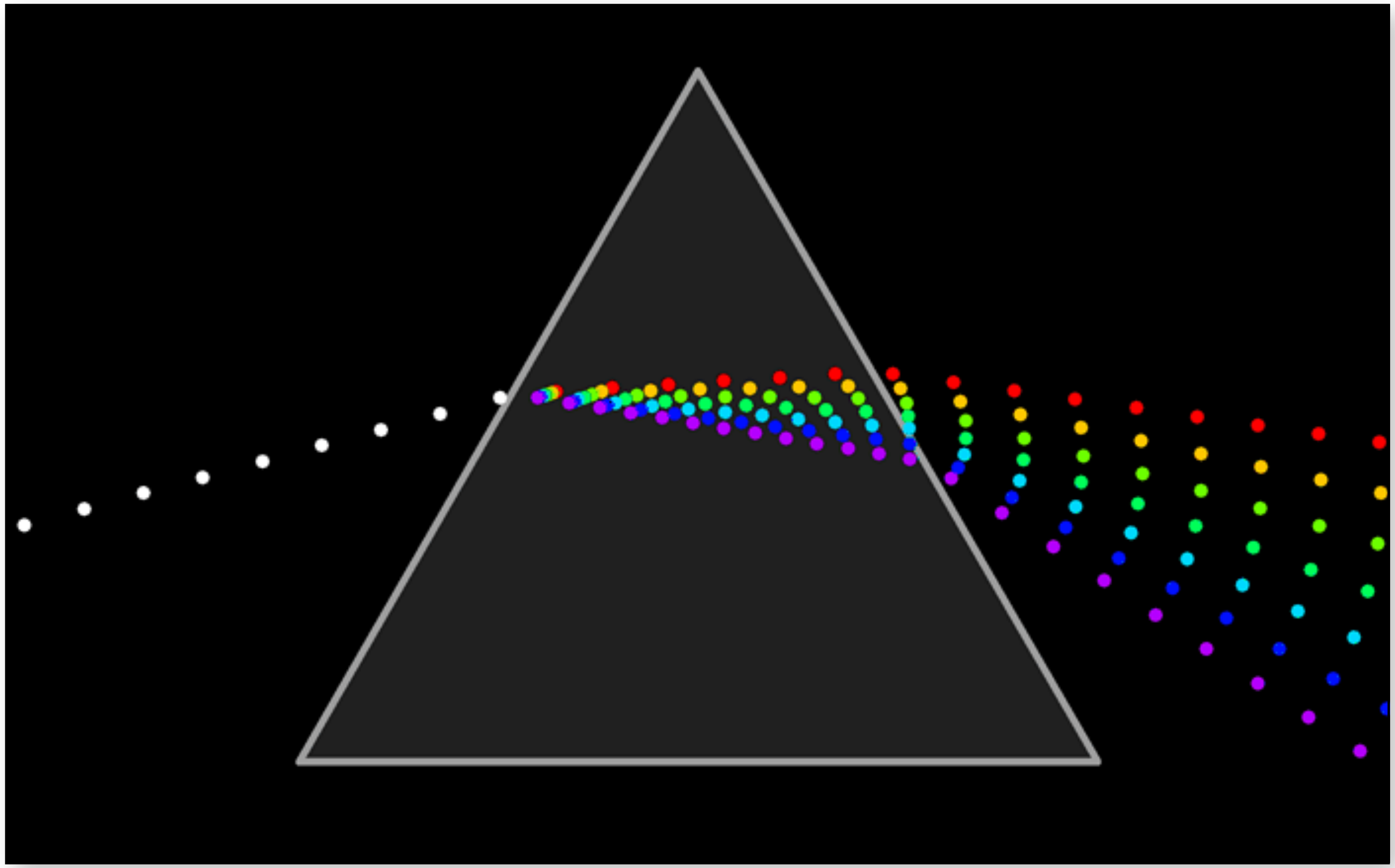
ECSE 446/546

IMAGE SYNTHESIS



RADIOMETRY

Prof. Derek Nowrouzezahrai
derek@cim.mcgill.ca

Today's Menu

How do we quantify light?

How does light interact with surfaces?

Quantifying Light

Radiometry

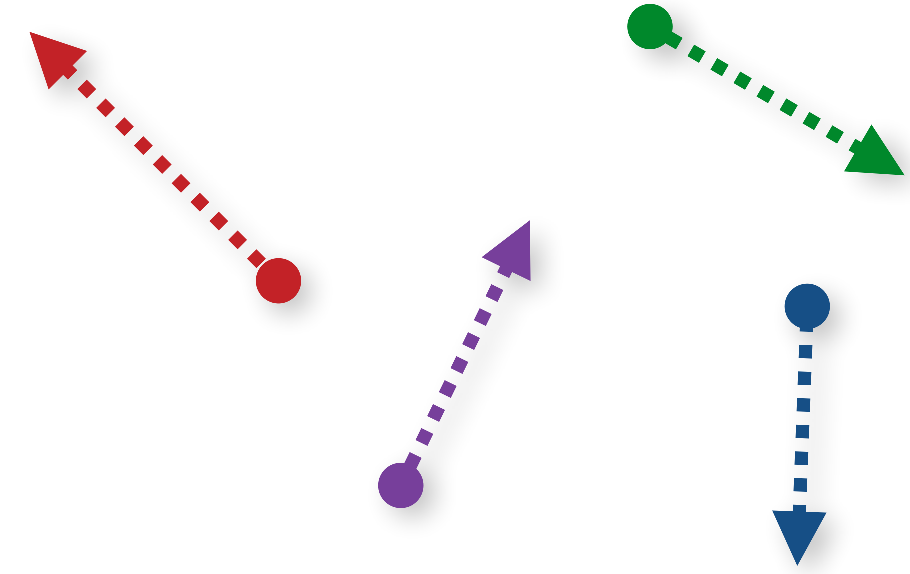
Radiometry studies the measurement of electromagnetic radiation, including visible light

Visible Light is any radiation that is capable of directly causing visual sensation
[International Lighting Vocabulary [CIE 1987]]

Radiometry

Assume light consists of photons with:

