An Under Voltage Load Shedding Optimization Method Based on the Online Voltage Stability Analysis

Yiping Yu, Liang Liu, Kunyuan Pei, Hongyu Li, Yaling Shen, and Weijuan Sun
College of Energy and Electrical Engineering, Hohai University
Nanjing, Jiangsu China
yyiping@hhu.edu.cn

***Abstract*-- The under voltage load shedding (UVLS) is an effective measure to prevent voltage collapse in power systems. The voltage stability varies with the system operation conditions and the under voltage load shedding strategies need to be adjusted in real time. In this paper, a strategy formulation method of UVLS is presented based on the on-line voltage stability indicator. First, the minimum singular value of the load flow Jacobian matrix is selected as the on-line voltage stability indicator. Then, the sensitivity of minimum singular with load is used to select the locations of load shedding in the premise of guaranteeing important load. Moreover, the Particle Swarm Optimization (PSO) is applied to determine the amount of load to be shed. Finally, the strategy of centralized under voltage load shedding is made. The simulation results of the New England 10-machine 39-bus system indicate that the system can be restored the voltage stability with the minimum amount of load shedding.**

***Index Terms*-- Voltage stability, on-line analysis, minimum singular value, under voltage load shedding**

I. INTRODUCTION

IN recent years, because of the frequent occurrence of large-scale blackout characterized by voltage collapse, the voltage stability has aroused high attention of power industry and academy [1][2]. UVLS is an important part of the defense to maintain the safe and stable operation of power systems, and it is an effective and economical method to prevent voltage collapse [3][4]. The accuracy location and the optimal amount of the load shedding is essential for improving the UVLS's efficiency and system's stability.

At present, the UVLS mainly exists in the distributed scheme and the centralized scheme. In most distributed schemes, local measurements are used to control UVLS [5][6]. In the paper [5], the load impedance and system Thevenin equivalent impedance are calculated by the local bus voltage and current, the relationship between load impedance and system Thevenin equivalent impedance determine whether it is necessary to control UVLS. But the accuracy of the method based on local system Thevenin equivalence is still in doubt. In many cases, the voltage stability index needs to be calculated by the whole network analysis, and it can be applied to the centralized UVLS schemes. For instance, the given minimum eigenvalue of the Jacobi matrix is used as voltage stability index to shed load, and the load shedding amount is optimized by the Hopfiled model, so that the system can return to the given voltage stable state [7]. In the paper [8], the load margin is used as the voltage stability index. Starting with the critical point of voltage collapse, the sensitivity of load margin to injection power is calculated, and the equivalent generator injecting power is reduced while some loads are shed. Considering a variety of constraints, the minimum load shedding amount can be solved by linear programming. The bus voltage is used as the voltage stability index in [9]. With the goal of restoring the bus voltage to specified value, UVLS scheme is designed. A set of buses, whose voltage below a certain limit, are selected as load standby buses to be shed, and shedding amount of the bus load is determined by the linear optimization algorithm. In fact, system voltage stability conditions often change with the operation state and the UVLS schemes are often need to adjust. The above methods are mainly based on off-line voltage stability analysis to set the threshold value of voltage stability index for solving the optimal UVLS scheme, which is difficult to meet the needs of determining UVLS scheme on-line according to the real-time status of power flow.

In the voltage stability analysis of power system, the static analysis method is relatively mature, and the singular value decomposition method is widely used [10]-[12]. In this paper, the minimum singular value of power flow Jacobian is used as on-line voltage stability index according to the real-time power flow analysis, and the load buses to be shed are selected with the sensitivity of the minimum singular value to load. The load shedding amount is optimized by particle swarm algorithm. Finally, the centralized UVLS scheme based on on-line voltage stability index is presented. The New England 10-machine 39-bus system simulation results verify the feasibility and effectiveness of this method.

This work is supported by National Natural Science Foundation of China (No. 51207046) and the 2015 Ordinary University Graduate Student Scientific Research Innovation Project of Jiangsu Province (No: KYZZ15_0480).

