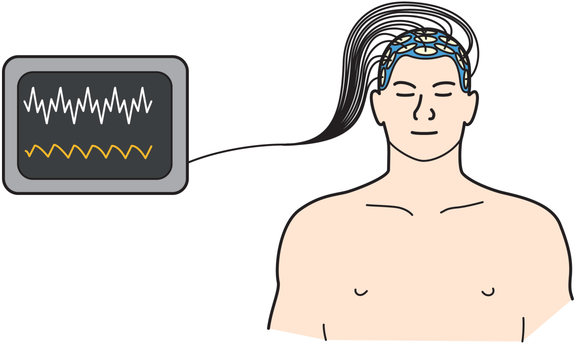
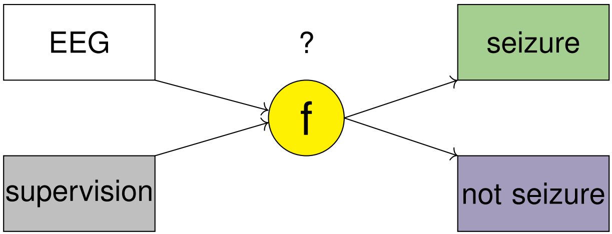


Figure 1: Electroencephalography (EEG) art.
From: The Clear Communication People. Licensed under: CC BY-NC-ND 2.0.

Background & Motivation



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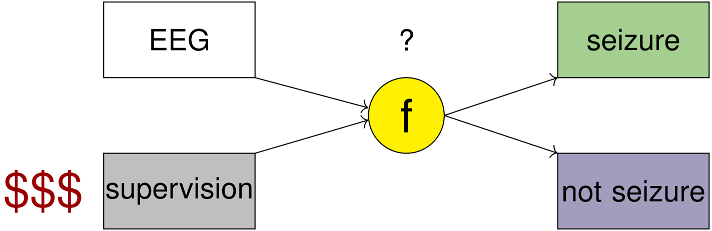

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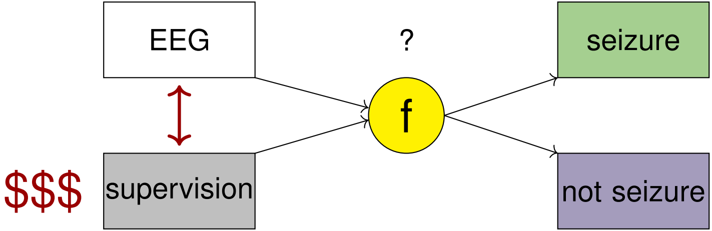

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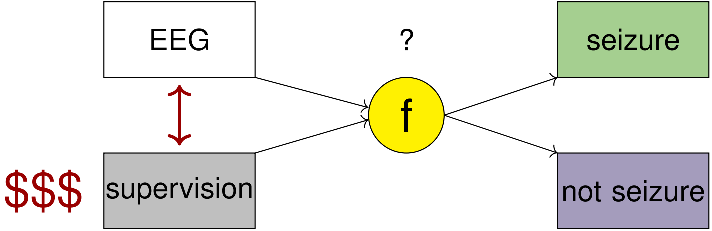

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Related works

1. Gardner et al. [2006] - unsupervised seizure detection.
2. Karoly et al. [2017] - circadian rhythms as priors.
3. Yang et al. [2022] - weak self-supervised learning.

Main contributions

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2. Demonstrating the approach on real-world data, achieving 0.88 AU-ROC with zero labels on a seizure detection task.
3. A weakly supervised version which is biased towards circadian rhythms is shown to improve detection in canines.

Research problem

Part one

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Solution

$$\underbrace{\mathbb{P}(S_t \mid E_t)}_{\text{probability of seizure given EEG}} = \frac{\mathbb{P}(S_t)\mathbb{P}(E_t \mid S_t)}{\mathbb{P}(E_t)} = \frac{\text{Prior} \cdot \text{Likelihood}}{\text{Normalizing factor}}$$

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Part two

How can we compute these values?

