ZIMPL User Guide

(Zuse Institute Mathematical Programming Language)

Thorsten Koch

for Version 3.3.8 June 28, 2019

Contents

1	Pre	face	2		
2	Inti	roduction	3		
3	Inv	ocation	5		
4	For	mat	5		
	4.1	Expressions	6		
	4.2	Tuples and sets	8		
	4.3	Parameters	10		
	4.4	Initializing sets and parameters from a file	12		
	4.5	sum-expressions	13		
	4.6	forall-statements	14		
	4.7	Function definitions	14		
	4.8	The do print and do check commands	14		
5	Mo	dels	14		
	5.1	Variables	15		
	5.2	Objective	15		
	5.3	Constraints	15		
6	Modeling examples				
	6.1	The diet problem	19		
	6.2	The traveling salesman problem	20		
	6.3	The capacitated facility location problem	21		
	6.4	The n -queens problem	23		
7	Err	or messages	27		

Abstract

ZIMPL is a little language in order to translate the mathematical model of a problem into a linear or (mixed-)integer mathematical program, expressed in LP or MPS file format which can be read and (hopefully) solved by a LP or MIP solver.

1 Preface

May the source be with you, Luke!

Many of the things in ZIMPL (and a lot more) can be found in the excellent book about the modeling language AMPL from Robert Fourer, David N. Gay and Brian W. Kernighan [FGK03]. Those interested in an overview of the current state-of-the-art in (commercial) modeling languages might have a look at [Kal04b]. On the other hand, having the source code of a program has its advantages. The possibility to run it regardless of architecture and operating system, the ability to modify it to suit the needs, and not having to hassle with license managers may make a much less powerful program the better choice. And so ZIMPL came into being.

By now ZIMPL has grown up and matured. It has been used in several industrial projects and university lectures, showing that it is able to cope with large-scale models and also with students. This would not have been possible without my early adopters Armin Fügenschuh, Marc Pfetsch, Sascha Lukac, Daniel Junglas, Jörg Rambau, and Tobias Achterberg. Thanks for their comments and bug reports. Special thanks to Tuomo Takkula for revising this manual.

ZIMPL is licensed under the GNU Lesser General Public License Version 3. For more information on free software see http://www.gnu.org. The latest version of ZIMPL can be found at http://zimpl.zib.de. If you find any bugs, then please send an email to mailto:koch@zib.de, and do not forget to include an example that shows the problem. If somebody extends ZIMPL, then I am interested in getting patches to include them into the main distribution.

The best way to refer to ZIMPL in a publication is to cite my PhD thesis [Koc04]

2 Introduction

Consider the LP formulation of the shortest s, t-path problem, applied to some directed graph (V, A) with cost coefficient c_{ij} for all $(i, j) \in A$:

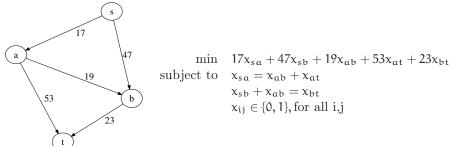
$$\min \sum_{\substack{(i,j)\in A\\ (i\nu)\in \delta^{-}(\nu)}} c_{ij}x_{ij}$$

$$\sum_{\substack{(i\nu)\in \delta^{-}(\nu)}} x_{i\nu} = \sum_{\substack{(\nu i)\in \delta^{+}(\nu)}} x_{\nu i} \quad \text{for all } \nu \in V \setminus \{s,t\}$$

$$\tag{1}$$

$$x_{ij} \in \{0,1\}, \text{for all i,j in A}$$

where $\delta^+(\nu):=\{(\nu,i)\in A\},\ \delta^-(\nu):=\{(i,\nu)\in A\}$ for $\nu\in V.$ For a given graph the instantiation is



The standard format used to feed such a problem into a solver is called MPS which was invented by IBM for the Mathematical Programming System/360 in the sixties [Kal04a, Spi04]. Nearly all available LP and MIP solvers can read this format. While MPS is a nice format to punch into a punch card and at least a reasonable format to read for a computer, it is quite unreadable for humans. For instance, the MPS file of the above linear program looks as follows:

NAME ROWS N Obj E c0 E c1 E c2 COLUMNS		shortestpa	th.l _l	0	
0020	INTSTART	'MARKER'		'INTORG'	
	×0	Obj	17	c0	1
	×0	c2	1		
	×1	Obj	47	c1	1
	×1	c2	1		
	×2	c1	1	c0	-1
	×2	Obj	19		
	x3	Obj	53	c0	-1
	×4	c1	-1	Obj	23
RHS					
	RHS	c2	1		
BOU					
UP	Bound	×0	1		
UP	Bound	×1	1		
UP	Bound	×2	1		
UP	Bound	x3	1		
UP	Bound	×4	1		
END	ATA				

Another possibility is the LP format [ILO02] which is more readable¹, quite similar to the instantiation, but only supported by a few solvers:

```
Minimize

Obj: +17 x0 +47 x1 +19 x2 +53 x3 +23 x4

Subject to

c0: -1 x3 -1 x2 +1 x0 = +0

c1: -1 x4 +1 x2 +1 x1 = +0

c2: +1 x1 +1 x0 = +1

Bounds

0 <= x0 <= 1

0 <= x1 <= 1

0 <= x2 <= 1

0 <= x3 <= 1

0 <= x4 <= 1

Generals

x0 x1 x2 x3 x4

End
```

Since each coefficient of the matrix A must be stated explicitly it is also not a desirable choice for the development of a mathematical model.

Abstract formulation

Now, with ZIMPL it is possible to write

- compare this with (1). Feeding the script into ZIMPL will automatically generate MPS or LP files.

The value of modeling languages like ZIMPL lies in their ability to work directly on the mathematical model in lieu of dealing merely with coefficients. Furthermore, more often than not instantiations (read LP or MPS-files) of models are generated from some external data. In some sense, these are the result of the model applied to this external data. In our example above, the external data is the graph together with the cost coefficients and the specification of s and t, and the model is the formulation of the shortest st-path problem. Of course, ZIMPL supports the initialization of models from files. For instance, the first lines from the ZIMPL script above can be replaced by

 $^{^1}$ The LP format has also some idiosyncratic restrictions. For example variables should not be named e12 or the like. and it is not possible to specify ranged constraints.

```
set     V:= {read "nodes.txt" as "<1s>"};
set     A:= {read "arcs.txt" as "<1s,2s>"};
param c[A] := read "arcs.txt" as "<1s,2s>3n";
```

and the ZIMPL script is ready produce instances of any shortest path problem defined by the files "nodes.txt" and "arcs.txt". The format of those files will be described in Subsection 4.4.

