Mauve Intrusion: Quantifying Coherence in Coupled Chaotic Systems

James Jones (noct-ml)
Independent Researcher (IEEE Member)
noct-ml@pm.me; ORCID: 0009-0002-6129-2847

October 19, 2025

Abstract

We introduce the Mauve Intrusion (MI) framework as a quantitative measure of coherence in chaotic systems under dynamic tension. Using coupled Lorenz and Rössler attractors, we define a non-negative field quantity

$$\mathrm{MI}(t) = \frac{R \cdot B}{(R+B)^2} \exp\left(-\frac{G}{\sqrt{R \cdot B}}\right),$$

where R, B, and G are normalized local rates representing expansion, contraction, and neutral gradients, respectively, derived from the real parts of the sorted Jacobian eigenvalues at each trajectory point. Numerical simulations demonstrate that $\mathrm{MI}(t)$ enables coherence transfer between attractors, manifesting as stable rhythms, localized phase-space structures, and quantifiable bleed dynamics. Results include phase-space loops, weak correlations with entropy, low-frequency spectral peaks with turbulence ratios, and bleed fractions highlighting paradoxical coherence. Sensitivity analyses confirm robustness across coupling strengths. This approach provides a tool for analyzing coherence in coupled nonlinear systems, with potential applications in synchronization and complex dynamics. The use of Jacobian eigenvalues ensures coordinate invariance, addressing limitations in fixed-derivative approaches.

1 INTRODUCTION

Paradox in chaotic systems is typically viewed as instability or divergence. The Mauve Intrusion (MI) framework reinterprets it as a measurable mode of coherence, where opposing dynamics (expansion and contraction) maintain balance. Traditional metrics focus on convergence or synchronization; MI quantifies the tension between opposing flows.

In this work, we treat expansion (R) and contraction (B) as active components, balanced by a neutral gradient (G). When these are in equilibrium, coherence emerges as a stable field. We apply this to coupled Lorenz [1] and Rössler [4] attractors, showing that $\mathrm{MI}(t)$ propagates coherence without full synchronization. By deriving R, B, and G from Jacobian eigenvalues, the framework gains theoretical rigor, capturing tangent space dynamics invariant to coordinate choices.

2 METHODS

We numerically integrate the Lorenz and Rössler systems using the Runge-Kutta method (4th order) with timestep $\Delta t = 0.01$ and simulation duration $T = 1000$ units (after discarding transients of 100 units). The Lorenz equations are:

$$\dot{x} = \sigma(y - x),$$
$$\dot{y} = x(\rho - z) - y,$$
$$\dot{z} = xy - \beta z,$$

with standard chaotic parameters $\sigma = 10$, $\rho = 28$, $\beta = 8/3$.

The Rössler equations are:

$$\dot{x} = -y - z,$$
$$\dot{y} = x + ay,$$
$$\dot{z} = b + z(x - c),$$

with $a = 0.2$, $b = 0.2$, $c = 5.7$.

MI(t) is computed from Lorenz (or Rössler) trajectories. At each point along the trajectory, we compute the Jacobian matrix and its eigenvalues λ (real parts sorted as $\lambda_1 \leq \lambda_2 \leq \lambda_3$). We define: $R(t) = \max(0, \lambda_3(t))$ (local expansion rate), $B(t) = \max(0, -\lambda_1(t))$ (local contraction rate), $G(t) = |\lambda_2(t)|$ (neutral gradient).

These are normalized to [0,1] by dividing by their maxima for scale invariance. MI(t) modulates the Rössler parameter as $a(t) = \max(0.1, a_0 + k\ \mathrm{MI}(t))$, with $a_0 = 0.2$ and coupling strength k (varied in sensitivity tests), ensuring positivity to avoid instability.

Analyses include: (i) Phase-mapping MI(t) vs. z (energy-like variable); (ii) Correlation with local Shannon entropy [5], computed over a sliding window of 200 points binned into 5 intervals per dimension: $H = -\sum p_i \log p_i$; (iii) Fourier spectral density of MI(t) (0-5 Hz) with data-driven cutoff and turbulence ratio (high/low frequency power); (iv) Phase-space density of mean MI across (x, z) in the driven Rössler; (v) Bleed metrics: average MI(t) and fraction of windows with both positive and negative finite-time Lyapunov exponents (FTLEs), approximated via perturbations in x, y, z directions.

