Noel Ball
HW8

20.1-1.) Computing the out-degrees and in-degrees both require iterating through all vertices and adjacency lists requiring $\Theta(V + E)$ time.

20.2-1.)

BFS	3	5	6	4	2	1
d	0	1	1	2	3	∞
π	NIL	3	5	5	4	NIL

20.2-2.)

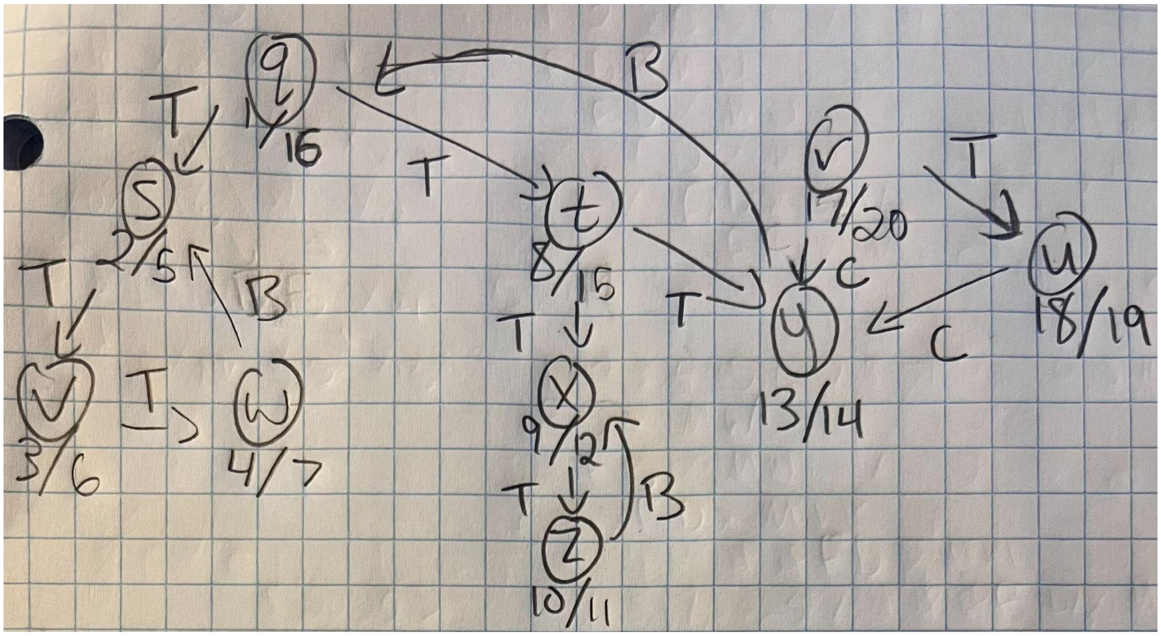
BFS	u	s	t	y	r	v	x	w	z
d	0	1	1	1	2	2	2	3	3
π	NIL	u	u	u	s	s	y	r	x

20.3-1.)

DIRECTED	white	gray	black
white	None	None	None
gray	Tree	Back, Forward, Cross	Forward, Cross
black	None	Back, Cross	Cross

UNDIRECTED	white	gray	black
white	None	Tree	None
gray	Tree	Back, Forward, Cross	Back, Forward, Cross
black	None	Back, Forward, Cross	Cross

20.3-2.)



20.3-3.) (u (v (y (x x) y) v) u) (w (z z) w)

