Optimization and Appliance Related Details

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1 Appliance Related Details

Our optimization approach controls the load consumption of Heating, Ventilation, Air-Conditioning (HVAC) and Electric Vehicle (EV). Therefore, we only provide explanations related to these appliances. We represent the estimated load consumption of all other appliances throughout the planning horizon by the vector $p_{i,\mathrm{U}}(t)$ for home i at time t and we assume that renewable energy resources provides power by $p^r(t)$ at time t. We control the load consumption of each home by selecting a trajectory among the comfort ensuring trajectories. We will briefly review the utilized HVAC trajectory generation mechanism.

1.1 HVAC

For the HVAC, we define the comfortable thermal zone for each home i by $T_{i,\mathrm{lower}}$ and $T_{i,\mathrm{upper}}$. Then, by using the HVAC model that our collaborators developed along with home specific parameters, we create various feasible load schedules that keep the room temperature within the comfortable thermal zone for each home i, and store those load schedules in the set $S_{i,\mathrm{HVAC}}$. The steps below summarize the workflow generating the lth feasible load schedule for the HVAC for home i:

- Suppose we have the l^{th} candidate set temperature T_l^{set}.
- Provide T_l^{set} to the HVAC model of home i, and generate a candidate load schedule for the HVAC $p_{i,\mathrm{HVAC}}^l$.
- If the generated $p_{i,\mathrm{HVAC}}^l$ is different enough than the vectors stored in $S_{i,\mathrm{HVAC}}$, add $p_{i,\mathrm{HVAC}}^l$ to the set $S_{i,\mathrm{HVAC}}$. Otherwise, repeat the steps above for another candidate set temperature T_l^{set}.

On the third item above, we use cosine similarity to check whether the generated $p_{i,\mathrm{HVAC}}^l$ is different than the vectors in $S_{i,\mathrm{HVAC}}$, or not. Although the mechanism above explains how we generate $p_{i,\mathrm{HVAC}}^l$ vector for a given T_l^{set}, it does not explain how we generate the candidate set temperature vector T_l^{set}.

Suppose the comfortable thermal region for user i, which is defined by $T_{i,\text{lower}}$ and $T_{i,\text{upper}}$, is known. By using these vectors and a fixed ψ value, we define the boundaries of the set temperature region we consider as follows:

$$T^{\text{set}}_{\text{lower}} = T_{i,\text{lower}} + \psi,$$
$$T^{\text{set}}_{\text{upper}} = T_{i,\text{upper}} - \psi.$$

Then, we randomly generate candidate set temperature vectors from this region. To give a flexibility to our optimization approach, we want to create different set temperature schedules as much as possible, and so far, we have developed four strategies to randomly sample set temperature vectors:

- We design a function, which sets $T^{\text{set}}(t)$ to either $T^{\text{set}}_{\text{lower}}(t)$ or $T^{\text{set}}_{\text{upper}}(t)$ by generating an independent Bernoulli random variable at each time point t.
- Another function samples T^{set} from a Gaussian with $\mu = \frac{T^{\text{set}}_{\text{lower}} + T^{\text{set}}_{\text{upper}}}{2}$ and $\sigma = \psi$. Then, we apply a clipping operation to randomly generated vector so that $T^{\text{set}}_{\text{lower}} \leq T^{\text{set}} \leq T^{\text{set}}_{\text{lower}}$ is satisfied.
- We also design a Markov based mixture algorithm, which decides increasing or decreasing the set temperature value at a given time point based on a Bernoulli as in item 1. However, instead of setting set temperature to the boundaries directly, it samples a random set temperature value from a uniform distribution.
- Finally, we design an algorithm, which creates random set temperature schedules by using piece-wise constant functions.

Suppose we fill the set $S_{i,\text{HVAC}}$ by using the approaches described above, and $S^l_{i,\text{HVAC}}$ denotes the $l-th$ feasible load schedule for home i. Then, we can define the HVAC related constraints that we are going to include for each home i as follows:

$$p_{i,\text{HVAC}} = \sum_{l=1}^{|S_{i,\text{HVAC}}|} x_{il} S^l_{i,\text{HVAC}}, \tag{1}$$

$$\sum_{l=1}^{|S_{i,\text{HVAC}}|} x_{il} = 1,\ x_{il} \in \{0,1\}. \tag{2}$$

1.2 EV

Given parameters such as arrival time α_i, duration of stay β_i, battery level at arrival $x(\alpha)$, target charge level by departure time T_i, and charging rate τ, we can define a set of linear constraints to maintain the user comfort for the EV. First, we have to check whether T_i is achievable or not, e.g.,

$$\tau * (\beta_i - \alpha_i) \geq T_i - x(\alpha).$$

.

