Link recommendations: Their impact on network structure and minorities

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ABSTRACT

Network-based people recommendation algorithms are widely employed on the Web to suggest new connections in social media or professional platforms. While such recommendations bring people together, the feedback loop between the algorithms and the changes in network structure may exacerbate social biases. These biases include rich-get-richer effects, filter bubbles, and polarization. However, social networks are diverse complex systems and recommendations may affect them differently, depending on their structural properties. In this work, we explore five people recommendation algorithms by systematically applying them over time to different synthetic networks. In particular, we measure to what extent these recommendations change the structure of bi-populated networks and show how these changes affect the minority group.

Our systematic experimentation helps to better understand when link recommendation algorithms are beneficial or harmful to minority groups in social networks. In particular, our findings suggest that, while all algorithms tend to close triangles and increase cohesion, all algorithms except Node2Vec are prone to favor and suggest nodes with high in-degree. Furthermore, we found that, especially when both classes are heterophilic, recommendation algorithms can reduce the visibility of minorities.

CCS CONCEPTS

 $\bullet \ Information \ systems \rightarrow Social \ networks; Recommender \ systems.$

KEYWORDS

Recommendation algorithms, friendship recommendations, network science, social networks, homophily, preferential attachment.

1 INTRODUCTION

Social networks are the infrastructure of our social and professional life. They impact, among others, our cooperation [18], our health [8], and our social perceptions [24]. The structure of modern online social networks is however not only shaped by well-studied social mechanisms (such as homophily or preferential attachment), but it is also affected by people recommender systems, complex algorithms that suggest new connections among social network users. How do these algorithms affect the structure of social networks over time? What are the consequences for different groups? In this paper, we aim to shed light on these questions.

Problem: Previous work has shown that recommendation algorithms are prone to reinforcing popularity bias [1]. A further subtle problem is that by matching users' preferences, these algorithms often lead to the formation of filter bubbles [7], echo chambers [5], and polarization [11]. In recent years, much attention has been paid to understanding when, and to what extent, such biases are being amplified. As an example, [15] and [14] have studied the correlation between network structure and the output of ranking algorithms in social networks. While these studies highlight that homophily—the tendency to connect to similar others—and preferential attachment—the tendency to connect to those that are already well-connected—are important structural factors that impact the visibility of nodes in algorithmic rankings, they do not compare effects over time. Feedback loops, instead, have been studied in [34] and [33], where they respectively analyze "rich-get-richer" and "glass ceiling" effects. Recently, also [16] and [9] have focused on feedback loops and long term effects of people recommender systems. The former analyzes inequalities in the exposure of minorities and the latter focuses on polarization and echo chambers. Our study integrates this body of research by providing a systematic analysis of how homophily and minority size relate to structural properties of the network and visibility of groups.

Approach: We systematically compare five recommendation algorithms and apply their recommendations to several synthetic

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networks. We focus on scale-free directed networks with adjustable homophily and minority group size [14] and quantify the global changes in network structure, as well as the changes in connectivity for the minority group over time. In particular, we assess whether certain types of links are created more often than others and whether the network becomes more cohesive or segregated. Similarly, we verify when, and at what rates, these algorithms put minorities at disadvantage by measuring the changes in their *visibility*, here defined as the fraction of minorities among the top most important nodes, based on their algorithmic ranking. To this end, we formulate the following research questions that will guide our analysis throughout this paper.

- RQ1: How do recommendation algorithms affect the structure of the network and the visibility of minorities?
- RQ2: To what extent is the change in visibility due to homophily?
- RQ3: Is the change in visibility inversely proportional to the size of the minority or proportional to the in-group links within the minority?

Contributions: Our contributions are the following: (1) We demonstrate that networks become more cohesive over time throughout multiple recommendations. However, the rate at which this cohesiveness gets stronger depends on the algorithm. (2) Not all algorithms suffer from the popularity bias problem, which means that certain algorithms may diversify their recommendations. (3) The visibility of the minority group gets affected differently depending on three main components: the algorithm, the initial conditions of homophily in the network, and the size of the minority group.

Moreover, our study sheds light on the weaknesses of algorithms under the initial conditions of network structure and can be used as key factors to improve recommendations, where necessary.

2 RELATED WORK

The related work is organized in two parts. First, we introduce the relevant literature on the mechanisms that drive the existence of biases in network structure. Then, we focus on the creation of new ties from link recommendations. In particular, we highlight the effects of recommendation algorithms on the network structure and the visibility of minorities.

Biases in network structure and related consequences: The rich-get-richer or Matthew effect [28] is one of the first mechanisms of edge formation discovered by sociologists to explain cumulative advantages in real-world networks. From the network perspective, the Matthew effect operates through the preferential attachment mechanism, that is the tendency of nodes to attach preferentially to those that are already well-connected [4]. This mechanism of edge formation and other structural characteristics may impact the visibility and importance of nodes, and thus, create and enlarge inequalities. For example, [3] and [23] propose mathematical models that integrate preferential attachment and homophily (the tendency to connect to similar others [27]) to explain the emergence of the "glass ceiling" effect in social networks. Glass ceiling, as defined by the US Federal Commission, is "the unseen, yet unbreakable barrier that keeps minorities and women from rising to the upper rungs of the corporate ladder, regardless of their qualifications or achievements". Studies on glass ceiling are expanded in [30],

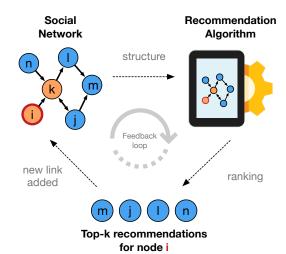
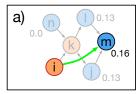


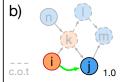
Figure 1: The recommendation cycle: A network-based recommendation algorithm uses the local or global structure of the network to recommend for each node i the top-k best matches with whom node i may want to connect. If node i accepts the recommendations, the structure of the network changes. This creates a feedback loop since the new structure is pre-processed by the algorithm to infer new recommendations.

