Nicholas Barrett
ID 110355429
AMS 582 Final Project Report
Project ID 5429 and Group Number 5

Introduction

This report is the summary of the design process for an experiment and the subsequent analysis to identify the independent variables that best predict the given results. We had many options in the requesting of our data, with the goal of minimizing cost and maximizing certainty about which factors generated the results. The resolution of the design is one of the most important aspects, as it determines the alias structure of the results. The higher the resolution of a design the more high-order interactions between variables can be investigated without aliasing. The cost of a higher resolution design is a higher number of runs. The number of levels the factors can take in an experiment also effects the alias structure and the complexity of the analysis required.

Methodology

For designing the experiment, we had the previously mentioned goal in mind. Given the cost structure of requesting runs, the grading structure of variable assessment, and the potential number of variables specified in the project, a two-level quarter fraction factorial design was selected. A full two-level design which for 10 factors would be 1024 runs would not be feasible given the number of points gained per variable identified, so a quarter fraction of 256 runs was specified. If there were at least 3 variables successfully identified, then the cost of these runs would be overshadowed by the points earned by identification. A base design of resolution V was folded on all factors for a resolution VI design. This has the advantage of all design generators being fourth order interactions and pushing all alias effects into fourth order or higher, allowing interactions of up to third order to be accurately evaluated. Also given the additional cost of requesting more runs of data, only 256 runs were planned to be requested. This design and the subsequent analysis were performed in the Minitab Software package as it contains many useful tools for designing and evaluating fractional factorial design experiments.

For analyzing the data, the goal is to gain as much confidence as possible that particular factors are involved in the generation of the dependent variables. Multiple factorial regression models were built to evaluate the factors. Stepwise, forward, and backward selection were all used to generate models from the possible range of models including up to third order interaction terms. The results of the F test and p values in the ANOVA table will be the determining factor for factor inclusion in the preliminary model selection. Since not all third order interaction terms can be evaluated simultaneously, first a forward selection and stepwise selection process generate models with up to second order terms. Any significant factors in these first models will then have their potential third order interactions added to the next round of models building. If any third order effects were found significant the fourth order effects involving those terms were also evaluated. Any factors that come out of this last selection process with an F value greater than 12 or p value less than .001 on the ANOVA table for the model are considered significant enough for inclusion. Although the F statistic and p values are the most important selection criteria at this stage, the R^2, adjusted R^2, and predicted R^2 values of the models are also of interest. The box cox transformation, and its effects on the normality of the residuals of the models was also evaluated. Some of this process was done on subsets of the requested data, but all tables and plots included in this report and appendix were generated using all requested observations.

Results

The results of the stepwise and forward selections are summarized in appendix 1 and 2. The most significant factors for further investigation at this stage are the factors A, C, E, G, H. Then the second stepwise selection process using a hierarchical model including the third and fourth order interactions with those factors of interest. This resulted in the most likely factors model shown in appendix 3. The factors and interactions being A, CE, CDG, CGI. The only factors or interactions with an F value greater than 12 are A and C*E. The final hierarchical model of A, C, E, CE is evaluated in the ANOVA table below.

Factorial Regression: y versus A, C, E

Method

Box-Cox transformation λ = 0

Analysis of Variance for Transformed Response

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	4	0.016220	0.004055	14.30	0.000
Linear	3	0.004062	0.001354	4.78	0.003
A	1	0.003258	0.003258	11.49	0.001
C	1	0.000744	0.000744	2.63	0.106
E	1	0.000060	0.000060	0.21	0.647
2-Way Interactions	1	0.012158	0.012158	42.88	0.000
C*E	1	0.012158	0.012158	42.88	0.000
Error	251	0.071172	0.000284		
Total	255	0.087392			

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	R-sq(pred)
0.0168390	18.56%	17.26%	15.28%

Coded Coefficients for Transformed Response

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		12.8734	0.0011	12231.93	0.000	
A	0.00713	0.00357	0.00105	3.39	0.001	1.00
C	0.00341	0.00171	0.00105	1.62	0.106	1.00
E	0.00097	0.00048	0.00105	0.46	0.647	1.00
C*E	0.01378	0.00689	0.00105	6.55	0.000	1.00

Regression Equation in Uncoded Units

ln(y) = 12.8734 + 0.00357 A + 0.00171 C + 0.00048 E + 0.00689 C*E

Table 1: ANOVA and Final Model

This is the final model with the variable y being transformed by the natural logarithm, as this transformation yielded the best results for the normality of the residuals. This is also the hierarchical model but based on the F/p values it is unlikely the C and E are included in addition to their interaction. The residual plots for this model are given in appendix 4. The ANOVA table and residual plots for the non-Hierarchical model is in appendix 5. The third order interaction terms CDG, CGI (shown as CGJ) were nearly

significant enough to include, but given the penalty for including incorrect variables, they were not included. The R^2 and Adjusted R^2 values decreases from ~.35 every time terms are removed from the model, but the predicted R squared value remains constant around .16 between all the models tested, including the final model of only 3 factors.

Variables	Model
A	In
B	Out
C	In
D	Out
E	In
F	Out
G	Out
H	Out
I	Out
J	Out

Table 2: Variable Inclusion

Conclusions and Discussion

The main conclusion of this report is that factors A, C, and E were used to generate the response variable y, with the CE interaction being the most significant factor.

