ACM-ICPC Team Reference Document
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Contents

1 Graphs

1.1 Topological Sort

```
// Theme: Topological Sort

vector<vector<int>> graph;
vector<int> used;

// Algorithm: Topological Sort
// Complexity: O(N + M)

vector<int> topsort;

void dfs_topsort(int cur, int p = -1) {
   used[cur] = 1;

   for (auto &to : graph[cur]) {
      if (to == p || used[to]) continue;
      dfs(to, cur);
   }

   topsort.push_back(cur);
}

for (int u = 0; u < n; u++)
   if (!used[u])
      dfs_topsort(u);

reverse(all(topsort));
```

1.2 Graph Implementation

```
// Theme: Graph Implementation

////////////////////////////////////////////////
// Adjacency List (Unoriented)
////////////////////////////////////////////////

int sz;

vector<vector<int>> graph;

graph.assign(n, {});

for (int i = 0; i < n; i++) {
   int u, v; cin >> u >> v; --u --v;
   graph[u].push_back(v);
```

```
   graph[v].push_back(u);
}

////////////////////////////////////////////////
// Adjacency List (Oriented)
////////////////////////////////////////////////

int sz;

vector<vector<int>> graph;
vector<vector<int>> rgraph;

graph.assign(n, {});
rgraph.assign(n, {});

for (int i = 0; i < n; i++) {
   int u, v; cin >> u >> v; --u --v;
   graph[u].push_back(v);
   rgraph[v].push_back(u);
}

////////////////////////////////////////////////
// Edges List (Unoriented)
////////////////////////////////////////////////

int sz;

vector<pair<int, int>> edges;
vector<vector<int>> graph;

graph.assign(n, {});

for (int i = 0; i < n; i++) {
   int u, v; cin >> u >> v; --u; --v;
   edges.push_back({ u, v });
   graph[u].push_back(i);
   graph[v].push_back(i);
}

////////////////////////////////////////////////
// Edges List + Structure (Unoriented)
////////////////////////////////////////////////

struct edge {
   int u, v, w;
   edge(int u, int v, int w = 0)
       : u(u), v(v), w(w) { }
};

int sz;

vector<edge> edges;
vector<vector<int>> graph;

graph.assign(n, {});

for (int i = 0; i < n; i++) {
   int u, v, w; cin >> u >> v >> w; --u; --v;
   edges.push_back({ u, v, w });
   graph[u].push_back(i);
   graph[v].push_back(i);
}

////////////////////////////////////////////////
// Edges List + Structure + Net Flows (Oriented)
////////////////////////////////////////////////

struct edge {
   int to, cap, flow, weight;
   edge(int to, int cap, int flow = 0, int weight = 0)
       : to(to), cap(cap), flow(flow), weight(weight) { }
   int res() {
      return cap - flow;
   }
};

int sz;

vector<edge> edges;
vector<vector<int>> fgraph;

fgraph.assign(n, {});

void add_edge(int u, int v, int limit, int flow = 0, int
     weight = 0) {
   fgraph[u].push_back(edges.size());
   edges.push_back({ v, limit, flow, weight });
   fgraph[v].push_back(edges.size());
   edges.push_back({ u, 0, 0, -weight });
}

////////////////////////////////////////////////
// Adjacency Matrix
////////////////////////////////////////////////

vector<vector<int>> graph;

for (int i = 0; i < n; i++)
   for (int j = 0; j < n; j++)
      cin >> graph[i][j];
```

1.3 Kruskal

```
#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

struct dsu {
   vector<int> p, size;

   dsu(int n) {
      p.assign(n, 0); size.assign(n, 0);
      for (int i = 0; i < n; i++) {
         p[i] = i;
         size[i] = 1;
      }
   }

   int get(int v) {
      if (p[v] != v) p[v] = get(p[v]);
      return p[v];
   }

   void unite(int u, int v) {
      auto x = get(u), y = get(v);
      if (x == y) return;
      if (size[x] > size[y]) swap(x, y);
      p[x] = y; size[y] += size[x];
   }
};

int sz;

struct edge {
   long long int u, v, w;
   edge(long long int uu, long long int vv, long long int
        ww) :u(uu), v(vv), w(ww) {};
};

vector<edge> edges;
vector<vector<int>> graph;

// Algorithm: Kruskal Algorithm
// Complexity: O(M)

vector<edge> mst;

void kruskal() {
   dsu d(sz);

   auto tedges = edges;
   sort(tedges.begin(), tedges.end(), [](edge& e1, edge& e2
        ) { return e1.w < e2.w; });

   for (auto& e : tedges) {
      if (d.get(e.u) != d.get(e.v)) {
         mst.push_back(e);
         d.unite(e.u, e.v);
      }
   }
}

int main() {
   long long int n, m, i, j, k,a,b,c;
   cin >> n >> m;
   for (i = 0; i < m; i++) {
      cin >> a >> b >> c;
      a--; b--;
      edge e(a, b, c);
      edges.push_back(e);
   }
```

```
    sz = n;
    kruskal();
    long long int ans = 0;
    for (auto it : mst)ans += it.w;
    cout << ans;
}
```

1.4 Lowest Common Ancestor

```
// Theme: Lowest Common Ancestor

// Algorithm: Binary Lifting
// Complextiry: O(N * log(N) * log(N))

vector<vector<int>> graph;
vector<vector<int>> up;
vector<int> tin, tout;

int timer;

// l == log(N) (~20)
int l;

void dfs(int cur, int p = -1) {
    tin[cur] = timer++;

    up[cur][0] = p;
    for (int i = 1; i <= l; i++)
        up[cur][i] = up[up[cur][i - 1]][i - 1];

    for (auto &to : graph[cur]) {
        if (to == p) continue;
        dfs(to, cur);
    }

    tout[cur] = timer++;
}

void preprocess(int u) {
    l = (int) ceil(log2(sz));
    up.assign(sz, vector<int>(l + 1));
    timer = 0;
    dfs(u, u);
}

bool is_anc(int u, int v) {
    return tin[u] <= tin[v] && tout[u] >= tout[v];
}

int lca(int u, int v) {
    if (is_anc(u, v))
        return v;
    if (is_anc(v, u))
        return v;
    for (int i = l; i >= 0; --i) {
        if (!is_anc(up[v][i], u))
            v = up[v][i];
    }
    return up[v][0];
}
```

1.5 2 Sat

```
// Theme: 2-SAT

// Algorithm: Adding Edges To 2-SAT

vector<vector<int>> ts_graph;
vector<vector<int>> ts_rgraph;
vector<int>used;
vector<int>top_sort;

// Vertex By Var Number
int to_vert(int x) {
    if (x < 0) {
        return ((abs(x) - 1) << 1) ^ 1;
    }
    else {
        return (x - 1) << 1;
    }
}

// Adding Implication
void add_impl(int a, int b) {
    ts_graph[a].insert(b);
    ts_rgraph[b].insert(a);
}

// Adding Disjunction
void add_or(int a, int b) {
    add_impl(a ^ 1, b);
    add_impl(b ^ 1, a);
}

//topsort
void dfs(int v){
    used[v] = 1;
    for(auto to:ts_graph[v]){
        if(!used[to])dfs(to);
    top_sort.push_back(v);
}

//scc
vector<vector<long long int>> scc;
void dfs_scc(long long int cur, long long int p = -1) {
    used[cur] = 1;
    scc.back().push_back(cur);
    for (auto to : rgr[cur]) {
        if (to == p || used[to]) continue;
        dfs_scc(to, cur);
    }
}

int main(){
    ...
    used.resize(n,0);
    for(i=0;i<n;i++){
        if(!used[i])dfs(i);
    }
    reverse(top_sort.begin(), top_sort.end());
    for(auto it:top_sort){
        if (!used[u]) {
            scc.push_back({});
            dfs_scc(u);
        }
    }
    vector<long long int>v_scc;
    v_scc.assign(2 * n, -1);

    for (int i = 0; i < scc.size(); i++)
        for (auto& u : scc[i])
            v_scc[u] = i;

    vector<long long int> values(2 * n, -1);

    for (int i = 0; i < 2 * n; i += 2)
        if (v_scc[i] == v_scc[i ^ 1]) {
            cout << "NO\n";
            return 0;
        }
        else {
            if (v_scc[i] < v_scc[i ^ 1]) {
                values[i] = 0;
                values[i ^ 1] = 1;
            }
            else {
                values[i] = 1;
                values[i ^ 1] = 0;
            }
        }
}
```

1.6 Kuhn

```
// Maximum Matching

// Algorithm: Kuhn Algorithm
// Complexity: O(|Left Part|^3)

vector<vector<int>> bigraph;
vector<int> used;

vector<int> mt;

bool kuhn(int u) {
    if (used[u]) return false;
```

```
    used[u] = 1;

    for (auto &v : bigraph[u]) {
        if (mt[v] == -1 || kuhn(mt[v])) {
            mt[v] = u;
            return true;
        }
    }

    return false;
}

int main() {
        ... чтениеграфа...

        mt.assign (k, -1);
        for (int v=0; v<n; ++v) {
                used.assign (n, false);
                try_kuhn (v);
        }

        for (int i=0; i<k; ++i)
                if (mt[i] != -1)
                        printf ("%d %d\n", mt[i]+1, i+1);
}
```

1.7 Eulerian Path

```
// Theme: Eulerian Path

int sz;

vector<vector<int>> graph;

// Algorithm: Eulerian Path
// Complexity: O(M)

vector<int> eul;

// 0 - path not exist
// 1 - cycle exits
// 2 - path exists
int euler_path() {
    vector<int> deg(sz);

    for (int i = 0; i < sz; i++)
        for (int j = 0; j < sz; j++)
            deg[i] += g[i][j];

    int v1 = -1, v2 = -1;
    for (int i = 0; i < sz; i++)
        if (deg[i] & 1)
            if (v1 == -1) v1 = i;
            else if (v2 == -1) v2 = i;
            else return 0;

    if (v1 != -1) {
        if (v2 == -1)
            return 0;
        graph[v1][v2]++;
        graph[v2][v1]++;
    }

    stack<int> st;

    for (int i = 0; i < sz; i++) {
        if (deg[i]) {
            st.push(i);
            break;
        }
    }

    while (st.size()) {
        int u = st.top();

        int ind = -1;

        for (int i = 0; i < sz; i++)
            if (graph[u][i]) {
                ind = i;
                break;
            }

        if (ind == -1) {
            eul.push_back(u);
            st.pop();
        }
        else {
            graph[u][ind]--;
            graph[ind][u]--;
            st.push(ind);
        }
    }

    int res = 2;
    if (v1 != -1) {
        res = 1;

        for (int i = 0; i < eul.size() - 1; i++)
            if (eul[i] == v1 && eul[i + 1] == v2 ||
            eul[i] == v2 && eul[i + 1] == v1) {
                vector<int> teul;
                for (int j = i + 1; j < eul.size(); j++)
                    teul.push_back(eul[j]);
                for (int j = 0; j <= i; j++)
                    teul.push_back(eul[j]);
                eul = teul;
                break;
            }
    }

    for (int i = 0; i < sz; i++)
        for (int j = 0; j < sz; j++)
            if (graph[i][j])
                return 0;

    return res;
}
```

