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The school bus routing problem: a case study

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This paper describes a case study of the school bus routing problem. It is formulated as a multi-objective combinatorial optimisation problem. The objectives considered include minimising the total number of buses required, the total travel time spent by pupils at all pick-up points, which is what the school and parents are concerned with most, and the total bus travel time. It also aims at balancing the loads and travel times between buses. A heuristic algorithm for its solution is proposed. The algorithm has been programmed and run efficiently on a PC. Numerical results are reported using test data from a kindergarten in Hong Kong. It has shown to be effective with a saving of 29% in total travelling times when comparing to current practice.
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Introduction

This research is motivated by developing a computer system for Hong Kong school bus services. In Hong Kong, there are 657 000 pupils in kindergartens and primary schools (collectively called 'school' below). Most of them are using the school bus service, which is usually operated by means of mini-buses or large school buses. The school may itself run the service, or just co-ordinate as a middle-man and contract out to individual drivers. In either case, it is presently based on just intuition to assign pupils to a certain bus and plan the routes of each bus. No scientific way has been adopted. It is hence not surprising to hear many parents complaining that it takes their children much more travelling time than expected.

Consider the following daily routine of school bus services in this real-world problem. In the morning a bus leaves its parking place (usually near the driver's home), arrives at the first pick-up point where the route starts, and travels along a predetermined route, picking up the pupils at pick-up points and taking them to school. If one bus cannot pick up all the pupils at one point owing to the capacity limit, the pupils left can be picked up by other buses. In the afternoon, the procedure is reversed: a bus picks up pupils at their school and drops them off at their pick-up points along the route, and returns to its overnight parking place.

This paper aims to provide techniques to plan the school bus routes. The morning problem will be considered. The afternoon problem is analogous, with minor modifications.

In the next section some of the literature on the general areas of vehicle routing and school bus routing is reviewed briefly. In the *Problem representation* section details of the formulation of the school bus routing problem in Hong Kong are discussed. The algorithm developed for solution is presented in the *Solution algorithm* section. To test the effectiveness of the algorithm, numerical results are reported using test data from a kindergarten in Hong Kong, which is in the *Experimental results* section.

Literature review

The school bus routing problem falls into a larger class of problem that is called the vehicle routing problem (VRP). VRP focuses on the efficient use of a fleet of vehicles (eg trucks, buses and cars) that must make a number of stops to pick up and/or deliver passengers or products. The problem requires one to specify which customers should be delivered by which vehicle and in what order so as to minimise total cost subject to a variety of constraints such as vehicle capacity and delivery time restrictions. The VRP has been analysed extensively in contemporary operational research journals. A survey of the literature may be found in Fisher,[1] Desrosiers *et al*,[2] Federgruen *et al*,[3] Laporte[4] and Bodin *et al*.[5] Because VRP is a well known hard problem, it is futile to search for an algorithm that gives the optimal solution in every instance. Therefore most of the research in this area concentrates on the development of heuristic algorithms.

The school bus routing problem has received some attention in the last two decades. Of those mentioned in the operational research literature, each problem has its peculiarities and may also have different objectives and constraints (see Table 1). Since there is no approach dominating all the others for every problem, many approaches seem to be problem dependent. A survey of some of these approaches may be found in Braca *et al*.[13]

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Table 1 Literature review on the school bus routing problem

Reference	*Problem type*	*Objective*	*Constraint*
Bennett and Gazis[6]	Single school	Minimise total student travel time	Bus capacity
Dulac *et al*[7]	Single school	Min {total distance × number of routes}	1. Bus capacity 2. The number of stops 3. The length of a route
Chen and Kallsen[8]	Single school	1. Minimise the number of buses required 2. Minimise fleet travel time 3. Balance the bus loads	1. Bus capacity 2. Student riding time 3. School time window
Bowerman *et al*[9]	Single school	1. Minimise the number of routes (buses) 2. Minimise total bus route length 3. Balance bus loads and route lengths 4. Minimise student walking distance	1. Bus capacity 2. Travel time on each route 3. The total travel time
Newton and Thomas[10]	Multi-school	1. Minimise total bus travel time 2. Minimise the number of routes required	1. Bus capacity 2. Student riding time
Angel *et al*[11]	Multi-school	1. Minimise the number of routes 2. Minimise total bus travel time	1. Bus capacity 2. Specified route time limit
Bodin and Bermen[12]	Multi-school	Minimise total bus travel time	1. Bus capacity 2. Allowable student travel time
Braca *et al*[13]	Multi-school	Minimise the number of buses needed	1. Upper and lower bounds on bus capacity 2. Student riding distance 3. School time window 4. Earliest pick-up time

Although many papers mention multiple objectives to be considered, only Bowerman *et al*[9] claim that their method examines the school bus routing problem from a multi-objective viewpoint.