II. The Analysis of Minimum Singular Value

Under the polar coordinate system, the linearized power flow correction equation can be written as [13]

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{p\theta} & J_{pv} \\ J_{q\theta} & J_{qv} \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta U/U \end{bmatrix} \tag{1}$$

The singular value decomposition of Jacobian matrix can be obtained

$$J = U \sum V^T \tag{2}$$

where J is Jacobian matrix of load flow equation, U and V are $n \times n$ orthonormal matrix composed by singular vectors.

The minimum singular value is defined as δ_n, the corresponding left and right singular vectors are defined respectively as u_n and v_n.

Equation (1) can be written as

$$\begin{bmatrix} \Delta \theta \\ \Delta U / U \end{bmatrix} = J^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = V \sum{}^{-1} U^T \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \tag{3}$$

when $\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = u_n$,

$$\begin{bmatrix} \Delta \theta \\ \Delta U/U \end{bmatrix} = \delta_n^{-1} v_n \tag{4}$$

According to (4), when δ_n tends to 0, power flow equation will no longer converge. Thus δ_n can be used as an online index to assess the system voltage stability.

In [14], a fast method for calculating the minimum singular value and the corresponding left and right singular vectors of the minimum value is presented.

III. The Analysis of Minimum Singular Value Sensitivity

A. The sensitivity of the minimum singular value to the state variable

The state variables are defined as x_i, the sensitivity of δ_n to x_i is [15]

$$\frac{\partial \delta_n}{\partial x_i} = \frac{u_n^T \dfrac{\partial J}{\partial x_i} v_n}{u_n^T u_n} \quad i = (3, 4, \cdots, 2n - r) \tag{5}$$

where, n is the total number of nodes. r is the number of PV nodes. The first node is the balance node. $\dfrac{\partial J}{\partial x_i}$ is a sparse matrix.

The specific calculation methods for each nonzero element are described in the paper [16].

The vector form of (5) is

$$\frac{\partial \delta_n}{\partial X} = \left[\frac{\partial \delta_n}{\partial \theta_2}, \cdots, \frac{\partial \delta_n}{\partial \theta_n}, \frac{\partial \delta_n}{\partial U_2} U_2, \cdots, \frac{\partial \delta_n}{\partial U_{n-r}} U_{n-r} \right]^T \tag{6}$$

where, X is the column vector of state variable. $\theta_i (i = 2, 3, \cdots, n)$ is the voltage phase angle of each node. $U_i (i = 2, 3, \cdots, n)$ is voltage magnitude of each PQ node.

B. The sensitivity of the minimum singular value to the control variable

In the UVLS analysis, the voltage and active power of PV buses and the active power and reactive power of PQ buses can be used as control variables. The sensitivity of minimum singular value to the load power is mainly considered in this paper. The voltage magnitude of the PV buses is assumed constant and the column vector of control variable is defined as $Y = [P_2, P_3, \cdots, P_n, Q_2, \cdots, Q_{n-r}]^T$.

Then the partial derivative of implicit function is worked out

$$\frac{\partial \delta_n}{\partial x_i} = \sum_{j=1}^{2n-r-2} \frac{\partial \delta_n}{\partial y_j} \frac{\partial y_j}{\partial x_i} \quad (i = 1, 2, \cdots, 2n - r - 2) \tag{7}$$

where, y_j is the control variable of the vector Y. $\dfrac{\partial y_j}{\partial x_i}$ is the element of Jacobian matrix Y_{ji}.

The vector form of (7) is:

$$\frac{\partial \delta_n}{\partial X} = J^T \frac{\partial \delta_n}{\partial Y} \tag{8}$$

When J is nonsingular, the sensitivity of δ_n to the control variable is

$$\frac{\partial \delta_n}{\partial Y} = (J^{-1})^T \frac{\partial \delta_n}{\partial X} \tag{9}$$

where, $\dfrac{\partial \delta_n}{\partial Y} = \left[\dfrac{\partial \delta_n}{\partial y_1}, \dfrac{\partial \delta_n}{\partial y_2}, \cdots, \dfrac{\partial \delta_n}{\partial y_{2n-r-2}} \right]^T$.

IV. The Analysis of the Location and Amount of UVLS

A. The selection of the load bus to be shed

The sensitivity of δ_n to the active power and reactive power of the bus is described as $\dfrac{\partial \delta_n}{\partial P_i} = a_i, \dfrac{\partial \delta_n}{\partial Q_i} = b_i$.