- Position $\mathbf{x}$
- Direction of travel $\vec{\omega}$
- Wavelength λ

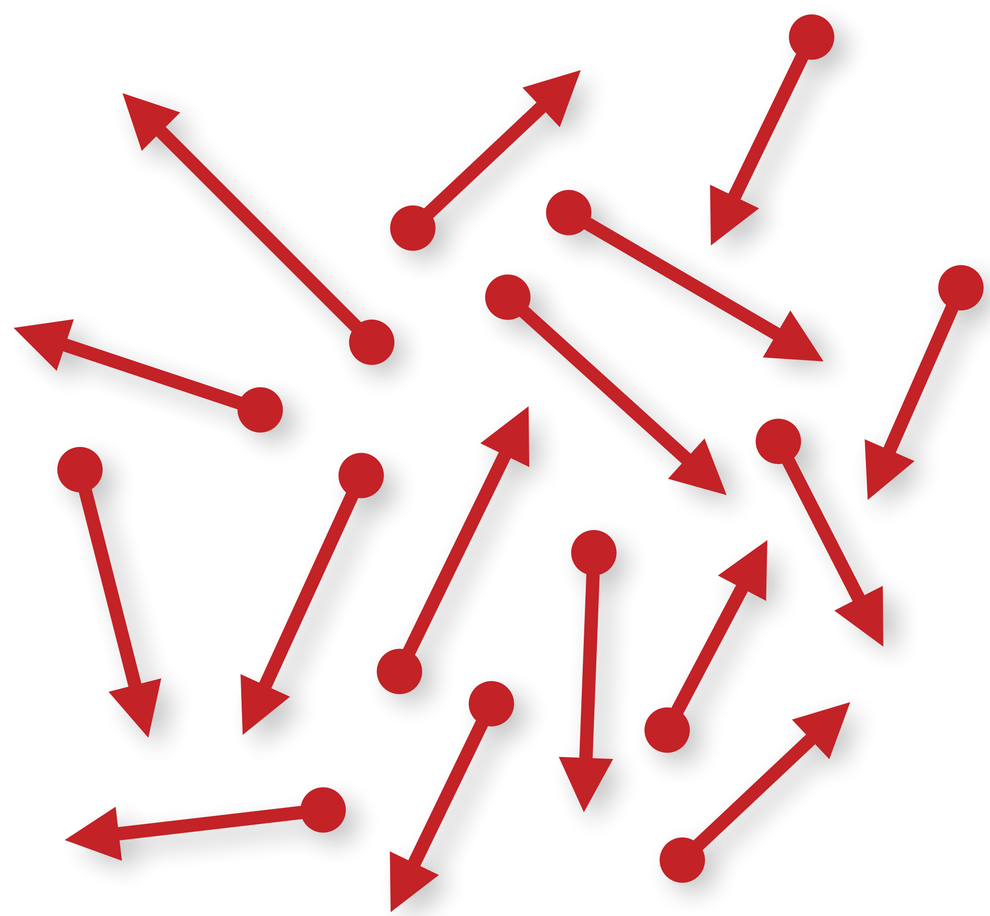


Each photon has an energy of: $\frac{h\,c}{\lambda}$

- Planck's constant $h \approx 6.63 \cdot 10^{-34}\ \mathrm{m}^2 \cdot \mathrm{kg/s}$
- speed of light in vacuum $c = 299,792,458\ \mathrm{m/s}$
- Unit of energy, Joule: $[\mathrm{J} = \mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}^2]$

Radiometry

How do we measure the energy flow?



Measuring energy means “counting photons”

Radiometry

Basic quantities (depend on wavelength*)

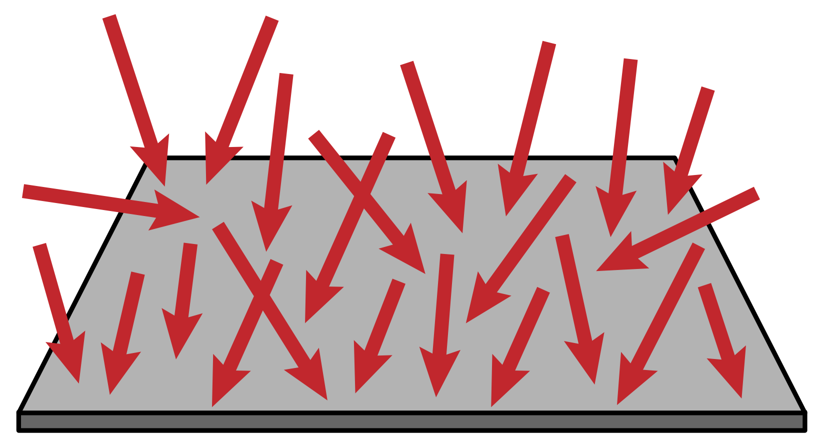
- flux Φ
- irradiance E
- radiosity B
- intensity I
- radiance L

will be the most important quantity for us

Flux (Radiant Flux, Power)

The total amount of radiant energy passing through surface or space *per unit time*

$$\Phi(A) \qquad \left[\frac{J}{s} = W\right]$$



examples:

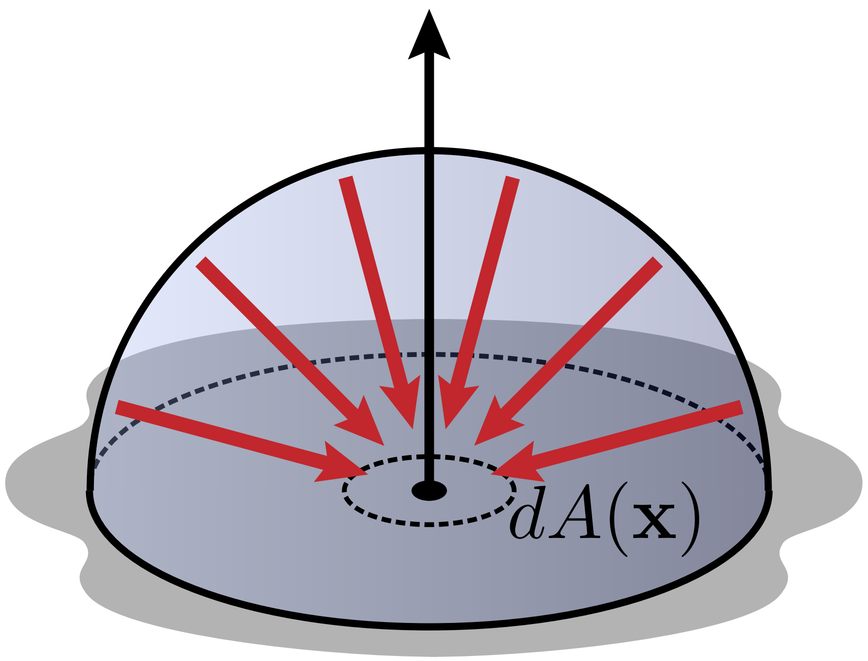
- number of photons hitting a wall per second
- number of photons leaving a lightbulb per second

Irradiance

The *area density* of flux

- flux per unit area **arriving** at a surface

$$E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})} \left[\frac{W}{m^2}\right]$$



example:

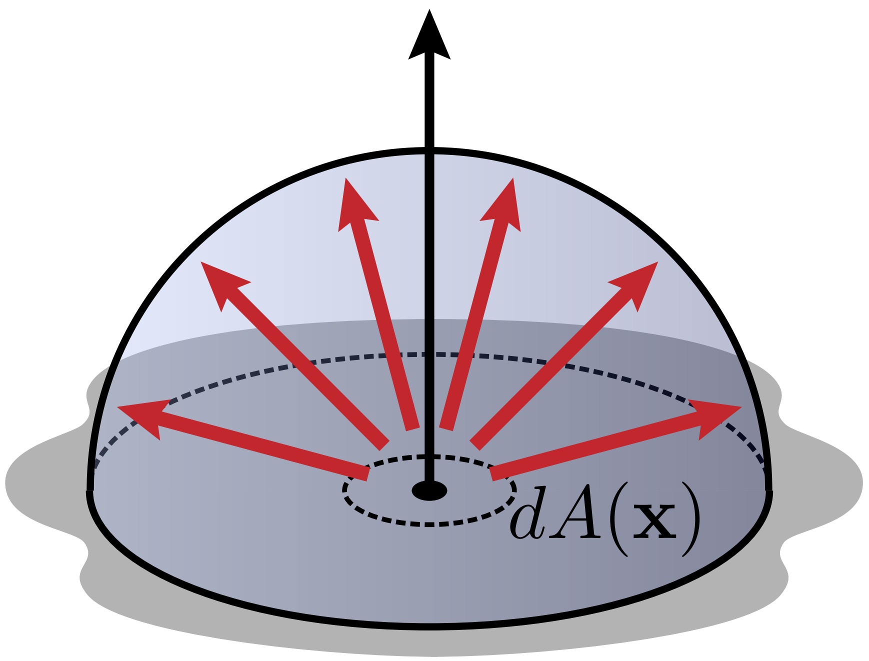
- number of photons **hitting** a small* patch of a wall per second, *divided* by the patch's size