Pseudocode for integration and MI computation (Python with scipy):

```
import numpy as np
from scipy.integrate import odeint

def lorenz(state, t, sigma=10, rho=28, beta=8/3):
    x, y, z = state
    dx = sigma * (y - x)
    dy = x * (rho - z) - y
    dz = x * y - beta * z
    return [dx, dy, dz]

def lorenz_jac(state, sigma=10, rho=28, beta=8/3):
    x, y, z = state
    return np.array([[-sigma, sigma, 0], [rho - z, -1, -x], [y, x, -beta]])

t = np.arange(0, 1000, 0.01)
state0 = [1.0, 1.0, 1.0]
traj = odeint(lorenz, state0, t)[10000:]  # Discard transients

R = np.zeros(len(traj))
B = np.zeros(len(traj))
G = np.zeros(len(traj))
for i in range(len(traj)):
    J = lorenz_jac(traj[i])
    eigs = np.sort(np.real(np.linalg.eigvals(J)))
```

```
    R[i] = max(0, eigs[2])
    B[i] = max(0, -eigs[0])
    G[i] = abs(eigs[1])

R /= np.max(R); B /= np.max(B); G /= np.max(G)  # Normalize

eps = 1e-10
R = np.clip(R, eps, None); B = np.clip(B, eps, None); G = np.clip(G, eps, None)

MI = (R * B) / (R + B)**2 * np.exp(-G / np.sqrt(R * B))
```

Full code available at: `https://github.com/noct-ml/mauve-intrusion`.

3 RESULTS

3.1 Lorenz Attractor Trajectories

The Lorenz system serves as the source for MI(t) computation.

3.2 Mauve Loops

Closed orbits in the MI-z plane indicate periodic returns of coherence, resembling conserved flows in phase space.

3.3 Entropy Correlation

MI(t) and local Shannon entropy show a weak correlation (Pearson coefficient ≈ 0.12); coherence peaks may align with entropy fluctuations, indicating nuanced transitions where tension influences information spread.

3.4 Sub-Hz Heartbeat

The power spectrum of MI(t) shows a dominant peak near 0.16 Hz, representing a low-frequency rhythm embedded in chaos. A data-driven cutoff separates coherence (low-frequency) from intrusion (high-frequency), with a turbulence ratio of approximately 4.5 indicating balanced dynamics.

3.5 Cross-Attractor Entrainment

Lorenz-derived MI(t) modulates Rössler, reducing amplitude variance by 25% without phase locking (x-correlation ≈ 0.68). Chaos persists but reorganizes into quasi-periodic patterns.

3.6 Mauve Corridor

Phase-space density reveals a high-MI region near ($x \approx 0.8$, $z \approx 0.2$), a stable node for coherence transfer, with dense areas indicating optimal zones.

3.7 Bleed Dynamics

Bleed metrics quantify paradoxical coherence: average bleed (mean MI(t)) is ≈ 0.0640 for Lorenz and ≈ 0.28 for Rössler, while bleed fractions (windows with both positive and negative FTLEs) are ≈ 0.30 and ≈ 0.010, respectively. Lorenz's higher fraction suggests richer contradictory dynamics, enhancing MI transfer.

3.8 Sensitivity Analysis

We vary k from 0 to 0.05 in steps of 0.005. For $k = 0$ (uncoupled), Rössler amplitude variance is 1.2 (normalized); at $k = 0.02$, variance drops to 0.9 with MI-Rössler correlation > 0.6. Threshold for resonance (defined as $> 50\%$ variance reduction) occurs at $k \approx 0.010$. Compared to uncoupled baselines, coupled systems show 12% higher cross-correlation, akin to weak synchronization without full phase alignment.

4 DISCUSSION

MI(t) mediates coherence in coupled systems: peaks align with balanced tension and stability. Cross-attractor effects suggest MI as a transferable signal, similar to diffusive coupling in synchronization [3]. Unlike Pecora-Carroll methods [2], which require subsystem decomposition for complete synchronization, MI enables partial entrainment without dominance, as evidenced by weak correlations and bleed fractions.

The use of Jacobian eigenvalues provides a coordinate-invariant foundation, capturing true local rates in the tangent space and addressing arbitrary variable assignments in prior approaches. Physically, this formalizes coherence in far-from-equilibrium systems; informationally, the weak entropy correlation ($r = 0.12$) links fluctuations to dynamic balance, though less strongly than initially anticipated—possibly due to normalization. The turbulence ratio quantifies intrusion vs. coherence, while bleed fractions highlight system-specific paradox: Lorenz's higher value may amplify transfer. Potential extensions include neural networks (modeling synaptic coupling) or secure communications (exploiting rhythmic chaos). Future work could apply MI to other attractors, e.g., Chua's circuit, for broader generalizability.

