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Optimal Design of Stewart Platform Safety Mechanism

Chen Hua*, Chen Weishan, Liu Junkao

School of Mechatronics Engineering, Harbin Institute of Technology, Harbin 150001, China

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Abstract

A safety mechanism capable of moving at will within the range of its whole link lengths is designed based on the link space. Sixteen extreme poses are obtained in a Stewart platform. The singular points of the extreme poses are solved by using homotopy method as well as the judgment condition of singular points, and thereby the maximum link lengths are achieved. The rotation angles of joints and the distances between two neighboring links are analyzed in a calculation example in which that the mechanism moves among the extreme poses is assumed. Then an algorithm to test the safety mechanism is presented taking the constraint conditions into account. A safety mechanism having optimal properties of global movement is worked out by optimizing all structural parameters through minimizing the average condition number of extreme poses.

Keywords: parallel manipulator; safety mechanism; optimal design; Stewart platform

1 Introduction

In comparison with a serial manipulator, the Stewart parallel manipulator, capable of providing 6-DOF movement, has advantages of high structural rigidity, high accuracy, fast dynamic response, and large load-to-weight ratio, which makes it find a wide application in flight simulation, spaceship aligning, radar and satellite antenna orientation, robots, parallel machine tools etc. In practice, the movements of a parallel manipulator are often controlled by the link lengths. For a Stewart platform with a given set of parameters of link length, it is possible to fall into a singular configuration, to run beyond the rotation angle limit of joints or to undergo link interference. Here arises the necessity to study the method of designing a proper safety mechanism which is required that its movable platform assume a feasible position and orientation corresponding to the way the six links move at will in their stroke ranges, which means the liberty the mechanism must possess to move in the space defined by the link lengths.

The objective of kinematic design of a parallel manipulator is to optimize the range of active joint variables and structural parameters. Proper mechanism parameters are of singular importance to ensure its workspace, improve its kinetic and dynamic performance. Merlet[1-3] proposed a methodology for designing a parallel manipulator with a specific workspace. It includes: from all of the 6-DOF parallel manipulators containing the given workspace that have been chosen, is searched out the one that best meets other performance indices. Stamper[4] optimized a 3-DOF translational parallel manipulator in pursuit of either total workspace maximization or global conditioning index optimization. Boudreau[5] presented a genetic algorithm to optimize a planar 3-DOF parallel manipulator having a workspace possibly closest to the demanded work-

*Corresponding author. Tel.: +86-451-86417891.
E-mail address: chenhua202@126.com

ing area. Li[6] analyzed optimally the parameter design of a three legged virtual axis machine tool that meets the required workspace. Chen[7] presented a genetic algorithm to synthesizing a Stewart platform inclusive of a desired workspace with orienting capabilities in a given 3D region. Sun[8] developed an optimal kinematic design method of a 2-DOF planar parallel robot based on the synthesis of workspaces, dexterity, velocity and geometric precision. There was also proposed a global index composed of conditioning index, velocity and precision so as to achieve a workspace with good performances. Arsenault[9] put forward a method for the optimal synthesis of planar parallel mechanisms on the ground of the performance criteria inclusive of dexterity, stiffness and the error between the real workspace and the demanded workspace. Also, singularity-free workspaces were obtained. From above-mentioned literatures, it is observed that researchers' attention was mainly centered on the optimization aimed at satisfying some performance indices for the purpose of optimal design of parallel manipulators. Much less efforts were made in the design of safety mechanisms capable of moving at will in the space defined by link lengths. On the other hand, Fang[10] designed a safety mechanism of a 3-6 Stewart platforms which, by introducing the vertex spaces, transformed the problem of designing a safety mechanism into that of the existence of a solution to the simultaneous equations. But this method was restricted to mere optimization of the extension ratio of links.

This paper suggests, based on the link space, an optimal design of safety mechanism capable of moving at will in the whole range defined by the link lengths and thereby presents a global kinematic performance index by analyzing the singular points of extreme poses, the structural constraints and the Jacobian condition number of the parallel manipulator. The global kinematic performance index is considered as an optimization goal to optimize all structural parameters which ensure the manipulator able to move at will in a link space.

