ACM-ICPC Team Reference Document
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Contents

1 Templates

1.1 C++ Template

```
#include <bits/stdc++.h>

using namespace std;



// DEFINES
#define precision(x) cout << fixed << setprecision(x);
#define fast cin.tie(0); ios::sync_with_stdio(0)
#define all(x) x.begin(), x.end()
#define rall(x) x.rbegin(), x.rend()
#define ff first
#define ss second
// #define nl endl
#define nl "\n"
#define sp " "
#define yes "Yes"
#define no "No"
#define int long long



// CONSTANTS
const int INF = 1e18;
// const int MOD = 1e9 + 7;
// const int MOD = 998244353;



// FSTREAMS
ifstream in("input.txt");
ofstream out("output.txt");



// RANDOM
const int RMIN = 1, RMAX = 1e9;

random_device rdev;
```

```
mt19937_64 reng(rdev());
uniform_int_distribution<mt19937_64::result_type> dist(RMIN
     , RMAX);



// CUSTOM HASH
struct custom_hash {
   static uint64_t splitmix64(uint64_t x) {
      // http://xorshift.di.unimi.it/splitmix64.c
      x += 0x9e3779b97f4a7c15;
      x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
      x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
      return x ^ (x >> 31);
   }

   size_t operator()(uint64_t x) const {
      static const uint64_t FIXED_RANDOM = chrono::
           steady_clock::now().time_since_epoch().count();
      return splitmix64(x + FIXED_RANDOM);
   }
};
// USAGE EXAMPLES:
//
// unordered_set<long long, custom_hash> safe_set;
// unordered_map<long long, int, custom_hash> safe_map;
// gp_hash_table<long long, int, custom_hash>
     safe_hash_table;
//
// for pairs might be used like `3 * a + b` or `a ^ (b >>
     1)`


// GLOBALS



// SOLUTION
void solve() {

}



// PREPROCESSING
void prepr() { }



// ENTRANCE
signed main() {
   precision(15);
   fast;
   prepr();
   int t = 1;
   cin >> t;
   while (t--) solve();
}
```

1.2 C++ Include

```
#include <iostream>
#include <iomanip>
#include <fstream>
#include <random>
#include <cmath>
#include <algorithm>
#include <string>
#include <vector>
#include <set>
#include <unordered_set>
#include <map>
#include <unordered_map>
#include <queue>
#include <deque>
#include <stack>
#include <list>
#include <bitset>
```

1.3 Py Template

```
from math import sqrt, ceil, floor, gcd
from random import randint
import sys


def inpt():
   return sys.stdin.readline().strip()


input = inpt


INF = int(1e18)
# MOD = int(1e9 + 7)
# MOD = 998244353


def solve():
   pass


t = 1
t = int(input())
for _ in range(t):
   solve()
```

2 Data Structures

2.1 Disjoint Set Union

```
// Theme: Disjoint Set Union

struct dsu {
   vector<int> p, size;

   dsu(int n) {
      p.assign(n, 0); size.assign(n, 0);
      for (int i = 0; i < n; i++) {
         p[i] = i;
         size[i] = 1;
      }
   }

   int get(int v) {
      if (p[v] != v) p[v] = get(p[v]);
      return p[v];
   }

   void unite(int u, int v) {
      auto x = get(u), y = get(v);
      if (x == y) return;
      if (size[x] > size[y]) swap(x, y);
      p[x] = y; size[y] += size[x];
   }
};
```

2.2 Segment Tree

```
// Theme: Segment Tree

struct segtree {
   int size;
   vector<int> tree;

   void init(int n) {
      size = 1;
      while (size < n) size <<= 1;
      tree.assign(2 * size - 1, 0);
   }

   void build(vector<int> &a, int x, int lx, int rx) {
      if (rx - lx == 1) {
         if (lx < a.size()) tree[x] = a[lx];
         return;
      }
      int m = (lx + rx) / 2;
      build(a, 2 * x + 1, lx, m);
      build(a, 2 * x + 2, m, rx);
      tree[x] = tree[2 * x + 1] + tree[2 * x + 2];
   }
```

```
    void build(vector<int> &a) {
        init(a.size());
        build(a, 0, 0, size);
    }

    // Complexity: O(log(n))
    void set(int i, int v, int x, int lx, int rx) {
        if (rx - lx == 1) {
            tree[x] = v;
            return;
        }
        int m = (lx + rx) / 2;
        if (i < m) set(i, v, 2 * x + 1, lx, m);
        else set(i, v, 2 * x + 2, m, rx);
        tree[x] = tree[2 * x + 1] + tree[2 * x + 2];
    }
    void set(int i, int v) {
        set(i, v, 0, 0, size);
    }

    // Complexity: O(log(n))
    int sum(int l, int r, int x, int lx, int rx) {
        if (l <= lx && rx <= r) return tree[x];
        if (l >= rx || r <= lx) return 0;
        int m = (lx + rx) / 2;
        return sum(l, r, 2 * x + 1, lx, m) +
            sum(l, r, 2 * x + 2, m, rx);
    }
    int sum(int l, int r) {
        return sum(l, r, 0, 0, size);
    }
};
```

2.3 Segment Tree Propagate

```
// Theme: Segment Tree With Propagation

struct segtree_prop {
    int size;
    vector<int> tree;

    void init(int n) {
        size = 1;
        while (size < n) size <<= 1;
        tree.assign(2 * size - 1, 0);
    }

    void build(vector<int> &a, int x, int lx, int rx) {
        if (rx - lx == 1) {
            if (lx < a.size()) tree[x] = a[lx];
            return;
        }
        int m = (lx + rx) / 2;
        build(a, 2 * x + 1, lx, m);
        build(a, 2 * x + 2, m, rx);
        tree[x] = tree[2 * x + 1] + tree[2 * x + 2];
    }
    void build(vector<int> &a) {
        init(a.size());
        build(a, 0, 0, size);
    }

    void push(int x, int lx, int rx) {
        if (rx - lx == 1) return;
        tree[2 * x + 1] += tree[x];
        tree[2 * x + 2] += tree[x];
        tree[x] = 0;
    }

    // Complexity: O(log(n))
    void add(int v, int l, int r, int x, int lx, int rx) {
        push(x, lx, rx);
        if (rx <= l || r <= lx) return;
        if (l <= lx && rx <= r) {
            tree[x] += v;
            return;
        }
        int m = (lx + rx) / 2;
        add(v, l, r, 2 * x + 1, lx, m);
        add(v, l, r, 2 * x + 2, m, rx);
    }
    void add(int v, int l, int r) {
        add(v, l, r, 0, 0, size);
    }
    // Complexity: O(log(n))
    int get(int i, int x, int lx, int rx) {
        push(x, lx, rx);
        if (rx - lx == 1) return tree[x];
        int m = (lx + rx) / 2;
        if (i < m) return get(i, 2 * x + 1, lx, m);
        else return get(i, 2 * x + 2, m, rx);
    }
    int get(int i) {
        return get(i, 0, 0, size);
    }
};
```

2.4 Treap

```
// Theme: Treap (Tree + Heap)

// Node
struct node {
    int key, priorty;
    shared_ptr<node> left, right;

    node(int key, int priorty = INF) :
        key(key),
        priorty(priorty == INF ?
        reng() : priorty) { }
};

// Treap
struct treap {
    shared_ptr<node> root;

    treap() { }

    treap(int root_key, int root_priorty = INF) {
        root = shared_ptr<node>(new node(root_key,
                root_priorty));
    }

    treap(shared_ptr<node> rt) {
        root = shared_ptr<node>(rt);
    }

    treap(const treap &tr) {
        root = shared_ptr<node>(tr.root);
    }

    // Complexity: O(log(N))
    pair<treap, treap> split(int k) {
        auto res = split(root, k);
        return { treap(res.ff), treap(res.ss) };
    }

    pair<shared_ptr<node>, shared_ptr<node>> split(
          shared_ptr<node> rt, int k) {
        if (!rt) return { nullptr, nullptr };
        else if (rt->key < k) {
            auto [rt1, rt2] = split(rt->right, k);
            rt->right = rt1;
            return { rt, rt2 };
        }
        else {
            auto [rt1, rt2] = split(rt->left, k);
            rt->left = rt2;
            return { rt1, rt };
        }
    }

    // Complexity: O(log(N))
    treap merge(const treap &tr) {
        root = shared_ptr<node>(merge(root, tr.root));
        return *this;
    }

    shared_ptr<node> merge(shared_ptr<node> rt1, shared_ptr<
          node> rt2) {
        if (!rt1) return rt2;
        if (!rt2) return rt1;
        if (rt1->priorty < rt2->priorty) {
            rt1->right = merge(rt1->right, rt2);
            return rt1;
        }
        else {
            rt2->left = merge(rt1, rt2->left);
            return rt2;
```

```
      }
   }
};
```

