Module 10 Homework - Noboru Hayashi

9.2.6

9.2.6 Let μ_x, μ_y be true mean age at death of authors noted/not noted for alcohol abuse

$H_0: \mu_x = \mu_y$

$H_a: \mu_x < \mu_y$

From the problem text & table:

$n = 9$, $\bar{x} = 65.2$

$m = 12$, $\bar{y} = 75.5$

$S_p = 13.9$

Test Statistic:

$$t = \frac{\bar{x} - \bar{y}}{S_p \sqrt{\frac{1}{n} + \frac{1}{m}}} = \frac{65.2 - 75.5}{13.9 \times \sqrt{\frac{1}{9} + \frac{1}{12}}} = -1.680$$

From t distibution table, $t_{\alpha, n+m-2} = t_{0.05, 19} = 1.7291$

Since $t > -t_{0.05, 19}$, Failed to reject Ho

9.3.4,

9.3.4 Let σ_x^2, σ_y^2 denote variances of weight gains for the first / second group

$H_0: \sigma_x^2 = \sigma_y^2$

$H_a: \sigma_x^2 \neq \sigma_y^2$

$\alpha = 0.05$

From table: $n = 10, \quad s_x = 5.67$
$m = 10, \quad s_y = 3.18$

Test Statistic: $F = \frac{s_Y^2}{s_X^2} = \frac{3.18^2}{5.67^2} = 0.3145$

From F dist table:

$F_{\alpha/2, m-1, n-1} = F_{0.025, 9, 9} = 0.248$

$F_{1-\alpha/2, m-1, n-1} = F_{0.975, 9, 9} = 4.03$

Since F is neighter $\leq F_{\alpha/2, m-1, n-1}$
or $\geq F_{1-\alpha/2, m-1, n-1}$

Fail to reject Ho

9.4.4,

9.4.4

$H_0: p_S = p_{NS}$

$H_a: p_S \neq p_{NS}$

$\alpha = 0.01$

From table: $n = 53+38 = 91$, $m = 705+412 = 1117$, $x = 53$, $y = 705$

$$p_e = \frac{53+705}{53+38+705+412} = 0.6275$$

Test Statistic:

$$z = \frac{\frac{x}{n} - \frac{y}{m}}{\sqrt{\frac{p_e(1-p_e)}{n} + \frac{p_e(1-p_e)}{m}}}$$

$$= \frac{53/91 - 705/1117}{\sqrt{\frac{0.6275(1-0.6275)}{91} + \frac{0.6275(1-0.6275)}{1117}}} = -0.925$$

From the table cumulative area under normal curve

$z_{\alpha/2} = z_{0.005} = 2.58$

Since $z < z_{0.005}$ & $z > -z_{0.005}$

So Fail to reject H_0

9.5.6,

9.5.6 95% CI for σ_X^2/σ_Y^2

$$\left(\frac{S_X^2}{S_Y^2} F_{\alpha/2,\, m-1,\, n-1},\ \frac{S_X^2}{S_Y^2} F_{1-\alpha/2,\, m-1,\, n-1}\right)$$

From case study 9.2.1 section

$n = 8, \quad m = 10$

$S_X^2 = 0.0002103$

$S_Y^2 = 0.0000955 \quad \Rightarrow \quad \frac{S_X^2}{S_Y^2} = 2.2021$

From F-dist table:

$F_{\alpha/2, m-1, n-1} = F_{0.025, 9, 7} = 0.238$

$F_{1-\alpha/2, m-1, n-1} = F_{0.975, 9, 7} = 4.82$

So $CI = (2.2021 \cdot 0.238,\ 2.2021 \cdot 4.82)$

$= (0.5241,\ 10.6141)$

Since ~~test~~ the ratio $\sigma_X^2/\sigma_Y^2 = 1$ contained in the CI, we cannot conclude about the population variances are equal or different

10.2.6,

10.2.6

Let X_1, X_2, X_3, X_4, X_5 be Out, Single, Double, Triple and homerun

P(2 outs, 2 singles, a double)

$$= P(X_1 = 2, X_2 = 2, X_3 = 1, X_4 = 0, X_5 = 0)$$

$$= \frac{5! \; \cancel{5!}}{2!2!1!0!0!} \cdot p_1^2 \cdot p_2^2 \cdot p_3^1 \cdot p_4^0 \cdot p_5^0$$

$$= \frac{5!}{2!2!1!0!0!} (0.713)^2 \cdot (0.270)^2 \cdot (.010)$$

$$= 0.0111$$

10.3.6,

10.3.6

From the problem:

number of outcomes $t = 2$ (born in the first quarter & not born in the first quarter)

