ECSE 446/546

IMAGE SYNTHESIS

Direct Illumination I

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(slides in part by W. Jarosz)

McGill

Recall

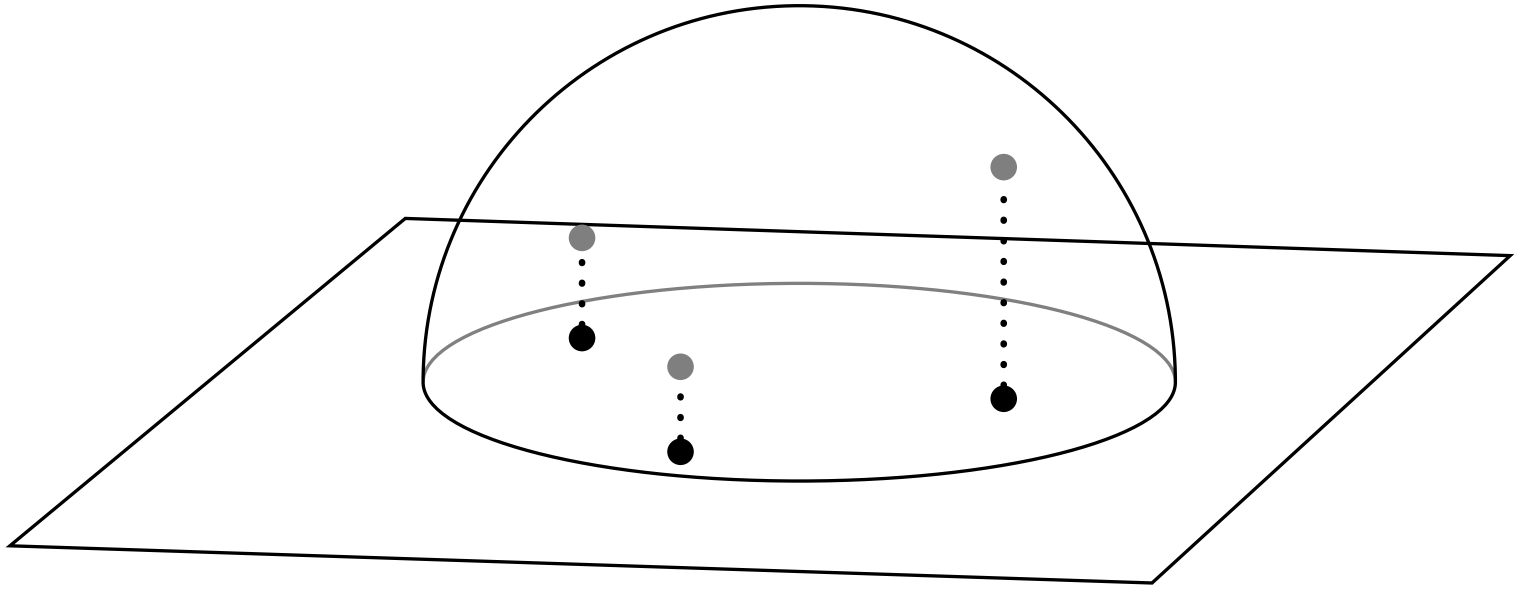
- Goal: evaluate integral $\int_a^b f(x)dx$
- Random variable $X_i \sim p(x)$
- Monte Carlo Estimator $F_N = \frac{1}{N} \sum_{i=1}^{N} \frac{f(X_i)}{p(X_i)}$
- Expectation $E[F_N] = \int_a^b f(x)dx$

Recall

- Sampling:
  - uniform disk
  - uniform sphere
  - uniform hemisphere
  - cosine-weighted hemisphere
- The inversion method
- Conditional and marginal distributions

Cosine-weighted Hemispherical Sampling

- Generate points uniformly on the disc, and then project these points to the surface of the hemisphere
  - Called the “Nusselt Analog”



Today's Menu

- *Direct* vs. *indirect* illumination
- *Hemispherical* vs. *area* formulation of the reflection equation
- Light source models
- Importance sampling
- Multiple importance sampling

Direct vs. Indirect Illumination

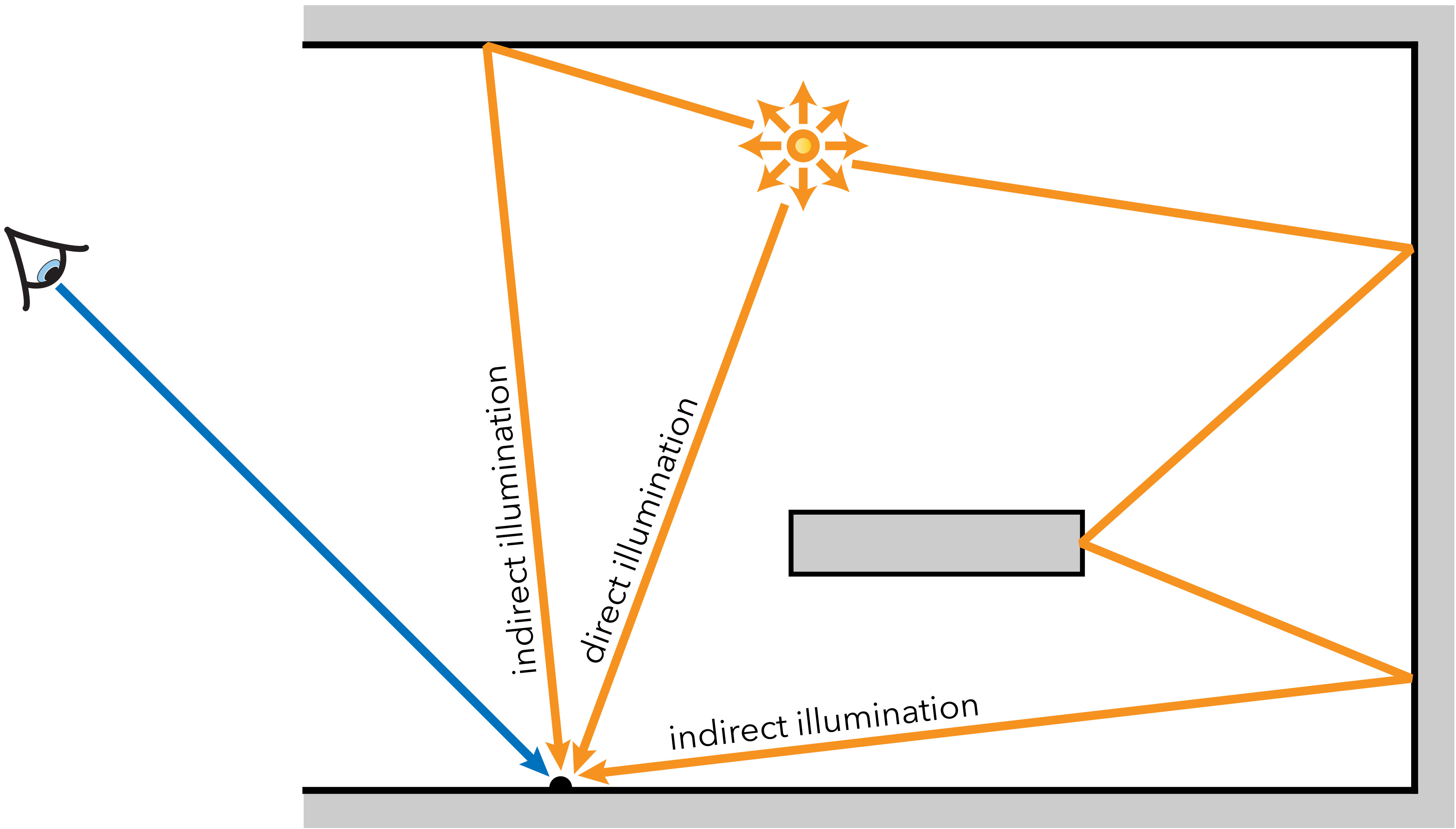
$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

- Where does L_i "come from"?
  - if directly from an emitter, we refer to it as *direct* illumination

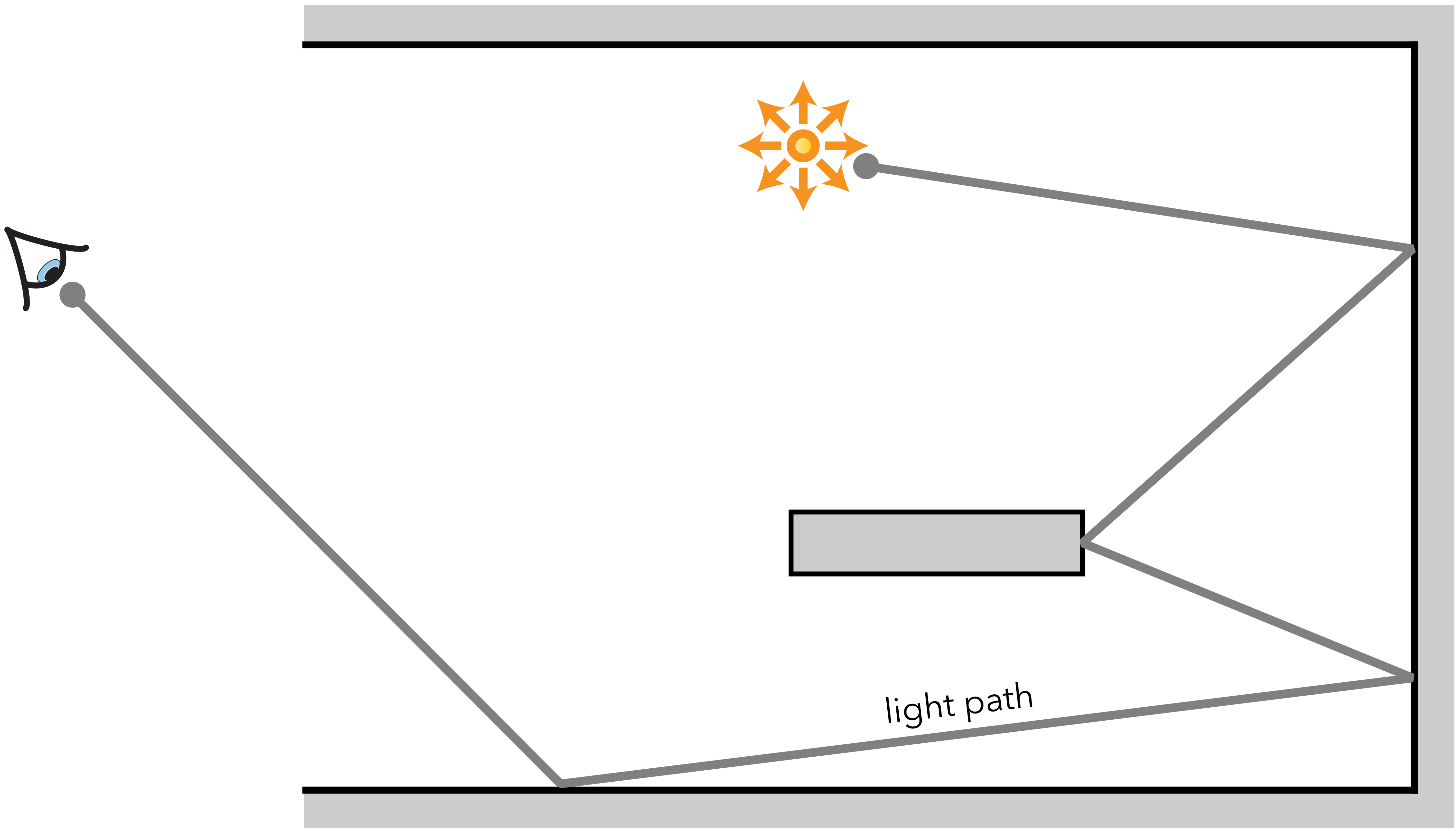
$$L_i(\mathbf{x}, \vec{\omega}) = L_e(r(\mathbf{x}, \vec{\omega}), -\vec{\omega})$$

