ml-ex1

August 4, 2017

0.1 Linear regression

This exercise is described in ex1.pdf.

In [76]:
```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.ticker as ticker

from mpl_toolkits.mplot3d import Axes3D
from sklearn.linear_model import LinearRegression
from sklearn.pipeline import Pipeline
from sklearn.preprocessing import StandardScaler

%matplotlib inline
```

0.1.1 Linear regression with one variable

In [77]:
```
# Load data from CSV file
# - column 1: population (x)
# - column 2: profit (y)
data = np.loadtxt('data/ml-ex1/ex1data1.txt', delimiter=',')
```

In [78]:
```
# Separate features (x) from target (y)
X, y = np.hsplit(data, 2)
```

In [79]:
```
# Number of samples
m = y.size
```

In [80]:
```
# Fit a linear regression model (without regularization)
model = LinearRegression()
model.fit(X, y)
```

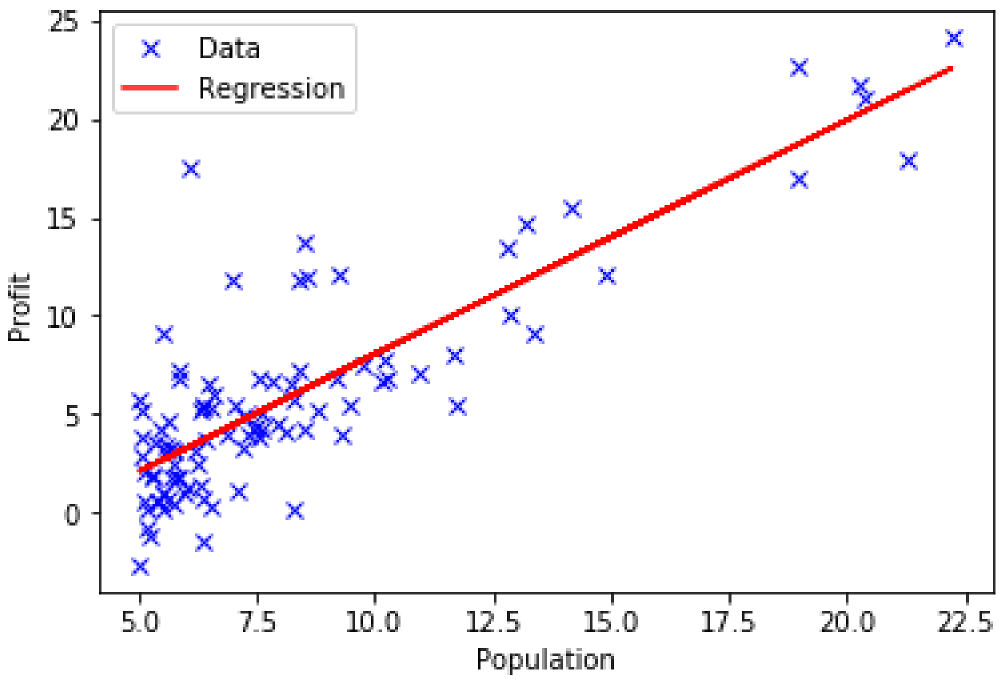
Out[80]:
```
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=1, normalize=False)
```

In [81]:
```
# Obtain coefficients theta0 and theta1 from model
theta0, theta1 = model.intercept_, model.coef_[0]
theta0, theta1
```

Out[81]:
```
(array([-3.89578088]), array([ 1.19303364]))
```

```
In [82]: # Plot data and regression line
         plt.plot(X, y, 'bx', label='Data')
         plt.plot(X, model.predict(X), 'r-', label='Regression')
         plt.xlabel('Population')
         plt.ylabel('Profit')
         plt.legend()
```

```
Out[82]: <matplotlib.legend.Legend at 0x10eb7a710>
```




```
In [83]: # Training mean squared error
         np.mean((model.predict(X) - y) ** 2)
```

```
Out[83]: 8.9539427519503558
```

```
In [84]: # Training score
         model.score(X, y)
```

```
Out[84]: 0.70203155378413973
```

```
In [85]: # Predict profit for populations of 35.000 and 70.000
         model.predict(np.array([[3.5], [7.0]])) * 10000
```