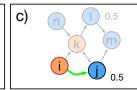
where the authors consider the effect of the perceived gender on the visibility of users on Twitter. In particular, they reveal how users perceived as women are hampered from attaining equal visibility. Furthermore, [23] shad light on how homophily can put minority groups at disadvantage by restricting their ability to establish links with the majority group and by limiting their access to information. Recently, [35] observed that PageRank [31] might unfairly allocate importance scores to different classes, and proposed alternative fair versions of the algorithm.

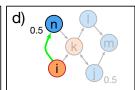
Our work is built upon this body of literature and integrates social biases in feedback loops of people recommendations. In particular, we analyze the tendencies of groups to connect to each other, how these tendencies or mechanisms of edge formation affect the recommendations, and ultimately how these recommendations affect the structure of networks and the visibility of minorities.

Effects of recommender systems on networks: [34] analyzes the "rich-get-richer" phenomenon through social recommendations. In particular, they study how the "Who-to-Follow" algorithm affects the structure of the follower network on Twitter. They found that most popular users profited substantially more than average from the user suggestions. They attributed this "rich-get-richer" effect to various factors, including the mismatch between users (being recommended proportional to their degree), and the baseline growth rate of users (whose asymptotic behavior is instead sub-linear in the degree). Users' centrality and clustering coefficient may also vary depending on the recommendation algorithm in "Social-Blue", an internal social networking site at IBM [10]. Similar effects have been found in Tumblr and Flickr, two social media platforms, where









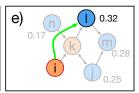


Figure 2: Recommendation algorithms: Given the network in Figure 1, here we explain the recommendation suggested to node i by each of our algorithms of interest. a) Personalized PageRank recommends the most visited nodes by performing random walks that restart from node i: $i \to m$. b) Who-to-follow builds the so called "circle of trust" (c.o.t.: nodes k, l, and m) for i and recommends nodes that are followed by the nodes in the c.o.t.: $i \to j$. c) Two-hops recommends nodes at a distance 2: $i \to j$. d) Common-followed suggests nodes with a similar set of out-links: $i \to n$. e) Node2Vec projects nodes into an euclidean space and recommends those with similar embeddings: $i \to l$. The values next to each node are the scores returned by each algorithm. The larger the value, the more important the node for i.

recommendations favor popular and well-connected nodes, and at the same time limit the growth of the diameter of the network [2].

In addition to these topological effects, social recommendations may also exacerbate the under-representation of certain demographic groups in the network. For instance, [14, 15, 33] show how the visibility of minorities can be amplified or mitigated by different levels of homophily within groups when using recommendation algorithms on scale-free networks. These inequalities have been also studied over time but only recently. [16] suggests that while the homophily level of the minority affects the speed of the growth of their disparate exposure, the relative size of the minority affects the magnitude of this effect.

One of the main differences between this body of research and our work is that we vary homophily systematically. This allows us to better understand the relationship between the initial homophily of the network and the long-term effects of the recommendations. In particular, to what extent they change network structure and the visibility of minorities over time. Moreover, we study Node2Vec [20], a more recent algorithm used to generate link recommendations through node embeddings.

3 METHODS

3.1 Directed networks

We consider attributed directed networks of the following form: let G = (V, E, C) be a node-attributed graph where $V = \{v_1, ..., v_n\}$ is a set of n nodes, $E \subseteq V \times V$ is a set of e unweighted directed edges, and $C: V \longrightarrow \{0, 1\}$ is a function that maps each node v_i into its group (or class) membership c_i . For the sake of simplicity we focus on binary group membership (e.g., black/white or male/non-male). The function C, hence, divides the nodes into two groups, a minority, called m, and a majority, called M. We refer to the fraction of the minority group in the network as f_m .

Further definitions, peculiar to the synthetic network generation model employed, are provided in Section 3.3.

3.2 Recommendation algorithms

In this section, we define the five recommendation algorithms of interest. All algorithms are class agnostic which means that their recommendations are solely based on topology. Note as well that for each node $v_i \in V$, the recommendation algorithm suggests a ranked

list of k nodes that v_i is not yet connected with. The ranked list is sorted in descending order in terms of relevance scores according to each algorithm. In the case of ties, where multiple nodes are equally relevant, nodes are chosen randomly. We refer the reader to Section 3.3 for the details on the configuration of hyper-parameters for each algorithm.

Personalized PageRank (PPR): It is an extension of PageRank to rank nodes in a network from the perspective of a seed node [31]. In principle, random walks are performed and restarted at the origin (or seed node) multiple times to update the importance score of all nodes, see Figure 2(a). We compute the PPR vector π_i with respect to each node $v_i \in V$ as follows:

$$\pi_i^T = (1 - \alpha)e_i^T + \alpha \pi_i^T W \tag{1}$$

where α is the probability of following links, e_i denotes the personalized one-hot vector¹, W is the transition matrix inferred from G and T represents the transpose operator. The ranking score given to node v_i is then the j^{th} component of π_i .

Who-to-follow (WTF): This algorithm, proposed by Twitter [21], suggests users who are followed by people that are similar to the one getting the recommendation, see Figure 2(b). For each user v_i , the algorithm looks for its *circle of trust*, which is the result of an egocentric random walk (similar to personalized PageRank [22]). Then, based on this circle-of-trust COT_i , WTF ranks (using the SALSA algorithm [25]) users that are not yet friends with v_i but are connected through the circle of trust $T_{COT_i}^{out}$.

$$WTF_i = SALSA(COT_i, \pi_{COT_i}^{out})$$
 (2)

Two-hops (2H): This algorithm follows the intuition behind *friends-of-friends*. In directed networks, the 2H algorithm recommends nodes v_j that are at a distance 2 from node v_i , see Figure 2(c). The more such paths, the more likely the recommendation. Calling Γ_a^{out} the set of nodes that v_a points towards (i.e., out-links), and Γ_a^{in} the set of nodes pointing to v_a (i.e., in-links), we define the 2H score function as the number of possible paths of length 2 from v_i to v_j :

$$2H(v_i, v_j) := |\Gamma_i^{out} \cap \Gamma_i^{in}| \tag{3}$$

Common-followed (CF): We extend the common neighbors approach [26], which is based on the idea that two nodes v_i and v_j are

