INSTITUTO POLITÉCNICO NACIONAL

ESCUELA SUPERIOR DE CÓMPUTO

Teoría de Comunicaciones y Señales
Problemario

Alumno: Silva Hernández Noé Jasiel

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Problema 1

Problemas 1

Señal e^{-t}

Sabemos que $t=1$ y $\omega_0 = 2\pi$

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \operatorname{sen} n\omega_0 t$$

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,dt \Rightarrow \int_0^1 e^{-t}\,dt = \left[-e^{-t}\right]\Big|_0^1$$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos n\omega_0 t\,dt \Rightarrow 2\int_0^1 e^{-t}\cos n2\pi t\,dt$$

$u = \cos 2\pi n t \qquad v = -e^{-t}$

$du = 2\pi n \operatorname{sen} \pi n t\,dt \qquad dv = e^{-t}$

$$\Rightarrow a_n = -e^{-t}\cos 2n\pi t - \int_0^1 2\pi n e^{-t} \operatorname{sen} \pi n t\,dt$$

$$a_n = -e^{-t}\cos 2\pi n t - \left[-2\pi n e^{-t}\operatorname{sen} 2\pi n t + 4\pi^2 n^2 \int_0^1 \cos 2\pi n t\,dt\right]$$

$$a_n = 2\left[\frac{2\pi n e^{-t}\operatorname{Sen} 2\pi n t - e^{-t}\cos 2\pi n t}{4\pi^2 n^2 + 1}\right]\Big|_0^1$$

$$a_n = 2\left[\frac{e^{-t}(2\pi n \operatorname{Sen} \pi n t - \cos 2n\pi t)}{4\pi^2 n^2 + 1}\right]\Big|_0^0$$

$$a_n = 2\left[\frac{e^{1}(-1)}{4\pi^2 n^2 + 1} - \frac{-1}{4\pi^2 n^2 + 1}\right] = 2\left[\frac{1 - e^{-1}}{4\pi^2 n^2 + 1}\right]$$

$$a_n = \frac{2 - 2e^{-1}}{4\pi^2 n^2 + 1}$$

$$\Rightarrow b_n = \frac{2}{T}\int_{t_0}^{t_0+t} f(t) \operatorname{Sen} n\omega_0 t\,dt = 2\int_0^1 e^{-t} \operatorname{sen} 2\pi n t\,dt$$

$u = \operatorname{Sen} \pi n t \qquad dv = e^{-t}$

$du = 2\pi n \cos 2\pi n t \qquad v = -e^{-t}$

$$b_n = 2\left[-e^{-t}\, sen\, 2\pi nt - \left(2\pi n e^{-t} \cos 2\pi nt + 4\pi^2 n^2 \int_0^1 e^{-t} sen\, \pi nt\, d\right)\right]$$

$$b_n = 2\left[\frac{-e^{-t}(sen\, 2\pi nt + 2\pi n \cos 2\pi nt)}{4\pi^2 n^2+1}\right]\Bigg|_0^1$$

$$b_n = 2\left[\frac{-e^{-1}(2\pi n)}{4\pi^2n^2+1} - \frac{(-1)(2\pi n)}{4\pi^2 n^2+1}\right]$$

$$b_n = \frac{4\pi n(1-e^{-1})}{4\pi^2 n^2+1}$$

$$f(t) = 1 - e^{-1} + \sum_{n=1}^{\infty} \frac{2-2e^{-1}}{4\pi^2n^2+1} \cos 2\pi nt + \frac{4\pi n(1-e^{-1})}{4\pi^2n^2+1} sen\, n\pi 2t$$

Señal $f(t) = t^2$

$T = 1 \quad \omega_0 = 2\pi \quad f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n\, sen\, \omega_0 t$

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,dt = \int_0^1 t^2\,dt = \left[\frac{t^3}{3}\right]\Bigg|_0^1$$

$$a_0 = \frac{1}{3} - \frac{0}{3} = \frac{1}{3}$$

$$a_n = 2\int_0^1 t^2 \cos 2n\pi t\, dt = \quad u = t^2 \quad dv = \cos 2\pi nt$$
$$du = 2t\,dt \quad v = \frac{sen\, \pi nt}{2\pi n}$$

$$a_n = 2\left[\frac{t^2\, sen\, 2\pi nt}{2\pi n} - \frac{t \cos 2\pi nt}{2\pi n} + \frac{1}{4\pi^2n^2}\int_0^1 \cos u\, du\right.$$

$$a_n = 2\left[\frac{sen 2\pi n}{2\pi n} - \frac{sen\, 2\pi n}{4\pi^2 n^2} + \frac{\cos 2\pi n}{2\pi^2 n^2} - \frac{sen\, 0}{2\pi n} + \frac{sen\, 0}{4\pi^3 n^3} - \frac{\cos 0}{2\pi^2 n^2}\right.$$

$$= 2\left[\frac{1}{2\pi^2 n^2}\right] = \frac{1}{\pi^2 n^2}$$

$$b_n = 2\int_0^1 t^2\, sen\, 2\pi nt\, dt = \quad u = t^2 \quad dv = sen\, 2\pi nt$$
$$du = 2t\,dt \quad v = \frac{\cos 2\pi nt}{2\pi n}$$

$$= 2\left[-\frac{t^2 \cos 2\pi n t}{2\pi n} + \frac{t \operatorname{sen} 2\pi n t}{2\pi n} - \int_0^1 \frac{\operatorname{Sen} 2\pi n t}{2\pi n} dt\right]$$

$$= 2\left[-\frac{t^2 \cos 2\pi n t}{2\pi n} + \frac{t \operatorname{sen} 2\pi n t}{2\pi n} - \frac{1}{4\pi^2 n^2}\int_0 \operatorname{sen} 2\pi n t \, dt\right.$$

$$= 2\left[\frac{t \operatorname{Sen} 2\pi n t}{2\pi^2 n^2} + \frac{t^2 \cos 2\pi n t}{2\pi n} + \frac{\cos 2\pi n t}{4\pi^2 n^3}\right]\Big|_0^1$$

$$= 2\left[\frac{\operatorname{Sen} 2\pi n}{2\pi n^2} + \frac{\cos 2\pi n}{2\pi n} + \frac{\cos 2\pi n}{4\pi^2 n^3} - \emptyset\right]$$

$$= 2\left[\frac{1}{2\pi n}\right] = \frac{1}{\pi n}$$

$$\Rightarrow F(t) = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{\cos 2\pi n t}{\pi^2 n^2} + \frac{\operatorname{Sen} 2\pi n t}{\pi n}$$

Señal $f(t) = 2t$

$$a_0 = \int_0^1 2t \, dt = t^2\Big|_0^1 = 1 - 0 = 1$$

$$a_n = 2\int_0^1 2t \cos 2\pi n t \, dt = 4\int_0^1 t \cos 2\pi n t \, dt$$

$$a_n = \left[t\frac{\operatorname{sen} 2\pi n t}{2\pi n} - \int_0^1 \frac{\operatorname{Sen} 2\pi n t}{2\pi n} dt\right] \qquad u = 2\pi n t, \; du = 2\pi n \, dt$$

$$a_n = 4\left[\frac{t \operatorname{sen} 2\pi n t}{2\pi n} + \frac{\cos 2\pi n t}{4\pi^2 n^2}\right]$$

$$a_n = \left[\left(\frac{2\operatorname{Sen} 2\pi n}{\pi n} + \frac{\cos 2\pi n}{\pi^2 n^2}\right) - \left(\frac{0}{\pi n} + \frac{\cos \emptyset}{\pi^2 n^2}\right)\right]$$

$$a_n = \left[\frac{1}{\pi^2 n^2} - \frac{1}{\pi^2 n^2}\right] = a_n = \emptyset$$

$$b_n = 2\int_0^1 2t \operatorname{Sen} 2\pi n t \, dt = 4\int_0^1 t \operatorname{Sen} 2\pi n t \, dt$$

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$$b_n = 4\left[-\frac{t\cos 2n\pi t}{2\pi n} + \frac{1}{4\pi^2 n^2}\int_0^1 \cos u \, du\right]$$

$$= 4\left[-\frac{t\cos 2\pi n t}{2\pi n} + \frac{\text{Sen } 2\pi n t}{4\pi^2 n^2}\right]$$

$$b_n = \left[\frac{\text{sen } 2\pi n t}{\pi^2 n^2} - \frac{2t\cos 2n\pi t}{\pi n}\right]\Big|_0^1$$

$$b_n = \left[\frac{\text{Sen } 2\pi n}{\pi^2 n^2} - \frac{2\cos 2n\pi}{\pi n} - \emptyset\right]$$

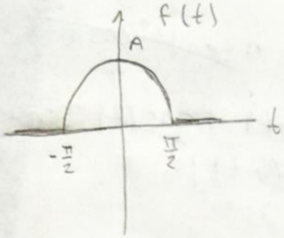
$$b_n = -\frac{2}{\pi n} \quad , \quad f(t) = 1 - \sum_{n=1}^{\infty} \frac{-2}{\pi n} \text{ Sen } 2\pi n t$$

Problema 2

Problemario

Problema 2

Encontrar la serie trigonometrica de furier de cada una de los señales de la sig figura en el intervalo $-\pi$ a π



$$f(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos n\omega_0 t + b_n \operatorname{sen} n\omega_0 t]$$

$f(t)$ es par , $b_n = 0$

$$f(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos n\omega_0 t]$$

$$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{1} = 2\pi \qquad f(t) \begin{cases} 0, & -\pi < t < \frac{\pi}{2} \\ A\cos t, & -\frac{\pi}{2} < t < \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < t < \pi \end{cases}$$

$$\Rightarrow f(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(nt)]$$

Calculando a_0

$$a_0 = \int_{t_0}^{t_0+T} f(t)\,dt = \frac{1}{2\pi}\left[\cancel{\int_{-\pi}^{-\frac{\pi}{2}} 0\,dt}^{\,0} + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} A\cos t\,dt + \cancel{\int_{\frac{\pi}{2}}^{\pi} 0\,dt}^{\,0}\right]$$

$$a_0 = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\pi/2} A\cos t\,dt = \frac{A}{2\pi}\int_{-\pi/2}^{\pi/2} \cos t\,dt = \frac{A}{2\pi} \operatorname{sen} t \Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{A}{2\pi}\left[\operatorname{Sen}\left(\frac{\pi}{2}\right) - \operatorname{sen}\left(-\frac{\pi}{2}\right)\right] = \frac{A}{2\pi}[1-(-1)] = \frac{2A}{2\pi} = \frac{A}{\pi}$$

Calculando a_n

$$a_n = \frac{2}{2\pi}\int_{-\pi/2}^{\pi/2} A\cos t \cdot \cos(nt)\,dt = \frac{2A}{2\pi}\left[\frac{\operatorname{sen}[(1-n)t]}{2(1-n)} + \frac{\operatorname{Sen}[(1+n)t]}{2(1+n)}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{A}{2\pi}\left[\frac{\operatorname{sen}[(1-n)t]}{(1-n)} + \frac{\operatorname{sen}[(1+n)t]}{(1+n)}\right]_{-\pi/2}^{\pi/2}$$

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$$a_n = \frac{A}{2\pi}\left[\frac{(1+n)\,sen(t-nt) + (1-n)\,Sen(t+nt)}{1-n^2}\right]$$

$$a_n = \frac{A}{2\pi(1-n^2)}\Big[(1+n)\left[Sen(t)\cos(nt) - \cos(t)\,sen(nt)\right] +$$
$$(1-n)\left[sen(t)\cos(nt) + \cos(t)\,sen(nt)\right]\Big]$$

$$a_n = \frac{A}{2\pi(1-n^2)}\Big[sen(t)\cos(nt) - \cancel{\cos(t)\,sen(nt)} + \cancel{n\,sen(t)\cos(nt)}$$
$$- n\cos(t)\,sen(nt) + sen(t)\cos(nt) + \cancel{\cos(t)}\,sen(nt)$$
$$- \cancel{n\,Sen(t)\cos(nt)} - n\cos(t)\,sen(nt)\Big]$$

$$a_n = \frac{A}{2\pi(1-n^2)}\left[2\,sen(t)\cos(nt) - 2n\cos(t)\,Sen(nt)\right]$$

$$a_n = \frac{A}{\pi(1-n^2)}\left[sen(t)\cos(nt) - n\cos(t)\,Sen(nt)\right]_{-\pi/2}^{\pi/2}$$

$$a_n = \frac{A}{\pi(1-n^2)}\Big[\Big(\overset{1}{\cancel{sen\left(\tfrac{\pi}{2}\right)}}\cos\frac{n\pi}{2} - n\overset{0}{\cancel{\cos\left(\tfrac{\pi}{2}\right)}}\,sen\left(\frac{n\pi}{2}\right)\Big) -$$
$$\Big(\overset{-1}{\cancel{Sen\left(\tfrac{\pi}{2}\right)}}\cos\left(\frac{-n\pi}{2}\right) - n\overset{0}{\cancel{\cos\left(-\tfrac{\pi}{2}\right)}}\,Sen\left(-\frac{n\pi}{2}\right)\Big)\Big]$$

$$a_n = \frac{A}{\pi(1-n^2)}\left[\cos\left(\frac{n\pi}{2}\right) + \cos\left(\frac{n\pi}{2}\right)\right] = \frac{2A}{\pi(1-n^2)}\left[\cos\left(\frac{n\pi}{2}\right)\right] \quad \forall n \neq 1$$

Obteniendo a_1

$$a_1 = \lim_{n\to 1}\left[\frac{2A\cos\left(\frac{n\pi}{2}\right)}{\pi(1-n^2)}\right] = \frac{2A\cos\left(\frac{\pi}{2}\right)}{\pi(1-1)} = \frac{2A(0)}{\pi(0)} = \frac{0}{0} \text{ Indeterminacion}$$

L Hopital

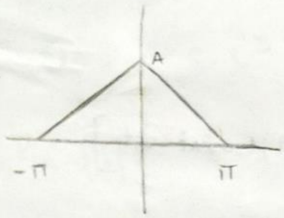
$$a_1 = \lim_{n\to 1}\left[\frac{-2A\,\frac{\pi}{2}\,Sen\left(\frac{n\pi}{2}\right)}{\pi(-2n)}\right] = \lim_{n\to 1}\left[\frac{\cancel{-}A\cancel{\pi}\,Sen\left(\frac{n\pi}{2}\right)}{\cancel{\pi}(\cancel{-}2n)}\right] =$$

$$\lim_{n\to 1}\left[\frac{A\,sen\left(\frac{n\pi}{2}\right)}{2n}\right] = \frac{A\,sen\left(\frac{\pi}{2}\right)}{2} = \frac{A}{2}$$

Por ultimo

$$f(t) = \frac{A}{\pi} + \frac{A}{2}\cos t = \sum_{n=2}^{\infty}\left[\frac{2A}{\pi(1-n^2)}\left[\cos\left(\frac{n\pi}{2}\right)\right]\cdot \cos(nt)\right]$$

2.-



$$f(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos n\omega_0 t + b_n \operatorname{Sen} n\omega_0 t\right]$$

$f(t)$ es par, $b_n = 0$

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos n\omega_0 t\right]$$

$$T = \frac{2\pi}{\omega_0}, \quad 2\pi = \frac{2\pi}{\omega_0}, \quad \omega_0 = 1$$

$$m = \frac{0-A}{-\pi-0} = \frac{-A}{-\pi} = \frac{A}{\pi}, \quad y - A = \frac{A}{\pi}(t-0)$$

$$y = \frac{At}{\pi} + A, \quad y = -\frac{At}{\pi} + A$$

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos nt\right]$$

$$f(t) = \begin{cases} 0, & t < -\pi \\ \frac{At}{\pi} + A, & -\pi < t < 0 \\ -\frac{At}{\pi} + A, & 0 < t < \pi \\ 0, & t > \pi \end{cases}$$

Calculando a_0

$$a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0}\left(\frac{At}{\pi} + A\right)dt + \int_{0}^{\pi}\left(-\frac{At}{\pi} + A\right)dt\right]$$

$$a_0 = \frac{1}{2\pi}\left[\left(\frac{A}{\pi}\cdot\frac{t^2}{2} + At\right)\Big|_{-\pi}^{0} + \left(-\frac{A}{\pi}\cdot\frac{t^2}{2} + At\right)\Big|_{0}^{\pi}\right]$$

$$a_0 = \frac{1}{2\pi}\left[\left(\cancel{\frac{A(0)^2}{2\pi} + A(0)}\right)^{0} - \left(\frac{A(-\pi)^2}{2\pi} + A(-\pi)\right) + \left(-\frac{A(\pi)^2}{2\pi} + A\pi\right) - \left(\cancel{-\frac{A(0)^2}{2\pi} + A(0)}\right)^{0}\right]$$

$$a_0 = \frac{1}{2\pi}\left[-\left(\frac{A\pi}{2} - A\pi\right) + \left(-\frac{A\pi}{2} + A\pi\right)\right]$$

$$a_0 = \frac{1}{2\pi}\left[-\frac{A\pi}{2} + A\pi - \frac{A\pi}{2} + A\pi\right] = \frac{1}{2\pi}\left[2A\pi - 2\left(\frac{A\pi}{2}\right)\right]$$

$$a_0 = \frac{1}{2\pi}\left[2A\pi - A\pi\right] = \frac{1}{2\pi}\left[A\pi\right] = \frac{A}{2}$$