The limitations of a fractional factorial design usually lie in the aliases of interactions of interest created by low resolution designs. Fortunately, the resolution 6 design aliases no second order effects with other second or third order effects. The limitation in this design is the aliasing of third and fourth order effects with each other, making distinguishing between the aliased interactions statistically impossible. In this case however no interactions higher than second order were found significant. The alias structure of the resolution VI design used is in appendix 5.

References

Montgomery, Douglas C. Design and Analysis of Experiments. New York :Wiley, 2017.

Appendix

Please note that Minitab did not like I as a factor name so factors (I, J) are displayed as (J, K in these tables and figures.

A.1 Stepwise Variable Selection

Factorial Regression: y versus A, B, C, D, E, F, G, H, J, K

The following terms are totally confounded with other terms and were removed:
F*H*K

Method
Box-Cox transformation λ = 0.5

Stepwise Selection of Terms
α to enter = 0.15, α to remove = 0.15
The stepwise procedure added terms during the procedure in order to maintain a hierarchical model at each step.

Analysis of Variance for Transformed Response

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	14	2269.75	162.12	6.26	0.000
Linear	6	542.48	90.41	3.49	0.002
A	1	317.44	317.44	12.26	0.001
B	1	0.05	0.05	0.00	0.964
C	1	71.82	71.82	2.77	0.097
E	1	6.10	6.10	0.24	0.628
G	1	46.03	46.03	1.78	0.184
H	1	101.03	101.03	3.90	0.049
2-Way Interactions	7	1671.47	238.78	9.22	0.000
A*G	1	186.49	186.49	7.20	0.008
B*G	1	78.26	78.26	3.02	0.083
C*E	1	1184.29	1184.29	45.73	0.000
C*G	1	2.85	2.85	0.11	0.740
C*H	1	92.40	92.40	3.57	0.060
E*G	1	123.94	123.94	4.79	0.030
G*H	1	3.24	3.24	0.13	0.724
3-Way Interactions	1	55.80	55.80	2.15	0.143
C*G*H	1	55.80	55.80	2.15	0.143
Error	241	6241.81	25.90		
Total	255	8511.56			

Model Summary for Transformed Response

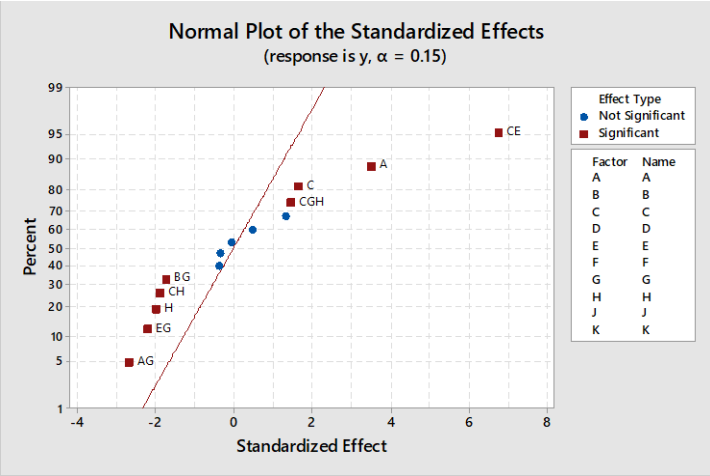
S	R-sq	R-sq(adj)	R-sq(pred)
5.08917	26.67%	22.41%	17.25%

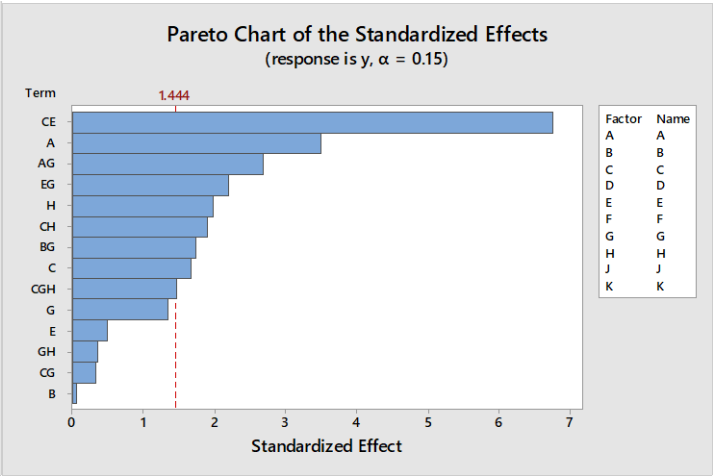
Coded Coefficients for Transformed Response

