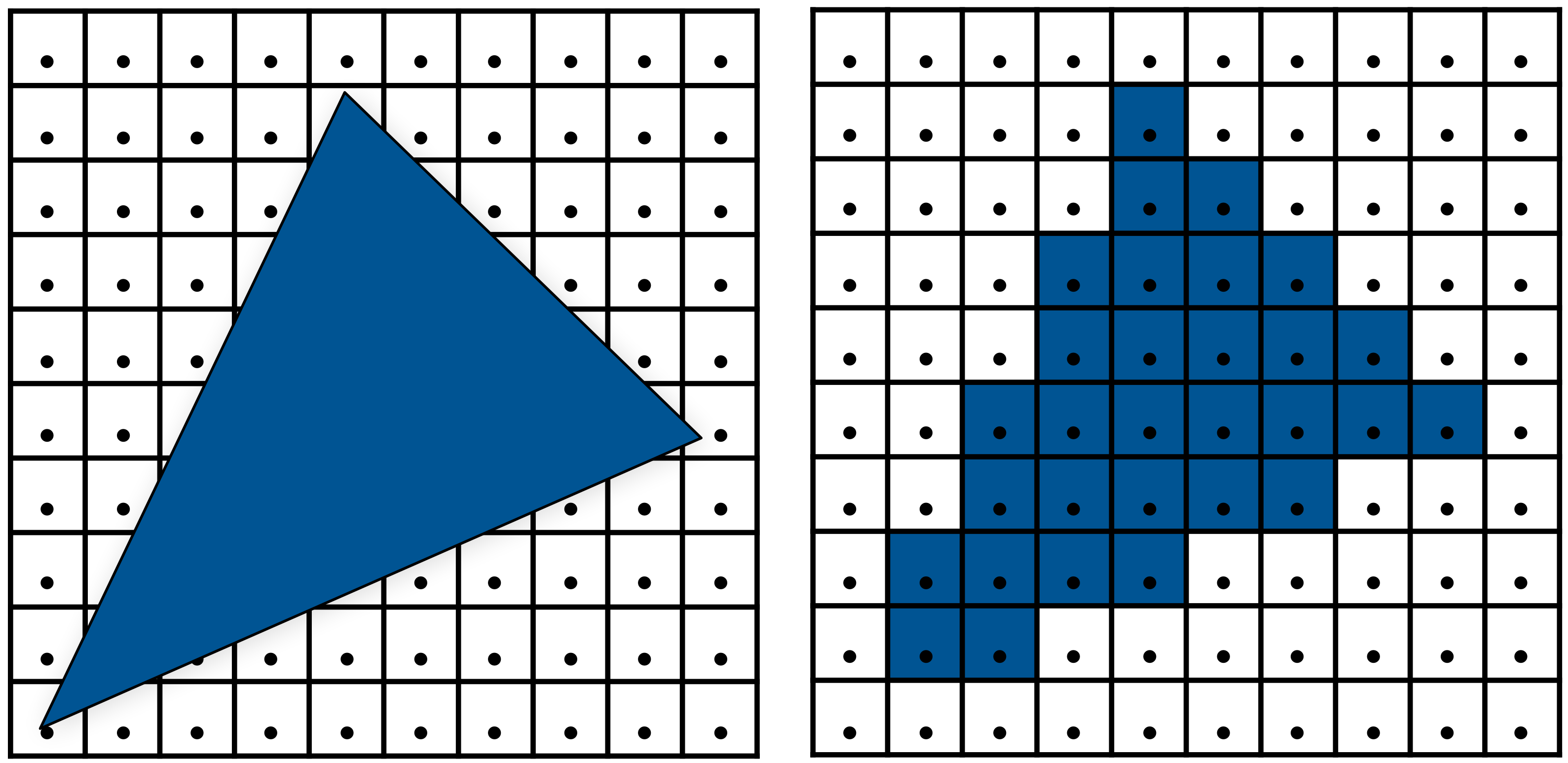
ECSE 446/546

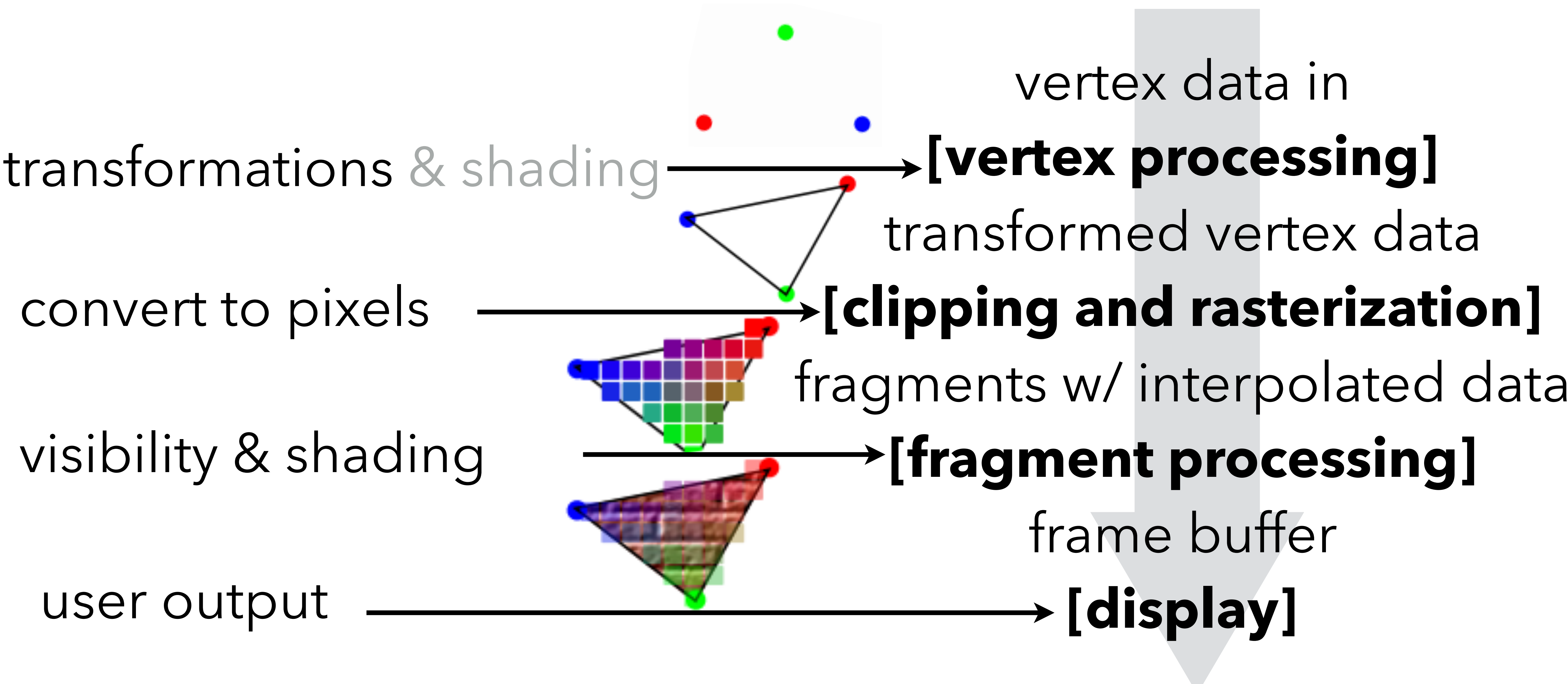
Image Synthesis



Systems 1 – Rasterization

Prof. Derek Nowrouzezahrai
derek@cim.mcgill.ca

Programmable Rasterization Pipeline™



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Programmable Rasterization Pipeline™

vertex data in

[vertex processing]

transformed vertex data

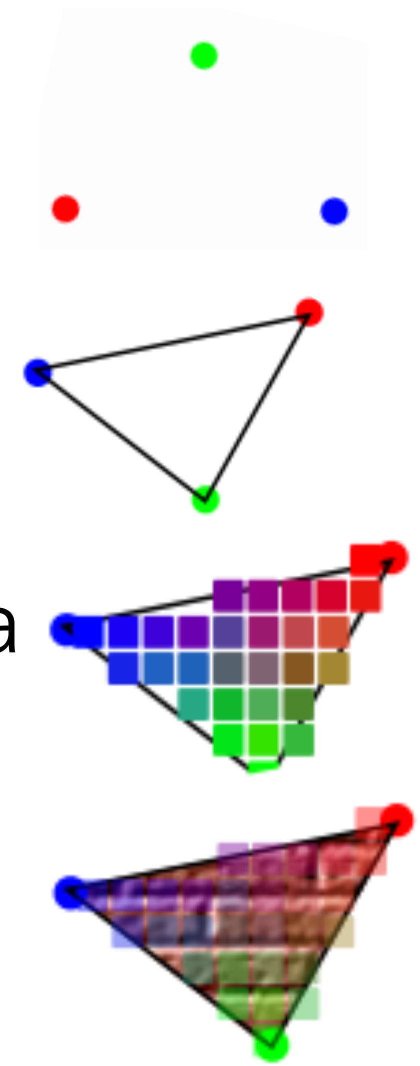
[clipping and rasterization]

fragments w/ interpolated data

[fragment processing]

frame buffer

[display]



```
for(each triangle)
    transform vertices into eye space
    project vertices to image space
    for(each pixel x,y)
        if(x,y in triangle)
```

Programmable Rasterization Pipeline™

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Programmable Rasterization Pipeline™

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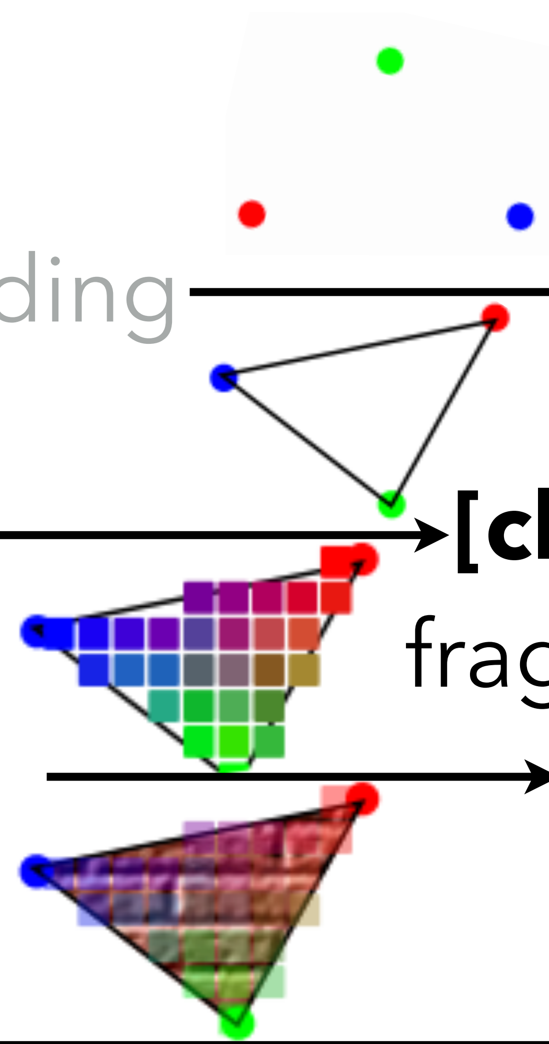
Programmable Rasterization Pipeline™



vertex data in

transformations & shading → **[vertex processing]**

transformed vertex data

convert to pixels → **[clipping and rasterization]**

fragments w/ interpolated data

visibility & shading → **[fragment processing]**

frame buffer

user output → **[display]**

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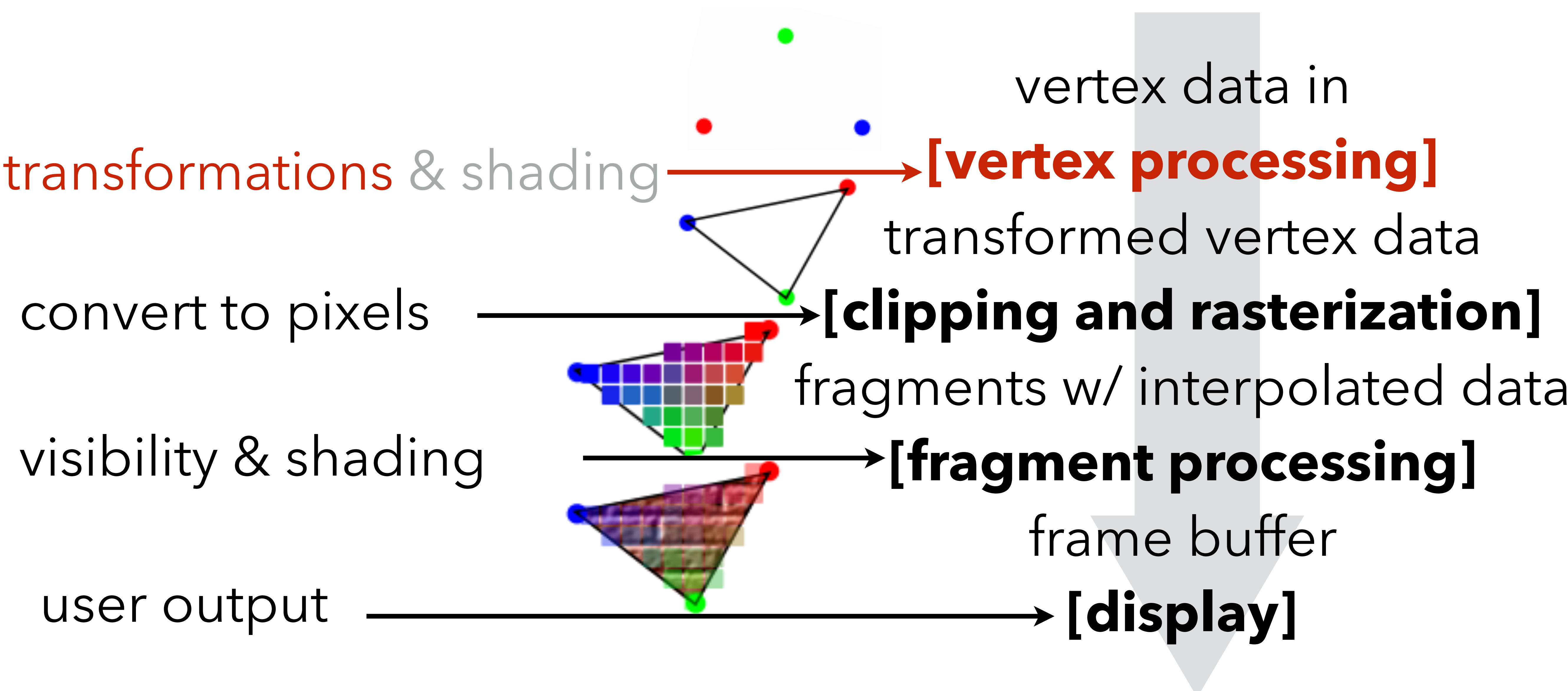
Programmable Rasterization Pipeline™



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Programmable Rasterization Pipeline™

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Programmable Rasterization Pipeline™

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Model/View/Projection (MVP) Transformations

