Exercício 2

October 25, 2020

1 Exercício PA2-1

Exercício com data de entrega para 26 de outubro de 2020.

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[1]:
```python
import numpy as np
import sympy as sp
import matplotlib.pyplot as plt
from IPython.display import display, Math, Image
sp.init_printing(use_latex='mathjax',latex_mode='equation*')
```

[2]:
```python
Image("Figuras/PA2-2-0.png")
```

[2]:

AVALIAÇÃO 3 – DEFLEXÃO DE VIGAS – EXERCÍCIO PA2-1

Para cada uma das vigas mostrados a seguir, determinar:

a) A equação dos momentos fletores;

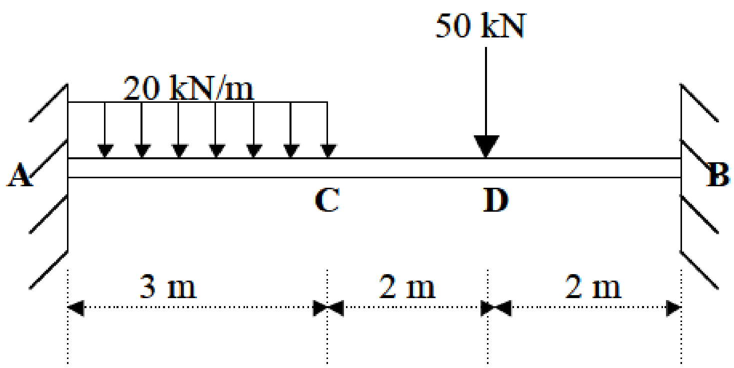
b) As condições de contorno

Utilizar Funções Singulares

2 Sistema Estrutural 1

[3]:
```python
Image("Figuras/PA2-2-1.png")
```

[3]:



2.1 a)

$$V(x) = V_A - 20 \cdot x + 20 \langle x-3 \rangle - 50 \langle x-5 \rangle^0$$

$$M(x) = V_A \cdot x - 10 \cdot x^2 + 10 \langle x-3 \rangle^2 - 50 \langle x-5 \rangle + M_A$$

$$EI \cdot \theta(x) = \frac{V_A}{2} \cdot x^2 - \frac{10}{3} \cdot x^3 + \frac{10}{3} \langle x-3 \rangle^3 - 25 \langle x-5 \rangle^2 + M_A \cdot x + C_1$$

$$EI \cdot y(x) = \frac{V_A}{6} \cdot x^3 - \frac{10}{12} \cdot x^4 + \frac{10}{12} \langle x-3 \rangle^4 - \frac{25}{3} \langle x-5 \rangle^3 + \frac{M_A}{2} \cdot x^2 + C_1 \cdot x + C_2$$

2.2 b)

- $y(0) = 0$
- $\theta(0) = 0$
- $y(7) = 0$
- $\theta(7) = 0$

Temos, portanto, que $C_1 = C_2 = 0$.

$$EI \cdot \theta(7) = \frac{V_A}{2} \cdot 7^2 - \frac{10}{3} \cdot 7^3 + \frac{10}{3} \langle 7-3 \rangle^3 - 25 \langle 7-5 \rangle^2 + M_A \cdot 7 = 0$$

$$EI \cdot y(7) = \frac{V_A}{6} \cdot 7^3 - \frac{10}{12} \cdot 7^4 + \frac{10}{12} \langle 7-3 \rangle^4 - \frac{25}{3} \langle 7-5 \rangle^3 + \frac{M_A}{2} \cdot 7^2 = 0$$

[4]:
```
V_A,M_A,EI,x = sp.symbols('V_A,M_A,EI,x')
```

[5]:
```
theta = sp.Function('theta')(x)
y = sp.Function('y')(x)
```

[6]:
```
theta = (1/EI) * ((V_A*sp.SingularityFunction(x,0,2)/2) - (10*sp.
↪SingularityFunction(x,0,3)/3) + (10*sp.SingularityFunction(x,3,3)/3) - 25*sp.
↪SingularityFunction(x,5,2) + M_A*sp.SingularityFunction(x,0,1))
y = sp.integrate(theta,x)
display(EI*theta.subs(x,7),EI*y.subs(x,7))
```

