The calculation of inelastic neutrino/dark matter nucleus scattering

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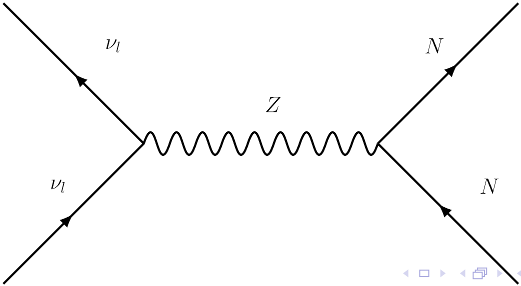
νN scattering: Elastic cross section

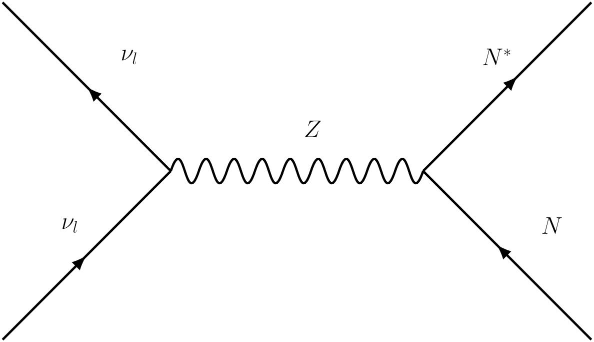
Standard Model CEνNS (coherent elastic neutrino-nucleus scattering) cross section **Drukier, Stodolsky, PRD, 1984** **Barranco, Miranda, Rashba, JHEP, 2005** **Patton, Engel, McLaughlin, Schunck, PRC 2012**

$$\frac{d\sigma}{dE_r} = \frac{G_F^2}{4\pi} m_N \left(1 - \frac{E_r}{E_i} - \frac{m_N E_r}{2E_i^2}\right)[(1 - 4\sin^2\theta_W)Z - N]^2 F_W^2$$

where m_N target nucleus mass, E_i incoming neutrino energy, E_r recoil energy, Z atomic number, N neutron number, F_W form factor, $\sin^2\theta_W \approx 0.23$ Weinberg angle



νN scattering: Inelastic cross section



$$\frac{d\sigma^{\nu}_{inel}}{d\Omega} = \frac{2G_F^2}{\pi(2J+1)} E_f^2 \; FF(2m_N E_r)$$

E_f outgoing neutrino energy

Walecka, Theoretical Nuclear and Subnuclear Physics, 2004

νN scattering: Inelastic cross section

$$FF(2m_N E_r) = \sum_{J\geqslant 1, spin} \left[\frac{1}{2}(\vec{l}\cdot\vec{l}^* - l_3 l_3^*)\left(|\langle J_f||\mathcal{T}^{mag}||J_i\rangle|^2 + |\langle J_f||\mathcal{T}^{el}||J_i\rangle|^2\right)\right.$$
$$\left. - i(\vec{l}\times\vec{l}^*)_3 Re(|\langle J_f||\mathcal{T}^{mag}||J_i\rangle||\langle J_f||\mathcal{T}^{el}||J_i\rangle|)^*\right] + \sum_{J\geqslant 0, spin}\left[l_0 l_0^* |\langle J_f||\mathcal{M}||J_i\rangle|^2\right.$$
$$\left. + l_3 l_3^* |\langle J_f||\mathcal{L}||J_i\rangle|^2 - 2Re\left(l_3 l_0^* |\langle J_f||\mathcal{L}||J_i\rangle||\langle J_f||\mathcal{M}||J_i\rangle|^*\right)\right]$$

Neutrino current $l_\mu = \bar{\nu}\gamma_\mu \frac{(1-\gamma_5)}{2}\nu$, recoil momentum q

