Revision

Monday, 4 August 2025 1:43 PM

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3-w3 hw ...

Mathematical Methods

Rev Sheet - T3-W3

Student name Ben Trim

Student number

Teacher Mr Mc^c^Clintock

Date Friday, Term 3 Week 3

Technique Homework Sheet

Unit Unit 4

Topic Topic 1, Topic 2, Topic 3, Topic 4, Topic 5

Time 1 week

Seen / unseen Take home questions

Other

Please ensure that you show **all** working out.

	Questions	Score	Marks
Topic 1	1 - 4		19
Topic 2	5 - 8		25
Topic 3	9 - 11		11
Topic 4	12 - 13		6
Topic 5	14 - 15		6
Total			67

Topic 1

Question 1 (TF) [4 marks]

The position, x metres, from a fixed origin at time t seconds of two particles travelling in a straight line are given by $x_1(t) = 6t^3 - 54t^2 + 6t - 10$ and $x_2(t) = (t-3)^4$ respectively for $0 \le t \le 10$.

(a) Determine each particle's acceleration at $t = 2$ [2]

$v_1(t) = 18t^2 - 108t + 6$, $v_2(t) = 4(t-3)^3$

$a_1(t) = 36t - 108$, $a_2(t) = 12(t-3)^2$

$a_1(2) = 72 - 108$

$= -36 \ m/s^2$

$a_2(2) = 12(-1)^2$

$= 12 \ m/s^2$

(b) Determine the times (if any) when the acceleration of both particles is equal. [2]

Let $a_1(t) = a_2(t)$,

$\cancel{12}(3t - 9) = \cancel{12}(t^2 - 6t + 9)$

$\therefore \ t^2 - 9t + 18 = 0$

$\therefore (t-3)(t-6) = 18$

$\therefore \ t = 3$ or $t = 6$.

Question 2 (TF) [5 marks]

Sketch the graph of the function $f(x) = (x+3)^2(x-5)$, clearly indicating all axis intercepts and using the second derivative test to determine the nature of any stationary points.

For intercepts: Let $x=0$, $y = 9 \times (-5) = -45$

Let $y=0$, clearly x-ints at $x=-3$ & $x=5$ ← repeated.

For turning points, $f(x) = (x^2+6x+9)(x-5)$

$= x^3+6x^2+9x-5x^2-30x-45$

$= x^3+x^2-21x-45$

$\therefore f'(x) = 3x^2+2x-21$,

$f''(x) = 6x+2$.

Let $f'(x)=0$: $3x^2+2x-21=0$

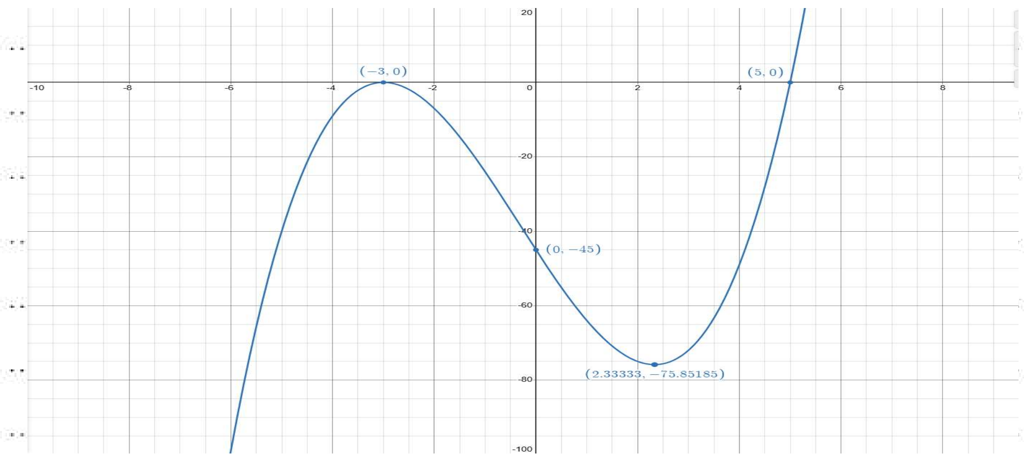
$x = \dfrac{-2 \pm \sqrt{4-4(3)(-21)}}{2(3)} = \dfrac{-2 \pm \sqrt{256}}{6} = \dfrac{-2 \pm 16}{6}$

$x = \frac{7}{3}$ or $x = -3$.