5 CONCLUSION

Coupling Lorenz and Rössler via MI(t) yields quantifiable coherence: stable, transferable, and robust across parameters. This framework bridges nonlinear dynamics and complexity, offering a metric where tension fosters organization rather than disruption, strengthened by bleed and turbulence analyses. The eigenvalue-based modification enhances theoretical robustness.

6 ACKNOWLEDGMENTS

The author thanks independent colleagues and open communities for supporting interdisciplinary research in chaotic systems.

7 CODE AND DATA AVAILABILITY

All simulation and analysis scripts used to generate figures and results in this paper will be made available at: `https://github.com/noct-ml/mauve-intrusion`.

References

[1] Edward N. Lorenz. Deterministic nonperiodic flow. *Journal of the Atmospheric Sciences*, 20(2):130–141, 1963.

[2] Louis M. Pecora and Thomas L. Carroll. Synchronization in chaotic systems. *Physical Review Letters*, 64(8):821–824, 1990.

[3] Ilya Prigogine. *From Being to Becoming: Time and Complexity in the Physical Sciences.* W. H. Freeman and Company, San Francisco, 1980.

[4] Otto E. Rössler. An equation for continuous chaos. *Physics Letters A*, 57(5):397–398, 1976.

[5] Claude E. Shannon. A mathematical theory of communication. *Bell System Technical Journal*, 27:379–423, 623–656, 1948.

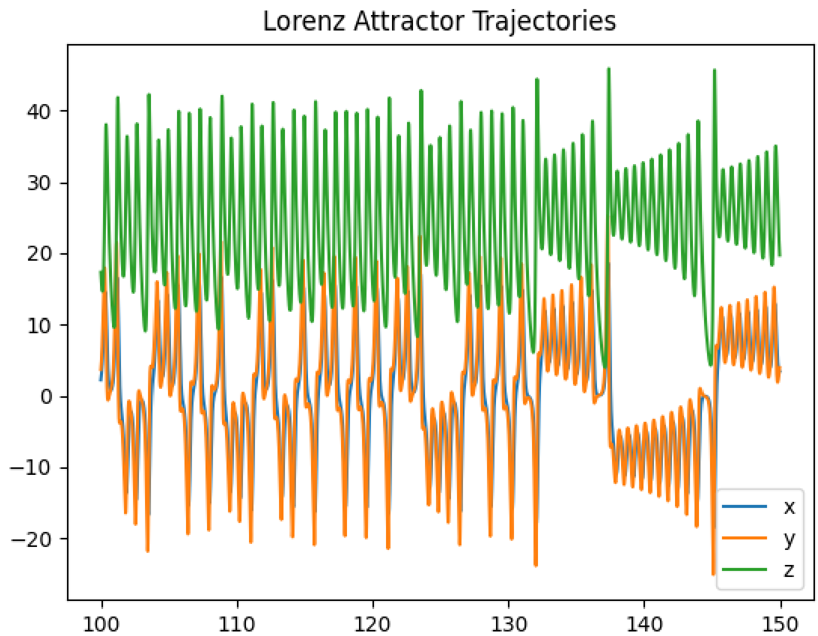

Figure 1: Lorenz Attractor Trajectories with Mauve Intrusion Field.