Programmable Rasterization Pipeline™

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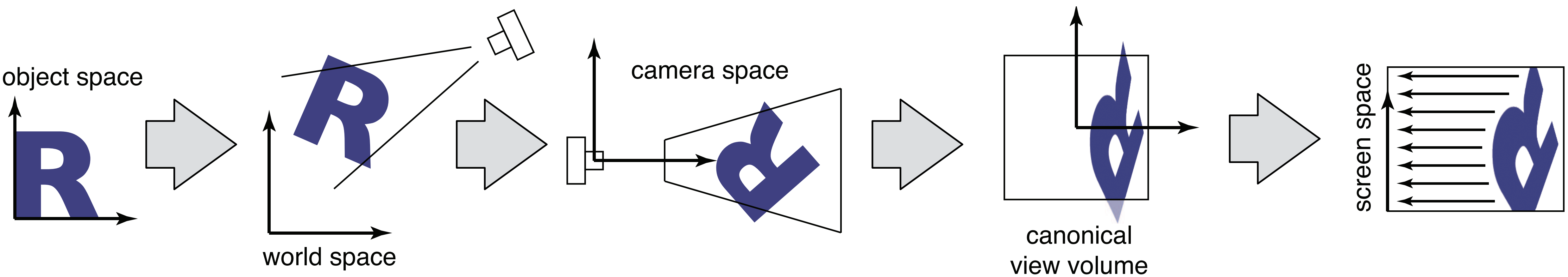
```

Rasterization

Goal: convert from object point to image plane

- start with a 3D object point
- apply a sequence of transforms
  - transforms can be specified by a matrix
- determine the 2D image plane point it projects to

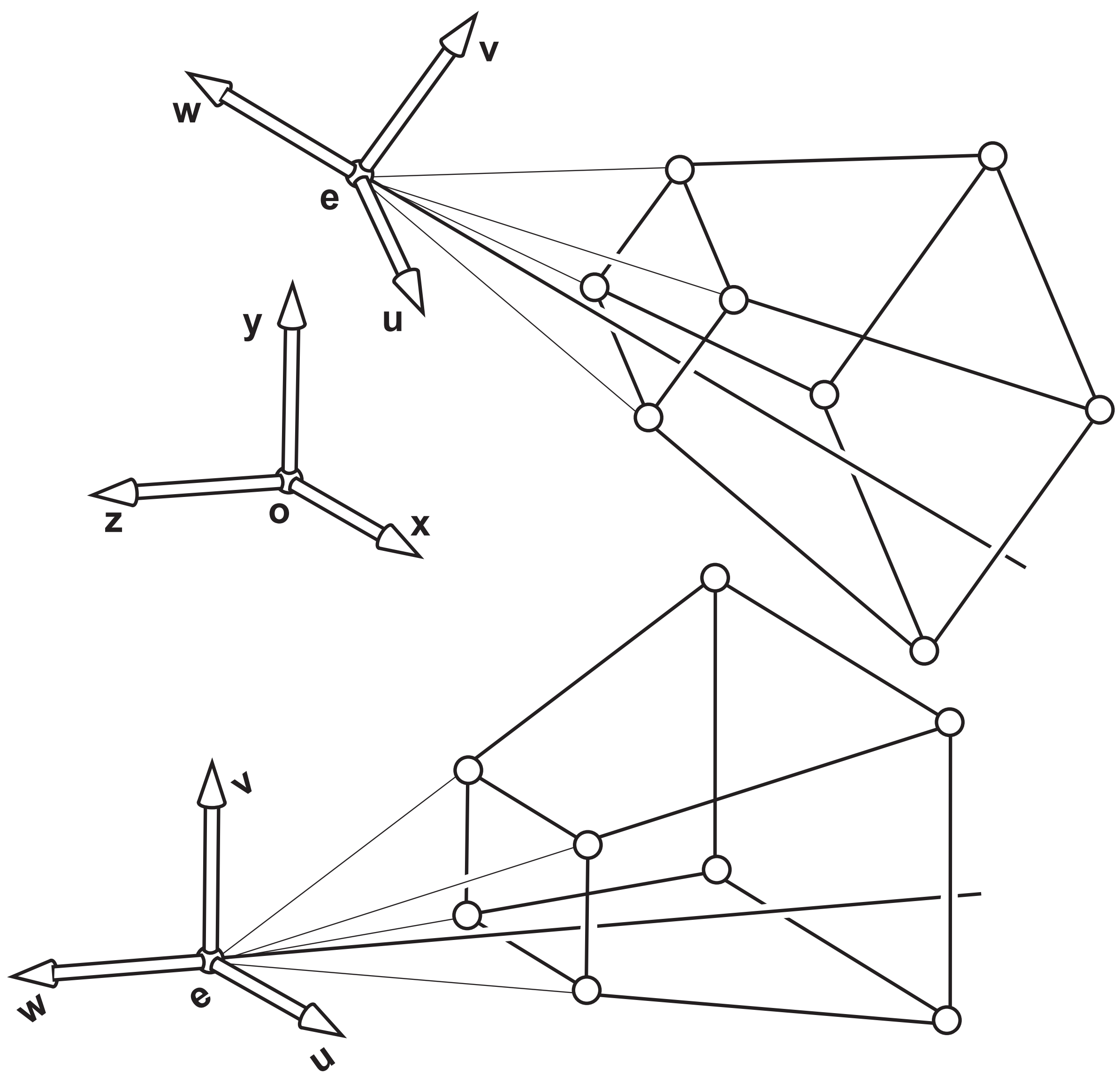
Pipeline of transformations

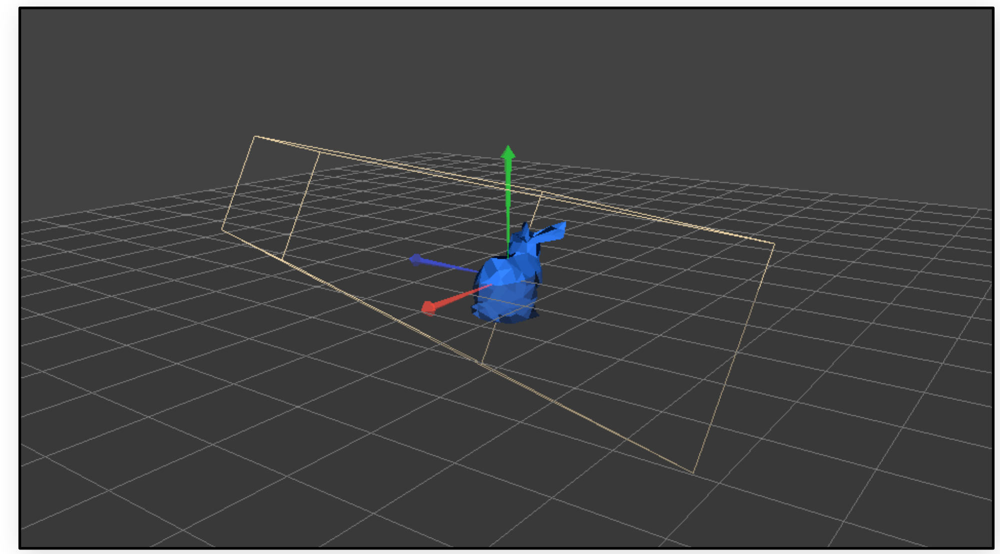


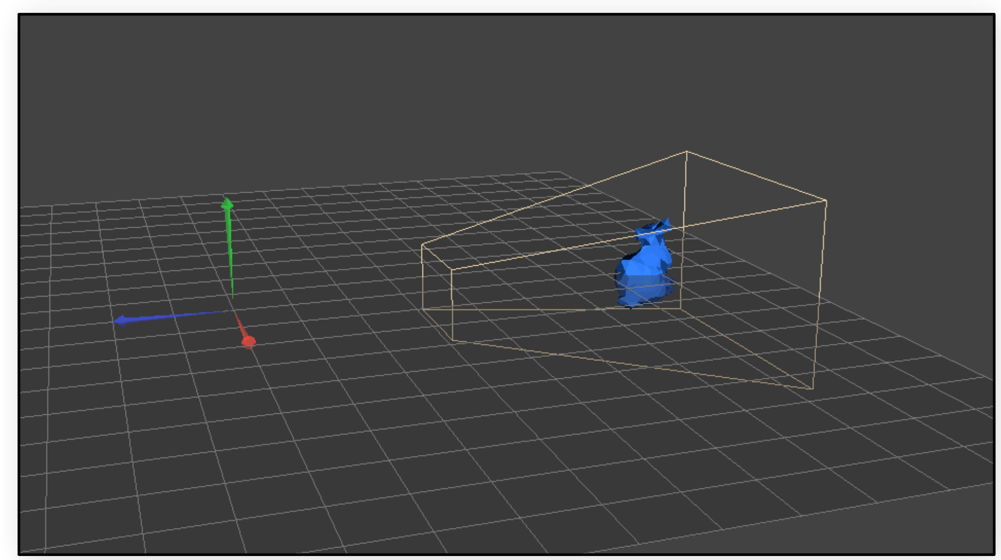
1. Model	2. Viewing	3. Projection	4. Viewport
map local object coords to world coords	map world coords to camera coords	map camera coords to canonical view volume	map canonical view volume to screen space

These two stages perform the actual 3D-to-2D projection

2. Viewing transform







The <u>view matrix</u> transforms all coordinates into “eye space”

2. Viewing transform

Many ways to construct a view transform (i.e., matrix)

For example, can specify an orthonormalized frame with the:

- **eye** point
- **up**-vector, and
- **look-at** point

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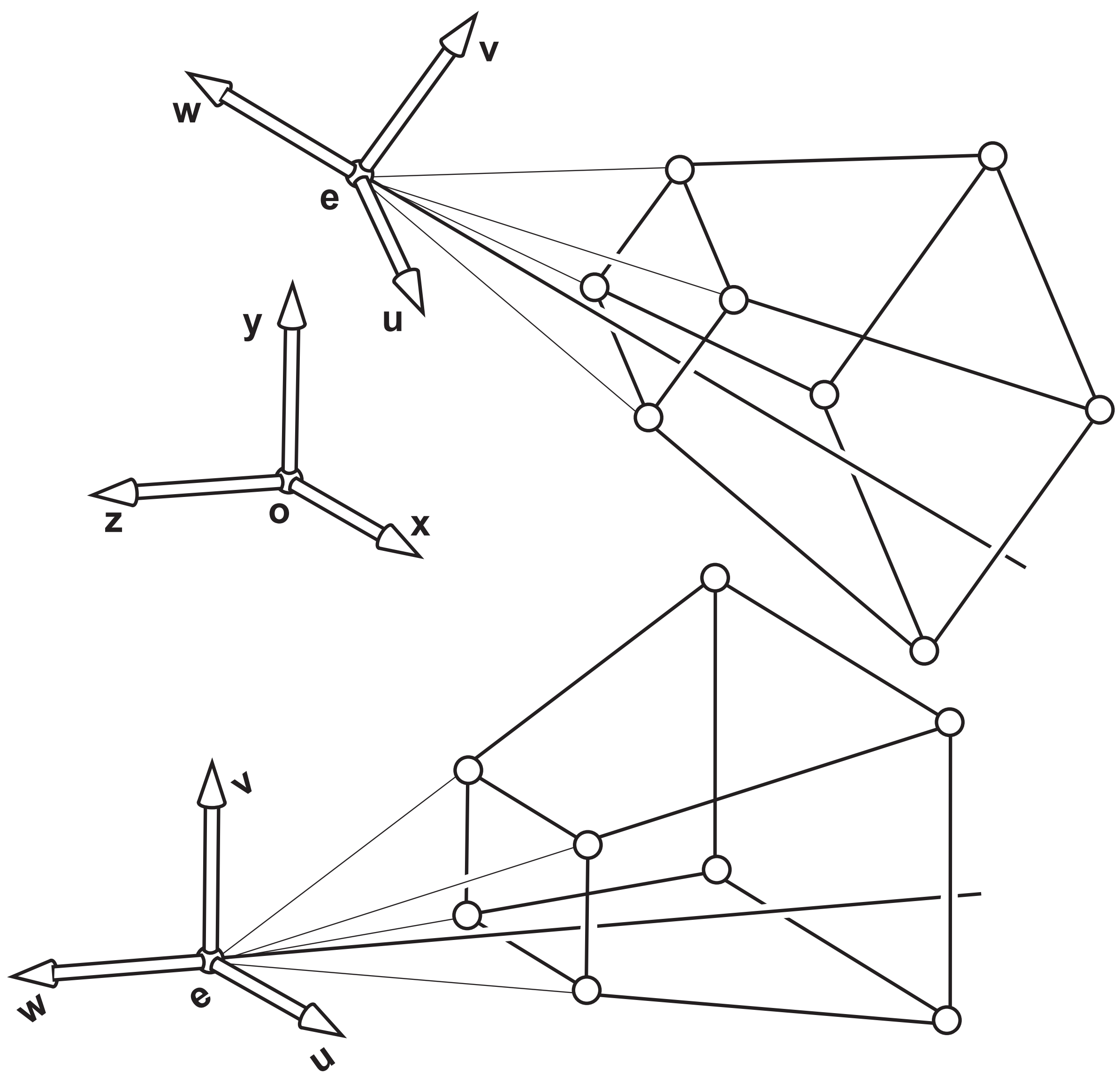
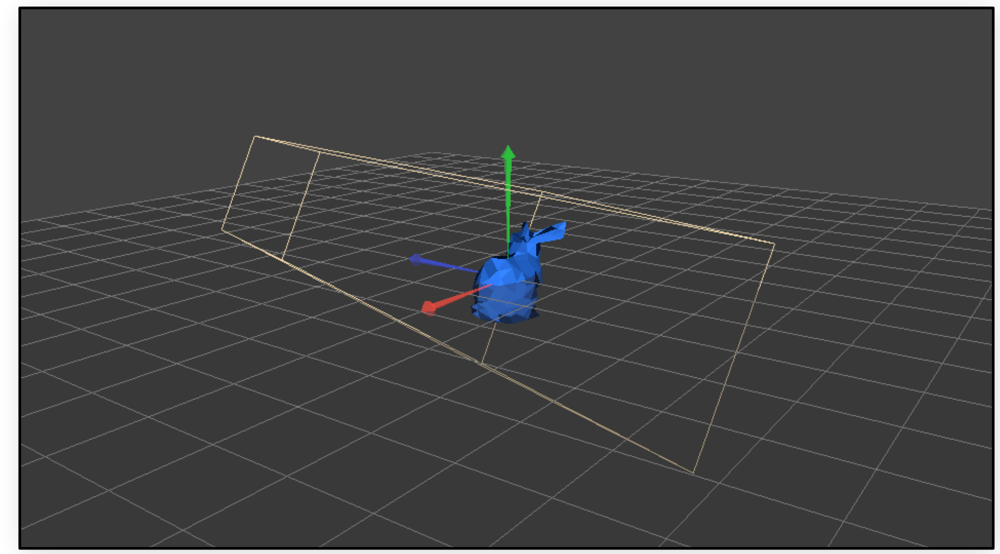
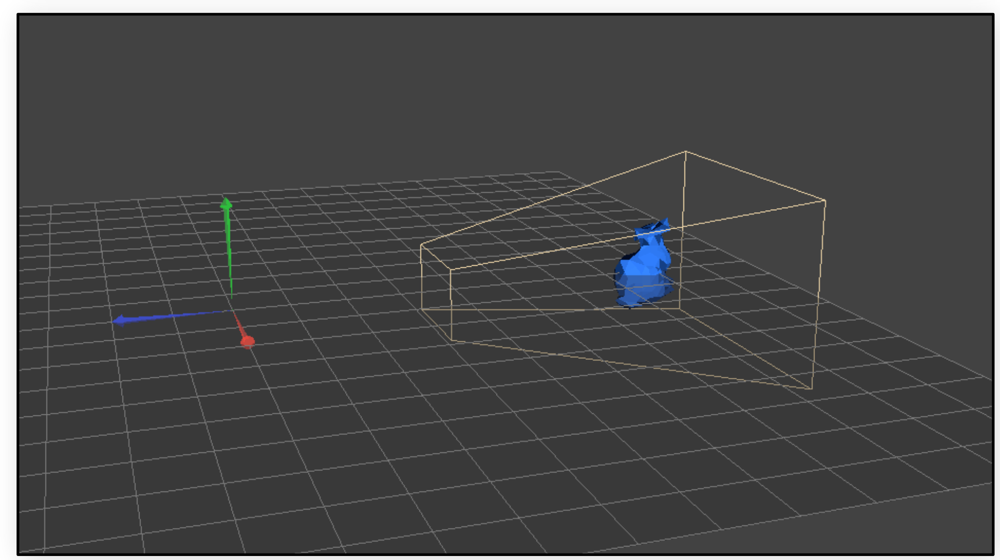
2. Viewing transform







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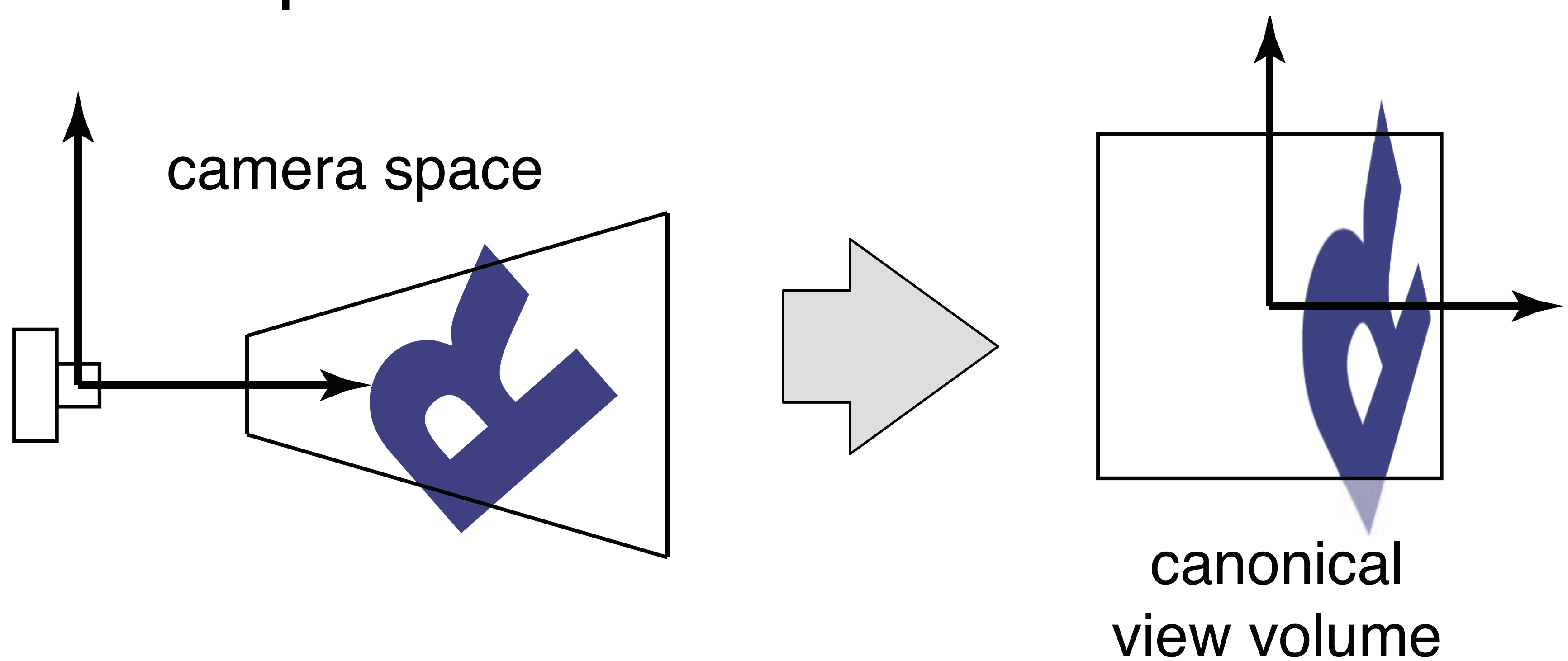
3. Projection

Generally, a function that transforms points from m- to n-space where $m > n$

In graphics, map 3D points to 2D image coordinates

- except we will keep around the third coordinate



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Mathematics of Projection

Always work in eye coords

- assume eye point at **0** and plane perpendicular to z

Orthographic case: just toss out the z-coord

Perspective case: scale diminishes with z

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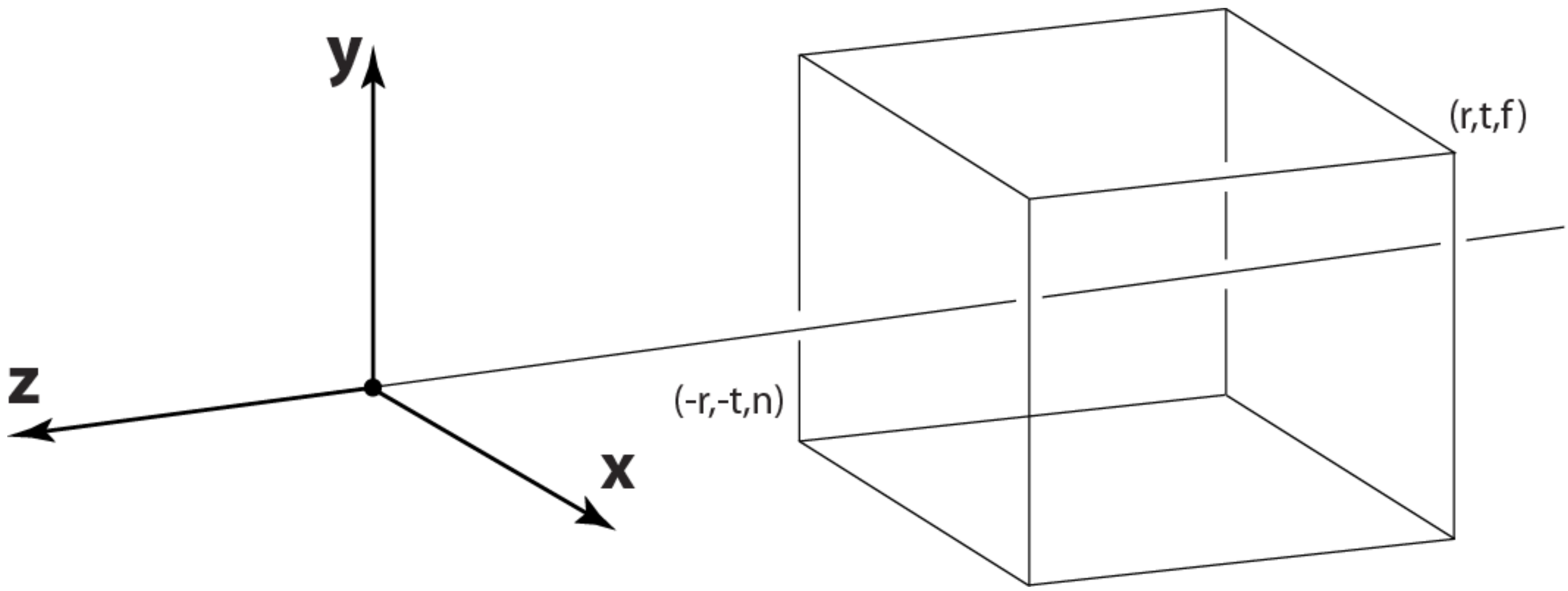
Orthographic projection

Box view volume



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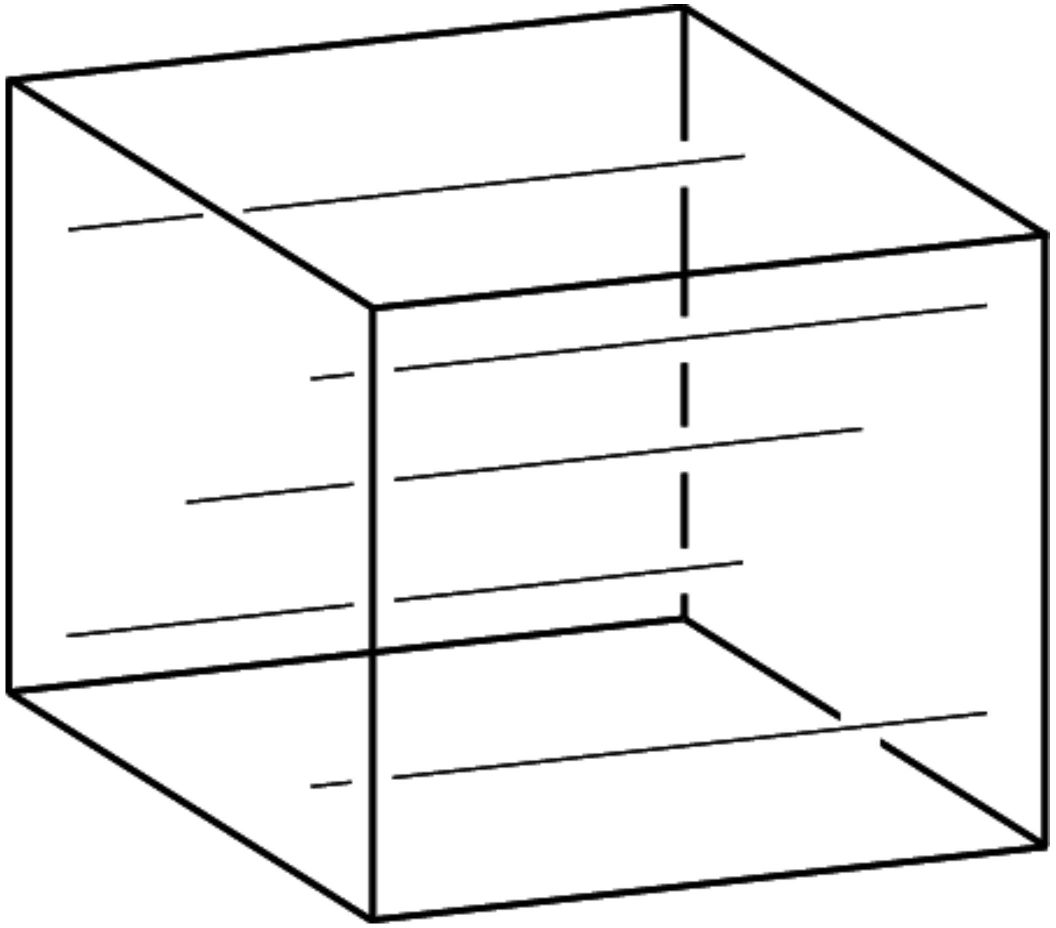
Orthographic projection

Viewing rays are parallel



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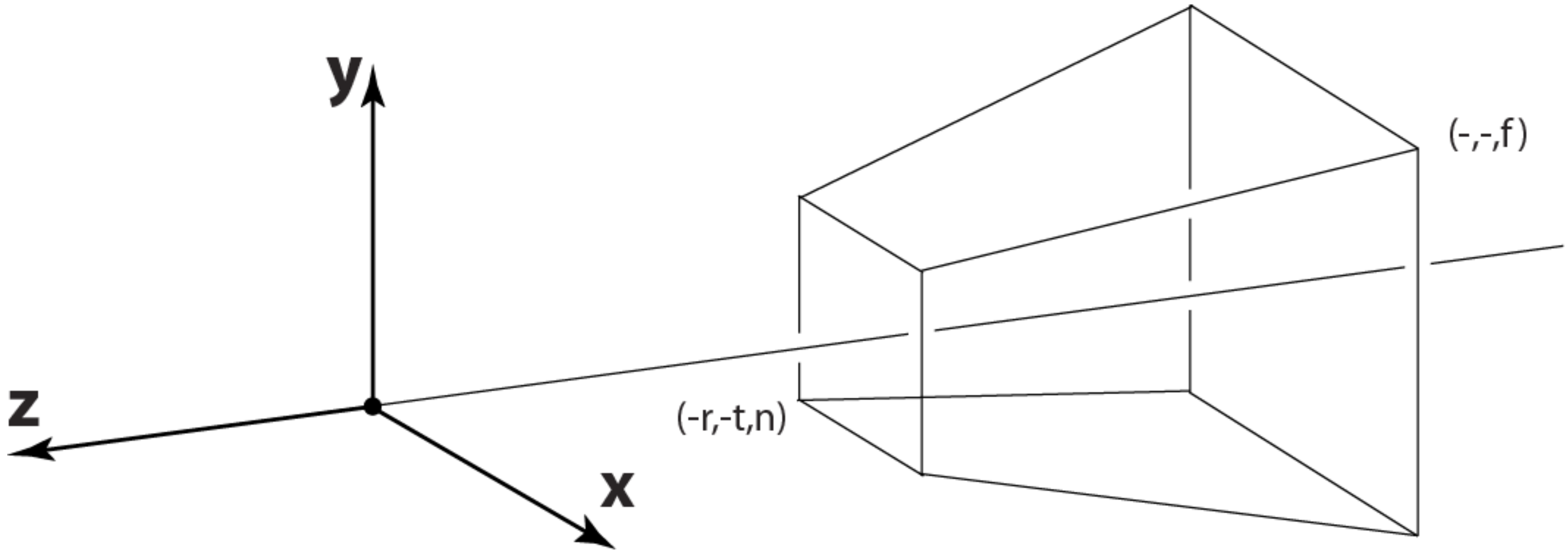
Perspective projection

Truncated pyramid view volume



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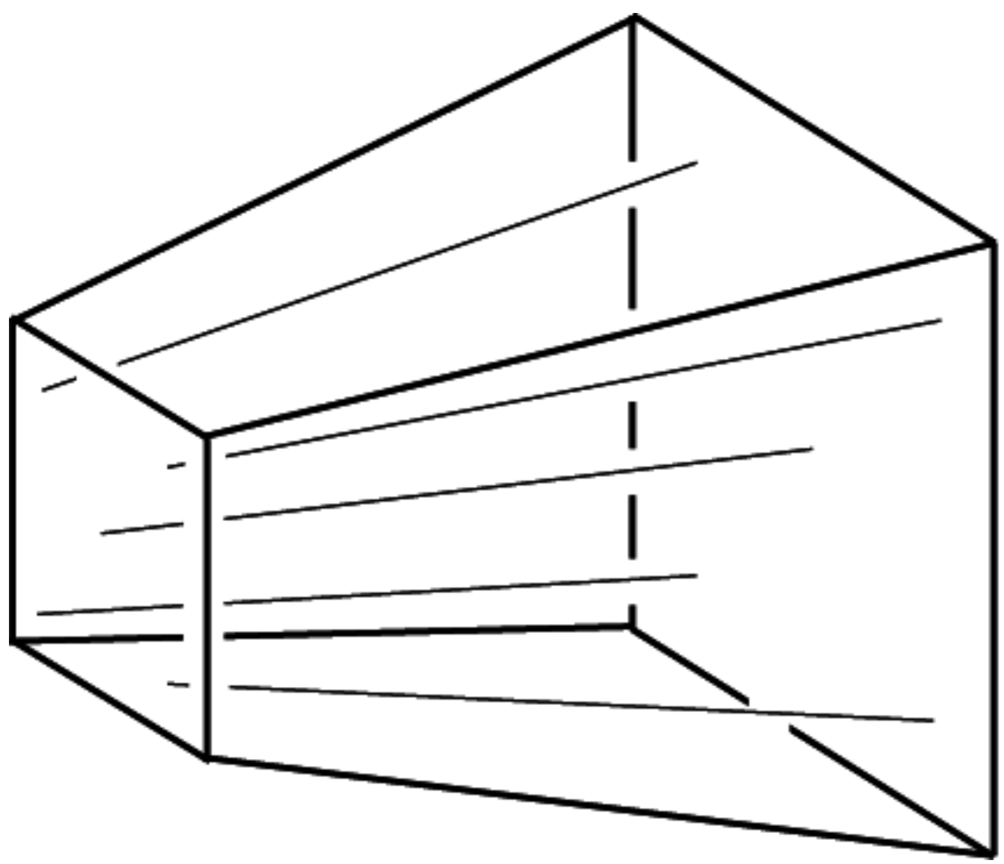
Perspective projection

Viewing rays converge to a point



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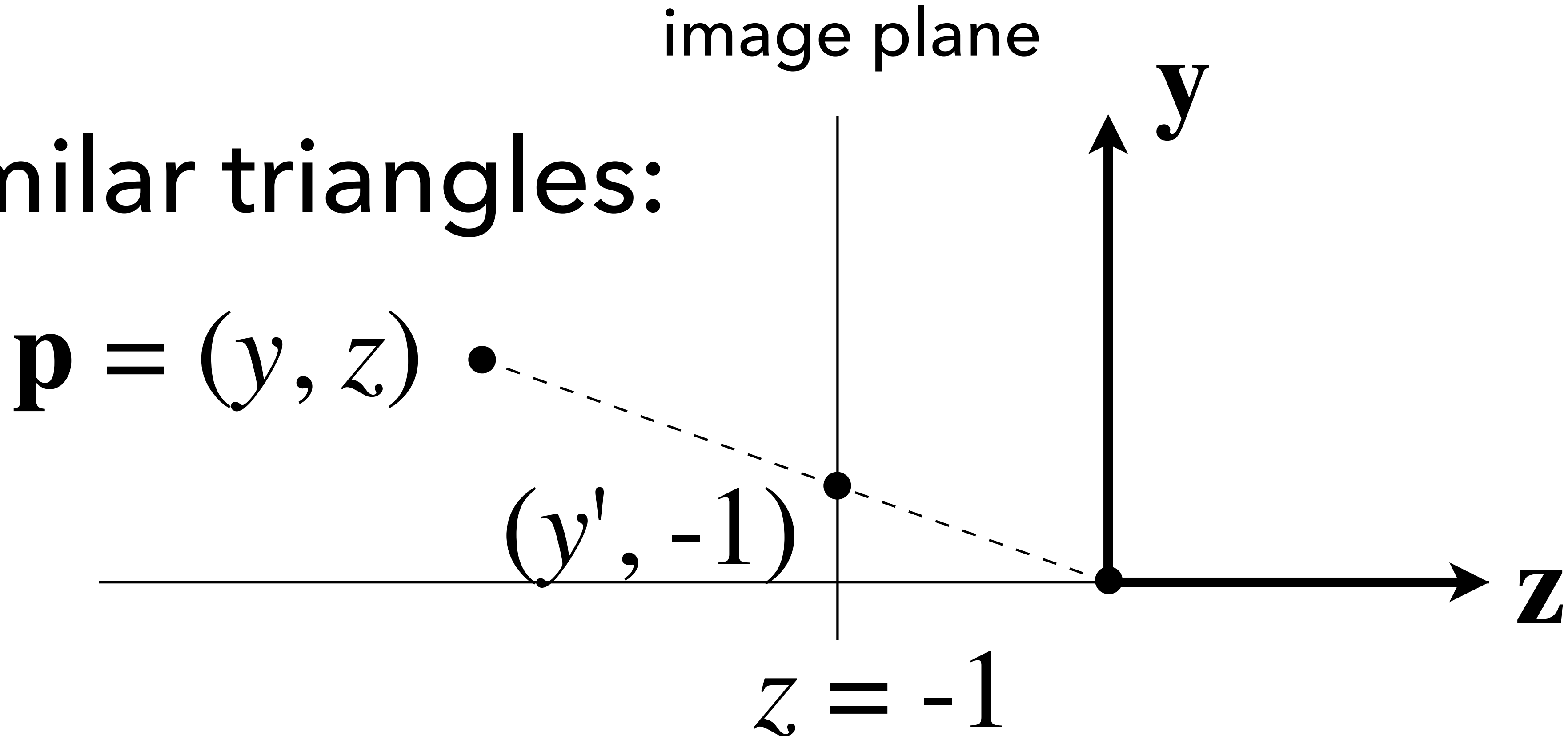
Perspective projection

Use similar triangles:



$$\frac{y'}{-1} = \frac{y}{z}$$

$$y' = -\frac{y}{z}$$

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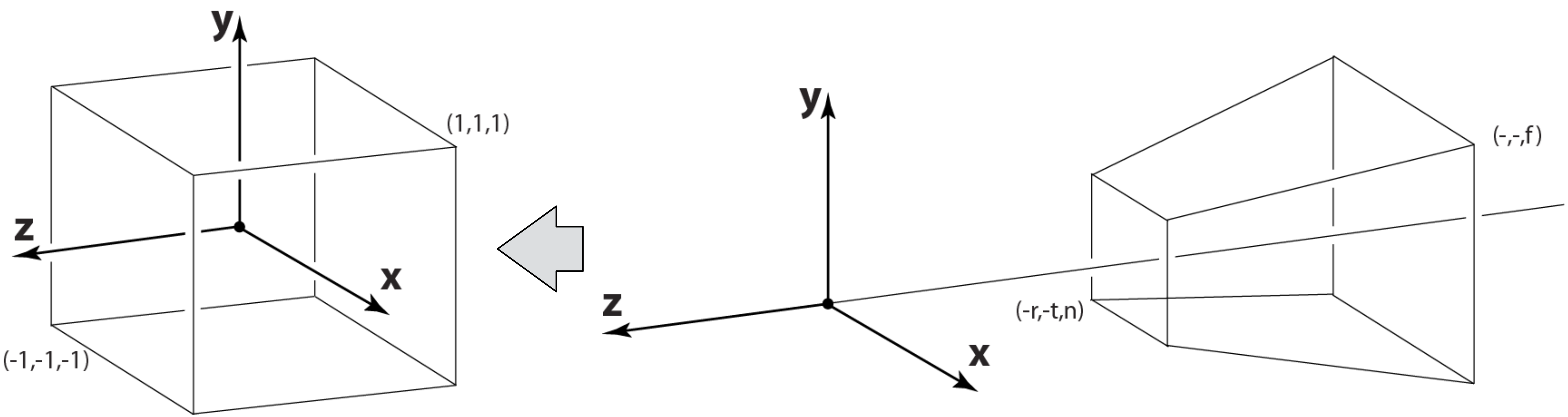
Remapping the view frustum

We also want the projection matrix to remap the space between **near** and **far** planes to a canonical view volume



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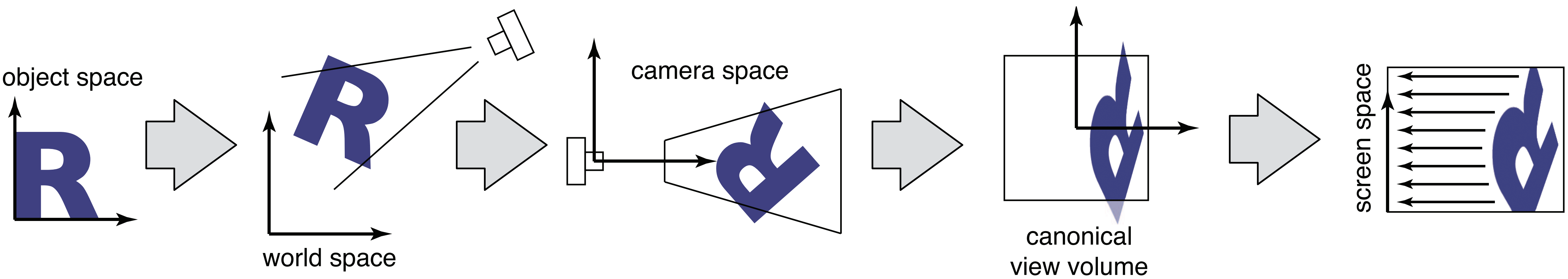
Pipeline of transformations



1. **Model**

map local object coords to world coords

2. **Viewing**

map world coords to camera coords

3. **Projection**

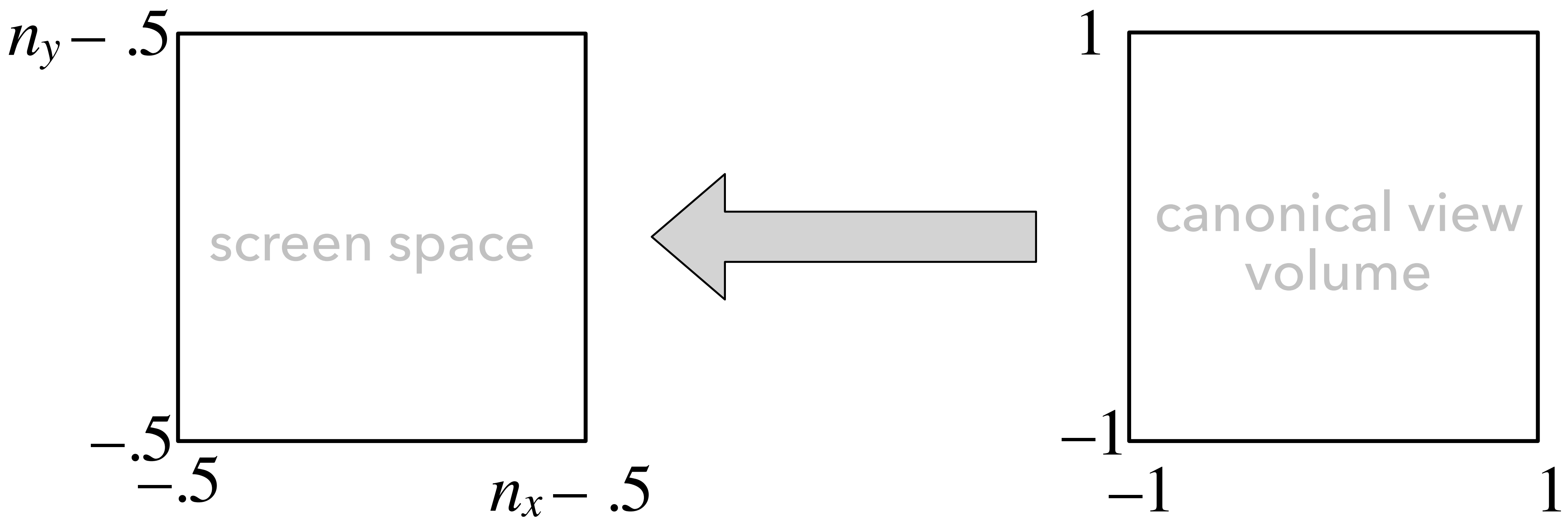
map camera coords to canonical view volume

4. **Viewport**

map canonical view volume to screen space

These two stages perform the actual 3D-to-2D projection

4. Viewport transformation



$$\begin{bmatrix} x_{\text{screen}} \\ y_{\text{screen}} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{n_x}{2} & 0 & \frac{n_x - 1}{2} \\ 0 & \frac{n_y}{2} & \frac{n_y - 1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{\text{canonical}} \\ y_{\text{canonical}} \\ 1 \end{bmatrix}$$

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Orthographic Transformation Chain

Start with coordinates in object's local coordinates

Transform into world coords (modeling transform, $\mathbf{M}_\mathrm{m}$)

Transform into eye coords (camera xf., $\mathbf{M}_\mathrm{cam} = \mathrm{F_c}^{-1}$)

Orthographic projection, $\mathbf{M}_\mathrm{orth}$

Viewport transform, $\mathbf{M}_\mathrm{vp}$

