ECSE 446/546

IMAGE SYNTHESIS

Direct Illumination I

image credit: feelgrafix

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(slides in part by W. Jarosz)

McGill

Light Sources

Light Sources

Point light | Directional light | Quad light | Sphere light | Mesh light

Delta lights
(create hard shadows)

Area/Shape lights
(create soft shadows)

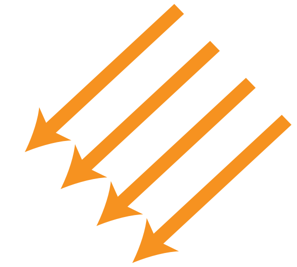
Point Light

- Typically defined using a point $\mathbf{p}$ and emitted power Φ (or radiant intensity)
- Omnidirectional emission from a single point
  - delta function

$$L_r(\mathbf{x}, \mathbf{z}) = \int_{A_e} f_r(\mathbf{x}, \mathbf{y}, \mathbf{z}) L_e(\mathbf{y}, \mathbf{x}) V(\mathbf{x}, \mathbf{y}) \frac{|\cos\theta_i|\,|\cos\theta_o|}{\|\mathbf{x} - \mathbf{y}\|^2} dA(\mathbf{y})$$

$$L_r(\mathbf{x}, \mathbf{z}) = \frac{\Phi}{4\pi} f_r(\mathbf{x}, \mathbf{p}, \mathbf{z}) V(\mathbf{x}, \mathbf{p}) \frac{|\cos\theta_i|}{\|\mathbf{x} - \mathbf{p}\|^2}$$

Directional Light

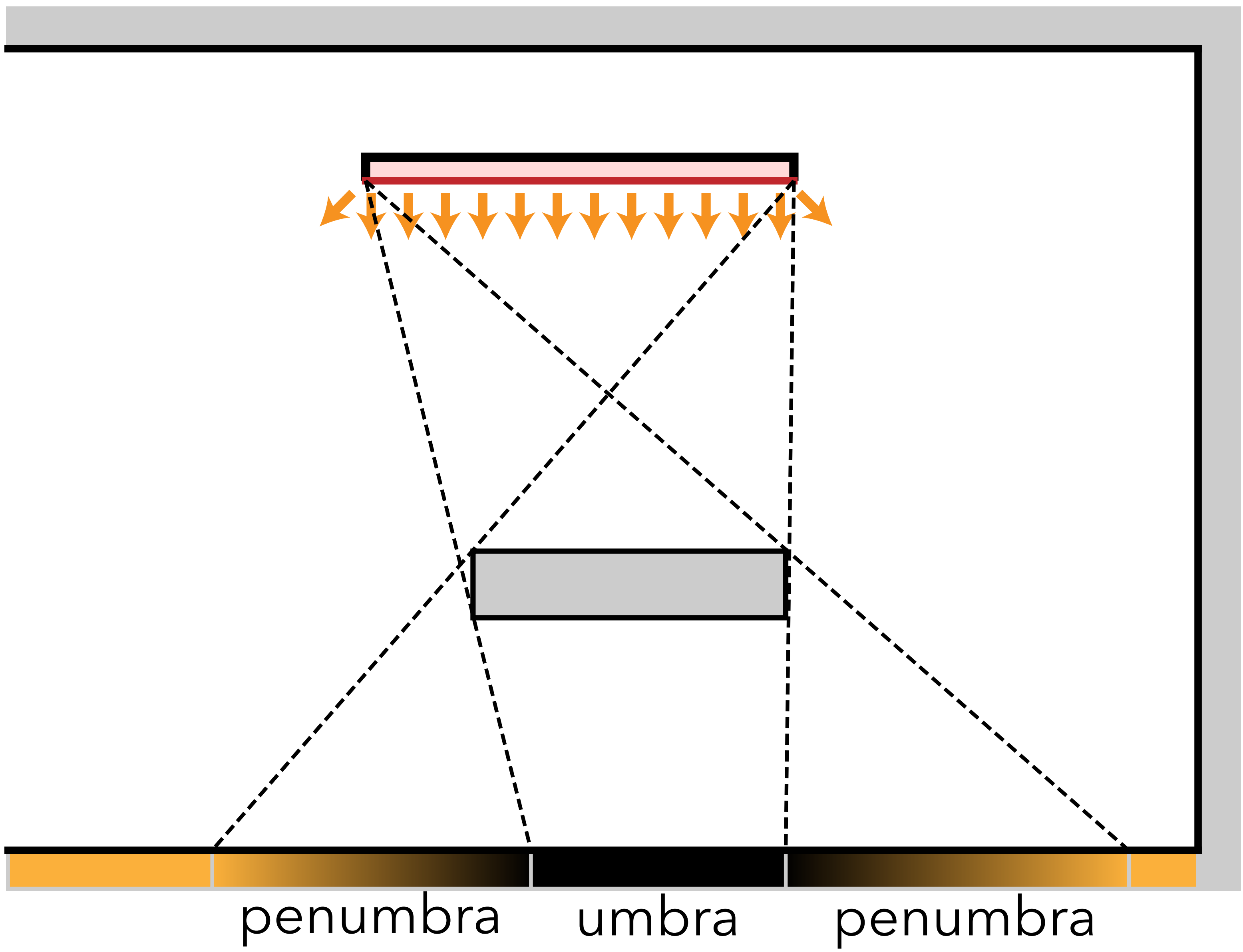


- Typically defined using direction $\vec{\omega}$ and radiance $L_d(\vec{\omega})$ coming *from* direction $\vec{\omega}$

$$L_r(\mathbf{x}, \vec{\omega}_r) = f_r(\mathbf{x}, \vec{\omega}, \vec{\omega}_r) V(\mathbf{x}, \vec{\omega}) L_d(\vec{\omega}) \cos\theta$$

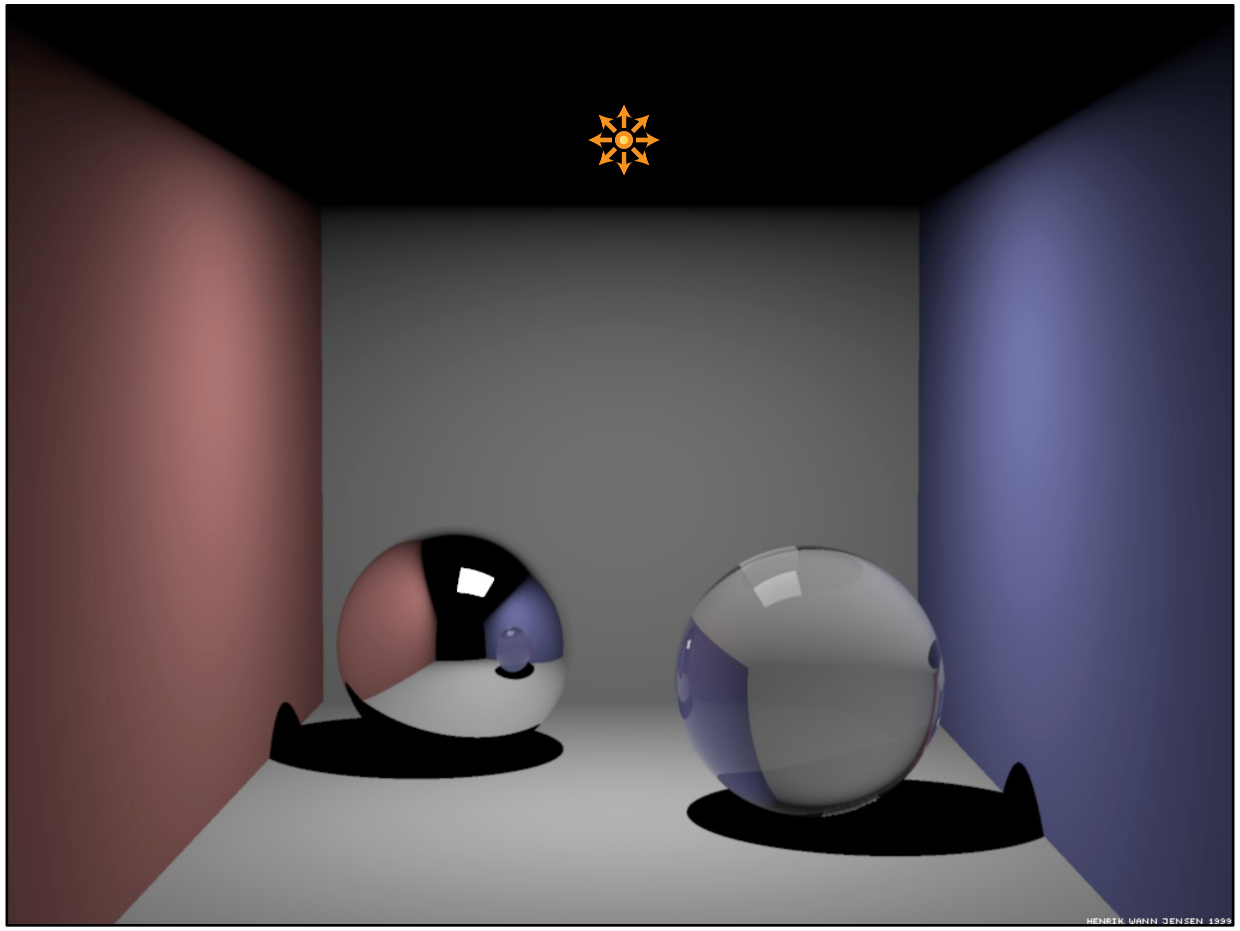
Quad Light

- Has finite area… creates soft shadows

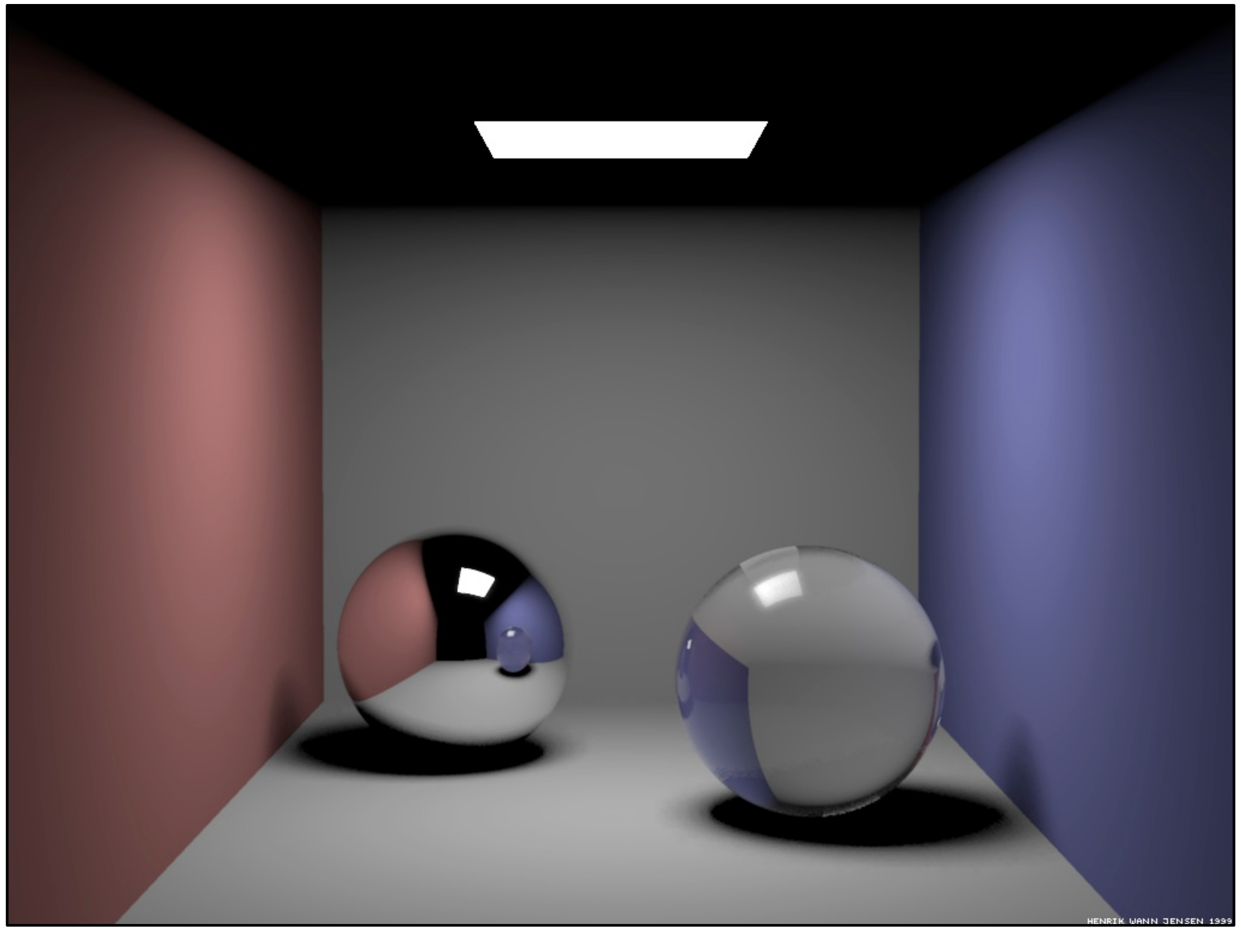


Quad Light

Point light



Quad light



Sphere Light

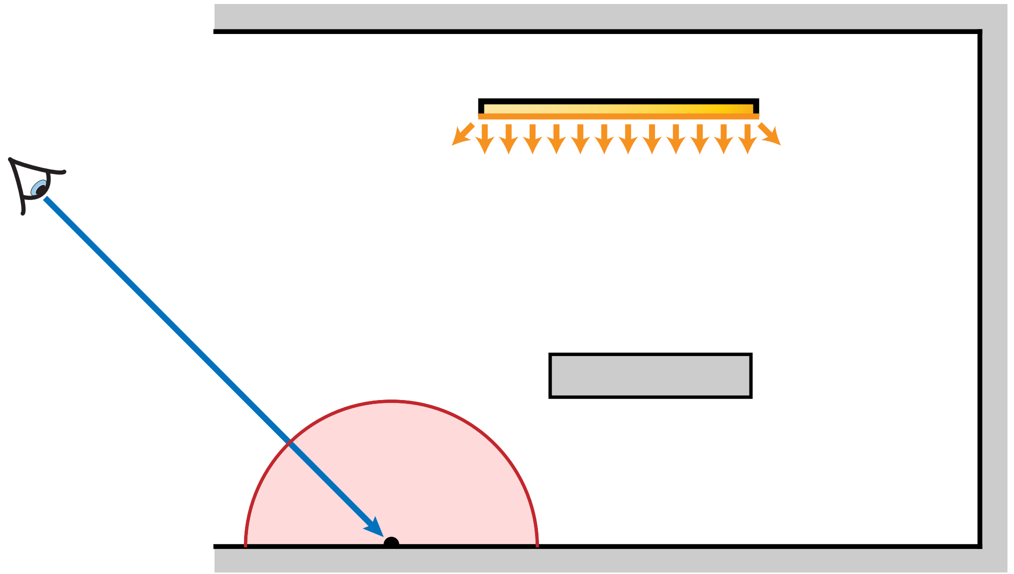
- Typically defined using a point $\mathbf{p}$, radius r and emitted power Φ (or emitted radiance L_e)
- Has finite area $4\pi r^2$

Sphere Light

- How to (importance) sample points emitted radiance from a sphere light?

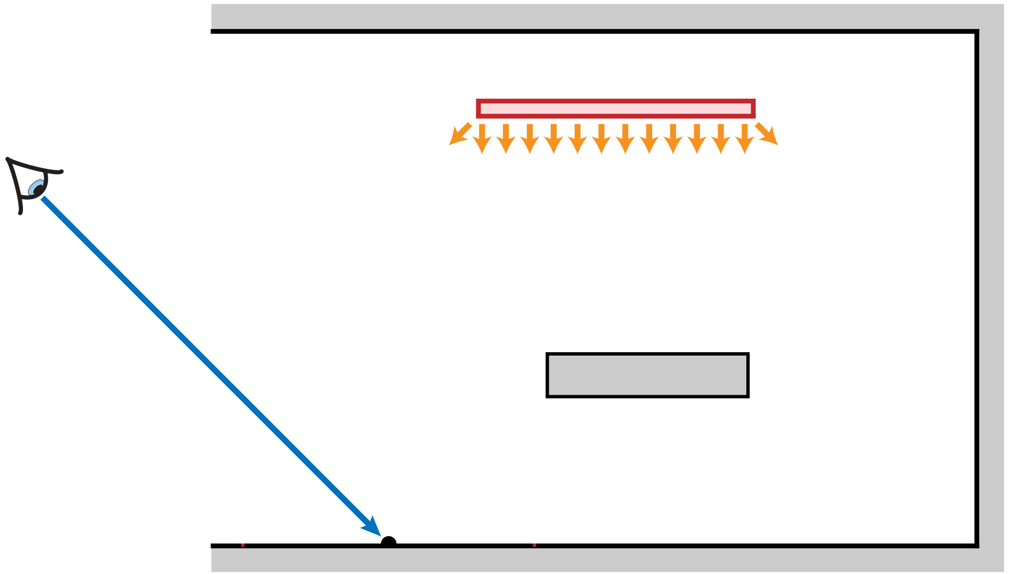
Forms of Reflection Equation

Hemispherical integration



$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

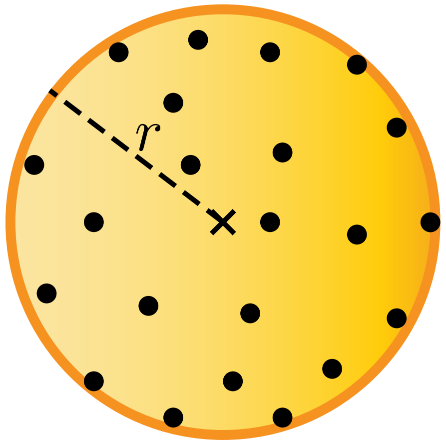
Surface Area integration



$$L_r(\mathbf{x}, \mathbf{z}) = \int_{A_e} f_r(\mathbf{x}, \mathbf{y}, \mathbf{z}) L_i(\mathbf{x}, \mathbf{y}) G(\mathbf{x}, \mathbf{y}) \, dA(\mathbf{y})$$

