Determining the relationship between masses in equilibrium and the angle of a frictionless plane

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Contents

1 Introduction

1.1 Research Question

When the mass of an on a frictionless plane is altered, and the mass of a hanging object adjusted so equilibrium is achieved, what is the accuracy and uncertainty of this method in comparison to conventional measuring techniques?

1.2 Rationale

The original experiment was conducted to determine the relationship between the mass of a carriage (C_m) and a hanging mass (H_m) when a frictionless plane was inclined at different angles. The results confirmed the theoretical relationship $H_m = C_m \sin(\theta)$.
It was noticed during the experiment that the angle measurement device, an 'angle gun', had a large uncertainty ($\pm 0.5\,\text{deg}$) compared to the scale used to measure masses (± 0.005 grams). It was questioned whether the relationship between the masses in equilibrium could be used to determine the angle of the plane with improved precision and uncertainty.

Considering Newtons first law, under equilibrium the net force is 0 (Britannica, 2023). If we consider the previously established linear relationship between H_m and C_m, then we can find θ in terms of its gradient.

$$
\begin{aligned}
H_m &= C_m \cdot \sin(\theta) \\
C_m &= H_m \cdot \frac{1}{\sin(\theta)} && \because \text{Make dependent variable subject} \\
\frac{C_m}{H_m} &= \text{gradient} && \because \text{gradient} = \frac{\text{rise}}{\text{run}} \\
\frac{H_m}{C_m} &= \frac{1}{\text{gradient}} = \sin(\theta) && \because \sin(\theta) = \frac{O}{H} \\
\therefore\ \theta &= \sin^{-1}\left(\frac{1}{\text{gradient}}\right)
\end{aligned}
$$

Since we now have the angle in terms of the gradient, we can use it to find the uncertainty in the angle. This implies the uncertainty in the gradient, found using the maximum and minimum slope, is equivalent to uncertainty in the angle.

1.3 Methodology

1.3.1 Modifications

The following modifications to the method were implemented

- The plane was kept at a constant angle throughout the entire duration of the experiment. This was done to isolate it from the independent and dependent variables and ensure that the results of all trials would point to the same relationship between them and the angle.
- The independent variable became the hanging mass (H_m). This was done to reduce uncertainty in its force via removing factors such as unaccounted for friction and unnecessary trigonometric calculations. This places as much of the uncertainty as possible on the dependent variable, therefore allowing uncertainty to be quantified.

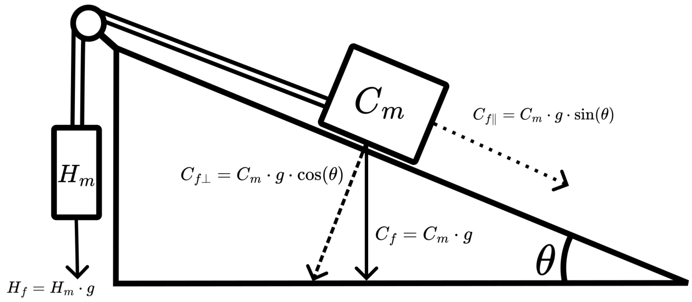


Figure 1: Diagram of experimental setup

- The dependent variable became the carriage mass (C_m) as the large area inside each carriage allowed for fine adjustment of its mass via the addition of brass weights.

1.3.2 Materials

- Angle gun
- Frictionless plane
- Brass weights
- Blue tack
- Scale
- Carriage

1.3.3 Method

1. Set up slope at a constant angle. It will remain at this angle for the entire duration of the experiment.
2. Set the hanging mass (H_m) to its minimum value initially.
3. Alter the mass of the carriage (C_m) until equilibrium with the H_m is achieved, i.e. The carriage remains stationary.
4. If the carriage does not have sufficient space for more weight, replace with a larger carriage or link an additional carriage to the chain.
5. Measure and record masses.
6. Repeat for 3 trials with current H_m value.
7. Increase H_m by 50 grams.
8. Repeat for each H_m value.

1.3.4 Risk Assessment

Frictionless plane

- Mishandling of heavy masses on the frictionless plane could result in them sliding down the slope at high speed. This could damage equipment of cause injury. The slope will be turned off not required, and one person will always be supporting the carriage whenever possible to prevent this.

- Using too low fan speed on the frictionless plane may not create enough of an air pocket to support heavy weights. This could create friction between the surfaces, which could damage equipment and introduce inaccuracies. The fan will be set to the highest possible speed throughout the experiment to negate the possibility of this occurring.

