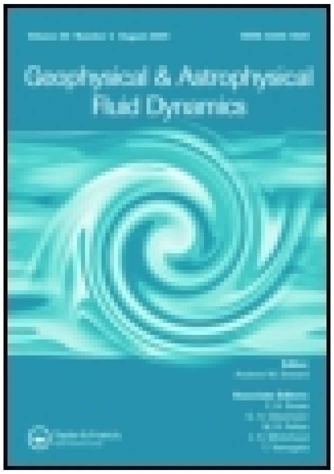

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Topographic Coupling at the Core-Mantle Interface†

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The model of Anufriyev and Braginskii (1975) and Moffatt and Dillon (1976) is further studied, with a view to calculating (i) the mean tangential stress on the core-mantle interface, and (ii) the level of the α-effect due to flow over the surface bumps. The stress calculated agrees in order of magnitude with the stress of 0.04 N m^{-2} that is inferred from the decade variations in the length of the day.

1. INTRODUCTION

It has been suggested by Hide (1969, 1977) that an important, and perhaps dominant, contribution to the tangential stress transmitted from the Earth's liquid core to the solid mantle may be associated with the presence of undulations, or "bumps", on the core-mantle interface. It is known from studies of the decade fluctuations in the length-of-day that the total mean tangential stress at the interface is approximately

$$F \approx 0.04\,\mathrm{N\,m^{-2}}. \tag{1}$$

This stress may be decomposed into viscous, electromagnetic and topographic ingredients:

$$F = F_V + F_E + F_T. \tag{2}$$

The viscous ingredient F_V is almost certainly negligible, due to the small estimated value of the kinematic viscosity in the core of the Earth. The electromagnetic ingredient F_E arises from the leakage of electric currents from the core into the lower mantle and is also estimated to be insufficient to

†Paper presented at British Geophysical Assembly, Edinburgh, April 1977.

account for the total stress (Rochester, 1973; Jacobs, 1975, §4.8). In this paper, we evaluate F_T on the basis of an idealised model previously treated by Anufriyev and Braginskii (1975) and by Moffatt and Dillon (1976, hereafter referred to as I) and we show that under reasonable estimates of the various physical parameters in the core of the Earth, topographic effects may indeed provide a contribution of the required order of magnitude.

The stress F_T has been expressed (Hide, 1977) in the form

$$F_T = C_T \Omega \rho_c U_0 h, \tag{3}$$

where Ω is the Earth's angular velocity, ρ_c the core density,† U_0 the relative velocity between core and mantle, h a measure of the bump amplitude, and C_T a drag coefficient, of order unity or less. C_T may however depend in a non-trivial way on the various dimensionless parameters controlling the flow over the bumps, and in particular on the magnetic Reynolds number.

$$R_{mT} = U_0 h/\lambda, \tag{4}$$

based on bump amplitude (λ being the magnetic diffusivity in the core), and it is important to determine this dependence before a result of the form (3) can be used with confidence.

Many of the results to be presented here are already implicit in the paper of Anufriyev and Braginskii (1975). It is hoped however that the discussion to be given here will clarify the various physical mechanisms involved.

2. ESSENTIAL FEATURES OF THE MODEL

Consider the problem depicted in Figure 1. The interface $z = \zeta(x)$ separates a non-conducting solid from a liquid of uniform density ρ_c and magnetic-

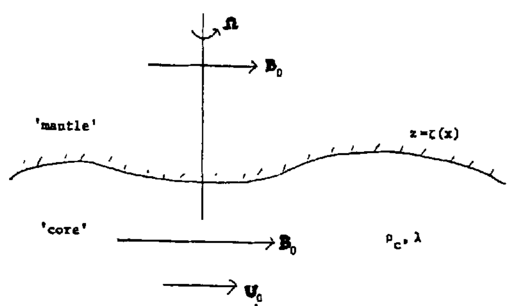

FIGURE 1. Sketch of the configuration considered.