1.8 Articulation Points

```
// Theme: Articulation Points And VCC

vector<pair<int, int>> edges;
vector<vector<int>> graph;
vector<int> used;

vector<int> height;
vector<int> up;

// Algorithm: Articulation Points
// Complexity: O(N + M)

set<int> art_points;

void dfs_artics(int cur, int p = -1) {
    used[cur] = 1;
    up[cur] = height[cur];

    int desc_count = 0;

    for (auto &ind : g[cur]) {
        int to = cur ^ edges[ind].ff ^ edges[ind].ss;
        if (to == p) continue;
        if (used[to]) {
            up[cur] = min(up[cur], height[to]);
        }
        else {
            desc_count++;
            height[to] = height[cur] + 1;
            dfs_artics(to, cur);
            up[cur] = min(up[cur], up[to]);
            if (up[to] >= height[cur] && p != -1)
                art_points.insert(cur);
        }
    }

    if (p == -1 && desc_count > 1) {
        art_points.insert(cur);
    }
}

// Algorithm: VCC
// Complexity: O(N + M)

vector<vector<int>> vcc;

void dfs_artics_comps(int cur, int p = -1) {
    used[cur] = 1;
    up[cur] = height[cur];
```

```
    for (auto &ind : g[cur]) {
        int to = cur ^ edges[ind].ff ^ edges[ind].ss;
        if (to == p) continue;
        if (used[to]) {
            up[cur] = min(up[cur], height[to]);
            if (height[to] < height[cur]) st.push_back(ind);
        }
        else {
            int sz = st.size();
            st.push_back(ind);
            height[to] = height[cur] + 1;
            dfs_artics_comps(to, cur);
            up[cur] = min(up[cur], up[to]);
            if (up[to] >= height[cur])
                vcc.push_back(add_comp(st, sz));
        }
    }
}
```

1.9 Graph Traversing

```
// Theme: Graph Traversing

vector<vector<int>> graph;
vector<int> used;

// Algorithm: Depth-First Search (Adjacency List)
// Complexity: O(N + M)

void dfs(int cur, int p = -1) {
    used[cur] = 1;

    for (auto &to : graph[cur]) {
        if (to == p || used[to]) continue;
        dfs(to, cur);
    }
}

// Algorithm: Breadth-First Search (Adjacency List)
// Complexity: O(N + M)

void bfs(int u) {
    queue<int> q; q.push(u);

    while (q.size()) {
        int cur = q.front(); q.pop();

        for (auto &to : graph[cur]) {
            if (used[to]) continue;
            q.push(to);
        }
    }
}
```

1.10 Maximum Flow

```
// Theme: Maximum Flow

int s, t, sz;

vector<edge> edges;
vector<vector<int>> fgraph;

vector<int> used;

// Algorithm: Ford-Fulkerson Algorithm
// Complexity: O(MF)

int dfs_fordfulk(int u, int bound, int flow = INF) {
    if (used[u]) return 0;
    if (u == t) return flow;

    used[u] = 1;

    for (auto &ind : fgraph[u]) {
        auto &e = edges[ind],
            &_e = edges[ind ^ 1];
        int to = e.to, res = e.res();

        if (res < bound) continue;

            int pushed = dfs_fordfulk(to, bound, min(res, flow))
                ;

            if (pushed) {
                e.flow += pushed;
                _e.flow -= pushed;
                return pushed;
            }
        }

    return 0;
}

// Algorithm: Edmonds-Karp Algorithm
// Complexity: O(N(M^2))

vector<int> p;
vector<int> pe;

void augment(int pushed) {
    int cur = t;
    while (cur != s) {
        auto &e = edges[pe[cur]],
            &_e = edges[pe[cur] ^ 1];
        e.flow += pushed;
        _e.flow -= pushed;
        cur = p[cur];
    }
}

int bfs_edmskarp(int u, int bound) {
    p.assign(sz, -1);
    pe.assign(sz, -1);

    int pushed = 0;

    queue<pair<int, int>> q;
    q.push({ u, INF });

    used[u] = 1;

    while (q.size()) {
        auto [v, f] = q.front(); q.pop();

        for (auto &ind : fgraph[v]) {
            auto &e = edges[ind];
            int to = e.to, res = e.res();

            if (used[to] || res < bound) continue;

            p[to] = v;
            pe[to] = ind;
            used[to] = 1;

            if (to == t) {
                pushed = min(f, res);
                break;
            }

            q.push({ to, min(f, res) });
        }
    }

    if (pushed)
        augment(pushed);

    return pushed;
}

// Algorithm: Dinic Algorithm
// Complexity: O((N^2)M)

vector<int> d;

bool bfs_dinic(int u, int bound) {
    d.assign(sz, INF); d[u] = 0;

    queue<int> q; q.push(u);

    while (q.size()) {
        int v = q.front(); q.pop();

        for (auto &ind : fgraph[v]) {
            auto &e = edges[ind];
            int to = e.to, res = e.res();

            if (d[v] + 1 >= d[to] || res < bound) continue;
```

```
            d[to] = d[v] + 1;
            q.push(to);
        }
    }

    return d[t] != INF;
}

vector<int> lst;

int dfs_dinic(int u, int mxf = INF) {
    if (u == t) return mxf;

    int smf = 0;

    for (int i = lst[u]; i < fgraph[u].size(); i++) {
        int ind = fgraph[u][i];

        auto &e = edges[ind],
            &_e = edges[ind ^ 1];
        int to = e.to, res = e.res();

        if (d[to] == d[u] + 1 && res) {
            int pushed = dfs_dinic(to, min(res, mxf - smf));

            if (pushed) {
                smf += pushed;
                e.flow += pushed;
                _e.flow -= pushed;
            }
        }

        lst[u]++;

        if (smf == mxf)
            return smf;
    }

    return smf;
}

int dinic(int u) {
    int pushed = 0;

    for (int bound = 1ll << 30; bound; bound >>= 1) {
        while (true) {
            bool bfs_ok = bfs_dinic(u, bound);
            if (!bfs_ok) break;

            lst.assign(sz, 0);

            while (true) {
                int dfs_pushed = dfs_dinic(u);
                if (!dfs_pushed) break;

                pushed += dfs_pushed;
            }
        }
    }

    return pushed;
}

// Algorithm: Maximum Flow Of Minimum Cost (SPFA)
// Complexity: ...


vector<int> d;
vector<int> p;
vector<int> pe;

void augment(int pushed) {
    int cur = t;
    while (cur != s) {
        auto &e = edges[pe[cur]],
            &_e = edges[pe[cur] ^ 1];
        e.flow += pushed;
        _e.flow -= pushed;
        cur = p[cur];
    }
}

int bfs_spfa(int u, int flow = INF) {
    d.assign(sz, INF); d[u] = 0;
    p.assign(sz, -1);
    pe.assign(sz, -1);

    queue<pair<int, int>> q; q.push({ u, flow });
    vector<int> in_q(sz, 0); in_q[u] = 1;

    int pushed = 0;

    while (q.size()) {
        auto [v, f] = q.front(); q.pop();

        in_q[v] = 0;

        if (v == t) {
            pushed = f;
            break;
        }

        for (auto &ind : fgraph[v]) {
            auto &e = edges[ind];
            int to = e.to, res = e.res(),
                w = e.weight;

            if (d[v] + w >= d[to] || !res) continue;

            d[to] = d[v] + w;
            p[to] = v;
            pe[to] = ind;

            if (!in_q[to]) {
                in_q[to] = 1;
                q.push({ to, min(f, res) });
            }
        }
    }

    if (pushed)
        augment(pushed);

    return pushed;
}
```

1.11 Connected Components

```
// Theme: Connectivity Components

vector<vector<int>> graph;
vector<int> used;

// Algorithm: Connected Components
// Complexity: O(N + M)

vector<vector<int>> cc;

void dfs_cc(int cur, int p = -1) {
    used[cur] = 1;
    cc.back().push_back(cur);

    for (auto &to : graph[cur]) {
        if (to == p || used[to]) continue;
        dfs_cc(to, cur);
    }
}

for (int u = 0; u < n; i++)
    if (!used[u])
        dfs_cc(u);

// Algorithm: Strongly Connected Components
// Complexity: O(N + M)

vector<vector<int>> rgraph;

vector<vector<int>> topsort;

vector<vector<int>> scc;

void dfs_scc(int cur, int p = -1) {
    used[cur] = 1;
    scc.back().push_back(cur);

    for (auto &to : rgraph[cur]) {
        if (to == p || used[to]) continue;
        dfs_scc(to, cur);
    }
}

for (auto &u: topsort)
    if (!used[u])
        dfs_scc(u);
```

1.12 Bridges

```
// Theme: Bridges And ECC

vector<pair<int, int>> edges;
vector<vector<int>> graph;
vector<int> used;

vector<int> height;
vector<int> up;

// Algorithm: Bridges
// Complexity: O(N + M)

vector<int> bridges;

void dfs_bridges(int cur, int p = -1) {
   used[cur] = 1;
   up[cur] = height[cur];
   for (auto &ind : g[cur]) {
      int to = cur ^ edges[ind].ff ^ edges[ind].ss;
      if (to == p) continue;
      if (used[to]) {
         up[cur] = min(up[cur], height[to]);
      }
      else {
         height[to] = height[cur] + 1;
         dfs_bridges(to, cur);
         up[cur] = min(up[cur], up[to]);
         if (up[to] > height[cur])
            bridges.push_back(ind);
      }
   }
}

// Algorithm: ECC
// Complexity: O(N + M)

vector<int> st;

vector<int> add_comp(vector<int> &st, int sz) {
   vector<int> comp;

   while (st.size() > sz) {
      comp.push_back(st.back());
      st.pop_back();
   }

   return comp;
}

vector<vector<int>> ecc;

void dfs_bridges_comps(int cur, int p = -1) {
   used[cur] = 1;
   up[cur] = height[cur];

   for (auto &ind : g[cur]) {
      int to = cur ^ edges[ind].ff ^ edges[ind].ss;
      if (to == p) continue;
      if (used[to]) {
         up[cur] = min(up[cur], height[to]);
      }
      else {
         int sz = st.size();
         st.push_back(to);
         height[to] = height[cur] + 1;
         dfs_bridges_comps(to, cur);
         up[cur] = min(up[cur], up[to]);
         if (up[to] > height[cur])
            ecc.push_back(add_comp(st, sz));
      }
   }
}
```

