# ml-ex5

August 4, 2017

## 0.1 Bias and variance

This exercise is described in ex5.pdf.

```
In [1]: import numpy as np
        import scipy.io as sio
        import matplotlib.pyplot as plt

        from sklearn.linear_model import Ridge
        from sklearn.model_selection import learning_curve, train_test_split, ShuffleSplit
        from sklearn.pipeline import Pipeline
        from sklearn.preprocessing import PolynomialFeatures, StandardScaler

        %matplotlib inline
```

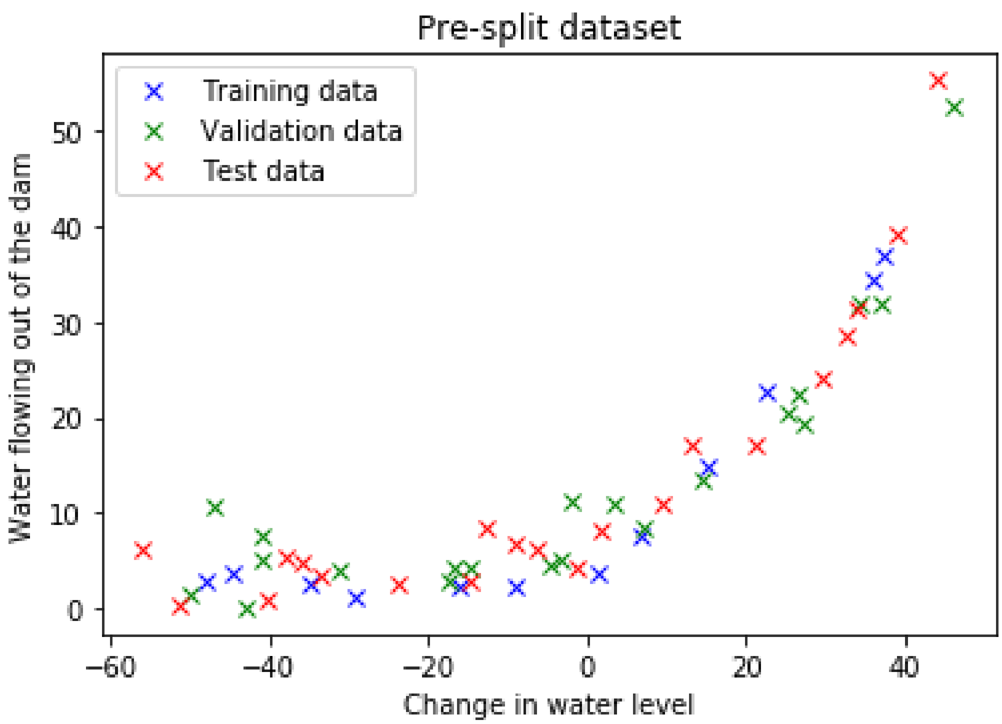
### 0.1.1 Prepare dataset

```
In [2]: # Load pre-split dataset provided by exercise
        data = sio.loadmat('data/ml-ex5/ex5data1.mat')
```

```
In [3]: # Training set
        X_train_0 = data['X']
        y_train_0 = data['y']

        # Validation set
        X_cval_0 = data['Xval']
        y_cval_0 = data['yval']

        # Test set
        X_test_0 = data['Xtest']
        y_test_0 = data['ytest']
```

```
In [4]: # Plot the three pre-split datasets
        plt.plot(X_train_0, y_train_0, 'bx', label='Training data')
        plt.plot(X_cval_0, y_cval_0, 'gx', label='Validation data')
        plt.plot(X_test_0, y_test_0, 'rx', label='Test data')
        plt.xlabel('Change in water level')
        plt.ylabel('Water flowing out of the dam')
        plt.title('Pre-split dataset')
        plt.legend()
```

