Modelling and Lighting Interior Spaces using Reflected Natural Light

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Contents

1 Introduction

1.1 Premise

This study aimed to find optimal mirror placement for minimum decrease in intensity as light travelled throughout a given cave. This was achieved by mapping the cave's geometry and simulating the possible paths of light as a vector throughout the cave with various mirror arrangements.

1.2 Assumptions

In Order for an accurate solution to be formed, a set of conditions had to be assumed in order to simplify modelling and remove ambiguity.

- The given specifications did not provide an orientation, so it was assumed that the cave described was traversed horizontally and depicted 'top-down'.
- Mirrors and walls of the cave were parallel to the floor and extended upwards with an undefined height. Light entered the cave parallel to the ground with uniform intensity. This simplified modelling and could easily be altered if height was defined.
- Either light did not decrease in intensity or increase in area according to the inverse square law, or the change was negligible. The suns rays have travelled so far that they can be considered effectively parallel and therefore did not diverge. (Yasuda, 2024)
- Mirrors reflected light across their entire surface. Light only intersected a mirror and reflected across the normal or did not intersect with a mirror at all. This meant that only mirror surfaces had to be accounted for
- Mirrors were perfectly flat and did not distort or affect light beams in any other way than reflecting them. While unrealistic to expect in the real world, such factors are out of the scope of this study.
- While the entry and exit vectors had no width, it was assumed that light entered the cave through the entire entrance, and could exit throughout the entire exit. Each point along the entrance was considered the beginning of its own vector, with equal direction to the entry vector. This allowed the entire beams width to be accounted for without introducing unnecessary calculations.
- The stimulus implies that the light entering the cave was from the sun, however the sun's position in the sky is constantly changing. Therefore it was assumed that the entry vector's was constant.

1.3 Observations

The following observations were made and their impacts on the solution's success criteria, or working were evaluated.

- Contaminants in the air may decrease the intensity of light. Distance light travels in cave must be minimised to avoid loss in intensity.
- Mirrors are imperfect and will only reflect a portion of light that hits them. Therefore the number of mirrors used must also be minimised to avoid loss in intensity.
- Only the edges of each beam of light needed to be found as any light that is within a beam was travelling the same direction as the light at the edges. This allowed intersections with mirrors and cave walls to be found without introducing unnecessary calculations.
- Clearly paths were not wide enough for a beam of 2 units, the size of the cave's entrance, to pass through them. Therefore the beam was required to be split in order for light to be passed through the cave efficiently.

- In the case of the beam being split, the edges of each beam were found and each considered the new edge for the split path.

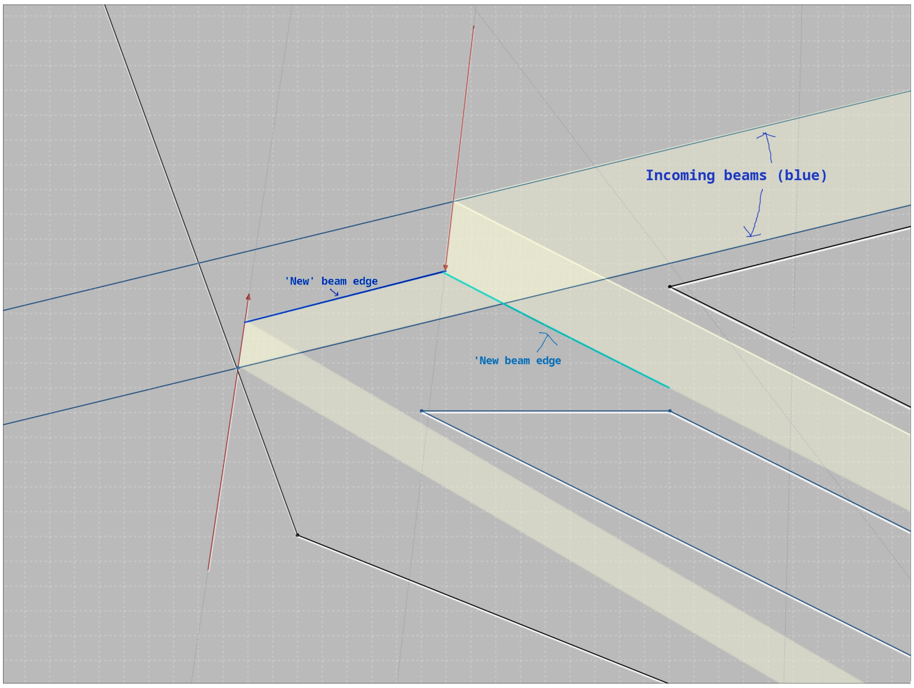


Figure 1: Expected and assumed behaviour when beam is split

1.4 Translation

Light can be modelled as a relative position vector with origin at a mirror surface on the Cartesian plane and translated without affecting its other properties. When a reflection occurs, angle of incidence = angle of refraction.