Hoferichter, Menéndez, Schwenk, PRD, 2020

$$\mathcal{M} = \mathcal{M}_{LM} + \mathcal{M}^5_{LM} = \left\{F_1^N M_L^M + \frac{\mathbf{q}^2}{4m_N^2}(F_1^N + 2F_2^N)(\Phi_L^{\prime\prime M} - \frac{1}{2}M_{LM})\right\} + \left\{-i\frac{|\mathbf{q}|}{m_N}G_A^N\left[\Omega_L^M + \frac{1}{2}\Sigma_L^{\prime\prime M}\right]\right\}$$

$$\mathcal{L} = \mathcal{L}_{LM} + \mathcal{L}^5_{LM} = \left\{\frac{q^0}{|\mathbf{q}|}\mathcal{M}\right\} + \left\{i\left[G_A^N(1-\frac{\mathbf{q}^2}{8m_N^2}) - \frac{\mathbf{q}^2}{4m_N^2}G_P^N\right]\Sigma_L^{\prime\prime M}\right\}$$

$$\mathcal{T}^{el} = \mathcal{T}^{el}_{LM} + \mathcal{L}^{el5}_{LM} = \left\{\frac{|\mathbf{q}|}{m_N}\left[F_1^N\Delta_L^{\prime M} + \frac{F_1^N + F_2^N}{2}\Sigma_L^M\right]\right\} + \left\{iG_A^N(1-\frac{\mathbf{q}^2}{8m_N^2})\Sigma_L^{\prime M}\right\}$$

$$\mathcal{T}^{mag} = \mathcal{T}^{mag}_{LM} + \mathcal{L}^{mag5}_{LM} = \left\{-i\frac{|\mathbf{q}|}{m_N}\left[F_1^N\Delta_L^M - \frac{F_1^N + F_2^N}{2}\Sigma_L^{\prime M}\right]\right\} + \left\{G_A^N(1-\frac{\mathbf{q}^2}{8m_N^2})\Sigma_L^M\right\}$$

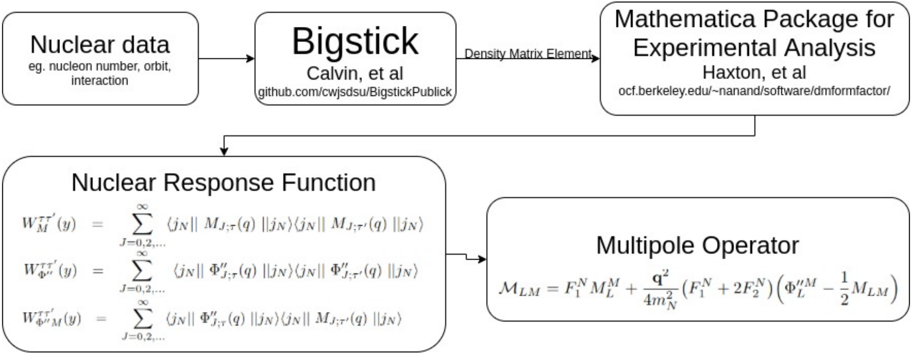
νN scattering: Inelastic cross section

$$\frac{d\sigma^{\nu}_{inel}}{d\Omega} = \frac{2G_F^2}{\pi(2J+1)} E_f^2 \cos^2\frac{\theta}{2}\Bigg\{ \sum_{J=0}^{\infty} |\langle J_f || \hat{\mathcal{M}}_J - \frac{q_0}{q}\hat{\mathcal{L}}_J || J_i\rangle|^2$$

$$+ \left[-\frac{q_\mu^2}{2q^2} + \tan^2\frac{\theta}{2} \right] \sum_{J=1}^{\infty} \left[|\langle J_f || \hat{\mathcal{T}}_J^{el} || J_i\rangle|^2 + |\langle J_f || \hat{\mathcal{T}}_J^{mag} || J_i\rangle|^2 \right]$$

$$\mp 2\tan\frac{\theta}{2} \left[-\frac{q_\mu^2}{q^2} + \tan^2\frac{\theta}{2} \right]^{1/2} \sum_{J=1}^{\infty} Re\left(\langle J_f || \hat{\mathcal{T}}_J^{mag} || J_i\rangle \langle J_f || \hat{\mathcal{T}}_J^{el} || J_i\rangle^* \right) \Bigg\}$$

