Dynamics of an SIS-like audience applause model

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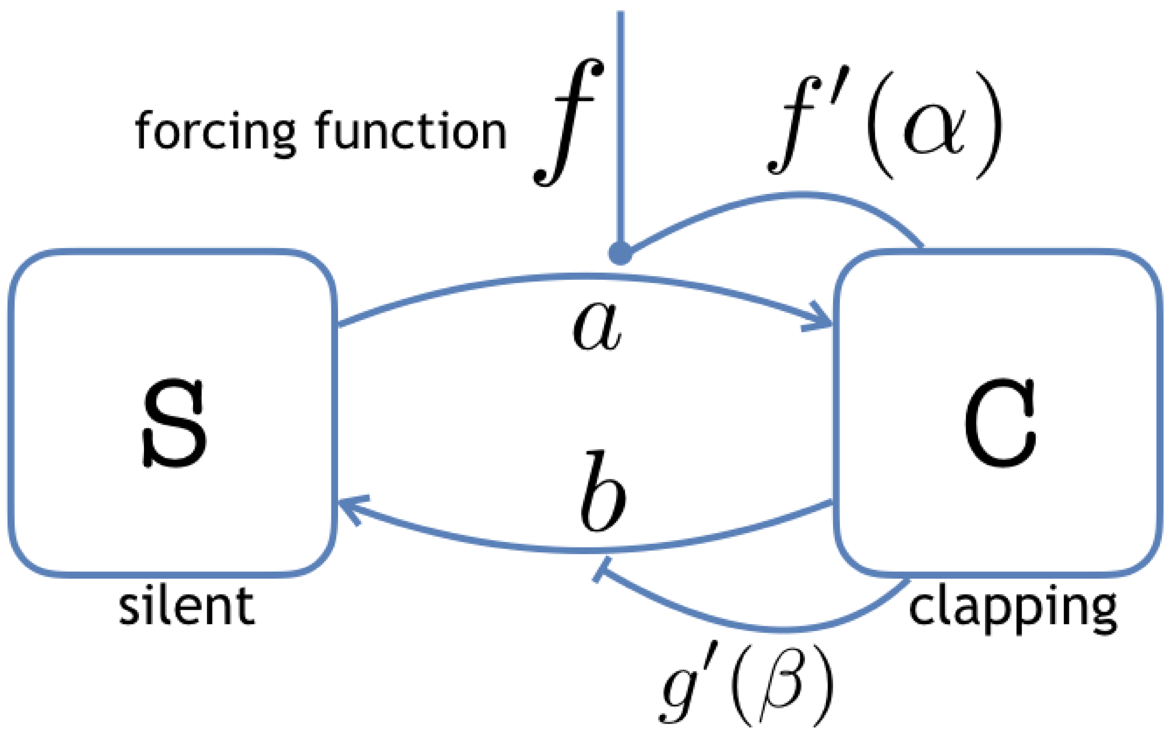
Objectives

- Create a model that simulates an audience applause
- Study the underlying dynamics of the system
- Find a correlation between the applause duration and audience size

Proposed Compartmental Model*



$$\frac{d}{dt}n_c = a(f + f - f'f)n_s - bg'n_c$$
$$\frac{d}{dt}n_s = bg'n_c + a(f + f - f'f)n_s$$

Differential equations of the system

$$f'(\alpha) = \frac{\alpha n_c}{N-1} \qquad g'(\beta) = \frac{1}{1+\frac{\beta n_c}{N-1}}$$

*based on the standard SIS-model [1]. g' is taken from the Michaelis-Menten equation[2].

