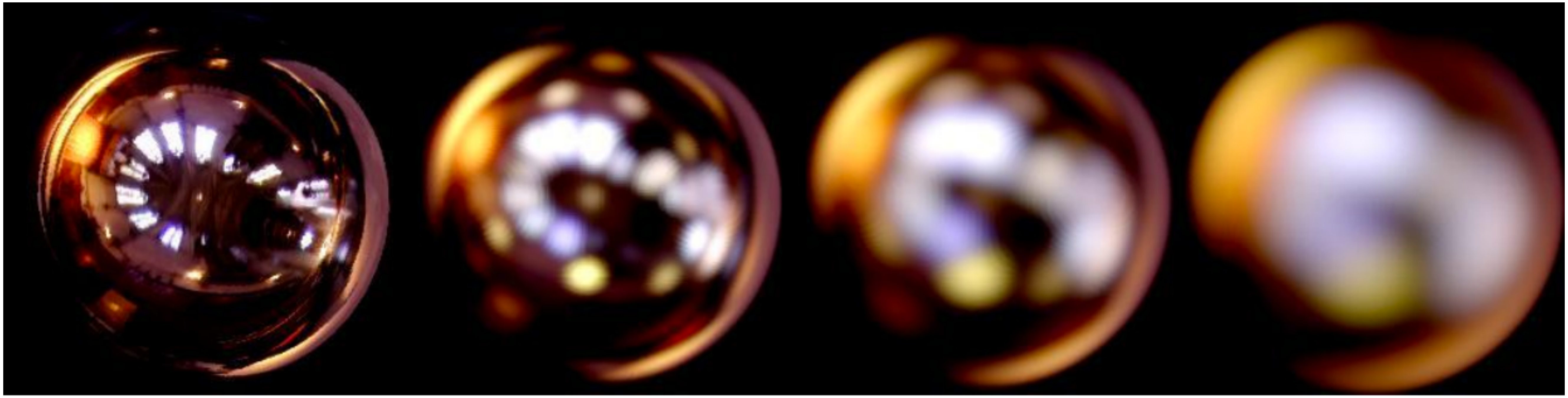
ECSE 446/546

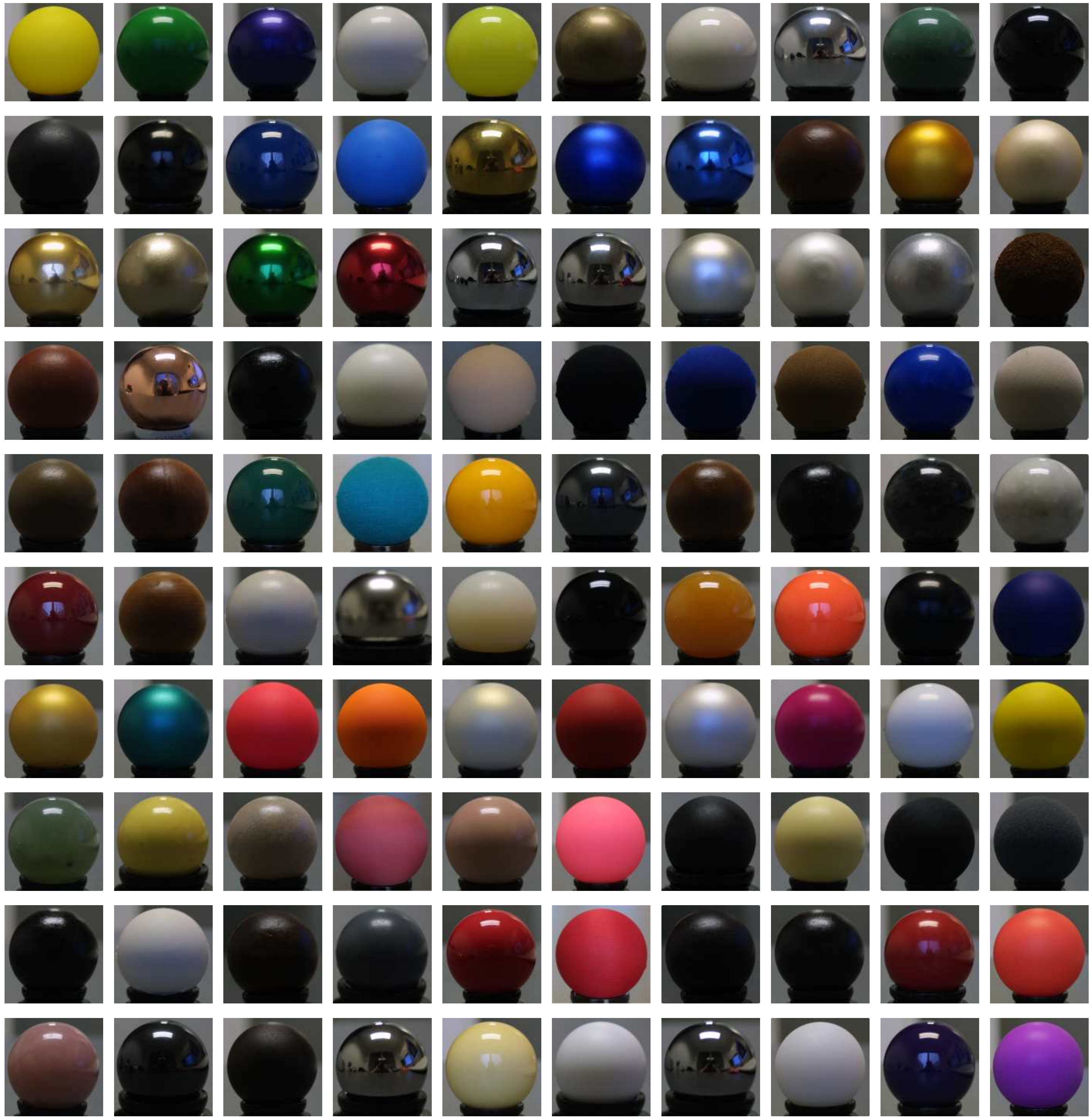
Image Synthesis



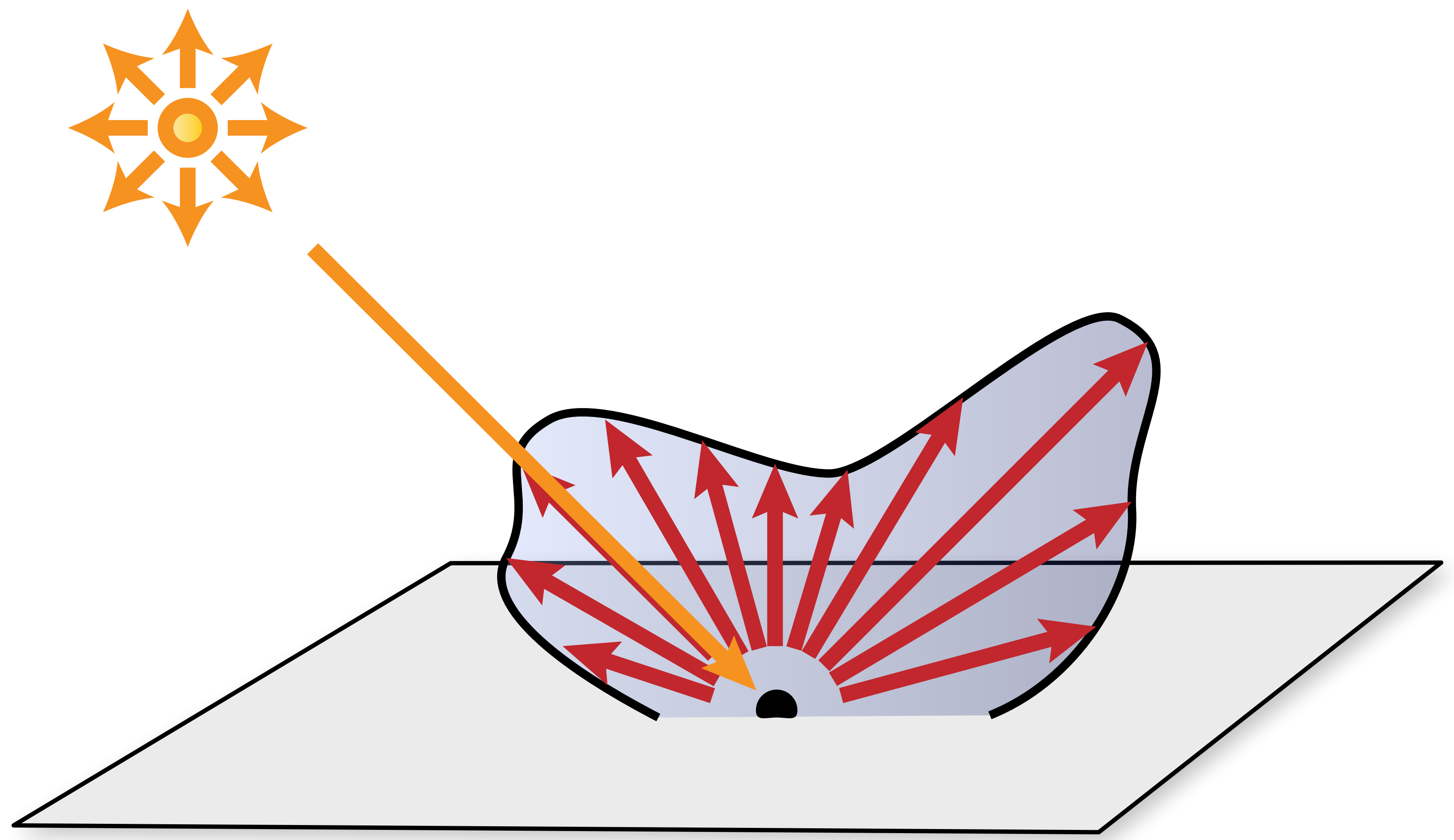
Appearance Models & Reflection

Prof. Derek Nowrouzezahrai
derek@cim.mcgill.ca

Light-Material Interactions



Light-Material Interactions



BRDF

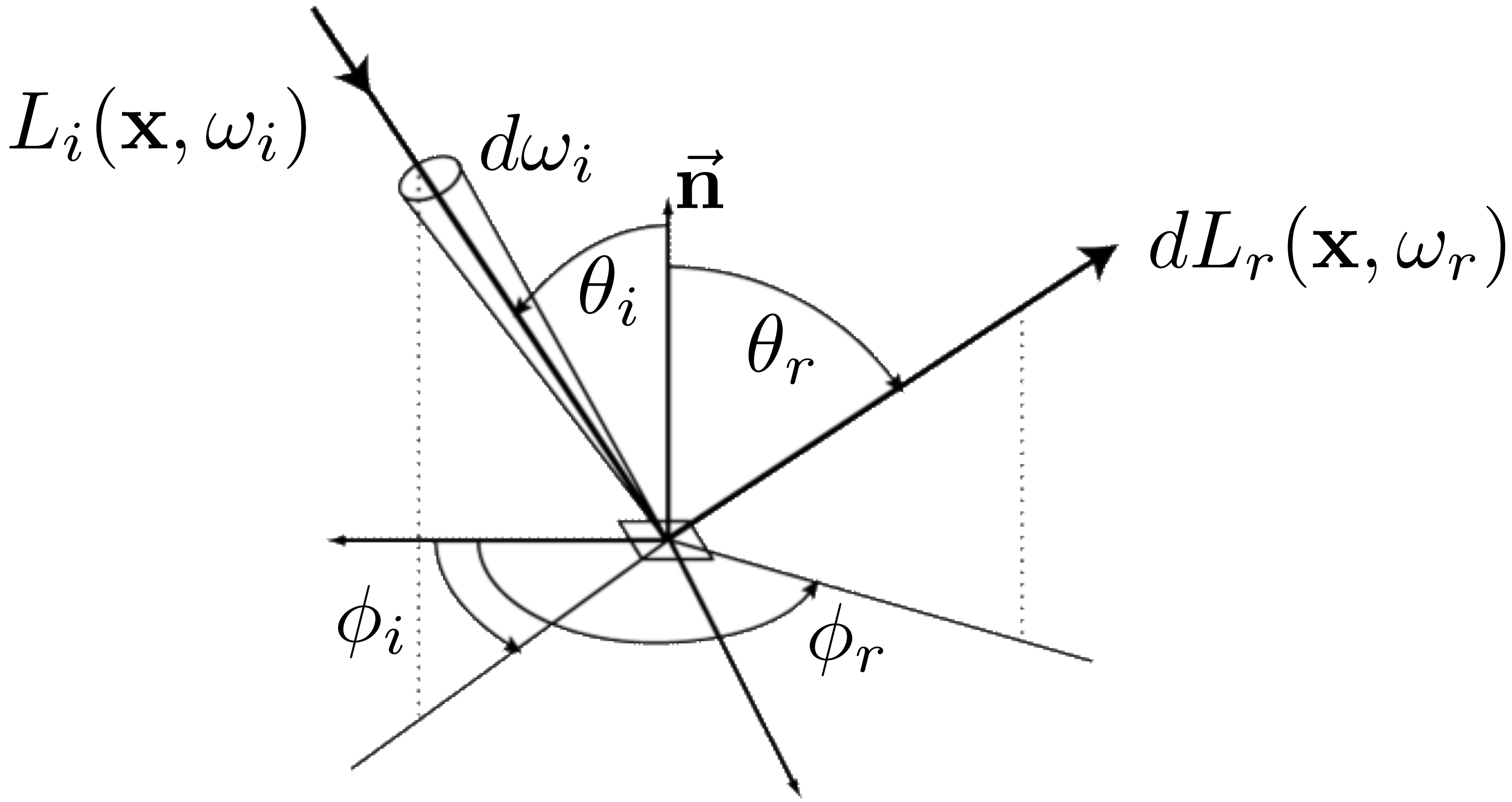
Bidirectional **R**eflectance **D**istribution **F**unction

- ratio of differential *reflected radiance* to differential *incident irradiance*

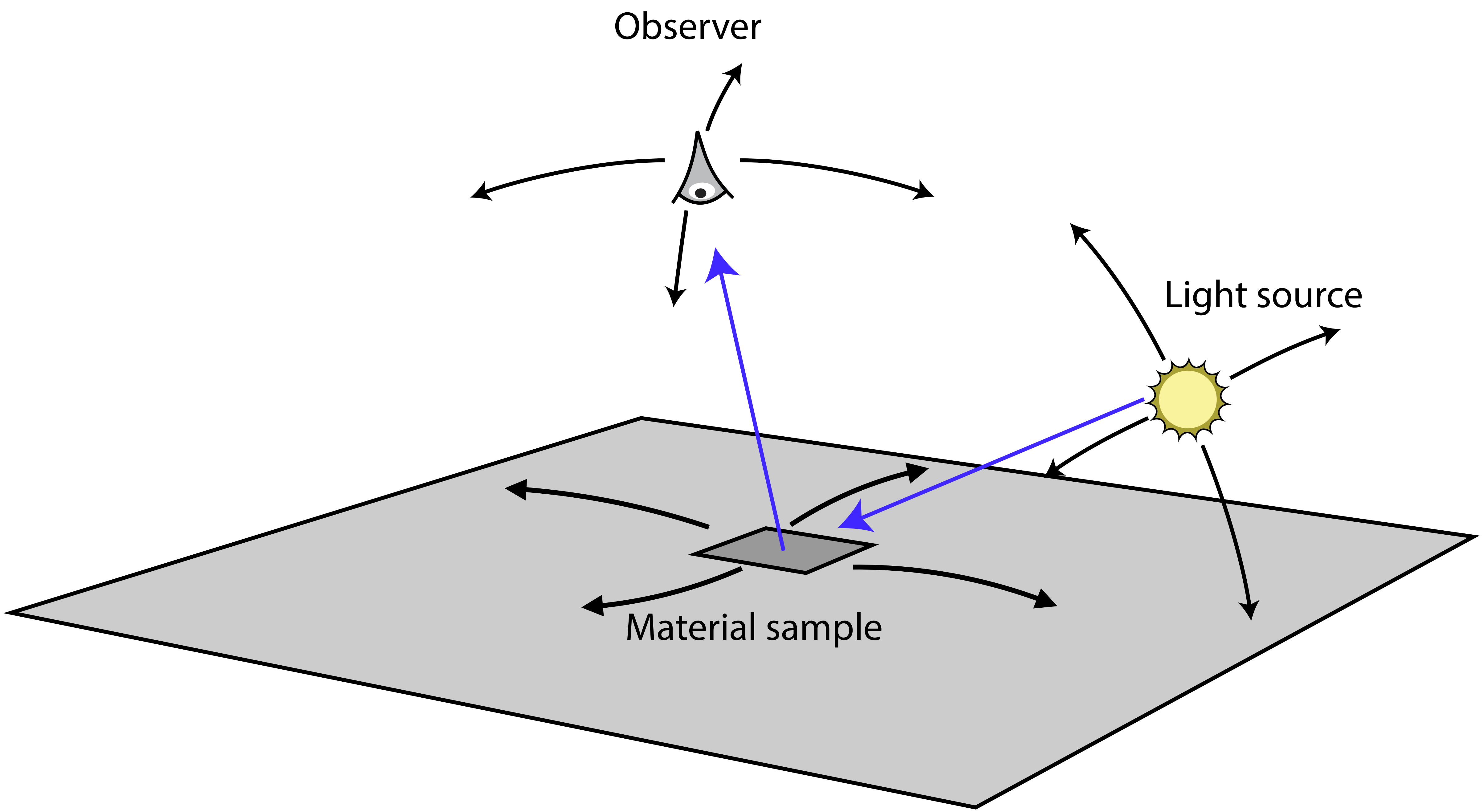
BRDF

Bidirectional **R**eflectance **D**istribution **F**unction

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = \frac{dL_r(\mathbf{x}, \vec{\omega}_r)}{dE_i(\mathbf{x}, \vec{\omega}_i)} = \frac{dL_r(\mathbf{x}, \vec{\omega}_r)}{L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i} \quad [1/sr]$$