Sample spaces

Let:

1. $e_t \sim E(t) \in \Omega_E = \mathbb{R}^{c \times N}$

a random EEG variable,

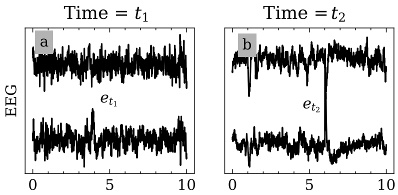

Figure 2: Samples from $E(t_1)$ and $E(t_2)$.

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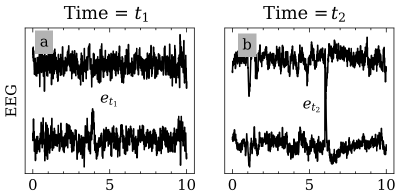
and:

2. $s_t \sim S(t) \in \Omega_S = \{0, 1\}$

a random Seizure variable.



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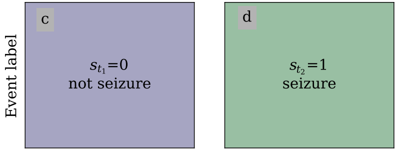

Figure 3: Samples from $S(t_1)$ and $S(t_2)$

Highest Density Regions

Adapted from Hyndman [1996]:

Definition (α-Highest Density Region - α-HDR)

Let $p(e)$ be the density function of a random variable E. For a fixed $0 < \alpha < 1$, and a sample space Ω_E:

$$L_\alpha\text{-HDR} := \{e : p(e) \geq L_\alpha\} \subseteq \Omega_E$$

where

$$L_\alpha \equiv \sup_{\mathbb{R}}\{L : \mathbb{P}(e \in L_\alpha\text{-HDR}) \geq 1 - \alpha\}$$

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Corollary

$$Vol(L_\alpha\text{-}HDR) \equiv \inf_{R \subseteq \Omega_E} \{Vol(R) : \mathbb{P}(e \in R) \geq \alpha\}$$

Theoretical Basis

Definition (Normal model - $\not{S}$)

For a fixed $0 < \alpha < 1$, a model for normal EEG in Ω_E is:

$$\not{S} := L_\alpha\text{-HDR} \subseteq \Omega_E$$

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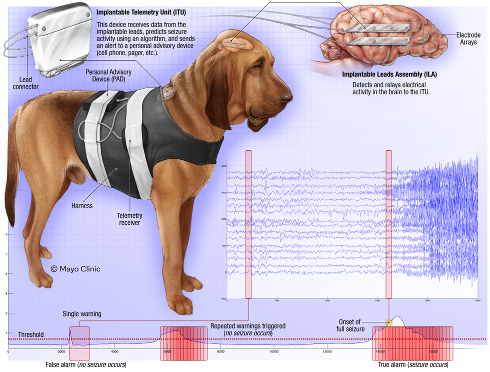
Research problem

$$\mathbb{P}(S_t \mid E_t) = 1 - \mathbb{P}(\not{S}_t \mid E_t) = 1 - \frac{\mathbb{P}(E_t \mid \not{S}_t)\mathbb{P}(\not{S}_t)}{\mathbb{P}(E_t)}$$

1. $\mathbb{P}(e_t)$ is the likelihood of observing e_t in general.
2. $\mathbb{P}(e_t \mid \not{S}_t)$ is the likelihood of observing $e_t \in \not{S}_t$.
3. $1 - \mathbb{P}(\not{S}_t)$ is the prior belief of a seizure at time t.

How do we calculate these values?

Data



Canine Epilepsy Dataset

Figure 4: From Coles et al. [2013]. With permission of Mayo Foundation for Medical Education and Research, all rights reserved.

Longitudinal data is scarce.

Invasive systems offer high SNR.

4 canines,
median 390 days iEEG,
made publicly available by NINDS.

We use 1 canine with 475 days iEEG.

