

# A comparison of Locational Marginal Prices and Locational Load Shedding Marginal Prices in a Deregulated Competitive Power Market

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**Abstract**— This paper presents a comparison of Locational Marginal Prices (LMP) and Locational Load Shedding Marginal Prices (LSMP), which are useful parameters for calculation of trading of active power in a deregulated power market. These parameters or indexes are well pictured through DC Optimal power flow (DC-OPF) and Shift factor (SF) method formulations, which are explained in a four bus system by taking the congestion management strategies to calculate the LMPs and LSMPs, solved by optimization techniques very similar to reverse form of Economic Dispatch (ED) problems. Non-linear constrained optimization problem relate to the load shedding problems. The LMPs and LSMPs which are calculated in the four bus test system are functions of active power. The LMP and LSMP curves reveal the practical advantages of the proposed index for the real-time problems in a competitive power market.

**Index Terms**— Congestion, DC-OPF, Electricity Market, LMP, LSMP, Load Shedding, Shift Factor, Shadow Price.

## I. INTRODUCTION

. Shedding of loads becomes an objective role and motive of the system operator due to some large disturbance in the power system, to maintain security and stability in the system. Transmission networks [1] play an essential role in supporting interaction between producers and customers to provide an unbiased environment for all the participants. A major obstacle for perfect competition among market participants [2] is bottlenecks in transmission lines. The drawback in transmission constraint is congestion. When both parties agree to produce and consume a particular amount of power that fail to do this because of transmission network exceeding power capability limits. This situation further impacts to exercise market power [3] that can cause price volatility beyond the marginal costs. Efficient congestion management become one of the

indicators to arrange the network capability in a competitive power market.

The key issue in a competitive market is the Market Clearing Price (MCP) [4] mechanism. Re-dispatch of generating units or even load shedding may be implemented as the power flow violates transmission constraints, settling different prices at every node. This action is called Locational Marginal Pricing (LMP). To satisfy market participants in trading electricity, [5] a transparent efficient nodal pricing is required which occur due to strong transmission management system which has a bondage with LMPs. LMP at any bus is defined as the marginal cost of supplying the next increment of electric energy while considering the cost of the marginal generators in the system maintaining the physical aspects of transmission system.

The LSMP (Load Shedding Marginal Prices) parameters that are to be determined depend on load shedding strategies, steps, delays proper voltage and frequency [6]. Power economics play an important role in the load shedding planning. High penalty costs will be borne by the generators for not supplying the expected energy to the customers and so proper load shedding decisions decrease penalty costs. Curtailing the expected energy from the customers who bought ancillary services through some contracts from the generator companies may increase the penalty costs for the generator companies. So optimally shedding the loads among the system of loads and minimization of total penalty costs becomes an objective which primarily focus on load shedding planning.

In a restructured power system there may be some uninterruptable loads and that the customers not to be debarred from the social benefits in the emergency condition, buy some contracts from generation companies and if they are unable to serve the customers due to disturbance should pay penalty to their non-served customers. Buying these services ensure of getting more penalty

payments [7] from generation companies when the loads are curtailed. The coordination between the allocation of shedding of loads and spinning reserve amount can reduce the total penalty costs that are to be paid by the generation companies under emergency conditions. Due to a large generation loss disturbance by some internal and external faults, or sudden load increase, in the peak durations, power system usually allot a portion of their loads as interruptible loads needed to be curtailed during the emergency condition. It is noteworthy that penalty price of loads differ from one load to another at different buses. So an optimal load shedding algorithm, optimizing the penalty prices by the loads to calculate the LSMPs in reducing the expected energy not supplied punishments (EENS) which are alike to optimization of generator bidding price cost to calculate the LMPs.

## II. SHIFT FACTOR (SF) AND SHADOW PRICE (SP)

### A. Shift Factor (SF)

A flow in branches with various nodes pattern in a network can be found in terms of a matrix of Shift Factors (SF) whose 'ij' element specifies the incremental flow induced on each transmission link

'ij' by injecting 1 MW at node 'i' and withdrawing it at reference node. For simplicity the transmission links are represented by pairs of indices representing the adjacent from/to nodes so that 'hk' represents the directional link from node 'h' to node 'k'. The SF matrix can be easily computed through simulation or directly from the susceptances of the transmission lines. For instance a flow on the line 1 to 6 resulting from injecting 1 MW at node 3 and withdrawing it at node 5 is given by

$$SF_{1-6, 3} - SF_{1-6, 5}$$

### B. Shadow Price (SP)

It is the maximum dispatch cost saved due to an increment increase of flow capacity in the line with violating transmission constraints.