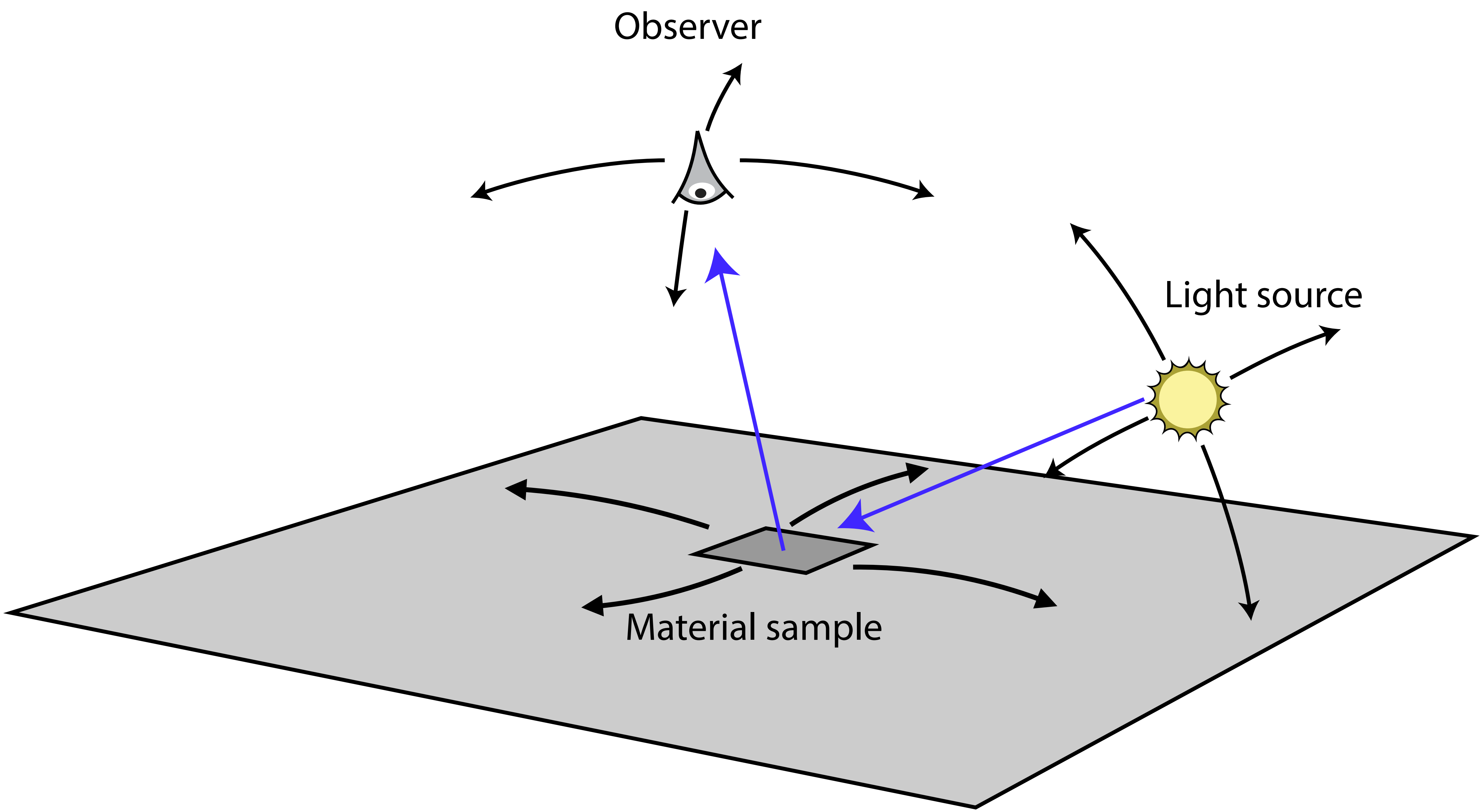


BRDF – Parameterization



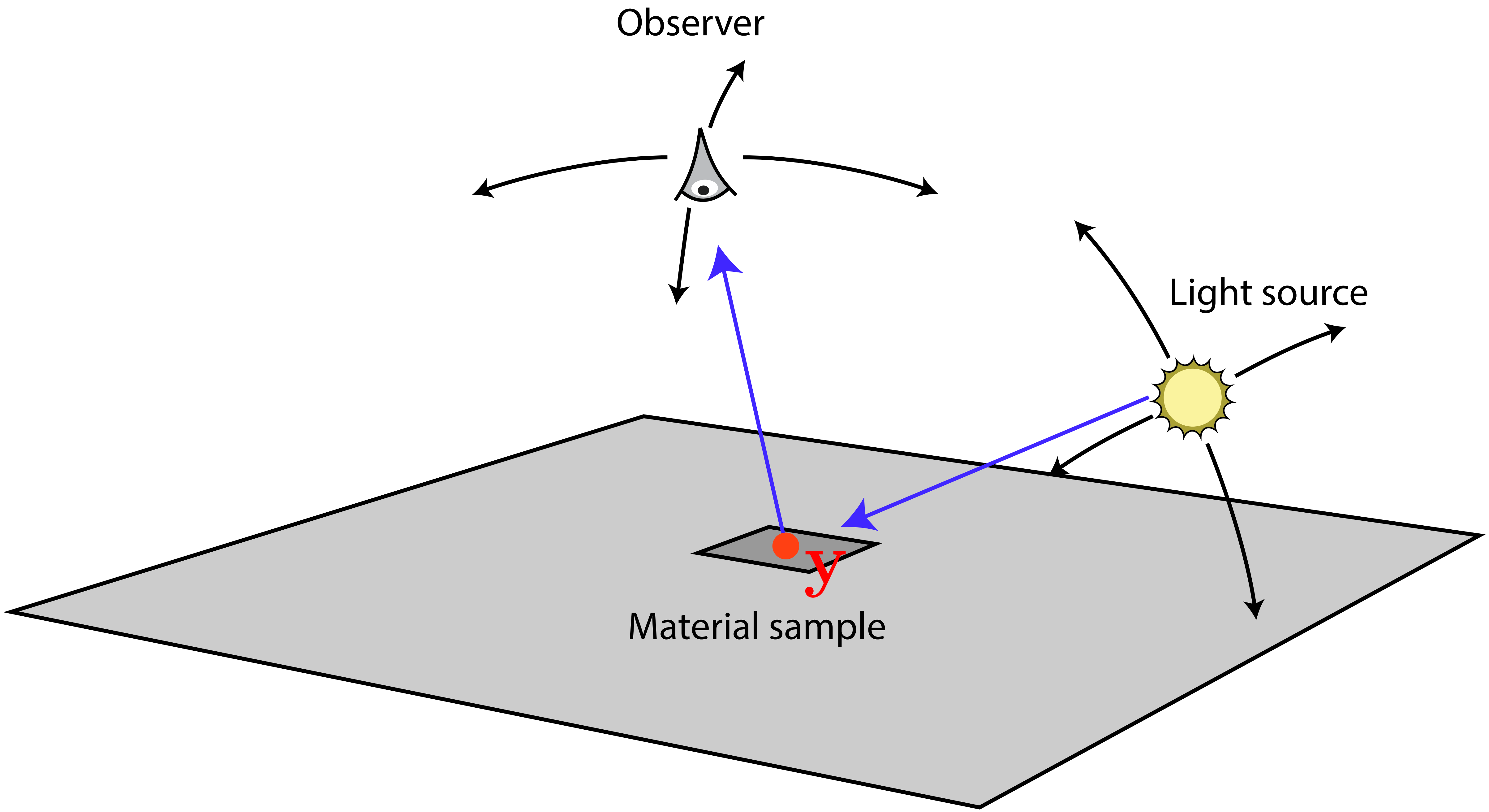
$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) : \mathcal{M} \times \mathcal{S}^2 \times \mathcal{S}^2 \to \mathbb{R}$$

BRDF – Parameterization



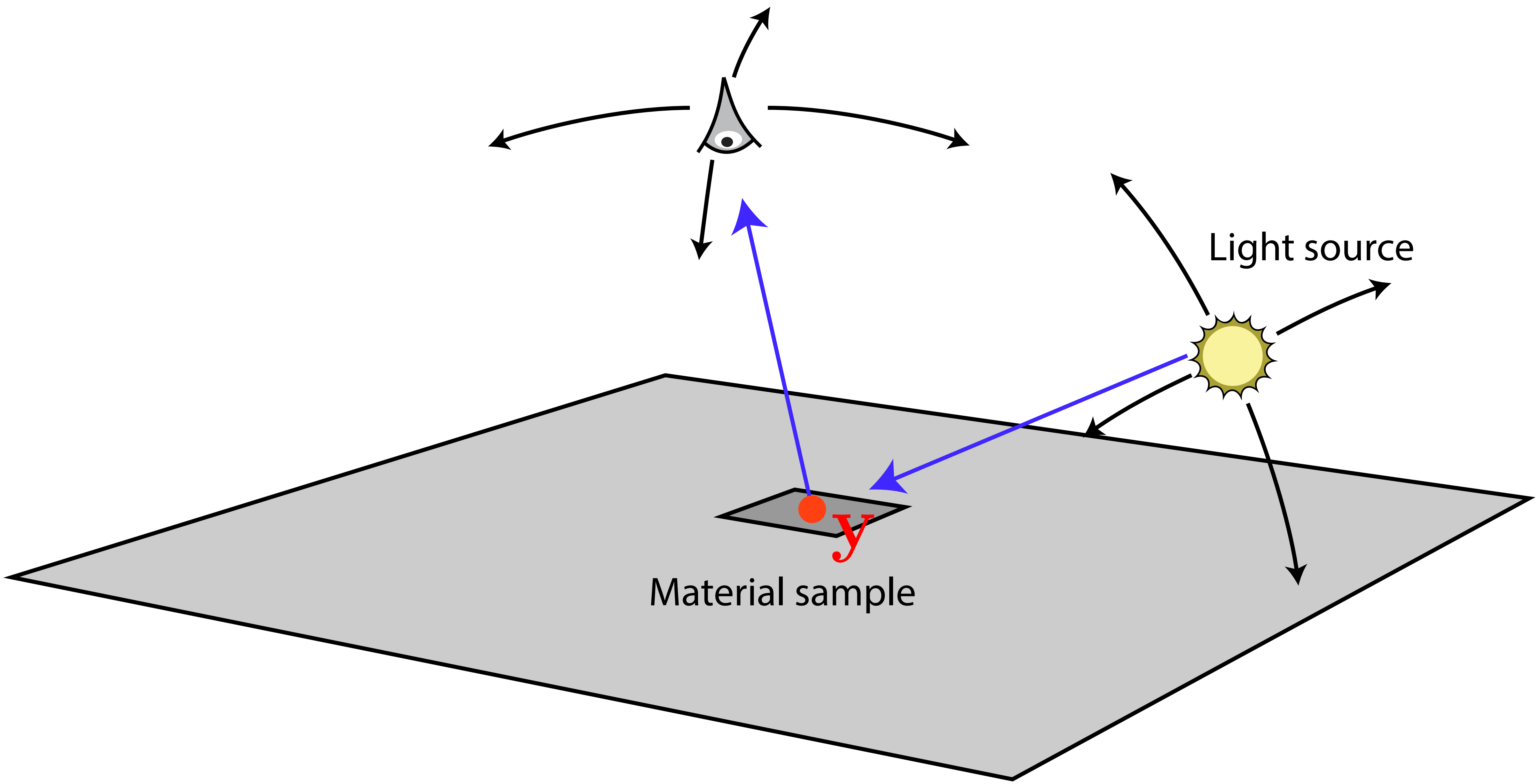
$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r)$$

BRDF – Parameterization



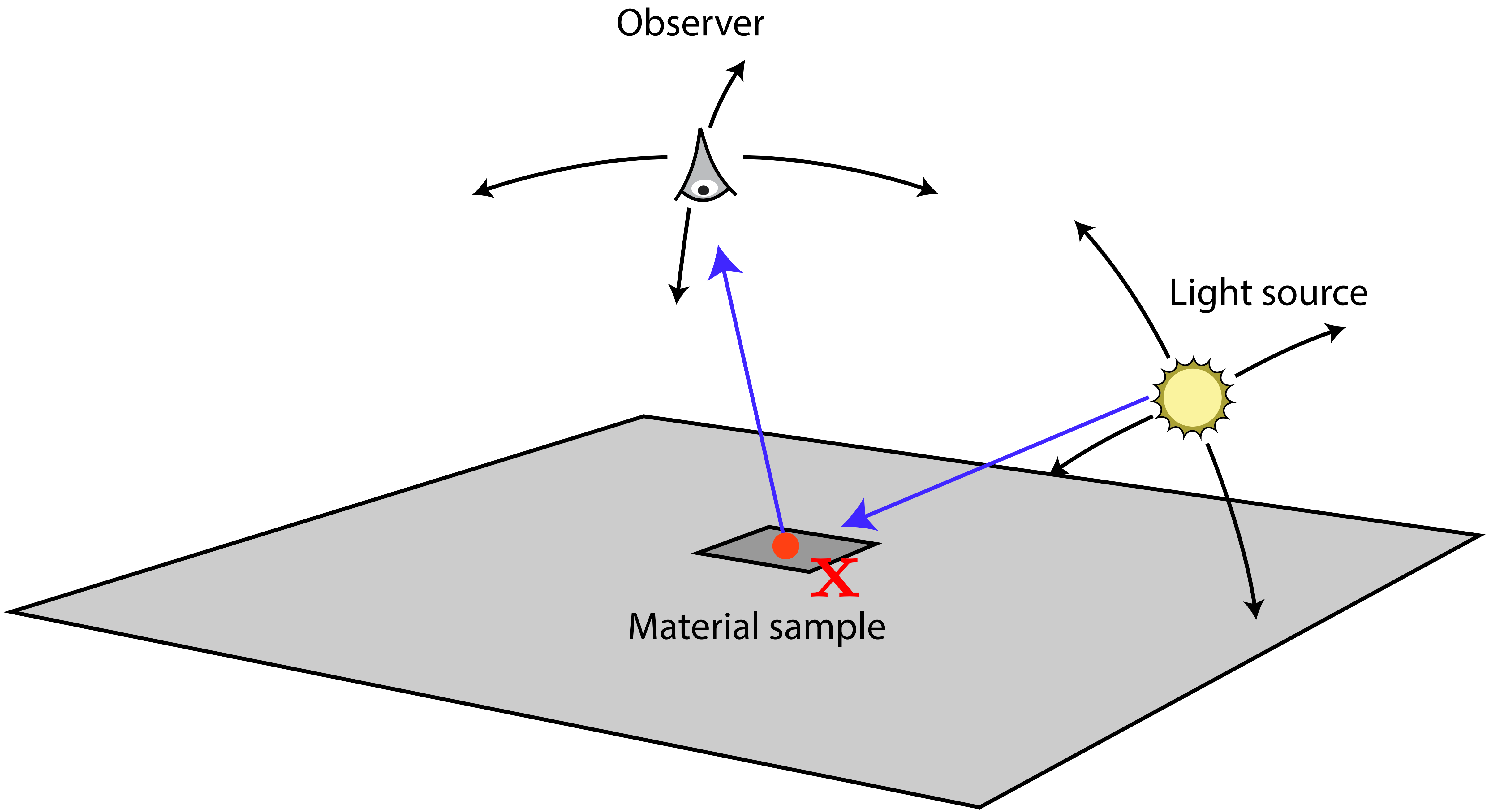
$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r)\Big|_{\mathbf{x}=\mathbf{y}} : \mathcal{S}^2 \times \mathcal{S}^2 \to \mathbb{R}$$