3 Invocation

In order to run ZIMPL on a model given in the file ex1.zpl type the command:

```
zimpl ex1.zpl
```

In general terms the command is:

```
zimpl [options] <input-files>
```

It is possible to give more than one input file. They are read one after the other as if they were all one big file. If any error occurs while processing, ZIMPL prints out an error message and aborts. In case everything goes well, the results are written into two or more files, depending on the specified options.

The first output file is the problem generated from the model in either CPLEXLP, MPS, or a "human readable" format, with extensions .lp, .mps, or .hum, respectively. The next one is the table file which has the extension .tbl. The table file lists all variable and constraint names used in the model and their corresponding names in the problem file. The reason for this name translation is the limitation of the length of names in the MPS format to eight characters. Also the LP format restricts the length of names. The precise limit is depending on the version. CPLEX 7.0 has a limit of 16 characters, and ignores silently the rest of the name, while CPLEX 9.0 has a limit of 255 characters, but will for some commands only show the first 20 characters in the output.

A complete list of all options understood by ZIMPL can be found in Table 1. A typical invocation of ZIMPL is for example:

```
zimpl -o solveme -t mps data.zpl model.zpl
```

This reads the files data.zpl and model.zpl as input and produces as output the files solveme.mps and solveme.tbl. Note that in case MPS-output is specified for a maximization problem, the objective function will be inverted, because the MPS format has no provision for stating the sense of the objective function. The default is to assume minimization.

4 Format

Each ZPL-file consists of six types of statements:

- ► Sets
- ▶ Parameters
- ► Variables
- ▶ Objective
- ► Constraints
- ▶ Function definitions

Each statement ends with a semicolon. Everything from a hash-sign #, provided it is not part of a string, to the end of the line is treated as a comment and is ignored. If a line starts with the word include followed by a filename in double quotation marks, then this file is read and processed instead of the line.

-t format	Selects the output format. Can be either 1p, which is default, or mps,
	or pip, which is polynomial IP, or hum, which is only human readable,
	or rlp, which is the same as lp but with rows and columns randomly
	permuted. The permutation is depending on the <i>seed</i> . Also possible is
	pip m which means Polynomial IP.
-o name	Sets the base-name for the output files.
	Defaults to the name of the first input file with its path and extension
_ ~	stripped off.
-F filter	The output is piped through a filter. A $\%$ s in the string is replaced by the
	output filename. For example -F "gzip -c >%s.gz" would compress
	all the output files.
-l length	Sets maximal length of variables and constraints for 1p format files to
	length.
-n <i>cform</i>	Select the format for the generation of constraint names. Can be cm
	which will number them 1n with a 'c' in front. cn will use the name
	supplied in the subto statement and number them 1n within the
	statement. cf will use the name given with the subto, then a 1n
	number like in cm and then append all the local variables from the forall
	statements.
-P filter	The input is piped through a filter. A %s in the string is replaced by
	the input filename. For example -P "cpp -DWITH_C1 %s" would pass
	the input file through the C-preprocessor.
-s seed	Positive seed number for the random number generator. For example,
3 3000	-s 'date +%N' will produce changing random numbers.
-v 05	Set the verbosity level. 0 is quiet, 1 is default, 2 is verbose, 3 and 4 are
-v 03	chatter, and 5 is debug.
-D name=val	
-D Hame=vai	Sets the parameter <i>name</i> to the specified value. This is equivalent to
	having this line in the ZIMPL program: param name:=val. If there is
	a declaration in the ZIMPL file and a -D setting for the same name, the
	latter takes precedent.
-b	Enables bison debug output.
-f	Enables flex debug output.
-h	Prints a help message.
-m	Writes a CPLEXmst (Mip STart) file.
-O	Tries to reduce the generated LP by doing some presolve analysis.
-r	Writes a CPLEXord branching order file.
-V	Prints the version number.

Table 1: Zimpl options

4.1 Expressions

ZIMPL works on its lowest level with two types of data: Strings and numbers. Wherever a number or string is required it is also possible to use a parameter of the corresponding value type. In most cases, expressions are allowed instead of just a number or a string. The precedence of operators is the usual one, but parentheses can always be used to specify the evaluation order explicitly.

Numeric expressions

A number in ZIMPL can be given in the usual format, e.g. as 2, -6.5 or 5.234e-12. Numeric expressions consist of numbers, numerically valued parameters, and any of

the operators and functions listed in Table 2. Additionally the functions shown in Table 3 can be used. Note that those functions are only computed with normal double precision floating-point arithmetic and therefore have limited accuracy. Examples on how to use the min and max functions can be found in Section 4.3 on page 10.

a^b, a**b	a to the power of b	α ^b , b must be integer
a+b	addition	a + b
a-b	subtraction	a - b
a*b	multiplication	$a \cdot b$
a/b	division	a/b
a mod b	modulo	a mod b
abs(a)	absolute value	a
sgn(a)	sign	$x > 0 \Rightarrow 1, x < 0 \Rightarrow -1, \text{else } 0$
floor(a)	round down	[a]
ceil(a)	round up	$\lceil \alpha \rceil$
round(a)	round towards zero	[a]
a!	factorial	a!, a must be nonnegative integer
min(S)	minimum of a set	$\min_{s \in S}$
min(s in S) e(s)	minimum over a set	$\min_{s \in S} e(s)$
max(S)	maximum of a set	$\max_{s \in S}$
max(s in S) e(s)	maximum over a set	$\max_{s \in S} e(s)$
min(a,b,c,,n)	minimum of a list	$\min(a, b, c, \ldots, n)$
max(a,b,c,,n)	maximum of a list	$\max(a, b, c, \dots, n)$
<pre>sum(s in S) e(s)</pre>	sum over a set	$\sum_{s \in S} e(s)$
<pre>prod(s in S) e(s)</pre>	product over a set	$\prod_{s \in S} e(s)$
card(S)	cardinality of a set	S
random(m,n)	pseudo random number	$\in [\mathfrak{m},\mathfrak{n}]$, rational
ord(A,n,c)	ordinal	c-th component of the n-th
		element of set A.
length(s)	length of a string	number of characters in s
if a then b	197	\int b, if $a = \text{true}$
else c end	conditional	c , if $\alpha = \text{false}$

Table 2: Rational arithmetic functions

gamt (a)	causes root	
sqrt(a)	square root	√a
log(a)	logarithm to base 10	$\log_{10} a$
ln(a)	natural logarithm	$\ln \alpha$
exp(a)	exponential function	e^{α}

Table 3: Double precision functions

String expressions

A string is delimited by double quotation marks ", e.g. "Hallo Keiken". Two strings can be concatenated using the + operator, i.e., "Hallo " + "Keiken" gives "Hallo Keiken". The function substr(string, begin, length) can be used to extract parts of a string. begin is the first character to be used, and counting starts with zero at first character. If begin is negative, then counting starts at the end of the string. length

is the number of charaters to extract starting at begin. The length of a string can be determined using the length function.

