Poisson Distribution Proving - 3

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Abstract

This is a LATEX project as well as a poisson distribution project, helping me practice the provings of poisson distributions I have learnt in my days as a statistics student of University of Ilorin.

1 Problem

Let X Poisson(λ)
where $\lambda > 0$.
Find the variance of X

2 Solution

Before we could solve for the variance of the poisson distribution, we first have to solve for the mean since the var(X) could be denoted by:
var(x) = E(x^2) $- \mu^2$
= E(x^2) $- [E(x)]^2$
We are also going to note the series expansion of $\exp(a)$ as:
$\exp(a) = \sum_{y=0}^{\infty} \frac{a^y}{y!}$

2.1 Solving for Mean

Using the Poisson probability mass function (PMF) formula:

$$f(x) = P(X = x) = \frac{\lambda^x \exp(-\lambda)}{x!}$$

and mean is $E(x)$ given by:

$$E(x) = \sum_{x=0}^{\infty} x f(x)$$
$$= \sum_{x=0}^{\infty} x \frac{\lambda^x \exp(-\lambda)}{x!}$$

We spread out the summation:

$$= \frac{(1\lambda^1 \exp(-\lambda))}{1!} + \frac{(2\lambda^2 \exp(-\lambda))}{2!} + \frac{(3\lambda^3 \exp(-\lambda))}{3!} + \frac{(4\lambda^4 \exp(-\lambda))}{4!} + \cdots + \frac{(\infty\lambda^\infty \exp(-\lambda))}{\infty!}$$

Simplifying the expression:

$$
\begin{aligned}
&= \lambda \exp(-\lambda) + \lambda^2 \exp(-\lambda) + \lambda^3 \exp(-\lambda) + \cdots + \lambda^\infty \exp(-\lambda) \\
&= \lambda \exp(-\lambda)(1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \frac{\lambda^4}{4!} + \dots) \\
&= \lambda \exp(-\lambda) \exp(\lambda) \\
&= \lambda \exp(-\lambda + \lambda) \\
&= \lambda
\end{aligned}
$$

2.2 Solving for Variance

Using the Poisson probability mass function (PMF) formula stated in section 2 above:

$$
\begin{aligned}
\mathrm{Var}(X) &= E((X - \mu)^2) \\
&= E(X^2) - \mu^2
\end{aligned}
$$

And as we've solved in the equation of mean above, $\mu = \lambda$, thus:

$$
= E(X^2) - \lambda^2
$$

We need to calculate $E(X^2)$ first.

$$E(X^2) = \sum_{x=0}^{\infty} x^2 f(x)$$
$$= \sum_{x=0}^{\infty} x^2 \frac{\lambda^x \exp(-\lambda)}{x!}$$

you'll agree with me

$$x^2 = x(x-1) + x$$

$$E(x^2) = E(x(x-1) + x)$$
$$= E(x(x-1)) + E(x)$$

and as we've already established$E(x) = \lambda$

so we'll focus on the first part: $E(x(x-1))$

$$E(x(x-1)) = \sum_{x=0}^{\infty} x(x-1) \frac{\lambda^x \exp(-\lambda)}{x!}$$
$$= \exp(-\lambda)\lambda^2 \sum_{x=2}^{\infty} x(x-1) \frac{\lambda^{x-2}}{(x-2)!}$$

let$y = x - 2$so,

$$= \exp(-\lambda)\lambda^2 \sum_{y=0}^{\infty} x(x-1) \frac{\lambda^y}{y!}$$

and once again we have our exponential expansion series

$$= \exp(-\lambda)\lambda^2 \exp(\lambda)$$
$$= \lambda^2 \exp(-\lambda + \lambda)$$
$$= \lambda^2 \exp(0)$$
$$E(x(x-1) = \lambda^2$$

then bringing it back up tox^2

$$E(x^2) = E(x(x-1) + x) = E(x(x-1)) + E(x) =^2 + \lambda$$

and back to variance

substituting this back into the variance formula:

$$\mathrm{Var}(X) = E(X^2) - \lambda^2$$
$$=^2 + \lambda - (\lambda)^2$$
$$= \lambda$$

Finally, substituting this back into the variance formula:
So, the variance of a Poisson distribution with parameter λ is too.

3 References:

[1]$https://proofwiki.org/wiki/Power_Series_Expansion_for_Exponential_Function$
[2]$http://www.math.com/tables/expansion/exp.htm$
[3]$https://en.wikipedia.org/wiki/Discrete_cosine_transform.$
[4]...
[5]....
[6]$https://www.overleaf.com.$