The change of minimum singular value can be obtained as

$$\Delta \delta_n = a_i \Delta P_i + b_i \Delta Q_i \tag{10}$$

In the load bus, the change of reactive power and active power has a certain ratio determined by the power factor of the bus

$$\Delta Q_i = c_i \Delta P_i \tag{11}$$

Substituting (11) into (10), equation (10) can be written as

$$\Delta\delta_n = (a_i + c_i b_i)\Delta P,\ \Delta\delta_n = k_i \Delta P \tag{12}$$

where, $k_i = a_i + c_i b_i$, $c_i = \tan\phi_i$, and ϕ_i is power factor angle of bus i.

The load bus which have the maximum impact on the minimum singular value should be selected firstly. Because of some important loads must be protected, the load bus whose k_i is maximum is chosen as the location of UVLS without the important loads.

B. The optimal amount of load shedding

After selecting the load buses, the optimal amount of load shedding is determined by the optimization method. The objective function is set as

$$\min\left\{\sum_{i\varepsilon LSB} \Delta P_i^{\,2}\right\} \tag{13}$$

The restriction conditions are

$$\begin{cases} \sum\limits_{i\in LSB} k_i \cdot \Delta P_i = \Delta\delta_{th} \\ \Delta P_i > 0 \end{cases} \tag{14}$$

where *LSB* is a bus set selected for load shedding. $\Delta\delta_{th} = \delta_{th} - \delta_n$, δ_{th} is the minimum singular value under a certain load safety margin.

The particle swarm optimization algorithm is selected to solve the above objective function and optimize the amount of load shedding. It is based on group optimization and can move the individuals to good region according to the environment. Firstly, initializing the system with a set of random solutions, then searching the optimal values by iteration [17].

C. The steps of the optimal load shedding strategy

The flow chart of the load shedding strategy is shown in Fig.1. The steps are as follows

(1) The online voltage stability index δ_n, based on the real-time power flow, is calculated.

(2) According to the load forecasting and power generation plan, the system real-time changing is determined and the voltage collapse point is calculated by the continuous power flow method.

(3) The minimum singular value of voltage collapse point, δ_m, is calculated.

(4) The minimum singular value δ_n is calculated. According to $\Delta\delta = \delta_n - \delta_m$, $\Delta\delta$ is determined by whether it satisfies the given stability margin, if it does not, continue to step (5), else, stop and give the final shedding node and optimal load shedding amount.

(5) The sensitivity of the minimum singular value to the load node, k_i, is calculated and the node which has bigger k_i and can be considered as load shedding node is selected.

(6) Taking the minimum shedding amount as the objective function and using the particle swarm optimization method, each particle represents specific load shedding amount of nodes, and the shedding amount of the selected nodes by optimization calculation is obtained.

(7) After UVLS, the power flow distribution is calculated, then turn to the step (4).

