

1. $T(n) = O(n^2)$

\hookrightarrow It takes 5 sec for $n=10$

Let's say $Kn^2 = 5$
 $K \cdot 100 = 5$
 $K = 5/100$

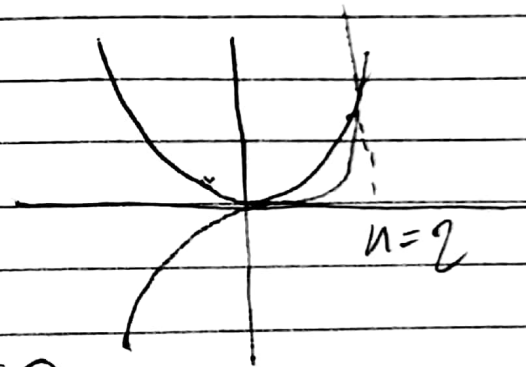
For $n=50$

Time = $K(50)^2 = 125$ seconds

2. $T(A_n) = n^3$

$T(B_n) = 2n^2$

So $n^3 = 2n^2$
 $n^2(n-2) = 0$
 $n=2$



($n=0$ not possible)

After $n=2$, they will deviate.

3. Using limit rule to check that $n2^n$ is in $O(4^n)$

$$\lim_{n \rightarrow \infty} \frac{4^n}{n2^n} = \lim_{n \rightarrow \infty} \frac{2^n}{n} \Rightarrow \lim_{n \rightarrow \infty} \frac{n2^{n-1}}{n} = 0$$

\therefore It means for large values of n , 4^n is much greater than $n2^n$.

4. The slope of the graph of log function is very less compared to functions of ~~other~~.

∴ We can say log has slowest growth rate compared to other polynomial functions.

5. Θ and O and Ω

$\Theta \Rightarrow$ Average Case time complexity i.e., it is the average of the time complexity that a function/algorithm performs.

$O \Rightarrow$ It is the worst time or worst performance ~~and~~ function/algorithm takes.

$\Omega \Rightarrow$ It is best case for an function or algorithm.

6(a) $n^4 + \log n + 17$

$$\lim_{n \rightarrow \infty} n^4 + \log n + 17 = \lim_{n \rightarrow \infty} n^4 \left(1 + \frac{\log n}{n^4} + \frac{17}{n^4} \right)$$

$$= \lim_{n \rightarrow \infty} n^4$$

We can say worst complexity for the given function is same as for n^4 .

∴ $O(n^4)$.

7 (a)

$K = 1$
while $K \leq n$
 $K = K + 1$
End while

$\Rightarrow O(n)$

(b)

for $i = 1$ to $n-1$
 for $j = i+1$ to n
 Swap

No. of times loop executes $\Rightarrow \frac{n(n-1)}{2}$

complexity $= O(n^2)$

8. Algorithm time complexity $= O(n^2)$

For Kn inputs $= \frac{K^2 n^2}{2n}$
 $= \frac{4n^2}{2n}$

for $K = \sqrt{2}$
If input size increase by 10^4 times then
that the time for running algorithm will
become twice.

g.

$$T_A = 100^n$$

$$T_B = n^4$$

$$\lim_{n \rightarrow \infty} \frac{100^n}{n^4} \Rightarrow \frac{100^n \log 100}{4n^3} = \infty$$

$$\lim_{n \rightarrow \infty} (100^n \log 100)$$

Since $T_A/T_B = \infty$,

$\therefore T_A$ is worst in terms of performance to T_B

11(a) $2^{n-1} + 4^{n+1}$

$$\frac{2^n}{2} + 2^{2n+2}$$

$$2^n \left(\frac{1}{2} + 2^n \cdot 4 \right)$$

Avg case $\Rightarrow 2^n \approx 4^n$

Worst case $\Rightarrow 4^{n+1}$

(b) Worst case for $(n^2 + 6)^8$

By binomial expansion term with high power is n^{16} . This term will affect the function largely for large values of n .

$\therefore O(n^{16})$

12.

$$T_1 = n^2$$

$$T_2 = n + 2$$

$$\text{Break point} \Rightarrow n^2 = n + 2$$

$$= n^2 - n - 2$$

$$= (n+1)(n-2)$$

$$\underline{\underline{n=2}}$$

After $n=2$, n^2 grows faster than $n+2$.