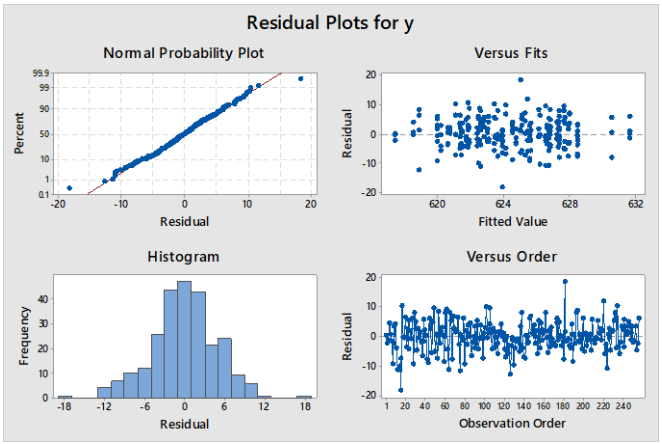
Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		624.355	0.318	1962.93	0.000	
A	2.227	1.114	0.318	3.50	0.001	1.00
B	-0.028	-0.014	0.318	-0.04	0.964	1.00
C	1.059	0.530	0.318	1.67	0.097	1.00
E	0.309	0.154	0.318	0.49	0.628	1.00
G	0.848	0.424	0.318	1.33	0.184	1.00
H	-1.256	-0.628	0.318	-1.98	0.049	1.00
A*G	-1.707	-0.854	0.318	-2.68	0.008	1.00
B*G	-1.106	-0.553	0.318	-1.74	0.083	1.00
C*E	4.302	2.151	0.318	6.76	0.000	1.00
C*G	-0.211	-0.106	0.318	-0.33	0.740	1.00
C*H	-1.202	-0.601	0.318	-1.89	0.060	1.00
E*G	-1.392	-0.696	0.318	-2.19	0.030	1.00
G*H	-0.225	-0.113	0.318	-0.35	0.724	1.00
C*G*H	0.934	0.467	0.318	1.47	0.143	1.00

Regression Equation in Uncoded Units

y^0.5 = 624.355 + 1.114 A - 0.014 B + 0.530 C + 0.154 E + 0.424 G - 0.628 H - 0.854 A*G
- 0.553 B*G + 2.151 C*E - 0.106 C*G - 0.601 C*H - 0.696 E*G - 0.113 G*H + 0.467 C*G*H







A.2 Stepwise Variable Selection with More Terms

Factorial Regression: y versus A, B, C, D, E, F, G, H, J, K

```
The following terms are totally confounded with other terms and were removed:
   F*H*K

Method
Box-Cox transformation  λ = 0.5

Stepwise Selection of Terms
α to enter = 0.15, α to remove = 0.15
The stepwise procedure added terms during the procedure in order to maintain a hierarchical
     model at each step.
```

Analysis of Variance for Transformed Response

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	32	3057.36	95.54	3.91	0.000
Linear	9	578.32	64.26	2.63	0.007
A	1	317.44	317.44	12.98	0.000
B	1	0.05	0.05	0.00	0.963
C	1	71.82	71.82	2.94	0.088
D	1	1.44	1.44	0.06	0.809
E	1	6.10	6.10	0.25	0.618
G	1	46.03	46.03	1.88	0.171
H	1	101.03	101.03	4.13	0.043
J	1	6.14	6.14	0.25	0.617
K	1	28.27	28.27	1.16	0.283
2-Way Interactions	17	1855.94	109.17	4.46	0.000
A*B	1	53.72	53.72	2.20	0.140
A*C	1	10.99	10.99	0.45	0.503
A*D	1	1.28	1.28	0.05	0.819
A*G	1	186.49	186.49	7.62	0.006
A*H	1	47.44	47.44	1.94	0.165
B*C	1	9.16	9.16	0.37	0.541
B*G	1	78.26	78.26	3.20	0.075
C*E	1	1184.29	1184.29	48.42	0.000
C*G	1	2.85	2.85	0.12	0.733
C*H	1	92.40	92.40	3.78	0.053
C*J	1	25.95	25.95	1.06	0.304
C*K	1	0.32	0.32	0.01	0.909
D*H	1	23.25	23.25	0.95	0.331
E*G	1	123.94	123.94	5.07	0.025
G*H	1	3.24	3.24	0.13	0.716
G*J	1	5.39	5.39	0.22	0.639
G*K	1	6.97	6.97	0.28	0.594
3-Way Interactions	6	623.10	103.85	4.25	0.000
A*B*C	1	153.26	153.26	6.27	0.013
A*C*G	1	51.05	51.05	2.09	0.150
A*D*H	1	95.02	95.02	3.88	0.050
C*G*H	1	55.80	55.80	2.28	0.132
C*G*J	1	168.72	168.72	6.90	0.009
C*G*K	1	99.26	99.26	4.06	0.045
Error	223	5454.20	24.46		
Total	255	8511.56			

Model Summary for Transformed Response

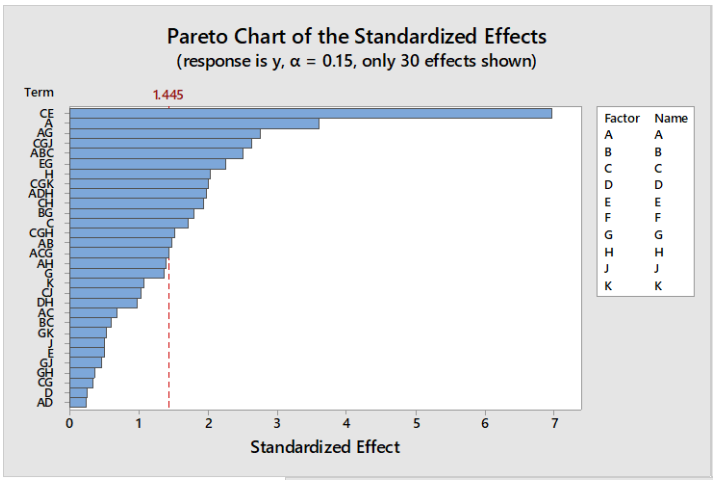
S	R-sq	R-sq(adj)	R-sq(pred)
4.94553	35.92%	26.72%	15.55%

