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DYNAMICS OF AN SIS-LIKE AUDIENCE APPLAUSE MODEL

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Abstract

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Chapter 1

Applause Dynamics

1.1 Nature of Applause

The audience applause is one of the most ubiquitous and timeless social phenomenons observed in human culture. People would normally applaud to express their approval over a social event, and may even jeer, shout, snap, and boo, on top of clapping. Such behavior is also exhibited by animals such as gorillas and chimpanzees. People collectively seem to know when to start clapping in any occasion, whether it is during or after a speech, performance, sport event, etc. People also unconsciously know whether to continue clapping or to stop. Audience members extend their applause to musicians playing an outro or to being addressed, and stop immediately when the musician proceeds with the next song. Lastly, it is historically known that people who have been hired to willingly and consciously applaud are inserted in an audience in hopes to extend the applause for a social event. These observations of an audience applause being self-organized and somehow connected give rise to the question of what the nature of applause is and how people decide when and how long to clap.

1.2 Social Dynamics

Social economics is an interdisciplinary study of the interrelationship between group and individual behavior. Individual decisions made interactively with others have been modelled formally to be able to understand the dynamics. The underlying assumption is that individuals are influenced by the choices of others. From this assumption, it can be said that feedback loops exist since the past decisions of a certain individuals may influence future decisions of others. Examples of such social phenomena are crime, teenage pregnancy, and high school dropout rates.[1][2]

1.3 Background

1.3.1 Networks

A network is a collection of points,called vertices or nodes, connected by lines, called edges. Networks are normally used to represent systems that contain individual parts that are somehow connected, such as the internet, big data, or social interactions. These are used to study the nature of individual components and their dynamics. A few properties of networks are topology and homogeneity. Network topology refers to how the network is connected. The different topologies are named after the physical shape of the network, such as a ring or a line. A key topology is a fully-connected network wherein all nodes are completely connected to each other. Network homogeneity refers to the nodes of the network. A homogeneous network has individual nodes of the same characteristics and properties while a heterogeneous network contains different nodes.

1.3.2 Complex Systems

By investigating the mechanisms that determine the topology of the networks of aforementioned systems (internet, big data, etc.) properties previously not observed when studying the components individually emerge. This has lead to a new field of study that focuses on how relationships between components give rise to its collective behaviors and how the system interacts and forms relationships with its environment, called complex systems. Intrinsic to complex systems is that they are hard to model due to the complexity of the interacting components. Over-simplifying the complexity may lead to the failure of the model. Included in as a property of complexity is the inclination of a large system to mutate after reaching a critical point or state. Methods on studying complex systems have been established and broken down to three parts, data acquisition, modelling, and measuring complexity. Data acquisition generally tends towards statistical learning and data mining. Modelling turns to mainly two methods, cellular automata and agent-based modelling. The cellular automata is a simple mathematical model used to investigate self-organization in statistical mechanics. This model represents spatial dynamics and highlights local interactions, spatial heterogeneity, and large-scale aggregate patterns. Measuring complexity has demanded an inclusion of new metrics. These metrics include average path length, clustering coefficient, degree distribution, and spectral properties.

1.4 The audience as a complex system

Human behavior is usually studied qualitatively (under psychology or sociology) and is notoriously hard to quantify due to all the possible parameters and the difficulty in creating a controlled environment. The audience applause is an example of human behavior that is a collective of interacting agents with underlying dynamics. This allows us to treat the audience as a complex system in order to study its complexity and dynamics.

1.5 Known Studies and Models

There have been very little studies made on applause, down to the sound, its rhythm and its dynamics. One particular study treats the applause as a contagion that propagates through the audience, allowing it to be modelled using an SIR-model. The SIR-model allows each unit in the model (for this case, each person henceforth referred to as an agent) to have 3 states, susceptible, infected, and recovered. Each agent is initally silent before the applause (susceptible). After which they start to applaud (infected). Finally stop clapping (recovered). Such a model would be appropriate if the agents no longer clap again after stopping, but cases exist where the agent my stop clapping prematurely, and then feel obliged to clap again due to the fact that the rest still continue to do so. With that, an SIS-like compartmental model with only two states(susceptible or infected) is adapted to properly account for such cases.

1.6 Problem Statement

The dynamics of the applause of an audience with N agents using an SIS epidemic model is presented and analyzed. *The same is done with an agent-based model with spatial effects.* The agents are connected via two dimensional lattice network and are initially fully connected. Later on, the agents will observe a modified, extended Moore neighborhood in order to incorporate spatial effects. The network is assumed to be homogeneous for simplicity; all agents share the same parameters for a given simulation. Agents are assumed to clap immediately after a performance, and may continue to do so depending on their given parameters. The research seeks only to find a correlation between the applause duration and the audience size, ultimately showing interdependence among the agents of the system. Why people applaud and to what are not investigated due to its unquantifiable nature.

