Homework #1

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1 First Problem

A.

$$\begin{bmatrix} u_{11} & 0 & 0 & \dots & 0 \\ 0 & u_{22} & 0 & \dots & 0 \\ 0 & 0 & u_{33} & \dots & \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & u_{nn} \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \dots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \dots \\ b_n \end{bmatrix}$$

Component-wise

$$u_{nn}x_n = b_n \Rightarrow x_n = \frac{b_n}{u_{nn}}$$

$$u_{n-1,n-1}x_{n-1} + u_{n-1,n}x_n = b_{n-1} \Rightarrow x_{n-1} = \frac{b_{n-1} - u_{n-1,n}x_n}{u_{n-1,n-1}}$$

$$u_{n-2,n-2}x_{n-2} + u_{n-2,n-1}x_{n-1} + u_{n-2,n}x_n = b_{n-2} \Rightarrow x_{n-2} = \frac{b_{n-2} - u_{n-2,n-1}x_{n-1} - u_{n-2,n}x_n}{u_{n-2,n-2}}$$

$$x_i = \frac{b_i - (u_{i,i+1}x_{i+1} + \dots + u_{i,n}x_n)}{u_{i,i}}$$

Row Oriented
$$x_i = \frac{b_i - u(i, i+1:n)x(i+1:n)}{u(i,i)}$$
 Column Oriented

$$\begin{bmatrix} u_{n-1,n-1} & u_{n-1,n} \\ 0 & u_{nn} \end{bmatrix} * \begin{bmatrix} x_{1:n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} b_{1:n-1} \\ b_n \end{bmatrix}$$

$$u_{nn}x_n = b_n \Rightarrow x_n = \frac{b_n}{a_{n-1}}$$

$$u_{nn}x_n = b_n \Rightarrow x_n = \frac{b_n}{u_{nn}}$$

$$u_{n-1,n-1}x_{1:n-1} + u_{n-1,n}x_n = b_{1:n-1} \Rightarrow x_i = \frac{b_i - \sum_{j=i+1}^n u_{ij}x_j}{u_{ii}}$$

Charts of time difference and errors shown on code outputs.