Boolean expressions

These evaluate either to true or to false. For numbers and strings the relational operators <, <=, ==, !=, >=, and > are defined. Combinations of Boolean expressions with and, or, and xor^2 and negation with not are possible. The expression tuple in set-expression (explained in the next section) can be used to test set membership of a tuple. Boolean expressions can also be in the then or else part of an if expression.

Variant expressions

The following is either a numeric, string or boolean expression, depending on whether *expression* is a string, boolean, or numeric expression:

if boolean-expression then expression else expression end

The same is true for the ord(set, tuple-number, component-number) function which evaluates to a specific element of a set (details about sets are covered below).

4.2 Tuples and sets

A tuple is an ordered vector of fixed dimension where each component is either a number or a string. Sets consist of (a finite number of) tuples. Each tuple is unique within a set. All sets in ZIMPL are internally ordered, but there is no particular order. Sets are delimited by braces, { and }, respectively. All tuples of a specific set have the same number of components. The type of the n-th component for all tuples of a set must be the same, i. e., they have to be either all numbers or all strings. The definition of a tuple is enclosed in angle brackets < and >, e. g. <1,2,"x">. The components are separated by commas. If tuples are one-dimensional, it is possible to omit the tuple delimiters in a list of elements, but in this case they must be omitted from all tuples in the definition, e. g. {1,2,3} is valid while {1,2,<3} is not.

Sets can be defined with the set statement. It consists of the keyword set, the name of the set, an assignment operator :=, and a valid set expression.

Sets are referenced by the use of a *template* tuple, consisting of placeholders which are replaced by the values of the components of the respective tuple. For example, a set S consisting of two-dimensional tuples could be referenced by <a,b> in S. If any of the placeholders are actual values, only those tuples matching these values will be extracted. For example, <1,b> in S will only get those tuples whose first component is 1. Please note that if one of the placeholders is the name of an already defined parameter, set or variable, it will be substituted. This will result either in an error or an actual value.

Examples

```
set A := { 1, 2, 3 };
set B := { "hi", "ha", "ho" };
set C := { <1,2,"x">, <6,5,"y">, <787,12.6,"oh"> };
```

For set expressions the functions and operators given in Table 4 are defined.

An example for the use of the if boolean-expression then set-expression else setexpression end can be found on page 10 together with the examples for indexed sets.

 $^{^{2}}a \text{ xor } b := a \wedge \neg b \vee \neg a \wedge b$

Examples

```
set D := A cross B;
set E := { 6 to 9 } union A without { 2, 3 };
set F := { 1 to 9 } * { 10 to 19 } * { "A", "B" };
set G := proj(F, <3,1>);
# will give: { <"A",1>, <"A",2"> ... <"B",9> }
```

```
A*B,
                        cross product
                                                     \{(x,y) \mid x \in A \land y \in B\}
A cross B
A+B.
                                                     \{x \mid x \in A \lor x \in B\}
                        union
A union B
union <i>
                        ditto, of indexed sets
                                                     \bigcup_{i \in I} S_i
 in I: S
                                                     \{x \mid x \in A \land x \in B\}
A inter B
                        intersection
inter <i>
                        ditto, of indexed sets
                                                     \bigcap_{i \in I} S_i
 in I: S
A \setminus B, A - B,
                        difference
                                                     \{x \mid x \in A \land x \notin B\}
A without B
A symdiff B
                        symmetric difference
                                                     \{x \mid (x \in A \land x \notin B) \lor (x \in B \land x \notin A)\}
\{n \dots m \ by \ s\},\
                        generate,
                                                     \{x \mid x = \min(n, m) + i |s| \leqslant \max(n, m),
                        (default s = 1)
                                                     i \in \mathbb{N}_0, x, n, m, s \in \mathbb{Z}
\{n \text{ to } m \text{ } by \text{ } s\}
                        generate
                                                     \{x \mid x = n + is \leq m, i \in \mathbb{N}_0, x, n, m, s \in \mathbb{Z}\}
                                                     The new set will consist of n-tuples, with
                        projection
proj(A, t)
                                                     the i-th component being the e_i-th com-
                        t = (e_1, \ldots, e_n)
                                                     ponent of A.
argmin <i>
                        minimum argument
                                                     \operatorname{argmin}_{\mathfrak{i}\in I} e(\mathfrak{i})
 in I : e(i)
argmin(n) <i>
                                                     The new set consists
                        n minimum
 in I : e(i)
                        arguments
                                                     of those n elemens of i for which e(i) was
                                                     smallest. The result can be ambiguous.
argmax <i>
                        maximum argument
 in I : e(i)
                                                     \operatorname{argmax}_{i \in I} e(i)
                                                     The new set consists
argmax(n) <i>
                        n maximum
 in I : e(i)
                                                     of those n elemens of i for which e(i) was
                        arguments
                                                     biggest. The result can be ambiguous.
if a then b
                                                         b, if a = true
                        conditional
else c end
                                                      c, if \alpha = \text{false}
```

Table 4: Set related functions

Conditional sets

It is possible to restrict a set to tuples that satisfy a Boolean expression. The expression given by the with clause is evaluated for each tuple in the set and only tuples for which the expression evaluates to *true* are included in the new set.

Examples

```
set F := { <i,j> in Q with i > j and i < 5 };
set A := { "a", "b", "c" };
set B := { 1, 2, 3 };
set V := { <a,2> in A*B with a == "a" or a == "b" };
# will give: { <"a",2>, <"b",2> }
set W := argmin(3) <i,j> in B*B : i+j;
# will give: { <1,1>, <1,2>, <2,1> }
```

Indexed sets

It is possible to index one set with another set resulting in a set of sets. Indexed sets are accessed by adding the index of the set in brackets [and], like S[7]. Table 5 lists the available functions. There are three possibilities how to assign to an indexed set:

- ▶ The assignment expression is a list of comma-separated pairs, consisting of a tuple from the index set and a set expression to assign.
- ▶ If an index tuple is given as part of the index, e.g. <i> in I, the assignment is evaluated for each value of the index tuple.
- ▶ By use of a function that returns an indexed set.

