Proof of Warlock DPS Equations

by Deadlord

Legend

$$D = \text{Total DPS over the fight}$$
$$T = \text{Fight length in seconds}$$
$$R_j = \text{Raw damage of the } j^{\text{th}} \text{ shadow bolt, } j \in \{1, ..., N\}$$
$$N = \text{Number of shadow bolts cast}$$

Total DPS Equation

First note that if there is no life tapping needed for the fight, we have

$$N = \left\lfloor \frac{T}{2.5} \right\rfloor$$

and the total damage over time will be the sum of all the random damage from each cast

$$D = \sum_{j=1}^{N} R_j$$

and each damage draw is

$$R_j = \left(B_j + \frac{3}{3.5} s \right) H_j \left(2C_j + 1 \right)$$

where B_j is the random base damage of the j^{th} cast, s is the spell power of the caster, H_j is the binary outcome of the spell hit occurring with probability $\min\{0.83+p, 0.99\}$ where p is the caster's hit chance, and C_j is likewise the binary event of a critical strike occurring with probability equal to the caster's crit chance.

Technically, the value of B_j is drawn uniformly, but the true distribution for the sum of these is difficult to work with so I'll approximate D as Normal in the limit of large N.

Extending to Life Tap

Longer fights require Life Tap, even with the use of Demonic Runes or Mana Potions. In this case, we have

$$N = \left\lfloor \frac{T - L}{2.5} \right\rfloor$$

where L is the time spent life tapping. If we have M total life taps during the fight, then

$$L = 1.5M$$

by the Global Cooldown (GCD).

Expected Value

$$\mathbb{E}[R_j] = \mathbb{E}\left[\left(B_j + \frac{3}{3.5}s\right) H_j\left(2C_j+1\right)\right] = \left(\mathbb{E}[B_j] + \frac{3}{3.5}s\right)\mathbb{E}[H_j]\left(2\mathbb{E}[C_j]+1\right)$$
$$= \left(\frac{a_k+b_k}{2} + \frac{3}{3.5}s\right) p\left(2q+1\right)$$

where a_k is the lower bound of the base shadow bolt damage of rank k. Likewise, b_j^k is this upper bound.

Variability

$$\mathbb{V}[R_j] = \mathbb{V}\left[\left(B_j + \frac{3}{3.5}s\right) H_j\left(2C_j+1\right)\right]$$
$$= \left(\mathbb{V}\left[B_j + \frac{3}{3.5}s\right] + \mathbb{E}^2\left[B_j + \frac{3}{3.5}s\right]\right)\left(\mathbb{V}\left[H_j\right] + \mathbb{E}^2\left[H_j\right]\right)\left(\mathbb{V}\left[2C_j+1\right] + \mathbb{E}^2\left[2C_j+1\right]\right)$$
$$-\mathbb{E}^2\left[B_j + \frac{3}{3.5}s\right]\mathbb{E}^2\left[H_j\right]\mathbb{E}^2\left[2C_j+1\right]$$
$$= \left(\mathbb{V}\left[B_j\right] + \left(\mathbb{E}\left[B_j\right] + \frac{3}{3.5}s\right)^2\right)\left(p\left(1-p\right)+p^2\right)\left(\left(1.5+0.5\mathbb{I}\left(\text{ruin}\right)\right)^2\mathbb{V}\left[C_j\right] + \left(\left(1.5+0.5\mathbb{I}\left(\text{ruin}\right)\right)\mathbb{E}\left[C_j\right]+1\right)^2\right)$$
$$-\left(\mathbb{E}\left[B_j\right] + \frac{3}{3.5}s\right)^2 p^2\left(\left(1.5+0.5\mathbb{I}\left(\text{ruin}\right)\right)\mathbb{E}\left[C_j\right]+1\right)^2$$
$$= \left(\frac{\left(b_k-a_k\right)^2}{12} + \left(\frac{a_k+b_k}{2} + \frac{3}{3.5}s\right)^2\right) p\left(\left(1.5+0.5\mathbb{I}\left(\text{ruin}\right)\right)^2 q\left(1-q\right) + \left(\left(1.5+0.5\mathbb{I}\left(\text{ruin}\right)\right)q+1\right)^2\right)$$
$$-\left(\frac{a_k+b_k}{2} + \frac{3}{3.5}s\right)^2 p^2\left(\left(1.5+0.5\mathbb{I}\left(\text{ruin}\right)\right)q+1\right)^2$$

where a_j^k is the lower bound of the base shadow bolt damage of rank k on the j^{th} cast. Likewise, b_j^k is this upper bound.