Out[4]: <matplotlib.legend.Legend at 0x109875f60>




In [5]:
```
# Number of training data
X_train_0.shape[0]
```

Out[5]: 12

In [6]:
```
# Number of validation data
X_cval_0.shape[0]
```

Out[6]: 21

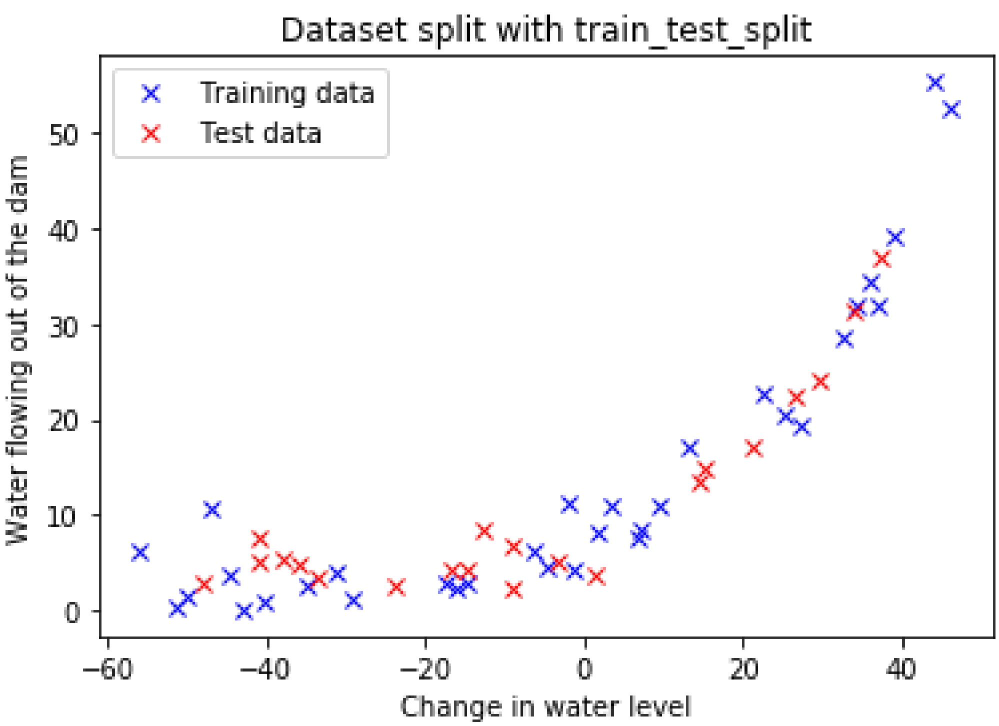
In [7]:
```
# Number of test data
X_test_0.shape[0]
```

Out[7]: 21

In [8]:
```
# Concatenate pre-split datasets into a single dataset
# (use scikit-learn utilities later to split datasets).
X = np.concatenate([X_train_0, X_cval_0, X_test_0])
y = np.concatenate([y_train_0, y_cval_0, y_test_0]).ravel()
```

In [9]:
```
# Randomly split into training and test set. Test set size = 21.
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=21, random_state=42)
```

```
In [10]: # Plot the datasets obtained with train_test_split
         plt.plot(X_train, y_train, 'bx', label='Training data')
         plt.plot(X_test, y_test, 'rx', label='Test data')
         plt.xlabel('Change in water level')
         plt.ylabel('Water flowing out of the dam')
         plt.title('Dataset split with train_test_split')
         plt.legend()

Out[10]: <matplotlib.legend.Legend at 0x109a564e0>
```