  - if indirectly, i.e. by bouncing off a scattering surface, we call it *indirect* illumination

Direct vs. Indirect Illumination

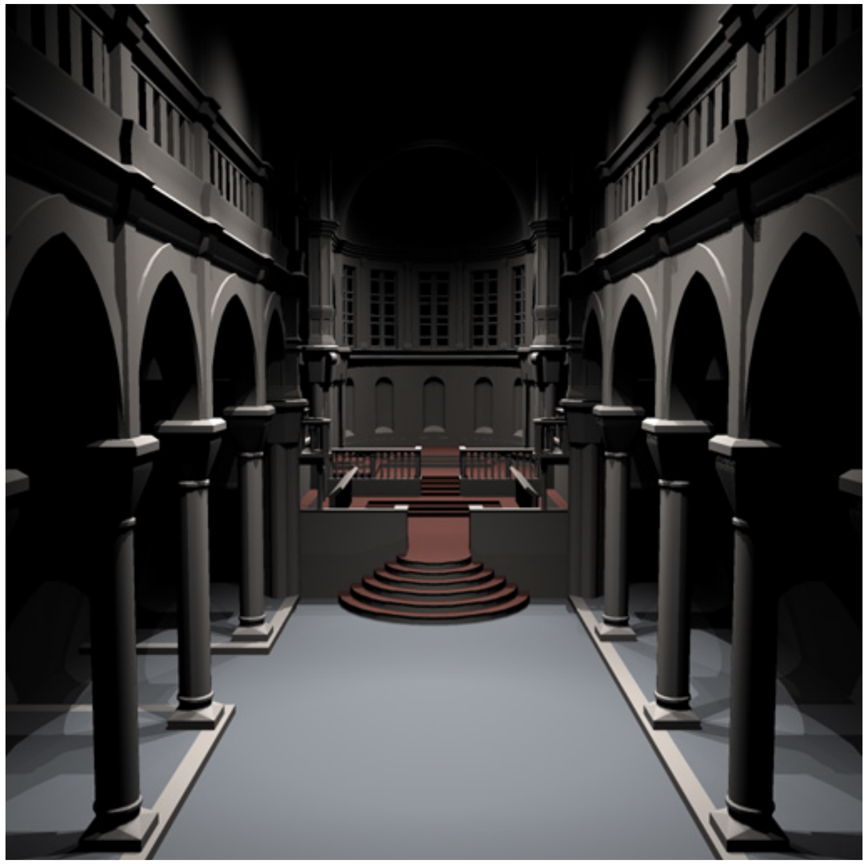


Light Path



Direct vs. Indirect Illumination

Direct illumination

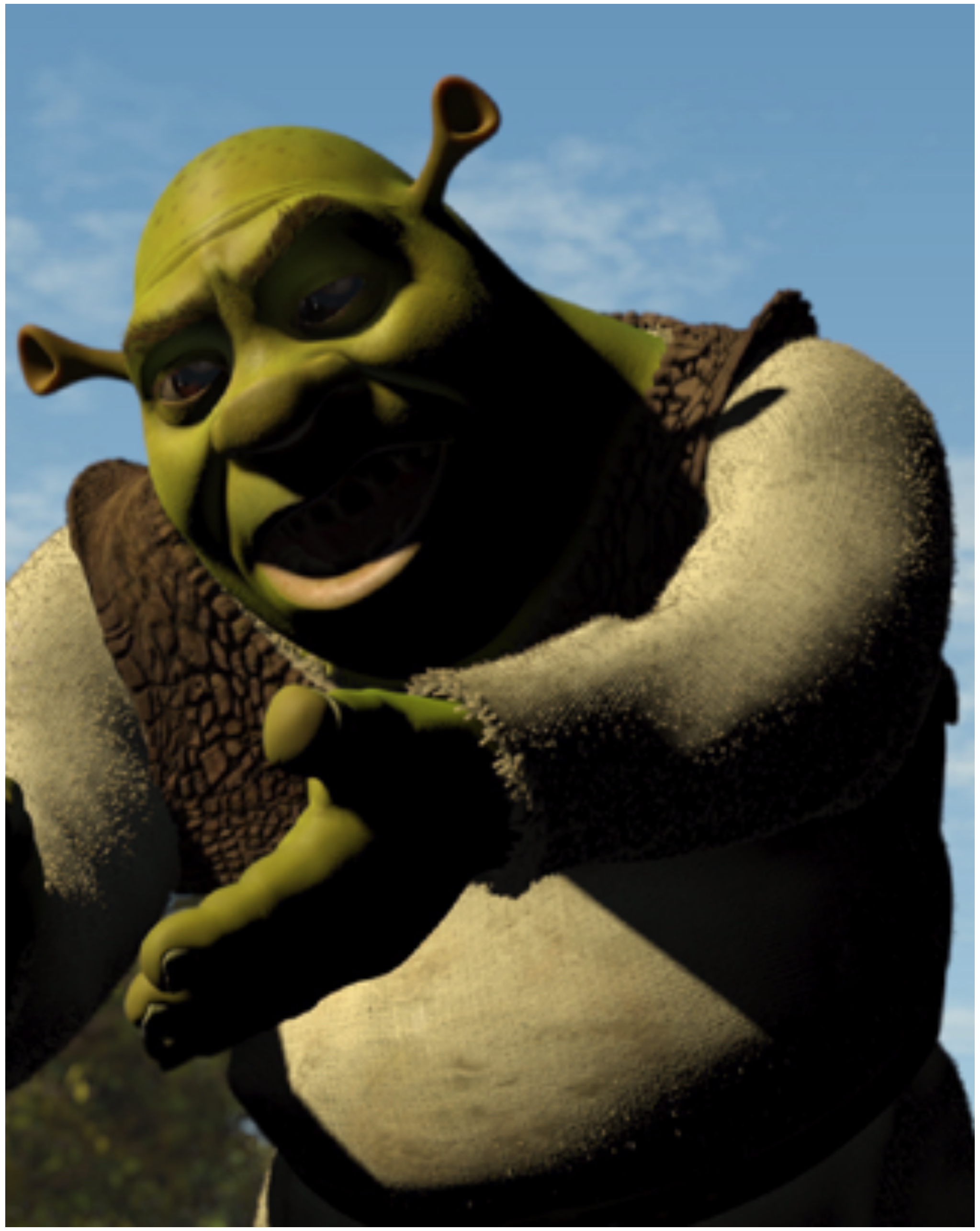


Indirect illumination

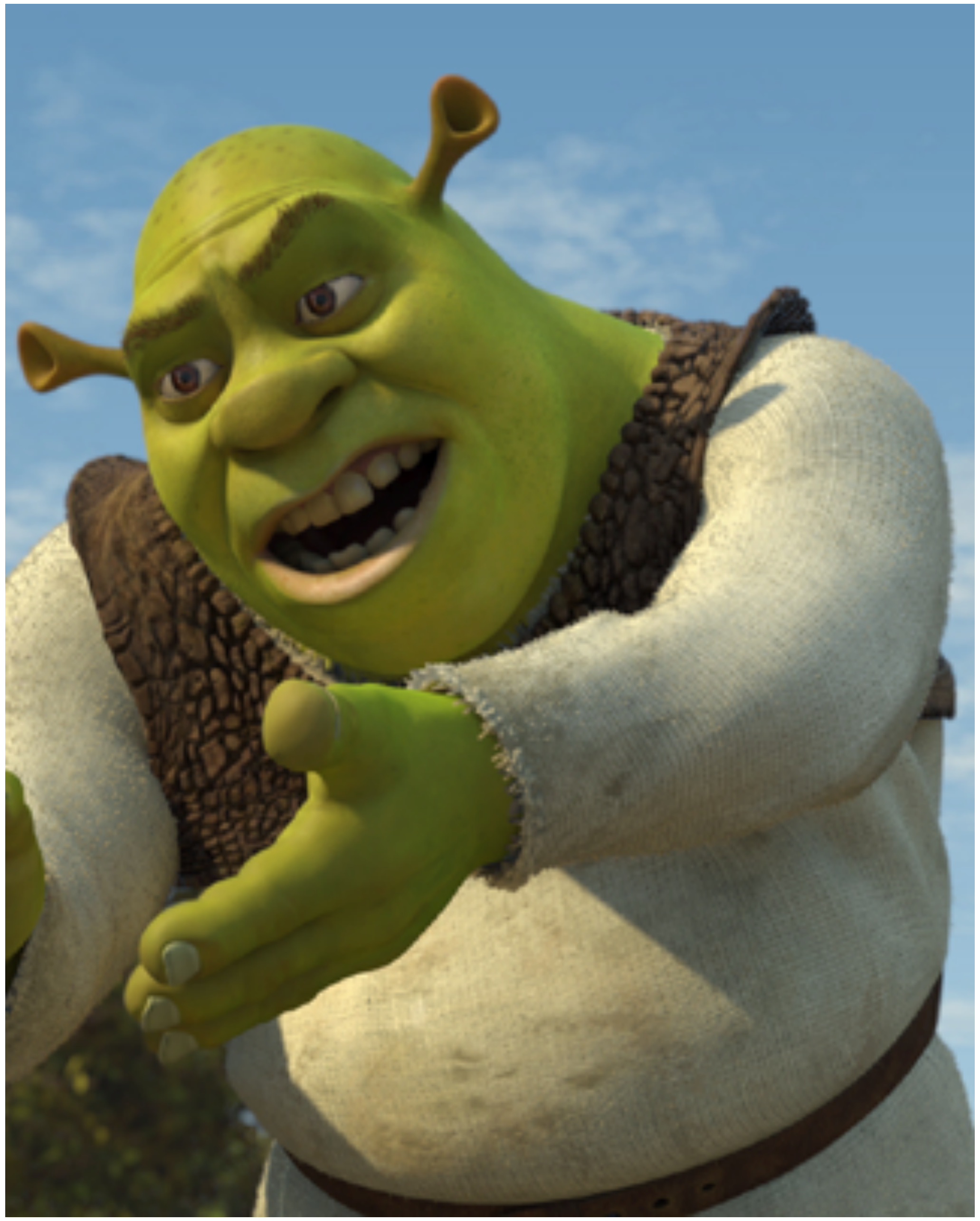
Direct + indirect illumination

Direct vs. Indirect Illumination

Direct illumination only



Direct + Indirect illumination



Images courtesy of PDI/DreamWorks

Direct Illumination

(indirect illumination will be covered later)

Direct Illumination

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

The general incident radiance L_i can be replaced by emission:

$$L_i(\mathbf{x}, \vec{\omega}_i) = L_e(r(\mathbf{x}, \vec{\omega}_i), -\vec{\omega}_i)$$

Direct Illumination

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_e(r(\mathbf{x}, \vec{\omega}_i), -\vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

How can we estimate the integral?

