Human Following Robot || Linear MPC

Problem Statement || Daksh Raval

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Contents

1 Optimal Control

Nutshell 1. *Find sequence of control actions that minimize control effort and tracking error.*

Remark 1. *All references (eg. [1]) in this file are clickable links.*

2 LQR

Nutshell 2. *Computed with deterministic matrix calcualation, constrains dynamics, but no control consraints (eg. max velocity 1 m/s, max steer angle 25°). [1]*

2.0.1 Problem Statement

Nutshell 3. *Minimize (regulator) weighted (linear) sum of squares (quadratic).*

$$\textbf{Cost Function:} \quad \min_{u_0,\ldots,u_T} \sum_{t=0}^{T} \underbrace{\mathbf{x}_t^T \cdot Q \cdot \mathbf{x}_t + \mathbf{u}_t^T \cdot R \cdot \mathbf{u}_t}_{Running\ Error\ +\ Effort\ Penalty} + \underbrace{\mathbf{x}_T^T \cdot F \cdot \mathbf{x}_T \, \Delta t}_{Final\ Error\ Penalty}$$

$$\tilde{\mathbf{x}}_\mathbf{t} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta \theta \end{bmatrix} \qquad \tilde{\mathbf{u}} = \begin{bmatrix} v \\ \delta \end{bmatrix} \qquad \textbf{subject to:} \ \tilde{\mathbf{x}}[t+1] = \underbrace{A\tilde{\mathbf{x}}[t] + B\tilde{\mathbf{u}}[t]}_{dynamics}$$

Remark 2. *The linearized and discretized state space model from the derivation.pdf is just this 'subject to' linear equation.*

Q and R just diagonal matrices with coefficients weighting $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{u}}$ entries squared. Square because $\pm$ irrelevant, only magnitude relevant

$$Q/R = \begin{bmatrix} q_1/r_1 & 0 & 0 \\ 0 & q_2/r_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix} \qquad \textbf{Cost Function} = q_1 \cdot \Delta x^2 + \dots + r_1 \cdot v^2 + \dots + f_1 \cdot \Delta x^2 + \dots$$

Remark 3. $\tilde{\mathbf{x}}$ *is the tracking ERROR, ie. pose and heading of ArUco code w.r.t car, minus some offset so it stops about 15cm away.*

2.1 Solution

Nutshell 4. *P feedback controller. Compute P gain matrix for n timesteps, which relates state/-tracking error vector to suggested control action*

$$\tilde{\mathbf{u}}_t = -P_t \cdot \tilde{\mathbf{x}}_t$$

Remark 4. *To see how P gain computed, see Finite Horizon Linear Quadratic Regulator in [2]*

3 Linear MPC

Nutshell 5. *Computed using numerical optimization (using CasADi), constrains dynamics and control.*

3.1 Problem Statement

Nutshell 6. *Same as LQR above, just also subject to control constraint*

$$\textbf{\textit{subject to:}}\ \tilde{\mathbf{u}}_{min} \leq \tilde{\mathbf{u}} \leq \tilde{\mathbf{u}}_{max}$$

4 Appendix

[1] MIT Underactuated Robotics Textbook - Chapter 3.4.3 (LQR Feedback)
[2] Drake Library C++ Ricatti Solver for LQR
[3] Youtuber Control Bootcamp by Steve Brunton