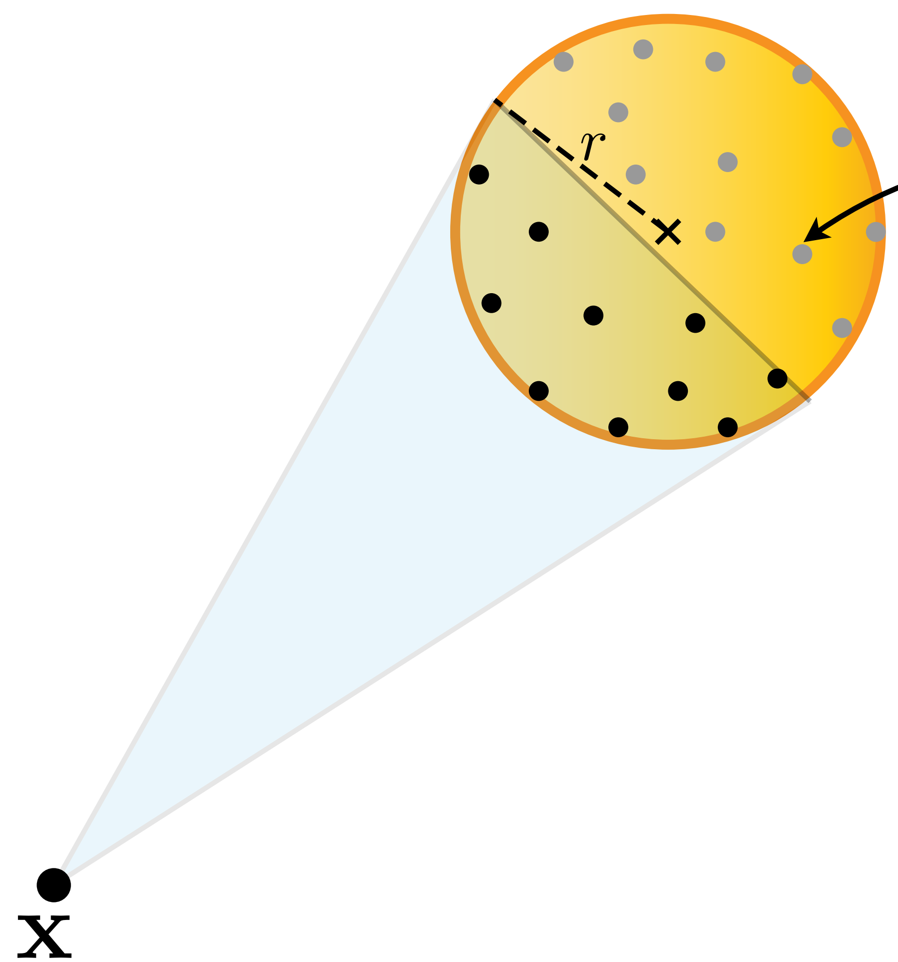
Sphere Light

- How to (importance) sample points emitted radiance from a sphere light?
- **Approach 1**: uniformly sample *surface area*



Sphere Light

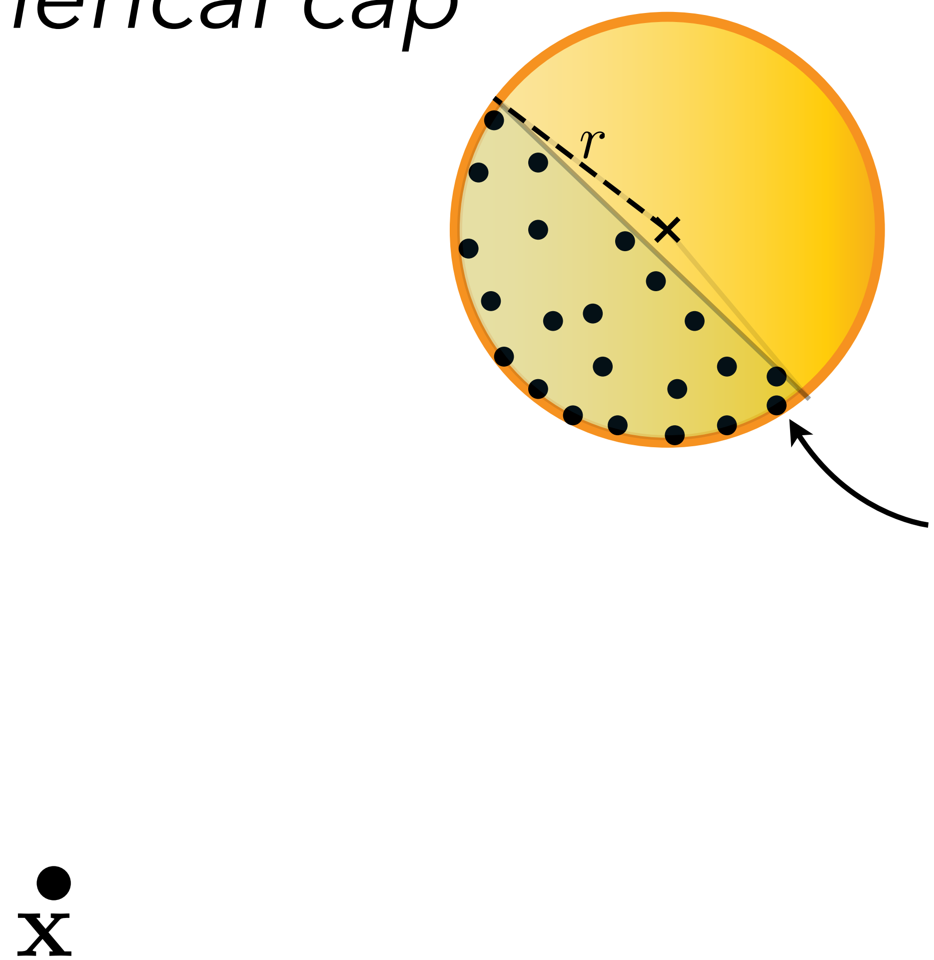
- How to (importance) sample points emitted radiance from a sphere light?
- **Approach 1**: uniformly sample *sphere* *area*



Many samples are not visible from the shading point!

Sphere Light

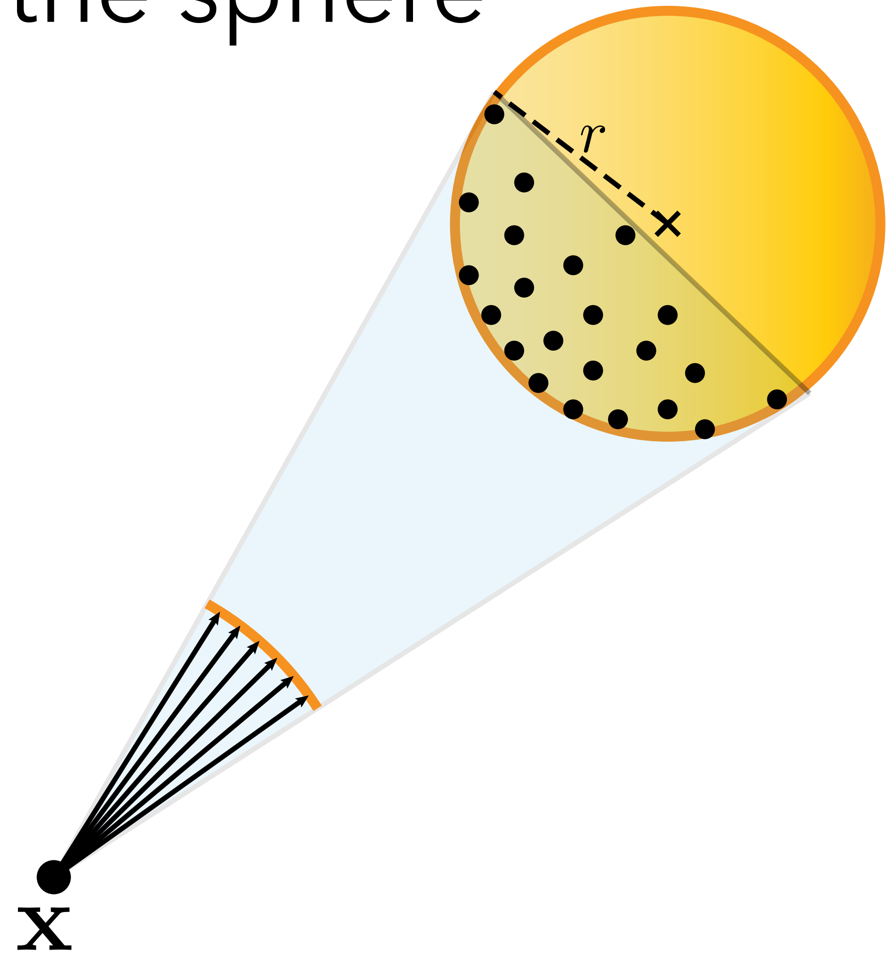
- How to (importance) sample points emitted radiance from a sphere light?
- **Approach 2**: uniformly sample <u>*area*</u> of the *visible spherical cap*



Uniform area-density is not ideal as emitted radiance is weighted by the cosine term
(recall the form factor in the G term)

Sphere Light

- How to (importance) sample points emitted radiance from a sphere light?
- **Approach 3**: uniformly sample *solid angle subtended* by the sphere

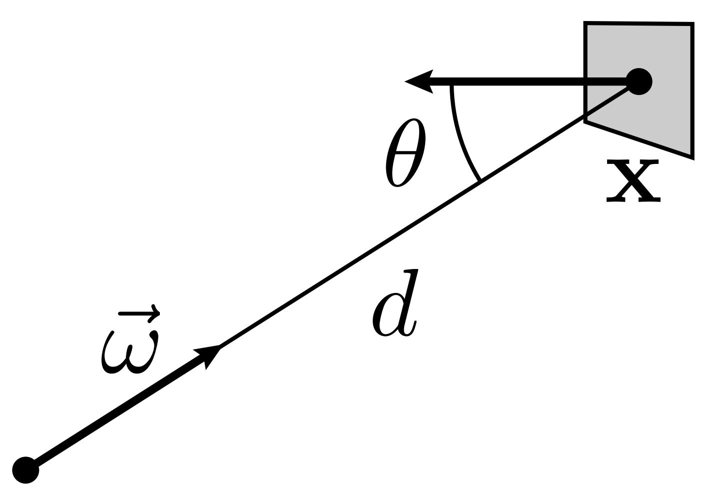


Sphere Light

- How to sample points on the sphere light?
- **Caution!**
  - Individual approaches use PDFs defined w.r.t. different measures
  - Make sure to convert the PDF into the measure of the integral!

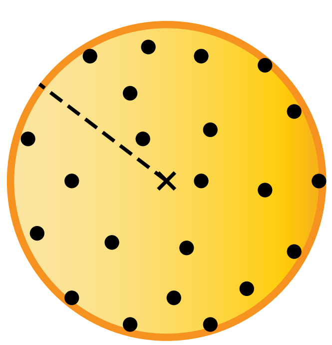
$$p_A(\mathbf{x}) = \frac{\cos\theta}{d^2} p_\Omega(\vec{\omega})$$

$$p_\Omega(\vec{\omega}) = \frac{d^2}{\cos\theta} p_A(\mathbf{x})$$



Sphere Light

- How to sample points on the sphere light?
- **Caution!**
  - Each approach uses a different measure
  - Make sure to convert the PDF into the measure of the integral
  - Example: using approach 1 for MC integration of the hemispherical formulation of the reflection eq.



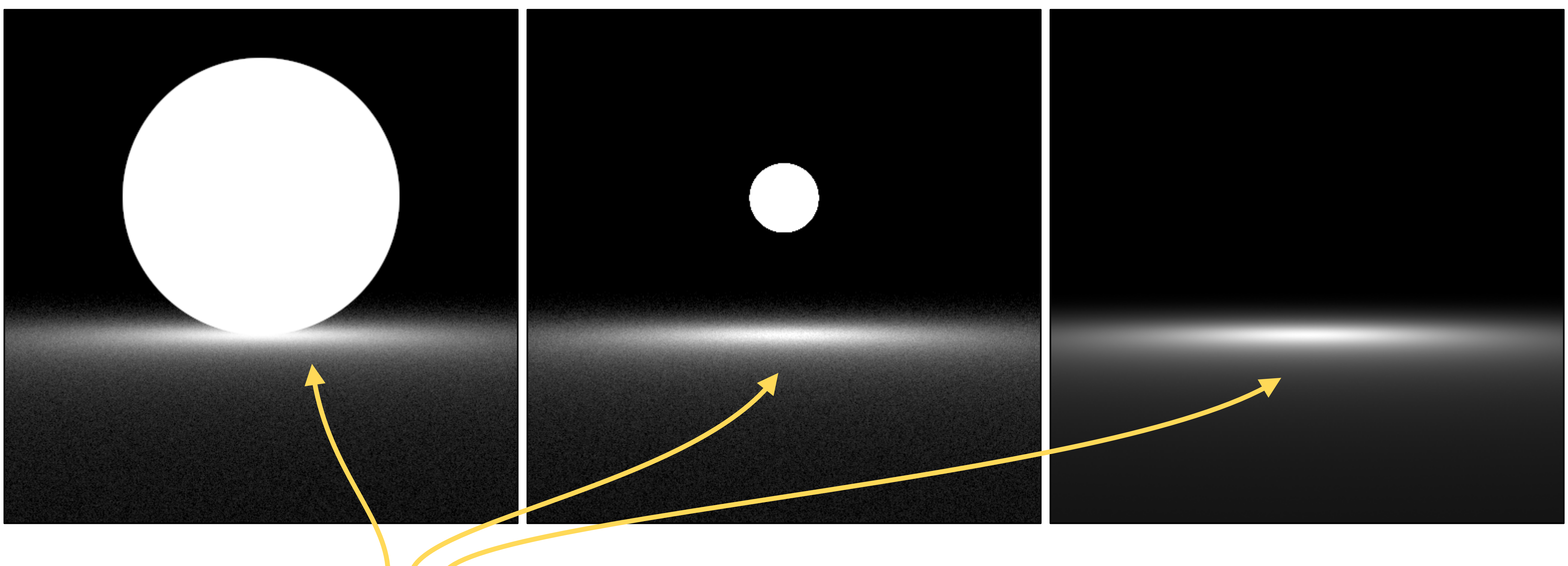
$$\langle L_r(\mathbf{x}, \vec{\omega}_r) \rangle = \frac{1}{N} \sum_{k=1}^{N} \frac{f_r(\mathbf{x}, \vec{\omega}_{i,k}, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_{i,k}) \cos\theta_{i,k}}{p_\Omega(\vec{\omega}_{i,k})}$$

$$p_A(\mathbf{y}) = \frac{1}{4\pi r^2} \qquad p_\Omega(\vec{\omega}_i) = \frac{\|\mathbf{x} - \mathbf{y}\|^2}{|-\omega_i \cdot \mathbf{n}_\mathbf{y}|\, 4\pi r^2}$$

Sphere Light

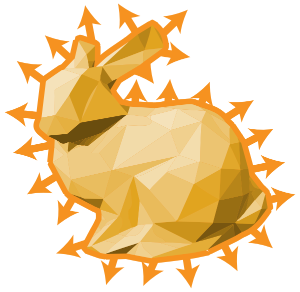
- **Validation**: irradiance is independent of radius (assuming it emits always the same power & no occluders)

A sphere light | A smaller sphere light | A point light



Identical irradiance profiles

Mesh Light



- An emissive mesh where every surface point emits given, constant radiance L_e
- Total area: $\sum A(k)$

Mesh Light

- How to importance sample?
- **Preprocess**:
  1. build a discrete PDF p_Δ for choosing polygons (triangles) *proportional to their area:*

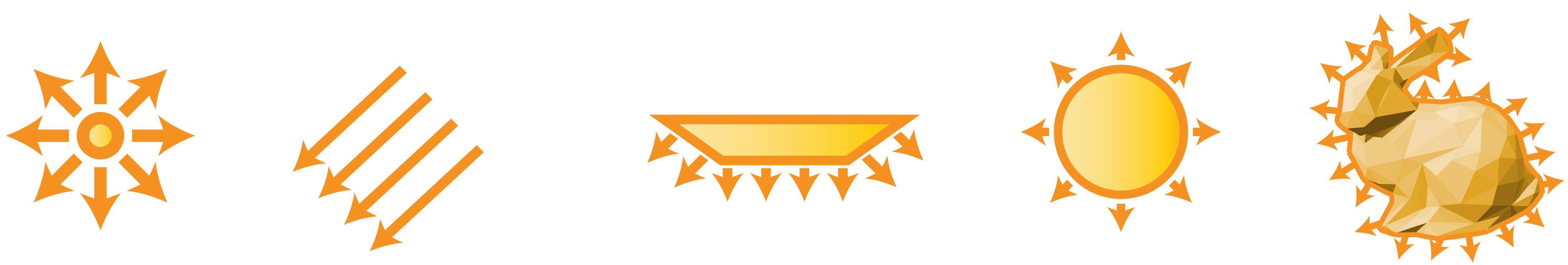
$$p_\Delta(i) = \frac{A(i)}{\sum A(k)}$$

- **Run-time**:
  2. sample a polygon i and a point $\mathbf{x}$ on i
  3. compute the PDF of choosing the point $\mathbf{x}$

$$p_A(\mathbf{x}) = p_\Delta(i) p_A(\mathbf{x}|i) = \frac{1}{\sum A(k)}$$

Light Sources

Point light | Directional light | Quad light | Sphere light | Mesh light



Delta lights
(create hard shadows)

Area/Shape lights
(create soft shadows)

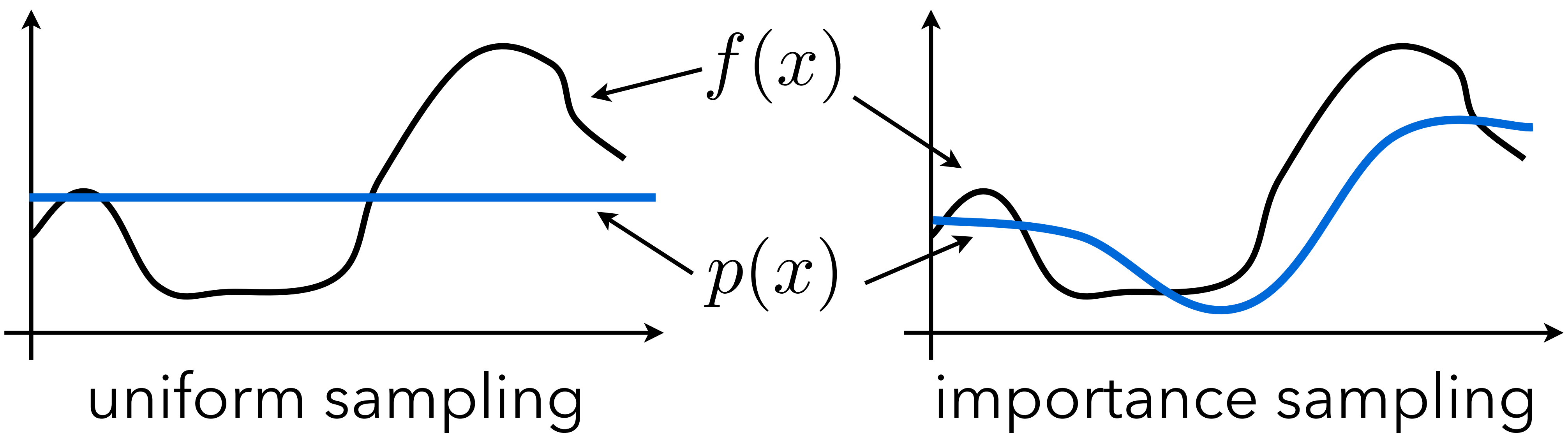
Importance Sampling

(recap)