Radiosity (Radiant Exitance)

Also measured as *area density of flux*

- flux per unit area leaving a surface

$$B(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})} \left[\frac{W}{m^2}\right]$$



example:

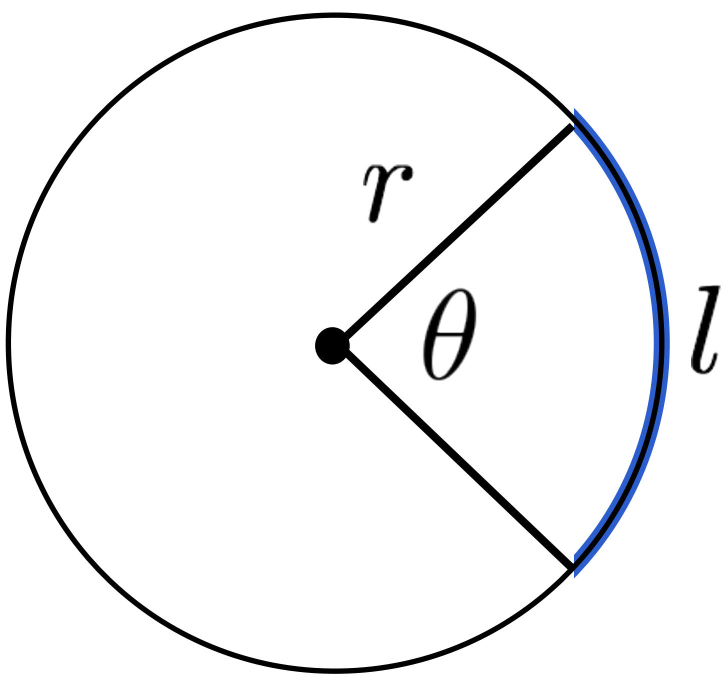
- number of photons reflecting off a small patch of a wall per second, divided by size of patch