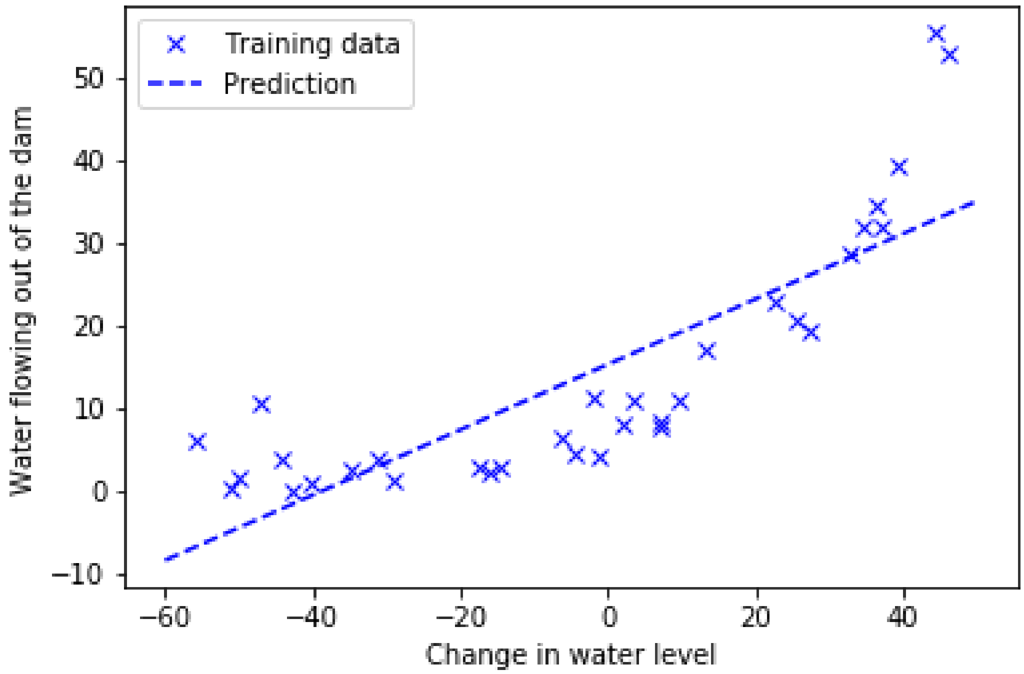

### 0.1.2 Utility functions

```
In [11]: # Plot training data and predictions by a trained model
         def plot_training_data_and_predictions(X_train, y_train, X_pred, y_pred, title=''):
             plt.plot(X_train, y_train, 'bx', label='Training data')
             plt.plot(X_pred, y_pred, 'b--', label='Prediction')
             plt.xlabel('Change in water level')
             plt.ylabel('Water flowing out of the dam')
             plt.title(title)
             plt.legend()

         # Plot learning curves obtained by training on different training set sizes
         def plot_learning_curves(train_sizes, train_scores, test_scores, y_min=-1, y_max=1, tit
             plt.plot(train_sizes, train_scores, 'b-', label='Training score')
```

