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1. $T C = O(n^2)$

\hookrightarrow It takes 5 sec for $n=10$

Let's say $K n^2 = 5$

$$K \cdot 100 = 5$$

$$K = 5/100$$

For $n=50$

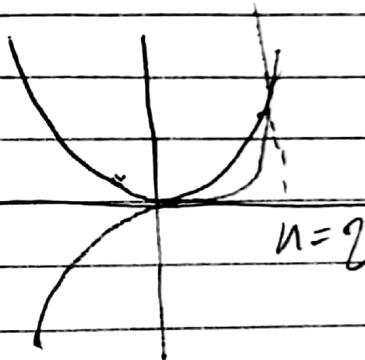
$$\text{Time} = K(50)^2 = 125 \text{ seconds}$$

2. $T(A_n) = n^3$
 $T(B_n) = 2n^2$

$$\text{So } n^3 = 2n^2$$

$$n^2(n-2) = 0$$

$$n=2$$



($n=0$ not possible)

After $n=2$, they will deviate.

3. Using limit rule to check that n^2 is in $O(4^n)$

$$\lim_{n \rightarrow \infty} \frac{4^n}{n^2} = \lim_{n \rightarrow \infty} \frac{2^n}{n} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^{2^n}}{n} = \infty$$

\therefore It means for large values of n , 4^n is much greater than n^2 .

4. The slope of the graph of log function is very less compared to logarithms of other functions.

∴ We can say log has slowest growth rate compared to other polynomial functions.

5. Θ and O and Ω

$\Theta \Rightarrow$ Average Case time complexity i.e., it is the average of the time complexity that a function / algorithm performs.

$O \Rightarrow$ It is the worst time or worst performance cost function / algorithm takes.

$\Omega \Rightarrow$ It is best case for an function or algorithm.

$$6(a) n^4 + \log n + 17$$

$$\lim_{n \rightarrow \infty} n^4 + \log n + 17 = \lim_{n \rightarrow \infty} n^4 \left(1 + \frac{\log n + 17}{n^4} \right)$$

$$= \lim_{n \rightarrow \infty} n^4$$

We can say worst complexity for the given function is same as for n^4 .

$$\therefore O(n^4)$$

3 (a)

$k = 1$
while $k \leq n$

$k = k + 1$

End while

$\Rightarrow O(n)$

(b) for $i = 1$ to $n-1$

for $j = i+1$ to n

Swap

No. of times loop executes $\Rightarrow \frac{n(n-1)}{2}$

complexity $= O(n^2)$

8. Algorithm time complexity $= O(n^2)$

For kn inputs $= \frac{n^2}{2}^2$
 $= 4n^2$

for $k = \sqrt{2}$

If input size increase by 10^{42} times from
that the time for running algorithm will
become twice.

$$g. \quad T_A = 100^n$$

$$T_B = n^4$$

$$\lim_{n \rightarrow \infty} \frac{100^n}{n^4} \rightarrow \text{mean } 100^n \log 100 = \infty$$

$$\lim_{n \rightarrow \infty} (100^n \log 100)$$

Since $T_A / T_B = \infty$,

$\therefore T_A$ is worst in terms of performance
to T_B

$$11(a) \quad 2^{n-1} + 4^{n+1}$$

$$\frac{2^n}{2} + 2^{2n+2}$$

$$2^n \left(\frac{1}{2} + 2^n \cdot 4 \right)$$

$$\text{Avg Case} \Rightarrow 2^n \approx 4^n$$

$$\text{Worst Case} \Rightarrow 4^{n+1}$$

$$(b) \quad \text{Worst case for } (n^2 + 6)^8$$

By binomial expansion term with high power is n^16 . This term will affect the function largely for large values of n .

$$\therefore O(n^k)$$

(2)

$$T_1 = n^2$$

$$T_2 = n+2$$

Break point $\Rightarrow n^2 = n+2$

$$= n^2 - n - 2$$

$$= (n+1)(n-2)$$

$$n=2$$

 \equiv

After $n=2$, n^2 grows faster than $n+2$