As will be seen in the next section, our problem is different from those previously reported in several aspects. The objectives considered include minimising the number of buses required, total travel time spent by pupils at all points, total bus travel time, and balancing the bus loads and travel times. The main constraint is the bus capacity.

Problem representation

Notation

K is the number of buses available for the school bus service.
C_k is the capacity of bus k, which may be the same or different for each bus.
n is the total number of pick-up points.
M is the total number of pupils to be served.
$p_1, p_2, \ldots, p_n$, are the n pick-up points. These are ordered by decreasing distance from the school. p_{n+1} denotes the school.
t_{ij} is the travel time from p_i to p_j.
f_i is the number of pupils to be picked up at p_i.
L is the average pick-up time at pick-up points.

$$x_{ijk} = \begin{cases} 1 & \text{if bus } k \text{ travels directly from } p_i \text{ to } p_j \\ 0 & \text{otherwise.} \end{cases}$$

$$z_{ik} = \begin{cases} 1 & \text{if bus } k \text{ picks up pupils at } p_i \\ 0 & \text{otherwise.} \end{cases}$$

y_{ik} is the number of pupils picked up by bus k at p_i.

Objectives

To evaluate any set of school bus routes, many different criteria must be examined simultaneously. Savas[14] provides three criteria for evaluating the provision of public services, namely efficiency, effectiveness and equity. Each criterion has its own unique set of considerations and objectives to satisfy, yet there are clear linkages between the criteria in terms of an overall assessment of service provision. Bowerman *et al*[9] explain the criteria in the situation of school bus transportation.

The objectives considered in the problem are listed and discussed first, then they are classified according to the relevant criteria.

(1) Minimise the total number of buses required. This is an objective related to the service cost. The minimum number of buses K (assume one bus for one route only) required to serve all points for a school can be determined by

$$K = \min(q) \text{ such that } \sum_{k=1}^{q} k \geqslant M$$

Schools usually do operate with only K buses for the service owing to the consideration of cost.

(2) Minimise the total travel time spent by pupils at all points. This objective is what the school and parents are concerned about most. To minimise this objective is to ensure providing higher quality service. It can be formulated as

$$\min \sum_{k=1}^{K} \left\{ \sum_{i=1}^{n} \left[\sum_{j=1}^{n+1} t_{ij} x_{ijk} \left(\sum_{l=1}^{i} z_{lk} \right) + L z_{ik} \left(\sum_{l=1}^{i} z_{lk} \right) \right] \right\}$$

Let d_i denote the shortest travel time from p_i to the school. The school and parents will compare the actual travel times spent with d_i to evaluate the service quality. Therefore

$$T = \sum_{i=1}^{n} d_i$$

is the lower bound of this objective.

(3) Minimise the total bus travel time. This is another objective related to the service cost. It consists of two parts: minimising the total bus loaded travel time (from route origins to the school) and minimising the total bus vacant travel (deadheading) time (from parking places to the route origins).

The former can be formulated as

$$\min \sum_{k=1}^{K} \left[\sum_{i=1}^{n} \left(\sum_{j=1}^{n+1} t_{ij} x_{ijk} + L z_{ik} \right) \right]$$

A solution resulting in the improvement of objective (2) usually leads to the improvement of this part of objective (3).

In the case of all buses with the same capacity, the latter can be formulated as the following assignment problem.

$$\begin{aligned} \text{Minimise} \quad & \sum_{i=1}^{K} \sum_{j=1}^{K} c_{ij} u_{ij} \\ \text{subject to} \quad & \sum_{i=1}^{K} u_{ij} = 1 \qquad j = 1, \ldots, K \\ & \sum_{j=1}^{K} u_{ij} = 1 \qquad i = 1, \ldots, K \\ & u_{ij} = 0 \text{ or } 1 \qquad i, j = 1, \ldots, K \end{aligned}$$

where c_{ij} is the travel time from parking place of bus i to the origin of route j.

$$u_{ij} = \begin{cases} 1 & \text{if bus } i \text{ is assigned to route } j \\ 0 & \text{otherwise.} \end{cases}$$

Otherwise, this assignment, or part of it, is given by the route planner.

