Module 1 Homework - Noboru Hayashi

2.2.4
For following combinations of two cards, the sum of them are 8:
A-7, 2-6, 3-5, 4-4, 5-3, 6-2, 7-A

Since there are 4 suits for each card in a standard 52-card playing card deck, for the outcomes A-7, 2-6, 3-5, 5-3, 6-2, 7-A (except 4-4), they can occur in 4*4 = 16 ways each.
While there are 4*3 ways to achieve the 4-4 outcome, so the total number of outcomes in A are:
6*16 + 4*3 = 96 + 12 = **108**.

2.2.28
(a) AC
= S-A
= {x: 0 ≤ x ≤ 10} - {x : 0 < x < 5} = {x : 5 ≤ x ≤ 10} ∪ {x: x = 0}

(b) A∩B
= {x : 0 < x < 5} ∩ {x : 3 ≤ x ≤ 7}
= {x : 3 ≤ x < 5}

(c) A∪B
= {x : 0 < x < 5} ∪ {x : 3 ≤ x ≤ 7}
= {x : 0 < x ≤ 7}

(d) A∩BC
= {x : 0 < x < 5} ∩ {x : 3 ≤ x ≤ 7}C
= {x : 0 < x < 5} ∩ ({x : 0 ≤ x ≤ 10} - {x : 3 ≤ x ≤ 7})
= {x : 0 < x < 5} ∩ ({x : 0 ≤ x < 3} ∪ {x : 7 < x ≤ 10})
= {x: 0 < x < 3}

(e) AC ∪ B
= {x : 0 < x < 5}C ∪ {x : 3 ≤ x ≤ 7}
= ({x : 0 ≤ x ≤ 10} - {x : 0 < x < 5}) ∪ {x : 3 ≤ x ≤ 7}
= ({x: x=0} ∪ {x: 5 ≤ x ≤ 10}) ∪ {x : 3 ≤ x ≤ 7}
= {x: x=0} ∪ {x : 3 ≤ x ≤ 10}

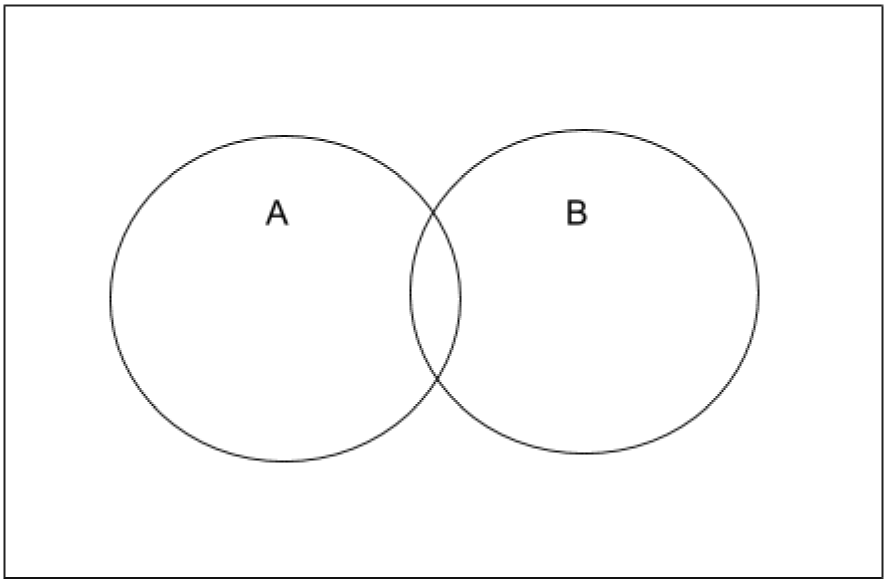
(f) AC∩BC
= {x : 0 < x < 5} C ∩ {x : 3 ≤ x ≤ 7} C
= ({x : 0 ≤ x ≤ 10} - {x : 0 < x < 5}) ∩ ({x : 0 ≤ x ≤ 10} - {x : 3 ≤ x ≤ 7})
= ({x: x=0} ∪ {x: 5 ≤ x ≤ 10}) ∩ ({x : 0 ≤ x < 3} ∪ {x : 7 < x ≤ 10})
= {x: x=0} ∪ {x : 7 < x ≤ 10}

2.3.2

$P(A) = 0.4$, $P(B) = 0.5$, $P(A \cap B) = 0.1$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.1 = 0.4 + 0.5 - 0.1 = 0.8$

P (A or B but not both occur) = $P(A \cup B) - P(A \cap B) = 0.8 - 0.1 =$ **0.7**



2.3.16

S= {(1,1),(1,2),(1,3), … , (6,5),(6,6)}, 6*6 = 36 outcomes

A = {(1,5),(2,4),(3,3),(4,2),(5,1)}, 5 outcomes

B = {(1,2), (2,1), (2,4), (4,2), (3,6), (6,3)}, 6 outcomes

$A \cap B$ = {(2,4),(4,2)}, 2 outcomes

$P(A \cap B^C) = P(A) - P(A \cap B) = 5/36 - 2/36 = 3/36 =$ **1/12**

2.4.23

P = P(a 7 of diamonds) * P(a jack of spades | 1st card: a 7 of diamonds) * P(a 10 of diamonds | 1st card: a 7 of diamonds, 2nd card: a jack of spade) * P(a 5 of hearts | 1st card: a 7 of diamonds, 2nd card: a jack of spade, 3rd card: a 10 of diamonds)

= 1/52 * 1/51 * 1/50 * 1/49 = **1/6497400**

2.5.16

Let events A, B, C and D each denote the event that the first(second, third, or fourth) light is green.

P(A) = P(B) = 40/60 = 2/3, P(C) = P(D) = 30/60 = 1/2.

And we assume each of 4 events is independent of the others.

For a commuter to stop at least 3 times, it means at most one light is green, and others are red lights (complement of the event A, B, C or D). The following are possible combinations:

(Ac, Bc, Cc, Dc), (A, Bc, Cc, Dc), (Ac, B, Cc, Dc), (Ac, Bc, C, Dc), (Ac, Bc, Cc, D).

For each combination, the probability is:

P(Ac, Bc, Cc, Dc) = $1/3 * 1/3 * 1/2 * 1/2 = 1/36$

P(A, Bc, Cc, Dc) = $2/3 * 1/3 * 1/2 * 1/2 = 2/36$

P(Ac, B, Cc, Dc) = $1/3 * 2/3 * 1/2 * 1/2 = 2/36$

P(Ac, Bc, C, Dc) = $1/3 * 1/3 * 1/2 * 1/2 = 1/36$

P(Ac, Bc, Cc, D) = $1/3 * 1/3 * 1/2 * 1/2 = 1/36$

Hence, P(the commuter has to stop at least three times) =

P(Ac, Bc, Cc, Dc) + P(A, Bc, Cc, Dc) + P(Ac, B, Cc, Dc) + P(Ac, Bc, C, Dc) + P(Ac, Bc, Cc, D)=

1/36 + 2/36 + 2/36 + 1/36 + 1/36 = **7/36**