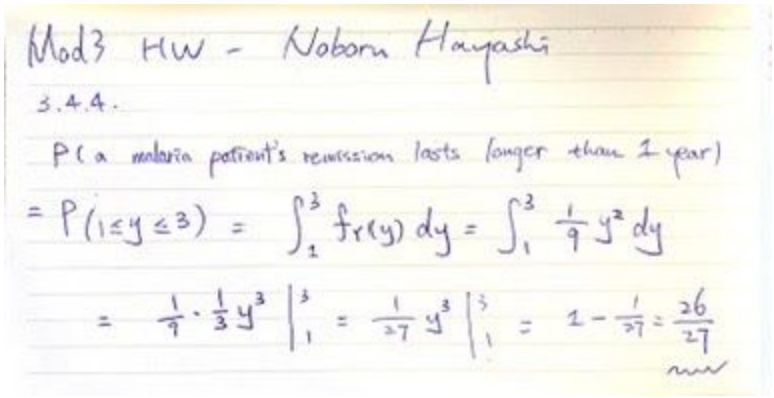
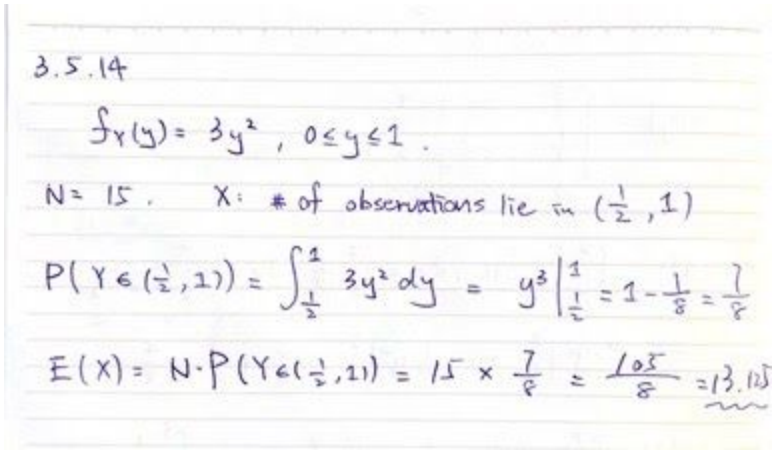
Module 3 Homework - Noboru Hayashi

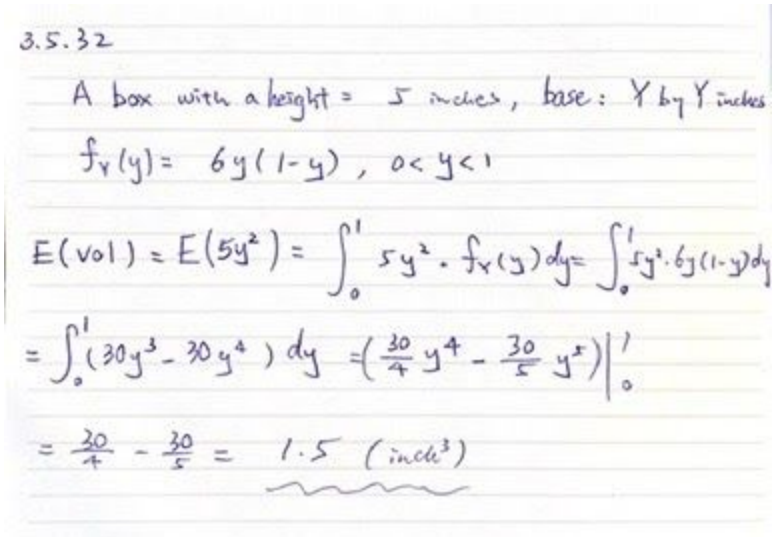
3.4.4



3.5.14



3.5.32



3.6.2

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$$f_Y(y) = \begin{cases} \frac{3}{4}, & 0 \le y \le 1 \\ \frac{1}{4}, & 2 \le y \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

$$E(y) = \int_{-\infty}^{\infty} y f_Y(y)\,dy = \int_0^1 \frac{3}{4} y\,dy + \int_2^3 \frac{1}{4} y\,dy$$

$$= \frac{3}{8} y^2 \Big|_0^1 + \frac{1}{8} y^2 \Big|_2^3 = \frac{3}{8} + \frac{9}{8} - \frac{4}{8} = 1$$

$$E(Y^2) = \int_{-\infty}^{\infty} y^2 f_Y(y)\,dy = \int_0^1 \frac{3}{4} y^2\,dy + \int_2^3 \frac{1}{4} y^2\,dy$$

$$= \frac{1}{4} y^3 \Big|_0^1 + \frac{1}{12} y^3 \Big|_2^3 = \frac{1}{4} + \frac{27}{12} - \frac{8}{12} = \frac{22}{12}$$

$$= \frac{11}{6}$$

$$\text{Var}(Y) = E(Y^2) - [E(Y)]^2$$

$$= \frac{11}{6} - 1^2 = \frac{5}{6}$$

3.6.10

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$f_Y(y) = 5y^4, \quad 0 \le y \le 1$

$\text{Var}(Y) = E(Y^2) - [E(Y)]^2$

$E(Y) = \int_0^1 y \cdot 5y^4 \, dy = \frac{5}{6} y^6 \Big|_0^1 = \frac{5}{6}$

$E(Y^2) = \int_0^1 y^2 \cdot 5y^4 \, dy = \frac{5}{7} y^7 \Big|_0^1 = \frac{5}{7}$

$\text{Var}(Y) = \frac{5}{7} - \left(\frac{5}{6}\right)^2 = \frac{5}{7} - \frac{25}{36} = \frac{5}{252}$