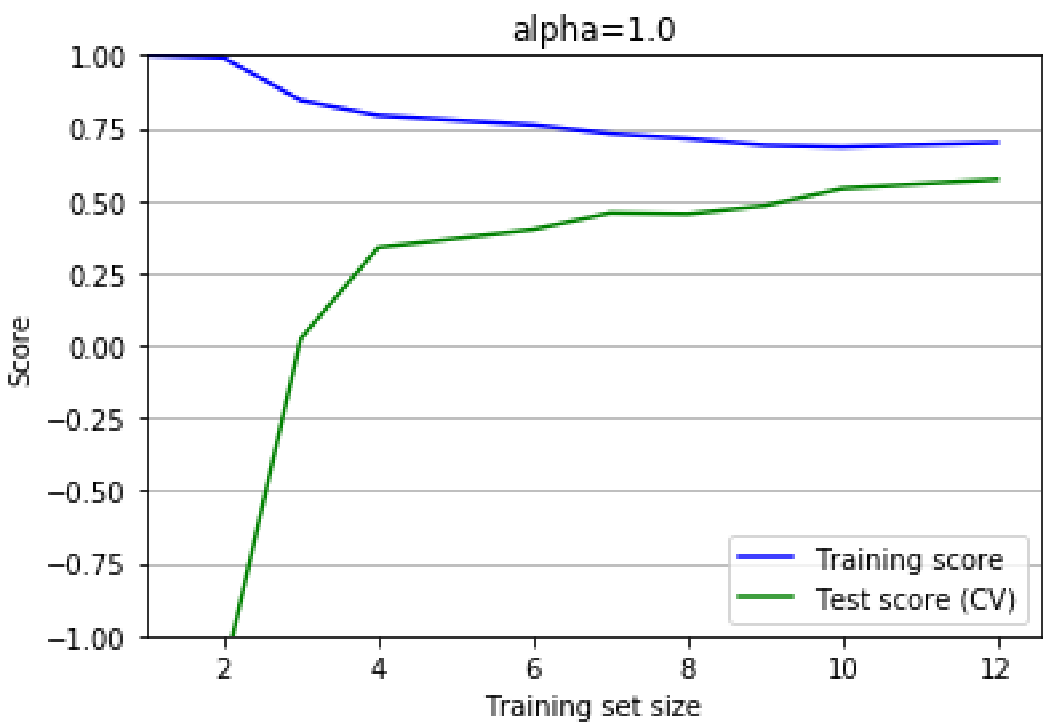
```
    plt.plot(train_sizes, test_scores, 'g-', label='Test score (CV)')

    plt.grid(True, axis='y')
    plt.xlabel('Training set size')
    plt.ylabel('Score')
    plt.xlim(xmin=1)
    plt.ylim(y_min, y_max)
    plt.title(title)
    plt.legend()

# Generate learning curves by training regressor on different training set sizes
def compute_learning_curves(regressor, X_train, y_train, test_size):
    # Train/test split using test_size examples
    cv = ShuffleSplit(n_splits=50, test_size=test_size, random_state=0)
    # Relative training set sizes
    train_sizes_rel = np.linspace(.1, 1.0, 10)
    # Generate learning curves
    train_sizes, train_scores, test_scores = learning_curve(regressor, X_train, y_train
    # Return training set sizes and average learning values
    return [train_sizes, np.mean(train_scores, axis=1), np.mean(test_scores, axis=1)]
```

### 0.1.3 Linear Regression

In [12]:
```
# Linear regression with default regularization strength (alpha=1.0)
alpha = 1.0

model = Ridge(alpha=alpha)
model.fit(X_train, y_train)

X_pred = np.array([[-60], [50]])
y_pred = model.predict(X_pred)

plot_training_data_and_predictions(X_train, y_train, X_pred, y_pred)
```




```
In [13]: train_sizes, train_scores, test_scores = compute_learning_curves(model, X_train, y_trai
         plot_learning_curves(train_sizes, train_scores, test_scores, title=f'alpha={alpha}')
```