2 Extreme Poses

It is unacceptable for a manipulator to sink into a state of singular configuration, exceed the rotation angle limit of joints, or even undergo link interferences while the six links move at will in their stroke ranges. Consequently, first the extreme poses should be analyzed assuming the lengths of six links being either minimum or maximum. Then, associated with a calculation example, the alteration of constraints is analyzed when the manipulator moves among these extreme poses.

2.1 Number of extreme poses

The Stewart platform is composed of a movable platform connected to a fixed base through six extendable links, as shown in Fig.1. With the upper ends of each link connected to the movable platform through a spherical joint, the six spherical joints constitute two equilateral triangles. The lower extremity of each link is connected to the base with a universal joint, and six universal joints also build up two equilateral triangles. As the lengths of six links change, the movable platform will move with six degrees of freedom.

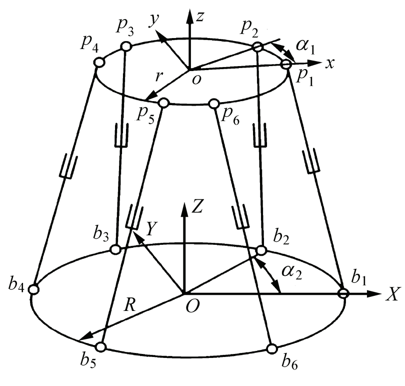

Fig.1 Stewart parallel manipulator

The workspace of a Stewart platform is defined by its structural parameters, the maximum and minimum length of links, the rotation angle limit of joints and the diameter of links. Once the structural parameters, the maximum rotation angle of joints and the diameter of links are given, the workspace is exclusively determined by the lengths of six links. The number of the extreme poses of a Stewart parallel manipulator is $2^6=64$. However, taking the symmetry into account, when the manipulator works within its workspace, there must be some extreme poses possessive of the same characteristic features. Therefore, after ruling out them, out of the 64 extreme poses, only sixteen typical ones are left to be discussed below.

(1) In the case of six links all having the minimum lengths, there is only one typical extreme pose.

(2) In the case of five links having the minimum lengths with the remaining one having the maximum, denoted by l_1, if the manipulator is rotated a given angle round the axis z, the links having the minimum lengths will take the same poses as that taken by the link with the maximum length l_1, which means only one typical extreme pose corresponding to this extreme situation.

(3) In the case of four links having the minimum lengths and the other two maximum, if their location relationship is characterized by the center angle ϕ that corresponds to the joints on the two links having the maximum lengths, after having treated as in the case of (2), the same location relationship between the two links having the maximum lengths can be regard as one extreme pose. In this way, each different value of ϕ will correspond to one typical extreme pose. It can be seen from Fig.1 that, corresponding to this extreme situation, exist four typical extreme poses, whose possible values of ϕ might be assumed as α_1, $120°-\alpha_1$, $120°$ and $120°+\alpha_1$.

(4) In the case of half of the six links having the minimum lengths and the others the maximum, if ϕ_1, ϕ_2, ϕ_3 express respectively the location relationships between every two of the three links having the maximum length, the number of the combinations of the values taken by ϕ_1, ϕ_2 and ϕ_3 will equal to the typical extreme poses. Under this extreme situation, thus exist four typical extreme poses because possible combinations of ϕ_1, ϕ_2, ϕ_3 are (α_1, 120°, 120°+α_1), (α_1, 120°$-\alpha_1$, 120°), (120°$-\alpha_1$, 120°+ α_1, 120°) and (120°, 120°, 120°).

(5) In the case of two links having the minimum lengths and the other four the maximum, it is only needed to contract the long links into the minimum and extend the short ones into the maximum, as is the case with (3), which results in four typical extreme poses under this extreme situation.

(6) In the case of one link having the minimum length and the others the maximum, there is only one typical extreme pose under this extreme situation, as is the case of (2).

(7) In the case of all six links having the maximum lengths, there is only one typical extreme pose under this extreme situation.