Coded Coefficients for Transformed Response

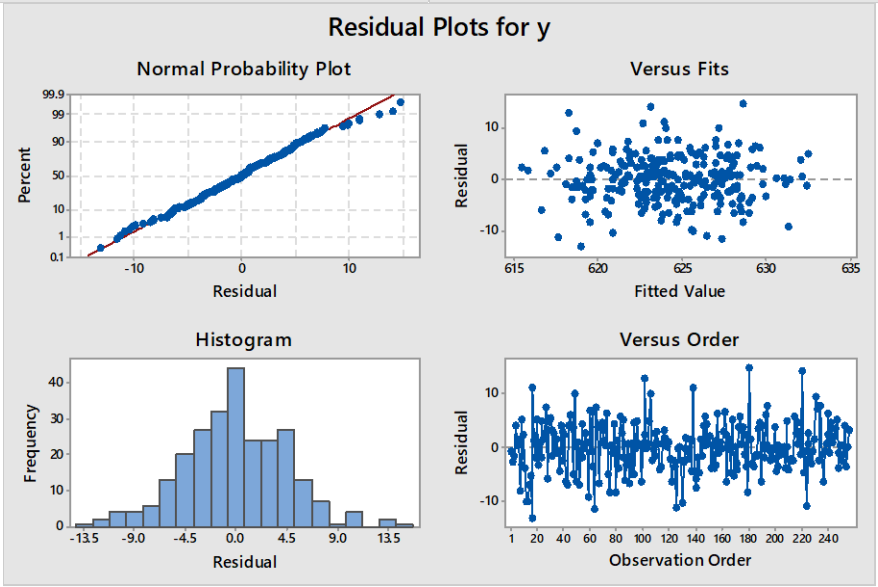
Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		624.355	0.309	2019.94	0.000	
A	2.227	1.114	0.309	3.60	0.000	1.00
B	-0.028	-0.014	0.309	-0.05	0.963	1.00
C	1.059	0.530	0.309	1.71	0.088	1.00
D	-0.150	-0.075	0.309	-0.24	0.809	1.00
E	0.309	0.154	0.309	0.50	0.618	1.00
G	0.848	0.424	0.309	1.37	0.171	1.00
H	-1.256	-0.628	0.309	-2.03	0.043	1.00
J	0.310	0.155	0.309	0.50	0.617	1.00
K	0.665	0.332	0.309	1.08	0.283	1.00
A*B	0.916	0.458	0.309	1.48	0.140	1.00
A*C	-0.414	-0.207	0.309	-0.67	0.503	1.00
A*D	0.141	0.071	0.309	0.23	0.819	1.00
A*G	-1.707	-0.854	0.309	-2.76	0.006	1.00
A*H	-0.861	-0.430	0.309	-1.39	0.165	1.00
B*C	-0.378	-0.189	0.309	-0.61	0.541	1.00
B*G	-1.106	-0.553	0.309	-1.79	0.075	1.00
C*E	4.302	2.151	0.309	6.96	0.000	1.00
C*G	-0.211	-0.106	0.309	-0.34	0.733	1.00
C*H	-1.202	-0.601	0.309	-1.94	0.053	1.00
C*J	0.637	0.318	0.309	1.03	0.304	1.00
C*K	-0.071	-0.035	0.309	-0.11	0.909	1.00
D*H	0.603	0.301	0.309	0.97	0.331	1.00
E*G	-1.392	-0.696	0.309	-2.25	0.025	1.00
G*H	-0.225	-0.113	0.309	-0.36	0.716	1.00
G*J	-0.290	-0.145	0.309	-0.47	0.639	1.00
G*K	-0.330	-0.165	0.309	-0.53	0.594	1.00
A*B*C	-1.547	-0.774	0.309	-2.50	0.013	1.00
A*C*G	-0.893	-0.447	0.309	-1.44	0.150	1.00
A*D*H	1.218	0.609	0.309	1.97	0.050	1.00
C*G*H	0.934	0.467	0.309	1.51	0.132	1.00
C*G*J	-1.624	-0.812	0.309	-2.63	0.009	1.00
C*G*K	1.245	0.623	0.309	2.01	0.045	1.00

Regression Equation in Uncoded Units

$$
\begin{aligned}
y^{0.5} = {} & 624.355 + 1.114\,A - 0.014\,B + 0.530\,C - 0.075\,D + 0.154\,E + 0.424\,G - 0.628\,H \\
& + 0.155\,J + 0.332\,K + 0.458\,A*B - 0.207\,A*C + 0.071\,A*D - 0.854\,A*G - 0.430\,A*H \\
& - 0.189\,B*C - 0.553\,B*G + 2.151\,C*E - 0.106\,C*G - 0.601\,C*H + 0.318\,C*J - 0.035\,C*K \\
& + 0.301\,D*H - 0.696\,E*G - 0.113\,G*H - 0.145\,G*J - 0.165\,G*K - 0.774\,A*B*C \\
& - 0.447\,A*C*G + 0.609\,A*D*H + 0.467\,C*G*H - 0.812\,C*G*J + 0.623\,C*G*K
\end{aligned}
$$