Calculando a_n

$$a_n = \frac{2}{2\pi}\left[\int_{-\pi}^{0}\left(\frac{At}{\pi} + A\right)\cos nt\, dt + \int_{0}^{\pi}\left(-\frac{At}{\pi} + A\right)\cos nt\, dt\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{A}{\pi}\int_{-\pi}^{0} t\cos nt\, dt + \int_{-\pi}^{0} A\cos nt\, dt - \frac{A}{\pi}\int_{0}^{\pi} t\cos nt\, dt + \int_{0}^{\pi} A\cos(nt)\, dt\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{A}{\pi}\left(\frac{1}{n}t\,\mathrm{Sen}(nt) + \frac{1}{n^2}\cos(nt)\right)\Big|_{-\pi}^{0} + A\left(\frac{1}{n}\mathrm{Sen}(nt)\right)\Big|_{-\pi}^{0} - \frac{A}{\pi}\left(\frac{1}{n}t\,\mathrm{Sen}(nt) + \frac{1}{n^2}\cos(nt)\right)\Big|_{0}^{\pi} + A\left(\frac{1}{n}\mathrm{Sen}(nt)\right)\Big|_{0}^{\pi}\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{A}{\pi}\left(\left(\cancel{\frac{1}{n}(0)\,\mathrm{Sen}(n(0))}^{0} + \frac{1}{n^2}\cos(n(0))\right) - \left(\cancel{\frac{1}{n}(-\pi)\,\mathrm{Sen}(n(-\pi))}^{0} + \frac{1}{n^2}\cos(n(-\pi))\right)\right) + A\left(\left(\frac{1}{n}\cancel{\mathrm{Sen}(n(0))}^{0}\right) - \left(\frac{1}{n}\cancel{\mathrm{Sen}(n(-\pi))}^{0}\right)\right)\right.$$
$$\left. - \frac{A}{\pi}\left(\left(\frac{1}{n}(\pi)\cancel{\mathrm{Sen}(n(\pi))}^{0} + \frac{1}{n^2}\cos(n\pi)\right) - \left(\frac{1}{n}(0)\cancel{\mathrm{Sen}(n(0))}^{0} + \frac{1}{n^2}\cos(n(0))\right)\right) + A\left(\left(\frac{1}{n}\cancel{\mathrm{Sen}(n\pi)}^{0}\right) - \left(\frac{1}{n}\cancel{\mathrm{Sen}(n(0))}^{0}\right)\right)\right]$$

$$a_0 = \frac{1}{\pi}\left[\frac{A}{\pi}\left(\frac{1}{n^2} - \left(\frac{1}{n^2}(-1)^n\right)\right] - \frac{A}{\pi}\left(\frac{1}{n^2}(-1)^n - \left(\frac{1}{n^2}\right)\right]\right]$$

$$a_0 = \frac{1}{\pi}\left[\frac{A}{\pi}\left(\frac{1}{n^2} - \frac{1}{n^2}(-1)^n\right) - \frac{A}{\pi}\left(\frac{1}{n^2}(-1)^n - \frac{1}{n^2}\right)\right]$$

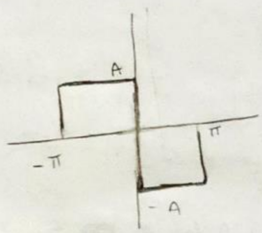
$$a_n = \frac{A}{\pi^2}\left[\frac{1}{n^2}\left(1-(-1)^n\right) - \frac{1}{n^2}\left((-1)^n - 1\right)\right]$$

$$a_n = \frac{A}{n^2\pi^2}\left[1-(-1)^n - (-1)^n + 1\right] = \frac{A}{n^2\pi^2}\left[2-2(-1)^n\right]$$

$$a_n = \frac{2A}{n^2\pi^2}\left[1-(-1)^n\right]$$

$$\Rightarrow f(t) = \frac{A}{2} + \sum_{n=1}^{\infty} \frac{2A}{n^2\pi^2}\left[1-(-1)^n\right] \cos nt \;//$$

2.3



$$f(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos n\omega_0 t + b_n \operatorname{Sen} n\omega_0 t\right]$$

$f(t)$ = es impar, $a_n = 0 = a_0 = 0$

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left[b_n \operatorname{Sen}(n\omega_0 t)\right]$$

$$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{1} = 2\pi, \quad \omega_0 = 1$$

$$f(t) = \begin{cases} 0, & x < -\pi \\ A, & -\pi < t < 0 \\ -A, & 0 < t < \pi \\ 0, & x > \pi \end{cases}$$

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left[b_n \operatorname{Sen}(nt)\right]$$

$$f(t) = \sum_{n=1}^{\infty}\left[b_n \operatorname{Sen}(nt)\right]$$

Calculando a_0

$$a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0} A\,dt - \int_{0}^{\pi} A\,dt\right] = \frac{1}{2\pi}\left[(At)\Big|_{-\pi}^{0} - (At)\Big|_{0}^{\pi}\right] =$$

$$\frac{1}{2\pi}\left[\left(\cancel{A(0)}^{0} - A(-\pi)\right) - \left(A(\pi) - \cancel{A(0)}^{0}\right)\right]$$

$$= \frac{1}{2\pi}\left[A\pi - A\pi\right] = \frac{1}{2\pi}(0) = 0$$

Calculando b_n

$$b_n = \frac{2}{2\pi}\left[\int_{-\pi}^{0} A \operatorname{sen}(nt)\,dt - \int_{0}^{\pi} A \operatorname{sen}(nt)\,dt\right]$$

$$b_n = \frac{1}{\pi}\left[A\left(-\frac{1}{n}\cos(nt)\right)\Big|_{-\pi}^{0} - A\left(-\frac{1}{n}\cos(nt)\right)\Big|_{0}^{\pi}\right]$$

$$b_n = \frac{1}{\pi}\left[A\left(\left(-\frac{1}{n}\cos(0)\right) - \left(-\frac{1}{n}\cos(-n\pi)\right)\right) - A\left(\left(-\frac{1}{n}\cos(n\pi)\right) - \left(-\frac{1}{n}\cos(0)\right)\right)\right]$$

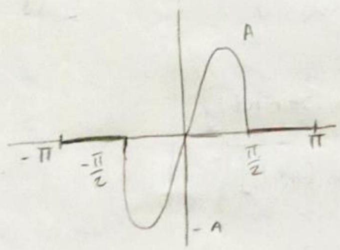
$$b_n = \frac{1}{\pi}\left[A\left(-\frac{1}{n} + \frac{1}{n}(-1)^n\right) - A\left(-\frac{1}{n}(-1)^n + \frac{1}{n}\right)\right]$$

$$b_n = \frac{A}{\pi}\left[\frac{1}{n}\left(-1 + (-1)^n\right) - \frac{1}{n}\left(-(-1)^n + 1\right)\right]$$

$$b_n = \frac{A}{\pi n}\left[-1 + (-1)^n + (-1)^n - 1\right]$$

$$b_n = \frac{A}{\pi n}\left[-2 + 2(-1)^n\right] = \frac{2A}{\pi n}\left[-1 + (-1)^n\right]$$

$$\Rightarrow f(t) = \sum_{n=1}^{\infty} \frac{2A}{\pi n}\left[-1 + (-1)^n\right] \operatorname{Sen}(nt) \;//$$



$$f(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n \operatorname{Cos} n\omega_0 t + b_n \operatorname{Sen} n\omega_0 t\right]$$

$f(t)$ es impar, $a_n = a_0 = 0$

$$f(t) = \sum_{n=1}^{\infty}\left[b_n \operatorname{Sen} n\omega_0 t\right]$$

$$T = \frac{2\pi}{\omega_0} \qquad 2\pi = \frac{2\pi}{\omega_0}, \quad \omega_0 = 1$$

$$f(t) = \begin{cases} 0, & -\pi < t < -\frac{\pi}{2} \\ A \operatorname{sen} \omega t & -\frac{\pi}{2} < t < \pi/2 \\ 0, & \pi/2 < t < \pi \end{cases} \qquad \omega = \frac{2\pi}{\pi} = 2$$

$$b_n = \frac{2}{2\pi}\int_{-\pi/2}^{\pi/2} A \operatorname{sen} 2t \operatorname{sen} nt \, dt$$

$$b_n = \frac{2A}{2\pi}\left[\frac{\operatorname{sen}[(2-n)t]}{2(2-n)} - \frac{\operatorname{Sen}[(2+n)t]}{2(2+n)}\right]_{-\pi/2}^{\pi/2}$$

$$b_n = \frac{A}{2\pi}\left[\frac{(2+n)\operatorname{Sen}[(2-n)t] - (2-n)\operatorname{Sen}[(2+n)t]}{4-n^2}\right]$$

$$bn = \frac{A}{2\pi(4-n^2)}\Big[2sen[2t-nt] + n\,sen[2t-nt] - \big[2sen(2t+nt) - n\,sen[2t+nt]\big]\Big]$$

$$bn = \frac{A}{2\pi(4-n^2)}\Big[2(sen(2t)\,Cos(nt) - cos(2t)\,Sen(nt)) - {}$$

$$+\,n(sen(2t)\,Cos(nt) - Cos(2t)\,Sen(nt)) -$$

$$\big[2(Sen(2t)\,Cos(nt) + Cos(2t)\,Sen(nt)) -$$

$$n(sen(2t)\,Cos(nt) + cos(2t)\,Sen(nt))\big]\Big]$$

$$bn = \frac{A}{2\pi(4-n^2)}\Big[2\,Sen(2t)\,\cancel{Cos(nt)} - 2\,cos(2t)\,Sen(nt) +$$

$$n\,sen(2t)\,Cos(nt) - \cancel{n\,Cos(2t)\,Sen(nt)} - \cancel{2\,Sen(2t)\,Cos(nt)} -$$

$$2\,cos(2t)\,Sen(nt) + n\,Sen(2t)\,Cos(nt) + n\,\cancel{Cos(2t)\,Sen(nt)}\Big]$$

$$bn = \frac{A}{\pi(4-n^2)}\Big[2n\,Sen(2t)\cos nt - 4\cos(2t)\,Sen\,nt\Big]$$

$$bn = \frac{A}{\pi(4-n^2)}\Big[n\,Sen(2t)\,Cos\,nt - 2\cos(2t)\,Sen(nt)\Big]_{-\pi/2}^{\pi/2}$$

$$bn = \frac{A}{\pi(4-n^2)}\Big[\Big(n\,Sen\left(\tfrac{2\pi}{2}\right)Cos\left(\tfrac{n\pi}{2}\right) - 2\cos\left(\tfrac{2\pi}{2}\right)Sen\left(\tfrac{n\pi}{2}\right)\Big)$$

$$-\Big(A\,Sen\left(-\tfrac{2\pi}{2}\right)Cos\left(-\tfrac{n\pi}{2}\right) - 2\cos\left(-\tfrac{2\pi}{2}\right)Sen\left(-\tfrac{n\pi}{2}\right)\Big)\Big]$$

$$bn = \frac{A}{\pi(4-n^2)}\Big[n\,\cancelto{0}{sen(\pi)}\,Cos\left(\tfrac{n\pi}{2}\right) - 2\,\cancelto{-1}{cos(\pi)}\,Sen\left(\tfrac{n\pi}{2}\right)$$

$$-\,n\,\cancelto{0}{sen(-\pi)}\,Cos\left(-\tfrac{n\pi}{2}\right) + 2\,\cancelto{-1}{cos(-\pi)}\,Sen\left(-\tfrac{n\pi}{2}\right)\Big]$$

$$bn = \frac{A}{\pi(4-n^2)}\Big[2sen\left(\tfrac{n\pi}{2}\right) - 2sen\left(-\tfrac{n\pi}{2}\right)\Big]$$

(12)

$$b_n = \frac{A}{\pi(4-n^2)}\left[2\,Sen\left(\frac{n\pi}{2}\right) - 2\,Sen\left(-\frac{n\pi}{2}\right)\right]$$

$$b_n = \frac{A}{\pi(4-n^2)}\left[2\,Sen\left(\frac{n\pi}{2}\right) + 2\,Sen\left(\frac{n\pi}{2}\right)\right]$$

$$b_n = \frac{A}{\pi(4-n^2)}\left[4\,Sen\left(\frac{n\pi}{2}\right)\right]$$

$$\Rightarrow f(t) = \sum_{n=1}^{\infty}\left[\frac{4A}{\pi(4-n^2)}\,Sen\left(\frac{n\pi}{2}\right)\right]Sen(nt) \quad \forall n \neq 2$$

Obteniendo b_2

$$b_2 = \lim_{n\to 2}\left[\frac{4A\,Sen\left(\frac{n\pi}{2}\right)}{\pi(4-n^2)}\right] = \frac{4A\,Sen\,\pi}{\pi(4-4)} = \frac{0}{0}$$

L'Hopital

$$b_2 = \lim_{n\to 2}\frac{4A.\frac{\pi}{2}\,Cos\left(\frac{n\pi}{2}\right)}{\pi(-2n)} = \lim_{n\to 2}\left[\frac{2A\cancel{\pi}\,Cos\left(\frac{n\pi}{2}\right)}{\cancel{\pi}(-2n)}\right]$$

$$= \lim_{n\to 2}\left[\frac{A\cos\left(\frac{n\pi}{2}\right)}{-n}\right] = \frac{A\cos(\pi)}{-2} = \frac{-A}{-2} = \frac{A}{2}$$

$b_1 \Rightarrow$

$$b_1 = \frac{4A\,Sen\left(\frac{\pi}{2}\right)}{\pi(4-1)} = \frac{4A\,Sen\left(\frac{\pi}{2}\right)}{3\pi} = \frac{4A}{3\pi}$$

$$\Rightarrow f(t) = \frac{4A}{3\pi}\,Sen\,t + \frac{A}{2}\,Sen\,2t + \sum_{n=3}^{\infty}\left[\frac{4A}{\pi(4-n^2)}\,Sen\,\frac{n\pi}{2}\right]$$

$$\cdot\,Sen\,nt$$

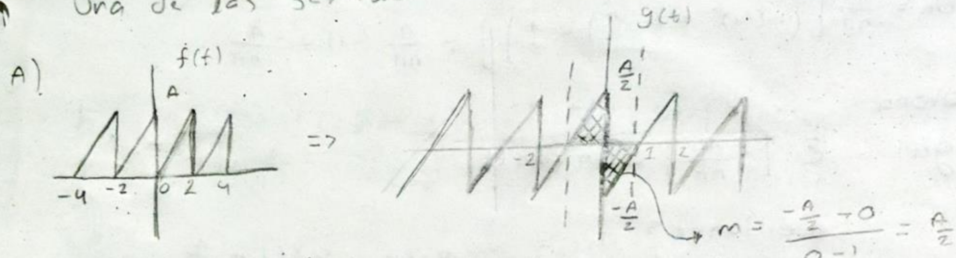
Problema 3

(13)

Problema 3

Determine la serie de fourier de cada una de las señales

A)



$$g(t) = f(t) - \frac{A}{2}$$

$$f(t) = \frac{A}{2}t + \frac{A}{2}$$

$$m = \frac{-\frac{A}{2} - 0}{0 - 1} = \frac{A}{2}$$

$$f(t) = \frac{A}{2}t - \frac{A}{2}$$

Como se desplazo ahora es funcion impar.

$$a_0 = a_n = 0$$

$$T = \frac{2\pi}{\omega_0}, \quad 2 = \frac{2\pi}{\omega_0}, \quad \omega_0 = \frac{2\pi}{2} = \pi$$