 $[\]overline{{}^{1}(e_i)_i} = 1 \text{ and } (e_i)_j = 0, \forall j \neq i$

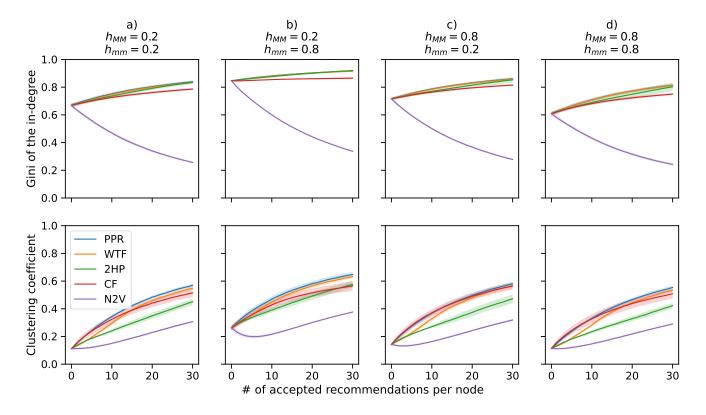


Figure 3: The evolution of network structure for different recommendation algorithms and different values of homophily in the initial network. One can see that, regardless of the type of network (columns), all algorithms except N2V have similar effects on the Gini coefficient of the in-degree distribution (top row) and the clustering coefficient (bottom row). Surprisingly, N2V reduces the Gini coefficient of the in-degree distribution over time (x-axis) and increases the global clustering coefficient at lower rates compared to the other algorithms.

more likely to connect to each other if they have multiple friends in common. In the context of directed networks, the *common-followed* algorithm will recommend node v_i to node v_i if they follow partially or fully the same set of nodes, see Figure 2(d). Then, the algorithm ranks all nodes v_j based on the number of common-followed nodes with v_i . Let Γ_a^{out} be the set of nodes that v_a follows. We define the set of common-followed nodes between v_i and v_j as:

$$CF(v_i, v_j) := |\Gamma_i^{out} \cap \Gamma_j^{out}| \tag{4}$$

Node2Vec (N2V): A popular embedding algorithm that maps nodes to a low-dimensional space of features, by maximizing the likelihood of preserving nodes' neighborhoods [20]. It has been used for link prediction by evaluating the cosine similarity between nodes in the embedding space, see Figure 2(e). Here we use N2V to recommend to each node v_i the most similar node in the embedding space, according to cosine similarity of the embedded vectors. Calling respectively v_i^p and v_j^p the embedded vector projections for v_i and v_j , the cosine similarity between these projections is defined as:

$$\operatorname{CosineSim}(v_i^p, v_j^p) := \frac{v_i^p \cdot v_j^p}{\|v_i^p\| \|v_j^p\|} \tag{5}$$

3.3 Experiments setup

Here we describe the networks employed in our experiments and explain how the recommendation algorithms are iteratively used to recommend new connections among nodes.

Synthetic networks: In order to systematically create networks as defined in Section 3.1, we employed the DPAH model [14]. This model allows to generate scale-free bi-populated directed networks with adjustable homophily (for each group), minority size, node activity, and edge density. DPAH is a growth model that generates networks as follows. First, n nodes are created and randomly assigned to one of two groups based on the fraction of minorities f_m . Then, the following steps are repeated until the desired edge density d is fulfilled. A source node v_i is drawn from a power-law distribution, modeled through the activity parameters γ_M and γ_m for the majority and the minority group, respectively. A target node v_j is drawn with a probability that is proportional to the product of its in-degree and the pair-wise homophily between the source and the target node. Lastly, a directed edge from v_i to v_j is created. Thus, the probability of creating a link from v_i to v_j is defined as:

$$\mathbb{P}(v_i \to v_j) = \frac{h_{ij} k_j^{in}}{\sum_{l=1}^n h_{il} k_l^{in}} \tag{6}$$

where k_j^{in} is the in-degree of v_j , and h_{ij} is the homophily between v_i and v_j and it is determined by their class membership.

In this work, we systematically modify the homophily within groups and the size of the minority, leaving the variation of node activity and edge density for a further study. In particular, in order to measure the influence of algorithms (RQ1) and homophily (RQ2) in the recommendations, we generate 4 networks for each combination of homophily parameters h_{mm} , $h_{MM} \in \{0.0, 0.1, \ldots, 1.0\}$ (h_{mM} and h_{Mm} are defined as $1-h_{mm}$ and $1-h_{MM}$, respectively) and fix the number of nodes n=1000, the size of the minority $f_m=0.3$, the node activity $\gamma_M=\gamma_m=2.5$ and the edge density d=0.03. We further adjust the size of the minority $f_m\in\{0.1,0.2,0.3,0.4\}$ to measure its influence in the visibility of minorities (RQ3).

Recommendation: Given an initial network G, we apply a recommendation algorithm R to suggest to each node v_i a node v_i to connect with. Then, we create a direct link $v_i \rightarrow v_j$ for each top-1 of these recommendations. By doing so, in what we call "one step", we create a new out-link for each node v_i . This decision is motivated by the fact that the employed acceptance policy plays only a marginal role in shaping the network [9, 16]. Then, for every addition, we remove a random out-link. This is a procedure previously employed in the literature, for example in [9]. One of the main reasons for this choice is to prevent a significant increase in the edge density of the network. The evaluation metrics considered in Section 3.4 are sensible to edge density. By removing a link every time a new one is created we ensure to keep the density constant on every step and make sure that the changes are due to the recommendations and not to an increase in the total amount of connections. The link removal procedure is also grounded on the social theory for which people exhibit a finite communication capacity and, thus, they have a limit on the number of ties that they can maintain active in time [12, 29].