Examples

```
set I
                := { 1..3 };
                := <1> {"a","b"}, <2> {"c","e"}, <3> {"f"};
set A[I]
set B(<i> in I) := { 3 * i };
                := powerset(I);
set P[]
                := indexset(P);
set J
                := subsets(I, 2);
set S[]
set T[]
                := subsets(I, 1, 2);
set K[<i> in I] := if i mod 2 == 0 then { i } else { -i } end;
                := union <i> in I : A[i];
set U
set IN
                := inter <j> in J : P[j]; # empty!
```

Table 5: Indexed set functions

4.3 Parameters

Parameters are the way to define constants within ZIMPL. They can be declared with or without an index set. Without index a parameter is just a single value which is either a number or a string. For indexed parameters there is one value for each member of the set. It is possible to declare a *default* value.

Parameters are declared in the following way: The keyword param is followed by the name of the parameter optionally followed by the index set in square brackets. Then, after the assignment operator, there is a list of pairs. The first element of each pair is a tuple from the index set, while the second element is the value of the parameter for this index.

Examples

Assignments do not need to be complete. In the example, no value is given for index <3,7,"z"> of parameter w. This is correct as long as it is never referenced.

Parameter tables

It is possible to initialize multi-dimensional indexed parameters from tables. This is especially useful for two-dimensional parameters. The data is put in a table structure with | signs on each margin. Then a headline with column indices has to be added, and one index for each row of the table is needed. The column index has to be one-dimensional, but the row index can be multi-dimensional. The complete index for the entry is built by appending the column index to the row index. The entries are separated by commas. Any valid expression is allowed here. As can be seen in the third example below, it is possible to add a list of entries after the table.

Examples

```
set I := \{ 1 ... 10 \};
set J := \{ "a", "b", "c", "x", "y", "z" \};
param h[I*J] := | "a", "c", "x", "z"
                  |1| 12, 17, 99,
                                            23 |
                             3,-17, 66*5.5 |
                       4,
                  |5| 2/3, -.4, 3, abs(-4)|
                              2, 0,
                         1,
                                             3 | default -99;
param g[I*I*I] :=
                       | 1, 2, 3 |
                     |1,3| 0, 0, 1 |
                     |2,1| 1, 0, 1 |;
param k[I*I] := | 7, 8, 9 |
                 |4| 89, 67, 55 |
                 |5| 12, 13, 14 |, <1,2> 17, <3,4> 99;
The last example is equivalent to:
param k[I*I] := \langle 4,7 \rangle \ 89, \ \langle 4,8 \rangle \ 67, \ \langle 4,9 \rangle \ 55, \ \langle 5,7 \rangle \ 12,
                  <5,8> 13, <5,9> 14, <1,2> 17, <3,4> 99;
```

4.4 Initializing sets and parameters from a file

It is possible to load the values for a set or a parameter from a file. The syntax is:

```
read filename as template [skip n] [use n] [match s] [comment s]
```

filename is the name of the file to read. template is a string with a template for the tuples to generate. Each input line from the file is split into fields. The splitting is done according to the following rules: All leading and trailing space and tab characters in the line are removed. Whenever a space, tab, comma, semicolon or colon is encountered a new field is started. Text that is enclosed in double quotes is not split and the quotes are always removed. When a field is split all space and tab characters around the splitting point are removed. If the split is due to a comma, semicolon or colon, each occurrence of these characters starts a new field.

Examples

All these lines have three fields:

```
Hallo;12;3
Moin 7 2
"Hallo, Peter"; "Nice to meet you" 77
,,2
```

For each component of the tuple, the number of the field to use for the value is given, followed by either ${\tt n}$ if the field should be interpreted as a number or ${\tt s}$ for a string. After the template, some optional modifiers can be given. The order does not matter. match s compares the regular expression s against the line read from the file. Only if the expression matches the line, it is processed further. POSIX extended regular expression syntax is used. comment s sets a list of characters that start comments in the file. Each line is ended when any of the comment characters is found. skip s instructs to skip the first s lines of the file. use s limits the number of lines to use to s. When a file is read, empty lines, comment lines, and unmatched lines are skipped and not counted for the use and skip clauses.

Examples

```
set P := { read "nodes.txt" as "<1s>" };
nodes.txt:
                             \rightarrow <"Hamburg">
    Hamburg
                             \rightarrow <"München">
    München
    Berlin
                             \rightarrow <"Berlin">
set Q := \{ \text{ read "blabla.txt" as "<1s,5n,2n>" skip 1 use 2 } \};
blabla.txt:
    Name; Nr; X; Y; No
                             \rightarrow skip
                             \rightarrow <"Hamburg",7,12>
    Hamburg; 12; x; y; 7
                             \rightarrow <"Bremen",5,4>
    Bremen; 4; x; y; 5
    Berlin;2;x;y;8
                             \rightarrow skip
param cost[P] := read "cost.txt" as "<1s> 2n" comment "#";
cost.txt:
    # Name Price
                             \rightarrow skip
    Hamburg 1000
                             \rightarrow <"Hamburg"> 1000
                             \rightarrow <"München"> 1200
    München 1200
```

```
Berlin 1400 \rightarrow <"Berlin"> 1400 
param cost[Q] := read "haha.txt" as "<3s,1n,2n> 4s"; haha.txt: 
 1:2:ab:con1 \rightarrow <"ab",1,2> "con1" 
 2:3:bc:con2 \rightarrow <"bc",2,3> "con2" 
 4:5:de:con3 \rightarrow <"de",4,5> "con3"
```

As with table format input, it is possible to add a list of tuples or parameter entries after a read statement.

Examples

```
set A := { read "test.txt" as "<2n>", <5>, <6> };
param winniepoh[X] :=
   read "values.txt" as "<1n,2n> 3n", <1,2> 17, <3,4> 29;
```

It is also possible to read a single value into a parameter. In this case, either the file should contain only a single line, or the read statement should be instructed by means of a use 1 parameter only to read a single line.

Examples

```
# Read the fourth value in the fifth line
param n := read "huhu.dat" as "4n" skip 4 use 1;
```

If all values in a file should be read into a set, this is possible by use of the "<s+>" template for string values, and "<n+>" for numerical values. Note, that currently at most 65536 values are allowed in a single line.

Examples

```
# Read all values into a set
set X := { read "stream.txt" as "<n+>" };
stream.txt:
1 2 3 7 9 5 6 23
63 37 88
4
87 27
# X := { 1, 2, 3, 7, 9, 5, 6, 23, 63, 37, 88, 4, 87, 27 };
```

$4.5 \quad sum$ -expressions

Sums are stated in the form:

```
sum index do term
```

It is possible to nest several sum instructions, but it is probably more convenient to simply merge the indices. The general form of index is:

```
tuple in set with boolean-expression
```

It is allowed to write a colon: instead of do and a vertical bar | instead of with. The number of components in the *tuple* and in the members of the *set* must match. The with part of an *index* is optional. The *set* can be any expression giving a set. Examples are given in the next section.