Steady-state solutions

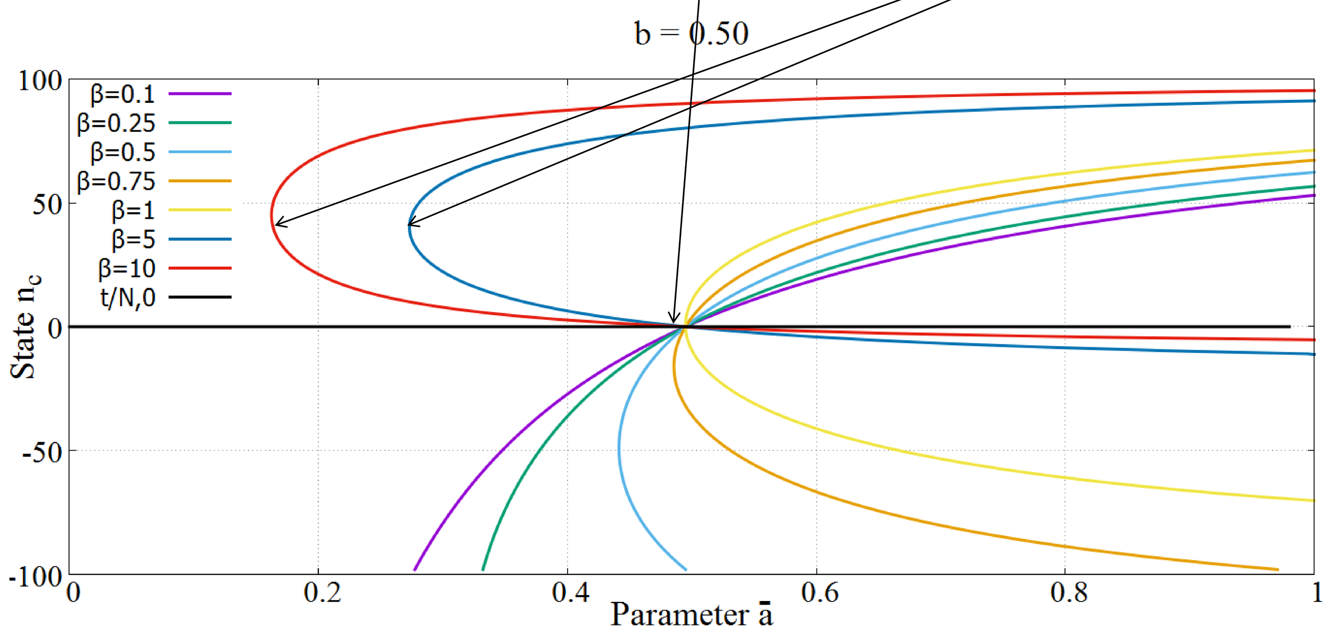
$$\bar{a}(N - n_c)(N - 1 + \beta n_c) = b(N-1)^2$$

Steady-state equation

$$\bar{a}_1 = \frac{b(N-1)}{N} \qquad \bar{a}_2 = \frac{b(N-1)^2}{(N - n_c^*)(N - 1 + \beta n_c^*)}$$

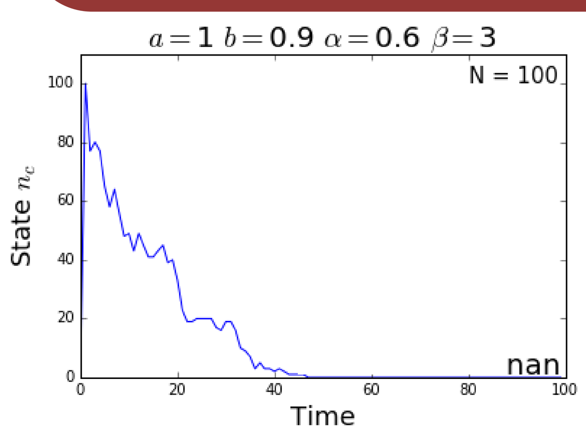
Critical Points

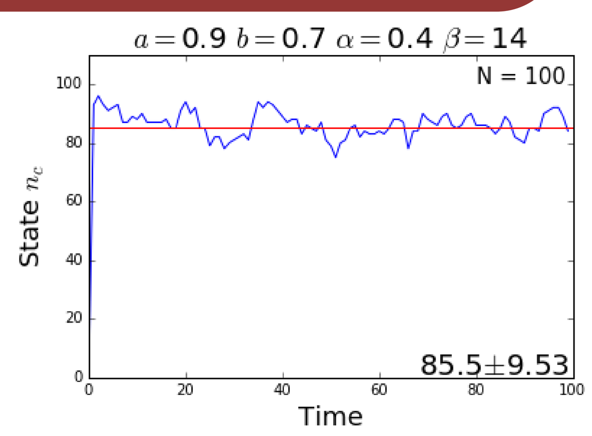
$N \rightarrow$ audience size
$n_c \rightarrow$ number of agents in state C
$\bar{a} = a\alpha$
$n^*_c = [1+(\beta - 1)N]/2\beta$