Direct Illumination

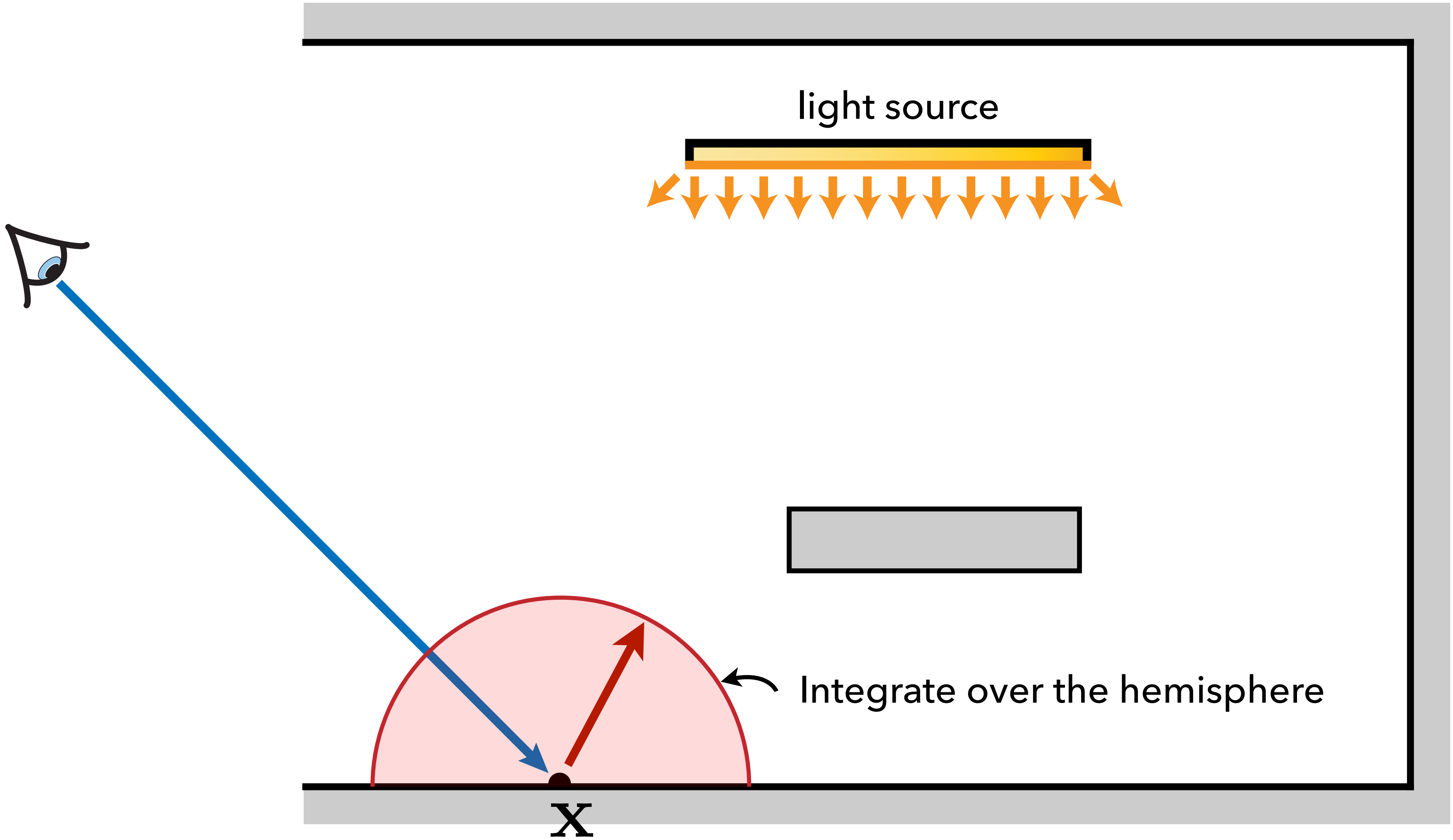
$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_e(r(\mathbf{x}, \vec{\omega}_i), -\vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

How can we estimate the integral?

$$\langle L_r(\mathbf{x}, \vec{\omega}_r)^N \rangle = \frac{1}{N} \sum_{k=1}^{N} \frac{f_r(\mathbf{x}, \vec{\omega}_{i,k}, \vec{\omega}_r) L_e(r(\mathbf{x}, \vec{\omega}_{i,k}), -\vec{\omega}_{i,k}) \cos\theta_{i,k} \, d\vec{\omega}_{i,k}}{p_\Omega(\vec{\omega}_{i,k})}$$

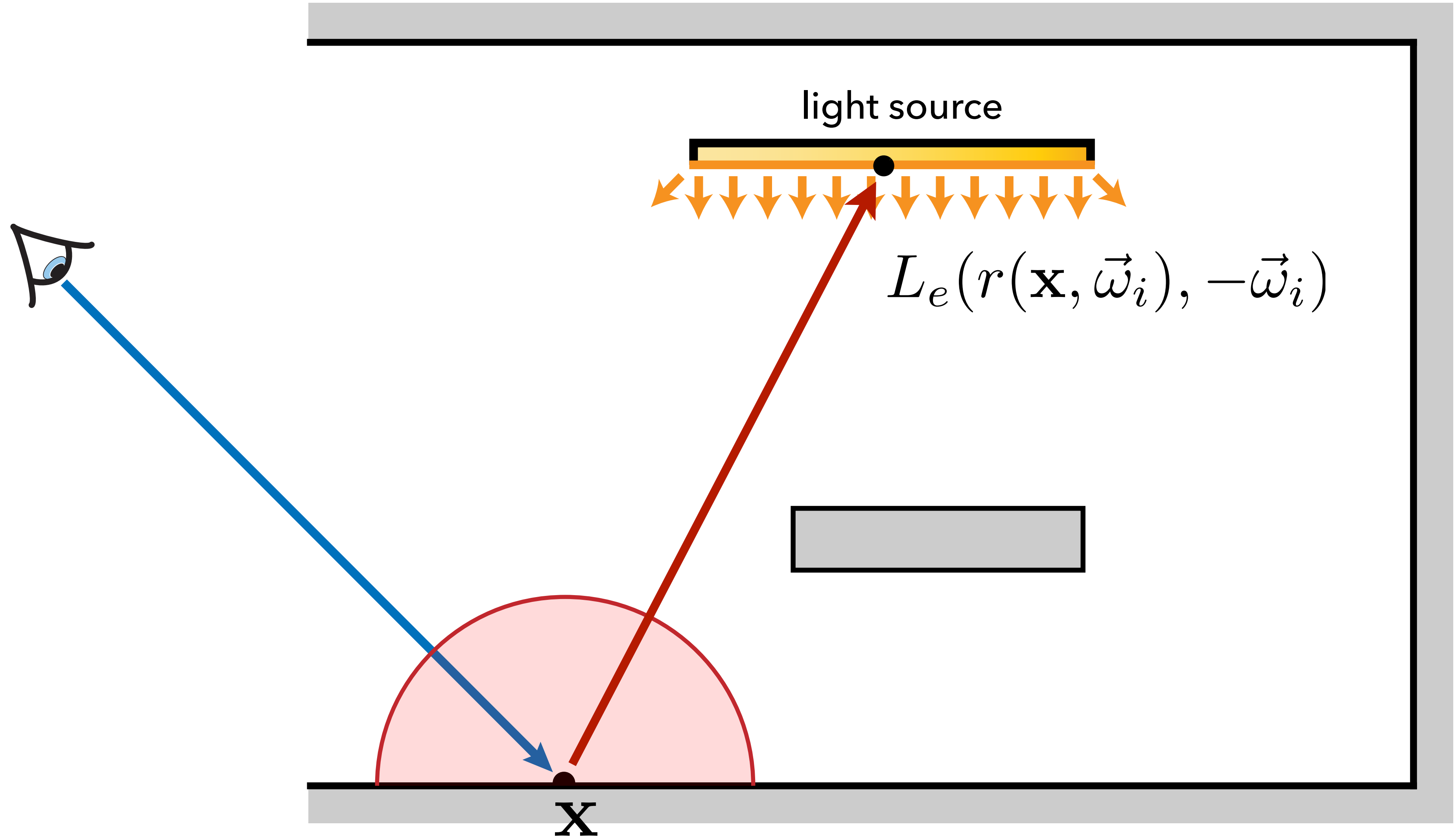
Direct Illumination

$$L_r(\mathbf{x}, \vec{\omega}_r) = \boxed{\int_{H^2}} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_e(r(\mathbf{x}, \vec{\omega}_i), -\vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$



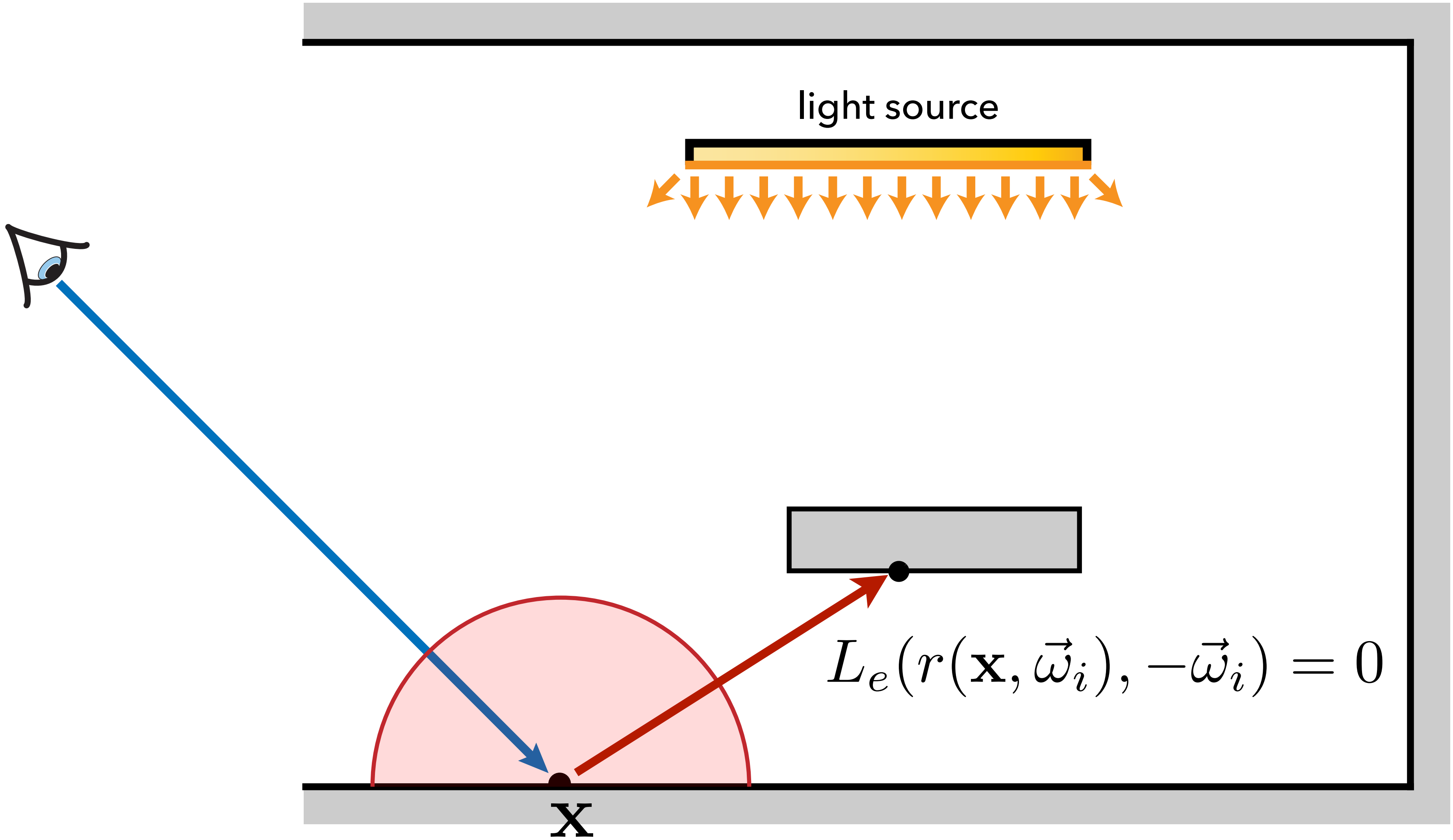
Direct Illumination

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) \boxed{L_e(r(\mathbf{x}, \vec{\omega}_i), -\vec{\omega}_i)} \cos\theta_i \, d\vec{\omega}_i$$



