Noel Ball
HW4

6.1-1.) A heap of height h can have a minimum of 2^h elements and a maximum of 2^{h+1}-1, for instance a heap with a height of 3 has at least 8 elements and at most 15.

6.1-2.) $\lfloor \lg n \rfloor$ results in h when n is in the range $2^h \leq n \leq 2^{h+1}-1$ due to the floor. For instance a heap with 8 to 15 elements has a height of 3 because $\lfloor \lg 8 \rfloor = 3$ and $\lfloor \lg 15 \rfloor = \lfloor 3.9 \rfloor = 3$.

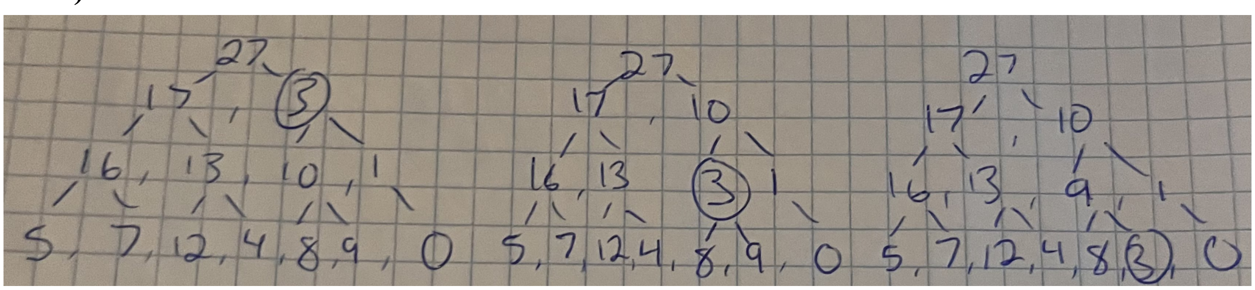
6.1-4.) The smallest element in a max heap must be on the final leaf of any branch, given that A[parent(i)] > A[i].

6.1-6.) A sorted array can be considered a min heap as long as it is ascending order, because each node will be less than their children and the property holds, however a descending sorted array is a max heap.

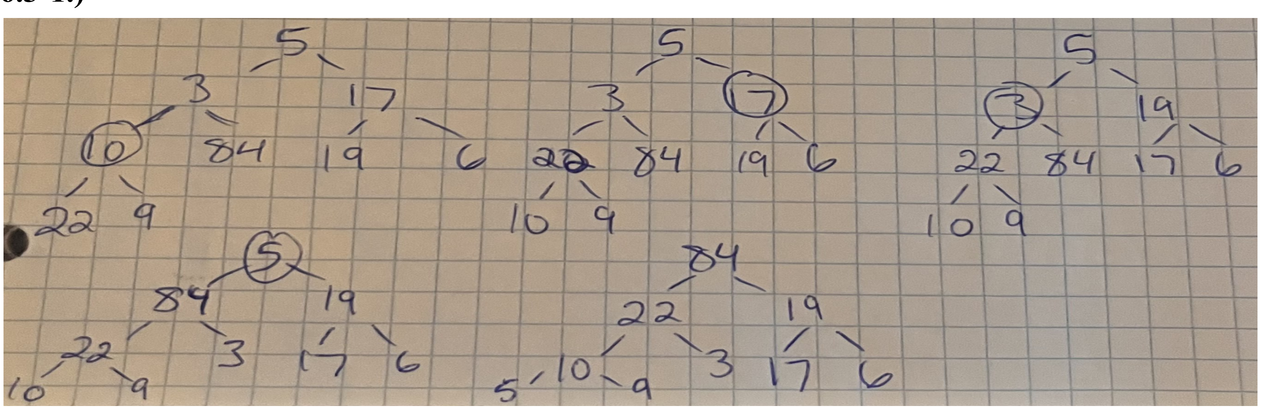
6.1-7.) Yes the array is a max heap because each parent is greater than their children.