Solid Angle

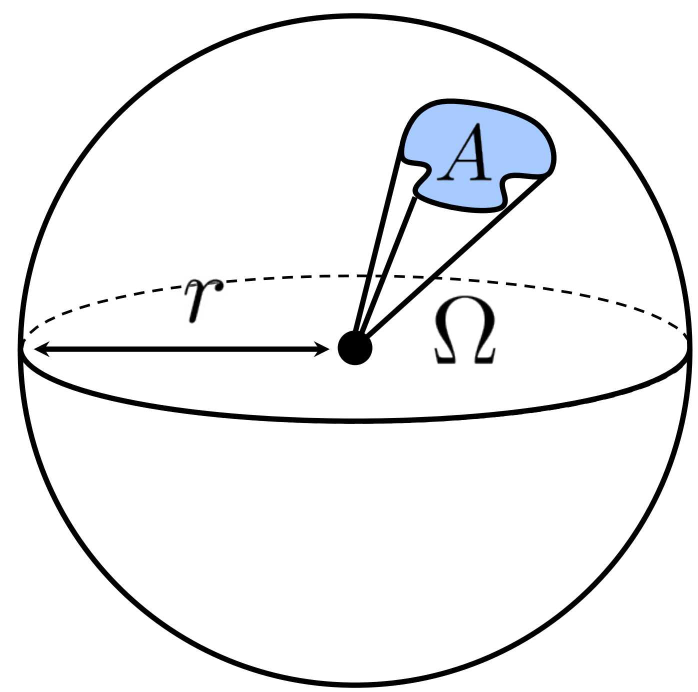
Angle

- circle: 2π radians

- Solid angle
  - sphere: 4π steradians



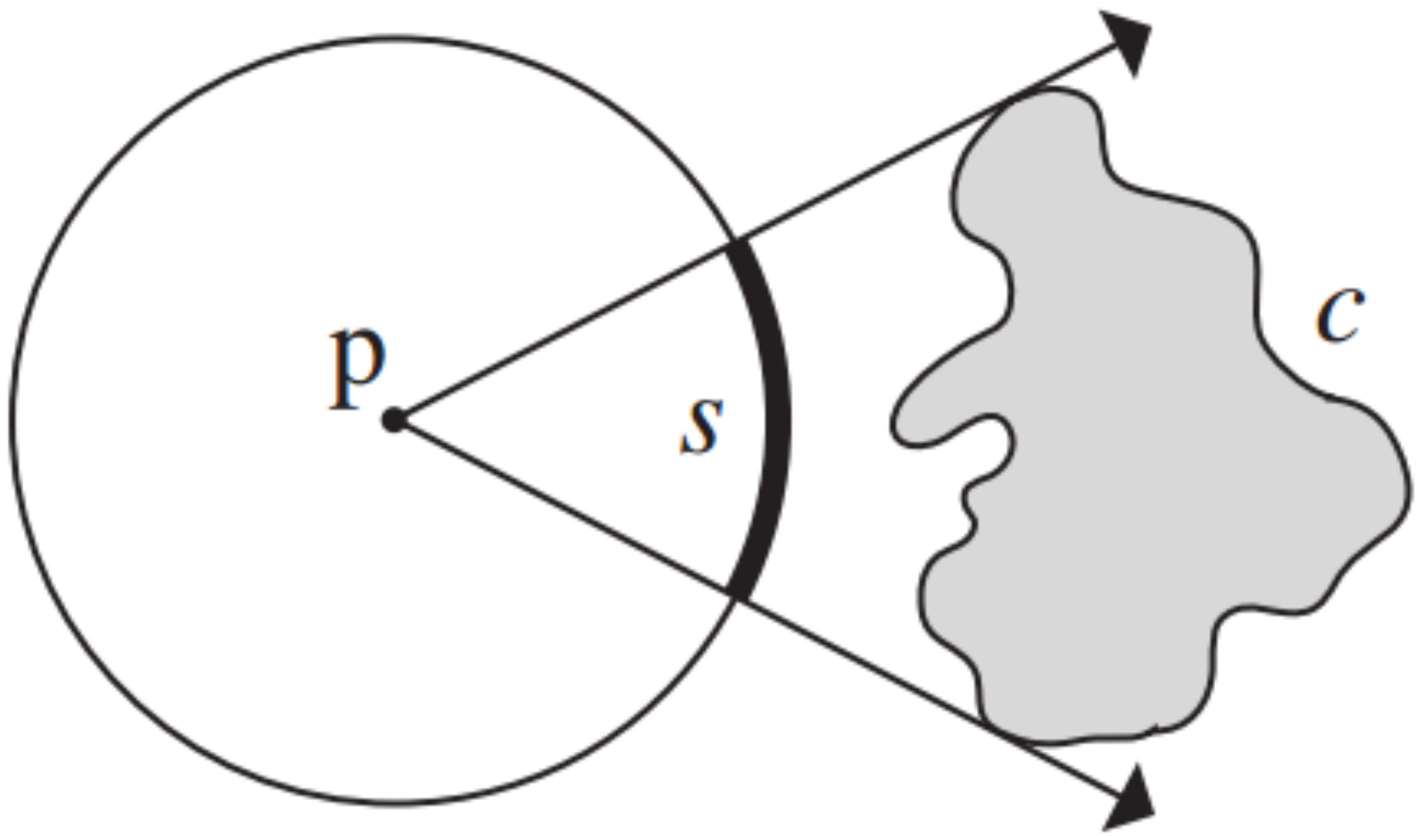
$$\theta = \frac{l}{r}$$



$$\Omega = \frac{A}{r^2}$$

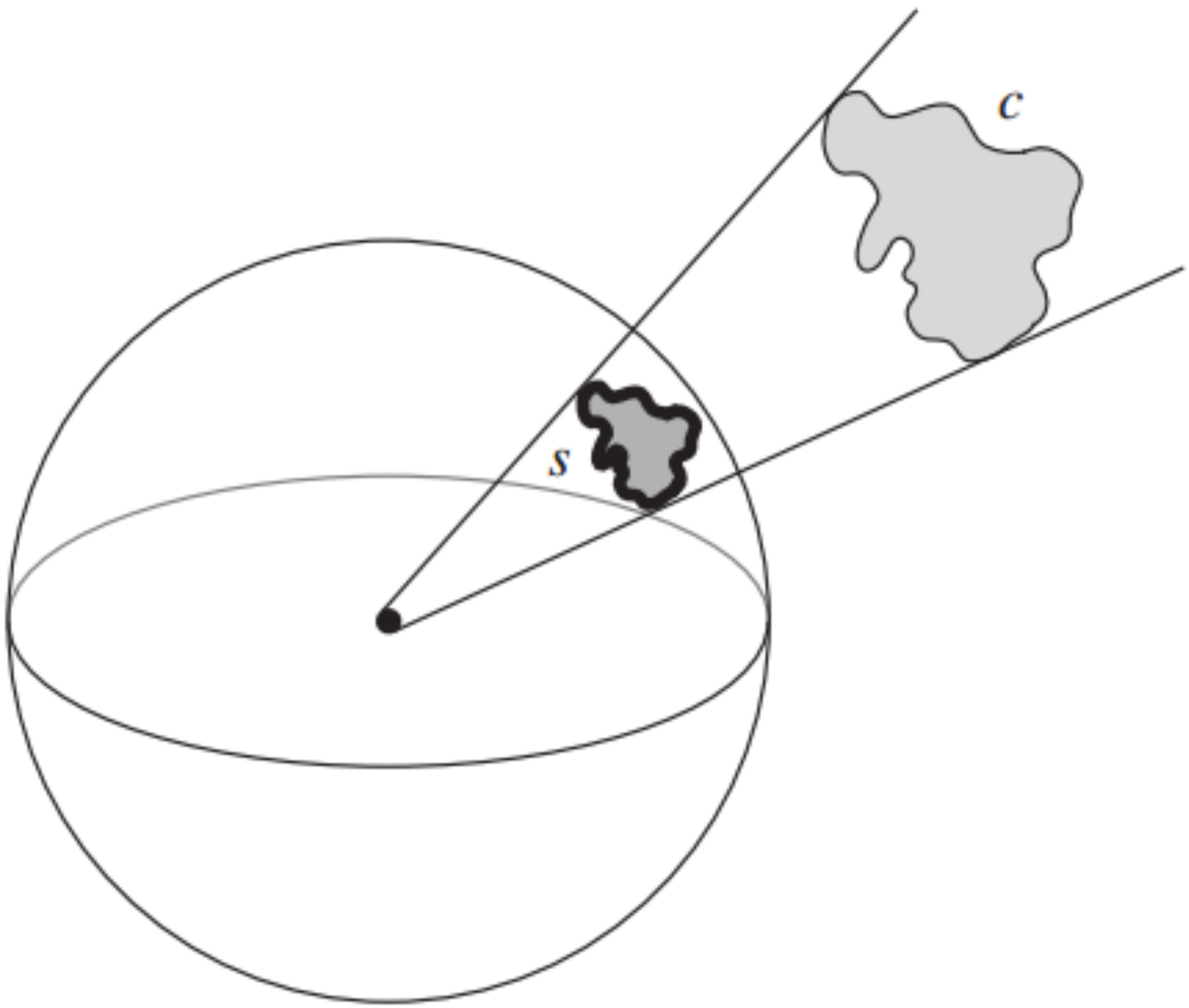
Subtended Angle

Length of object's projection onto a unit circle



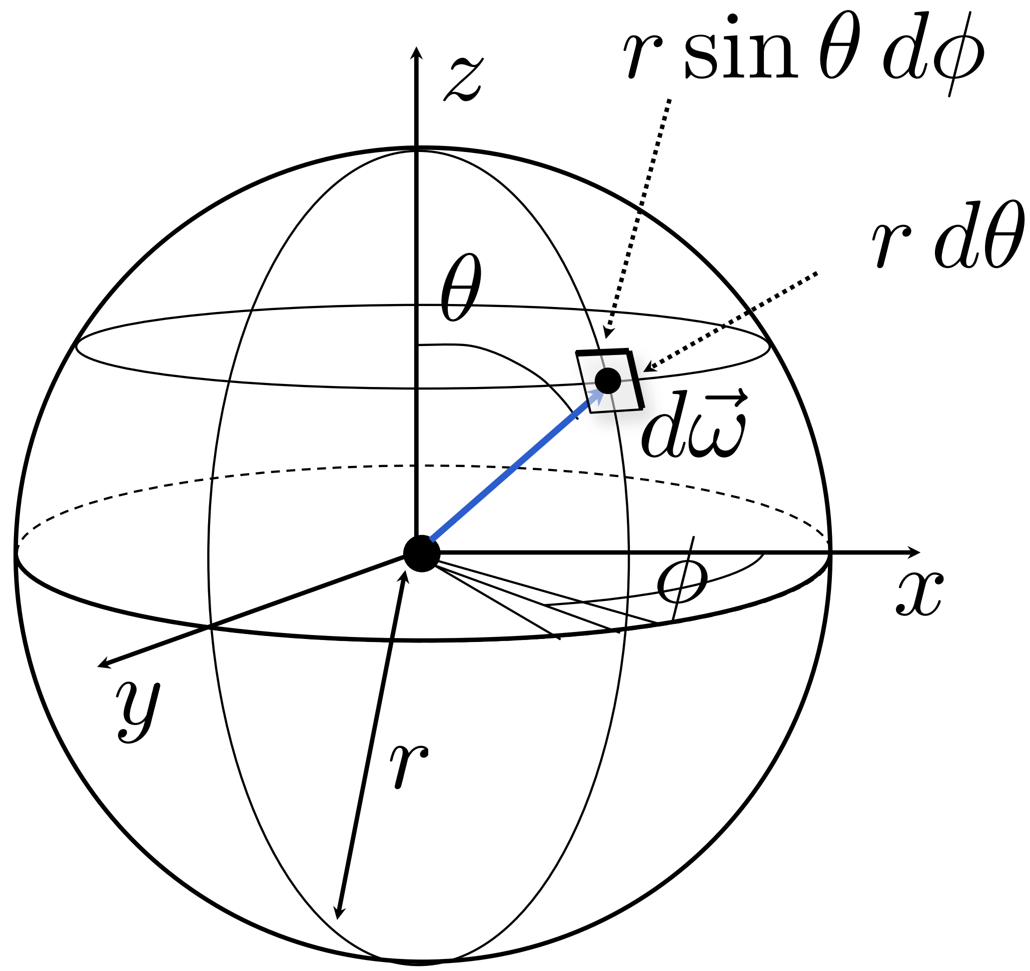
Subtended Solid Angle

Area of object's projection onto a unit **sphere**



Differential Solid Angle

Differential area on the unit sphere around direction $\vec{\omega}$