We repeat the above procedure 30 times to simulate an equal amount of recommendations per node.

Hyper-parameters: For PPR, we set the probability of following links to $\alpha=0.85$, as suggested by Brin and Page [6] and widely used in many applications. In N2V, we use the default values for the dimensions of the embedding space dimensions=64, the number of visited nodes in each random walk $walk_length=10$, and the number of random walks to be generated from each node in the graph $num_walks=200$. For WTF, we constrain the circle of trust to include only the top-10 nodes.

Additional assumption: We assume that the recommendations of different algorithms are similarly relevant, as our goal is not to evaluate which algorithm performs better in terms of utility metrics, but rather to study their effects on the structure and their impact on the visibility of the minorities (see Section 3.4).

3.4 Evaluation metrics

We use the global *clustering coefficient* [17] of the network and the *Gini coefficient* [19] of the in-degree distribution as proxies of network structure, and the fraction of minorities among the most important nodes as *visibility*. We measure these metrics before and after each round of recommendations to verify whether certain types of networks change these metrics faster or slower and by how much.

Clustering coefficient: This metric allows to verify whether the recommendations are making the network more cohesive by closing more triangles. The clustering coefficient of node v_i is defined as:

$$c_{v_i} = \frac{2}{\deg^{tot}(v_i)\left(\deg^{tot}(v_i) - 1\right) - 2\deg^{\leftrightarrow}(v_i)}T(v_i) \tag{7}$$

where $T(v_i)$ is the number of directed triangles through node v_i , $\deg^{tot}(v_i)$ is the sum of in-degree and out-degree of v_i , and $\deg^{\leftrightarrow}(v_i)$ is the reciprocal degree of v_i . The global clustering coefficient of the network is then obtained by taking the mean across all nodes: $c = 1/n \sum_{i=1}^{n} c_{v_i}$.

Gini coefficient of the in-degree distribution: Popularity bias is a well-known issue reinforced by certain recommendation algorithms [1]. The Gini coefficient [19] allows us to demonstrate whether this bias is exacerbated by the algorithms regardless of the initial conditions of the network structure, or whether certain types of networks are exempt from this bias. The Gini coefficient of the in-degree distribution π^{in} , sorted in ascending order, is defined as follows:

$$Gini(\pi^{in}) = \frac{\sum_{i=1}^{n} (2i - n - 1)\pi_i^{in}}{n \sum_{i=1}^{n} \pi_i^{in}}$$
(8)

The higher the Gini coefficient, the more skewed or unequal the in-degree distribution across all nodes.

Visibility of the minority group: First, we measure the importance of nodes by computing their PageRank [31]. Then, out of the top-10% highest-scored nodes, we measure the fraction of nodes that belong to the minority group and refer to this fraction as the visibility of the minority group \hat{f}_m . We use the relative visibility $\hat{f}_m^* = \hat{f}_m - f_m$ to verify how far the visibility of the minority is from statistical parity [13] before the recommendations. Finally, we measure the change in visibility by computing \hat{f}_m after and before the recommendations to verify whether the minority group is gaining or losing visibility:

$$\delta_{f_m} = \hat{f_m}(after) - \hat{f_m}(before) \tag{9}$$

In-group links: We also look at the fraction of links within groups to see what type of edges are being recommended more often by the algorithms. The in-group link ratio for group *a* is defined as:

$$I_a = \frac{e_{aa}}{e_{aa} + e_{ab}} \tag{10}$$

where $a, b \in \{m, M\}$ and $a \neq b$.

4 RESULTS

Here we address our three research questions and present the results obtained after applying the recommendation algorithms iteratively to the simulated directed networks described in Sections 3.2 and 3.3, respectively. First, we show the consequences of these recommendations on the structure of the network and on the visibility of the minority group (RQ1). Second, we explain the changes in structure and visibility as a function of homophily (RQ2). Third, we further investigate the role of the size of the minority group and in-group links in the effects of the recommendations (RQ3).

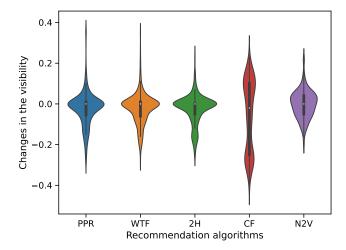


Figure 4: Changes in the visibility of minorities for different recommendation algorithms. After 30 recommendations for each node (in networks with different values of homophily and fixed minority size $f_m=0.3$), we see that all algorithms may increase (positive change), decrease (negative change) or keep constant (zero change) the visibility of the minority group. In particular, PPR, WTF and 2H mostly keep the visibility of minorities unchanged. However, their tails are asymmetric and denser in the negative direction, in correspondence with a decrease in visibility. CF maintains the initial visibility of minorities in a few cases, and otherwise it may drastically change the visibility in both directions. N2V generates less extreme changes in both directions.

4.1 RQ1: How do recommendation algorithms affect the structure of the network and the visibility of minorities?

Changes in network structure: To address this question, we first assess the changes in network structure in terms of global clustering coefficient and Gini coefficient of the in-degree distribution. The idea is to verify whether the algorithms (while connecting people together) make the network more cohesive and whether popularity bias increases at the same rate for all algorithms. Figure 3 shows the results for both metrics (top/bottom) on different types of networks (columns) across multiple rounds of recommendations (x-axis). Note that the x-axis reflects the iteration or step of recommendation, e.g., at step=20, each algorithm has independently recommended 20 connections to each node in the network. First, we see that overall, the evolution of these metrics is consistent across types of networks (columns) and recommendation algorithms (colors). Second, all recommendation algorithms increase the clustering coefficient of the network which means that the networks are becoming more cohesive as more triangles are getting closed. However, the rate at which this clustering increases, differs across algorithms, especially for N2V which, surprisingly, is the slowest. Third, we corroborate that PPR, WTF, 2H and CF reinforce the popularity bias issue since the Gini increases over time. This means that these algorithms make popular people (in terms of high in-degree) more popular.