2.5 Treap K

```
// Theme: Treap With Segments

// Node
struct node_k {
   int key, priorty, size;
   shared_ptr<node_k> left, right;

   node_k(int key, int priorty = INF) :
      key(key),
      priorty(priorty == INF ?
      reng() : priorty),
      size(1) { }

   friend int sz(shared_ptr<node_k> nd) {
      return (nd ? nd->size : 0ll);
   }

   void upd() {
      size = sz(left) + sz(right) + 1;
   }
};

// Treap
struct treap_k {
   shared_ptr<node_k> root;

   treap_k() { }

   treap_k(int root_key, int root_priorty = INF) {
      root = shared_ptr<node_k>(new node_k(root_key,
           root_priorty));
   }

   treap_k(shared_ptr<node_k> rt) {
      root = shared_ptr<node_k>(rt);
   }

   treap_k(const treap_k &tr) {
      root = shared_ptr<node_k>(tr.root);
   }

   // Complexity: O(log(N))
   pair<treap_k, treap_k> split_k(int k) {
      auto res = split_k(root, k);
      return { treap_k(res.ff), treap_k(res.ss) };
   }

   pair<shared_ptr<node_k>, shared_ptr<node_k>> split_k(
        shared_ptr<node_k> rt, int k) {
      if (!rt) return { nullptr, nullptr };
      else if (sz(rt) <= k) return { rt, nullptr };
      else if (sz(rt->left) + 1 <= k) {
         auto [rt1, rt2] = split_k(rt->right, k - sz(rt->
              left) - 1);
         rt->right = rt1;
         rt->upd();
         return { rt, rt2 };
      }
      else {
         auto [rt1, rt2] = split_k(rt->left, k);
         rt->left = rt2;
         rt->upd();
         return { rt1, rt };
      }
   }

   // Complexity: O(log(N))
   treap_k merge_k(const treap_k &tr) {
      root = shared_ptr<node_k>(merge_k(root, tr.root));
      return *this;
   }

   shared_ptr<node_k> merge_k(shared_ptr<node_k> rt1,
        shared_ptr<node_k> rt2) {
      if (!rt1) return rt2;
      if (!rt2) return rt1;
      if (rt1->priorty < rt2->priorty) {
         rt1->right = merge_k(rt1->right, rt2);
         rt1->upd();
         return rt1;
      }
      else {
         rt2->left = merge_k(rt1, rt2->left);
         rt2->upd();
         return rt2;
      }
   }
};
```

2.6 Treap Universal

```
// Theme: Treap (Tree + Heap)
// Supports both explicit and implicit keys (not
    simultaniously ofc)
// Core operations are all O(log n) average

mt19937 rng(378);

struct Node {
   int x, y, size; // "x" is key or payload, "y" is
        priority
   Node* left, * right;

   Node(int val): x(val), y(rng() % 1'000'000'000), size(1)
        , left(nullptr), right(nullptr) {}
};

int get_size(Node* root) {
    if (root == nullptr) return 0;
    return root->size;
}
void update(Node* root) {
    if (root == nullptr) return;
    root->size = get_size(root->left) + 1 + get_size(root->
     right);
}

// split by value (for explicit keys)
pair<Node*, Node*> split(Node* root, int v) {
    if (root == nullptr) return {nullptr, nullptr};
    if (root->x <= v) {
        auto res = split(root->right, v);
        root->right = res.first;
        update(root);
        return {root, res.second};
    } else {
        auto res = split(root->left, v);
        root->left = res.second;
        update(root);
        return {res.first, root};
    }
}

// split by size (for implicit keys)
pair<Node*, Node*> split_k(Node* root, int k) {
    if (root == nullptr) return {nullptr, nullptr};
    if (get_size(root) <= k) return {root, nullptr};
    if (k == 0) return {nullptr, root};

    int left_size = get_size(root->left);
    if (left_size >= k) {
        auto res = split_k(root->left, k);
        root->left = res.second;
        update(root);
        return {res.first, root};
    } else {
        auto res = split_k(root->right, k - left_size - 1);
        root->right = res.first;
        update(root);
        return {root, res.second};
    }
}

// merge for both explicit and implicit keys
Node* merge(Node* root1, Node* root2) {
    if (root1 == nullptr) return root2;
    if (root2 == nullptr) return root1;

    if (root1->y < root2->y) {
        root1->right = merge(root1->right, root2);
        update(root1);
        return root1;
    } else {
        root2->left = merge(root1, root2->left);
```

```
        update(root2);
        return root2;
    }
}

// insert for explicit keys (use split_k for implicit keys)
Node* insert(Node* root, int v) {
    auto subs = split(root, v);
    return merge(merge(subs.first, new Node(v)), subs.
     second);
}

// debug helper
void print_node(Node* root, bool end = false) {
    if (root->left != nullptr) print_node(root->left);
    cout << root->x << " ";
    if (root->right != nullptr) print_node(root->right);
    if (end) cout << "\n";
}
```

2.7 Fenwick Tree

```
// Theme: Fenwick Tree
// Core operations are O(log n)

struct Fenwick {
   vector<int> data;

   explicit Fenwick(int n) {
      data.assign(n + 1, 0);
   }
   explicit Fenwick(vector<int>& arr): Fenwick(arr.size())
        {
      for (int i = 1; i <= arr.size(); ++i) {
         add(i, arr[i - 1]);
      }
   }

   // Nested loops (also vector) for multi-dimensional.
        Also in add().
   // (x & -x) = last non-zero bit
   int sum(int right) {
      int res = 0;
      for (int i = right; i > 0; i -= (i & -i)) {
         res += data[i];
      }
      return res;
   }
   int sum(int left, int right) {
      return sum(right) - sum(left - 1); // inclusion-
           exclusion principle
   }

   void add(int idx, int x) {
      for (int i = idx; i < data.size(); i += (i & -i)) {
         data[i] += x;
      }
   }

   // CONCEPT (didn't test it). Should work if all real
        values are non-negative.
   int lower_bound(int s) {
      int k = 0;
      int logn = (int)(log2(data.size() - 1) + 1); //
           maybe rewrite this line
      for (int b = logn; b >= 0; --b) {
         if (k + (1 << b) < data.size() && data[k + (1 <<
              b)] < s) {
            k += (1 << b);
            s -= data[k];
         }
      }
      return k;
   }
};
```

3 Algebra

3.1 Primes Sieve

```
// Theme: Prime Numbers

// Algorithm: Eratosthenes Sieve
// Complexity: O(N*log(log(N)))

// = 0 - Prime,
// != 0 - Lowest Prime Divisor
auto get_sieve(int n) {
   vector<int> sieve(n); // Sieve
   sieve[0] = sieve[1] = 1;

   for (int i = 2; i * i < n; i++)
      if (!sieve[i])
         for (int j = i * i; j < n; j += i)
            sieve[j] = i;

   return sieve;
}

// Algorithm: Prime Numbers With Sieve
// Complexity: O(N*log(log(N)))

auto get_primes(int n) {
   vector<int> primes, sieve = get_sieve(n);

   for (int i = 2; i < sieve.size(); i++)
      if (!sieve[i])
         primes.push_back(i);

   return primes;
}

// Algorithm: Linear Algorithm
// Complexity: O(N)

// lp[i] = Lowest Prime Divisor
auto get_sieve_primes(int n, vector<int> &primes) {
   vector<int> lp(n);
   lp[0] = lp[1] = 1;

   for (int i = 2; i < n; i++) {
      if (!lp[i]) {
         lp[i] = i;
         primes.push_back(i);
      }
      for (int j = 0; j < primes.size() &&
               primes[j] <= lp[i] &&
               i * primes[j] < n; j++)
         lp[i * primes[j]] = primes[j];
   }

   return lp;
}
```

3.2 Factorization

```
// Theme: Factorization

// Algorithm: Trivial Algorithm
// Complexity: O(sqrt(N))

auto factors(int n) {
  vector<int> factors;

  for (int i = 2; i * i <= n; i++) {
    if (n % i)
      continue;
    while (n % i == 0)
      n /= i;
    factors.push_back(i);
  }

  if (n != 1)
    factors.push_back(n);

  return factors;
}

// Algorithm: Factorization With Sieve
// Complexity: O(N*log(log(N)))

auto factors_sieve(int n) {
  vector<int> factors,
      sieve = get_sieve(n + 1);
```

```
  while (sieve[n]) {
    factors.push_back(sieve[n]);
    n /= sieve[n];
  }

  if (n != 1)
    factors.push_back(n);

  return factors;
}

// Algorithm: Factorization With Primes
// Complexity: O(sqrt(N)/log(sqrt(N)))

auto factors_primes(int n) {
  vector<int> factors,
      primes = get_primes(n + 1);

  for (auto &i : primes) {
    if (i * i > n)
      break;
    if (n % i)
      continue;
    while (n % i == 0)
      n /= i;
    factors.push_back(i);
  }

  if (n != 1)
    factors.push_back(n);

  return factors;
}

// Algorithm: Ferma Test
// Complexity: O(K*log(N))

bool ferma(int n) {
  if (n == 2)
    return true;

  uniform_int_distribution<int> distA(2, n - 1);

  for (int i = 0; i < 1000; i++) {
    int a = distA(reng);
    if (gcd(a, n) != 1 ||
        binpow(a, n - 1, n) != 1)
      return false;
  }

  return true;
}

// Algorithm: Pollard Rho Algorithm
// Complexity: O(N^(1/4))

int get_random_number(int l, int r) {
  random_device random_device;
  mt19937 generator(random_device());
  uniform_int_distribution<int> distribution(l, r);

  return distribution(generator);
}

int f(int x, int c, int n) {
  return ((x * x + c) % n);
}

int ff(int n) {
  int g = 1;
  for (int i = 0; i < 5; i++) {
    int x = get_random_number(1, n);
    int c = get_random_number(1, n);
    int h = 0;
    while (g == 1) {
      x = f(x, c, n) % n;
      int y = f(f(x, c, n), c, n) % n;
      g = gcd(abs(x - y), n);
      if (g == n) {
        g = 1;
      }
      h++;

      if (h > 4 * (int)pow(n, 1.0 / 4)) {
        break;
      }
    }
    if (g > 1) {
      return g;
    }
  }

  return -1;
}

signed main() {
  // ...read n...
  vector<int> a;
  while (n > 1) {
    int m = ff(n);
    if (m > 0) {
      n = n / m;
      a.push_back(m);
    } else {
      break;
    }
  }

  vector<int> ans;
  a.push_back(n);

  for (auto &it : a) {
    int i = 2;
    int m = it;
    while (i * i <= m) {
      if (m % i == 0) {
        ans.push_back(i);
        m = m / i;
      } else {
        i += 1;
      }
    }

    ans.push_back(m);
  }

  sort(all(ans));
}
```

