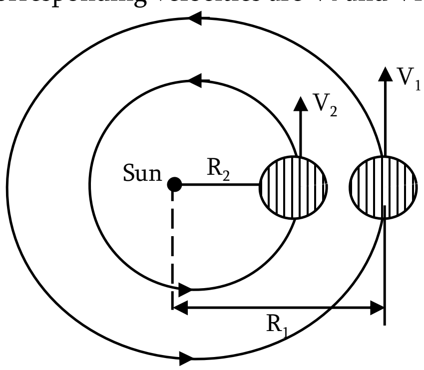
## NUMERICAL EXAMPLES

1. The distance of the two planets from the sun are $10^{13}$m and $10^{12}$m respectively. Find the ratio of the time periods and the speeds of the two planets.

**Solution**

$R_1 = 10^{13}M$, $R_2 = 10^{12}m$

Let $T_1$ and $T_2$ be periodic times of first and second planet. Respectively and their corresponding velocities are $V_1$ and $V_2$




According to the Kepler's third law

$$\frac{T^2}{R^3} = \text{Constant}$$

$$\frac{T_1^2}{R_1^3} = \frac{T_2^2}{R_2^3}$$

$$\frac{T_1}{T_2} = \left[\frac{R_1}{R_2}\right]^{3/2} = \left[\frac{10^{13}}{10^{12}}\right]^{3/2}$$

$$\frac{T_1}{T_2} = 10\sqrt{10}$$

Again $\frac{V_1}{V_2} = ?$

$$V_1 = \frac{2\pi R_1}{T_1}, \quad V_2 = \frac{2\pi R_2}{T_3}$$

$$\frac{V_1}{V_2} = \left(\frac{R_1}{R_2}\right)\left(\frac{T_2}{T_1}\right) = \frac{10^{13}}{10^{12}} \times \frac{1}{10\sqrt{10}}$$

$$\frac{V_1}{V_2} = \frac{1}{\sqrt{10}}$$

2. (a) A Saturn year is 29.5 times the Earth's year. How far is the Saturn from the sun if the Earth is $1.5 \times 10^8$km away from the sun.
   (b) calculate the period of Neptune around the sun. Given that diameter of its orbit is 30times the diameter of the Earth's orbits around the sun, both orbits assume circular.

**Solution**

(a) Given that $T_S = 29.5T_E$

$R_E = 1.5 \times 10^8$Km, $R_S = ?$

According to the Kepler's third law

$T^2 \alpha R^3$

$$\therefore \frac{T_S^2}{T_E^2} = \frac{R_S^3}{R_E^3}$$

$$R_S = R_E\left[\frac{T_S}{T_E}\right]^{2/3} = 1.5\times10^8\left[29.5\right]^{2/3}$$

$$R_S = 14.32\times10^{11}\,\text{Km}$$

(b) Let $T_2$ = Periodic time of Neptune

$T_1$ = Periodic time of Earth

$$\frac{T_2^2}{R_2^3} = \frac{T_1^2}{R_1^3}$$

$$T_2 = T_1\left[\frac{R_2}{R_1}\right]^{2/3} = 1\text{yr}\left[\frac{30R_1}{R_1}\right]^{3/2}$$

$$T_2 = 164.3\text{years}$$

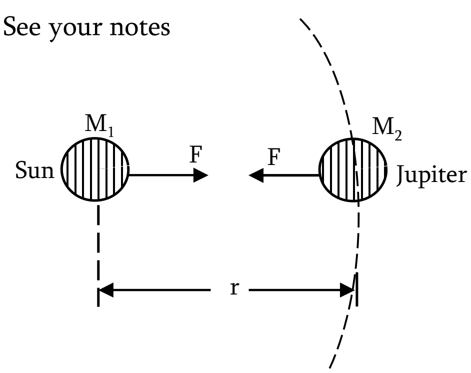
3. The moon has a period of 28days and an orbital radius $3.8 \times 10^8$km. What is orbital radius of a satellite that has a period of one day? Answer $4.1 \times 10^4$km.

4. (a) State the Newton's universal law of gravitation.
   (b) The mass of planet Jupiter is $1.9 \times 10^{27}$kg and that of the sun is $1.99 \times 10^{30}$kg. The mean distance of the sun from the Jupiter is $7.8 \times 10^{11}$m, calculate the gravitational force in which the sun exerts on Jupiter. Take $G = 6.67 \times 10^{-11}Nm^{-2}kg^{-2}$.

**Solution**

(a) See your notes

(b)



Apply Newton's universal law

$$F = \frac{GM_1M_2}{r^2}$$

$$F = \frac{6.67\times10^{-11}\times1.99\times10^{30}\times1.9\times10^{27}}{\left(7.8\times10^{11}\right)^2}$$

$$F = 4.15\times10^{23}\,N$$

5. How fast (in m2/s) is area swept out
   (i) The radius from sun to Earth?
   (ii) The radius from Earth to moon? Sun to Earth distance = 1.496 × $10^{11}$m, Earth distance = 3.845 × $10^8$m and period of revolution of moon $= 271\!\!\diagup_{3}$ days (27 1/3 days).

**Solution**

(i) T = 365 days = 365 × 24v× 60 × 60

$$\frac{dA}{dt} = \frac{\pi R^2}{T} = \frac{\pi\left(1.496\times10^{11}\right)^2}{365\times24\times60\times60}$$

$$\frac{dA}{dt} = 2.23\times10^{15}\,m^2/s$$

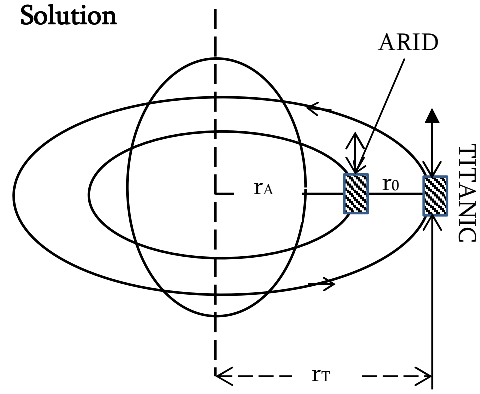
(ii) $T = \frac{82}{3}\times24\times60\times60$

$$\frac{dA}{dt} = \frac{\pi\left(3.845\times10^{8}\right)^2}{\frac{82}{3}\times24\times60\times60}$$

$$\frac{dA}{dt} = 1.97\times10^{11}\,m^2/s$$

6. The satellite ARID describe a very nearly circular orbit of radius 9.0 × $10^8$m round the planet Uranus with period of 2.16 × $10^5$sec, calculate.
   (i) The mass of Uranus
   (ii) The radius of the orbits TITANIC (another satellite of Uranus) if its period of revolution is 7.49 × $10^5$sec.
   (iii) The distance of closest approach between the two satellite during their motion
   G = 6.67 × 10-11Nm-2kg-2

**Solution**




$r_A$ = orbital radius of ARID satellite
$r_T$ = orbital radius of TITANIC satellite

Given that;

$r_A = 1.9\times10^8\,m$ , $T_A = 2.16\times10^5\,Sec$

$T_T = 7.49\times10^5\,Sec$

$G = 6.67\times10^{-11}\,Nm^{-2}kg^{-2}$

Let M = Mass of Uranus

$M_1$ = Mass of ARID satellite

Consider the motion of ARID around the Uranus.

Gravitation force = Centripetal force

$$\frac{GMM_1}{r_A^2} = M_1\omega^2 r_A$$

$$M = \frac{\omega^2 r_A^3}{G}$$

$$M = \left(\frac{2\pi}{T_A}\right)^2\cdot\frac{r_A^3}{G}$$

$$= \left(\frac{2\pi}{2.16\times10^5}\right)^2\cdot\left(\frac{1.9\times10^8}{6.67\times10^{-11}}\right)^2$$

$$M = 8.69\times10^{25}\,Kg$$

(ii) $r_T$ = ?

Apply Kepler's third law

$$\frac{T^2}{T^3} = \text{a constant}$$

$$\frac{T_A^2}{r_A^3} = \frac{T_T^2}{r_T^3}$$

$$r_T = r_A\left[\frac{T_T}{T_A}\right]^{2/3}$$

$$= 1.9\times10^8\left[\frac{7.49\times10^5}{2.16\times10^5}\right]^{2/3}$$

$$r_T = 4.35\times10^8\,m$$

(iii) $r_O$ = closest distance of approach between TITANIC and ARID satellite.

$$r_0 = r_T - r_A$$

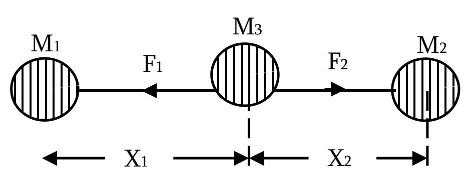
$$= (4.35-1.9)\times10^8$$

$$r_0 = 2.45\times10^8\,m$$

7. Two particles of mass 0.20kg and 0.30kg are placed 0.15m apart. A third particle of mass 0.050kg is placed between them on the line joining the first two particles calculate.
   (a) The gravitational force acting on the third particle if it is placed 0.05m from 0.30kg and
   (b) Where along the line it should be placed for no gravitational force be exerted on it.

**Solution**

(a) $M_1$ = Mass of the 1st particle
$M_2$ = Mass of the 2nd particle
$M_3$ = Mass of the 3rd particle




Apply the Newton's universal law of gravitation

$$F_1 = \frac{GM_1M_3}{X_1^2}\ ,\ F_2 = \frac{GM_2M_3}{X_2^2}$$

Let F = Resultant gravitational force on M3

$$F = F_2 - F_1$$

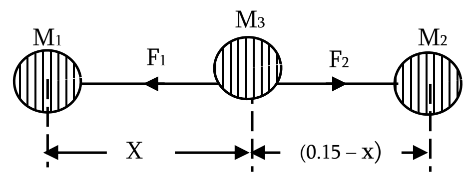
$$= \frac{GM_2M_3}{X_2^2} - \frac{GM_1M_3}{X_1^2}$$

$$= GM_3\left[\frac{M_2}{X_2^2} - \frac{M_1}{X_1^2}\right]$$

$$= 6.67\times10^{-11}\times0.05\left[\frac{0.3}{(0.05)^2} - \frac{0.2}{(0.1)^2}\right]$$

$F = 33.5\times10^{-11}\,N$ towards to 0.30kg

(b) Let X be the position of mass $M_3$ from $M_1$ where no gravitational force




For no resultant gravitational force on $M_3$

$$F_1 = F_2$$

$$\frac{GM_1M_3}{X^2} = \frac{GM_2M_3}{(0.15-X)^2}$$

$$\frac{M_1}{X^2} = \frac{M_2}{(0.15-X)^2}$$

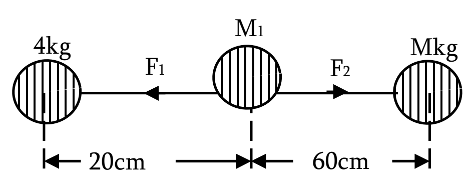
$$\frac{0.2}{X^2} = \frac{0.3}{(0.15-X)^2}$$

On solving X = 0.067m

∴ The particle of mass $M_3$ must be placed at a distance of 0.067m from $M_1$.

8. Two small spheres of mass 4.0kg and M kg are placed 80cm apart. If the gravitational force is zero at a point 20cm from 4kg mass along the line between the two masses, calculate the value of M.

**Solution**




For no resultant gravitational force

$$F_1 = F_2$$

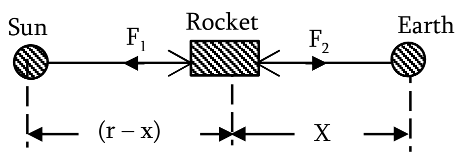
$$\frac{GM_1 \times 4}{(20)^2} = \frac{GM_1 M}{(60)^2}$$

$$\frac{4}{(20)^2} = \frac{M}{(60)^2}$$

$$M = 36\text{kg}$$

9. A rocket is fired from the Earth towards the sun. at what point on its path is the gravitational force on the rocket is zero? Mass of sun Ms = $2 \times 10^{30}$kg ; mass of Earth = $M_E$ = $6.0 \times 10^{24}$kg. Neglecting the effect of other planets orbital radius of Earth r = $1.5 \times 10^{11}$m.