The scalar product between 2 vectors is a measure of how closely a set of two vectors match each others direction. To find the scalar product, the equation $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| \cdot |\mathbf{b}| \cdot \cos(\theta)$ was used. Clearly for this application, a and b would be listed in traditional component form, When rearranged as $\cos^{-1}(\frac{x_1 \cdot x_2 + y_1 \cdot y_2}{|\mathbf{a}| \cdot |\mathbf{b}|}) = \theta$ the acute angle between 2 vectors, when arranged tail-to-tail, could be found

Tools such as desmos mapped vectors as an arrows that span 2 points. The x and y values that usually refer to a direction and magnitude were converted into 2 discrete sets points, in which one defines the starting point of a vector, and one defines the end. It must be noted that unless explicitly stated, the latter format is what is being used to describe vector position, not regular component form.

The proportion of light remaining after n number of reflections was modelled as the exponential $I = \text{Efficiency}^x$, where I is a scalar representation of intensity with respect to the original intensity , n is the number of mirrors, and Efficiency is the percentage efficiency of the mirror in decimal form. Clearly, adding more mirrors will lead to diminishing losses in intensity.

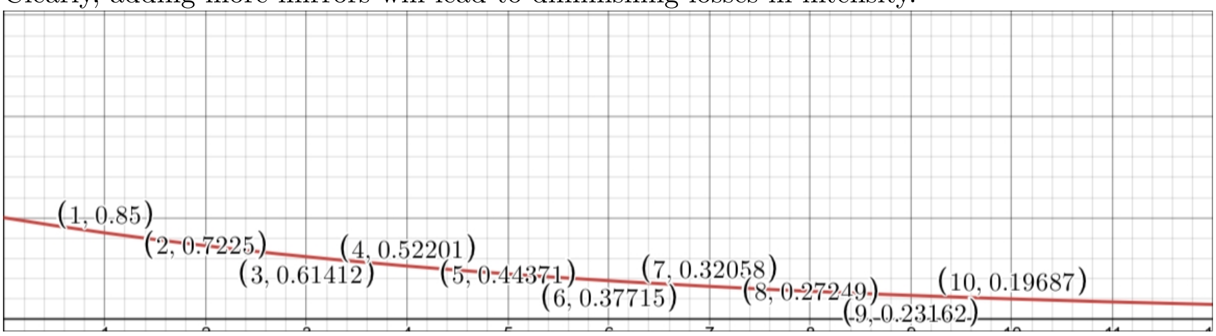


Figure 2: The exponential 0.85^x showing number of mirrors on x-axis, and scalar intensity on y-axis

2 Solve

To map out the cave and its obstacles, the list of conjoined vectors had to be converted into Cartesian co-ordinates. This was achieved through considering the vector sum of the first n_{th} vectors that made up an object as tail of a new vector, then doing the same with the $(n+1)_{th}$ vector sum, and considering it the head. This gave the relative position vector of the new wall. These calculations were completed in Excel with vectors in component form.

The co-ordinates of the start and end points of the left wall were arranged into a chain of interconnected x, and y co-ordinates and inserted into desmos as lists X_1, and Y_1 respectively. These lists were then plotted as connected points. The same was repeated for the remaining wall, and obstacle geometry.

These lists were also manually inserted into the phydemo.app light ray simulator using the text based editor. When the lists were initially imported the scale was imperceptibly small and the y-axis was inverted. To solve this, all points had their x components scaled by a factor of 100, and their y components by a factor of -100. Henceforth any mention of points in the context of the phydemo.app simulator will be in reference to these scaled points.

In the phydemo simulator, light is emitted from the normal of objects. Therefore the entry vector was required to be rotated by 90° by swapping its **i** and **j** components, then multiplying the new j component by -1. It was then offset so that its tail was positioned at the edge of the cave's right entrance, at $(16, 0)$, and plotted at a relative position vector, by adding its components to the co-ordinates of its tail. It was found that the width of its emitted beam matched that of the width of the entrance. No further intervention was required.