νN scattering: Numerical calculation



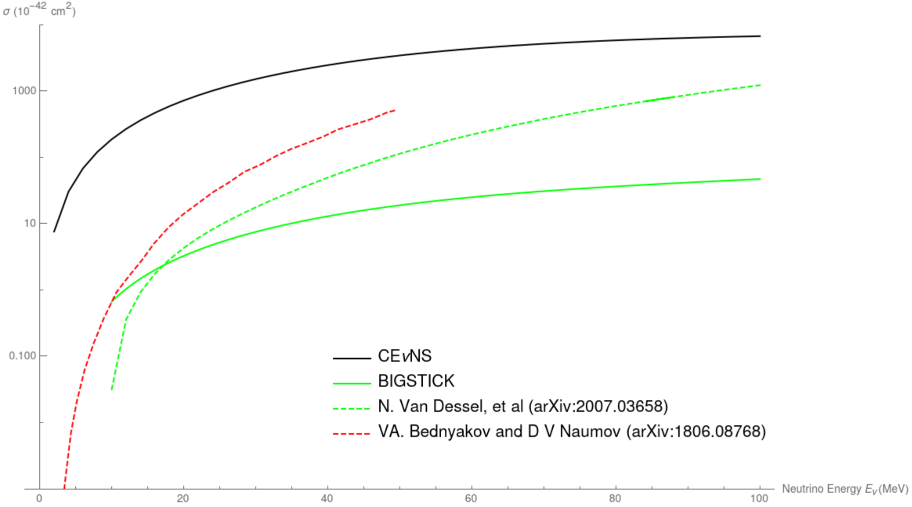
Bigstick is based on nuclear shell model.

Anand, Fitzpatrick, Haxton, PRC. 2014

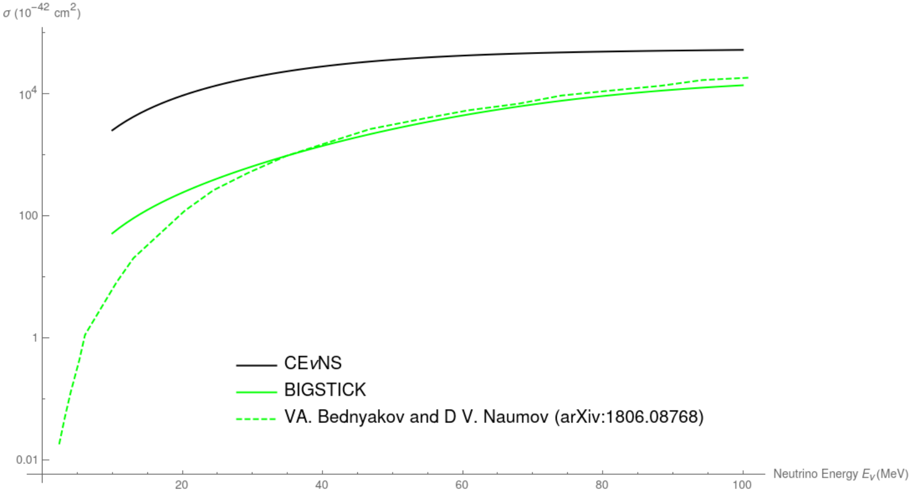
Johnson, Ormand, McElvain, Shan, 2018

νN scattering: Calculation result (Ar40)

Dessel, et al use quasielastic scattering. Bednyakov, et al use single nucleon model.

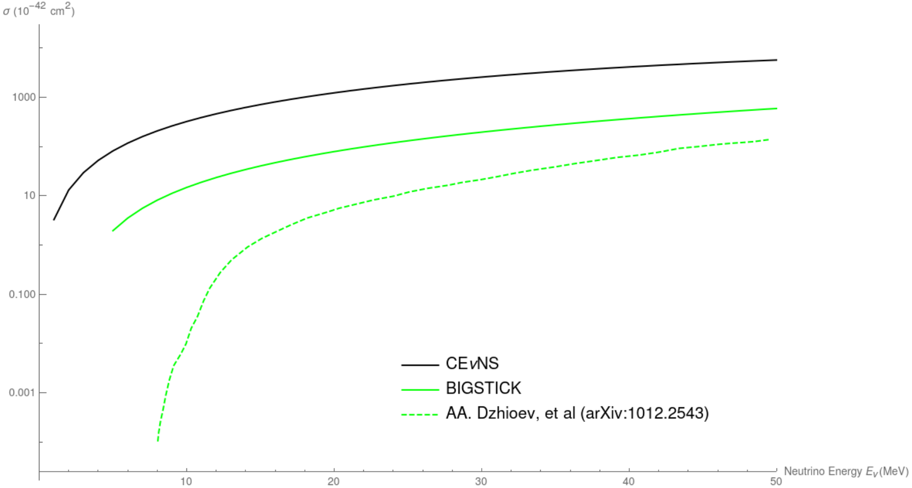


νN scattering: Calculation result (Cs133)



νN scattering: Calculation result (Fe54)

They use shell model as well, but with different parameters.