**Solution**




For no gravitational force on the rocket

$$F_1 = F_2$$

$$\frac{GM_E M}{X^2} = \frac{GM_S M}{(r-x)^2}$$

$$\left[\frac{r-x}{x}\right]^2 = \frac{M_S}{M_E}$$

$$\frac{r-x}{x} = \sqrt{\frac{M_S}{M_E}}$$

$$r = x\left[1+\sqrt{\frac{M_S}{M_E}}\right]$$

$$7.8\times10^{11} = x\left[1+\sqrt{\frac{2\times10^{30}}{6\times10^{24}}}\right]$$

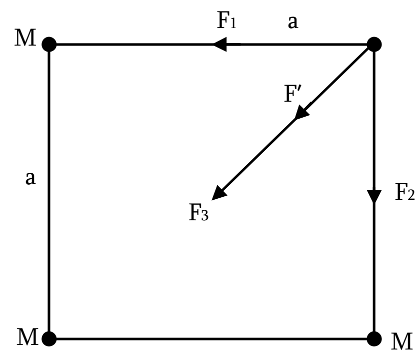
$$x = 2.59\times10^{8}\,\text{m}$$

10. (a) What is meant by gravitational force?
(b) Three masses, each equal to M are placed at the three corners of a square of side 'a'. calculate the force of attraction on a unit mass at the fourth corner.

**Solution**

(a) Gravitation force – is force of attraction between any two material bodies in the universe.

(b)




$$F_1 = F_2 = \frac{GM}{a^2}$$

Let $F'$ = Resultant force between $F_1$ and $F_2$

$$F' = \sqrt{F_1^2 + F_2^2} \quad \text{(Pythagoras theorem)}$$

$$= \sqrt{2}\,F_1 = \sqrt{2}\,\frac{GM}{a^2}$$

Again $r^2 = a^2 + a^2 = 2a^2$ (Pythagoras theorem).

Since $F_3$ and $F'$ acts in the same direction.

The resultant force

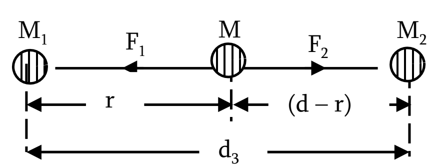
$$F = F_3 + F'$$

$$= \frac{\sqrt{2}GM}{a^2} + \frac{GM}{2a^2}$$

$$F = \frac{GM}{a^2}\left[\sqrt{2}+\frac{1}{2}\right]$$

11. Two stationary particles of masses $M_1$ and $M_2$ are a distance d apart. A third particle, lying on the line joining the particles, experiences no resultant gravitational force. What is the distance of this particle from $M_1$?

**Solution**



According to the Newton's universal law of gravitation.

$$F_1 = \frac{GMM_1}{r^2}$$

$$F_2 = \frac{GMM_2}{(d-r)^2}$$

For no resultant gravitational force

$$F_1 = F_2$$

$$\frac{GMM_1}{r^2} = \frac{GMM_2}{(d-r)^2}$$

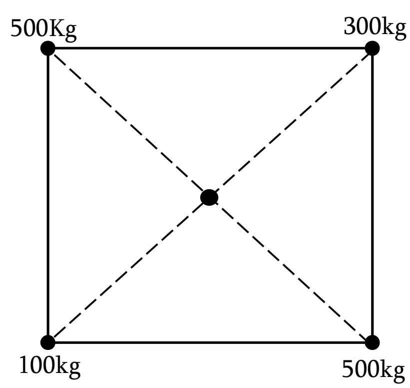
$$\left(\frac{d-r}{r}\right)^2 = \frac{M_2}{M_1} \Rightarrow \frac{d}{r} - 1 = \frac{\sqrt{M_2}}{\sqrt{M_1}}$$

$$\frac{d}{r} = \frac{\sqrt{M_2}+\sqrt{M_1}}{\sqrt{M_1}}$$

$$r = d\left[\frac{\sqrt{M_1}}{\sqrt{M_1}+\sqrt{M_2}}\right]$$

**EXERCISE NO 1**

12. Four sphere forms the corner of a square whose side is 2.0cm long. What are the magnitude and direction of the net gravitational force from them on a central sphere with mass =, Ms = 250kg?




Answer : 0.017N towards the 300kg sphere.

13. Consider two solid uniform spherical objects of the same density ρ. One has a radius R and other a radius 2R. They are in outer space where the gravitational fields from other objects are negligible. If they are at rest with their surfaces touching, what is the contact force between the objects due to their gravitational attraction?

Answer : $\frac{128}{81}G\pi^2R^4\rho^2$

14. (a) What are gravitation and gravity?
    (b) The mass of the Earth is $6 \times 10^{24}$kg and that of moon $7.4 \times 10^{22}$kg. If the distance between their centre is $3.8 \times 10^8$m. Calculate at what point on the line joining their centres there is no gravitational force. Neglecting the effect of the planets and the sun. Answer $3.4 \times 10^8$m from the Earth.

15. The gravitational force on a mass of 1kg at the Earth's surface is 10N. Assuming the Earth is a sphere of radius R, Calculate the gravitational force on a satellite of mass 100kg in a circular orbit of radius 2R from the centre of the Earth. ($g = 10Nkg^{-1}$). Answer 250N.

**NUMERICAL EXAMPLES**

16. In an experiment using covendish balance, the smaller sphere have a mass of $5.0 \times 10^{-3}$kg each , the larger sphere have a mass of 12kg each, the length of the bar is 100cm, the torsional constant of the fibre is $3.56 \times 10^{-8}$Nm per radian, the angle of twist is $4.86 \times 10^{-3}$ radian and the distance between the centre of each pair of heavy and light sphere is 15cm. compute the value of gravitational constant G from this data.

**Solution**

$$C\theta = \frac{GM_ML}{r^2}$$

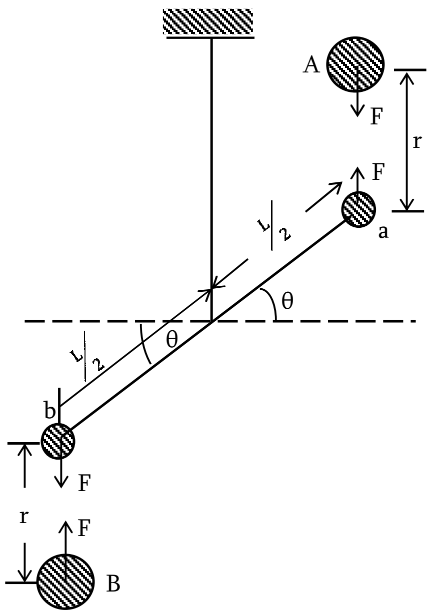
$$G = \frac{C\theta r^2}{MmL}$$

$$G = \frac{(3.56\times10^{-8})(4.86\times10^{-3})\times(0.15)^2}{12\times5\times10^{-3}\times1}$$

$$G = 6.488\times10^{-11}\,Nm^2kg^{-2}$$

17. NECTA 2007/P2/1(b)
Two small sphere each of mass 10gm are attached to a light rod 50cm long. The system is set into oscillation and the period of torsion oscillation is found to be 7700seconds. To produce maximum Torsion to the system two large sphere each of mass 10kg are placed near each suspended sphere; if the angular is $3.96 \times 10^{-3}$rad and the distance between the centre of the large spheres and small sphere is 10cm. determine the value of the universal gravtitaional constant, G from the given information. (06 marks)

**Solution**




According to the Newton's universal law of gravitation.

$$F = \frac{GMm}{r^2}$$

Gravitation torque, $\tau_g = \frac{GMmL}{r^2}$ .....(i)

Restoring torque $\tau = c\theta$ ..........(ii)

(i) = (ii)

$$c\theta = \frac{GMmL}{r^2}$$

$$G = \frac{c\theta r^2}{MmL}$$

Periodic time of oscillation of the Torsional pendulum.

$$T = 2\pi\sqrt{\frac{I}{C}}$$

$$C = \frac{4\pi^2 I}{T^2}$$

Since $I = m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{2}$

$$C = \frac{2\pi^2 mL^2}{T^2}$$

Now

$$G = \left(\frac{2\pi^2 mL^2}{T^2}\right)\left(\frac{\theta r^2}{MmL}\right)$$

$$G = \frac{2\pi^2\theta r^2 L}{T^2 M}$$

$$G = \frac{2\times(3.14)^2\times3.96\times10^{-3}\times(0.5)^2\times0.1}{(7700)^2\times10}$$

$$G = 6.67\times10^{-11}\,Nm^2kg^{-2}$$

18. A sphere of mass 40kg is attracted by another sphere of mass 15kg with a force of $1/10$ mgwt. Find the value of constant of gravitation if centes of sphere are 0.2m apart.

**Solution**

$M_1 = 40kg$ , $M_2$ 15kg

$$F = \frac{1}{10}mgwt = \frac{1}{10}\times10^{-3}gwt$$

$$= \frac{1}{10}\times10^{-6}kgwt$$

(1kgwt = 9.8 Newton's)

$$F = \frac{GM_1M_2}{r^2}$$

$$G = \frac{Fr^2}{M_1M_2} = \frac{9.8\times10^{-7}\times(0.2)^2}{40\times15}$$

$$G = 6.533\times10^{-11}\,Nm^2kg^{-2}$$

19. (a) G is called the universal gravitational constant. Why?

(b) Two bodies on the surface of the Earth do not move towards each other due to force of attraction given by Newton's law of gravitation. Why?

(c) Why Earth does not fall towards sun due to its attraction?

**Solution**

(a) G has the same value for microscopic as well as macroscopic bodies found anywhere in the universe it does not depend on the nature of medium between bodies and also on the nature of the bodies.

(b) The masses involved are small so the force of attraction is very small to cause any acceleration. The force of attraction between 1kg masses separated by 1m is only $6.67 \times 10^{-11}$N.

(c) The force of attraction provides the centripetal force and Earth moves in a stable orbit with a velocity perpendicular to the force so it does not fall.

20. A mass M is broken into two parts m and (M − m). How m and M related so that gravitational force between the two parts is maximum?

**Solution**

According to the Newton's universal law of gravitation.

$$F = \frac{Gm(M-m)}{r^2}$$

$$F = \frac{GMm}{r^2} - \frac{Gm^2}{r^2}$$

Differentiate F w.r.t m

$$\frac{dF}{dm} = \frac{d}{dm}\left[\frac{GMm}{r^2} - \frac{Gm^2}{r^2}\right]$$

$$\frac{dF}{dm} = \frac{GM}{r^2} - \frac{2Gm}{r^2}$$

When F = Fmax; $\frac{dF}{dm} = 0$

$$0 = \frac{GM}{r^2} - 2\frac{Gm}{r^2}$$

$$\frac{2Gm}{r^2} = \frac{GM}{r^2}$$

$$2M = m \ , \ m = \frac{M}{2}$$

## NUMERICAL EXAMPLES

21. The acceleration due to gravity at the moon's surface is $1.67 m/s^2$. If the radius of the moon is $1.74 \times 10^6$m, calculate the mass of the moon, $G = 6.67 \times 10^{-11} Nm2kg^{-2}$

**Solution**

Since $GM = Gr^2$

$$M = \frac{gR^2}{G} = \frac{1.67\times(1.74\times10^6)}{6.67\times10^{-11}}$$

$$M = 7.58\times10^{22}Kg$$

22. If the radius of the Earth were increased by a factor of 3, by what factor would its density have to be changed to keep 'g' the same?

**Solution**

$$g = \frac{GM}{R^2}$$

Let: $\rho$ = density of the Earth

$$\rho = \frac{M}{4/3\,\pi R^3} \quad \text{or} \quad M = \frac{4}{3}\pi R^3\rho$$

Now: $g = \frac{G}{R^2} \times \frac{4}{3}\pi R^3 \rho$

$$g = \frac{4}{3}\pi G\rho R$$

$4/3$ , $\pi$ , G are constant

For no change in value of g

$$R \ \alpha \ \frac{1}{\rho}$$

$\therefore$ If R is made 3R, $\rho$ must become $\rho/3$

23. (a) What will be the acceleration due to gravity on the surface of moon if its radius is ¼ th the radius of the Earth and its mass $1/80$ th t of the mass of the Earth?

(b) A man can jump 1.5m on the Earth. calculate the maximum approximate height he might be able to jump on a planet whose density is one quarter that of the Earth and whose radius is one – third that of the Earth.

**Solution**

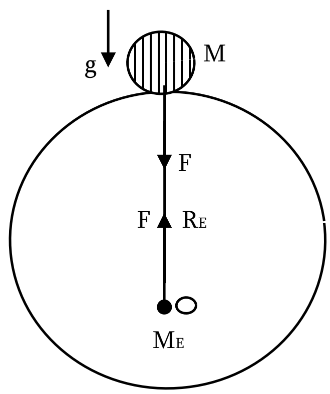
(a) Let $M_E$ = Mass of the Earth

$R_E$ = Earth radius

Mm = Mass of the moon

$R_m$ = Moon radius

Consider the body of mass M placed on the Earth – surface.