Evaluation plan - non-informative prior

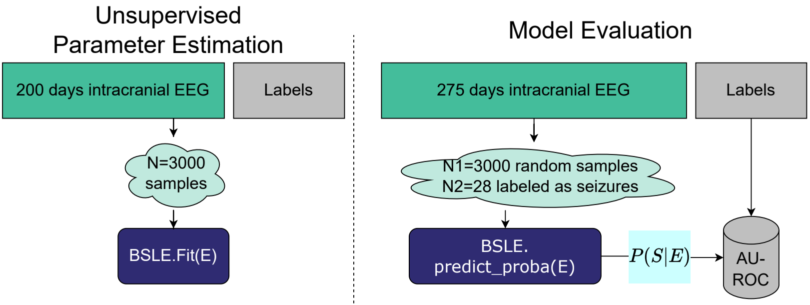


Figure 5: Simulating a Prospective Study - Unsupervised Model

Evaluation plan - informative prior

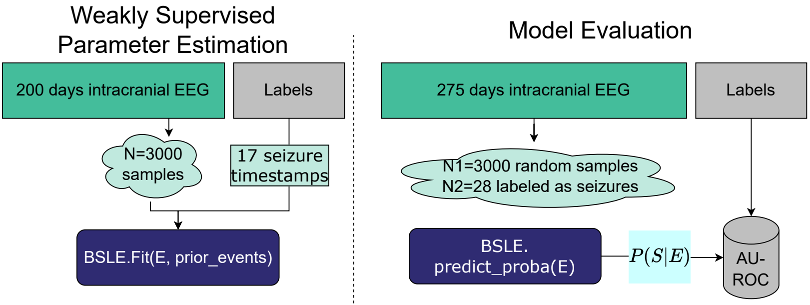


Figure 6: Simulating a Prospective Study - Weakly Supervised Model

Methods

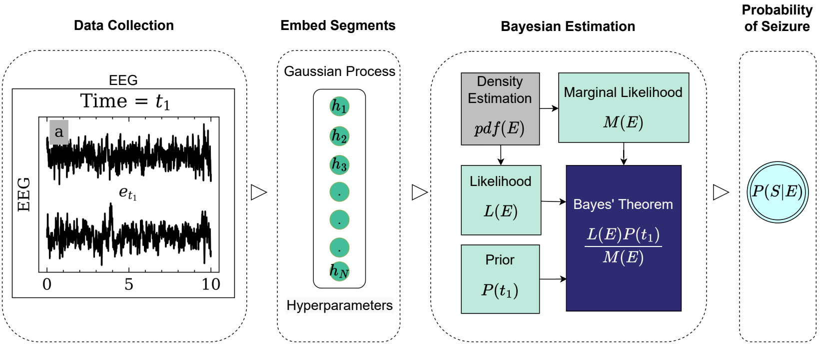


Figure 7: Bayesian Seizure Likelihood Estimation (BSLE) pipeline

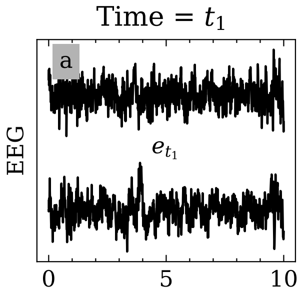
Embedding an EEG segment, by

maximizing the likelihood of:

$$e(t_1) \sim \mathcal{GP}(m(t), k(t, t'))$$

$$m(t) := \mathbb{E}[e(t)]$$

$$k(t, t') := \mathbb{E}[(e(t) - m(t))(e(t') - m(t'))]$$



subject to:

- $e(t_1)$ is a given z-scored EEG segment.
- $m \equiv 0$.
- $k \equiv K_{Matern}(\frac{3}{2})$.

