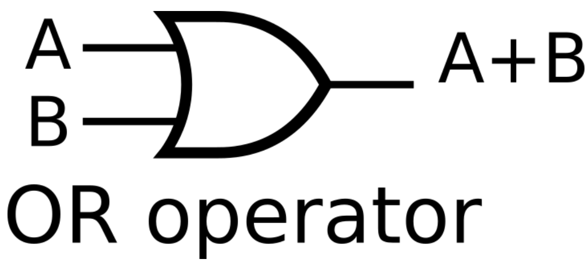
San Quentin Math Circle // Spring 2019

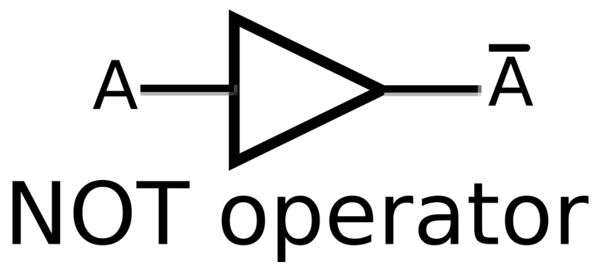
Logic Gates 2 - Electric Boogaloo

Before we get started on the new logic gates, let's have a quick refresher on what we covered last week: *AND*, *OR*, and *NOT*.
Below is a visual representation and truth table of each operator

Input		Output
A	B	A and B
0	0	0
1	0	0
0	1	0
1	1	1



Input		Output
A	B	A or B
0	0	0
1	0	1
0	1	1
1	1	1

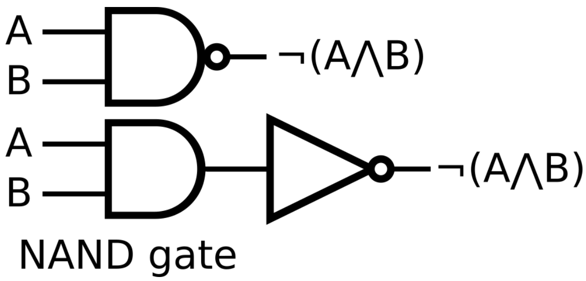


Input	Output
A	NOT A
1	0
0	1

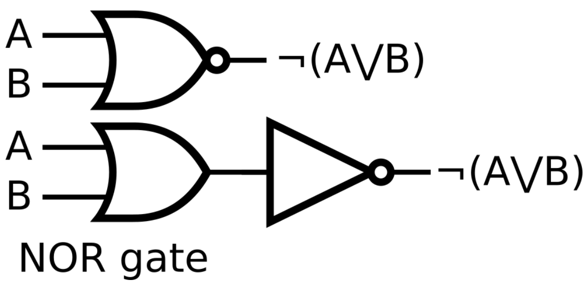
Commonly Used Symbols

A and B $\rightarrow$ A $\wedge$ B or A $\cdot$ B
A or B $\rightarrow$ A $\vee$ B or A + B
Not A $\rightarrow$ $\bar{A}$ or $\neg$ A

Today, we'll introduce some new logic gates, which are actually just combinations of previous logic gates we've seen so far! The first two of these, are NAND and NOR. These are made simply by tacking on the NOT logic gate after the AND logic gate, or the OR logic gate, which is where we get the names from, N(ot)AND, and N(ot)OR.



Input		Output
A	B	A nand B
0	0	
1	0	
0	1	
1	1	



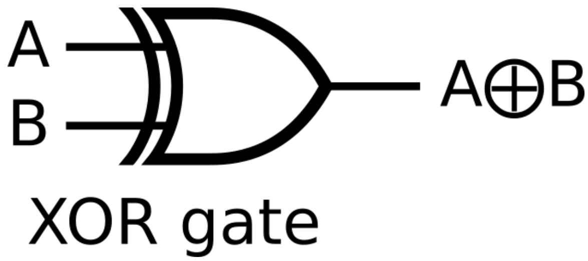
Input		Output
A	B	A nor B
0	0	
1	0	
0	1	
1	1	

Problems

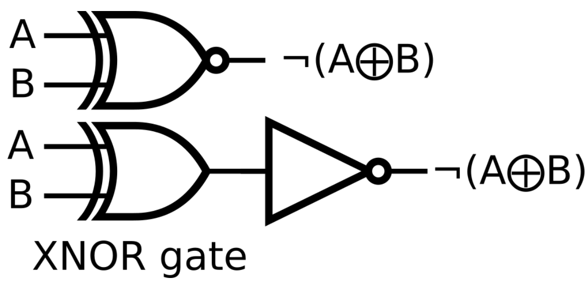
1. Can you think of a real life example of NAND or NOR?