$$7M_A + \frac{49V_A}{2} - 1030$$

$$\frac{49M_A}{2} + \frac{343V_A}{6} - \frac{11125}{6}$$

[7]:
```
K = sp.Matrix([[7,49/2],[49/2,343/6]])
f = sp.Matrix([[1030],[11125/6]])
display(K.solve(f))
```

$$\begin{bmatrix} -67.2448979591836 \\ 61.2536443148688 \end{bmatrix}$$

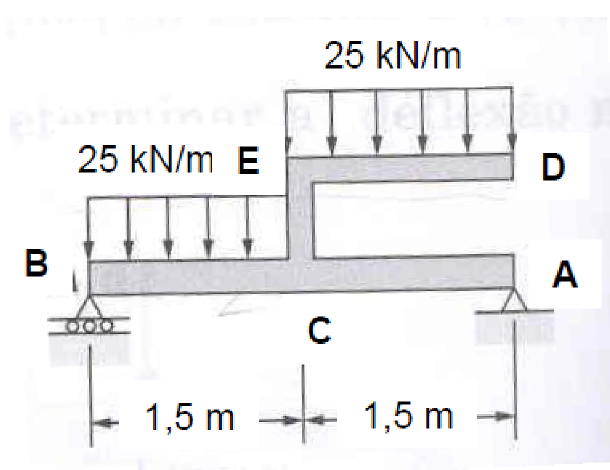
Assim, pelas condições de contorno, temos $M_A = 67,24\ kN \cdot m$ e $V_A = 61,25\ kN$. Estes valores permitem calcular $V_B = 20 \times 3 + 50 - 61,25 = 48,75\ kN$.

Logo, $M_B = 67,24 + 61,25 \times 7 - 20 \times 3 \times 5,5 - 50 \times 2 = 66,02\ kN \cdot m$

3 Sistema Estrutural 2

[8]:
```
Image("Figuras/PA2-2-2.png")
```

[8]:



3.1 a)

$$V\left(x\right) = V_B - 25 \cdot x + 25 \left\langle x - \frac{3}{2} \right\rangle - \frac{75}{2} \left\langle x - \frac{2}{3} \right\rangle^0$$

$$M\left(x\right) = V_B \cdot x - \frac{25}{2} \cdot x^2 + \frac{25}{2} \left\langle x - \frac{3}{2} \right\rangle^2 - \frac{75}{2} \left\langle x - \frac{3}{2} \right\rangle - \frac{225}{8} \left\langle x - \frac{3}{2} \right\rangle^0$$

$$EI \cdot \theta\left(x\right) = \frac{V_B}{2} \cdot x^2 - \frac{25}{6} \cdot x^3 + \frac{25}{6} \left\langle x - \frac{3}{2} \right\rangle^3 - \frac{75}{4} \left\langle x - \frac{3}{2} \right\rangle^2 - \frac{225}{8} \left\langle x - \frac{3}{2} \right\rangle + C_1$$

$$EI \cdot y\left(x\right) = \frac{V_B}{6} \cdot x^3 - \frac{25}{24} \cdot x^4 + \frac{25}{24} \left\langle x - \frac{3}{2} \right\rangle^4 - \frac{75}{12} \left\langle x - \frac{3}{2} \right\rangle^3 - \frac{225}{16} \left\langle x - \frac{3}{2} \right\rangle^2 + C_1 \cdot x + C_2$$

3.2 b)

- $y(0) = 0$
- $y(3) = 0$

Temos, portanto, que $C_2 = 0$.

[9]:
```
V_B,C_1 = sp.symbols('V_B,C_1')
M = sp.Function('M')(x)
M = V_B*sp.SingularityFunction(x,0,1) - 25*sp.SingularityFunction(x,0,2)/2 +
↪25*sp.SingularityFunction(x,3/2,2)/2 - 75*sp.SingularityFunction(x,3/2,1)/2
↪- 225*sp.SingularityFunction(x,3/2,0)/8
theta = M.integrate(x) + C_1
y = theta.integrate(x)
display(sp.solve(y.subs(x,3),C_1))
```

