

Module 3 Homework - Noboru Hayashi

3.4.4

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3.4.4.

$P(\text{a malaria patient's remission lasts longer than 1 year})$

$$= P(1 \leq Y \leq 3) = \int_1^3 f_Y(y) dy = \int_1^3 \frac{1}{9} y^2 dy$$
$$= \left. \frac{1}{9} \cdot \frac{1}{3} y^3 \right|_1^3 = \left. \frac{1}{27} y^3 \right|_1^3 = 1 - \frac{1}{27} = \frac{26}{27}$$

3.5.14

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$f_Y(y) = 3y^2, 0 \leq y \leq 1.$

$N = 15.$ X : # of observations lie in $(\frac{1}{2}, 1)$

$$P(Y \in (\frac{1}{2}, 1)) = \int_{\frac{1}{2}}^1 3y^2 dy = y^3 \Big|_{\frac{1}{2}}^1 = 1 - \frac{1}{8} = \frac{7}{8}$$
$$E(X) = N \cdot P(Y \in (\frac{1}{2}, 1)) = 15 \times \frac{7}{8} = \frac{105}{8} \approx 13.125$$

3.5.32

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A box with a height = 5 inches, base: Y by Y inches

$f_Y(y) = 6y(1-y), 0 < y < 1$

$$E(\text{Vol}) = E(5y^2) = \int_0^1 5y^2 \cdot f_Y(y) dy = \int_0^1 5y^2 \cdot 6y(1-y) dy$$
$$= \int_0^1 (30y^3 - 30y^4) dy = \left(\frac{30}{4} y^4 - \frac{30}{5} y^5 \right) \Big|_0^1$$
$$= \frac{30}{4} - \frac{30}{5} = 1.5 \text{ (inch}^3\text{)}$$

3.6.2

$$f_X(y) = \begin{cases} \frac{3}{4}, & 0 \leq y \leq 1 \\ \frac{1}{4}, & 2 \leq y \leq 3 \\ 0, & \text{elsewhere} \end{cases}$$

$$E(Y) = \int_{-\infty}^{\infty} y f_X(y) dy = \int_0^1 \frac{3}{4} y dy + \int_2^3 \frac{1}{4} y dy$$

$$= \left. \frac{3}{8} y^2 \right|_0^1 + \left. \frac{1}{8} y^2 \right|_2^3 = \frac{3}{8} + \frac{9}{8} - \frac{4}{8} = 1$$

$$E(Y^2) = \int_{-\infty}^{\infty} y^2 f_X(y) dy = \int_0^1 \frac{3}{4} y^2 dy + \int_2^3 \frac{1}{4} y^2 dy$$

$$= \left. \frac{1}{4} y^3 \right|_0^1 + \left. \frac{1}{12} y^3 \right|_2^3 = \frac{1}{4} + \frac{27}{12} - \frac{8}{12} = \frac{22}{12} = \frac{11}{6}$$

$$\text{Var}(Y) = E(Y^2) - [E(Y)]^2$$

$$= \frac{11}{6} - 1^2 = \frac{5}{6}$$

3.6.10

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$$f_Y(y) = 5y^4, \quad 0 \leq y \leq 1$$

$$\text{Var}(Y) = E(Y^2) - [E(Y)]^2$$

$$E(Y) = \int_0^1 y \cdot 5y^4 dy = \left. \frac{5}{6} y^6 \right|_0^1 = \frac{5}{6}$$

$$E(Y^2) = \int_0^1 y^2 \cdot 5y^4 dy = \left. \frac{5}{7} y^7 \right|_0^1 = \frac{5}{7}$$

$$\text{Var}(Y) = \frac{5}{7} - \left(\frac{5}{6}\right)^2 = \frac{5}{7} - \frac{25}{36} = \frac{5}{252}$$