BRDF – Parameterization



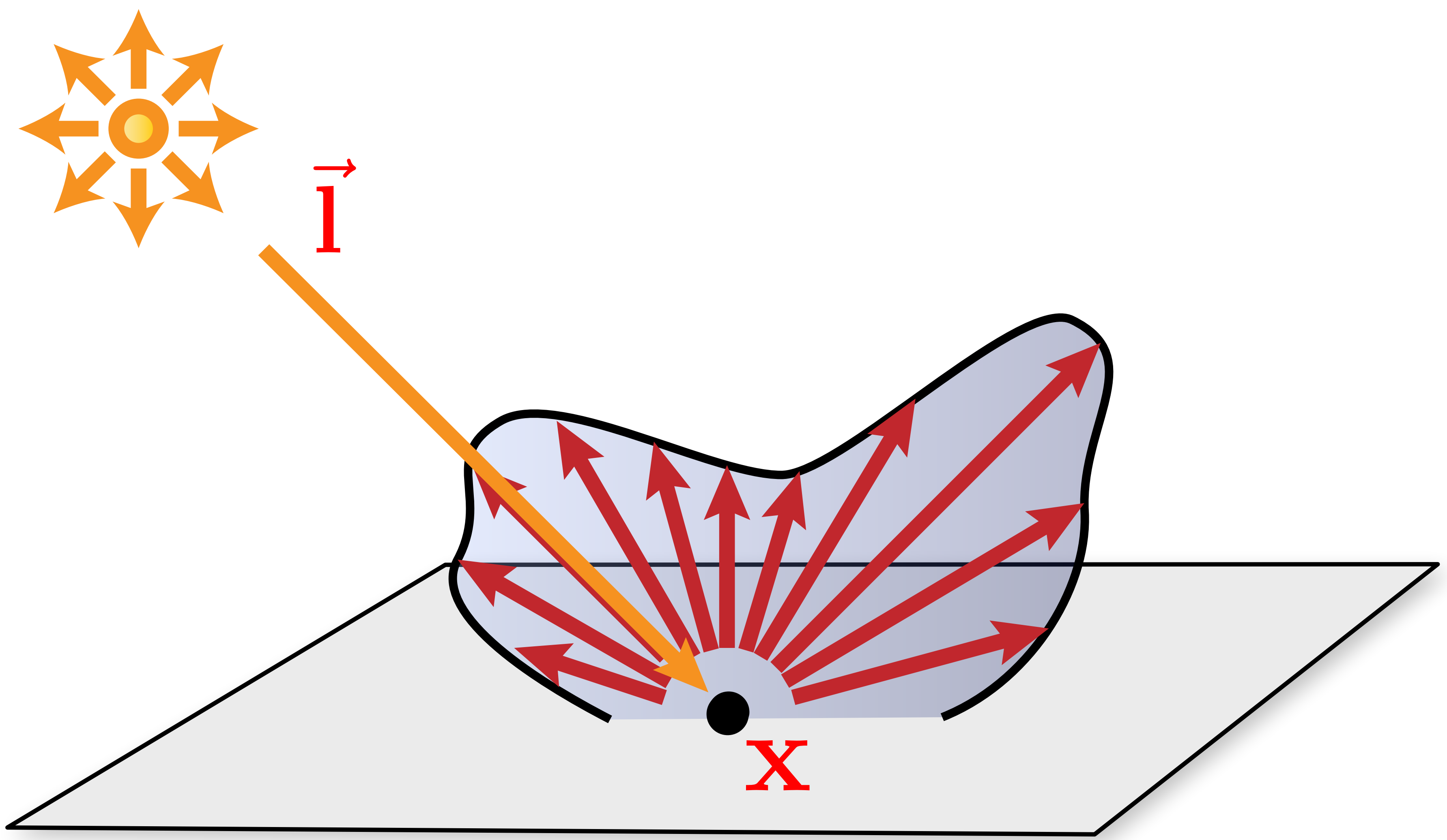
$$f_r(\mathbf{y}, \vec{\omega}_i, \vec{\omega}_r)$$

BRDF – Parameterization



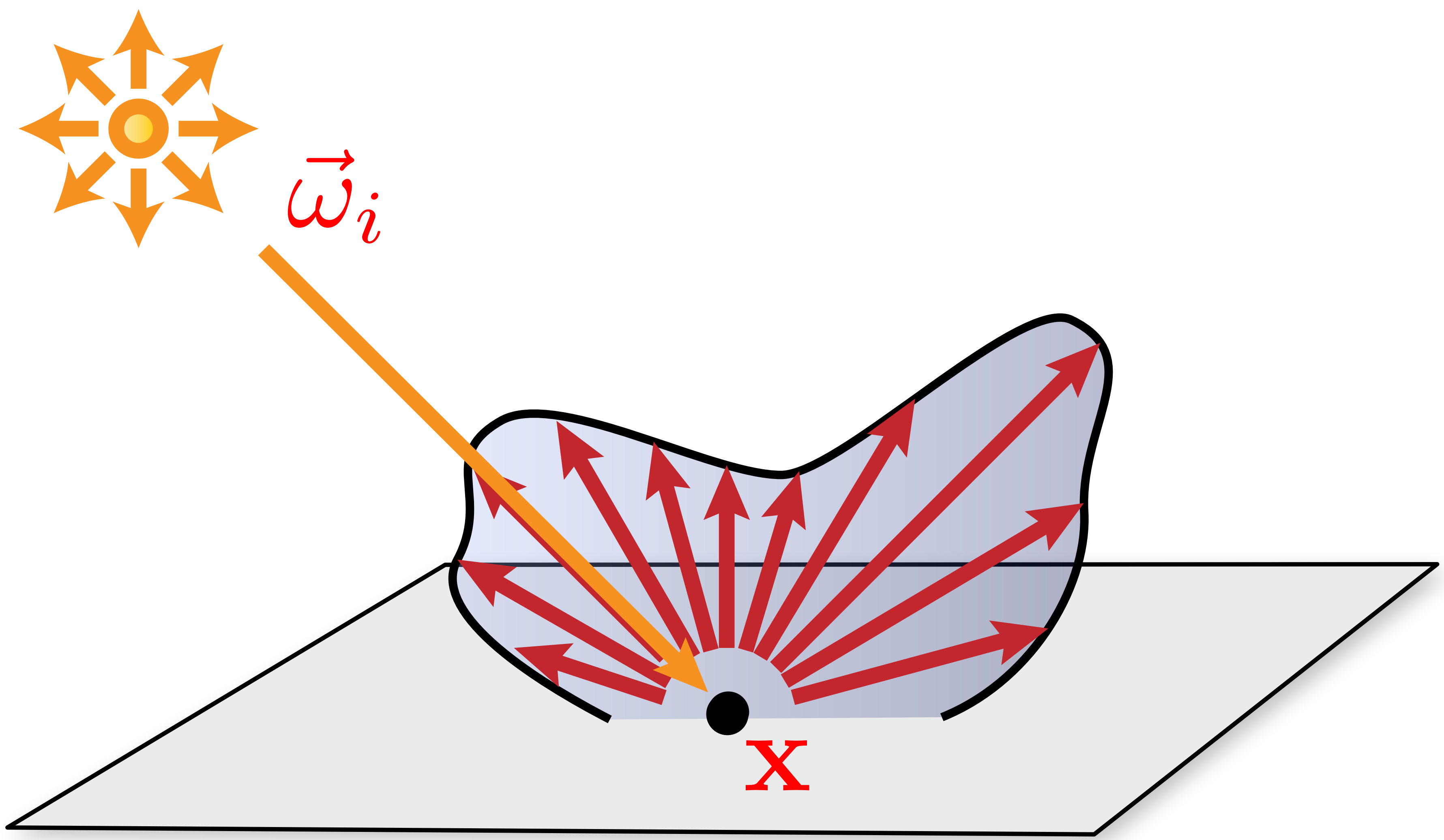
$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r)$$

BRDF — Parameterization



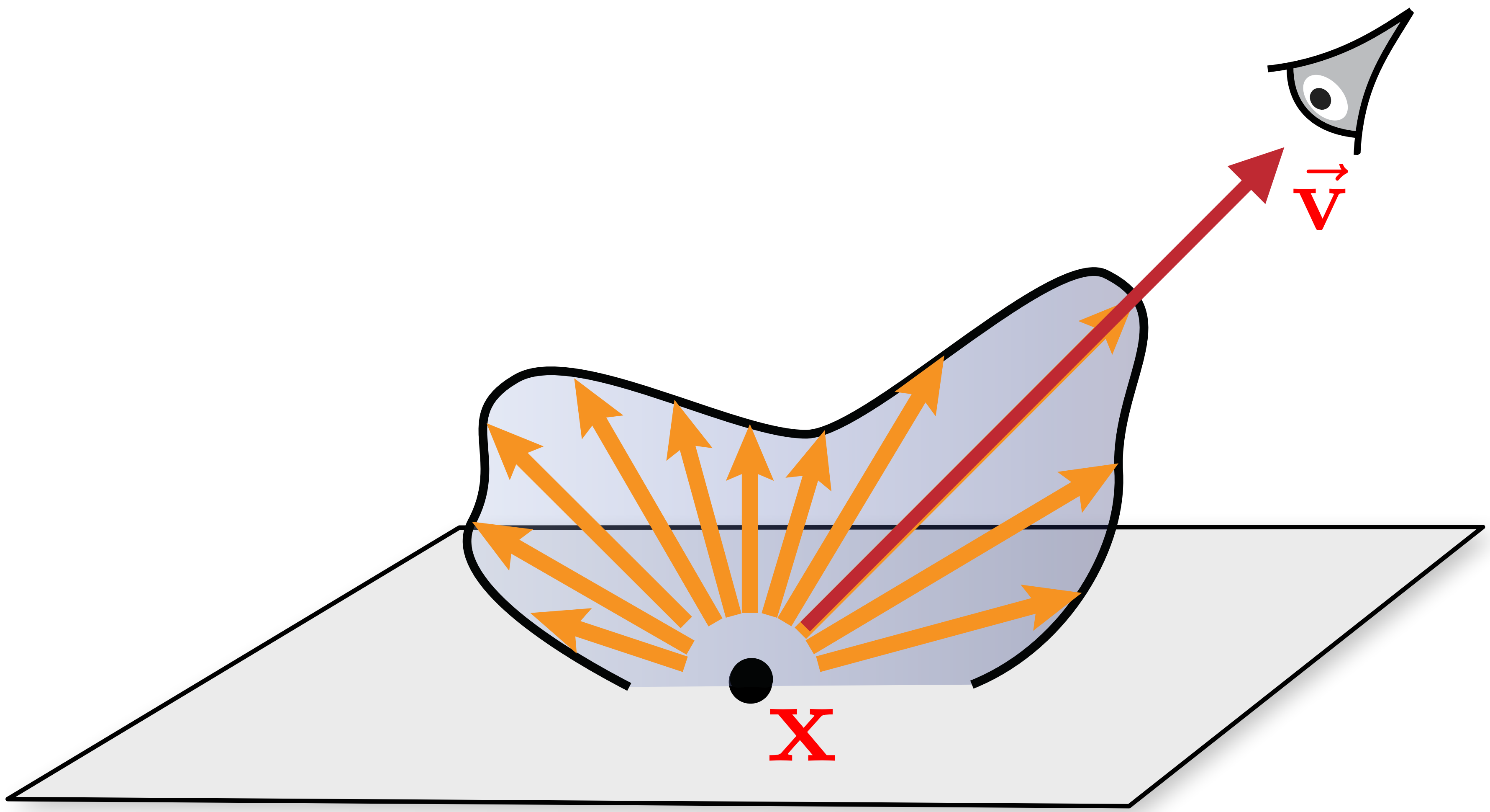
$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r)\Big|_{\vec{\omega}_i = \vec{\mathbf{l}}} \;:\; \mathcal{S}^2 \rightarrow \mathbb{R}$$

BRDF — Parameterization



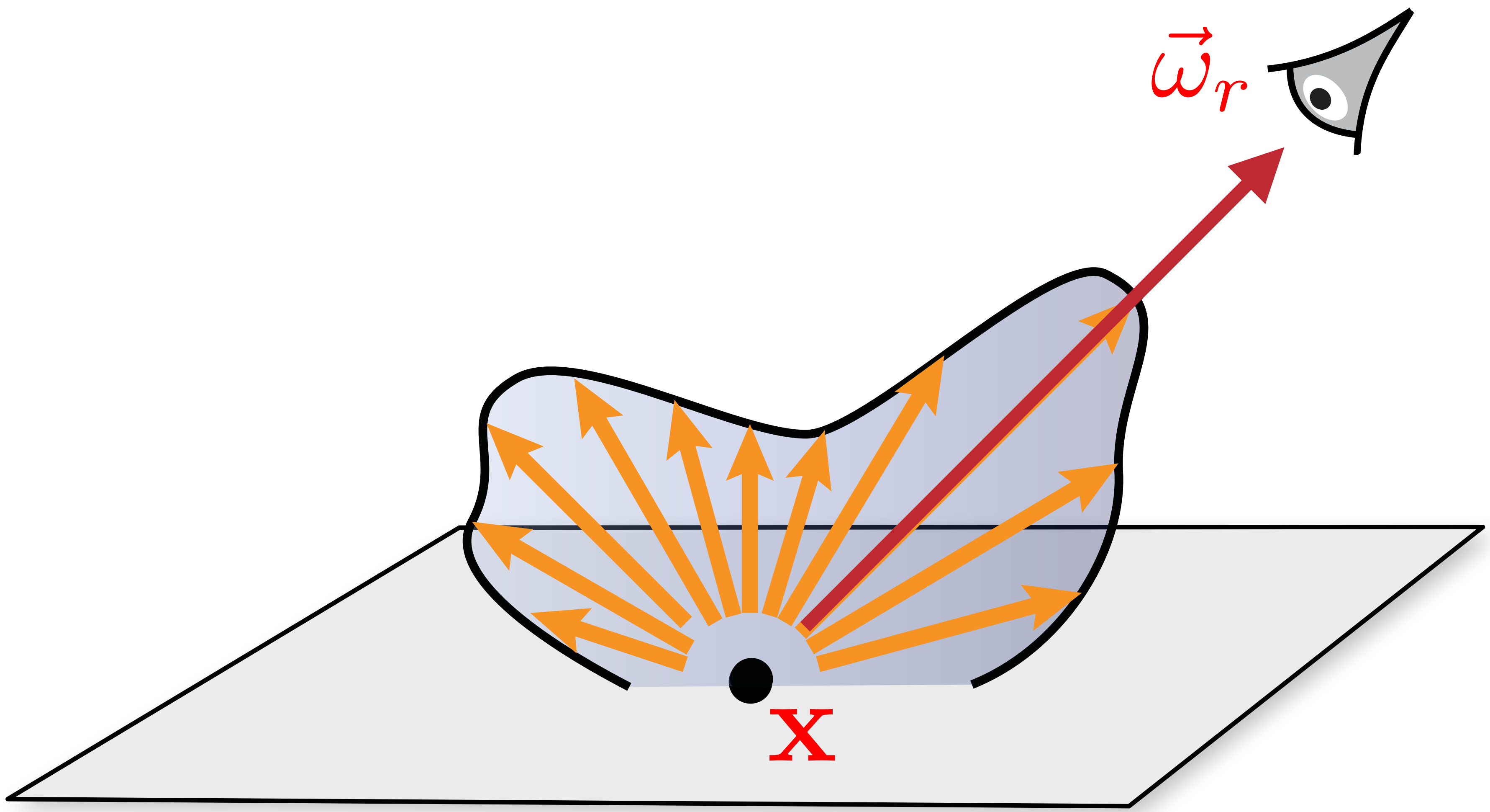
$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) \; : \; \mathcal{S}^2 \rightarrow \mathbb{R}$$

