Modeling and Simulation of Earth's Average Global Temperature

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Abstract

The present report outlines the findings from a series of simulations designed to model the Earth's average global temperature. Utilizing a range of models, from a basic Energy Balance Model (EBM) to more complex variations incorporating factors like albedo and emissivity, this study aims to provide insights into the dynamics of Earth's climate system and the factors influencing its temperature.

1 Introduction

Climate change, characterized by alterations in Earth's average temperature, is a critical global issue. This report focuses on modeling the Earth's temperature using various simulation models. The objective is to understand how different parameters affect global temperature. We explore several models, including the Basic EBM and its extensions, to simulate and analyze temperature changes under different conditions.

2 Methodology

The methodology employed in this project involves simulating Earth's temperature using several models. The Basic EBM is the starting point, which is then expanded to include factors like albedo variation and emissivity. The simulations are conducted using the Octave software, leveraging its numerical computation capabilities to solve complex differential equations inherent in the models.

3 Results

3.1 Energy Balance Model

The equilibrium temperature of the system in the basic Energy Balance Model (EBM) is calculated by balancing the incoming solar radiation with the outgoing longwave radiation. This balance is represented by the equation:

$$Q(1-\alpha) = \sigma T_{eq}^4 \tag{1}$$

where:

- Q is the average global solar radiation (342 W/m^2).
- α is the albedo of the planet (0.3).
- σ is the Stefan-Boltzmann constant (5.67×10^{-8} W/m^2/K^4).
- T_{eq} is the equilibrium temperature in Kelvin.

3.1.1 Basic EBM

Rearranging the equation to solve for T_{eq}, we get:

$$T_{eq} = \left(\frac{Q(1-\alpha)}{\sigma}\right)^{\frac{1}{4}} \tag{2}$$

Substituting the values:

$$T_{eq} = \left(\frac{342 \times (1-0.3)}{5.67 \times 10^{-8}}\right)^{\frac{1}{4}} \tag{3}$$

Calculating this, we find:

$$T_{eq} \approx 254.91\,\mathrm{K} \tag{4}$$

This is the theoretical equilibrium temperature of the Earth according to this simple model, which does not account for the greenhouse effect.

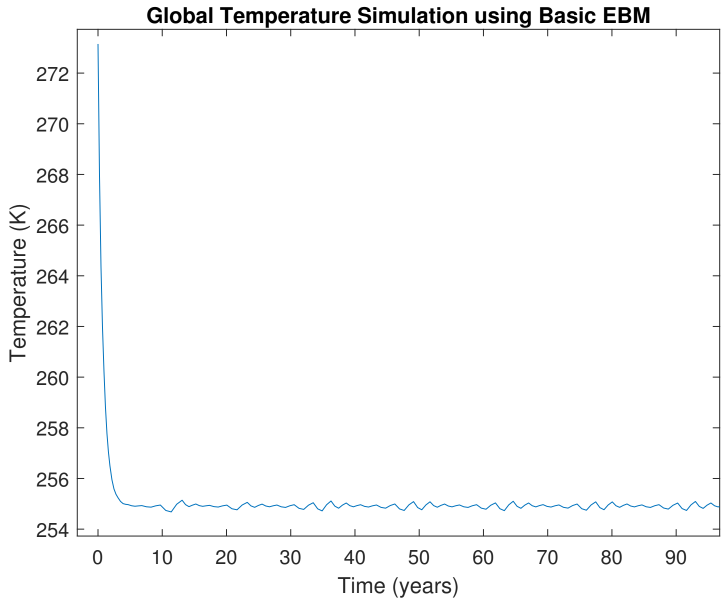


Figure 1: Simulation results for the Basic EBM model.

3.1.2 Albedo Variation

The albedo of the Earth is not constant, and varies with the seasons. The albedo is higher in the winter due to the presence of snow and ice, and lower in the summer due to the presence of vegetation.

For an albedo (α) of 0 (absorbing all incoming solar radiation), the equilibrium temperature is approximately 278.68 Kelvin. For an albedo of 1 (reflecting all incoming solar radiation), the equilibrium temperature is 0 Kelvin. These results highlight the significant impact of albedo on the Earth's temperature. An albedo of 1 is an extreme and theoretical scenario where all incoming solar radiation is reflected, leading to no energy absorption and hence a theoretical equilibrium temperature of absolute zero. In contrast, an albedo of 0 leads to higher temperatures due to the absorption of all incoming solar radiation.