$f(\frac{7}{3}) = -\frac{2048}{27}$, $f(-3)=0$

$f''(\frac{7}{3}) = 6(\frac{7}{3})+2 > 0$, $\therefore$ min at $(\frac{7}{3}, -\frac{2048}{27})$

$f''(-3) = 6(-3)+2 < 0$, $\therefore$ max at $(-3, 0)$.



Question 3 [4 marks]

For each of the following, determine the absolute maximum and minimum values over the specified domain.

(a) $f(x) = -2(x+1)(x-2) + 5, \quad x \in [-3, 2]$. [2]

$f(x) = -2(x^2 - x - 2) + 5 = -2x^2 + 2x + 9$

$f'(x) = -4x + 2$

Let $f'(x) = 0$, $0 = -4x + 2$, $x = \frac{1}{2}$

Then $f(-3) = -15$, $f(\frac{1}{2}) = 9.5$, $f(2) = 5$.

$\therefore$ Abs max at $(\frac{1}{2}, 9\frac{1}{2})$,

Abs min at $(-3, -15)$.

(b) $f(x) = 3(x+1)(x-2)(x-4) + 5, \quad x \in [-5, 2]$. [2]

$f(x) = 3(x^2 - x - 2)(x - 4) + 5 = 3(x^3 - x^2 - 2x - 4x^2 + 4x + 8) + 5$

$= 3(x^3 - 5x^2 + 2x + 8) + 5 = 3x^3 - 15x^2 + 6x + 29$

$f'(x) = 9x^2 - 30x + 6$

Let $f'(x) = 0$, $3x^2 - 10x + 2 = 0$

$x = \frac{10 \pm \sqrt{(10)^2 - 4(3)(2)}}{6} = \frac{10 \pm \sqrt{76}}{6} = \frac{10 \pm 2\sqrt{19}}{6}$

$\therefore x = \frac{1}{3}(5 \pm \sqrt{19})$

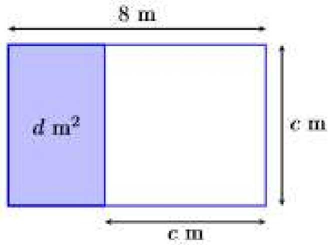
$f(-5) = -751$, $f(2) = 5$, $f\left(\frac{5+\sqrt{19}}{3}\right) = -7.18$, $f\left(\frac{5-\sqrt{19}}{3}\right) = 29.63$

$\therefore$ Abs max at $\left(\frac{5-\sqrt{19}}{3}, 29.63\right)$,

Abs min at $(-5, -751)$.

Question 4 **[6 marks]**

The following diagram shows a rectangle with side lengths 8 m and c m. The area of the shaded region is d m^2. Determine the maximum possible value of d and the corresponding value of c.



Clearly $0 \le c \le 8$.

$d(c) = c \times (8-c) = 8c - c^2$.

$d'(c) = 8 - 2c$,

$d''(c) = -2$. $\therefore$ any stationary point is a max.

Let $d'(c) = 0$, $0 = 8 - 2c$

$c = 4$.

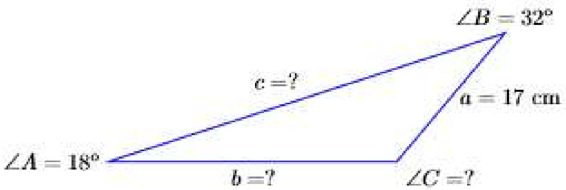
$\therefore$ for max d, $d = 4(8-4) = 16 \text{ m}^2$.

Topic 2

Question 5 [9 marks]

Use the sine rule to determine the unknown value(s) in each of the following.