Calcular b_n

$$b_n = 2\left[\frac{2}{2}\int_0^1 \left(\frac{A}{2}t - \frac{A}{2}\right) \operatorname{sen}(n\pi t)\,dt\right]$$

$$b_n = 2\int_0^1 \left[\frac{A}{2} t \operatorname{sen}(n\pi t) - \frac{A}{2}\operatorname{sen}(n\pi t)\right] dt$$

$$b_n = A\left[\int_0^1 t \operatorname{sen}(n\pi t)\,dt - \int_0^1 \operatorname{sen}(n\pi t)\,dt\right]$$

$$b_n = A\left[-\frac{1}{n\pi} t\cos(n\pi t) + \frac{1}{n^2\pi^2}\operatorname{sen}(n\pi t) + \frac{1}{n\pi}\cos(n\pi t)\right]_0^1$$

$$b_n = \frac{A}{n\pi}\left[-t\cos(n\pi t) + \frac{1}{n\pi}\operatorname{sen}(n\pi t) + \cos(n\pi t)\right]_0^1$$

$$b_n = \frac{A}{n\pi}\left[\left(-\cos(n\pi) + \frac{1}{n\pi}\operatorname{Sen}(n\pi) + \cos(n\pi)\right) - \left((0)\cos\xrightarrow{0}(n\pi(0)) - \frac{1}{n\pi}\operatorname{Sen}(n\pi(0)) + \cos(n\pi(0))\right)\right]$$ (14)

$$b_n = \frac{A}{n\pi}\left[\left(-(-1)^n + (-1)^n\right) - 1\right] = \frac{A}{n\pi}(-1) = -\frac{A}{n\pi}$$

ahora

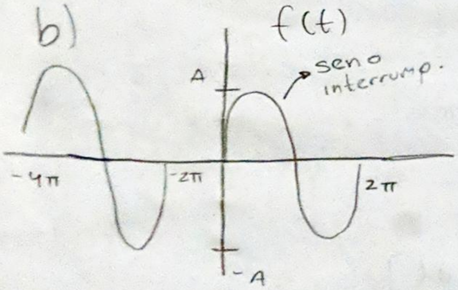
$$g(t) = \sum_{n=1}^{\infty}\left[-\frac{A}{n\pi}\right]\operatorname{Sen}(n\pi t)$$

entonces

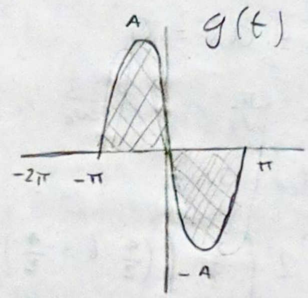
$$g(t) = f(t) - \frac{A}{2}, \quad f(t) = g(t) + \frac{A}{2}$$

$$f(t) = \frac{A}{2} + \sum_{n=1}^{\infty}\left[-\frac{A}{n\pi}\right]\operatorname{Sen}(n\pi t)$$

b)



=>



$$g(t) = \begin{cases} -A\operatorname{sen}(t), & -\pi < t < \pi \\ 0, & \text{otro caso} \end{cases}$$

$w_0 = \frac{2\pi}{T}$ $\quad T = 2\pi$, $w_0 = \frac{2\pi}{2\pi} = 1$

entonces

$$f(t)\Big|_{t = t-\pi} = g(t) \Rightarrow f(t-\pi) = g(t)$$ $\quad T = 4\pi$, $w_0 = \frac{2\pi}{4\pi} = \frac{1}{2}$

$g(t)$ es impar. $a_0 = a_n = 0$

Calculando b_n

$$b_n = -2\left[\frac{2}{4\pi}\int_0^{\pi} A\operatorname{sen}(t)\operatorname{sen}\left(\frac{nt}{2}\right)dt\right] = -\frac{A}{\pi}\int_0^{\pi}\operatorname{sen}(t)\operatorname{sen}\left(\frac{nt}{2}\right)dt$$

$$b_n = -\frac{A}{\pi}\left[\frac{\operatorname{Sen}\left[\left(1-\frac{n}{2}\right)t\right]}{2\left(1-\frac{n}{2}\right)} - \frac{\operatorname{Sen}\left[\left(1+\frac{n}{2}\right)t\right]}{2\left(1+\frac{n}{2}\right)}\right]_0^{\pi}$$

$$b_n = -\frac{A}{2\pi}\left[\frac{\left(1+\frac{n}{2}\right)\operatorname{Sen}\left[\left(1-\frac{n}{2}\right)t\right] - \left[\left(1-\frac{n}{2}\right)\operatorname{Sen}\left[\left(1+\frac{n}{2}\right)t\right]\right]}{1-\frac{n^2}{4}}\right]$$

$$b_n = -\frac{A}{2\pi}\left[\frac{\operatorname{sen}\left(t-\frac{nt}{2}\right) + \frac{n}{2}\operatorname{Sen}\left[t-\frac{nt}{2}\right] - \operatorname{Sen}\left[t+\frac{nt}{2}\right] + \frac{n}{2}\operatorname{Sen}\left[t+\frac{nt}{2}\right]}{1-\frac{n^2}{4}}\right]$$

$$b_n = \frac{-A}{2\pi\left(1-\frac{n^2}{4}\right)}\left[\operatorname{Sen}(t)\cancel{\operatorname{Cos}\left(\frac{nt}{2}\right)} - \operatorname{Cos}(t)\operatorname{Sen}\left(\frac{nt}{2}\right) + \frac{n}{2}\left(\operatorname{Sen}(t)\operatorname{Cos}\left(\frac{nt}{2}\right) - \right.\right.$$

$$\left.\cancel{\operatorname{Cos}(t)\operatorname{Sen}\left(\frac{nt}{2}\right)}\right) - \left(\cancel{\operatorname{sen}(t)\operatorname{Cos}\left(\frac{nt}{2}\right)} + \operatorname{Cos}(t)\operatorname{Sen}\left(\frac{nt}{2}\right)\right]$$

$$\left.+ \frac{n}{2}\left(\operatorname{Sen}(t)\operatorname{Cos}\left(\frac{nt}{2}\right) + \operatorname{Cos}(t)\operatorname{Sen}\left(\frac{nt}{2}\right)\right)\right]$$

$$b_n = \frac{-A}{2\pi\left(1-\frac{n^2}{4}\right)}\left[-2\operatorname{Cos}(t)\operatorname{Sen}\left(\frac{nt}{2}\right) + n\operatorname{Sen}(t)\operatorname{Cos}\left(\frac{nt}{2}\right)\right]_0^{\pi}$$

$$b_n = \frac{-A}{2\pi\left(1-\frac{n^2}{4}\right)}\left[\left(-2\operatorname{Cos}(\pi)\operatorname{Sen}\left(\frac{n\pi}{2}\right) + \cancel{n\operatorname{Sen}(\pi)\operatorname{Cos}\left(\frac{n\pi}{2}\right)}\right)\right.$$

$$\left. - \left(-2\cancel{\operatorname{Cos}(0)\operatorname{Sen}(0)}^{0} + n\operatorname{Sen}(0)\operatorname{Cos}(0)\right)\right]$$

$$b_n = \frac{-A}{2\pi\left(1-\frac{n^2}{4}\right)}\left[2\operatorname{Sen}\left(\frac{n\pi}{2}\right)\right], \quad b_n = \frac{-A}{\pi\left(1-\frac{n^2}{4}\right)}\left[\operatorname{sen}\left(\frac{n\pi}{2}\right)\right] \quad \forall n \neq 2$$

Obteniendo b_2

$$\lim_{n\to 2} \frac{-A\left[\operatorname{Sen}\left(\frac{n\pi}{2}\right)\right]}{\pi\left(1-\frac{n^2}{4}\right)} = -\frac{A\operatorname{Sen}\pi}{\pi(1-1)} = \left(\frac{0}{0}\right)$$

L'Hopital

$$\lim_{n\to 2} \frac{-A\,\frac{\pi}{2}\operatorname{Cos}\left(\frac{n\pi}{2}\right)}{\pi\left(-\frac{2n}{4}\right)} = \lim_{n\to 2} \frac{-\frac{A\pi}{2}\operatorname{Cos}\left(\frac{n\pi}{2}\right)}{\pi\left(-\frac{n}{2}\right)} = \frac{-A\pi\operatorname{Cos}(\pi)}{-\pi}$$

$$= \frac{\frac{A\pi}{2}}{\frac{-\pi}{1}} = \frac{A\pi}{2\pi} = -\frac{A}{2}$$

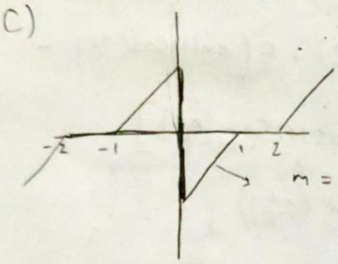
$$\Rightarrow b_1 = \frac{-A\left[\operatorname{Sen}\left(\frac{\pi}{2}\right)\right]}{\pi\left(1-\frac{1}{4}\right)} = \frac{-A}{\frac{3\pi}{4}} = \frac{-4A}{3\pi}$$

$$g(t) = -\frac{4A}{3\pi}\operatorname{Sen}\left(\frac{t}{2}\right) - \frac{A}{2}\operatorname{Sen}(t) + \sum_{n=3}^{\infty}\left[\frac{-A\operatorname{Sen}\frac{n\pi}{2}}{\pi\left(1-\frac{n^2}{4}\right)}\right]\operatorname{Sen}\left(\frac{nt}{2}\right)$$

pero $g(t) = f(t-\pi)$

$$f(t) = -\frac{4A}{3\pi}\operatorname{Sen}\left(\frac{t-\pi}{2}\right) - \frac{A}{2}\operatorname{Sen}(t-\pi) + \sum_{n=3}^{\infty}\left[\frac{-A\operatorname{Sen}\left(\frac{n\pi}{2}\right)}{\pi\left(1-\frac{n^2}{4}\right)}\right]\operatorname{Sen}\left(\frac{n(t-\pi)}{2}\right)$$

C)



$$m = \frac{0-(-A)}{1-0} = A \qquad y + A = A(t-0)$$
$$y = At - A$$

funcion impar

$a_0 = a_n = 0$, $T = \frac{2\pi}{\omega_0} \Rightarrow 4 = \frac{2\pi}{\omega_0} \Rightarrow \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

$$f(t) = \sum_{n=1}^{\infty}\left[b_n \operatorname{Sen}\left(\frac{n\pi t}{2}\right)\right]$$

Calculando b_n

$$b_n = 2\left[\frac{2}{4}\int_0^1 [At - A]\operatorname{Sen}\left(\frac{n\pi t}{2}\right)dt\right]$$

$$b_n = \int_0^1 At\operatorname{sen}\left(\frac{n\pi t}{2}\right)dt - \int_0^1 A\operatorname{Sen}\left(\frac{n\pi t}{2}\right)dt$$

$$b_n = A\left[\int_0^1 t\operatorname{sen}\left(\frac{n\pi t}{2}\right)dt - \int_0^1 \operatorname{Sen}\left(\frac{n\pi t}{2}\right)dt\right]$$

$$b_n = A\left[-\frac{1}{\frac{n\pi}{2}}t\cos\left(\frac{n\pi t}{2}\right) + \frac{1}{\frac{n^2\pi^2}{4}}\operatorname{Sen}\left(\frac{n\pi t}{2}\right) + \frac{1}{\frac{n\pi}{2}}\operatorname{Cos}\left(\frac{n\pi}{2}t\right)\right]_0^1$$

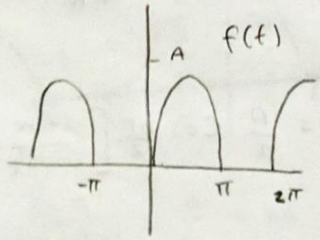
$$b_n = \frac{2A}{n\pi}\left[-(1)\cancel{\cos\left(\frac{n\pi}{2}\right)} + \frac{2}{n\pi}\operatorname{Sen}\left(\frac{n\pi}{2}\right) + \cancel{\cos\left(\frac{n\pi}{2}\right)}\right) - ($$

$$\left[-(0)\overset{0}{\cancel{\cos(0)}} + \frac{2}{n\pi}\overset{0}{\cancel{\operatorname{Sen}(0)}} + \overset{1}{\cancel{\cos(0)}}\right)\Big]$$

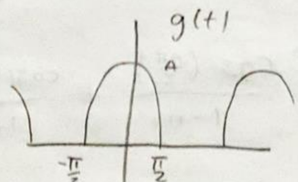
$$b_n = \frac{2A}{n\pi}\left[\frac{2}{n\pi}\operatorname{Sen}\left(\frac{n\pi}{2}\right) - 1\right]$$

$$\Rightarrow f(t) = \sum_{n=1}^{\infty}\left[\frac{2A}{n\pi}\left[\frac{2}{n\pi}\operatorname{Sen}\left(\frac{n\pi}{2}\right) - 1\right]\right]\operatorname{Sen}\left(\frac{n\pi t}{2}\right)$$

d)



$\Rightarrow$



$$g(t)\Big|_{t = t-\pi} = f(t) \Rightarrow g\left(t - \frac{\pi}{2}\right) = f(t)$$

$$w_0 = \frac{2\pi}{T}\ ;\ w_0 = \frac{2\pi}{2\pi} = 1$$

a_0

$$a_0 = \frac{2}{T}\int_0^{T/2} g(t)\,dt = \frac{1}{\pi}\left[\int_0^{\pi/2} A\cos t\,dt + \cancel{\int_{\pi/2}^{\pi} 0\,dt}\right]$$

$$a_0 = \frac{A}{\pi}\int_0^{\pi/2}\cos t\,dt = \frac{A}{\pi}\left[\operatorname{Sen} t\Big|_0^{\pi/2} = \frac{A}{\pi}\left(\overset{1}{\cancel{\operatorname{Sen}\frac{\pi}{2}}} - \operatorname{Sen} 0\right) = \frac{A}{\pi}\right.$$

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$$a_n = \frac{4}{2\pi}\left[\int_0^{\pi/2} A\cos t \, \cos nt \, dt + \int_{\pi/2}^{\pi} \cancel{0 \cdot \cos nt \, dt}^{\,0}\right]$$

$$a_n = \frac{2A}{\pi}\int_0^{\pi/2} \cos t \, \cos nt \, dt = \frac{2A}{\pi}\left[\frac{\text{Sen}((1-n)t)}{2(1-n)} + \frac{\text{Sen}((1+n)t)}{2(1+n)}\right]_0^{\pi/2}$$

$$= \frac{A}{\pi}\left[\frac{\text{Sen}(t-nt)}{1-n} + \frac{\text{Sen}(t+nt)}{1+n}\right]$$

$$= \frac{A}{\pi}\left[\frac{\text{Sen}\,t \cos nt - \cos t \,\text{Sen}\,nt}{1-n} + \frac{\text{Sen}\,t \cos nt + \cos t \,\text{Sen}\,nt}{1+n}\right]$$

$$= \frac{A}{\pi}\left[\frac{\cos\left(\frac{n\pi}{2}\right)}{1-n} + \frac{\cos\left(\frac{n\pi}{2}\right)}{1+n}\right] = \frac{A}{\pi}\cos\frac{n\pi}{2}\left[\frac{1+n+1-n}{(1-n^2)}\right]$$

$$a_n = \frac{2A\cos\left(\frac{n\pi}{2}\right)}{\pi(1-n^2)} \qquad \forall n \neq 1$$

Obteniendo a_1

$$\lim_{n\to 1}\left[\frac{2A\cos\left(\frac{n\pi}{2}\right)}{\pi(1-n^2)}\right] = \frac{2A(0)}{\pi(0)} = \frac{0}{0}$$

L' Hopital

$$\lim_{n\to 1}\left[\frac{-2A\,\text{Sen}\left(\frac{n\pi}{2}\right)\cdot\frac{\pi}{2}}{\pi(-2n)}\right] = \frac{-2A\,\text{Sen}\left(\frac{\pi}{2}\right)\frac{\pi}{2}}{-2\pi}$$

$$= \frac{\frac{-2A\cdot\pi}{2}}{-2\pi} = \frac{-2A\pi}{-4\pi} = \frac{A}{2},$$

10

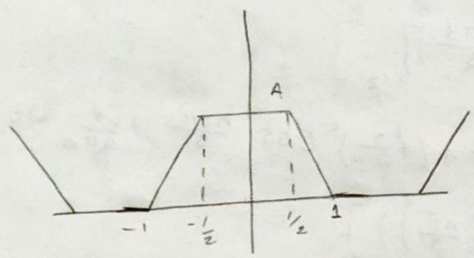
$$g(t) = \frac{A}{\pi} + \frac{A}{2}\cos t + \sum_{n=2}^{\infty}\left[\frac{2A\cos\frac{n\pi}{2}}{\pi(1-n^2)}\right]\cos nt$$