If the inequality above does not hold, there is no need to formulate constraints for home i, because we are going to schedule charging from arrival to departure directly. Otherwise, we fill write a set of constraints to generate EV charging schedules for home i. Suppose $T^{\text{EV}}(\alpha_i, \beta_i)$ denotes the time intervals the car is at home.

- $T^{\text{EV}} = \{\alpha_i, \alpha_i + 1, ..., \alpha_i + \beta_i - 1\}$ set storing the indices for time intervals when the EV is at home.

$$
\begin{aligned}
x(\beta_i) &\geq T_i, && && (3)\\
x(t) &\leq 1, && \forall t, && (4)\\
x(t+1) &= x(t) + p_{i,\text{EV}}(t), && t = \{\alpha, ..., \alpha + \beta - 1\}, && (5)\\
p_{i,\text{EV}}(t) &= 0, && t \in \{0, ..., K-1\} \setminus T^{\text{EV}}, && (6)\\
p_{i,\text{EV}}(t) &\leq \tau, && \forall t. && (7)
\end{aligned}
$$

Due to randomness in $\alpha_i, \beta_i, x(\alpha)$, we can sample these values by using available historical data, and we can define our problem by using these sampled values. The constraints in (3)-(7) define a convex polyhedron, we can integrate those constraints into our optimization problem directly. However, in our implementation, instead of adding these constraints into our problem, we employ the trajectory selection idea that we introduce for the HVAC. Suppose we store the comfort maintaining load schedules in the set $S_{i,\text{EV}}$ and $S^l_{i,\text{EV}}$ denotes the $l-th$ feasible load schedule for home i. Then, we can define the EV related constraints that we are going to include for each home i as follows:

$$
p_{i,\text{EV}} = \sum_{l=1}^{|S_{i,\text{EV}}|} x_{il} S^l_{i,\text{EV}}, \tag{8}
$$

$$
\sum_{l=1}^{|S_{i,\text{EV}}|} x_{il} = 1, \; x_{il} \in \{0, 1\}. \tag{9}
$$

2 Optimization

The constraints defined in Equations (1)-(2) and (8)-(9) represent the comfort related constraints for appliances HVAC and EV. Hence, we can maintain the user comfort as long as $p_i \in \mathcal{H}_i$. The optimization level we formulate at the neighborhood level is as follows:

$$\begin{aligned}
\min_{\boldsymbol{p}_1,\cdots,\boldsymbol{p}_N} \quad & \sum_{t=0}^{K-1} s(t) && && (10)\\
\text{s.t.} \quad & s(t) - a(t) \geq 0, && \forall\ t, && (11)\\
& s(t) + a(t) \geq 0, && \forall\ t, && (12)\\
& a(t) = q + p^r(t) - \sum_{i=1}^{N} \left(p_{i,\text{HVAC}}(t) + p_{i,\text{EV}}(t)\right) && && \\
& \qquad\qquad - \sum_{i=1}^{N} p_{i,\text{U}}(t), && \forall\ t, && (13)\\
& p_i \in \mathcal{H}_i, && \forall\ i, && (14)
\end{aligned}$$

where q denotes the amount of power that the aggregator provides from external generators at each time period t. We describe how to solve the problem given in (10)-(14) in a distributed way in [Ozcan and Paschalidis, 2023]. We can apply the same idea to solve this problem in a distributed way.

2.1 Automatic Selection of q

The problem given in Equations (10)-(14 assumes that q is provided to the problem as a parameter by the coordination agent. However, finding an appropriate q value ensuring $\sum_{i=1}^{N} \left(p^*_{i,\text{HVAC}}(t) + p^*_{i,\text{EV}}(t)\right) - p^r(t) \sim q$ can be problematic. To address this issue, we implement an algorithm finding the optimal $q*$ maintaining balance between load supply and demand. In this version, q becomes a non-negative decision variable, and its value is set by the optimization algorithm automatically.

References

[Ozcan and Paschalidis, 2023] Ozcan, E. C. and Paschalidis, I. C. (2023). A distributed optimization framework to regulate the electricity consumption of a residential neighborhood. arXiv preprint arXiv:2306.09954.