(a) [3]



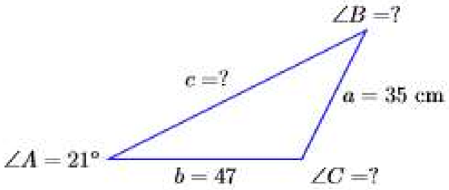
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, $\therefore b = \frac{17\sin(32)}{\sin(18)}$

≈ 29.15 cm

$c = \frac{17\sin(130)}{\sin 18} = 11.76$ cm

$C = 180 - 32 - 18 \approx 130°$

(b) [3]



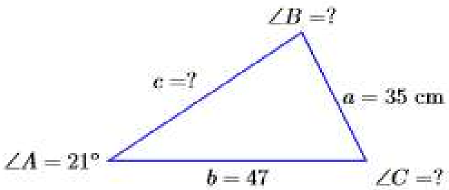
$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$

$\frac{\sin(21)}{35} = \frac{\sin(B)}{47}$ $c =$

$B = \sin^{-1}\left(\frac{47\sin(21)}{35}\right) \approx 28.77°$,

$\angle C = 180 - 21 - 28.77 \approx 130.23°$

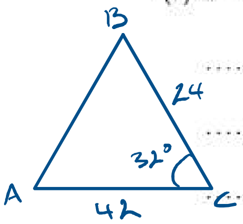
(c) [3]



Question 6 [6 marks]

For triangle ABC with:

(a) $a = 24$, $b = 42$, $C = 32°$, determine c. [1]



$c^2 = a^2 + b^2 - 2ab\cos(C)$

$= 24^2 + 42^2 - 2(24)(42)\cos(32°)$

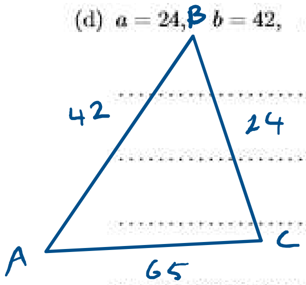
≈ 630.34

$\therefore c = \sqrt{630.34} \approx 25.11$

(b) $b = 35$, $c = 49$, $A = 39°$, determine a. [1]

(c) $c = 22$, $a = 44$, $B = 59°$, determine b. [1]

(d) $a = 24$, $b = 42$, $c = 65$, determine A, B and C. [3]



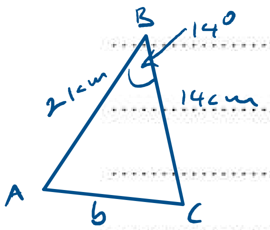
$c^2 = a^2 + b^2 - 2ab\cos C$

$\therefore C = \cos^{-1}\left(\frac{c^2 - a^2 - b^2}{2ab}\right)$

Question 7 [4 marks]

A triangle has vertices A, B and C. Side AB is 21 cm and side BC is 14 cm. $\angle ABC = 41°$.

(a) Determine the length of the third side AC, correct to the nearest centimetre. [1]



cosine rule:

$b^2 = a^2 + c^2 - 2ac\cos(B)$

$= 14^2 + 21^2 - 2(14)(21)\cos(14°)$

≈ 66.47

$\therefore b \approx \sqrt{66.47} = 8.15\text{cm}$

(b) Determine the size of the other two angles, correct to the nearest degree. [2]

sine rule: $\frac{\sin B}{b} = \frac{\sin A}{a} = \frac{\sin C}{c}$

$\therefore \frac{\sin(14)}{8.15} = \frac{\sin A}{14}$ $\therefore A = \sin^{-1}\left(\frac{14\sin(14°)}{8.15}\right) \approx 25^0$

and $C = 180 - 14 - 25 = 141^0$.

(c) Determine the area of the triangle, correct to 1 decimal place. [1]

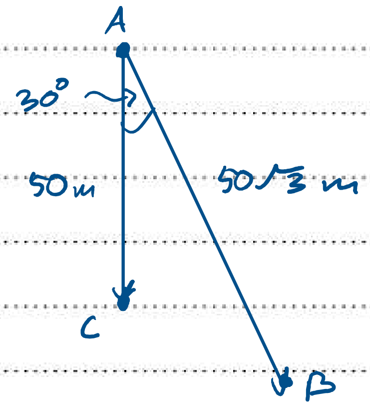
Area $= \frac{1}{2}ac\sin B$

$= \frac{1}{2} \times 14 \times 21 \sin 14$

$\approx 35.56\ \text{cm}^2$

Question 8 (TF) [6 marks]

Three ships are placing a triangular shark net near a beach and are located at points A, B and C. Ship B is $50\sqrt{3}$ m from ship A, on a bearing of 150°. Ship C is 50 m due south of ship A. Determine the distance between ships B and C, and the area of the shark net.