The exception is N2V, which after several recommendations makes the in-degree distribution less skewed (i.e., the recommendations are more diverse). One possible explanation is that similarity in the embedding space is only partially sensible to popularity bias.

Changes in the visibility: Now, we explore to what extent each recommendation algorithm changes the visibility of the minority group after several recommendations. We show the results in Figure 4. Each violin refers to one algorithm and the distribution of the violin represents the variation across a multiplicity of networks with different initial homophily values, fixed number of nodes and minority size (see Section 3.3 for details). PPR, WTF and 2H show similar patterns: they have median close to zero but denser tails in the negative direction. This indicates that these algorithms mostly keep the visibility of the minority unchanged, but, in certain cases, they decrease this visibility. CF shows the opposite behavior. First, it keeps the visibility unchanged for a few cases, but most of the time it drastically changes this visibility in either direction. Among all, N2V reveals more symmetric and smaller effects. Summarizing, Figure 4 suggests that four out of five algorithms are more prone to keep the visibility of the minority unchanged. Nevertheless, in certain regimes (explored next in RQ2) this visibility can be increased or reduced depending on the levels of homophily.

4.2 RQ2: To what extent is the change in visibility due to homophily?

To understand how the initial levels of homophily in the network affect the recommendations, we compare the visibility of the minority before and after the recommendations for each algorithm, see Figure 5. We control for the number of nodes and the fraction of minorities by keeping them fixed, and vary homophily values (see Section 3.3 for more details). As defined in Section 3.4, visibility measures the fraction of nodes that belong to a particular group and make it to the top-10% of the rank. This rank reflects the importance of nodes in the network and it is assessed through their PageRank [31].

Visibility before the recommendations: Figure 5(a) shows the relative visibility of the minority before the recommendations. White regions (neutral visibility) represent statistical parity [13], in which the fraction of the minority in the top-10% is equal to the fraction of minority populating the whole network. Orange regions (positive visibility) represent higher amount of minority nodes at the top of the rank compared to the statistical parity condition. Blue regions (negative visibility), instead, represent underrepresentation of minorities in top ranks. We see that the minority is over-represented mostly when the majority is heterophilic $h_{MM} < 0.5$ or when the minorities are more homophilic than the majorities $h_{mm} > h_{MM}$.

Changes in the visibility after the recommendations: Figures 5(b) to 5(f) show the change in visibility after 30 recommendations per node. A positive change (orange) indicates that the visibility of the minority increased after the recommendations (relative to the initial visibility they had before the recommendations). Actual values in each cell denote the magnitude of this change. Conversely, a negative change (blue) indicates that the majority increased its visibility at the cost of reducing the visibility of the minority. No changes (white) indicate that the visibility did not vary across time.

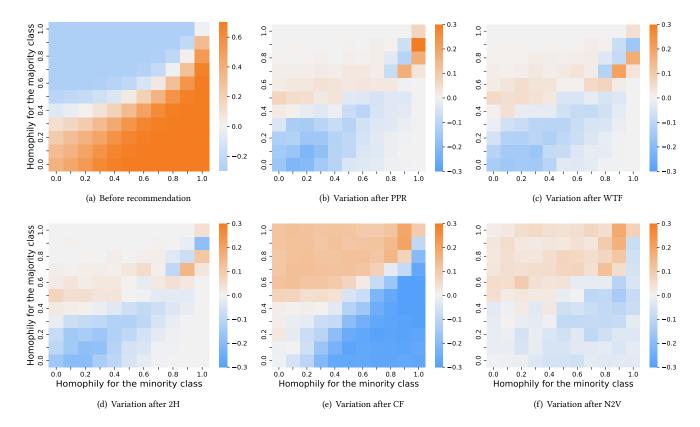


Figure 5: Visibility of the minority group as a function of homophily. Heatmaps show the visibility of the minority group before and after the recommendations for different algorithms and different combinations of homophily within the majority (y-axis) and the minority (x-axis) groups. The visibility of the minority group is measured by the fraction of minorities in the top-10% of nodes ranked by their PageRank. In (a), colors show the relative visibility of the minority group w.r.t., the fraction of minorities in the network $f_m = 0.3$ before the recommendations. Positive visibility means that the minority is overrepresented (orange), and negative visibility means that the minority is under-represented or the majority is over-represented (blue). Zero visibility refers to those cases where the top rank does not include any node from the minority group. In (b-f), colors represent the variation in the visibility due to different recommendation algorithms. For PPR, WTF and 2H one can see that the minority loses more visibility than the majority (especially in the heterophilic regime), while CF and N2V show more symmetric effects on the visibility of the minority and majority. Notice that the homophily values shown in the x- and y-axis of all plots represent the initial levels of homophily in the network before the recommendations.

At first glance, we see that the visibility gets affected differently depending on the algorithm and the initial values of homophily. We further notice that there are slightly more blue than orange regions in almost all plots (i.e., the majority increases its visibility more often than the minority across all regimes).

Among all the algorithms CF produces the strongest changes, while N2V is the most balanced. PPR, WTF and 2H, on the other hand, show a similar behavior. They penalize minorities especially in the heterophilic regimes for both classes, i.e., $h_{**} < 0.5$, bottom-left corners of Figures 5(b) to 5(d).

Furthermore, when only one group is homophilic, PPR, WTF and 2H do not change the initial over-representation of the homophilic group, see top-left and bottom-right corners in Figures 5(b) to 5(d).

4.3 RQ3: Is the change in visibility inversely proportional to the size of the minority or proportional to the in-group links within the minority?

Size of the minority: To answer RQ1 and RQ2, we kept the size of the minority fixed ($f_m=0.3$) to study the effects of homophily on the visibility of minorities after the recommendations. However, it is unclear whether the changes in visibility are inversely proportional to the size of the minority (e.g., larger changes for smaller minorities), or whether these are steady-state changes that appear regardless of the size of the minority. In Figure 6, we show how the change in visibility (y-axis) is affected by multiple factors including the size of the minority. First, we see a concordance among algorithms when the majority is heterophilic, Figures 6(a) and 6(b).