3.3 Euler Totient Function

```
// Theme: Euler Totient Function

// Algorithm: Euler Product Formula
// Complexity: O(sqrt(N))

// phi = n(1 - 1 / pi), i = 1,...
int phi(int n) {
   if (n == 1) return 1;

   auto f = factors(n);

   int res = n;
   for (auto &p : f)
      res -= res / p;

   return res;
}
```

3.4 Greatest Common Divisor

```
// Theme: Greatest Common Divisor

// Algorithm: Simple Euclidean Algorithm
// Complexity: O(log(N))

int gcd(int a, int b) {
   while (b) {
      a %= b;
      swap(a, b);
   }
   return a;
}

// Algorithm: Extended Euclidean Algorithm
// Complexity: O(log(N))

// d = gcd(a, b)
// x * a + y * b = d
```

```
// returns {d, x, y}
vector<int> euclid(int a, int b) {
   if (!a) return { b, 0, 1 };
   auto v = euclid(b % a, a);
   int d = v[0], x = v[1], y = v[2];
   return { d, y - (b / a) * x, x };
}
```

3.5 Binary Operations

```
// Theme: Binary Operations

// Algorithm: Binary Multiplication
// Complexity: O(log(b))

int binmul(int a, int b, int p = 0) {
   int res = 0;
   while (b) {
      if (b & 1) res = p ? (res + a) % p : (res + a);
      a = p ? (a + a) % p : (a + a);
      b >>= 1;
   }
   return res;
}

// Algorithm: Binary Exponentiation
// Complexity: O(log(b))

int binpow(int a, int b, int p = 0) {
   int res = 1;
   while (b) {
      if (b & 1) res = p ? (res * a) % p : (res * a);
      a = p ? (a * a) % p : (a * a);
      b >>= 1;
   }
   return res;
}
```

3.6 Matrices

```
// Theme: Matrix Opeations

template <typename T>
using row = vector<T>;
template <typename T>
using matrix = vector<vector<T>>;

// Algorithm: Matrix-Matrix Multiplication
// Complexity: O(N*K*M)

auto m_prod(matrix<int> &a, matrix<int> &b, int p = 0) {
   int n = a.size(), k = a[0].size(), m = b[0].size();

   matrix<int> res(n, row<int>(m));

   for (int i = 0; i < n; i++)
      for (int j = 0; j < m; j++)
         for (int z = 0; z < k; z++)
            res[i][j] = p ? (res[i][j] + a[i][z] * b[z][j]
                  % p) % p
            : (res[i][j] + a[i][z] * b[z][j]);

   return res;
}

// Algorithm: Matrix-Vector Multiplication
// Complexity: O(N*M)

auto m_prod(matrix<int> &a, row<int> &b, int p = 0) {
   int n = a.size(), m = b.size();

   row<int> res(n);

   for (int i = 0; i < n; i++)
      for (int j = 0; j < m; j++)
         res[i] = p ? (res[i] + a[i][j] * b[j] % p) % p
         : (res[i] + a[i][j] * b[j]);

   return res;
}

// Algorithm: Fast Matrix Exponentiation
// Complexity: O(N^3*log(K))

auto m_binpow(matrix<int> a, int x, int p = 0) {
   int n = a.size();

   matrix<int> res(n, row<int>(n));
   for (int i = 0; i < n; i++) res[i][i] = 1;

   while (x) {
      if (x & 1) res = m_prod(res, a, p);
      a = m_prod(a, a, p);
      x >>= 1;
   }

   return res;
}
```

3.7 Fibonacci

```
// Theme: Fibonacci Sequence

// Algorithm: Fibonacci Numbers With Matrix Exponentiation
// Complexity: O(log(N))

int fibonacci(int n) {
   row<int> first_two = { 1, 0 };
   if (n <= 2) return first_two[2 - n];

   matrix<int> fib(2, row<int>(2, 0));
   fib[0][0] = 1; fib[0][1] = 1;
   fib[1][0] = 1; fib[1][1] = 0;

   fib = m_binpow(fib, n - 2);

   row<int> last_two = m_prod(fib, first_two);

   return last_two[0];
}
```

3.8 Baby Step Giant Step

```
// Theme: Descrete Logarithm

// Algorithm: Baby-Step Giant-Step Algorithm
// Complexity: O(sqrt(p)*log(p))

// a ^ (x) = b (mod p), (a, p) = 1
// a ^ (i * m + j) = b (mod p), m = ceil(sqrt(p))
// a ^ (i * m) = b * a ^ (-j) (mod p)
int baby_giant_step(int a, int b, int p) {
   // a ^ (-1) = a ^ (p - 2) mod p
   int m = ceil(sqrt(p)), _a = binpow(a, p - 2, p);

   // s[b * a ^ (-j)] = j
   unordered_map<int, int> s;
   for (int j = 0, t = b; j < m; j++, t = t * _a % p) s[t]
       = j;

   for (int i = 0; i < m; i++) {
      // s.find(a ^ (i * m))
      auto f = s.find(binpow(a, i * m, p));
      // i * m + j
      if (f != s.end()) return i * m + f->ss;
   }

   return -1;
}
```

3.9 Combinations

```
// Theme: Combination Number

// Algorithm: Online Multiplication-Division
// Complexity: O(k)

// C_n^k - from n by k
int C(int n, int k) {
   int res = 1;
```

```
    for (int i = 1; i <= k; i++) {
        res *= n - k + i;
        res /= i;
    }

    return res;
}

// Algorithm: Pascal Triangle Preprocessing
// Complexity: O(N^2)

auto pascal(int n) {
    // C[i][j] = C_i+j^i
    vector<vector<int>> C(n + 1, vector<int>(n + 1, 1));
    for (int i = 1; i < n + 1; i++)
        for (int j = 1; j < n + 1; j++)
            C[i][j] = C[i - 1][j] + C[i][j - 1];

    return C;
}
```

3.10 Permutation

```
// Theme: Permmutations

// Algorithm: Next Lexicological Permutation
// Complexity: O(N)

bool perm(vector<int> &v) {
    int n = v.size();

    for (int i = n - 1; i >= 1; i--) {
        if (v[i - 1] < v[i]) {
            reverse(v.begin() + i, v.end());

            int j = distance(v.begin(),
            upper_bound(v.begin() + i, v.end(), v[i - 1]));

            swap(v[i - 1], v[j]);
            return true;
        }
    }

    return false;
}
```

3.11 Fast Fourier Transform

```
// Theme: Fast Fourier Transform

// Algorithm: Fast Fourier Transform (Complex)
// Complexity: O(N*log(N))

using cd = complex<double>;
const double PI = acos(-1);


auto fft(vector<cd> a, bool invert = 0) {
    // n = 2 ^ x
    int n = a.size();

    // Bit-Reversal Permutation (0000, 1000, 0100, 1100,
        0010, ...)
    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j >= bit; bit >>= 1) j -= bit;
        j += bit;
        if (i < j) swap(a[i], a[j]);
    }

    for (int len = 2; len <= n; len <<= 1) {
        // Complex Root Of One
        double ang = 2 * PI / len * (invert ? -1 : 1);
        cd lroot(cos(ang), sin(ang));

        for (int i = 0; i < n; i += len) {
            cd root(1);
            for (int j = 0; j < len / 2; j++) {
                cd u = a[i + j], v = a[i + j + len / 2] * root
                    ;
                a[i + j] = (u + v);
                a[i + j + len / 2] = (u - v);
                root = (root * lroot);
            }

        }
    }

    if (invert) {
        for (int i = 0; i < n; i++) a[i] /= n;
    }

    return a;
}

// Module (7340033 = 7 * (2 ^ 20) + 1)
// Primiive Root (5 ^ (2 ^ 20) == 1 mod 7340033)
// Inverse Primitive Root (5 * 4404020 == 1 mod 7340033)
// Maximum Degree Of Two (2 ^ 20)

const int mod = 7340033;
const int proot = 5;
const int proot_1 = 4404020;
const int pw = 1 << 20;

// Algorithm: Discrete Fourier Transform (Inverse Roots)
// Complexity: O(N*log(N))

auto fft(vector<int> a, bool invert = 0) {
    // n = 2 ^ x
    int n = a.size();

    // Bit-Reversal Permutation (0000, 1000, 0100, 1100,
        0010, ...)
    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j >= bit; bit >>= 1) j -= bit;
        j += bit;
        if (i < j) swap(a[i], a[j]);
    }

    for (int len = 2; len <= n; len <<= 1) {
         // Primitive Root Or Inverse Root (Inverse
             Transform)
        int lroot = invert ? proot_1 : proot;

        // Current Primitive Root
        lroot = binpow(lroot, pw / len, mod);

        for (int i = 0; i < n; i += len) {
            int root = 1;
            for (int j = 0; j < len / 2; j++) {
                int u = a[i + j], v = a[i + j + len / 2] *
                    root % mod;
                a[i + j] = (u + v) % mod;
                a[i + j + len / 2] = (u - v + mod) % mod;
                root = (root * lroot) % mod;
            }

        }
    }

    if (invert) {
        int _n = binpow(n, mod - 2, mod);
        for (int i = 0; i < n; i++) a[i] = (a[i] * _n) % mod
            ;
    }

    return a;
}

// Algorithm: Discrete Fourier Transform
// Complexity: O(N*log(N))

auto fft(vector<int> &a, bool invert = 0) {
    // n = 2 ^ x
    int n = a.size();

    // Bit-Reversal Permutation (0000, 1000, 0100, 1100,
        0010, ...)
    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j >= bit; bit >>= 1) j -= bit;
        j += bit;
        if (i < j) swap(a[i], a[j]);
    }

    for (int len = 2; len <= n; len <<= 1) {
        // Current Primitive Root
        int lroot = binpow(proot, pw / len, mod);
```

```
    for (int i = 0; i < n; i += len) {
      int root = 1;
      for (int j = 0; j < len / 2; j++) {
        int u = a[i + j], v = a[i + j + len / 2] *
             root % mod;
        a[i + j] = (u + v) % mod;
        a[i + j + len / 2] = (u - v + mod) % mod;
        root = (root * lroot) % mod;
      }

    }
  }

  if (invert) {
    reverse(a.begin() + 1, a.end());
    int _n = binpow(n, mod - 2, mod);
    for (int i = 0; i < n; i++) a[i] = (a[i] * _n) % mod
         ;
  }

  return a;
}
```