Masses

- Heavy masses or items containing many brass weights may cause injury if dropped or mishandled. participants will wear enclosed footwear to negate injury if this occurs.

2 Results and Evaluation

2.1 Results

2.1.1 Raw Data

Carriage mass					Hanging mass			
1	2	3	Uncertainty	Average	1	2	3	Average
138.550	139.200	138.540	0.330	138.763	50.180	50.160	50.140	50.160
277.270	278.870	278.010	0.800	278.050	100.240	101.230	100.220	100.563
418.050	418.480	418.500	0.225	418.343	150.160	150.390	150.270	150.273
554.870	555.460	554.890	0.295	555.073	200.180	200.180	200.170	200.177
696.860	696.990	697.860	0.500	697.237	250.180	250.210	250.200	250.197
848.190	849.060	849.060	0.435	848.770	300.300	300.200	300.240	300.247

Figure 2: Raw results with additional calculations

2.1.2 Sample Calculations

Absolute uncertainty for C_m when $H_m = 50.160$

$$\begin{aligned}\sigma(C_m) &= \pm\frac{\max - \min}{2}\\ &= \pm\frac{139.20 - 138.55}{2}\\ &= \pm\, 0.325\end{aligned}$$

Average mass of C_m when $H_m = 50.16$

$$\begin{aligned}\bar{C_m} =& \frac{\Sigma_{i=1}^{n} C_m}{n}\\ =& \frac{138.55 + 139.20 + 138.54}{3}\\ =& 137.76\end{aligned}$$