3.2 Emissivity

The equilibrium temperature of the system, considering emissivity in the Energy Balance Model (EBM), is calculated by modifying the original equation

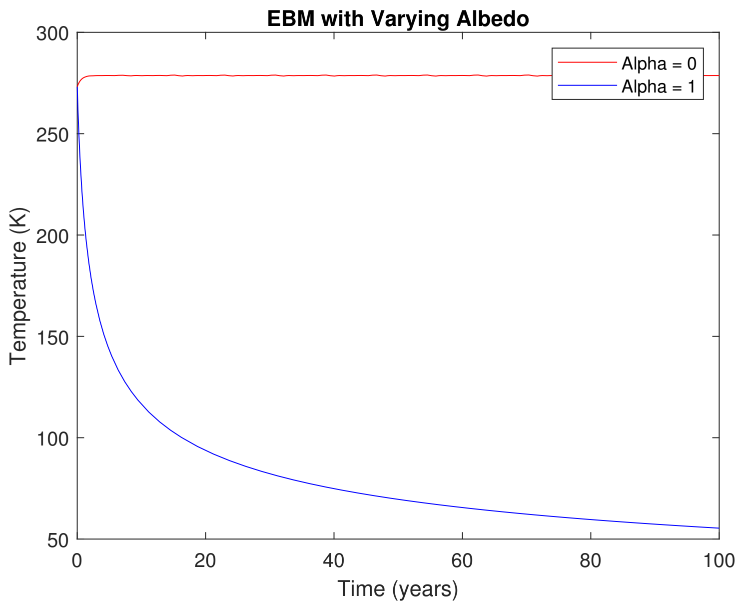


Figure 2: Simulation results for the Basic EBM model with albedo extremes.

to include the emissivity factor.

The modified equation at equilibrium is:

$$0 = Q(1-\alpha) - \epsilon\sigma T_{eq}^4 \tag{5}$$

Rearranging this equation to solve for the equilibrium temperature (T_{eq}):

$$\epsilon\sigma T_{eq}^4 = Q(1-\alpha) \tag{6}$$

$$T_{eq}^4 = \frac{Q(1-\alpha)}{\epsilon\sigma} \tag{7}$$

$$T_{eq} = \left(\frac{Q(1-\alpha)}{\epsilon\sigma}\right)^{\frac{1}{4}} \tag{8}$$

Substituting the values:

- $Q = 342\,\mathrm{W/m^2}$ (Average global solar radiation).
- $\alpha = 0.3$ (Albedo).

- $\sigma = 5.67 \times 10^{-8}\,\mathrm{W/m^2/K^4}$ (Stefan-Boltzmann constant).
- $\epsilon = 0.61$ (Emissivity, a typical value for Earth).

The equilibrium temperature (T_{eq}) is calculated as:

$$T_{eq} = \left(\frac{342 \times (1 - 0.3)}{0.61 \times 5.67 \times 10^{-8}} \right)^{\frac{1}{4}} \approx 288.44\,\mathrm{K} \tag{9}$$

Therefore, the equilibrium temperature of approximately 288.44 Kelvin is equivalent to about 15.29°C (Celsius).

This equilibrium temperature is much closer to the actual average surface temperature of the Earth, demonstrating the significant impact of emissivity in the energy balance model.

3.3 Temperature Dependent OLR

In this model, the Outgoing Longwave Radiation (OLR) is considered to be a function of temperature, represented by $A + BT$, where A and B are constants. This approach acknowledges the fact that the Earth's radiation into space varies with its surface temperature.

The equilibrium temperature under this model, represented by the equation $R\frac{dT}{dt} = Q(1 - \alpha) - (A + BT)$, aligns more closely with empirical observations when compared to the basic EBM. This model also accounts for feedback mechanisms that are temperature-dependent, offering a more nuanced understanding of the Earth's climate system.

3.4 Temperature Dependent Albedo

The model incorporating temperature-dependent albedo reflects the dynamic nature of the Earth's surface. As the temperature varies, so does the albedo, due to factors like melting ice and snow cover. This is represented by the equation $\alpha(T) = 0.5 + 0.2 \tanh(0.1(265 - T - 273.5))$.

This model introduces a feedback mechanism where rising temperatures can lead to decreased albedo, further accelerating warming—a phenomenon observed in the melting of polar ice caps.

4 Discussion

4.1 Analysis of Emissivity and Temperature-Dependent OLR Models

The results from the simulations raise interesting points of discussion, particularly regarding the relative accuracy of the models in predicting Earth's average temperature. The model incorporating emissivity (Question 3) yielded a temperature closer to the real-world average of 14.84°C, whereas the temperature-dependent OLR model (Question 4) deviated more significantly from this value.

4.1.1 Impact of Emissivity in Climate Models

The inclusion of emissivity in the energy balance model significantly enhances its realism. Emissivity, representing the Earth's ability to emit radiation, is a key factor in the Earth's energy balance. Our simulation using an emissivity factor close to Earth's real value (approximately 0.61) resulted in a temperature estimation that aligns more closely with the actual average global temperature. This underscores the importance of emissivity in modeling Earth's climate and suggests that it captures essential aspects of the Earth's radiation dynamics.