1.13 Shortest Paths

```
// Theme: Shortest Paths

int sz;

vector<edge> edges;
vector<vector<int>> graph;

// Algorithm: Dijkstra Algorithm
// Complexity: O(M*log(N))

vector<int> d;
vector<int> p;

void dijkstra(int u) {
   d.assign(sz, INF); d[u] = 0;
   p.assign(sz, -1);

   priority_queue<pair<int, int>> q;
   q.push({ 0, u });

   while (q.size()) {
      int dist = -q.top().ff, v = q.top().ss; q.pop();

      if (dist > d[v]) continue;

      for (auto &ind : graph[v]) {
         int to = v ^ edges[ind].u ^ edges[ind].v,
            w = edges[ind].w;
         if (d[v] + w < d[to]) {
            d[to] = d[v] + w;
            p[to] = v;
            q.push({ -d[to], -to });
         }
      }
   }
}

// Algorithm: Shortest Path Faster Algorithm
// Complexity: ...

vector<int> d;

void bfs_spfa(int u) {
   d.assign(sz, INF); d[u] = 0;

   queue<int> q; q.push(u);
   vector<int> in_q(sz, 0); in_q[u] = 1;

   while (q.size()) {
      auto [v, f] = q.front(); q.pop();

      in_q[v] = 0;

      for (auto &ind : graph[v]) {
         int to = v ^ edges[ind].u ^ edges[ind].v,
            w = edges[ind].w;
         if (d[v] + w < d[to]) {
            d[to] = d[v] + w;
            if (!in_q[to]) {
               in_q[to] = 1;
               q.push( to );
            }
         }
      }
   }
}

// Algorithm: Belman-Ford Algorithm
// Complexity: (N*M)

vector<int> d;

void bfa(int u) {
   d.assign(sz, INF); d[u] = 0;

   for (;;) {
      bool any = false;

      for (auto &e : edges) {
         if (d[e.u] != INF && d[e.u] + e.w < d[e.v]) {
            d[e.v] = d[e.u] + e.w;
            any = true;
         }
         if (d[e.v] != INF && d[e.v] + e.w < d[e.u]) {
            d[e.u] = d[e.v] + e.w;
            any = true;
         }
      }

      if (!any) break;
   }
}

// Algorithm: Floyd-Warshall Algorithm
// Complexity: O(N^3)
```

```
vector<vector<int>> d;

void fwa() {
   d.assign(sz, vector<int>(sz, INF));

   for (int i = 0; i < sz; i++)
      for (int j = 0; j < sz; j++)
         for (int k = 0; k < sz; k++)
            if (d[i][k] != INF && d[k][j] != INF)
               d[i][j] = min(d[i][j], d[i][k] + d[k][j]);
}
```

2 Geometry

2.1 Planimetry

```
// Theme: Planimetry Objects

// Point
template <typename T>
struct point {
   T x, y;

   point() : x(0), y(0) { }
   point(T x, T y) : x(x), y(y) { }
};


// Rectangle
template <typename T>
struct rectangle {
   point<T> ld, ru;

   rectangle(const point<T> &ld, const point<T> &ru) :
      ld(ld), ru(ru) { }
};
```

2.2 Graham

```
// Theme: Convex Hull

// Algorithm: Graham Algorithm
// Complexity: O(N*log(N))

auto graham(const vector<vec<int>> &points) {
   vec<int> p0 = points[0];

   for (auto p : points)
      if (p.y < p0.y ||
      p.y == p0.y && p.x > p0.x)
         p0 = p;

   for (auto &p : points) {
      p.x -= p0.x;
      p.y -= p0.y;
   }

   sort(all(points), [] (vec<int> &p1, vec<int> &p2) {
      return (p1 ^ p2).z > 0 ||
         (p1 ^ p2).z == 0 && p1.norm() > p2.norm(); });

   vector<vec<int>> hull;
   for (auto &p : points) {
      while (hull.size() >= 2 &&
      (((p - hull.back()) ^ (hull[hull.size() - 1] - hull[
          hull.size() - 2]))).z >= 0)
         hull.pop_back();
      hull.push_back(p);
   }

   for (auto &p : hull) {
      p.x += p0.x;
      p.y += p0.y;
   }

   return hull;
}
```

2.3 Vector

```
// Theme: Methematical 3-D Vector

template <typename T>
struct vec {
   T x, y, z;
   vec(T x = 0, T y = 0, T z = 0) : x(x), y(y), z(z) { }
   vec<T> operator+(const vec<T> &v) const {
      return vec<T>(x + v.x, y + v.y, z + v.z);
   }
   vec<T>operator-(const vec<T> &v) const {
      return vec<T>(x - v.x, y - v.y, z - v.z);
   }
   vec<T>operator*(T k) const {
      return vec<T>(k * x, k * y, k * z);
   }
   friend vec<T> operator*(T k, const vec<T> &v) {
      return vec<T>(v.x * k, v.y * k, v.z * k);
   }
   vec<T> operator/(T k) {
      return vec<T>(x / k, y / k, z / k);
   }
   T operator*(const vec<T> &v) const {
      return x * v.x + y * v.y + z * v.z;
   }
   vec<T> operator^(const vec<T> &v) const {
      return { y * v.z - z * v.y, z * v.x - x * v.z, x * v
           .y - y * v.x };
   }
   auto operator<=>(const vec<T> &v) const = default;
   bool operator==(const vec<T> &v) const = default;
   T norm() const {
      return x * x + y * y + z * z;
   }
   double abs() const {
      return sqrt(norm());
   }
   double cos(const vec<T> &v) const {
      return ((*this) * v) / (abs() * v.abs());
   }
   friend ostream &operator<<(ostream &out, const vec<T> &v
        ) {
      return out << v.x << sp << v.y << sp << v.z;
   }
   friend istream &operator>>(istream &in, vec<T> &v) {
      return in >> v.x >> v.y >> v.z;
   }
};
```

2.4 Formulae

Triangles.
Radius of circumscribed circle:
$R = \frac{abc}{4S}$.
Radius of inscribed circle:
$r = \frac{S}{p}$.
Side via medians:
$a = \frac{2}{3}\sqrt{2(m_b^2 + m_c^2) - m_a^2}$.
Median via sides:
$m_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$.
Bisector via sides:
$l_a = \frac{2\sqrt{bcp(p-a)}}{b+c}$.
Bisector via two sides and angle:
$l_a = \frac{2bc\cos\frac{\alpha}{2}}{b+c}$.
Bisector via two sides and divided side:
$l_a = \sqrt{bc - a_b a_c}$.

Right triangles.
a, b - cathets, c - hypotenuse.
h - height to hypotenuse, divides c to c_a and c_b.

$$\begin{cases} h^2 = c_a \cdot c_b, \\ a^2 = c_a \cdot c, \\ b^2 = c_b \cdot c. \end{cases}$$

Quadrangles.
Sides of circumscribed quadrangle:
$a + c = b + d$.
Square of circumscribed quadrangle:
$S = \frac{Pr}{2} = pr$.
Angles of inscribed quadrangle:
$\alpha + \gamma = \beta + \delta = 180°$.
Square of inscribed quadrangle:
$S = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

Circles.
Intersection of circle and line:

$$\begin{cases} (x - x_0)^2 + (y - y_0)^2 = R^2 \\ y = ax + b \end{cases}$$

Task comes to solution of $\alpha x^2 + \beta x + \gamma = 0$, where

$$\begin{cases} \alpha = (1 + a^2), \\ \beta = (2a(b - y_0) - 2x_0), \\ \gamma = (x_0^2 + (b - y_0)^2 - R^2). \end{cases}$$

Intersection of circle and circle:

$$\begin{cases} (x - x_0)^2 + (y - y_0)^2 = R_0^2 \\ (x - x_1)^2 + (y - y_1)^2 = R_1^2 \end{cases}$$

$y = \frac{1}{2} \frac{(R_1^2 - R_0^2) + (x_0^2 - x_1^2) + (y_0^2 - y_1^2)}{y_0 - y_1} - \frac{x_0 - x_1}{y_0 - y_1} x$
Task comes to intersection of circle and line.

3 Stringology

3.1 Z Function

```
// Theme: Z-Function

// Algorithm: Linear Algorithm
// Complexity: O(N)

auto z_func(const string &s) {
   int n = s.size();
   vector<int> z(n);

   for (int i = 1, l = 0, r = 0; i < n; i++) {
      if (i <= r) z[i] = min(r - i + 1, z[i - l]);

      while (i + z[i] < n && s[z[i]] == s[i + z[i]]) z[i
          ]++;

      if (i + z[i] - 1 > r) {
         l = i;
         r = i + z[i] - 1;
      }
   }

   return z;
}
```

3.2 Manacher

```
// Theme: Palindromes

// Algorithm: Manacher Algorithm
// Complexity: O(N)

int manacher(const string &s) {
   int n = s.size();
   vector<int> d1(n), d2(n);

   for (int i = 0, l = 0, r = -1; i < n; i++) {
      int k = i > r ? 1 : min(d1[l + r - i], r - i + 1);
      while (i + k < n && i - k >= 0 && s[i + k] == s[i -
          k]) k++;
      d1[i] = k;
      if (i + k - 1 > r) {
         l = i - k + 1;
         r = i + k - 1;
      }
   }

   for (int i = 0, l = 0, r = -1; i < n; i++) {
      int k = i > r ? 0 : min(d2[l + r - i + 1], r - i +
          1);
      while (i + k < n && i - k - 1 >= 0 && s[i + k] == s[
          i - k - 1]) k++;
      d2[i] = k;
      if (i + k - 1 > r) {
         l = i - k;
         r = i + k - 1;
      }
   }

   int res = 0;
   for (int i = 0; i < n; i++) {
      res += d1[i] + d2[i];
   }

   return res;
}
```

3.3 Suffix Array

```
// Theme: Suffix array

// Algorithm: Binary Algorithm With Count Sort
// Complexity: O(N*log(N))

void count_sort(vector<int> &p, vector<int> &c) {
   int n = p.size();
   vector<int> cnt(n), p_new(n), pos(n);

   for (auto &x : c) cnt[x]++;

   pos[0] = 0;
   for (int i = 1; i < n; i++)
      pos[i] = pos[i - 1] + cnt[i - 1];

   for (auto &x : p) {
      int i = c[x];
      p_new[pos[i]] = x;
      pos[i]++;
   }

   p = p_new;
}

auto suffix_array(const string &str) {
   string s = str + '$';
   int n = s.size();

   vector<int> p(n), c(n);
   vector<pair<char, int>> a(n);

   for (int i = 0; i < n; i++) a[i] = { str[i], i };

   sort(a.begin(), a.end());

   for (int i = 0; i < n; i++) p[i] = a[i].second;

   c[p[0]] = 0;
   for (int i = 1; i < n; i++)
      c[p[i]] = c[p[i - 1]] + (a[i].first != a[i - 1].
          first);

   int k = 0;
   while ((1 << k) < n) {
      for (int i = 0; i < n; i++)
         p[i] = (p[i] - (1 << k) + n) % n;
```

```
        count_sort(p, c);

        vector<int> c_new(n);

        c_new[p[0]] = 0;
        for (int i = 1; i < n; i++) {
            pair<int, int> prev = { c[p[i - 1]], c[(p[i - 1]
                 + (1 << k)) % n] };
            pair<int, int> now = { c[p[i]], c[(p[i] + (1 << k
                 )) % n] };
            c_new[p[i]] = c_new[p[i - 1]] + (now != prev);
        }

        c = c_new;
        k++;
    }

    return p;
}
```