Pareto Chart of the Standardized Effects
(response is y, α = 0.15, only 30 effects shown)
Term
1.445
CE
A
AG
CGJ
ABC
EG
H
CGK
ADH
CH
BG
C
CGH
AB
ACG
AH
G
K
CJ
DH
AC
BC
GK
J
F
GJ
GH
CG
D
AD
0 1 2 3 4 5 6 7
Standardized Effect
Factor Name
A A
B B
C C
D D
E E
F F
G G
H H
J J
K K


Normal Plot of the Standardized Effects
(response is y, α = 0.15)
Percent
99 95 90 80 70 60 50 40 30 20 10 5 1
CE
A
CGK
ADH
C
CGH
AB
ACG
CGJ
CH
H
EG
ABC
CGJ
AG
-4 -2 0 2 4 6 8
Standardized Effect
Effect Type
Not Significant
Significant
Factor Name
A A
B B
C C
D D
E E
F F
G G
H H
J J
K K


Residual Plots for y
Normal Probability Plot
Percent
99.9 99 90 50 10 1 0.1
-10 0 10
Residual
Versus Fits
Residual
10 0 -10
615 620 625 630 635
Fitted Value
Histogram
Frequency
40 30 20 10 0
-13.5 -9.0 -4.5 0.0 4.5 9.0 13.5
Residual
Versus Order
Residual
10 0 -10
1 20 40 60 80 100 120 140 160 180 200 220 240
Observation Order

A.3 Most Likely Terms

Factorial Regression: y versus A, C, D, E, G, J

Method

Box-Cox transformation λ = 0

Analysis of Variance for Transformed Response

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	14	0.021480	0.001534	5.61	0.000
Linear	6	0.004625	0.000771	2.82	0.011
A	1	0.003258	0.003258	11.91	0.001
C	1	0.000744	0.000744	2.72	0.100
D	1	0.000014	0.000014	0.05	0.819
E	1	0.000060	0.000060	0.22	0.641
G	1	0.000487	0.000487	1.78	0.183
J	1	0.000062	0.000062	0.23	0.634
2-Way Interactions	6	0.012786	0.002131	7.79	0.000
C*D	1	0.000007	0.000007	0.02	0.876
C*E	1	0.012158	0.012158	44.45	0.000
C*G	1	0.000031	0.000031	0.11	0.738
C*J	1	0.000269	0.000269	0.98	0.322
D*G	1	0.000269	0.000269	0.98	0.323
G*J	1	0.000053	0.000053	0.19	0.661
3-Way Interactions	2	0.004069	0.002035	7.44	0.001
C*D*G	1	0.002324	0.002324	8.50	0.004
C*G*J	1	0.001745	0.001745	6.38	0.012
Error	241	0.065911	0.000273		
Total	255	0.087392			

Coded Coefficients for Transformed Response

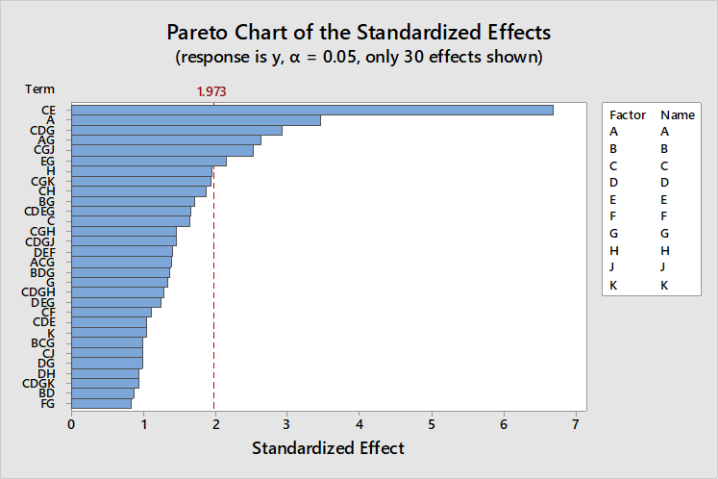
Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		12.8734	0.0010	12454.93	0.000	
A	0.00713	0.00357	0.00103	3.45	0.001	1.00
C	0.00341	0.00171	0.00103	1.65	0.100	1.00
D	-0.00047	-0.00024	0.00103	-0.23	0.819	1.00
E	0.00097	0.00048	0.00103	0.47	0.641	1.00
G	0.00276	0.00138	0.00103	1.33	0.183	1.00
J	0.00098	0.00049	0.00103	0.48	0.634	1.00
C*D	-0.00032	-0.00016	0.00103	-0.16	0.876	1.00
C*E	0.01378	0.00689	0.00103	6.67	0.000	1.00
C*G	-0.00069	-0.00035	0.00103	-0.34	0.738	1.00
C*J	0.00205	0.00102	0.00103	0.99	0.322	1.00
D*G	-0.00205	-0.00102	0.00103	-0.99	0.323	1.00
G*J	-0.00091	-0.00045	0.00103	-0.44	0.661	1.00
C*D*G	-0.00603	-0.00301	0.00103	-2.92	0.004	1.00
C*G*J	-0.00522	-0.00261	0.00103	-2.53	0.012	1.00