Importance Sampling

- Placing samples intelligently reduces variance

$$\langle L_r(\mathbf{x}, \vec{\omega}_r)^N \rangle = \frac{1}{N} \sum_{k=1}^{N} \frac{f_r(\mathbf{x}, \vec{\omega}_{i,k}, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_{i,k}) \cos\theta_{i,k} \, d\vec{\omega}_{i,k}}{p_\Omega(\vec{\omega}_{i,k})}$$



Reflection Equation

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

- What terms can we importance sample?
  - BRDF
  - incident (or emitted) radiance
  - cosine term

Reflection Equation

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \boxed{\cos\theta_i} d\vec{\omega}_i$$

- What terms can we importance sample?
  - BRDF
  - incident radiance
  - **cosine term**

Sampling the Cosine Term

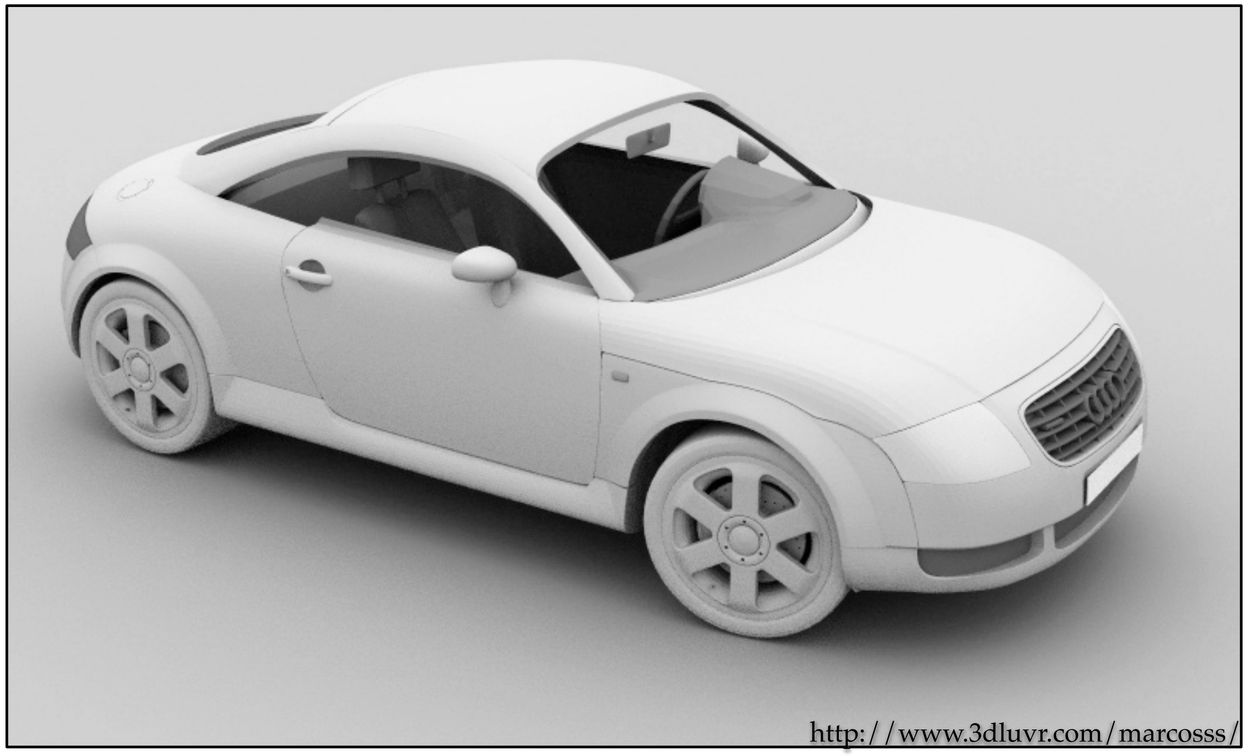
- Let's consider a simplified setup: diffuse objects illuminated by an ambient white sky

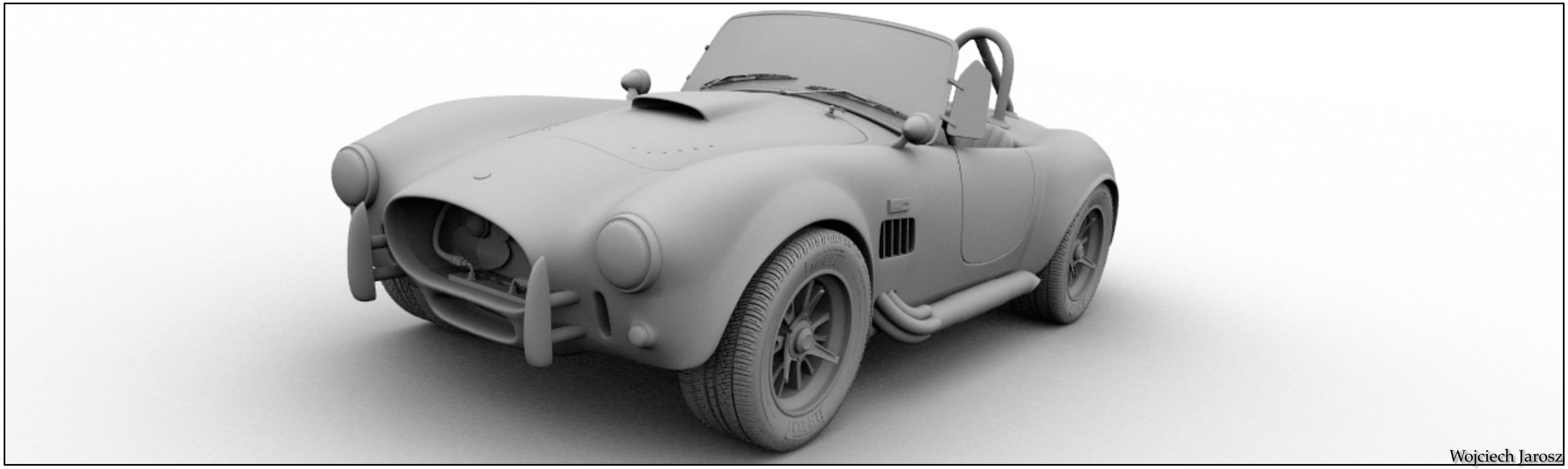
$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

$$L_r(\mathbf{x}) = \frac{\rho}{\pi} \int_{H^2} V(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

- a.k.a. *ambient occlusion*

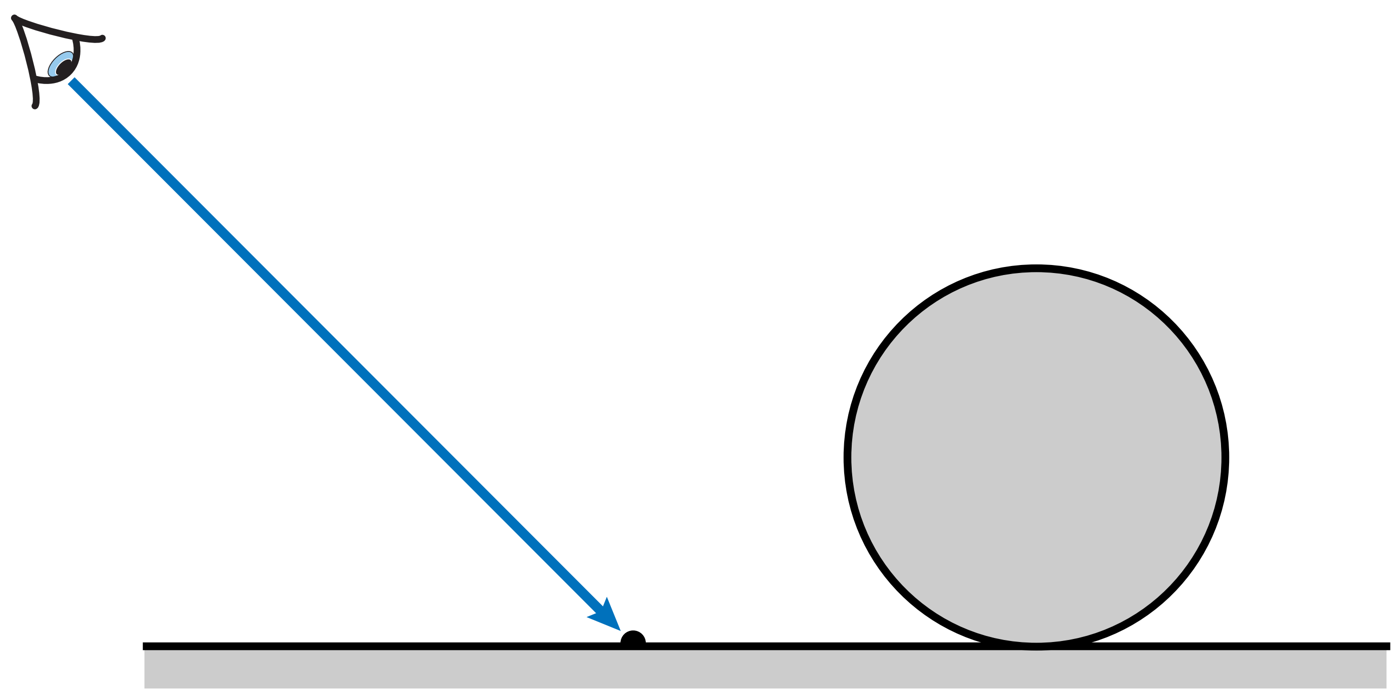
Ambient Occlusion





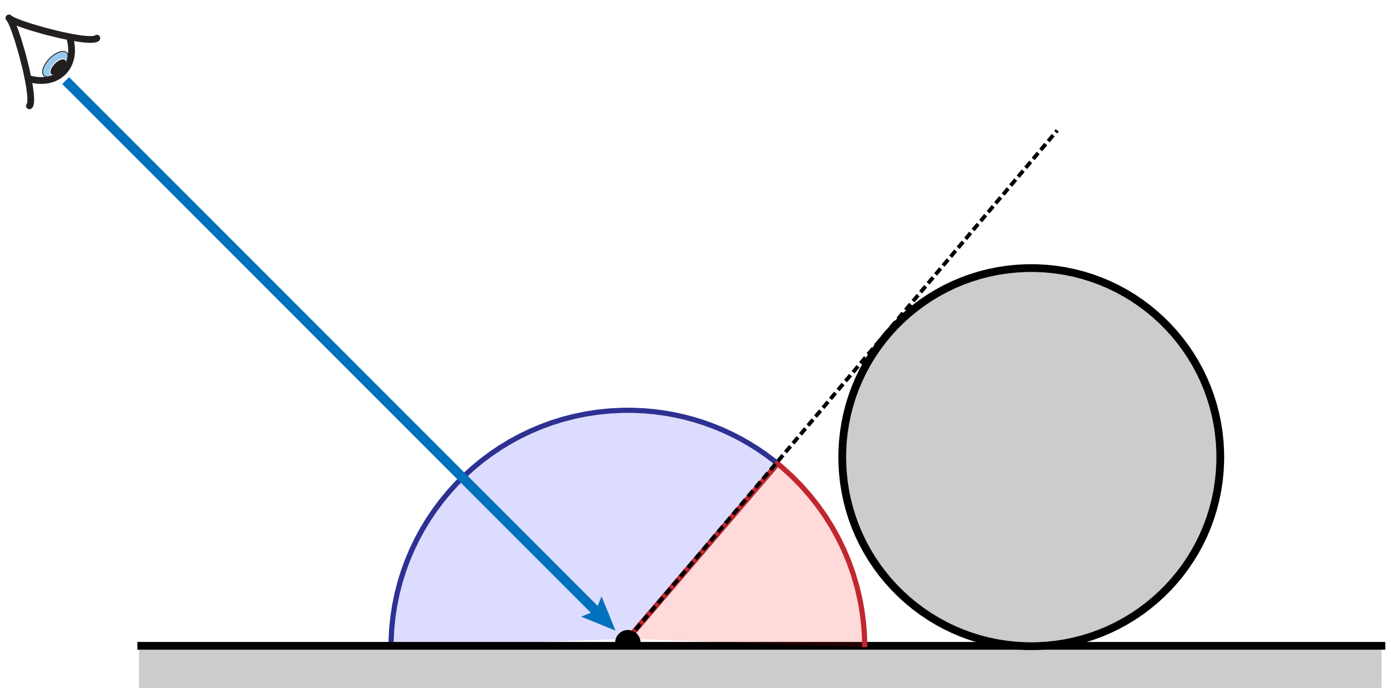
Ambient Occlusion

$$L_r(\mathbf{x}) = \frac{\rho}{\pi} \int_{H^2} V(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$



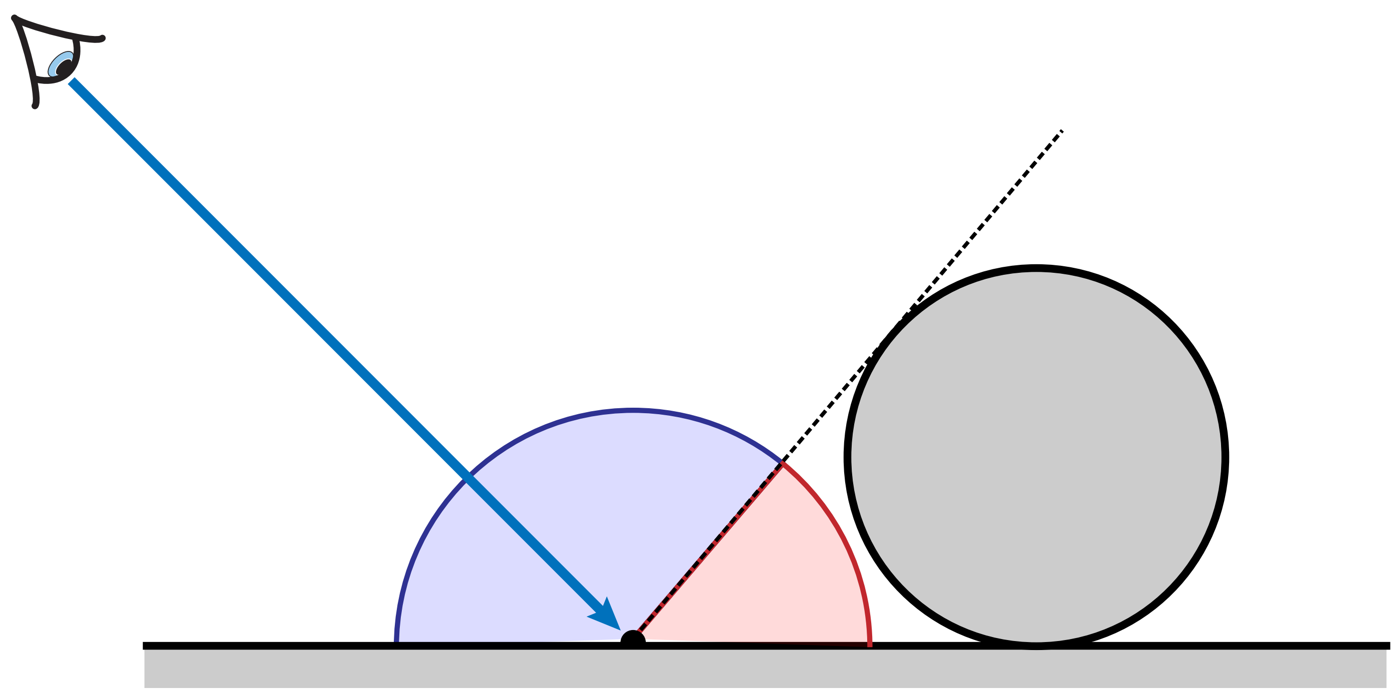
Ambient Occlusion

$$L_r(\mathbf{x}) = \frac{\rho}{\pi} \int_{H^2} V(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$



Ambient Occlusion

$$L_r(\mathbf{x}) \approx \frac{\rho}{\pi N} \sum_{k=1}^{N} \frac{V(\mathbf{x}, \vec{\omega}_{i,k}) \cos \theta_{i,k}}{p(\vec{\omega}_{i,k})}$$



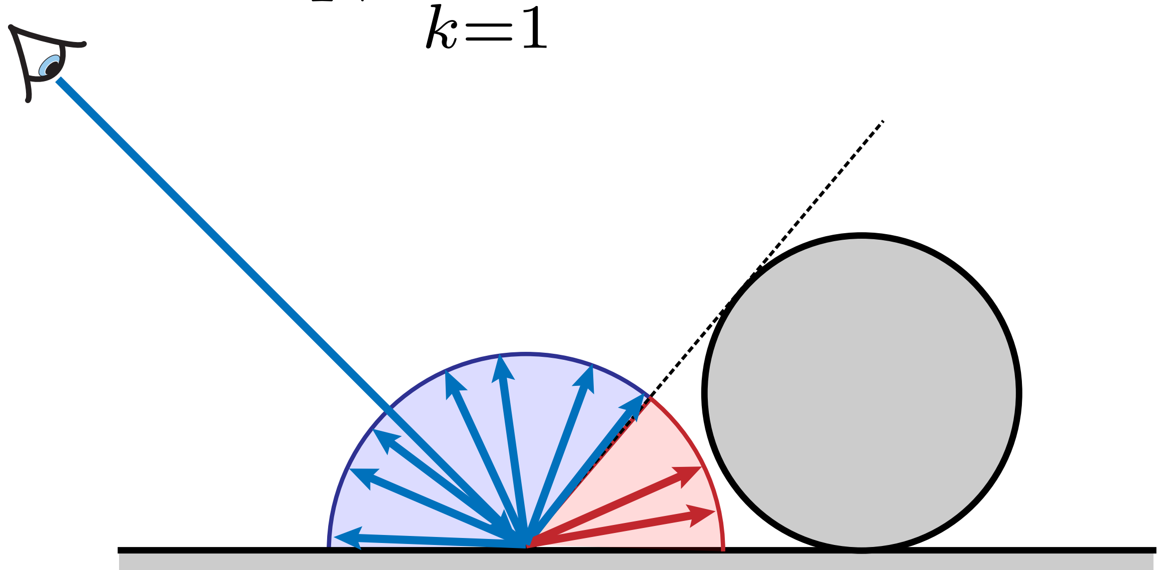
Ambient Occlusion

$$L_r(\mathbf{x}) \approx \frac{\rho}{\pi N} \sum_{k=1}^{N} \frac{V(\mathbf{x}, \vec{\omega}_{i,k}) \cos\theta_{i,k}}{p(\vec{\omega}_{i,k})}$$