BRDF – Parameterization



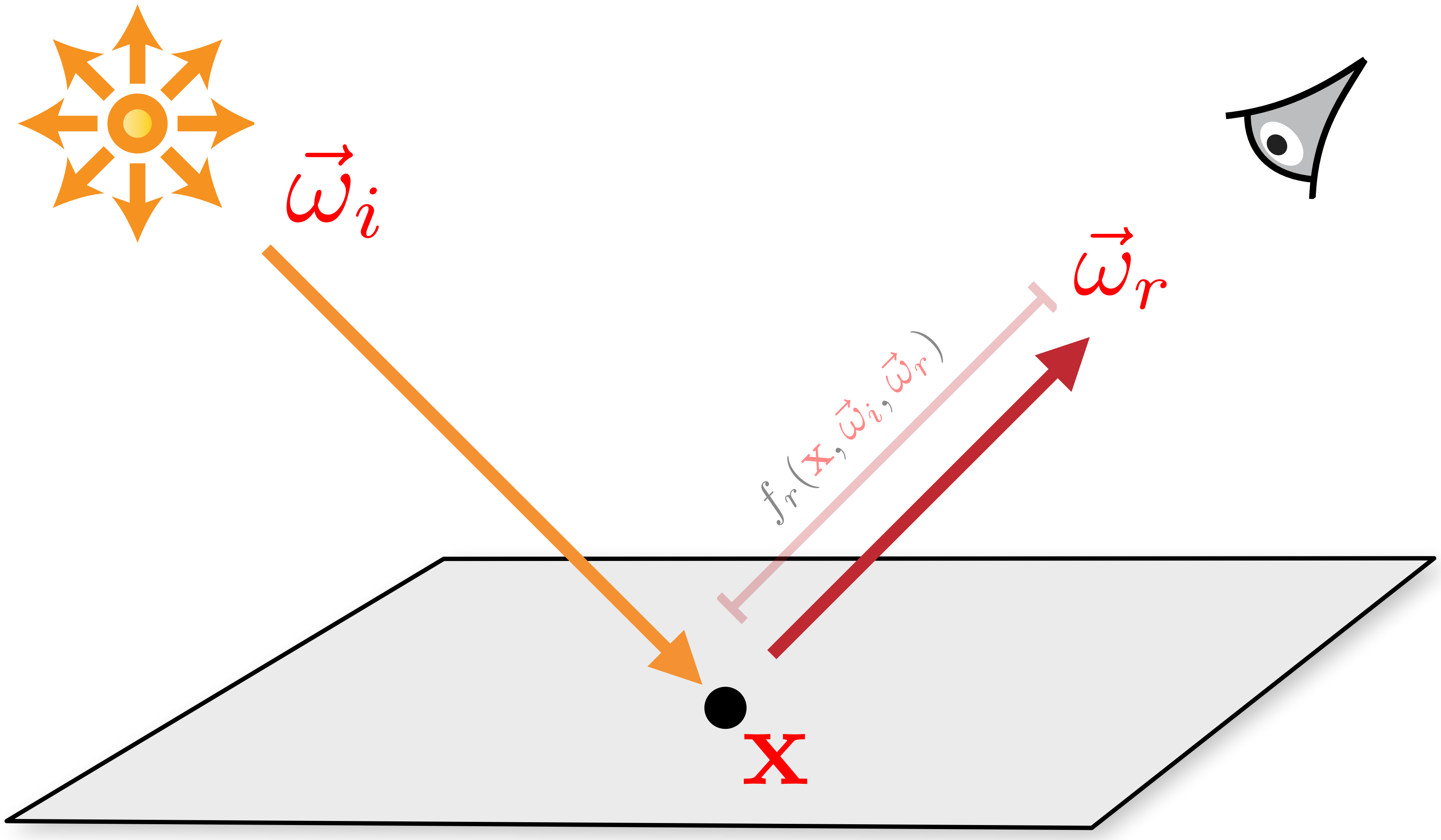
$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r)\Big|_{\vec{\omega}_r = \vec{\mathbf{v}}} \; : \; \mathcal{S}^2 \to \mathbb{R}$$

BRDF – Parameterization



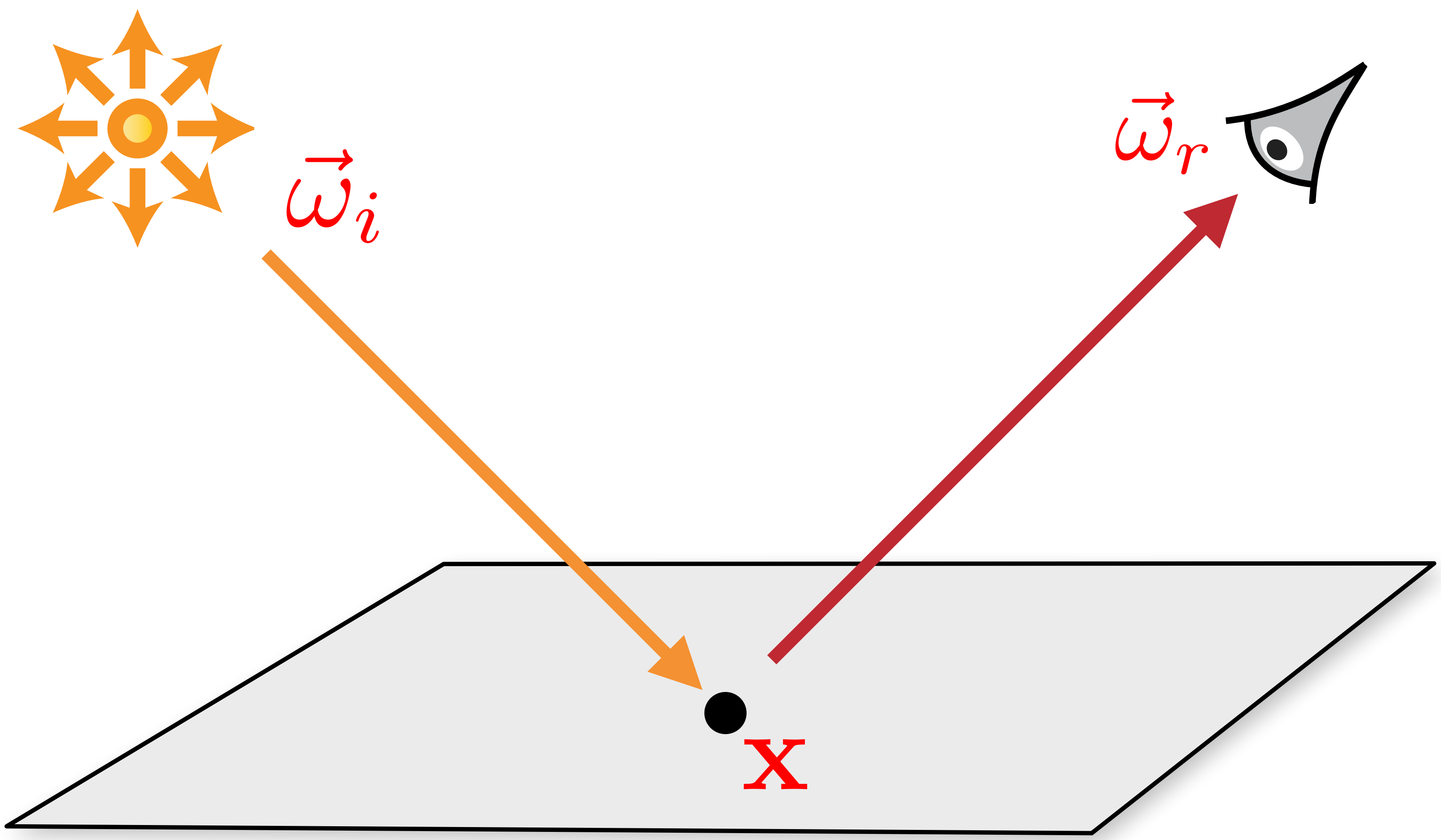
$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) \ : \ \mathcal{S}^2 \to \mathbb{R}$$

BRDF – Parameterization



$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) \in \mathbb{R}$$

BRDF – Parameterization



$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) \in \mathbb{R}$$

BRDFs Properties

Real/physically-plausible BRDFs obey:

- Helmholtz reciprocity

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = f_r(\mathbf{x}, \vec{\omega}_r, \vec{\omega}_i)$$

- Energy conservation

$$\int_{H^2} f_r(\vec{\omega}_i, \vec{\omega}_r) \cos\theta_i \, d\omega_i \leq 1, \quad \forall \, \vec{\omega}_r$$

BRDFs Properties

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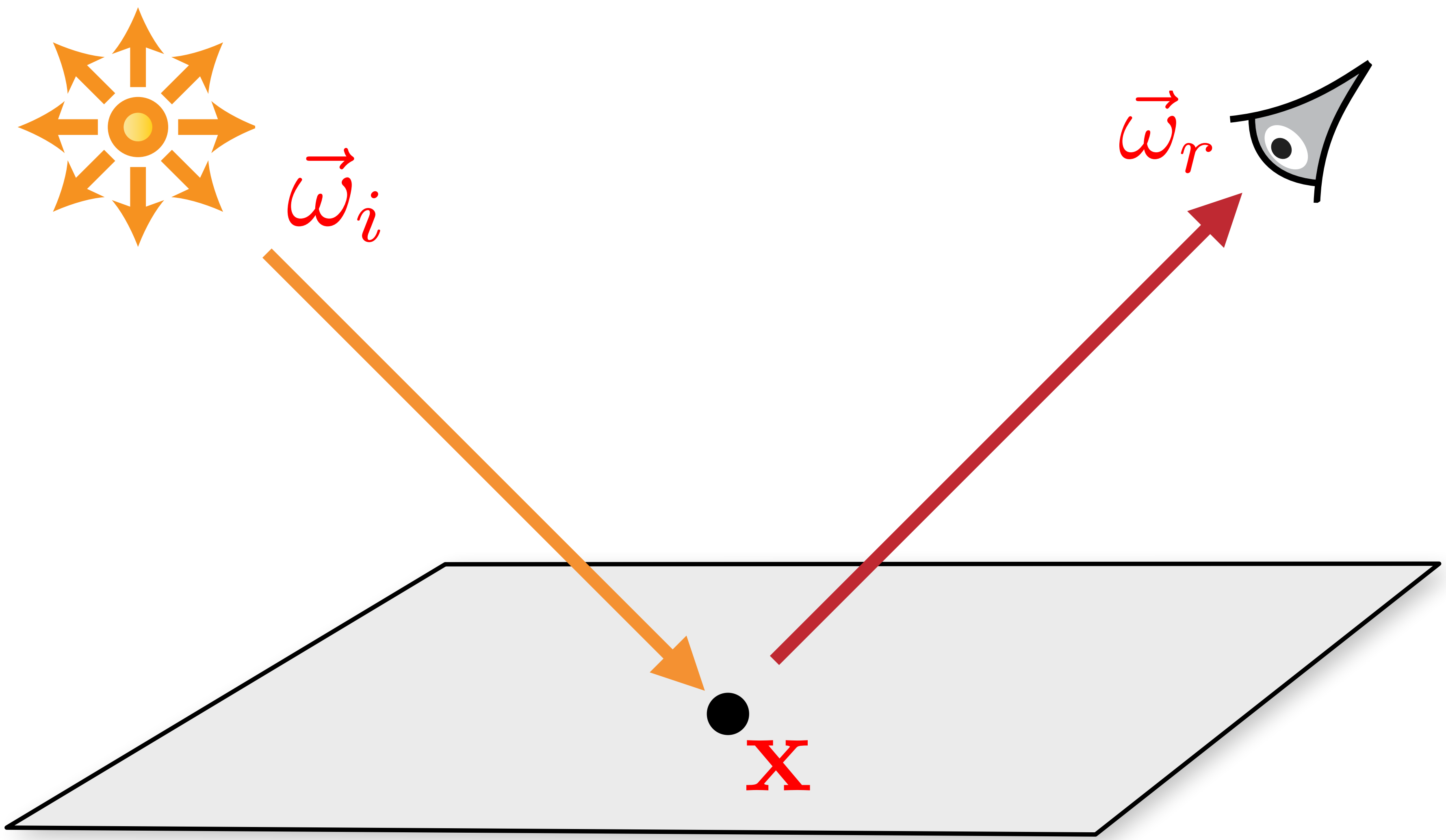
$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = f_r(\mathbf{x}, \vec{\omega}_r, \vec{\omega}_i)$$



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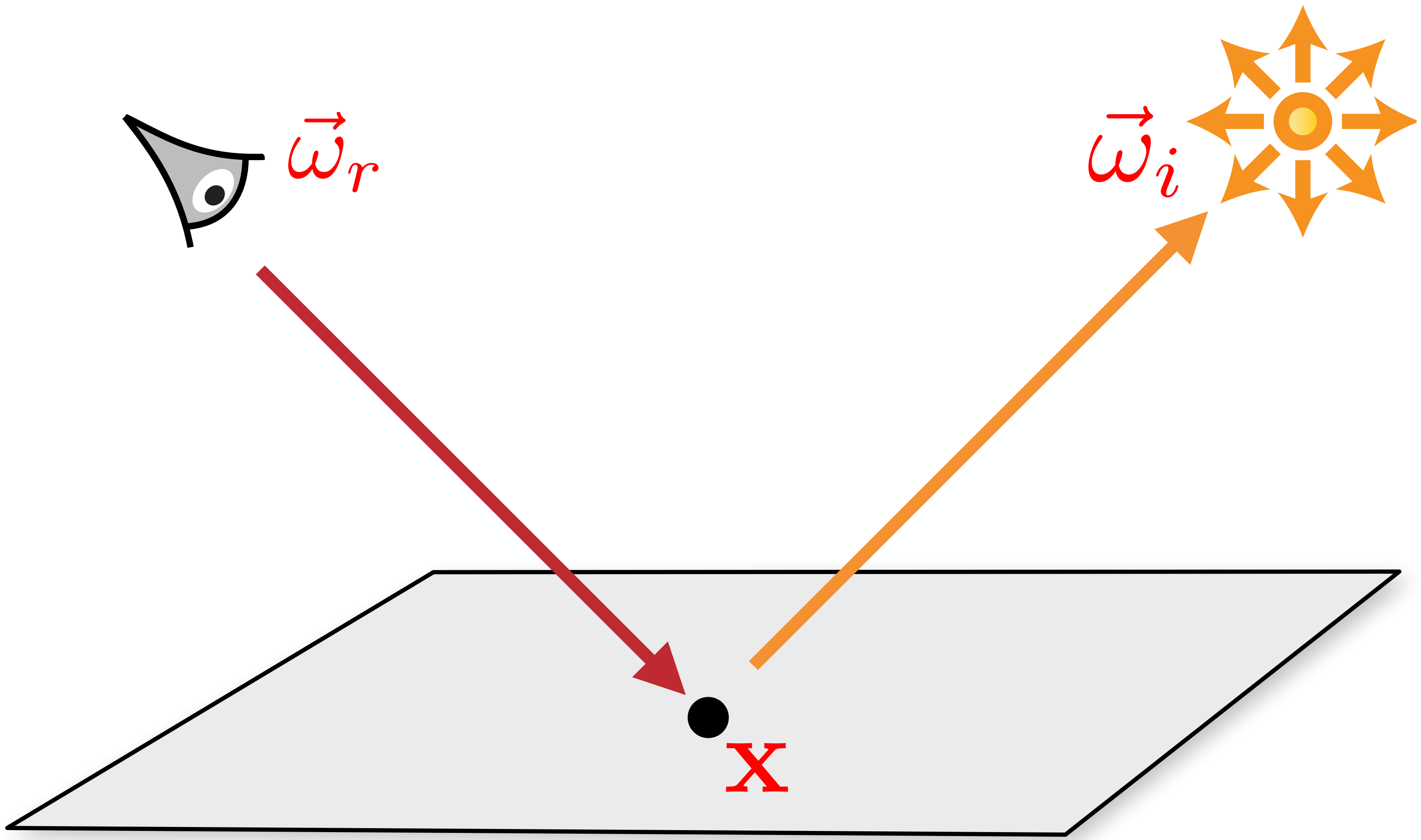