4.1.2 Challenges with Temperature-Dependent OLR

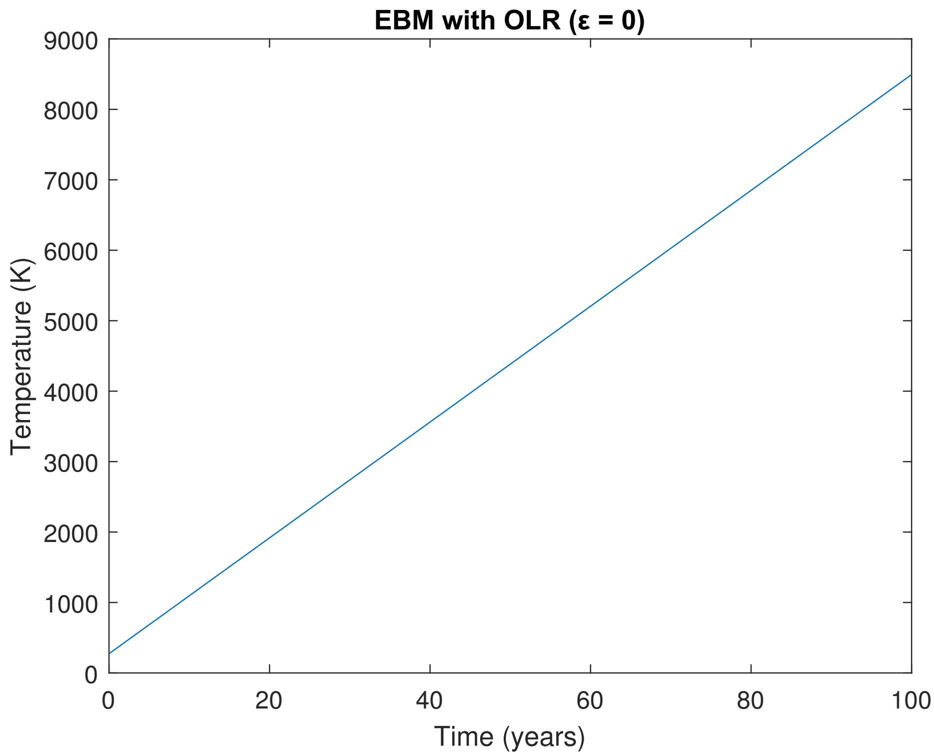
The model with temperature-dependent OLR, despite adding complexity, may not have been as effective in mimicking the actual climate system. The parameters A and B in the $A + BT$ formulation of OLR are crucial. If these parameters do not accurately reflect the Earth's real radiation properties, the model's predictions can deviate from real-world temperatures. This indicates the challenge of accurately calibrating climate models to match the complexities of the Earth's climate system.

4.2 Model Limitations and Realism

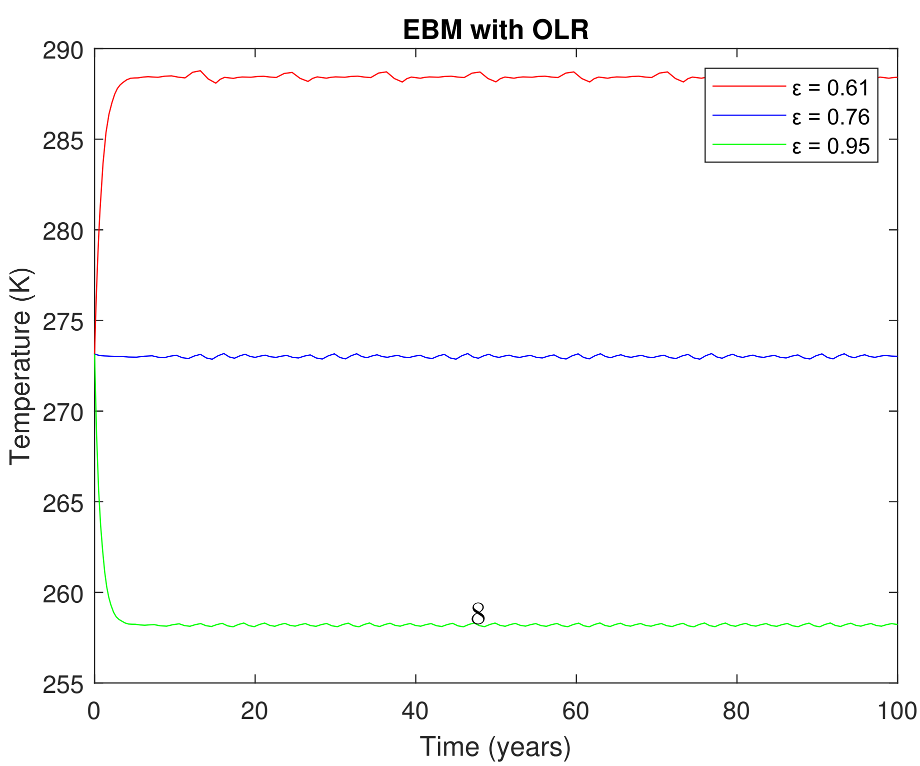
Both models are simplified representations of the actual climate system. The differences in their predictions highlight the impact of various climatic factors on Earth's temperature. It also emphasizes the need for precise parameter selection and calibration in climate modeling. The closer alignment of the emissivity model with the actual average temperature suggests that, for our models, the emissivity factor is more crucial in capturing the key aspects of Earth's thermal radiation balance compared to the linear relationship of temperature-dependent OLR.