Pero

$$g\left(t-\frac{\pi}{2}\right) = f(t)$$

$$g(t) = \frac{A}{\pi} + \frac{A}{2}\cos\left(t-\frac{\pi}{2}\right) + \sum_{n=2}^{\infty}\left[\frac{2A\cos\frac{n\pi}{2}}{\pi(1-n^2)}\right]\cos\left(n\left(t-\frac{\pi}{2}\right)\right)$$

e)



$$m = \frac{A-0}{-\frac{1}{2}-(-1)} = \frac{A}{\frac{1}{2}} = 2A$$

$$w_0 = \frac{2\pi}{T}, \quad w_0 = \frac{2\pi}{3}$$

$$y - 0 = 2A(t-(-1))$$

$f(t)$ es par $\quad b_n = 0$

$$f(t) = 2A(t+1)$$

$$a_0 = \frac{2}{T}\int_0^{T/2} f(t)\,dt = \frac{2}{3}\left[\int_0^{\frac{1}{2}} A\,dt + \int_{1/2}^{1} -2A(t-1)\,dt + \int^{3/2} \cancel{0}\,dt\right] = \frac{2}{3}\left[A\left(\frac{1}{2}\right) - 2A\left(\frac{t^2}{2}-t\right)\Big|_{1/2}^{1}\right] =$$

$$\frac{2}{3}\left[\frac{A}{2} - 2A\left(\left(\frac{1}{2}-1\right) - \left(\frac{\frac{1}{4}}{2} - \frac{1}{2}\right)\right)\right]$$

$$= \frac{2}{3}\left[\frac{A}{2} - 2A\left(-\cancel{\frac{1}{2}} - \frac{1}{8} + \cancel{\frac{1}{2}}\right)\right] = \frac{2}{3}\left[\frac{A}{2} - 2A\left(-\frac{1}{2}\right)\right]$$

$$= \frac{2}{3}\left[\frac{A}{2} + \frac{A}{4}\right] = \frac{2}{3}\left[\frac{3A}{4}\right] = \frac{A}{2}$$

Calcolando

$$a_n = \frac{4}{3}\int_0^{3/2} f(t)\cos\left(\frac{n2\pi t}{3}\right)dt = \frac{4}{3}\left[\int_0^{1/2} A\cos\left(\frac{n2\pi t}{3}\right)dt\right.$$

$$\left. -2A\int_{1/2}^{1}(t-1)\cos\left(\frac{n2\pi t}{3}\right)dt + \int_1^{3/2}\cancel{0}\,dt^{\,0}\right]$$

$$= \frac{4}{3}\left[\frac{3A}{2\pi n}\operatorname{Sen}\left(\frac{n2\pi t}{3}\right)\Big|_0^{1/2} - 2A\left[\frac{3}{2\pi n}(t-1)\operatorname{Sen}\left(\frac{n2\pi t}{3}\right)\right.\right.$$

$$\left.\left. + \left(\frac{3}{2\pi n}\right)^2\cos\left(\frac{n2\pi t}{3}\right)\right]_{1/2}^{1}\right]$$

$$= \frac{4}{3}\left[\frac{3A\operatorname{Sen}\left(\frac{n\pi}{3}\right)}{2\pi n} - 2A\left(\left(\frac{3}{2\pi n}\right)^2\cos\frac{n2\pi}{3} + \frac{3}{4\pi n}\operatorname{Sen}\frac{n\pi}{3}\right.\right.$$

$$\left.\left. - \left(\frac{3}{2\pi n}\right)^2\cos\left(\frac{n\pi}{3}\right)\right)\right]$$

$$= \frac{4}{\cancel{3}}\left(\frac{\cancel{3}}{2\pi n}\right)\left[A\operatorname{Sen}\left(\frac{n\pi}{3}\right) - 2A\left(\frac{3}{2\pi n}\right)\cos\frac{n2\pi}{3} - \frac{\cancel{2}A}{\cancel{2}}\operatorname{Sen}\frac{n\pi}{3}\right.$$

$$\left. + 2A\left(\frac{3}{2\pi n}\right)\cos\left(\frac{n\pi}{3}\right)\right]$$

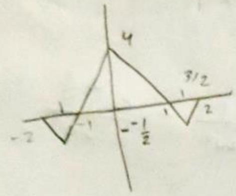
$$= \frac{2}{\pi n}\left(2A\left(\frac{3}{2\pi n}\right)\right)\left[\cos\frac{n\pi}{3} - \cos\frac{n2\pi}{3}\right]$$

$$= \frac{6A}{(\pi n)^2}\left[\cos\frac{n\pi}{3} - \cos\frac{n2\pi}{3}\right]$$

$$\Rightarrow f(t) = \frac{A}{2} + \sum_{n=1}^{\infty}\left[\frac{6A}{(\pi n)^2}\left[\cos\left(\frac{n\pi}{3}\right) - \cos\frac{n2\pi}{3}\right]\right]\cos\left(\frac{n2\pi t}{3}\right)$$

Problema 5

Problemas (5) (21)



$$f(t)\begin{cases} -t-2 & -2<t<-\frac{3}{2} \\ 3t+4 & -\frac{3}{2}<t<0 \\ -3t+4 & 0<t<\frac{3}{2} \\ t-2 & \frac{3}{2}<t<2 \end{cases}$$

$$F\{f(t)\} = \int_{-2}^{2} f(t)e^{-i\omega t}\,dt = \int_{-2}^{-3/2}(-t-2)e^{-i\omega t}\,dt + \int_{-3/2}^{0}(3t+4)\,e^{-i\omega t}\,dt$$

$$+ \int_{0}^{3/2}(-3t+4)e^{-i\omega t}\,dt + \int_{3/2}^{2}(t-2)e^{-i\omega t}\,dt$$

$$= -\int_{-2}^{-3/2} t\,e^{-i\omega t}\,dt - 2\int_{-2}^{0} e^{-i\omega t}\,dt + 3\int_{-3/2}^{3/2} + 3\int_{-3/2}^{3/2} t\,e^{-i\omega t}\,dt + 4\int_{-3/2}^{0} e^{-i\omega t}\,dt$$

$$-\int_{-2}^{-3/2} t\,e^{-it\omega}\,dt = -\frac{1}{i\omega}\left(t\,e^{-i\omega t}\Big|_{-2}^{-3/2}\right) + \frac{1}{i\omega}\int_{-2}^{-3/2} e^{-i\omega t}\,dt$$

$$\Rightarrow -\frac{1}{i\omega}\left(-\frac{3}{2}e^{i\frac{3}{2}\omega} + 2e^{i\omega 2}\right) + \frac{1}{i\omega}\left(-\frac{1}{i\omega}e^{-i\omega t}\Big|_{-2}^{-\frac{3}{2}}\right)$$

$$= \frac{3e^{i\omega\frac{3}{2}}}{2i\omega} - \frac{2e^{i\omega 2}}{i\omega} + \frac{1}{\omega^2}\left(e^{i\omega\frac{3}{2}} - e^{i\omega 2}\right)$$

$$= \frac{3e^{i\omega\frac{3}{2}}}{2i\omega} - \frac{2e^{i\omega 2}}{i\omega} + \frac{e^{i\omega\frac{3}{2}}}{\omega^2} - \frac{e^{i\omega 2}}{\omega^2}$$

$$\Rightarrow \int_{-2}^{-\frac{3}{2}} e^{-i\omega t}\,dt = -\frac{1}{i\omega}\left(e^{-i\omega t}\Big|_{-2}^{-3/2}\right) = \frac{-1}{i\omega}\left(e^{i\omega\frac{3}{2}} - e^{i\omega 2}\right)$$

$$= \frac{-e^{i\omega 3/2}}{i\omega} + \frac{e^{i\omega 2}}{i\omega}$$

$$\int_{-3/2}^{0} t e^{-i\omega t}\, dt = -\frac{1}{i\omega}\left(t e^{-i\omega t} \Big|_{-3/2}^{0} \right) + \frac{1}{i\omega}\int_{-3/2}^{0} e^{-i\omega t}\, dt = \frac{-1}{i\omega}\left(0 + \frac{3}{2} e^{i\omega \frac{3}{2}}\right)$$

$$+ \frac{1}{i\omega}\left(\frac{-1}{i\omega} \; e^{-i\omega t}\Big|_{-\frac{3}{2}}^{0} \right)$$

$$= \frac{-3 e^{i\omega \frac{3}{2}}}{2 i\omega} + \frac{1}{\omega^2}\left(1 - e^{i\omega\frac{3}{2}}\right) = \frac{-3e^{i\omega\frac{3}{2}}}{2i\omega} + \frac{1}{\omega^2} - \frac{e^{i\omega\frac{3}{2}}}{\omega^2}$$

$$\int_{0}^{3/2} t e^{-i\omega t}\, dt = -\frac{1}{i\omega}\left(t e^{-i\omega t} \Big|_{0}^{3/2} \right) + \frac{1}{i\omega}\int_{0}^{3/2} e^{-i\omega t}\, dt$$

$$= -\frac{3e^{-i\omega 3/2}}{2i\omega} + \frac{e^{-i\omega 3/2}}{\omega^2} - \frac{1}{\omega^2}$$

$$\Rightarrow \int_{3/2}^{2} e^{-i\omega t}\, dt = -\frac{1}{i\omega}\left(e^{-i\omega t}\Big|_{3/2}^{2} \right) = -\frac{1}{i\omega}\left(e^{-i\omega 2} - e^{-i\omega 3/2} \right)$$

$$= -\frac{e^{-i\omega 2}}{i\omega} + \frac{e^{-3/2\, i\omega}}{i\omega}$$

$$\mathcal{F}\{f(t)\} = \frac{-3e^{i\omega 3/2}}{2i\omega} + \frac{2e^{i\omega 2}}{i\omega} - \frac{e^{i\omega 3/2}}{\omega^2} + \frac{e^{i\omega 2}}{\omega^2} - \frac{2e^{i\omega 2}}{\omega i}$$

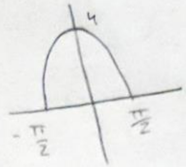
$$+ \frac{3e^{-i\omega\frac{3}{2}}}{2i\omega} + \frac{e^{-i\omega 2}}{\omega^2} - \frac{e^{-i\omega 3/2}}{\omega^2}$$

$$\Rightarrow -\frac{3}{\omega}\left(\frac{e^{i\omega 3/2} - e^{-i\omega 3/2}}{2}\right) + \frac{2}{\omega}\left(\frac{e^{i\omega 2} - e^{-i\omega 2}}{2}\right) - \frac{1}{\omega^2}\left(e^{i\omega 3/2} + e^{-i\omega 3/2}\right)$$

$$+ \frac{1}{\omega^2}\left(e^{i\omega 2} + e^{-i\omega 2}\right)$$

$$= -\frac{8}{\omega^2}\cos\left(\frac{3\omega}{2}\right) + \frac{2}{\omega^2}\cos(2\omega) + \frac{6}{\omega^2}$$

$$= \frac{1}{\omega^2}\left(8\cos\left(\frac{3\omega}{2}\right) - 2\cos(2\omega - 6\right)$$

b) 

$$f(t) = A\cos t \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

(23)

$$F(\omega) = \int_{-\infty}^{\infty} A\cos t\, e^{-i\omega t}\,dt = A\int_{-\frac{\pi}{2}}^{\pi/2}\left(\frac{e^{it}+e^{-it}}{2}\right)e^{-i\omega t}\,dt$$

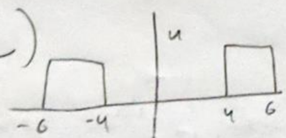
$$= \frac{A}{2}\left[\int_{-\frac{\pi}{2}}^{\pi/2} e^{it(1-\omega)}\,dt + \int_{-\pi/2}^{\pi/2} e^{-it(1+\omega)}\,dt\right]$$

$$= \frac{A}{2}\left[\frac{1}{i(1-\omega)}e^{it(1-\omega)}\Big|_{-\frac{\pi}{2}}^{\pi/2} + \left(-\frac{1}{i(1+\omega)}e^{-it(1+\omega)}\Big|_{-\pi/2}^{\pi/2}\right)\right]$$

$$= \frac{A}{2}\cdot\left[\frac{1}{i(1-\omega)}\left(e^{i\frac{\pi}{2}(1-\omega)} - e^{-i\frac{\pi}{2}(1-\omega)}\right) + \left(-\frac{1}{i(1+\omega)}\left(e^{-i\frac{\pi}{2}(1+\omega)} - e^{i\frac{\pi}{2}(1+\omega)}\right)\right)\right]$$

$$= A\left[\frac{1}{1-\omega}\,\mathrm{sen}\left(\frac{\pi}{2}-\frac{\pi}{2}\omega\right) + \frac{1}{1+\omega}\,\mathrm{sen}\left(\frac{\pi}{2}+\frac{\pi\omega}{2}\right)\right]$$

$$\Rightarrow F(\omega) = \frac{A\pi}{2}\left[\mathrm{Sa}\,\frac{\pi}{2}(1-\omega) + \mathrm{Sa}\,\frac{\pi}{2}(1+\omega)\right]$$

C) 

$$f(t)\begin{cases} A & -6 < t < -4 \\ 0 & -4 < t < 4 \\ A & 4 < t < 6 \end{cases}$$

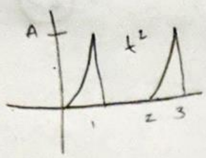
$$F(\omega) = \int_{-6}^{6} f(t)\,e^{-i\omega t}\,dt \Rightarrow A\int_{-6}^{-4} e^{-it\omega}\,dt + A\int_{4}^{6} e^{-it\omega}\,dt$$

$$= A\left[-\frac{1}{i\omega}\left(e^{i4\omega} - e^{i6\omega}\right) - \frac{1}{i\omega}\left(e^{-i6\omega} - e^{-i4\omega}\right)\right]$$

$$\Rightarrow -\frac{A}{i\omega}\left[e^{i4\omega} - e^{i6\omega} + e^{-i6\omega} - e^{-i4\omega}\right]$$

$$= -\frac{2A}{\omega}\left[\left(\frac{e^{i4\omega} - e^{-i4\omega}}{2i}\right) - \left(\frac{e^{i6\omega} - e^{-i6\omega}}{2}\right)\right] = -\frac{2A}{\omega}\left[\mathrm{sen}(4\omega) - \mathrm{sen}(6\omega)\right)$$

D)



$$f(t)\begin{cases} At^2 & 0<t<1 \\ \emptyset & 1<t<2 \\ At^2-4At+4A & 2<t<3 \end{cases}$$

$$F(\omega) = \int_0^1 At^2\, e^{-i\omega t}\,dt + \int_2^3 (At^2-4At+4A)\,|\,e^{-i\omega t}\,dt$$

$$\Rightarrow A\left[\int_0^1 t^2 e^{-i\omega t}\,dt\right] = A\left[-\frac{1}{i\omega}\left(t^2 e^{-i\omega t}\right)\Big|_0^1\right) + \frac{2}{i\omega}\int_0^1 t\, e^{-i\omega t}\,dt\Big]$$

$$= A\left[-\frac{e^{-i\omega}}{i\omega} - \frac{2e^{-i\omega}}{i^2\omega^2} + \frac{2}{i^2\omega^2}\left[\frac{1}{i\omega} - \frac{e^{-i\omega}}{i\omega}\right]\right]$$

$$= -\frac{Ae^{-i\omega}}{i\omega} + \frac{2Ae^{-i\omega}}{\omega^2} + -\frac{2}{i\omega^3} + \frac{2e^{-i\omega}}{i\omega^3}$$

$$= -4A\left[\int_2^3 t e^{-i\omega t}\,dt\right] = -4A\left[\frac{-1}{i\omega}\, t\, e^{-i\omega t}\Big|_2^3\right] + \frac{1}{i\omega}\int_2^3 e^{-i\omega t}\,dt\Big]$$

$$-4A\left[\frac{-3e^{-i3\omega}}{i\omega} + \frac{2e^{-i2\omega}}{i\omega} + \frac{e^{-i3\omega}}{\omega^2} + \frac{e^{-i2\omega}}{\omega^2}\right]$$

$$= 4A\left[\int_2^3 e^{-i\omega t}\,dt\right] = 4A\left[\frac{-1}{\omega i}\, e^{-i\omega t}\Big|_2^3\right] = \frac{-4A}{i\omega}\left(e^{-i3\omega} - e^{-i3\omega}\right)$$

$$\Rightarrow -\frac{4Ae^{-i3\omega}}{i\omega} + \frac{4Ae^{-i2\omega}}{i\omega}$$

$$F(\omega) = A\Big[\frac{1}{i\omega}\left(e^{-i\omega} + 4e^{-i2\omega} + 9e^{-i3\omega} - 4e^{-i3\omega} + 12e^{-3i\omega} - 8e^{-2i\omega}\right) +$$

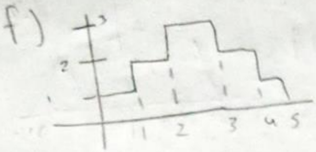
$$\frac{1}{\omega^2}\left(2e^{-i\omega} + 6e^{-3i\omega} - 4e^{-2i\omega} - 4e^{-3i\omega} + 4e^{-2i\omega}\right) + \frac{1}{i\omega^3}\Big(-2 + 2e^{-i\omega}$$

$$+ 2e^{-i\omega} + 2e^{-3i\omega} - 2e^{-i\omega}\Big)\Big]$$

$$F(\omega) = A\left[e^{-i3\omega}\left(\frac{17}{i\omega} + \frac{2}{\omega^2} + \frac{2}{i\omega^2}\right) + e^{-i\omega}\left(\frac{-1}{i\omega} + \frac{2}{\omega^2}\right) - \frac{2}{i\omega^3}\right]$$