## III. MATHEMATICAL FOUNDATION

To obtain a secure and economic operation of power systems, the optimal power flow (OPF) is commonly been applied. LMP or LSMP can be obtained at different nodes through the OPF model. Mathematically, LMP or LSMP at any bus signifies the Lagrangian multiplier (also called a shadow price) for the bus equality constraint in OPF. Nodal power balance is the sum of injections and withdrawals at that bus and is equal to zero. Physically, the Lagrangian multipliers are the cost of re dispatching one additional MW at a certain bus.

Mathematically LMPs are the Lagrange multipliers for the bus equality constraints.

We opt to calculate LMPs by DC-Optimal power flow (DC-OPF) since AC-Optimal Power Flow (AC-OPF) is highly

non-linear and time consuming to solve. LMPs are obtained on-line by DCOPF for quick action in the system. System losses are not considered in a DC-OPF which is a lossless network model.

As a result, the marginal loss component is not included in LMP. However, in order to obtain more accurate LMP values, system losses are taken into account by using a loss factor. In the following, we introduce LMP calculations in dc models with system losses.

### A. Calculation based on the Shift Factors (SF), Lagrangian

### B. Multipliers while considering system losses

$$\min c^T * P - b^T * P_D$$

subject to

$$P_L + \sum_j P_{Dj} - \sum_i P_i = 0 \quad \lambda$$

$$LF^T * (A * P - B * P_D) + \text{offset} - P_L = 0 \quad \psi$$

$$P_L = SF * (A * P - B * P_D) \leq P_{L\max} \quad \pi^+$$

$$-P_L = -SF * (A * P - B * P_D) \leq P_{L\max} \quad \pi^-$$

$$P_{\min} \leq P \leq P_{\max}$$

$$P_{D\min} \leq P_D \leq P_{D\max}$$

$$LMP = \lambda - \psi * LF - SF^T * (\pi^+ - \pi^-)$$

According to Kuhn-Tucker (KT) conditions,  $\lambda = \psi$  and LMP is represented by three components that are

$$LMP_{\text{energy}} = \lambda,$$

$$LMP_{\text{loss}} = -\psi * LF, \text{ and}$$

$$LMP_{\text{congestion}} = -SF^T * (\pi^+ - \pi^-).$$

## IV. A FOUR BUS NETWORK TO ILLUSTRATE TRADING

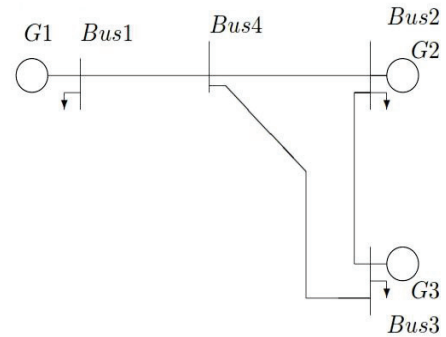


Fig. 1.

TABLE I. GENERATING BID UNITS INFORMATION

| Bus | c  | PC | P <sub>min</sub> | P <sub>max</sub> | P <sub>Dmax</sub> |
|-----|----|----|------------------|------------------|-------------------|
| 1   | 10 | 10 | 0                | 100              | 100               |
| 2   | 30 | 30 | 0                | 200              | 200               |
| 3   | 45 | 45 | 0                | 300              | 300               |
| 4   | -  | -  | -                | -                | -                 |

c, Load Bid, b, Penalty Costs, PC in \$/MW h.

Where the units of 'c', 'b' are in \$/MW h. P<sub>D</sub> and P<sub>Lmax</sub> denotes load and Thermal Limit respectively in MW.