2.1.3 Prerequisite trigonometric measurements

Hypotenuse	Height	Length
2.65	2.50	0.98

Figure 3: Table showing the side lengths of triangle formed by incline plane

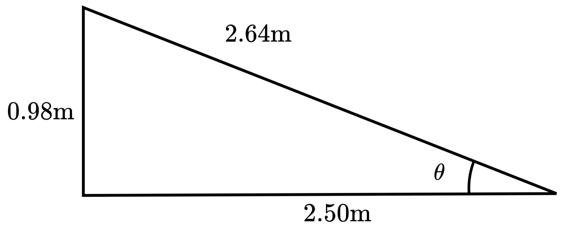


Figure 4: Diagram showing the side lengths of triangle formed by incline plane

2.1.4 Plotting

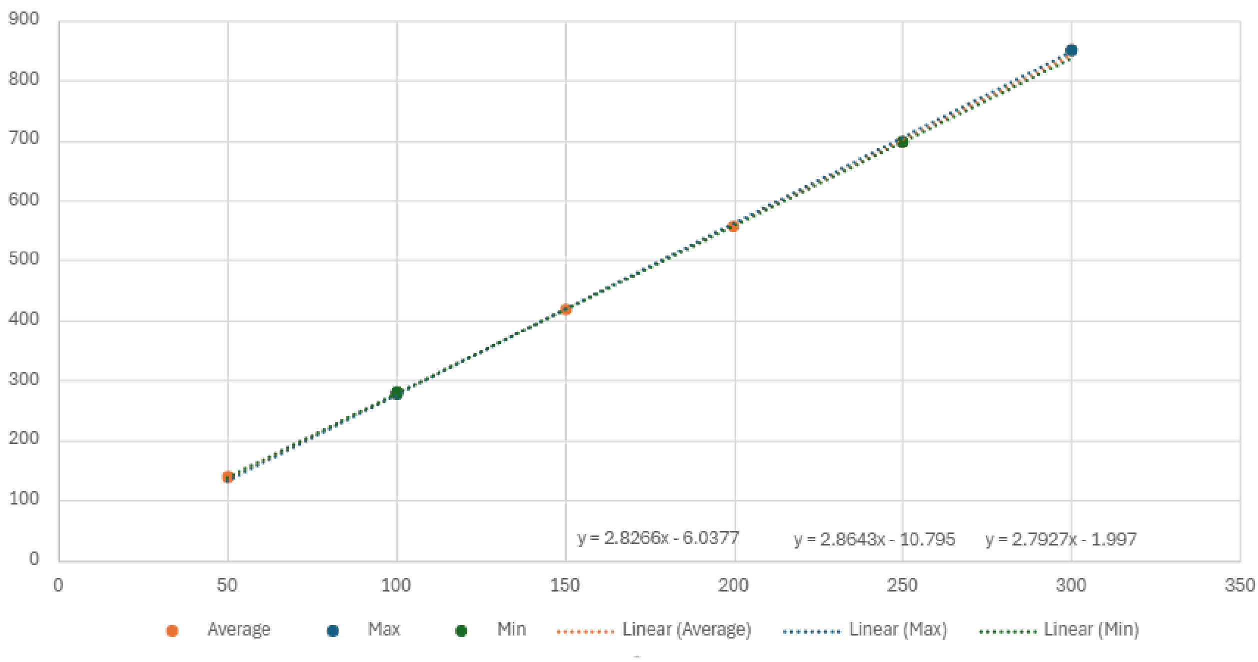


Figure 5: Average H_m and C_m values

3 Discussion

3.1 Analysis of evidence

Figure 2 depicts a clear linear relationship formed by the data plotted. This supports the previously investigated relationship $H_m = C_m \cdot \sin(\theta)$. However, clearly this graph represents the rearranged equation $C_m = H_m \cdot \frac{1}{\sin(\theta)}$.

The presence of vertical shift implies there is some inaccuracy in the data. In theory the relationship should be directly proportional. Considering $\sin^{-1}\left(\frac{1}{\text{gradient}}\right) = \theta$, clearly $\theta = \sin^{-1}\left(\frac{1}{2.8266}\right) = 20.71881007°$.

Since the value of H_m was deliberately set, any uncertainty was due to the inherent random error in the scale used to measure ($\pm 0.005g$)it. By averaging H_m across trials, a more reliable value can be found, however it should be noted that although this negates the effect of random error, the systematic uncertainty is not affected as it is determined by the scale's precision.
To find the absolute uncertainty in the gradient, the maximum and minimum slopes were required. By considering the two average carriage masses with the largest uncertainties, say m_1, m_2, then plotting the line between $m_1 + \sigma_1$ and $m_2 - \sigma_2$, then $m_1 - \sigma_1$ and $m_2 + \sigma_2$, the equations of the maximum and minimum slopes could be solved as the line of best fit.

$$\begin{aligned} \sigma(\text{gradient}) &= \pm \frac{\max - \min}{2} \\ \therefore \ \sigma(\text{gradient}) &= \pm \frac{2.8643 - 2.7927}{2} \\ &= \pm \frac{0.0716}{2} \\ &= \pm\, 0.0358 \end{aligned}$$

This can be used to find the absolute uncertainty in the angle

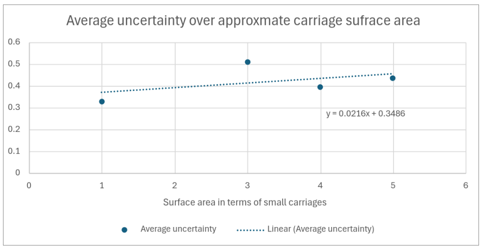
$$\begin{aligned} \sigma(\text{angle}) &= \pm \frac{\max - \min}{2} \\ \sigma(\text{angle}) &= \pm \frac{\sin^{-1}(\frac{1}{2.8643}) - \sin^{-1}(\frac{1}{2.7927})}{2} \\ &= \pm\, 0.274138198466^\circ \end{aligned}$$

4 Evaluation

4.1 Reliability and validity

4.2 Errors and limitations

Angle consistency
Since the experiment was conducted over multiple class periods, the experimental setup was dismantled and reassembled mid way through. Additionally the plane was bumped on multiple occasions, requiring it to be returned back to its original location and angle. Despite the angle of the plane measuring the same throughout the experiment, the uncertainty in the tool used to set the angle of the plane ($\pm 0.5^\circ$) meant that the angle was likely inconsistent. This was not accounted for in any way.
Friction
It was assumed that the frictionless plane was completely frictionless. This was not the case, and multiple factors likely altered the degree of friction across it throughout the experiment. It was noticed that when masses were in equilibrium, pushing the cart would cause it to move for a while, then slow down and stop. If the plane was truly frictionless then the plane would not have slowed down, but continued until it reached the end stop. It was theorised that the surface area of the carriage on the plane determined how much of an air pocket could form underneath it. This was briefly investigated as the type of carriage used was recorded for each trial. Considering one big carriage having the approximate equivalent surface area of two small carriages, the average uncertainty for each carriage surface area was graphed. However, rather than a reduction in uncertainty as surface area increased, an increase was observed.



4.3 Extension

5 Conclusion

References

Britannica, E. (2023). *Newton's laws of motion.* Retrieved from https://www.britannica.com/science/Newtons-laws-of-motion