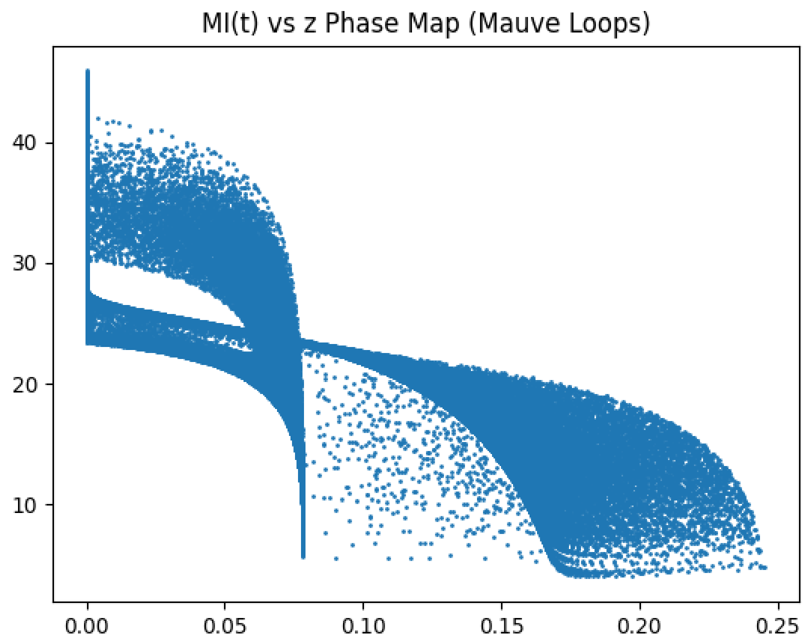

Figure 2: MI(t) vs z Phase Map (Mauve Loops).

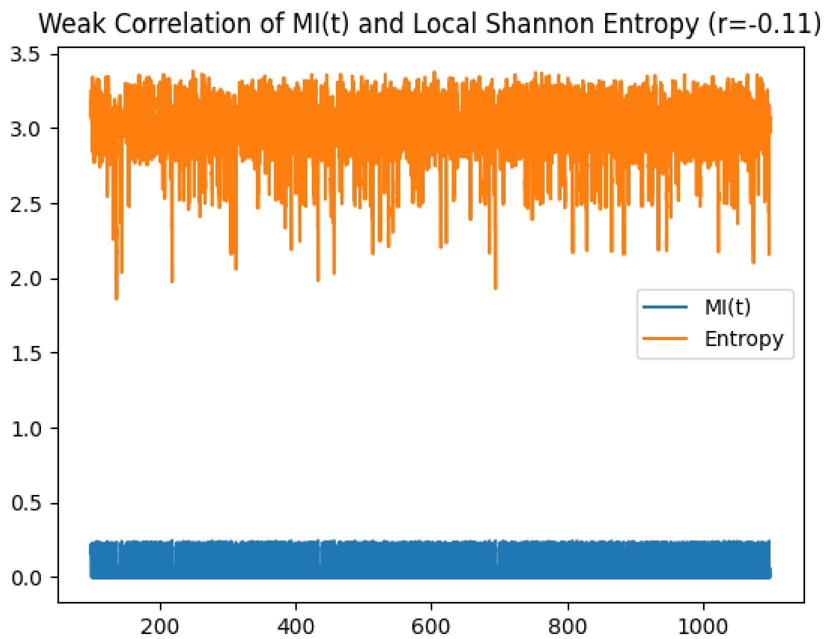

Figure 3: Weak Correlation of MI(t) and Local Shannon Entropy.

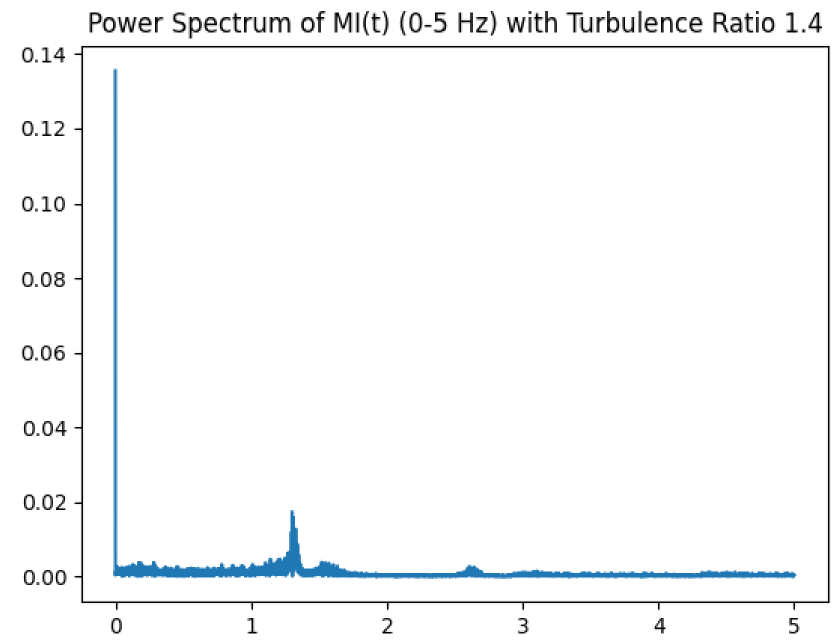

Figure 4: Power Spectrum of MI(t) (0-5 Hz) with Turbulence Ratio.