4.6 forall-statements

The general forms are:

```
forall index do term
```

It is possible to nest several forall instructions. The general form of *index* equals that of *sum*-expressions. Examples are given in the next section.

4.7 Function definitions

It is possible to define functions within ZIMPL. The value a function returns has to be either a number, a string, a boolean, or a set. The arguments of a function can only be numbers or strings, but within the function definition it is possible to access all otherwise declared sets and parameters.

The definition of a function has to start with defnumb, defstrg, defbool, or defset, depending on the return value. Next is the name of the function and a list of argument names put in parentheses. An assignment operator := has to follow and a valid expression or set expression.

Examples

```
defnumb dist(a,b) := sqrt(a*a + b*b);
defstrg huehott(a) := if a < 0 then "hue" else "hott" end;
defbool wirklich(a,b) := a < b and a >= 10 or b < 5;
defset bigger(i) := { <j> in K with j > i };
```

4.8 The do print and do check commands

The do command is special. It has two possible incarnations: print and check. print will print to the standard output stream whatever numerical, string, Boolean or set expression, or tuple follows it. This can be used for example to check if a set has the expected members, or if some computation has the anticipated result. check always precedes a Boolean expression. If this expression does not evaluate to true, the program is aborted with an appropriate error message. This can be used to assert that specific conditions are met. It is possible to use a forall clause before a print or check statement. For string and numeric values, it is possible to give a comma-separated list to the print statement.

Examples

```
set I := { 1..10 };
do print I;
do print "Cardinality of I:", card(I);
do forall <i> in I with i > 5 do print sqrt(i);
do forall  in P do check sum <p,i> in PI : 1 >= 1;
```

5 Models

In this section, we use the machinery developed up to now to formulate mathematical programs. Apart from the usual syntax to declare variables, objective functions and constraints there is special syntax available that permits the easy formulation of special constraints, such as special ordered sets, conditional constraints, and extended functions.

5.1 Variables

Like parameters, variables can be indexed. A variable has to be one out of three possible types: Continuous (called real), binary or integer. The default type is real. Variables may have lower and upper bounds. Defaults are zero as the lower and infinity as the upper bound. Binary variables are always bounded between zero and one. It is possible to compute the value of the lower or upper bounds depending on the index of the variable (see the last declaration in the example). Bounds can also be set to infinity and -infinity. Binary and integer variables can be declared implicit, i. e., the variable can be assumed to have an integral value in any optimal solution to the integer program, even if it is declared continuous. Use of implicit is only useful if used together with a solver that take advantage of this information. As of this writing, only SCIP (http://scip.zib.de) with linked in ZIMPL can do so. In all other cases, the variable is treated as a continuous variable. It is possible to specify initial values for integer variables by use of the startval keyword. Furthermore, the branching priority can be given using the priority keyword. Currently, these values are written to a CPLEXord branching order file if the -r command line switch is given.

Examples

5.2 Objective

There must be at most one objective statement in a model. The objective can be either minimize or maximize. Following the keyword is a name, a colon: and then a linear term expressing the objective function.

If there is an objective offset, i. e., a constant value in the objective function, ZIMPL automatically generates an internal variable <code>@QObjOffset</code> set to one. This variable is put into the objective function with the appropriate coefficient.³

Example

5.3 Constraints

The general format for a constraint is:

```
subto name: term sense term
```

Alternatively, it is also possible to define ranged constraints which have the form:

³The reason for this is that there is no portable way to put an offset into the objective function in neither LP nor MPS-format.

```
subto name: expr sense term sense expr
```

name can be any name starting with a letter. term is defined as in the objective. sense is one of <=, >= and ==. In case of ranged constraints both senses have to be equal and may not be ==. expr is any valid expression that evaluates to a number.

Additional to linear constraints as in the objective it is also possible to state terms with higher degree for constraints.

Many constraints can be generated with one statement by the use of the forall instruction, as shown below.

Examples

```
subto time: 3 * x1 + 4 * x2 <= 7;

subto space: 50 >= sum <a> in A: <math>2 * u[a] * y[a] >= 5;

subto weird: forall <a> in A: sum <a,b,c> in C: z[a,b,c]==55;

subto c21: 6*(sum <i> in A: x[i] + sum <j> in B: y[j]) >= 2;

subto c40: x[1] == a[1] + 2 * sum <i> in A do <math>2*a[i]*x[i]*3+4;

subto frown: forall <i,j> in X cross \{1 \text{ to } 5\} without \{<2,3>\}

with i>5 and j<2 do

sum <i,j,k> in X cross <math>\{1 \text{ to } 3\} cross Z do

p[i] * q[j] * w[j,k] >= if i == 2 \text{ then } 17 \text{ else } 53 \text{ end};
subto nonlin: 3 * x * y * z + 6 <= x^3 + z * y^2 + 3;
```

Note that in the example i and j are set by the forall instruction. So they are fixed in all invocations of sum.

if in terms

Part of a constraint can be put into an if-then-else-end. In this particular case, the else part is mandatory and both parts need to include some variables in the term.

Examples

```
subto c2: sum <i> in I :
    if (i mod 2 == 0) then 3 * x[i] else -2 * y[i] end <= 3;</pre>
```

if in constraints

It is possible to put two variants of a constraint into an if-statement. A forall statement inside the result part of an if is also possible. The else part is optional.

Examples

Combining constraints with and

It is possible to group constraints by concatenating them with and. This group will then always be processed together. This operator is particularly useful in combination with forall and if.

Examples

```
subto c1: forall <i> in I:
  if (i > 3)
     then if (i == 4)
        then z[1] + z[2] == i
        else z[2] + z[3] == i
     end and
     z[3] + z[4] == i and
     z[4] + z[5] == i
  else
     if (i == 2)
        then z[1] + z[2] == i + 10
        else z[2] + z[3] == i + 10
     end and
     z[3] + z[4] == i + 10 and
     z[4] + z[5] == i + 10
  end;
```

Constraint attributes

It is possible to specify special attributes for a constraint regarding how it should be handled later on.

scale Before the constraint is written to a file, it is scaled by $1/\max|c|$ with c being the coefficients of the constraint.

separate Do not include the constraint in the initial LP, but separate it later on. The constraint need not to be checked for feasibility. When written to an LP file it will be written into a USER CUTS section, in SCIP separate will be set to true.

checkonly Do not include the constraint in the initial LP, but separate only to check for feasibility. When written to an LP file it will be written into a LAZY CUTS section, in SCIP check will be set to true.

indicator If vif (see below) is part of the constraint then it is modeled as an indicator constraint and not by a big-M formulation. If you use this attribute, you can no longer write MPS-format, as indicator variables are not supported by this format.

The attributes are given after the constraint, before the semi-colon and are separated by commas.