3.12 Primitive Roots

```
// Module (7 == 3 * (2 ^ 1) + 1)
// Primitive Root (3)
// Primitive Root {2 ^ 1} (6)
// Inverse Root {2 ^ 1} (6)
// Degree Of Two (2)

// const int mod = 7
// const int proot = 6
// const int proot_1 = 6
// const int pw = 1 << 1

// Module (13 == 3 * (2 ^ 2) + 1)
// Primitive Root (2)
// Primitive Root {2 ^ 2} (8)
// Inverse Root {2 ^ 2} (5)
// Degree Of Two (4)

// const int mod = 13
// const int proot = 8
// const int proot_1 = 5
// const int pw = 1 << 2
```

3.13 Number Decomposition

```
// Theme: Integer Numbers Decomposition With Composite
     Module

// Module
// m = (p1 ^ m1) * (p2 ^ m2) * ... * (pn ^ mn)
int m;
// Prime Divisors Of Module
vector<int> p;


struct num {
    // GCD(x, m) = 1
  int x;
    // Powers Of Primes
  vector<int> a;

  num() : x(0), a(vector<int>(p.size())) { }

    // n = (p1 ^ a1) * (p2 ^ a2) * ... * (pn ^ an) * x
  num(int n) : x(0), a(vector<int>(p.size())) {
    if (!n) return;
    for (int i = 0; i < p.size(); i++) {
      int ai = 0;
      while (n % p[i] == 0) {
        n /= p[i];
        ai++;
      }
      a[i] = ai;
    }
    x = n;
  }

  num operator*(const num &nm) {
    vector<int> new_a(p.size());
    for (int i = 0; i < p.size(); i++)
      new_a[i] = a[i] + nm.a[i];
    num res; res.a = new_a;
    res.x = x * nm.x % m;
    return res;
  }

  num operator/(const num &nm) {
    vector<int> new_a(p.size());
    for (int i = 0; i < p.size(); i++)
      new_a[i] = a[i] - nm.a[i];
    num res; res.a = new_a;
    int g = euclid(nm.x, m)[1];
    g += m; g %= m;
    res.x = x * g % m;
    return res;
  }

  int toint() {
    int res = x;
    for (int i = 0; i < p.size(); i++)
      res = res * binpow(p[i], a[i], m) % m;
    return res;
  }
};
```

3.14 Formulae

Combinations.
$C_n^k = \frac{n!}{(n-k)!k!}$
$C_n^0 + C_n^1 + ... + C_n^n = 2^n$
$C_{n+1}^{k+1} = C_n^{k+1} + C_n^k$
$C_n^k = \frac{n}{k} C_{n-1}^{k-1}$

Striling approximation.
$n! \approx \sqrt{2\pi n} \frac{n}{e}^n$

Euler's theorem.
$a^{\phi(m)} \equiv 1 \bmod m$, $gcd(a, m) = 1$

Ferma's little theorem.
$a^{p-1} \equiv 1 \bmod p$, $gcd(a, p) = 1$, p - prime.

Catalan number.
$$C_0 = 0, C_n = \sum_{i=0}^{n-1} C_i C_{n-1-i}$$
$$C_n = \frac{2(2n-1)}{n+1} C_{n-1}$$
$$C_n = \frac{(2n)!}{n!(n+1)!}$$

Arithmetic progression.
$S_n = \frac{a_1 + a_n}{2} n = \frac{2a_1 + d(n-1)}{2} n$

Geometric progression.
$S_n = \frac{b_1(1-q^n)}{1-q} n$

Infinitely decreasing geometric progression.
$S_n = \frac{b_1}{1-q} n$

Sums.
$\sum_{i=1}^n i = \frac{n(n+1)}{2}$,
$\sum_{i=1}^n i^2 = \frac{n(2n+1)(n+1)}{6}$,
$\sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$,

$\sum_{i=1}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$,
$\sum_{i=a}^{b} c^i = \frac{c^{b+1}-c^a}{c-1}, c \neq 1.$

4 Geometry

4.1 Vector

```
// Theme: Methematical 3-D Vector

template <typename T>
struct vec {
   T x, y, z;
   vec(T x = 0, T y = 0, T z = 0) : x(x), y(y), z(z) { }
   vec<T> operator+(const vec<T> &v) const {
      return vec<T>(x + v.x, y + v.y, z + v.z);
   }
   vec<T>operator-(const vec<T> &v) const {
      return vec<T>(x - v.x, y - v.y, z - v.z);
   }
   vec<T>operator*(T k) const {
      return vec<T>(k * x, k * y, k * z);
   }
   friend vec<T> operator*(T k, const vec<T> &v) {
      return vec<T>(v.x * k, v.y * k, v.z * k);
   }
   vec<T> operator/(T k) {
      return vec<T>(x / k, y / k, z / k);
   }
   T operator*(const vec<T> &v) const {
      return x * v.x + y * v.y + z * v.z;
   }
   vec<T> operator^(const vec<T> &v) const {
      return { y * v.z - z * v.y, z * v.x - x * v.z, x * v
           .y - y * v.x };
   }
   auto operator<=>(const vec<T> &v) const = default;
   bool operator==(const vec<T> &v) const = default;
   T norm() const {
      return x * x + y * y + z * z;
   }
   double abs() const {
      return sqrt(norm());
   }
   double cos(const vec<T> &v) const {
      return ((*this) * v) / (abs() * v.abs());
   }
   friend ostream &operator<<(ostream &out, const vec<T> &v
        ) {
      return out << v.x << sp << v.y << sp << v.z;
   }
   friend istream &operator>>(istream &in, vec<T> &v) {
      return in >> v.x >> v.y >> v.z;
   }
};
```

4.2 Planimetry

```
// Theme: Planimetry Objects

// Point
template <typename T>
struct point {
   T x, y;

   point() : x(0), y(0) { }
   point(T x, T y) : x(x), y(y) { }
};


// Rectangle
template <typename T>
struct rectangle {
   point<T> ld, ru;

   rectangle(const point<T> &ld, const point<T> &ru) :
      ld(ld), ru(ru) { }
};
```

4.3 Graham

```
// Theme: Convex Hull

// Algorithm: Graham Algorithm
// Complexity: O(N*log(N))

auto graham(const vector<vec<int>> &points) {
   vec<int> p0 = points[0];

   for (auto p : points)
      if (p.y < p0.y ||
      p.y == p0.y && p.x > p0.x)
         p0 = p;

   for (auto &p : points) {
      p.x -= p0.x;
      p.y -= p0.y;
   }

   sort(all(points), [] (vec<int> &p1, vec<int> &p2) {
      return (p1 ^ p2).z > 0 ||
         (p1 ^ p2).z == 0 && p1.norm() > p2.norm(); });

   vector<vec<int>> hull;
   for (auto &p : points) {
      while (hull.size() >= 2 &&
      (((p - hull.back()) ^ (hull[hull.size() - 1] - hull[
           hull.size() - 2]))).z >= 0)
         hull.pop_back();
      hull.push_back(p);
   }

   for (auto &p : hull) {
      p.x += p0.x;
      p.y += p0.y;
   }

   return hull;
}
```

4.4 Formulae

Triangles.
Radius of circumscribed circle:
$R = \frac{abc}{4S}$.
Radius of inscribed circle:
$r = \frac{S}{p}$.
Side via medians:
$a = \frac{2}{3}\sqrt{2(m_b^2 + m_c^2) - m_a^2}$.
Median via sides:
$m_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$.
Bisector via sides:
$l_a = \frac{2\sqrt{bcp(p-a)}}{b+c}$.
Bisector via two sides and angle:
$l_a = \frac{2bc\cos\frac{\alpha}{2}}{b+c}$.
Bisector via two sides and divided side:
$l_a = \sqrt{bc - a_b a_c}$.

Right triangles.
a, b - cathets, c - hypotenuse.
h - height to hypotenuse, divides c to c_a and c_b.

$$\begin{cases} h^2 = c_a \cdot c_b, \\ a^2 = c_a \cdot c, \\ b^2 = c_b \cdot c. \end{cases}$$

Quadrangles.
Sides of circumscribed quadrangle:
$a + c = b + d$.