†We shall use a suffix c to denote liquid core quantities, and a suffix m to denote mantle quantities.

diffusivity λ; viscous effects will be neglected. Both solid and liquid have magnetic permeability μ_0, and are permeated by a magnetic field $\mathbf{B}$ which tends to the uniform field $\mathbf{B}_0=(B_0,0,0)$ far from the interface. The liquid flows steadily relative to the boundary, with velocity $\mathbf{U}$ tending to $\mathbf{U}_0=(U_0,0,0)$ far from the interface. The whole configuration (including the magnetic field) rotates with angular velocity $\boldsymbol{\Omega}=(0,0,\Omega)$. We shall suppose that $|\zeta'(x)|\ll 1$ so that all perturbations associated with the bumps are small. Let L be the horizontal scale of the bumps, i.e. $|\zeta/\zeta'|=O(L)$.

There are three independent dimensionless numbers that arise in the analysis of this situation, viz.

$$A=U_0/H_0, \quad Q=\Omega\lambda/H_0^2, \quad R_0=U_0/\Omega L, \tag{5}$$

where $H_0=(\mu_0\rho_c)^{-1/2}B_0$ is the Alfvén velocity associated with the field $\mathbf{B}_0$. With the estimates (see I)

$$U_0\approx 10^{-4}\,\mathrm{m/s},\ H_0\approx 0.4\,\mathrm{m/s},\ \Omega=7.3\,10^{-5}\mathrm{s}^{-1},\ \lambda\approx 3\,\mathrm{m^2/s},\ L\approx 10^3\,\mathrm{km} \tag{6}$$

in the geophysical context, we have

$$A\approx 3.10^4, \quad Q\approx 2.10^{-3}, \quad R_0\approx 1\cdot 5.10^{-6}, \tag{7}$$

and it is reasonable to base the analysis on the assumptions†

$$A^2\ll R_0\ll 1, \quad Q\ll 1. \tag{8}$$

The condition $R_0\ll 1$ allows us to neglect the inertia term $D\mathbf{U}/Dt$ in the equation of motion relative to the Coriolis term $2\Omega\wedge\mathbf{U}$. Under steady conditions, the governing equations in the fluid region are then

$$2\boldsymbol{\Omega}\wedge\mathbf{U}=-\nabla P+\mathbf{H}\cdot\nabla\mathbf{H}, \tag{9}$$

$$\mathbf{U}\cdot\nabla\mathbf{H}=\mathbf{H}\cdot\nabla\mathbf{U}+\lambda\nabla^2\mathbf{H}, \tag{10}$$

$$\nabla\cdot\mathbf{U}=\nabla\cdot\mathbf{H}=0, \tag{11}$$

where $\mathbf{H}=(\mu_0\rho_c)^{-1/2}\mathbf{B}$, and

$$P=\rho_c^{-1}(p_f+\mu_0^{-1}\mathbf{B}^2). \tag{12}$$

p_f is the fluid pressure including modifications associated with gravity and centrifugal acceleration. Note that the stress tensor (including the Maxwell

†The importance of the particular combination A^2/R_0 is emphasised by Anufriyev and Braginskii (1975).

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electromagnetic ingredient) is given by

$$\sigma_{ij} = -p_f\delta_{ij} + \mu_0^{-1}(B_iB_j - \tfrac{1}{2}B^2\delta_{ij}) = \rho_c(-P\delta_{ij} + H_iH_j). \tag{13}$$

Writing

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{h}, \quad \mathbf{U} = \mathbf{U}_0 + \mathbf{u}, \quad P = P_0 + p, \tag{14}$$

the linearised form of Eqs. (9)–(11) becomes

$$2\boldsymbol{\Omega} \wedge \mathbf{u} = -\nabla p + \mathbf{H}_0 \cdot \nabla \mathbf{h}, \tag{15}$$

$$\mathbf{U}_0 \cdot \nabla \mathbf{h} = \mathbf{H}_0 \cdot \nabla \mathbf{u} + \lambda \nabla^2 \mathbf{h}, \tag{16}$$

$$\nabla \cdot \mathbf{u} = \nabla \cdot \mathbf{h} = 0. \tag{17}$$

In the solid region $z > \zeta(x)$, the magnetic field is a potential field, i.e.

$$\mathbf{h} = -\nabla \psi_m \quad \text{for} \quad z > \zeta, \tag{18}$$

and evidently $\psi_m \to 0$ as $z \to \infty$. Moreover

$$\mathbf{h}, \mathbf{u} \to 0 \quad \text{as} \quad z \to -\infty, \tag{19}$$

and we have also the matching conditions on the interface

$$\mathbf{U} \cdot \mathbf{n} = 0, \quad [\mathbf{h}] = 0 \quad \text{on} \quad z = \zeta. \tag{20}$$