Gravitation force = weight of the body

$$\frac{GM_EM}{R_E^2} = Mg$$

$$g = \frac{GM_E}{R_E^2} \text{.......(i)}$$

Similarly, when the body lies on the moon surface.

$$g_m = \frac{GM_m}{R_m^2}$$

$$\text{(ii)}/\text{(i)}$$

$$\frac{g}{g_m} = \frac{GM_E}{R_E^2} \Big/ \frac{GM_m}{R_m^2}$$

$$\frac{g_m}{g} = \left(\frac{M_m}{M_E}\right)\left(\frac{R_E}{R_m}\right)^2$$

$$g_m = g\left(\frac{M_m}{M_E}\right)\left(\frac{R_E}{R_m}\right)^2$$

$$R_m = \frac{R_E}{4}, \ R_E = 4R_m$$

$$M_m = M_E/80, \ M_E = 80M_m$$

$$g_m = 9.8\left[\frac{M_m}{80M_m}\right]\left[\frac{4R_m}{R_m}\right]^2$$

$$g_m = 1.96\text{m}/\text{s}^2$$

(b) h = 1.5m , hp = ?

$\rho_P = \frac{1}{4}\rho_E$ , $\rho_E = 4\rho_P$

$R_P = \frac{1}{3}R_E$ , $R_E = 3R_P$

Let $R_P$ and $R_E$ are radii of planet and Earth respectively. Assumption: initial kinetic energy of the man when start to jump on the Earth and given planet are the same.

p.e of man on the Earth = p.e of man on the planet

$$Mg_ph_p = Mg_Eh$$

$$h_p = \left(\frac{g_E}{g_p}\right)h$$

Acceleration due to gravity on the Earth – surface. $g_E = \frac{GM_E}{R_E^2}$

Planet surface: $g_p = \frac{GM_p}{R_p^2}$

$$\frac{g_E}{g_P} = \frac{GM_E}{R_E^2} \Big/ \frac{GM_P}{R_P^2}$$

$$\frac{g_E}{g_P} = \left(\frac{M_E}{M_P}\right)\left(\frac{R_P}{R_E}\right)^2$$

Also $M_E = \frac{4}{3}\pi R_E^3\rho_E$

$$M_p = \frac{4}{3}\pi R_p^3 \rho_p$$

$$\frac{M_E}{M_p} = \frac{\frac{4}{3}\pi R_E^3 \rho_E}{\frac{4}{3}\pi R_p^3 \rho_p} = \left(\frac{\rho_E}{\rho_p}\right)\left(\frac{R_E}{R_p}\right)^3$$

$$\text{Now } \frac{g_E}{g_p} = \left(\frac{M_E}{M_p}\right)\left(\frac{R_p}{R_E}\right)^2$$

$$= \left(\frac{\rho_E}{\rho_p}\right)\left(\frac{R_E}{R_p}\right)^3 \cdot \left(\frac{R_p}{R_E}\right)^2$$

$$h_p = \left(\frac{\rho_E}{\rho_p}\right)\left(\frac{R_E}{R_p}\right)h$$

$$= \left(\frac{4\rho_p}{\rho_p}\right)\left(\frac{3R_p}{R_p}\right)\times 1.5\text{m}$$

$$h_p = 1.5\times 12 = 18\text{m}$$

$$h_p = 18\text{m}$$

24. If the radius of the Earth shrinks by 1.5% (mass remaining the same) then how would the value of acceleration due to gravity changes?

**Solution**

$$\text{Since} \quad g = \frac{GM}{R^2} = GMR^{-2}$$

$$\log_e^g = \log_e\left[GMR^{-2}\right]$$

$$= \log G + \log_e^M \mp 2\log_e^R$$

(On differentiate both side)

$$\frac{dg}{g} = -2\frac{dR}{R}$$

Percentage error

$$\frac{dg}{g}\times 100\% = -2\left[\frac{dR}{R}\times 100\%\right]$$

$$\frac{dg}{g}\times 100\% = 3\%$$

25. (a) What is the acceleration due to gravity on the surface of a planet that has a radius half that of Earth and the same average density as the Earth?

(b) The mass of the planet Jupiter is $1.9 \times 10^{27}$kg and that of the sun is $1.99 \times 10^{30}$kg the mean distance of the Jupiter from the sun is $7.8 \times 10^{11}$m. Calculate the gravitational force which the sun exert on the Jupiter. Assume that the Jupiter moves in circular orbit around the sun. Calculate the speed of Jupiter.

**Answer**

(a) $4.9\text{m/s}^2$

(b) $4.14\times 10^{23}$N, $1.304\times 10^4$m/s

## NUMERICAL EXAMPLES

26. Assuming the Earth to be a sphere of uniform mass density how much would a body weight half way down to the centre of the Earth. If its weighted 250N on the surface?

**Solution**

$$Mg = 250\text{N}, \; h = \frac{R}{2}$$

Acceleration due to gravity below the Earth – surface

$$g' = g\left(1-\frac{h}{R}\right)$$

$$Mg' = Mg\left(1-\frac{h}{R}\right)$$

$$= 250\left(1-\frac{1}{2}\right)$$

$$Mg' = 125\text{N}$$

27. (a) Calculate the height above the Earth's surface at which the value of acceleration due to gravity reduces to half its value of the Earth's surface. Assume the Earth to be sphere of radius 6400km.

(b) If the radius of the Earth were increased by a factor of 3, by what factor would its density have to be changed to keep 'g' the same?

**Solution**

(a) $$g' = g\left[\frac{R}{R+h}\right]^2, \quad g' = \frac{g}{2}$$

$$\frac{1}{2} = \left(\frac{R}{R+h}\right)^2 \text{ or } \frac{R}{R+h} = \frac{1}{\sqrt{2}}$$

$$\frac{R+h}{R} = \sqrt{2}$$

$$\frac{h}{2} = \sqrt{2}-1 = 0.414$$

$$h = 0.414R = 0.414\times6400$$

$$h = 2649.6Km$$

(b) $g = \frac{G}{R^2}\cdot\frac{4}{3}\pi R^3\rho = \frac{4}{3}\pi G\rho R$

$$g = \frac{G}{R^2}\cdot\frac{4}{3}\pi R^3\rho = \frac{4}{3}\pi G\rho R$$

$4/3$ , $\pi$ , G are constant

For no change in value of g

$$R \propto \frac{1}{\rho}$$

$\therefore$ If R is made 3R, $\rho$ must become $\rho/3$

28. (a) Calculate the imaginary angular velocity of the Earth for which the effective acceleration due to gravity at the equator becomes zero. In this condition what will be the length (in hours) of the day?

(b) How far above the Earth – surface does the value of g becomes 16% value on the surface ($g = 10m/s^2$).

**Solution**

(a) Since $g_e = g-\omega^2 R$ but $g_e = 0$

$$0 = g-\omega^2 R$$

$$\omega^2 R = g \ , \ \omega = \sqrt{\frac{g}{R}}$$

$$\omega = \sqrt{\frac{10}{6.4\times10^6}} = 1.25\times10^{-3}$$

$$\omega = 1.25\times10^{-3}\,rad/s$$

Length of the day

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{1.25\times10^{-3}}$$

$$T = 5024\,sec = 1.4hr$$

(b) $\frac{g'}{g}\times100\% = 16\%$ , $\frac{g'}{g} = 0.16$

Since $g' = g\left(\frac{R}{R+h}\right)^2$

$$\frac{g'}{g} = \left(\frac{R}{R+h}\right)^2 = \frac{16}{100}$$

$$h = \frac{3}{2}R = \frac{3}{2}\times6400km$$

$$h = 9600km$$

29. (a) Calculate the effect of rotation of Earth on the weight of the body at a place at latitude 45°. Take radius of Earth = $6.37\times10^6$m.

(b) Find the percentage decrease in the weight of a body when taken to a depth of 32km below the surface of Earth. Radius of Earth is 6400km.

**Solution**

(a) $g' = g-R\omega^2\cos^2\theta$

$$\omega = \frac{2\pi}{24\times3600} = 7.2\times10^{-5}\,rads^{-1}$$

$$g-g' = R\omega^2\cos^2\theta$$

$$= 6.37\times10^6\times\left(7.27\times10^{-5}\right)^2\left(\cos45^\circ\right)^2$$

$$g-g' = 0.0168m/s^2$$

It shows that the value of acceleration due to gravity at the pole is greater than at latitude 45° by $0.0168m/s^2$. Hence the weight of a body of 1kg mass at the poles will be greater than at 45° latitude by 0.0168N.

(b) Since $g' = g\left(1-\frac{d}{R}\right)$

$$Mg' = Mg\left(1-\frac{d}{R}\right)$$

$$\frac{Mg'-Mg}{Mg} = \frac{d}{R}$$

$$\left(\frac{Mg'-Mg}{Mg}\right)\times100\% = \frac{d}{R}\times100\%$$

$$= \frac{32}{6400}\times100\%$$

% age decreases in weight = 0.5%

30. (a) How much faster than its present rate should the Earth rotate about its axis so that the weight of a body at equator becomes zero?
(b) Also calculate the new length of the day?
(c) What should happen if the rotation becomes still faster?

**Solution**

(a) Since $g' = g\left(1-\frac{R\omega^2}{g}\cos^2\theta\right)$

At the equator $\theta = 0^\circ$ , $\cos 0^\circ = 1$

$$g_e = g-\omega^2 R = g\left(1-\frac{\omega^2 R}{g}\right)$$

It can be shown that

$$\frac{R\omega^2}{g} = \frac{1}{288}$$

In order that the weight of a body at the equator to be zero $g_e = 0$. This means that $\frac{R\omega^2}{g} = 1$ so, $R\omega^2$ should be 288times greater. Since R is constant, therefore $\omega^2$ should be 288times more than its present value thus $\omega$ should be $\sqrt{288}$ times i.e 16.97 times greater. So the Earth should rotate 17times faster than its present value so that the weight of a body at the equator becomes zero.

**Alternative**

Since $g' = g\left(1-\frac{R\omega^2}{g}\cos^2\theta\right)$

At the equator , $\theta = 0^\circ$ , $\cos 0^\circ = 1$

$$g_e = g-R\omega^2$$

Suppose when $\omega$ is changed to $\omega'$, ge (and hence $mg_e$) becomes zero.

$$0 = g-R\omega'^2 \text{ OR } \omega' = \sqrt{\frac{g}{R}}$$

$$\frac{\omega'}{\omega} = \frac{1}{\omega}\sqrt{\frac{g}{R}} = \frac{1}{\omega}\sqrt{\frac{981}{6.4\times 10^8}}$$

(b) Since the Earth then rotates 17times faster, therefore, the Earth makes 17rotations in 24hours and hence the length of the day would be $\frac{24}{17}\text{hr} = 1.412\text{hours}$.

(c) If the rotation becomes still faster i.e faster than 17times its present value, the increased centripetal acceleration on the bodies will be greater than acceleration due to gravity. Therefore, there will now be a resultant force actin on them outwards and all objects kept loose on the equator will start leaving the Earth's surface.

## NUMERICAL EXAMPLES

31. (a) (i) What is meant by Gravitational field?
(i) Outline five (5) properties of gravitation field lines.
(b) With what velocity must a body be thrown upward from the surface of the Earth so that it reaches height 10R? Radius of the Earth = $6.4 \times 10^6$m, mass of the Earth $M = 6 \times 10^{24}$kg, $G = 6.67 \times 10^{-11}\text{Nm}^2\text{Kg}^{-2}$.