$$dA = (r d\theta)(r \sin\theta d\phi)$$

$$d\vec{\omega} = \frac{dA}{r^2} = \sin\theta d\theta d\phi$$

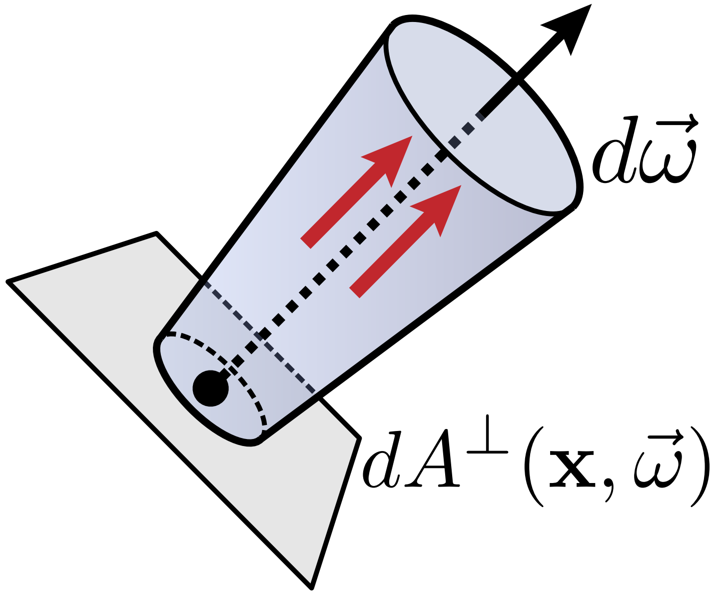
$$\Omega = \int_{S^2} d\vec{\omega} = \int_0^{2\pi}\int_0^{\pi} \sin\theta d\theta d\phi = 4\pi$$

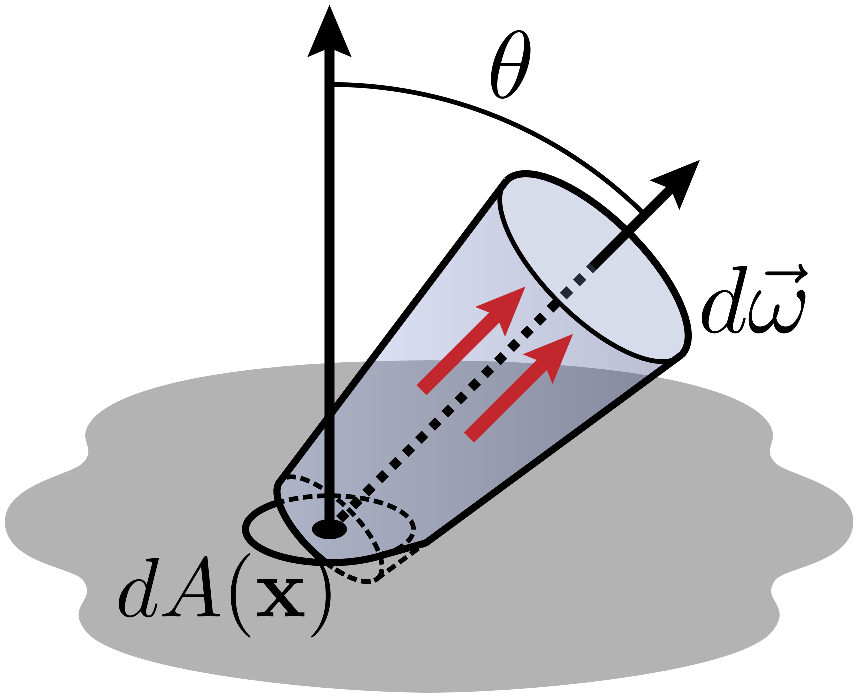
Radiance

The flux density per unit solid angle, per perpendicular unit area

$$L(\mathbf{x},\vec{\omega}) = \frac{d^2\Phi(A)}{d\vec{\omega}\, dA^{\perp}(\mathbf{x},\vec{\omega})} \qquad \left[\frac{W}{m^2 sr}\right]$$