3.4 Trie

```
// Theme: Trie

// Algorithm: Aho-Corasick
// Complexity: O(N)

struct trie {
    // Vertex
    struct vertex {
        vector<int> next;
        bool leaf;
    };

    // Alphabet size
    static const int N = 26;
    // Maximum Vertex Number
    static const int MX = 2e5 + 1;

    // Vertices Vector
    vector<vertex> t;
    int sz;

    trie(): sz(1) {
        t.resize(MX);
        t[0].next.assign(N, -1);
    }

    void add_str(const string &s) {
        int v = 0;
        for (int i = 0; i < s.length(); i++) {
            char c = s[i] - 'a';
            if (t[v].next[c] == -1) {
                t[sz].next.assign(N, -1);
                t[v].next[c] = sz++;
            }
            v = t[v].next[c];
        }
        t[v].leaf = true;
    }
};
```

3.5 Prefix Function

```
// Theme: Prefix function

// Algorithm: Knuth-Morris-Pratt Algorithm
// Complexity: O(N)

auto pref_func(const string &s) {
    int n = s.size();
    vector<int> pi(n);

    for (int i = 1; i < n; i++) {
        int j = pi[i - 1];

        while (j > 0 && s[i] != s[j]) j = pi[j - 1];

        if (s[i] == s[j]) j++;

        pi[i] = j;
    }
    return pi;
}
```

4 Data Structures

4.1 Segment Tree

```
// Theme: Segment Tree

struct segtree {
    int size;
    vector<int> tree;

    void init(int n) {
        size = 1;
        while (size < n) size <<= 1;
        tree.assign(2 * size - 1, 0);
    }

    void build(vector<int> &a, int x, int lx, int rx) {
        if (rx - lx == 1) {
            if (lx < a.size()) tree[x] = a[lx];
            return;
        }
        int m = (lx + rx) / 2;
        build(a, 2 * x + 1, lx, m);
        build(a, 2 * x + 2, m, rx);
        tree[x] = tree[2 * x + 1] + tree[2 * x + 2];
    }
    void build(vector<int> &a) {
        init(a.size());
        build(a, 0, 0, size);
    }

    // Complexity: O(log(n))
    void set(int i, int v, int x, int lx, int rx) {
        if (rx - lx == 1) {
            tree[x] = v;
            return;
        }
        int m = (lx + rx) / 2;
        if (i < m) set(i, v, 2 * x + 1, lx, m);
        else set(i, v, 2 * x + 2, m, rx);
        tree[x] = tree[2 * x + 1] + tree[2 * x + 2];
    }
    void set(int i, int v) {
        set(i, v, 0, 0, size);
    }

    // Complexity: O(log(n))
    int sum(int l, int r, int x, int lx, int rx) {
        if (l <= lx && rx <= r) return tree[x];
        if (l >= rx || r <= lx) return 0;
        int m = (lx + rx) / 2;
        return sum(l, r, 2 * x + 1, lx, m) +
            sum(l, r, 2 * x + 2, m, rx);
    }
    int sum(int l, int r) {
        return sum(l, r, 0, 0, size);
    }
};
```

4.2 Segment Tree Propagate

```
// Theme: Segment Tree With Propagation

struct segtree_prop {
    int size;
    vector<int> tree;

    void init(int n) {
        size = 1;
        while (size < n) size <<= 1;
        tree.assign(2 * size - 1, 0);
    }

    void build(vector<int> &a, int x, int lx, int rx) {
        if (rx - lx == 1) {
            if (lx < a.size()) tree[x] = a[lx];
```

```
            return;
        }
        int m = (lx + rx) / 2;
        build(a, 2 * x + 1, lx, m);
        build(a, 2 * x + 2, m, rx);
        tree[x] = tree[2 * x + 1] + tree[2 * x + 2];
    }
    void build(vector<int> &a) {
        init(a.size());
        build(a, 0, 0, size);
    }

    void push(int x, int lx, int rx) {
        if (rx - lx == 1) return;
        tree[2 * x + 1] += tree[x];
        tree[2 * x + 2] += tree[x];
        tree[x] = 0;
    }

    // Complexity: O(log(n))
    void add(int v, int l, int r, int x, int lx, int rx) {
        push(x, lx, rx);
        if (rx <= l || r <= lx) return;
        if (l <= lx && rx <= r) {
            tree[x] += v;
            return;
        }
        int m = (lx + rx) / 2;
        add(v, l, r, 2 * x + 1, lx, m);
        add(v, l, r, 2 * x + 2, m, rx);
    }
    void add(int v, int l, int r) {
        add(v, l, r, 0, 0, size);
    }

    // Complexity: O(log(n))
    int get(int i, int x, int lx, int rx) {
        push(x, lx, rx);
        if (rx - lx == 1) return tree[x];
        int m = (lx + rx) / 2;
        if (i < m) return get(i, 2 * x + 1, lx, m);
        else return get(i, 2 * x + 2, m, rx);
    }
    int get(int i) {
        return get(i, 0, 0, size);
    }
};
```

4.3 Disjoint Set Union

```
// Theme: Disjoint Set Union

struct dsu {
    vector<int> p, size;

    dsu(int n) {
        p.assign(n, 0); size.assign(n, 0);
        for (int i = 0; i < n; i++) {
            p[i] = i;
            size[i] = 1;
        }
    }

    int get(int v) {
        if (p[v] != v) p[v] = get(p[v]);
        return p[v];
    }

    void unite(int u, int v) {
        auto x = get(u), y = get(v);
        if (x == y) return;
        if (size[x] > size[y]) swap(x, y);
        p[x] = y; size[y] += size[x];
    }
};
```

4.4 Treap K

```
// Theme: Treap With Segments

// Node
struct node_k {
    int key, priorty, size;
    shared_ptr<node_k> left, right;

    node_k(int key, int priorty = INF) :
        key(key),
        priorty(priorty == INF ?
        reng() : priorty),
        size(1) { }

    friend int sz(shared_ptr<node_k> nd) {
        return (nd ? nd->size : 0ll);
    }

    void upd() {
        size = sz(left) + sz(right) + 1;
    }
};

// Treap
struct treap_k {
    shared_ptr<node_k> root;

    treap_k() { }

    treap_k(int root_key, int root_priorty = INF) {
        root = shared_ptr<node_k>(new node_k(root_key,
            root_priorty));
    }

    treap_k(shared_ptr<node_k> rt) {
        root = shared_ptr<node_k>(rt);
    }

    treap_k(const treap_k &tr) {
        root = shared_ptr<node_k>(tr.root);
    }

    // Complexity: O(log(N))
    pair<treap_k, treap_k> split_k(int k) {
        auto res = split_k(root, k);
        return { treap_k(res.ff), treap_k(res.ss) };
    }

    pair<shared_ptr<node_k>, shared_ptr<node_k>> split_k(
        shared_ptr<node_k> rt, int k) {
        if (!rt) return { nullptr, nullptr };
        else if (sz(rt) <= k) return { rt, nullptr };
        else if (sz(rt->left) + 1 <= k) {
            auto [rt1, rt2] = split_k(rt->right, k - sz(rt->
                left) - 1);
            rt->right = rt1;
            rt->upd();
            return { rt, rt2 };
        }
        else {
            auto [rt1, rt2] = split_k(rt->left, k);
            rt->left = rt2;
            rt->upd();
            return { rt1, rt };
        }
    }

    // Complexity: O(log(N))
    treap_k merge_k(const treap_k &tr) {
        root = shared_ptr<node_k>(merge_k(root, tr.root));
        return *this;
    }

    shared_ptr<node_k> merge_k(shared_ptr<node_k> rt1,
        shared_ptr<node_k> rt2) {
        if (!rt1) return rt2;
        if (!rt2) return rt1;
        if (rt1->priorty < rt2->priorty) {
            rt1->right = merge_k(rt1->right, rt2);
            rt1->upd();
            return rt1;
        }
        else {
            rt2->left = merge_k(rt1, rt2->left);
            rt2->upd();
            return rt2;
        }
    }
};
```

4.5 Fenwick Tree

```
// Theme: Fenwick Tree
// Core operations are O(log n)

struct Fenwick {
   vector<int> data;

   explicit Fenwick(int n) {
      data.assign(n + 1, 0);
   }
   explicit Fenwick(vector<int>& arr): Fenwick(arr.size())
        {
      for (int i = 1; i <= arr.size(); ++i) {
         add(i, arr[i - 1]);
      }
   }

   // Nested loops (also vector) for multi-dimensional.
        Also in add().
   // (x & -x) = last non-zero bit
   int sum(int right) {
      int res = 0;
      for (int i = right; i > 0; i -= (i & -i)) {
         res += data[i];
      }
      return res;
   }
   int sum(int left, int right) {
      return sum(right) - sum(left - 1); // inclusion-
           exclusion principle
   }

   void add(int idx, int x) {
      for (int i = idx; i < data.size(); i += (i & -i)) {
         data[i] += x;
      }
   }

   // CONCEPT (didn't test it). Should work if all real
        values are non-negative.
   int lower_bound(int s) {
      int k = 0;
      int logn = (int)(log2(data.size() - 1) + 1); //
           maybe rewrite this line
      for (int b = logn; b >= 0; --b) {
         if (k + (1 << b) < data.size() && data[k + (1 <<
              b)] < s) {
            k += (1 << b);
            s -= data[k];
         }
      }
      return k;
   }
};
```

4.6 Treap

```
// Theme: Treap (Tree + Heap)

// Node
struct node {
   int key, priorty;
   shared_ptr<node> left, right;

   node(int key, int priorty = INF) :
      key(key),
      priorty(priorty == INF ?
      reng() : priorty) { }
};

// Treap
struct treap {
   shared_ptr<node> root;

   treap() { }

   treap(int root_key, int root_priorty = INF) {
      root = shared_ptr<node>(new node(root_key,
           root_priorty));
   }

   treap(shared_ptr<node> rt) {
      root = shared_ptr<node>(rt);
   }

   treap(const treap &tr) {
      root = shared_ptr<node>(tr.root);
   }

   // Complexity: O(log(N))
   pair<treap, treap> split(int k) {
      auto res = split(root, k);
      return { treap(res.ff), treap(res.ss) };
   }

   pair<shared_ptr<node>, shared_ptr<node>> split(
        shared_ptr<node> rt, int k) {
      if (!rt) return { nullptr, nullptr };
      else if (rt->key < k) {
         auto [rt1, rt2] = split(rt->right, k);
         rt->right = rt1;
         return { rt, rt2 };
      }
      else {
         auto [rt1, rt2] = split(rt->left, k);
         rt->left = rt2;
         return { rt1, rt };
      }
   }

   // Complexity: O(log(N))
   treap merge(const treap &tr) {
      root = shared_ptr<node>(merge(root, tr.root));
      return *this;
   }

   shared_ptr<node> merge(shared_ptr<node> rt1, shared_ptr<
        node> rt2) {
      if (!rt1) return rt2;
      if (!rt2) return rt1;
      if (rt1->priorty < rt2->priorty) {
         rt1->right = merge(rt1->right, rt2);
         return rt1;
      }
      else {
         rt2->left = merge(rt1, rt2->left);
         return rt2;
      }
   }
};
```