25

f)



$$f(t)\begin{cases} 1 & 0<t<1 \\ 2 & 1<t<2 \\ 3 & 2<t<3 \\ 2 & 3<t<4 \\ 1 & 4<t<5 \end{cases}$$

$$f(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt \; \int_0^5 f(t)\, e^{-i\omega t}\, dt$$

$$= \int_0^1 e^{-i\omega t}\, dt + \int_1^2 2e^{-i\omega t}\, dt + \int_2^3 3e^{-i\omega t}\, dt + \int_3^4 4e^{-i\omega t}\, dt +$$

$$\int_4^5 5t^{-i\omega t}\, dt$$

$$= -\frac{1}{i\omega}\left(e^{-i\omega t}\Big|_0^1\right) - \frac{2}{i\omega}\left(e^{-i\omega t}\Big|_1^2\right) - \frac{3}{i\omega}\left(e^{-i\omega t}\Big|_2^3\right) - \frac{2}{i\omega}\left(e^{-i\omega t}\Big|_3^4\right)$$

$$-\frac{1}{i\omega}\left(e^{-i\omega t}\Big|_4^5\right)$$

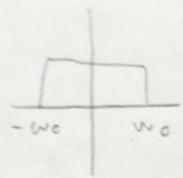
$$= \frac{1}{2\omega} - \frac{e^{-i\omega}}{i\omega} + \frac{2e^{-i\omega}}{i\omega} - \frac{2e^{-2i\omega}}{i\omega} + \frac{3e^{-2i\omega}}{i\omega} + -\frac{3e^{-3i\omega}}{i\omega} + \frac{2e^{-3i\omega}}{i\omega} -$$

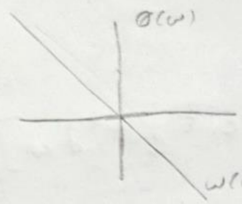
$$\frac{2e^{-4i\omega}}{i\omega} + \frac{e^{-4i\omega}}{i\omega} - \frac{e^{-5i\omega}}{i\omega}$$

$$f(\omega) = \frac{1}{i\omega}\left(1 + e^{-i\omega} + e^{-2i\omega} - e^{-3i\omega} - e^{-4i\omega} - e^{-5i\omega}\right.$$

Problema 6

Problemas 6





$$f(\omega) = x + iy \Rightarrow f(\omega) = |f(\omega)|\, e^{i\theta\omega}$$

$$\theta\omega = -\omega t_0 \qquad f(\omega) = \begin{cases} 0 & \omega < -\omega_0 \\ A e^{-i\omega t} & -\omega_0 < \omega < \omega_0 \\ \emptyset & \omega > \omega_0 \end{cases}$$

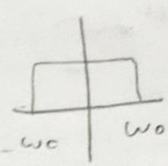
$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{i\omega t}\, dt = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} A e^{-i\omega t_0} e^{i\omega t}\, d\omega$$

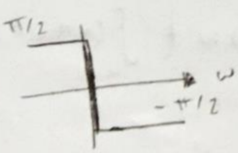
$$\Rightarrow \frac{A}{2\pi}\int_{-\omega_0}^{\omega_0} e^{i(t-t_0)\omega}\, d\omega \Rightarrow \frac{A}{2\pi}\left(\frac{e^{i(t-t_0)\omega}}{i(t-t_0)} \Big|_{-\omega_0}^{\omega_0} \right)$$

$$\Rightarrow \frac{A}{\pi(t-t_0)} \operatorname{Sen}((t-t_0)\omega_0) = \frac{A\omega_0}{\pi(t-t_0)} \operatorname{Sen}(t-t_0)\omega_0$$

$$\frac{A\omega_0}{\pi} \operatorname{Sa}(t-t_0)\omega_0$$

b)





$$F(\omega) = x + iy \Rightarrow f(\omega) = |F(\omega)|\, e^{i\theta\omega}$$

$$F(\omega) = \begin{cases} \pi e^{i\frac{\pi}{2}} & -\omega_0 < \omega < 0 \\ \pi e^{-i\frac{\pi}{2}} & 0 < \omega < \omega_0 \\ \emptyset & \end{cases}$$

$$f(t) = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} F(\omega)\, e^{i\omega t}\, dt$$

$$= \frac{1}{2\pi}\left[\int_{-\omega_0}^{0} \pi e^{i\frac{\pi}{2}} e^{i\omega t}\, d\omega + \int_{0}^{\omega_0} \pi e^{i\frac{\pi}{2}} e^{i\omega t}\, d\omega\right]$$

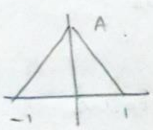
$$= \frac{1}{2}\left[e^{i\frac{\pi}{2}}\left(\frac{1}{it} e^{i\omega t}\Big|_{-\omega_0}^{0}\right) + e^{i\frac{\pi}{2}}\left(\frac{1}{it} e^{i\omega t}\Big|_{0}^{\omega_0}\right)\right]$$

$$e^{\pm i\frac{\pi}{2}} = \cos\frac{\pi}{2} \pm \operatorname{sen}\frac{\pi}{2} \Rightarrow \left[\frac{i}{it}\left(1 - e^{-i\omega_0 t}\right) - \frac{i}{it}\left(e^{i\omega_0 t} - 1\right)\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{1 - e^{-i\omega_0 t} - e^{i\omega_0 t} + 1}{t}\right] = \frac{1}{2}\left[\frac{2 - \left(e^{i\omega_0 t} + e^{-i\omega_0 t}\right)}{t}\right]$$

$$= \frac{1}{t}\left[1 - \cos\omega_0 t\right]$$

C)



$$m = \frac{-A}{1-0}(\omega - \omega_1) + y_1 = -A\omega + A$$

$$A\omega + A$$

$$F(\omega) \begin{cases} A\omega + A & -1<\omega<0 \\ -A\omega + A & 0<\omega<1 \end{cases}$$

$$f(t) = \frac{1}{2\pi}\left[\int_{-1}^{0}(A\omega + A)\, e^{-i\omega t}\, d\omega + \int_{0}^{1}(-A\omega + A)\, e^{i\omega t}\, d\omega\right]$$

$$f(t) = \frac{A}{2\pi}\left[\int_{-1}^{0}\omega e^{-i\omega t}\, d\omega + \int_{-1}^{0} e^{i\omega t}\, d\omega - \int_{0}^{1}\omega e^{i\omega t}\, d\omega + \int_{0}^{1} e^{i\omega t}\, d\omega\right]$$

$$= \frac{e^{-it}}{it} - \frac{1}{i^2t^2}(1-e^{-it}) = \frac{e^{-it}}{it} + \frac{1}{t^2}(1-e^{-it})$$

$$= \frac{e^{-it}}{it} + \frac{1}{t^2} - \frac{e^{-it}}{t^2}$$

$$= \int_0^1 \omega e^{i\omega t}\, d\omega = \frac{1}{it}\left(\omega e^{i\omega t}\Big|_0^1\right) - \frac{1}{it}\int_0^1 e^{i\omega t}\, d\omega$$

$$= \frac{e^{it}}{it} - \frac{1}{i^2t^2}(e^{it}-1) = \frac{e^{it}}{it} + \frac{1}{t^2}(e^{it}-1)$$

$$= \frac{e^{it}}{it} + \frac{e^{it}}{t^2} - \frac{1}{t^2}$$

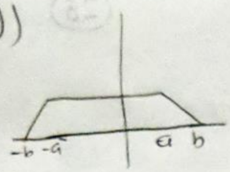
$$= \int_0^1 e^{i\omega t}\, d\omega = \frac{1}{it}\left(e^{i\omega t}\Big|_0^1\right) = \frac{e^{it}}{it} - \frac{1}{it}$$

$$f(t) = \frac{A}{2\pi}\left[\frac{e^{-it}}{it} + \frac{1}{t^2} - \frac{e^{-it}}{t^2} + \frac{1}{it} - \frac{e^{it}}{it} - \frac{e^{it}}{it} - \frac{e^{it}}{t^2} + \frac{e^{it}}{it} - \frac{1}{it}\right]$$

$$= \frac{A}{\pi}\left[-\frac{1}{t}\,\mathrm{sen}\,t + \frac{1}{t}\,\mathrm{sen}\,t - \frac{1}{t}\cos t + \frac{1}{t^2}\right]$$

$$f(t) = \frac{A}{\pi}\left[\frac{1}{t^2}(1-\cos t)\right] = \frac{A}{t^2\pi}(1-\cos t)$$

d)



$$F(\omega)\begin{cases}\dfrac{A\omega}{b-a}+\dfrac{Ab}{b-a} & -b<\omega<-a\\ A & -a<\omega<a\\ \dfrac{-A\omega}{b-a}+\dfrac{Ab}{b-a} & a<\omega<b\end{cases}$$

$$f(t)=\frac{1}{2\pi}\left[\int_{-b}^{-a}\left(\frac{A\omega}{b-a}+\frac{Ab}{b-a}\right)e^{i\omega t}\,d\omega+\int_{-a}^{a}Ae^{i\omega t}\,d\omega+\int_{a}^{b}\left(\frac{-A\omega}{b-a}+\frac{Ab}{b-a}\right)e^{i\omega t}\,d\omega\right]$$

$$=\int_{-b}^{-a}\omega e^{-i\omega t}\,d\omega=\frac{1}{it}\,\omega e^{i\omega t}\Big|_{-b}^{-a}-\frac{1}{it}\int_{-b}^{-a}e^{i\omega t}\,d\omega$$

$$=\frac{1}{it}\left(-ae^{-iat}+be^{-ibt}\right)-\frac{1}{it}\left(\frac{1}{it}e^{i\omega t}\Big|_{-b}^{-a}\right)$$

$$=\frac{-ae^{-iat}}{it}+\frac{be^{-ibt}}{it}+\frac{e^{-iat}}{t^2}-\frac{e^{-ibt}}{t^2}$$

$$\Rightarrow\int_{-a}^{a}e^{i\omega t}\,d\omega=\frac{1}{it}\left(e^{i\omega t}\Big|_{-a}^{a}\right)=\frac{1}{it}\left(e^{iat}-e^{-iat}\right)=\frac{e^{iat}}{it}-\frac{e^{at}}{it}$$

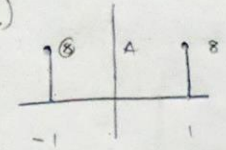
$$=\int_{a}^{b}e^{i\omega t}\,d\omega=\frac{1}{it}\left(e^{i\omega t}\Big|_{a}^{b}\right)=\frac{1}{it}\left(e^{ibt}-e^{iat}\right)=\frac{e^{ibt}}{it}-\frac{e^{iat}}{it}$$

$$=\frac{A}{2\pi}\left[\frac{1}{b-a}\left(\frac{1}{t}\left(2a\left(\frac{e^{iat}-e^{-iat}}{2i}\right)-2b\left(\frac{e^{ibt}-e^{-ibt}}{2i}\right)-2b\left(\frac{e^{iat}-e^{-iat}}{2i}\right)\right.\right.\right.$$
$$\left.\left.+2b\left(\frac{e^{ibt}-e^{-ibt}}{2i}\right)+\frac{1}{t^2}\left(\frac{e^{iat}+e^{-ait}}{2}-\frac{2\left(e^{ibt}+e^{-ibt}\right)}{2}\right)\right)\right)+$$
$$\left.\frac{2}{t}\,\text{Sen}\,at\right]$$

$$\Rightarrow\frac{A}{2\pi}\left[Sa(at)\left(\frac{2a^2}{b-a}-\frac{2ab}{b-a}+2a\right)+\frac{2}{(b-a)t^2}\left(\cos(at)-\cos bt\right)\right]$$

$$= \frac{A}{\pi}\left[Sa(at)\left(\frac{a^2}{b-a} - \frac{ab}{b-a} + a \right) + \frac{1}{(b-a)\,t^2}\left(\cos at - \cos bt \right) \right]$$

e)



$$F(\omega) \begin{cases} 8\delta(\omega+1) & \omega = -1 \\ 8\delta(\omega-1) & \omega = 1 \end{cases}$$

$$f(t) = \frac{1}{2\pi}\left[8\int_{-1}^{1} \delta(\omega+1)\, e^{i\omega t}\, d\omega + 8\int_{-1}^{1^+} \delta(\omega-1)\, e^{i\omega t}\, d\omega \right]$$

$$f(t) = \frac{1}{2\pi}\left[8(e^{-it}) + 8(e^{it}) \right] = \frac{1}{2\pi}\left(8e^{-it} + 8e^{it} \right)$$

$$= \frac{8}{2\pi}\left(e^{it} + e^{-it} \right) \Rightarrow \frac{8}{\pi}\left(\frac{e^{it} + e^{-it}}{2} \right) = \frac{8}{\pi}\cos t$$

Problema 7

Problemas (7)

a) $\int_{-\infty}^{\infty} \delta(t-5)\,\mathrm{sen}\,2t\,dt = \int$

$= \int_{-\infty}^{\infty} \mathrm{Sen}\,2t\;\delta(t-5)\,dt = \mathrm{Sen}(2(5)) = \mathrm{Sen}\,10$

b) $\int_{-\infty}^{\infty} \delta(2-t)(t^5-3)\,dt = \int_{-\infty}^{\infty} \delta(-(t-2))(t^5-3)\,dt$

$= \int_{-\infty}^{\infty} \delta(t-2)(t^5-3)\,dt = \int_{-\infty}^{\infty} \delta(t-2)\,t^5\,dt - \int_{-\infty}^{\infty} \delta(t-2)\,3\,dt$

$\Rightarrow 32-3 = \underline{29}$

c) $\int_1^x e^{-x^2} f(x)\,dx \Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = \int_{0^-}^{0^+} f(x)\,dx = 1$

$\Rightarrow \int_1^x e^{-x^2} \delta(x)\,dx = \int_1^x e^{-x^2}\,0\,dx = 0$

d) $\int_{-\infty}^{\infty} \delta(t-2)\cos(\pi(t-3))\,dt = \int_{-\infty}^{\infty} \delta(t-2)\cos(\pi t-3\pi)\,dt$

$= \cos(2\pi-3\pi) = \cos -\pi = \cos\pi = 1$

e) $\int_{-\infty}^{\infty} e^{\cos t}\,\delta(t-\pi)\,dt = e^{\cos\pi} = e^{-1} = \frac{1}{e}$

h) $t \frac{d}{\,} f(t)$ i) ...

e) $\int_{-\infty}^{\infty} \delta(t+2)\, e^{-2t}\, dt = e^{-2(-2)} = e^{4}$

g) $\int_{1}^{100} \log_{10} t\ \delta(t-10)\, dt = \log(10) = \underline{1}$

Problema 8

Problema 8 Considerando $f(t)$ y $F(\omega)$ forman un par de transformados usando las propiedades de la transformación de fourier encontrar transformada de las sig. Expresiones

a) $f(2-t)$

se tiene

$f(t) \leftrightarrow F(\omega)$

desplazamiento en t

$f(t-2) \longleftrightarrow F(\omega)e^{-i2\omega}$

escalamiento

$f((-1)(t-2)) \leftrightarrow \frac{1}{|-1|} F\left(\frac{\omega}{-1}\right)e^{-i2\left(\frac{\omega}{-1}\right)}$

$f(2-t) \leftrightarrow F(-\omega)\, e^{i2\omega}$

b) $f[(t-3)-3]$

se tiene

$f(t) \longleftrightarrow F(\omega)$

Desplazamiento en t

$f(t-3) \longleftrightarrow F(\omega)e^{-i3\omega}$

Desp. en t

$f(t-3)\Big|_{t=t-3}$

$f((t-3)-3) \longleftrightarrow$

$[F(\omega)e^{-i3\omega}]e^{-i3\omega}$

c) $\left(\frac{d}{dt} f(t)\right) \operatorname{Sen} t$

$f(t) \leftrightarrow F(\omega)$

Dif de t

$\frac{d}{dt} f(t) \leftrightarrow (i\omega) F(\omega)$

$\left(\frac{d}{dt} f(t)\right)\operatorname{Sen} t \leftrightarrow i\,\frac{i}{2}\left[(\omega+1)F(\omega+1) - (\omega-1)F(\omega-1)\right]$

$\left(\frac{d}{dt} f(t)\right)\operatorname{Sen} t \longleftrightarrow \frac{1}{2}\left[(\omega+1)F(\omega+1) - (\omega-1)F(\omega-1)\right]$

d) $\frac{d}{dt}\left[f(-2t)\right]$

$f(t) \longleftrightarrow F(\omega)$

escalamiento

$f(-2t) \leftarrow \frac{1}{|-2|} F\left(\frac{\omega}{-2}\right)$

Dif en t

$\frac{d}{dt}\left[f(-2t) \longleftrightarrow (i\omega)\left[\frac{1}{2} F\left(-\frac{\omega}{2}\right)\right]\right.$