**Solution**

(a) (i) Refer to your notes
(ii)
- They are parallel
- They are perpendicular to the surface when they enter or leave.
- They are closer together near the Earth and far distant points.
- They are field strength determinant.
- They determine the direction of force of attraction

(b) From the equation of

$$\Delta P.e = GMm\left[\frac{1}{R}-\frac{1}{R+h}\right]$$

But h = 10R

$$= GMm\left[\frac{1}{R}-\frac{1}{R+10R}\right]$$

$$\Delta P.e = \frac{GMm\left[11R-R\right]}{11R^2} = \frac{10GMmR}{11R^2}$$

$$\Delta P.e = \frac{10GMm}{11R} \quad \text{.........(i)}$$

Also change in K.E

$$\Delta K.e = \frac{1}{2}M\left(V^2 - U^2\right)$$

$$\Delta K.e = \frac{1}{2}M\left[V^2 - 0^2\right] = \frac{1}{2}MV^2$$

Apply the law of conservation of energy

$$\Delta k.e = \Delta p.e$$

$$\frac{1}{2}MV^2 = \frac{10GMm}{11R}$$

$$V = \sqrt{\frac{20GM}{11R}}$$

$$V = \sqrt{\frac{20\times 6.67\times 10^{-11}\times 6\times 10^{24}}{11\times 6.4\times 10^{6}}}$$

$$V = 1.0663\times 10^{4}\,m/s$$

32. Calculate the gravitational field intensity on the surface of Mars assuming it to be uniform sphere. Given that the mass of Mars is $6.420\times 10^{23}$kg and its radius is $3.375\times 10^{6}$m. $G = 6.67\times 10^{-11}Nm^2kg^{-2}$.

**Solution**

Gravitational field strength

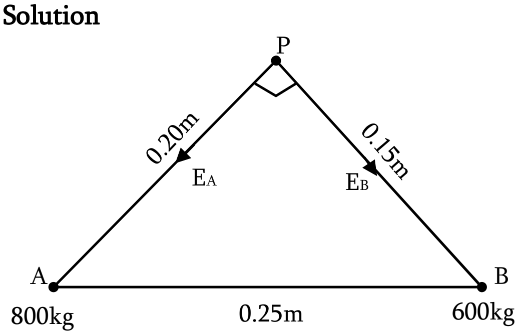
$$g'_m = \frac{GM}{r^2}$$

$$= \frac{6.67\times 10^{-11}\times 6.420\times 10^{23}}{\left(3.375\times 10^{6}\right)^2}$$

$$g'_m = 3.76Nkg^{-1}$$

33. Two masses 800kg and 600kg are at a distance of 0.25m apart. Compute the magnitude of intensity of gravitational field at a point distant 0.2m from 800kg mass and 0.15m from the 600kg mass.

**Solution**




Gravitational field intensity at P due to the mass at A.

$$E_A = \frac{GM_1}{r_1^2} = \frac{G\times 800}{(0.2)^2}$$

$$E_A = 20{,}000\,G \text{ along PA}$$

Also

$$E_B = \frac{GM_2}{r_2^2} = \frac{G\times 600}{(0.15)^2}$$

$$E_B = \frac{80{,}000G}{3} \text{ along PB}$$

In $\Delta APB$, $\overline{PA}^2 + \overline{PB}^2 = \overline{AB}^2$

$\therefore \angle APB = 90^{\circ}$

Magnitude of resultant gravitational field at P.

$$E = \sqrt{E_A^2 + E_B^2}$$

$$E = G\sqrt{(20000)^2 + \left(\frac{80{,}000}{3}\right)^2}$$

$$= 6.67\times 10^{-11}\times \frac{10{,}000}{3}$$

$$E = 6.67\times 10^{-11}\times \frac{10{,}000}{3}$$

$$E = 2.22\times 10^{-6}NKg^{-1}$$

34. Calculate the gravitational potential due a body of a mass 10kg at a distance of
(i) 10m and
(ii) 20m from the body. Given that $G = 6.67\times 10^{-11}Nm^2Kg^{-2}$.

**Solution**

Gravitational potential is given by $V = \frac{-GM}{r}$

(i) $V = ?$, $r = 10m$

$$V = \frac{-6.67\times 10^{-11}\times 10}{10}$$

$$V = -6.67\times 10JKg^{-1}$$

(ii) $$V = \frac{-6.67\times 10^{-11}\times 10}{20}$$

$$V = -3.33\times 10^{-12}Jkg^{-1}$$

35. (a) Two bodies of masses $M_1$ and $M_2$ are placed distant d apart. Show that the position where the gravitational field due to them is zero, the gravitational potential is given by

$$V = \frac{-G}{d}\left[M_1 + M_2 + 2\sqrt{M_1 M_2}\right]$$

(b) Assuming that the Earth is a uniform sphere of radius $6.4 \times 10^6$m and mass $6.0 \times 10^{24}$kg, find the gravitational field strength at a point.

(i) On the surface

(ii) On height 0.50times its radius above the Earth's surface.

**Solution**

(a) Refer to your notes

(b) (i) $$g_E' = \frac{GM}{R^2}$$

$$= \frac{6.67\times10^{-11}\times6.0\times10^{24}}{\left(6.4\times10^{6}\right)^2}$$

$$g_E' = 9.8\text{Nkg}^{-1}$$

(ii) $R_1 = 1.5R$

$$g' \propto \frac{1}{r^2}, \quad g' = \frac{K}{R^2}$$

$$g_1' = \frac{K}{R_1^2}, \quad g_E = \frac{K}{R^2}$$

$$g_1' = g_E'\left[\frac{R}{R_1}\right]^2 = 9.8\left[\frac{R}{1.5R}\right]^2$$

$$g_1' = 4.36\text{Nkg}^{-1}$$

36. (a) (i) State the Newton's law of gravitational and show that the speed V of a particle in an orbit of radius r round a planet of mass M is given by

$$V = \left[\frac{GM}{r}\right]^{\frac{1}{2}}$$

(ii) Define gravitational field strength and gravitational potential at a point on the Earth's gravitational field. How are they related?

(iii) Given a graph of the variation of gravitational potential with distance away from the Earth; how could the graph of gravitational field strength with distance be derived?

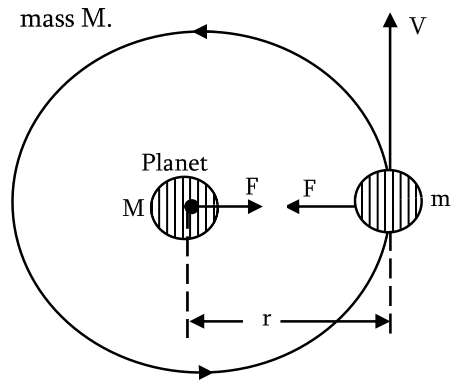
(b) (i) At one point on a line between the Earth and the moon the gravitational field caused by the two bodies is zero. Explain briefly why this so?

(ii) The mass of moon is $1/81$ the mass of Earth and its radius $1/4^{th}$ that of the Earth. if the acceleration due to gravity at the surface of the Earth is $9.8\text{m/s}^2$. What is its value at the surface of the moon.

**Solution**

(a) (i) Refer to your notes

Consider the motion of the particle of mass m revolves around the planet of mass M.




Centripetal force On the particle = Gravitation force

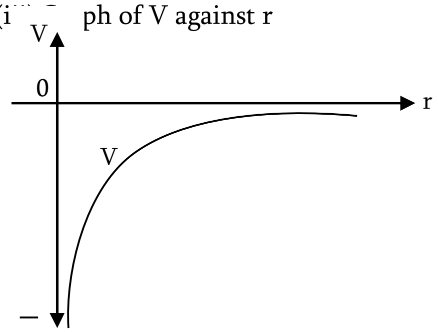
$$\frac{MV^2}{r} = \frac{GMm}{r^2}$$

$$V^2 = \frac{GM}{r}$$

$$V = \left[\frac{GM}{r}\right]^{\frac{1}{2}}$$

(ii) See your notes

(i ph of V against r




The negative of the gradient at any point of the graph of V against r is the gravitational field strength at that distance.

$$V = \frac{-GM}{r}$$

$$\frac{dv}{dr} = \frac{d}{dr}\left[\frac{-GM}{r}\right] = \frac{GM}{r^2}$$

$$E = \frac{-dv}{dr} = \frac{GM}{r^2}$$

Therefore the graph of gravitational field strength can be derived from the graph of potential against distance at any point , negative the value of those slopes and plotting the graph if the obtained slope versus distance. The resulting graph will be gravitational field strength versus distance.

(b) (i) see your notes

(ii) On the Earth – surface

$$g_e = \frac{GM}{R^2}$$

On the moon

$$g_m = \frac{GMm}{R_m^2}$$

$$\frac{g_m}{g_e} = \frac{GMm}{R_m^2} \Big/ \frac{GM}{R^2}$$

$$g_m = g_e\left[\frac{Mm}{M}\right]\left[\frac{R}{R_m}\right]^2$$

But $\frac{Mm}{M} = \frac{1}{81}$ , $M = 81Mm$

$$\frac{R_m}{R} = \frac{1}{4} \text{ , } R = 4R_m$$

$$g_m = 9.8\left[\frac{Mm}{81Mm}\right]\left[\frac{4Rm}{R_m}\right]$$

$$g_m = 1.93m/s^2$$

37. NECTA 2009/P2/4

(a) (i) State Kepler's laws of planetary motion.

(iii) Explain the variation of acceleration due to gravity, g inside and outside of the Earth.

(b) Derive the formulae of mass and density of the Earth.

38. (a) (i) The gravitational potential energy of a body on the Earth surface is $-6.5 \times 10^6$J. What do you mean by this statement.

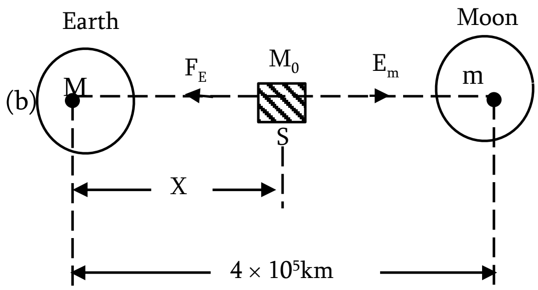
(ii) The gravitational potential energy of a body at the surface of Earth is negative. What does it mean?

(b) The mass of the Earth is 81times that of the moon and the distance from the centre of the Earth to that of moon is about $4.0 \times 10^5$km. calculates the distance from the centre of the Earth where the resultant gravitational force becomes zero when a space craft is launched from the Earth to the moon. Draw a sketch showing roughly how the gravitational force on the space craft varies in its journey.

**Solution**

(a) (i) it means that $6.5 \times 10^6$J of energy is required on the surface of Earth to send the body outside the gravitational field of the Earth.

(ii) It means that the gravitational potential energy of the body at the surface of the Earth is less than at infinity (p.e of the body at infinity is zero). It follows that a body at infinity would fall towards the Earth; a body on the Earth does not fall to infinity.




Gravitational force on the space craft s due to the Earth is in opposite direction to that of the moon.

$$F_E = F_M$$

$$\frac{GMm}{X^2} = \frac{GmM}{\left(4\times10^5 - X\right)^2}$$

$$\frac{M}{m} = \frac{X^2}{\left(4\times10^5 - X\right)^2}$$

But: $$\frac{M}{m} = \frac{81}{1} = 81$$

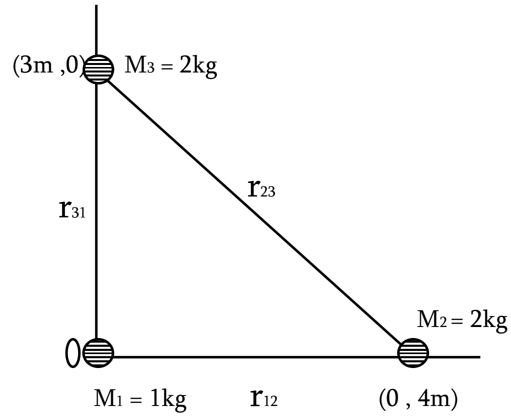
$$81 = \left[\frac{X}{4\times10^5 - X}\right]^2$$

$$9 = \frac{X}{4\times10^5 - X}$$

On solving $X = 3.6 \times 10^5$Km

The resultant F on Mo due to the Earth acts towards the Earth until S is reached. It then acts towards the moon. So F changes in direction after S is passed.

39. Three masses are in the configuration as shown in the figure below. What is the total gravitational potential energy of the configuration?




**Solution**

By using principle of superposition, the total gravitational potential energy of the system is

$$U = \left(\frac{-GM_1M_2}{r_{12}}\right) + \left(\frac{-GM_2M_3}{r_{23}}\right) + \left(\frac{-GM_1M_3}{r_{31}}\right)$$

$$= -G\left[\frac{M_1M_2}{r_{12}} + \frac{M_2M_3}{r_{23}} + \frac{M_1M_3}{r_{31}}\right]$$

$$U = -1.31\times10^{-10}\,J$$

$$= -6.67\times10^{-11}\left[\frac{1\times2}{4} + \frac{2\times2}{5} + \frac{2\times1}{3}\right]$$

$$U = -1.31\times10^{-10}\,J$$

40. At a point above the surface of Earth, the gravitational potential is $-5.12 \times 10^7 Jkg^{-1}$ and the acceleration due to gravity is $6.4 m/s^2$. Assuming the mean radius of Earth to be 6400km; calculate the height of this point from the surface of the Earth?