4.7 Treap Universal

```
// Theme: Treap (Tree + Heap)
// Supports both explicit and implicit keys (not
     simultaniously ofc)
// Core operations are all O(log n) average

mt19937 rng(378);

struct Node {
   int x, y, size; // "x" is key or payload, "y" is
        priority
   Node* left, * right;

   Node(int val): x(val), y(rng() % 1'000'000'000), size(1)
        , left(nullptr), right(nullptr) {}
};

int get_size(Node* root) {
    if (root == nullptr) return 0;
    return root->size;
}
void update(Node* root) {
    if (root == nullptr) return;
    root->size = get_size(root->left) + 1 + get_size(root->
     right);
}

// split by value (for explicit keys)
pair<Node*, Node*> split(Node* root, int v) {
    if (root == nullptr) return {nullptr, nullptr};
    if (root->x <= v) {
        auto res = split(root->right, v);
        root->right = res.first;
        update(root);
        return {root, res.second};
    } else {
```

```
        auto res = split(root->left, v);
        root->left = res.second;
        update(root);
        return {res.first, root};
    }
}

// split by size (for implicit keys)
pair<Node*, Node*> split_k(Node* root, int k) {
    if (root == nullptr) return {nullptr, nullptr};
    if (get_size(root) <= k) return {root, nullptr};
    if (k == 0) return {nullptr, root};

    int left_size = get_size(root->left);
    if (left_size >= k) {
        auto res = split_k(root->left, k);
        root->left = res.second;
        update(root);
        return {res.first, root};
    } else {
        auto res = split_k(root->right, k - left_size - 1);
        root->right = res.first;
        update(root);
        return {root, res.second};
    }
}

// merge for both explicit and implicit keys
Node* merge(Node* root1, Node* root2) {
    if (root1 == nullptr) return root2;
    if (root2 == nullptr) return root1;

    if (root1->y < root2->y) {
        root1->right = merge(root1->right, root2);
        update(root1);
        return root1;
    } else {
        root2->left = merge(root1, root2->left);
        update(root2);
        return root2;
    }
}

// insert for explicit keys (use split_k for implicit keys)
Node* insert(Node* root, int v) {
    auto subs = split(root, v);
    return merge(merge(subs.first, new Node(v)), subs.
     second);
}

// debug helper
void print_node(Node* root, bool end = false) {
    if (root->left != nullptr) print_node(root->left);
    cout << root->x << " ";
    if (root->right != nullptr) print_node(root->right);
    if (end) cout << "\n";
}
```

5 Miscellaneous

5.1 Ternary Search

```
// Theme: Ternary Search

// Algorithm: Continuous Search With Golden Ratio
// Complexity: O(log(N))

// Golden Ratio
// Phi = 1.618...
double phi = (1 + sqrt(5)) / 2;

double cont_tern_srch(double l, double r) {
  double m1 = l + (r - l) / (1 + phi),
    m2 = r - (r - l) / (1 + phi);

  double f1 = f(m1), f2 = f(m2);

  int count = 200;
  while (count--) {
    if (f1 < f2) {
      r = m2;
      m2 = m1;
      f2 = f1;
      m1 = l + (r - l) / (1 + phi);
      f1 = f(m1);
    }
    else {
      l = m1;
      m1 = m2;
      f1 = f2;
      m2 = r - (r - l) / (1 + phi);
      f2 = f(m2);
    }
  }

  return f((l + r) / 2);
}

// Algorithm: Discrete Search
// Complexity: O(log(N))

double discr_tern_srch(int l, int r) {
  while (r - l > 2) {
    int m1 = l + (r - l) / 3,
      m2 = r - (r - l) / 3;
    if (f(m1) < f(m2))
      r = m2;
    else
      l = m1;
  }

  return min(f(l), min(f(l + 1), f(r)));
}
```

6 Dynamic Programming

6.1 Increasing Subsequence

```
// Theme: Longest Increasing Subsequence

// Algorithm: Binary Search Algorithm
// Complexity: O(N*log(N))

auto inc_subseq(const vector<int> &a) {
   int n = a.size();
   vector<int> dp(n + 1, INF), pos(n + 1), prev(n), subseq;

   int len = 0;
   dp[0] = -INF;
   pos[0] = -1;

   for (int i = 0; i < n; i++) {
      int j = distance(dp.begin(), upper_bound(all(dp), a[
          i]));
      if (dp[j - 1] < a[i] && a[i] < dp[j]) {
         dp[j] = a[i];
         pos[j] = i;
         prev[i] = pos[j - 1];
         len = max(len, j);
      }
   }

   int p = pos[len];
   while (p != -1) {
      subseq.push_back(a[p]);
      p = prev[p];
   }
   reverse(subseq.begin(), subseq.end());

   return subseq;
}
```

7 Algebra

7.1 Greatest Common Divisor

```
// Theme: Greatest Common Divisor

// Algorithm: Simple Euclidean Algorithm
// Complexity: O(log(N))
```

```
int gcd(int a, int b) {
   while (b) {
      a %= b;
      swap(a, b);
   }
   return a;
}

// Algorithm: Extended Euclidean Algorithm
// Complexity: O(log(N))

// d = gcd(a, b)
// x * a + y * b = d
// returns {d, x, y}
vector<int> euclid(int a, int b) {
   if (!a) return { b, 0, 1 };
   auto v = euclid(b % a, a);
   int d = v[0], x = v[1], y = v[2];
   return { d, y - (b / a) * x, x };
}
```

7.2 Primes Sieve

```
// Theme: Prime Numbers

// Algorithm: Eratosthenes Sieve
// Complexity: O(N*log(log(N)))

// = 0 - Prime,
// != 0 - Lowest Prime Divisor
auto get_sieve(int n) {
   vector<int> sieve(n); // Sieve
   sieve[0] = sieve[1] = 1;

   for (int i = 2; i * i < n; i++)
      if (!sieve[i])
         for (int j = i * i; j < n; j += i)
            sieve[j] = i;

   return sieve;
}

// Algorithm: Prime Numbers With Sieve
// Complexity: O(N*log(log(N)))

auto get_primes(int n) {
   vector<int> primes, sieve = get_sieve(n);

   for (int i = 2; i < sieve.size(); i++)
      if (!sieve[i])
         primes.push_back(i);

   return primes;
}

// Algorithm: Linear Algorithm
// Complexity: O(N)

// lp[i] = Lowest Prime Divisor
auto get_sieve_primes(int n, vector<int> &primes) {
   vector<int> lp(n);
   lp[0] = lp[1] = 1;

   for (int i = 2; i < n; i++) {
      if (!lp[i]) {
         lp[i] = i;
         primes.push_back(i);
      }
      for (int j = 0; j < primes.size() &&
               primes[j] <= lp[i] &&
               i * primes[j] < n; j++)
         lp[i * primes[j]] = primes[j];
   }

   return lp;
}
```

7.3 Fibonacci

```
// Theme: Fibonacci Sequence

// Algorithm: Fibonacci Numbers With Matrix Exponentiation
// Complexity: O(log(N))

int fibonacci(int n) {
   row<int> first_two = { 1, 0 };
   if (n <= 2) return first_two[2 - n];

   matrix<int> fib(2, row<int>(2, 0));
   fib[0][0] = 1; fib[0][1] = 1;
   fib[1][0] = 1; fib[1][1] = 0;

   fib = m_binpow(fib, n - 2);

   row<int> last_two = m_prod(fib, first_two);

   return last_two[0];
}
```

7.4 Factorization

```
// Theme: Factorization

// Algorithm: Trivial Algorithm
// Complexity: O(sqrt(N))

auto factors(int n) {
   vector<int> factors;

   for (int i = 2; i * i <= n; i++) {
      if (n % i) continue;
      while (n % i == 0) n /= i;
      factors.push_back(i);
   }

   if (n != 1)
      factors.push_back(n);

   return factors;
}

// Algorithm: Factorization With Sieve
// Complexity: O(N*log(log(N)))

auto factors_sieve(int n) {
   vector<int> factors,
      sieve = get_sieve(n + 1);

   while (sieve[n]) {
      factors.push_back(sieve[n]);
      n /= sieve[n];
   }

   if (n != 1)
      factors.push_back(n);

   return factors;
}

// Algorithm: Factorization With Primes
// Complexity: O(sqrt(N)/log(sqrt(N)))

auto factors_primes(int n) {
   vector<int> factors,
      primes = get_primes(n + 1);

   for (auto &i : primes) {
      if (i * i > n) break;
      if (n % i) continue;
      while (n % i == 0) n /= i;
      factors.push_back(i);
   }

   if (n != 1)
      factors.push_back(n);

   return factors;
}

// Algorithm: Ferma Test
// Complexity: O(K*log(N))

bool ferma(int n) {
   if (n == 2) return true;

   uniform_int_distribution<int> distA(2, n - 1);
```

```
    for (int i = 0; i < 1000; i++) {
        int a = distA(reng);
        if (gcd(a, n) != 1 ||
        binpow(a, n - 1, n) != 1)
            return false;
    }

    return true;
}

// Algorithm: Pollard Rho Algorithm
// Complexity: O(N^(1/4))

int f(int x, int c, int n) {
    return ((x * x) % n + c) % n;
}

int pollard_rho(int n) {
    if (n % 2 == 0) return 2;

    uniform_int_distribution<int> distC(1, n), distX(1, n);

    int c = distC(reng), x = distX(reng);
    int y = x;

    int g = 1;

    while (g == 1) {
        x = f(x, c, n);
        y = f(f(y, c, n), c, n);
        g = gcd(abs(x - y), n);
    }

    return g;
}

// Algorithm: Pollard Rho Factorization + Ferma Test
// Complexity: O(N^(1/4)*log(N))

void factors_pollard_rho(int n, vector<int> &factors) {
    if (n == 1) return;

    if (ferma(n)) {
        factors.push_back(n);
        return;
    }

    int d = pollard_rho(n);

    factors_pollard_rho(d, factors);
    factors_pollard_rho(n / d, factors);
}
```