In [14]:
```
# Score on test set
model.score(X_test, y_test)
```

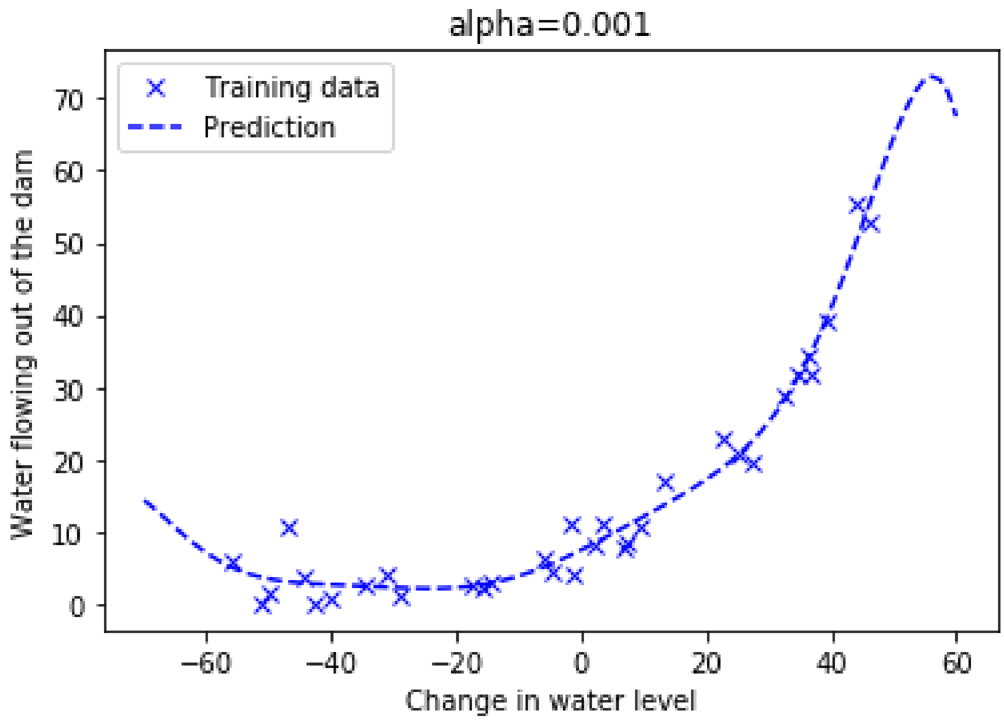
Out[14]:
```
0.60084127499786522
```

### 0.1.4 Polynomial regression

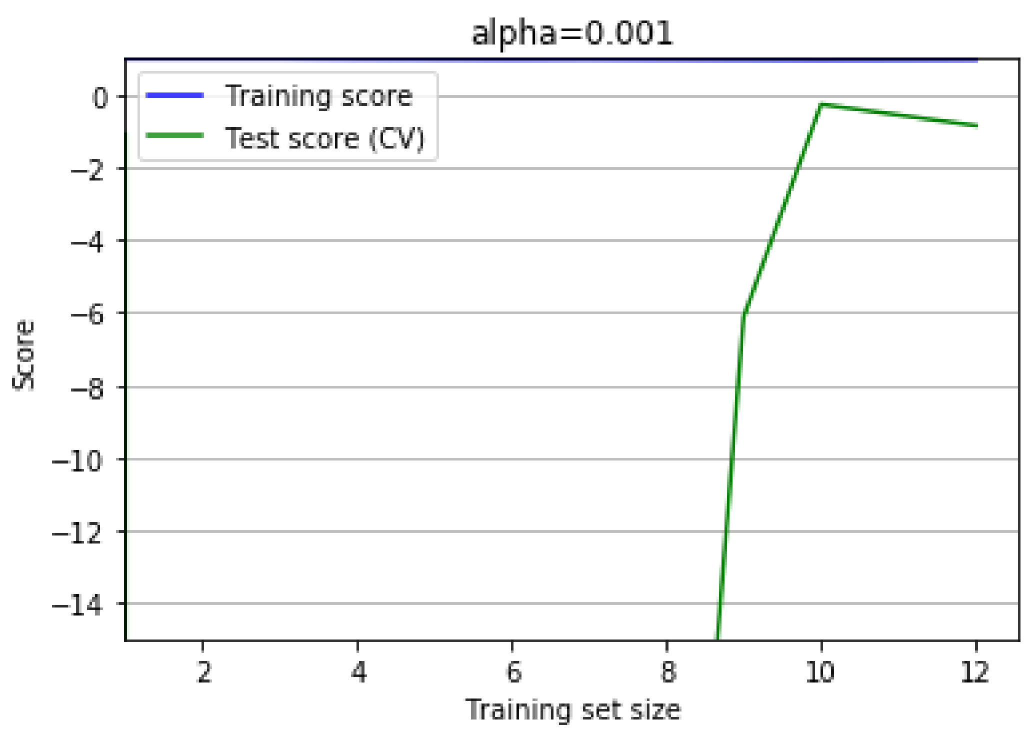
In [15]:
```
# Creates a linear regressor with given regularization strength alpha.
# Ploynomial features of degree 8 are added and scaled before running
# regulaized linear regression.
def regressor(alpha):
    return Pipeline(steps=[
        ('poly', PolynomialFeatures(degree=8)),
        ('scaler', StandardScaler()),
        ('lreg', Ridge(alpha=alpha))
    ])
```