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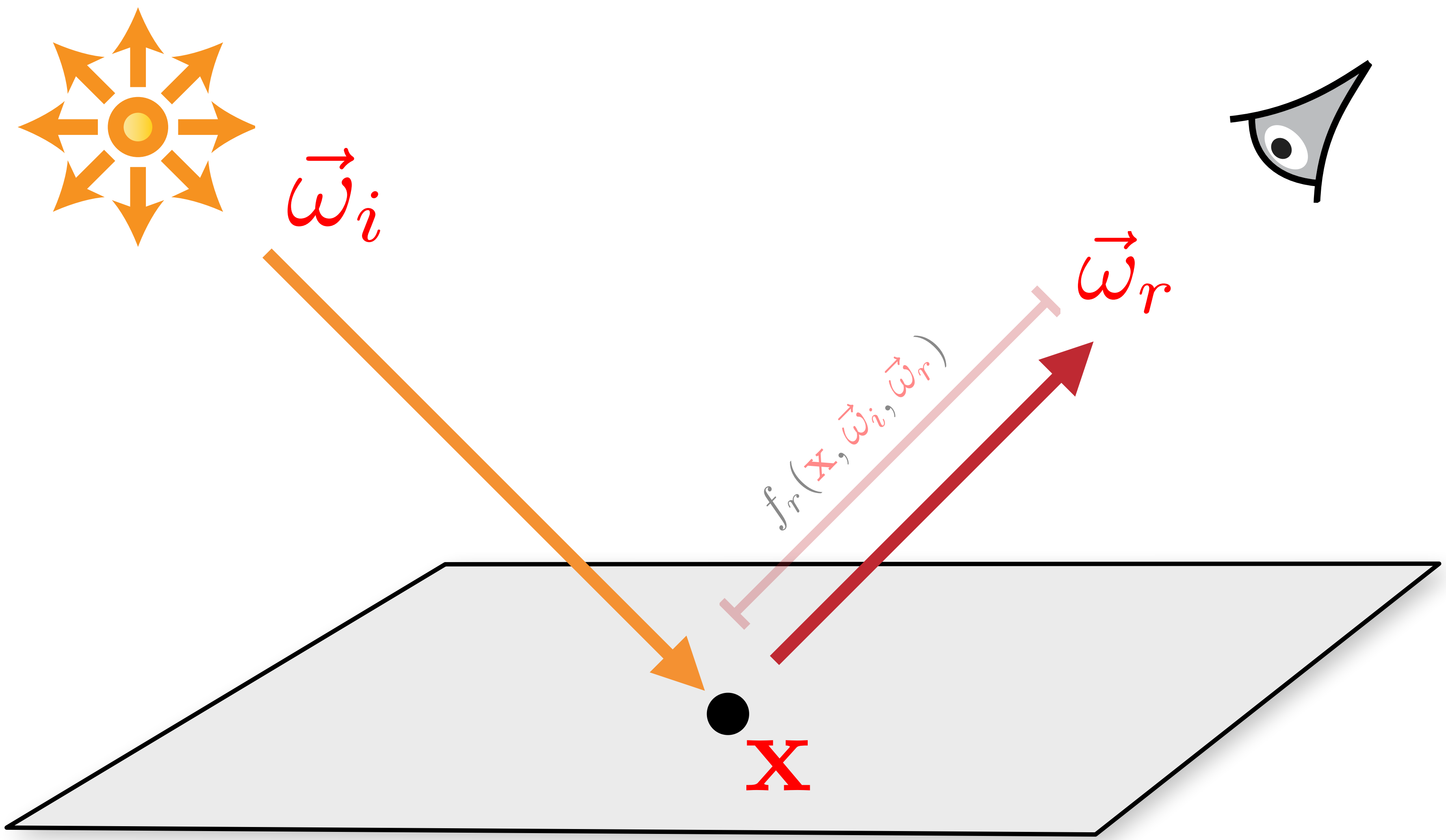

BRDFs Properties

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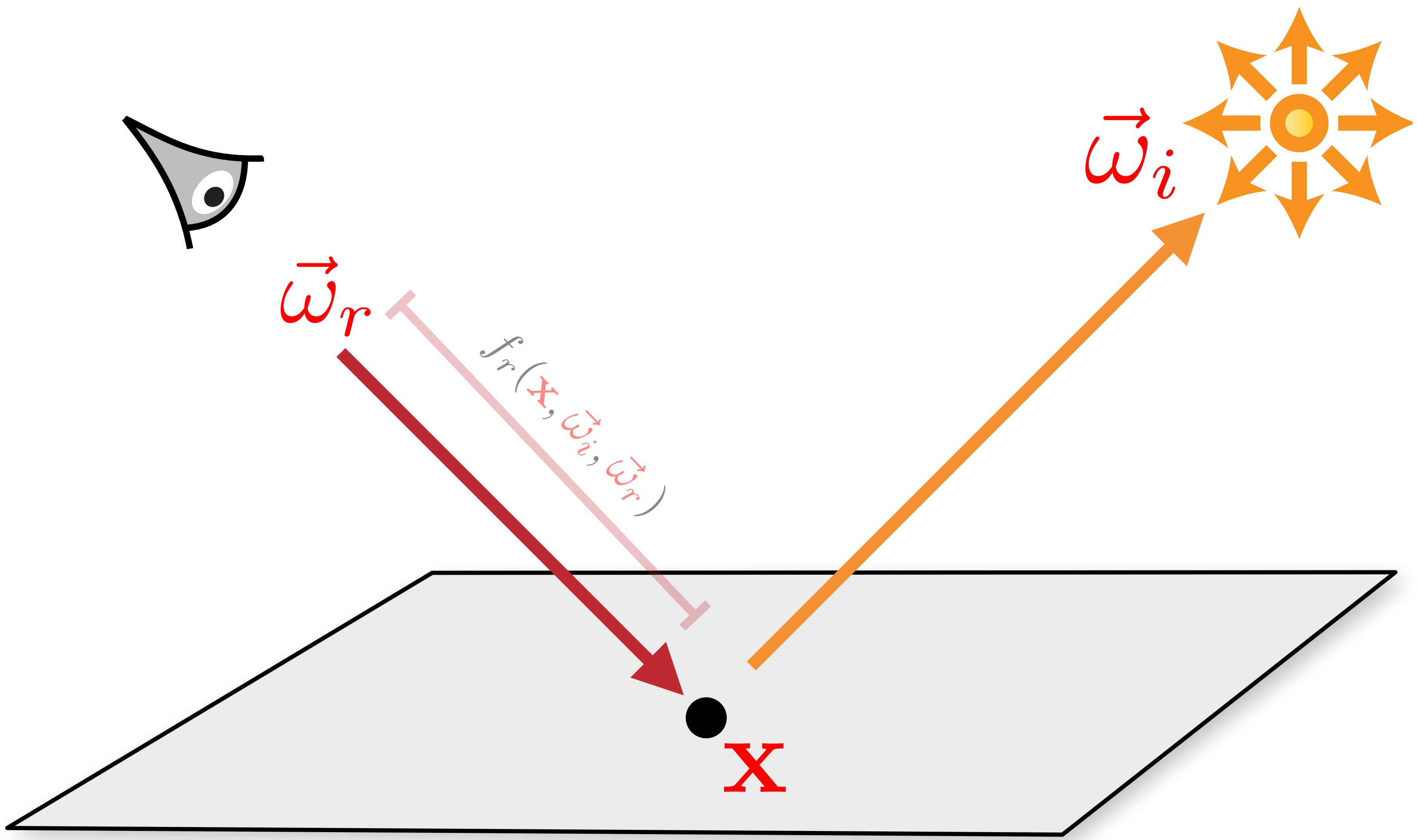
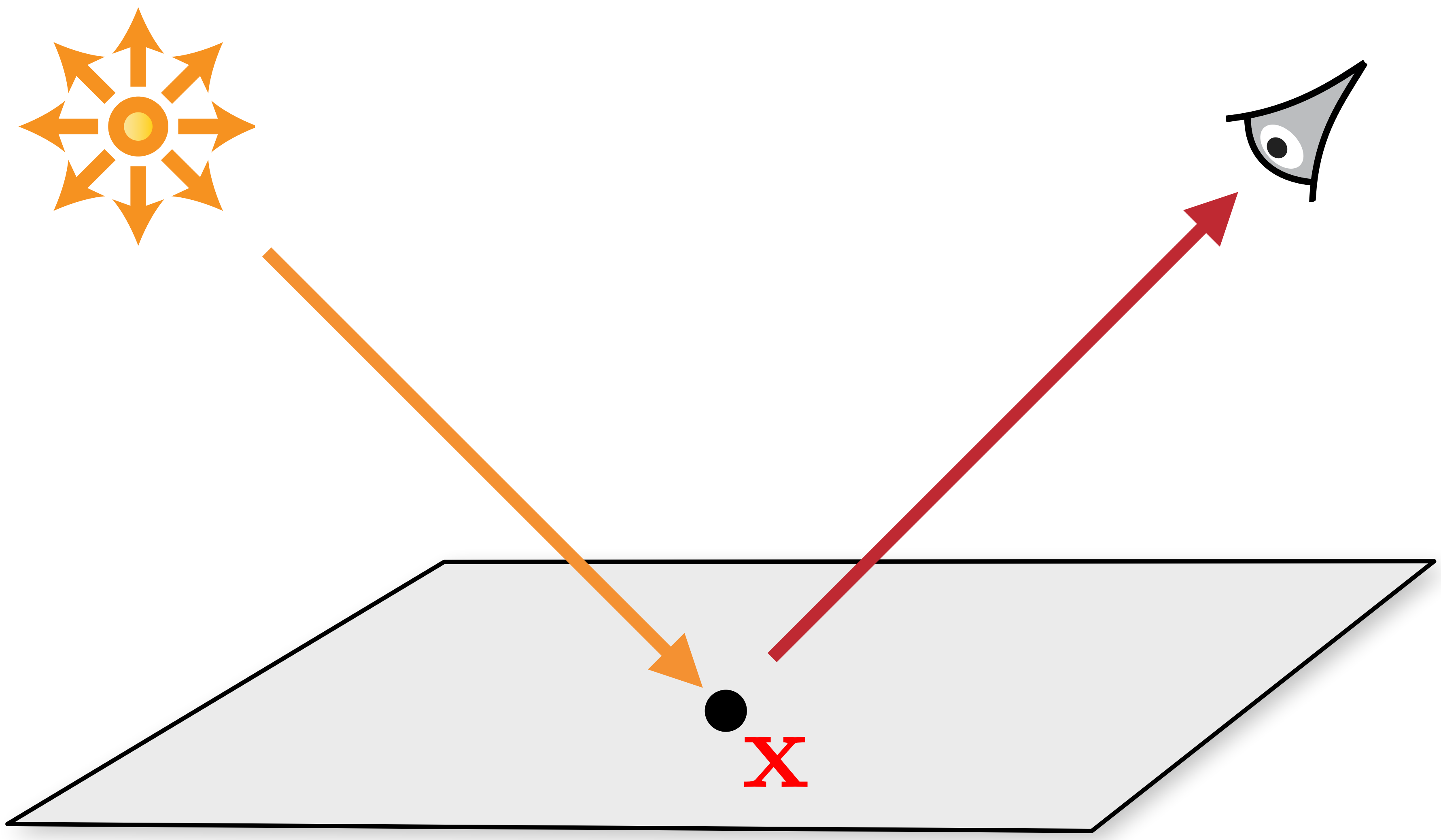


BRDFs Properties

- Helmholtz reciprocity

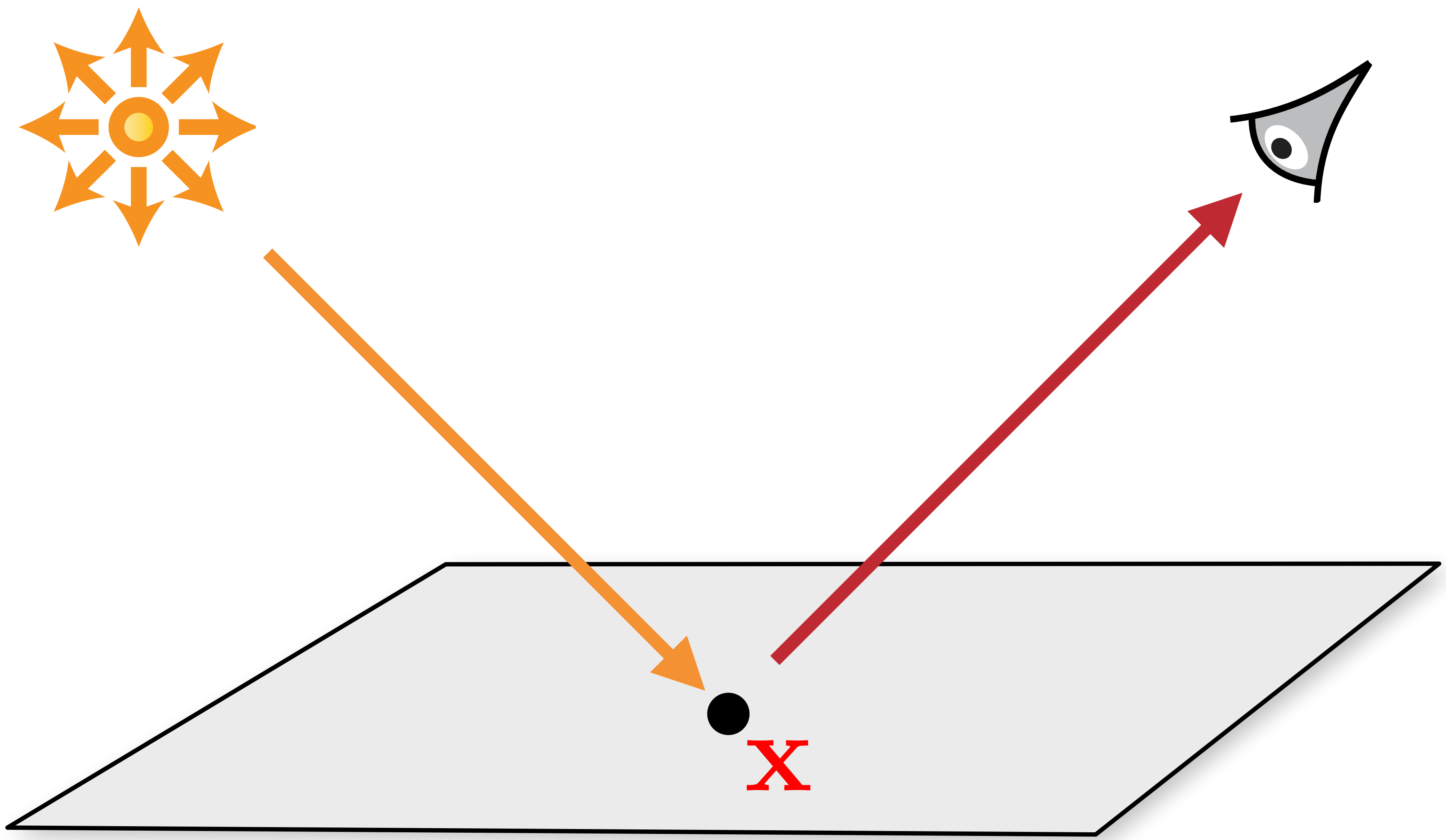
$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = f_r(\mathbf{x}, \vec{\omega}_r, \vec{\omega}_i)\ , \forall \vec{\omega}_i\ , \forall \vec{\omega}_r$$