7.5 Number Decomposition

```
// Theme: Integer Numbers Decomposition With Composite
     Module

// Module
// m = (p1 ^ m1) * (p2 ^ m2) * ... * (pn ^ mn)
int m;
// Prime Divisors Of Module
vector<int> p;


struct num {
        // GCD(x, m) = 1
    int x;
        // Powers Of Primes
    vector<int> a;

    num() : x(0), a(vector<int>(p.size())) { }

        // n = (p1 ^ a1) * (p2 ^ a2) * ... * (pn ^ an) * x
    num(int n) : x(0), a(vector<int>(p.size())) {
        if (!n) return;
        for (int i = 0; i < p.size(); i++) {
            int ai = 0;
            while (n % p[i] == 0) {
                n /= p[i];
                ai++;
            }
            a[i] = ai;
        }
        x = n;
    }

    num operator*(const num &nm) {
        vector<int> new_a(p.size());
        for (int i = 0; i < p.size(); i++)
            new_a[i] = a[i] + nm.a[i];
        num res; res.a = new_a;
        res.x = x * nm.x % m;
        return res;
    }

    num operator/(const num &nm) {
        vector<int> new_a(p.size());
        for (int i = 0; i < p.size(); i++)
            new_a[i] = a[i] - nm.a[i];
        num res; res.a = new_a;
        int g = euclid(nm.x, m)[1];
        g += m; g %= m;
        res.x = x * g % m;
        return res;
    }

    int toint() {
        int res = x;
        for (int i = 0; i < p.size(); i++)
            res = res * binpow(p[i], a[i], m) % m;
        return res;
    }
};
```

7.6 Euler Totient Function

```
// Theme: Euler Totient Function

// Algorithm: Euler Product Formula
// Complexity: O(sqrt(N))

// phi = n(1 - 1 / pi), i = 1,...
int phi(int n) {
    if (n == 1) return 1;

    auto f = factors(n);

    int res = n;
    for (auto &p : f)
        res -= res / p;

    return res;
}
```

7.7 Permutation

```
// Theme: Permmutations

// Algorithm: Next Lexicological Permutation
// Complexity: O(N)

bool perm(vector<int> &v) {
    int n = v.size();

    for (int i = n - 1; i >= 1; i--) {
        if (v[i - 1] < v[i]) {
            reverse(v.begin() + i, v.end());

            int j = distance(v.begin(),
            upper_bound(v.begin() + i, v.end(), v[i - 1]));

            swap(v[i - 1], v[j]);
            return true;
        }
    }

    return false;
}
```