Now

$$\mathbf{n} = (-\zeta'(x), 0, 1)/\sqrt{1 + \zeta'^2}, \tag{21}$$

so that the linearised form of (20) is

$$u_z = U_0\zeta'(x), \quad [\mathbf{h}] = 0 \quad \text{on} \quad z = 0. \tag{22}$$

The problem as specified above has been treated exactly in I, and it emerges that under the conditions (8), the disturbance modes available are of two types: (i) a current-free irrotational mode, and (ii) a mode having a boundary-layer structure, the boundary layer thickness being of order $Q^{1/2}$. In the following two sections, we discuss further the structure of these modes and the manner in which their amplitudes and phases are determined by the boundary conditions (22).

3. THE CURRENT-FREE IRROTATIONAL MODE

We shall suppose for simplicity that

$$\zeta(x)=\mathrm{Re}\,(\hat{\zeta}\,e^{ikx}), \qquad k>0, \qquad |k\hat{\zeta}|\ll 1. \tag{23}$$

The linearity of the problem allows more general interface profiles to be treated by Fourier synthesis.

In the formal limit $\lambda\to 0$ (or $Q\to 0$) Eq. (16) becomes

$$(\mathbf{H}_0\cdot\nabla)(\mathbf{u}_1-A\mathbf{h}_1)=0, \qquad \text{i.e.} \qquad \mathbf{u}_1=A\mathbf{h}_1 \tag{24}$$

(where we use the suffix 1 to denote this dissipationless type of disturbance), and Eq. (15) then becomes

$$2A\boldsymbol{\Omega}\wedge\mathbf{h}_1=-\nabla p_1+\mathbf{H}_0\cdot\nabla\mathbf{h}_1. \tag{25}$$

Now

$$\frac{|A\boldsymbol{\Omega}\wedge\mathbf{h}_1|}{|\mathbf{H}_0\cdot\nabla\mathbf{h}_1|}=\mathrm{O}\left(\frac{A\Omega}{H_0/L}\right)=\mathrm{O}\left(\frac{A^2}{R_0}\right), \tag{26}$$

so that, under the condition $A^2\ll R_0$ (see (8) above), Eq. (25) takes the approximate form

$$\mathbf{H}_0\cdot\nabla\mathbf{h}_1\approx\nabla p_1, \tag{27}$$

or equivalently

$$\mathbf{h}_1\approx(ikH_0)^{-1}\nabla p_1=-\nabla\psi_c, \text{ say}. \tag{28}$$

The field $\mathbf{h}_1$ (and so $\mathbf{u}_1$ from (24)) is therefore irrotational. The normal component of this field must match the normal component of $-\nabla\psi_m$ across $z=\zeta$; since both ψ_c and ψ_m are harmonic fields, they may then be easily obtained in the form

$$\psi_c=\hat{\psi}\,e^{ikx+kz}, \qquad \psi_m=-\hat{\psi}e^{ikx-kz}, \tag{29}$$

where $\hat{\psi}$ is a complex amplitude to be determined. Note that

$$[h_{1x}]=-\frac{\partial\psi_m}{\partial x}+\frac{\partial\psi_c}{\partial x}=2ik\hat{\psi}\,e^{ikx}. \tag{30}$$

There is a corresponding current sheet on $z=0$, the surface current being

$$\mathbf{J}_s=(\rho_c/\mu_0)^{1/2}(0,2ik\hat{\psi}\,e^{ikx},0). \tag{31}$$

The structure of this current sheet can be resolved only by restoring the effects of magnetic diffusion.

