Module 2 Homework - Noboru Hayashi

2.3.10
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
= P(the number is divisible by 2) + P(the number is divisible by 3) - P(the number is divisible by 2 &3)
$= 1/2 + 1/3 - 1/6 = 3/6+2/6-1/6 = 4/6 =$ **2/3**

2.4.4
$P((A \cup B)C) = 0.6$, $P (A \cap B) = 0.1$
E = either A or B but not both occur.

$P(A \cup B) = 1 - 0.6 = 0.4$
$P (E) = P(A \cup B) - P (A \cap B) = 0.4 - 0.1 = 0.3$
$P(E|A \cup B) = P(E) - P(A \cup B) = 0.3/0.4 = 3/4 =$ **0.75**

2.4.36
Let G be an event that a jury returns a guilty verdict,
Let D be an event that the defense can discredit the police department

$P(G \mid D) = 0.15$
$P(G \mid Dc) = 0.8$

$P (D) = 0.7$

$P (G) = P (G \cap D) + P (G \cap Dc) = P(G \mid D)*P(D) + P(G \mid Dc)*P(Dc) = 0.15*0.7 + 0.8*(1-0.7)$
$= 0.105 + 0.24 = 0.345 =$ **34.5%**

2.5.20

Since players A, B and C toss a fair coin in order, for the first round of the tosses, the expected set of outcomes is (H for head, T for tail) :

{HHH, HHT, HTH, HTT, THT, THH, TTH, TTT}.

Total number of outcomes for the first round is 8.
And let X, Y, Z be the events that player A, B, or C wins at the first round.

X = {HHH, HHT, HTH, HTT}, P(X) = 4/8 = 0.5
Y = {THT, THH} , P(Y) = 2/8 = 0.25
Z = {TTH}, P(Z) = 1/8 = 0.125
And P(repeat) = 1/8 = 0.125

Since P(A wins) = P(X) + P(repeat once then X) + P(repeat twice then X) + …
So, P(A wins) =
$0.5 + 0.5 * 1/8 + 0.5 * 1/8^2 + ... = 0.5(1 + 1/8 + 1/8^2 + ...) = 0.5 * (1 - 1/8)^{-1} = 0.5 * 8/7$ = **4/7**

Similarly,
P(B wins) = P(Y) + P(repeat once then Y) + P(repeat twice then Y) + …
$0.25 + 0.25 * 1/8 + 0.25 * 1/8^2 + ... = 0.25(1 + 1/8 + 1/8^2 ...) = 0.25 * (1 - 1/8)^{-1} = 1/4 * 8/7$ = **2/7**

P(C wins) = P(Z) + P(repeat once then Z) + P(repeat twice then Z) + …
$0.125 + 0.125 * 1/8 + 0.125 * 1/8^2 + ... = 0.125(1 + 1/8 + 1/8^2 ...) = 0.125 * (1 - 1/8)^{-1} = 1/8 * 8/7$ = **1/7**

2.6.12

Given the international morse code represents any letter with a series of dots and dashes, for each digit of the code, it can be a dot or dashes.
Therefore, for the code with a length n, there are 2^n ways to arrange it. And total numbers of the combinations of dots and dashes for the n-digit code is: 2 + 4 + … + 2^n

If n = 3, the # of combinations is 2 + 4 + 8 = 14.
If n = 4, the # of combinations is 2 + 4 + 8 + 16 = 30.

Since the number of alphabet is 26, the minimum number of digits for morse codes to represent any letter in the alphabet is **4**.

Simulation:
Consider a Baseball World Series (best of 7 game series) in which team A theoretically has a 0.55 chance of winning each game against team B. Simulate the probability that team A would win a World Series against team B by simulating 1000 World Series. You may use any software to conduct the simulation.

Theoretically, P(Team A win a World Series | team A has a 0.55 chance of winning each game)
$= 0.55^4 + 4C3 * 0.55^4 * 0.45 + 5C3 * 0.55^4 * 0.45^2 + 6C3 * 0.55^4 * 0.45^3$
$= 0.55^4(1 + 4 * 0.45 + 10 * 0.45^2 + 20 * 0.45^3) = 0.608287797$

For simulation in R:

```
> # init count = 0
> count = 0
>
> # init p(A wins each game)
> pA = 0.55
>
> # simulate 1000 world series
> for (i in 1:1000){
+         series = runif(7)
+         aWins = series < pA
+         aWin = sum(aWins) >= 4
+         count = count + sum(aWin)
+ }
> count/1000
[1] 0.604
```

Therefore, by simulation, the probability that team A would win a World Series against team B is: **0.604**