2. Can you think of any benefits to making these extra gates?

The next and final gates we'll introduce are XOR and XNOR. XOR is the exclusive or gate, and can be used to express that two things are mutually exclusive. This is in line with the standard English usage of or, rather than the math version of or we used last week. Note that we'll introduce a new symbol for XOR. Following the convention of the previous gates, we'll see that XNOR is the negation of XOR.



Input		Output
A	B	A xor B
0	0	
1	0	
0	1	
1	1	



Input		Output
A	B	A xor B
0	0	
1	0	
0	1	
1	1	

Problems

1. Can you make a combination of AND and OR gates that replicate the behavior of the XOR gate?
2. Looking back on all of the new gates, can you see a common reason why there are this many gates?
3. Can you make a logic diagram with the following truth tables?

(a)

Input				Output
A	B	C	D	
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

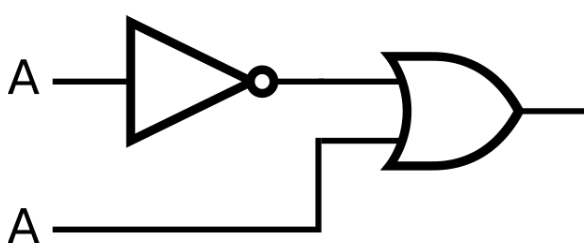
(b)

Input				Output
A	B	C	D	
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

(c)

Input				Outputs	
A	B	C	D		
0	0	0	0	1	0
0	0	0	1	0	1
0	0	1	0	1	1
0	0	1	1	0	0
0	1	0	0	1	0
0	1	0	1	1	0
0	1	1	0	1	1
0	1	1	1	0	0
1	0	0	0	1	1
1	0	0	1	1	0
1	0	1	0	1	0
1	0	1	1	1	1
1	1	0	0	1	1
1	1	0	1	0	0
1	1	1	0	0	1
1	1	1	1	0	1

4. For the following excercises, make your own truth tables, then share with a neighbor for them to make a logic diagram that matches the truth table you made!
   (a) Make a truth table with 2 inputs and 1 output
   (b) Make a truth table with 3 inputs and 2 output
   (c) Make a truth table with 4 inputs and 1 output
5. Make a logic diagram to determine if you had a good day, try to have multiple inputs if possible!
6. A tautology or something that is vacuously true is a diagram that only gives a true output, no matter what inputs you give it. An example from the previous worksheet was,



   Can you make your own tautology? Can you think of an English sentence that's a tautology?
7. Are there any more logic gates that could possibly exist? Explain your reasoning, thoughts or musings on the matter. I encourage
8. These gates are made to give logic some kind of notation or language which we typically refer to as abstraction. Are there any problems with using these to describe any given logical argument or sequence of thoughts? Keep in mind that when we do this kind of thing, our variables like A and B from the input columns of each of these diagrams represent a truth, condition or state of existence for some arbitrary thing, like last week when we talked about AND in terms of having a cat AND having a dog. Consider the impact of things like varying perspective, and things that are not true or not untrue.

9. All of these questions are optional, but this one is *super optional.* If we consider only the symbols like $\wedge$, $\vee$, $\neg$, $\oplus$, can we give them any of the arithmetic properties we've come to know and love, like commutativity, associativity, and distributivity? Which of these properties work with which of these symbols? Recall,

$$\begin{aligned} a + b &= b + a, && \text{commutativity of addition} \\ a + (b + c) &= (a + b) + c, && \text{associativity of addition} \\ a(b + c) &= ab + ac, && \text{distributivity of multiplication over addition} \end{aligned}$$

Use the diagrams if they help!