$$= \frac{d^2\Phi(A)}{d\vec{\omega}\, dA(\mathbf{x})\cos\theta}$$



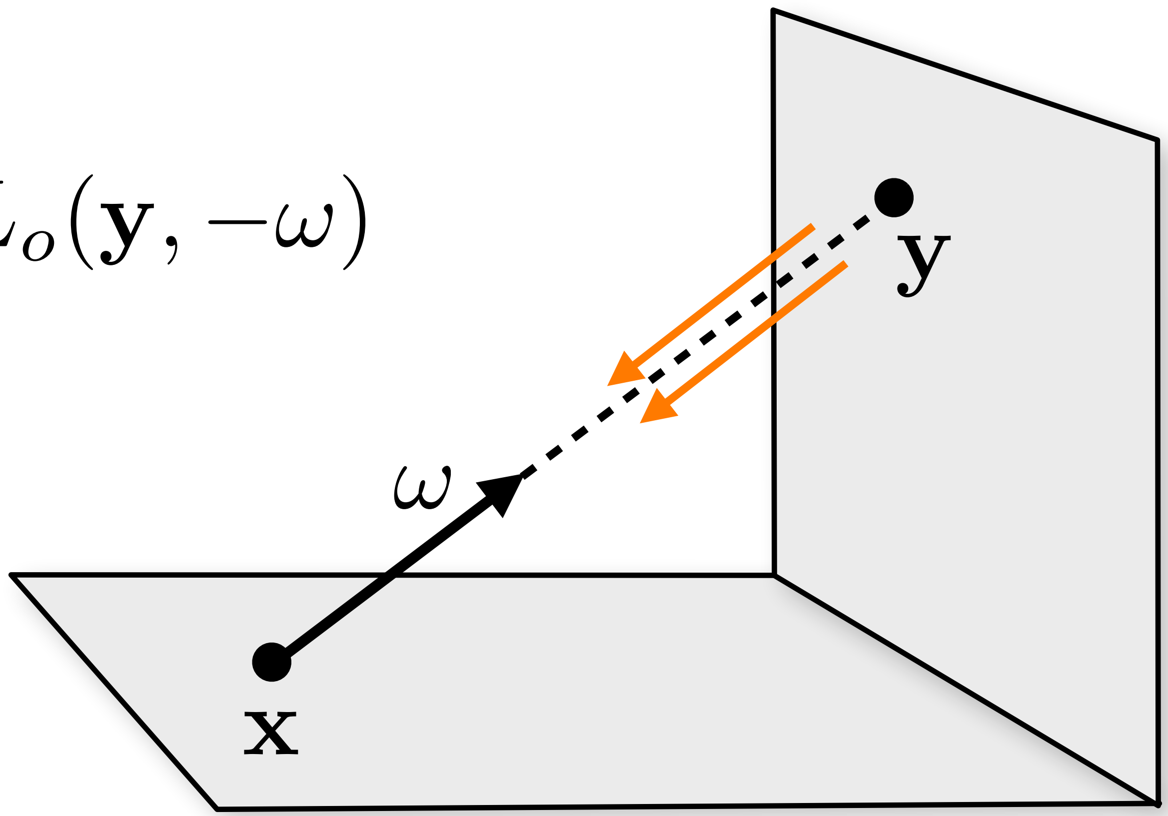


Radiance

The fundamental quantity for ray tracing

- remains constant along a ray (in vacuum only!)
- incident radiance L_i at one point can be expressed as outgoing radiance L_o at another point

$$L_i(\mathbf{x}, \omega) = L_o(\mathbf{y}, -\omega)$$



Radiance

How to express *irradiance* in terms of radiance:

$$L(\mathbf{x}, \vec{\omega}) = \frac{d^2\Phi(A)}{\cos\theta dA(\mathbf{x}) d\vec{\omega}} \qquad E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$$

$$L(\mathbf{x}, \vec{\omega}) = \frac{dE(\mathbf{x})}{\cos\theta d\vec{\omega}}$$

$$L(\mathbf{x}, \vec{\omega}) \cos\theta \, d\vec{\omega} = dE(\mathbf{x})$$

$$\int_{H^2} L(\mathbf{x}, \vec{\omega}) \cos\theta \, d\vec{\omega} = E(\mathbf{x})$$

Integrate radiance over **hemisphere**

Radiance

How to express *flux* in terms of radiance:

$$\int_{H^2} L(\mathbf{x}, \vec{\omega}) \cos\theta \, d\vec{\omega} = E(\mathbf{x}) \qquad E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$$

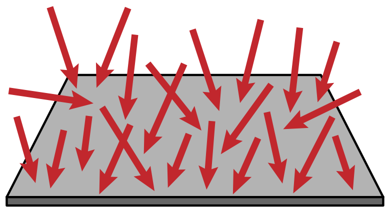
$$\int_A E(\mathbf{x}) \, dA(\mathbf{x}) = \Phi(A)$$

$$\int_A \int_{H^2} L(\mathbf{x}, \vec{\omega}) \cos\theta \, d\vec{\omega} dA(\mathbf{x}) = \Phi(A)$$

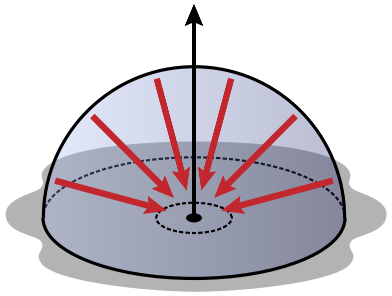
Integrate radiance over **hemisphere** and **area**

Overview of Quantities

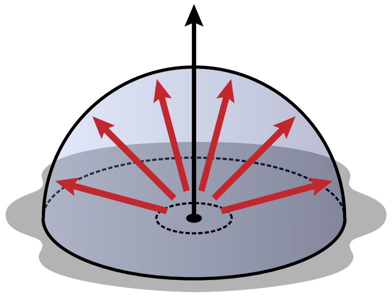
- flux: $\Phi(A)$ $\left[\frac{J}{s} = W\right]$



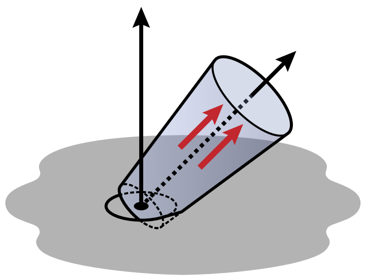
- irradiance: $E(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$ $\left[\frac{W}{m^2}\right]$



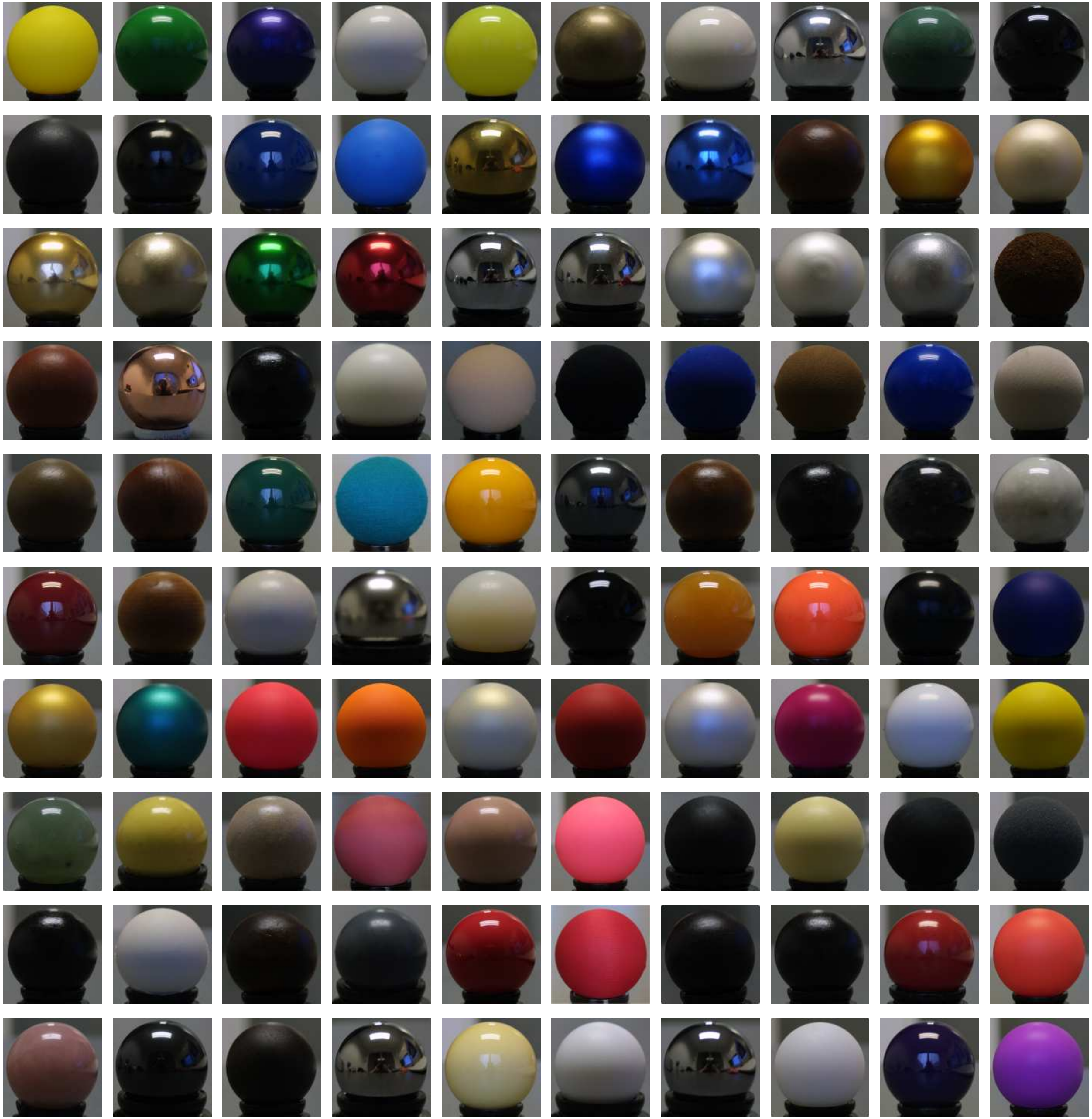
- radiosity: $B(\mathbf{x}) = \frac{d\Phi(A)}{dA(\mathbf{x})}$ $\left[\frac{W}{m^2}\right]$