For distance BC,

$a^2 = b^2 + c^2 - 2bc\cos(A)$

$= 50^2 +$

Topic 3

Question 9 (TA) [2 marks]

Determine the exact values of the mean and variance for each of the following Bernoulli distributions, correct to 2 decimal places.

Please note: We incorrectly wrote down the formula for variance in 15B. I have fixed this in OneNote. It should have been $\text{var}(X) = p(1-p)$, which is exactly the same for a binomial distribution, but with $n = 1$.

(a) Scoring a one on a roll of a die. [1]

mean $= p = \frac{1}{6}$, $var(X) = \frac{1}{6}\left(\frac{5}{6}\right) = \frac{5}{36}$

(b) Three coins are flipped, and you record whether there are at most two heads. [1]

2^3 possible outcomes, 7 matching outcomes

$\therefore p = \frac{7}{8}$, $Var(X) = \frac{7}{8}\left(\frac{1}{8}\right)$

$= \frac{7}{64}$

Question 10 (TF) [4 marks]

For each of the following, calculate the exact value of any unknown n or p.
Please note: Our textbook did not use this terminology. $X \sim B(n,p)$ is a shorthand way of writing that X is a binomial random variable, with the specified n and p values (in that order).

(a) $X \sim B(21,p)$, $E(X) = 7$ [1]

$E(X) = np$, $\therefore$ $7 = 21p$,
$p = \frac{1}{3}$

(b) $X \sim B(n, 1)$, $E(X) = 1$ [1]

0.1

$E(X) = np$, $1 = n \times 0.1$
$n = 10$

(c) A binomial random variable, X has a mean of 36 and a variance of 9 (calculate n and p). [2]

$np = 36$, $Var(X) = np(1-p) = 9$
$\therefore$ $36(1-p) = 9$
$\therefore$ $1-p = \frac{1}{4}$
$\therefore$ $p = \frac{3}{4}$.

$\therefore$ $n = \frac{36}{\frac{3}{4}} = 48$

Question 11 (TA) [5 marks]

The probability, p, of winning a prize in a game of chance was modelled by a binomial distribution. After n turns, this distribution was found to have mean 38 and variance 34.2.

Calculate the values of n and p, hence determine the number of turns that would be required to have a 99% chance of winning at least 3 prizes.

Topic 4

Question 12 (TF) [2 marks]

Our textbook did not use this terminology. $X \sim N(\mu, \sigma)$ is a shorthand way of writing that X is a normal random variable, with the specified μ and σ values (in that order).

(a) If $X \sim N(20, 4)$ determine the exact z-value corresponding to $x = 19$. [1]

$z = \frac{x-\mu}{\sigma} = \frac{19-20}{4} = -\frac{1}{4}$

(b) A normal random variable X has a mean of 60. The value 45 has a standardised value of -3, determine the standard deviation. [1]

$z = \frac{x-\mu}{\sigma}$, $-3 = \frac{45-60}{\sigma}$, $\therefore \sigma = \frac{-15}{-3} = 5$

Question 13 (TA) [4 marks]

The useful life of a school laptop is known to be normally distributed with a mean life of 4 years and a variance of 0.4.

(a) What is the probability that a laptop will have a useful life of less than 3 years? Give your answer correct to 4 decimal places. [1]

(b) Determine the probability that a laptop will have a useful life between 4 and 4.5 years. Give your answer correct to 4 decimal places. [1]

(c) Historically 55% of laptops have a useful life that is less than the manufacturer's advertised life. Determine the manufacturer's advertised life as a whole number of months. [1]

Topic 5

Question 14 (TA) [3 marks]

A 95% confidence interval is used to obtain an estimate for a population with a sample proportion of 0.9.

(a) Determine the margin of error if the sample size is 650. [1.5]

(b) Determine the sample size that will generate a margin of error of 2%. [1.5]

Question 15 (TA) [3 marks]

In a survey, it was found that 43% of 90 people perferred Mexican food to Asian food.

(a) Calculate the expected value, correct to 2 decimal places. [1]

(b) Calculate the margin of error, correct to 3 decimal places. [1]

(c) Assess the normality. [1]