Setup

Box of atoms m
n = $\frac{N}{V} \to r_0 \approx n^{-1/3}$
$\lambda_T = \frac{h}{\sqrt{2m\pi k_B T}}$
dimensionless quantity $n\lambda_T^3$ or λ_t/r_0
if we fixed the density n, then there must exist low tempreture T_{deg} when $\lambda_T \approx r_0$.

Analysis

- For $T > T_{deg}$: Classical regime$\to$ $U/N \propto \frac{3}{2}k_B T$, where Tempreature sets the energy scale.
- For SHO where $E_n = (n = \frac{1}{2}\hbar\omega_0)$:
  - Occupancy up to high n s.t. $E_n \approx n\hbar\omega_0 \approx k_B T$
  - Average Occupancy of a particular level < 1 where $n(E) \approx e^{-\beta(E-\mu)}$
  - We then must have $\sum_i n(E_i) = N$, and $\mu(T) = k_B T \ln(n\lambda_T^3) = k_B T \ln(\frac{T_{deg}}{T})^{3/2} = -\frac{3}{2}k_B T \ln(\frac{T}{T_{deg}})$
- For $T = 0$, the overlap comes from **Quantum Statistics** instead of other physical potential interactions. Thus our Hamiltonian is still $H = \sum_i \frac{p_i^2}{2m}$
  - For SHO with Fermeions, we have $n_i = 0, 1, H = (N + 1/2)\hbar\omega$, and there is only one accessible microstate so $S = 0$, and $\mu = \frac{E_{n+1} - E_N}{N+1-N} = (N + \frac{3}{2})\hbar\omega_0 \approx N\hbar\omega_0$
- For $T < T_{deg}$, we will derive that
  - $n^F(E_i) = \frac{1}{e^{\beta(E_i - \mu)+1}}$, and $\sum_i n^F(E_i) = N$

Distribution function $n^{F,B}(E)$

Start with the Grand Paritition Function T, V, μ
For a microstate, we must have $H = \sum_i E_j n_j$ and $N = \sum_j n_j$ Where $n_j = 0, 1$ for Fermions, $n_j \in [0, \infty)$ for Bozons
We then write our partition function
$Z = \sum_N \sum_{n_j} e^{-\beta[E - \mu N]}$, where $< N >= \frac{1}{\beta}\frac{1}{Z}(\frac{\partial Z}{\partial \mu})_{T,V}$
Then, we can follow theese steps to solve for physical systems exactly:

- Simplify for noninteracting particles
- include Fermi or Bozon statistics

To be proven:

- $\langle N\rangle = \sum_i \frac{1}{e^{\beta(E_i-\mu)+1}}$ for Fermions
- $\langle N\rangle = \sum_i \frac{1}{e^{\beta(E_i-\mu)-1}}$ for Bozons