$$\mathbf{p}_s = \mathbf{M}_\mathrm{vp}\mathbf{M}_\mathrm{orth}\mathbf{M}_\mathrm{cam}\mathbf{M}_\mathrm{m}\mathbf{p}_o$$

$$\begin{bmatrix} x_s \\ y_s \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x-1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{w} & \mathbf{e} \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \mathbf{M}_\mathrm{m} \begin{bmatrix} x_o \\ y_o \\ z_o \\ 1 \end{bmatrix}$$

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Perspective Transformation Chain

Start with coordinates in object's local coordinates

Transform into world coords (modelling transform, $\mathbf{M}_{\mathrm{m}}$)

Transform into eye coords (camera xf., $\mathbf{M}_{\mathrm{cam}} = \mathrm{F}_{\mathrm{c}}^{-1}$)

Perspective projection, $\mathbf{M}_{\mathrm{persp}}$

Viewport transform, $\mathbf{M}_{\mathrm{vp}}$

$$\mathbf{p}_s = \mathbf{M}_{\mathrm{vp}}\mathbf{M}_{\mathrm{persp}}\mathbf{M}_{\mathrm{cam}}\mathbf{M}_{\mathrm{m}}\mathbf{p}_o$$

$$\begin{bmatrix} x_s \\ y_s \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x-1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{l+r}{l-r} & 0 \\ 0 & \frac{2n}{t-b} & \frac{b+t}{b-t} & 0 \\ 0 & 0 & \frac{f+n}{n-f} & \frac{2fn}{f-n} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{w} & \mathbf{e} \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \mathbf{M}_{\mathrm{m}} \begin{bmatrix} x_o \\ y_o \\ z_o \\ 1 \end{bmatrix}$$

After a slide by Steve Marschner

Recap: Transformation Pipeline

Perform rotation/translation/other transforms to put viewpoint at origin and view direction along z axis

- combination of model and view matrix called "modelview" matrix (e.g., in OpenGL)

Combine with projection matrix (perspective or orthographic)

- this is the OpenGL "projection" matrix

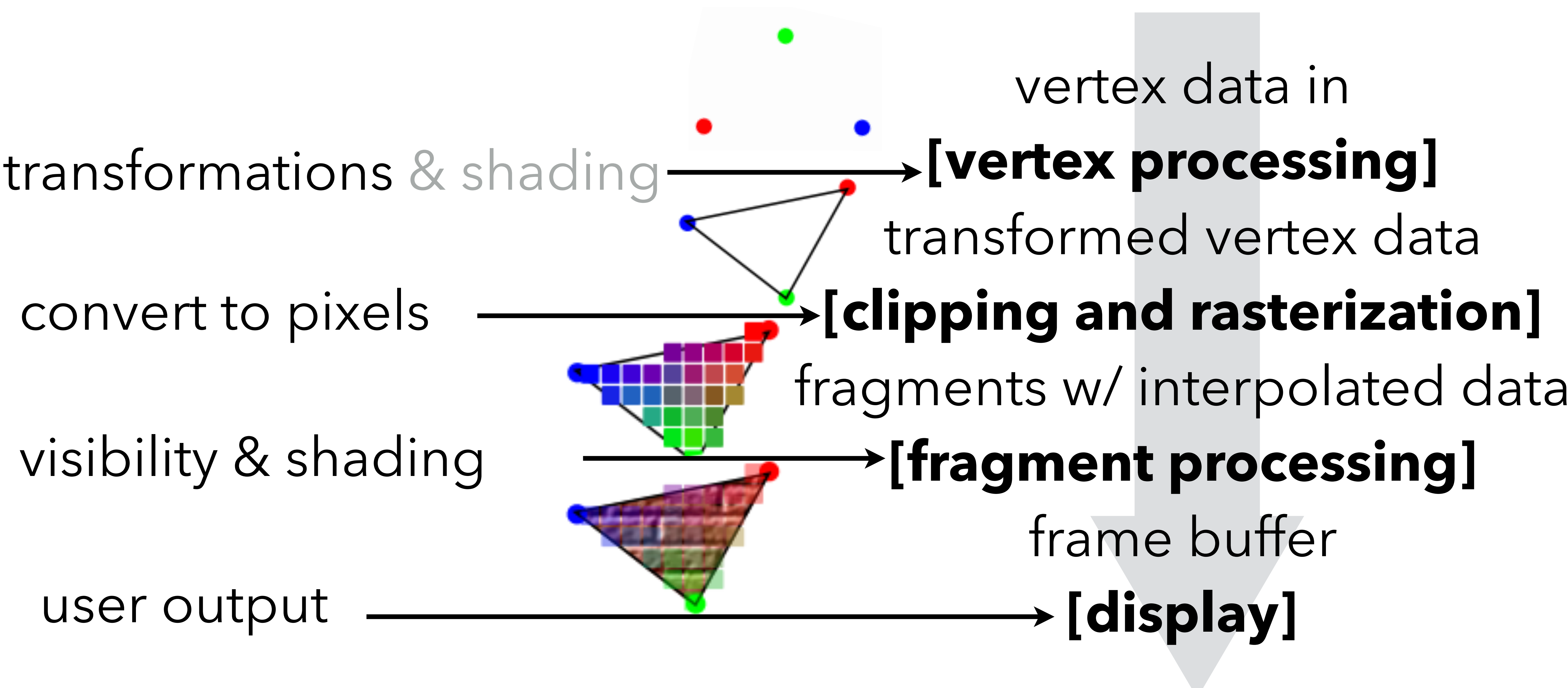
Convert canonical view volume $[-1,1]^3$ to screen space

Corollary: The entire transformation from local object space to screen space is a single 4x4 matrix!

After a slide by Frédo Durand

Questions?

Programmable Rasterization Pipeline™



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Programmable Rasterization Pipeline™

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transformations & shading → **[vertex processing]**

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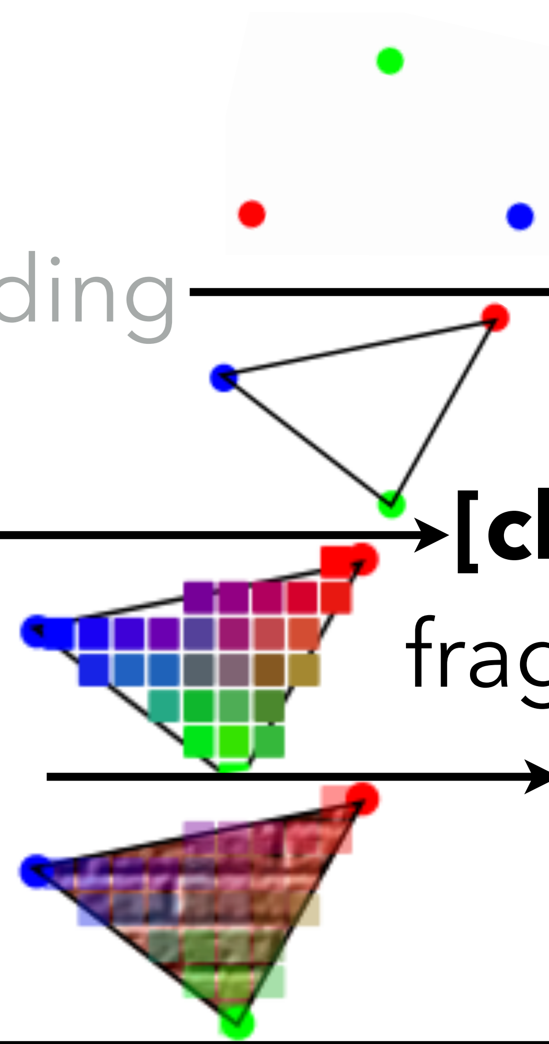

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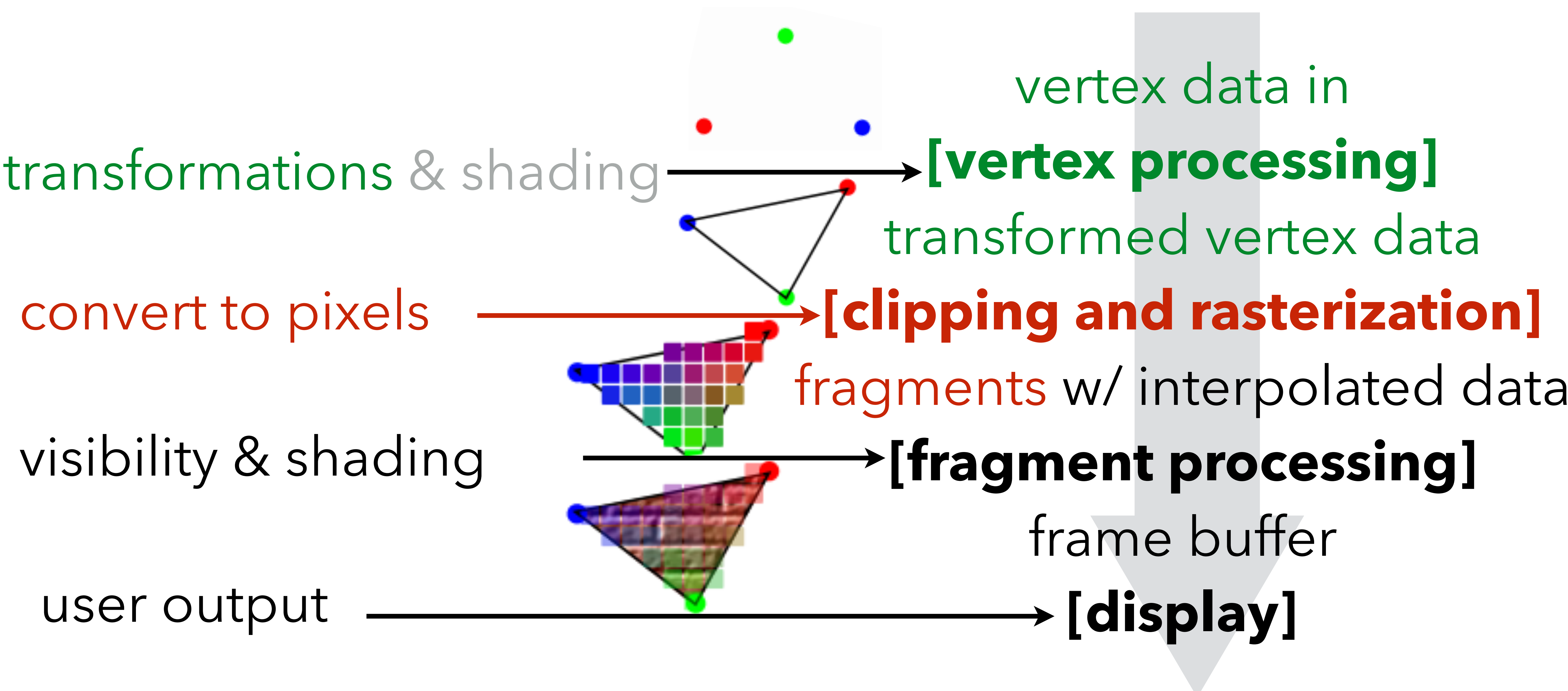

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Programmable Rasterization Pipeline™

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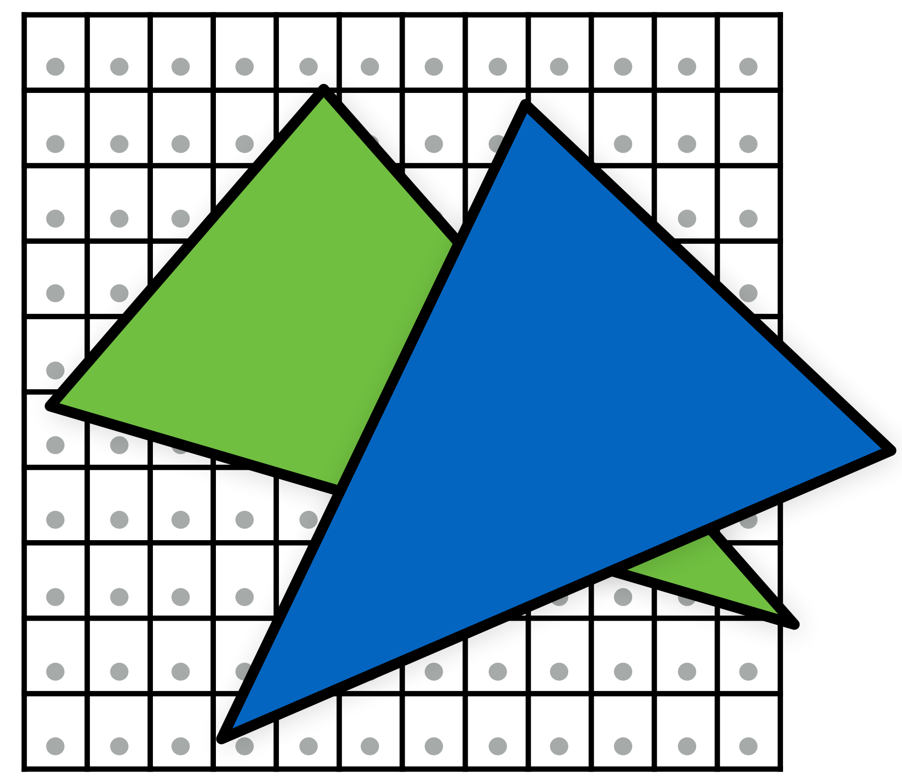
Triangle Rasterization

Triangle Rasterization

Primitives are "continuous" geometric objects; screen is discrete (pixels)



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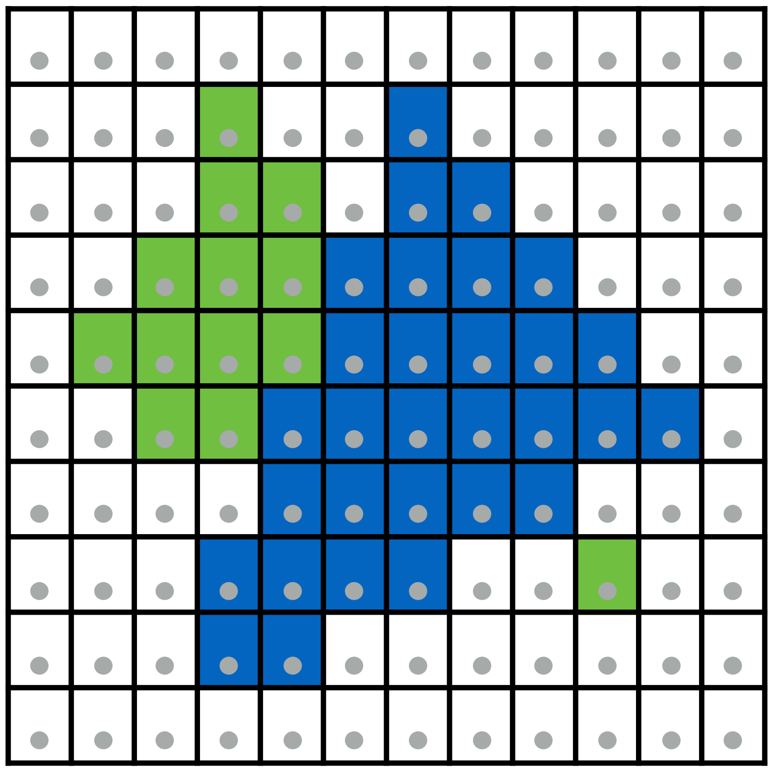
Rasterization

Primitives are "continuous" geometric objects; screen is discrete (pixels)

Rasterization computes a discrete approximation in terms of pixels



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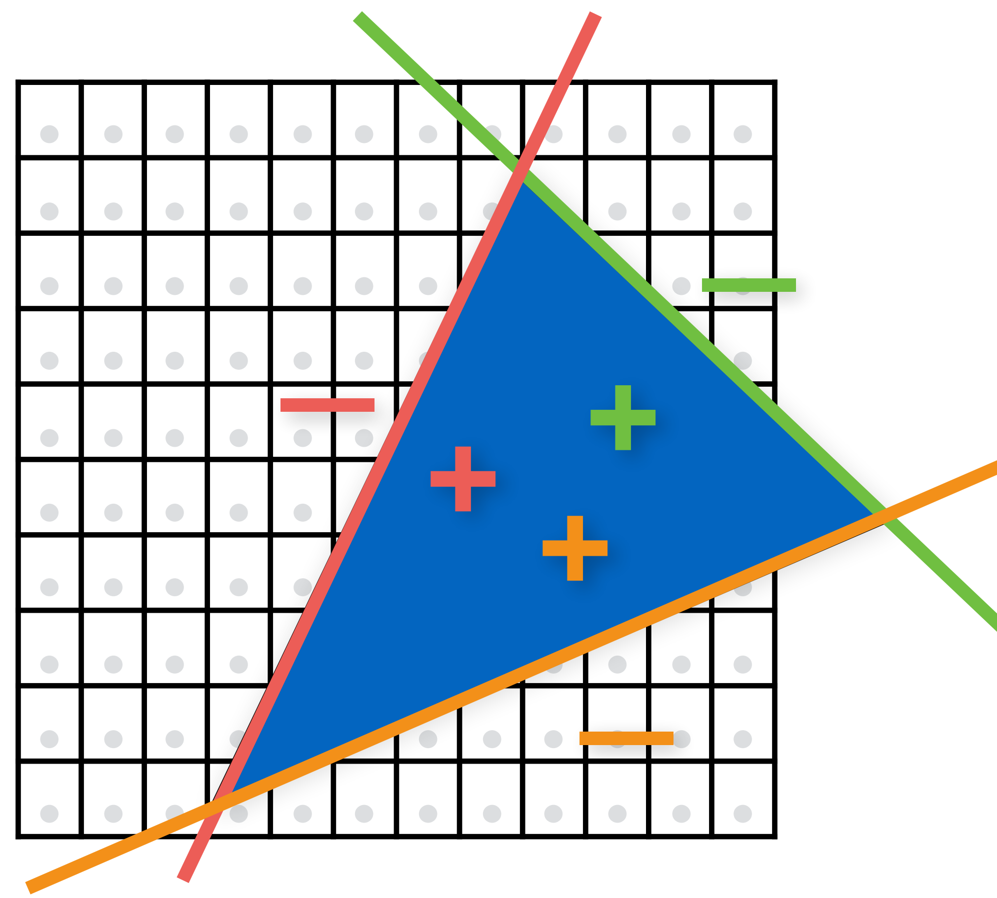
Edge Functions

A triangle's 3D edges project to line segments in the image (thanks to planar perspective)

- lines map to lines, not curves



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Edge Functions

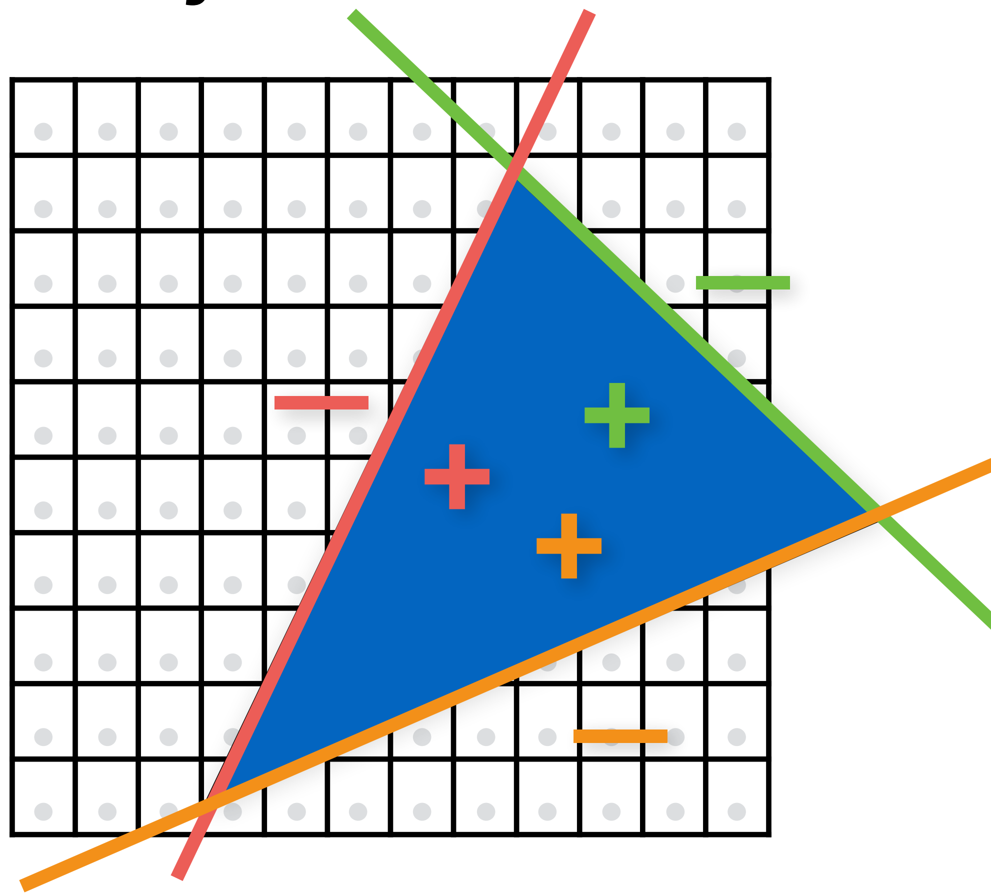
A triangle's 3D edges project to line segments in the image (thanks to planar perspective)

- interior of the triangle is the set of points lie inside **all** 3 half-spaces defined by these lines

After a slide by Frédo Durand

$$E_i(x, y) = a_i x + b_i y + c_i$$

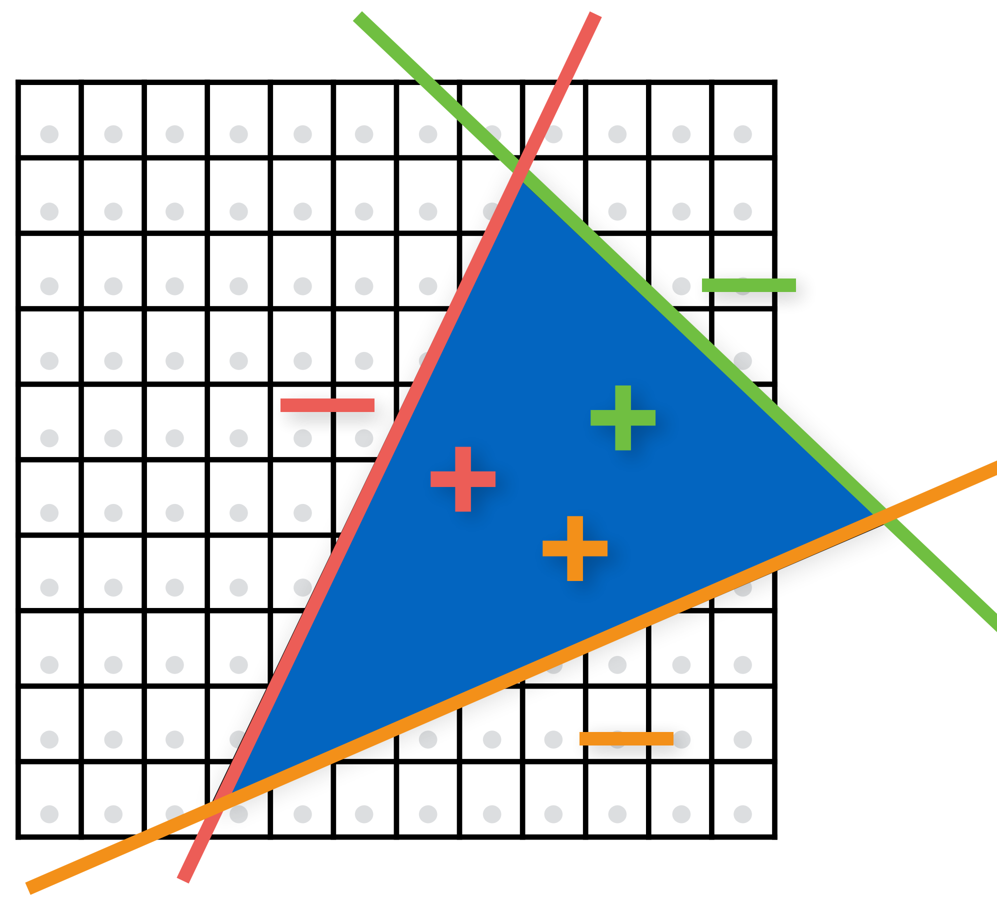
$$(x, y) \text{ within triangle iff}$$
$$E_i(x, y) \geq 0, \forall i = 1, 2, 3$$



Brute Force Triangle Rasterizer

Compute E_1, E_2, E_3 coefficients from projected vertices

- called "triangle setup": yields a_i, b_i, c_i for $i = 1,2,3$



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Brute Force Triangle Rasterizer