Uniform hemispherical sampling

$$p(\vec{\omega}_{i,k}) = 1/2\pi$$

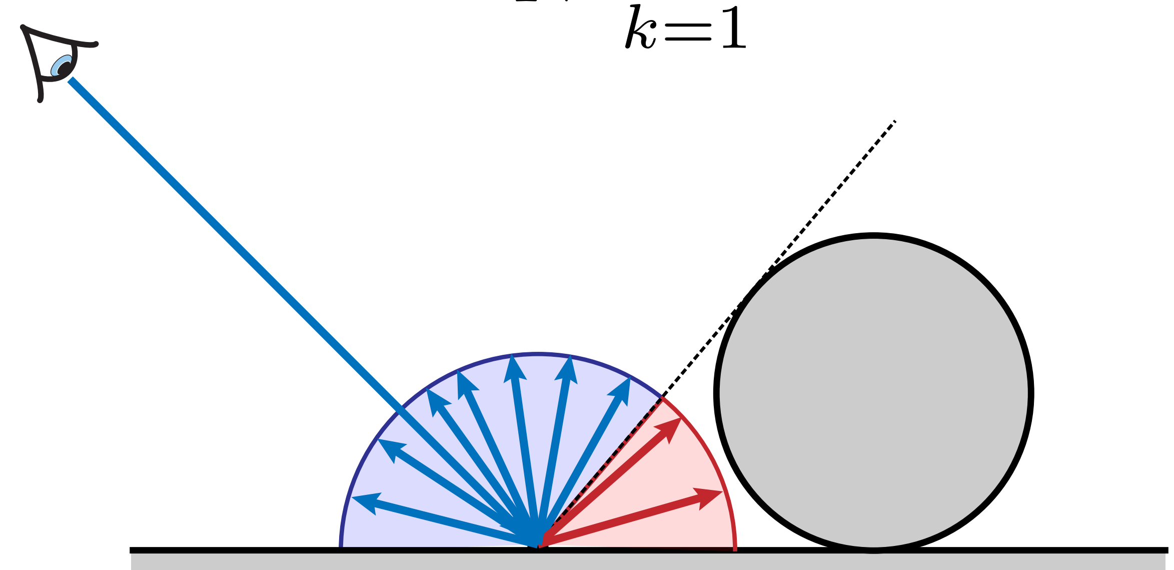
$$L_r(\mathbf{x}) \approx \frac{2\rho}{N} \sum_{k=1}^{N} V(\mathbf{x}, \vec{\omega}_{i,k}) \cos\theta_{i,k}$$



Cosine-weighted importance sampling

$$p(\vec{\omega}_{i,k}) = \cos\theta_{i,k}/\pi$$

$$L_r(\mathbf{x}) \approx \frac{\rho}{N} \sum_{k=1}^{N} V(\mathbf{x}, \vec{\omega}_{i,k})$$

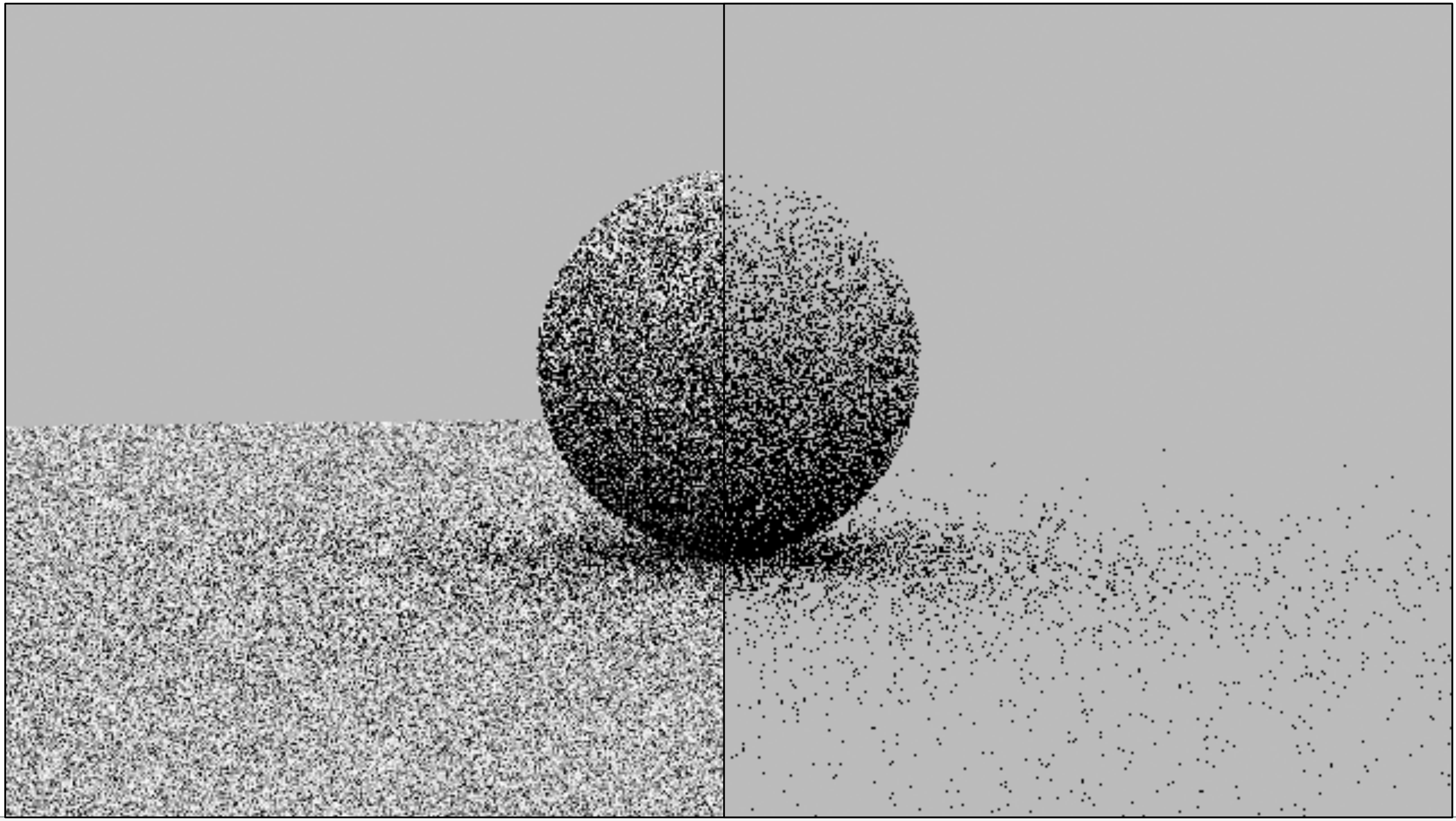


Ambient Occlusion

Uniform hemispherical sampling

Cosine-weighted importance sampling

1 sample/pixel

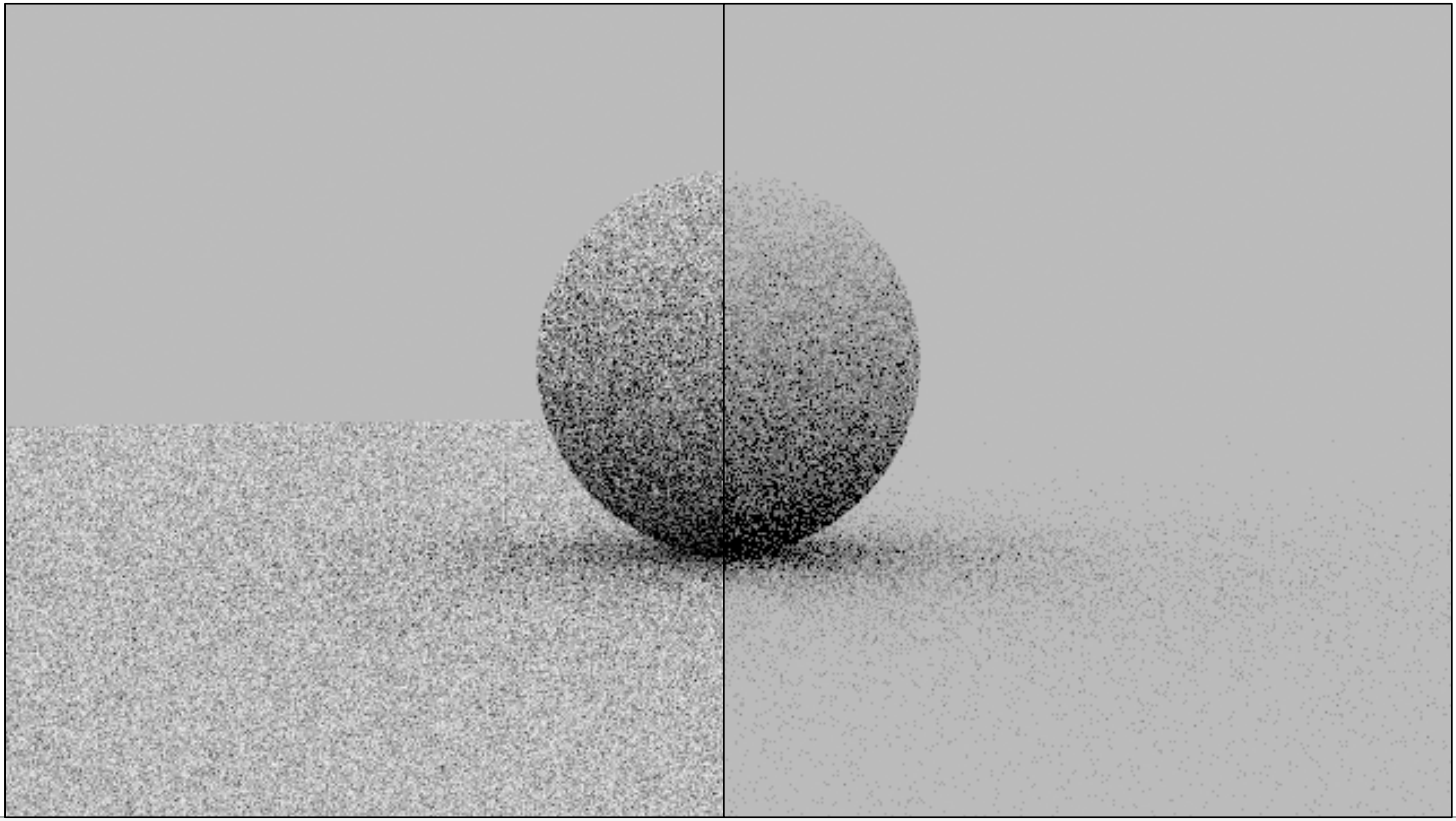


Ambient Occlusion

Uniform hemispherical sampling

Cosine-weighted importance sampling

4 samples/pixel

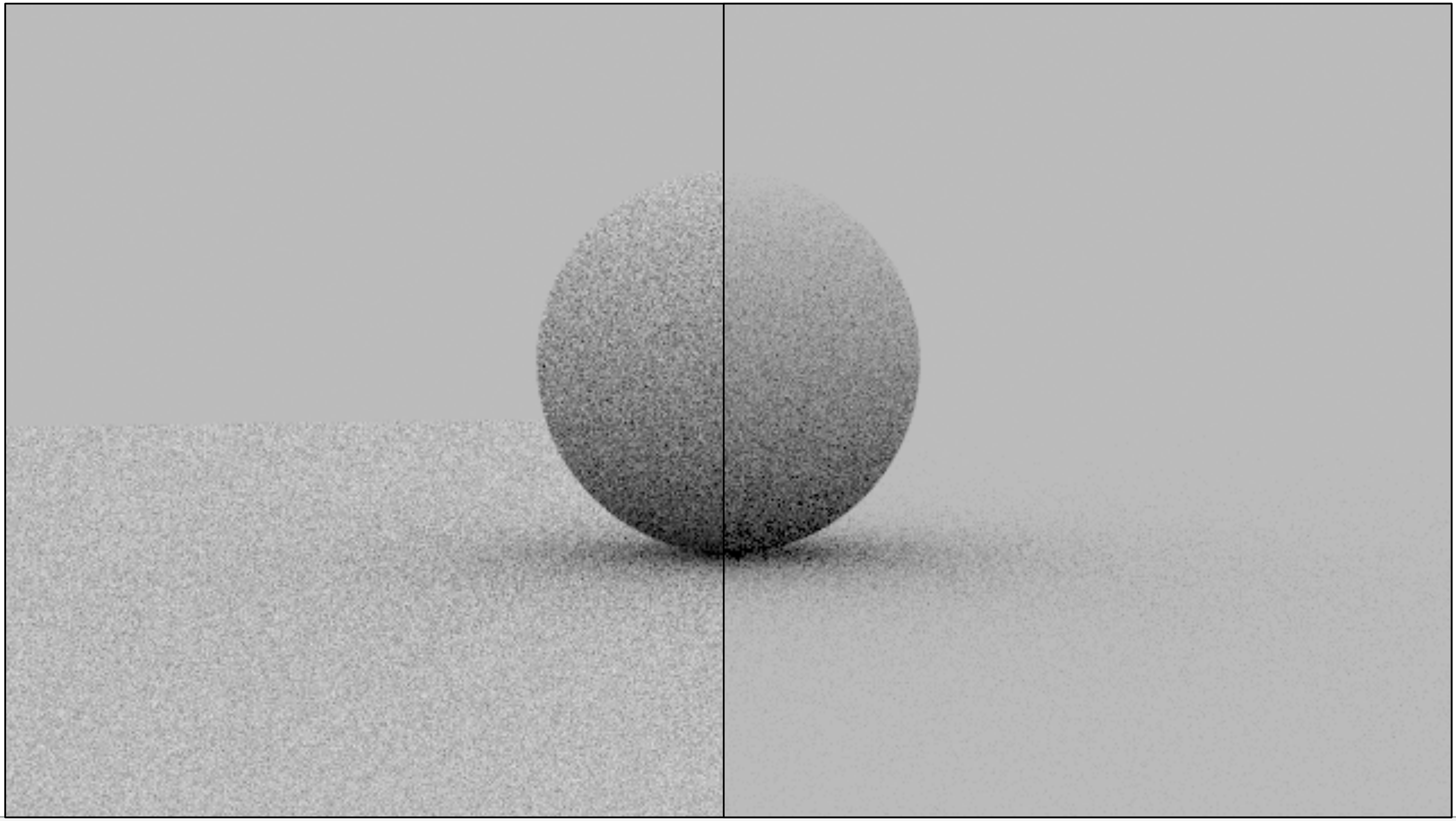


Ambient Occlusion

Uniform hemispherical sampling

Cosine-weighted importance sampling

16 samples/pixel

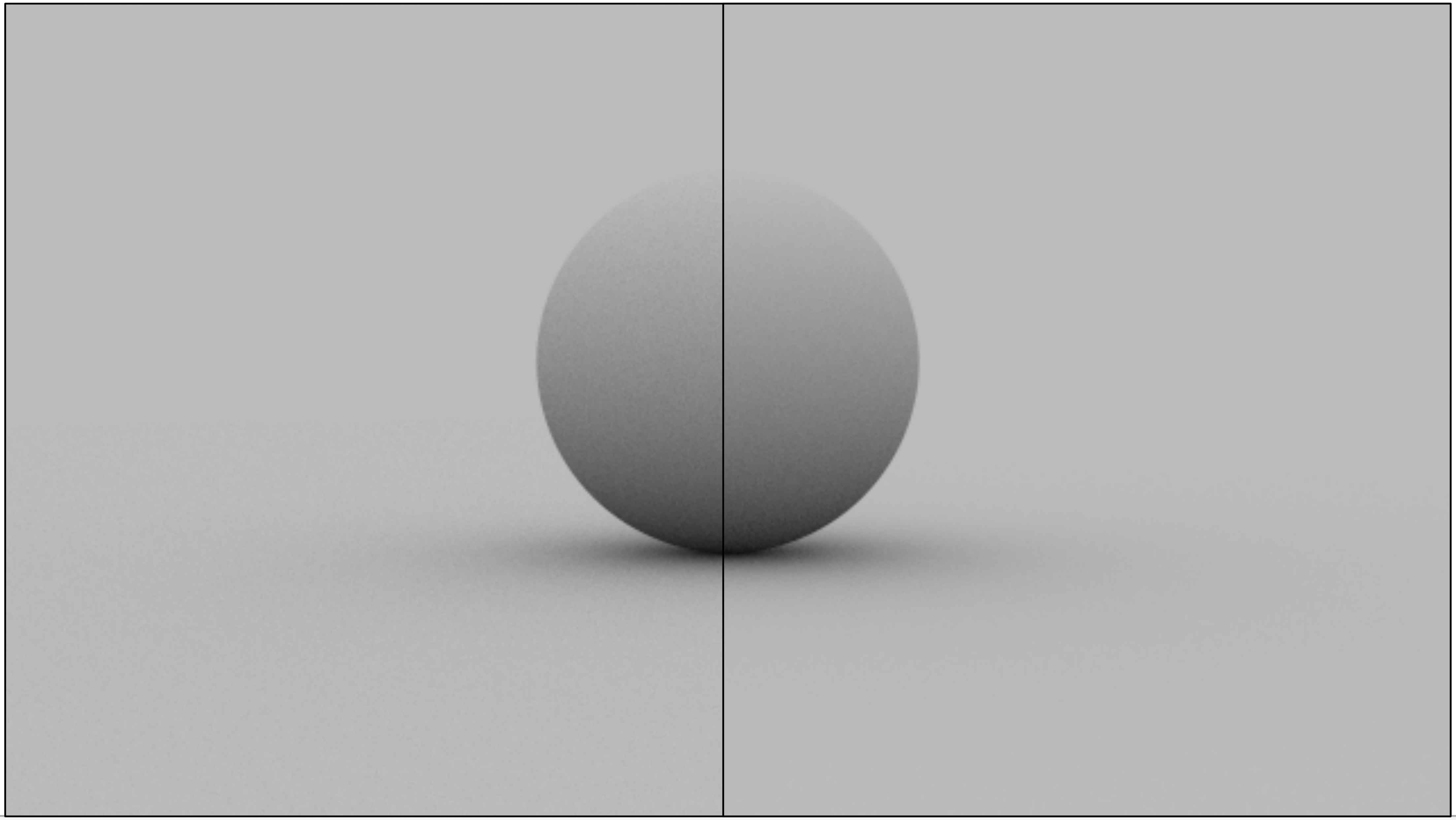


Ambient Occlusion

Uniform hemispherical sampling

Cosine-weighted importance sampling

1024 samples/pixel



Reflection Equation

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \boxed{\cos\theta_i} d\vec{\omega}_i$$