$\frac{d}{dt}\left[f(-2t)\right] \leftarrow \frac{i\omega}{2} F\left(-\frac{\omega}{2}\right)$

e) $t f(3t)$

$f(t) \longleftrightarrow F(\omega)$

Ampliacion

$f(3t) \longleftrightarrow \frac{1}{|3|} F\left(\frac{\omega}{3}\right)$

$-it f(3t) \longleftrightarrow \frac{\partial}{\partial t}\left[\frac{1}{3} F\left(\frac{\omega}{3}\right)\right]$

$t f(3t) \longleftrightarrow \frac{1}{i} \frac{d}{d\omega}\left[\frac{1}{3} F\left(\frac{\omega}{3}\right)\right]$

$t f(3t) \longleftrightarrow i \frac{d}{d\omega}\left[\frac{1}{3} F\left(\frac{\omega}{3}\right)\right]$

f) $(t-s) f(t)$

$\underbrace{t f(t)}_{\mathrm{I}} - \underbrace{s f(t)}_{\mathrm{II}}$

I) $f(t) \longleftrightarrow F(\omega)$

$(-it)' f(t) \longleftrightarrow \frac{d}{d\omega} F(\omega)$

$-it f(t) \longleftrightarrow \frac{d}{d\omega} F(\omega)$

$t f(t) \longleftrightarrow \frac{1}{i} \frac{d}{d\omega} F(\omega)$

$t f(t) \longleftrightarrow i \frac{d}{d\omega} F(\omega)$

II) $f(t) \longleftrightarrow F(\omega)$

$s f(t) \longleftrightarrow s F(\omega)$

$t f(t) - s f(t)$

$\longleftrightarrow i \frac{d}{d\omega} F(\omega) - s F(\omega)$

g) $(t-3) f(-3t)$

$\underbrace{t f(-3t)}_{\mathrm{I}} - \underbrace{3 f(-3t)}_{\mathrm{II}}$

I) $f(t) \longleftrightarrow F(\omega)$

Escalamiento

$f(-3t) \longleftarrow \frac{1}{|-3|} F\left(\frac{\omega}{-3}\right)$

$f(-3t) \longleftrightarrow \frac{1}{3} F\left(-\frac{\omega}{3}\right)$

$(-it) f(-3t) \longleftrightarrow \frac{d}{d\omega}\left[\frac{1}{3} F\left(-\frac{\omega}{3}\right)\right]$

$t f(-3t) \longleftrightarrow i \frac{d}{d\omega}\left[\frac{1}{3} F\left(-\frac{\omega}{3}\right)\right]$

II) $f(t) \longleftrightarrow F(\omega)$

$f(-3t) \longleftrightarrow \frac{1}{3} F\left(-\frac{\omega}{3}\right)$

$3 f(-3t) \longleftrightarrow \cancel{\frac{3}{3}} F\left(-\frac{\omega}{3}\right)$

$t f(-3t) - 3 f(-3t)$

$\longleftrightarrow i \frac{d}{d\omega}\left[\frac{1}{3} F\left(-\frac{\omega}{3}\right)\right] - F\left(-\frac{\omega}{3}\right)$

h) $t \frac{d}{dt} f(t)$

$f(t) \leftrightarrow F(\omega)$

$\frac{d}{dt} f(t) \leftrightarrow i\omega' F(\omega)$

$\frac{d}{dt} f(t) \leftrightarrow i\omega F(\omega)$

$(-it)' \frac{d}{dt} f(t) \longleftrightarrow \frac{d}{d\omega}\left[i\omega F(\omega) \right]$

$t \frac{d}{dt} f(t) \leftrightarrow \frac{d}{d\omega} \left[\omega f(\omega) \right]$

i) $f(6-t)$

$f(t) \longleftrightarrow F(\omega)$

$f(t) \Big|_{t = t-6}'$

$f(t-6) \longleftrightarrow F(\omega) e^{-i6\omega}$

$f((-1)(t-6)) \leftrightarrow \frac{1}{|-1|} F\left(\frac{\omega}{-1}\right) e^{-i6\frac{\omega}{-1}}$

$f(6-t) \leftrightarrow F(\omega) e^{i6\omega}$

i) $(2-t) f(8-t)$

$$\underbrace{2 f(8-t)}_{\mathrm{I}} - \underbrace{t f(8-t)}_{\mathrm{II}} \leftrightarrow 2F(-\omega) e^{i8\omega} - i \frac{d}{dt} \left[F(-\omega) e^{i8\omega} \right]$$

I) $f(t) \leftrightarrow F(\omega$

$f(t) \Big|_{t = t-8}$

$f(t-8) \leftrightarrow F(\omega) e^{-i8\omega}$

$f((-1)(t-8)) \leftrightarrow \frac{1}{|-1|} F(-\omega) e^{i8\omega}$

$f(8-t) \leftrightarrow F(-\omega) e^{i8\omega}$

$2 f(8-t) \longleftrightarrow 2F(-\omega) e^{i8\omega}$

II)

$(-it)' f(8-t) \longleftrightarrow \frac{d}{dt} \left[F(-\omega) e^{i8\omega} \right]$

$t f(8-t) \leftrightarrow i \frac{d}{dt} \left[F(-\omega) e^{i8\omega} \right]$

Problema 9

Problema 9 Completa en tiempo o frecuencia el par de transferencia solicitado usando propiedades de la transferencia de furier

① $5\delta(t-1) \leftrightarrow ?$

$\Rightarrow \delta(t) \leftrightarrow 1$

Desplazo en tiempo

$\delta(t)\big|_{t=t-1} \leftrightarrow ?$

$\delta(t-1) \leftrightarrow 1e^{-i\omega}$

Linealidad

$5\delta(t-1) \leftrightarrow 5e^{-i\omega}$

3) $t \leftrightarrow ?$

$\Rightarrow \delta(t) \leftrightarrow 1$

$1 \leftrightarrow 2\pi\delta(-\omega)$

$1 \leftrightarrow 2\pi\delta(\omega)$

$(-it)' \leftrightarrow 2\pi \frac{d'}{d\omega}\delta(\omega)$

$-it \leftrightarrow 2\pi \frac{d}{d\omega}\delta(\omega)$

$t \leftrightarrow 2\pi \frac{d}{d\omega}\delta(\omega) \frac{-1}{i}$

$t \leftrightarrow 2\pi\,\delta(\omega)\, i$

2) $? \leftrightarrow \underbrace{8\delta(\omega+1)}_{I} + \underbrace{8\delta(\omega-1)}_{II}$

(I) $e^{\pm i\omega_0 t} \leftrightarrow 2\pi\delta(\omega\mp\omega_0)$

$\omega_0 = 1$

$e^{-it} \leftrightarrow 2\pi\delta(\omega+1)$

linealidad

$\frac{e^{-it}}{\pi} \leftrightarrow 2\delta(\omega+1)$

$\frac{4e^{-it}}{\pi} \leftrightarrow 8\delta(\omega+1)$

II) $e^{\pm i\omega_0 t} \leftrightarrow 2\pi\delta(\omega\mp\omega_0)$

$\omega_0 = 1,\ e^{it} \leftrightarrow 2\pi\delta(\omega-1)$

$\frac{e^{it}}{\pi} \leftrightarrow 2\delta(\omega-1)$

$\frac{4e^{it}}{\pi} \leftrightarrow 8\delta(\omega-1)$

$\frac{4e^{-it}}{\pi} + \frac{4e^{it}}{\pi} \leftrightarrow 8\delta(\omega+1) + 8\delta(\omega-1)$

4) $t^2 \leftrightarrow ?$

$\Rightarrow \delta(t) \leftrightarrow 1$

$1 \leftrightarrow 2\pi\delta(-\omega)$

$1 \leftrightarrow 2\pi\delta(\omega)$

$(-it)^2 \leftrightarrow 2\pi \frac{d^2}{d\omega}\delta(\omega)$

$-t^2 \leftrightarrow 2\pi \frac{d^2}{d\omega^2}\delta(\omega)$

$t^2 \leftrightarrow -2\pi \frac{d^2}{d\omega^2}\delta(\omega)$

Problema 10

Problemas 10

a) $? \longleftrightarrow 3\,sgn(4\omega - 2)$

$sgn(t) \longleftrightarrow \frac{2}{i\omega}$

Escalamiento

$sgn(t)\Big|_{t=-4t} \longleftrightarrow ?$

$sgn(t)\Big|_{t=-4t} \longleftrightarrow \frac{1}{|-4|}\,\frac{2}{i(\frac{\omega}{-4})}$

Simetria

$\frac{1}{4}\left(\frac{2}{i(-\frac{t}{4})}\right) \longleftrightarrow 2\pi\,sgn(-4(-\omega))$

$\frac{1}{2i(-\frac{t}{4})} \longleftrightarrow 2\pi\,sgn(4\omega)$

Desplazamiento de frecuencia

$\frac{-4}{2it}\,e^{i\frac{t}{2}} \longleftrightarrow 2\pi\,sgn\left(4(\omega - \frac{1}{2})\right)$

$-\frac{2}{it}\,e^{it/2} \longleftrightarrow 2\pi\,sgn(4\omega - 2)$

$-\frac{3}{2\pi it}\,e^{it/2} \longleftrightarrow 3\,sgn(4\omega - 2)$

b) $C_2(\frac{2}{3}t) \longleftrightarrow ?$

Si $AC_d \longleftrightarrow A\,d\,Sa\left(\frac{\omega d}{2}\right)$

$C_2(t) \longleftrightarrow 2\,Sa\,\omega$

$C_2(t)\Big|_{t=\frac{2}{3}t} \longleftrightarrow \frac{3}{2}\cdot 2\,Sa\,\frac{3\omega}{2}$

$\Rightarrow C_2\,\frac{2}{3}t \longleftrightarrow 3\,Sa\left(\frac{3\omega}{2}\right)$

(38)

C) $2C_2(t)\cos 250t \longleftrightarrow ?$

$AC_d(t) \longleftrightarrow A d\, Sa\left(\frac{\omega d}{2}\right)$

$2C_2(t) \longleftrightarrow 4\, Sa\,\omega$

$2C_2(t)\cos(250t) \longleftrightarrow 2\left[Sa(\omega+250) + Sa[\omega-250]\right]$

$2C_2(t)\cos(250t) \longleftrightarrow 2\left[Sa(\omega+250) + Sa(\omega-250)\right]$

d) $u(10t-1)t \longleftrightarrow ?$

$u(t) \longleftrightarrow \pi\,\delta(\omega) + \frac{1}{i\omega}$

$u(t-1) \longleftrightarrow \pi\,f(\omega) + \frac{1}{\omega i}\Big|\, e^{-i\omega}$

$u(10t-1) \longleftrightarrow \frac{1}{10}\,\pi\,\delta\left(\frac{\omega}{10}\right) + \frac{1}{i\frac{\omega}{10}}\; e^{-i\frac{\omega}{10}}$

$(-it)\,u(10t-1) \longleftrightarrow \frac{1}{10}\,\frac{\partial}{\partial\omega}\left[\pi\,\delta\left(\frac{\omega}{10}\right) + \frac{10}{i\omega}\; e^{-i\frac{\omega}{10}}\right.$

$u(10t-1)t \longleftrightarrow \frac{i}{10}\,\frac{\partial}{\partial\omega}\left[\left(\pi\,\delta\left(\frac{\omega}{10}\right) + \frac{1}{i\frac{\omega}{10}}\; e^{-\frac{\omega}{10}}\right.\right.$

e) $e^{i7}\,\delta(6t-1)\; t^3\, e^{i5t}$

$\delta(t) \longleftrightarrow 1$

$\delta(t-1) \longleftrightarrow e^{-i\omega}$

$\delta(6t-1)\,e^{i7t} \longleftrightarrow e^{-\frac{i(\omega-7)}{6}}$

$\delta(6t-1)\,e^{i7t}\,e^{i5t} \longleftrightarrow e^{-\frac{i(\omega-12)}{6}}$

$\delta(6t-1)\,e^{i7t}\,e^{i5t} \longleftrightarrow e^{-\frac{i(\omega-12)}{6}}$

$= (-it)^3\,\delta(6t-1)\,e^{i7t}\,e^{i5t} \longleftrightarrow \frac{d^3}{d\omega^3}\left(e^{-i(\omega-12)/6}\right)$

$e^{i7t}\,\delta(6t-1)\,t^3\,e^{i5t} \longleftrightarrow -\frac{i\,d^3}{d\omega^3}\left(e^{-\frac{i}{6}(\omega-12)}\right)$

f) $? \longleftrightarrow \frac{4}{\pi} sa(4w - 2$

Si $A C_d(t) \longleftrightarrow A d\, sa\left(\frac{wd}{2}\right)$

$\frac{1}{2} C_8(t) \longleftrightarrow 4\, sa(4w)$

$\frac{1}{2} C_8(t)\, e^{it/2} \longleftrightarrow 4\, sa(4w - 2)$

$\frac{1}{2\pi} C_8(t) e^{it/2} \longleftrightarrow \frac{4}{\pi} sa(4w-2)$

h) $C_{4/3}(t+6) \longleftrightarrow ?$

Si $A C_d(t) \longleftrightarrow A d\, sa\left(\frac{wd}{2}\right)$

$C_{\frac{4}{3}}(t) \longleftrightarrow \frac{4}{3} sa\left(\frac{4w}{6}\right)$

$= C_{\frac{4}{3}}(t+6) \longleftrightarrow \frac{4}{3} sa\left(\frac{2w}{3}\right) e^{i6w}$

i) $[3\delta(t-1) - 3\delta(t+1)] \cos 18t \longleftrightarrow ?$

$\delta(t) \longleftrightarrow 1$

$3\delta(t) \longleftrightarrow 3$

$3\delta(t-1) \longleftrightarrow 3e^{-iw}$

$3\delta(t+1) \longleftrightarrow 3e^{iw}$

$3\delta(t-1) - 3\delta(t+1) \longleftrightarrow -6i\left(\frac{e^{iw} - e^{-iw}}{2i}\right)$

$3\delta(t-1) - 3\delta(t+1) \longleftrightarrow -6i\, \text{sen}\, w$

$3\delta(t-1) - 3\delta(t+1)$

$[3\delta(t-1) - 3\delta(t+1)] \cos 18t \longleftrightarrow -3i[\text{sen}(w+18) + \text{sen}(w-18)]$

j) $? \longleftrightarrow 2\cos 5w$

$\delta(t) \longleftrightarrow 1$

$2\delta(t) \longleftrightarrow 2$

$[\delta(t+t_0) + \delta(t-t_0)] \longleftrightarrow 2\cos t_0 w$

$[\delta(t+500) + \delta(t-500)] \longleftrightarrow 2\cos 500w$

(40)

l) $i\frac{5}{t} \longleftrightarrow ?$

$\text{sgn}(t) \longleftrightarrow \frac{2}{i\omega}$

$\frac{2}{it} \longleftrightarrow 2\pi\, \text{sgn}(-\omega)$

$\frac{5}{it} \longleftrightarrow -2\pi\, \text{sng}(\omega)$

$\frac{i5}{t} \longleftrightarrow -2\pi\, \text{sgn}(\omega)$

$\frac{i5}{t} \longleftrightarrow 2\pi\, \text{Sgn}(\omega)$

m) $2 \longleftrightarrow \frac{1}{\omega}$

S, $\text{sgn}(t) \longleftrightarrow \frac{2}{i\omega}$

$\frac{1}{2}\,\text{sgn}(t) \longleftrightarrow \frac{1}{\omega}$

n) $? \longleftrightarrow \frac{1}{\omega} e^{-i4\omega}$

$\text{sgn}(t) \longleftrightarrow \frac{2}{i\omega}$

$\frac{i}{2}\,\text{sgn}(t) \longleftrightarrow \frac{1}{\omega}$

$\frac{i}{2}\,\text{Sgn}(t-4) \longleftrightarrow \frac{1}{\omega} e^{-i4\omega}$

ñ) $5e^{-i\frac{7}{8}(t-3)} \longleftrightarrow ?$

$\delta(t) \longleftrightarrow 1$

$1 \longleftrightarrow \delta(-\omega)$

$1 \longleftrightarrow \delta(\omega)$

$5 \longleftrightarrow \delta(\omega)5$

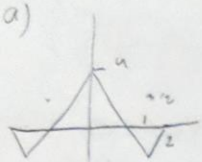
$5e^{-i7t} \longleftrightarrow \delta(\omega + \frac{7}{8})5$

$5e^{-i\frac{7}{8}(t-3)} \longleftrightarrow 5\delta(\omega + \frac{7}{8})e^{-i3\omega}$