Square of circumscribed quadrangle:
$S = \frac{Pr}{2} = pr$.
Angles of inscribed quadrangle:
$\alpha + \gamma = \beta + \delta = 180°$.
Square of inscribed quadrangle:
$S = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

Circles.
Intersection of circle and line:
$$\begin{cases}(x-x_0)^2 + (y-y_0)^2 = R^2\\ y = ax + b\end{cases}$$
Task comes to solution of $\alpha x^2 + \beta x + \gamma = 0$, where
$$\begin{cases}\alpha = (1 + a^2),\\ \beta = (2a(b - y_0) - 2x_0),\\ \gamma = (x_0^2 + (b - y_0)^2 - R^2).\end{cases}$$
Intersection of circle and circle:
$$\begin{cases}(x-x_0)^2 + (y-y_0)^2 = R_0^2\\ (x-x_1)^2 + (y-y_1)^2 = R_1^2\end{cases}$$
$y = \frac{1}{2}\frac{(R_1^2 - R_0^2) + (x_0^2 - x_1^2) + (y_0^2 - y_1^2)}{y_0 - y_1} - \frac{x_0 - x_1}{y_0 - y_1}x$
Task comes to intersection of circle and line.

5 Stringology

5.1 Z Function

```
// Theme: Z-Function

// Algorithm: Linear Algorithm
// Complexity: O(N)

auto z_func(const string &s) {
   int n = s.size();
   vector<int> z(n);

   for (int i = 1, l = 0, r = 0; i < n; i++) {
      if (i <= r) z[i] = min(r - i + 1, z[i - l]);

      while (i + z[i] < n && s[z[i]] == s[i + z[i]]) z[i
          ]++;

      if (i + z[i] - 1 > r) {
         l = i;
         r = i + z[i] - 1;
      }
   }

   return z;
}
```

5.2 Manacher

```
// Theme: Palindromes

// Algorithm: Manacher Algorithm
// Complexity: O(N)

int manacher(const string &s) {
   int n = s.size();
   vector<int> d1(n), d2(n);

   for (int i = 0, l = 0, r = -1; i < n; i++) {
      int k = i > r ? 1 : min(d1[l + r - i], r - i + 1);
      while (i + k < n && i - k >= 0 && s[i + k] == s[i -
          k]) k++;
      d1[i] = k;
      if (i + k - 1 > r) {
         l = i - k + 1;
         r = i + k - 1;
      }
   }

   for (int i = 0, l = 0, r = -1; i < n; i++) {
      int k = i > r ? 0 : min(d2[l + r - i + 1], r - i +
          1);
      while (i + k < n && i - k - 1 >= 0 && s[i + k] == s[
          i - k - 1]) k++;
      d2[i] = k;
      if (i + k - 1 > r) {
         l = i - k;
         r = i + k - 1;
      }
   }

   int res = 0;
   for (int i = 0; i < n; i++) {
      res += d1[i] + d2[i];
   }

   return res;
}
```

5.3 Trie

```
// Theme: Trie

// Algorithm: Aho-Corasick
// Complexity: O(N)

struct trie {
   // Vertex
   struct vertex {
      vector<int> next;
      bool leaf;
   };

   // Alphabet size
   static const int N = 26;
   // Maximum Vertex Number
   static const int MX = 2e5 + 1;

   // Vertices Vector
   vector<vertex> t;
   int sz;

   trie(): sz(1) {
      t.resize(MX);
      t[0].next.assign(N, -1);
   }

   void add_str(const string &s) {
      int v = 0;
      for (int i = 0; i < s.length(); i++) {
         char c = s[i] - 'a';
         if (t[v].next[c] == -1) {
            t[sz].next.assign(N, -1);
            t[v].next[c] = sz++;
         }
         v = t[v].next[c];
      }
      t[v].leaf = true;
   }
};
```

5.4 Prefix Function

```
// Theme: Prefix function

// Algorithm: Knuth-Morris-Pratt Algorithm
// Complexity: O(N)

auto pref_func(const string &s) {
   int n = s.size();
   vector<int> pi(n);

   for (int i = 1; i < n; i++) {
      int j = pi[i - 1];

      while (j > 0 && s[i] != s[j]) j = pi[j - 1];
```

```
        if (s[i] == s[j]) j++;

        pi[i] = j;
    }

    return pi;
}
```

5.5 Suffix Array

```
// Theme: Suffix array

// Algorithm: Binary Algorithm With Count Sort
// Complexity: O(N*log(N))

void count_sort(vector<int> &p, vector<int> &c) {
    int n = p.size();
    vector<int> cnt(n), p_new(n), pos(n);

    for (auto &x : c) cnt[x]++;

    pos[0] = 0;
    for (int i = 1; i < n; i++)
        pos[i] = pos[i - 1] + cnt[i - 1];

    for (auto &x : p) {
        int i = c[x];
        p_new[pos[i]] = x;
        pos[i]++;
    }

    p = p_new;
}

auto suffix_array(const string &str) {
    string s = str + '$';
    int n = s.size();

    vector<int> p(n), c(n);
    vector<pair<char, int>> a(n);

    for (int i = 0; i < n; i++) a[i] = { str[i], i };

    sort(a.begin(), a.end());

    for (int i = 0; i < n; i++) p[i] = a[i].second;

    c[p[0]] = 0;
    for (int i = 1; i < n; i++)
        c[p[i]] = c[p[i - 1]] + (a[i].first != a[i - 1].
            first);

    int k = 0;
    while ((1 << k) < n) {
        for (int i = 0; i < n; i++)
            p[i] = (p[i] - (1 << k) + n) % n;

        count_sort(p, c);

        vector<int> c_new(n);

        c_new[p[0]] = 0;
        for (int i = 1; i < n; i++) {
            pair<int, int> prev = { c[p[i - 1]], c[(p[i - 1]
                + (1 << k)) % n] };
            pair<int, int> now = { c[p[i]], c[(p[i] + (1 << k
                )) % n] };
            c_new[p[i]] = c_new[p[i - 1]] + (now != prev);
        }

        c = c_new;
        k++;
    }

    return p;
}
```

6 Dynamic Programming

6.1 Longest Increasing Subsequence

```
// Theme: Longest Increasing Subsequence

// Algorithm: LIS via binary search
// Complexity: O(N*log(N))

vector<int> lis(vector<int>& arr) {
    int n = arr.size();
    vector<int> dp(n + 1, INF); dp[0] = -INF;
    vector<int> pos(n + 1, -1), previous(n, -1);
    int length = 0;
    for (int i = 0; i < n; ++i) {
        int j = lower_bound(dp.begin(), dp.end(), arr[i]) -
            dp.begin();
        if (dp[j - 1] < arr[i] && arr[i] < dp[j]) {
            dp[j] = arr[i];
            pos[j] = i;
            previous[i] = pos[j - 1];
            length = max(length, j);
        }
    }

    vector<int> res;
    int p = pos[length];
    while (p != -1) {
        res.push_back(arr[p]);
        p = previous[p];
    }
    reverse(res.begin(), res.end());
    return res;
}
```

6.2 Longest Common Subsequence

```
// Theme: Longest Common Subsequence

// Algorithm: LCS via simple dynamic
// Complexity: O(n*m)

vector<int> lis(vector<int>& arr) {
    int n = arr.size();
    vector<int> dp(n + 1, INF); dp[0] = -INF;
    vector<int> pos(n + 1, -1), previous(n, -1);
    int length = 0;
    for (int i = 0; i < n; ++i) {
        int j = lower_bound(dp.begin(), dp.end(), arr[i]) -
            dp.begin();
        if (dp[j - 1] < arr[i] && arr[i] < dp[j]) {
            dp[j] = arr[i];
            pos[j] = i;
            previous[i] = pos[j - 1];
            length = max(length, j);
        }
    }

    vector<int> res;
    int p = pos[length];
    while (p != -1) {
        res.push_back(arr[p]);
        p = previous[p];
    }
    reverse(res.begin(), res.end());
    return res;
}
```