5 Conclusion

The comparison between the two models demonstrates the nuanced nature of climate modeling. It highlights the importance of choosing and calibrating model parameters accurately to reflect the complexities of the real-world climate system.



(a) This plot demonstrates the hypothetical scenario where the Earth does not emit any longwave radiation, with $\epsilon = 0$, showing a temperature increase of nearly 85 Kelvin per year.



(b) EBM simulation with typical $\epsilon = 0.61$, $\epsilon = 0.76$, showing negligible temperature change, and near maximum $\epsilon = 0.95$, showing a small temperature decrease.

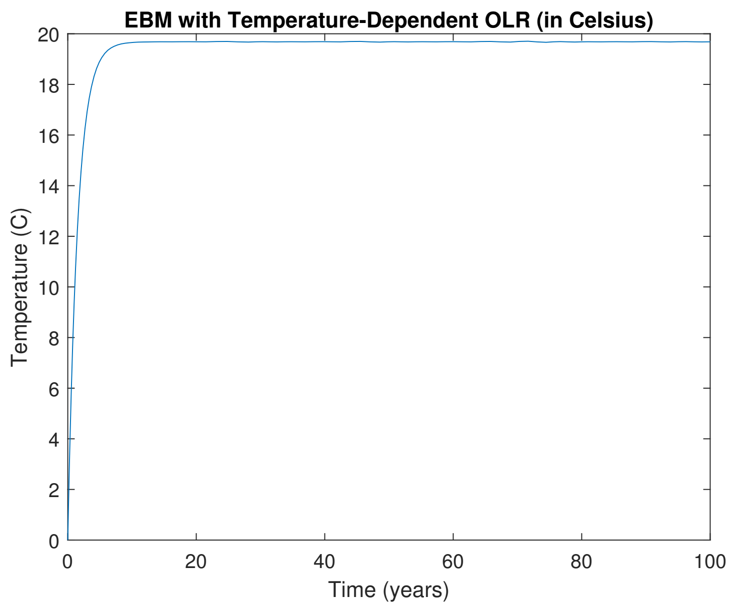


Figure 4: Simulation results showing the effect of temperature-dependent OLR on Earth's climate.

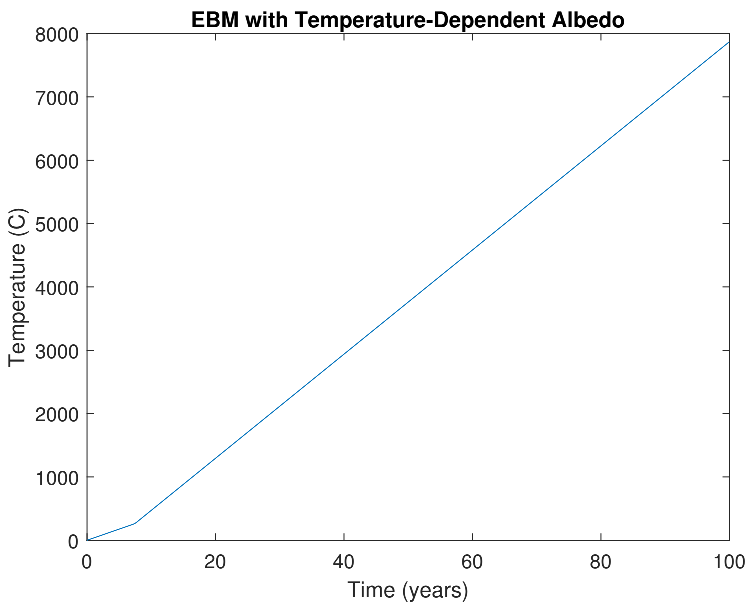


Figure 5: Simulation results for the EBM with temperature-dependent albedo, demonstrating the feedback loop between temperature and albedo.