Examples

```
subto c1: 1000 * x + 0.3 * y \le 5, scale, checkonly; subto c2: x + y + z == 7, separate; subto c3: vif x == 1 then y == 7 end, indicator;
```

Special ordered sets

ZIMPL can be used to specify special ordered sets (SOS) for an integer program. If a model contains any SOS a sos file is written together with the 1p or mps file. The general format of a special ordered set is:

```
sos name: [type1|type2] priority expr : term
```

name can be any name starting with a letter. SOS use the same namespace as constraints. term is defined as in the objective. type1 or type2 indicate whether a type-1

or type-2 special ordered set is declared. The priority is optional and equal to the priority setting for variables. Many sos can be generated with one statement by the use of the forall instruction, as shown above.

Examples

```
sos s1: type1: 100 * x[1] + 200 * x[2] + 400 * x[3];
sos s2: type2 priority 100 : sum < i > in I: a[i] * x[i];
sos s3: forall < i > in I with i > 2:
type1: (100 + i) * x[i] + i * x[i-1];
```

Extended constraints

ZIMPL can generate systems of constraints that mimic conditional constraints. The general syntax is as follows (note that the else part is optional):

```
vif boolean-constraint then constraint [else constraint] end
```

where boolean-constraint consists of a linear expression involving variables. All these variables have to be bounded integer or binary variables. It is not possible to use any continuous variables or integer variables with infinite bounds in a boolean-constraint. All comparison operators $(<, \leq, ==, !=, \geq, >)$ are allowed. Also, the combination of several terms with and, or, and xor and negation with not is possible. The conditional constraints (those which follow after then or else) may include bounded continuous variables. Be aware that using this construct will lead to the generation of several additional constraints and variables.

Examples

```
var x[I] integer >= 0 <= 20;
subto c1: vif 3 * x[1] + x[2] != 7
    then sum <i> in I : y[i] <= 17
    else sum <k> in K : z[k] >= 5 end;
subto c2: vif x[1] == 1 and x[2] > 5 then x[3] == 7 end;
subto c3: forall <i> in I with i < max(I) :
    vif x[i] >= 2 then x[i + 1] <= 4 end;</pre>
```

Extended functions

It is possible to use special functions on terms with variables that will automatically be converted into a system of inequalities. The arguments of these functions have to be linear terms consisting of bounded integer or binary variables. At the moment only the function vabs(t) that computes the absolute value of the term t is implemented, but functions like the minimum or the maximum of two terms, or the sign of a term can be implemented in a similar manner. Again, using this construct will lead to the generation of several additional constraints and variables.

Examples

```
var x[I] integer >= -5 <= 5;
subto c1: vabs(sum <i> in I : x[i]) <= 15;
subto c2: vif vabs(x[1] + x[2]) > 2 then x[3] == 2 end;
```

7 Error messages

Here is a (hopefully) complete list of the incomprehensible error messages ZIMPL can produce:

101 Bad filename

The name given with the -o option is either missing, a directory name, or starts with a dot.

102 File write error

Some error occurred when writing to an output file. A description of the error follows on the next line. For the meaning consult your OS documentation.

103 Output format not supported, using LP format

You tried to select another format than lp, mps, hum, rlp, or pip.

104 File open failed

Some error occurred when trying to open a file for writing. A description of the error follows on the next line. For the meaning consult your OS documentation.

105 Duplicate constraint name "xxx"

Two subto statements have the same name.

106 Empty LHS, constraint trivially violated

One side of your constraint is empty and the other not equal to zero. Most frequently this happens, when a set to be summed up is empty.

107 Range must be $l \le x \le u$, or $u \ge x \ge l$

If you specify a range you must have the same comparison operators on both sides.

108 Empty Term with nonempty LHS/RHS, constraint trivially violated

The middle of your constraint is empty and either the left- or right-hand side of the range is not zero. This most frequently happens, when a set to be summed up is empty.

109 LHS/RHS contradiction, constraint trivially violated

The lower side of your range is bigger than the upper side, e.g. $15 \le x \le 2$.

110 Division by zero

You tried to divide by zero. This is not a good idea.

111 Modulo by zero

You tried to compute a number modulo zero. This does not work well.

112 Exponent value xxx is too big or not an integer

It is only allowed to raise a number to the power of integers. Also trying to raise a number to the power of more than two billion is prohibited.¹⁰

113 Factorial value xxx is too big or not an integer

You can only compute the factorial of integers. Also computing the factorial of a number bigger then two billion is generally a bad idea. See also Error 115.

114 Negative factorial value

To compute the factorial of a number it has to be positive. In case you need it for a negative number, remember that for all even numbers the outcome will be positive and for all odd number negative.

¹⁰The behavior of this operation could easily be implemented as for(;;) or in a more elaborate way as void f(){f();}.

115 Timeout!

You tried to compute a number bigger than 1000!. See also the footnote to Error 112.

116 Illegal value type in min: xxx only numbers are possible

You tried to build the minimum of some strings.

117 Illegal value type in max: xxx only numbers are possible

You tried to build the maximum of some strings.

118 Comparison of different types

You tried to compare apples with oranges, i.e, numbers with strings. Note that the use of an undefined parameter can also lead to this message.

119 xxx of sets with different dimension

To apply Operation xxx (union, minus, intersection, symmetric difference) on two sets, both must have the same dimension tuples, i. e., the tuples must have the same number of components.

120 Minus of incompatible sets

To apply Operation xxx (union, minus, intersection, symmetric difference) on two sets, both must have tuples of the same type,i.e., the components of the tuples must have the same type (number, string).

121 Negative exponent on variable

The exponent to a variable was negative. This is not supported.

123 "from" value xxx is too big or not an integer

To generate a set, the "from" number must be an integer with an absolute value of less than two billion.

124 "upto" value xxx is too big or not an integer

To generate a set, the "upto" number must be an integer with an absolute value of less than two billion.

125 "step" value xxx is too big or not an integer

To generate a set, the "step" number must be an integer with an absolute value of less than two billion.

126 Zero "step" value in range

The given "step" value for the generation of a set is zero. So the "upto" value can never be reached.

127 Illegal value type in tuple: xxx only numbers are possible

The selection tuple in a call to the proj function can only contain numbers.

128 Index value xxx in proj too big or not an integer

The value given in a selection tuple of a proj function is not an integer or bigger than two billion.

129 Illegal index xxx, set has only dimension yyy

The index value given in a selection tuple is bigger than the dimension of the tuples in the set.

131 Illegal element xxx for symbol

The index tuple used in the initialization list of a index set, is not member of the index set of the set. E.g, set A[{ 1 to 5 }] := <1> { 1 }, <6> { 2 };

132 Values in parameter list missing, probably wrong read template

Probably the template of a read statement looks like "<1n>" only having a tuple, instead of "<1n> 2n".