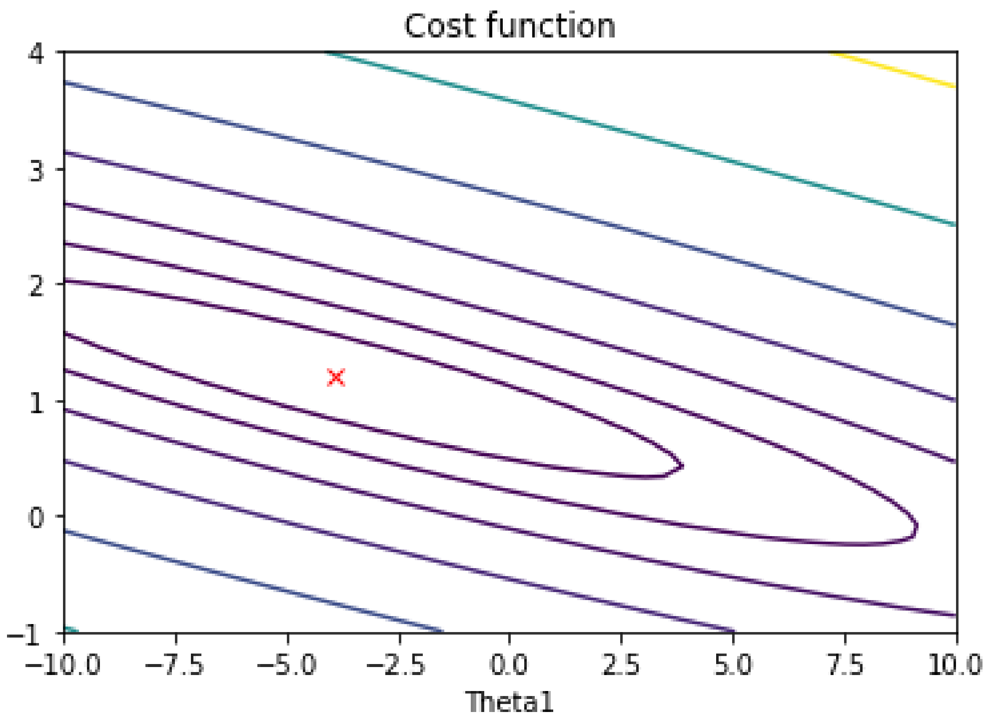
```
Out[85]: array([[  2798.36876352],
                [ 44554.54631015]])
```

See also this Stackoverflow entry for the following calculations:

```
In [86]: # LinearRegression above internally added a column vector of 1s.
         # For the following calculations, this must be done explicitly.
         X = np.c_[np.ones(m), X]

In [87]: # Compute least-squares solution with np.linalg.lstsq.
         # This is what LinearRegression actually does under the hood.
         Theta = np.linalg.lstsq(X, y)[0]
         Theta

Out[87]: array([[-3.89578088],
                [ 1.19303364]])

In [88]: # Solve analytically (without explicit inverse)
         Theta = np.linalg.solve(X.T.dot(X), X.T.dot(y))
         Theta

Out[88]: array([[-3.89578088],
                [ 1.19303364]])

In [89]: # Solve normal equation (explicit inverse)
         Theta = np.linalg.inv(X.T.dot(X)).dot(X.T).dot(y)
         Theta

Out[89]: array([[-3.89578088],
                [ 1.19303364]])

In [90]: # Solve normal equation (explicit pseudo-inverse)
         Theta = np.linalg.pinv(X.T.dot(X)).dot(X.T).dot(y)
         Theta

Out[90]: array([[-3.89578088],
                [ 1.19303364]])

In [91]: # Cost function (mean squared error)
         def cost(X, y, Theta):
             diff = X.dot(Theta) - y
             return np.sum(diff * diff, axis=0) / y.size

In [92]: # Create a 50*50 grid for Theta0 and Theta1 ranges
         Theta_grid = np.mgrid[-10:10:50j, -1:4:50j]

         # Theta values in an array of shape (2500,2)
         Thetas = np.c_[Theta_grid[0].ravel(), Theta_grid[1].ravel()]

         # Compute costs on grid points
         costs = cost(X, y, Thetas.T)

         # Cost values reshaped to grid
         cost_grid = costs.reshape(Theta_grid[0].shape)
```

```
# Contour plot of cost function
plt.contour(Theta_grid[0], Theta_grid[1], cost_grid, levels=[20, 40, 80, 160, 320, 640,

# Plot optmimum
plt.plot(Theta[0], Theta[1], 'rx')

# Title and axis labels
plt.xlabel('Theta0')
plt.xlabel('Theta1')
plt.title('Cost function')
```

Out[92]: <matplotlib.text.Text at 0x10ec5f8d0>



0.1.2 Linear regression with multiple variables

In [93]:
```
# Load data from CSV file
# - column 1: house size in feet**2 (x1)
# - column 2: number of bedrooms (x2)
# - column 3: house price (y)
data = np.loadtxt('data/ml-ex1/ex1data2.txt', delimiter=',')
```

In [94]:
```
# Separate features (x1, x2) from target (y)
X, y = np.hsplit(data, np.array([2]))
```