```
for each pixel (x,y)
    evaluate edge functions at pixel center
    if all non-negative, pixel is in
```

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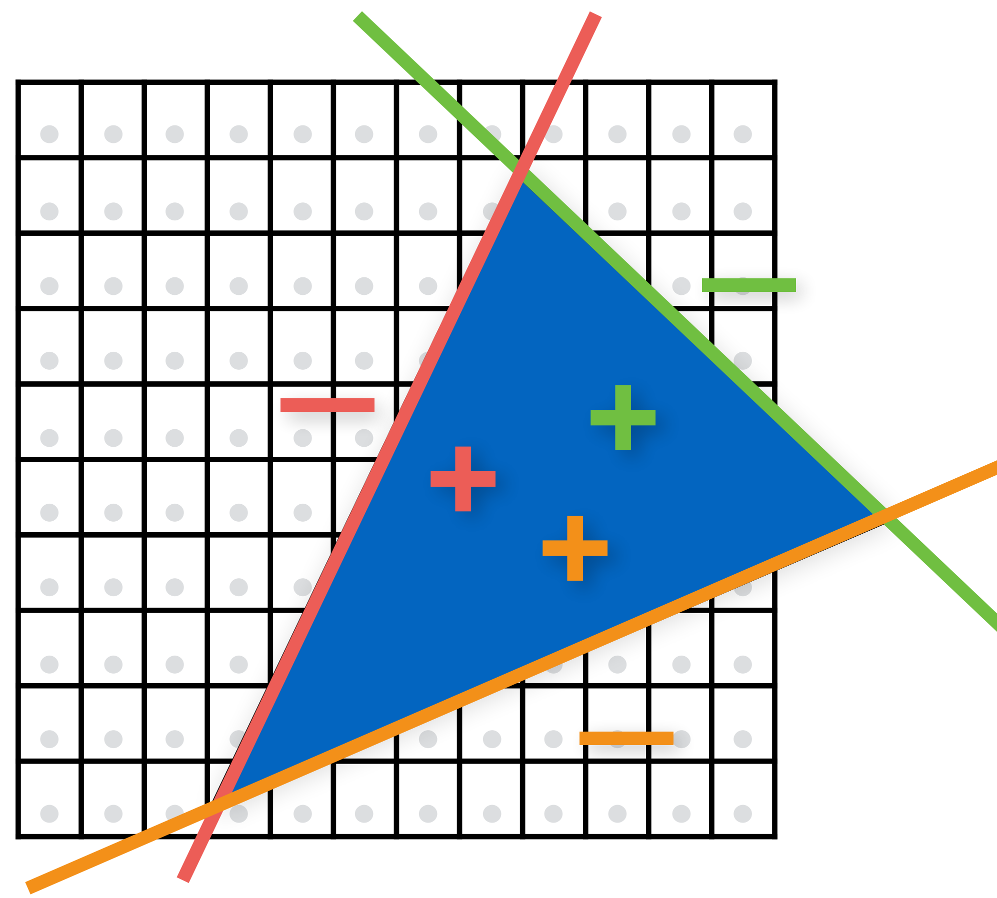
Problem?



Brute Force Triangle Rasterizer