$$\left[43.9453125 - 1.5V_B\right]$$

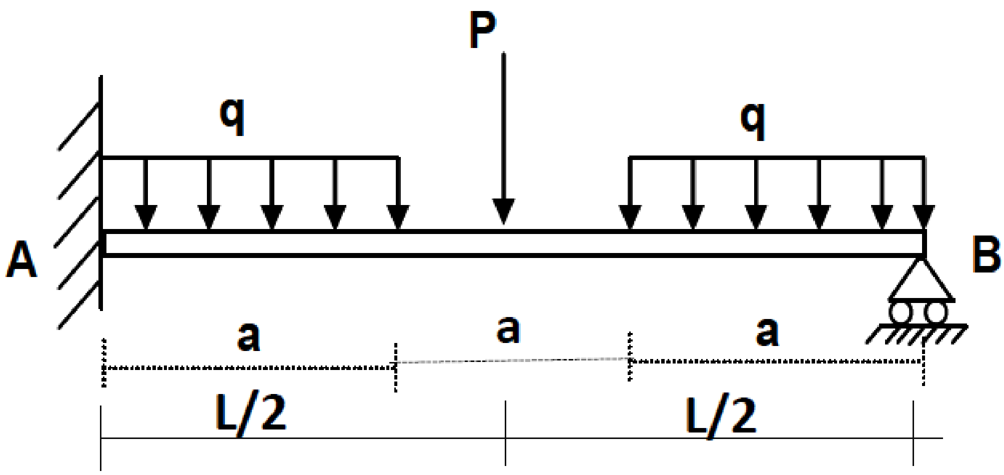
$C_1 = 43,95 - 1,5V_B$, Logo,

$$EI \cdot y\left(x\right) = \frac{V_B}{6} \cdot x^3 - \frac{25}{24} \cdot x^4 + \frac{25}{24} \left\langle x - \frac{3}{2} \right\rangle^4 - \frac{75}{12} \left\langle x - \frac{3}{2} \right\rangle^3 - \frac{225}{16} \left\langle x - \frac{3}{2} \right\rangle^2 + \left(43,95 - \frac{3}{2}V_B\right) \cdot x$$

4 Sistema Estrutural 3

[10]:
```
Image("Figuras/PA2-2-3.png")
```

[10]:



4.1 a)

$$V\left(x\right) = V_A - q \cdot x + q\left\langle x - \frac{L}{3}\right\rangle - P\left\langle x - \frac{L}{2}\right\rangle^0 - q\left\langle x - \frac{2L}{3}\right\rangle$$

$$M\left(x\right) = V_A \cdot x - \frac{q}{2} \cdot x^2 + \frac{q}{2}\left\langle x - \frac{L}{3}\right\rangle^2 - P\left\langle x - \frac{L}{2}\right\rangle - \frac{q}{2}\left\langle x - \frac{2L}{3}\right\rangle^2 + M_A$$

$$EI \cdot \theta\left(x\right) = \frac{V_A}{2} \cdot x^2 - \frac{q}{6} \cdot x^3 + \frac{q}{6}\left\langle x - \frac{L}{3}\right\rangle^3 - \frac{P}{2}\left\langle x - \frac{L}{2}\right\rangle^2 - \frac{q}{6}\left\langle x - \frac{2L}{3}\right\rangle^3 + M_A \cdot x + C_1$$

$$EI \cdot y\left(x\right) = \frac{V_A}{6} \cdot x^3 - \frac{q}{24} \cdot x^4 + \frac{q}{24}\left\langle x - \frac{L}{3}\right\rangle^4 - \frac{P}{6}\left\langle x - \frac{L}{2}\right\rangle^3 - \frac{q}{24}\left\langle x - \frac{2L}{3}\right\rangle^4 + \frac{M_A}{2} \cdot x^2 + C_1 \cdot x + C_2$$

4.2 b)

- $y(0) = 0$
- $\theta(0) = 0$
- $M(L) = 0$
- $y(L) = 0$

Temos, portanto, que $C_1 = C_2 = 0$.

[11]:
```
q,L,P = sp.symbols('q,L,P')
```

Abaixo, segue uma forma de encontrar M_A através da condição $M(L) = 0$.