Problema 11

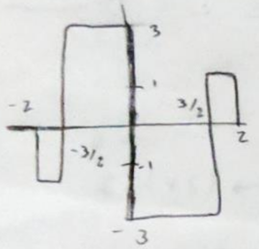
Problema 11

a)

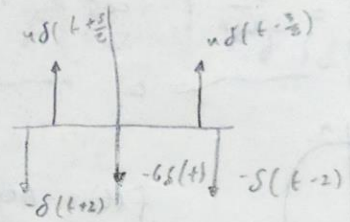


$$f(t)\begin{cases} -t-2 & -2<t<-\frac{3}{2} \\ 3t+4 & -\frac{3}{2}<t<0 \\ -3t+4 & 0<t<\frac{3}{2} \\ t-2 & \frac{3}{2}<t<2 \end{cases}$$

drivando $f(t)$



derivando $\frac{\partial}{\partial w} f(t)$



entonces

$$\frac{\partial^2 f(t)}{\partial t^2} = -\delta(t+2) + 4\delta(t+\tfrac{3}{2}) - 6\delta(t) + 4\delta(t-\tfrac{3}{2}) - \delta(t-2)$$

$$\mathcal{F}\left\{\frac{\partial^2 f(t)}{\partial t^2}\right\} = -\mathcal{F}\{f(t+2)\} + 4\mathcal{F}\{f(t+\tfrac{3}{2})\} - 6\mathcal{F}\{f(t)\} + 4\mathcal{F}\{\delta(t-\tfrac{3}{2})\} - \mathcal{F}\{f(t-2)\}$$

Si
$$\left.\begin{aligned} \delta(t) &\longleftrightarrow 1 \\ \delta(t+2) &\longleftrightarrow e^{i2w} \\ \delta(t+\tfrac{3}{2}) &\longleftrightarrow e^{i\frac{3}{2}w} \\ \delta(t-3/2) &\longleftrightarrow e^{-i3/2 w} \\ \delta(t-2) &\longleftrightarrow e^{-i2w} \end{aligned}\right\} \quad \frac{\partial^2 f(t)}{\partial t^2} = -e^{i2w} + 4e^{i\frac{3}{2}w} - 6 + 4e^{-i\frac{3}{2}w} - e^{-i2w}$$

$$\frac{\partial^2 f(t)}{\partial t^2} \longleftrightarrow -2\cos 2w + 8\cos \tfrac{3}{2}w - 6$$

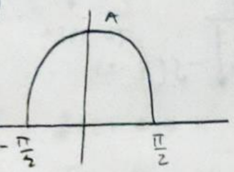
Pero

$$\frac{d^2 f(t)}{dt^2} \longleftrightarrow (i\omega)^n F(\omega)$$

$$(i\omega)^2 F(\omega) = -2\cos 2\omega + 8\cos \tfrac{3}{2}\omega - 6$$

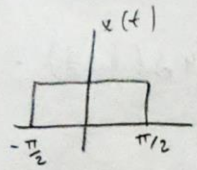
$$F(\omega) = -\frac{1}{\omega^2}\left(-2\cos 2\omega + 8\cos \tfrac{3}{2}\omega - 6\right)$$

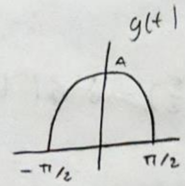
b)



$$f(t) = A\cos t \quad \forall\ -\tfrac{\pi}{2} < t < \tfrac{\pi}{2}$$

$$g(t) = f(t) \cdot x(t)$$





$$g(t) \longleftrightarrow G(\omega)$$

$$f(x)x(t) \longleftrightarrow ?$$

$$x(t) = A\,C_d(t) \Rightarrow x(t) = C_\pi(t)$$

$$C_\pi(t)\, A\cos(t) \longleftrightarrow ?$$

$$\text{si } A\,C_d(t) \longleftrightarrow A d \,Sa\, \frac{\omega d}{2}$$

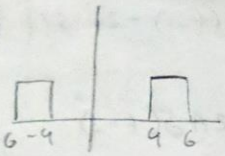
$$C_\pi(t) \longleftrightarrow \pi \,Sa\, \frac{\pi}{2}\omega$$

$$C_\pi(t)\cos(\omega_0 t) \longleftrightarrow \frac{\pi}{2}\left[Sa\left(\frac{\pi}{2}(\omega+\omega_0)\right) + Sa\left(\frac{\pi}{2}(\omega-\omega_0)\right)\right]$$

$$A\,C_\pi(t)\,\cos(t) \longleftrightarrow \frac{A\pi}{2}\left[Sa\left(\frac{\pi}{2}(\omega+1)\right) + Sa\left(\frac{\pi}{2}(\omega-1)\right)\right]$$

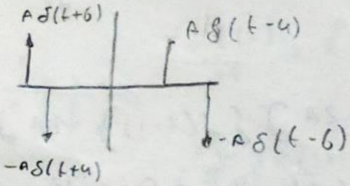
(43)

c)



$$f(t)\begin{cases} A & -6<t<-4 \\ A & 4<t<6 \end{cases}$$

Derivando



$$\mathcal{F}\left\{\frac{\partial}{\partial\omega} f(t)\right\} = A\mathcal{F}\{\delta(t+6)\} - A\mathcal{F}\{\delta(t+4)\} + A\mathcal{F}\{f(t-4)\} - A\mathcal{F}\{\delta(t-6)\}$$

Si $\delta(t) \longleftrightarrow 1$ $\qquad \delta(t-6) \longleftrightarrow e^{-i\omega 6}$

$\delta(t+6) \longleftrightarrow e^{i\omega 6}$ $\qquad \delta(t+4) \longleftrightarrow e^{i\omega 6}$

$\delta(t-4) \longleftrightarrow e^{-i\omega 4}$

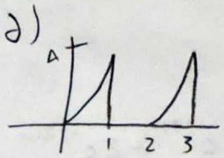
$$\mathcal{F}\left\{\frac{\partial}{\partial t} f(t)\right\} = A\left[e^{i\omega 6} - e^{i\omega 4} + e^{-i\omega 4} - e^{-i\omega 6}\right]$$

$$\frac{\partial}{\partial t} f(t) \longleftrightarrow 2Ai\left[\operatorname{sen}(6\omega - \operatorname{sen} 4\omega\right]$$

Si $\dfrac{d^n f(t)}{d t^n} \longleftrightarrow (i\omega)^n F(\omega)$

$$F(\omega) = \frac{2A}{\omega}\left[\operatorname{sen} 6\omega - \operatorname{sen} 4\omega\right]$$

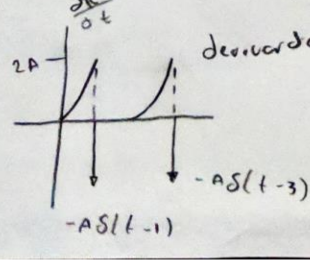
d)



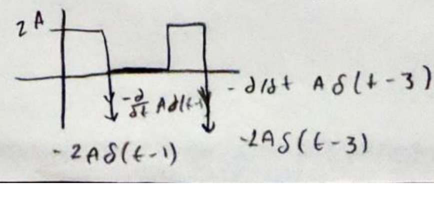
$$f(t)\begin{cases} At^2 & 0<t<1 \\ A(t-2)^2 & 2<t<3 \end{cases}$$

$$A(t-2)^2 = A(t^2-4t+4) = At^2 - 4At + 4A$$

$\dfrac{\partial f(t)}{\partial t}$ derivando



Derivando $\partial^2/\partial\omega^2$ $\qquad \dfrac{\partial f(t)}{\partial t}$

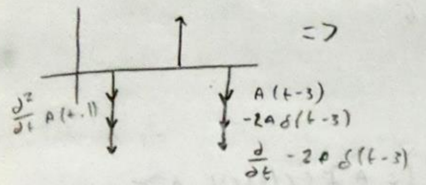


derivando $\frac{d^2}{d t^2}$

$$\frac{d^3 f(t)}{dt^3} = -\frac{d^2}{dt^2} A(t-1) - 2A\,\delta(t-1)$$

$$-\frac{d}{dt} 2A\,\delta(t-1) + 2A\,\delta(t-2)$$

$$-\frac{d^2}{dt^2} A\,\delta(t-3) - 2A\,\delta(t-3) - \frac{d}{dt} 2A\,\delta(t-3)$$



=>

$$\mathcal{F}\left\{\frac{d^3 f(t)}{dt^3}\right\} = -A\,\mathcal{F}\left\{\frac{d^2}{dt^2}(t-1)\right\} - 2A\,\mathcal{F}\{\delta(t-1)\} - 2A\,\mathcal{F}\left\{\frac{d}{dt}\delta(t-1)\right\} + 2\,\mathcal{F}\{\delta(t-2)\} - A\,\mathcal{F}\left\{\frac{d^2}{dt^2}\delta(t-3)\right\}$$

$$-2A\{\delta(t-3)\} - 2A\,\mathcal{F}\{f(t-3)\}$$

$\delta(t) \leftrightarrow 1$

$\delta(t-1) \leftrightarrow e^{-iw}$

$\frac{d}{dt}\delta(t-1) \leftrightarrow iw\,e^{-iw}$

$\delta(t-2) \leftrightarrow e^{-i2w}$

$\delta(t-3) \leftrightarrow e^{-i3w}$

$\frac{d}{dt}\delta(t-3) \leftrightarrow iw$

$\frac{d^2}{dt^2}\delta(t-1) \leftrightarrow iw\,e^{-iw}$

$\frac{d^3}{dt^3}\delta(t-5) \leftrightarrow (iw)^2 e^{-i3w}$

$$\frac{d^3}{dt^3} f(t) \leftrightarrow -A(iw)^2 e^{-iw} - 2A e^{-iw} - 2A(iw)e^{-iw} + 2A e^{-i2w} - A(iw)^2 e^{-i3w}$$

$$-2A\,e^{-i3w} - 2A(iw)e^{-i3w}$$

$f(t) \longleftrightarrow F(w)$

$\frac{d^n}{dt^n} f(t) \longleftrightarrow (iw)^n F(w)$

$$F(w) = \frac{Ai}{w^3}\left[w^2 e^{-iw} - 2e^{-iw} - 2iwe^{-iw} + 2e^{-iw} + w^2 e^{-i3w} - 2e^{-i2w} - 2iwe^{-i3}\right]$$

44

46

$f(t) = x(t) - x(-t)$

$\Rightarrow x(t) = h(t) + g(t)$

$F(\omega) = X(\omega) - X(-\omega)$

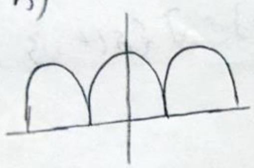
$$F(\omega) = \frac{-4}{\omega^2}\left[i\omega e^{i8\omega} - i\omega e^{-i7\omega} + i\omega e^{i2\omega} - e^{i\omega} + 4\right] + 20\,Sa\left(\frac{5}{2}\omega\right) e^{i\frac{9}{2}\omega}$$

$$+ 5\left[Sa\left(\tfrac{5}{2}(\omega+100)\right) e^{i\frac{9}{2}(\omega+100)} + Sa\left(\tfrac{5}{2}(\omega-100)\right) e^{i\frac{9}{2}(\omega-100)}\right]$$

$$+ \frac{4}{\omega^2}\left[-i\omega e^{-i3\omega} + i\omega e^{-i7\omega} - i\omega e^{-i2\omega} - e^{-i\omega} + 4\right] - 20\,Sa\left(-\frac{5}{2}\omega\right)$$

$$e^{-i\frac{9}{2}\omega} - 5\left[Sa\left(\tfrac{5}{2}(100-\omega)\right) e^{i\frac{9}{2}(100\omega)} + Sa\left(\tfrac{5}{2}(\omega+100)\right) e^{-i\frac{9}{2}\omega+100}\right]$$

h)



$f(t) = A\cos t \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}$

$T = \pi \qquad \omega_0 = 2$

$$\mathcal{F}\{f(t)\} = \sum_{n=-\infty}^{\infty} C_n 2\pi\,\delta(\omega - n\omega_0)$$

$$C_n = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\pi/2} A\cos t\, e^{-i2nt}\,dt = \frac{A}{\pi}\int_{-\pi/2}^{\pi/2}\left(\frac{e^{it}+e^{-it}}{2}\right) e^{-i2nt}\,dt$$

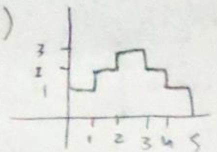
$$= \frac{A}{2\pi}\left[\int_{-\pi/2}^{\pi/2} e^{it(1-2n)} + \int_{-\pi}^{\pi} e^{-it(1+2n)}\right]$$

$$\Rightarrow \frac{A}{2\pi}\left[\frac{1}{i(1-2n)} e^{it(1-2n)}\Big|_{-\pi/2}^{\pi/2} + \left(\frac{-1}{i(1+2n)} e^{-it(1+2n)}\Big|_{\pi/2}^{\pi/2}\right)\right]$$

$$\Rightarrow \frac{A}{2\pi}\left[\frac{\operatorname{sen}\pi/2(1-2n)}{1-2n} + \frac{\operatorname{sen}\pi/2(1+2n)}{1+2n}\right] = \frac{A}{2\pi}\left[Sa\,\frac{\pi}{2}(1-2n) + Sa\,\frac{\pi}{2}(1+2n)\right]$$

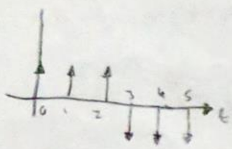
$$F(\omega) = \sum_{n=-\infty}^{\infty} A\pi\left[Sa\,\frac{\pi}{2}(1-2n) + Sa\,\frac{\pi}{2}(1+2n)\right]\delta(\omega - 2n)$$

f.)



$$f(t)\begin{cases} 1 & 0<t<1 \\ 2 & 1<t<2 \\ 3 & 2<t<3 \\ 2 & 3<t<4 \\ 1 & 4<t<5 \end{cases}$$

Deriando



$$\frac{\partial f(t)}{\partial t} = \delta(t)+\delta(t-1)+\delta(t-2)-\delta(t-3)-\delta(t-4)-\delta(t-5)$$

$$\mathcal{F}\left\{\frac{\partial f(t)}{\partial t}\right\} = \mathcal{F}\{\delta(t)\}+\mathcal{F}\{\delta(t-1)\}+\mathcal{F}\{\delta(t-2)\}$$
$$-\mathcal{F}\{\delta(t-3)\}-\mathcal{F}\{\delta(t-4)\}-\mathcal{F}\{\delta(t-5)\}$$

S. $\delta(t) \leftrightarrow 1$ $\quad\delta(t-2) \leftrightarrow e^{-i2w}$

$\delta(t-1) \leftrightarrow e^{-iw}$ $\quad\delta(t-4) \leftrightarrow e^{-i4w}$

$\delta(t-3) \leftrightarrow e^{-i3w}$

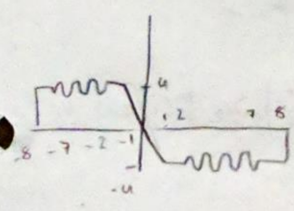
$\partial(t-5) \leftrightarrow e^{-i5w}$

$$\frac{d\,f(t)}{dt} \leftrightarrow 1+e^{-iw}+e^{-i2w}+e^{-i3w}+e^{-i4w}+e^{-i5w}$$

$$iwF(w) = 1+e^{-iw}+e^{-iw}+e^{-i3w}+e^{i4w}+e^{-i5w}$$

$$F(w) = \frac{1}{w}\left(1+e^{-iw}+e^{-i2w}+e^{-i3w}+e^{i4w}+e^{-i5w}\right)$$

g)



$$f(t) = \begin{cases} 4 & -8<t<-7 \\ 4+2\cos t100 & -7<t<-2 \\ 4 & -2<t<-1 \\ -4t & -1<t<0 \end{cases}$$

(46)

$f(t) = x(t) - x(-t)$

$\Rightarrow x(t) = h(t) + g(t)$

$F(\omega) = X(\omega) - X(-\omega)$

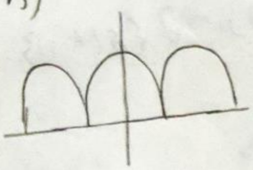
$$F(\omega) = -\frac{4}{\omega^2}\left[i\omega e^{i8\omega} - i\omega e^{-i7\omega} + i\omega e^{i2\omega} - e^{i\omega} + 4\right] + 20\,Sa\left(\frac{5}{2}\omega\right) e^{i\frac{9}{2}\omega}$$

$$+ 5\left[Sa\left(\frac{5}{2}(\omega+100)\right) e^{i\frac{9}{2}(\omega+100)} + Sa\left(\frac{5}{2}(\omega-100)\right) e^{i\frac{9}{2}(\omega-100)}\right]$$

$$+ \frac{4}{\omega^2}\left[-i\omega e^{-i3\omega} + i\omega e^{-i7\omega} - i\omega e^{-i2\omega} - e^{-i\omega} + 4\right] - 20\,Sa\left(-\frac{5}{2}\omega\right)$$

$$e^{-i\frac{9}{2}\omega} - 5\left[Sa\left(\frac{5}{2}(100-\omega)\right) e^{i\frac{9}{2}(100\omega)} + Sa\left(\frac{5}{2}(\omega+100)\right) e^{-i\frac{9}{2}(\omega+100)}\right]$$

h)