#### Overfit example

In [16]:
```
alpha = 0.001

model = regressor(alpha=alpha)
model.fit(X_train, y_train)

X_pred = np.linspace(-70, 60, 100).reshape(-1,1)
y_pred = model.predict(X_pred)

plot_training_data_and_predictions(X_train, y_train, X_pred, y_pred, f'alpha={alpha}')
```

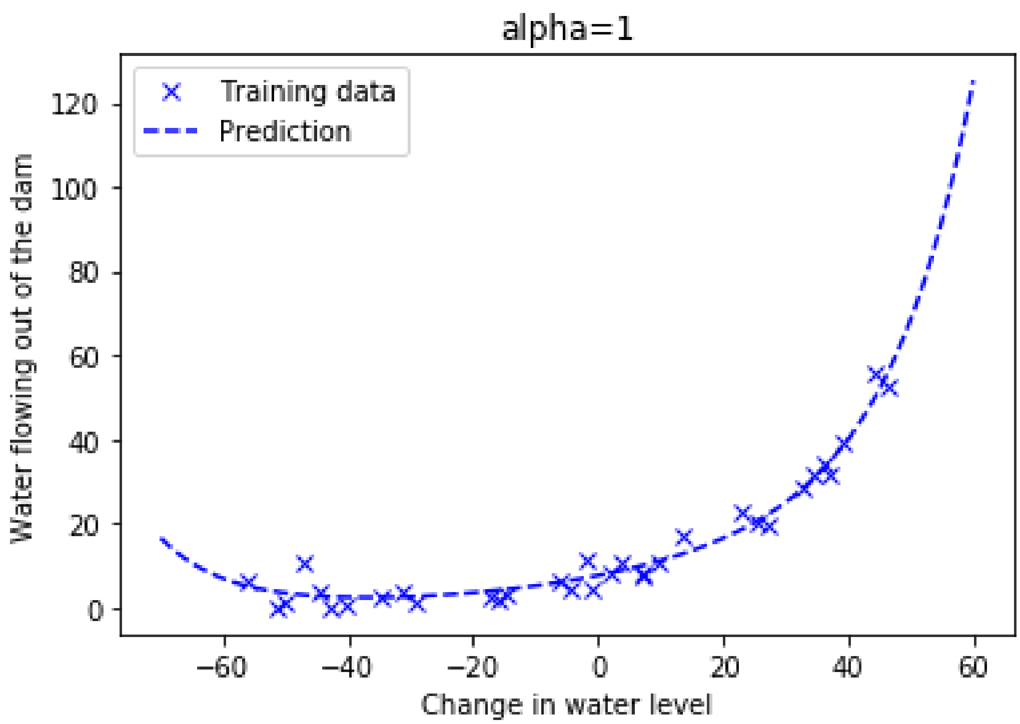



```
In [17]: train_sizes, train_scores, test_scores = compute_learning_curves(model, X_train, y_trai
         plot_learning_curves(train_sizes, train_scores, test_scores, y_min=-15, title=f'alpha={
```