It can be concluded that a Stewart platform proves to be possessive of 16 typical extreme poses. In order to present them clearer, the length vectors of the six links are used to express these typical extreme poses, where 0 marks the minimum length and 1 marks the maximum length, as follows

$$\begin{bmatrix} 0&1&1&1&1&1&1&1&1&1&0&0&0&0&0&1\\ 0&0&1&0&0&0&1&1&0&0&0&1&1&1&1&1\\ 0&0&0&1&0&0&1&0&1&1&1&0&1&1&1&1\\ 0&0&0&0&1&0&0&1&0&0&1&1&0&1&1&1\\ 0&0&0&0&0&0&0&0&1&0&1&1&1&1&1&1\\ 0&0&0&0&0&1&0&0&0&1&1&1&1&0&1&1 \end{bmatrix}$$

2.2 Singular points of extreme poses

When the parameters of a manipulator are to be determined, singular points of extreme poses should be avoided as they exert negative effects on the performances of the manipulator. The approach of solving singular points of a parallel manipulator often ends up in the solution of a nonlinear polynomial algebra system, which can be dealt with the numerical iterative method.

Suppose the length vector of links is $\boldsymbol{L} = [l_1\ \ l_2\ \ l_3\ \ l_4\ \ l_5\ \ l_6]^{\mathrm{T}}$, the position and orientation of the movable platform $\boldsymbol{P} = [\theta_x\ \ \theta_y\ \ \theta_z\ \ x_0\ \ y_0\ \ z_0]^{\mathrm{T}}$, where l_i is the length of the ith link, orientation variables (θ_x, θ_y, θ_z) are the rotation angles round the fixed coordinate axes, position variables (x_0, y_0, z_0) are the center coordinates of the movable platform in the fixed coordinate system. For convenience, a constraint equation could be worked out using the center coordinates of the movable platform and the coordinates (x_1, y_1, z_1), (x_2, y_2, z_2) of the two spherical joints on it[11]. The orientation variables $(\theta_x, \theta_y, \theta_z)$ of the movable platform could be acquired by the above three coordinates. According to the constraint conditions of the link lengths and the geometric relations of the movable platform, can be obtained a set of constraint equations with nine dimensions as follows

$$\left.\begin{aligned} &\boldsymbol{F}(\boldsymbol{x},\boldsymbol{L})=0 \\ &\boldsymbol{x}=[x_0\ \ y_0\ \ z_0\ \ x_1\ \ y_1\ \ z_1\ \ x_2\ \ y_2\ \ z_2]^{\mathrm{T}} \\ &\boldsymbol{L}=[l_1\ \ l_2\ \ l_3\ \ l_4\ \ l_5\ \ l_6]^{\mathrm{T}} \end{aligned}\right\} \tag{1}$$

Assuming that, under the initial situation, every link has the minimum length, the movable platform is horizontally located over the fixed base platform, and the coordinates of the movable platform center, x and y, are equal to zero. With the alteration of the link length variables, the manipulator will take one of the other 15 extreme poses. Thus, using the numerical iterative method, could be graphed the curves that describe the positions and orientations of the movable platform corresponding to extreme poses changing with the length of the extended links μ. Then, the singular points on the curves could be obtained with the following judgment condition:

$$\left.\begin{aligned} &\boldsymbol{F}(\boldsymbol{x},\boldsymbol{L})=0 \\ &\det(\boldsymbol{F}_{\boldsymbol{x}}(\boldsymbol{x},\boldsymbol{L}))=0 \end{aligned}\right\} \tag{2}$$

where $\boldsymbol{F}_{\boldsymbol{x}}(\boldsymbol{x},\boldsymbol{L})$ is a 9×9 matrix, a one-order derivative of $\boldsymbol{F}$ for every element of $\boldsymbol{x}$.

Here is a numerical example to illustrate an analysis of the singular points of extreme poses for a parallel manipulator. With the structural parameters of the Stewart platform being r = 750 mm, R = 1 000 mm, $l_{\min}$=1 000 mm, α_1 = 100°, α_2 = 25°, the singular points of typical extreme poses are listed in Table 1.

