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HW6

9.1-1.) Organizing the *n* elements into a binary tournament tree, it takes $\lceil \lg n \rceil$-1 comparisons to find the smallest element due to the height of the tree, and another *n*-1 comparisons to find the second smallest, for a total of $n + \lceil \lg n \rceil - 2$ comparisons.

9.1-2.) Finding the minimum and maximum elements takes 2*n*-2 comparisons. With *n*-2 elements remaining, it takes *n*-2 comparisons to find the second minimum and maximum each, for a total of 2*n*-2 + 2*n*-4 = 4*n* - 6 comparisons.

9.2-1.) Randomized-Partition makes recursive calls on subarrays that are smaller than the original array, by dividing it in two parts with at least one element in each part, therefore it will never make a recursive call to a 0-length array.

11.1-1.) To find the maximum element of the dynamic set S represented by a direct-address table T of length m, a max_key value can be updated by iterating through the direct-address table and comparing non-empty keys to find the max, with worst case performance of O(m).

11.2-1.) The probability that two distinct keys k_1 and k_2 collide is the probability that both keys hash to the same slot, being $\frac{1}{m}$ given the length of the array T. All possible pairs of distinct keys is $\frac{n(n-1)}{2}$ out of *n* keys. Therefore the expected number of collisions E = $\frac{1}{m} * \frac{n(n-1)}{2} = \frac{n(n-1)}{2m}$.

11.2-2.) Given the hash function h(*k*) = *k* mod 9, a 9 slot table, and chaining on collisions
0:
1: 28 → 19 → 10
2: 20
3: 12
4:
5: 5,
6: 15 → 33
7:
8: 17

11.3-1.) Hash values can be taken advantage of to search a linked list by obtaining the hash value for the *k* being searched for, traversing the linked list sequentially and comparing the other hash values with the target, then comparing the key of the element with *k*.

14.1-1.) To show that $T(n) = 2^n$, we can assume for some $k < n$, $T(k) = 2^k$. The base case $T(0) = 2^0 = 1$ holds, and replacing $T(j)$ with $T(k)$ in the sum $T(n) = 1 + \sum_{j=0}^{n-1} 2^k \rightarrow T(n) = 1+(1+2^1+2^2+...+2^{n-1})$. Rewriting the geometric series, $T(n) = 1 + \frac{2^k-1}{2-1} = 1 + 2^k - 1 = 2^k$. Therefore by induction $T(n) = 2^n$.

14.1-2.)

14.1-6.)