6.1-8.) The index $\lfloor n/2 \rfloor$ in an n element heap array is the last parent, therefore every element that follows $\lfloor n/2 \rfloor + 1$, $\lfloor n/2 \rfloor + 2$, … , n are leaf nodes.

6.2-1.)

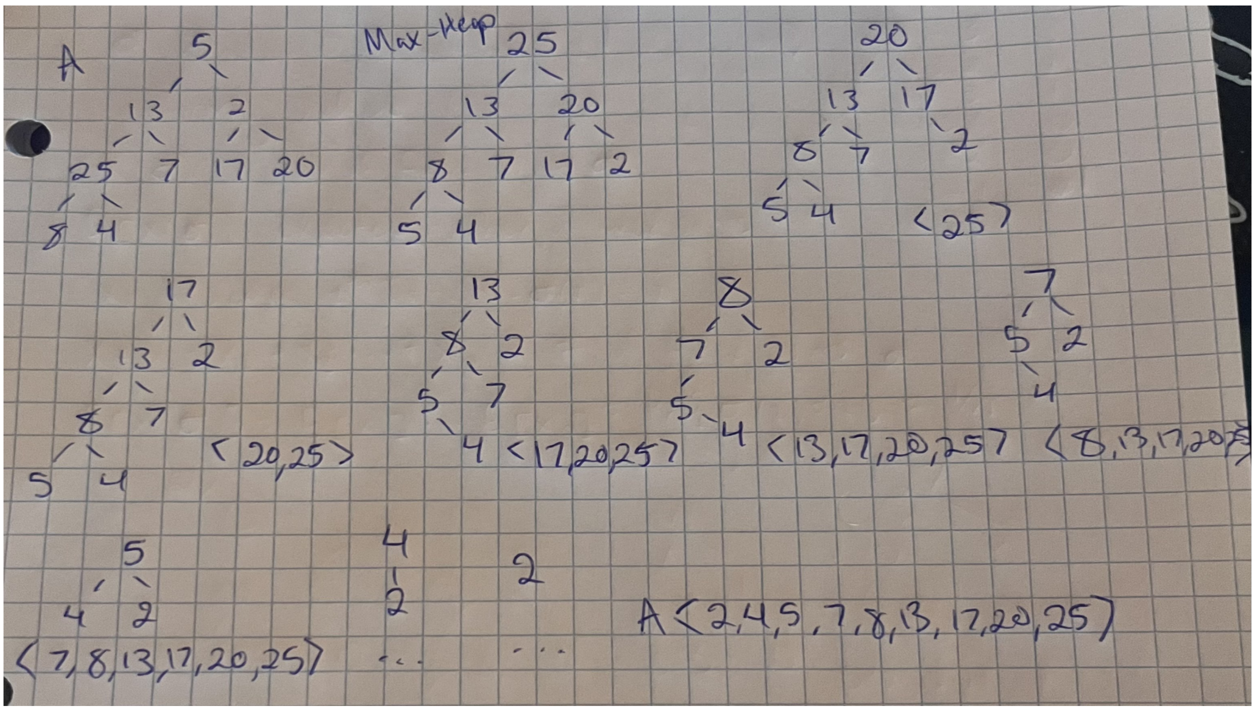


6.3-1.)



6.3-2.) A heap with $h = 0$ must have $n = 1$, so $\lceil \frac{n}{2^{h+1}} \rceil \geq \frac{1}{2}$ and the base case holds. For some height $h = k$ such that $0 \leq k \leq \lfloor \lg n \rfloor$, and $\lceil \frac{n}{2^{k+1}} \rceil \geq \frac{1}{2}$, when $h = k+1$ there must be at least 2^{k+1} nodes $n \geq 2^{k+1} \Rightarrow \frac{n}{2^{k+2}} \geq \frac{1}{2} \Rightarrow \lceil \frac{n}{2^{k+2}} \rceil \geq \frac{1}{2}$ demonstrating the statement holds for $h = k+1$, proving the statement by induction.

6.4-1.)



6.4-4.)
6.5-2.)