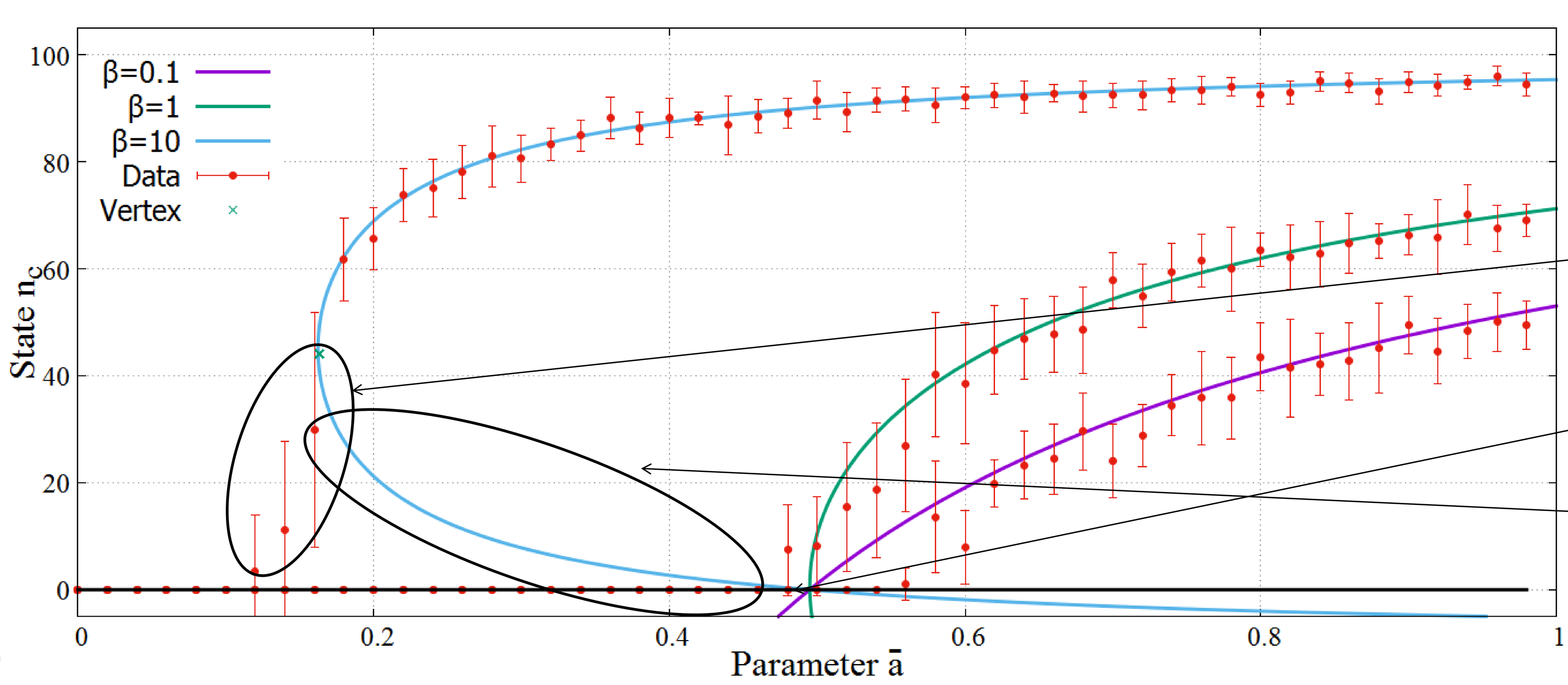


The phase space plot of the steady-state solutions with a set $b = 0.5$ and varying β. Included is the trivial steady-state $n_c = 0$. Values below 0 are extraneous.