Computed embeddings

$$\underset{d=8000}{e_{t_1}} \xrightarrow[V.I.]{} \underset{d=8}{h_{t_1}}$$

For each sample, estimate:

- signal covar. K_{data}
- task covar. K_{tasks}
- obs. noise $\vec{\epsilon}$

parameters which maximize:

$$p(f(e_{t_1}) \mid h_{t_1})$$

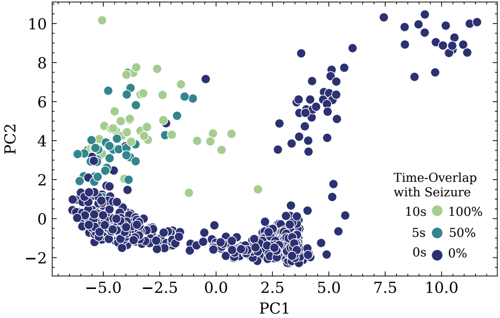
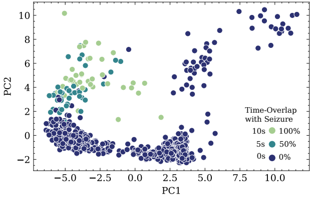


Figure 8: EEG embeddings. Eight dimensional embeddings preserve class separability.

Calibrating Internal Model Parameters



Embeddings

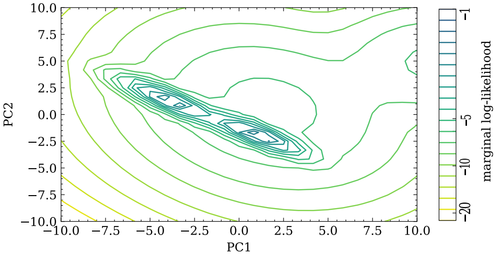


Figure 9: Estimated $\hat{p}(h)$ using Gaussian mixture model

Model of Normal EEG

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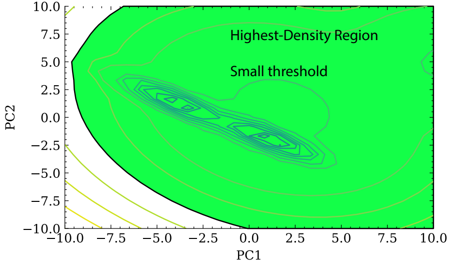


Figure 10: Illustration of $\not{S}$ with small α.

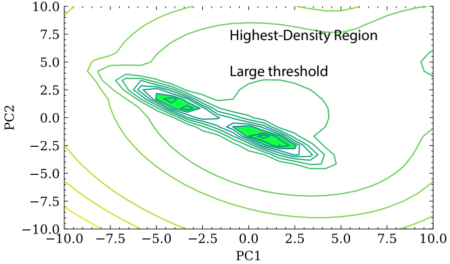
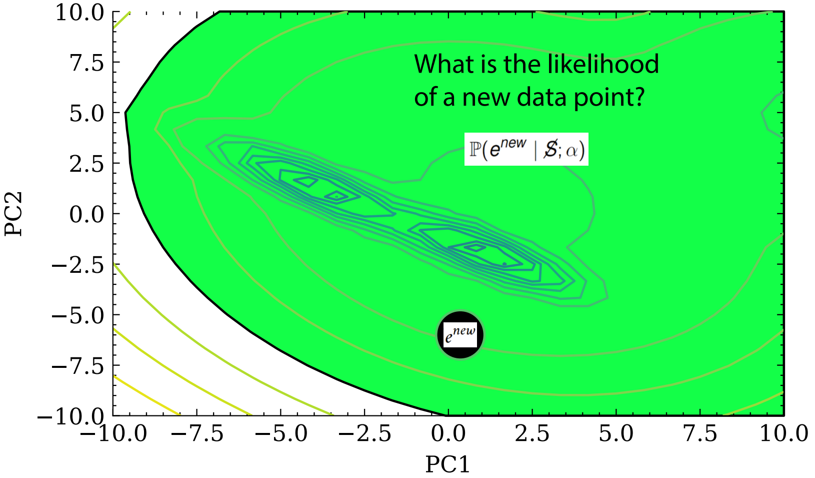


Figure 11: Illustration of $\not{S}$ with large α.