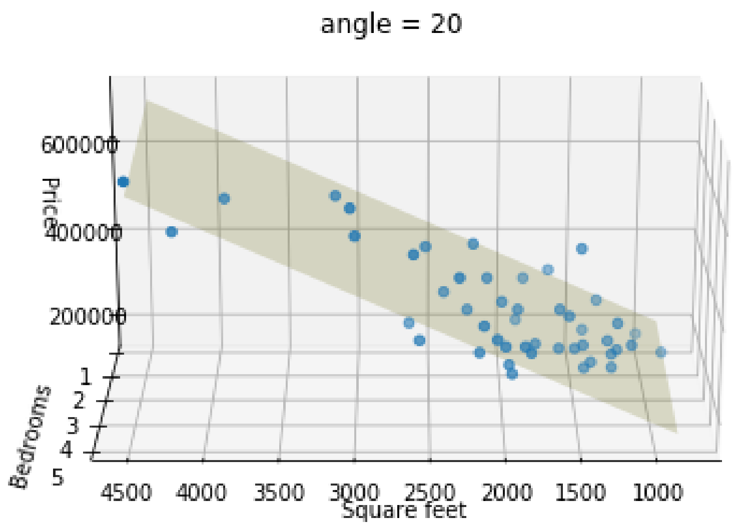
In [95]:
```
# Number of samples
m = y.size
```

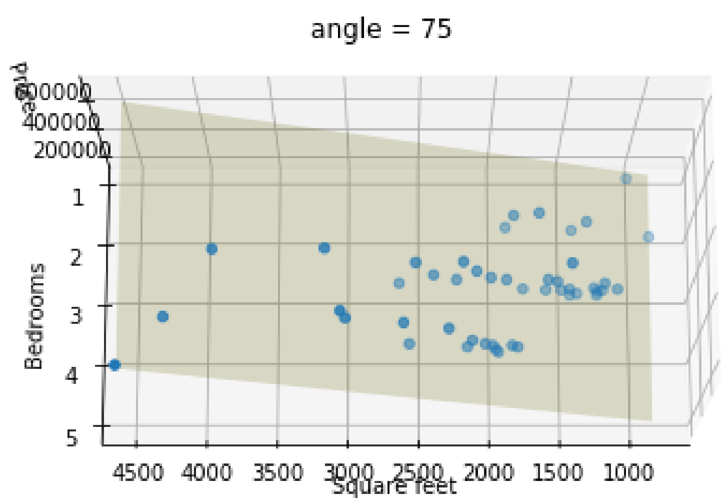
In [96]:
```
# Pipeline components
scaler = StandardScaler()
regr = LinearRegression()

# Pipeline of feature scaler and linear regressor
model = Pipeline([('scaler', scaler), ('regr', regr)])
```

In [97]:
```
# Run linear regression on scaled features
model.fit(X, y)
```

Out[97]:
```
Pipeline(steps=[('scaler', StandardScaler(copy=True, with_mean=True, with_std=True)), (
```

In [98]:
```
# Create a grid from feature min and max values
grid_range = np.vstack([X.min(axis=0), X.max(axis=0)])
grid = np.meshgrid(grid_range[:,0], grid_range[:,1])

# Compute predictions from grid values
X_grid = np.c_[grid[0].ravel(), grid[1].ravel()]
y_grid = model.predict(X_grid).reshape(grid[0].shape)

def plot_data_and_regression(fig, fignum, view_angle):
    sp = fig.add_subplot(2, 1, fignum, projection='3d')
    sp.view_init(view_angle, 90)
    sp.set_xlabel('Square feet')
    sp.set_ylabel('Bedrooms')
    sp.set_zlabel('Price')
    sp.set_title(f'angle = {view_angle}')
    # Customize tick locators
    sp.yaxis.set_major_locator(ticker.MultipleLocator(1.00))
    sp.zaxis.set_major_locator(ticker.MultipleLocator(200000))
    # Plot samples
    sp.scatter(X[:,0], X[:,1], y)
    # Plot regression area
    sp.plot_surface(grid[0], grid[1], y_grid, color='y', alpha=0.2)

# Create new figure
fig = plt.figure(2, figsize=[9, 10])

# Plot data and regression from two different angles
plot_data_and_regression(fig, fignum=1, view_angle=20)
plot_data_and_regression(fig, fignum=2, view_angle=75)
```






```
In [99]: # Training mean squared error
         np.mean((model.predict(X) - y) ** 2)
```

```
Out[99]: 4086560101.2056565
```

```
In [100]: # Training score
          model.score(X, y)
```

```
Out[100]: 0.7329450180289143 2
```

See also this Stackoverflow entry for the following calculations:

In [101]:
```
# Add a column of 1s
X = np.c_[np.ones(m), X]
```

In [102]:
```
# Solve analytically (without explicit inverse)
Theta = np.linalg.solve(X.T.dot(X), X.T.dot(y))
Theta
```

Out[102]:
```
array([[ 89597.9095428 ],
       [   139.21067402],
       [ -8738.01911233]])
```

In [103]:
```
# Solve normal equation (explicit pseudo-inverse)
Theta = np.linalg.pinv(X.T.dot(X)).dot(X.T).dot(y)
Theta
```

Out[103]:
```
array([[ 89597.90954355],
       [   139.21067402],
       [ -8738.01911255]])
```

In [104]:
```
# Compute cost of solution
cost(X, y, Theta)
```

Out[104]:
```
array([  4.08656010e+09])
```