- What terms can we importance sample?
  - BRDF
  - incident radiance
  - **cosine term**

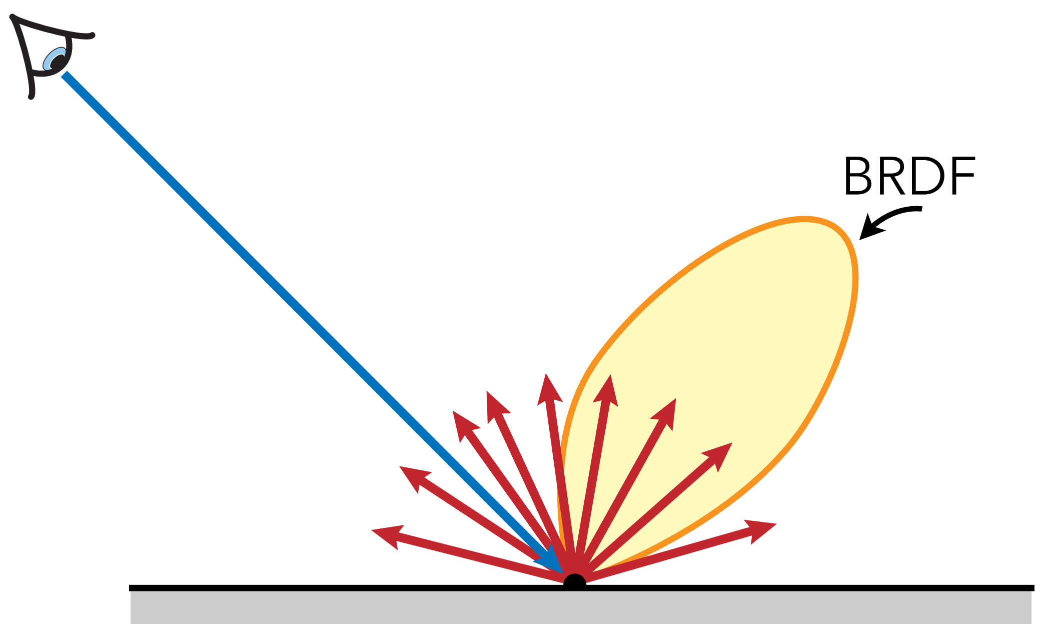
Reflection Equation

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} \boxed{f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r)} L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

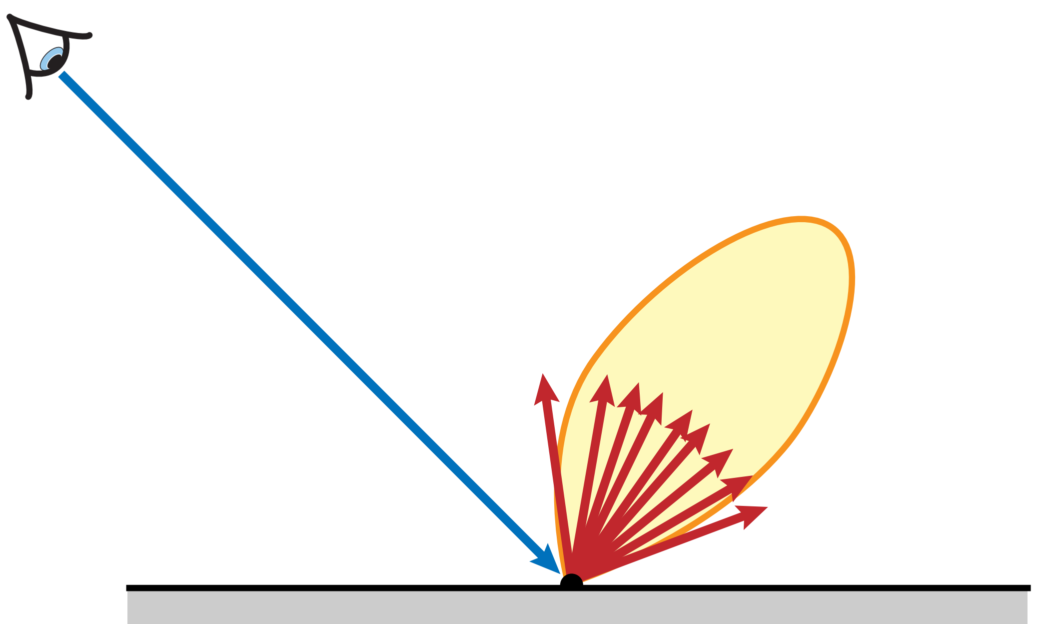
- What terms can we importance sample?
  - **BRDF**
  - incident radiance
  - cosine term

Importance Sampling the BRDF

Cosine-weighted importance sampling

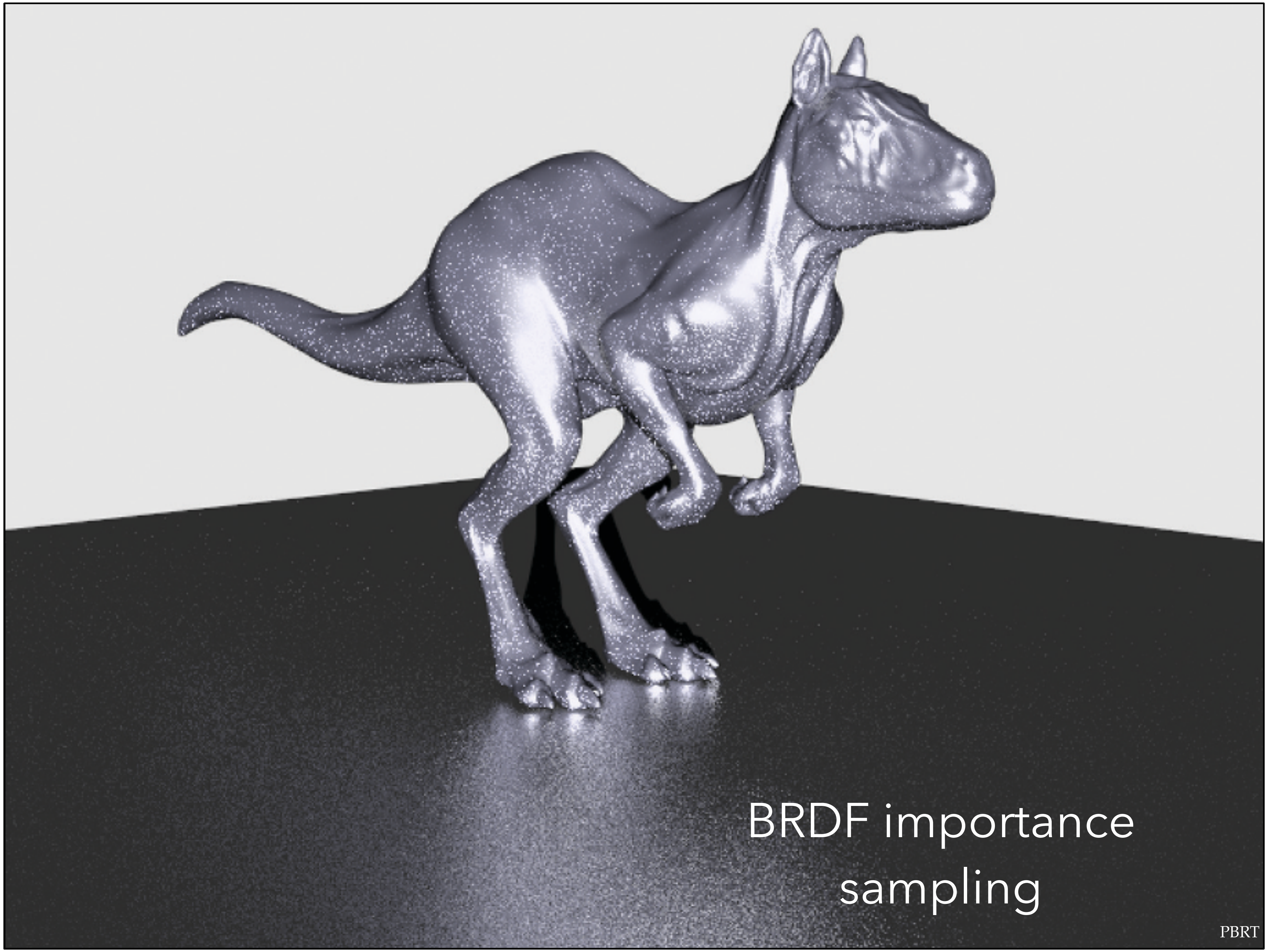


BRDF importance sampling



$$p(\vec{\omega}_i) \underset{\sim}{\propto} f(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r)$$

Importance Sampling the BRDF



Importance Sampling the BRDF

Recipe:

1. Express the desired distribution in a convenient coordinate system
   - might require computing the determinant of the Jacobian matrix
2. Compute marginal and conditional 1D PDFs
3. Sample 1D PDFs using the inversion method

Glossy Phong

- Normalized Phong-like $\cos^n$ lobe:

$$p(\theta, \phi) = \frac{n+2}{2\pi} \cos^n \theta$$
$$(\theta, \phi) = \left(\cos^{-1} \left((1 - \xi_1)^{\frac{1}{n+2}} \right), 2\pi\xi_2 \right)$$

BRDFs with Multiple Lobes

- Typically, each lobe has a scaling coefficient
  - $k_d \to \rho_d$ diffuse coefficient
  - $k_s \to \rho_s$ specular/glossy coefficient
- Can treat the sum of the lobes as a single function, or can importance sample the lobes:
  - probabilistically choose a lobe, e.g. proportional to the coefficient $p_\Omega(\omega) = p_\mathrm{l}(l) p_\Omega(\omega|l)$
  - sample a direction proportional to the probabilistically chosen lobe

*or use multiple importance sampling (preferred)

Reflection Equation

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} \boxed{f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r)} L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

- What terms can we importance sample?
  - **BRDF**
  - incident radiance
  - cosine term

Reflection Equation

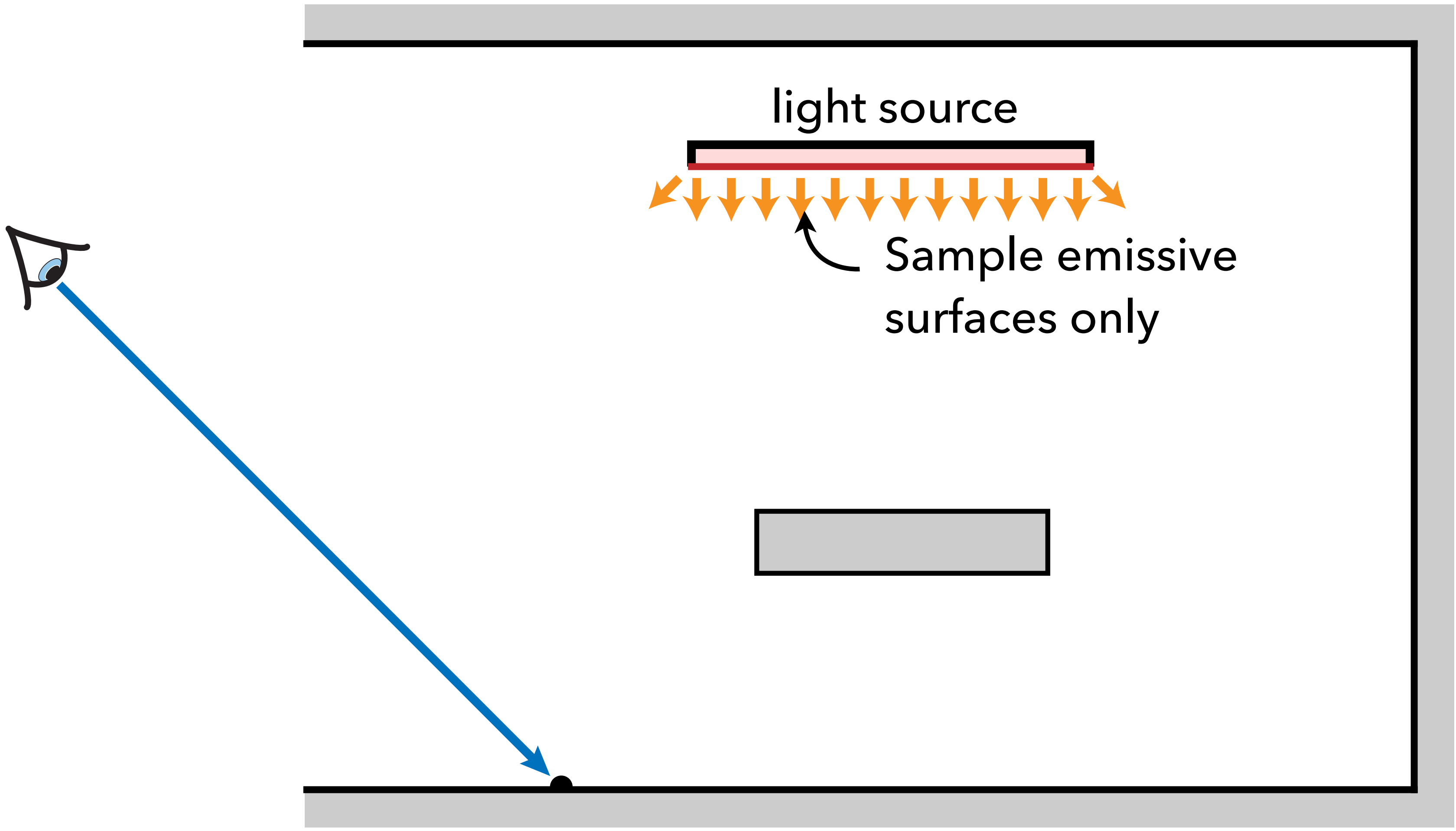
$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) \boxed{L_i(\mathbf{x}, \vec{\omega}_i)} \cos\theta_i \, d\vec{\omega}_i$$

- What terms can we importance sample?
  - BRDF
  - **incident radiance**
  - cosine term

Importance Sampling Incident Radiance

- Generally impossible, but…

  for direct illumination we can explicitly sample emissive surfaces



Importance Sampling Incident Radiance

- Generally impossible, but…

  for direct illumination we can explicitly sample emissive surfaces

- Use e.g. the area form of the reflection eq.:

$$L_r(\mathbf{x}, \mathbf{z}) = \int_{A_e} f_r(\mathbf{x}, \mathbf{y}, \mathbf{z}) L_e(\mathbf{y}, \mathbf{x}) V(\mathbf{x}, \mathbf{y}) \frac{|\cos\theta_i|\,|\cos\theta_o|}{\|\mathbf{x} - \mathbf{y}\|^2} dA(\mathbf{y})$$

Integrate over emissive surfaces only

Reflection Equation

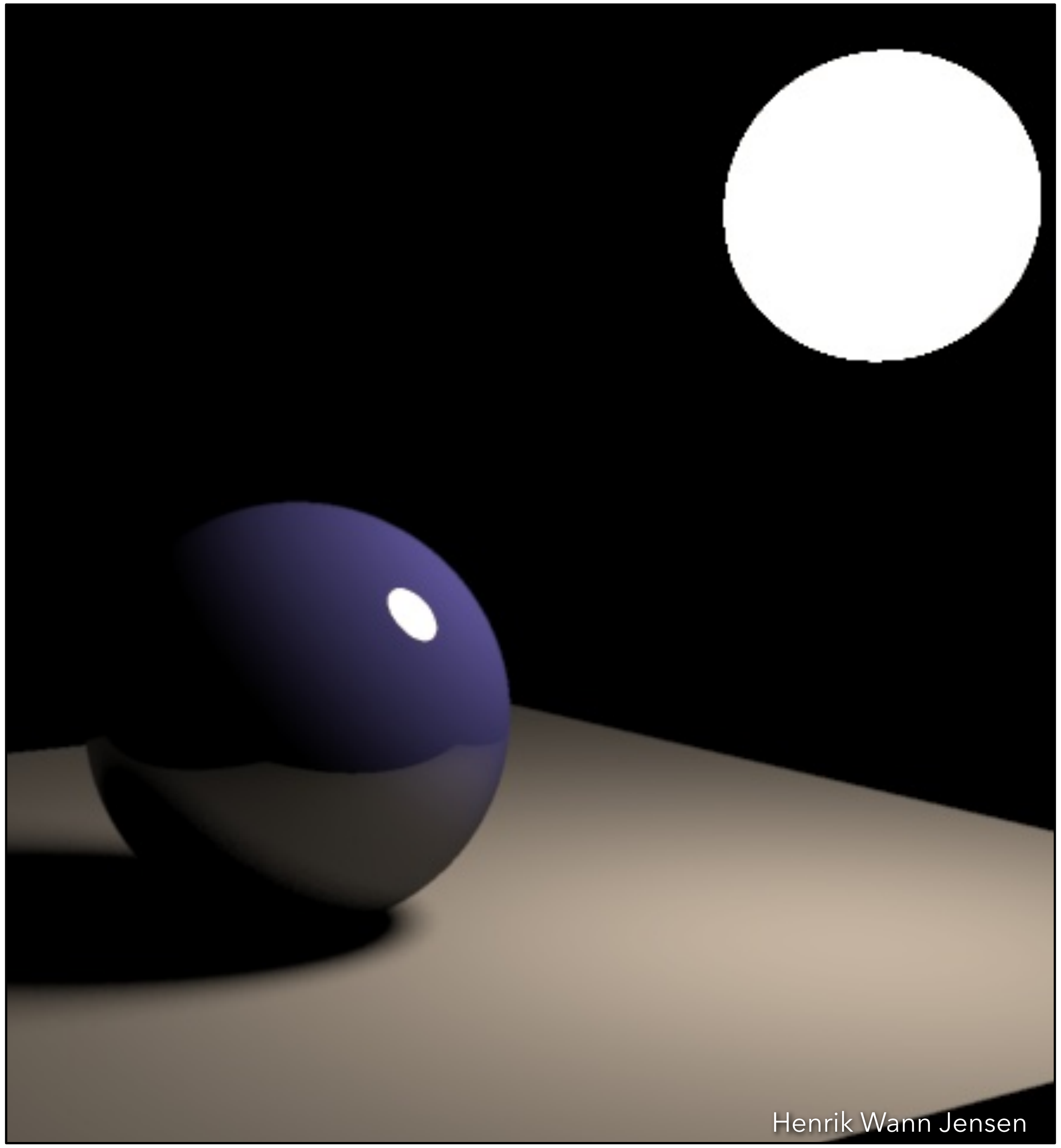
$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