TABLE II. LINE PARAMETERS INFORMATION

| L  | Fr | To | R <sub>pu</sub> | X <sub>pu</sub> | PL <sub>max</sub> |
|----|----|----|-----------------|-----------------|-------------------|
| L1 | 1  | 4  | 0.05            | 0.25            | 40                |
| L2 | 2  | 3  | 0.05            | 0.25            | 50                |
| L3 | 2  | 4  | 0.05            | 0.25            | 50                |
| L4 | 3  | 4  | 0.05            | 0.25            | 50                |

## V. RESULTS AND DISCUSSIONS

### A. For Unconstrained Case

(Taking Bus 3 as Ref Bus)

#### 1) The LMPs in \$/MWh are

| LMP <sub>1</sub> | LMP <sub>2</sub> | LMP <sub>3</sub> | LMP <sub>4</sub> |
|------------------|------------------|------------------|------------------|
| 30               | 30               | 30               | 30               |

### B. For Constrained Case

(Taking Bus 3 as Ref Bus)

#### 1) The generator dispatch, load in MW and cost in \$/h Are

| Load | Cost | P <sub>1</sub> | P <sub>2</sub> | P <sub>3</sub> |
|------|------|----------------|----------------|----------------|
| 150  | 3375 | 60             | 85             | 5              |

#### 2) The LMPs and Shadow prices (SP) in congested lines (1-4) and (2-3) in \$/MWh respectively are

| LMP <sub>1</sub> | LMP <sub>2</sub> | LMP <sub>3</sub> | LMP <sub>4</sub> | SP   | SP   |
|------------------|------------------|------------------|------------------|------|------|
| 10               | 30               | 45               | 37.5             | 27.5 | 22.5 |

C. Considering generation of (G<sub>1</sub>=20MW, G<sub>2</sub>=30MW, G<sub>3</sub>=100MW) at the buses and uniform load distribution of 150MW,  
(taking bus 3 as the reference)

$$1) \text{ SF matrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/3 & 2/3 & 0 & 1/3 \\ -1/3 & 1/3 & 0 & -1/3 \\ -2/3 & -1/3 & 0 & -2/3 \end{bmatrix}$$

2) Conventional flows calculated through load flow programs under normal generator operating conditions are

| FR-TO   | 4-1 | 2-4 | 3-2  | 3-4  |
|---------|-----|-----|------|------|
| Flow-MW | 30  | 3.5 | 23.5 | 26.5 |

D. Considering line unconstrained case, for minimum penalty costs and no generation loss, loads and LSMPs to be satisfied are

| PC   | L <sub>1</sub> | L <sub>2</sub> | L <sub>3</sub> |
|------|----------------|----------------|----------------|
| 2500 | 100            | 50             | 0              |

At bus 1, extra load of 50MW can be added but there is a load curtailment of 50MW at bus 3.

| BUS  | 1  | 2  | 3  | 4  |
|------|----|----|----|----|
| LSMP | 30 | 30 | 30 | 30 |

Mathematically, LSMP in \$/MWh at any bus is the Lagrangian multiplier for the bus equality constraint in Optimal Power Flow (OPF). LSMP is also defined as the marginal load penalty cost supplied by one decrement of generation at a specific bus taking into consideration the physical aspects of the transmission system.

E. Considering line constrained case, for minimum penalty costs and no generation loss, loads to be satisfied are

| PC   | L <sub>1</sub> | L <sub>2</sub> | L <sub>3</sub> |
|------|----------------|----------------|----------------|
| 2500 | 100            | 50             | 0              |

The penalty cost remain same to 2500\$/MW h. At bus 1, increase of load of 50MW, and load curtailment of 50MW at bus 3.

| BUS  | 1  | 2  | 3  | 4  |
|------|----|----|----|----|
| LSMP | 30 | 30 | 30 | 30 |

The LSMPs are also similarly calculated.

