

Human Following Robot || Linear MPC

Problem Statement || Daksh Raval

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1 Optimal Control

Nutshell 1. Find sequence of control actions that minimize control effort and tracking error.

Remark 1. All references (eg. [1]) in this file are clickable links.

2 LQR

Nutshell 2. Computed with deterministic matrix calculation, constrains dynamics, but no control constraints (eg. max velocity 1 m/s, max steer angle 25°). [1]

2.0.1 Problem Statement

Nutshell 3. Minimize (regulator) weighted (linear) sum of squares (quadratic).

$$\text{Cost Function: } \min_{u_0, \dots, u_T} \sum_{t=0}^T \underbrace{\mathbf{x}_t^T \cdot Q \cdot \mathbf{x}_t + \mathbf{u}_t^T \cdot R \cdot \mathbf{u}_t}_{\text{Running Error} + \text{Effort Penalty}} + \underbrace{\mathbf{x}_T^T \cdot F \cdot \mathbf{x}_T}_{\text{Final Error Penalty}} \Delta t$$

$$\tilde{\mathbf{x}}_t = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta \theta \end{bmatrix} \quad \tilde{\mathbf{u}} = \begin{bmatrix} v \\ \delta \end{bmatrix} \quad \text{subject to: } \tilde{\mathbf{x}}[t+1] = \underbrace{A\tilde{\mathbf{x}}[t] + B\tilde{\mathbf{u}}[t]}_{\text{dynamics}}$$

Remark 2. *The linearized and discretized state space model from the derivation.pdf is just this 'subject to' linear equation.*

Q and R just diagonal matrices with coefficients weighting $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{u}}$ entries squared. Square because \pm irrelevant, only magnitude relevant

$$Q/R = \begin{bmatrix} q_1/r_1 & 0 & 0 \\ 0 & q_2/r_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix} \quad \text{Cost Function} = q_1 \cdot \Delta x^2 + \dots + r_1 \cdot v^2 + \dots + f_1 \cdot \Delta x^2 + \dots$$

Remark 3. $\tilde{\mathbf{x}}$ is the tracking ERROR, ie. pose and heading of ArUco code w.r.t car, minus some offset so it stops about 15cm away.

2.1 Solution

Nutshell 4. *P feedback controller. Compute P gain matrix for n timesteps, which relates state/-tracking error vector to suggested control action*

$$\tilde{\mathbf{u}}_t = -P_t \cdot \tilde{\mathbf{x}}_t$$

Remark 4. *To see how P gain computed, see Finite Horizon Linear Quadratic Regulator in [2]*

3 Linear MPC

Nutshell 5. *Computed using numerical optimization (using CasADi), constrains dynamics and control.*

3.1 Problem Statement

Nutshell 6. *Same as LQR above, just also subject to control constraint*

$$\text{subject to: } \tilde{\mathbf{u}}_{min} \leq \tilde{\mathbf{u}} \leq \tilde{\mathbf{u}}_{max}$$

4 Appendix

- [1] MIT Underactuated Robotics Textbook - Chapter 3.4.3 (LQR Feedback)
- [2] Drake Library C++ Ricatti Solver for LQR
- [3] Youtuber Control Bootcamp by Steve Brunton