Module 13 Homework - Noboru Hayashi

13.3.2

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Let X, Y denotes the number of trials to learn depth perception for mothered & un mothered lambs.

and $d = X - Y$

$H_0: \mu_d = 0$

$H_a: \mu_d \neq 0$

From Q.8.2.6: $\sum_{i=1}^{13} d_i = -42$

$\sum_{i=1}^{13} d_i^2 = 216$

$\Rightarrow \bar{d} = \frac{-42}{13} = -3.2308$

$s_D^2 = \frac{13 \times 216 - (-42)^2}{13 \times 12} = 6.6923$

$s_D = \sqrt{6.6923} = 2.5869$

t-value: $\frac{\bar{d} - \mu_D}{s_D / \sqrt{n}} = \frac{-3.2308}{2.5869/\sqrt{13}} = -4.503$

$t_{\alpha/2, n-1} = t_{0.025, 13-1} = 2.1788$

Since $-4.503 < -t_{0.025, 12}$

Reject H_0

13.3.6

13.3.6 Given μ_D & σ_D^2 for b pairs of difference

$$P\left(-t_{\alpha/2,b-1} \le \frac{\bar{D}-\mu_D}{\sigma_D/\sqrt{b}} \le t_{\alpha/2,b-1}\right) = 1-\alpha$$

$$\Rightarrow P\left(-t_{\alpha/2,b-1}\cdot\frac{\sigma_D}{\sqrt{b}} \le \bar{D}-\mu_D \le t_{\alpha/2,b-1}\cdot\frac{\sigma_D}{\sqrt{b}}\right) = 1-\alpha$$

$$\Rightarrow P\left(-t_{\alpha/2,b-1}\cdot\frac{\sigma_D}{\sqrt{b}} - \bar{D} \le -\mu_D \le t_{\alpha/2,b-1}\cdot\frac{\sigma_D}{\sqrt{b}} - \bar{D}\right) = 1-\alpha$$

$$\Rightarrow P\left(\bar{D} - t_{\alpha/2,b-1}\cdot\frac{\sigma_D}{\sqrt{b}} \le \mu_D \le \bar{D} + t_{\alpha/2,b-1}\cdot\frac{\sigma_D}{\sqrt{b}}\right) = 1-\alpha$$

So, CI for μ_D is: $\left[\bar{D} - t_{\alpha/2,b-1}\cdot\frac{\sigma_D}{\sqrt{b}},\ \bar{D} + t_{\alpha/2,b-1}\cdot\frac{\sigma_D}{\sqrt{b}}\right]$

For Case Study 13.3.1:

$\bar{D} = 0.47$, $\sigma_D = \sqrt{0.662} = 0.8136$, $b = 10$

$t_{0.05/2,\,10-1} = t_{0.025,\,9} = 2.2622$

$\Rightarrow$ CI for μ_D is:

$$\left[0.47 - 2.2622\cdot\frac{0.8136}{\sqrt{10}},\ 0.47 + 2.2622\cdot\frac{0.8136}{\sqrt{10}}\right]$$

$$= [-0.1120,\ 1.0520]$$

14.2.6

From Case Study 7.4.2, $n=20$.

Set $H_0: \tilde{\mu} = 0.618$

$H_a: \tilde{\mu} \neq 0.618$

For $\tilde{\mu}_0 = 0.618$, there are 11 samples over $\tilde{\mu}_0$

$$\Rightarrow z = \frac{k - n/2}{\sqrt{n}/4} = \frac{11 - 20/2}{\sqrt{20}/4} = 0.4472$$

With ~~$z_{\alpha=0.05}$~~ $\alpha = 0.05$ and two-tail test,

$$z_{\alpha/2} = 1.96$$

Since $z < z_{\alpha/2}$, fail to reject H_0.