(4) Balance the loads and travel times between buses. It is very important in practice to do so. The acceptable level of load and travel time balance depends on the route planner, parents and bus drivers.

The priority of objectives is the same order as above, ie giving first priority to objective (1), second to objective (2), then objective (3), and finally objective (4).

The classification of the objectives considered is given below.

A measure of efficiency is the ratio of the level of a service compared to the cost of the resources required to provide the service. Objectives (1) and (3) belong to the efficiency criterion.

The effectiveness of a service is measured by how good the quality of service is. Objective (2) reflects the total time spent to travel to school, hence a measurement of the service quality.

Equity consideration assess the fairness or impartiality of the provision of the service in question. Balancing the bus loads and travel times, objective (4), is one of the objectives of the equity criterion.

Constraints

The constraints are shown in the appendix.

According to the above analysis, the school bus routing problem considered in this paper can be stated as: given p_i, f_i, t_{ij}, L and C_k, our aim is to assign school bus routes with the optimisation of objectives (1), (2), (3) and (4), subject to constraints (1)–(8). It is clear that the problem is a multi-objective combinatorial one.

Solution algorithm

Henceforth it is assumed that $K > 1$ and $f_i < C_k$ ($i = 1, 2, \ldots, n$). If $K = 1$, then the problem can be transformed into a Travelling Salesman Problem (TSP) and solved by using one of the available algorithms.

In multi-objective problems, it is very difficult to find a solution which optimises all objectives. Sometimes no such solution exists. The preceding mathematical formulation cannot be used directly to solve the problem because there is no known polynomial time algorithm for the solution of such a multi-objective problem. Furthermore, the approaches previously reported cannot be applied directly to this problem owing to the differences in problem formulations. Therefore, a new algorithm must be developed.

In developing the algorithm, two points must be considered. First, the algorithm should be flexible enough to handle multiple objectives. Second, it must have some interactive components that allow an experienced route planner to help construct routes using his/her prior knowledge when necessary. The heuristic algorithm proposed below is shown to satisfy the above considerations and give a ‘good’ solution to the problem. Some available efficient optimisation algorithms are adopted in some parts of this heuristic.

The heuristic algorithm is divided into 5 stages.

Stage 1. Find the optimal solution K for objective (1), where

$$K = \min(q) \qquad \text{such that } \sum_{k=1}^{q} C_k \geqslant M$$

Stage 2. At this stage the solution building strategy is applied to find an initial feasible solution by constructing the routes one by one. The flowchart of stage 2 is shown in Figure 1. If the bus capacities in the fleet are different, the route planner may determine randomly or according to own preference which bus to serve this route interactively and hence its capacity is known. The strategy of not assigning a new bus until the bus currently under consideration is full ensures that the solution keeps objective (1) optimal. Therefore, after the implementation of Stage 2, only the last bus considered is possibly not yet full. Furthermore, selecting the shortest, second shortest, ..., kth shortest route from p_i to the school to test in turn is to try to keep objectives (2) as well as (3) optimal. The procedure due to Lawler[15] can be used to find the kth shortest route from p_i to the school.

Stage 3. The improvement strategy is adopted at this stage to improve the initial solution. A similar procedure has been adopted in an earlier study by one of the authors (see Fu and Wright[16]). The aim is to make the shorter-distance buses as full as possible by withdrawing some pupils from the longer-distance buses. In this way the number of pick-up points (and so the total pick-up time) of longer-distance buses can be reduced, leading to the reduction of objectives (2) and (3). It is also a strategy to optimise objective (4). The flowchart of this stage is shown in Figure 2.