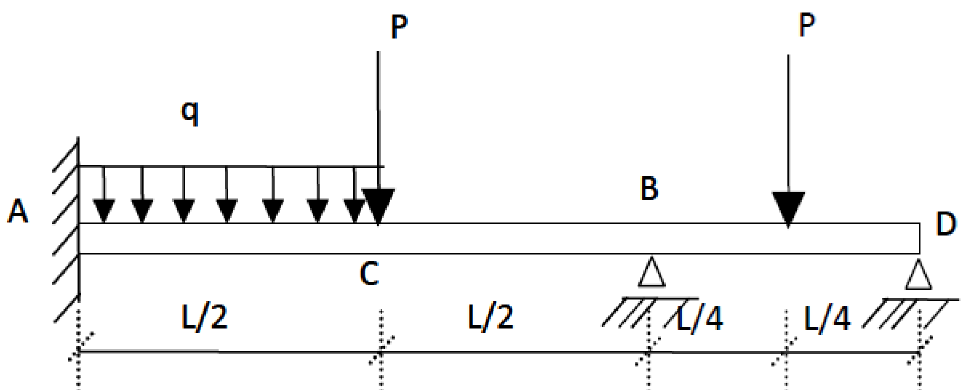
[12]:
```
M = V_A*sp.SingularityFunction(x,0,1) - q*sp.SingularityFunction(x,0,2)/2 +
↪q*sp.SingularityFunction(x,L/3,2)/2 - P*sp.SingularityFunction(x,L/2,1) -
↪q*sp.SingularityFunction(x,2*L/3,2)/2 + M_A
display(sp.solve(sp.Eq(M.subs(x,L),0),M_A)[0])
```

$$P\left\langle \frac{L}{2} \right\rangle^1 - V_A \langle L \rangle^1 + \frac{q\langle L\rangle^2}{2} - \frac{q\left\langle \frac{2L}{3}\right\rangle^2}{2} + \frac{q\left\langle \frac{L}{3}\right\rangle^2}{2}$$

5 Sistema Estrutural 4

[13]:
```
Image("Figuras/PA2-2-4.png")
```

[13]:



5.1 a)

$$V(x) = V_A - q\cdot x + q\left\langle x - \frac{L}{2}\right\rangle - P\left\langle x - \frac{L}{2}\right\rangle^0 + V_B\langle x - L\rangle^0 - P\left\langle x - \frac{5L}{4}\right\rangle^0$$

$$M(x) = V_A\cdot x - \frac{q}{2}\cdot x^2 + \frac{q}{2}\left\langle x - \frac{L}{2}\right\rangle^2 - P\left\langle x - \frac{L}{2}\right\rangle + V_B\langle x - L\rangle - P\left\langle x - \frac{5L}{4}\right\rangle + M_A$$

$$EI\cdot\theta(x) = \frac{V_A}{2}\cdot x^2 - \frac{q}{6}\cdot x^3 + \frac{q}{6}\left\langle x - \frac{L}{2}\right\rangle^3 - \frac{P}{2}\left\langle x - \frac{L}{2}\right\rangle^2 + \frac{V_B}{2}\langle x - L\rangle^2 - \frac{P}{2}\left\langle x - \frac{5L}{4}\right\rangle^2 + M_A\cdot x + C_1$$

$$EI\cdot y(x) = \frac{V_A}{6}\cdot x^3 - \frac{q}{24}\cdot x^4 + \frac{q}{24}\left\langle x - \frac{L}{2}\right\rangle^4 - \frac{P}{6}\left\langle x - \frac{L}{2}\right\rangle^3 + \frac{V_B}{6}\langle x - L\rangle^3 - \frac{P}{6}\left\langle x - \frac{5L}{4}\right\rangle^3 + \frac{M_A}{2}\cdot x^2 + C_1\cdot x + C_2$$

5.2 b)

- $y(0) = 0$
- $\theta(0) = 0$
- $y(L) = 0$
- $M\left(\frac{3L}{2}\right) = 0$
- $y\left(\frac{3L}{2}\right) = 0$

Temos, portanto, que $C_1 = C_2 = 0$.

$$EI \cdot y\left(x\right) = \frac{V_A}{6} \cdot x^3 - \frac{q}{24} \cdot x^4 + \frac{q}{24}\left\langle x - \frac{L}{2}\right\rangle^4 - \frac{P}{6}\left\langle x - \frac{L}{2}\right\rangle^3 + \frac{V_B}{6}\left\langle x - L\right\rangle^3 - \frac{P}{6}\left\langle x - \frac{5L}{4}\right\rangle^3 + \frac{M_A}{2} \cdot x^2$$