BRDFs Properties

- Helmholtz reciprocity

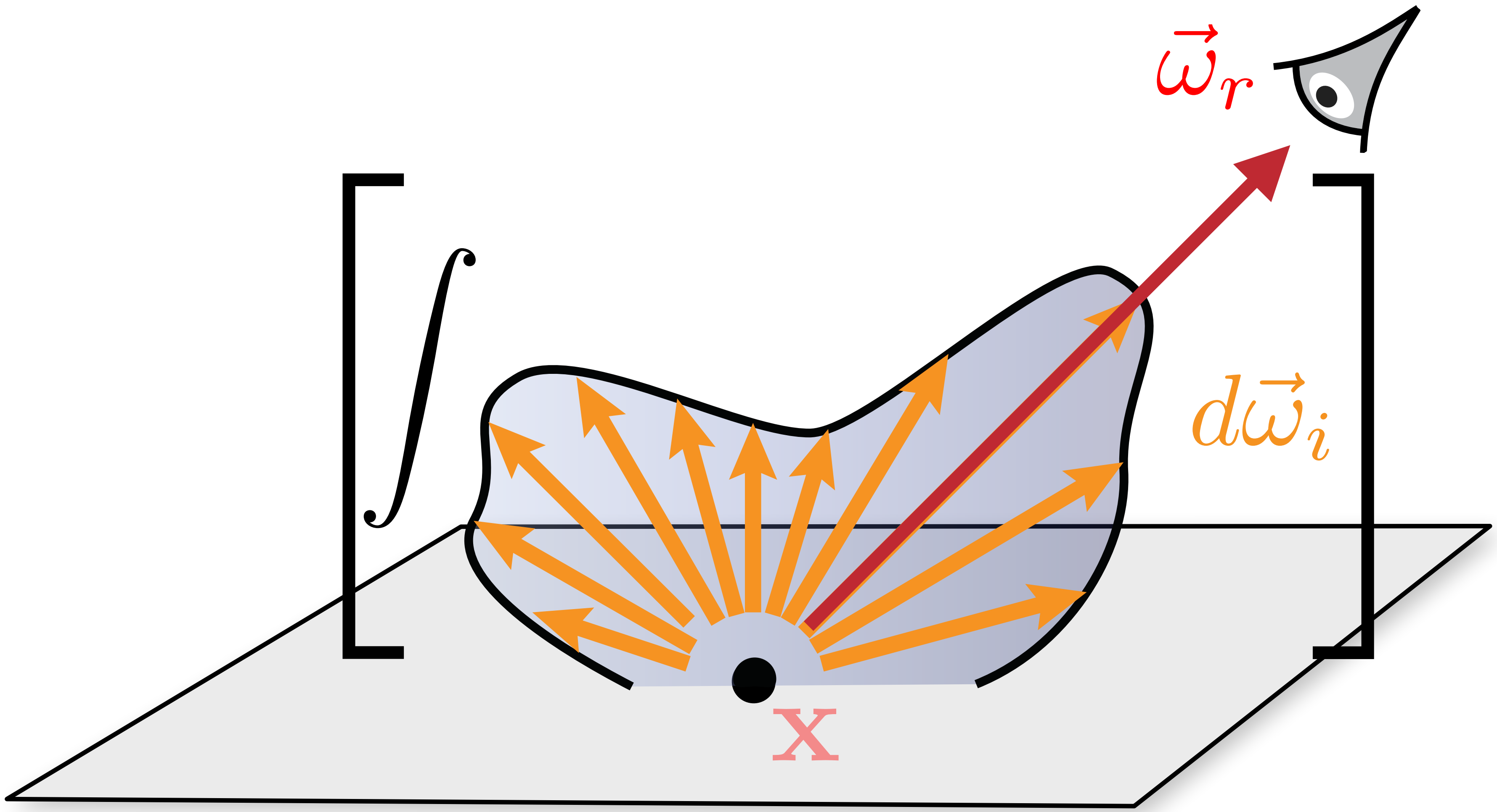
$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = f_r(\mathbf{x}, \vec{\omega}_r, \vec{\omega}_i)$$



BRDFs Properties

- Energy conservation

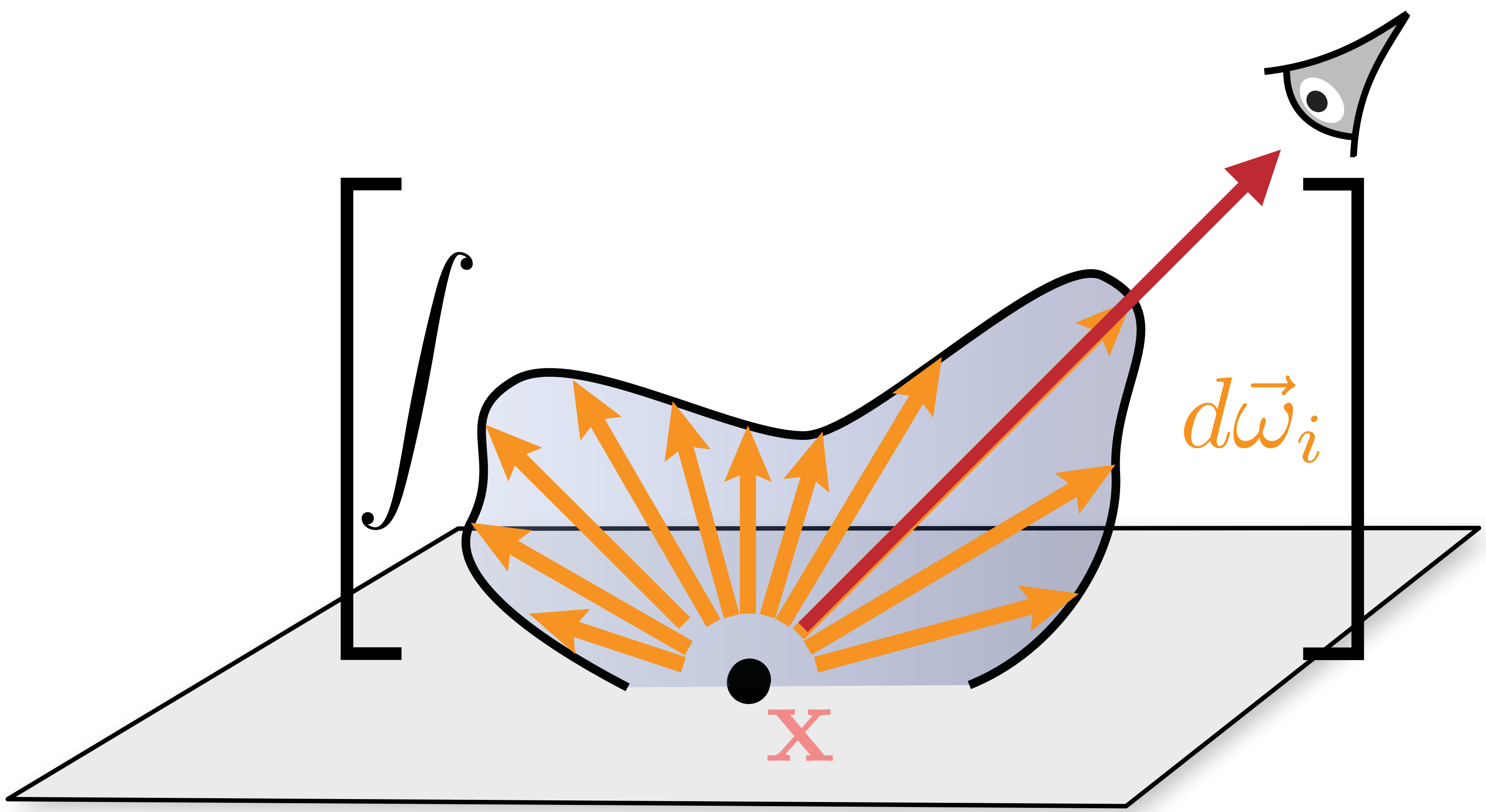
$$\int_{H^2} f_r(\vec{\omega}_i, \vec{\omega}_r) \cos\theta_i \, d\omega_i \leq 1, \quad \forall\, \vec{\omega}_r$$



BRDFs Properties

- Energy conservation

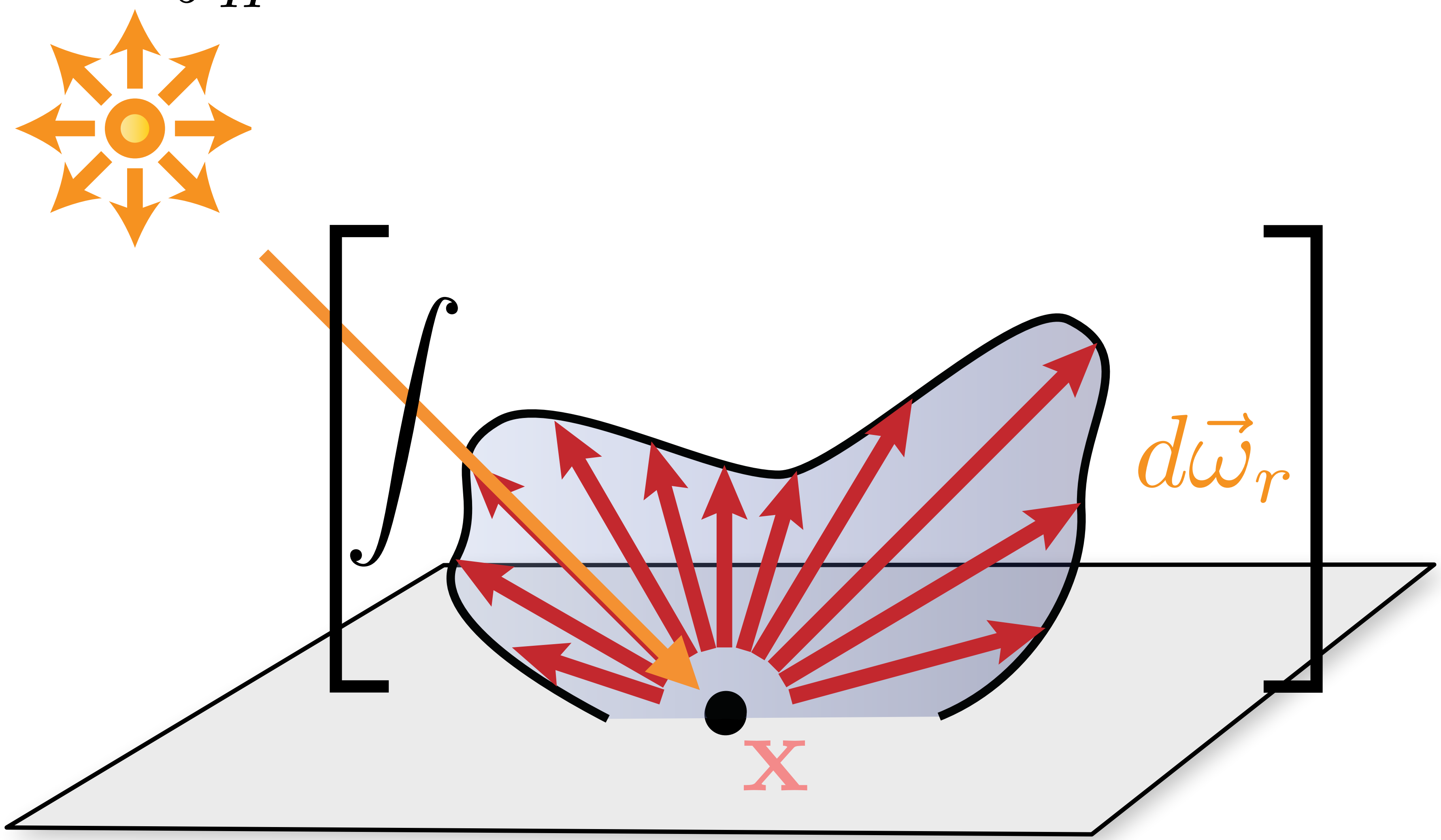
$$\int_{H^2} f_r(\vec{\omega}_i, \vec{\omega}_r) \cos\theta_i \, d\omega_i \leq 1, \quad \forall\, \vec{\omega}_r$$



BRDFs Properties

- Energy conservation (reciprocal)

$$\int_{H^2} f_r(\vec{\omega}_i, \vec{\omega}_r) \cos\theta_r \, d\vec{\omega}_r \leq 1, \quad \forall \, \vec{\omega}_i$$



Reflection Equation

The BRDF relates the incident radiance and (differential) reflected radiance at a point

- from this we can derive the **Reflection Equation**:

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = \frac{dL_r(\mathbf{x}, \vec{\omega}_r)}{L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i}$$

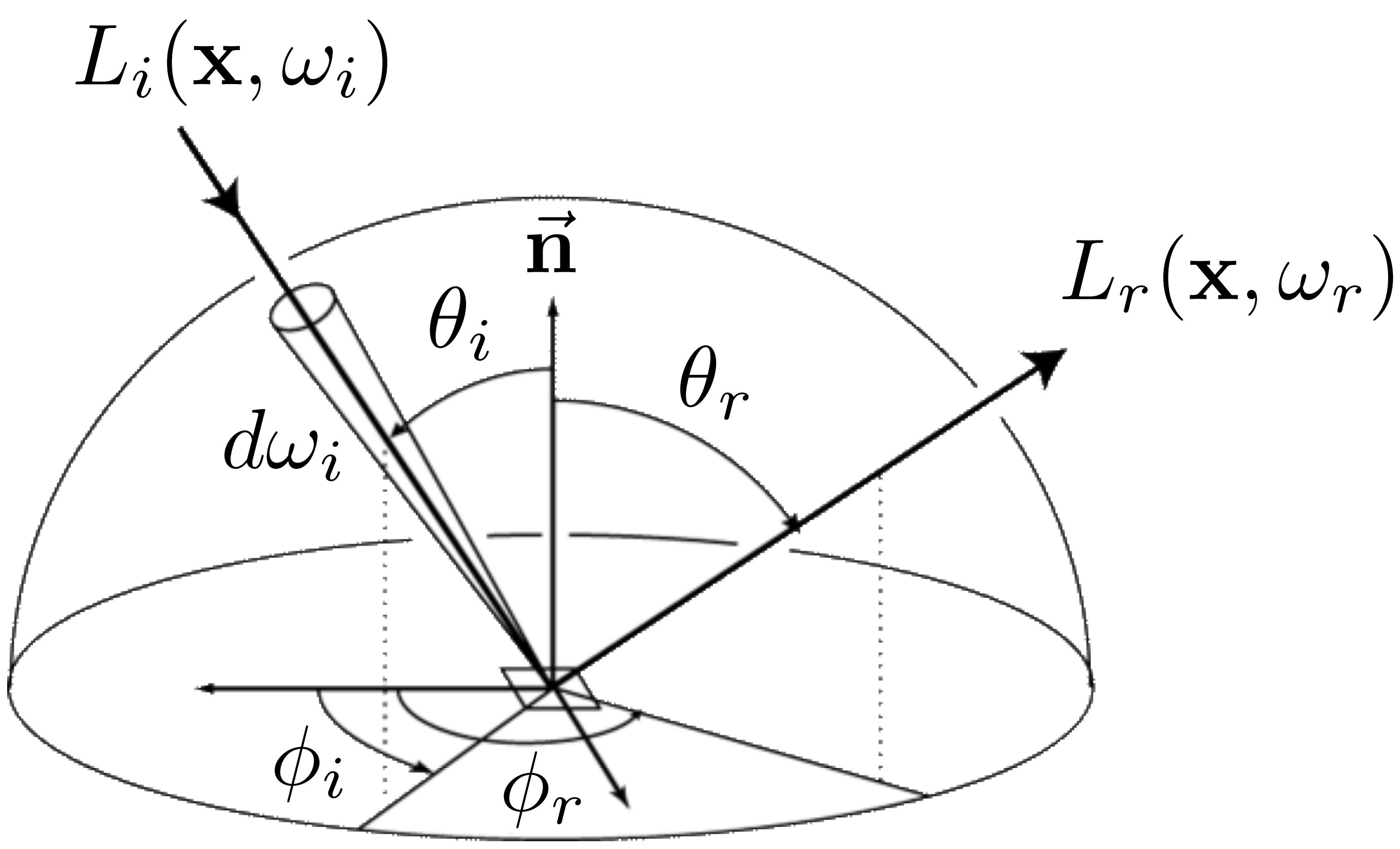
$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i = \frac{dL_r(\mathbf{x}, \vec{\omega}_r)}{d\vec{\omega}_i}$$

$$\int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i = L_r(\mathbf{x}, \vec{\omega}_r)$$

Reflection Equation

The Reflection Equation describes a *local illumination* model

- what's the reflected radiance at a shade point, given the incident illumination arriving from every direction around it