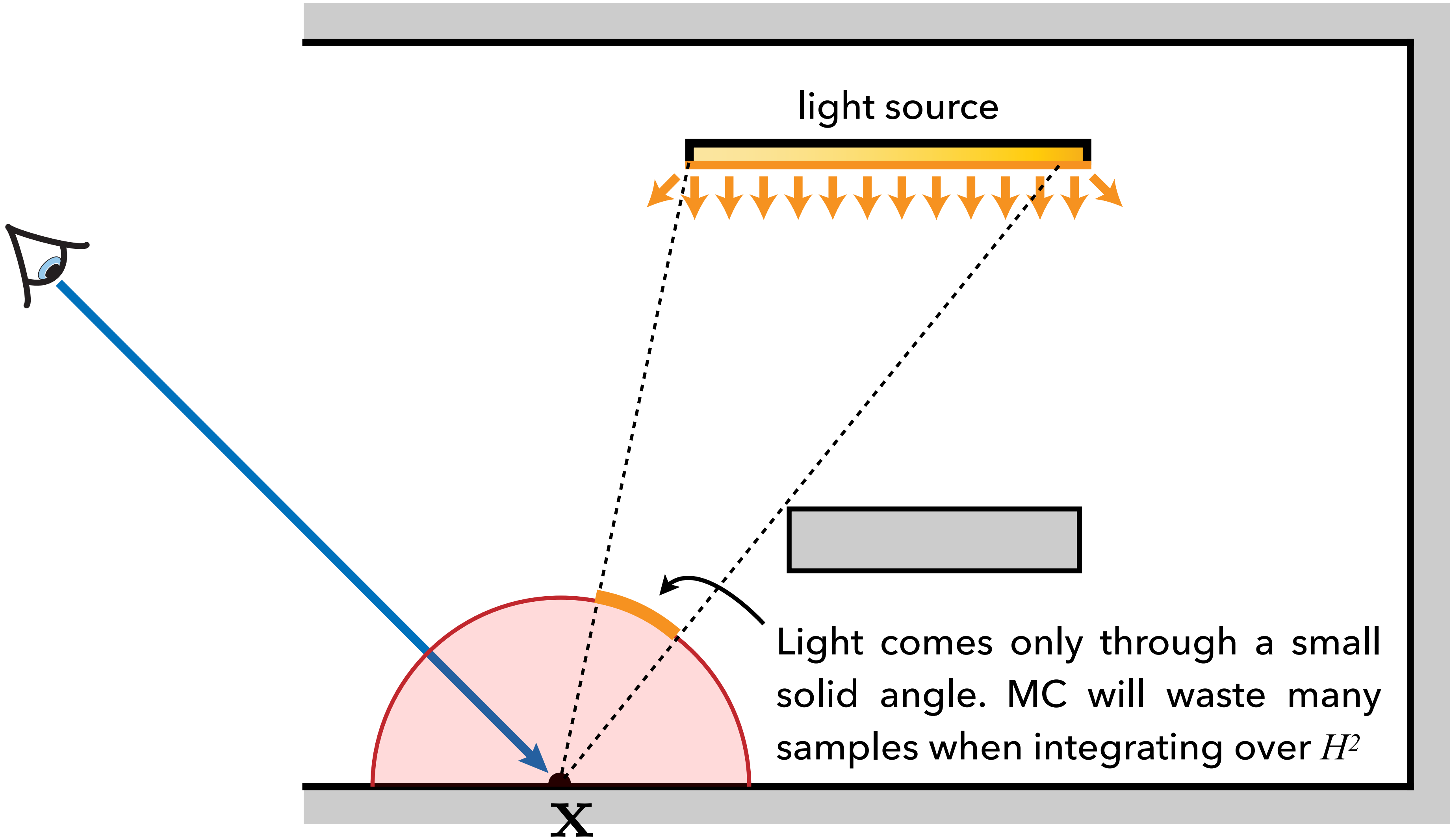
Direct Illumination

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) \boxed{L_e(r(\mathbf{x}, \vec{\omega}_i), -\vec{\omega}_i)} \cos\theta_i \, d\vec{\omega}_i$$



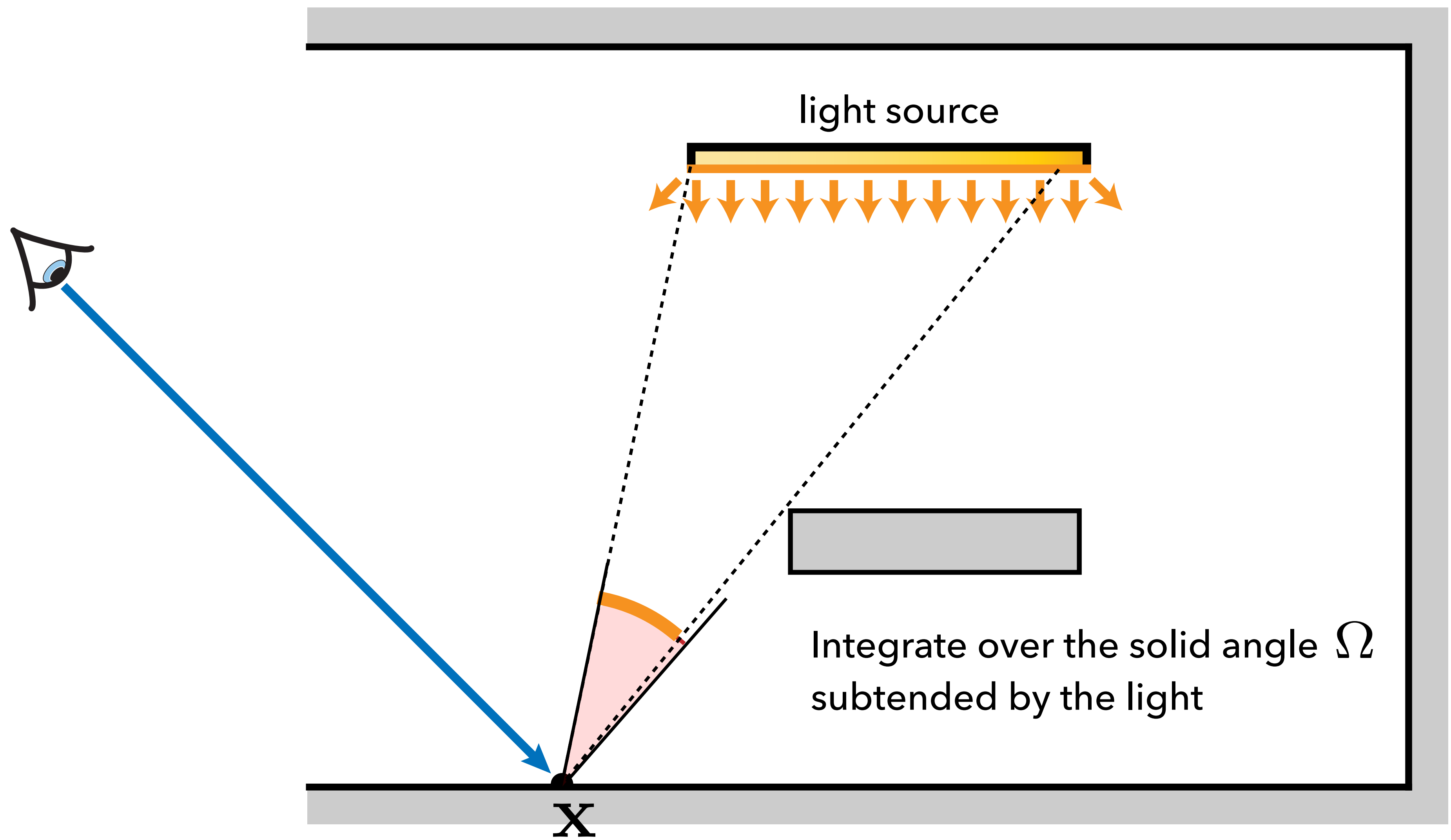
Direct Illumination

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_e(r(\mathbf{x}, \vec{\omega}_i), -\vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$



Direct Illumination

$$L_r(\mathbf{x},\vec{\omega}_r) = \int_{\boxed{\Omega}} f_r(\mathbf{x},\vec{\omega}_i,\vec{\omega}_r)\,L_e(r(\mathbf{x},\vec{\omega}_i),-\vec{\omega}_i)\,|\cos\theta_i|\,d\vec{\omega}_i$$

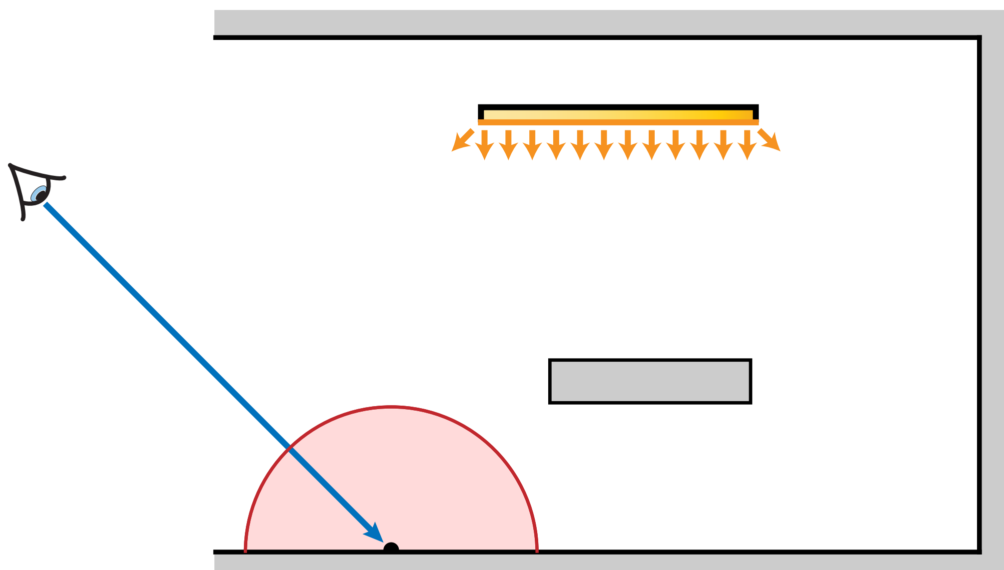


Integration over Solid Angle

- Can significantly improve efficiency of MC estimation
- Difficult for anything but spheres and disks
- Can we do better for light sources of arbitrary shapes?