νN scattering: Calculation result Ar40 photon production

State transition	ΔE energy difference	Rate (COHERENT LAr)	Rate (CCM)
$2 \to 1(gs)$	1460.82	4.57×10^{-3}	5.52×10^{-2}
$3 \to 2$	660.1	10.73	1.30×10^{2}
$4 \to 3$	403	5.84×10^{-6}	7.07×10^{-5}
$4 \to 2$	1063.1	3.33×10^{-4}	$4.0. \times 10^{-3}$
$4 \to 1$	2524.1	2.45×10^{-4}	2.97×10^{-3}
$5 \to 4$	369	0	0
$5 \to 2$	1431.82	0	0
$6 \to 5$	315	4.66×10^{-6}	5.64×10^{-5}
$6 \to 3$	1087.6	1.40×10^{-5}	1.69×10^{-4}
$6 \to 2$	1746.5	4.05×10^{-4}	4.90×10^{-3}
$6 \to 1$	3208.2	4.66×10^{-5}	5.64×10^{-4}

where ΔE (or equivalently emitted photon energy E_γ) is in keV, the rate is the total number of photon. (Assume CCM operates for 3 years)

ΔE is larger than recoil energy threshold.

χN scattering: Elastic and inelastic cross section

Pions $(\pi^0, \pi^\pm)$ are produced after protons hit a target and dark photon A' is produce and decays to 2 DM (χ). We set a constant mass ratio $\frac{m_{A'}}{m_\chi} = 3$

- $A' \to 2\chi$ Dutta, Kim, Liao, Park, Shin, Strigari, Thompson, PRL, 2020
- $\pi^0 \to \gamma + A'$ Deniverville, Pospelov, Ritz, PRD, 2015, Ge, Shoemaker, JHEP, 2018
- $\pi^{-/+} + p/n \to n/p + A'$
- Dark Bremsstrahlung: $e^{\pm *} \to e^\pm + A'$
- Lagrangian $\mathcal{L} \supset g_D A'_\mu \bar{\chi}\gamma^\mu\chi + e\epsilon Q_q A'_\mu \bar{q}\gamma^\mu q$

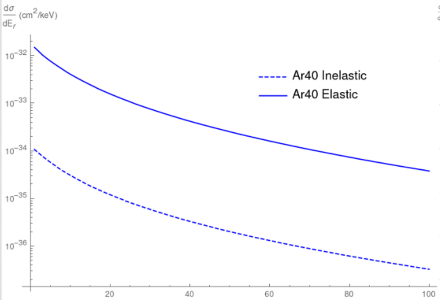
$$\frac{d\sigma}{dE_r}\bigg|_{el} = \frac{e^2\epsilon^2 g_D^2 Z^2}{4\pi(E_\chi^2 - m_\chi^2)(2m_N E_r + m_{A'}^2)^2}\Big[2E_\chi^2 m_N\Big(1 - \frac{E_r}{E_\chi} - \frac{m_N^2 E_r + m_\chi^2 E_r}{2m_N E_\chi^2}\Big) + E_r^2 m_N\Big]|F(2m_N E_r)|^2$$

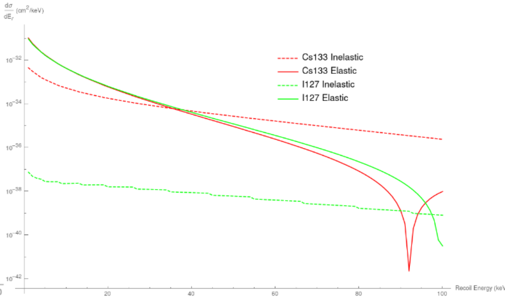
$$\frac{d\sigma}{dE_r}\bigg|_{inel} = \frac{2e^2\epsilon^2 g_D^2}{(1 - \frac{m_\chi^2}{E_\chi^2})(2m_N E_r + m_{A'}^2)^2}\frac{m_N}{2\pi}\frac{4\pi}{2J+1}\, FF(2m_N E_r)$$