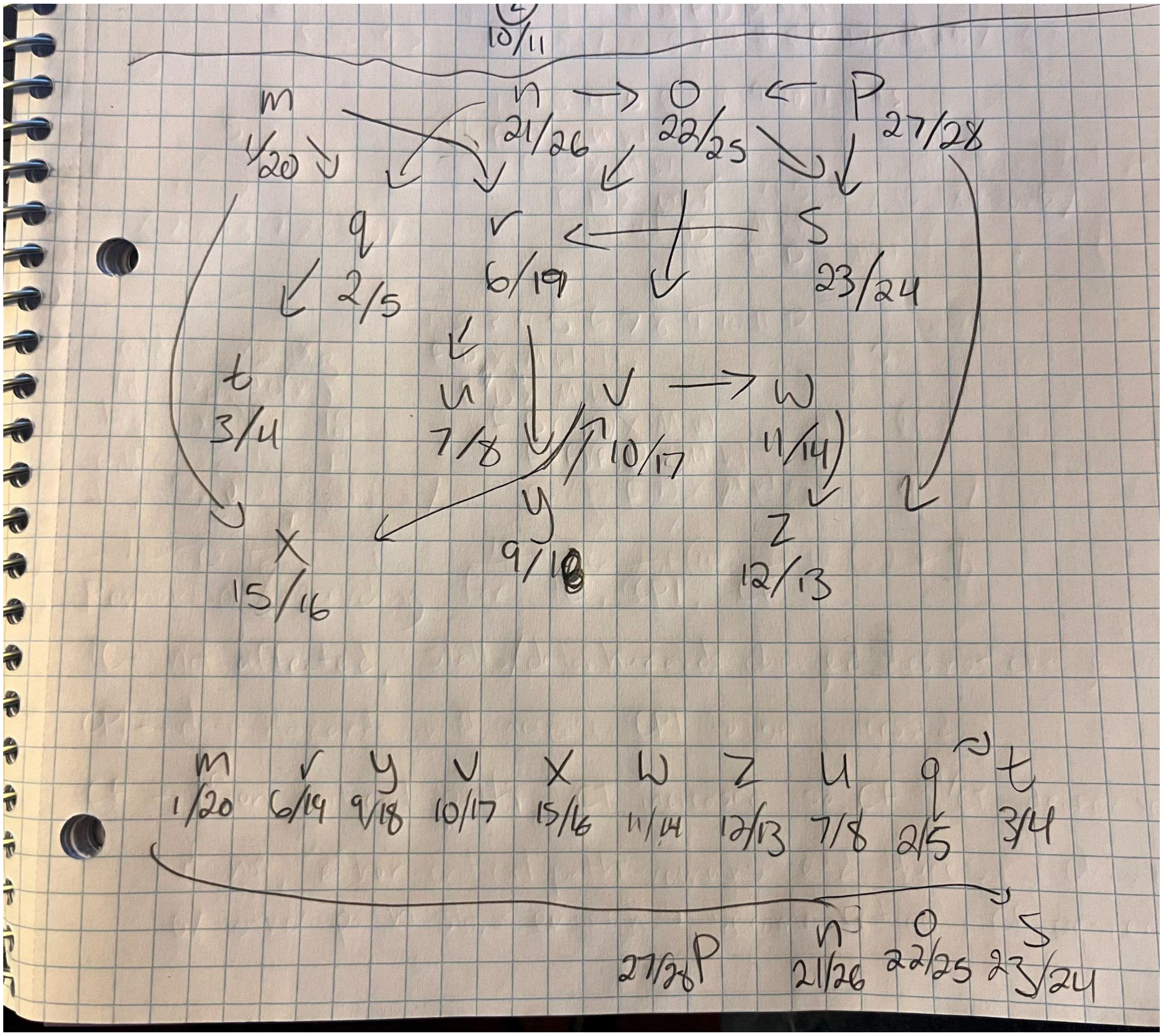
20.3-5a.) Since u.d < v.d < v.f < u.f, v is both discovered and finished after u is discovered and before u is finished, so v must be a descendant of u. In order to discover a new node and make a tree edge or to connect a proper descendant and make a forward edge, v must be a descendant of u, therefore tree edges and forward edges are formed if and only if u.d < v.d < v.f < u.f.

20.3-5b.) Since v.d ≤ u.d < u.f ≤ v.f, u is both discovered and finished at or after v is discovered and at or before v is finished, so u must be a descendant of v or u = v. In order to connect a vertex to its ancestor or itself and create a back edge, u must be a descendant or equal to v, therefore back edges are formed if and only if v.d ≤ u.d < u.f ≤ v.f.

20.3-5c.) Since v.d < v.f < u.d < u.f, v is both discovered and finished after u is discovered and finished, so v and u must be distinct from each other. In order to connect non-ancestors in the same tree, or two nodes in different trees, u and v must be distinct from each other, therefore cross edges are formed if and only if v.d < v.f < u.d < u.f.

20.3-8.) There is no counter-example, in a directed graph G with an edge (u, v), there is no way for u to finish before v is discovered, because in order for the node u to be finished all of the paths from u must be discovered.

20.4-1.)



20.5-1.) Adding an edge that does not create a new cycle or change the existing cycles, the number of SCC remains unchanged. If the new edge connects two nodes creating a cycle, and merging two SCC's into a single larger SCC, then the number of SCC can be reduced. Lastly, adding an edge can create a new SCC, increasing the total amount.