4. THE MAGNETIC DIFFUSION LAYER

Let us now focus attention on solutions of Eqs. (15)–(17) having a scale of variation δ in the z- direction small compared with $L\ (\approx k^{-1})$. As mentioned above, the scale that emerges from an exact analysis is $\delta = O(Q^{1/2})k^{-1}$. When $Q \ll 1$, it follows that $\nabla^2\mathbf{h} \approx \partial^2\mathbf{h}/\partial z^2$ in (16), and moreover

$$\frac{|U_0 \cdot \nabla\mathbf{h}|}{|\lambda\partial^2\mathbf{h}/\partial z^2|} = O\left(\frac{U_0\delta^2}{L\lambda}\right) = O\left(\frac{U_0 LQ}{\lambda}\right) = O\left(\frac{A^2}{R_0}\right). \tag{32}$$

Hence, in the boundary layer modes considered, $\mathbf{U}_0 \cdot \nabla\mathbf{h}$ is negligible. Moreover, horizontal pressure gradients are negligible, and the vertical pressure gradient merely serves to maintain the incompressibility condition $\nabla \cdot \mathbf{u} = 0$.

Denoting the boundary layer modes by a subscript δ, the horizontal (x and y) components of (15) and (16) reduce to

$$2\mathbf{\Omega} \wedge \mathbf{u}_\delta = (\mathbf{H}_0 \cdot \nabla)\mathbf{h}_\delta, \tag{33}$$

$$0 = (\mathbf{H}_0 \cdot \nabla)\mathbf{u}_\delta + \lambda\partial^2\mathbf{h}_\delta/\partial z^2. \tag{34}$$

With $\mathbf{u}_\delta, \mathbf{h}_\delta \propto e^{ikx}$, we easily obtain from (33) and (34) the equation

$$2\mathbf{i}_z \wedge \frac{\partial^2\mathbf{h}_\delta}{\partial z^2} = -\frac{H_0^2k^2}{\lambda\Omega}\mathbf{h}_\delta = -\frac{k^2}{Q}\mathbf{h}_\delta, \tag{35}$$

where $\hat{\mathbf{i}}_z$ is a unit vector in the z-direction. This may be most easily treated by defining the quantities

$$\mathscr{H}_\pm = h_{\delta x} \pm ih_{\delta y}, \tag{36}$$

which satisfy the equations

$$\frac{\partial^2\mathscr{H}_\pm}{\partial z^2} = \pm\frac{ik^2}{2Q}\mathscr{H}_\pm. \tag{37}$$

Defining δ by†

$$\delta = 2Q^{1/2}k^{-1} = 2(\Omega\lambda)^{1/2}/kH_0, \tag{38}$$

†This form for δ requires that $\Omega > 0$ and $H_0 > 0$; if either Ω or H_0 is negative, then $|\Omega|$ and $|H_0|$ must be used.

the relevant solutions of (37), vanishing as $z \to -\infty$, are

$$\mathscr{H}_+ = \mathscr{H}_+ e^{ikx} e^{(1+i)z/\delta}, \qquad \mathscr{H}_- = \mathscr{H}_- e^{ikx} e^{(1-i)z/\delta}, \tag{39}$$

and we then have

$$h_{\delta x} = \tfrac{1}{2}(\mathscr{H}_+ + \mathscr{H}_-), \qquad h_{\delta y} = \frac{1}{2i}(\mathscr{H}_+ - \mathscr{H}_-). \tag{40}$$

The conditions of continuity of the x and y components of $\mathbf{h} = \mathbf{h}_1 + \mathbf{h}_\delta$ across $z=0$ can now be satisfied. Since $h_{1y}=0$, the condition $[h_y]=0$ gives simply $\mathscr{H}_+ = \mathscr{H}_-$. The condition $[h_x]$ then gives $-ik\hat{\psi} + \frac{1}{2}(\mathscr{H}_+ + \mathscr{H}_-) = ik\hat{\psi}$. Hence

$$\mathscr{H}_+ = \mathscr{H}_- = 2ik\hat{\psi}, \tag{41}$$

and so, determining $h_{\delta z}$ from the condition $\nabla \cdot \mathbf{h}_\delta = 0$, we obtain

$$\mathbf{h}_\delta = 2ik\hat{\psi} e^{ikx} e^{z/\delta} \left(\cos\frac{z}{\delta}, \sin\frac{z}{\delta}, -iQ^{1/2}\left(\cos\frac{z}{\delta} + \sin\frac{z}{\delta}\right)\right). \tag{42}$$

Similarly, from (33), and $\nabla \cdot \mathbf{u}_\delta = 0$,

$$\mathbf{u}_\delta = -\frac{H_0 k^2}{\Omega} \hat{\psi} e^{ikx} e^{z/\delta} \left[\sin\frac{z}{\delta}, -\cos\frac{z}{\delta}, iQ^{1/2}\left(\cos\frac{z}{\delta} - \sin\frac{z}{\delta}\right)\right]. \tag{43}$$

Note that $|h_{\delta z}| = O(Q^{1/2})|h_{1z}|$ on $z=0$, so that to leading order the condition of continuity of $\mathbf{h} \cdot \mathbf{n}$ across the interface is unaffected by the boundary layer mode.