Table 1 Singular points of extreme poses

No.	μ /mm	x_0/mm	y_0/mm	z_0/mm	θ_x/(°)	θ_y/(°)	θ_z/(°)
2	1 808.0	– 440.1	–514.1	604.2	–14.8	–23.7	–60.4
3	2 440.6	– 991.5	–219.8	–369.4	156.4	–18.6	–113.6
4	1 751.5	195.0	–568.3	681.1	–16.1	–18.4	–87.0
5	1 487.8	– 206.0	–653.2	706.8	–25.3	–8.8	–35.5
6	2 346.2	– 442.0	482.4	754.5	–175.3	–25.1	121.4
7	2 140.0	– 222.4	–514.6	1 167.7	59.3	2.2	–96.1
8	1 748.6	– 889.4	–639.6	468.5	–14.2	–37.4	–7.5
9	1 681.2	0	0	592.2	0	0	–127.5
10	1 629.9	477.5	73.7	935.9	20.7	–54.7	–71.4
11	2 082.2	1 547.7	343.1	379.1	–40.1	28.4	– 48.0
12	1 779.3	– 25.8	488.0	938.7	–28.9	43.9	26.4
13	1 928.4	190.4	603.8	1 112.9	111.4	18.7	–10.3
14	1 587.6	– 223.4	243.8	1 156.4	10.8	46.8	–32.8
15	1 839.1	374.6	732.2	1 249.0	9.9	50.9	– 0.3

From Table 1, the value μ of the singular point of the 5th extreme pose is found to be the minimum, which means it is possible for a manipulator with the maximum length of links more than 1 487.8 mm to work abnormally if the rotation angle limit of joints and the interference between any two links are not taken into account.

2.3 Structural constraints

In the above-mentioned manipulator, supposing the maximum length of links $l_{\max}$ = 1 400 mm, a variety of constraints will be discussed when the manipulator moves in turns among these extreme poses. Fig.2 shows the rotation angles of the spherical joints, and Fig.3 shows the universal joints. As

As shown in Fig.2 and Fig.3, the maximum rotation angle of joints is achieved at an extreme pose. The alteration of distances between two links is shown in Fig.4, from which the minimum distance is also found at an extreme pose.

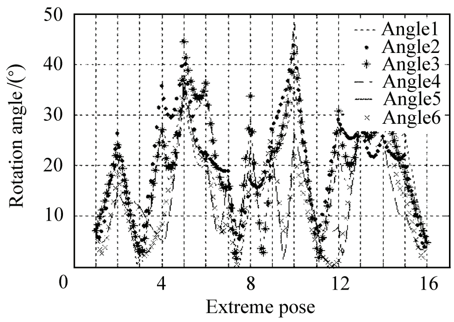

Fig.2 Rotation angles of the spherical joints.

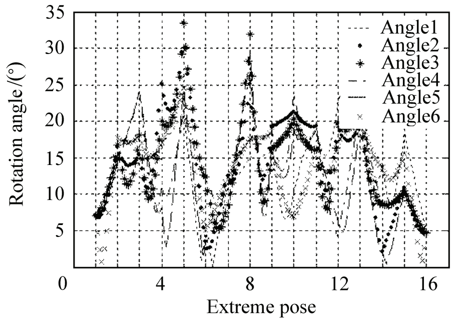

Fig.3 Rotation angles of the universal joints.

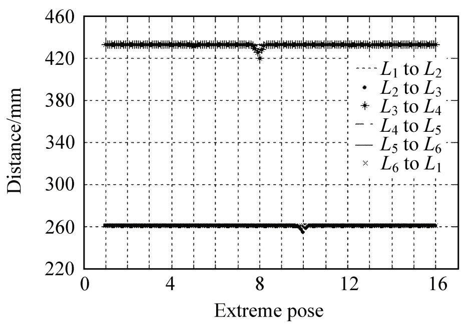

Fig.4 Distances between two links.

2.4 Condition numbers of extreme poses

With a parallel manipulator in a singular configuration, a serious problem will be caused by its failure of bearing any load at that time. The Jacobian condition number serves to judge the proximity to singularity. In the case of a large condition number, a small relative error in the input will produce a large relative error in the output which means the mechanism configuration is closer to singularity[12-13]. The condition number is defined by

$$\mathrm{Cond}(\boldsymbol{J}) = \|\boldsymbol{J}\| \cdot \|\boldsymbol{J}^{-1}\| \tag{3}$$

where $\|\cdot\|$ refers to the Euclidean 2-norm.

The variation of condition numbers of extreme poses are shown in Fig.5, from which the maximum value of condition numbers is seen at an extreme pose.

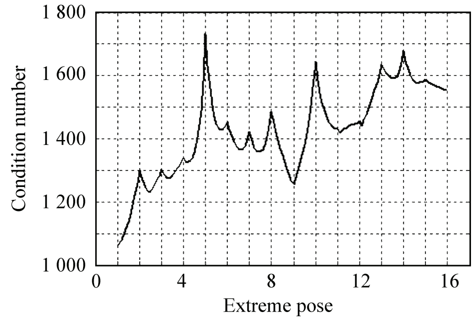

Fig.5 Condition numbers.