```
for each pixel (x,y)
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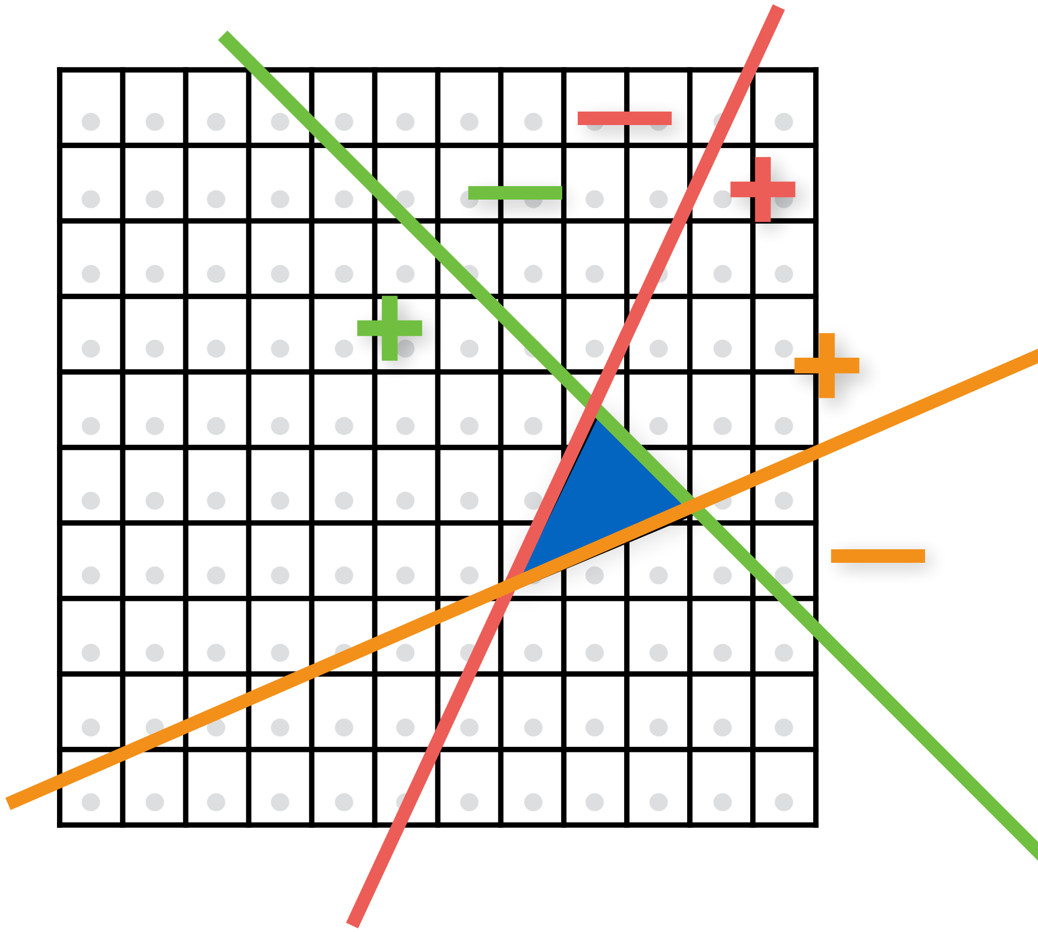
```

If the triangle is small, lots of useless computation (if we really test all pixels)



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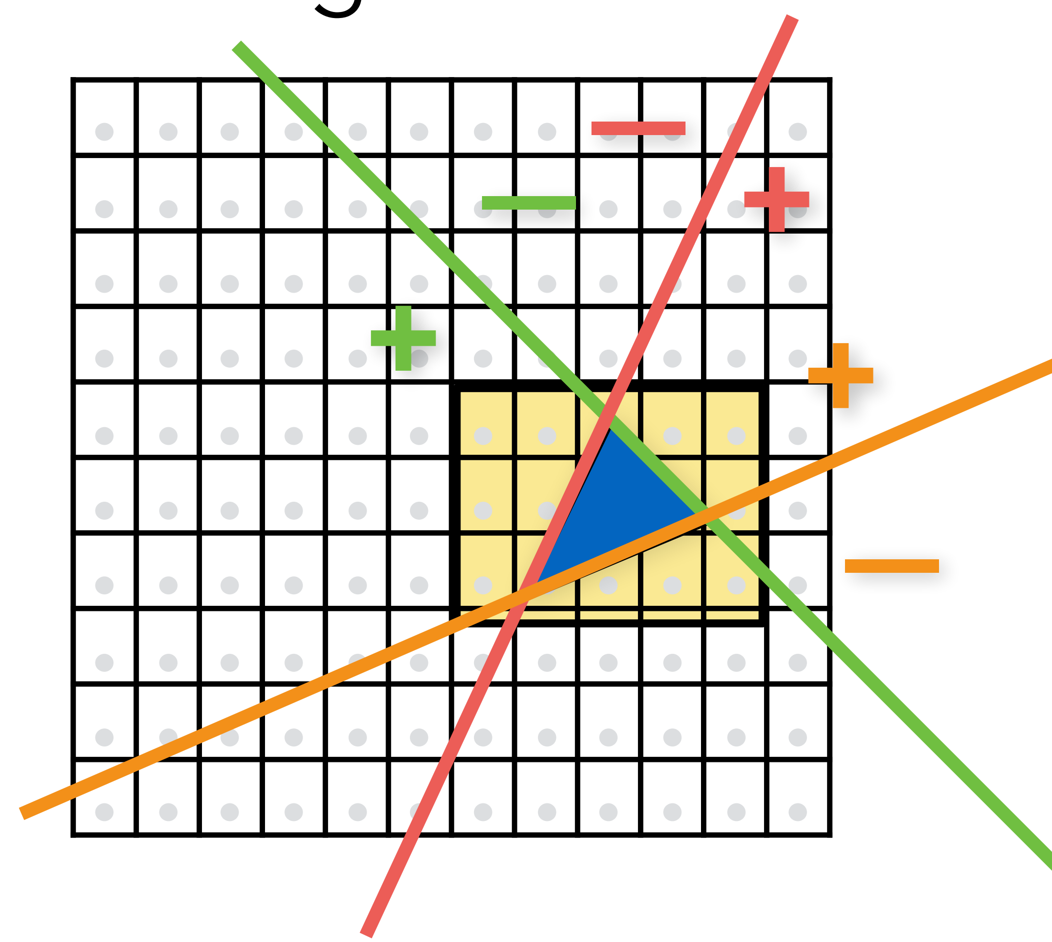
Easy Optimization

Improvement: only scan over pixels overlapping the **screen bounding box** (BBox) of the triangle

- how do we get such a bounding box?
  - $x_{min}, x_{max}, y_{min}, y_{max}$ of projected triangle vertices



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Triangle Rasterization Pseudocode