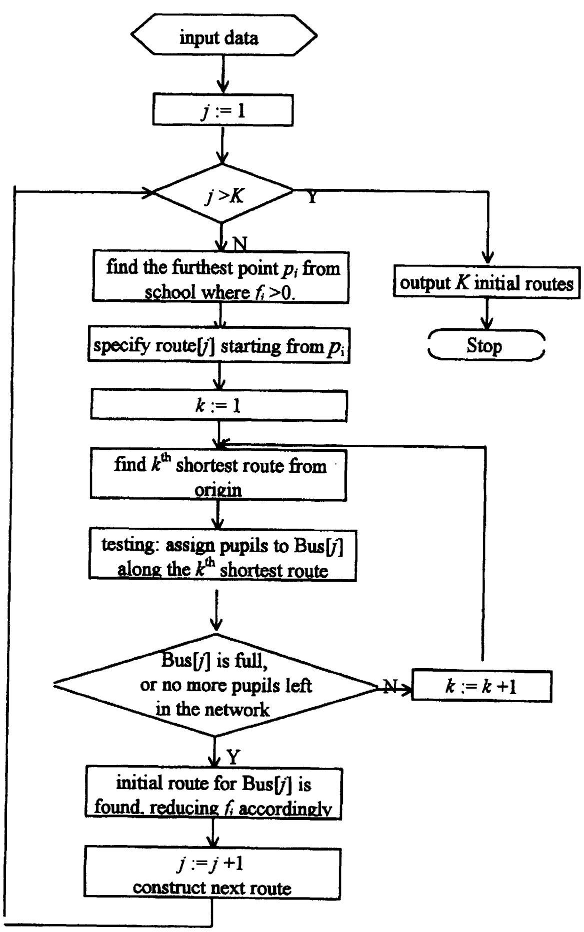

Figure 1 Stage 2 of the algorithm.

First of all, working in reverse order of the routes formation in Stage 2, ie consider shorter routes first, find the first route not yet examined where its bus has spare capacity. If one is found, call it Route [j] (its bus called Bus [j]). Then working in the order of the routes formation in Stage 2, ie consider longer routes first, find a Route [h] (its bus called Bus [h]) formed before Route [j]. For such a Bus [h] the following two cases are examined.

Case 1. Bus [h] picks up those pupils whose pick-up point is the same as some pupils in Bus [j], ie pupils at this pick-up point are assigned to both Buses [j] and [h]. In this case, move as many pupils picked up at this point as possible from Bus [h] to Bus [j].

Case 2. Bus [h] picks up those pupils whose pick-up point is on the same route as Bus [j]. In this case, move all pupils at that pick-up point from Bus [h] to Bus [j] if and only if Bus [j] has enough capacity left for all of them. Otherwise all the pupils are still kept in Bus [h] so that pick-up time will not be duplicated.

In both cases when searching for such pick-up points, those near the school are considered first.

Stage 4. Owing to the use of improvement strategy in Stage 3, the pick-up points of some buses have possibly been changed. Therefore all sub-routes from a pick-up point to the next one or to the school must be checked and altered if necessary, to make sure that buses go along the shortest path. This step contributes to objectives (2) and (3).

Stage 5. Finally, for those buses that have the same capacity, solve the appropriate assignment problem, with the objective of minimising bus vacant-travel (deadheading) time, using the Hungarian algorithm.[17]

Experimental results

The algorithm is applied to a kindergarten in Hong Kong to test its effectiveness. The existing school bus routes of the kindergarten are planned manually.

Data input

The following (see Table 2) are the data used to find the solution.

The total number of pupils to be served is 86. All buses have the same capacity, 36. Then $K = 3$. Average pick-up time $L = 25$ (s). There are 54 pick-up points, where each has 1–8 pupils.

The shortest distances from every pick-up point to the school are found by using the program codes of Dijkstra's algorithm in Syslo *et al.*[18] All pick-up points p_i are ordered by decreasing distance to the school. There are both one-way streets and two-way streets on the street network.