7 Graphs

7.1 Graph Implementation

```
// Theme: Graph Implementation
```

```
/////////////////////////////////////////////////
// Adjacency List (Unoriented)
/////////////////////////////////////////////////

int sz;

vector<vector<int>> graph;

graph.assign(n, {});

for (int i = 0; i < n; i++) {
   int u, v; cin >> u >> v; --u --v;
   graph[u].push_back(v);
   graph[v].push_back(u);
}

/////////////////////////////////////////////////
// Adjacency List (Oriented)
/////////////////////////////////////////////////

int sz;

vector<vector<int>> graph;
vector<vector<int>> rgraph;

graph.assign(n, {});
rgraph.assign(n, {});

for (int i = 0; i < n; i++) {
   int u, v; cin >> u >> v; --u --v;
   graph[u].push_back(v);
   rgraph[v].push_back(u);
}

/////////////////////////////////////////////////
// Edges List (Unoriented)
/////////////////////////////////////////////////

int sz;

vector<pair<int, int>> edges;
vector<vector<int>> graph;

graph.assign(n, {});

for (int i = 0; i < n; i++) {
   int u, v; cin >> u >> v; --u; --v;
   edges.push_back({ u, v });
   graph[u].push_back(i);
   graph[v].push_back(i);
}

/////////////////////////////////////////////////
// Edges List + Structure (Unoriented)
/////////////////////////////////////////////////

struct edge {
   int u, v, w;
   edge(int u, int v, int w = 0)
       : u(u), v(v), w(w) { }
};

int sz;

vector<edge> edges;
vector<vector<int>> graph;

graph.assign(n, {});

for (int i = 0; i < n; i++) {
   int u, v, w; cin >> u >> v >> w; --u; --v;
   edges.push_back({ u, v, w });
   graph[u].push_back(i);
   graph[v].push_back(i);
}

/////////////////////////////////////////////////
// Edges List + Structure + Net Flows (Oriented)
/////////////////////////////////////////////////

struct edge {
   int to, cap, flow, weight;
   edge(int to, int cap, int flow = 0, int weight = 0)
       : to(to), cap(cap), flow(flow), weight(weight) { }
   int res() {
      return cap - flow;
   }
};

int sz;

vector<edge> edges;
vector<vector<int>> fgraph;

fgraph.assign(n, {});

void add_edge(int u, int v, int limit, int flow = 0, int
     weight = 0) {
   fgraph[u].push_back(edges.size());
   edges.push_back({ v, limit, flow, weight });
   fgraph[v].push_back(edges.size());
   edges.push_back({ u, 0, 0, -weight });
}

/////////////////////////////////////////////////
// Adjacency Matrix
/////////////////////////////////////////////////

vector<vector<int>> graph;

for (int i = 0; i < n; i++)
   for (int j = 0; j < n; j++)
      cin >> graph[i][j];
```

7.2 Graph Traversing

```
// Theme: Graph Traversing

vector<vector<int>> graph;
vector<int> used;

// Algorithm: Depth-First Search (Adjacency List)
// Complexity: O(N + M)

void dfs(int cur, int p = -1) {
   used[cur] = 1;

   for (auto &to : graph[cur]) {
      if (to == p || used[to]) continue;
      dfs(to, cur);
   }
}

// Algorithm: Breadth-First Search (Adjacency List)
// Complexity: O(N + M)

void bfs(int u) {
   queue<int> q; q.push(u);

   while (q.size()) {
      int cur = q.front(); q.pop();

      for (auto &to : graph[cur]) {
         if (used[to]) continue;
         q.push(to);
      }
   }
}
```

7.3 Topological Sort

```
// Theme: Topological Sort

vector<vector<int>> graph;
vector<int> used;

// Algorithm: Topological Sort
// Complexity: O(N + M)

vector<int> topsort;

void dfs_topsort(int cur, int p = -1) {
   used[cur] = 1;

   for (auto &to : graph[cur]) {
      if (to == p || used[to]) continue;
      dfs(to, cur);
   }

   topsort.push_back(cur);
```

```
}

for (int u = 0; u < n; u++)
   if (!used[u])
      dfs_topsort(u);

reverse(all(topsort));
```

7.4 Connected Components

```
// Theme: Connectivity Components

vector<vector<int>> graph;
vector<int> used;

// Algorithm: Connected Components
// Complexity: O(N + M)

vector<vector<int>> cc;

void dfs_cc(int cur, int p = -1) {
   used[cur] = 1;
   cc.back().push_back(cur);

   for (auto &to : graph[cur]) {
      if (to == p || used[to]) continue;
      dfs_cc(to, cur);
   }
}

for (int u = 0; u < n; i++)
   if (!used[u])
      dfs_cc(u);

// Algorithm: Strongly Connected Components
// Complexity: O(N + M)

vector<vector<int>> rgraph;

vector<vector<int>> topsort;

vector<vector<int>> scc;

void dfs_scc(int cur, int p = -1) {
   used[cur] = 1;
   scc.back().push_back(cur);

   for (auto &to : rgraph[cur]) {
      if (to == p || used[to]) continue;
      dfs_scc(to, cur);
   }
}

for (auto &u: topsort)
   if (!used[u])
      dfs_scc(u);
```

7.5 2 Sat

```
// Theme: 2-SAT

// Algorithm: Adding Edges To 2-SAT

vector<vector<int>> ts_graph;
vector<vector<int>> ts_rgraph;
vector<int>used;
vector<int>top_sort;

// Vertex By Var Number
int to_vert(int x) {
   if (x < 0) {
      return ((abs(x) - 1) << 1) ^ 1;
   }
   else {
      return (x - 1) << 1;
   }
}

// Adding Implication
void add_impl(int a, int b) {
   ts_graph[a].insert(b);
   ts_rgraph[b].insert(a);
}

// Adding Disjunction
void add_or(int a, int b) {
   add_impl(a ^ 1, b);
   add_impl(b ^ 1, a);
}

//topsort
void dfs(int v){
   used[v] = 1;
   for(auto to:ts_graph[v]){
      if(!used[to])dfs(to);
   top_sort.push_back(v);
}

//scc
vector<vector<long long int>> scc;
void dfs_scc(long long int cur, long long int p = -1) {
   used[cur] = 1;
   scc.back().push_back(cur);
   for (auto to : rgr[cur]) {
      if (to == p || used[to]) continue;
      dfs_scc(to, cur);
   }
}

int main(){
   ...
   used.resize(n,0);
   for(i=0;i<n;i++){
      if(!used[i])dfs(i);
   }
   reverse(top_sort.begin(), top_sort.end());
   for(auto it:top_sort){
      if (!used[u]) {
         scc.push_back({});
         dfs_scc(u);
      }
   }
   vector<long long int>v_scc;
   v_scc.assign(2 * n, -1);

   for (int i = 0; i < scc.size(); i++)
      for (auto& u : scc[i])
         v_scc[u] = i;

   vector<long long int> values(2 * n, -1);

   for (int i = 0; i < 2 * n; i += 2)
      if (v_scc[i] == v_scc[i ^ 1]) {
         cout << "NO\n";
         return 0;
      }
      else {
         if (v_scc[i] < v_scc[i ^ 1]) {
            values[i] = 0;
            values[i ^ 1] = 1;
         }
         else {
            values[i] = 1;
            values[i ^ 1] = 0;
         }
      }
}
```

7.6 Bridges

```
// Theme: Bridges And ECC

vector<pair<int, int>> edges;
vector<vector<int>> graph;
vector<int> used;

vector<int> height;
vector<int> up;

// Algorithm: Bridges
// Complexity: O(N + M)

vector<int> bridges;

void dfs_bridges(int cur, int p = -1) {
   used[cur] = 1;
   up[cur] = height[cur];
```

```
   for (auto &ind : g[cur]) {
      int to = cur ^ edges[ind].ff ^ edges[ind].ss;
      if (to == p) continue;
      if (used[to]) {
         up[cur] = min(up[cur], height[to]);
      }
      else {
         height[to] = height[cur] + 1;
         dfs_bridges(to, cur);
         up[cur] = min(up[cur], up[to]);
         if (up[to] > height[cur])
            bridges.push_back(ind);
      }
   }
}

// Algorithm: ECC
// Complexity: O(N + M)

vector<int> st;

vector<int> add_comp(vector<int> &st, int sz) {
   vector<int> comp;

   while (st.size() > sz) {
      comp.push_back(st.back());
      st.pop_back();
   }

   return comp;
}

vector<vector<int>> ecc;

void dfs_bridges_comps(int cur, int p = -1) {
   used[cur] = 1;
   up[cur] = height[cur];

   for (auto &ind : g[cur]) {
      int to = cur ^ edges[ind].ff ^ edges[ind].ss;
      if (to == p) continue;
      if (used[to]) {
         up[cur] = min(up[cur], height[to]);
      }
      else {
         int sz = st.size();
         st.push_back(to);
         height[to] = height[cur] + 1;
         dfs_bridges_comps(to, cur);
         up[cur] = min(up[cur], up[to]);
         if (up[to] > height[cur])
            ecc.push_back(add_comp(st, sz));
      }
   }
}
```

7.7 Articulation Points

```
// Theme: Articulation Points And VCC

vector<pair<int, int>> edges;
vector<vector<int>> graph;
vector<int> used;

vector<int> height;
vector<int> up;

// Algorithm: Articulation Points
// Complexity: O(N + M)

set<int> art_points;

void dfs_artics(int cur, int p = -1) {
   used[cur] = 1;
   up[cur] = height[cur];

   int desc_count = 0;

   for (auto &ind : g[cur]) {
      int to = cur ^ edges[ind].ff ^ edges[ind].ss;
      if (to == p) continue;
      if (used[to]) {
         up[cur] = min(up[cur], height[to]);
      }
      else {
         desc_count++;
         height[to] = height[cur] + 1;
         dfs_artics(to, cur);
         up[cur] = min(up[cur], up[to]);
         if (up[to] >= height[cur] && p != -1)
            art_points.insert(cur);
      }
   }

   if (p == -1 && desc_count > 1) {
      art_points.insert(cur);
   }
}

// Algorithm: VCC
// Complexity: O(N + M)

vector<vector<int>> vcc;

void dfs_artics_comps(int cur, int p = -1) {
   used[cur] = 1;
   up[cur] = height[cur];

   for (auto &ind : g[cur]) {
      int to = cur ^ edges[ind].ff ^ edges[ind].ss;
      if (to == p) continue;
      if (used[to]) {
         up[cur] = min(up[cur], height[to]);
         if (height[to] < height[cur]) st.push_back(ind);
      }
      else {
         int sz = st.size();
         st.push_back(ind);
         height[to] = height[cur] + 1;
         dfs_artics_comps(to, cur);
         up[cur] = min(up[cur], up[to]);
         if (up[to] >= height[cur])
            vcc.push_back(add_comp(st, sz));
      }
   }
}
```