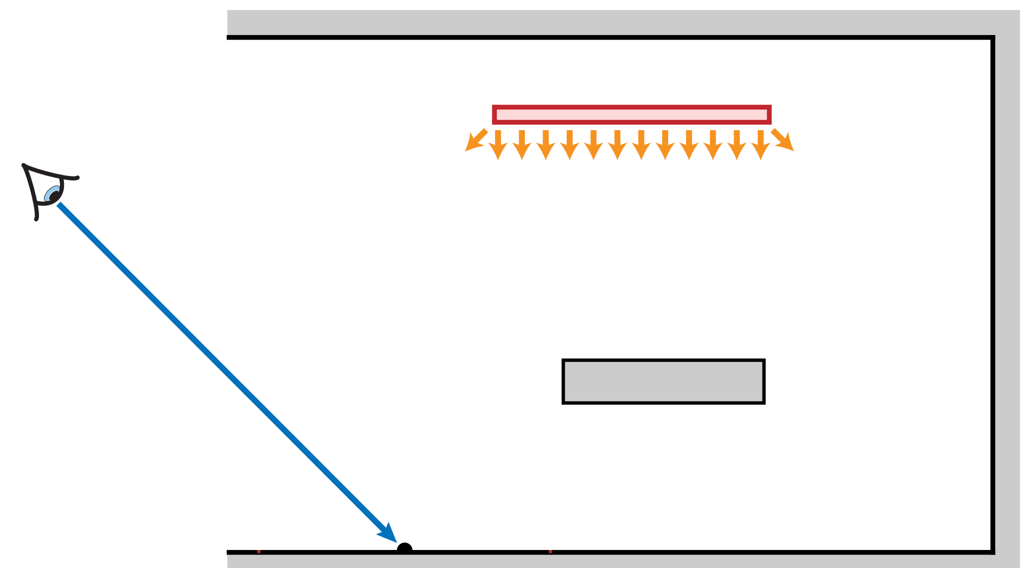
Forms of Reflection Equation

Hemispherical integration



$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

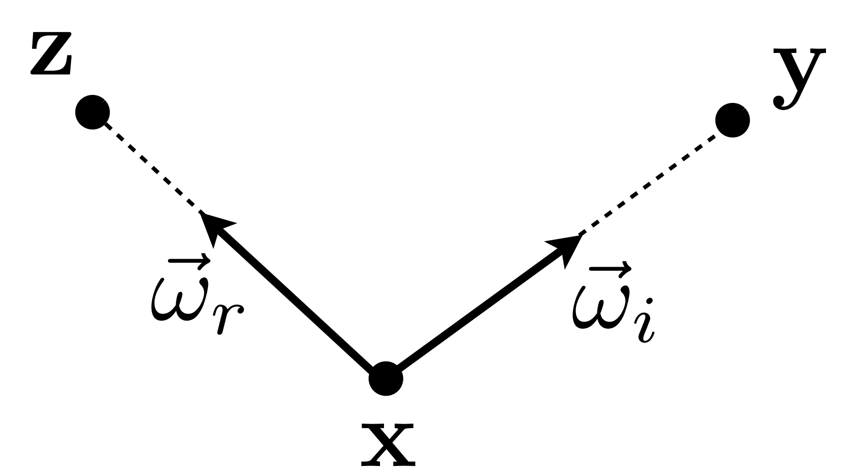
Surface Area integration



Forms of Reflection Equation

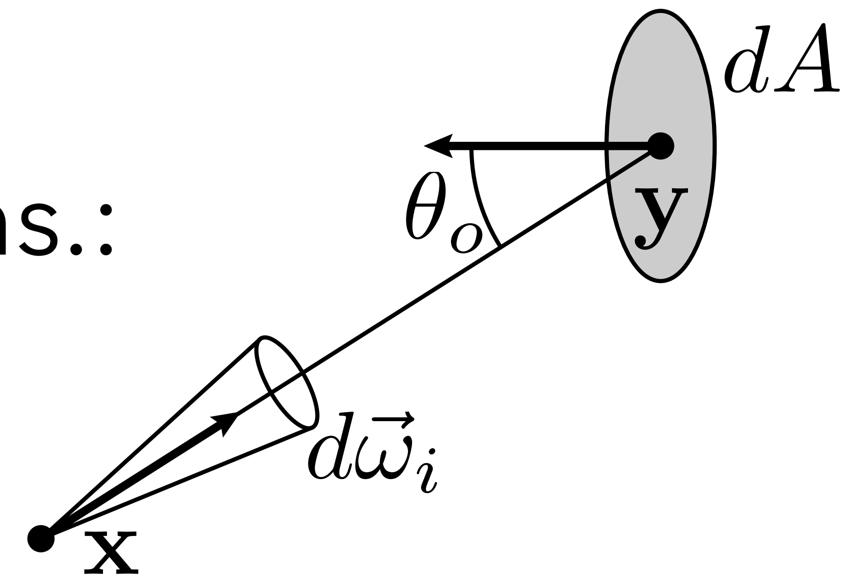
- Change in notation:

$$L_i(\mathbf{x}, \vec{\omega}_i) = L_i(\mathbf{x}, \mathbf{y})$$
$$L_r(\mathbf{x}, \vec{\omega}_r) = L_r(\mathbf{x}, \mathbf{z})$$
$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = f_r(\mathbf{x}, \mathbf{y}, \mathbf{z})$$



- Transform integral over directions into integral over surface area.
- Jacobian determinant of the trans.:

$$d\vec{\omega}_i = \frac{|\cos\theta_o|}{\|\mathbf{x}-\mathbf{y}\|^2} dA$$



Forms of Reflection Equation

$$L_i(\mathbf{x}, \vec{\omega}_i) = L_i(\mathbf{x}, \mathbf{y})$$

$$L_r(\mathbf{x}, \vec{\omega}_r) = L_r(\mathbf{x}, \mathbf{z})$$

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = f_r(\mathbf{x}, \mathbf{y}, \mathbf{z})$$

$$d\vec{\omega}_i = \frac{|\cos\theta_o|}{\|\mathbf{x} - \mathbf{y}\|^2} dA$$

Hemispherical form:

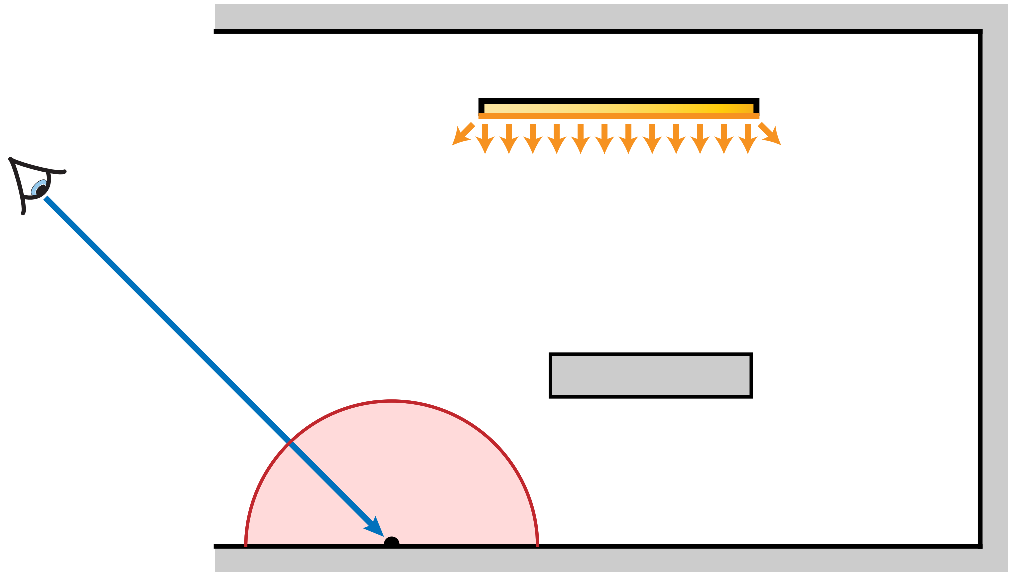
$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

Surface area form:

$$L_r(\mathbf{x}, \mathbf{z})$$

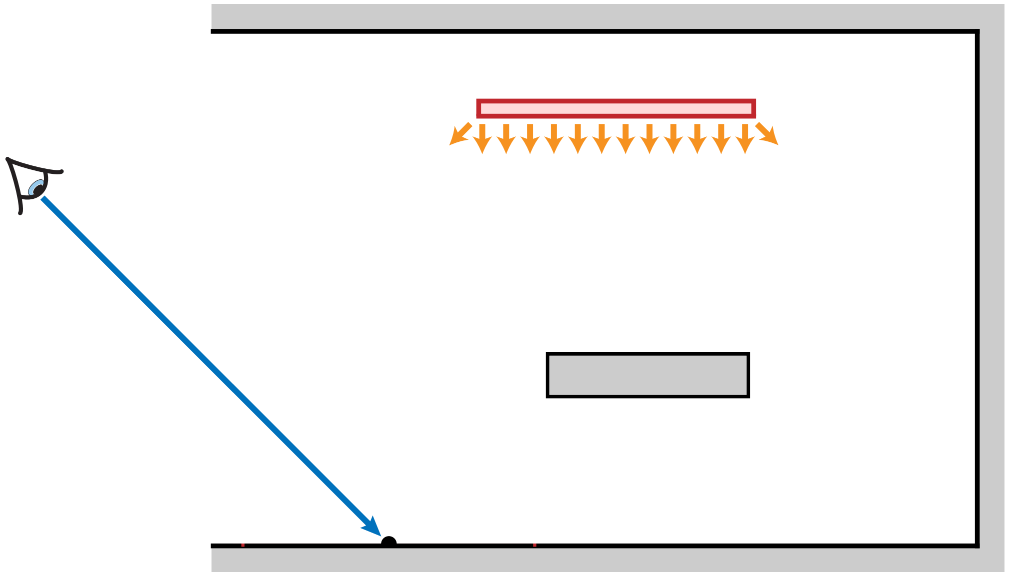
Forms of Reflection Equation

Hemispherical integration



$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

Surface Area integration



$$L_r(\mathbf{x}, \mathbf{z}) = \int_{A_e} f_r(\mathbf{x}, \mathbf{y}, \mathbf{z}) L_i(\mathbf{x}, \mathbf{y}) G(\mathbf{x}, \mathbf{y}) \, dA(\mathbf{y})$$

Area Form of the Reflection Eq.

$$L_r(\mathbf{x}, \mathbf{z}) = \int_{A_e} f_r(\mathbf{x}, \mathbf{y}, \mathbf{z}) L_i(\mathbf{x}, \mathbf{y}) \boxed{G(\mathbf{x}, \mathbf{y})} \, dA(\mathbf{y})$$

Geometry term:

$$G(\mathbf{x}, \mathbf{y}) = \boxed{V(\mathbf{x}, \mathbf{y})} \, \frac{|\cos\theta_i| \, |\cos\theta_o|}{\|\mathbf{x} - \mathbf{y}\|^2}$$

Visibility term:

$$V(\mathbf{x}, \mathbf{y}) = \begin{cases} 1: & \text{visible} \\ 0: & \text{not visible} \end{cases}$$

Area Form of the Reflection Eq.

$$L_r(\mathbf{x},\mathbf{z}) = \int_A f_r(\mathbf{x},\mathbf{y},\mathbf{z}) L_i(\mathbf{x},\mathbf{y}) \boxed{G(\mathbf{x},\mathbf{y})}\, dA(\mathbf{y})$$

Original foreshortening term

Geometry term:

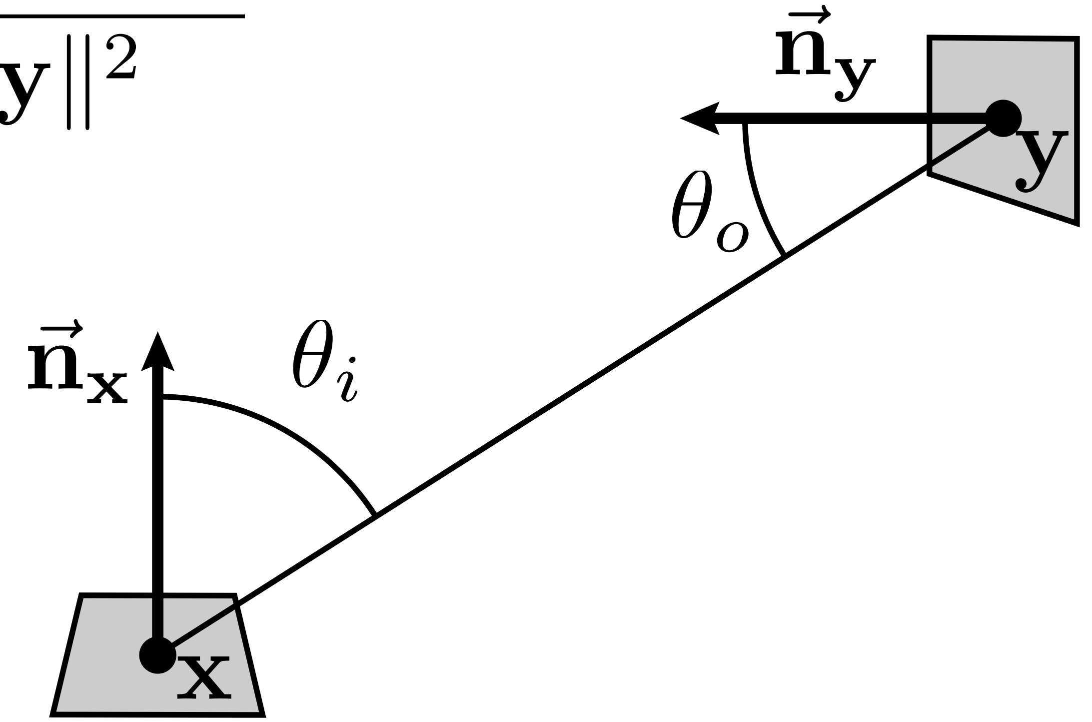
$$G(\mathbf{x},\mathbf{y}) = V(\mathbf{x},\mathbf{y}) \ \frac{\boxed{|\cos\theta_i|}\ \boxed{|\cos\theta_o|}}{\boxed{\|\mathbf{x}-\mathbf{y}\|^2}}$$

Jacobian determinant
of the transform

$$d\vec{\omega}_i = \frac{|\cos\theta_o|}{\|\mathbf{x}-\mathbf{y}\|^2} dA$$

Area Form of the Reflection Eq.