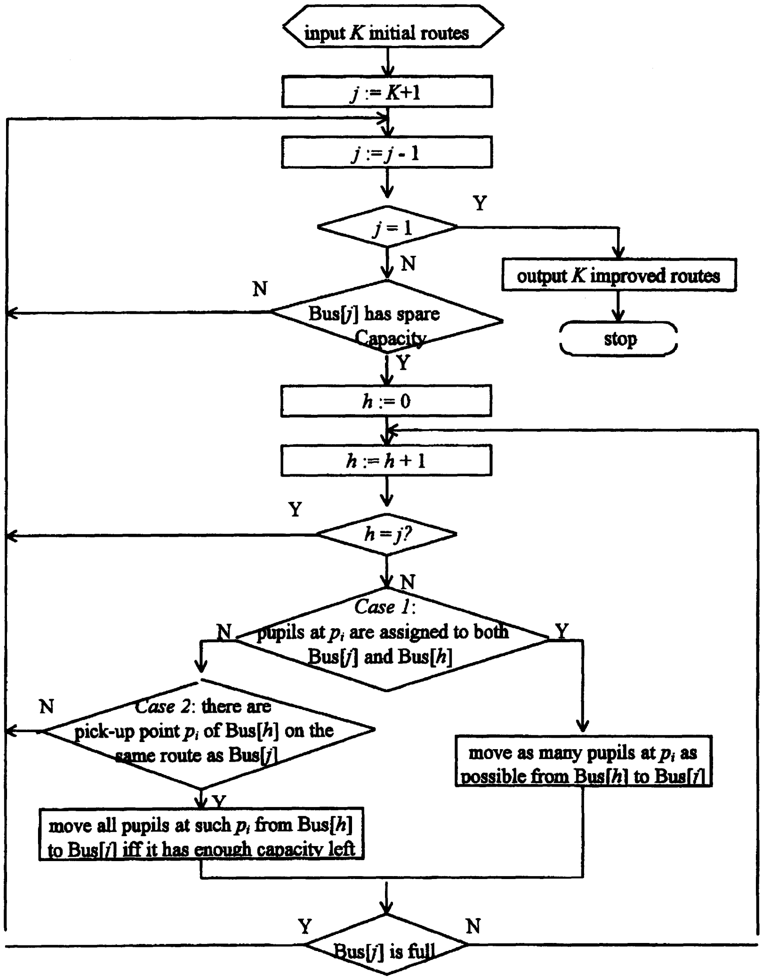

Figure 2 Stage 3 of the algorithm.

Result analysis

The algorithm has been programmed using Turbo Pascal. After inputting the above data, the algorithm requires only a few seconds on a PC to output the results. The results are shown in Table 3.

In this example the load of bus 3 is much less than that of buses 1 and 2. The reason is that the improvement of load balance in Stage 3 is limited owing to priorities been given to objectives (2) and (3). However, one may obtain the desirable level of load balance simply by decreasing the bus capacity to a suitable level interactively and forcing the algorithm to get a result with desirable balance loads. The closer to the average bus load the bus capacity, the more balanced the actual bus loads are. In this case, however, the values for objectives (2) and (3) will possibly increase, resulting in lower service quality. This is a trade-off to be made.

A comparison of objective (2) (excluding pick-up time) is shown in Table 4. The result is just 17.4% above the lower bound $T = \sum_{i=1}^{n} d_i$. Notice that the lower bound is just a very loose one. The actual gap between the optimum and the solution found by the algorithm should be much narrower. The new routes produced by the algorithm save 29% total pupil travel time in comparison with that of the existing routes.

A comparison of the bus loaded-travel times (objective (3)) (including pick-up times) is shown in Table 5. 13.2%, 32.5% and 23.4% are shortened respectively for each route.

Conclusions

In this paper, a case study of the school bus routing problem is described. It is formulated as a multi-objective combinatorial optimisation problem. A heuristic algorithm for its solution is proposed. The algorithm is developed by a combination of heuristic and optimisation methods (Hungarian algorithm, Lawler's kth shortest route algorithm, and Dijkstra's shortest route algorithm), and runs efficiently on a PC. The objectives of the school bus routing problem considered include minimising the total number of buses required, the total travel time spent by pupils at all pick-up points, which is what the school and parents are concerned

Table 2 Data input

p_i	f_i	p_j	t_{ij} (s)	p_i	f_i	p_j	t_{ij} (s)
1	1	2	30	29	1	27, 31	110, 30
2	1	3	60	30	8	33, 32	20, 80
3	1	4	585	31	1	29, 30, 37, 42, 55	30, 85, 75, 205, 595
4	1	5	240	32	1	35, 36	45, 150
5	1	7	420	33	2	30, 34	20, 15
6	2	21	825	34	2	33, 35	15, 25
7	1	6, 8	300, 150	35	1	34, 40, 32	25, 60, 45
8	1	9	15	36	1	32, 41	150, 75
9	1	10	30	37	2	31, 30, 39	75, 130, 15
10	2	11	15	38	1	44, 54, 53, 55, 42, 31	425, 535, 535, 510, 720, 865
11	1	12	15	39	3	37, 40	15, 20
12	1	14	35	40	1	39, 35, 32, 43, 42, 55	20, 60, 70, 80, 90, 480
13	1	19	90	41	5	36, 43, 45	75, 60, 65
14	1	15	15	42	1	46	30
15	1	16	15	43	2	40, 49, 47	80, 60, 40
16	1	17	15	44	1	50	75
17	2	18	15	45	1	41, 43, 48	65, 35, 30
18	1	7, 25	215, 545	46	1	49	30
19	1	20	210	47	3	43, 53	40, 105
20	1	22	30	48	3	45, 51	30, 30
21	2	23	180	49	3	40, 47, 53, 55	173, 95, 110, 353
22	1	38	740	50	1	52	45
23	1	24	96	51	3	48, 54	30, 60
24	1	26	45	52	1	55	280
25	1	27	235	53	2	47, 55, 44, 54	105, 265, 140, 90
26	2	27	220	54	2	51, 53, 55, 44	60, 90, 265, 140
27	1	28, 29	70, 110	55	—	—	—
28	1	29	70				