It remains to satisfy the condition

$$u_z = u_{1z} + u_{\delta z} = U_0 \zeta'(x) = ikU_0\hat{\zeta} \qquad \text{on} \quad z=0. \tag{44}$$

On $z=0$, we have

$$u_{1z} = Ah_{1z} = -Ak\hat{\psi} e^{ikx}, \qquad u_{\delta z} = -\frac{iH_0}{\Omega} k^2 Q^{1/2} \hat{\psi}. \tag{45}$$

Hence (44) gives

$$\left(-A - i\frac{H_0}{\Omega} Q^{1/2} k\right)\hat{\psi} = iU_0\hat{\zeta}, \tag{46}$$

or

$$\hat{\psi} = \frac{-iH_0}{1 + ik/k_0}\hat{\zeta}, \tag{47}$$

where

$$k_0 = (U_0/H_0)(\Omega/\lambda)^{1/2}. \tag{48}$$

Note that

$$u_{1z}/u_{\delta z} = -ik_0 k \quad \text{on} \quad z=0, \tag{49}$$

and that the ratio k/k_0 is crucial in determining the phase shift of $\hat{\psi}$ relative to $\hat{\zeta}$. If $k \ll k_0$, then the boundary layer mode makes negligible contribution in the condition (44), while if $k \gg k_0$ it makes a dominant contribution. In the geophysical context, $k_0^{-1} \approx 700\,\text{km}$, and values of k/k_0 of order unity are of particular interest; in this situation the boundary-layer and irrotational modes make comparable contributions in (44).

5. THE MEAN STRESS ON THE BOUNDARY

From the expression (13) for the stress tensor, it follows that the mean x-component of stress on $z=\zeta$ is

$$F_T = \langle \sigma_{1j} n_j \rangle = \rho_c \langle -Pn_1 + \mathbf{H}_x \mathbf{H} \cdot \mathbf{n} \rangle . \tag{50}$$

Now, $P = P_0 + p_1 + p_\delta$, and from (27)

$$p_1 = H_0 h_{1x}, \tag{51}$$

while, from (15), within the boundary layer,

$$p_\delta = 0(\delta H_0 k h_{\delta z}) = \mathrm{O}(Q) p_1 . \tag{52}$$

Hence, to order ζ^2, and neglecting the $\mathrm{O}(Q)$ contribution from p_δ, we have from (50)

$$F_T = \rho_c \langle H_0 h_{1x} \zeta' - H_x^2 \zeta' + H_x H_z \rangle_{z=0} . \tag{53}$$

Now both H_x and H_z are continuous across $z=0$, and H_x and H_z are exactly out of phase† on the mantle side $(z=0+)$; hence $\langle H_x H_z \rangle_{z=0} = 0$. With $H_x = H_0 + h_x$, (53) then gives

$$\begin{aligned} F_T &= \rho_c H_0 \langle (h_{1x} - 2h_x)\zeta' \rangle_{z=0-} \\ &= -3\rho_c H_0 \langle h_x \zeta' \rangle_{z=0+} , \end{aligned} \tag{54}$$

using (30). Hence

$$F_T = 3\rho_c H_0 \cdot \tfrac{1}{2} \operatorname{Re}(ik\hat{\psi})(ik\hat{\zeta}^*),$$

†If finite mantle conductivity is taken into account, this result no longer holds, and the resulting contribution is just the term F_E referred to in the introduction.

and, using (47), this reduces to

$$F_T = \frac{3}{2} \rho_c H_0^2 k^2 \frac{k/k_0}{1+(k/k_0)^2} |\hat{\zeta}|^2 . \tag{55}$$

A similar, though not identical, expression has been obtained by Anufriyev and Braginskii (1975).