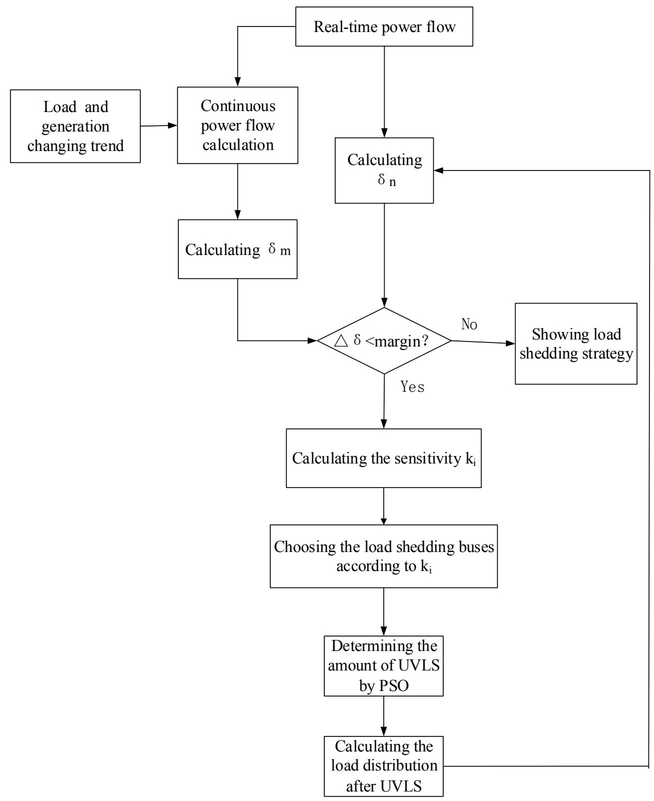

Figure 1 The load shedding strategy flow chart

V. THE ANALYSIS OF EXAMPLES

In this paper, the New England 10-machine 39-bus system is chosen to verify the feasibility of the UVLS method, and the system is shown in Fig.2. Node 31 is the balance node, and 30, 32, 33, 34, 35, 36, 37, 38 and 39 are PV nodes.

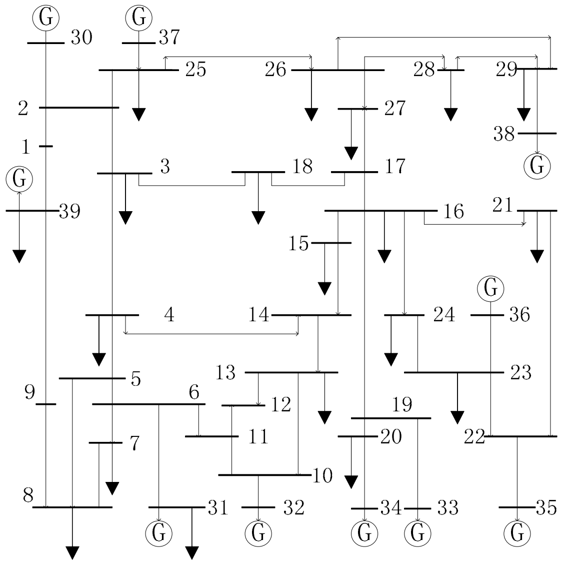

Figure 2 The New England 10-machine 39-bus system

The active and reactive power of PQ nodes are gradually increased according to the constant power factor, until the system is close to the voltage collapse. The relationship between the load power and the minimum singular value is calculated by the continuation power flow method, as shown in Fig.3.

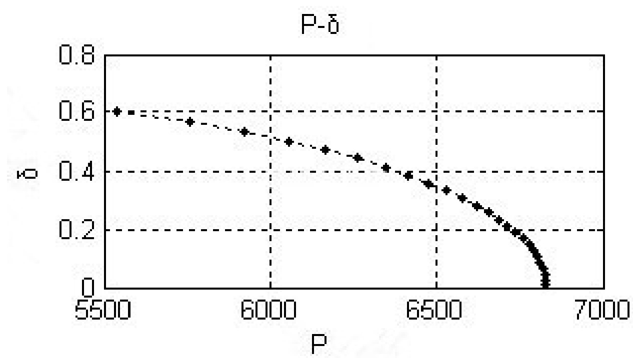

Figure 3 The P-δ curve

Based on the real-time power flow, $\delta_n = 0.0235$ is calculated. Only 6 high sensitivity nodes are listed in the paper, and the maximum sensitivity is used as the reference values for normalization. The obtained data are shown in Table I.

TABLE I. THE MINIMUM SINGULAR VALUE LOAD SENSITIVITY

Node	7	15	21	23	27	29
U	0.7598	0.8956	0.9713	1.0098	0.9662	1.0283
a_i	0.6889	0.6423	0.5504	0.5708	0.6584	0.7374
b_i	0.7230	0.7484	0.7159	0.7184	0.7064	0.6986
k_i	0.9488	1.0000	0.8508	0.8164	0.8481	0.8037

From Table I, the sensitivity of the node 7 and node 15 are the maximum, thus the node 7 and node 15 are chosen as the load shedding nodes. The P-V curve of node 7 is shown in Fig.4.