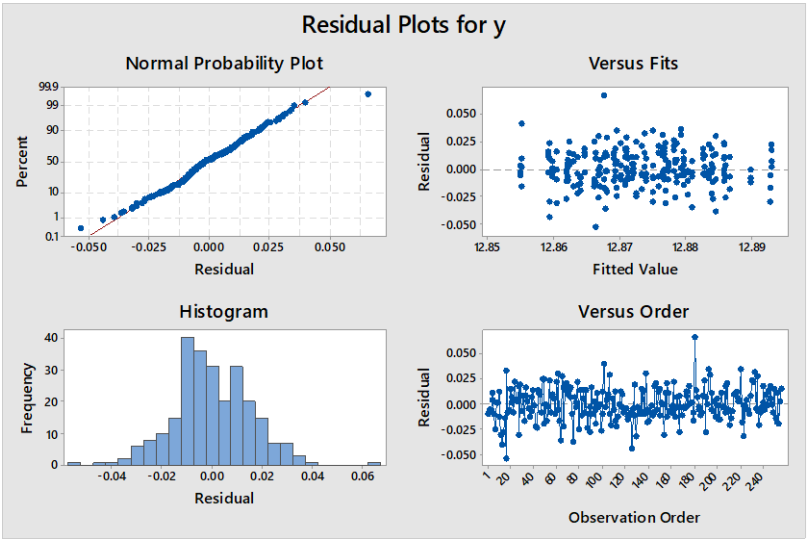
Regression Equation in Uncoded Units

$$\ln(y) = 12.8734 + 0.00357\,A + 0.00171\,C - 0.00024\,D + 0.00048\,E + 0.00138\,G + 0.00049\,J - 0.00016\,C*D + 0.00689\,C*E - 0.00035\,C*G + 0.00102\,C*J - 0.00102\,D*G - 0.00045\,G*J - 0.00301\,C*D*G - 0.00261\,C*G*J$$

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	R-sq(pred)
0.0165375	24.58%	20.20%	14.90%





A.4 Hierarchical Final Model

Factorial Regression: y versus A, C, E

Method

Box-Cox transformation λ = 0

Analysis of Variance for Transformed Response

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	4	0.016220	0.004055	14.30	0.000
Linear	3	0.004062	0.001354	4.78	0.003
A	1	0.003258	0.003258	11.49	0.001
C	1	0.000744	0.000744	2.63	0.106
E	1	0.000060	0.000060	0.21	0.647
2-Way Interactions	1	0.012158	0.012158	42.88	0.000
C*E	1	0.012158	0.012158	42.88	0.000
Error	251	0.071172	0.000284		
Total	255	0.087392			

Coded Coefficients for Transformed Response

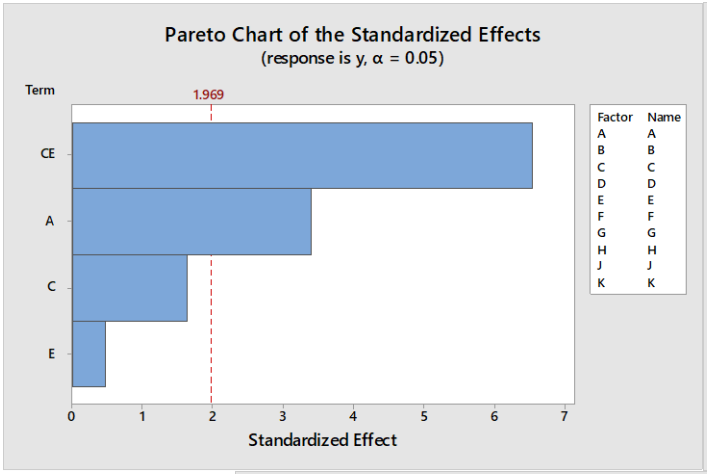
Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		12.8734	0.0011	12231.93	0.000	
A	0.00713	0.00357	0.00105	3.39	0.001	1.00
C	0.00341	0.00171	0.00105	1.62	0.106	1.00
E	0.00097	0.00048	0.00105	0.46	0.647	1.00
C*E	0.01378	0.00689	0.00105	6.55	0.000	1.00