3 Optimal Design of Safety Mechanism

It is required that a safety mechanism be able to move at will in a link space without any limitation from rotation angles of joints and link interference. By analyzing the constraint variation when the Stewart parallel manipulator moves among the extreme poses, it can be concluded that the extremum of every constraint occurs at an extreme pose. If the fact that the extremum of every constraint of Stewart platforms with different parameters must occur at an extreme pose could be ascertained, the constraint conditions will be satisfied at any poses no matter how long the links are on condition that the constraint conditions are satisfied at all extreme poses. However, the above-mentioned hypothesis is very difficult to be proved by analytical solution. Therefore, considering the continuity of the mechanical motion, a test algorithm is applied to check the safety of the mechanism at the 16 typical extreme poses.

3.1 Test algorithm of safety mechanism

From the foregoing discussion, it can be found that if, in a specific initial assembled configuration with a set of given manipulator parameters, the maximum link length is less than the length of extended links defined by the singular point at all extreme poses which are determined starting from the position and orientation of the initial configuration, there exists a feasible forward kinematics solution corresponding to each extreme pose. Therefore, it is necessary only to solve the forward kinematics solutions of 16 typical extreme poses determined by the specific initial assembled configuration. The existence of a feasible forward kinematics solution corresponding to each extreme pose will ascertain the feasibility of this set of manipulator parameters. In order to solve this problem, the following arithmetic is used, which, simple and convenient, obviates the need for solving all the singular points of all extreme poses.

Suppose $\boldsymbol{P}_0$ is the initial value of the position and orientation corresponding to the initial configuration, then, by inverse kinematics method, the corresponding initial link length vector is $\boldsymbol{L}_0$. The difference between the actual length vector and the initial length vector is $\Delta\boldsymbol{L} = \boldsymbol{L} - \boldsymbol{L}_0$.

Let $\boldsymbol{J}$ be the one-order influence coefficient matrix and $\boldsymbol{G}$ the transform matrix of angle velocity, the position and orientation increment can be obtained through the link length increment when $\boldsymbol{J}$ and $\boldsymbol{G}$ are non-singular

$$\Delta\boldsymbol{P} = (\boldsymbol{JG})^{-1}\,\Delta\boldsymbol{L} \tag{4}$$

Then, suppose $\boldsymbol{P}_0 + \Delta\boldsymbol{P}$ is the new position and orientation, repeat the above-described process until every element of $\Delta\boldsymbol{L}$ is less than an extremely tiny positive value e, from which the position and orientation of the movable platform is obtained. This solution must be exclusive due to the continuity of mechanical motion. Moreover, with the iterative method applied, if the forward kinematics solution lies just on or close to a singular point, and the iterative result is close to the forward kinematics solution, vibrations will take place because of the abnormal Jacobian and the errors caused by the iterative calculation. This would result in failure to meet the requisite precision and achieve the feasible forward kinematics solution.

In order to consider the rotation angle limit of joints and the interference between any two links, it is only needed to judge whether an extreme pose satisfies the rotation angle limit of joints and the constraint of link interference if the extreme pose has feasible forward kinematics solution.

The constraints of rotation angel of the spherical joint and the universal joint are expressed respectively as follows

$$\theta_{1i} = \arccos\frac{\boldsymbol{l}_i \cdot (\boldsymbol{T}\boldsymbol{l}_{ni})}{|\boldsymbol{l}_i||\boldsymbol{l}_{ni}|} < \theta_{1\max} \tag{5}$$

$$\theta_{2i} = \arccos\frac{\boldsymbol{l}_i \cdot \boldsymbol{l}_{ni}}{|\boldsymbol{l}_i||\boldsymbol{l}_{ni}|} < \theta_{2\max} \tag{6}$$

where $\boldsymbol{l}_{ni}$ is the initial fixing vector of the joint, $\theta_{1\max}$ is the maximum rotation angle of the spherical joint, and $\theta_{2\max}$ is the universal joint.