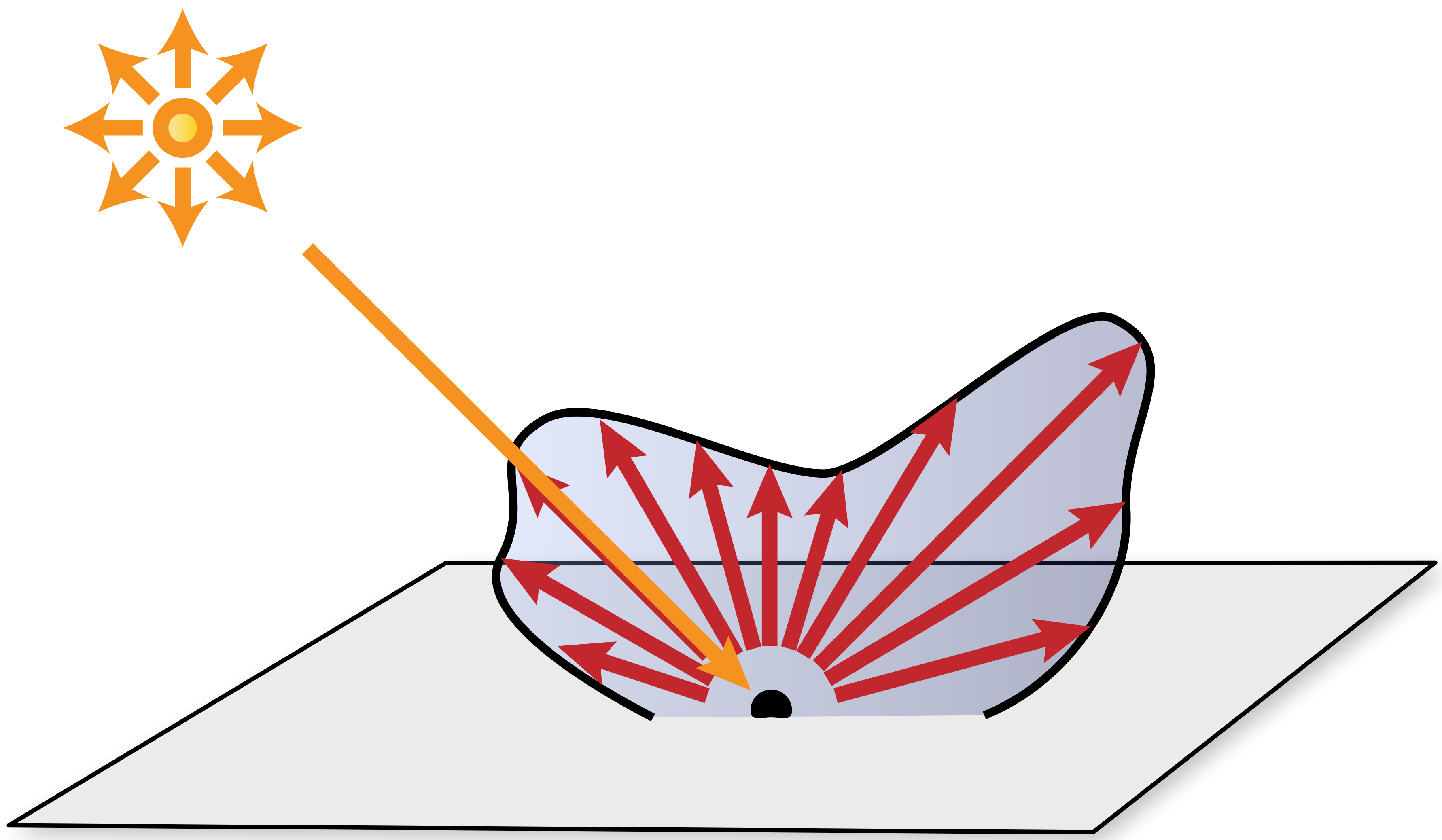
- radiance: $L(\mathbf{x},\vec{\omega}) = \frac{d^2\Phi(A)}{\cos\theta dA(\mathbf{x})d\vec{\omega}}$ $\left[\frac{W}{m^2 sr}\right]$



