Steady-state dynamics of an audience applause model

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Abstract

We propose a compartmental model for the dynamics of audience applause consisting of agents that are either silent (S) or clapping (C). The model includes two mechanisms. The first is a feedback mechanism that is parametrized by α which enhances the probability a that agents in state C influence those in state S to transition to C. Parameters α and a are simplified to $\bar{a}$, where $\bar{a} = a\alpha$ The second is a modulating mechanism that is parametrized by β which limits the probability b that those in state C transition to S. The steady-state dynamics of the resulting equations show that three possible solutions can exist, which includes a trivial case when all agents rest at S. Since negative steady-state values are extraneous solutions, only two of these solutions exist for $\bar{a}$ greater than a critical parameter value $\bar{a}_1$, For $\beta > 1$, we find a second critical parameter value $\bar{a}_2$ such that a second non-trivial non-extraneous but unstable solution exists. We confirm our results by agent-based Monte Carlo simulations.

Keywords: [89.65.-s] Social systems, [87.23.Ge] collective dynamics, [02.70.Uu] Monte Carlo

1 Introduction

An audience applauding after an event or performance is considered as a complex system where each audience member is an agent and the influence each member has among the others to applaud is the interaction among agents. Studies have been made to quantify this phenomenon from analyzing its sound[1], to the physics of its rhythm[2], and to its dynamics. It has been proposed that the dynamics of audience applause can be quantified as a contagion that spreads in the audience and can be modeled after a susceptible-infected-recovered (SIR) compartmental model [3]. In this case, applauding audience members are initially assigned to state S ("susceptible"), transition to state I ("infected") when they start to applaud, and then transition to state R ("recovered") once they cease to applaud.

The SIR model is appropriate for situations where previously infected agents are no longer susceptible. However, it is possible for audience members who have ceased clapping to applaud once more due to the duration of the audience applause, making the SIS model more appropriate. In the SIS model, agents in state I that transition to state S may transition back to state I and so on, given the appropriate parameters. This research adapts this approach to create a new model based on the standard SIS model and studies it's steady state dynamics.

2 Compartmental model of audience applause

Figure 1(a) shows a compartmental model that may be used to analyze the dynamics of audience applause. This is inspired by a simplistic dynamical SIR model[3]. Here, we propose an SCS model based on the standard SIS model[4] where each agent in the system is either in state S ("silent") or in state C ("clapping"). State S replaces the susceptible state and state C replaces the infected state I. The state of the system is given by the number of agents in each corresponding state, $\mathbf{n} \equiv (n_c, n_s)$. Since we assume that the total number of agents is fixed at $N = n_c + n_s$ such that $n_s = N - n_c$, $\mathbf{n}$ will be fully specified by n_c alone.

The parameters a and b are the transition probabilities for the respective transitions:

$$\mathrm{R}_1 \quad : \quad \mathrm{S} \xrightarrow{a} \mathrm{C} \tag{1}$$

$$\mathrm{R}_2 \quad : \quad \mathrm{C} \xrightarrow{b} \mathrm{S}. \tag{2}$$

The parameter f is a function that forces the transition R_1 for an indicated time interval such that when $f = 0$, the system behaves freely. We assume that agents in state S may be encouraged by the agents already in state C to spontaneously undergo R_1.

The function f' incorporates this feedback mechanism parametrized by α:

$$f'(\alpha) = \alpha \frac{n_c}{N-1}, \tag{3}$$

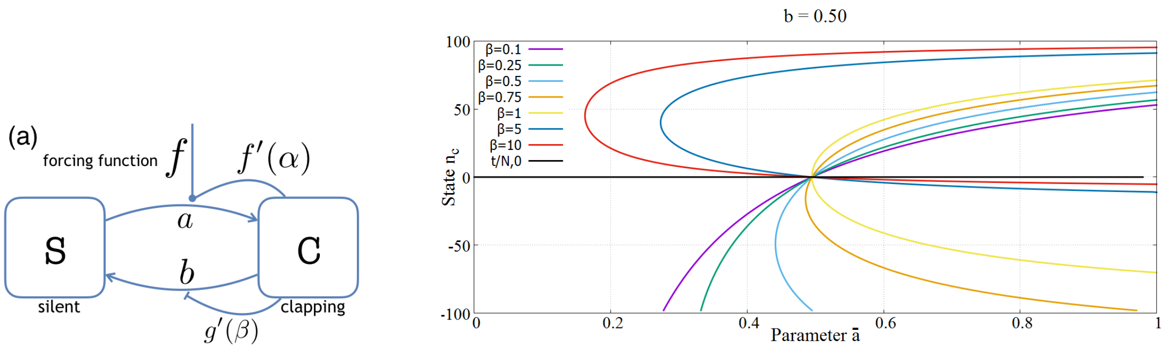

Figure 1: (a) The compartmental model for the audience applause where each agent is either silent (S) or clapping (C). The rates a and b and the functions f, f' and g' are explained in the text. (b) The parametric plot for $b = 0.5$ for different β-values. All solutions pass through the point $(\bar{a}, n_c) = (b, 0)$ for all values of β. The trivial solution $n_c = 0$ is also shown (black horizontal line).

where $0 \leq \alpha \leq 1$ for consistency with probabilistic interpretations. The probability for a spontaneous R_1 transition is directly proportional to around a fraction of the population in state C and α. The denominator is set to $N-1$ because an agent cannot spontaneously influence itself; it is only influenced by the rest of the population. The function is more effective when there are more agents in state C.

Another assumption is that the presence of a large applauding audience inhibits those already clapping to undergo R_2. The factor g' incorporates a modulation function for this inhibition and is parametrized by β:

$$g'(\beta) = \frac{1}{1 + \beta\, n_C/(N-1)} \tag{4}$$

where $\beta \geq 0$. This equation is taken from the Michaelis-Menten equation, which aims to model enzyme kinetics[5]. This completes the differential equations for the reactions (1) and (2) as follows.

$$\frac{d}{dt} n_c = a(f + f' - f'f)n_s - bg'n_c \tag{5}$$

$$\frac{d}{dt} n_s = bg'n_c - a(f + f' - f'f)n_s \tag{6}$$

These equations are consistent with the assumption that the total audience size is fixed, that is $dn_c/dt = -dn_s/dt$.

3 Steady-state solutions

The steady-state conditions are when the state $\mathbf{n}$ is fixed and when the forcing function expires ($d\mathbf{n}/dt = 0$ and $f = 0$) Once these conditions are achieved, (5) and (6) are simplified to the steady-state equation for n_c:

$$\bar{a}(N - n_c)(N - 1 + \beta n_c) = b(N-1)^2 \tag{7}$$

where $\bar{a} \equiv a\alpha$. The values of steady state n_c provided by this equation does not include the trivial solution $n_c = 0$. Figure 1(b) shows the possible steady-states n_c for different β values and for $b = 0.5$. Since $n_c > 0$, all curves below the $n_c = 0$ axis are extraneous. A similar pattern is observed for other values of b where all non-zero curves intersect the $n_c = 0$ line at around $\bar{a} = b$. Setting $n_c = 0$ in (7) gives us the critical $\bar{a} = \bar{a}_1$:

$$\bar{a}_1 = \frac{b(N-1)}{N} \tag{8}$$

which for $N \gg 1$ results to the expected $\bar{a}_1 \approx b$.

The non-trivial solution (7) is quadratic, with n_c resulting to two solutions for a given $\bar{a}$-value and parameters b and β. For $\beta \leq 1$, the steady-state solutions for $n_c > 0$ appear uniquely for $\bar{a} > \bar{a}_1$. However, when $\beta > 1$, two non-trivial solutions for which $n_c > 0$ appear between a new critical value of $\bar{a} = \bar{a}_2$, corresponding to the vertex of (7), and $\bar{a}_1$. This $\bar{a}_2$ is given by

$$\bar{a}_2 = \frac{b(N-1)^2}{(N - n_c^*)(N - 1 + \beta n_c^*)} \tag{9}$$

where $n_c^* \equiv [1 + (\beta - 1)N]/2\beta$. This non-trivial solution however, is unstable upon substitution with $\ddot{\mathbf{n}}$ resulting to $\ddot{\mathbf{n}} < 0$. Thus, the middle branch in the range $\bar{a} \in (\bar{a}_2, \bar{a}_1)$ is an unstable steady-state.