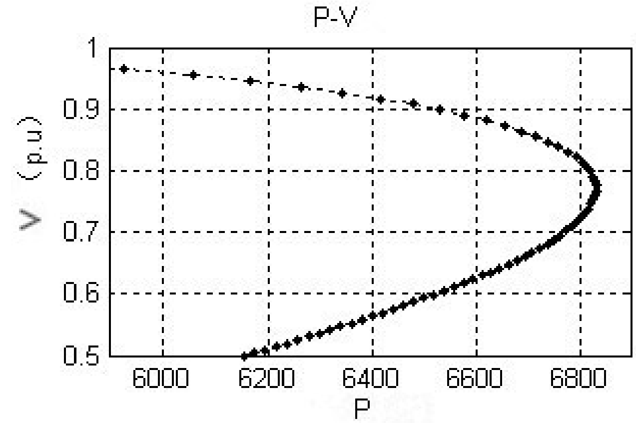

Figure 4 The node 7 P-V curve

The power margin must be not less than 5% to maintain the system voltage stability, then the load capacity is obtained from Fig.4. According to the load capacity, $\delta_n = 0.3525$, is obtained from Fig.3. The δ_n is set to the limit value. The optimal shedding amount of the selected nodes is determined by the particle swarm optimization algorithm. Because the sensitivity of constraints is constant, according to Fig.3, the sensitivity will be reduced when UVLS, thus the calculated shedding amount by optimization algorithm is smaller. Many times iterations are needed to meet the requirements.

After the first iteration, the load shedding amount is $\Delta P_7^1 = 40.0567\text{MW}$, $\Delta Q_7^1 = 14.3916\text{Mvar}$, $\Delta P_{15}^1 = 42.2188\text{MW}$, $\Delta Q_{15}^1 = 20.1859\text{Mvar}$. The sensitivity after the load shedding is shown in Table II. At this point, $\delta_n = 0.1866$.

TABLE II. THE MINIMUM SINGULAR VALUE LOAD SENSITIVITY

Node	7	15	21	23	27	29
U	0.8288	0.9274	0.9854	1.0176	0.9822	1.0325
k_i	0.9625	1.0000	0.8708	0.8347	0.8625	0.8153

Since the minimum singular value has not reached the limit value, it is necessary to carry out the second iteration. From Table II, the sensitivity of the node 7 and node 15 is also the maximum.

After the second iteration, the load shedding amount is $\Delta P_7^2 = 115.1247\text{MW}$, $\Delta Q_7^2 = 41.3622\text{Mvar}$, $\Delta P_{15}^2 = 119.6091\text{MW}$, $\Delta Q_{15}^2 = 57.1881\text{Mvar}$. The sensitivity after the load shedding is shown in Table III. At this point, $\delta_n = 0.3492$.

TABLE III. THE MINIMUM SINGULAR VALUE LOAD SENSITIVITY

Node	7	15	21	23	27	29
U	0.8966	0.9643	1.0012	1.0264	0.9996	1.0371
k_i	0.9690	1.0000	0.8894	0.8584	0.8761	0.8319

Since the minimum singular value has not reached the limit value yet, it is necessary to carry out the third iteration. From table III, the sensitivity of the node 7 and node 15 is also the maximum.

After the third iteration, the load shedding amount is

$\Delta P_7^3 = 3.5381\text{MW}$, $\Delta Q_7^3 = 1.2712\text{Mvar}$, $\Delta P_{15}^3 = 3.6511\text{MW}$, $\Delta Q_{15}^3 = 1.7457\text{Mvar}$. At this point, $\delta_n = 0.3527$, and the minimum singular value has reached the limit value. The sensitivity after the load shedding is shown in Table IV.

TABLE IV. THE MINIMUM SINGULAR VALUE LOAD SENSITIVITY

Node	7	15	21	23	27	29
U	0.8981	0.9652	1.0016	1.0266	1.0000	1.0372
k_i	0.9710	1.0000	0.8906	0.8594	0.8772	0.8326

Finally, the UVLS strategy is that the selected nodes are node 7 and node 15, and the shedding load amount is $\Delta P_7 = 158.7195\text{MW}$ $\Delta Q_7 = 57.0250\text{Mvar}$ $\Delta P_{15} = 165.4790\text{MW}$ $\Delta Q_{15} = 79.1197\text{Mvar}$. The voltage stability level is improved, while the shedding amount is the minimum and the minimum singular value reaches the limit value.