```
for every triangle
    project vertices, compute the Ei
    compute bbox, clip box to screen limits
    for all pixels in box
        evaluate Ei functions
        if all > 0
            framebuffer[x,y] = c;
```

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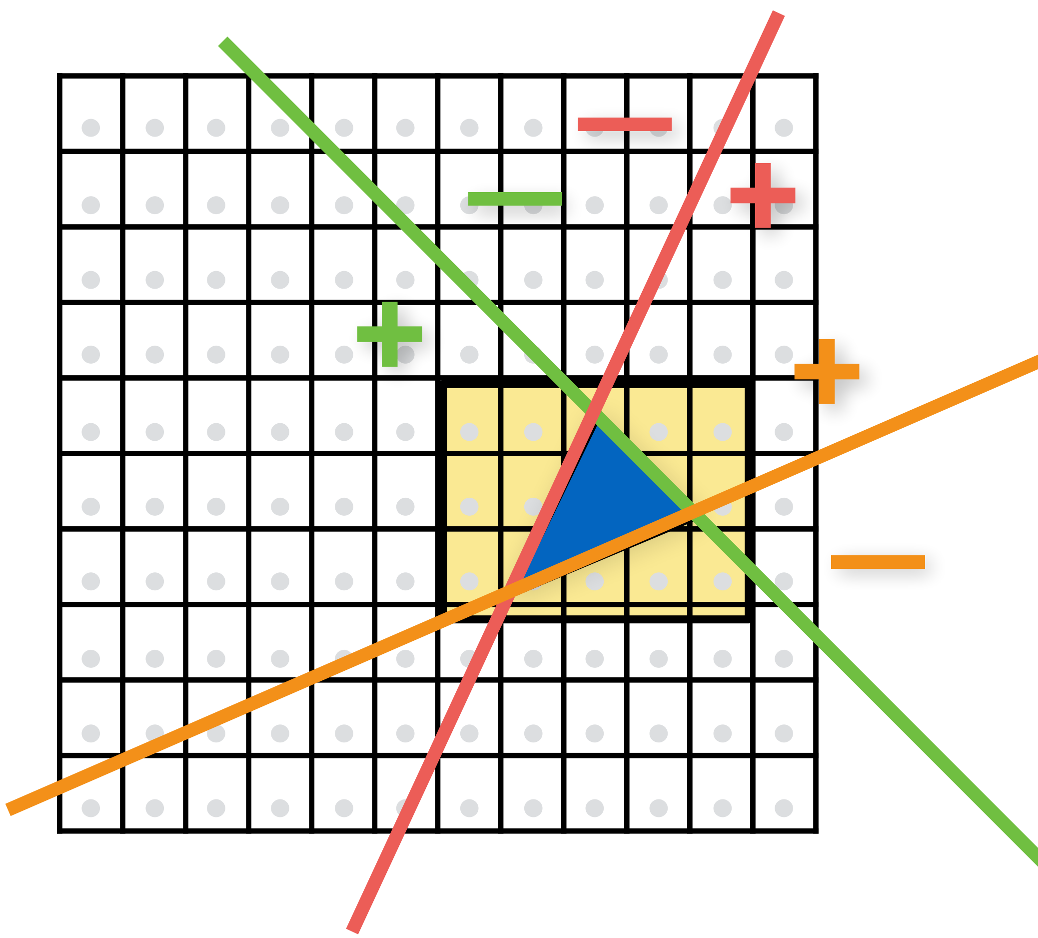
Bounding box clipping is easy, just clamp the coordinates to the screen rectangle



Note: no visibility!

Triangle Rasterization Pseudocode

Jaakko Lehtinen
@jaakkolehtinen

Follow

My most succinct rasterizer yet. Still kind of readable.

```
% triangle vertices in [-1,1]^2
P = [-1 -0.51; -0.3 0.77; -0.1 -0.32];
% pixel samples 0.1 units apart
[px,py] = meshgrid(-1+0.1/2:0.1:1, -1+0.1/2:0.1:1);
% form edge functions
E = [P([2 3 1],2), P([1 2 3],1)] - [P([1 2 3],2), P([2 3 1],1)];
E(:,3) = -diag(E*P');
% evaluate edge functions at sample points
res = E*[px(:)'; py(:)'; ones(1,length(px(:)))];
% which ones are in?
inside = min(res,[],1) > 0;
```

7:32 AM - 13 Nov 2015

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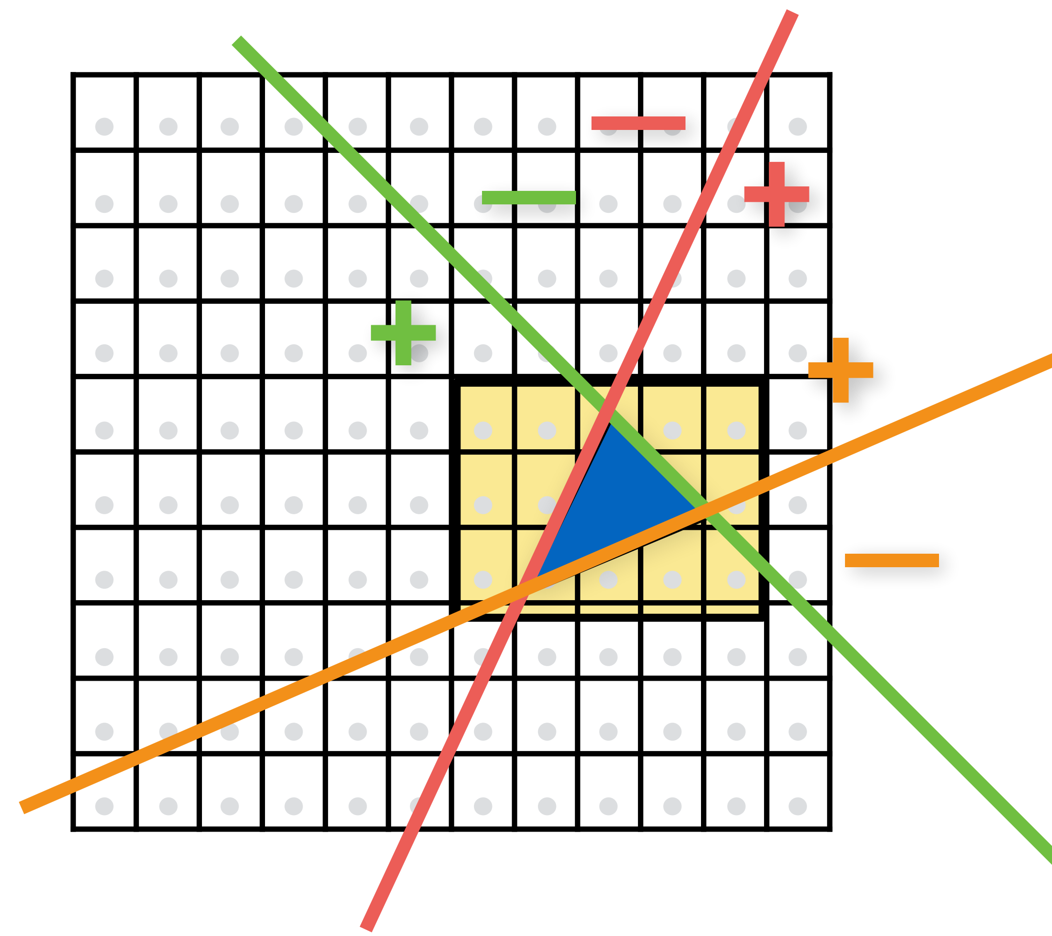

Options, Optimizations and Caveats

Could use barycentric coordinates instead of edge functions

Can operate on blocks of pixels before going per-pixel

Incremental computation possible, but...

- ... parallelism most important for hardware

Be careful about pixels on triangle edges!

- pixels should only be drawn once, but no holes between adjacent triangles

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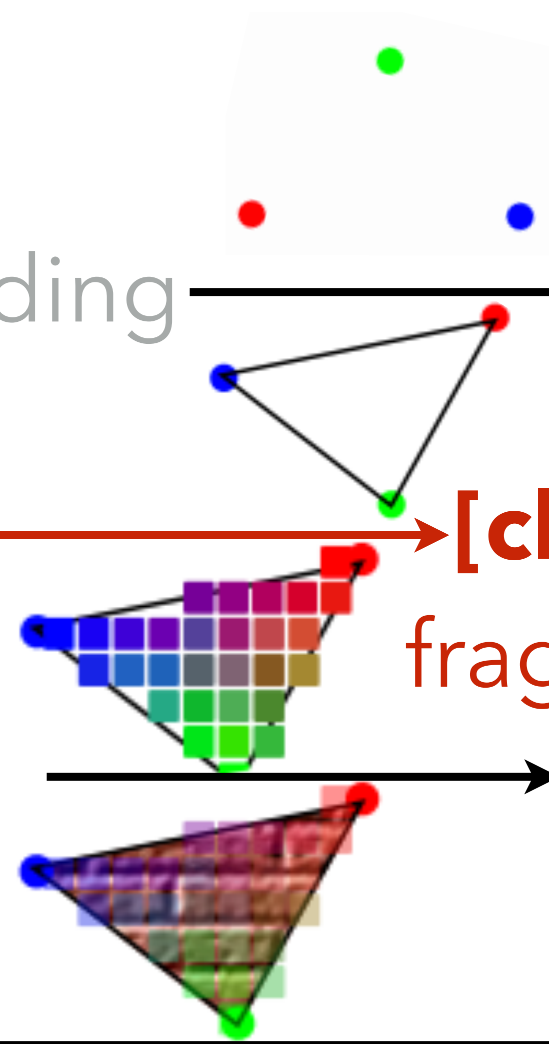

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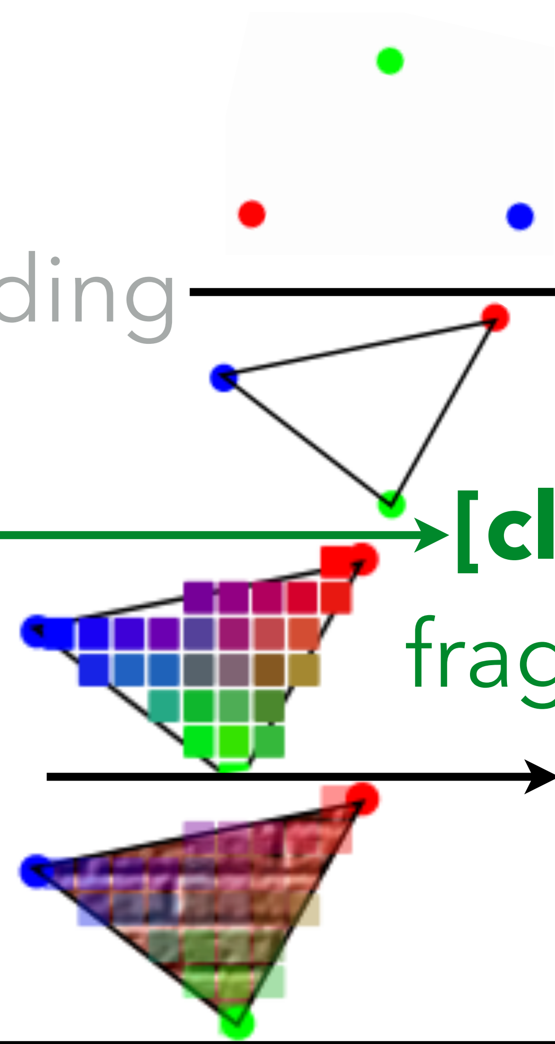
Programmable Rasterization Pipeline™



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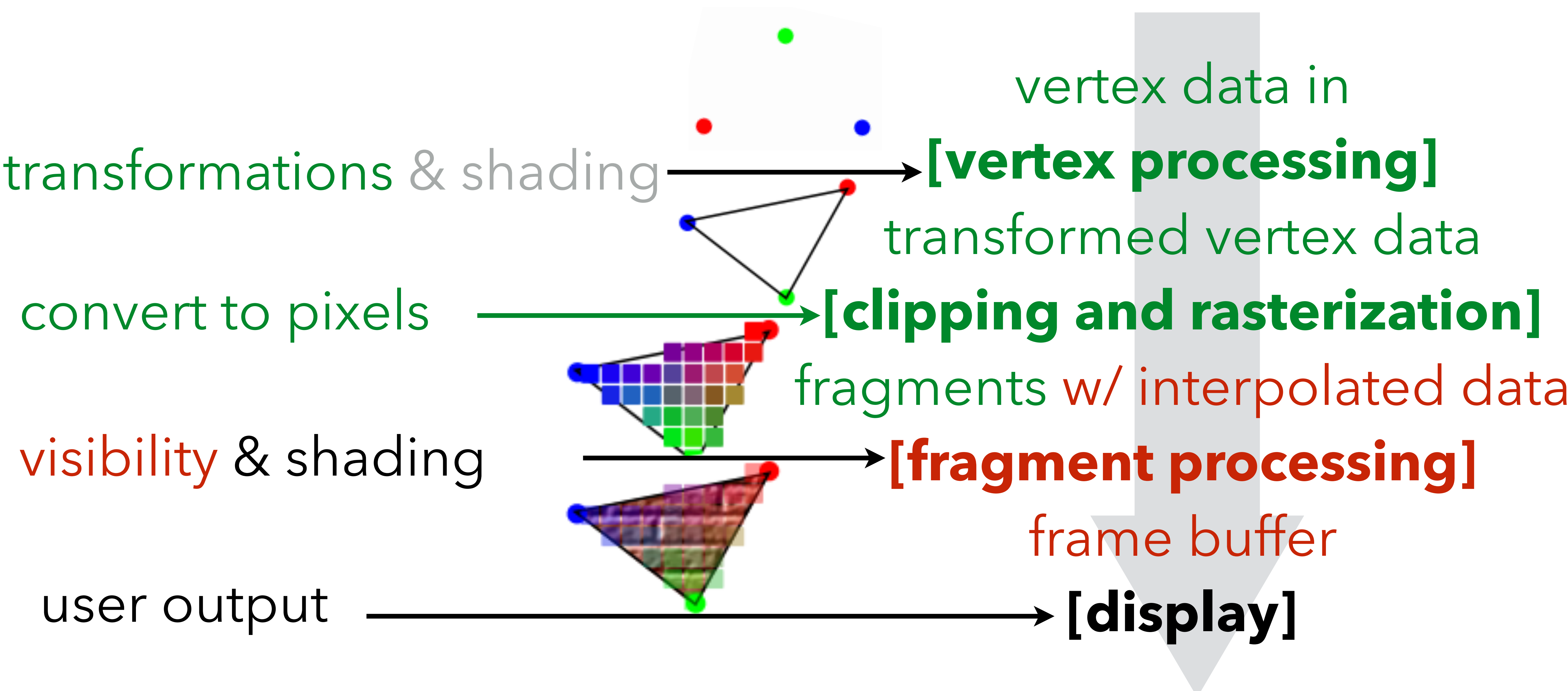

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                framebuffer[x,y] = shade()
```