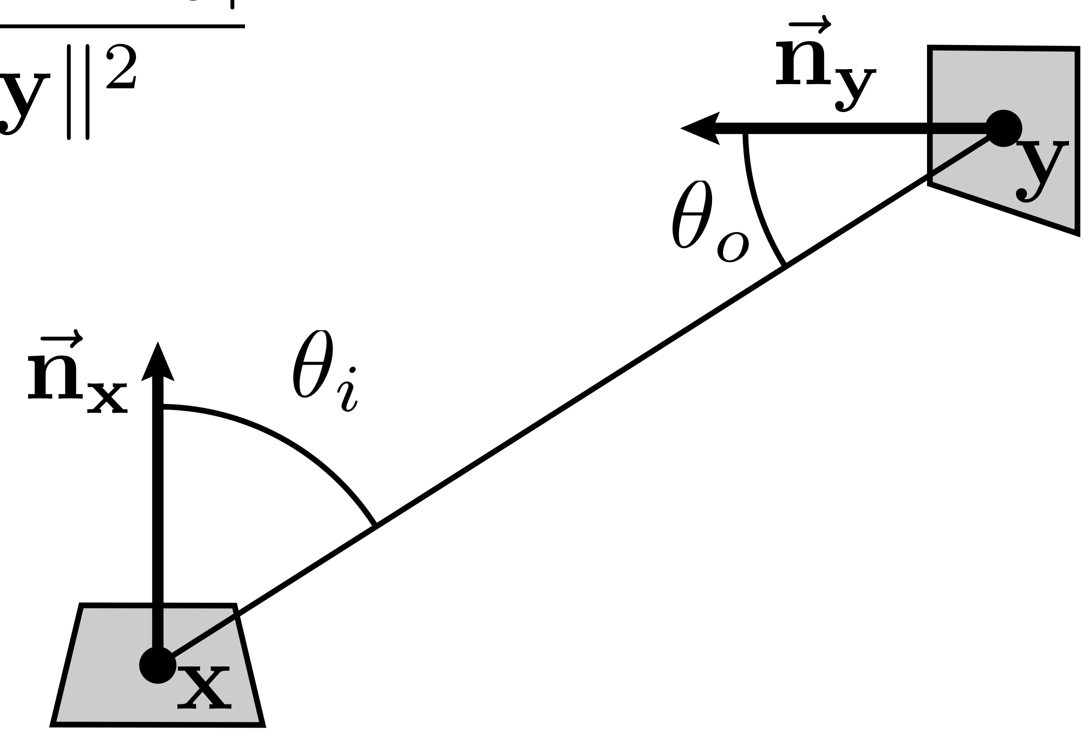
- Interpreting $\dfrac{|\cos\theta_i|\,|\cos\theta_o|}{\|\mathbf{x}-\mathbf{y}\|^2}$



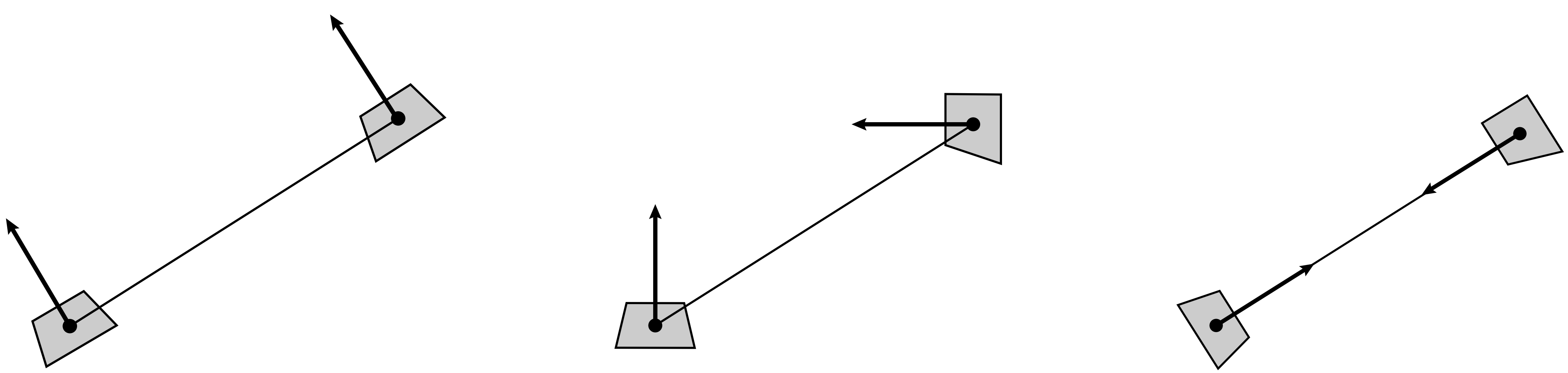
- The chance that a photon emitted from a differential patch will hit another diff patch decreases as:
  - the patches face away from each other (numerator)
  - the patches move away from each other (denominator)

Area Form of the Reflection Eq.

- Interpreting $\dfrac{|\cos\theta_i|\,|\cos\theta_o|}{\|\mathbf{x}-\mathbf{y}\|^2}$