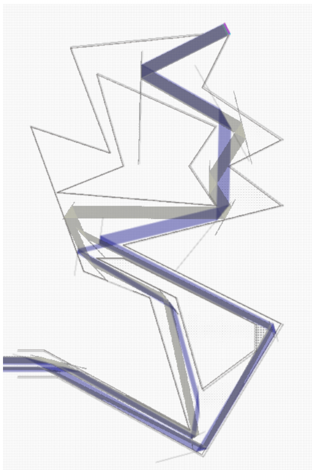


Figure 3: A comparison of an early (black) and final (purple) path iterations

Once the entry vector was positioned, mirrors were manually placed and adjusted so that light travelled throughout the cave and exited parallel. Then began an iterative process with the goal of (in order of priority) reducing light absorbed into the cave walls/resolving collisions, decreasing reflections that the light underwent before exiting, and/or reducing the distance that light had to travel before exiting.

The initial drafted solution required light to undergo 9 reflections in order to reach the exit. This was reduced to 7 in the final revision. The differences between these solutions were the removal of a mirror beside the first obstacle,and a mirror directly before the beam was split. These changes not only reduced the number of mirrors required, but the distance that the beam travelled. No solutions that did not split the beam of light were considered as intentionally losing light was counter intuitive to the task.

Once a satisfactory mirror arrangement was achieved, the start and end points of each mirror were extracted from the phydemo.app simulator and converted back into Cartesian co-ordinates and inserted into desmos as separate lists so the n_{th} element of each list corresponded to the x or y co-ordinate, of either the start or and of the n_{th} mirror. These lists were defined as x_1, y_1, x_2, and y_2.

By defining a vector from the n_{th} element of each corresponding list, the mirrors could be depicted in desmos. The same was done but with the 'line' function, rather than the 'vector' function, as to create an object compatible with desmos's intersection and angle measurement tools.

For each entry vector (originating from the left and right sides of the cave's entrance), the path of the incoming beam of light to the mirror was found and defined as a new vector with components $(x_2 - x_1)\mathbf{i}$ and $(y_2 - y_1)\mathbf{j}$. This value was used to find the acute angle between the incoming ray, and the mirror (θ_1). The angle between the mirror and the negative x-axis (θ_2) was also found.

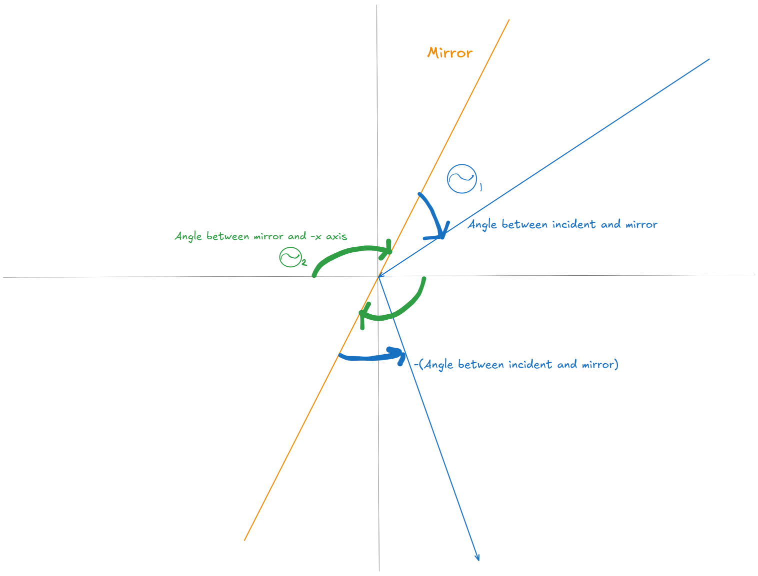


Figure 4: A diagram showing how θ_1 and θ_2 are used to find the angle of the reflected beam.

By performing the operation $-\theta_2+\theta_1$, the angle of the new beam could be found. This line was then plotted in polar form with infinite magnitude. This process was repeated for remaining reflections across all paths until the angle of the exiting beam could be found. Mirror positions were redefined so that they started and ended at the points in which their respective beams intersected with them so that they were as small as physically possible. This change was implemented in Excel, however, diagrams and depictions were not updated.

3 Evaluate and Verify

To verify that the solution was valid, all light vectors were visually inspected for unintended collisions in desmos. Comparing the intended exit vector to the actual exit vectors showed that the ray of light to the left of the second obstacle, exited at $-180.140271880705°$, while the other exited at $-180.001018112731°$. This was considered acceptable as both values were within $0.15°$ of the intended exit vector: $-180°$.
If the efficiency of each mirror was considered 85%, the intensity after 7 reflections would have been $0.85^7 \approx 0.32$. This was considered acceptable as it was $\approx 9\%$ better than initial solutions and the direction of further optimisation was unclear.