133 Unknown symbol xxx

A name was used that is not defined anywhere in scope.

134 Illegal element xxx for symbol

The index tuple given in the initialization is not member of the index set of the parameter.

135 Index set for parameter xxx is empty

The attempt was made to declare an indexed parameter with the empty set as index set. Most likely the index set has a with clause which has rejected all elements.

139 Lower bound for integral var xxx truncated to yyy (warning)

An integral variable can only have an integral bound. So the given non integral bound was adjusted.

140 Upper bound for integral var xxx truncated to yyy (warning)

An integral variable can only have an integral bound. So the given non integral bound was adjusted.

141 Infeasible due to conflicting bounds for var xxx

The upper bound given for a variable was smaller than the lower bound.

142 Unknown index xxx for symbol yyy

The index tuple given is not member of the index set of the symbol.

143 Size for subsets xxx is too big or not an integer

The cardinality for the subsets to generate must be given as an integer smaller than two billion.

144 Tried to build subsets of empty set

The set given to build the subsets of, was the empty set.

145 Illegal size for subsets xxx, should be between 1 and yyy

The cardinality for the subsets to generate must be between 1 and the cardinality of the base set.

146 Tried to build powerset of empty set

The set given to build the powerset of, was the empty set.

147 use value xxx is too big or not an integer

The use value must be given as an integer smaller than two billion.

148 use value xxx is not positive

Negative or zero values for the use parameter are not allowed.

149 skip value xxx is too big or not an integer

The skip value must be given as an integer smaller than two billion.

150 skip value xxx is not positive

Negative or zero values for the skip parameter are not allowed.

151 Not a valid read template

A read template must look something like "<1n,2n>". There have to be a < and a > in this order.

152 Invalid read template syntax

Apart from any delimiters like <, >, and commas a template must consists of number character pairs like 1n, 3s.

153 Invalid field number xxx

The field numbers in a template have to be between 1 and 255.

154 Invalid field type xxx

The only possible field types are n and s.

155 Invalid read template, not enough fields

There has to be at least one field inside the delimiters.

156 Not enough fields in data

The template specified a field number that is higher than the actual number of field found in the data.

157 Not enough fields in data (value)

The template specified a field number that is higher than the actual number of field found in the data. The error occurred after the index tuple in the value field.

159 Type error, expected xxx got yyy

The type found was not the expected one, e.g. subtracting a string from a number would result in this message.

160 Comparison of elements with different types xxx / yyy

Two elements from different tuples were compared and found to be of different types.

161 Line xxx: Unterminated string

This line has an odd number of " characters. A String was started, but not ended.

162 Line xxx: Trailing "yyy" ignored (warning)

Something was found after the last semicolon in the file.

163 Line xxx: Syntax Error

A new statement was not started with one of the keywords: set, param, var, minimize, maximize, subto, or do.

164 Duplicate element xxx for set rejected (warning)

An element was added to a set that was already in it.

165 Comparison of different dimension sets (warning)

Two sets were compared, but have different dimension tuples. (This means they never had a chance to be equal, other than being empty sets.)

166 Duplicate element xxx for symbol yyy rejected (warning)

An element that was already there was added to a symbol.

167 Comparison of different dimension tuples (warning)

Two tuples with different dimensions were compared.

168 No program statements to execute

No ZIMPL statements were found in the files loaded.

169 Execute must return void element

This should not happen. If you encounter this error please email the .zpl file to mailto:koch@zib.de.

170 Uninitialized local parameter xxx in call of define yyy

A define was called and one of the arguments was a "name" (of a variable) for which no value was defined.

171 Wrong number of arguments (xxx instead of yyy) for call of define zzz

A define was called with a different number of arguments than in its definition.

172 Wrong number of entries (xxx) in table line, expected yyy entries

Each line of a parameter initialization table must have exactly the same number of entries as the index (first) line of the table.

173 Illegal type in element xxx for symbol

A parameter can only have a single value type. Either numbers or strings. In the initialization both types were present.

174 Numeric field xxx read as "yyy". This is not a number

It was tried to read a field with an 'n' designation in the template, but what was read is not a valid number.

175 Illegal syntax for command line define "xxx" – ignored (warning)

A parameter definition using the command line -D flag, must have the form name=value. The name must be a legal identifier, i.e., it has to start with a letter and may consist only out of letters and numbers including the underscore.

176 Empty LHS, in Boolean constraint (warning)

The left hand side, i. e., the term with the variables, is empty.

177 Boolean constraint not all integer

No continuous (real) variables are allowed in a Boolean constraint.

178 Conditional always true or false due to bounds (warning)

All or part of a Boolean constraint are always either true or false, due to the bounds of variables.

179 Conditional only possible on bounded constraints

A Boolean constraint has at least one variable without finite bounds.

180 Conditional constraint always true due to bounds (warning)

The result part of a conditional constraint is always true anyway. This is due to the bounds of the variables involved.

181 Empty LHS, not allowed in conditional constraint

The result part of a conditional constraint may not be empty.

182 Empty LHS, in variable vabs

There are no variables in the argument to a vabs function. Either everything is zero, or just use abs.

183 vabs term not all integer

There are non integer variables in the argument to a vabs function. Due to numerical reasons continuous variables are not allowed as arguments to vabs.

184 vabs term not bounded

The term inside a vabs has at least one unbounded variable.

185 Term in Boolean constraint not bounded

The term inside a vif has at least one unbounded variable.

186 Minimizing over empty set – zero assumed (warning)

The index expression for the minimization was empty. The result used for this expression was zero.

187 Maximizing over empty set – zero assumed (warning)

The index expression for the maximization was empty. The result used for this expression was zero.

188 Index tuple has wrong dimension

The number of elements in an index tuple is different from the dimension of the tuples in the set that is indexed. This might occur, when the index tuple of an entry in a parameter initialization list has not the same dimension as the indexing set of the parameter. This might occur, when the index tuple of an entry in a set initialization list has not the same dimension as the indexing set of the set. If you use a powerset or subset instruction, the index set has to be one dimension.

189 Tuple number xxx is too big or not an integer

The tuple number must be given as an integer smaller than two billion.

190 Component number xxx is too big or not an integer

The component number must be given as an integer smaller than two billion.

191 Tuple number xxx is not a valid value between 1..yyy

The tuple number must be between one and the cardinality of the set.

192 Component number xxx is not a valid value between 1..yyy

The component number must be between one and the dimension of the set.

193 Different dimension tuples in set initialization

The tuples that should be part of the list have different dimension.

195 Genuine empty set as index set (warning)

The set of an index set is always the empty set.