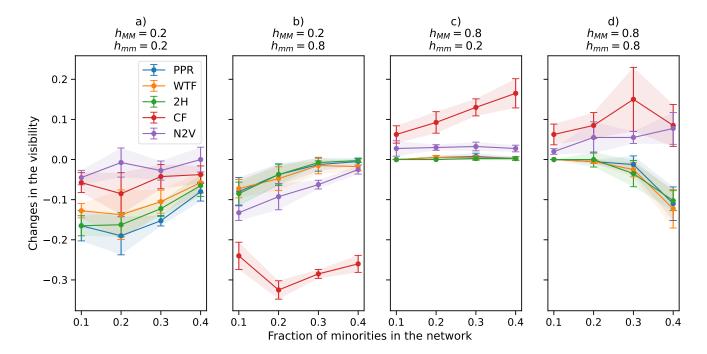


Figure 6: Changes in the visibility of minorities as a function of the minority size. The y-axis shows the change in visibility for the minority group after the recommendations. A positive (negative) change indicates that minorities appeared more (less) often in the top-10% compared to their initial representation before the recommendations. If this change is around zero, the visibility of minorities remained constant or invariant. The x-axis shows the size of the minority group as a fraction of all nodes in the network. In general, we see that larger minorities get penalized less than smaller ones when the majority is heterophilic (a,b). When the majority is homophilic, however, the changes in visibility not only depend on the fraction of the minority but also on its homophily level. For instance, when the minority is heterophilic (c), its visibility remains mostly constant for all algorithms except CF, and when the minority is homophilic (d), its visibility drops for larger-size minorities, unless N2V and CF are used.

In these cases, the larger the minority, the smaller the change in the visibility of the minority, except for CF which drastically reduces this visibility when the minority is more homophilic than the majority, Figure 6(b). When only the majority is homophilic, Figure 6(c), i.e., most out-links point to nodes in the majority group, the size of minorities has almost no effect on their final visibility in algorithmic rankings unless CF is used as recommendation algorithm. When both groups are homophilic, Figure 6(d), however, only CF and N2V increase the visibility of larger minorities more than the visibility of smaller minorities.

In-group links: As we have seen previously, the visibility of the minority can be affected by different factors, including the initial homophily of the network. Since homophily depends on the mixing of types of edges (see [14] for a detailed derivation of homophily in DPAH networks), we further investigate the evolution of in-group links over time, see Figure 7. Here, we found two main patterns. First, results from PPR, WTF and 2H are consistent in each type of network (columns). These algorithms mostly increase the number of in-group minority links, see Figures 7(a), 7(b) and 7(e). Surprisingly, this advantage does not guarantee an increase in visibility for the minority group. On the contrary, they lose visibility, see Figures 6(a) and 6(b) for $f_m = 0.3$. Second, results from CF and N2V are also

consistent in each type of network. We see in Figures 7(a) and 7(b) that these two algorithms increase the in-group majority links when the majority is initially heterophilic, and reduce them when the majority is initially homophilic, see Figures 7(c), 7(d) and 7(e).

Now, we analyze in details different possible homophily configurations.

When one class is homophilic and the other class is heterophilic, the links coming from both classes are mostly directed to nodes in the homophilic class. Let us consider PPR, WTF and 2H where the values of homophily are $h_{MM} = 0.2$ for the majority and $h_{mm} = 0.8$ for the minority and vice-versa, see Figures 7(b) and 7(c), respectively. In these situations, these recommendation algorithms will keep increasing the in-group proportions of the homophilic group since the recommended links mostly point to nodes in this group. These correspond to situations in the white regions at the top-left and bottom-right of Figures 5(b) to 5(d). Hence, this shows that the absence of variation in the fraction of minority is due to the fact that PPR, WTF and 2H do not modify connections between classes in these cases. This does not hold for CF and N2V. In fact, under the same homophily conditions, these methods make the in-group links for both classes more similar, decreasing structural differences between classes, see Figures 7(b) and 7(c).

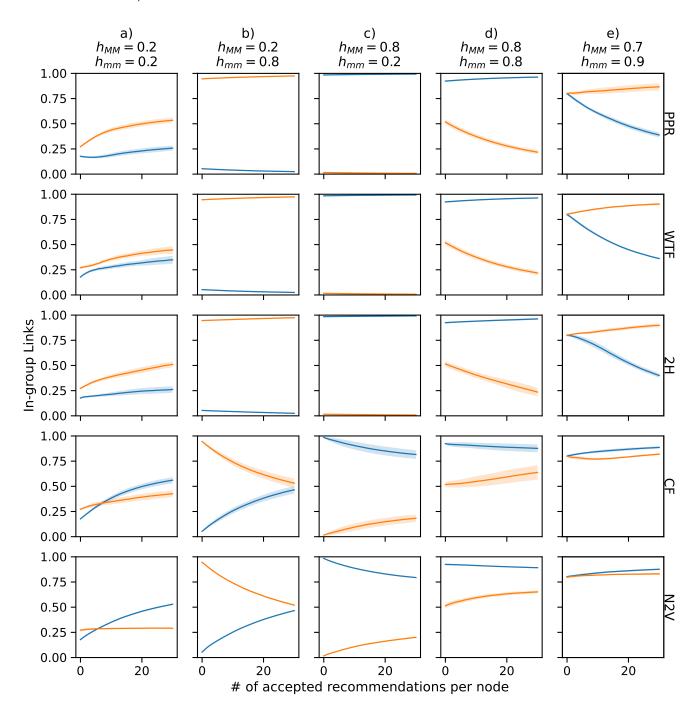


Figure 7: The evolution of the in-group links within majority nodes (blue) and within minority nodes (orange) for different recommendation algorithms (rows) and different values of homophily (columns) in the initial network. These networks posses a fixed number of nodes and fraction of minorities $f_m = 0.3$.