7.8 Primitive Roots

```
// Module (7 == 3 * (2 ^ 1) + 1)
// Primitive Root (3)
// Primitive Root {2 ^ 1} (6)
// Inverse Root {2 ^ 1} (6)
// Degree Of Two (2)

// const int mod = 7
// const int proot = 6
// const int proot_1 = 6
// const int pw = 1 << 1



// Module (13 == 3 * (2 ^ 2) + 1)
// Primitive Root (2)
// Primitive Root {2 ^ 2} (8)
// Inverse Root {2 ^ 2} (5)
// Degree Of Two (4)

// const int mod = 13
// const int proot = 8
// const int proot_1 = 5
// const int pw = 1 << 2



// Module (19 == 9 * (2 ^ 1) + 1)
// Primitive Root (2)
// Primitive Root {2 ^ 1} (18)
// Inverse Root {2 ^ 1} (18)
// Degree Of Two (2)

// const int mod = 19
// const int proot = 18
// const int proot_1 = 18
// const int pw = 1 << 1



// Module (37 == 9 * (2 ^ 2) + 1)
// Primitive Root (2)
// Primitive Root {2 ^ 2} (31)
// Inverse Root {2 ^ 2} (6)
// Degree Of Two (4)

// const int mod = 37
// const int proot = 31
// const int proot_1 = 6
// const int pw = 1 << 2



// Module (73 == 9 * (2 ^ 3) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 3} (10)
// Inverse Root {2 ^ 3} (22)
// Degree Of Two (8)

// const int mod = 73
// const int proot = 10
// const int proot_1 = 22
// const int pw = 1 << 3



// Module (97 == 3 * (2 ^ 5) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 5} (28)
// Inverse Root {2 ^ 5} (52)
// Degree Of Two (32)

// const int mod = 97
// const int proot = 28
// const int proot_1 = 52
// const int pw = 1 << 5



// Module (109 == 27 * (2 ^ 2) + 1)
// Primitive Root (6)
// Primitive Root {2 ^ 2} (33)
// Inverse Root {2 ^ 2} (76)
// Degree Of Two (4)

// const int mod = 109
// const int proot = 33
// const int proot_1 = 76
// const int pw = 1 << 2



// Module (163 == 81 * (2 ^ 1) + 1)
// Primitive Root (2)
// Primitive Root {2 ^ 1} (162)
// Inverse Root {2 ^ 1} (162)
// Degree Of Two (2)

// const int mod = 163
// const int proot = 162
// const int proot_1 = 162
// const int pw = 1 << 1



// Module (193 == 3 * (2 ^ 6) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 6} (125)
// Inverse Root {2 ^ 6} (105)
// Degree Of Two (64)

// const int mod = 193
// const int proot = 125
// const int proot_1 = 105
// const int pw = 1 << 6



// Module (433 == 27 * (2 ^ 4) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 4} (238)
// Inverse Root {2 ^ 4} (282)
// Degree Of Two (16)

// const int mod = 433
// const int proot = 238
// const int proot_1 = 282
// const int pw = 1 << 4



// Module (487 == 243 * (2 ^ 1) + 1)
// Primitive Root (3)
// Primitive Root {2 ^ 1} (486)
// Inverse Root {2 ^ 1} (486)
// Degree Of Two (2)

// const int mod = 487
// const int proot = 486
// const int proot_1 = 486
// const int pw = 1 << 1



// Module (577 == 9 * (2 ^ 6) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 6} (557)
// Inverse Root {2 ^ 6} (375)
// Degree Of Two (64)

// const int mod = 577
// const int proot = 557
// const int proot_1 = 375
// const int pw = 1 << 6



// Module (769 == 3 * (2 ^ 8) + 1)
// Primitive Root (11)
// Primitive Root {2 ^ 8} (562)
// Inverse Root {2 ^ 8} (26)
// Degree Of Two (256)

// const int mod = 769
// const int proot = 562
// const int proot_1 = 26
// const int pw = 1 << 8



// Module (1153 == 9 * (2 ^ 7) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 7} (1096)
// Inverse Root {2 ^ 7} (445)
// Degree Of Two (128)

// const int mod = 1153
```

```
// const int proot = 1096
// const int proot_1 = 445
// const int pw = 1 << 7



// Module (1297 == 81 * (2 ^ 4) + 1)
// Primitive Root (10)
// Primitive Root {2 ^ 4} (355)
// Inverse Root {2 ^ 4} (464)
// Degree Of Two (16)

// const int mod = 1297
// const int proot = 355
// const int proot_1 = 464
// const int pw = 1 << 4



// Module (1459 == 729 * (2 ^ 1) + 1)
// Primitive Root (3)
// Primitive Root {2 ^ 1} (1458)
// Inverse Root {2 ^ 1} (1458)
// Degree Of Two (2)

// const int mod = 1459
// const int proot = 1458
// const int proot_1 = 1458
// const int pw = 1 << 1



// Module (2593 == 81 * (2 ^ 5) + 1)
// Primitive Root (7)
// Primitive Root {2 ^ 5} (1997)
// Inverse Root {2 ^ 5} (335)
// Degree Of Two (32)

// const int mod = 2593
// const int proot = 1997
// const int proot_1 = 335
// const int pw = 1 << 5



// Module (2917 == 729 * (2 ^ 2) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 2} (2863)
// Inverse Root {2 ^ 2} (54)
// Degree Of Two (4)

// const int mod = 2917
// const int proot = 2863
// const int proot_1 = 54
// const int pw = 1 << 2



// Module (3457 == 27 * (2 ^ 7) + 1)
// Primitive Root (7)
// Primitive Root {2 ^ 7} (540)
// Inverse Root {2 ^ 7} (685)
// Degree Of Two (128)

// const int mod = 3457
// const int proot = 540
// const int proot_1 = 685
// const int pw = 1 << 7



// Module (3889 == 243 * (2 ^ 4) + 1)
// Primitive Root (11)
// Primitive Root {2 ^ 4} (1925)
// Inverse Root {2 ^ 4} (1396)
// Degree Of Two (16)

// const int mod = 3889
// const int proot = 1925
// const int proot_1 = 1396
// const int pw = 1 << 4



// Module (10369 == 81 * (2 ^ 7) + 1)
// Primitive Root (13)
// Primitive Root {2 ^ 7} (5758)
// Inverse Root {2 ^ 7} (6762)
// Degree Of Two (128)

// const int mod = 10369
// const int proot = 5758
// const int proot_1 = 6762
// const int pw = 1 << 7



// Module (12289 == 3 * (2 ^ 12) + 1)
// Primitive Root (11)
// Primitive Root {2 ^ 12} (1331)
// Inverse Root {2 ^ 12} (7968)
// Degree Of Two (4096)

// const int mod = 12289
// const int proot = 1331
// const int proot_1 = 7968
// const int pw = 1 << 12



// Module (17497 == 2187 * (2 ^ 3) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 3} (14518)
// Inverse Root {2 ^ 3} (7565)
// Degree Of Two (8)

// const int mod = 17497
// const int proot = 14518
// const int proot_1 = 7565
// const int pw = 1 << 3



// Module (18433 == 9 * (2 ^ 11) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 11} (17660)
// Inverse Root {2 ^ 11} (18123)
// Degree Of Two (2048)

// const int mod = 18433
// const int proot = 17660
// const int proot_1 = 18123
// const int pw = 1 << 11



// Module (39367 == 19683 * (2 ^ 1) + 1)
// Primitive Root (3)
// Primitive Root {2 ^ 1} (39366)
// Inverse Root {2 ^ 1} (39366)
// Degree Of Two (2)

// const int mod = 39367
// const int proot = 39366
// const int proot_1 = 39366
// const int pw = 1 << 1



// Module (52489 == 6561 * (2 ^ 3) + 1)
// Primitive Root (7)
// Primitive Root {2 ^ 3} (37459)
// Inverse Root {2 ^ 3} (37783)
// Degree Of Two (8)

// const int mod = 52489
// const int proot = 37459
// const int proot_1 = 37783
// const int pw = 1 << 3



// Module (65537 == 1 * (2 ^ 16) + 1)
// Primitive Root (3)
// Primitive Root {2 ^ 16} (3)
// Inverse Root {2 ^ 16} (21846)
// Degree Of Two (65536)

// const int mod = 65537
// const int proot = 3
// const int proot_1 = 21846
// const int pw = 1 << 16



// Module (139969 == 2187 * (2 ^ 6) + 1)
```

```
// Primitive Root (13)
// Primitive Root {2 ^ 6} (8104)
// Inverse Root {2 ^ 6} (40191)
// Degree Of Two (64)

// const int mod = 139969
// const int proot = 8104
// const int proot_1 = 40191
// const int pw = 1 << 6



// Module (147457 == 9 * (2 ^ 14) + 1)
// Primitive Root (10)
// Primitive Root {2 ^ 14} (94083)
// Inverse Root {2 ^ 14} (163)
// Degree Of Two (16384)

// const int mod = 147457
// const int proot = 94083
// const int proot_1 = 163
// const int pw = 1 << 14



// Module (209953 == 6561 * (2 ^ 5) + 1)
// Primitive Root (10)
// Primitive Root {2 ^ 5} (198463)
// Inverse Root {2 ^ 5} (179931)
// Degree Of Two (32)

// const int mod = 209953
// const int proot = 198463
// const int proot_1 = 179931
// const int pw = 1 << 5



// Module (331777 == 81 * (2 ^ 12) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 12} (100795)
// Inverse Root {2 ^ 12} (281060)
// Degree Of Two (4096)

// const int mod = 331777
// const int proot = 100795
// const int proot_1 = 281060
// const int pw = 1 << 12



// Module (472393 == 59049 * (2 ^ 3) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 3} (407677)
// Inverse Root {2 ^ 3} (406705)
// Degree Of Two (8)

// const int mod = 472393
// const int proot = 407677
// const int proot_1 = 406705
// const int pw = 1 << 3



// Module (629857 == 19683 * (2 ^ 5) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 5} (255372)
// Inverse Root {2 ^ 5} (55433)
// Degree Of Two (32)

// const int mod = 629857
// const int proot = 255372
// const int proot_1 = 55433
// const int pw = 1 << 5



// Module (746497 == 729 * (2 ^ 10) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 10} (371573)
// Inverse Root {2 ^ 10} (21163)
// Degree Of Two (1024)

// const int mod = 746497
// const int proot = 371573
// const int proot_1 = 21163
// const int pw = 1 << 10



// Module (786433 == 3 * (2 ^ 18) + 1)
// Primitive Root (10)
// Primitive Root {2 ^ 18} (1000)
// Inverse Root {2 ^ 18} (710149)
// Degree Of Two (262144)

// const int mod = 786433
// const int proot = 1000
// const int proot_1 = 710149
// const int pw = 1 << 18



// Module (839809 == 6561 * (2 ^ 7) + 1)
// Primitive Root (7)
// Primitive Root {2 ^ 7} (500841)
// Inverse Root {2 ^ 7} (2262)
// Degree Of Two (128)

// const int mod = 839809
// const int proot = 500841
// const int proot_1 = 2262
// const int pw = 1 << 7



// Module (995329 == 243 * (2 ^ 12) + 1)
// Primitive Root (7)
// Primitive Root {2 ^ 12} (712513)
// Inverse Root {2 ^ 12} (946681)
// Degree Of Two (4096)

// const int mod = 995329
// const int proot = 712513
// const int proot_1 = 946681
// const int pw = 1 << 12



// Module (1179649 == 9 * (2 ^ 17) + 1)
// Primitive Root (19)
// Primitive Root {2 ^ 17} (612074)
// Inverse Root {2 ^ 17} (1093705)
// Degree Of Two (131072)

// const int mod = 1179649
// const int proot = 612074
// const int proot_1 = 1093705
// const int pw = 1 << 17



// Module (1492993 == 729 * (2 ^ 11) + 1)
// Primitive Root (7)
// Primitive Root {2 ^ 11} (143225)
// Inverse Root {2 ^ 11} (1126252)
// Degree Of Two (2048)

// const int mod = 1492993
// const int proot = 143225
// const int proot_1 = 1126252
// const int pw = 1 << 11



// Module (1769473 == 27 * (2 ^ 16) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 16} (374254)
// Inverse Root {2 ^ 16} (643391)
// Degree Of Two (65536)

// const int mod = 1769473
// const int proot = 374254
// const int proot_1 = 643391
// const int pw = 1 << 16



// Module (199065 == 24883 * (2 ^ 3) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 3} (18290)
// Inverse Root {2 ^ 3} (115385)
// Degree Of Two (8)

// const int mod = 199065
// const int proot = 18290
```

```
// const int proot_1 = 115385
// const int pw = 1 << 3



// Module (2654209 == 81 * (2 ^ 15) + 1)
// Primitive Root (11)
// Primitive Root {2 ^ 15} (1985530)
// Inverse Root {2 ^ 15} (2369076)
// Degree Of Two (32768)

// const int mod = 2654209
// const int proot = 1985530
// const int proot_1 = 2369076
// const int pw = 1 << 15



// Module (5038849 == 19683 * (2 ^ 8) + 1)
// Primitive Root (29)
// Primitive Root {2 ^ 8} (4318906)
// Inverse Root {2 ^ 8} (2727143)
// Degree Of Two (256)

// const int mod = 5038849
// const int proot = 4318906
// const int proot_1 = 2727143
// const int pw = 1 << 8



// Module (5308417 == 81 * (2 ^ 16) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 16} (3305774)
// Inverse Root {2 ^ 16} (3708247)
// Degree Of Two (65536)

// const int mod = 5308417
// const int proot = 3305774
// const int proot_1 = 3708247
// const int pw = 1 << 16



// Module (8503057 == 531441 * (2 ^ 4) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 4} (4589209)
// Inverse Root {2 ^ 4} (2906831)
// Degree Of Two (16)

// const int mod = 8503057
// const int proot = 4589209
// const int proot_1 = 2906831
// const int pw = 1 << 4



// Module (11337409 == 177147 * (2 ^ 6) + 1)
// Primitive Root (7)
// Primitive Root {2 ^ 6} (3744116)
// Inverse Root {2 ^ 6} (9616850)
// Degree Of Two (64)

// const int mod = 11337409
// const int proot = 3744116
// const int proot_1 = 9616850
// const int pw = 1 << 6



// Module (14155777 == 27 * (2 ^ 19) + 1)
// Primitive Root (7)
// Primitive Root {2 ^ 19} (2742784)
// Inverse Root {2 ^ 19} (1606624)
// Degree Of Two (524288)

// const int mod = 14155777
// const int proot = 2742784
// const int proot_1 = 1606624
// const int pw = 1 << 19



// Module (19131877 == 4782969 * (2 ^ 2) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 2} (19127503)
// Inverse Root {2 ^ 2} (4374)
// Degree Of Two (4)

// const int mod = 19131877
// const int proot = 19127503
// const int proot_1 = 4374
// const int pw = 1 << 2



// Module (28311553 == 27 * (2 ^ 20) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 20} (4493789)
// Inverse Root {2 ^ 20} (13207632)
// Degree Of Two (1048576)

// const int mod = 28311553
// const int proot = 4493789
// const int proot_1 = 13207632
// const int pw = 1 << 20



// Module (57395629 == 14348907 * (2 ^ 2) + 1)
// Primitive Root (10)
// Primitive Root {2 ^ 2} (19864209)
// Inverse Root {2 ^ 2} (37531420)
// Degree Of Two (4)

// const int mod = 57395629
// const int proot = 19864209
// const int proot_1 = 37531420
// const int pw = 1 << 2



// Module (63700993 == 243 * (2 ^ 18) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 18} (48698706)
// Inverse Root {2 ^ 18} (16386043)
// Degree Of Two (262144)

// const int mod = 63700993
// const int proot = 48698706
// const int proot_1 = 16386043
// const int pw = 1 << 18



// Module (71663617 == 2187 * (2 ^ 15) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 15} (37080182)
// Inverse Root {2 ^ 15} (7507216)
// Degree Of Two (32768)

// const int mod = 71663617
// const int proot = 37080182
// const int proot_1 = 7507216
// const int pw = 1 << 15



// Module (86093443 == 43046721 * (2 ^ 1) + 1)
// Primitive Root (2)
// Primitive Root {2 ^ 1} (86093442)
// Inverse Root {2 ^ 1} (86093442)
// Degree Of Two (2)

// const int mod = 86093443
// const int proot = 86093442
// const int proot_1 = 86093442
// const int pw = 1 << 1



// Module (102036673 == 1594323 * (2 ^ 6) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 6} (50805973)
// Inverse Root {2 ^ 6} (42074539)
// Degree Of Two (64)

// const int mod = 102036673
// const int proot = 50805973
// const int proot_1 = 42074539
// const int pw = 1 << 6



// Module (113246209 == 27 * (2 ^ 22) + 1)
// Primitive Root (7)
```

```
// Primitive Root {2 ^ 22} (58671006)
// Inverse Root {2 ^ 22} (62639419)
// Degree Of Two (4194304)

// const int mod = 113246209
// const int proot = 58671006
// const int proot_1 = 62639419
// const int pw = 1 << 22


// Module (120932353 == 59049 * (2 ^ 11) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 11} (40826043)
// Inverse Root {2 ^ 11} (93710416)
// Degree Of Two (2048)

// const int mod = 120932353
// const int proot = 40826043
// const int proot_1 = 93710416
// const int pw = 1 << 11


// Module (998244353 == 119 * (2 ^ 23) + 1)
// Primitive Root (3)
// Primitive Root {2 ^ 23} (15311432)
// Inverse Root {2 ^ 23} (469870224)
// Degree Of Two (8388608)

// const int mod = 998244353
// const int proot = 15311432
// const int proot_1 = 469870224
// const int pw = 1 << 23


// Module (1000000007 == 500000003 * (2 ^ 1) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 1} (1000000006)
// Inverse Root {2 ^ 1} (1000000006)
// Degree Of Two (2)

// const int mod = 1000000007
// const int proot = 1000000006
// const int proot_1 = 1000000006
// const int pw = 1 << 1
```

7.9 Formulae

Combinations.

$C_n^k = \frac{n!}{(n-k)!k!}$

$C_n^0 + C_n^1 + ... + C_n^n = 2^n$

$C_{n+1}^{k+1} = C_n^{k+1} + C_n^k$

$C_n^k = \frac{n}{k} C_{n-1}^{k-1}$

Striling approximation.

$n! \approx \sqrt{2\pi n} \frac{n}{e}^n$

Euler's theorem.

$a^{\phi(m)} \equiv 1 \bmod m$, $gcd(a, m) = 1$

Ferma's little theorem.

$a^{p-1} \equiv 1 \bmod p$, $gcd(a, p) = 1$, p - prime.

Catalan number.

$$C_0 = 0, C_n = \sum_{i=0}^{n-1} C_i C_{n-1-i}$$

$$C_n = \frac{2(2n-1)}{n+1} C_{n-1}$$

$$C_n = \frac{(2n)!}{n!(n+1)!}$$

Arithmetic progression.

$S_n = \frac{a_1 + a_n}{2} n = \frac{2a_1 + d(n-1)}{2} n$

Geometric progression.

$S_n = \frac{b_1(1-q^n)}{1-q} n$

Infinitely decreasing geometric progression.

$S_n = \frac{b_1}{1-q} n$

Sums.