7.8 Kuhn

```
// Maximum Matching

// Algorithm: Kuhn Algorithm
// Complexity: O(|Left Part|^3)

vector<vector<int>> bigraph;
vector<int> used;

vector<int> mt;

bool kuhn(int u) {
   if (used[u]) return false;

   used[u] = 1;

   for (auto &v : bigraph[u]) {
      if (mt[v] == -1 || kuhn(mt[v])) {
         mt[v] = u;
         return true;
      }
   }

   return false;
}

int main() {
      ... чтениеграфа...

      mt.assign (k, -1);
      for (int v=0; v<n; ++v) {
            used.assign (n, false);
            try_kuhn (v);
      }

      for (int i=0; i<k; ++i)
            if (mt[i] != -1)
                  printf ("%d %d\n", mt[i]+1, i+1);
}
```

7.9 Kruskal

```
#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

struct dsu {
   vector<int> p, size;

   dsu(int n) {
      p.assign(n, 0); size.assign(n, 0);
      for (int i = 0; i < n; i++) {
         p[i] = i;
         size[i] = 1;
      }
   }

   int get(int v) {
      if (p[v] != v) p[v] = get(p[v]);
      return p[v];
   }

   void unite(int u, int v) {
      auto x = get(u), y = get(v);
      if (x == y) return;
      if (size[x] > size[y]) swap(x, y);
      p[x] = y; size[y] += size[x];
   }
};

int sz;

struct edge {
   long long int u, v, w;
   edge(long long int uu, long long int vv, long long int
        ww) :u(uu), v(vv), w(ww) {};
};

vector<edge> edges;
vector<vector<int>> graph;

// Algorithm: Kruskal Algorithm
// Complexity: O(M)

vector<edge> mst;

void kruskal() {
   dsu d(sz);

   auto tedges = edges;
   sort(tedges.begin(), tedges.end(), [](edge& e1, edge& e2
        ) { return e1.w < e2.w; });

   for (auto& e : tedges) {
      if (d.get(e.u) != d.get(e.v)) {
         mst.push_back(e);
         d.unite(e.u, e.v);
      }
   }
}

int main() {
   long long int n, m, i, j, k,a,b,c;
   cin >> n >> m;
   for (i = 0; i < m; i++) {
      cin >> a >> b >> c;
      a--; b--;
      edge e(a, b, c);
      edges.push_back(e);
   }
   sz = n;
   kruskal();
   long long int ans = 0;
   for (auto it : mst)ans += it.w;
   cout << ans;
}
```

7.10 LCA (binary Lifting)

```
// Theme: Lowest Common Ancestor

// Algorithm: Binary Lifting
// Complextiry: Preprocessing O(N * log(N)) and request O(
     log(N))

vector<vector<int>> g;
vector<int> d;
vector<vector<int>> dp;
vector<int> used;

void dfs(int v, int p = -1) {
  if (p == -1) {
    p = v;
    d[v] = 0;
  } else {
    d[v] = d[p] + 1;
  }

  dp[0][v] = p;
  for (int i = 1; i < dp.size(); i++) {
    dp[i][v] = dp[i - 1][dp[i - 1][v]];
  }
  for (int to : g[v]) {
    if (to != p) {
      dfs(to, v);
    }
  }
}

int lca(int a, int b) {
  if (d[a] > d[b]) {
    swap(a, b);
  }

  for (int i = dp.size() - 1; i >= 0; i--) {
    if (d[dp[i][b]] >= d[a]) {
      b = dp[i][b];
    }
  }

  if (a == b) {
    return a;
  }

  for (int i = dp.size() - 1; i >= 0; i--) {
    if (dp[i][a] != dp[i][b]) {
      a = dp[i][a];
      b = dp[i][b];
    }
  }

  return dp[0][a];
}

signed main() {
  int n = 0, m = 0; // n - vertex count, m - requests
  g.resize(n);
  d.resize(n);
  dp.resize(((int)log2(n) + 1));
  used.assign(n, 0);
  for (int i = 0; i < dp.size(); i++) {
    dp[i].resize(n);
  }

  // ...reading graph...

  dfs(0);

  // ...lca(u -1, v - 1) + 1
}
```

7.11 LCA RMQ (seqtree)

```
// Theme: Lowest Common Ancestor

// Algorithm: RMQ (seqtree)
// Complextiry: Preprocessing O(N) and request O(log(N))

typedef vector<vector<int>> graph;
typedef vector<int>::const_iterator const_graph_iter;

vector<int> lca_h, lca_dfs_list, lca_first, lca_tree;
vector<char> lca_dfs_used;

void lca_dfs(const graph &g, int v, int h = 1) {
  lca_dfs_used[v] = true;
  lca_h[v] = h;
```

```
  lca_dfs_list.push_back(v);
  for (const_graph_iter i = g[v].begin(); i != g[v].end();
       ++i)
    if (!lca_dfs_used[*i]) {
      lca_dfs(g, *i, h + 1);
      lca_dfs_list.push_back(v);
    }
}

void lca_build_tree(int i, int l, int r) {
  if (l == r)
    lca_tree[i] = lca_dfs_list[l];
  else {
    int m = (l + r) >> 1;
    lca_build_tree(i + i, l, m);
    lca_build_tree(i + i + 1, m + 1, r);
    if (lca_h[lca_tree[i + i]] < lca_h[lca_tree[i + i + 1]])
      lca_tree[i] = lca_tree[i + i];
    else
      lca_tree[i] = lca_tree[i + i + 1];
  }
}

void lca_prepare(const graph &g, int root) {
  int n = (int)g.size();
  lca_h.resize(n);
  lca_dfs_list.reserve(n * 2);
  lca_dfs_used.assign(n, 0);
  lca_dfs(g, root);
  int m = (int)lca_dfs_list.size();
  lca_tree.assign(lca_dfs_list.size() * 4 + 1, -1);
  lca_build_tree(1, 0, m - 1);
  lca_first.assign(n, -1);
  for (int i = 0; i < m; ++i) {
    int v = lca_dfs_list[i];
    if (lca_first[v] == -1)
      lca_first[v] = i;
  }
}

int lca_tree_min(int i, int sl, int sr, int l, int r) {
  if (sl == l && sr == r)
    return lca_tree[i];
  int sm = (sl + sr) >> 1;
  if (r <= sm)
    return lca_tree_min(i + i, sl, sm, l, r);
  if (l > sm)
    return lca_tree_min(i + i + 1, sm + 1, sr, l, r);
  int ans1 = lca_tree_min(i + i, sl, sm, l, sm);
  int ans2 = lca_tree_min(i + i + 1, sm + 1, sr, sm + 1, r)
       ;
  return lca_h[ans1] < lca_h[ans2] ? ans1 : ans2;
}

int lca(int a, int b) {
  int left = lca_first[a],
     right = lca_first[b];
  if (left > right)
    swap(left, right);
  return lca_tree_min(1, 0, (int)lca_dfs_list.size() - 1,
       left, right);
}

signed main() {
  int n = 0, m = 0; // n - vertex count, m - requests
  graph g(n);

  // ...reading graph...

  lca_prepare(g, 0);

  // ...lca(u -1, v - 1) + 1
}
```