197 Empty index set for set

The index set for a set is empty.

198 Incompatible index tuple

The index tuple given had fixed components. The type of such a component was not the same as the type of the same component of tuples from the set.

199 Constants are not allowed in SOS declarations

When declaring an SOS, weights are only allowed together with variables. A weight alone does not make sense.

200 Weights are not unique for SOS xxx (warning)

All weights assigned to variables in an special ordered set have to be unique.

201 Invalid read template, only one field allowed

When reading a single parameter value, the read template must consist of a single field specification.

202 Indexing over empty set (warning)

The indexing set turns out to be empty.

203 Indexing tuple is fixed (warning)

The indexing tuple of an index expression is completely fixed. As a result only this one element will be searched for.

204 Random function parameter minimum= xxx >= maximum= yyy

The second parameter to the function random has to be strictly greater than the first parameter.

205 xxx excess entries for symbol yyy ignored (warning)

When reading the data for symbol yyy there were xxx more entries in the file than indices for the symbol. The excess entries were ignored.

206 argmin/argmax over empty set (warning)

The index expression for the argmin or argmax was empty. The result is the empty set.

207 "size" value xxx is too big or not an integer

The size argument for an argmin or argmax function must be an integer with an absolute value of less than two billion.

208 "size" value xxx not >= 1

The size argument for an argmin or argmax function must be at least one, since it represents the maximum cardinality of the resulting set.

209 MIN of set with more than one dimension

The expressions min(A) is only allowed if the elements of set A consist of 1-tuples containing numbers.

210 MAX of set with more than one dimension

The expressions max(A) is only allowed if the elements of set A consist of 1-tuples containing numbers.

211 MIN of set containing non number elements

The expressions min(A) is only allowed if the elements of set A consist of 1-tuples containing numbers.

212 MAX of set containing non number elements

The expressions max(A) is only allowed if the elements of set A consist of 1-tuples containing numbers.

213 More than 65535 input fields in line xxx of yyy (warning)

Input data beyond field number 65535 in line xxx of file yyy are ignored. Insert some newlines into your data!

214 Wrong type of set elements – wrong read template?

Most likely you have tried read in a set from a stream using "n+" instead of "<n+>" in the template.

215 Startvals violate constraint, ... (warning)

If the given startvals are summed up, they violate the constraint. Details about the sum of the LHS and the RHS are given in the message.

216 Redefinition of parameter xxx ignored

A parameter was declared a second time with the same name. The typical use would be to declare default values for a parameter in the ZIMPL file and override them by command-line defined.

217 begin value xxx in substr too big or not an integer

The begin argument for an substr function must be an integer with an absolute value of less than two billion.

218 length value xxx in substr too big or not an integer

The length argument for an substr function must be an integer with an absolute value of less than two billion.

219 length value xxx in substr is negative

The length argument for an substr function must be greater or equal to zero.

220 Illegal size for subsets xxx, should be between yyy and zzz

The cardinality of the subsets to generate must be between the given lower bound and the cardinality of the base set.

221 The objective function has to be linear

Only objective functions with linear constraints are allowed.

222 Term inside a then or else constraint not linear

The term inside a then or else is not linear.

223 Objective function xxx overwrites existing one (warning)

Another objective function declaration has been encountered, superceeding the previous one.

301 variable priority has to be integral (warning)

If branching priorities for variables are given, these have to be integral.

302 SOS priority has to be integral (warning)

If SOS priorities are given, these have to be integral.

600 File format can only handle linear and quadratic constraints (warning)

The chosen file format can currently only handle linear and quadratic constraints. Higher degree constraints were ignored.

$700 \log()$: OS specific domain or range error message

Function log was called with a zero or negative argument, or the argument was too small to be represented as a double.

701 sqrt(): OS specific domain error message

Function sqrt was called with a negative argument.

$702 \, \ln()$: OS specific domain or range error message

Function ln was called with a zero or negative argument, or the argument was too small to be represented as a double.

800 parse error: expecting xxx (or yyy)

Parsing error. What was found was not what was expected. The statement you entered is not valid.

801 Parser failed

The parsing routine failed. This should not happen. If you encounter this error, please email the .zpl file to mailto:koch@zib.de.

802 Regular expression error

A regular expression given to the match parameter of a read statement was not valid. See error messages for details.

803 String too long xxx > yyy

The program encountered a string which is larger than 1 GB.

900 Check failed!

A check instruction did not evaluate to true.

References

- [Chv83] Vašek Chvátal. Linear Programming. H.W. Freeman and Company, New York, 1983.
- [Dan 90] Georg B. Dantzig. The diet problem. Interfaces, 20:43–47, 1990.
- [FGK03] R. Fourier, D. M. Gay, and B. W. Kernighan. *AMPL: A Modelling Language for Mathematical Programming*. Brooks/Cole—Thomson Learning, 2nd edition, 2003.

- [GNU03] GNU multiple precision arithmetic library (GMP), version 4.1.2., 2003. Code and documentation available at http://gmplib.org.
- [IBM97] IBM optimization library guide and reference, 1997.
- [ILO02] ILOG CPLEX Division, 889 Alder Avenue, Suite 200, Incline Village, NV 89451, USA. ILOG CPLEX 8.0 Reference Manual, 2002. Information available at http://www.ilog.com/products/cplex.
- [Kal04a] Josef Kallrath. Mathematical optimization and the role of modeling languages. In Josef Kallrath, editor, *Modeling Languages in Mathematical Optimization*, pages 3–24. Kluwer, 2004.
- [Kal04b] Josef Kallrath, editor. Modeling Languages in Mathematical Optimization. Kluwer, 2004.
- [Koc04] Thorsten Koch. Rapid Mathematical Programming. PhD thesis, Technische Universität Berlin, 2004.
- [Sch03] Alexander Schrijver. Combinatorial Optimization. Springer, 2003.
- [Sch04] Hermann Schichl. Models and the history of modeling. In Josef Kallrath, editor, *Modeling Languages in Mathematical Optimization*, pages 25–36. Kluwer, 2004.
- [SG91] Rok Sosič and Jun Gu. 3,000,000 million queens in less than a minute. $SIGART\ Bulletin,\ 2(2):22-24,\ 1991.$
- [Spi04] Kurt Spielberg. The optimization systems MPSX and OSL. In Josef Kallrath, editor, *Modeling Languages in Mathematical Optimization*, pages 267–278. Kluwer, 2004.
- [vH99] Pascal van Hentenryck. *The OPL Optimization Programming Language*. MIT Press, Cambridge, Massachusetts, 1999.
- [XPR99] XPRESS-MP Release 11 Reference Manual. Dash Associates, 1999. Information available at http://www.dashoptimization.com.