Inference Procedure



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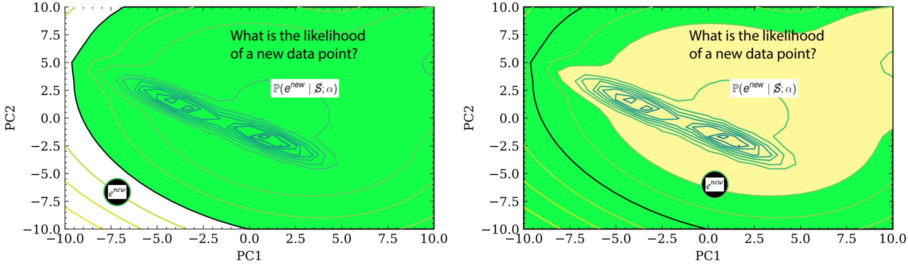


Computing Likelihood

$$\mathbb{P}(e^{new} \mid \not{S}; \alpha) = \begin{cases} 0 & \text{if } e^{new} \in S \\ \mathrm{Vol}(L_{e^{new}}\text{-LDR}) & \text{if } e^{new} \in \not{S} \end{cases}$$

Inference Procedure

Computing Marginal Likelihood

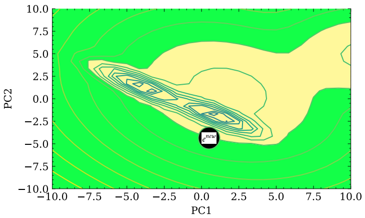
$$\mathbb{P}(e^{new}) = \mathrm{Vol}(L_{e^{new}}\text{-LDR})$$

Estimating w/ Monte Carlo

For a new sample:

$$\mathbb{P}(e^{new}) \leftarrow \frac{|\{e_i \mid \hat{p}(e_i) \leq \hat{p}(e^{new})\}|}{|\{e_i\}|}$$

where $\{e_i\}$ is the training set.



Priors

Non-informative prior

$$\mathbb{P}(\cancel{S_t}) = C$$

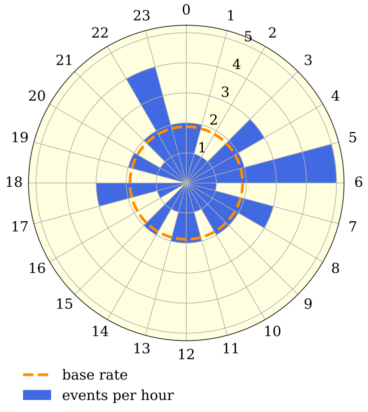
Informative prior

$$f(x|\mu, \kappa; \omega) = \frac{\exp(\kappa \cos(\omega(x - \mu)))}{2\pi I_0(\kappa)}$$

$$\mathbb{P}(S_t) = \frac{1}{K}\sum_{i=0}^{23} a_i f(t \mid i, k)$$

where a_i are the histogram values and K is calculated numerically.

Empirical Circadian Profile



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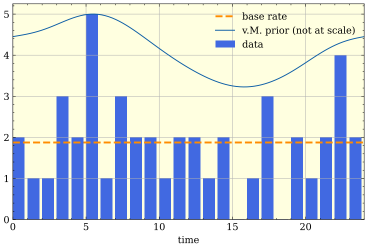
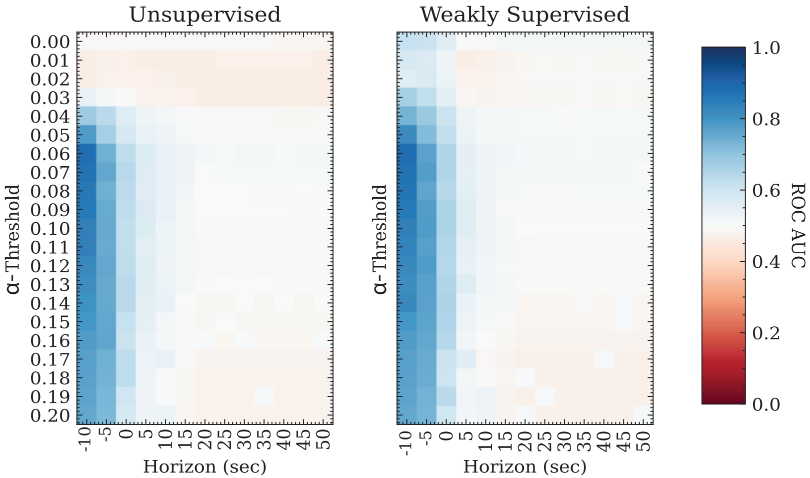