$\sum_{i=1}^n i = \frac{n(n+1)}{2}$,

$\sum_{i=1}^n i^2 = \frac{n(2n+1)(n+1)}{6}$,

$\sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$,

$\sum_{i=1}^n i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$,

$\sum_{i=a}^b c^i = \frac{c^{b+1} - c^a}{c-1}, c \neq 1.$

7.10 Baby Step Giant Step

```
// Theme: Descrete Logarithm

// Algorithm: Baby-Step Giant-Step Algorithm
// Complexity: O(sqrt(p)*log(p))

// a ^ (x) = b (mod p), (a, p) = 1
// a ^ (i * m + j) = b (mod p), m = ceil(sqrt(p))
// a ^ (i * m) = b * a ^ (-j) (mod p)
int baby_giant_step(int a, int b, int p) {
   // a ^ (-1) = a ^ (p - 2) mod p
   int m = ceil(sqrt(p)), _a = binpow(a, p - 2, p);

   // s[b * a ^ (-j)] = j
   unordered_map<int, int> s;
   for (int j = 0, t = b; j < m; j++, t = t * _a % p) s[t]
        = j;

   for (int i = 0; i < m; i++) {
      // s.find(a ^ (i * m))
      auto f = s.find(binpow(a, i * m, p));
      // i * m + j
      if (f != s.end()) return i * m + f->ss;
   }

   return -1;
}
```

7.11 Matrices

```
// Theme: Matrix Opeations

template <typename T>
using row = vector<T>;
template <typename T>
using matrix = vector<vector<T>>;

// Algorithm: Matrix-Matrix Multiplication
// Complexity: O(N*K*M)

auto m_prod(matrix<int> &a, matrix<int> &b, int p = 0) {
   int n = a.size(), k = a[0].size(), m = b[0].size();

   matrix<int> res(n, row<int>(m));

   for (int i = 0; i < n; i++)
      for (int j = 0; j < m; j++)
         for (int z = 0; z < k; z++)
            res[i][j] = p ? (res[i][j] + a[i][z] * b[z][j]
                  % p) % p
            : (res[i][j] + a[i][z] * b[z][j]);

   return res;
}
```

```
// Algorithm: Matrix-Vector Multiplication
// Complexity: O(N*M)

auto m_prod(matrix<int> &a, row<int> &b, int p = 0) {
   int n = a.size(), m = b.size();

   row<int> res(n);

   for (int i = 0; i < n; i++)
      for (int j = 0; j < m; j++)
         res[i] = p ? (res[i] + a[i][j] * b[j] % p) % p
         : (res[i] + a[i][j] * b[j]);

   return res;
}

// Algorithm: Fast Matrix Exponentiation
// Complexity: O(N^3*log(K))

auto m_binpow(matrix<int> a, int x, int p = 0) {
   int n = a.size();

   matrix<int> res(n, row<int>(n));
   for (int i = 0; i < n; i++) res[i][i] = 1;

   while (x) {
      if (x & 1) res = m_prod(res, a, p);
      a = m_prod(a, a, p);
      x >>= 1;
   }

   return res;
}
```

7.12 Fast Fourier Transform

```
// Theme: Fast Fourier Transform

// Algorithm: Fast Fourier Transform (Complex)
// Complexity: O(N*log(N))

using cd = complex<double>;
const double PI = acos(-1);


auto fft(vector<cd> a, bool invert = 0) {
   // n = 2 ^ x
   int n = a.size();

   // Bit-Reversal Permutation (0000, 1000, 0100, 1100,
       0010, ...)
   for (int i = 1, j = 0; i < n; i++) {
      int bit = n >> 1;
      for (; j >= bit; bit >>= 1) j -= bit;
      j += bit;
      if (i < j) swap(a[i], a[j]);
   }

   for (int len = 2; len <= n; len <<= 1) {
      // Complex Root Of One
      double ang = 2 * PI / len * (invert ? -1 : 1);
      cd lroot(cos(ang), sin(ang));

      for (int i = 0; i < n; i += len) {
         cd root(1);
         for (int j = 0; j < len / 2; j++) {
            cd u = a[i + j], v = a[i + j + len / 2] * root
                ;
            a[i + j] = (u + v);
            a[i + j + len / 2] = (u - v);
            root = (root * lroot);
         }

      }
   }

   if (invert) {
      for (int i = 0; i < n; i++) a[i] /= n;
   }

   return a;
}

// Module (7340033 = 7 * (2 ^ 20) + 1)
// Primiive Root (5 ^ (2 ^ 20) == 1 mod 7340033)
// Inverse Primitive Root (5 * 4404020 == 1 mod 7340033)
// Maximum Degree Of Two (2 ^ 20)

const int mod = 7340033;
const int proot = 5;
const int proot_1 = 4404020;
const int pw = 1 << 20;

// Algorithm: Discrete Fourier Transform (Inverse Roots)
// Complexity: O(N*log(N))

auto fft(vector<int> a, bool invert = 0) {
   // n = 2 ^ x
   int n = a.size();

   // Bit-Reversal Permutation (0000, 1000, 0100, 1100,
       0010, ...)
   for (int i = 1, j = 0; i < n; i++) {
      int bit = n >> 1;
      for (; j >= bit; bit >>= 1) j -= bit;
      j += bit;
      if (i < j) swap(a[i], a[j]);
   }

   for (int len = 2; len <= n; len <<= 1) {
       // Primitive Root Or Inverse Root (Inverse
           Transform)
      int lroot = invert ? proot_1 : proot;

      // Current Primitive Root
      lroot = binpow(lroot, pw / len, mod);

      for (int i = 0; i < n; i += len) {
         int root = 1;
         for (int j = 0; j < len / 2; j++) {
            int u = a[i + j], v = a[i + j + len / 2] *
                root % mod;
            a[i + j] = (u + v) % mod;
            a[i + j + len / 2] = (u - v + mod) % mod;
            root = (root * lroot) % mod;
         }

      }
   }

   if (invert) {
      int _n = binpow(n, mod - 2, mod);
      for (int i = 0; i < n; i++) a[i] = (a[i] * _n) % mod
          ;
   }

   return a;
}

// Algorithm: Discrete Fourier Transform
// Complexity: O(N*log(N))

auto fft(vector<int> &a, bool invert = 0) {
   // n = 2 ^ x
   int n = a.size();

   // Bit-Reversal Permutation (0000, 1000, 0100, 1100,
       0010, ...)
   for (int i = 1, j = 0; i < n; i++) {
      int bit = n >> 1;
      for (; j >= bit; bit >>= 1) j -= bit;
      j += bit;
      if (i < j) swap(a[i], a[j]);
   }

   for (int len = 2; len <= n; len <<= 1) {
      // Current Primitive Root
      int lroot = binpow(proot, pw / len, mod);

      for (int i = 0; i < n; i += len) {
         int root = 1;
         for (int j = 0; j < len / 2; j++) {
            int u = a[i + j], v = a[i + j + len / 2] *
                root % mod;
            a[i + j] = (u + v) % mod;
            a[i + j + len / 2] = (u - v + mod) % mod;
            root = (root * lroot) % mod;
         }

      }
   }
```

```
    if (invert) {
        reverse(a.begin() + 1, a.end());
        int _n = binpow(n, mod - 2, mod);
        for (int i = 0; i < n; i++) a[i] = (a[i] * _n) % mod
            ;
    }

    return a;
}
```

7.13 Combinations

```
// Theme: Combination Number

// Algorithm: Online Multiplication-Division
// Complexity: O(k)

// C_n^k - from n by k
int C(int n, int k) {
    int res = 1;

    for (int i = 1; i <= k; i++) {
        res *= n - k + i;
        res /= i;
    }

    return res;
}

// Algorithm: Pascal Triangle Preprocessing
// Complexity: O(N^2)

auto pascal(int n) {
    // C[i][j] = C_i+j^i
    vector<vector<int>> C(n + 1, vector<int>(n + 1, 1));
    for (int i = 1; i < n + 1; i++)
        for (int j = 1; j < n + 1; j++)
            C[i][j] = C[i - 1][j] + C[i][j - 1];

    return C;
}
```

7.14 Binary Operations

```
// Theme: Binary Operations

// Algorithm: Binary Multiplication
// Complexity: O(log(b))

int binmul(int a, int b, int p = 0) {
    int res = 0;
    while (b) {
        if (b & 1) res = p ? (res + a) % p : (res + a);
        a = p ? (a + a) % p : (a + a);
        b >>= 1;
    }
    return res;
}

// Algorithm: Binary Exponentiation
// Complexity: O(log(b))

int binpow(int a, int b, int p = 0) {
    int res = 1;
    while (b) {
        if (b & 1) res = p ? (res * a) % p : (res * a);
        a = p ? (a * a) % p : (a * a);
        b >>= 1;
    }
    return res;
}
```

8 Templates

8.1 C++ Include

```
#include <iostream>
#include <iomanip>
#include <fstream>
#include <random>
#include <cmath>
#include <algorithm>
#include <string>
#include <vector>
#include <set>
#include <unordered_set>
#include <map>
#include <unordered_map>
#include <queue>
#include <deque>
#include <stack>
#include <list>
#include <bitset>
```

8.2 C++ Template

```
#include <bits/stdc++.h>

using namespace std;



// DEFINES
#define precision(x) cout << fixed << setprecision(x);
#define fast cin.tie(0); ios::sync_with_stdio(0)
#define all(x) x.begin(), x.end()
#define rall(x) x.rbegin(), x.rend()
#define ff first
#define ss second
// #define nl endl
#define nl "\n"
#define sp " "
#define yes "Yes"
#define no "No"
#define int long long



// CONSTANTS
const int INF = 1e18;
// const int MOD = 1e9 + 7;
// const int MOD = 998244353;



// FSTREAMS
ifstream in("input.txt");
ofstream out("output.txt");



// RANDOM
const int RMIN = 1, RMAX = 1e9;

random_device rdev;
mt19937_64 reng(rdev());
uniform_int_distribution<mt19937_64::result_type> dist(RMIN
    , RMAX);



// CUSTOM HASH
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        // http://xorshift.di.unimi.it/splitmix64.c
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }

    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM = chrono::
            steady_clock::now().time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
};
// USAGE EXAMPLES:
//
// unordered_set<long long, custom_hash> safe_set;
// unordered_map<long long, int, custom_hash> safe_map;
```

```
// gp_hash_table<long long, int, custom_hash>
     safe_hash_table;
//
// for pairs might be used like `3 * a + b` or `a ^ (b >>
     1)`


// GLOBALS



// SOLUTION
void solve() {

}



// PREPROCESSING
void prepr() { }



// ENTRANCE
signed main() {
   precision(15);
   fast;
   prepr();
   int t = 1;
   cin >> t;
   while (t--) solve();
}
```

8.3 Py Template

```
from math import sqrt, ceil, floor, gcd
from random import randint
import sys


def inpt():
   return sys.stdin.readline().strip()


input = inpt


INF = int(1e18)
# MOD = int(1e9 + 7)
# MOD = 998244353


def solve():
   pass


t = 1
t = int(input())
for _ in range(t):
   solve()
```