VI. CONCLUSION

The UVLS is an economical and effective solution to prevent the system voltage collapse, and it is also the important component of the last defense method against the voltage collapse. The minimum singular value of load flow Jacobian has been selected as online voltage stability index in this paper. The system will be restored to a safe voltage stability margin by optimizing the location and amount of under voltage load shedding. On the premise of guaranteeing important load, the sensitivity index of load node has been used for selecting nodes to be shed, and the particle swarm is used to optimize the concrete amount of load to be shed. Then the strategy of centralized under voltage load shedding is determined. The voltage meets the stability margin, while the shedding amount is the minimum.

The simulation results of the New England 10-machine 39-bus system show that the scheme is simple and feasible. It can effectively improve the voltage level and stability. In the paper, the on-line centralized UVLS scheme is proposed. It is developed in matlab and need more calculation time. The method will satisfy the actual application in the future.

VII. REFERENCES

[1] LIU Bei, WANG Feng, CHEN Chun., "A phased fault restoration algorithm for distribution system," *Power System Protection and Control*, 2014, 42(3): 24-30.

[2] LU Jinling, BAI Lili, YANG Rengang, "Research on voltage self-healing strategy for grid under fault condition," *Power System Protection and Control*, 2013, 41(9): 7-12.

[3] YUAN Zhichang, XIA Tao, "An overview of setting method of under voltage load shedding," *Power System Protection and Control*, 2012, 40(15): 136-142.

[4] LIU Bin, CHEN Lei, MIN Yong. "Determination of the location of under-voltage load shedding considering transient voltage stability," *Automation of Electric Power Systems*, 2008, 5(32): 11-14.

[5] VU K, BEGOVIC M M, NOVOSEL D, "Use of local measurements to estimate voltage stability margin," *IEEE Trans on Power Systems*, 1999, 14(3): 1029-1035.

[6] GREGOR V, FERDINAND G, "A novel scheme of local protection against voltage collapse based on the apparent-power losses," *Electrical Power and Energy Systems*, 2004, 26(5): 341-347.

[7] ARYA L D, PANDE V S, KOTHAN D P, "A technique for load-shedding based on voltage stability consideration," *Electrical Power and Energy Systems*, 2005, 27(7): 506-5174.

[8] ECHAVARREN F M, LOBATO E, ROUCO L, "A load shedding algorithm for improvement of load margin to voltage collapse," *Power Tech Conference Proceedings*, 2003, 1: 23-26.

[9] TUAN T Q, FANDLNO J, SANBONNADLERE J C, et al. "Determination of load shed using linear programming to avoid voltage instability," *Athens Power Tech*, 1993, 2: 553-557.

[10] CHEN Min, ZHANG Buhan, DUAN Xianzhong, et al. "Study on weak buses of voltage stability based on sensitivity of minimum singular value." *Power System Technology*, 2006, 30(24): 36-40.

[11] DI Tao, LI Huaqiang, FAN Pei, "Reactive power optimization of AC/DC power system based on singular value decomposition and interior point method," *Transactions of China Electrotechnical Society*, 2009, 24(2): 158-163.

[12] DU Lin, DAI Bin, LU Guojun, "Overvoltage features extraction based on S transform and local singular value decomposition," *Transactions of China Electrotechnical Society*, 2010, 25(12): 147-153.

[13] WANG xifan, FANG wanliang, DU zhengchun, *Modern power system analysis*, Beijing: Science Press 2003.

[14] LOF P A, SMED T, ANDERSSON G, "Fast calculation of a voltage stability index," *IEEE Trans on Power Systems*, 1992, 7(1): 54-64.

[15] FENG Zhihong, LIU Qu, NI Yixin, "Analysis of steady-state voltage stability in multimachine power systems by singular value decomposition method," *Proceedings of the CSEE*, 1992, 12(3): 10-19.

[16] ZHOU shuangxi, ZHU lingzhi, GUO xijiu, *Stability and control of power system voltage*, Beijing: Electrical Power Press of China 2003.

[17] LI Xinbin, ZHU Qingjun, "Application of improved particle swarm optimization algorithm to multi-objective reactive power optimization," *Transactions of China Electrotechnical Society*, 2010, 25(7): 137-143.