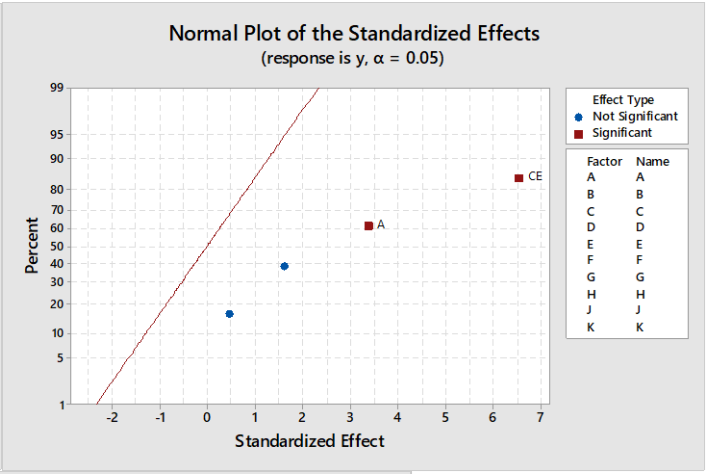
Regression Equation in Uncoded Units

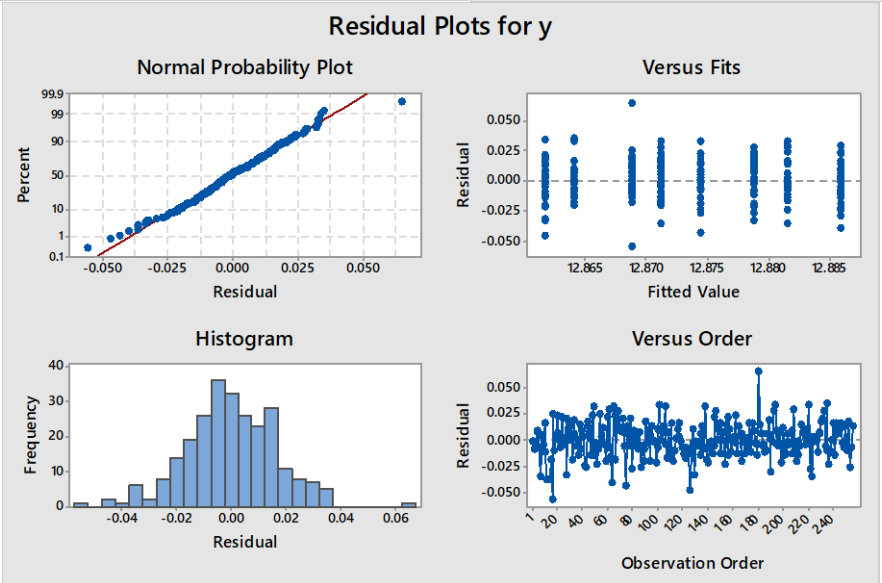
$$\ln(y) = 12.8734 + 0.00357\ A + 0.00171\ C + 0.00048\ E + 0.00689\ C*E$$

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	R-sq(pred)
0.0168390	18.56%	17.26%	15.28%







A.5 Non-Hierarchical Final Model

Factorial Regression: y versus A, C, E

Method

Box-Cox transformation λ = 0

Analysis of Variance for Transformed Response

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	2	0.015416	0.007708	27.09	0.000
Linear	1	0.003258	0.003258	11.45	0.001
A	1	0.003258	0.003258	11.45	0.001
2-Way Interactions	1	0.012158	0.012158	42.74	0.000
C*E	1	0.012158	0.012158	42.74	0.000
Error	253	0.071976	0.000284		
Total	255	0.087392			

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	R-sq(pred)
0.0168668	17.64%	16.99%	15.68%

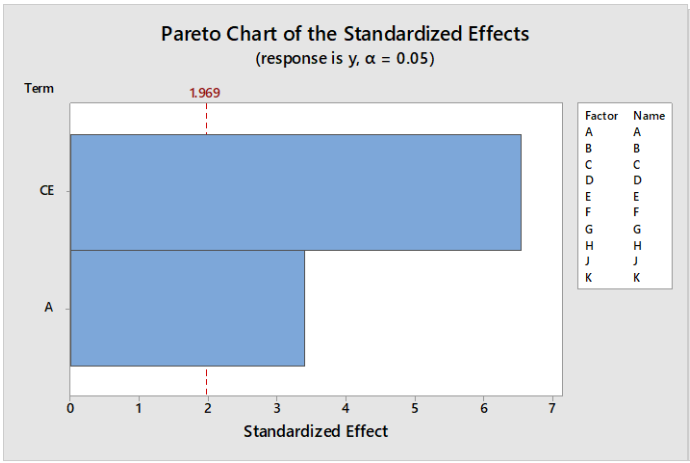
Coded Coefficients for Transformed Response

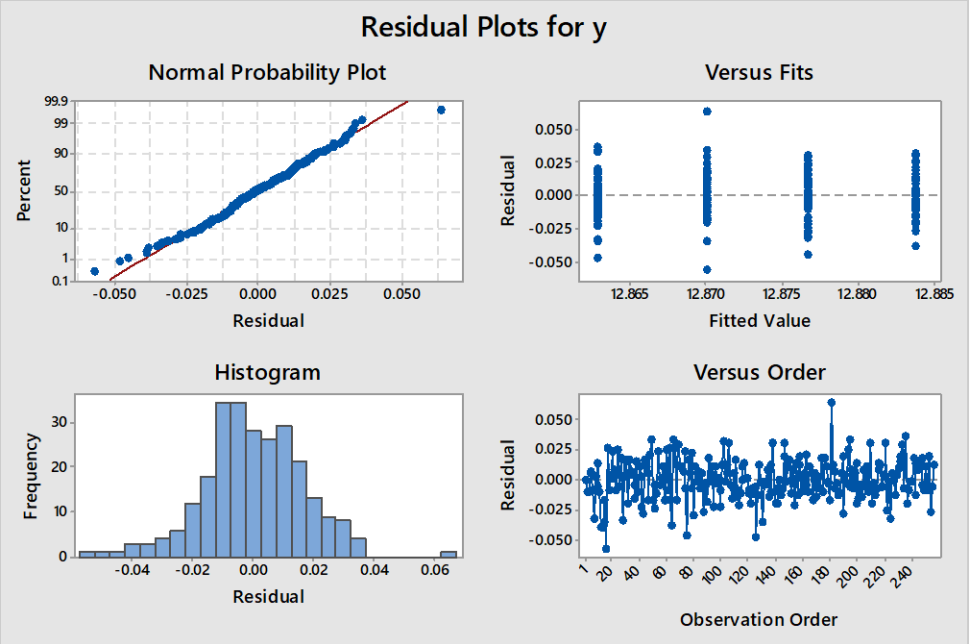
Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		12.8734	0.0011	12211.77	0.000	
A	0.00713	0.00357	0.00105	3.38	0.001	1.00
C*E	0.01378	0.00689	0.00105	6.54	0.000	1.00