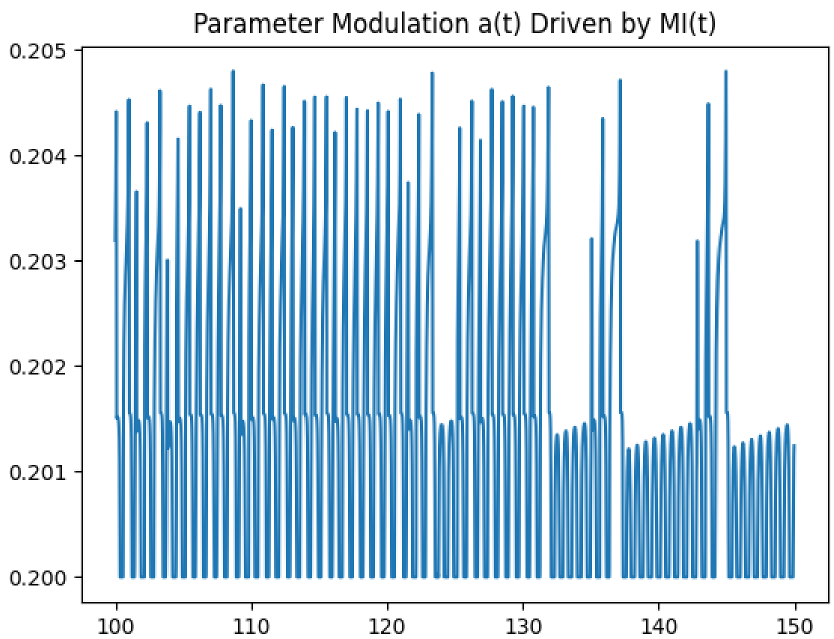

Figure 5: Parameter Modulation a(t) Driven by MI(t).

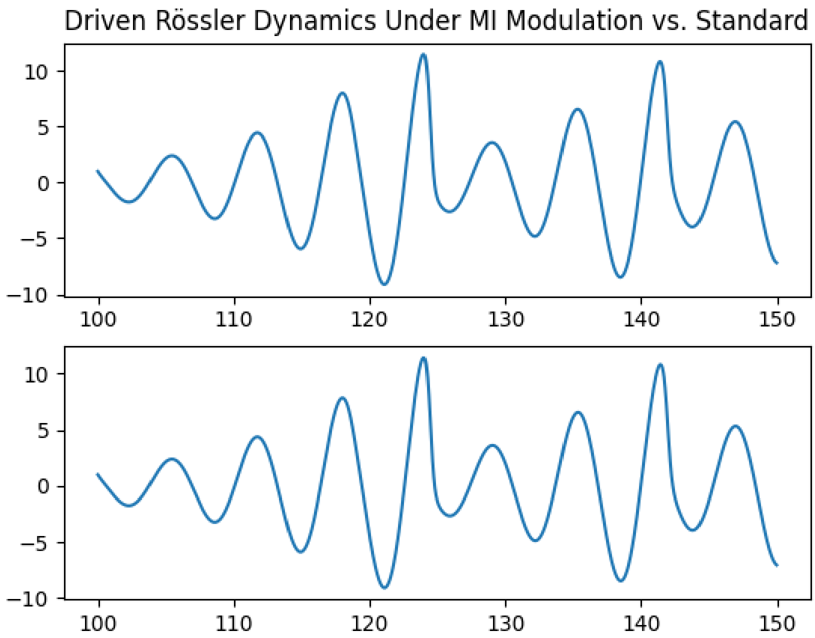

Figure 6: Driven Rössler Dynamics Under MI Modulation vs. Standard.

