Formulaire opérateurs différentielles

Nabla

Cartésien	Cylindrique	Sphérique
$\overrightarrow{\nabla} = \begin{bmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \\ \dfrac{\partial}{\partial z} \end{bmatrix}$	$\overrightarrow{\nabla} = \begin{bmatrix} \dfrac{\partial}{\partial r} \\ \dfrac{1}{r}\dfrac{\partial}{\partial \theta} \\ \dfrac{\partial}{\partial z} \end{bmatrix}$	$\overrightarrow{\nabla} = \begin{bmatrix} \dfrac{\partial}{\partial r} \\ \dfrac{1}{r}\dfrac{\partial}{\partial \theta} \\ \dfrac{1}{r\sin(\theta)}\dfrac{\partial}{\partial \varphi} \end{bmatrix}$

Opérateurs différentielles

div	$\overrightarrow{grad}$	$\overrightarrow{rot}$	Δ
$\overrightarrow{\nabla}\cdot$	$\overrightarrow{\nabla}$	$\overrightarrow{\nabla}\wedge$	$\overrightarrow{\nabla}\cdot\overrightarrow{\nabla} = div\ \overrightarrow{grad}$

divergence

Cartésien	Cylindrique	Sphérique
$\frac{\partial}{\partial x}A_x + \frac{\partial}{\partial y}A_y + \frac{\partial}{\partial z}A_z$	$\frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial}{\partial \theta}A_\theta + \frac{\partial}{\partial z}A_z$	$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial \varphi}A_\varphi$

gradient d'un champ scalaire

<u>Cartésien</u>	<u>Cylindrique</u>	<u>Sphérique</u>
$\begin{bmatrix} \frac{\partial}{\partial x} A \\ \frac{\partial}{\partial y} A \\ \frac{\partial}{\partial z} A \end{bmatrix}$	$\begin{bmatrix} \frac{\partial}{\partial r} A \\ \frac{1}{r} \frac{\partial}{\partial \theta} A \\ \frac{\partial}{\partial z} A \end{bmatrix}$	$\begin{bmatrix} \frac{\partial}{\partial r} A \\ \frac{1}{r} \frac{\partial}{\partial \varphi} A \\ \frac{1}{r\sin(\theta)} \frac{\partial}{\partial \varphi} A \end{bmatrix}$

rotationnel

<u>Cartésien</u>	<u>Cylindrique</u>	<u>Sphérique</u>
$\vec{\nabla} = \begin{bmatrix} \frac{\partial}{\partial y} A_z - \frac{\partial}{\partial z} A_y \\ \frac{\partial}{\partial z} A_x - \frac{\partial}{\partial x} A_z \\ \frac{\partial}{\partial x} A_y - \frac{\partial}{\partial y} A_x \end{bmatrix}$	$\vec{\nabla} = \begin{bmatrix} \frac{1}{r} \frac{\partial}{\partial \theta} A_z - \frac{\partial}{\partial z} A_\theta \\ \frac{\partial}{\partial z} A_r - \frac{\partial}{\partial r} A_z \\ \frac{1}{r} \frac{\partial}{\partial r} (r A_\theta) - \frac{1}{r} \frac{\partial}{\partial \theta} A_r \end{bmatrix}$	$\vec{\nabla} = \begin{bmatrix} \frac{1}{r\sin\theta} \left(\frac{\partial}{\partial \theta} (\sin\theta\, A_\varphi) - \frac{\partial}{\partial \varphi} A_\theta \right) \\ \frac{1}{r} \left(\frac{1}{\sin\theta} \frac{\partial}{\partial \varphi} A_r - \frac{\partial}{\partial r} (r A_\varphi) \right) \\ \frac{1}{r} \left(\frac{\partial}{\partial r} (r A_\theta) - \frac{\partial}{\partial \theta} A_r \right) \end{bmatrix}$

laplacien scalaire

<u>Cartésien</u>	<u>Cylindrique</u>	<u>Sphérique</u>
$\frac{\partial^2}{\partial x^2} A + \frac{\partial^2}{\partial y^2} A + \frac{\partial^2}{\partial z^2} A$	$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} A \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} A + \frac{\partial^2}{\partial z^2} A$	$\frac{1}{r} \frac{\partial^2}{\partial r^2} (rA) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial}{\partial \theta} A \right) + \frac{1}{r^2 \sin^2(\theta)} \frac{\partial^2}{\partial \varphi^2} A$

Résultats vraiment utiles

$$\overrightarrow{rot}\ (\overrightarrow{grad}\ \ A) = \overrightarrow{0}$$

$\longrightarrow$ **Consequence :** On a que si A est tq $\overrightarrow{rot}\overrightarrow{A} = \overrightarrow{0}$, alors on a l'$\exists$ d'un potentiel, càd on a affaire à un champ de gradient

$$div\ (\overrightarrow{rot}\ \ \overrightarrow{A}) = 0$$

$\longrightarrow$ **Consequence :** On a que, si on a un champ $\overrightarrow{U}$ est tq $div\overrightarrow{U} = 0$, alors il est **équivalent** que $\exists$ A tq $\overrightarrow{U} = \overrightarrow{rot}\ \overrightarrow{A}$

$$\overrightarrow{grad}\ (AB) = A\ \overrightarrow{grad}(B) + B\ \overrightarrow{grad}(A)$$

$$\overrightarrow{rot}\ (A\overrightarrow{B}) = A\ \overrightarrow{rot}(\overrightarrow{B}) + \overrightarrow{grad}(A) \wedge \overrightarrow{B}$$

$$div\ (A\overrightarrow{B}) = A\ div(\overrightarrow{B}) + \overrightarrow{grad}(A) \cdot \overrightarrow{B}$$

$$div\ (\overrightarrow{A} \wedge \overrightarrow{B}) = \overrightarrow{B} \cdot \overrightarrow{rot}(\overrightarrow{A}) - \overrightarrow{A} \cdot \overrightarrow{rot}\overrightarrow{B}$$

$$\Delta A = div(\overrightarrow{grad}\ A)$$

$$\overrightarrow{rot}(\overrightarrow{rot}\overrightarrow{A}) = \overrightarrow{grad}(div\ \ \overrightarrow{A}) - \Delta\overrightarrow{A}$$

$$\overrightarrow{grad}(\overrightarrow{A} \cdot \overrightarrow{B}) = \overrightarrow{A} \wedge \overrightarrow{rot}(\overrightarrow{B}) + (\overrightarrow{A} \cdot \overrightarrow{grad})(\overrightarrow{B}) + \overrightarrow{B} \wedge \overrightarrow{rot}(\overrightarrow{A}) + (\overrightarrow{B} \cdot \overrightarrow{grad})\overrightarrow{A}$$

$$\overrightarrow{rot}(\overrightarrow{A} \wedge \overrightarrow{B}) = \overrightarrow{A}div(\overrightarrow{B}) - (\overrightarrow{A} \cdot \overrightarrow{grad})(\overrightarrow{B}) - \overrightarrow{B}div(\overrightarrow{A}) + (\overrightarrow{B} \cdot \overrightarrow{grad})\overrightarrow{A}$$