QM Stark Effect Feb 3

Stark Effect

H_0: H-atom

$H_1 = eEr\cos\theta$

$$H_1 \doteq \begin{pmatrix} 0 & \Delta & 0 & 0 \\ \Delta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Note that $\langle nlm| H_1 |n'l'm'\rangle \neq 0 \iff m = m', l = l \pm 1$

The proof of the above state uses the fact that :

$Y_0^0 \approx 1, Y_1^{\pm 1} \approx \sin\theta e^{\pm i\phi}, \text{and} Y_1^0 \approx \cos\theta$

Due to this selection rule, the only non-zero elements are

$\langle 210| H_1 |200\rangle = \langle 200| H_1 |210\rangle^* = \Delta$

(do integral to very this): $\Delta = -3eEa_0$

Next step: FInd the Eigensystem of the matrix

Eigenvalue: $\pm\Delta, 0, 0$, Eigenvector:$\frac{1}{\sqrt{2}}(|200\rangle \pm |210\rangle), |211\rangle, |21-1\rangle$

for $n = 2$ state, our result tells us the degenerate energy level splits into 3 energy levels $E_2^0 + 3ea_0E, E_2^0, \text{and} E_2^0 - 3ea_0E$

Corollary to the Stark Effect

This is imporant because it shows that perterbation *may* break degenercy depending on the problem's setup. This is because many times perterbation might change/break the symmertry of the system.

Note that if the state has not definite parity, we then **cannot** use the parity arguement to elimate first order perterbation. This gives our $n=2$ states linear perterbation term. Note that this is **different** from $n=0$ state.

Fine Structure of Hydrogen Spectrum

$H_0 = \frac{p^2}{2m} - \frac{e^2}{r}$

The velocity of the lectron (e.g. 1s state) $mv^2 \approx \frac{me^4}{\hbar^2} \Rightarrow \frac{v}{c} \approx \frac{e^2}{\hbar c} \approx \frac{1}{137} << 1$. Therefore, the velocity of the electron is small compared to the speed of light, so we can use perterbation theory:

- Write down Dirac theory of relativistic electron
- epxpand in $\frac{v}{c} << 1$

We then get $H = mc^2 + \frac{p^2}{2m} - \frac{e^2}{r} + H_{\text{relativistic}}$

Where the first 3 terms are the non-relativistic hydrogen ateom hamiltonian $\approx 10\text{eV}$, and
$R_y = \frac{1}{2}\frac{me^4}{\hbar^2} = \frac{1}{2}(\frac{e^2}{\hbar c})^2 mc^2 = \frac{1}{2}\alpha^2 mc^2$

Note that $H_{\text{relativistic}} \propto O(\alpha^4 mc^2)$ and

$H_{\text{relativistic}} = -\frac{p^4}{8m^3c^2} + \frac{e^2}{2m^2c^2}\frac{1}{r^3}\hat{L}\cdot\hat{S} + \frac{\pi e^2\hbar^2}{2m^2c^2}\delta(r)$
$H_{\text{relativistic}} = H_K + H_{SO} + H_D$

1. H_K come from expanding
   - $E = \sqrt{p^2c^2 + m^2c^4} = mc^2\sqrt{1 + \frac{p^2}{m^2c^2}} \approx mc^2 + \frac{p^2}{2m} - \frac{p^4}{8m^3c^2}$.
   - It's magnitude $\frac{H_k}{H_0} \approx \frac{\frac{p^4}{8m^3c^2}}{\frac{p^2}{2m}} \approx \alpha^2$
2. H_D
   - Compton wavelength $\approx \frac{\hbar}{mc}$
   - This means interaction becomes none local at this order of magnitude
   - $V(r) \to \int \mathrm{d}^3\rho f(|\rho|)$
     $\int \mathrm{d}^3 f(|\rho|) = 1$ and $f = 0$ for $|\rho| \le \frac{\hbar}{mc}$
     We then taylor expand $V(r)$
     - $\delta(\vec{r}) \to$ only $l = 0$ state is affected by H_D
   - Order of magnitude of H_D
     - $H_D \approx \frac{e^2\hbar^2}{m^2c^2|\psi(0)^2|}$, and $\frac{H_D}{H_0} \approx \alpha^2$