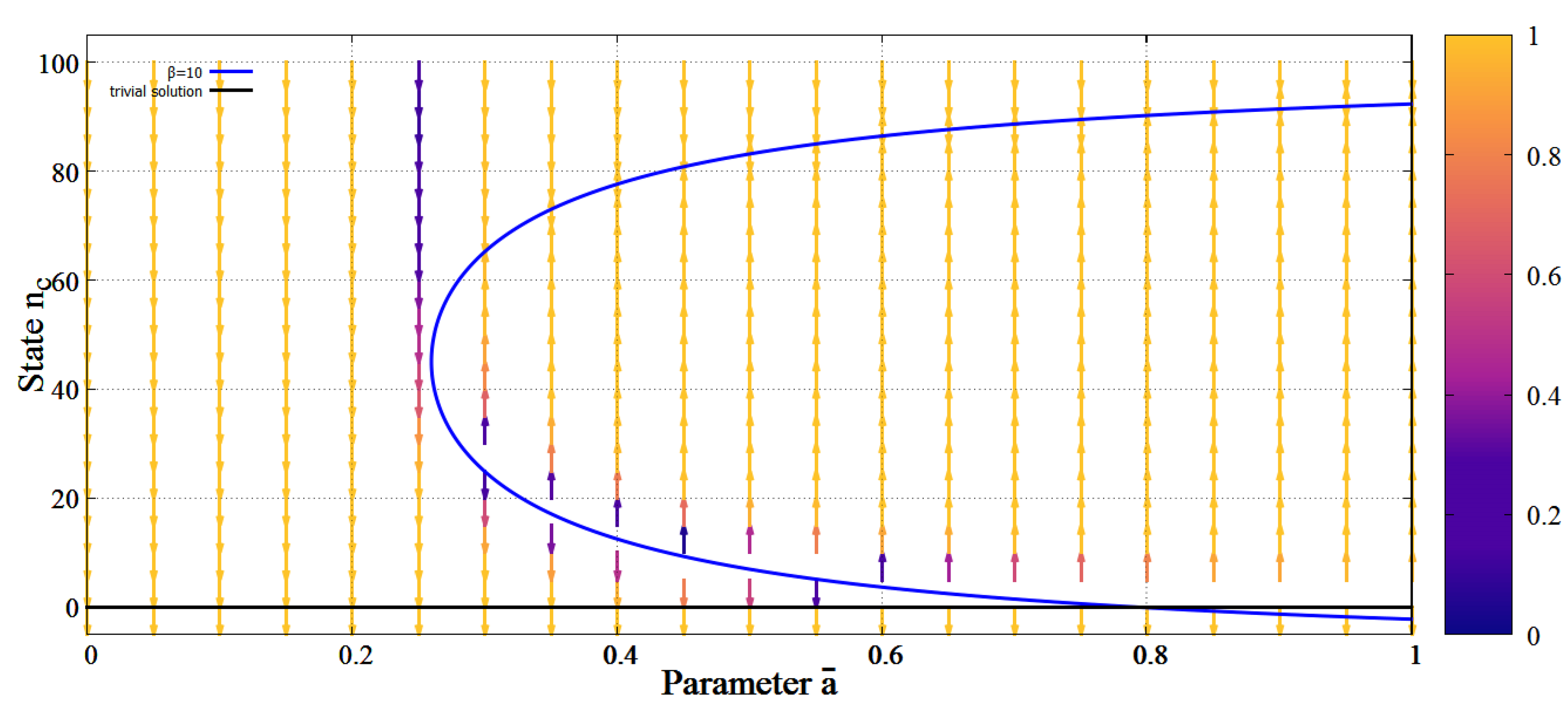
Analytical solutions are confirmed using an agent based Monte-Carlo method. Shown below are sample simulations with a trivial (left) and non-trivial (right) steady-state.







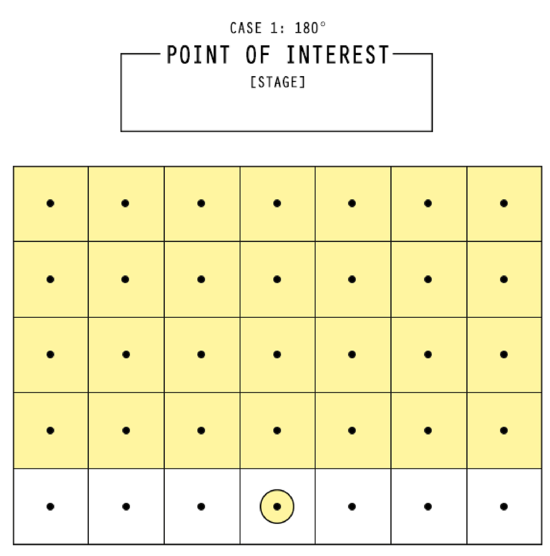
Bifurcation from the trivial to non-trivial solution occurs before the critical point $\bar{a}_2$, for $\beta > 1$, and at $\bar{a}_1$, for $\beta < 1$. Steady-state does not occur at lower branches of $\beta > 1$ solutions