Chapter 2

Monte Carlo Compartmental Method

2.1 The applause as a contagion

Previous studies have treated the applause as a contagion that spreads across the audience. Doing this allows the use of an SIR compartmental model to simulate the complex system. A more appropriate model would be an SIS model since this would account for the people who stop clapping but may do so again.

2.2 Compartmental Model

2.2.1 States

An SIS-like model is proposed where each agent in the system is either in state S (silent) or state C (clapping). State S replaces the susceptible state while state C replaces the infected state. The state of the system is given by the number of agents in each corresponding state, $\vec{n} \equiv (n_c, n_s)$. Since we assume that the total number of agents is fixed at $N = n_c + n_s$ such that $n_s = N - n_c$, $\vec{n}$ will be fully specified by n_c alone.

2.2.2 Parameters

The parameters a and b are the transition probabilities for the respective transitions:

$$\mathrm{R}_1 \; : \; \mathrm{S} \xrightarrow{a} \mathrm{C} \tag{2.1}$$

$$\mathrm{R}_2 \; : \; \mathrm{C} \xrightarrow{b} \mathrm{S}. \tag{2.2}$$

The parameter f is a function that forces the transition R_1 for an indicated time interval such that when $f = 0$, the system behaves freely. We assume that agents in state S may be encouraged by the agents already in state C to spontaneously undergo R_1.

2.2.3 Functions

The function f' incorporates this feedback mechanism parametrized by α:

$$f'(\alpha) = \alpha \frac{n_c}{N-1}, \tag{2.3}$$

where $0 \leq \alpha \leq 1$ for consistency with probabilistic interpretations. The probability for a spontaneous R_1 transition is directly proportional to around a fraction of the population in state C and α. The denominator is set to $N-1$ because an agent cannot spontaneously influence itself; it is only influenced by the rest of the population. The function is more effective when there are more agents in state C.

Another assumption is that the presence of a large applauding audience inhibits those already clapping to undergo R_2. The factor g' incorporates a modulation function for this inhibition and is parametrized by β:

$$g'(\beta) = \frac{1}{1 + \beta \, n_C/(N-1)} \tag{2.4}$$

where $\beta \geq 0$. This equation is taken from the Michaelis-Menten equation, which aims to model enzyme kinetics[**?**]. This completes the differential equations for the reactions (2.1) and (2.2) as follows.

$$\frac{d}{dt} n_c = a(f + f' - f'f)n_s - bg'n_c \tag{2.5}$$

$$\frac{d}{dt} n_s = bg'n_c - a(f + f' - f'f)n_s \tag{2.6}$$

These equations are consistent with the assumption that the total audience size is fixed, that is $dn_c/dt = -dn_s/dt$.

2.3 Simulation Alogrithm

Chapter 3

Steady-state dynamics of the audience applause

3.1 Steady-state equation for different cases

The steady-state conditions are when the state $\vec{n}$ is fixed and when the forcing function expires ($d\vec{n}/dt = 0$ and $f = 0$) Once these conditions are achieved, (2.5) and (2.6) are simplified to the steady-state equation for n_c:

$$\bar{a}(N - n_c)(N - 1 + \beta n_c) = b(N - 1)^2 \tag{3.1}$$

where $\bar{a} \equiv a\alpha$. The values of steady state n_c provided by this equation does not include the trivial solution $n_c = 0$. Figure **??**(b) shows the possible steady-states n_c for different β values and for $b = 0.5$. Since $n_c > 0$, all curves below the $n_c = 0$ axis are extraneous. A similar pattern is observed for other values of b where all non-zero curves intersect the $n_c = 0$ line at around $\bar{a} = b$.

3.1.1 Case 1

Setting $n_c = 0$ in (3.1) gives us the critical $\bar{a} = \bar{a}_1$:

$$\bar{a}_1 = \frac{b(N-1)}{N} \tag{3.2}$$

which for $N \gg 1$ results to the expected $\bar{a}_1 \approx b$.

The non-trivial solution (3.1) is quadratic, with n_c resulting to two solutions for a given $\bar{a}$-value and parameters b and β. For $\beta \leq 1$, the steady-state solutions for $n_c > 0$ appear uniquely for $\bar{a} > \bar{a}_1$. However, when $\beta > 1$, two non-trivial solutions for which $n_c > 0$ appear between a new critical value of $\bar{a} = \bar{a}_2$, corresponding to the vertex of (3.1), and $\bar{a}_1$. This $\bar{a}_2$ is given by

$$\bar{a}_2 = \frac{b(N-1)^2}{(N-n_c^*)(N-1+\beta n_c^*)} \tag{3.3}$$

where $n_c^* \equiv [1 + (\beta - 1)N]/2\beta$. This non-trivial solution however, is unstable upon substitution with $\ddot{\vec{n}}$ resulting to $\ddot{\vec{n}} < 0$. Thus, the middle branch in the range $\bar{a} \in (\bar{a}_2, \bar{a}_1)$ is an unstable steady-state.