Now, we will consider regimes where both classes are heterophilic, $h_{mm} = h_{MM} = 0.2$, see Figure 7(a). Here, CF and N2V are the only algorithms in which the initial conditions of in-group links are flipped. Note that the proportion of links within the majority group

gets larger than the proportion of links within the minority after multiple rounds of recommendations. Consequently, the majority increases its visibility even further by pushing minorities to lower

ranks, see Figures 5(e) and 5(f). Interestingly, the visibility of minorities decreases even if the flip does not occur in these heterophilic settings for PPR, WTF and 2H, see bottom-left of Figures 5(b) to 5(d).

On the other extreme of homophily, when both groups are homophilic, $h_{mm} = h_{MM} = 0.8$, we found two main patterns, see Figure 7(d). First, PPR, WTF and 2H tend to strengthen the connections towards the majority group by either recommending majority-to-majority or minority-to-majority links. This in turn penalizes the minorities at the top of the rank, see $h_{mm} = h_{MM} = 0.8$ in Figures 5(b) to 5(d). In contrast, CF and N2V slowly increase the number of connections within the minority group. For N2V, one possible explanation is that the homophily levels are high enough so that the two classes (especially the minority class), are represented in the embeddings as, at least partially, separated clusters.

Lastly, the possibility to systematically vary the initial levels of homophily for both classes allows us to identify tipping points. For instance, in a homophilic regime, where both groups have the same level of initial homophily, $h_{MM} = h_{mm} = 0.8$, we found that PPR, WTF and 2H increase the number of links within the majority group after multiple recommendations, see Figure 7(d). However, the same algorithms may also increase the number of links within the minorities, and thus their visibility, if the minority group is initially more homophilic than the majority, $h_{MM} = 0.7$ and $h_{mm} = 0.9$, see Figure 7(e). CF and N2V, on the other hand, do not show this tipping effect when both groups of nodes are initially homophilic. In either case, these two algorithms keep increasing the proportion of in-group links which induces segregation.

5 LIMITATIONS AND FUTURE WORK

We have limited our study to five recommendation algorithms, and in future work we aim to include more algorithms into this investigation, especially recent versions of popular algorithms that have been developed with the goal to increase fairness.

Furthermore, we focused on scale-free directed networks with homophily which represent a plausible configuration of online social networks. As next steps, we would like to include in our analysis different network simulation models that include other factors in the network generation process, such as multiple node-attributes, heterogeneous group mixing, the presence of communities, and triadic closure. We also acknowledge the fact that our analysis is theoretical and has not been validated with real data. We plan to extend our study by considering empirical networks.

Importantly, link recommendation algorithms and datasets are generally proprietary. This is why simulation-based approaches are often necessary for this kind of investigations. In addition, the simulation approach enables us to examine different scenarios which might not occur in one instance of the data [32].

6 CONCLUSIONS

In this work, we systematically studied five link recommendation algorithms and quantified their feedback loop effects on bi-populated scale-free directed networks with homophily. In particular, we assessed two types of changes in these networks due to multiple link recommendations. First, we measured the changes in network structure in terms of clustering and in-degree distribution. Second, we measured the changes in the visibility of minorities at the top-10%

of the rank with respect to their PageRank (importance) scores, highlighting the effects of homophily, minority size, and in-group links.

Our results show that four out of the five algorithms reduced on average the visibility of minorities more often than to the majority counterpart. In particular, PPR, WTF and 2H when both groups are initially heterophilic, and CF when the minority is initially more homophilic than the majority.

We also found that while all algorithms tend to close triangles and increase the clustering coefficient, all algorithms except N2V are prone to favor and suggest nodes with high in-degree. This is known as popularity bias, rich-get-richer effect or cumulative advantage, a well-known mechanism that contributes to inequality.

Link recommendations based on N2V rely on the proximity of nodes in the embedding space, which does not necessarily imply closeness to nodes with high in-degree. Consequently, N2V is a promising alternative to other link recommendation algorithms since it mitigates cumulative advantage.

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REFERENCES

- Himan Abdollahpouri. 2019. Popularity bias in ranking and recommendation. In Proceedings of the 2019 AAAI/ACM Conference on AI, Ethics, and Society. 529–530.
- [2] Luca Maria Aiello and Nicola Barbieri. 2017. Evolution of ego-networks in social media with link recommendations. In Proceedings of the Tenth ACM International Conference on Web Search and Data Mining. 111–120.
- [3] Chen Avin, Barbara Keller, Zvi Lotker, Claire Mathieu, David Peleg, and Yvonne-Anne Pignolet. 2015. Homophily and the glass ceiling effect in social networks. In Proceedings of the 2015 conference on innovations in theoretical computer science. 41–50.
- [4] Albert-László Barabási and Réka Albert. 1999. Emergence of scaling in random networks. science 286, 5439 (1999), 509–512.
- [5] Fabian Baumann, Philipp Lorenz-Spreen, Igor M Sokolov, and Michele Starnini. 2020. Modeling echo chambers and polarization dynamics in social networks. Physical Review Letters 124, 4 (2020), 048301.
- [6] Sergey Brin and Lawrence Page. 1998. The anatomy of a large-scale hypertextual web search engine. Computer networks and ISDN systems 30, 1-7 (1998), 107–117.
- [7] Uthsav Chitra and Christopher Musco. 2020. Analyzing the impact of filter bubbles on social network polarization. In Proceedings of the 13th International Conference on Web Search and Data Mining. 115–123.
- [8] Nicholas A. Christakis and James H. Fowler. 2013. Social contagion theory: examining dynamic social networks and human behavior. Statistics in Medicine 32, 4 (2013), 556–577. https://doi.org/10.1002/sim.5408 arXiv:https://onlinelibrary.wiley.com/doi/pdf/10.1002/sim.5408
- [9] Federico Cinus, Marco Minici, Corrado Monti, and Francesco Bonchi. 2021. The Effect of People Recommenders on Echo Chambers and Polarization. arXiv preprint arXiv:2112.00626 (2021).
- [10] Elizabeth M Daly, Werner Geyer, and David R Millen. 2010. The network effects of recommending social connections. In Proceedings of the fourth ACM conference on Recommender systems. 301–304.
- [11] Pranav Dandekar, Ashish Goel, and David T Lee. 2013. Biased assimilation, homophily, and the dynamics of polarization. Proceedings of the National Academy of Sciences 110, 15 (2013), 5791–5796.
- [12] Robin IM Dunbar. 1992. Neocortex size as a constraint on group size in primates. Journal of human evolution 22, 6 (1992), 469–493.