- What terms can we importance sample?
  - BRDF
  - incident radiance
  - cosine term
- What terms **should** we importance sample?
  - depends on the context, hard to make a general statement

Importance Sampling the Light

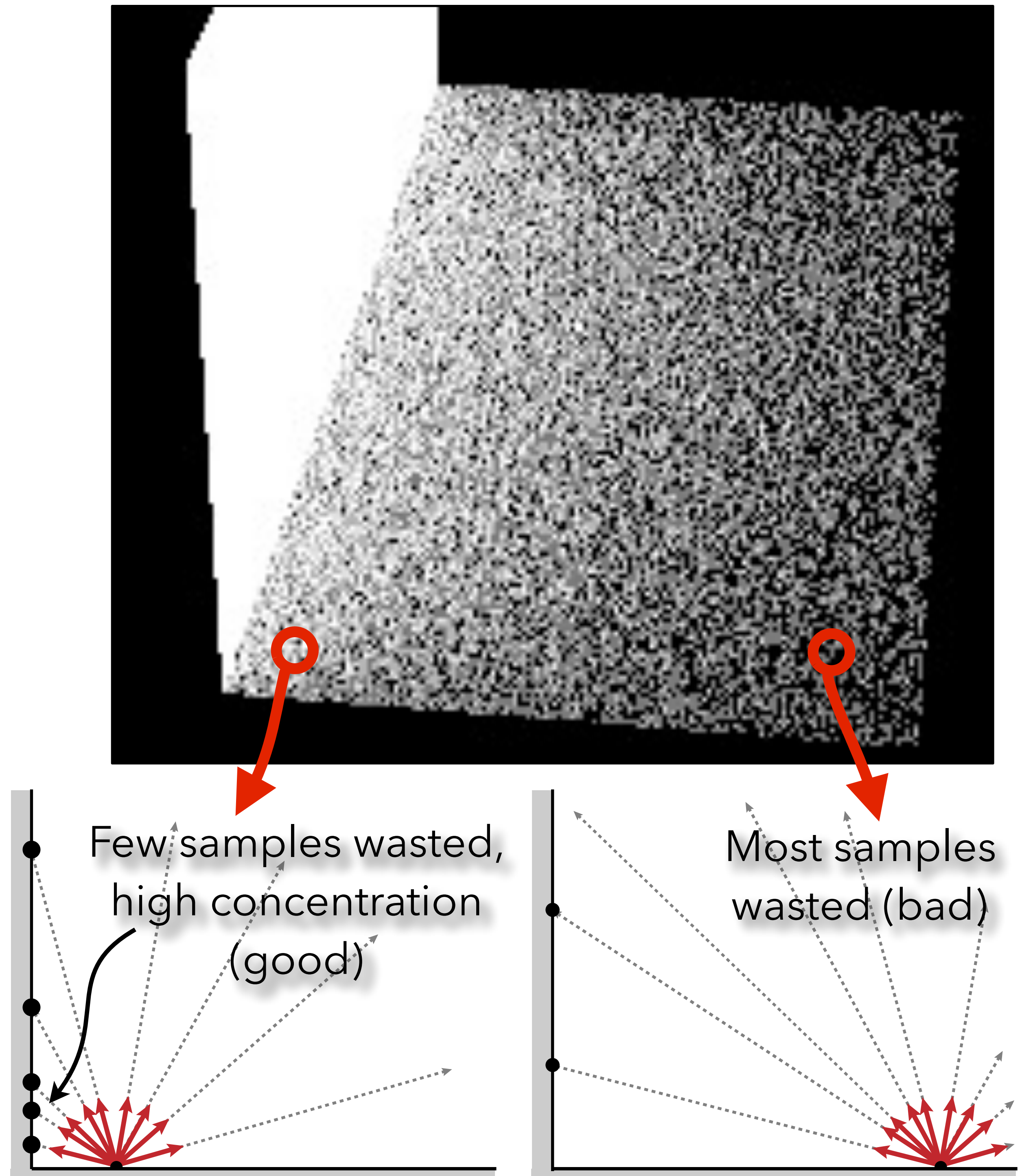
Uniform hemispherical sampling

Uniform sampling of subtended solid angle

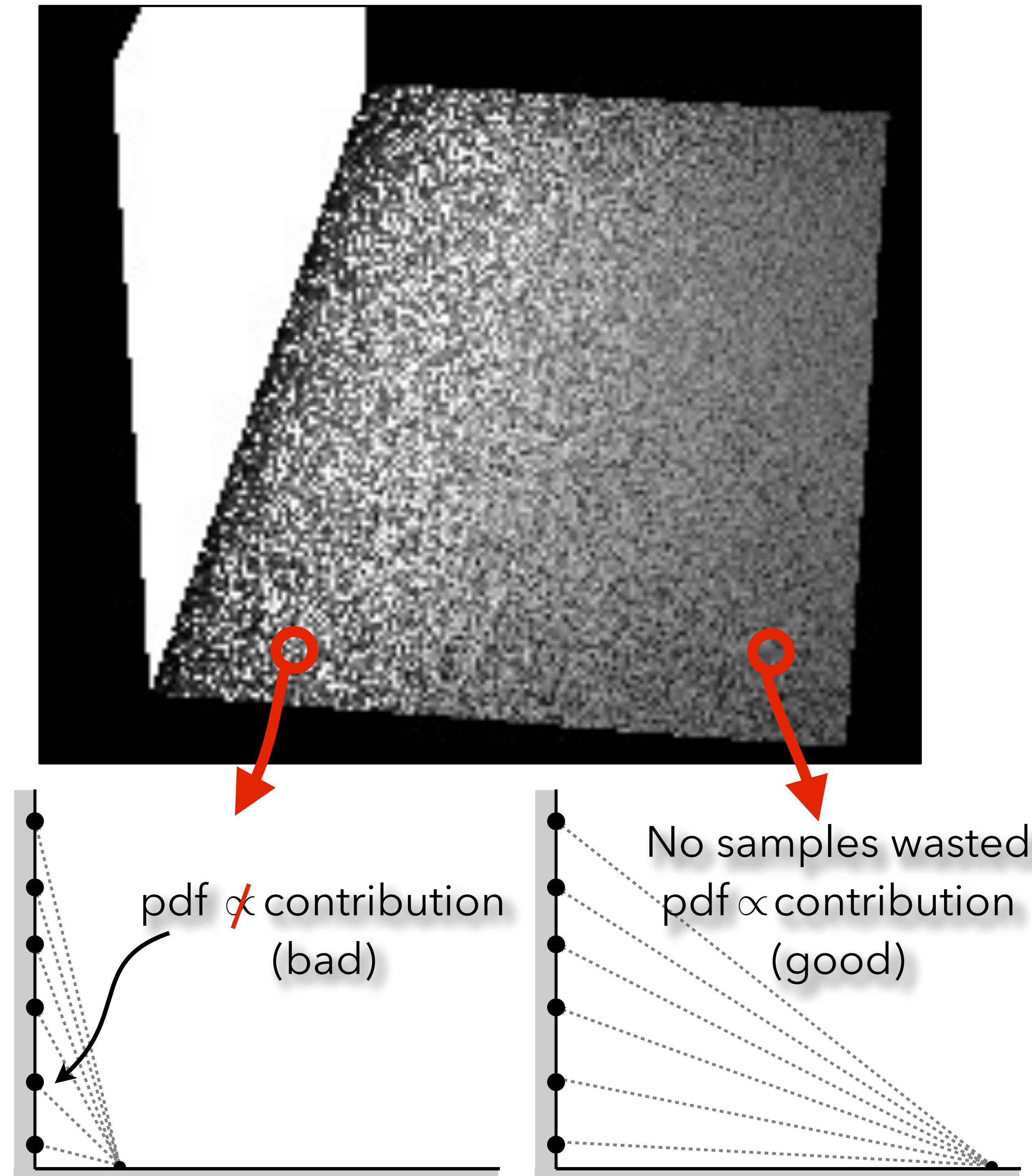


Multiple Strategies

Cosine-weighted hemisphere

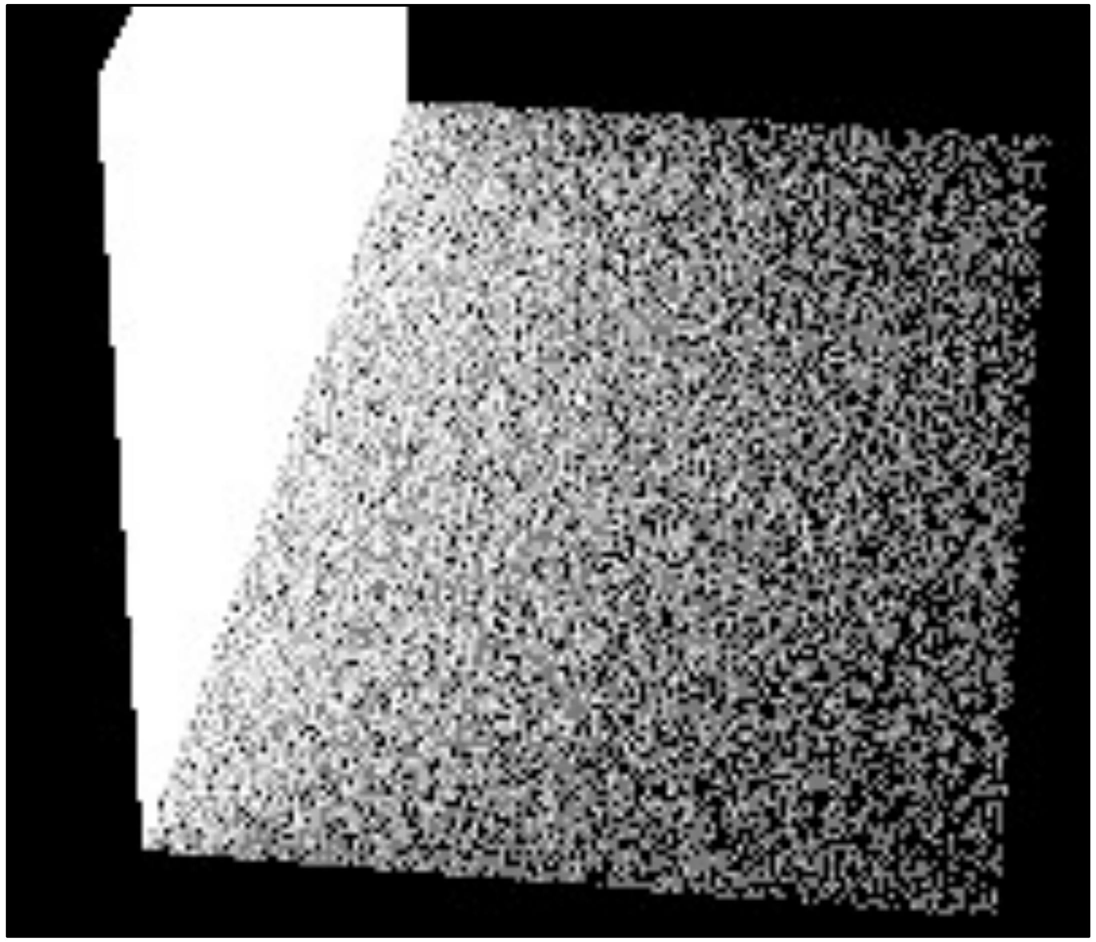


Uniform surface area



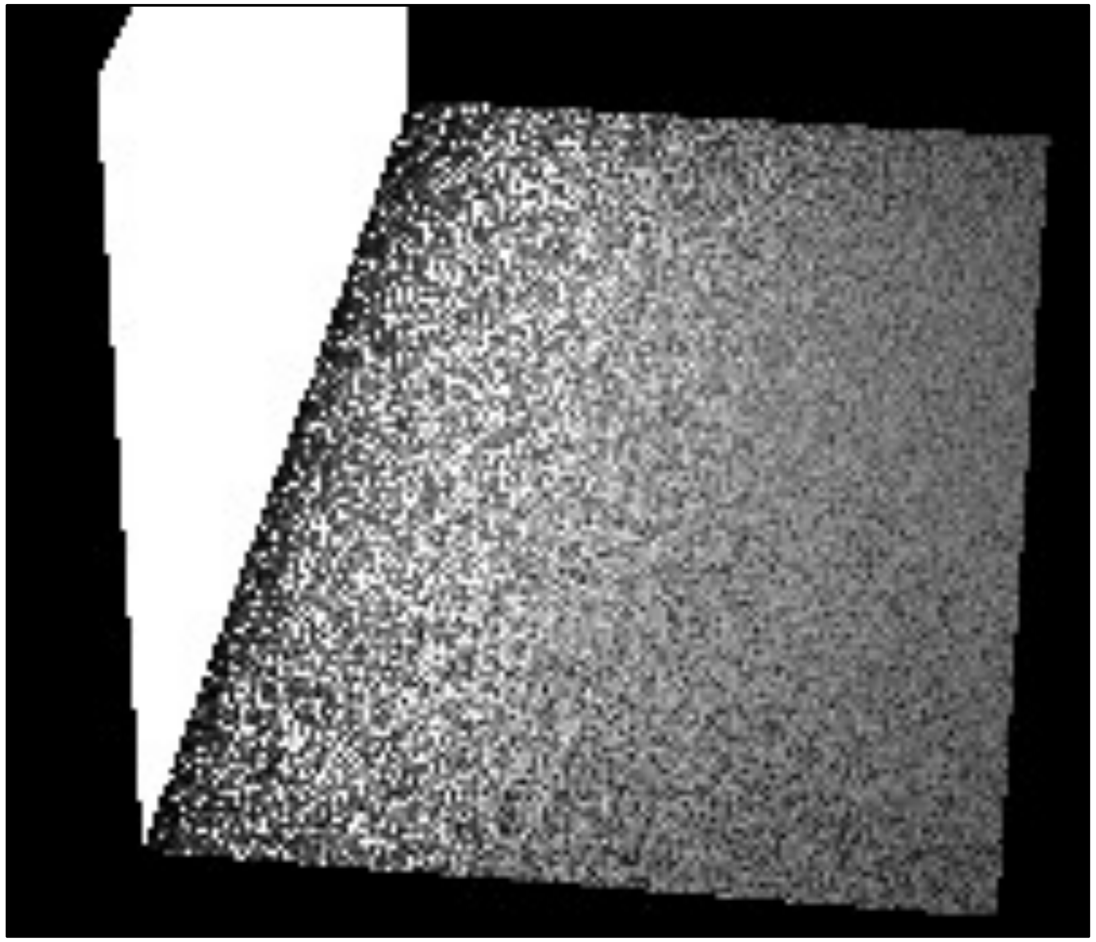
Combining Multiple Strategies

Cosine-weighted hemisphere



$$p_1(\vec{\omega}) = \frac{\cos\theta}{\pi}$$

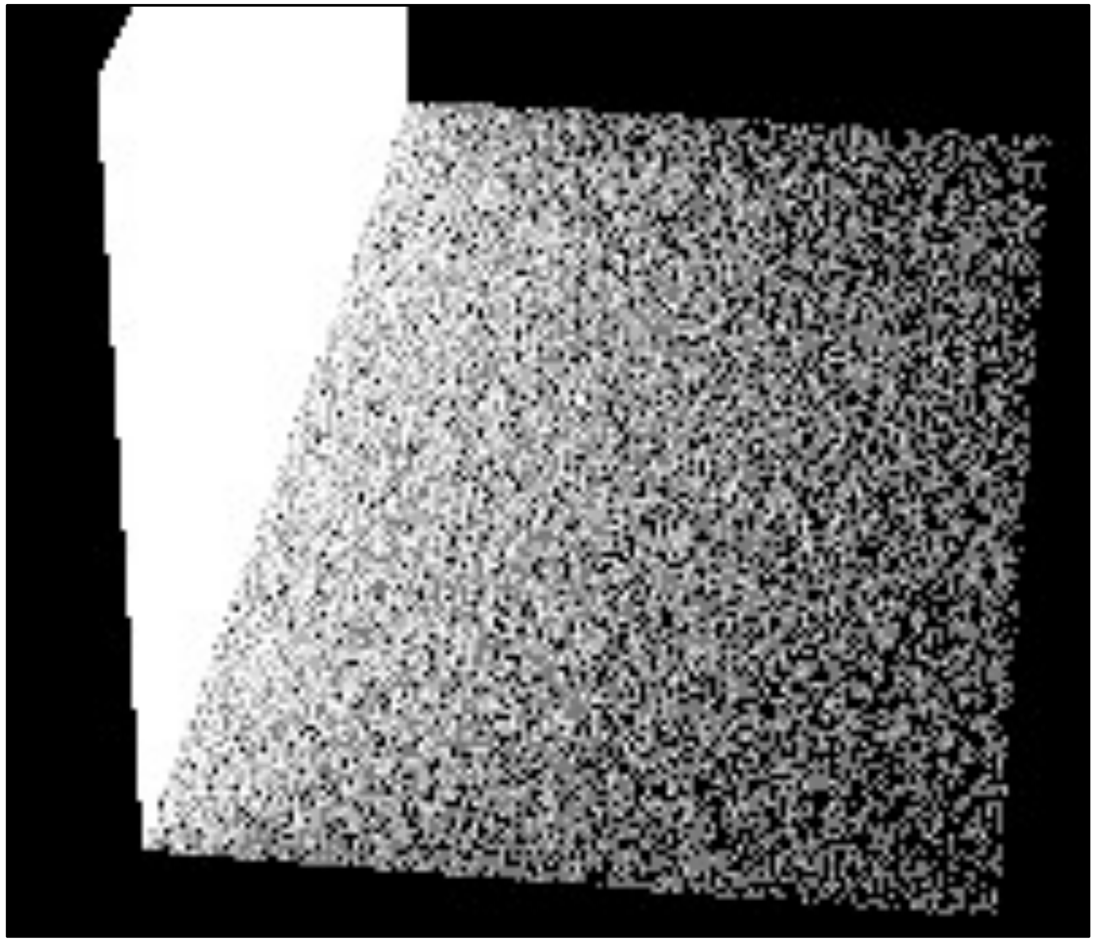
Uniform surface area



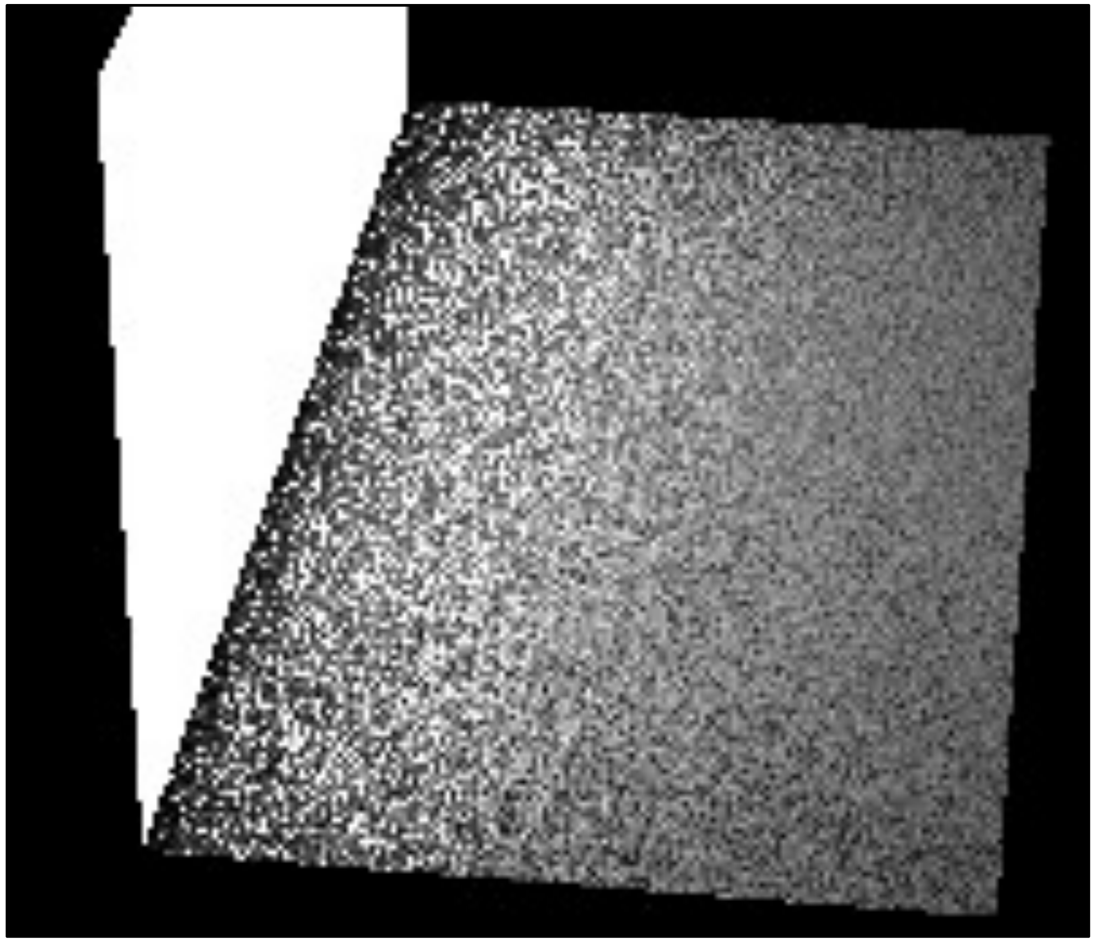
$$p_2(\mathbf{x}) = \frac{1}{A}$$

Combining Multiple Strategies

Cosine-weighted hemisphere



Uniform surface area



$$p_1(\vec{\omega}) = \frac{\cos\theta}{\pi}$$

$$p_2(\mathbf{x}) = \frac{1}{A} \quad p_2(\vec{\omega}) = \frac{1}{A}\frac{d^2}{\cos\theta}$$