E_χ incoming DM energy, J target nucleus spin, $e \approx 0.303$

χN scattering: Calculation result

$m_{A'} = 30\text{MeV},\ m_\chi = 10\text{MeV}$





- Plots don't change significantly when mass of dark photon $m_{A'}$ changed

Conclusion

- We calculate the inelastic cross-section for neutrino and DM nucleus scattering.
- We calculate inclusive the rate of photon production.
- The calculation results give us an insight about inelastic scattering cross section compared to CEνNS, which has been observed by COHERENT experiment.
- For neutrino-nucleus scattering, the inelastic contribution is 1% compared to CEνNS for COHERENT and CCM measurements.
- The DM inelastic contribution for COHERENT CsI can be significant for larger recoil energy.

Backup

$$\langle f|\hat{H}_W|i\rangle = \frac{G_F}{\sqrt{2}} \int d^3x \langle f| j_\mu^{lep} \hat{\mathcal{J}}^\mu(\vec{x})|i\rangle$$

$$= \frac{G_F}{\sqrt{2}} \int d^3x \, e^{-i\vec{q}\cdot\vec{x}} \left(l_0 \mathcal{J}^0(\vec{x}) - \vec{l} \cdot \boldsymbol{\mathcal{J}}(\vec{x}) \right)$$

Spherical decompsition

Multipole operator $\hat{\mathcal{M}}_{JM}(q) \equiv \int d^3x \, [j_J(qx) Y_{JM}(\Omega_x)] \, \hat{\mathcal{J}}^0(\vec{x})$

Nuclear response function $M_{JM}(q\vec{x}_i) \equiv j_J(qx_i) Y_{JM}(\Omega_{x_i})$

where Y_{JM} is Bessel spherical harmonics

Backup

Multipole operators are defined by

$$
\begin{aligned}
\hat{\mathcal{M}}_{JM} &= M_{JM} + M^5_{JM} &&= \int d^3x [j_J(qx) Y_{JM}(\Omega_x)] \hat{\mathcal{J}}_0(x) \\
\hat{\mathcal{L}}_{JM} &= L_{JM} + L^5_{JM} &&= \frac{i}{q} \int d^3x \, [\nabla [j_J(qx) Y_{JM}(\Omega_x)]] \cdot \hat{\vec{\mathcal{J}}}(x) \\
\hat{\mathcal{T}}^{el}_{JM} &= T^{el}_{JM} + T^{el5}_{JM} &&= \frac{1}{q} \int d^3x [\nabla \times j_J(qx) \mathbf{Y}^M_{JJ1}(\Omega_x)] \cdot \hat{\vec{\mathcal{J}}}(x) \\
\hat{\mathcal{T}}^{mag}_{JM} &= T^{mag}_{JM} + T^{mag5}_{JM} &&= \int d^3x [j_J(qx) \mathbf{Y}^M_{JJ1}(\Omega_x)] \cdot \hat{\vec{\mathcal{J}}}(x)
\end{aligned}
$$

Backup

Nuclear response functions are defined by

$$
\begin{aligned}
M_{JM} &= F_1^{(1)}(q_\mu^2) M_J^M \\
L_{JM} &= \frac{q_0}{q} M_{JM} \\
T_{JM}^{el} &= \frac{q}{m_n}\left(F_1^{(1)}(q_\mu^2)\Delta_J'^M + \tfrac{1}{2}\mu^{(1)}(q_\mu^2)\Sigma_J^M\right) \\
T_{JM}^{mag} &= -\frac{iq}{m_n}\left(F_1^{(1)}(q_\mu^2)\Delta_J^M - \tfrac{1}{2}\mu^{(1)}(q_\mu^2)\Sigma_J'^M\right) \\
M_{JM}^5 &= \frac{iq}{m_n}\left(F_A^{(1)}(q_\mu^2)\Omega_J'^M + \tfrac{1}{2}q_0 F_P^{(1)}(q_\mu^2)\Sigma_J''^M\right) \\
L_{JM}^5 &= i\left(F_A^{(1)}(q_\mu^2) - \frac{q^2}{2m_n}F_P^{(1)}(q_\mu^2)\right)\Sigma_J''^M \\
T_{JM}^{el5} &= iF_A^{(1)}(q_\mu^2)\Sigma_J'^M \\
T_{JM}^{mag5} &= F_A^{(1)}(q_\mu^2)\Sigma_J^M
\end{aligned}
$$

