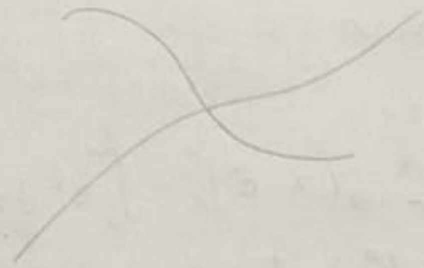
1) Determina si convergen las sig series

i) $\frac{1}{3}+\frac{1\cdot 2}{3\cdot 5}+\frac{1\cdot 2\cdot 3}{3\cdot 5\cdot 7}+\frac{1\cdot 2\cdot 3\cdot 4}{3\cdot 5\cdot 7\cdot 9}+\frac{1\cdot 2\cdot 3\cdot 4\cdot 5}{3\cdot 5\cdot 7\cdot 9}+\dots$

$\Sigma \frac{n!}{(2n+1)!} =$



2) Evaluar limite

$$\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{bx} = \lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{bx} = \lim_{x\to\infty} e^{\ln\left(1+\frac{a}{x}\right)^{bx}}$$

$$= e^{\lim_{x\to\infty}\ln\left(1+\frac{a}{x}\right)^{bx}} = \lim_{x\to\infty}\ln\left(1+\frac{a}{x}\right)^{bx} = \lim_{x\to\infty} bx\ln\left(1+\frac{a}{x}\right)$$

$$= \lim_{x\to\infty}\frac{\ln\left(1+\frac{a}{x}\right)}{\frac{1}{bx}} = \frac{0}{0}$$

Por l'hopital

$$\lim_{x\to\infty}\frac{\ln\left(1+\frac{a}{x}\right)}{\frac{1}{bx}} = \lim_{x\to\infty}\frac{\frac{1}{1+\frac{a}{x}}\cdot\cancel{-}\frac{a}{\cancel{x^2}}}{\cancel{-}\frac{1}{b\cancel{x^2}}}$$

$$= \lim_{x\to\infty}\frac{ab}{\left(1+\cancel{\frac{a}{x}}^{0}\right)} = \frac{ab}{1} = ab$$

$$\Rightarrow \lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{bx} = e^{ab}$$

3) integre si es posible los sig funciones

i) $\int_{-\infty}^{0} xe^{x}\,dx = \lim_{a\to-\infty} \int_{a}^{0} xe^{x}\,dx$

Integrar x partes

$u = x \qquad dv = e^{x}\,dx$

$du = dx \qquad v = e^{x}$

$\Rightarrow \lim_{a\to-\infty} \int_{a}^{0} xe^{x}\,dx = \lim_{a\to-\infty}\left(x e^{x}\Big|_{a}^{0} - \int_{a}^{0} e^{x}\,dx\right)$

$= \lim_{a\to-\infty}\left(xe^{x} - e^{x}\Big|_{a}^{0}\right) = \lim_{a\to-\infty}\left(-e^{0} - (ae^{a} - e^{a})\right)$

$= \lim_{a\to-\infty}\left(-ae^{a} - \cancel{e^{0}}^{1} + e^{a}\right)$

$= -1 + \lim_{a\to-\infty} ae^{a} + \cancel{\lim_{a\to-\infty} e^{a}}^{0} = -1 + \lim_{a\to-\infty} \frac{a}{e^{a}} =$

$= -1 + \frac{\infty}{\infty}$

Por l'hopital

$\lim_{a\to-\infty} \frac{a}{e^{a}} = \lim_{a\to-\infty} \frac{1}{e^{a}} = 0$

$-1 + 0 = -1$

$\Rightarrow \int_{-\infty}^{0} xe^{x}\,dx = -1$

3 iii) $\int_{-\infty}^{1} \frac{dx}{4x^2-12x+9} = \lim_{a\to\infty} \int_{-a}^{1} \frac{dx}{4x^2-12x+9}$

$I = \int \frac{dx}{4x^2-12x+9}$

Obteniendo raiz del Polinomio por formula gral

$x = \frac{12 \pm \sqrt{12^2-4(4)(9)}}{8} = \frac{12}{4} = \frac{3}{2}$ → es la raiz

$= \int \frac{dx}{(2x-3)^2} = \frac{1}{2}\int \frac{du}{u^2} = \frac{1}{2}\int u^{-2}du = \frac{1}{2} \cdot \frac{u^{-1}}{-1} = -\frac{1}{2u}$

$u = 2x-3$

$\frac{du}{dx} = 2 \quad \frac{du}{2} = dx$

$\Rightarrow \lim_{x\to\infty} \int_{-a}^{1} \frac{dx}{4x^2-12x+9} = \lim_{a\to\infty} \left(-\frac{1}{2(2x-3)} \Big|_{a}^{1} \right)$