**Solution**

Gravitational potential at this point

$$V = \frac{-GM}{r} = -5.12\times10^7 \text{ .......(i)}$$

Acceleration due to gravity at this point

$$g = \frac{GM}{r^2} = 6.4 \text{ ................(ii)}$$

(i) / (ii)

$$\frac{5.12\times10^7}{6.4} = \frac{GM}{r} \Big/ \frac{GM}{r^2}$$

$$r = 8\times10^6 m = 8000km$$

Since $R + h = r$

$$h = r - R$$
$$= 8000 - 6400$$
$$h = 1600km$$

**NUMERICAL EXAPLES**

41. NECTA 2013/P1/3(C)
   (i) With the aid of a labeled diagram, sketch the possible orbits for a satellite launched from the Earth (3 marks)
   (ii) From the diagram in (c) (i) above, write down an expression for the velocity of a satellite corresponding to each orbit.

42. (a) The gravitational potential energy of a body on the surface of Earth is -6.5 × $10^6$J. What do you mean by this statement.
   (b) A spaceship is launched in a circular orbit closed to Earth's surface. What additional velocity ha now to be imported to the space ship in the orbit to overcome the gravitational pull?
   ($R = 6400Km$, $g = 9.8m/s^2$)

**Solution**

(a) It means that 6.5 × $10^6$J of energy is required on the surface of Earth to send the body outside the gravitational field of the Earth.

(b) Orbital velocity of spaceship in the circular orbit. $V = \sqrt{\dfrac{gR^2}{R+h}}$

When the satellite is closed to the Earth surface, $h \simeq 0$

$$V = \sqrt{gR} = \sqrt{9.8 \times 6.4 \times 10^6}$$
$$V = 7.9195 \times 10^3 m/s$$
$$V = 7.9195km/s$$

Escape velocity

$$V_e = \sqrt{2gR} = \sqrt{2}V = \sqrt{2} \times 7.9195$$
$$V_e = 11.200km/s$$

Additional velocity required

$$V_a = V_e - V$$
$$= 11.2 - 7.9195$$
$$V_a = 3.2805km/s$$

43. (a) On a planet whose size is the same and mass 4 times as that of Earth, find the amount of work done to lift 6kg of mass vertically upward though a distance of 3m on the planet. The value of g on the planet. The value of g on the surface of Earth is $10m/s^2$.
   (b) Two metal sphere of the same material and equal radius R are touching each other. Show that force of attraction between them is directly proportional to $R^4$.

**Solution**

(a) On the planet, $g_p = \dfrac{GM_p}{R_p^2}$

On the Earth, $g_E = \dfrac{GM_E}{R_E^2}$

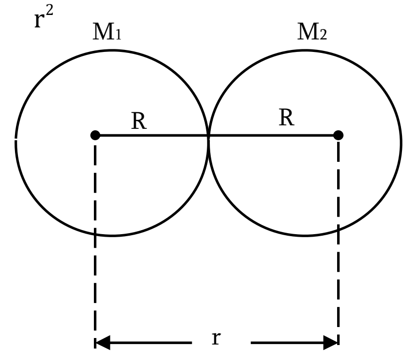
$$\frac{g_p}{g_E} = \left(\frac{M_p}{M_E}\right)\left(\frac{R_E}{R_p}\right)^2 = 4 \times 1^2 = 4$$

$$g_p = 4g_E = 4 \times 10 = 40m/s^2$$

Work done $W = Mg_p h = 6 \times 40 \times 3$

$$w = 720J$$

(b) $F = \dfrac{GM_1M_2}{r^2}$




If d is the density of the material of the sphere.

$$M_1 = \frac{4}{3}\pi R^3 d \ , \ M_2 = \frac{4}{3}\pi R^3 d$$
$$r = 2R$$

$$\text{Now, } F = \frac{G\left(\frac{4}{3}\pi R^3 d\right)\left(\frac{4}{3}\pi R^3 d\right)}{(2R)^2}$$

$$F = \frac{4}{9}G\pi^2 d^2 R^4$$

$F \alpha R^4$ Hence shown.

44. NECTA 2016/P1/5
   (a) (i) Mention one application of parking orbit. (01 mark)
   (ii) Briefly explain how parking orbit of a satellite is achieved? (1.5 marks)
   (b) The Earth satellite revolves in a circular orbit at a height of 300km above the Earth's surface. Find the
   (i) Velocity of the satellite (2 marks)
   (ii) Period of the satellite (1.5 marks)
   (c) (i) Why are space rockets usually launched from west to east? (1.5 marks)
   (ii) A spaceship is launched into a circular orbit close to the Earth's surface. What additional velocity has to be imparted to the spaceship in order to overcome the gravitational pull? (2.5 marks)
   Given that $g = 9.8m/s^2$
   $G = 6.67 \times 10^{-11}Nm2kg^{-2}$
   $R = 6.4 \times 10^6 m$

**Solution**

(a) Refer to your notes

(b) (i) since $\frac{mv^2}{r} = \frac{GMm}{r^2}$

$$V = \left[\frac{GM}{r}\right]^{\frac{1}{2}} \quad \text{but} \quad r = R + h, \quad Gm = gR^2$$

$$V = \left[\frac{gR^2}{R+h}\right]^{\frac{1}{2}}$$

$$V = \left[\frac{9.8\left(6.4\times10^6\right)^2}{6.4\times10^6\times300\times10^3}\right]^{\frac{1}{2}}$$

V =

(ii) $T = \frac{2\pi(R+h)}{V}$

$= 2\times3.14\left(6.4\times10^6 + 300\times10^3\right)$

T =

(c) (i) We know that Earth rotates about its axis from west to east. Therefore, any point on the Earth's surface has linear velocity from west to east. When a rocket is launched from west to East, the linear velocity of Earth is added to the launching velocity of the rocket

(ii) See solution example 42 (b)

45. NECTA 2015/P1/6
   (a) (i) State Newton's law of gravitation (01 mark)
   (ii) Use the law started in (a) (i) to derive Keppler's law third law (1.5 marks)
   (b) (i) Briefly explain why Newton's equation of universal gravitation does not hold for bodies falling near the surface of the Earth? (1.5 marks)
   (ii) Show that the total energy of a satellite in a circular orbit equal half its potential energy? (1.5 marks)
   (c) (i) What would be the length of a day if the rate of rotation of the Earth were such that the acceleration due to gravity g = 0 at the equator? (2.5 marks)
   (ii) Calculate the height above the Earth's surface for a satellite in a parking orbit (02 marks)
   $g = 9.8m/s^2$, $G = 6.67 \times 10^{-11}Nm^2kg^2$
   Radius of the Earth, $Re = 6.5 \times 10^6 m$.

46. (a) The escape velocity of a body from Earth is 11.2km/s. if the radius of a planet be half the radius of Earth and its mass be one – fourth of Earth. What will be the escape velocity for the planet?

(b) With what velocity should a body be projected horizontally at a height of 30km from the ground so that it becomes the satellite of the Earth? Neglecting friction. Also calculate the time period of revolution of the satellite radius of the Earth = 6370km.

**Solution**

(a) Escape velocity for Earth

$$V_e = \sqrt{\frac{2GM}{R}}$$

Escape velocity for planet

$$V_P = \sqrt{\frac{2GM_P}{R_P}}$$

$$\frac{V_P}{V_e} = \sqrt{\left(\frac{M_P}{M}\right)\left(\frac{R}{R_P}\right)}$$

$$\frac{V_P}{V_e} = \sqrt{\left(\frac{m/4}{m}\right)\left(\frac{R}{R/2}\right)} = \frac{1}{\sqrt{2}}$$

$$V_P = \frac{V_e}{\sqrt{2}} = \frac{11.2\text{km/s}}{\sqrt{2}}$$

$$V_P = 8\text{km/s}$$

(b) Suppose the mass of the body is m. Let radius of Earth be R and the projection velocity to be V.

Centripetal force = weight of a body

$$\frac{mV^2}{R+h} = mg'$$

$$V^2 = g'(R+h)$$

$$\text{But } g' = g\left[1-\frac{2h}{R}\right]$$

$$V^2 = g(R+h)\left(1-\frac{2h}{R}\right)$$

$$V = \sqrt{g(R+h)\left(1-\frac{2h}{R}\right)}$$

$$V = \sqrt{9.8(6370+30)\times10^3\times\left(1-\frac{2\times30}{6370}\right)}$$

$$V = 7882\text{m/s} = 7.882\text{km/s}$$

$$\text{Time period} \quad T = \frac{2\pi(R+h)}{V}$$

$$T = \frac{2\times3.14(6370+30)}{7.882}$$

$$T = 5099\text{sec} = 1.416\text{h}$$

47. (a) What is the difference between Geostationary and Ordinary satellite?

(b) A rocket is launched vertically upwards from the surface of the Earth with initial velocity, Vo. Show that is velocity V at a height h is given by

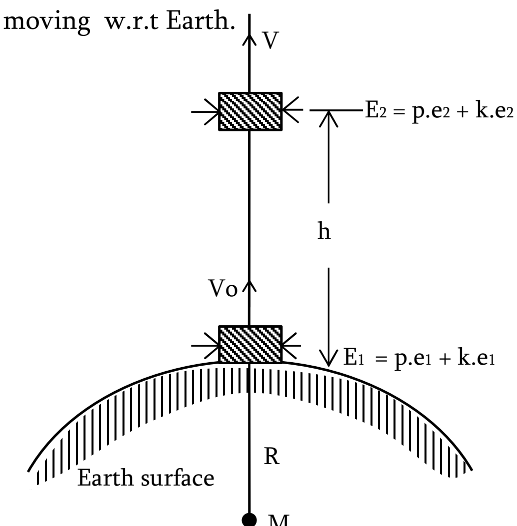
$$V_0^2 - V^2 = \frac{2gh}{1+\frac{h}{2}}$$

Where R is the radius of the Earth and g is the acceleration due to gravity at Earth's surface. Hence find the maximum height reached by a rocket fired with 90% escape velocity.

**Solution**

(a) A geostationary satellite orbits around the Earth at a height of about 3600km above Earth's surface and its motion is synchronous with Earth rotation about its axis. Therefore, it has a time period of 24 hours and stays over the same place relative to the observer on the Earth. An ordinary satellite has a time period other than 24 hours and hence appears to be moving w.r.t Earth.

(b)



According to the principle of conservation of mechanical energy.

$$E_1 = E_2$$

$$p.e_1 + k.e_1 = p.e_2 + k.e_2$$

$$\frac{-GMm}{R} + \frac{1}{2}mV_0^2 = \frac{-GMm}{R+h} + \frac{1}{2}mV^2$$

$$V_0^2 - V^2 = \frac{2GM}{R} - \frac{2GM}{R+h}$$

$$= 2GM\left[\frac{1}{R} - \frac{1}{R+h}\right]$$

$$= 2gR^2\left[\frac{R+h-h}{R(R+h)}\right]$$

$$= \frac{2gRh}{R+h} = \frac{2gRh/R}{\frac{R+h}{R}}$$

$$V_0^2 - V^2 = \frac{2gh}{1+\frac{h}{2}} \quad \text{Hence shown}$$

Now $V_O$ = 90% of Ve but $V_e = \sqrt{2gR}$

$$V_0 = \frac{90}{100}.\sqrt{2gR} \;,\; V = 0$$

$$V_0^2 = \left(\frac{90}{100}\right)^2 \cdot 2gR$$

$$\text{Then } \left(\frac{90}{100}\right)^2 \cdot 2gR - 0^2 = \frac{2gh}{1+{}^h\!/\!_R}$$

$$\text{On solving} \quad h = \frac{81R}{19}$$

48. NECTA 2005/P2/2

(b) (i) List two (2) ways of describe 'g' as applied to gravitation. Give its appropriate units in each case. Assuming the Earth to be a uniform sphere of radius 6.4 × $10^6$m and Me = 6 × $10^{24}$kg, calculate the.

(ii) Gravitational potential at a point 6 × $10^5$m above the Earth's surface.

(iii) Work done in taking a 5.0kg mass from the Earth's surface to a point where the gravitational field of the Earth is negligible.

(c) What is the binding energy of neglecting the presence of the other planets or satellites, calculating the binding energy of this system. (Take mass of the sun Ms = 3.3 × $10^5$Me, Mass of the Earth, Me = 6 × $10^{24}$kg, and radius of Earth – sun orbit r = 1.5 × $10^{11}$m.