The expression (55) is maximal as a function of k/k_0 when $k/k_0 = 1$, and then

$$F_{T\max} = \frac{3}{4} \rho_c H_0^2 k_0^2 |\hat{\zeta}|^2 = \frac{3\rho_c \Omega U_0^2}{4\lambda} |\hat{\zeta}|^2 . \tag{56}$$

It is interesting that this maximal stress, although entirely of magnetic origin, does not depend on the field strength H_0, [although of course (56) is valid only if H_0 is sufficiently large for (8) to be satisfied]. With the values of Ω, U_0 and λ given in (6) above, and with

$$\rho_c \approx 10^4 \,\mathrm{kg/m^3}, \qquad |\hat{\zeta}| \approx 5.10^3 \mathrm{m}, \tag{57}$$

(56) gives

$$F_{T\max} \approx 0.045 \,\mathrm{N\,m^{-2}}, \tag{58}$$

which is of the right order of magnitude to account for the total stress given by (1).

If $\zeta(x)$ is a stationary random function of x with spectrum function $W(k)$, then the appropriate generalisation of (55) is clearly

$$F_T = \frac{3}{2} \rho_c H_0^2 \int_0^\infty \frac{k^3/k_0}{1+(k/k_0)^2} W(k)\, dk. \tag{59}$$

The appropriate expression for C_T may be inferred from (3), and in general there appears to be no guarantee that this will be of order unity.

6. THE α-EFFECT ASSOCIATED WITH THE BOUNDARY LAYER FLOW

As observed in I, the boundary layer mode is strongly helical in character, and a strong α-effect is therefore to be expected (see, for example, Moffatt, 1976). The mean electromotive force $\mathscr{E}$ associated with the perturbation fields is given by

$$(\mu_0 \rho_c)^{-1/2} \mathscr{E} = \langle \mathbf{u} \wedge \mathbf{h} \rangle = \langle \mathbf{u}_1 \wedge \mathbf{h}_\delta \rangle + \langle \mathbf{u}_\delta \wedge \mathbf{h}_1 \rangle + \langle \mathbf{u}_\delta \wedge \mathbf{h}_\delta \rangle . \tag{60}$$

Of particular interest is the component $\mathscr{E}_x$ in the direction of H_0, given by

$$(\mu_0\rho_c)^{-1/2}\mathscr{E}_x = -\langle u_{1z}h_{\delta y}\rangle + \langle u_{\delta y}h_{1z}\rangle + \langle u_{\delta y}h_{\delta z} - u_{\delta z}h_{\delta y}\rangle . \qquad (61)$$

Now u_{1z} and $h_{\delta y}$ are exactly out of phase so that $\langle u_{1z}h_{\delta y}\rangle = 0$; on the other hand $u_{\delta y}$ and h_{1z} are exactly in phase, and

$$\langle u_{\delta y}h_{1z}\rangle = -\frac{H_0 k^3}{2\Omega}\cos\frac{z}{\delta}e^{z(1/\delta+k)}|\psi|^2. \qquad (62)$$

Further, from (42) and (43),

$$\langle u_{\delta y}h_{\delta z} - u_{\delta z}h_{\delta y}\rangle = \frac{2H_0k^3}{\Omega}Q^{1/2}\left(\cos\frac{2z}{\delta} + \sin\frac{2z}{\delta}\right)e^{2z/\delta}|\hat{\psi}|^2. \qquad (63)$$

The expression (62) clearly provides the dominant contribution, and we have in fact $\mathscr{E}_x = \alpha B_0$ where (since $k \ll \delta^{-1}$)

$$\alpha(z) = -\frac{k^3}{2\Omega}\frac{H_0^2}{1+(k/k_0)^2}|\zeta|^2 \cos\frac{z}{\delta}e^{z/\delta}. \qquad (64)$$

Again it must be emphasised that this result holds only when H_0 is strong enough for (8) to be satisfied; the dependence of α on H_0 is indicative of the strong dynamic interactions taking place. When $H_0 \to \infty$, $k_0 \to 0$ and $\alpha(z)$ tends to the saturation level given by

$$\alpha_{\text{sat}}(z) \sim -\frac{kU_0^2}{2\lambda}|\zeta|^2 \cos\frac{z}{\delta}e^{z/\delta}. \qquad (65)$$

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