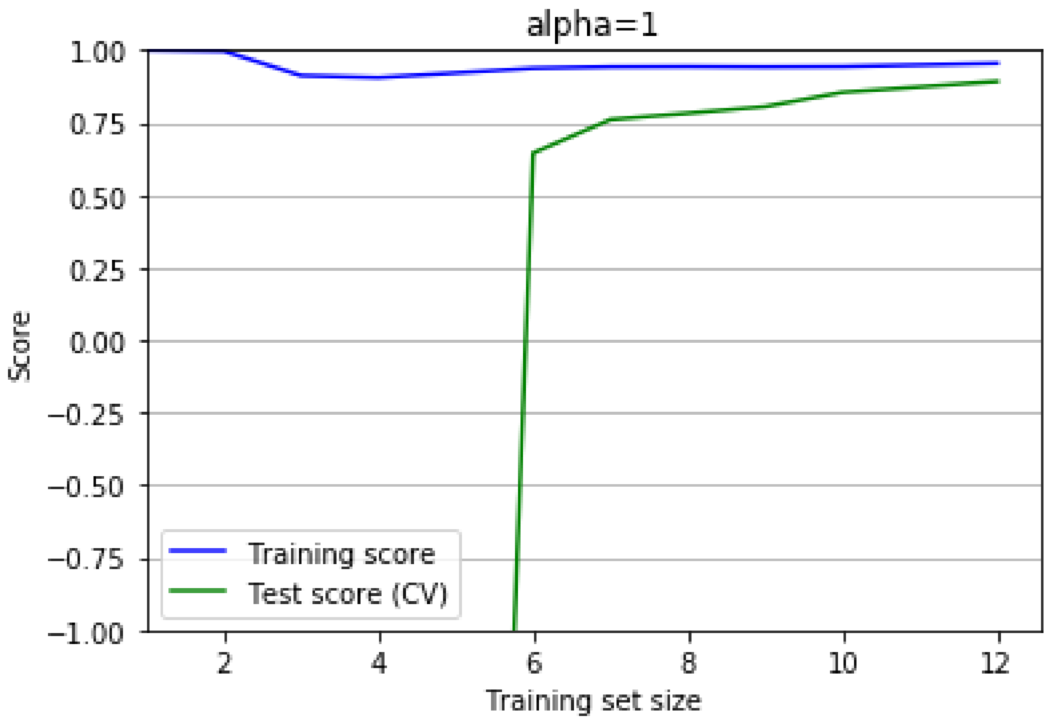


**Good fit example**

In [18]:
```
alpha = 1

model = regressor(alpha=alpha)
model.fit(X_train, y_train)

X_pred = np.linspace(-70, 60, 100).reshape(-1,1)
y_pred = model.predict(X_pred)

plot_training_data_and_predictions(X_train, y_train, X_pred, y_pred, f'alpha={alpha}')
```




In [19]:
```
train_sizes, train_scores, test_scores = compute_learning_curves(model, X_train, y_trai
plot_learning_curves(train_sizes, train_scores, test_scores, title=f'alpha={alpha}')
```

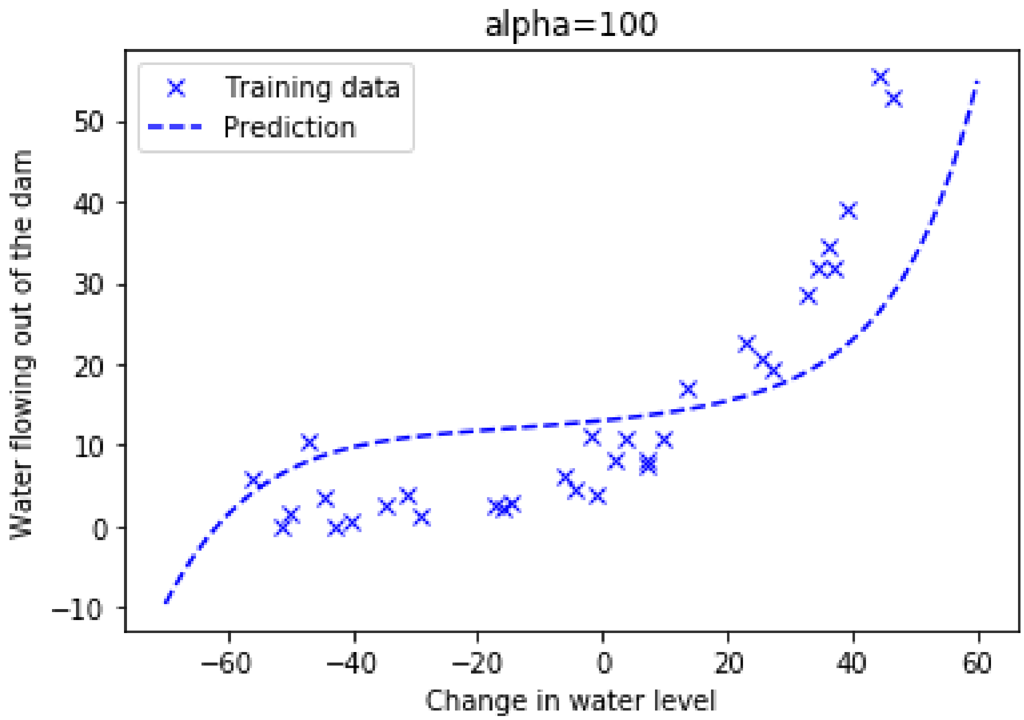



```
In [20]: # Score on test set
         model.score(X_test, y_test)
```