Suppose that the six links are bar-shaped each with a diameter D. If $D_{i,i+1}$ is the minimum distance between two neighboring links $\boldsymbol{l}_i$ and $\boldsymbol{l}_{i+1}$, then the condition, under which link interference would not happen, is

$$D \leqslant D_{i,i+1} \qquad i, i+1 = 1, 2, \cdots, 6 \tag{7}$$

3.2 Optimization goal

Different optimization goals might be adopted to meet different work requests. Based on the above analysis, an average Jacobian condition number of 16 typical extreme poses is used as a means to measure the global kinematics performance. And the optimization goal is set to acquire a minimal average condition number. As a result, the optimization problem can be formulated as follows

$$\min\left(\frac{1}{16}\sum_{i=1}^{16}\mathrm{cond}(\boldsymbol{J}_i)\right) \tag{8}$$

where $\boldsymbol{J}_i$ represents the Jacobian number of the ith extreme pose.

3.3 Calculation example

In this example, the rotation angle limits of spherical joints and universal joints are assumed to be 35° and 45° respectively, and the diameter of all links is 100 mm. In addition, the mechanical constraint $l_{min} / l_{max} = 0.7$.

With a population size and the maximum number of generations being 25 and 100 separately, a genetic algorithm with bit string encoding is used to optimize the safety mechanism. The fitness function is as follows

$$f(k)=\begin{cases} -\min(\frac{1}{16}\sum_{i=1}^{16}\mathrm{cond}(\boldsymbol{J}_i)) & \text{if safety} \\ -10\,000 & \text{if not safety} \end{cases} \tag{9}$$

The above test algorithm could be used to verify whether the manipulator is of a safety mechanism.

In genetic algorithm, each individual consists of five parameters, and a search domain must be specified for each parameter to create an initial population. Here the search domains are set to be $r \in [800,1\,000]$, $R \in [800,1\,400]$, $l_{min} \in [600,1\,200]$, $\alpha_1 \in [10,30]$ and $\alpha_2 \in [90,115]$.

Table 2 shows the best structural parameters and the best fitness after 100 generations. Fig.6 shows the condition numbers of typical extreme poses of the manipulator with thus obtained best structural parameters. It follows that the slightly narrow variation of the condition numbers of 16 extreme poses ensures a relatively better kinematical performance of the manipulator in the workspace determined by the link lengths.

Table 2 Best structural parameters and best fitness

r /mm	R /mm	l_{min} /mm	α_1 /(°)	α_2 /(°)	Fitness
800	835.29	635.02	15.149	90.175	−1 214.4

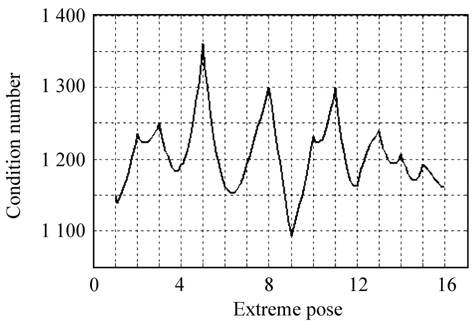

Fig.6 Condition numbers.

4 Conclusions

Based on a link space, the method of designing safety mechanism is of great value and feasibility in practice, for it transforms the problems relating to singularity, constraints of rotation angle limit of joints and link interference into the problem of link length constraints. Associated with an optimization goal, the genetic algorithm can be utilized to design a safety mechanism to the requirement.

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Biographies:

Chen Hua Born in 1974, she received a M.S. degree from Harbin Institute of Technology in 2003, and now is a Ph.D. candidate in Mechatronics Engineering, Harbin Institute of Technology. Her academic interests cover parallel robot kinematics and dynamics, etc.

E-mail: chenhua202@126.com

Chen Weishan Born in 1965, he received a Ph.D. degree from Harbin Institute of Technology in 1997, and now works as a professor in Mechatronics Engineering Department, Harbin Institute of Technology. His academic interests cover ultrasonic driving and control, smart material and structure, parallel mechanism, etc.

E-mail: cws@hit.edu.cn

Liu Junkao Born in 1973, he received a Ph.D. degree from Harbin Institute of Technology in 2001 and now works as an associate professor in Mechatronics Engineering Department, Harbin Institute of Technology. His academic interests cover aircraft hard-ware-in-the-loop simulation, biomimetic robot and simulation system of multi-degree freedom of parallel manipulator, etc.

E-mail: jkliu@hit.edu.cn