3.1 Strengths and Limitations of Solution

3.1.1 Strengths

- No light was lost through absorption into the cave walls as beam was able to be split. Clearly this had a large impact on the quantity of exiting light.
- Light entering throughout the entire entrance was considered and reflected to the exit as opposed to a single vector or light ray.
- Multiple iterations show direct improvement in solution.

3.1.2 Limitations

- Beams never come together as they were at the entrance, and exit at incorrect angles.
- Low tolerance for error in mirror placement in case of physical installation, which is imminent considering the context of the task
- No investigation into alternative methods of directing light, such as curved mirrors, lenses, or beam expansion/compression.
- Solution not easily optimisable nor mathematically proven to be the best. The process of synthesising new light paths is not automated nor easily optimisable therefore to iterate on the solution, a significant amount of manual intervention is required

4 Conclusion

The given solution provided a valid mirror arrangement that successfully reflected light through the cave while minimising losses and utilising multiple alternative routes.

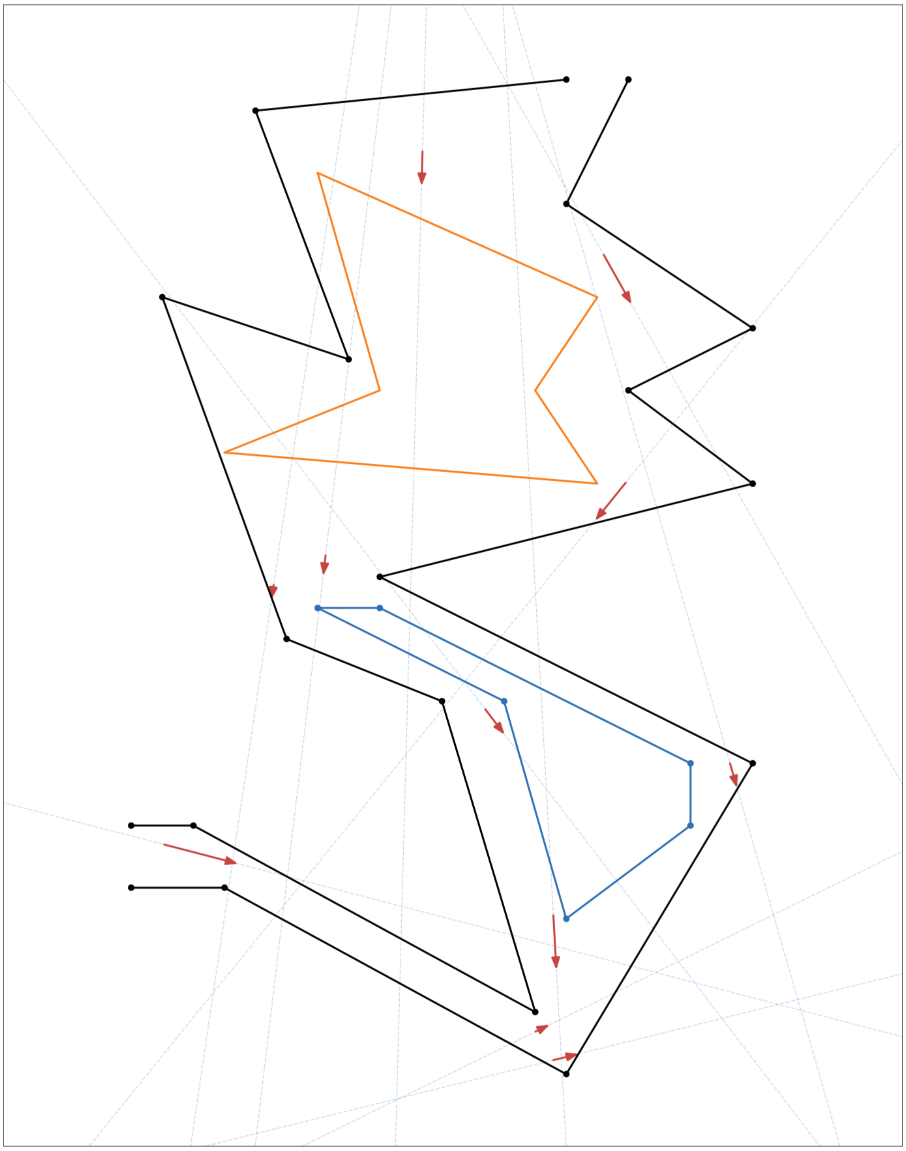
5 Appendices

Object	x1	y1	neg y1	x2	y2	neg y2
Mirror 1	9.37239	-2.3138	2.3138	9.34481	-3.32759	3.32759
Mirror 2	15.20404	-5.63765	5.63765	16.05624	-7.1486	7.1486
Mirror 3	15.90875	-12.97034	12.97034	14.98217	-14.11637	14.11637
Mirror 4	6.25553	-15.30591	15.30591	6.188	-15.872	15.872
Mirror 5	4.58262	-16.25859	16.25859	4.52708	-16.63408	16.63408
Mirror 6	11.3956	-20.26068	20.26068	11.96353	-21.00241	21.00241
Mirror 7	19.26916	-21.99586	21.99586	19.47144	-22.70065	22.70065
Mirror 8	13.58552	-26.89374	26.89374	13.67728	-28.54813	28.54813
Mirror 9	13.58371	-31.5464	31.5464	14.31294	-31.36601	31.36601
Mirror 10	1.063757	-24.61176	24.61176	3.34964	-25.20674	25.20674
Mirror 11	13.01008	-30.63217	30.63217	13.38482	-30.44808	30.44808

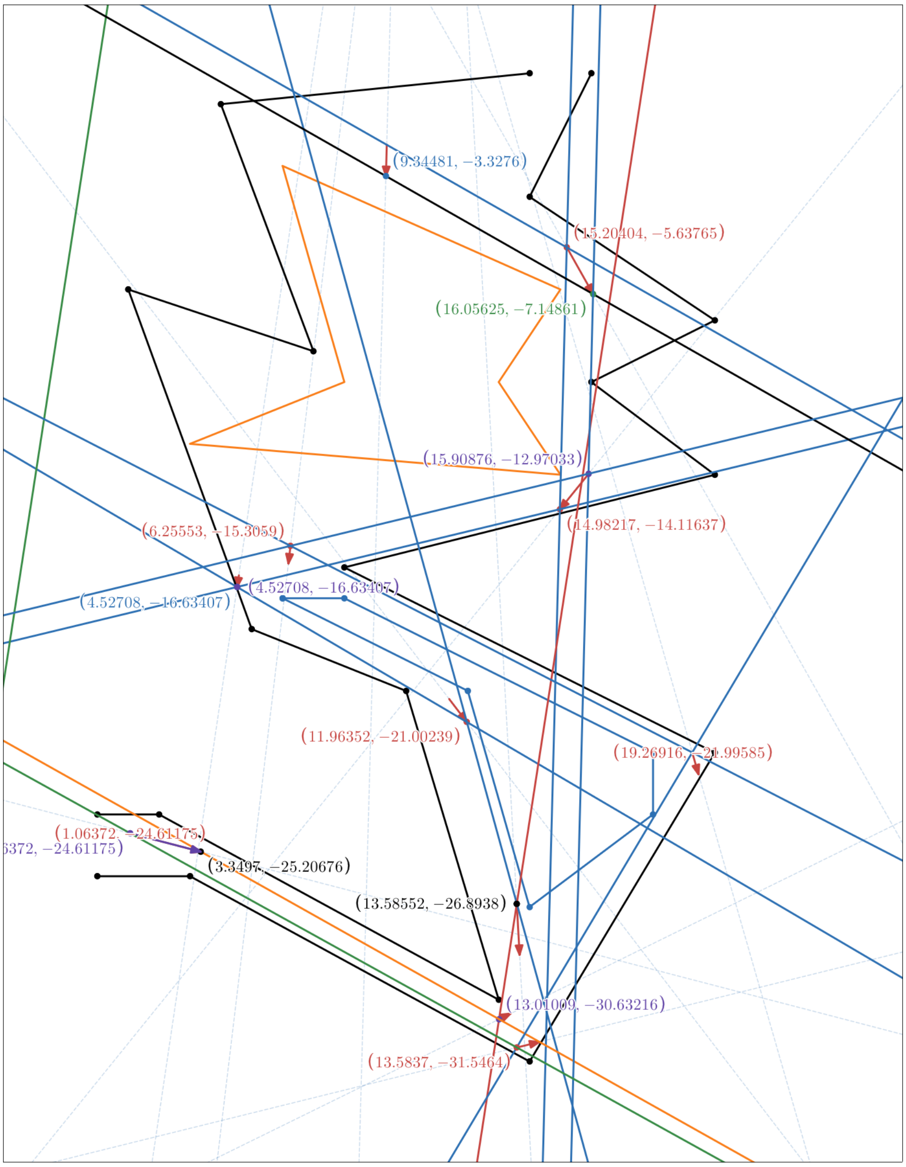
Appendix 1: A table showing the final, trimmed positions of each mirror