Combining Multiple Strategies

- Could just average two different estimators:

$$\frac{0.5}{N_1}\sum_{i=1}^{N_1}\frac{f(x_i)}{p_1(x_i)}+\frac{0.5}{N_2}\sum_{i=1}^{N_2}\frac{f(x_i)}{p_2(x_i)}$$

  - doesn't really help: *variance is additive*

- Instead, sample from the average PDF

$$\frac{1}{N}\sum_{i=1}^{N}\frac{f(x_i)}{0.5(p_1(x_i)+p_2(x_i))}$$

Sample from Average PDF

- You are given two sampling functions and their corresponding pdfs:

```
float sample1(float rnd); float pdf1(float x);
float sample2(float rnd); float pdf2(float x);
```

- Create a new function:

```
float sampleAvg(float rnd);
```

- which has the corresponding pdf:

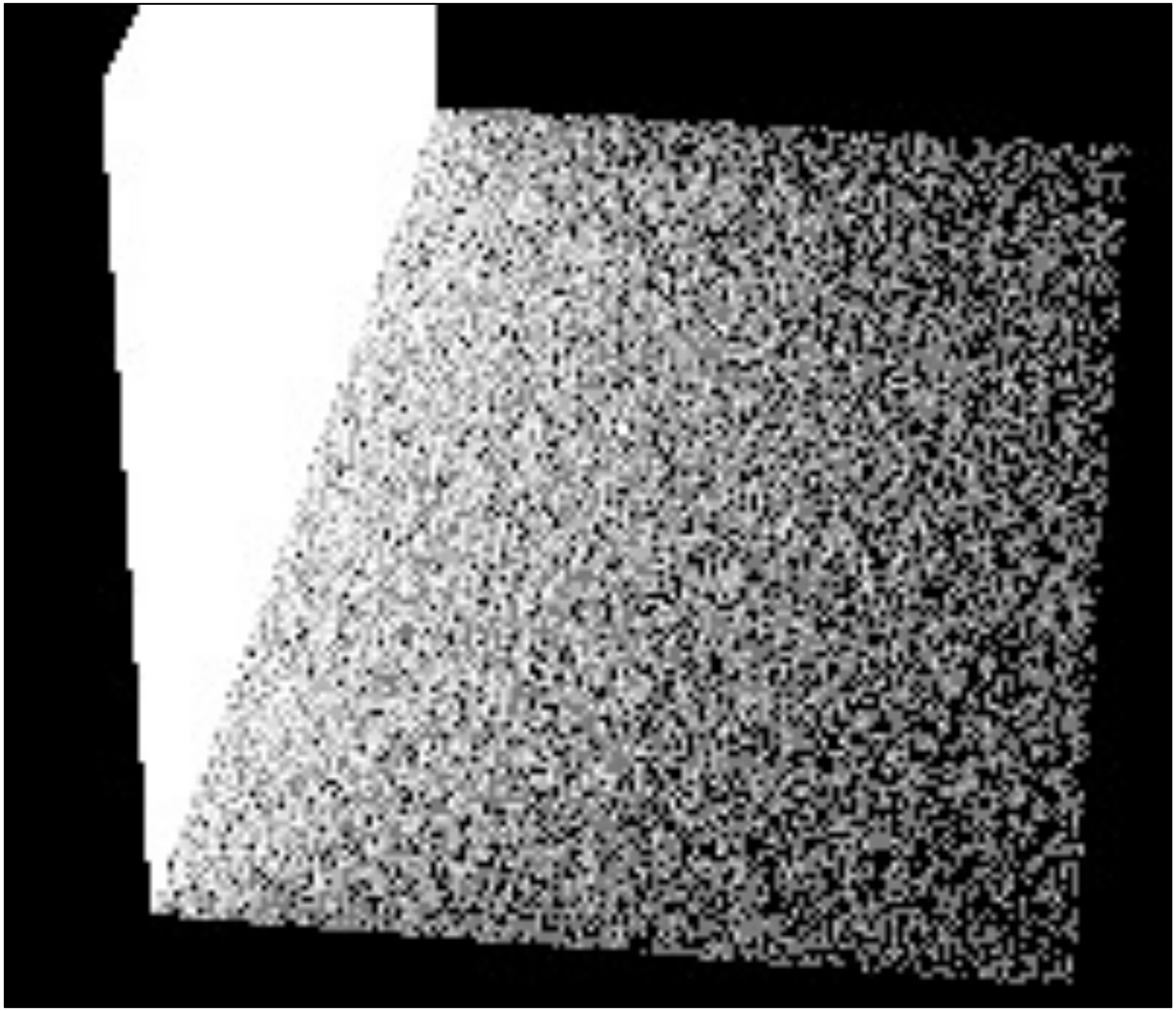
```
float pdfAvg(float x)
{
    return 0.5 * (pdf1(x) + pdf2(x));
}
```

Sample from Average PDF

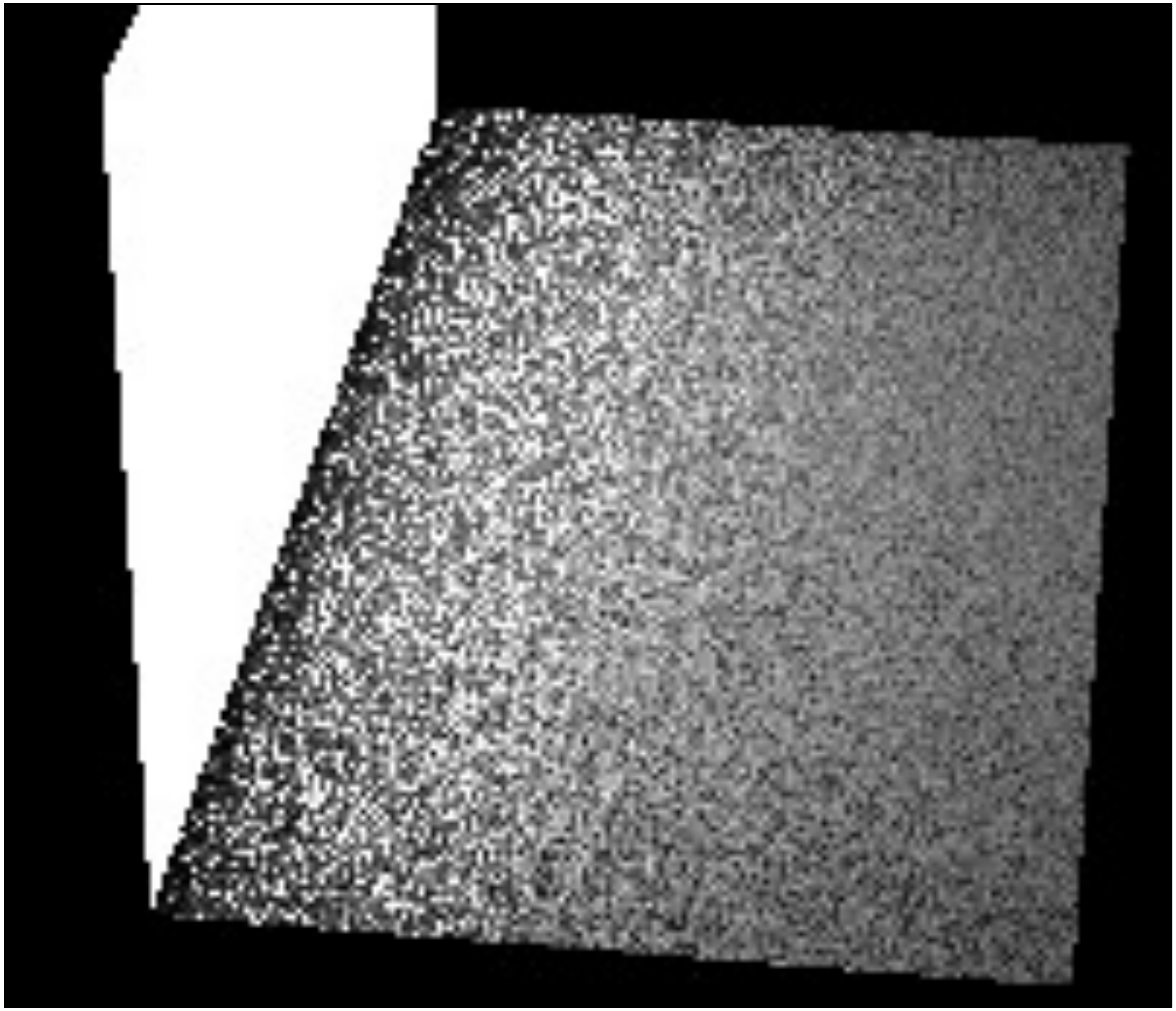
```
float sampleAvg(float rnd)
{
   if (rand.nextFloat() < 0.5)
      return sample1(rnd);
   else
      return sample2(rnd);
}
```