Light-Material Interactions



Light-Material Interactions



BRDF

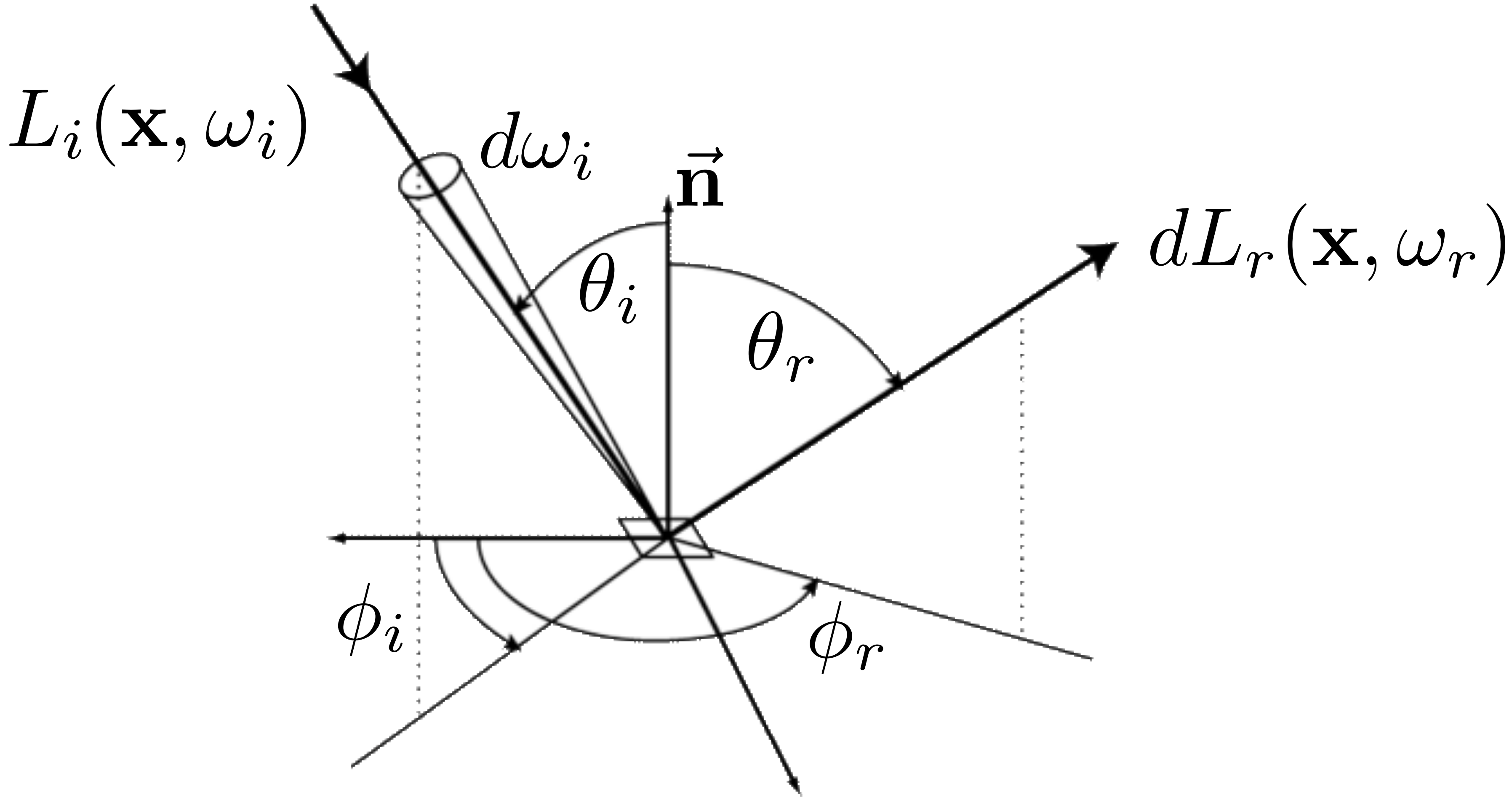
Bidirectional **R**eflectance **D**istribution **F**unction

- ratio of differential *reflected radiance* to differential *incident irradiance*

BRDF

Bidirectional **R**eflectance **D**istribution **F**unction

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = \frac{dL_r(\mathbf{x}, \vec{\omega}_r)}{dE_i(\mathbf{x}, \vec{\omega}_i)} = \frac{dL_r(\mathbf{x}, \vec{\omega}_r)}{L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i} \quad [1/sr]$$



Reflection Equation

The BRDF relates the incident radiance and (differential) reflected radiance at a point

- from this we can derive the **<u>Reflection Equation</u>**:

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = \frac{dL_r(\mathbf{x}, \vec{\omega}_r)}{L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i}$$

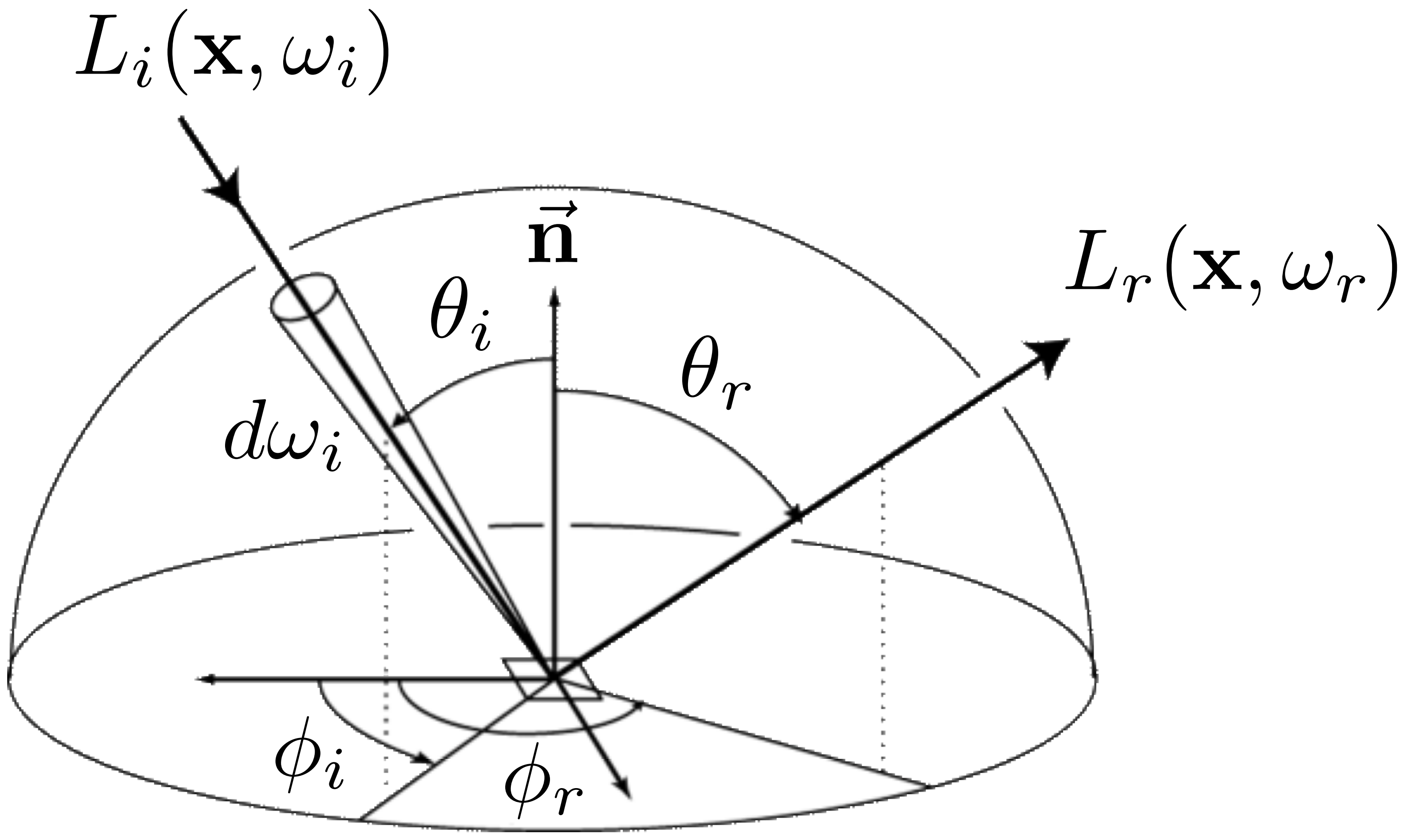
$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i = \frac{dL_r(\mathbf{x}, \vec{\omega}_r)}{d\vec{\omega}_i}$$

$$\int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i = L_r(\mathbf{x}, \vec{\omega}_r)$$

Reflection Equation

The Reflection Equation describes a *local illumination* model

- what's the reflected radiance at a shade point, given the incident illumination arriving from every direction around it



$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

BRDFs Properties

Real/physically-plausible BRDFs obey:

- Helmholtz reciprocity

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = f_r(\mathbf{x}, \vec{\omega}_r, \vec{\omega}_i)$$

- Energy conservation

$$\int_{H^2} f_r(\vec{\omega}_i, \vec{\omega}_r) \cos\theta_i \, d\omega_i \leq 1, \quad \forall\, \vec{\omega}_r$$