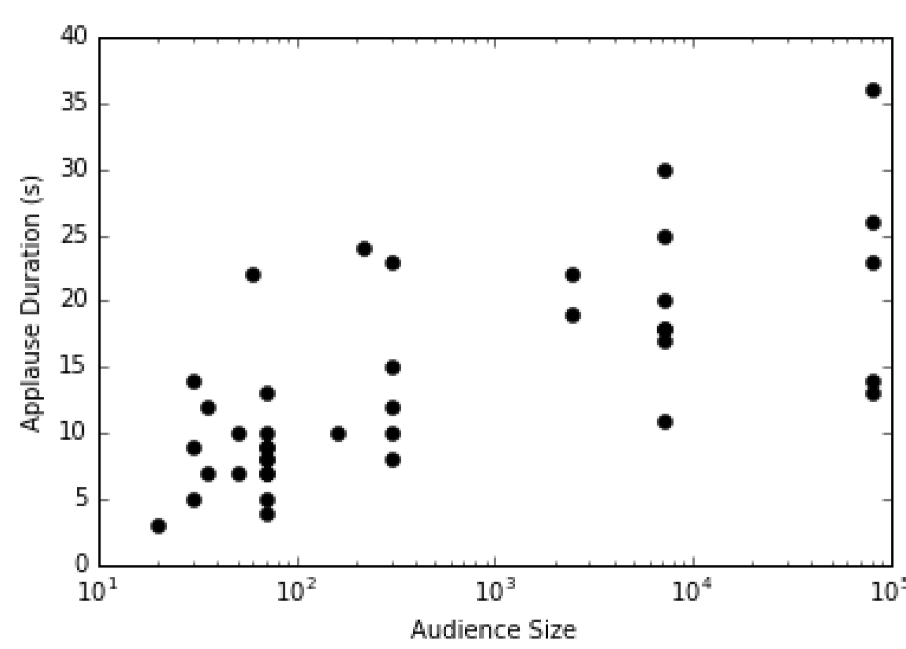


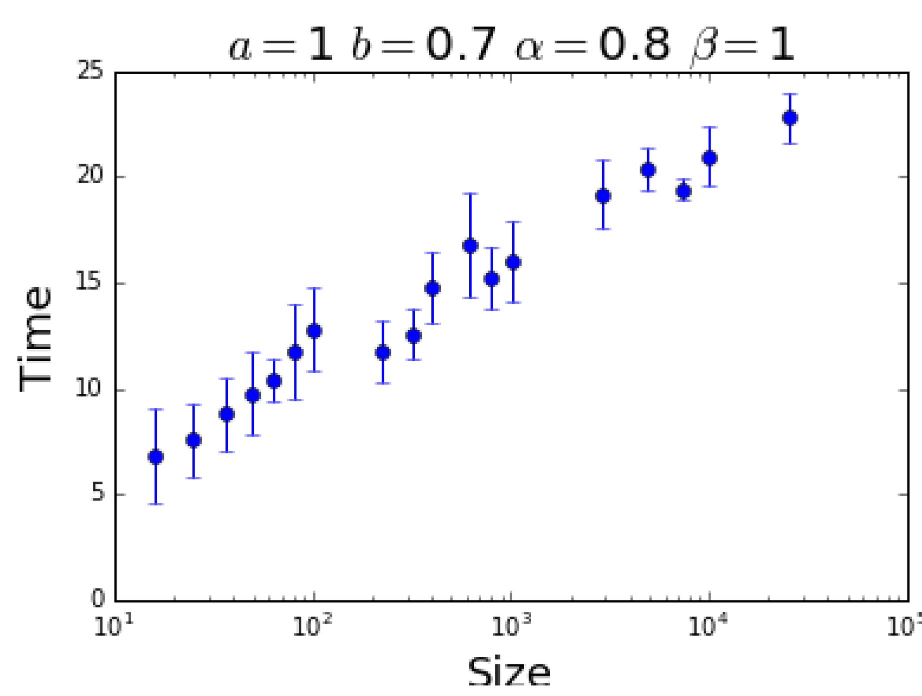
Removing the forcing function allows the system to act freely. The vectors point to the steady-state of the coordinate point ($\bar{a}$, n_c). The heat map represents the probability, 1 being 100% and 0 being 50%. Coordinate points on the lower branch are unstable and may settle to either trivial or non-trivial steady-states.

Incorporating Spatial Effects

Limiting the agents that can influence the reference agent allows simulations to have a finite applause time. This configuration was used to find parameter sets the best emulated that of real-life applause.







References

[1] P. Dodds and D. Watts, A generalized model of social and biological contagion, *J. Theoret. Biol.* **232**, 587 (2005).
[2] L. Johnson and R. Goody, The original michaelis constant: Translation of the 1913 michaelismenten paper, *Biochemistry* **50**, 8264 (2011).