Homework #3

Noa Shadmon 999765980

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Files attached:
p6.m –> this produces the output for 6.
polyinterp.m, piecelin.m, pchiptx.m, splinetx.m, graphs.m,interpolation.m –> for 2 and 3

1 First Problem

We will use use the first fundamental theorem of algebra to prove this. This theorem states that every polynomial with n degree greater than 0 has exactly n roots that can be either real or complex.
To Prove uniqueness we first have our n distinct points (x1, x2, ..., xn). Thus, there must exist two polynomials p1 and p2 (both of these go through the same points) that have a degree less than or equal to n-1 with p1(xi) = f(i) for 1$\leq i \leq n$. Since the number of roots of any non zero polynomial is its degree we can say that z = p1 - p2 = 0. Since we set z to be 0, p1 = p2 and we proved uniqueness for any order less than or equal to n.

2 Second Problem

A. Files attached!
B. For pciptx i would use x = -.3. This is because at -.3 is within the accepted range (median values) of -.65 and .1. Any increase in y is relative to x which makes so there isn't any unnecessary increase or decrease.
However, polyinterp and splinetx are dynamic and have increases and decreases. Both of these are below the median values (-.65 and .1) as well. Thus I would not choose those interpolants.
Between pchiptx and picelin, pchiptx is a lot smoother so I think that that is the best

interpolant for this.
C. x = [-1 -0.96 -0.65 0.1 0.4 1]' ;
y = [-1 -0.1512 0.3860 0.4802 0.8838 1]';
V_sum = vander(x);
coeffs = $\frac{V_sum}{y}$;
coeffs
coeffs = 16.0018 0.0007 -20.0022 -0.0007 5.0004 0.0000
If we take the clean integer values then we get 16, -20, and 5.
Thus our polynomial would be f(x)=$16x^5 - 20x^3 + 5x$

3 Third Problem

Files attached!
Pchiptx interpolation is a lot smoother around the interpolated points so the graph is more accurate. The splinetx function however has many edges and doesn't seem smooth for the given range so pchiptx is the better model.

4 Fourth Problem

With the equation provided we can step through different values of N. In the interval [-.5,.5] the values converge and fit the function but points outside these two points they aren't close to the function.

5 Fifth Problem

X is a sparse matrix with indices from the overdetermined system, r is the rank matrix, and y is the matrix of points won.

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \end{bmatrix} * \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 6 \\ 3 \\ 7 \end{bmatrix}$$

$$Then X^T = \begin{bmatrix} 1 & -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 \end{bmatrix}$$

Let M=X^TX

$$M = \begin{bmatrix} 1 & -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 \end{bmatrix} * \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}$$

Let P = X^Ty. Then,

$$P = \begin{bmatrix} 1 & -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 \end{bmatrix} * \begin{bmatrix} 4 \\ 9 \\ 6 \\ 3 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 12 \\ -16 \end{bmatrix}$$

Let P = Mr. Then,

$$\begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix} * \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 12 \\ -16 \end{bmatrix}$$

From here we have 4 unknowns and 4 systems of linear equations.
Solving for each rank we get

$$r = \begin{bmatrix} \frac{17}{4} \\ \frac{29}{8} \\ \frac{65}{8} \\ 0 \end{bmatrix}$$

However since they add up to 16, we multiply both sides by 100/16 and get
$r_1 = 25.5625, r_2 = 22.65625, r_3 = 8.125, r_4 = 0$. Thus $r_3 > r_1 > r_2 > r_4$

6 Sixth Problem

The program is attached.
When we split into 100 equally spaced points we get that QR decomposition and normal equations are very similar, however the normal equations values are slighly larger. However, if we split it into 10 equally spaced points, we get some larger values for both qr and normal equation values. However, the normal equaiton values are way more off than the normal equation ones. This must mean that the larger the interval the more error there is in calculating the polynomial coefficents.

7 Seventh Problem

Pchip and spline are most affected by the outlier. Their curves both dip down to go through the extra point. Polynomial with a high n degree also dips. This makes sense because polynomials with degree n need to have n-1 mins/max. So when n is 2, it'll be a parabola that isn't affected too much by the outlier. But as n increases there will be dips which will result in the curve skewing towards the outlier.

8 Eighth Problem

$$x = \begin{bmatrix} 9 \\ 2 \\ 6 \end{bmatrix}$$

$$Hx = \begin{bmatrix} -11 \\ 0 \\ 0 \end{bmatrix}$$

α = ||x|| = 11
β = 11(11 - x_1) = 11(11-9) = 11 * 2 = 22
v = $(x_1 - \alpha, x_2, x_3)^T$ = $(-2, 2, 6)^T$
H = I - $\frac{1}{\beta} vv^T$ = I - $\frac{1}{22}(-2, 2, 6)^T$(-2, 2, 6)=

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \frac{1}{22} \begin{bmatrix} 4 & -4 & -12 \\ -4 & 4 & 12 \\ -12 & 12 & 36 \end{bmatrix} = \frac{-1}{11} \begin{bmatrix} 9 & 2 & 6 \\ 2 & -9 & 6 \\ 6 & -6 & -7 \end{bmatrix}$$

u = $\frac{1}{\sqrt[2]{2\beta}}$v = $\frac{1}{\sqrt[2]{44}}[-2, 2, 6]^T$
v was solved for above