Evaluating BSLE on validation data



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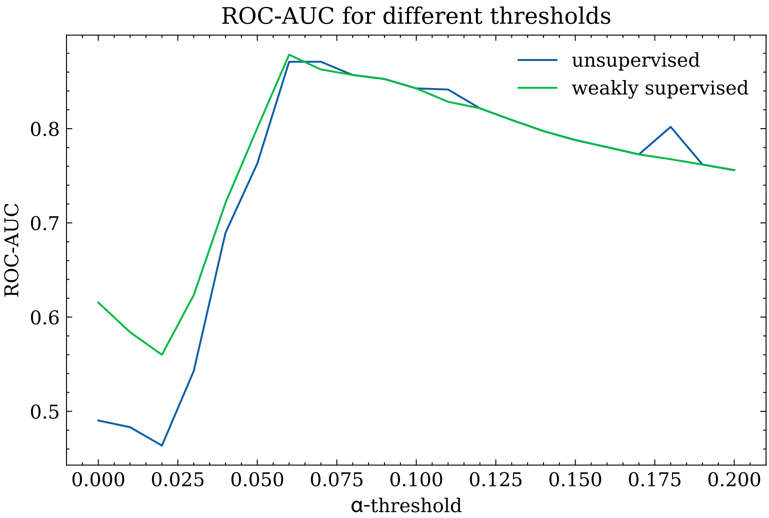

Conclusion

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BSLE can be used for unsupervised seizure alerts, and in addition learn from untaped seizures.

More work is required to statistically validate findings. Further work could use BSLE as a pseudo-labeler module in self-supervised learning.

End

Thank You

References I

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Apx. 1 - Gaussian processes

Definition (Gaussian process)

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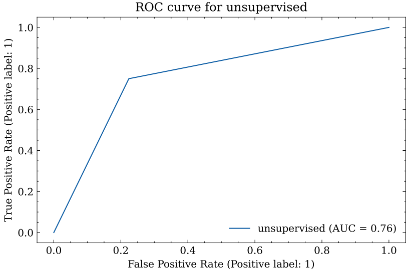
Definition (Matérn class of covariance functions)

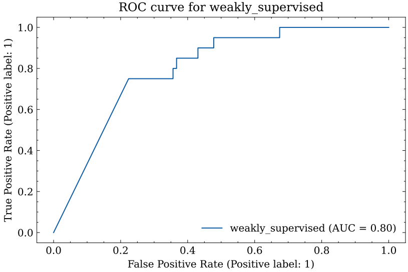
$$k_{Matern}(t, t') := \frac{2^{1-\nu}}{\Gamma(\nu)}(\sqrt{2\nu}d)^{\nu} K_{\nu}(\sqrt{2\nu}d)$$

where

$$d = (t - t')^T \Phi^{-2}(t - t')$$

Apx 2. - ROC curves





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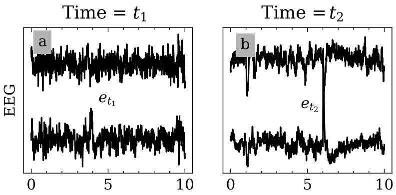

Figure 12: Samples from $E(t_1)$ and $E(t_2)$.

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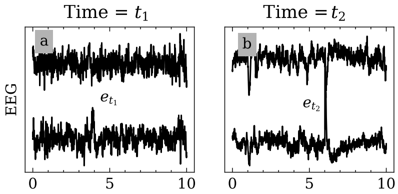

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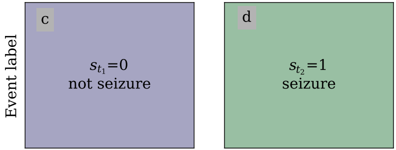

Figure 13: Samples from $S(t_1)$ and $S(t_2)$