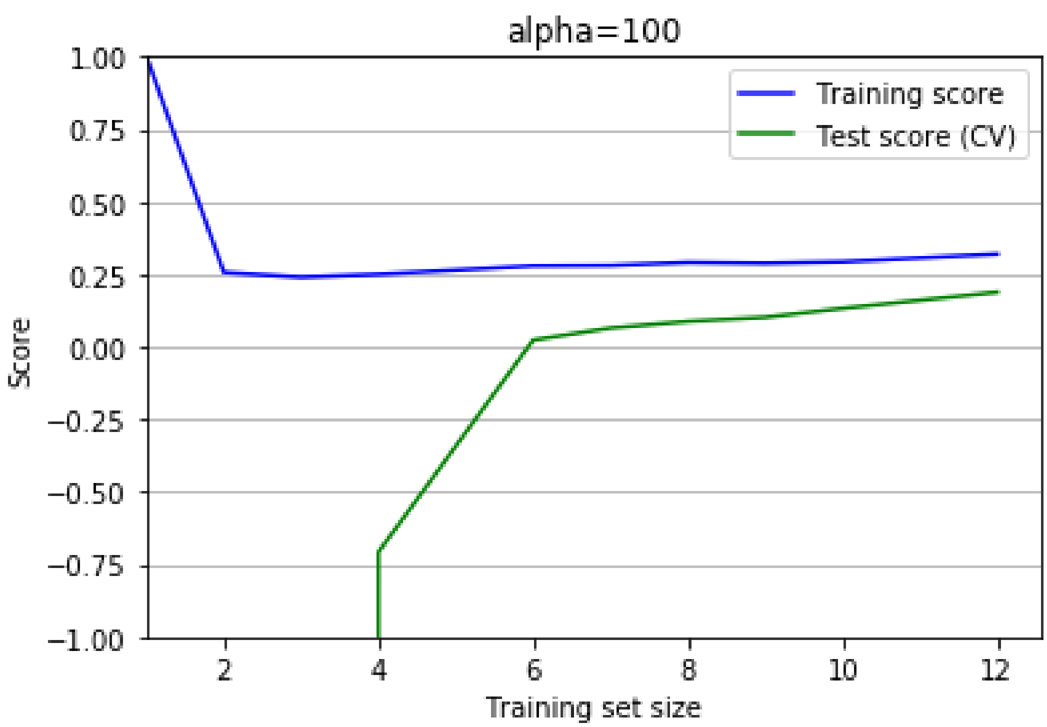
```
Out[20]: 0.95207578600535314
```

**Underfit example**

```
In [21]: alpha = 100

         model = regressor(alpha=alpha)
         model.fit(X_train, y_train)

         X_pred = np.linspace(-70, 60, 100).reshape(-1,1)
         y_pred = model.predict(X_pred)

         plot_training_data_and_predictions(X_train, y_train, X_pred, y_pred, f'alpha={alpha}')
```




In [22]:
```
train_sizes, train_scores, test_scores = compute_learning_curves(model, X_train, y_trai
plot_learning_curves(train_sizes, train_scores, test_scores, title=f'alpha={alpha}')
```



In [23]:
```
# Score on test set
model.score(X_test, y_test)
```

Out[23]:
```
0.45166186769096572
```