3.2 Phase space for varying parameters

By parametrizing the steady-state functions as functions of time, t, we can plot the steady-state population, nC, versus $\bar{a}$. This phase space gives us the steady-state value for any given $\bar{a}$, b, and β. For the graphs shown, b is typically constant, and can be identified in the title, while β is varied, shown by the various colored curves. Both trivial and non-trivial steady-state solutions are shown in the graph. In order to determine which steady-state solution is correct, simulations are performed and their results plotted against the curves.

3.3 Simulation experiments

We confirm our analytical results by simulating the processes R_1 and R_2 via an agent-based Monte Carlo method. For each iteration, each agent is assigned a random number which is then compared to one of the transition probabilities:

$$P(\mathrm{R}_1) = a(f + f' - f'f) \tag{3.4}$$

$$P(\mathrm{R}_2) = bg' \tag{3.5}$$

Agents at state S transition to state C via R_1 with probability (3.4) and those at state C transition to state S via R_2 with probability (3.5) Monte Carlo procedure is done by drawing a uniform random number $u \in [0, 1]$ from a random number generator and comparing it with the corresponding probability P. If $u \leq P$ the chosen transition is allowed to occur.

Figure **??**(a) and **??**(b) show sample simulations given specific parameters that settle to a certain n_c as the simulation time t progresses. Since the values settles around some value of n_c beyond $t > 50$ iterations for this value of N, we can safely assume that the n_c for $t = 100$ is equivalent to n_c for $t \to \infty$. We took 10 sample runs with different initial random number generator states and recorded the final n_c value for each iteration. The mean and standard deviation of the steady-state values are then computed for each parameter space $(\bar{a}, b, \beta)$. The data points were then plotted with the corresponding parametrized differential curve shown in Fig. **??**(c). For $\beta \leq 1$, the simulation is consistent with the trivial steady state solution until it reaches the critical point $b = \bar{a}$, after which follows non-trivial, non-extraneous solution consistent with (3.1). For $\beta > 1$, the simulation follows the trivial steady-state and then breaks away and approaches the vertex of (3.1) at $\bar{a}_2$, after which, continues to follow the upper branch of (3.1).

3.3.1 Confirmation of model by simulation

Shown are the results of the simulation experiments plotted against the steady-state solutions. For all cases, the simulation initially follows the trivial solution until it reaches a certain point and bifurcates to the non-trivial point. For $\beta < 1$ the point of bifurcation is the critical point $\bar{a}$. For $\beta > 1$, the point of bifurcation is an unknown point before $\bar{a}_2$, the vertex of the curve. The simulations jump from the trivial solution to points unpredictable by the analytics and then arrive somewhere near the vertex at $\bar{a}_2$. After arriving at the critical point, the simulations then follow the upper branch of the non-trivial solution curve. At this point, the lower branch of the non-trivial solution is unknown and will be further investigated.

3.3.2 Unstable points

For the previous simulation experiments, all agents start at state S and then forced to transition to state C. To investigate the properties of the lower branch on the non-trivial solution, we vary the number of agents that start the simulation in state C and remove the forcing function. This allows us to determine where the system will settle given the specific starting nC value, more particularly for nC values near the lower branch of the non-trivial solution. Shown are the new phase space graphs that include arrows that point to where that nC value for the given $\bar{a}$ settles. The color of the arrows represents the probability of going towards that direction. Intuitively for $\beta < 1$, all nC values point towards the trivial solution for all $\bar{a}$ values less than $\bar{a}_1$. All nC with $\bar{a}$ values greater than $\bar{a}_1$ point towards the non-trival solution. For $\beta > 1$, what differs is the behavior at the vertex and the lower branch of the non-trivial solution. nC values above the vertex have an estimated 50% to either settle at the vertex or zero, while nC values below the vertex absolutely settle to zero. Points slightly above the lower branch tend more towards settling to the upper branch but still have a chance to settle at 0. Likewise for points below the lower branch. Points within the lower branch have a 50% of settling to either trivial or non trivial solution.

Chapter 4

Incorporating Spatial Effects

4.1 Real-life applause and the standing ovation effect

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4.2 Different configurations for field of vision

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Chapter 5

Conclusions and Recommendations

In this thesis, we have reviewed the definition of the revival time associated with the wave packet revival phenomena. We have obtained approximate analytical expressions for the classical time scale T_{cl} and quantum revival time scale T_{rev} for the single- and double- δ-perturbed infinite square well system.

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