Regression Equation in Coded Units

ln(y) = 12.8734 + 0.00357 A + 0.00689 C*E

Uncoded coefficients are not available with non-hierarchical model.





A.6 Fractional Factorial Design and Aliases

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Fractional Factorial Design

Factors:   10   Base Design:          10, 128   Resolution:   VI
Runs:     256   Replicates:                 1   Fraction:    1/4
Blocks:     1   Center pts (total):         0

Design Generators (before folding): H = ABCG, J = BCDE, K = ACDF
Folded on Factors: A, B, C, D, E, F, G, H, J, K

Alias Structure (up to order 4)
```

Alias

I

A	BD + FGHK	EF + ABJK	ABH	AEK + BFJ	BDG + FHK	CEG	EGK	ABEH + BD	ACFH	AHJK + BE	CEFG
B	BE + AFJK	EG + ADHJ	ABJ + EFK	AFG	BDH + FGK	CEH	EHK	ABFG + AD	ACFJ + BC	BCDE	CEFH
C	BF + AEJK	EH + ADGJ	ABK + EFJ	AFH	BDJ	CEJ	FGJ	ABFH + AD	ACFK + BC	BCDF + CG	CEGK
D	BG + DFHK	EJ + ABFK	ACD	AFJ + BEK	BDK + FGH	CEK	FHJ	ABGH + AD	ACGH + CD	BCDG + CF	CEHK
E	BH + DFGK	EK + ABFJ	ACE	AFK + BEJ	BEG	CFG	GJK	ABGJ + BD	ACGJ + CD	BCDH + CF	CFGJ
F	BJ + AEFK	FG + BDHK	ACF	AGH + DEJ	BEH	CFH	HJK	ABGK + AD	ACGK	BCDJ	CFHJ
G	BK + AEFJ	FH + BDGK	ACG	AGJ + DEH	BFG + DHK	CFJ	ABCD	ABHJ + BD	ACHJ + CD	BCDK + CF	CGJK
H	CD	FJ + ABEK	ACH	AGK	BFH + DGK	CFK	ABCE + CF	ABHK + AD	ACHK	BCEG	CHJK
J	CE	FK + ABEJ	ACJ	AHJ + DEG	BGH + DFK	CGH	ABCF + CE	ACDE + CG	ACJK + BC	BCEH	
K	CF	GH + ADEJ	ACK	AHK	BGJ	CGJ	ABCG	ACDF	ADEF + BD	BCFG + CD	
AB + EFJK	CG	GJ + ADEH	ADE + GHJ	AJK + BEF	BGK + DFH	CGK	ABCH	ACDG + CE	ADEK + BD	BCFH + CD	
AC	CH	GK + BDFH	ADF	BCD	BHJ	CHJ	ABCJ + CE	ACDH + CE	ADFJ + BD	BCGH + CD	
AD + EGHJ	CJ	HJ + ADEG	ADG + EHJ	BCE	BHK + DFG	CHK	ABCK + CE	ACDJ + CE	ADJK + BD	BCGJ	
AE + BFJK	CK	HK + BDFG	ADH + EGJ	BCF	CDE	CJK	ABDE + BG	ACDK	AEFG + BG	BCGK + CD	
AF + BEJK	DE + AGHJ	JK + ABEF	ADJ + EGH	BCG	CDF	DEF	ABDF + AG	ACEF + BC	AEFH + BH	BCHJ	
AG + DEHJ	DF + BGHK	ABC	ADK	BCH	CDG	DEK	ABDG + AF	ACEG + CD	AEGK + BF	BCHK + CD	
AH + DEGJ	DG + AEHJ	ABD	AEF + BJK	BCJ	CDH	DFJ	ABDH + AF	ACEH + CD	AEHK + BF	CDEF	
AJ + BEFK	DH + AEGJ	ABE + FJK	AEG + DHJ	BCK	CDJ	DJK	ABDJ + BE	ACEJ + BC	AFGJ + BE	CDEK	
AK + BEFJ	DJ + AEGH	ABF + EJK	AEH + DGJ	BDE	CDK	EFG	ABDK + AF	ACEK + BC	AFHJ + BE	CDFJ	
BC	DK + BFGH	ABG	AEJ + BFK	BDF + GHK	CEF	EFH	ABEG + BD	ACFG	AGJK + BE	CDJK	

Note: This is a long list folded on itself, it is not indicating that they are aliased column wise