F. Considering line constrained case, for minimum penalty costs and a generation loss of G<sub>2</sub>=30MW, due to disturbance, loads to be satisfied maintaining total generation of 120MW only then

| PC   | L <sub>1</sub> | L <sub>2</sub> | L <sub>3</sub> |
|------|----------------|----------------|----------------|
| 1000 | 100            | 20             | 0              |

The penalty cost decreases to 1600 \$/MW h. At bus1, increase of load of 50MW, and load curtailment of

30MW at bus 2, and 50MW at bus 3.

| BUS  | 1  | 2  | 3  | 4  |
|------|----|----|----|----|
| LSMP | 30 | 30 | 30 | 30 |

G. Considering line constrained case, for minimum penalty costs and a generation loss of  $G_2=30\text{MW}$ , due to disturbance, loads to be satisfied taking minimum load at bus3=75MW maintaining total generation of 120MW only then

| PC   | $L_1$ | $L_2$ | $L_3$ |
|------|-------|-------|-------|
| 3825 | 45    | 0     | 75    |

| BUS  | 1  | 2  | 3  | 4  |
|------|----|----|----|----|
| LSMP | 10 | 10 | 10 | 10 |

H. Graphical representation of load variation at buses with LSMPs, penalty costs and load demands at buses.

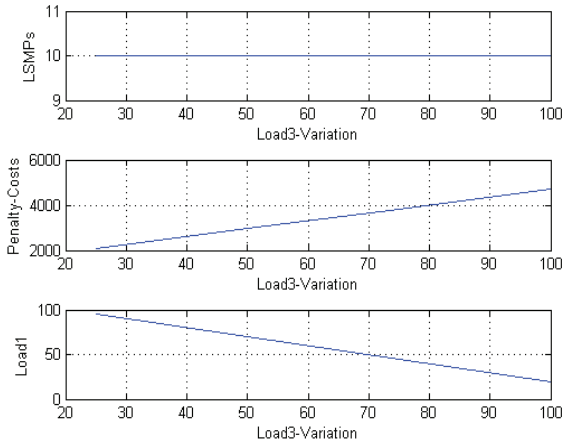


Fig. 2.

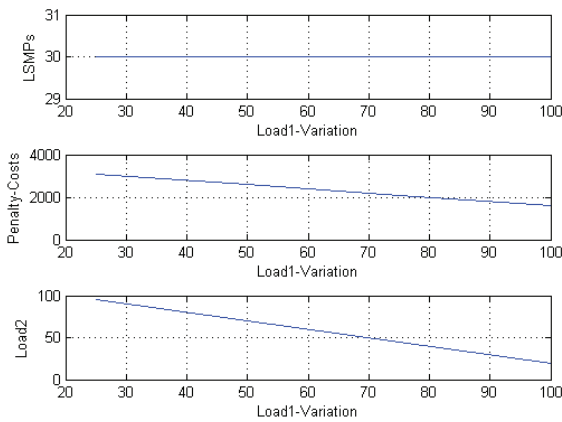


Fig. 3.

Figures 1 and 2 are the graphical representation of load variation of  $L_3$  and  $L_1$  with LSMPs, penalty costs and load demands  $L_1$  and  $L_2$ .

Assuming a generator loss of  $G_2$ , plots between different sizes of loads at bus 3 and bus 1 are compared with LSMPs of different buses, penalty costs and optimal loads of bus 1 and bus 2, supporting the security of the system are shown in Figure 1, Figure 2 respectively.

## VI. CONCLUSIONS

LMPs represent the different price indicators at various buses due to congestion in transmission lines and it provide an efficient mechanism for managing the competitive power market at different periods. The transmission congestion leads to market inefficiency owing to maximum load ability of the network and is the possibility of exercising market power.

In order to analyze LSMP behavior, plots are simulated in terms of generation loss magnitudes, to LSMP values at different buses, penalty costs and optimum loads at other buses. This innovation of load shedding price index (LSMP) is a boon to allocate real time penalty factors for generator buses and participation factors for load buses by electricity market holders in potential load shedding situations

## APPENDIX

### NOMENCLATURE

- c Bidding price vector of sellers (Generation).
- b Buyers Bidding price vector (Load).
- P Bidding energy vectors of the sellers.
- P<sub>D</sub> Bidding energy vector of the buyers.
- P<sub>L</sub> System Losses at time t.
- P L<sub>max</sub> Power flow upper limit in vector form.
- P L Real power flow in vector form
- A Bus-unit incidence matrix.
- B Bus-load incidence matrix.
- SF Shift Factor.
- $\lambda \pi \psi$  Lagrangian multipliers.
- R p.u Resistance of the line.
- X p.u Reactance of the line.

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