

$$1. \quad \text{Advanced}(S) = \begin{cases} \text{Manhattan distance between goal piece and the goal} \\ \text{(the original heuristic)} \\ + \\ \text{number of non-goal pieces at the } 2 \times 2 \text{ goal} \\ \text{(where the goal piece should be)} \end{cases}$$

2. It is admissible since for goal piece to arrive at the goal, it must travel at least the manhattan distance # of steps. Further, the non-goal pieces that were originally at the goal must step aside for the goal piece to fit in, so they all must move at least 1 step.

$\therefore \forall \text{ state } S, \text{ Advanced}(S) \leq \text{actual cost to get to goal}$
Hence admissible.

3. For each S , $\text{Advanced}(S)$ dominates $f(S)$ (the original heuristic), since the # of non-goal pieces at the goal is always ≥ 0 (let this number be n), so:

$$\text{Advanced}(S) = h(S) + n \geq h(S)$$

In particular, for ANY state where there exists non-goal piece at the goal:
 $n > 0$, so $\text{Advanced}(S) = h(S) + n > h(S)$, hence dominating $h(S)$.