$= \lim_{a\to\infty} \left[-\frac{1}{2(2\cdot1-3)} - \left(-\frac{1}{2(2(-a)-3)} \right) \right]$

$= \lim_{a\to\infty} \left[-\frac{1}{2(-1)} + \frac{1}{-4a-6} \right] = \frac{1}{2} + 0 = \frac{1}{2}$

3 iii) $\int_0^{\pi} \frac{dx}{1+x^2}$

$$\Rightarrow \int_0^{\pi} \frac{dx}{1+x^2} = \text{arctag}\, x \Big|_0^{\pi} = \text{arctg}\, \pi - \text{arctg}\, 0 = 1{,}2626$$

iv) $\int_1^{\infty} \frac{\ln x}{x^2}\, dx = \int_1^{\infty} \frac{\ln x}{x^2}\, dx = \lim_{a \to \infty} \int_1^{a} \frac{\ln x}{x^2}\, dx$

$$= \lim_{a \to \infty} \left[-\frac{\ln x}{x} \Big|_1^{a} (-x^{-1}) \frac{dx}{x} \right]$$

Sea

$u = \ln x \qquad dv\ x^{-2}\, dx \qquad du = \frac{1}{x}\, dx$

$\frac{du}{dx} = \frac{1}{x} \qquad v = -x^{-1}$

$$= \lim_{a \to \infty} \left[-\frac{\ln x}{x} \Big|_1^{a} + \int_1^{a} x^{-2}\, dx \right]$$

$$= \lim_{a \to \infty} \left[-\left(\frac{\ln x}{x} + \frac{1}{x} \right) \Big|_1^{a} \right]$$

$$= \lim_{a \to \infty} \left(\frac{\ln x}{x} + \frac{1}{x} \Big|_a^1 \right) = \lim_{a \to \infty} \left[\cancel{\frac{\ln 1}{1}}^{0} + \cancel{\frac{1}{x}}^{1} - \left(\frac{\ln a}{a} + \frac{1}{a} \right) \right.$$

$$= \lim_{a \to \infty} \left[1 - \frac{\ln a}{a} + \frac{1}{a} \right] = 1 - \lim_{a \to \infty} \frac{\ln a}{a} + \cancel{\lim_{a \to \infty} \frac{1}{a}}$$

$$= 1 - \lim_{a \to \infty} \frac{\ln a}{a} = 1 - \frac{\infty}{\infty} \Big\} \to \text{L'hopital}$$

Aplicando L'hopital

$$1 - \lim_{a \to \infty} \frac{\ln a}{a} = \lim_{a \to \infty} \frac{\frac{1}{a}}{1} = \lim_{a \to \infty} \frac{1}{a} = 1 - 0 \ //$$

$$\Rightarrow \int_1^{\infty} \frac{\ln x}{x^2}\, dx = 1$$

3 v) $\int_0^e \frac{dx}{x\sqrt{\ln x}} = \lim_{a\to 0} \int_a^e \frac{dx}{x\sqrt{\ln x}} = \lim_{a\to 0} 2\int_{x=a}^{x=e} du = \lim_{a\to 0}\left(2\sqrt{\ln x}\Big|_a^e\right)$

$\lim_{a\to 0}\left(2\sqrt{\cancel{\ln e}} - 2\sqrt{\ln a}\right) = 2 - 2\lim_{a\to 0}\sqrt{\ln a}$

$u = \sqrt{\ln x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{\ln x}} \cdot \frac{1}{x} \Rightarrow 2du = \frac{dx}{x\sqrt{\ln x}}$

No se puede calcular el limite ya que diverge

4) $a_n = 3x^n$

$\sum_{n=0}^{\infty} 3x^n = 3\sum_{n=0}^{\infty} x^n$

$\lim_{n\to\infty} \frac{x^{n+1}}{x^n} = \lim_{x\to\infty} \frac{\cancel{x^n}\cdot x}{\cancel{x^n}} = \lim_{x\to\infty} x = x$

y para que converga

$x < 1$

entonces si $x < 1$ la serie converge

de lo contrario diverge

5) se deja caer una pelota desde una altura de
con cada vez de rebota alcanza 3/4 de la
altura anterior ¿cual es la distancia recorrida
al tocar el suelo por 5ta vez?

Se observa

Primer recorrido 10

Segundo $20(3/4)$
(ida y vuelta)

tercer recorrido $20(3/4)(3/4) = 20(3/4)^2$

nos queda como, sea D distancia recorrida
y n el numero de rebotes despues de tocar por
primera vez el suelo

$$D = 10 + 20 \sum_{i=1}^{n} (3/4)^n$$

Para $n = 5$

$$D = 10 + 20(3/4) + 20(3/4)^2 + 20(3/4)^3 + 20(3/4)^4 + 20(3/4)^5$$

$$D = 55\left(\frac{195}{256}\right) = 41.89 \text{ metros}$$