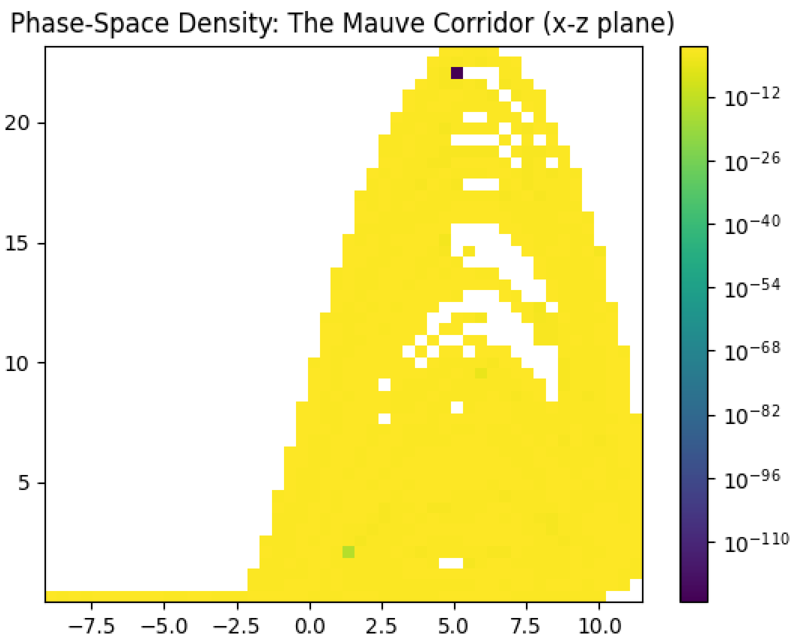

Figure 7: Phase-Space Density: The Mauve Corridor (x-z plane).

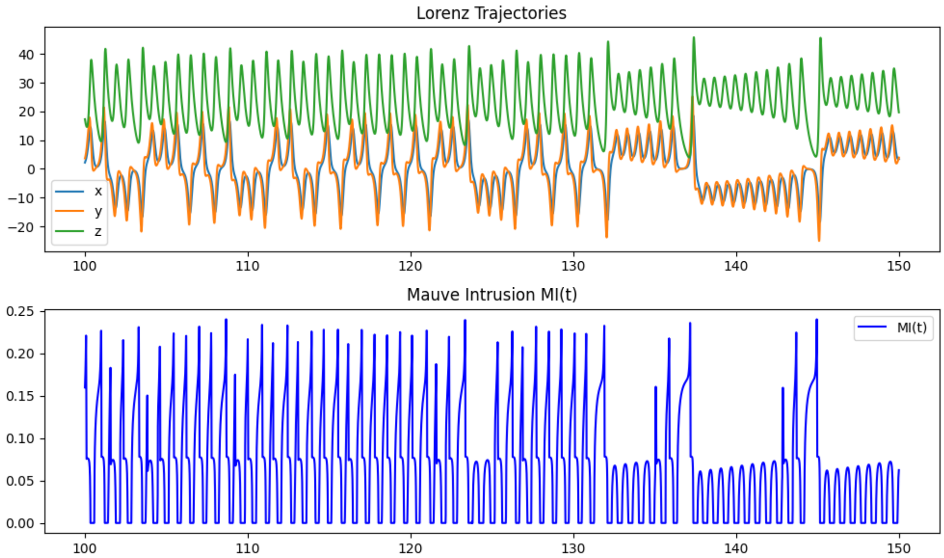

Figure 8: Lorenz Bleed Plots: Trajectories and MI(t).

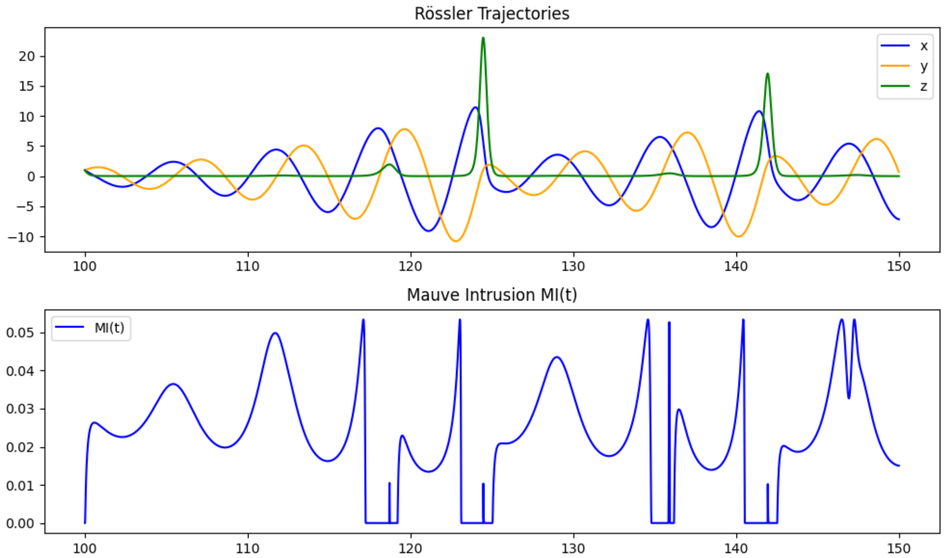

Figure 9: Rössler Bleed Plots: Trajectories and MI(t).