Backup

In previous slide m_n is nucleon mass and $\mu^{(1)}(q_\mu^2) = F_1^{(1)}(q_\mu^2) + 2m_n F_2^{(1)}(q_\mu^2)$. In low recoil energy limit $F_1^{(1)}(0) = 1$, $F_A^{(1)}(0) \sim -1.26$, $F_p^{(1)}(0) = \frac{2M_n F_A^{(1)}(0)}{m_\pi}$, $\mu^{(1)}(0) \sim 4.706$.

We use mathematica package *SevenOperators* (0706.2210) and nuclear shell model code *BIGSTICK*(1303.0905) to calculate the nuclear response functions.

Backup

Dirac form factor $F_1^N = Q^N + \frac{\langle r_1^2 \rangle^N}{6} q^2$

Pauli form factor $F_2^N = \kappa^N$

with charge Q^N, magnetic moment $\kappa^p \approx 1.796, \kappa^n \approx -1.913$

and charge radius $\langle r_1^2 \rangle^N = \langle r_E^2 \rangle^N - \frac{3\kappa_N}{2m_N^2}$

with $\langle r_E^2 \rangle^p \approx 0.707 fm^2, \langle r_E^2 \rangle^n \approx -0.116 fm^2$

Pseudoscalar form factor $G_P = -\frac{4 m_N g_{\pi NN} F_\pi}{q^2 - M_\pi^2} - \frac{2}{3} g_A m_N^2 \langle r_A^2 \rangle$

Axial vector form factor $G_A = \frac{g_A}{(1 - q^2/M_A^2)^2}$ with $F_\pi \approx 92.28 MeV$

$\frac{g_{\pi NN}^2}{4\pi} \approx 13.7, \langle r_A^2 \rangle \approx 0.46 fm^2, g_A \approx 1.276, M_A \approx 1 GeV$

Backup

Spin sum of dark matter current $l_\mu = \bar{\chi}\gamma^\mu\chi$ is given by

$$\sum_{s_i,s_f} l_\mu l_\nu^* = \sum_{s_i,s_f} \bar{\chi}(p_f)\gamma^\mu\chi(p_i)\bar{\chi}(p_i)\gamma^\mu\chi(p_f)$$

$$\sum_{s_i,s_f} l_0 l_0^* = 2 - \frac{m_N E_r}{E_\chi^2} + \frac{m_\chi^2}{4E_\chi^2}$$

$$\sum_{s_i,s_f} l_3 l_3^* = 2 - 3\frac{m_N E_r}{E_\chi^2} - \frac{m_\chi^2}{4E_\chi^2}$$

$$\sum_{s_i,s_f} l_3 l_0^* = 2 - \frac{m_N E_r}{E_\chi^2}$$

$$\sum_{s_i,s_f} \vec{l}\cdot\vec{l}^* = 2 + \frac{m_N E_r}{E_\chi^2} - \frac{3m_\chi^2}{4E_\chi^2}$$

$$\sum_{s_i,s_f} (\vec{l}\times\vec{l}^*)_3 = 0$$

- Coherent Captain-Mills (CCM) experiment (LANL). [800 MeV protons hit a tungsten target, total 7 tons (fiducial) LAr of detector 20m from the target. $\sim 10^{22}$ POT (protons-on-target) per year, currently ongoing] **Aguilar-Arevalo, et al, 2021**
- COHERENT experiment (ORNL) [1 GeV protons hit a mercury target, 14.6kg CsI of detector 19.3m from the target, $\sim 10^{23}$ POT per year] **Akimov, et al, 2017**

$m_{A'} = 30MeV,\ m_\chi = 10MeV$