**Solution**

(b) (i) Acceleration due to gravity is the acceleration that is experienced by the free falling body. It is S.I. Unit is m/s². Gravitational force per unit mass on the region of gravitational field. Its S.I. Unit is N/kg.

(ii) Since $V = \frac{-GM}{R+h}$

$$= \frac{-6.67\times10^{-11}\times6\times10^{24}}{\left(6.4\times10^6+6\times10^5\right)}$$

$$V = -5.7428\times10^7\,\text{Jkg}^{-1}$$

(iii) $dw = \frac{GMmdr}{r^2}$

$$\int_0^w dw = GMm\int_a^b r^{-2}dr$$

$$w = GMm\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$= 6.67\times10^{-11}\times6\times10^{24}\times5\left[\frac{1}{6.4\times10^6} - \frac{1}{\alpha}\right]$$

w =

(c) In this case the Binding energy of the Earth – sun system is the amount of energy required in order to set the Earth free from the sun's gravitational field that is the energy to put the Earth out of control of sun's gravitational field.

$$M_1 = 3.3\times10^5\,Me \;,\; M_2 = 6\times10^{24}\,kg$$

$$b = \alpha \qquad a = 1.5\times10^{11}\,m$$

$$B.E = GM_1M_2\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$=6.67\times10^{-11}\times3.3\times10^{5}\times\left(6.0\times10^{24}\right)^{2}\left[\frac{1}{1.5\times10^{11}}-\frac{1}{\alpha}\right]$$

$$B.E = 5.3064\times10^{33}J$$

49. NECTA 2002/P2/1

(a) (i) State Newton's law of gravitational. Use the law to derive Kepler's third law.

(ii) Explain why Newton's equation of universal gravitation does not hold for bodies falling near the surface of the Earth?

(b) (i) With regard to the Earth – moon system discuss the formation of tides.

(ii) A satellite of mass 600kg is in a circular orbit at a height of 2 × 10³km above the Earth's surface. Calculate the orbit speed, the kinetic energy and its gravitation potential energy.

(c) Jupiter has a mass 318times that of the Earth, its radius is 11.2 times the Earth's radius. Using this information to estimate the escape velocity of a body from Jupiter's surface, if the escape velocity from the Earth's surface is 11.2km/s

**Solution**

(a) (i) Refer to your notes

(ii) Reason:

- Universal gravitational constant, G is very small.
- The gravitational forces between the bodies on Earth's surface is small. These are reasons which makes why this law does not hold for the bodies falling near to the Earth – surface.

(b) (i) Refer to your notes

(ii) Orbital speed, $V = \left[\frac{gR^2}{R+h}\right]^{\frac{1}{2}}$

$$V = \left[\frac{9.8\times\left(6\times10^{6}\right)^{2}}{\left(6.4+2\right)\times10^{6}}\right]^{\frac{1}{2}}$$

$$V = 6912.79m/s$$

- K.e of the satellite

$$k.e = \frac{GMm}{2(R+h)} = \frac{gR^2m}{2(R+h)}$$

$$= \frac{9.8\left(6.4\times10^{6}\right)^{2}\times600}{2\left(6.4+2\right)\times10^{6}}$$

$$k.e = 1.4336\times10^{10}J$$

- P.e of the satellite

$$p.e = \frac{-gR^2m}{R+h}$$

$$= \frac{-9.8\left(6.4\times10^{6}\right)^{2}\times600}{\left(6..4+2\right)\times10^{6}}$$

$$p.e = -2.8672\times10^{10}J$$

(c) Escape velocity on the surface of the Earth

$$V_e = \sqrt{\frac{2GM}{R}}$$

Escape velocity on the Jupiter

$$V_J = \sqrt{\frac{2GM_J}{R_J}}$$

$$\frac{V_J}{V_e} = \sqrt{\left(\frac{M_J}{M}\right)\left(\frac{R}{R_J}\right)}$$

$$\frac{R_J}{R} = 11.2 \text{ , } R_J = 11.2R$$

$$\frac{M_J}{M} = 318 \text{ , } M_J = 318M$$

$$V_J = V_e\cdot\sqrt{\left(\frac{M_J}{M}\right)\left(\frac{R}{R_J}\right)}$$

$$= 11.2\sqrt{\left(\frac{318M}{M}\right)\left(\frac{R}{11.2R}\right)}$$

$$V_J = 59.68km/s$$

50. NECTA 2001/P2/1

(b) Taking the Earth to be a uniform sphere of radius 6,400km and the value of g at the surface to be 9.8m/s2. Calculate the total energy needed to raise a satellite of mass 2000kg into an orbit at an altitude of 8,000km.

(c) (i) Explain the term parking orbit of a satellite.

(ii) Explain briefly how the satellite is sent into orbit when the intended altitude has been reached. What would happen if this procedure of a putting satellite in an orbit failed to overcome into effect?

**Solution**

(b) Total energy of the satellite ate the Earth surface.

$$E_1 = p.e = \frac{-GMm}{R} = \frac{-gR^2m}{R}$$

$$E_1 = -gRm$$

Total energy of the satellite at the altitude, h

$$E_2 = p.e + k.e = \frac{-GMm}{2(R+h)}$$

$$E_2 = \frac{-gR^2m}{2(R+h)}$$

$$\Delta E = E_1 - E_2 = -gRm - \frac{-gR^2m}{2(R+h)}$$

$$\Delta E = gRm\left[\frac{R+2h}{2(R+h)}\right]$$

$$= 9.8 \times 6.4 \times 10^6 \times 2000\left[\frac{6.4\times10^6 + 2\times8000\times10^3}{2(6.4\times10^6 + 8000\times10^3)}\right]$$

$$\Delta E = 9.76 \times 10^{10}\,J$$

(c) Refer to your notes.

51. A satellite orbits the Earth at height 500km from its surface compute its;

(a) Kinetic energy

(b) Potential energy

(c) Total energy if mass of the satellite = 300kg, mass of the Earth = 6.0 × 106m, G = 6.67 × 10-11Nm2kg-2.

**Solution**

(a) $$K.E = \frac{GMm}{2(R+h)}$$

$$= \frac{6.67\times10^{-11}\times6\times10^{24}\times300}{2(6.4\times10^6 + 500\times10^3)}$$

$$K.E = 8.7\times10^9\,J$$

(b) $$P.E = \frac{-GMm}{R+h}$$

$$= \frac{-6.67\times10^{-11}\times6\times10^{24}\times300}{6.4\times10^6 + 500\times10^3}$$

$$P.E = -17.4\times10^9\,J$$

(c) $$E = P.E + K.E$$

$$= 8.7\times10^9 - 17.4\times10^9$$

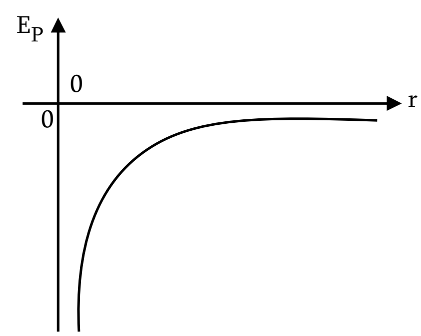
$$E = -8.7\times10^9\,J$$

Note: Will your answer alter if the Earth were to shrink suddenly to half its size?

Hints : When the Earth shrinks to half the size

R will become $\frac{6.4\times10^6}{2} = 3.2\times10^6\,m$

52. (a) The gravitational potential energy $E_P$ of a body varies with its distance r from the centre of a planet as shown in the figure below. What does the gradient at any point on the curve represent?



(b) A satellite of mass 100kg moves in a circular orbit of radius 8000km round the Earth, assumed to be a sphere of radius 6400km. Calculate the total energy to place the satellite in orbit from the Earth, assuming that g = 9.8Nkg$^{-1}$ at the Earth – surface.

**Solution**

(a) Gravitational potential energy

$$U = E_p = \frac{-GMm}{r}$$

$$\frac{dU}{dr} = \frac{d}{dr}\left[\frac{-GMm}{r}\right] = \frac{GMM}{r^2}$$

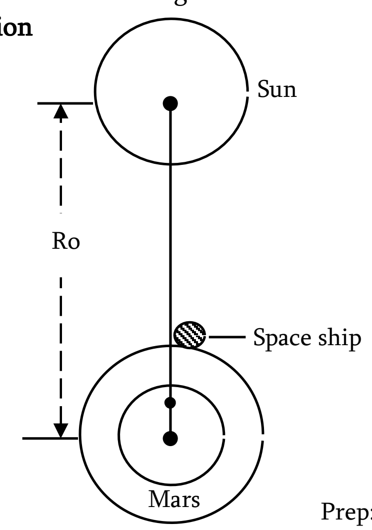
$$\text{Force of attraction} = \frac{dU}{dr} = \frac{+GMm}{r^2}$$

Thus, the gradient at any point on the gravitational potential energy curve represent the force pulling the body towards the planet i.e gravitational force of attraction.

(b) $E = 3.76 \times 10^{10}$J

53. A spaceship is stationed on Mars. How much energy must be expended on the spaceship to rocket it out of the solar system?
Mars of the spaceship = 1000kg
Mass of the sun = $2 \times 10^{30}$kg
Mass of the Mars = $6.4 \times 10^{23}$kg
Radius of mars = 3395km
Radius of the orbit of mars = $2.28 \times 10^{8}$km
G = $6.67 \times 10^{-11}$Nm$^2$kg$^{-2}$.

**Solution**




Energy required to pull out the spaceship = total energy .

Total energy of space ship = p.e due to attraction of mars + p.e due to attraction of sun + k.e

$$\text{p.e due to attraction of mars} = \frac{-GM_mM}{R_m}$$

k.e of spaceship is zero

$$\text{p.e due to attraction of sun} = \frac{-GM_sM}{R_0}$$

$$E = \frac{-GM_mM}{R_m} + \frac{-GM_sM}{R_0}$$

Energy required to pull out the spaceship'

$$\Delta E = E\alpha - E = GM\left[\frac{M_m}{R_m} + \frac{M_s}{R_0}\right]$$

$$\Delta E = 6.67\times10^{-11}\times1000\left[\frac{6.4\times10^{23}}{3395\times10^{3}} + \frac{2\times10^{30}}{2.28\times10^{11}}\right]$$

$$\Delta E = 5.976\times10^{11}\text{ J}$$

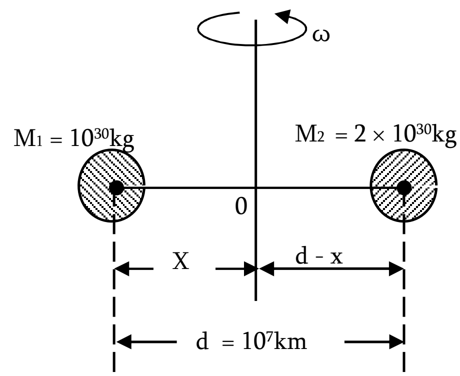
54. (a) (i) Explain the weightless of a man in a satellite.
(ii) Explain why moon has no atmosphere?
(b) A binary stars consists of two dense spherical masses of $10^{30}$kg and $2 \times 10^{30}$kg whose centres are $10^7$km apart and which rotate together with a uniform angular velocity ω about an axis intersects the line joining their centres. Assuming that only forces acting on the stars arise from their mutual gravitational attraction and that each mass may be taken to acts at its centre. Show that the axis of rotation passes through the centre of the system and find the value of ω.

**Solution**

(a) (i) The weight of the person provides the necessary centripetal force. so he feels weightless weightlessness does not mean absence of gravity. It is a situation where the person feels that he is not attracted by any force.