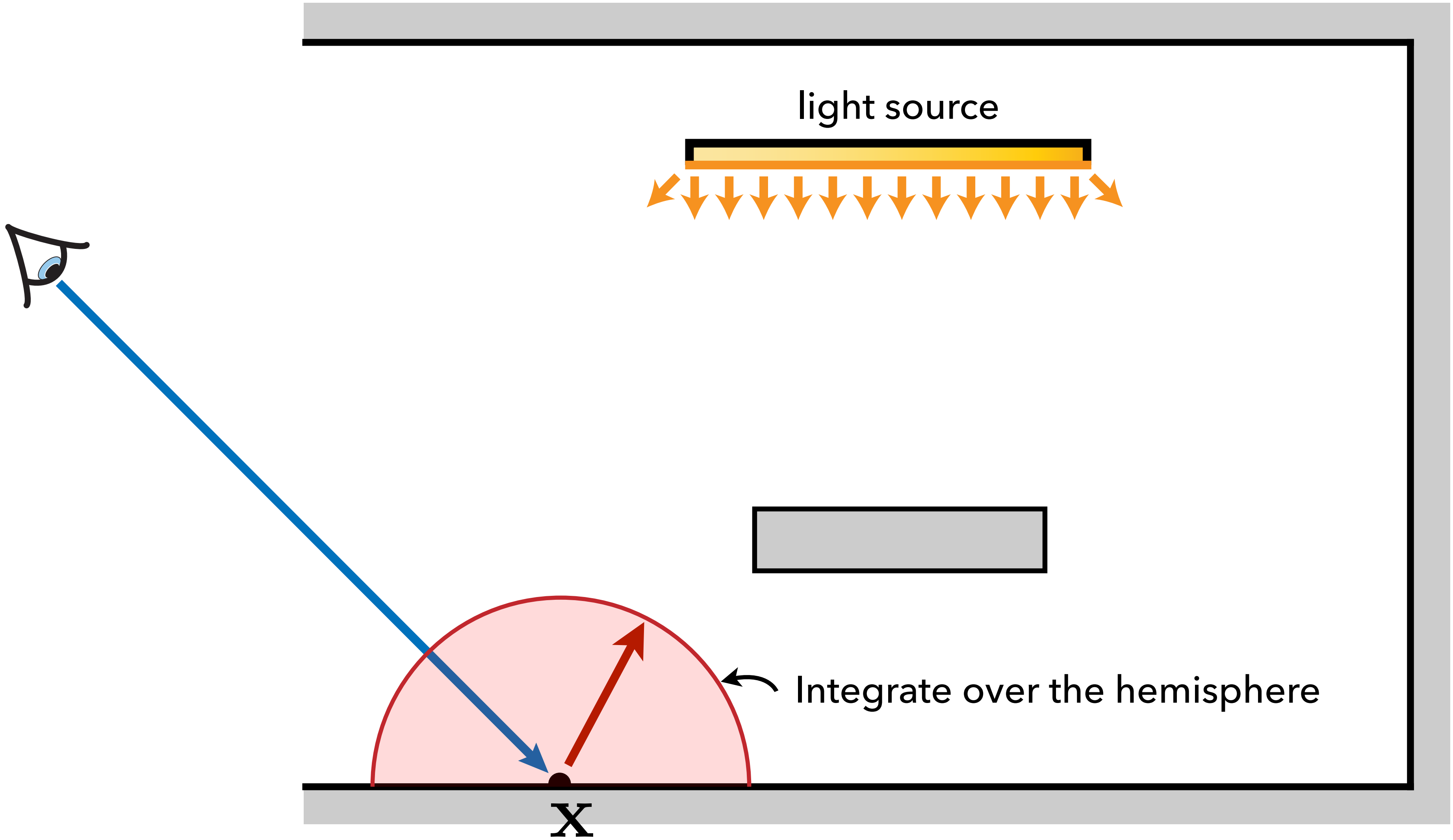


numerator = 0 | 0 < numerator < 1 | numerator = 1



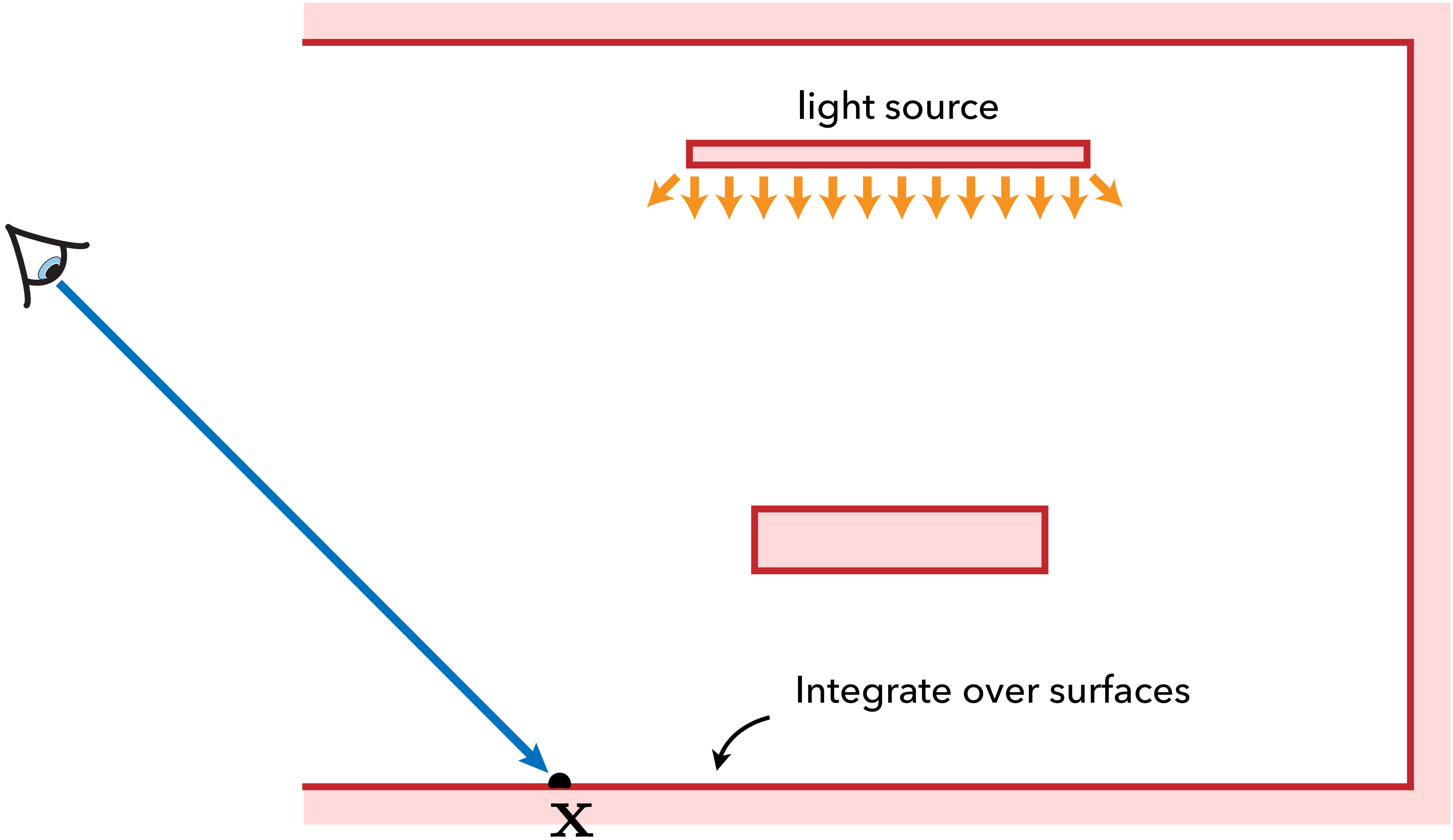
Direct Illumination

$$L_r(\mathbf{x},\vec{\omega}_r) = \boxed{\int_{H^2}} f_r(\mathbf{x},\vec{\omega}_i,\vec{\omega}_r) L_e(r(\mathbf{x},\vec{\omega}_i), -\vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$



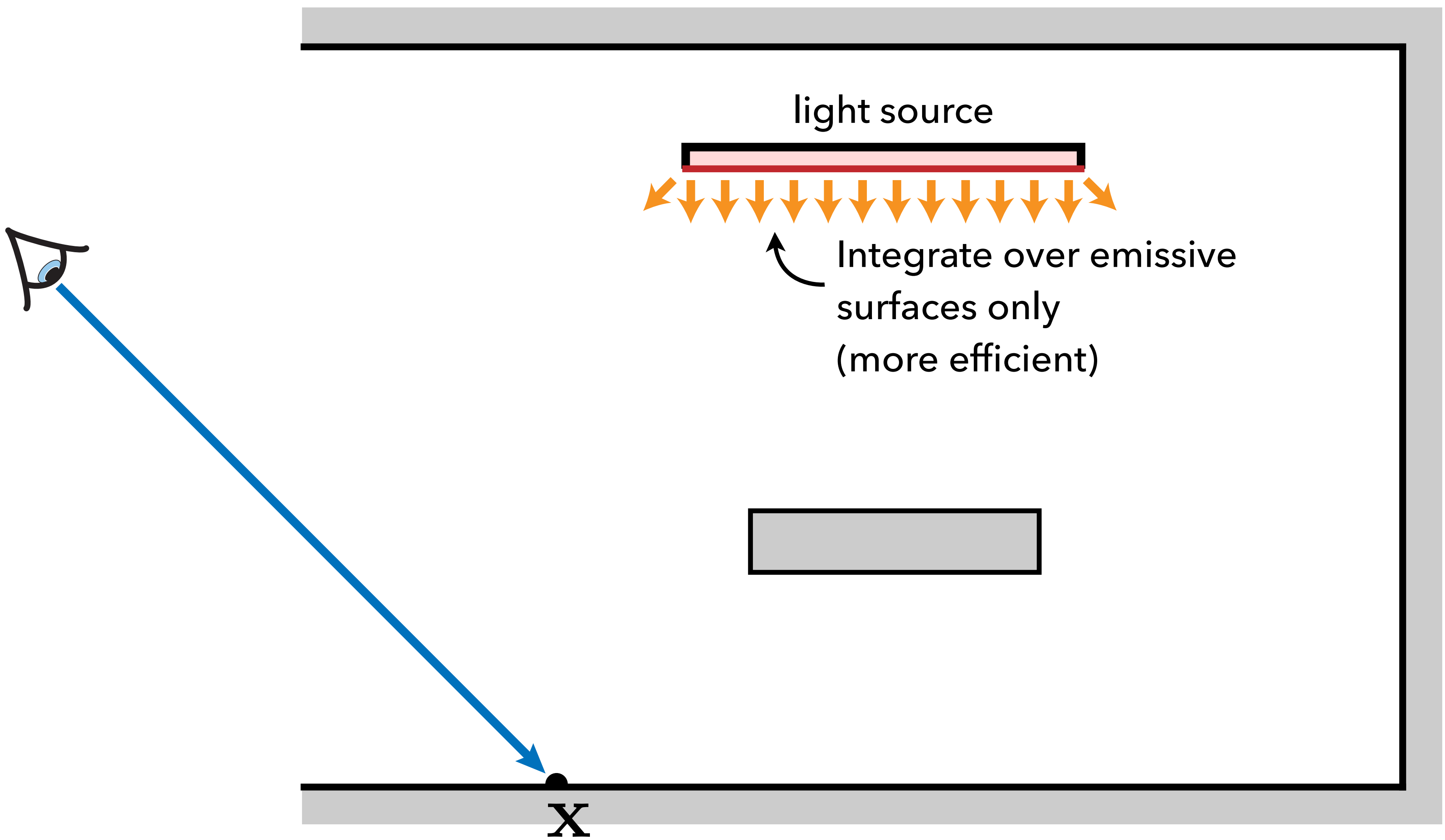
(Full) Global Illumination

$$L_r(\mathbf{x},\mathbf{z}) = \boxed{\int_A} f_r(\mathbf{x},\mathbf{y},\mathbf{z}) L_e(\mathbf{y},\mathbf{x}) V(\mathbf{x},\mathbf{y}) \frac{|\cos\theta_i|\,|\cos\theta_o|}{\|\mathbf{x}-\mathbf{y}\|^2} dA(\mathbf{y})$$



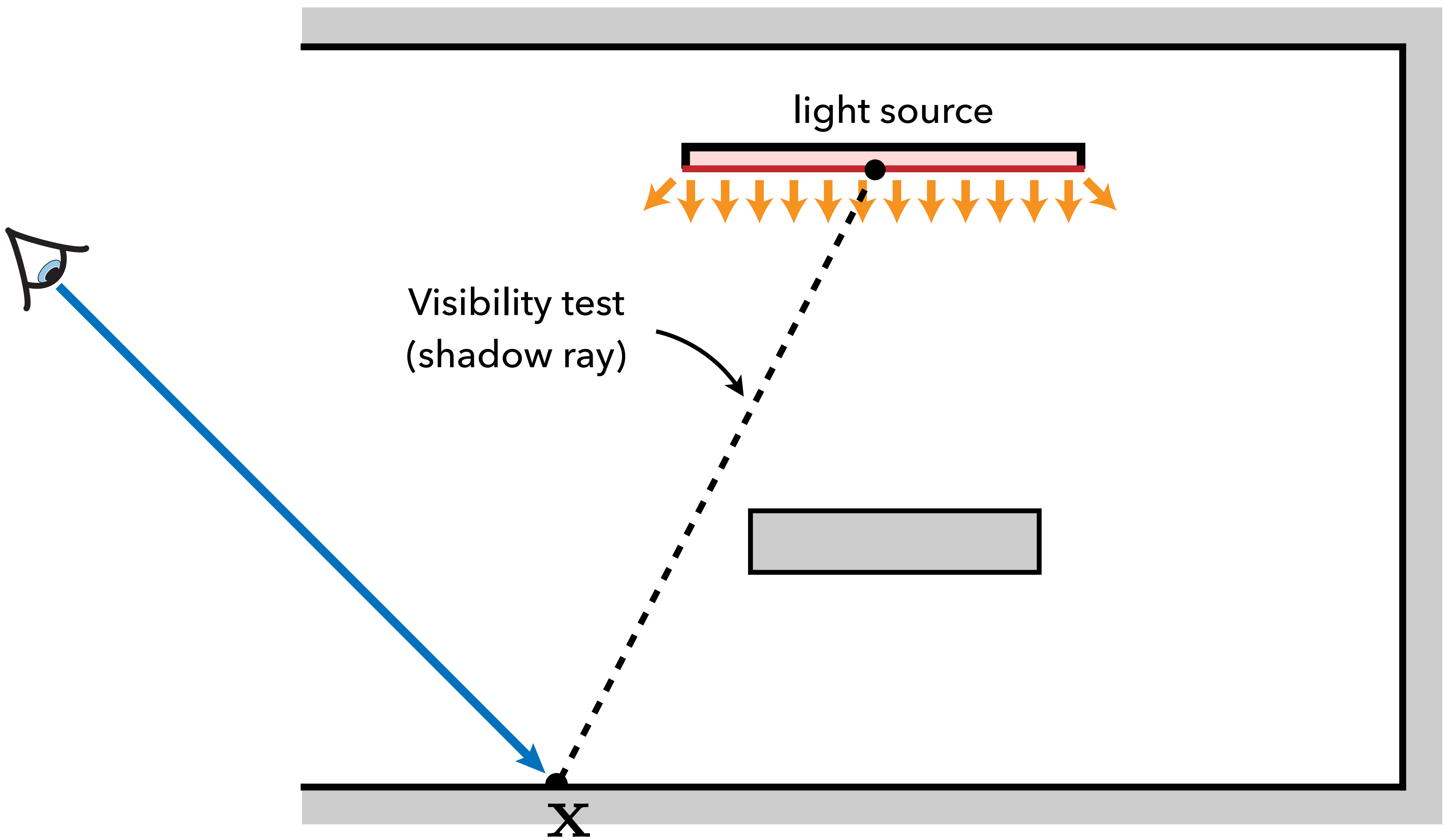
Direct Illumination

$$L_r(\mathbf{x},\mathbf{z}) = \boxed{\int_{A_e}} f_r(\mathbf{x},\mathbf{y},\mathbf{z}) L_e(\mathbf{y},\mathbf{x}) V(\mathbf{x},\mathbf{y}) \frac{|\cos\theta_i|\,|\cos\theta_o|}{\|\mathbf{x}-\mathbf{y}\|^2} dA(\mathbf{y})$$