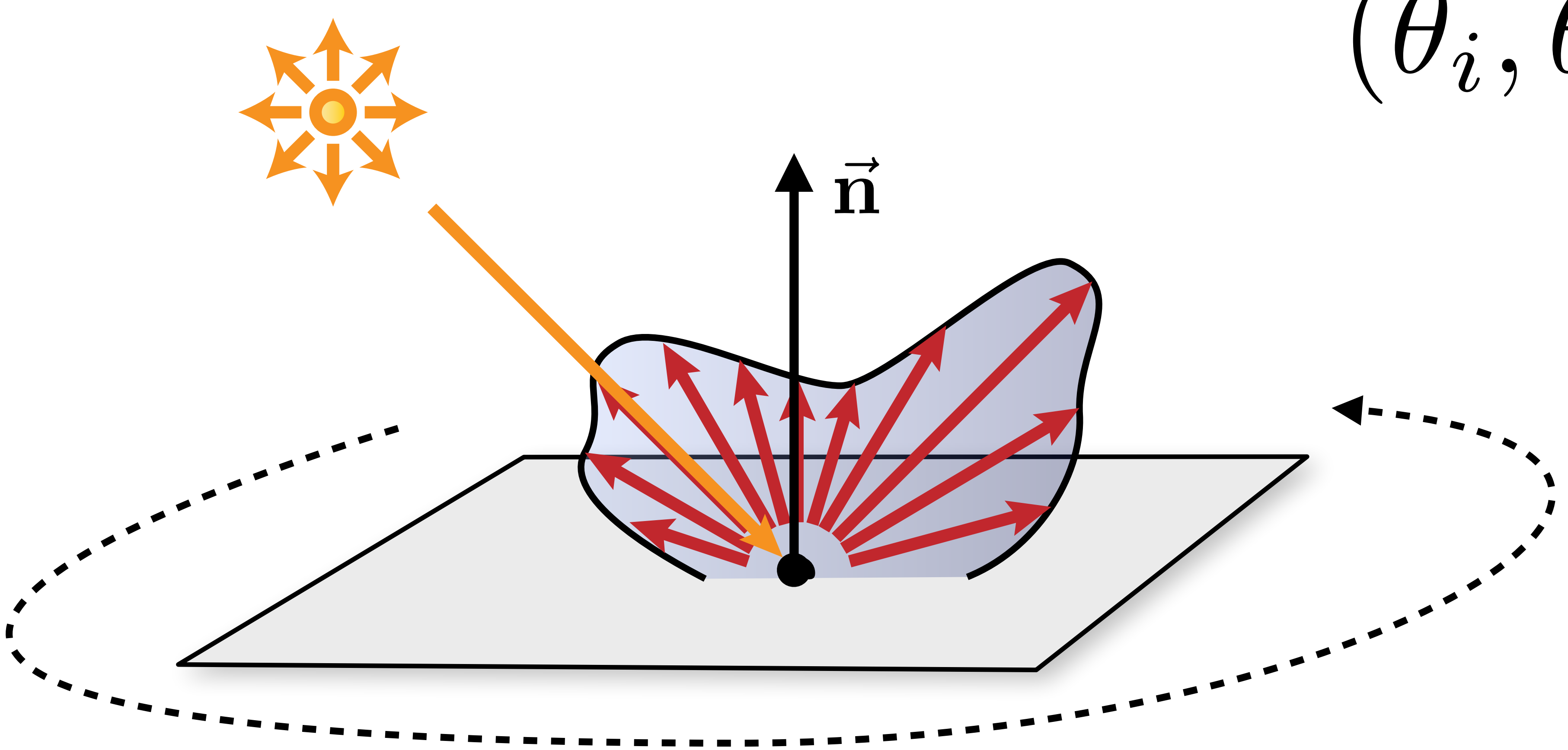
$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

BRDFs Properties

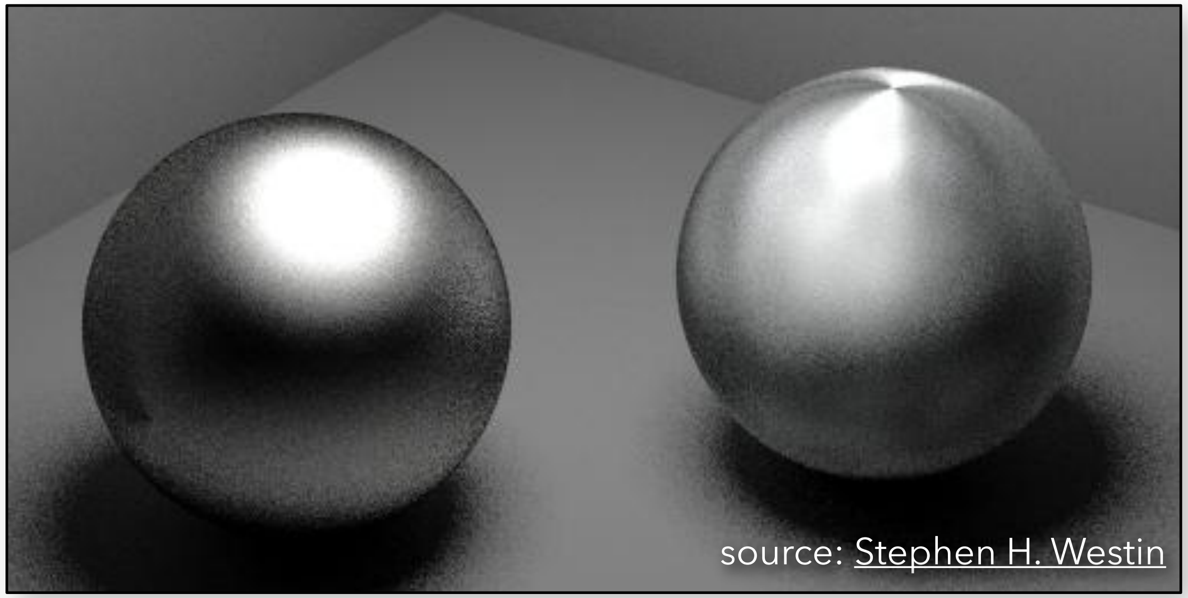
If the BRDF is unchanged as the material is rotated around the shading/surface normal, then the BRDF is ***isotropic***; otherwise, it's ***anisotropic***

- isotropic BRDFs are functions of just 3 variables

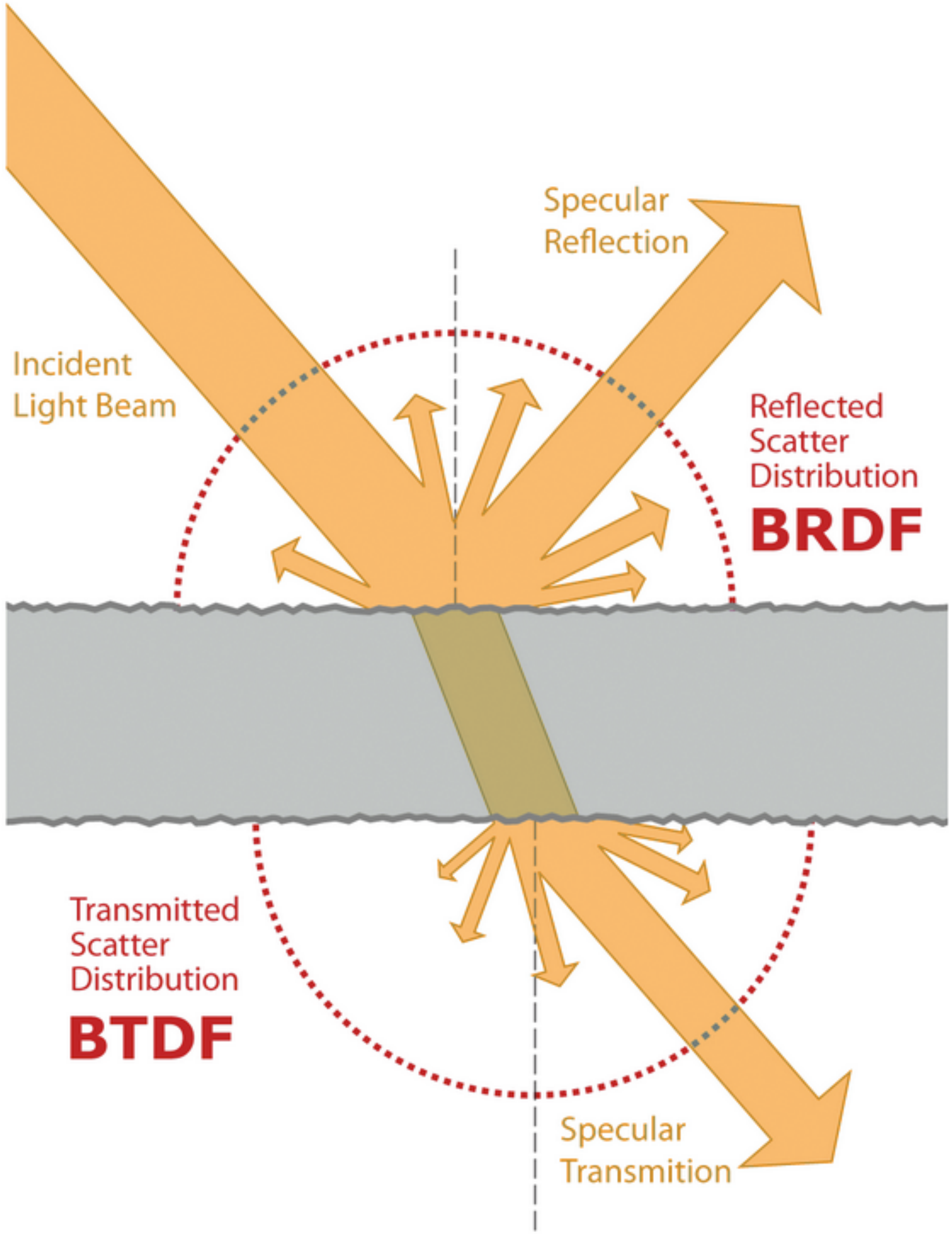
$$(\theta_i, \theta_r, \Delta\phi)$$



Isotropic vs. Anisotropic Reflection

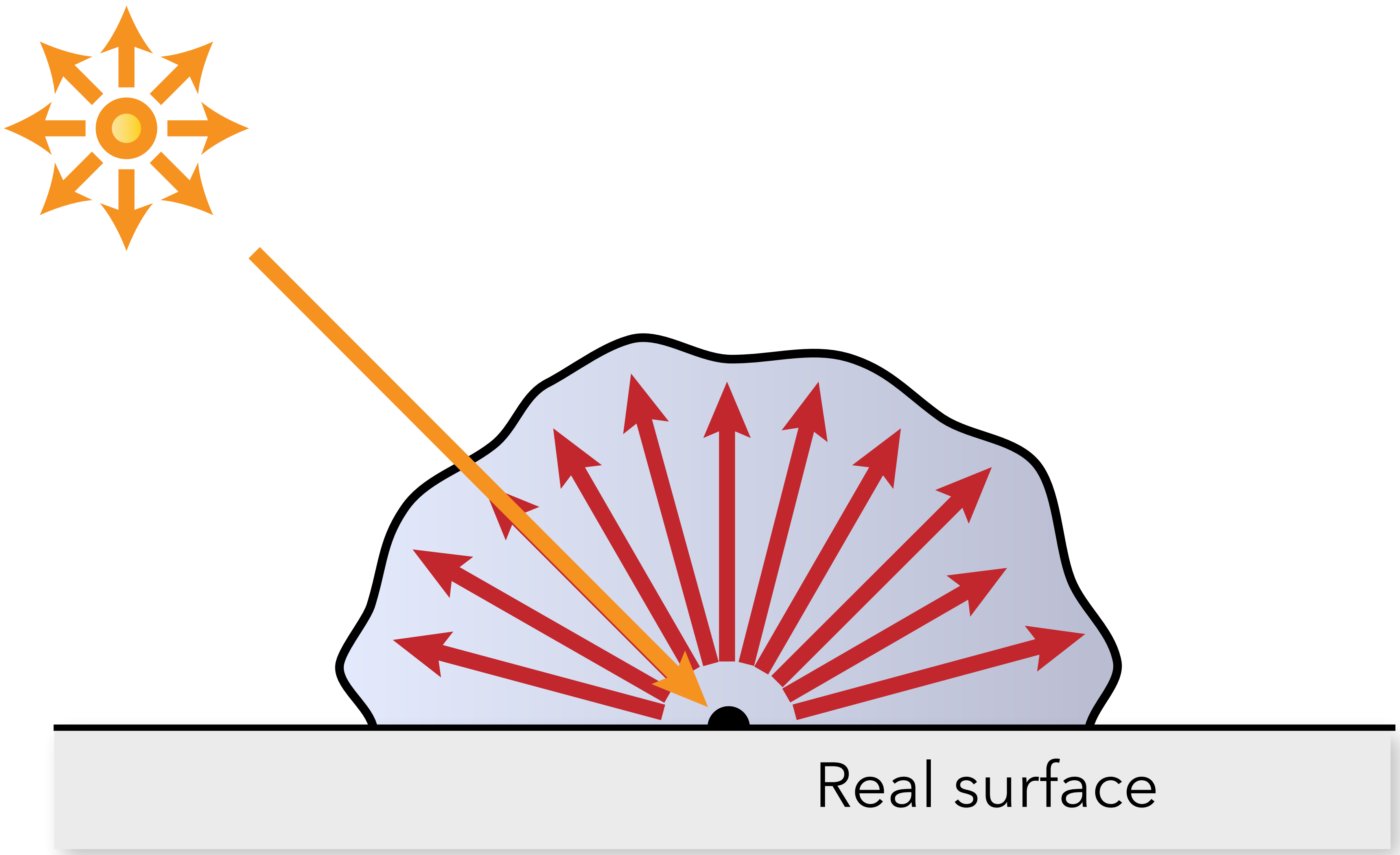


Reflection vs. Refraction

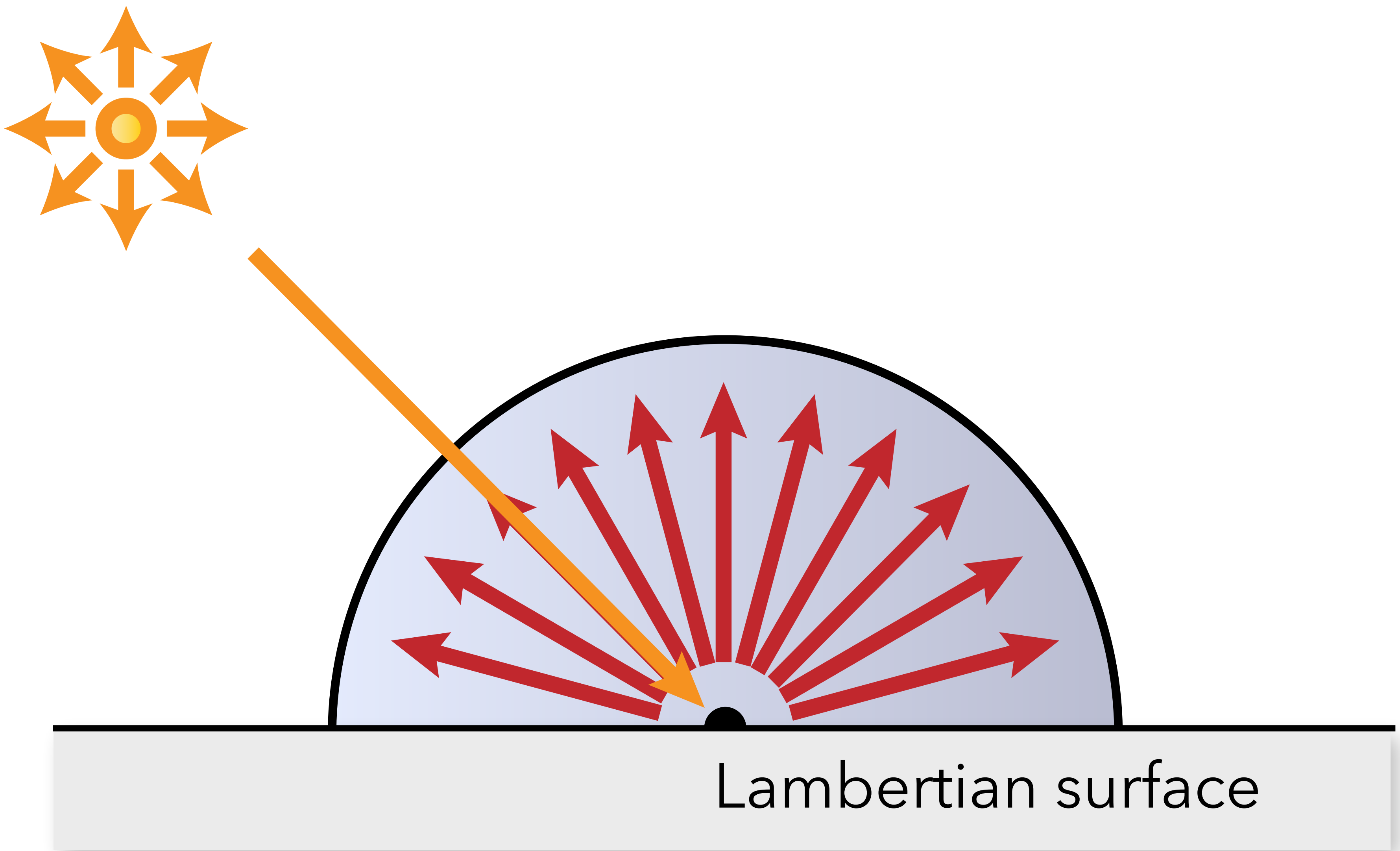


Simple BRDF Models
(diffuse, perfectly specular)