$f(t) = A\cos t \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}$

$T = \pi \qquad \omega_0 = 2$

$$\mathcal{F}\{f(t)\} = \sum_{n=-\infty}^{\infty} C_n 2\pi\,\delta(\omega - n\omega_0)$$

$$C_n = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\pi/2} A\cos t\, e^{-i2nt}\,dt = \frac{A}{\pi}\int_{-\pi/2}^{\pi/2}\left(\frac{e^{it}+e^{-it}}{2}\right)e^{-i2nt}\,dt$$

$$= \frac{A}{2\pi}\left[\int_{-\pi/2}^{\pi/2} e^{it(1-2n)} + \int_{\pi}^{\pi} e^{-it(1+2n)}\right]$$

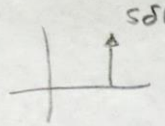
$$\Rightarrow \frac{A}{2\pi}\left[\frac{1}{i(1-2n)} e^{it(1-2n)}\Big|_{-\pi/2}^{\pi/2} + \left(\frac{-1}{i(1+2n)} e^{-it(1+2n)}\Big|_{\pi/2}^{\pi/2}\right)\right]$$

$$\Rightarrow \frac{A}{2\pi}\left[\frac{\operatorname{sen}\pi/2(1-2n)}{1-2n} + \frac{\operatorname{sen}\pi/2(1+2n)}{1+2n}\right] = \frac{A}{2\pi}\left[Sa\,\frac{\pi}{2}(1-2n) + Sa\,\frac{\pi}{2}(1+2n)\right]$$

$$F(\omega) = \sum_{n=-\infty}^{\infty} A\pi\left[Sa\,\frac{\pi}{2}(1-2n) + Sa\,\frac{\pi}{2}(1+2n)\right]\delta(\omega - 2n)$$

47

j)



$F(w) = 5\delta(w-1) \quad w=1$

$\delta t \to 1$

$1 \longleftrightarrow 2\pi\,\delta(w)$

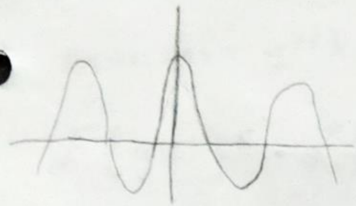
$\frac{5}{2\pi} \longleftrightarrow 5\,\delta(w)$

$\frac{5e^{it}}{2\pi} \longleftrightarrow 5\,\delta(w-1)$

Problema 12

Problemas (12)

$f(t) = A\cos 20t \quad -\frac{\pi}{5} < t < \frac{\pi}{5}$



$f(t) \longleftrightarrow ?$

$A\, C_d(t)\, A\cos 20t \longleftrightarrow ?$

Solo si

$A C_d(t) \longleftrightarrow A d\, Sa\, \frac{\omega d}{2}$

$C_{\frac{2\pi}{5}}(t) \longleftrightarrow \frac{2\pi}{5}\, Sa\left(\frac{\omega \frac{2\pi}{5}}{5}\right)$

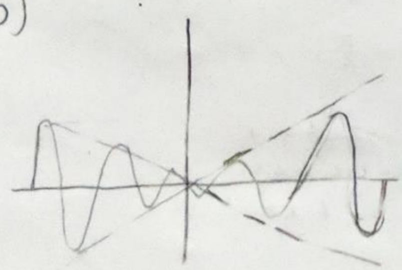
$C_{\frac{2\pi}{5}}(t) \longleftrightarrow \frac{2\pi}{5}\, Sa\left(\omega \frac{\pi}{5}\right)$

$C_{\frac{2\pi}{5}}(t)\, Cos(\omega_0 t) \longleftrightarrow \frac{2\pi}{5} \cdot \frac{1}{2}\left[Sa\left(\frac{\omega+\omega_0}{5}\pi\right) + Sa\left(\frac{\pi}{5}(\omega-\omega_0)\right)\right.$

$C_{\frac{2\pi}{5}}(t)\, Cos(20t) \longleftrightarrow \frac{\pi}{5}\left[Sa\left(\frac{\pi}{5}(\omega+20)\right) + Sa\left(\frac{\pi}{5}(\omega-20)\right)\right]$

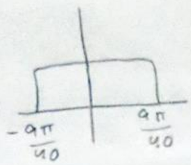
$C_{\frac{2\pi}{8}}(t)\, A\cos 20t \longleftrightarrow \frac{A\pi}{5}\left[Sa\left(\frac{\pi}{5}(\omega+20)\right) + Sa\left(\frac{\pi}{5}(\omega-20)\right)\right.$

b)



$$f(t) \begin{cases} -\frac{40A}{9\pi} t\cos 20t & \left(-\frac{9\pi}{40} < t < \frac{9\pi}{40}\right) \\ 0 & \text{otro Caso} \end{cases}$$

teniendo



$\Rightarrow f(t) \longleftrightarrow ?$

$-\frac{40A}{9\pi} t \cos 20t \left(C_{\frac{3\pi}{40}} t \right) \longleftrightarrow ?$

Si $A\, C_d(t) \longleftrightarrow A d\, Sa \frac{\omega d}{2}$

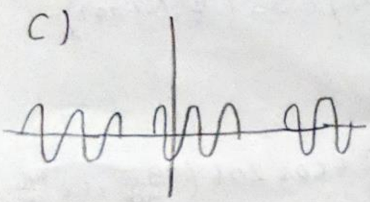
$C_{\frac{9\pi}{20}}(t) \longleftrightarrow \frac{9\pi}{20} Sa \left(\frac{9\pi\omega}{40} \right)$

$(-it) C_{\frac{9\pi}{20}}(t) \longleftrightarrow \frac{9\pi}{20} \frac{d}{d\omega} \left[Sa \left(\frac{9\pi\omega}{40} \right) \right]$

$-\frac{40A}{9\pi} t\, C_{\frac{9\pi}{20}}(t) \longleftrightarrow \frac{40A}{20i} \frac{d}{d\omega} \left[Sa \left[\frac{9\pi\omega}{40} \right] \right]$

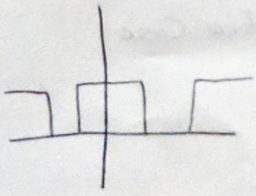
$= -\frac{40A}{9\pi} t\; C_{\frac{9\pi}{20}}\, t \cos 20t \longleftrightarrow -4i \frac{d}{d\omega} \left[Sa \left(\frac{9\pi}{40} (\omega+20) \right) + Sa \left(\frac{9\pi}{40} (\omega-20) \right) \right]$ //

C)



$f(t) = g(t)\, A \cos 200\pi t$

$T = 4 \qquad \omega = \frac{\pi}{2}$



$$g(t) = \begin{cases} 0 & -2 < t < -1 \\ 1 & -1 < t < 1 \\ 0 & 1 < t < 2 \end{cases}$$

50

$f(t) \longleftrightarrow ?$

$A\cos 200\pi\, g(t) \longleftrightarrow ?$

$$\mathcal{F}\{g(t)\} = \sum_{n=-\infty}^{\infty} C_n\, 2\pi\, \delta(\omega - n\omega_0)$$

$$C_n = \frac{1}{4}\int_{-2}^{2} g(t)\, e^{-in\frac{\pi}{2}t}\, dt = \frac{1}{4}\int_{-1}^{1} e^{-in\frac{\pi}{2}t}\, dt$$

$$= \frac{1}{4}\left(\frac{-2}{in\pi} e^{-in\frac{\pi}{2}t}\Big|_{-1}^{1}\right) = \frac{1}{n\pi}\left(\frac{e^{-in\frac{\pi}{2}} - e^{-in\frac{\pi}{2}}}{2}\right)$$

$$= \frac{1}{2}\, Sa\, \frac{n\pi}{2}$$

$$g(t) \longleftrightarrow \sum_{-\infty}^{\infty} \frac{1}{2}\, Sa\, \frac{n\pi}{2}\, 2\pi\, \delta\left(\omega - \frac{n\pi}{2}\right)$$

$$g(t) \longleftrightarrow \sum_{-\infty}^{\infty} \pi\, Sa\, \frac{n\pi}{2}\, \delta\left(\omega - \frac{n\pi}{2}\right)$$

$$\cos 200\pi t\, g(t) \longleftrightarrow \sum_{-\infty}^{\infty} \frac{\pi}{2}\, Sa\, \frac{n\pi}{2}\left[\delta\left(\omega + 200\pi - \frac{n\pi}{2}\right) + \delta\left(\omega - 200\pi - \frac{n\pi}{2}\right)\right]$$

$$= A\cos 200\pi t\, g(t) \longleftrightarrow \sum_{-\infty}^{\infty} \left[\frac{A\pi}{2}\, Sa\, \frac{n\pi}{2}\left[\delta\left(\omega + 200\pi - \frac{n\pi}{2}\right) + \delta\left(\omega - 200\pi - \frac{n\pi}{2}\right)\right]\right]$$

//

50

$f(t) \longleftrightarrow ?$

$A\cos 200\pi\, g(t) \longleftrightarrow ?$

$$\mathcal{F}\{g(t)\} = \sum_{n=-\infty}^{\infty} C_n\, 2\pi\, \delta(\omega - n\omega_0)$$

$$C_n = \frac{1}{4}\int_{-2}^{2} g(t)\, e^{-in\frac{\pi}{2}t}\, dt = \frac{1}{4}\int_{-1}^{1} e^{-in\frac{\pi}{2}t}\, dt$$

$$= \frac{1}{4}\left(\frac{-2}{jn\pi} e^{-in\frac{\pi}{2}t}\Big|_{-1}^{1}\right) = \frac{1}{n\pi}\left(\frac{e^{-in\frac{\pi}{2}} - e^{-in\frac{\pi}{2}}}{2}\right)$$

$$= \frac{1}{2}\, Sa\, n\frac{\pi}{2}$$

$$g(t) \longleftrightarrow \sum_{-\infty}^{\infty} \frac{1}{2}\, Sa\, n\frac{\pi}{2}\, 2\pi\, \delta(\omega - n\frac{\pi}{2})$$

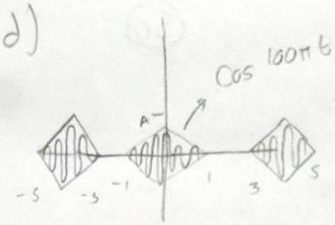
$$g(t) \longleftrightarrow \sum_{-\infty}^{\infty} \pi\, Sa\, n\frac{\pi}{2}\, \delta(\omega - \frac{n\pi}{2})$$

$$\cos 200\pi t\, g(t) \longleftrightarrow \sum_{-\infty}^{\infty} \frac{\pi}{2}\, Sa\, \frac{n\pi}{2}\Big[\delta(\omega + 200\pi - \frac{n\pi}{2}) + \delta(\omega - 200\pi - \frac{n\pi}{2})\Big]$$

$$= A\cos 200\pi t\, g(t) \longleftrightarrow \sum_{-\infty}^{\infty} \Big[\frac{A\pi}{2}\, Sa\, \frac{n\pi}{2}\Big[\delta(\omega + 200\pi - \frac{n\pi}{2}) + \delta(\omega - 200\pi - \frac{n\pi}{2})\Big]$$

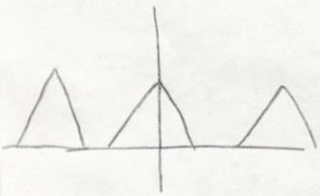
//

d) (51)



$f(t) = x(t)\cos \pi t$

$$x(t)\begin{cases} 0 & -2<t<-1 \\ At+A & -1<t<0 \\ -At+A & 0<t<1 \\ 0 & 1<t<2 \end{cases}$$



$f(t) \longleftrightarrow ?$

$x(t)\cos 100\pi t \longleftrightarrow ?$

$$\mathcal{F}\{x_s(t)\} = \sum_{n=-\infty}^{\infty} C_n\, 2\pi\, \delta(\omega - n\omega_0)$$

$$C_n = \frac{1}{4}\int_{-1}^{1} x(t)\, e^{-in\omega_0 t}\,dt = \frac{A}{4}\left[\int_{-1}^{0}(t+1)e^{-in\frac{\pi}{2}t}\,dt + \int_0^1(-t+1)e^{-in\frac{\pi}{2}t}\,dt\right.$$

$$= \frac{A}{4}\left[\int_{-1}^{0} t e^{-in\frac{\pi}{2}t}\,dt + \int_{-1}^{0} e^{-in\frac{\pi}{2}t}\,dt - \int_0^1 t e^{-in\frac{\pi}{2}t}\,dt + \int_0^1 e^{-in\pi/2\, t}\,dt\right]$$

$$= \int_{-1}^{0} t e^{-in\frac{\pi}{2}t}\,dt = \frac{-2}{in\pi}\left(t e^{-in\frac{\pi}{2}t}\Big|_{-1}^{0}\right) + \frac{2}{in\pi}\int_{-1}^{0} e^{-in\pi/2\, t}\,dt$$

$$= \frac{-2e^{jn\pi/2}}{in\pi} - \frac{4}{i^2n^2\pi^2}\left(1 - e^{in\pi/2}\right) = \frac{-2e^{in\pi/2}}{in\pi} + \frac{4}{n^2\pi^2} - \frac{4e^{in\pi/2}}{n^2\pi^2}$$

$$\Rightarrow \int_{-1}^{0} e^{-in\frac{\pi}{2}t}\,dt = \frac{-2}{in\pi}\left(e^{-in\pi/2\, t}\Big|_{-1}^{0}\right) = \frac{-2}{in\pi}\left(1 - e^{in\pi/2}\right)$$

$$= \frac{-2}{in\pi} + \frac{2e^{in\frac{\pi}{2}}}{in\pi}$$

Continuacion d) (52)

$$\int_0^1 t e^{-in\pi/2 t}\,dt = \frac{-2}{in\pi}\left(t e^{-in\pi/2 t}\right) + \frac{2}{in\pi}\int_0^1 e^{-in\pi/2 t}\,dt$$

$$\Rightarrow \frac{-2}{in\pi}\left(e^{-in\pi/2} - \varnothing\right) + \frac{2}{in\pi}\left(\frac{-2}{in\pi} e^{-in\pi/2 t}\Big|_0^1\right)$$

$$= \frac{-2e^{-in\pi/2}}{in\pi} + \frac{4e^{-in\pi/2}}{n^2\pi^2} - \frac{4}{n^2\pi^2}$$

$$\Rightarrow \int_0^1 e^{-i\frac{n\pi}{2}t}\,dt = \frac{-2}{in\pi}\left(e^{-in\pi/2 t}\Big|_0^1\right) = \frac{-2}{in\pi}\left(e^{-in\pi/2} - 1\right)$$

$$= \frac{-2e^{-in\pi/2}}{in\pi} + \frac{2}{in\pi}$$

$$C_n = \frac{A}{4}\left[\frac{-2}{n\pi}\left(\frac{e^{in\pi/2} - e^{-in\pi/2}}{i}\right) - \frac{4}{n^2\pi^2}\left(\frac{e^{in\pi/2} - e^{-in\pi/2}}{i}\right) + \frac{8}{n^2\pi^2} + \frac{2}{n\pi}\left(\frac{e - e}{3}\right)\right.$$

$$C_n = \frac{A}{4}\left[\frac{8}{n^2\pi^2}\left(1 - \cos\frac{n\pi}{2}\right)\right] = \frac{2A}{n^2\pi^2}\left(1 - \cos\frac{\pi n}{2}\right)$$

$$x(t)\cos 100\pi t \longleftrightarrow \frac{A\pi}{4}\left[\delta(\omega\, 100\pi) + \delta(\omega - 100\pi)\right]$$
$$+ \sum \frac{4A}{2\pi^2 n^2}\left(1 - \cos\frac{n\pi}{2}\right)\left[\delta\left(\omega + 100\pi - \frac{n\pi}{2}\right) + \delta\left(\omega - 100\pi - \frac{n}{2}\pi\right)\right.$$

$$x(t)\cos 100\pi t \longleftrightarrow \frac{A\pi}{4}\left[\delta(\omega + 100\pi) + \delta(\omega - 100\pi)\right] + \frac{2A}{\pi}\sum\frac{1}{n^2}\left(1 - \cos\frac{n\pi}{2}\right)$$
$$\left[\delta\left(\omega + 100\pi - \frac{n\pi}{2}\right) + \delta\left(\omega - 100\pi - \frac{n\pi}{2}\right)\right.$$

//