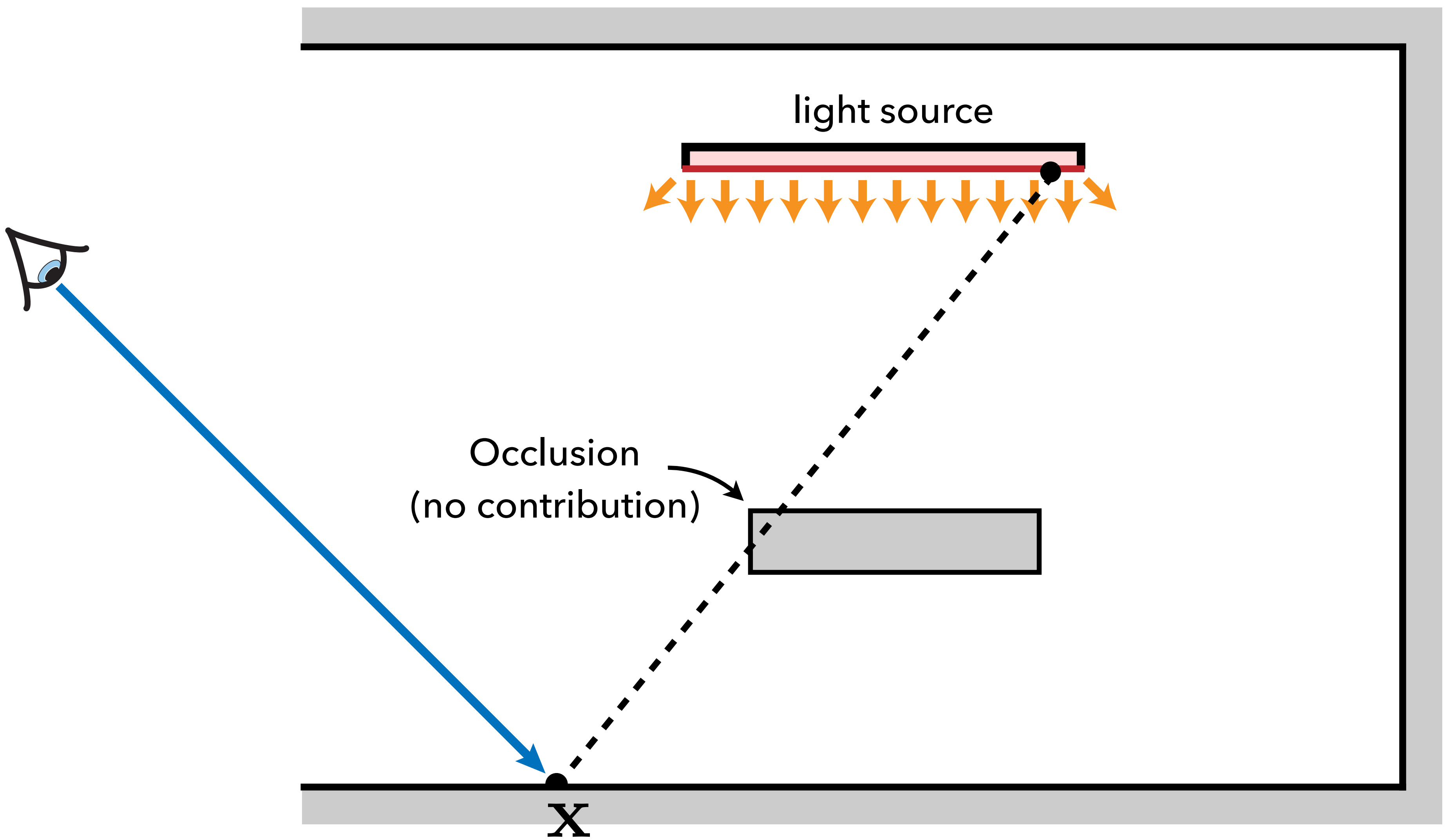
Direct Illumination

$$L_r(\mathbf{x},\mathbf{z}) = \int_{A_e} f_r(\mathbf{x},\mathbf{y},\mathbf{z}) L_e(\mathbf{y},\mathbf{x}) \boxed{V(\mathbf{x},\mathbf{y})} \frac{|\cos\theta_i|\,|\cos\theta_o|}{\|\mathbf{x}-\mathbf{y}\|^2} dA(\mathbf{y})$$

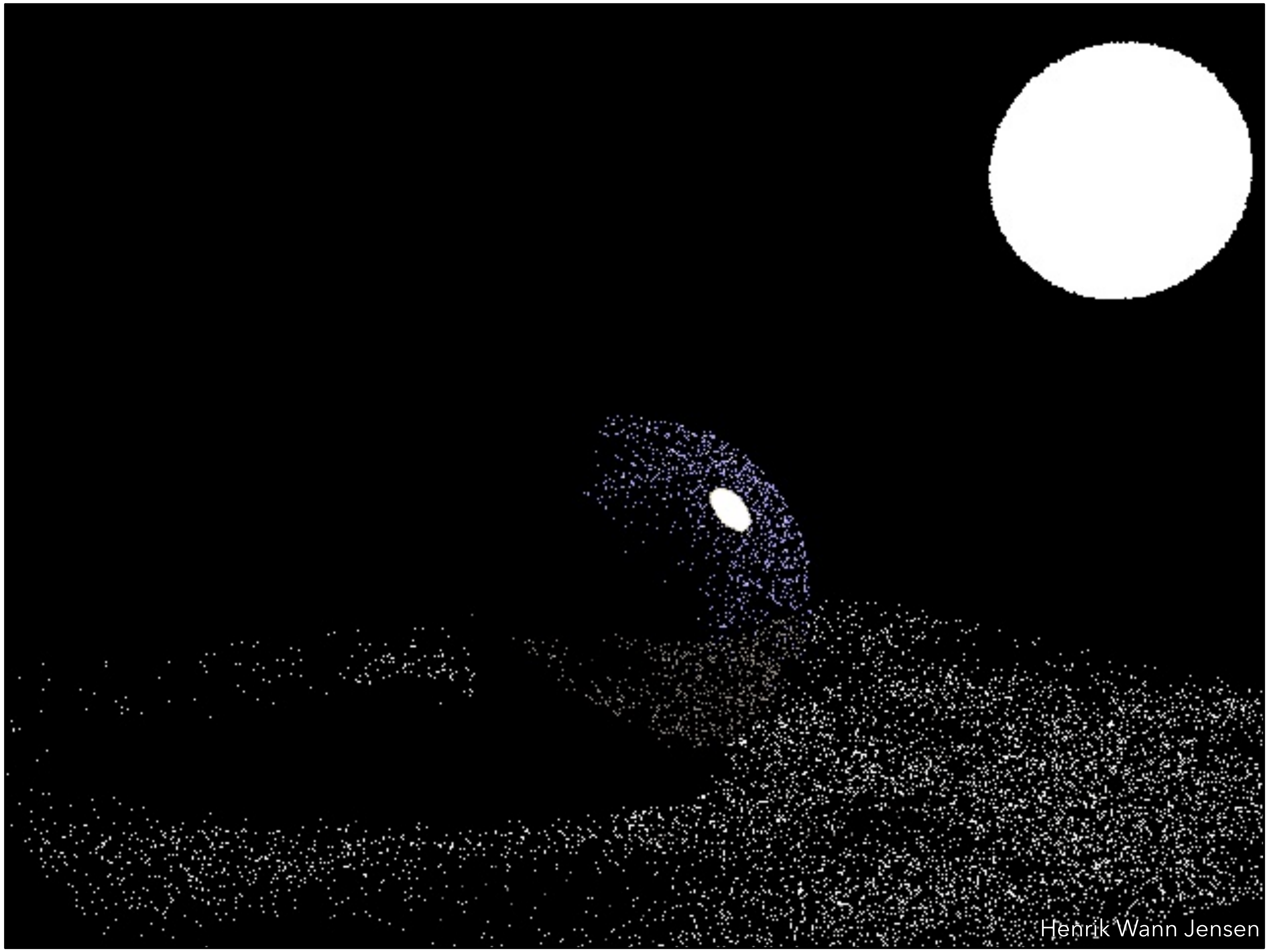


Direct Illumination

$$L_r(\mathbf{x},\mathbf{z}) = \int_{A_e} f_r(\mathbf{x},\mathbf{y},\mathbf{z}) L_e(\mathbf{y},\mathbf{x}) \boxed{V(\mathbf{x},\mathbf{y})} \frac{|\cos\theta_i|\,|\cos\theta_o|}{\|\mathbf{x}-\mathbf{y}\|^2}\, dA(\mathbf{y})$$

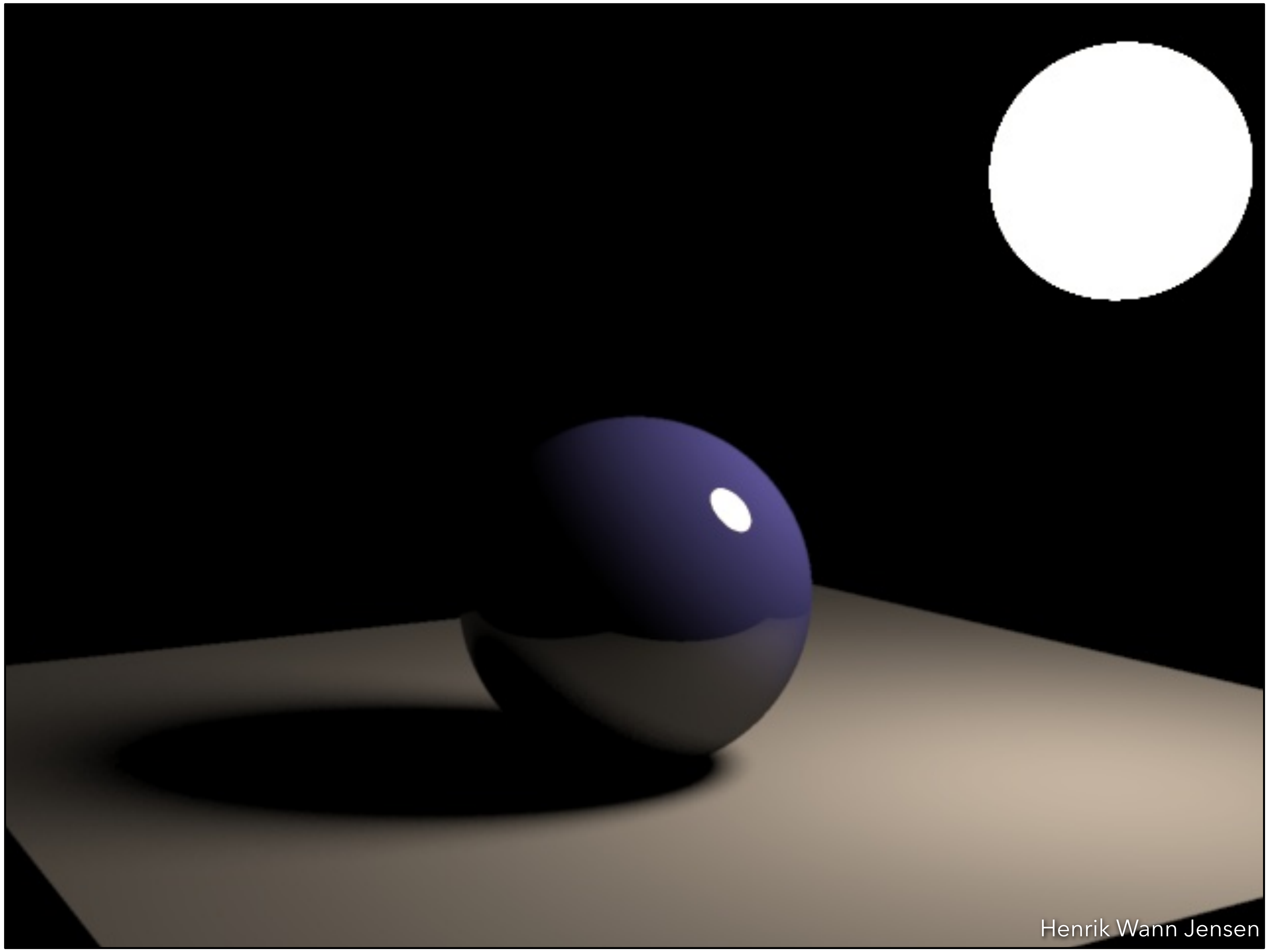


Direct Illumination



Sampling the hemisphere

Direct Illumination



Sampling the area of the light