Object Type	x1	y1	neg y1	x2	y2	neg y2
Mirror 1	9.4	1.299	-1.299	9.168	9.827	-9.827
Mirror 2	17.471	9.657	-9.657	13.365	2.377	-2.377
Mirror 3	16.116	12.714	-12.714	11.877	17.957	-17.957
Mirror 4	6.423	13.902	-13.902	6.188	15.872	-15.872
Mirror 5	4.283	18.284	-18.284	4.612	16.06	-16.06
Mirror 6	12.16	21.259	-21.259	10.696	19.347	-19.347
Mirror 7	19.225	21.842	-21.842	19.636	23.274	-23.274
Mirror 8	13.572	26.65	-26.65	13.705	29.048	-29.048
Mirror 9	10.411	32.423	-32.423	14.41	31.342	-31.342
Mirror 10	5.36	25.73	-25.73	-0.353	24.243	-24.243
Mirror 11	12.986	30.644	-30.644	14.138	30.083	-30.083

Appendix 2: A table showing raw imported values of each mirror



Appendix 3: A screenshot of the desmos environment with final mirror placements



Appendix 4: A screenshot of the desmos environment with final mirror placements and connecting lines

Cave lambda

- The entry vector is $-\frac{4}{5}\boldsymbol{i} - \frac{2}{5}\boldsymbol{j}$ at position $(15, 0)$.
- The exit vector is $-\boldsymbol{i}$ at position $(0, -25)$.
- One cave wall begins at position $(14, 0)$ and consists of the following vectors.
  - $-10\boldsymbol{i} - \boldsymbol{j}$
  - $3\boldsymbol{i} - 8\boldsymbol{j}$
  - $-6\boldsymbol{i} + 2\boldsymbol{j}$
  - $4\boldsymbol{i} - 11\boldsymbol{j}$
  - $5\boldsymbol{i} - 2\boldsymbol{j}$
  - $3\boldsymbol{i} - 10\boldsymbol{j}$
  - $-11\boldsymbol{i} + 6\boldsymbol{j}$
  - $-2\boldsymbol{i}$
- The other cave wall begins at position $(16, 0)$ and consists of the following vectors.
  - $-2\boldsymbol{i} - 4\boldsymbol{j}$
  - $6\boldsymbol{i} - 4\boldsymbol{j}$
  - $-4\boldsymbol{i} - 2\boldsymbol{j}$
  - $4\boldsymbol{i} - 3\boldsymbol{j}$
  - $-12\boldsymbol{i} - 3\boldsymbol{j}$
  - $12\boldsymbol{i} - 6\boldsymbol{j}$
  - $-6\boldsymbol{i} - 10\boldsymbol{j}$
  - $-11\boldsymbol{i} + 6\boldsymbol{j}$
  - $-3\boldsymbol{i}$
- The cave has an obstacle that begins at position $(7, -3)$ and consists of the following vectors.
  - $2\boldsymbol{i} - 7\boldsymbol{j}$
  - $-5\boldsymbol{i} - 2\boldsymbol{j}$
  - $12\boldsymbol{i} - \boldsymbol{j}$
  - $-2\boldsymbol{i} + 3\boldsymbol{j}$
  - $2\boldsymbol{i} + 3\boldsymbol{j}$
  - $-9\boldsymbol{i} + 4\boldsymbol{j}$
- The cave has an obstacle that begins at position $(6, -17)$ and consists of the following vectors.
  - $2\boldsymbol{i}$
  - $10\boldsymbol{i} - 5\boldsymbol{j}$
  - $-2\boldsymbol{j}$
  - $-4\boldsymbol{i} - 3\boldsymbol{j}$
  - $-2\boldsymbol{i} + 7\boldsymbol{j}$
  - $-6\boldsymbol{i} + 3\boldsymbol{j}$

Appendix 5: The provided stimulus

References

Yasuda, M. (2024). *Why the earth receives parallel rays of light from the sun - earthguide online classroom.* Retrieved 2024-11-20, from `https://earthguide.ucsd.edu/eoc/special_topics/teach/sp_climate_change/p_sunlight_parallel.html`