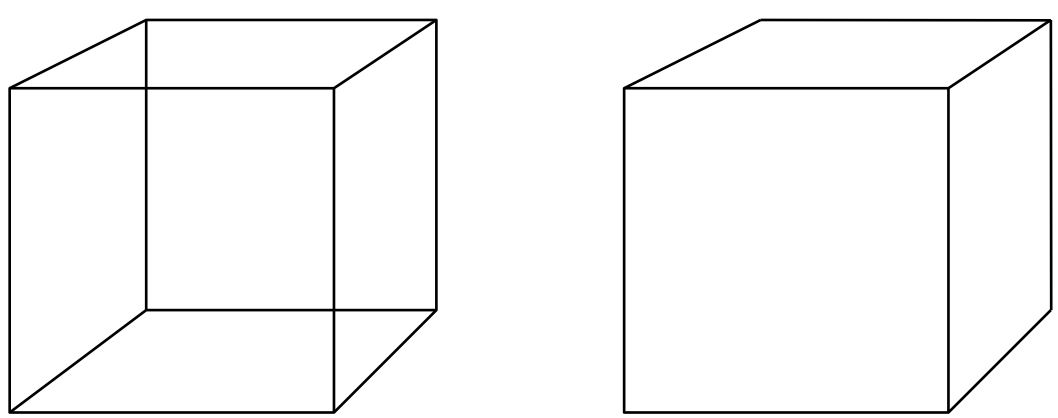
Hidden Surface Elimination

Visible Surface Determination

Hidden Surface Elimination

We have discussed how to map primitives to image space

- projection and perspective are depth cues
- occlusion is another very important cue



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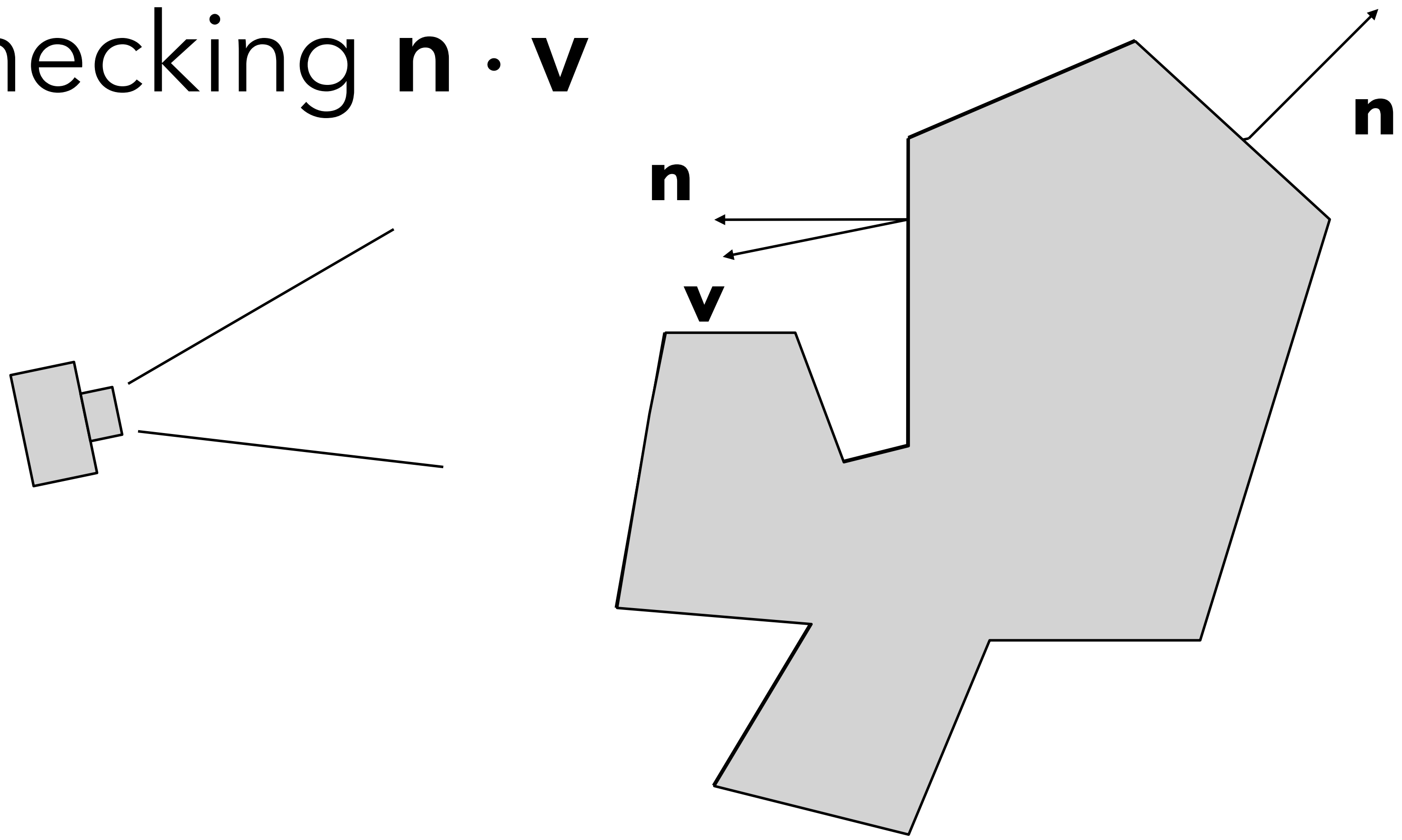
Backface Culling

For closed shapes you will never see the inside of the shape

- therefore, only draw surfaces that face the camera
- implement by checking $\mathbf{n} \cdot \mathbf{v}$



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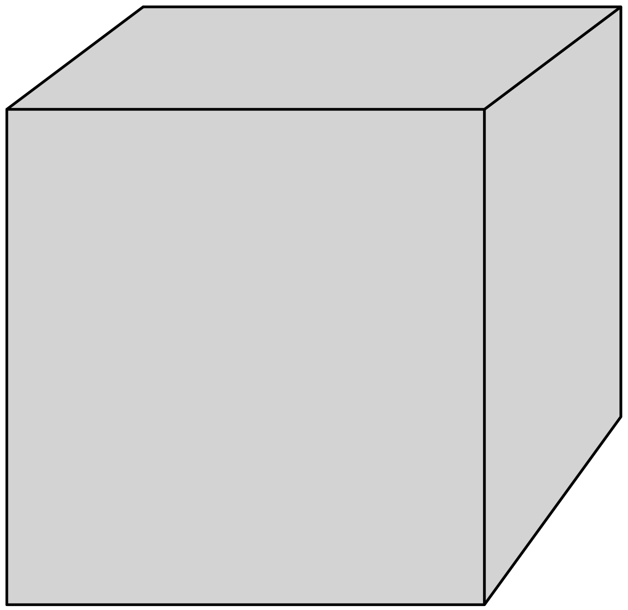
Painter's Algorithm

Simplest way to treat hidden surfaces:

- draw from back to front, overwriting the framebuffer along the way



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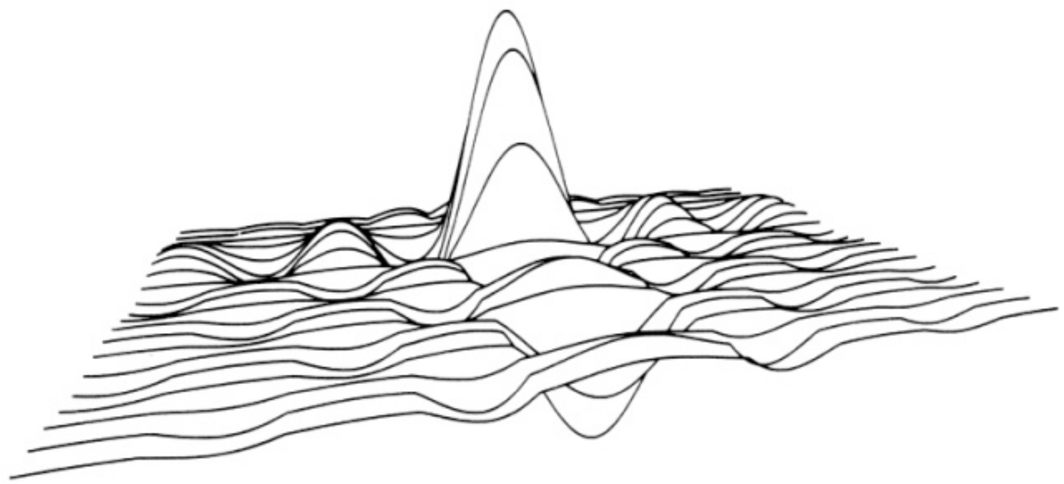
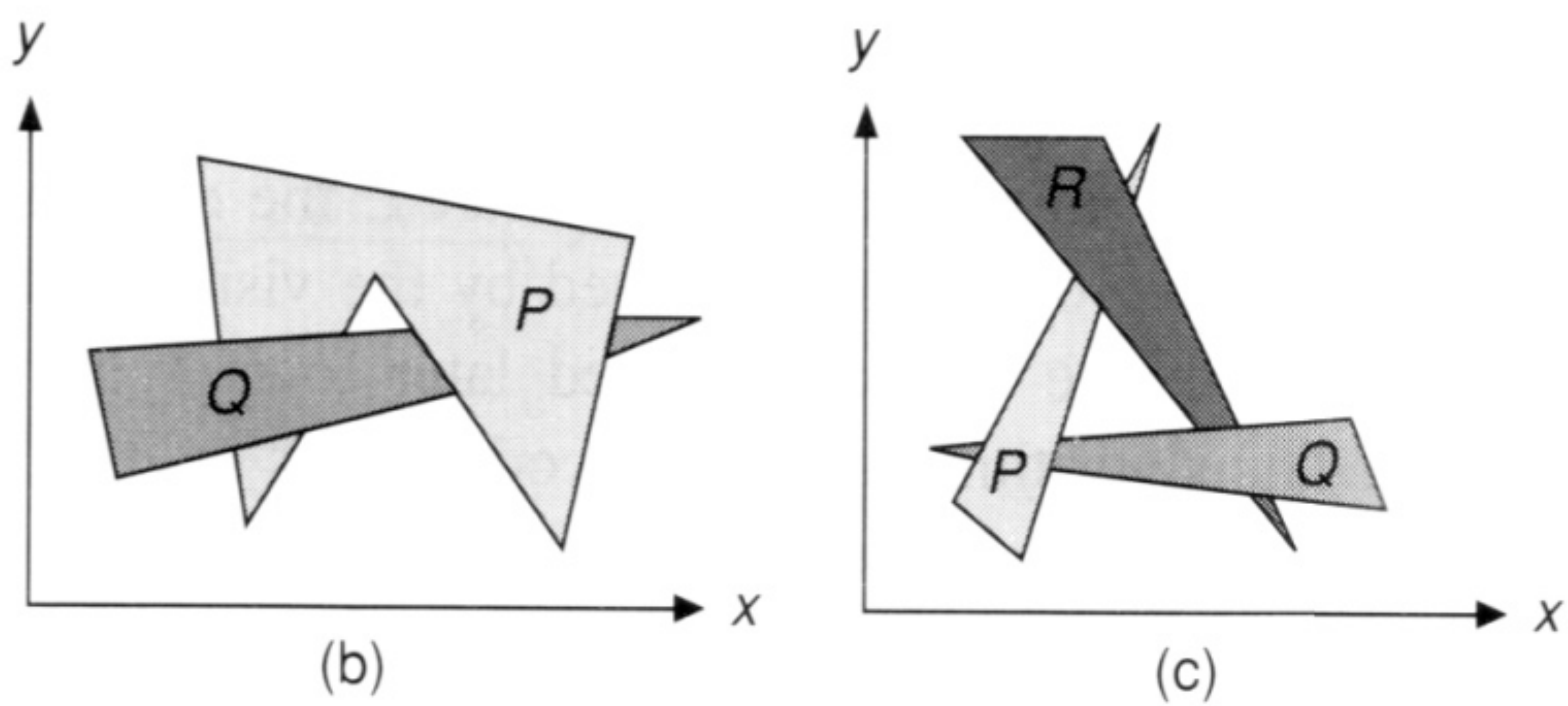
Painter's Algorithm

Useful when a valid order is easy to come by





[Foley et al.]

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The z-buffer (a.k.a. depth buffer)

In many/most applications, maintaining a z sort is too expensive

- changes all the time as the view changes
- many data structures exist, but complex

Solution: draw in any order, keep track of closest

- allocate extra channel per pixel to keep track of **closest depth so far**
- when drawing, compare object's depth to current closest depth, and discard if greater*
- works just like any other compositing operation

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The z-buffer

Another example of a memory-intensive, brute force approach that works and has become the standard

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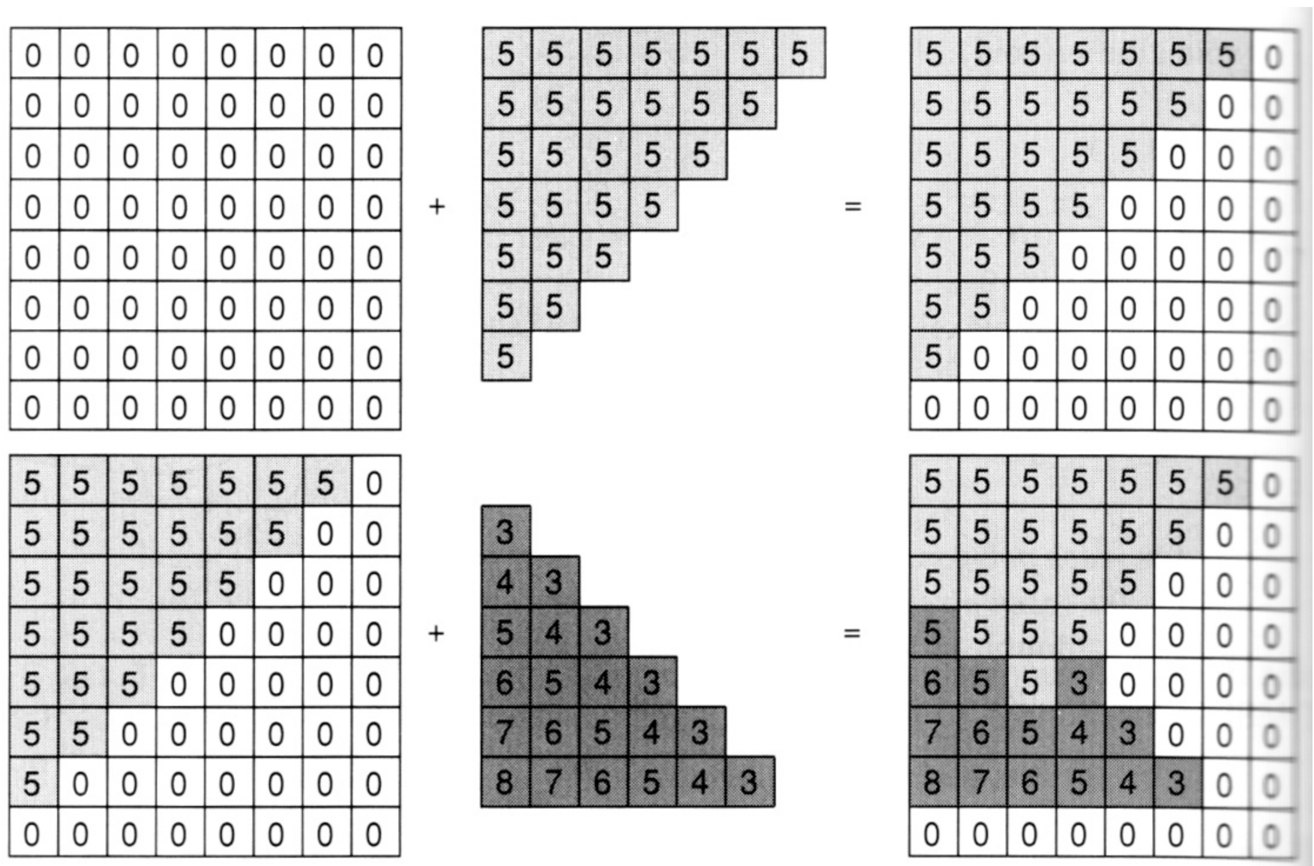
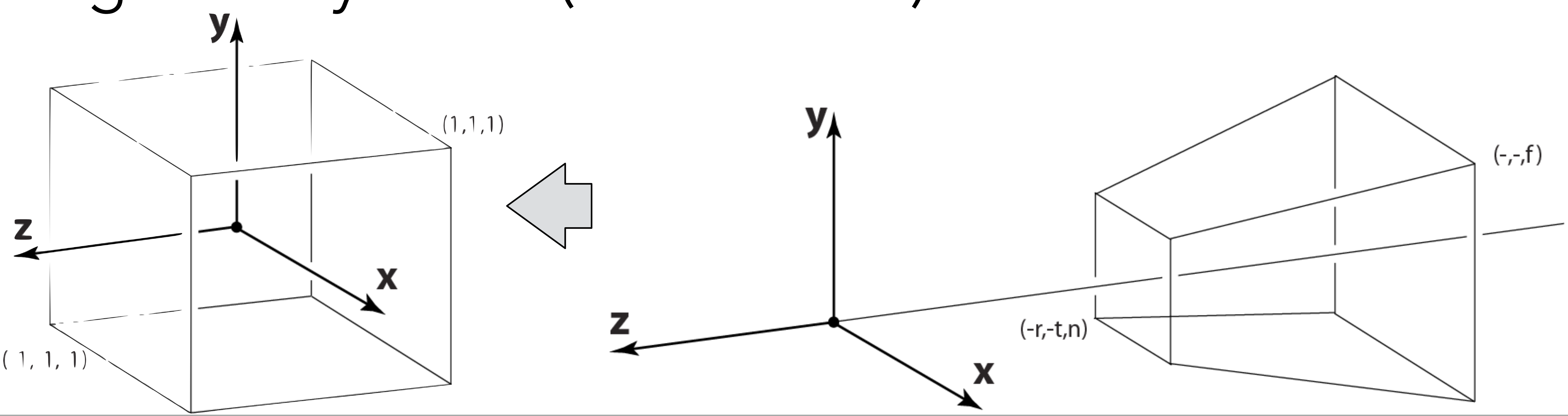


[Foley et al.]