Let $k_1 = 1383$, $k_2 = 5139 - 1383 = 3756$

$np_{10} = 1292.1$, $np_{20} = 5139 - 1292.1 = \cancel{3847}$ 3846.9

H_0: the births of who are admitted to a physchiatric ward with a diagnosis of schizophrenia are uniformly occurring ($p_{10} = p_1$, $p_{20} = p_2$)

H_a: $p_{10} \neq p_1$, $p_{20} \neq p_2$

Test Statistics:

$$d = \sum_{i=1}^{2} \frac{(k_i - np_{i0})^2}{np_{i0}} = \frac{(1383-1292)^2}{1292} + \frac{(3756-3846.9)^2}{\cancel{3847}\ 3846.9}$$

$$= 8.5428$$

From χ^2 dist table.

$$\chi^2_{1-\alpha, t-1} = \chi^2_{0.95, 1} = 3.841$$

Since $d > \chi^2_{1-\alpha, t-1}$, reject H_0

10.4.10,

10.4.10 From table: average turnovers per game

$$\lambda = \frac{800}{440} = 1.8182$$

H_0: the data fits a Poisson dist with $\lambda = 1.8182$
($p_X(k) = p_0(k)$)

H_0 : $p_X(k) \neq p_0(k)$

Caculate $n\hat{p}_{io}$ ~~from~~ (estimated frequencies) with $p_X(k) = \frac{\lambda^k e^{-\lambda}}{k!}$

Turnovers	observed k_i	estimated $n\hat{p}_{io}$	$(k_i - n\hat{p}_{io})^2 / n\hat{p}_{io}$
0	75	71.42	0.1795
1	125	129.86	0.1819
2	126	118.05	0.5354
3	60	71.55	1.8645
4	34	32.52	0.0674
5	13	11.83	0.1157
6+	7	4.77	1.0425
Total	440	440	3.9869

Test Stat : $d_1 = \sum \frac{(k_i - n\hat{p}_{io})^2}{n\hat{p}_{io}} = 3.9869$

From χ^2 - dist table

$$\chi^2_{1-\alpha, t-1-s} = \chi^2_{0.95, 6-1-1} = 9.488 > d_1$$

Therefore, fail to reject H_0

10.5.6

10.5.6

H_0: Blood pressure of the child and father are independent

H_0: Blood pressure of the child and father are not independent

Calculate ~~Expe~~ expected values by $\left(\frac{r_i}{n} \times \frac{c_i}{n}\right) n$:

Father \ Child	Lower	Middle	Upper	Tot
Lower	11.12	11.48	10.40	33
Middle	10.45	10.78	9.77	31
Upper	9.43	9.74	8.83	28
Total	31	32	29	92

Calculate ~~observed~~ $(k_{ij} - np_iq_j)^2 / np_iq_j$:

Father \ Child	lower	Middle	Upper
Lower	0.746	0.200	0.554
Middle	0.029	0.004	0.061
Upper	1.248	0.007	1.138

Test Stat: $d_2 = \sum\sum \frac{(k_{ij} - np_iq_j)^2}{np_iq_j} = 3.99$

$df = (r-1)(c-1) = 2 \times 2 = 4$

From χ^2 dist table: $\chi^2_{1-\alpha, df} = \chi^2_{0.95, 4} = 9.488 > d_2$

$\Rightarrow$ fail to reject H_0

Randomization Test in R:

```
> # init
> tyf <- c(3.525, 3.625, 3.383, 3.625, 3.661, 3.791, 3.941, 3.781, 3.660, 3.733)
> arm <- c(2.923, 3.385, 3.154, 3.363, 3.226, 3.283, 3.427, 3.437, 3.746, 3.438)
> cmb <- c(tyf, arm)
>
> # observed Test statistic
> Tobs = mean(tyf) - mean(arm)
>
> # Randomization Test
> Ts = vector(,1000)
> for (i in 1:1000){
+     smpl = sample(cmb, 20)
+     smpl1 = smpl[1:10]
+     smpl2 = smpl[11:20]
+     Ts[i] = mean(smpl1) - mean(smpl2)
+ }
>
> # plot
> hist(Ts)
> abline(v=c(-1,1)*Tobs,lwd=2,col="red")
> plot(density(Ts))
> abline(v=c(-1,1)*Tobs,lwd=2,col="red")
>
> # calculate p-value proportion of samples have test statistics less than -1 * Tobs, or greater than Tobs
> sum(Ts > Tobs | Ts < -1*Tobs)/1000
[1] 0.002
```

⇒ P value is 0.002