- [13] Cynthia Dwork, Moritz Hardt, Toniann Pitassi, Omer Reingold, and Richard Zemel. 2012. Fairness through awareness. In Proceedings of the 3rd innovations in theoretical computer science conference. 214–226.
- [14] Lisette Espín-Noboa, Claudia Wagner, Markus Strohmaier, and Fariba Karimi. 2022. Inequality and inequity in network-based ranking and recommendation algorithms. Scientific Reports 12, 1 (2022).
- [15] Francesco Fabbri, Francesco Bonchi, Ludovico Boratto, and Carlos Castillo. 2020. The effect of homophily on disparate visibility of minorities in people recommender systems. In Proceedings of the International AAAI Conference on Web and Social Media. Vol. 14. 165–175.
- [16] Francesco Fabbri, Maria Luisa Croci, Francesco Bonchi, and Carlos Castillo. 2021. Exposure Inequality in People Recommender Systems: The Long-Term Effects. arXiv preprint arXiv:2112.08237 (2021).
- [17] Giorgio Fagiolo. 2007. Clustering in complex directed networks. Physical Review E 76, 2 (2007), 026107.
- [18] James H. Fowler and Nicholas A. Christakis. 2010. Cooperative behavior cascades in human social networks. Proceedings of the National Academy of Sciences 107, 12 (2010), 5334–5338. https://doi.org/10.1073/pnas.0913149107 arXiv:https://www.pnas.org/content/107/12/5334.full.pdf
- [19] Corrado Gini. 1912. Variabilità e mutabilità. Reprinted in Memorie di metodologica statistica (Ed. Pizetti E (1912).
- [20] Aditya Grover and Jure Leskovec. 2016. node2vec: Scalable Feature Learning for Networks. CoRR abs/1607.00653 (2016). arXiv:1607.00653 http://arxiv.org/abs/ 1607.00653
- [21] Pankaj Gupta, Ashish Goel, Jimmy Lin, Aneesh Sharma, Dong Wang, and Reza Zadeh. 2013. Wtf: The who to follow service at twitter. In Proceedings of the 22nd international conference on World Wide Web. 505–514.
- [22] Glen Jeh and Jennifer Widom. 2003. Scaling personalized web search. In Proceedings of the 12th international conference on World Wide Web. 271–279.
- [23] Fariba Karimi, Mathieu Génois, Claudia Wagner, Philipp Singer, and Markus Strohmaier. 2018. Homophily influences ranking of minorities in social networks. Scientific reports 8, 1 (2018), 1–12.
- [24] Eun Lee, Fariba Karimi, Claudia Wagner, Hang-Hyun Jo, Markus Strohmaier, and Mirta Galesic. 2019. Homophily and minority-group size explain perception biases in social networks. *Nature human behaviour* 3, 10 (2019), 1078–1087.

- [25] Ronny Lempel and Shlomo Moran. 2001. SALSA: the stochastic approach for link-structure analysis. ACM Transactions on Information Systems (TOIS) 19, 2 (2001), 131–160.
- [26] David Liben-Nowell and Jon Kleinberg. 2007. The link-prediction problem for social networks. Journal of the American society for information science and technology 58, 7 (2007), 1019–1031.
- [27] Miller McPherson, Lynn Smith-Lovin, and James M Cook. 2001. Birds of a feather: Homophily in social networks. Annual review of sociology 27, 1 (2001), 415–444.
- [28] Robert K Merton. 1988. The Matthew effect in science, II: Cumulative advantage and the symbolism of intellectual property. isis 79, 4 (1988), 606–623.
- [29] Giovanna Miritello, Rubén Lara, Manuel Cebrian, and Esteban Moro. 2013. Limited communication capacity unveils strategies for human interaction. Scientific reports 3, 1 (2013), 1–7.
- [30] Shirin Nilizadeh, Anne Groggel, Peter Lista, Srijita Das, Yong-Yeol Ahn, Apu Kapadia, and Fabio Rojas. 2016. Twitter's glass ceiling: The effect of perceived gender on online visibility. In Proceedings of the International AAAI Conference on Web and Social Media, Vol. 10.
- [31] Lawrence Page, Sergey Brin, Rajeev Motwani, and Terry Winograd. 1999. The PageRank citation ranking: Bringing order to the web. Technical Report. Stanford Infol ab.
- [32] Mitja Steinbacher, Matthias Raddant, Fariba Karimi, Eva Camacho Cuena, Simone Alfarano, Giulia Iori, and Thomas Lux. 2021. Advances in the agent-based modeling of economic and social behavior. SN Business & Economics 1, 7 (2021), 1–24.
- [33] Ana-Andreea Stoica, Christopher Riederer, and Augustin Chaintreau. 2018. Algorithmic Glass Ceiling in Social Networks: The effects of social recommendations on network diversity. In Proceedings of the 2018 World Wide Web Conference. 923–932
- [34] Jessica Su, Aneesh Sharma, and Sharad Goel. 2016. The effect of recommendations on network structure. In Proceedings of the 25th international conference on World Wide Web. 1157–1167.
- [35] Sotiris Tsioutsiouliklis, Evaggelia Pitoura, Panayiotis Tsaparas, Ilias Kleftakis, and Nikos Mamoulis. 2021. Fairness-Aware PageRank. In Proceedings of the Web Conference 2021. 3815–3826.