Precision in the z-buffer

The precision is distributed between the near and far clipping planes

- this is why these planes have to exist
- also why you can't always just set them to very small and very large distances
- generally use z′ (not world z) in z buffer



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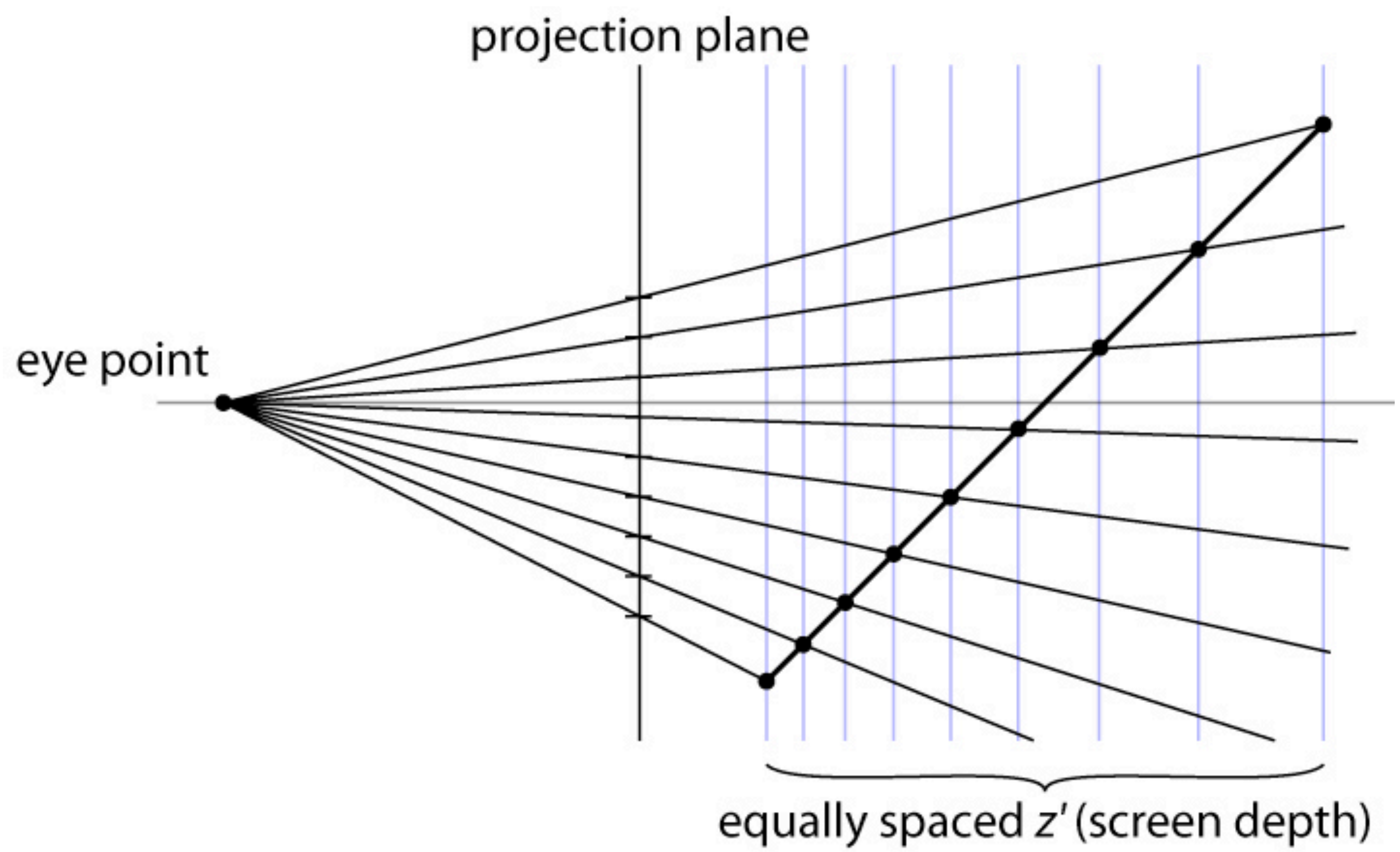
Interpolating in Projection



linear interp. in screen space ≠ linear interp. in world (eye) space

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Pipeline for Basic z-buffer

Vertex stage (input: position / vtx; color / tri)

- transform position (object to screen space)
- pass through color

Rasterizer

- interpolated parameter: z′ (screen z)
- pass through color

Fragment stage (output: color, z′)

- write to frame buffer only if interpolated z′ < current z′

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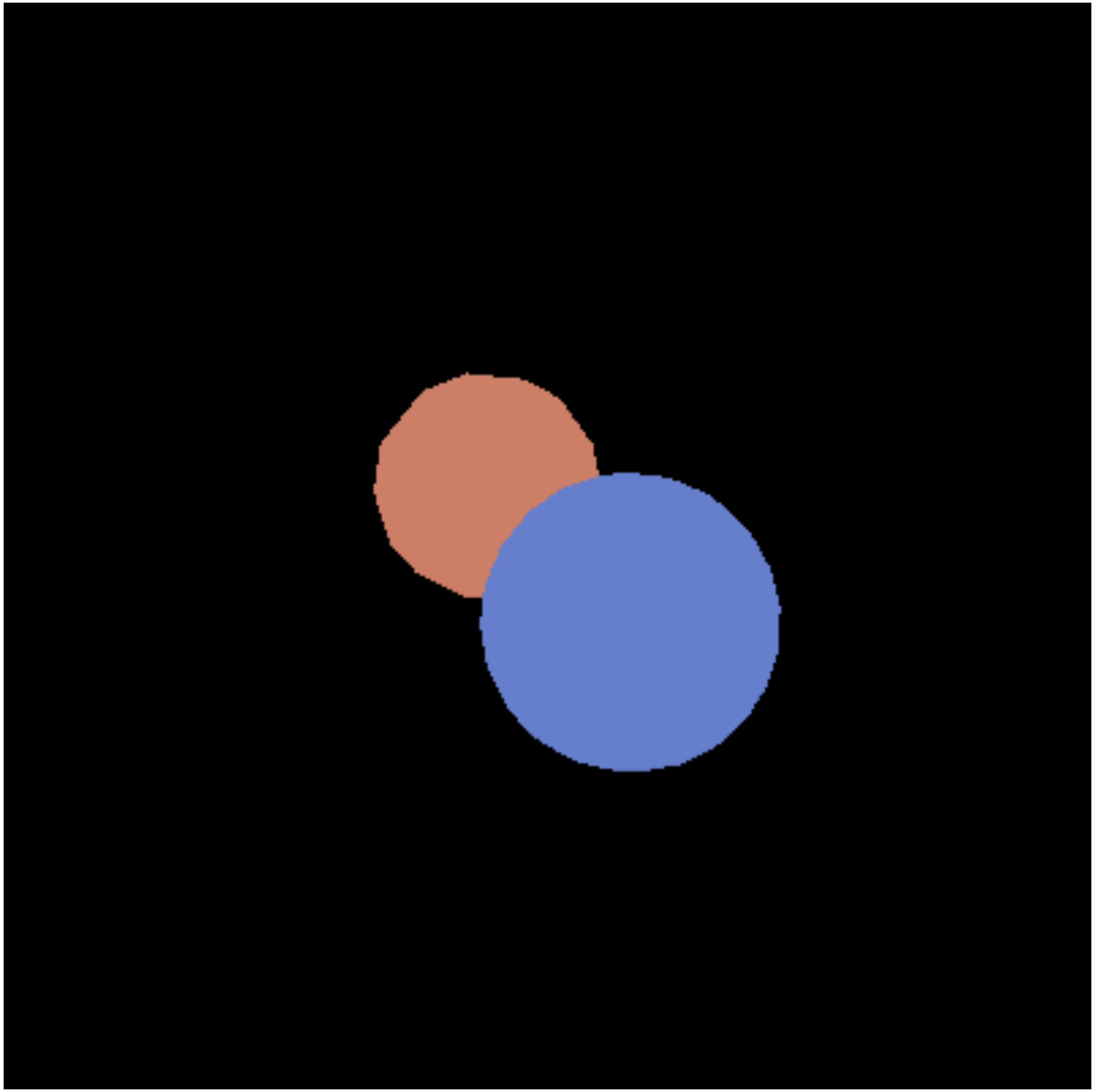
Result of Basic z-buffer Pipeline



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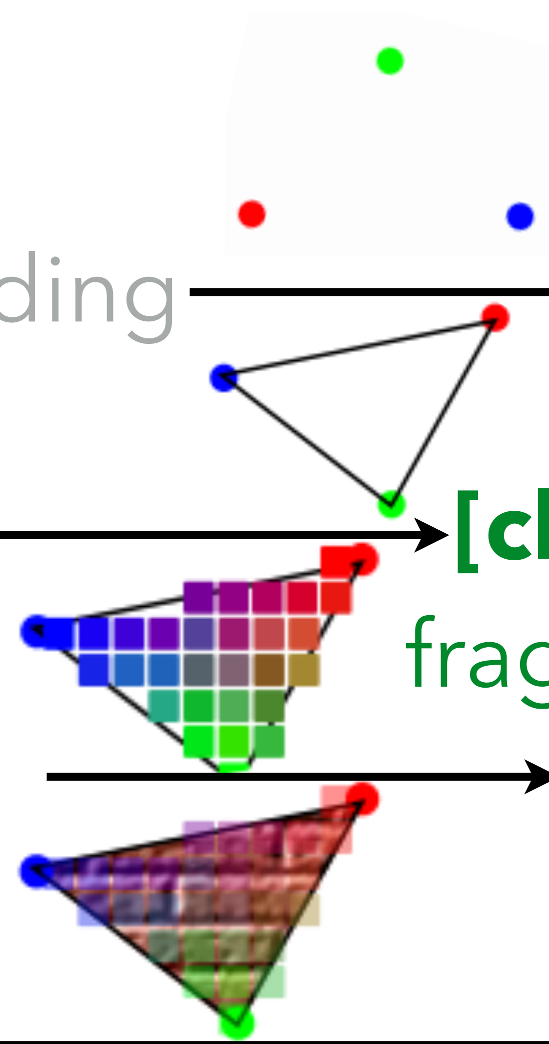
Programmable Rasterization Pipeline™



vertex data in

transformations & shading → **[vertex processing]**

transformed vertex data

convert to pixels → **[clipping and rasterization]**

fragments w/ interpolated data

visibility & shading → **[fragment processing]**

frame buffer

user output → **[display]**

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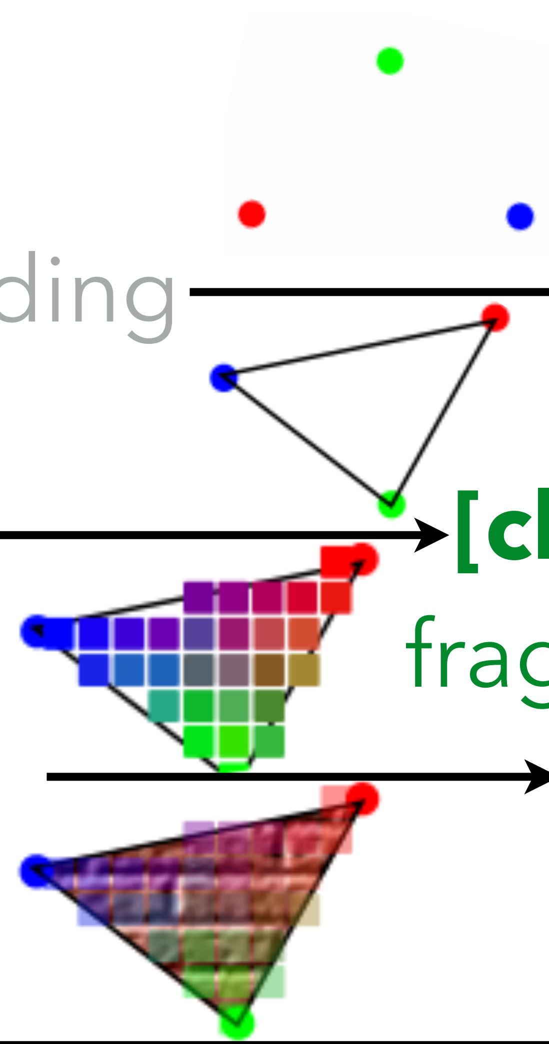
Programmable Rasterization Pipeline™



vertex data in
transformations & shading → **[vertex processing]**
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frame buffer
user output → **[display]**

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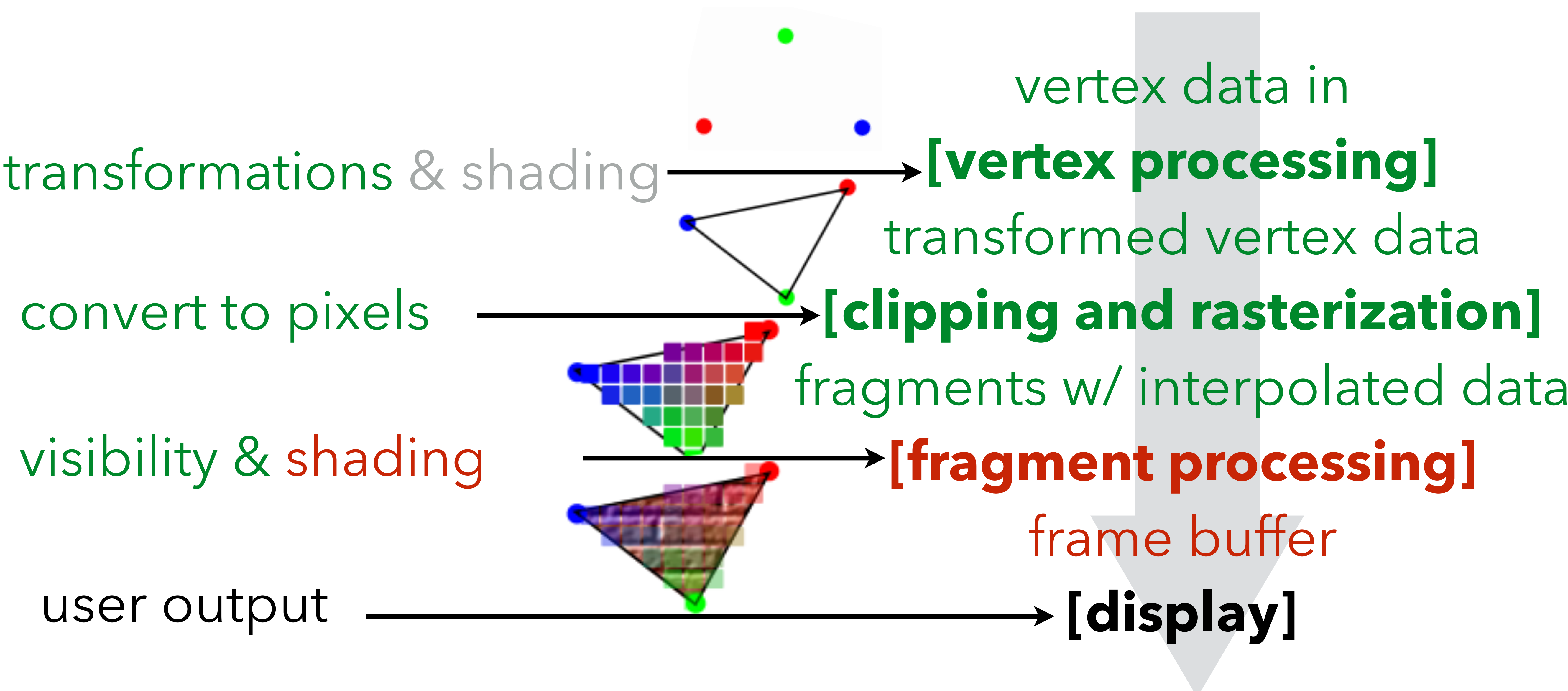
Programmable Rasterization Pipeline™



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Programmable Rasterization Pipeline™

```
for(each triangle)
    transform vertices into eye space
    project vertices to image space
    for(each pixel x,y)
        if(x,y in triangle)
            compute z
            if(z < zbuffer[x,y])
                zbuffer[x,y] = z
                framebuffer[x,y] = shade()
```