p_j are those points which can be reached directly from p_i. The relevant t_{ij} are arranged in the same order. p_{55} denotes the kindergarten.

Table 3 School bus routes

Bus	*Pick-up*	*Route*
Bus 1	33	1,2,3,4,5,7,8,9,10,11,12,14,15,16,17,18,25,27,28, 29,31,37,39,40,43,47,53,55
Bus 2	36	6,21,23,24,26,27,29,31,30,33,34,35,32,36,41,45, 48,51,54,55
Bus 3	17	13,19,20,22,38,42,46,49,53,54,44,50,52,55

about most, and the total bus travel time. It also aims at balancing the loads and travel times between buses. Numerical results are reported using test data from a kindergarten in Hong Kong, which have shown to be much better than those achieved by current manual methods.

This study may also shed light on other VRPs such as refuse collection at specified locations, delivery of goods to outlets, and so on.

Further research work may include finding a larger scale problem to test the effectiveness of this algorithm. This algorithm is developed mainly by using some classical

Table 5 Bus loaded-travel times (min)

	Existing route	*New route*	*Shortened*	*Shortened (%)*
Bus 1	73.5	63.8	9.7	13.2%
Bus 2	68.4	46.2	22.2	32.5%
Bus 3	63.3	48.5	14.8	23.4%
Total	205.2	158.5	46.7	22.8%

Table 4 Total travel times (s) spent by pupils at all points

	Existing routes		*New routes*		
Lower bound $T = \sum_{i=1}^{n} d_i$	*Travel time*	*% above T*	*Travel time*	*% above T*	*Savings from existing one*
57 827	95 703	65.5%	67 908	17.4%	29.0%

ideas. New algorithms with modern heuristics may also be developed and compared.

Appendix

Constraints of the mathematical formulation

Assume that the school is the dummy starting point 0 for every route, and let $t_{0j} = 0$ $(j = 1, 2, \ldots, n)$, then the constraints of the problem can be formulated as

$$\sum_{i=1}^{n} y_{ik} \leqslant C_k \qquad (k = 1, 2, \ldots, K) \tag{1}$$

$$\sum_{k=1}^{K} y_{ik} = f_i \qquad (i = 1, 2, \ldots, n) \tag{2}$$

$$\sum_{i=0}^{n} x_{ilk} = \sum_{j=1}^{n+1} x_{ljk} \quad (l = 1, 2, \ldots, n;\ k = 1, 2, \ldots, K) \tag{3}$$

$$\sum_{i=1}^{n} x_{i(n+1)k} = \sum_{j=1}^{n} x_{0jk} = 1 \qquad (k = 1, 2, \ldots, K) \tag{4}$$

$$\sum_{j=1}^{n+1} x_{ijk} \geqslant z_{ik} \qquad (i = 1, 2, \ldots, n;\ k = 1, 2, \ldots, K) \tag{5}$$

$$y_{ik} \leqslant f_i z_{ik} \qquad (i = 1, 2, \ldots, n;\ k = 1, 2, \ldots, K) \tag{6}$$

$$x_{ijk}, z_{ik} = 0 \text{ or } 1 \tag{7}$$

$$y_{ik} \geqslant 0 \text{ and an integer} \tag{8}$$

Constraint (1) is the bus capacity constraint. Constraint (2) ensures that all pupils are picked up. Constraint (3) requires that a bus visiting a point also leaves that point. Constraint (4) ensures that all buses visit the school, and Constraint (5) ensures that if a bus picks up pupils at a point, it must also visit that point. Constraint (6) ensures that if any pupil is picked up at point i by bus k, z_{ik} will be set to 1.

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