(ii) The escape velocity of moon is 2km/s. the root mean square velocity (r.m.s) for hydrogen is 2km/s. Thus hydrogen can easily escape from the moon atmosphere. Further , the molecule of other gases having high velocity will also leak from its atmosphere. Hence the moon has no atmosphere similarly for other smaller planets

(b)




Taking the moment about O

$$M_1X = M_2(d-x)$$

$$(M_1+M_2)X = M_2d$$

$$X = \frac{M_2d}{M_1+M_2}, \quad \frac{1}{X} = \frac{M_1+M_2}{M_2d}$$

Centripetal force = Gravitational force
On $M_1$ between $M_1$ and $M_2$

$$M_1\omega^2 = \frac{GM_1M_2}{d^2}$$

$$\omega = \sqrt{\frac{GM_2}{d^2X}}$$

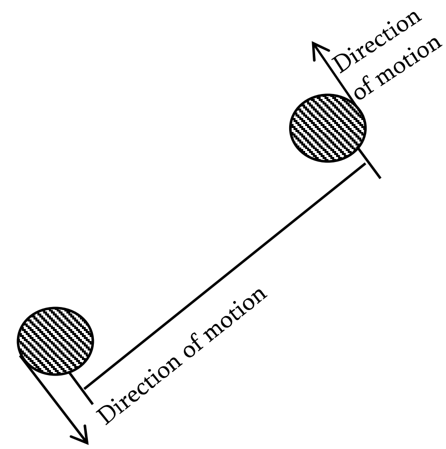
$$\omega = \sqrt{\frac{GM_2}{d^2}\cdot\frac{(M_1+M_2)}{M_2d}}$$

$$\omega = \sqrt{\frac{G(M_1+M_2)}{d^3}}$$

$$\omega = \left[\frac{6.7\times10^{-11}(10^{30}+2\times10^{30})}{(7\times10^3)^3}\right]^{1/3}$$

$$\omega = 1.4\times10^{-5}\,rad/sec(approx)$$

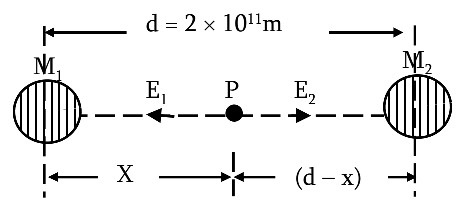
55. Two stars each of mass $4 \times 10^{30}$kg separated by $2 \times 10^{11}$m. The stars rotates about the centre of mass of the system.




(i) Determine the gravitational potential at a point where gravitational field strength is zero ($G = 6.67 \times 10^{-11}Nm^2kg^{-2}$)
(ii) Calculate the linear speed of each star in the system.
(iii) Determine the time period of rotation
(iv) Calculate the force on each star in the system.

**Solution**

(i) Let X be a distance from one of star at which gravitational field strength be equal to zero.




$$M_1 = M_2 = 4\times10^{30}\,kg$$

At the neutral point

$$E_1 = E_2$$

$$\frac{GM_1}{X^2} = \frac{GM_2}{(d-X)^2} \qquad [GM_1 = GM_2]$$

$$X^2 = (d-X)^2$$

$$X = d-X$$

$$X = \frac{d}{2} = \frac{2\times10^{11}\,m}{2}$$

$$X = 1.0\times10^{11}\,m$$

Gravitational potential at the neutral point

$$U = \frac{-GM_1}{X} + \frac{-GM_2}{d-X}$$

$$= -GM_1\left[\frac{1}{X} + \frac{1}{d-X}\right]$$

$$= -6.67\times10^{-11}\times4\times10^{30}\left[\frac{1}{1.0\times10^{11}} + \frac{1}{(2-1)\times10^{11}}\right]$$

$$U = -5.36\times10^{9}\,JKg^{-1}$$

(ii) Centripetal force = Gravitation force

$$\frac{M_1V^2}{X} = \frac{GM_1M_2}{d^2}$$

$$V = \sqrt{\frac{GM_2X}{d^2}}$$

$$= \sqrt{\frac{6.7\times10^{-11}\times4\times10^{30}\times10^{11}}{\left(2\times10^{11}\right)^2}}$$

$$V = 2.5\times10^{4}\,m/s$$

(iii) $\frac{GM_1M_2}{d^2} = M_1\omega^2X \quad \left[\omega = \frac{2\pi}{T}\right]$

$$T = 2\pi\sqrt{\frac{d^2X}{GM_2}}$$

$$T = 2\times3.14\sqrt{\frac{\left(2\times10^{11}\right)^2\times1\times10^{11}}{6.7\times10^{-11}\times4\times10^{3}}}$$

$$T = 2.43\times10^{7}\,sec$$

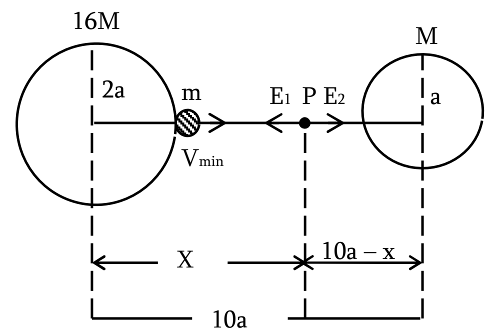
(iv) $F = \frac{GM_1M_2}{d^2}$

$$= \frac{6.7\times10^{-11}\times\left(4\times10^{30}\right)^2}{\left(2\times10^{11}\right)^2}$$

$$F = 2.68\times10^{28}\,N$$

56. Distance between the centres of the two stars is 10a. The masses of these stars are M and 16M and their radii a and 2a respectively. A body of mass m is fired straight from the surface of the larger star towards the smaller star. What would be its minimum initial speed to reach the surface of the smaller star? Obtain the expression in terms of G, M and a.

**Solution**




Initial total energy

$$E = p.e + k.e$$

$$= \frac{1}{2}mM_{mm}^2 + \frac{-G(16)m}{2a} + \frac{-GM_m}{8a}$$

$$E = \frac{1}{2}mV_{min}^2 - \frac{8GM_m}{a} - \frac{GM_m}{8a} \;\;.......(1)$$

At zero gravitational field (i.e at point P)

$$E_1 = E_2$$

$$\frac{G(16M)}{X^2} = \frac{GM}{(10a-X)^2}$$

$$\frac{16}{X^2} = \frac{1}{(10a-X)^2}$$

$$\frac{10a-X}{X} = \sqrt{\frac{1}{16}} = \frac{1}{4}$$

$$40a - 4X = X$$

$$40a = 5X$$

$$X = 8a$$

But at point P, the gravitational force cancels each other, so the particle will be at rest and the velocity will be zero. The particle will gain minimum speed when it starts moving again. Total energy at P.

$$E_p = \frac{-G(16M)m}{8a} + \frac{-GM_m}{2a}$$

Apply the law of conservation of energy (1) = (2)

$$\frac{1}{2}mV_{min}^2 + \frac{-8GM_m}{a} + \frac{-GM_m}{8a} = \frac{-G(16M)m}{8a} + \frac{-GM_m}{2a}$$

$$V_{min}^2 = \frac{45GM}{4a} = \frac{9\times 5GM}{4a}$$

$$V_{min} = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

57. Explain what is meant by the 'constant of gravitation'. A proposed communication satellite would revolve round Earth in a circular orbit in the equatorial plane at a height of 35880km above the Earth's surface. Find the period of revolution of the satellite in hours and comment on the result. Take mean radius of earth = 6370km, mass of the Earth = 5.98 × $10^{24}$kg, constant of gravitation = 6.67 × $10^{-11}$Nm²kg⁻².

**Solution**

$$T = \frac{2\pi(R+h)}{V} \quad \text{but } V = \sqrt{\frac{GM}{R+h}}$$

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

$$T = 2\times 3.14\sqrt{\frac{(6.37\times 10^6 + 3.588\times 10^7)^3}{6.67\times 10^{-11}\times 5.98\times 10^{24}}}$$

$$T = 86398.5\,\text{sec} = 24\,\text{hours}$$

Since the time period of the satellite is 24hours, it is clear that the satellite is geostationary.

58. (a) A satellite is in a circular orbit about a planet of radius R. If the altitude of the satellite is h and its period is T, show that the density of the planet is

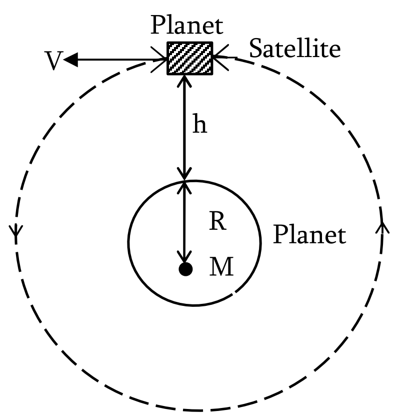
$$\rho = \frac{3\pi}{GT^2}\left[1+\frac{h}{R}\right]^3$$

(b) Two stars each of mass 2 × $10^{30}$kg are approaching each other for a head – on collision when they are at a distance 109km apart their speeds are negligible. What is the speed with which they collide? The radius of each star is $10^4$km. assume the stars to collide,

G = 6.67 × $10^{-11}$Nm2kg⁻².

**Solution**

(a) Consider the motion of satellite around the planet.




Centripetal force On satellite = gravitational force

$$m\omega^2(R+h) = \frac{GMm}{(R+h)^2}$$

$$\omega^2(R+h)^3 = GM$$

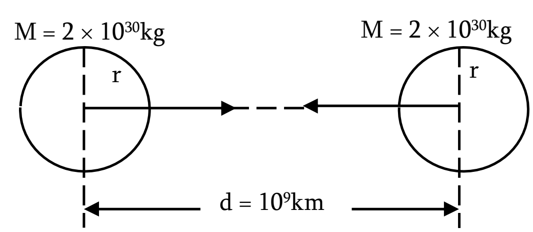
$$M = \frac{\omega^2}{G}(R+h)^3$$

But $M = \frac{4}{3}\pi R^3\rho$ , $\omega = \frac{2\pi}{T}$

$$\frac{4}{3}\pi R^3\rho = \frac{4\pi^2}{GT^2}(R+h)^3$$

$$\rho = \frac{3\pi}{GT^2}\left[1+\frac{h}{R}\right]^3 \quad \text{Hence shown.}$$

(b) Before the collisions, stars approaching each other.



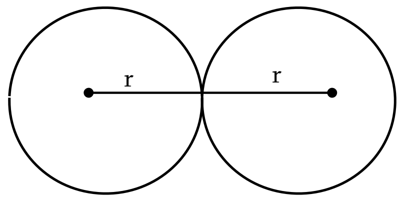
Total energy before collision

$$E_0 = p.e + k.e$$

$$= \frac{-GMM}{d} + 0$$

$$E_0 = \frac{-GM^2}{d} \text{.......(1)}$$

During collision




Total energy of the system during collision

$$E = p.e + k.e$$

$$= \frac{-GM^2}{2r} + \frac{1}{2}MV^2 + \frac{1}{2}MV^2$$

$$E = \frac{-GM^2}{2r} + MV^2 \text{......(2)}$$

Apply the law of conservation of energy

$$E = E_0$$

$$\frac{-GM^2}{2r} + MV^2 = \frac{-GM^2}{d}$$

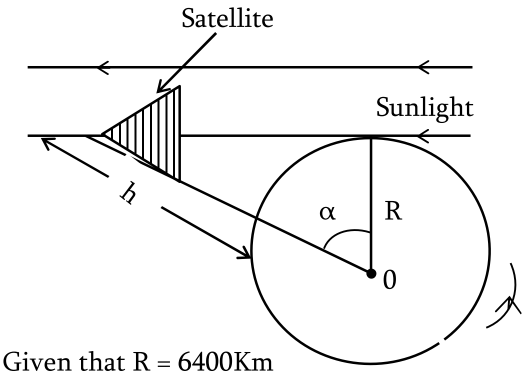
$$V^2 = \frac{GM}{2r} - \frac{GM}{d}$$

$$V = \left[GM\left(\frac{1}{2r} - \frac{1}{d}\right)\right]^{\frac{1}{2}}$$

$$V = \left[6.67\times10^{-11}\times2\times10^{30}\left(\frac{1}{2\times10^7} - \frac{1}{10^{12}}\right)\right]^{\frac{1}{2}}$$

$$V = 2.58\times10^6 \, m/s$$

59. An artificial satellite can often be seen as bright high in the sky long after sun set shown in the figure below. What must be the minimum altitude in metres if satellite moving above the Earth's equator for it to appear visible directly overhead two hours after sunset? Satellite.




Given that R = 6400Km

**Solution**

h = ?

Assume that the satellite is on the parking orbit

t = 2hr

T = 24hrs

$$\alpha = \left(\frac{t}{T}\right)\times360^0$$

$$= \left(\frac{2}{24}\right)\times360^\circ$$

$$\alpha = 30^\circ$$

From the figure above

$$\cos\alpha = \frac{R}{R+h}, \quad R+h = \frac{R}{\cos\alpha}$$

$$h = R\left[\frac{1}{\cos\alpha} - 1\right] = 6400\left[\frac{1}{\cos30^0 - 1}\right]$$

$$h = 990.30km$$

60. (a) (i) State Kepler's laws of planetary motion
(ii) Suppose that the radius of the Earth was to shrink by 1%, its mass remaining the same would the acceleration due to gravity (g) on the Earth's surface increase or decrease and by what percentage? (03 marks)

(b) (i) Define the term Escape velocity ( ½ mark)
(ii) The escape velocity of projectile on the Earth's surface is 11.2km/s thrice this speed. What is the speed of the body far away from the Earth? ignore the presence of the sun and other planets (3½ marks)

(iii) A satellite orbits the Earth at height of 500km from its surface. Calculate its kinetic energy, given that the mass of satellite is 300kg.