Diffuse Reflection



Lambertian Reflection



Also called ideal diffuse reflection

Ideal Diffuse BRDF

For Lambertian reflection, the BRDF is a constant:

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

$$L_r(\mathbf{x}) = f_r \int_{H^2} L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

$$L_r(\mathbf{x}) = f_r \, E(\mathbf{x})$$

If *all* incoming light is reflected:

$$E(\mathbf{x}) = B(\mathbf{x})$$

$$E(\mathbf{x}) = \int_{H^2} L_r(\mathbf{x}, \vec{\omega}) \cos\theta \, d\vec{\omega}$$

$$E(\mathbf{x}) = L_r(\mathbf{x}) \int_{H^2} \cos\theta \, d\vec{\omega}$$

$$E(\mathbf{x}) = L_r(\mathbf{x}) \, \pi$$

$$f_r = \frac{1}{\pi}$$

Diffuse BRDF

For Lambertian reflection, the BRDF is a constant:

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

$$L_r(\mathbf{x}) = \frac{\rho}{\pi} \int_{H^2} L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

ρ : Diffuse reflectance (albedo) [0..1)

Why Divide by π?

Real/physically-plausible BRDFs obey:

- Helmholtz reciprocity $f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = f_r(\mathbf{x}, \vec{\omega}_r, \vec{\omega}_i)$

- Energy conservation

$$\int_{H^2} f_r(\vec{\omega}_i, \vec{\omega}_r) \cos\theta_i \, d\omega_i \leq 1, \quad \forall\, \vec{\omega}_r$$

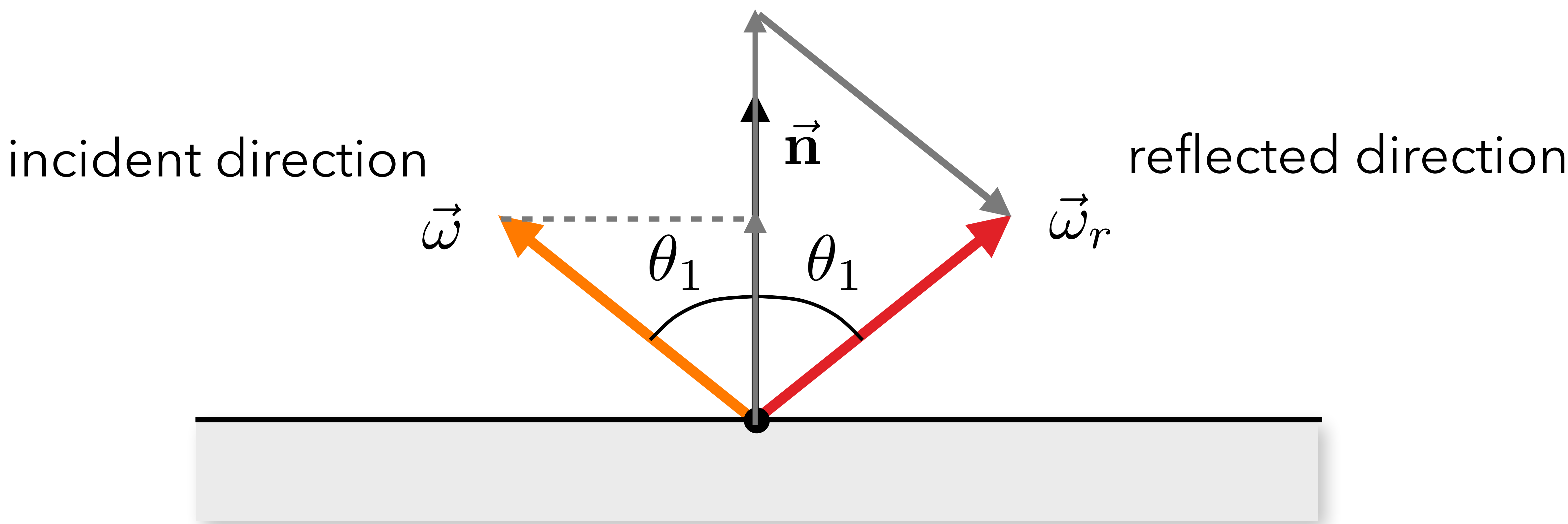
$$= \int_{H^2} (\alpha\,\rho)\ \cos\theta_i \, d\omega_i \leq 1, \quad \forall\, \vec{\omega}_r$$

$$= (\alpha\,\rho) \int_{H^2} \cos\theta_i \, d\omega_i \leq 1$$

$$= (\alpha\,\rho)\,\pi \leq 1$$

Lambertian Reflection

Ideal Specular Reflection



$$\vec{\omega}_r = 2\vec{\mathbf{n}}\cos\theta - \vec{\omega} = 2(\vec{\mathbf{n}}\cdot\vec{\omega})\vec{\mathbf{n}} - \vec{\omega}$$

Ideal Specular Reflection

Ideal Specular Reflection BRDF

What is the BRDF for (perfect) specular reflection?

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

Ideal Specular Reflection BRDF

What is the BRDF for (perfect) specular reflection?

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

Dirac delta

Fresnel reflection

mirroring function
(flips about normal)

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = F_r(\vec{\omega}_i) \frac{\delta(\vec{\omega}_r - R(\vec{\omega}_i, \vec{\mathbf{n}}))}{\cos\theta_i}$$

to cancel the cosine term
in the reflection equation
(Fresnel eqs. account for it)

Ideal Specular Refraction BRDF

What about perfect specular refraction?

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{S^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

Dirac delta

refraction function

Fresnel reflection

$$f_t(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = \frac{\eta_1^2}{\eta_2^2}(1 - F_r(\vec{\omega}_i)) \frac{\delta(\vec{\omega}_r - T(\vec{\omega}_i, \vec{\mathbf{n}}))}{\cos\theta_i}$$

to cancel the cosine term
in the reflection equation
(Fresnel eqs. account for it)

Rendering with <u>Delta</u> BRDFs

Specular BRDFs contain delta functions

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = F_r(\vec{\omega}_i) \frac{\delta(\vec{\omega}_r - R(\vec{\omega}_i, \vec{\mathbf{n}}))}{\cos\theta_i}$$

The integral simplifies to an evaluation of the integrand:

$$L_r(\mathbf{x}, \vec{\omega}_r) = F_r(R(\vec{\omega}_i, \vec{\mathbf{n}}))\, L_i(\mathbf{x}, R(\vec{\omega}_i, \vec{\mathbf{n}}))$$

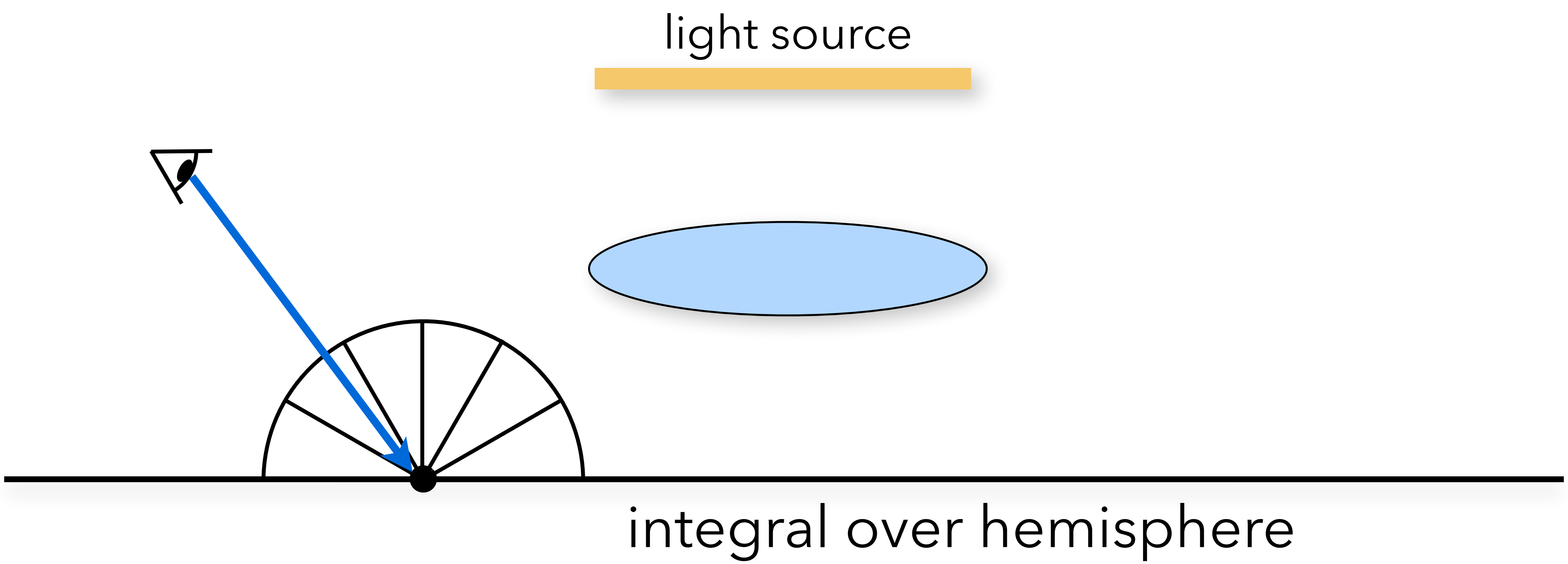
- specular interactions are handled explicitly

From Reflection to Direct Lighting

Direct illumination (a.k.a. direct lighting) is a special-case of reflection where incident lighting is due **only to emitters**

$$L_r(\mathbf{x}, \vec{\omega}_r) = \boxed{\int_{H^2}} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

$$L_o(\mathbf{x}, \vec{\omega}_o) = \boxed{\int_{H^2}} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_o) L_e(\mathbf{x}, \vec{\omega}_i)\, V(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$



Rough Surfaces

Many surfaces are not perfectly smooth

- Phong came up with an empirical model to account for this
- Simple and fast to evaluate

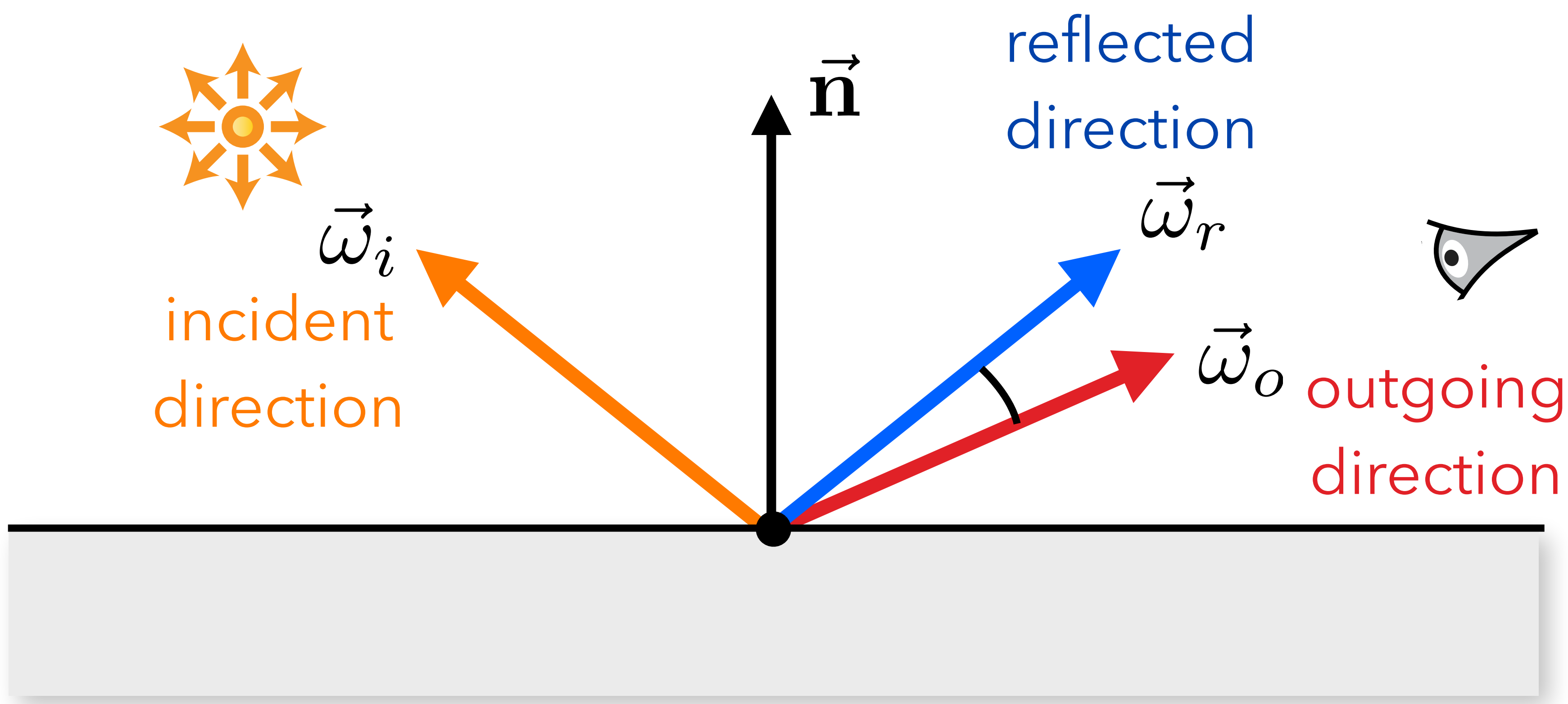
"In trying to improve the quality of the synthetic images, we do not expect to be able to display the object exactly as it would appear in reality, with texture, overcast shadows, etc. We hope only to **display an image that approximates the real object closely** enough to provide a certain degree of realism." – Bui Tuong Phong, 1975

Normalized Phong BRDF

Normalized exponentiated cosine lobe:

$$f_r(\vec{\omega}_o, \vec{\omega}_i) = \frac{e+2}{2\pi}(\vec{\omega}_r \cdot \vec{\omega}_o)^e$$

$$\vec{\omega}_r = (2\vec{\mathbf{n}}(\vec{\mathbf{n}} \cdot \vec{\omega}_i) - \vec{\omega}_i)$$



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Normalized Phong

Normalized exponentiated cosine lobe:

$$f_r(\vec{\omega}_o, \vec{\omega}_i) = \frac{e+2}{2\pi}(\vec{\omega}_r \cdot \vec{\omega}_o)^e$$

$$\vec{\omega}_r = (2\vec{\mathbf{n}}(\vec{\mathbf{n}} \cdot \vec{\omega}_i) - \vec{\omega}_i)$$

Interpretations:

- blur reflection rays in a cone about mirror direction
- perfect mirror reflection of a blurred light

Normalized Phong

- Interpretation
  - blur reflection rays in a cone about mirror direction
  - perfect mirror reflection of a blurred light

Next

Monte Carlo Integration

- applying statistics to solve integration problems
- rendering images using integration

Diffuse Shading – Complex Lighting

Diffuse Shading – Complex Lighting