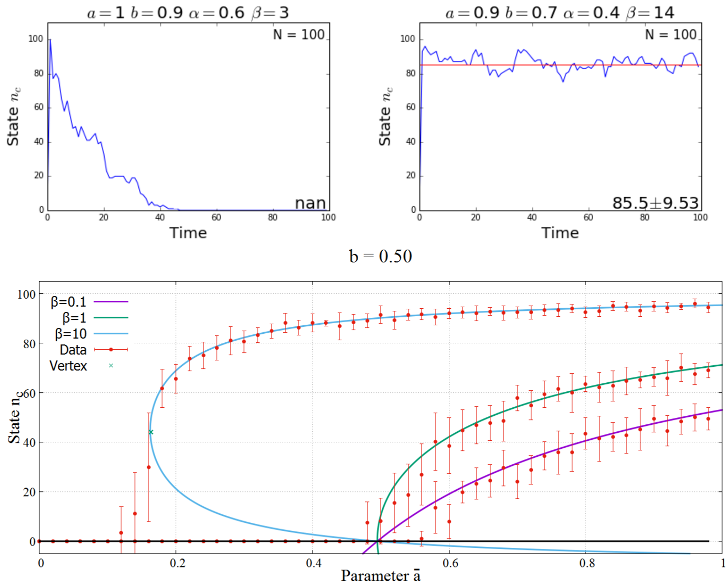

Figure 2: Sample simulations for $N = 100$ with the parameters: (a) $\bar{a} = 0.6$, $b = 0.9$ and $\beta = 3$ and (b) $\bar{a} = 0.36$, $b = 0.7$ and $\beta = 14$. The simulation in (a) settles to the trivial steady-state ($n_c = 0$) while the results in (b) show an average of $\langle n_c \rangle \approx 85.5 \pm 9.53$. (c) The average of the simulated steady-state values for different parameters are shown together with the theoretical predictions. The error bars provide the extent of the sample standard deviations computed. The cross indicates the vertex given by (9), i.e., $\bar{a}_2$

4 Simulation experiments

We confirm our analytical results by simulating the processes R_1 and R_2 via an agent-based Monte Carlo method. For each iteration, each agent is assigned a random number which is then compared to one of the transition probabilities:

$$P(\mathrm{R}_1) = a(f + f' - f'f) \tag{10}$$

$$P(\mathrm{R}_2) = bg' \tag{11}$$

Agents at state S transition to state C via R_1 with probability (10) and those at state C transition to state S via R_2 with probability (11) Monte Carlo procedure is done by drawing a uniform random number $u \in [0, 1]$ from a random number generator and comparing it with the corresponding probability P. If $u \leq P$ the chosen transition is allowed to occur.

Figure 2(a) and 2(b) show sample simulations given specific parameters that settle to a certain n_c as the simulation time t progresses. Since the values settles around some value of n_c beyond $t > 50$ iterations for this value of N, we can safely assume that the n_c for $t = 100$ is equivalent to n_c for $t \to \infty$. We took 10 sample runs with different initial random number generator states and recorded the final n_c value for each iteration. The mean and standard deviation of the steady-state values are then computed for each parameter space $(\bar{a}, b, \beta)$. The data points were then plotted with the corresponding parametrized differential curve shown in Fig. 2(c). For $\beta \leq 1$, the simulation is consistent with the trivial steady state solution until it reaches the critical point $b = \bar{a}$, after which follows non-trivial, non-extraneous solution consistent with (7). For $\beta > 1$, the simulation follows the trivial steady-state and then breaks away and approaches the vertex of (7) at $\bar{a}_2$, after which, continues to follow the upper branch of (7).

5 Conclusion

Simulations successfully emulate the compartmental model in accordance to the differential equation for the steady-state of the given system. A first critical point $\bar{a}_1$ is obtained such that one cannot obtain a steady-state applauding audience for $\bar{a}$ less than this critical value. Since a and α are average characteristics of the agents in the system, this implies that the sustenance of applause also depends on their b-value. This b-value is related to how generous they are in giving the applause. One can say that b is low for "die-hard" fans or for audience members who appreciated the act.

The feedback mechanism parameter β is an agent character related to how they are affected by already-applauding audience. Agents with higher β-values tend to generate very high steady-state values and create a critical number required to approach this steady-state value. In reality, this β-parameter may be time-dependent since the energy of applauding agents should be finite. It will be interesting how our model checks out with real-world applause dynamics.

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