7.12 Maximum Flow

```
// Theme: Maximum Flow

int s, t, sz;

vector<edge> edges;
vector<vector<int>> fgraph;

vector<int> used;

// Algorithm: Ford-Fulkerson Algorithm
// Complexity: O(MF)

int dfs_fordfulk(int u, int bound, int flow = INF) {
    if (used[u]) return 0;
    if (u == t) return flow;

    used[u] = 1;

    for (auto &ind : fgraph[u]) {
        auto &e = edges[ind],
            &_e = edges[ind ^ 1];
        int to = e.to, res = e.res();

        if (res < bound) continue;

        int pushed = dfs_fordfulk(to, bound, min(res, flow))
             ;

        if (pushed) {
            e.flow += pushed;
            _e.flow -= pushed;
            return pushed;
        }
    }

    return 0;
}

// Algorithm: Edmonds-Karp Algorithm
// Complexity: O(N(M^2))

vector<int> p;
vector<int> pe;

void augment(int pushed) {
    int cur = t;
    while (cur != s) {
        auto &e = edges[pe[cur]],
            &_e = edges[pe[cur] ^ 1];
        e.flow += pushed;
        _e.flow -= pushed;
        cur = p[cur];
    }
}

int bfs_edmskarp(int u, int bound) {
    p.assign(sz, -1);
    pe.assign(sz, -1);

    int pushed = 0;

    queue<pair<int, int>> q;
    q.push({ u, INF });

    used[u] = 1;

    while (q.size()) {
        auto [v, f] = q.front(); q.pop();

        for (auto &ind : fgraph[v]) {
            auto &e = edges[ind];
            int to = e.to, res = e.res();

            if (used[to] || res < bound) continue;

            p[to] = v;
            pe[to] = ind;
            used[to] = 1;

            if (to == t) {
                pushed = min(f, res);
                break;
            }

            q.push({ to, min(f, res) });
        }
    }

    if (pushed)
        augment(pushed);

    return pushed;
}

// Algorithm: Dinic Algorithm
// Complexity: O((N^2)M)
```

```
vector<int> d;

bool bfs_dinic(int u, int bound) {
   d.assign(sz, INF); d[u] = 0;

   queue<int> q; q.push(u);

   while (q.size()) {
      int v = q.front(); q.pop();

      for (auto &ind : fgraph[v]) {
         auto &e = edges[ind];
         int to = e.to, res = e.res();

         if (d[v] + 1 >= d[to] || res < bound) continue;

         d[to] = d[v] + 1;
         q.push(to);
      }
   }

   return d[t] != INF;
}

vector<int> lst;

int dfs_dinic(int u, int mxf = INF) {
   if (u == t) return mxf;

   int smf = 0;

   for (int i = lst[u]; i < fgraph[u].size(); i++) {
      int ind = fgraph[u][i];

      auto &e = edges[ind],
         &_e = edges[ind ^ 1];
      int to = e.to, res = e.res();

      if (d[to] == d[u] + 1 && res) {
         int pushed = dfs_dinic(to, min(res, mxf - smf));

         if (pushed) {
            smf += pushed;
            e.flow += pushed;
            _e.flow -= pushed;
         }
      }

      lst[u]++;

      if (smf == mxf)
         return smf;
   }

   return smf;
}

int dinic(int u) {
   int pushed = 0;

   for (int bound = 1ll << 30; bound; bound >>= 1) {
      while (true) {
         bool bfs_ok = bfs_dinic(u, bound);
         if (!bfs_ok) break;

         lst.assign(sz, 0);

         while (true) {
            int dfs_pushed = dfs_dinic(u);
            if (!dfs_pushed) break;

            pushed += dfs_pushed;
         }
      }
   }

   return pushed;
}

// Algorithm: Maximum Flow Of Minimum Cost (SPFA)
// Complexity: ...


vector<int> d;
vector<int> p;
vector<int> pe;

void augment(int pushed) {
   int cur = t;
   while (cur != s) {
      auto &e = edges[pe[cur]],
         &_e = edges[pe[cur] ^ 1];
      e.flow += pushed;
      _e.flow -= pushed;
      cur = p[cur];
   }
}

int bfs_spfa(int u, int flow = INF) {
   d.assign(sz, INF); d[u] = 0;
   p.assign(sz, -1);
   pe.assign(sz, -1);

   queue<pair<int, int>> q; q.push({ u, flow });
   vector<int> in_q(sz, 0); in_q[u] = 1;

   int pushed = 0;

   while (q.size()) {
      auto [v, f] = q.front(); q.pop();

      in_q[v] = 0;

      if (v == t) {
         pushed = f;
         break;
      }

      for (auto &ind : fgraph[v]) {
         auto &e = edges[ind];
         int to = e.to, res = e.res(),
            w = e.weight;

         if (d[v] + w >= d[to] || !res) continue;

         d[to] = d[v] + w;
         p[to] = v;
         pe[to] = ind;

         if (!in_q[to]) {
            in_q[to] = 1;
            q.push({ to, min(f, res) });
         }
      }
   }

   if (pushed)
      augment(pushed);

   return pushed;
}
```

7.13 Eulerian Path

```
// Theme: Eulerian Path

int sz;

vector<vector<int>> graph;

// Algorithm: Eulerian Path
// Complexity: O(M)

vector<int> eul;

// 0 - path not exist
// 1 - cycle exits
// 2 - path exists
int euler_path() {
   vector<int> deg(sz);

   for (int i = 0; i < sz; i++)
      for (int j = 0; j < sz; j++)
         deg[i] += g[i][j];

   int v1 = -1, v2 = -1;
   for (int i = 0; i < sz; i++)
      if (deg[i] & 1)
         if (v1 == -1) v1 = i;
         else if (v2 == -1) v2 = i;
         else return 0;

   if (v1 != -1) {
```

```
        if (v2 == -1)
            return 0;
        graph[v1][v2]++;
        graph[v2][v1]++;
    }

    stack<int> st;

    for (int i = 0; i < sz; i++) {
        if (deg[i]) {
            st.push(i);
            break;
        }
    }

    while (st.size()) {
        int u = st.top();

        int ind = -1;

        for (int i = 0; i < sz; i++)
            if (graph[u][i]) {
                ind = i;
                break;
            }

        if (ind == -1) {
            eul.push_back(u);
            st.pop();
        }
        else {
            graph[u][ind]--;
            graph[ind][u]--;
            st.push(ind);
        }
    }

    int res = 2;
    if (v1 != -1) {
        res = 1;

        for (int i = 0; i < eul.size() - 1; i++)
            if (eul[i] == v1 && eul[i + 1] == v2 ||
            eul[i] == v2 && eul[i + 1] == v1) {
                vector<int> teul;
                for (int j = i + 1; j < eul.size(); j++)
                    teul.push_back(eul[j]);
                for (int j = 0; j <= i; j++)
                    teul.push_back(eul[j]);
                eul = teul;
                break;
            }
    }

    for (int i = 0; i < sz; i++)
        for (int j = 0; j < sz; j++)
            if (graph[i][j])
                return 0;

    return res;
}
```

7.14 Eulerian Path Oriented

```
// Theme: Eulerian Path

// Algorithm: Eulerian Path
// Complexity: O(M)

// Algo doesn't validate the path to be correct.
// If the path exists, it will be found.
// Result way has to be reversed after execution.

struct Edge {
    int v, u;
    bool deleted = false;
};

vector<Edge> edges;

vector<deque<int>> graph;
vector<int> way;

void euler(int v, int last = -1) {
    while (!graph[v].empty()) {
        int idx = graph[v].front(); graph[v].pop_front();
        auto& e = edges[idx];
        if (e.deleted) continue;
        e.deleted = true; // If graph is not oriented, also
            delete reverse edge[v ^ 1]
        euler(e.u, idx);
    }
    if (last != -1) way.push_back(last);
}
```

7.15 Shortest Paths

```
// Theme: Shortest Paths

int sz;

vector<edge> edges;
vector<vector<int>> graph;

// Algorithm: Dijkstra Algorithm
// Complexity: O(M*log(N))

vector<int> d;
vector<int> p;

void dijkstra(int u) {
    d.assign(sz, INF); d[u] = 0;
    p.assign(sz, -1);

    priority_queue<pair<int, int>> q;
    q.push({ 0, u });

    while (q.size()) {
        int dist = -q.top().ff, v = q.top().ss; q.pop();

        if (dist > d[v]) continue;

        for (auto &ind : graph[v]) {
            int to = v ^ edges[ind].u ^ edges[ind].v,
                w = edges[ind].w;
            if (d[v] + w < d[to]) {
                d[to] = d[v] + w;
                p[to] = v;
                q.push({ -d[to], -to });
            }
        }
    }
}

// Algorithm: Shortest Path Faster Algorithm
// Complexity: ...

vector<int> d;

void bfs_spfa(int u) {
    d.assign(sz, INF); d[u] = 0;

    queue<int> q; q.push(u);
    vector<int> in_q(sz, 0); in_q[u] = 1;

    while (q.size()) {
        auto [v, f] = q.front(); q.pop();

        in_q[v] = 0;

        for (auto &ind : graph[v]) {
            int to = v ^ edges[ind].u ^ edges[ind].v,
                w = edges[ind].w;
            if (d[v] + w < d[to]) {
                d[to] = d[v] + w;
                if (!in_q[to]) {
                    in_q[to] = 1;
                    q.push( to );
                }
            }
        }
    }
}

// Algorithm: Belman-Ford Algorithm
// Complexity: (N*M)

vector<int> d;

void bfa(int u) {
```

```
   d.assign(sz, INF); d[u] = 0;

   for (;;) {
      bool any = false;

      for (auto &e : edges) {
         if (d[e.u] != INF && d[e.u] + e.w < d[e.v]) {
            d[e.v] = d[e.u] + e.w;
            any = true;
         }
         if (d[e.v] != INF && d[e.v] + e.w < d[e.u]) {
            d[e.u] = d[e.v] + e.w;
            any = true;
         }
      }

      if (!any) break;
   }
}

// Algorithm: Floyd-Warshall Algorithm
// Complexity: O(N^3)

vector<vector<int>> d;

void fwa() {
   d.assign(sz, vector<int>(sz, INF));

   for (int i = 0; i < sz; i++)
      for (int j = 0; j < sz; j++)
         for (int k = 0; k < sz; k++)
            if (d[i][k] != INF && d[k][j] != INF)
               d[i][j] = min(d[i][j], d[i][k] + d[k][j]);
}
```

8 Miscellaneous

8.1 Ternary Search

```
// Theme: Ternary Search

// Algorithm: Continuous Search With Golden Ratio
// Complexity: O(log(N))

// Golden Ratio
// Phi = 1.618...
double phi = (1 + sqrt(5)) / 2;

double cont_tern_srch(double l, double r) {
  double m1 = l + (r - l) / (1 + phi),
    m2 = r - (r - l) / (1 + phi);

  double f1 = f(m1), f2 = f(m2);

  int count = 200;
  while (count--) {
    if (f1 < f2) {
      r = m2;
      m2 = m1;
      f2 = f1;
      m1 = l + (r - l) / (1 + phi);
      f1 = f(m1);
    }
    else {
      l = m1;
      m1 = m2;
      f1 = f2;
      m2 = r - (r - l) / (1 + phi);
      f2 = f(m2);
    }
  }

  return f((l + r) / 2);
}

// Algorithm: Discrete Search
// Complexity: O(log(N))

double discr_tern_srch(int l, int r) {
  while (r - l > 2) {
    int m1 = l + (r - l) / 3,
      m2 = r - (r - l) / 3;
    if (f(m1) < f(m2))
      r = m2;
    else
      l = m1;
  }

  return min(f(l), min(f(l + 1), f(r)));
}
```