requires extra random number

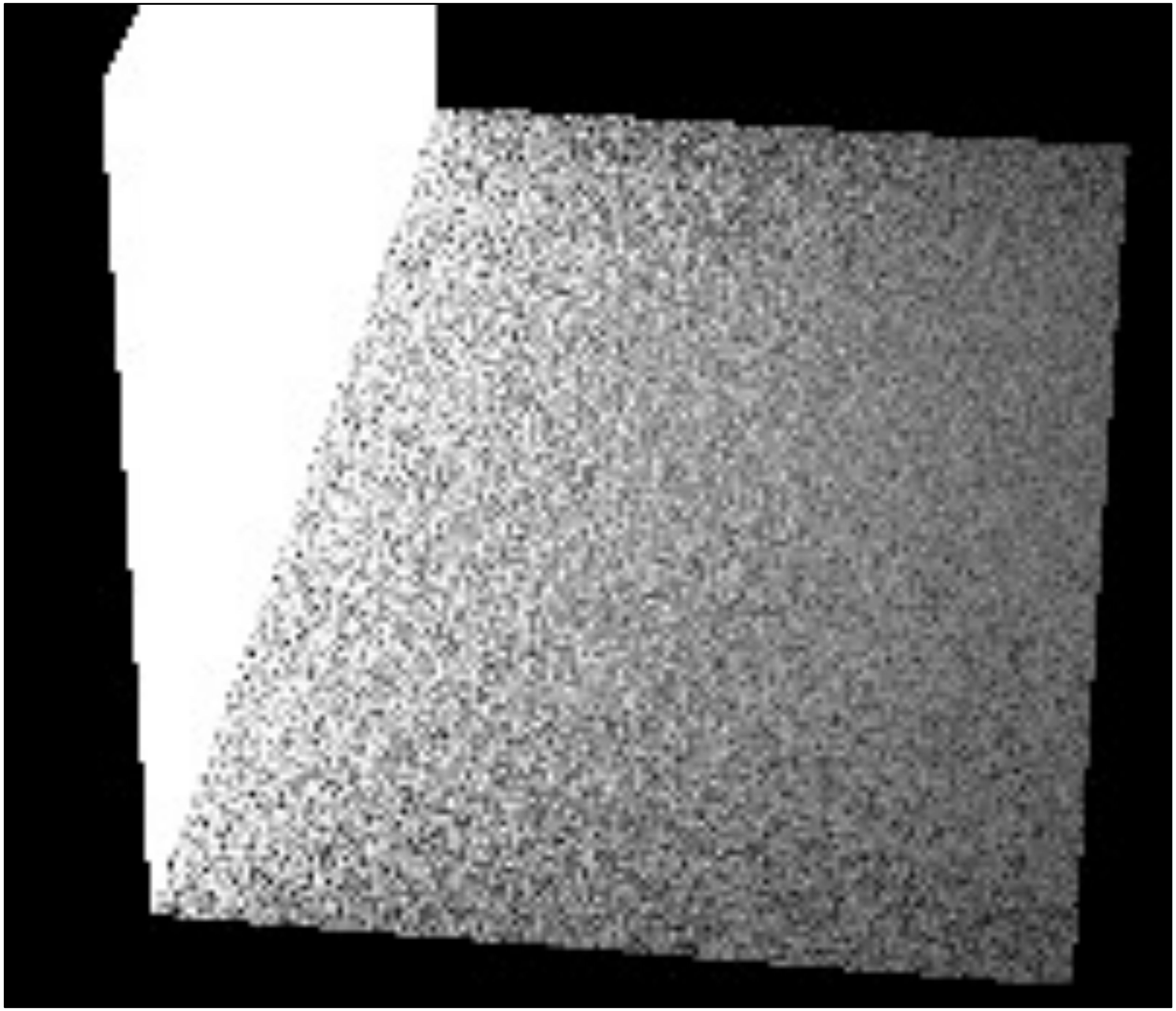
Cosine-weighted Hemisphere



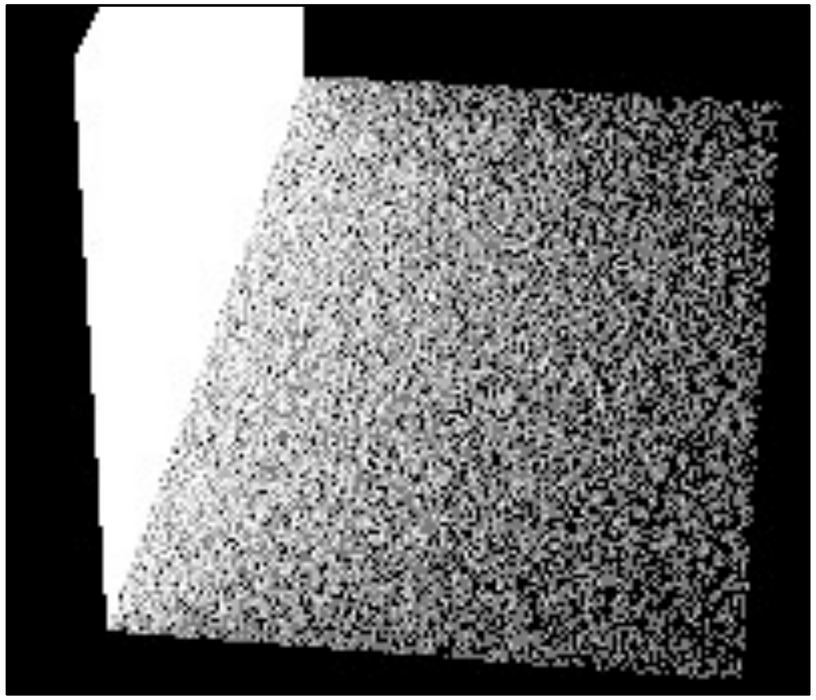
Uniform Surface Area



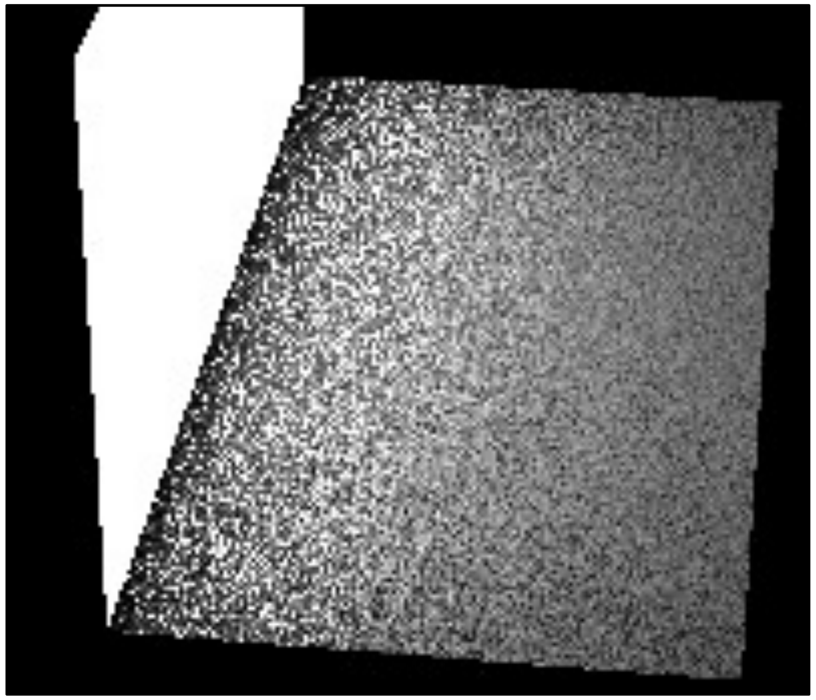
Average PDF



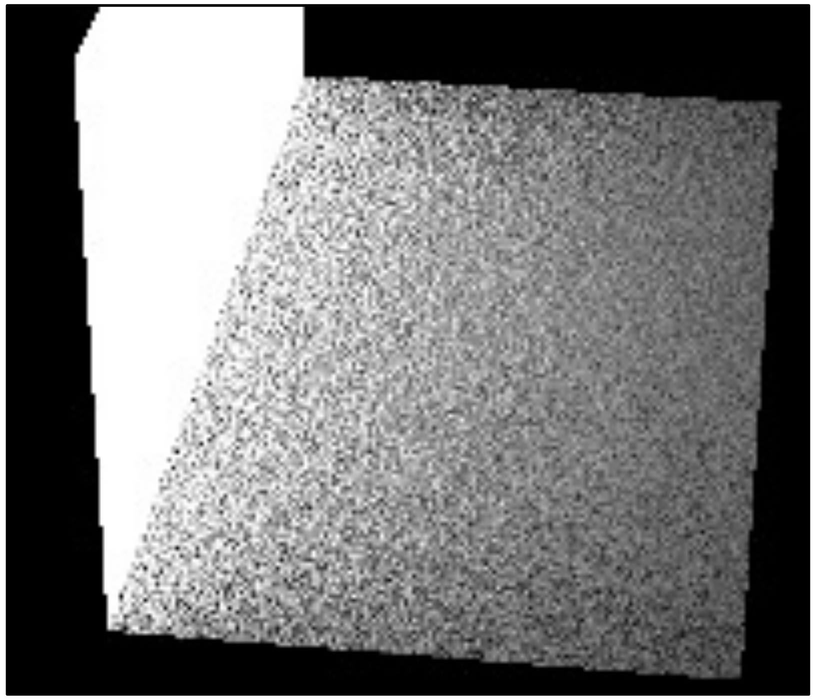
Comparison



Cosine-weighted Hemisphere



Uniform Surface Area



Average

Visual Break

Multiple Importance Sampling (MIS)

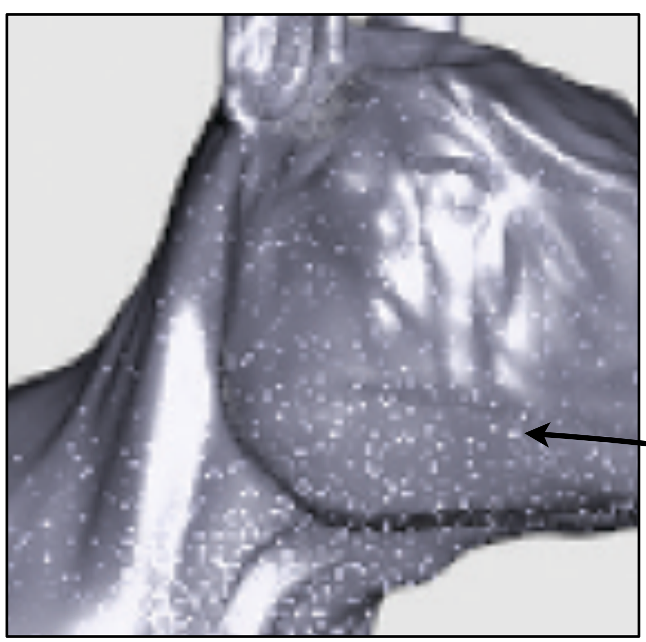
Motivation

- In MC integration, variance is high when the PDF is not proportional to the integrand
- Worst case: *rare* samples with *huge* contributions

$$\langle F^N \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(x_i)}{p(x_i)}$$

large value

small value



"fireflies"

Motivation

- In MC integration, variance is high when the PDF is not proportional to the integrand
- Worst case: *rare* samples with *huge* contributions

$$\langle F^N \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(x_i)}{p(x_i)}$$

large value

small value

- We often have multiple sampling strategies
- If at least one covers each part of the integrand well, then combining them should reduce fireflies

Multiple Importance Sampling

- Weighted combination of 2 strategies

$$\langle F^{N_1+N_2} \rangle = \frac{1}{N_1} \sum_{i=1}^{N_1} w_1(x_i) \frac{f(x_i)}{p_1(x_i)} + \frac{1}{N_2} \sum_{i=1}^{N_2} w_2(x_i) \frac{f(x_i)}{p_2(x_i)}$$

  - where:

$$w_1(x) + w_2(x) = 1$$

Multiple Importance Sampling

- Weighted combination of M strategies

$$\langle F^{\sum N_s} \rangle = \sum_{s=1}^{M} \frac{1}{N_s} \sum_{i=1}^{N_s} w_s(x_i) \frac{f(x_i)}{p_s(x_i)}$$

  - where:

$$\sum_{s=1}^{M} w_s(x) = 1$$

- How to choose the weights?

Multiple Importance Sampling

- Balance heuristic (provably good):

$$w_s(x) = \frac{N_s\, p_s(x)}{\sum_j N_j\, p_j(x)}$$

- Power heuristic (more aggressive, can be better):

$$w_s(x) = \frac{(N_s\, p_s(x))^\beta}{\sum_j (N_j\, p_j(x))^\beta}$$

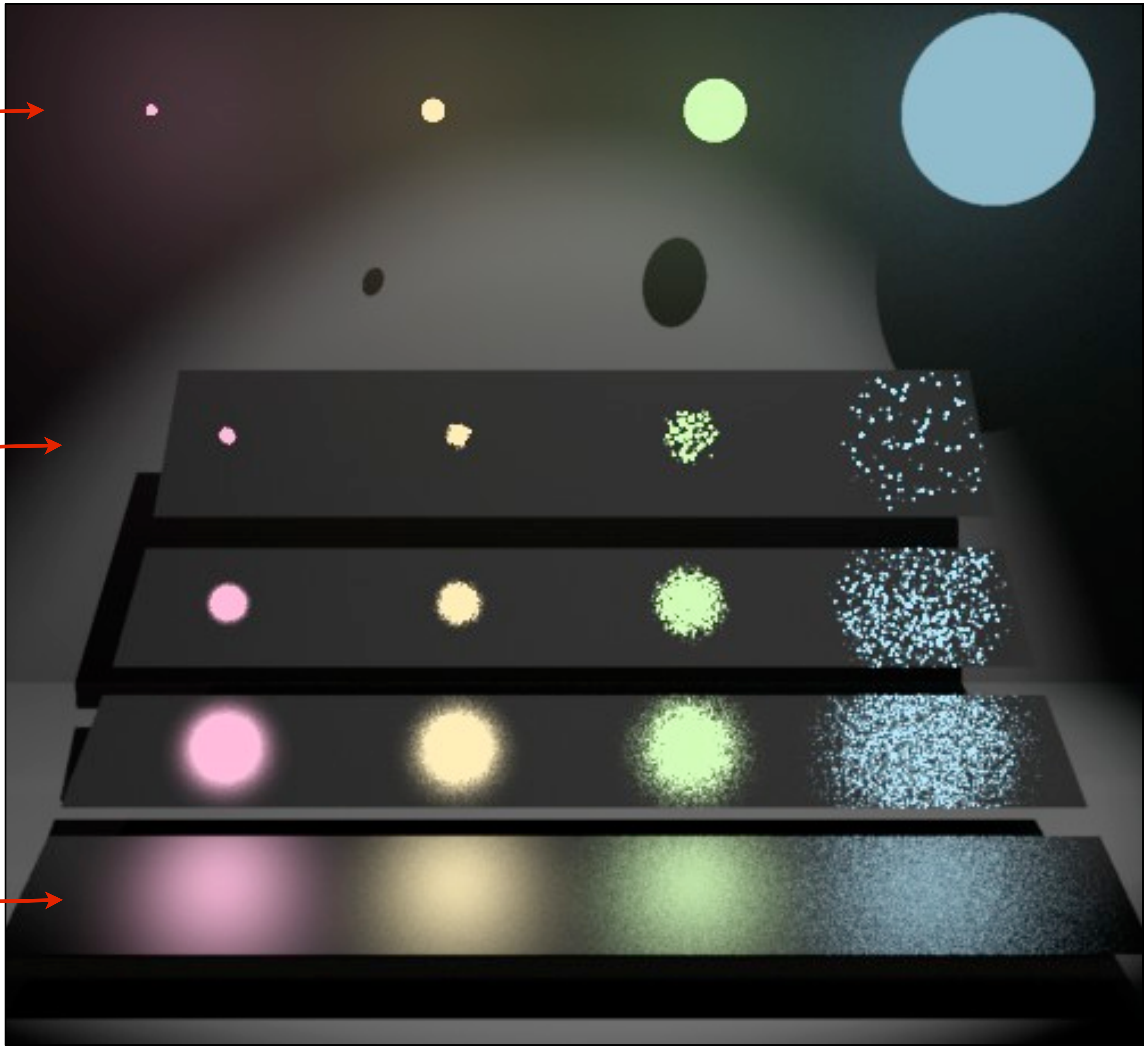
- Other heuristics exist (e.g. cutoff heuristic, maximum heuristic,…)

Sampling the Light

sphere lights

highly glossy surface

moderately glossy surface



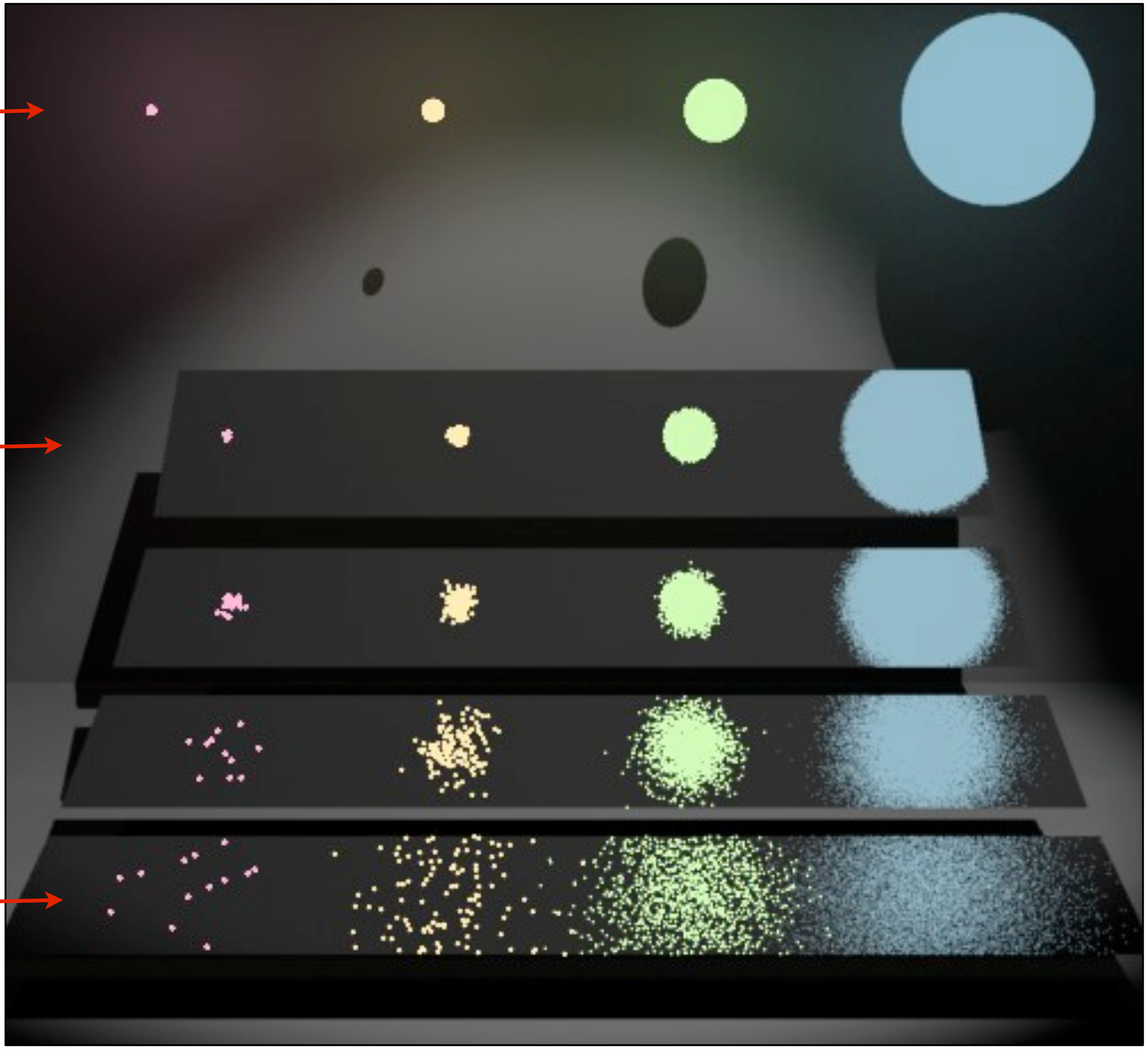
Eric Veach and Leonidas J. Guibas 1995.

Sampling the BRDF

sphere lights

highly glossy surface

moderately glossy surface



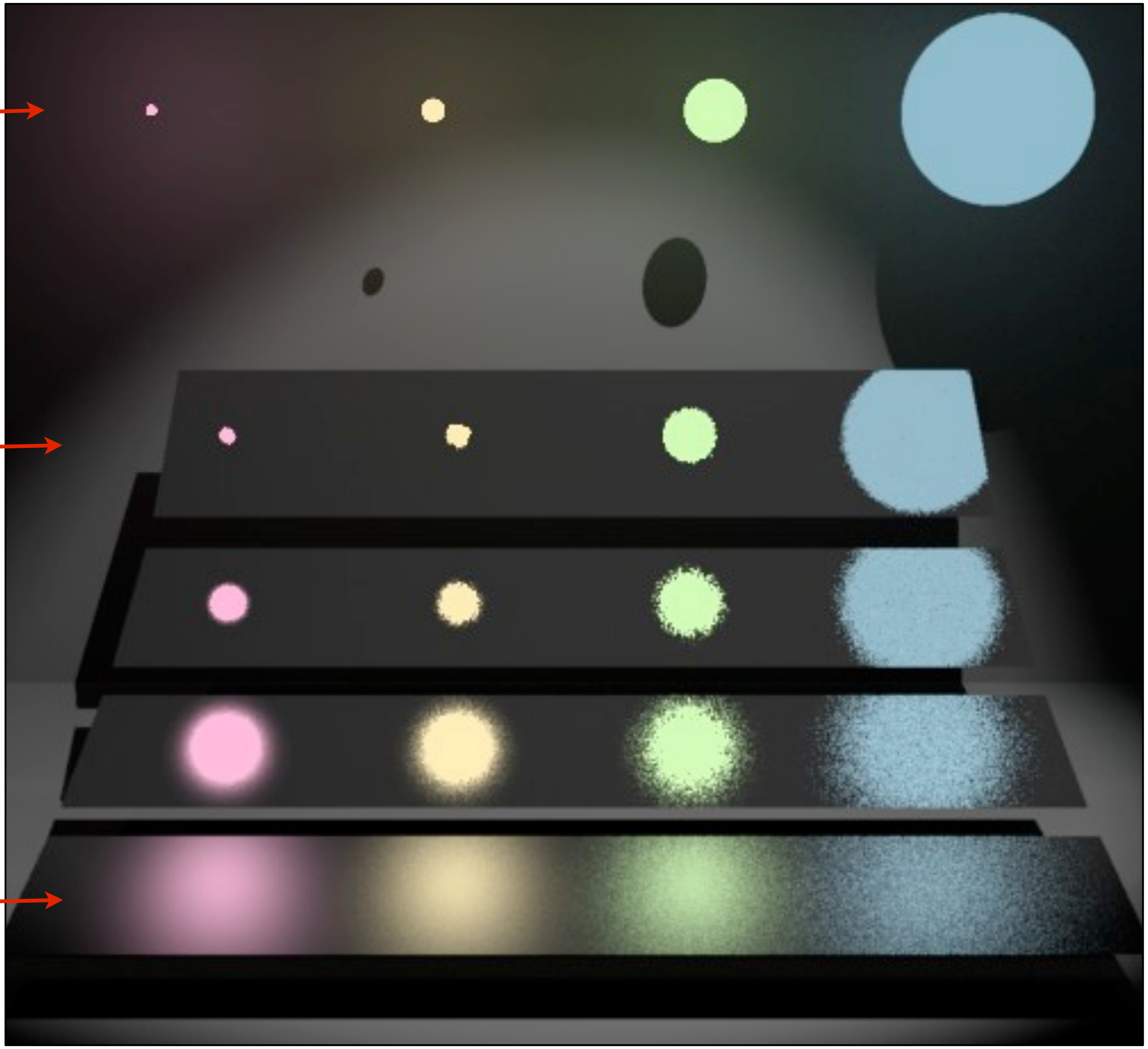
Eric Veach and Leonidas J. Guibas 1995.

Multiple Importance Sampling

sphere lights

highly glossy surface

moderately glossy surface



Eric Veach and Leonidas J. Guibas 1995.

Multiple Importance Sampling

- See PBRe3 14.3.1 for more details

Eric Veach and Leonidas J. Guibas 1995.

Next: More MC & Direct Lighting

Eric Veach and Leonidas J. Guibas 1995.