**Solution**

(a) (i) Refer to your notes

(ii) Given that $\frac{dR}{R}\times 100\% = -1\%$

Since $G = \frac{GM}{R^2} = GMR^{-2}$

Apply natural logarithm both side

$$\log_e^g = \log_e\left[GMR^{-2}\right]$$

$$\log_e^g = \log_e^G + \log_e^M \mp 2\log_e^R$$

On differentiating both side

$$\frac{dg}{g} = 0+0-2\frac{dR}{R}$$

$$\frac{dg}{g}\times 100\% = -2\left[\frac{dR}{R}\times 100\%\right]$$

$$= -2X-1\%$$

$$\frac{dg}{g}\times 100\% = 2\%$$

(b) (i) Refer to your notes

(ii) Ve = 11.2km/s

Velocity of projection V = 3Ve

Let Vo = Velocity of projectile after escaping the gravitational pull.

Apply the law of conservation of energy

$$\frac{1}{2}MV^2 = \frac{1}{2}MV_0^2 + \frac{1}{2}MV_e^2$$

$$V_0^2 = V^2 - V_e^2$$

$$V_0^2 = (3V_e)^2 - V_e^2 = 8V_e^2$$

$$V_0 = \sqrt{8}V_e = \sqrt{8}\times 11.2$$

$$V_0 = 31.68km/s$$

$$V_0 = 31.68km/s$$

61. (a) Explain why any resistance to the forward motion of an artificial satellite result in an increase in its speed?

(b) Show that the free fall acceleration of a body at the Earth's surface, g can be expressed as

$$g = \frac{4\pi\rho GR}{3}$$

Where ρ and R are the average density and radius of the Earth respectively.

(c) The moon moves around the Earth in orbit which is approximately circular and of radius 60R, g = 9.8m/s².

(i) Calculate the moon acceleration towards the Earth.

(ii) Estimate the speed of the moon relative to the Earth.

(iii) Explain how the value of g, G and R can be used to determine the mean density of the Earth

**Solution**

(a) If the satellites forward motion is reduced it will go into a lower orbit thus losing g.p.e and gaining k.e. its speeds up as result despite some energy turning into heat.

(b) Since $mg = \frac{GMm}{R^2}$

$$g = \frac{GM}{R^2} = \frac{G}{R^2}\left[\frac{4}{3}\pi R^3\rho\right]$$

$$g = \frac{4}{3}\pi\rho GR$$

(c) (i) since $g = \frac{GM}{R^2} = \frac{K}{R^2}$ (On Earth)

$$g_m = \frac{K}{R_m^2}$$

$$\frac{g_m}{g} = \left(\frac{R}{R_m}\right)^2, \quad g_m = g\left[\frac{R}{R_m}\right]^2$$

$$= 9.81\left[R/\right]^2$$

$$g_m = 2.7\times 10^{-3} m/s^2$$

(ii) Weight of an Object = Centripetal force of an object

$$mg_m = \frac{MV^2}{R_m}$$

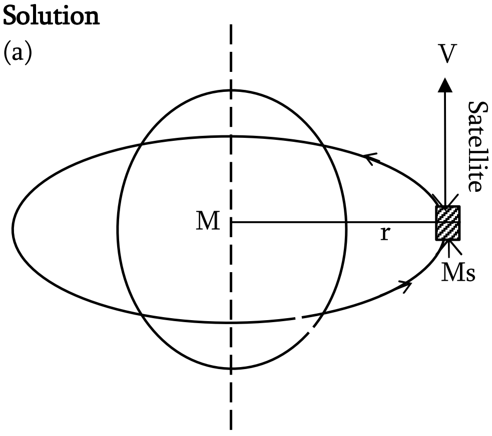
$$V = \sqrt{g_m R_m}$$
$$= \sqrt{60 \times 6.4 \times 10^6 \times 2 \times 10^{-3}}$$
$$V = 1018.23 m/s$$

62. (a) Prove that the angular momentum of a satellite of mass Ms revolving round the Earth of mass M in an orbit of radius r is equal to $\left[GMM_S^2 r\right]^{1/2}$

(b) The escape velocity of a projectile on the Earth's surface is 11.2km/s. A body is projected out with 4 times this speed. What is the speed of the body for away from the Earth?

**Solution**

(a)




$$F_C = F_g$$
$$\frac{M_S V^2}{r} = \frac{GMMs}{r^2}$$
$$V = \sqrt{\frac{GM}{r}}$$

Angular momentum

$$L = MsVr$$
$$= Msr\sqrt{\frac{GM}{r}}$$
$$L = \sqrt{GMM_S^2 r}$$

(b) Apply the principle of conservation of energy

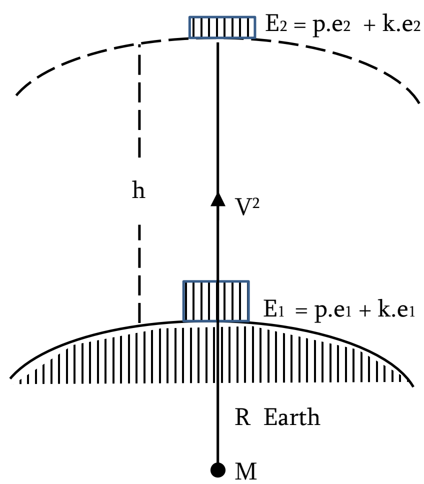
$$\frac{1}{2}MV^2 = \frac{1}{2}MV_e^2 + \frac{1}{2}MV_0^2$$
$$V_0^2 = V^2 - V_e^2$$
$$= \left(4V_e\right)^2 - V_e^2 = 15V_e^2$$
$$V_0 = V_e\sqrt{15} = 11.2\sqrt{15}$$
$$V_0 = 43.4 km/s$$

63. (a) a projectile is fired vertically from the Earth surface with an initial velocity of 10km/s. Neglecting atmospheric retardation how far above the surface of the Earth would it go? Take the Earth's radius as 6400km.

(b) The escape velocity of a body on the surface of the Earth is 11.2km/s. A body is projected away with twice this speed. What is the speed if the body at infinity. Ignore the presence of other heavy bodies.

**Solution**

(a)




Apply the law of conservation of energy

$$p.e_1 + k.e_1 = p.e_2 + k.e_2$$
$$\frac{1}{2}MV^2 + \frac{-GMm}{R} = \frac{-GMm}{R+h} + 0$$
$$\frac{1}{2}MV^2 = \frac{GMmh}{R\left(R+h\right)} \qquad \left[gm = gR^2\right]$$
$$h = \frac{V^2R^2}{2GM - V^2R} = \frac{V^2R^2}{2gR^2 - V^2R}$$

$$h = \frac{V^2R}{2gR - V^2}$$

$$= \frac{(10^4)^2 \times 6.4\times10^6}{2\times9.8\times6.4\times10^6 - (10^4)^2}$$

$$h = 2.5\times10^7\,\text{m} = 2.5\times10^4\,\text{km}$$

(b) If V is the velocity of projection and Vo is the velocity at infinity.
Apply the principle of conservation of energy.

$$\frac{1}{2}mV^2 + \frac{-GMm}{R} = \frac{1}{2}mV_0^2 + 0$$

But $V = 2V_e$ , $V_e = \sqrt{\frac{2GM}{R}}$

$$V_e^2 = \frac{2GM}{R}$$

Now $\frac{1}{2}mV^2 - \frac{GMm}{R} = \frac{1}{2}mV_0^2$

$$V^2 - \frac{2GM}{R} = V_0^2$$

$$V^2 - V_e^2 = V_0^2$$

$$(2V_e)^2 - V_e^2 = V_0^2$$

$$3V_e^2 = V_0^2$$

$$V_0 = \sqrt{3}V_e$$

$$= \sqrt{3}\times11.2$$

$$V_0 = 19.4\,\text{km/s}$$

64. A projectile is fired upwards from the surface of the Earth with a velocity KVe. Where Ve is the escape velocity and K < 1. Neglecting air resistance, show that the maximum height to which it will rise, measured from the centre of the Earth, is $\frac{R}{1-K^2}$ where R is the radius of the Earth.

**Solution**

Let r be maximum distance from the centre of the Earth to which the projectile rises

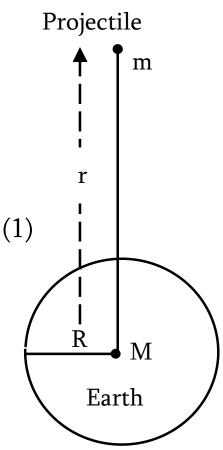
Initial total energy of projectile on Earth surface.

$$E_0 = p.e + k.e$$

$$= \frac{-GM_m}{R} + \frac{1}{2}mV^2$$

$$= \frac{-GMm}{R} + \frac{1}{2}m(kV_e)^2$$

$$E_0 = \frac{-GMm}{R} + mK^2V_e^2 \text{.......(1)}$$




Total final energy

$$E = p.e + k.e$$

$$= \frac{-GM_m}{r} + 0$$

$$E = \frac{-GM_m}{r} \text{........(2)}$$

Apply the law of conservation of Energy

$$E = E_0$$

$$\frac{-GM_m}{R} + \frac{1}{2}mK^2V_e^2 = \frac{-GM_m}{r}$$

But $V_e^2 = \frac{2GM}{R}$

$$\frac{-GM_m}{R} + \frac{1}{2}mK^2\frac{2GM}{R} = \frac{-GM_m}{r}$$

$$\frac{-1}{R} + \frac{K^2}{R} = \frac{-1}{r}$$

$$\frac{1}{r} = \frac{1}{R} - \frac{K^2}{R}$$

$$\frac{1}{r} = \frac{1-K^2}{R}$$

$$r = \frac{R}{1-K^2}$$

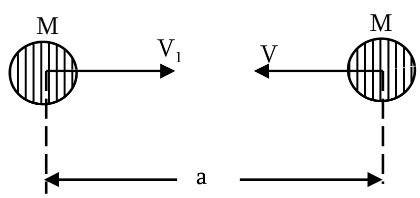
65. Two particles having masses m and M attract each other according to the law of gravitation initially they are at rest at an infinite distance apart. Show that their velocity of approach is

$$\sqrt{\frac{2G(M+m)}{a}}$$

Where $a$ is their separation.

**Solution**

If $V_1$ and $V$ are velocities of m and M at separation, a




Apply the principle of conservation of linear momentum.

$$0 = mV_1 - MV$$
$$mV_1 = MV$$
$$V_1 = \frac{MV}{m} \ldots\ldots\ldots\ldots(1)$$

Apply the law of conservation of energy

$$\frac{1}{2}mV_1^2 + \frac{1}{2}MV^2 - \frac{GMm}{a} = 0$$
$$\frac{1}{2}mV_1^2 + \frac{1}{2}MV^2 = \frac{GMm}{a}$$
$$mV_1^2 + MV^2 = \frac{2GMm}{a} \ldots\ldots\ldots(2)$$

Putting equation (1) into (2)

$$m\left[\frac{MV}{m}\right]^2 + MV^2 = \frac{2GMm}{a}$$
$$\frac{M^2V^2}{m} + MV^2 = \frac{2GMm}{a}$$
$$V = m\sqrt{\frac{2G}{a(M+m)}}$$

Also $V_1 = M\sqrt{\frac{2G}{a(M+m)}}$

Relative velocity of approach

$$V_0 = V_1 + V$$
$$= M\sqrt{\frac{2G}{a(M+m)}} + m\sqrt{\frac{2G}{a(M+m)}}$$
$$= (M+m)\sqrt{\frac{2G}{a(M+m)}}$$
$$V_0 = \sqrt{\frac{2G(M+m)}{a(M+m)}}$$
$$V_0 = \sqrt{\frac{2G(M+m)}{a}}$$

66. Sun and the Earth revolve around a common centre of mass with common period of revolution T. If $M_1$ and $M_2$ are the masses of the sun and Earth respectively and 'a' is the separation between them, then prove that

$$M_1 + M_2 = \frac{4\pi^2}{G}\cdot\frac{a^3}{T^2}$$

Where G is the universal constant of gravitation.

**Solution**