MODULE 3 :PROJECTILE MOTION

PROJECTILE: Is the body or particles thrown to move in space under the influence of gravity alone.

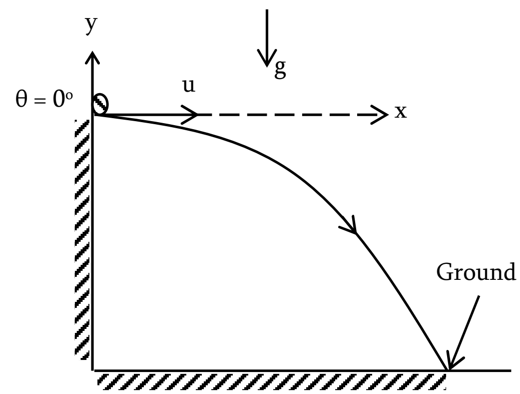
PROJECTILE MOTION
Is the motion of any object acted upon by the force of gravity only and air resistance can neglected

EXAMPLES (APPLICATIONS) OF PROJECTILE MOTION.
1. An arrow released from bow
2. Thrown of base ball.
3. A football can be kicked to the desired team made accurately.
4. A stone shot out from a catapult
5. An althlete doing the high jump
6. A javelin thrown by an ar row
7. A bullet shot from a rifle.

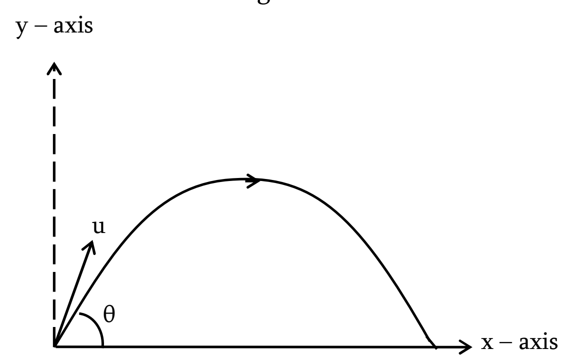
PRACTICAL APPLICATION (USES) OF THE PROJECTILE MOTION.
1. In military science.
   projectile motions are used in wars when heavy shells are required to be projected from certain position onto the position of the enemy.
2. In rescure of the enemy
   Projectile motions are used in distributing relief supplies from flying planes to the areas isolated by flood or any natural disasters such as flood , drought for fairly long time.
3. Some hunters use the knowledge of projectile to shoot their arrows so that they fall on a chosen position.
4. Rocket propulsion
5. Projectile motion is used in setting satellites into orbits by throwing them horizontally at larger speed so that they fall around the earth.

TYPES OF PROJECTILE
There are two types of projectile:-
(i) HORIZONTAL PROJECTILE
(ii) OBLIQUE PROJECTILE.

HORIZONTAL PROJECTILE
Is the kind of projectile in which the body is projected horizontally from a certain height with a certain velocity.

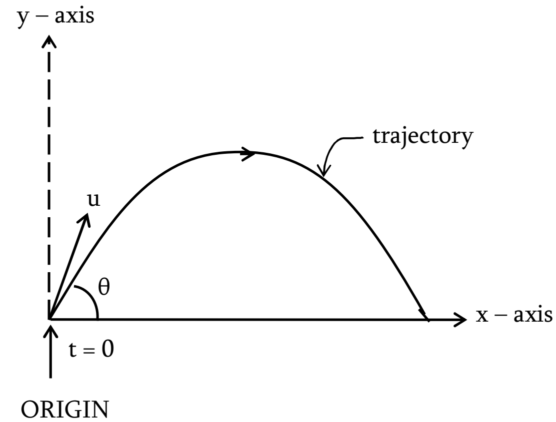


OBLIQUE PROJECTILE
Is the kind of projectile in which the body is projected at a certain angle with the horizontal.



PARAMETERS OF PROJECTILE MOTION
1. ORIGIN , O
   Is the point at which projectile starts its flight i.e is the starting point of launching or projection at time t = 0.

2. TRAJECTORY
   Is the path following or described by a projectile.



3. Assumption made in the treatment of projectile motion.
   (i) Air resistance to the motion is negligible.
   (ii) The variation in the direction and magnitude of acceleration due to gravity , g is constant throughout the motion.
   (iii) The effect due to curvature of the earth is negligible.
   (iv) The effect due to rotation of the earth is negligible.

Limitations of projectile motion

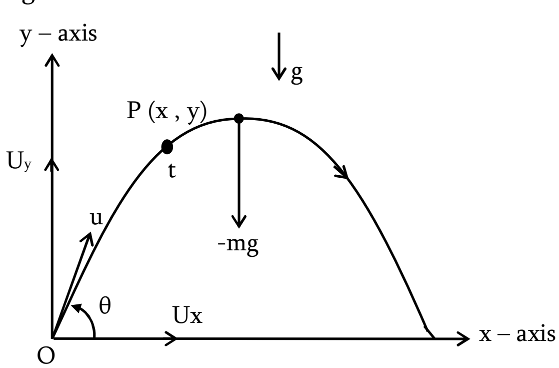
The study of projectile motion is limited by:-

(i) Variation of acceleration due to gravity
(ii) Air resistance or air drag
(iii) Curvature and rotation of the earth.

Different cases of projectile motion

Case 1: OBLIQUE PROJECTILE:

Mathematical treatment of the oblique projectile motion. Consider a projectile is projected from point , O with initial velocity U at an angle θ with horizontal as shown on the figure below:-



Minus sign on weight shows that the direction of projectile motion initially is in opposite to the direction of the acceleration due to gravity (weight).

Vector quantity	Sign of the quantity
Initial velocity , U_y	Points toward +y, hence +.
Vertical distance , S_y	Points towards to +y , hence +
Acceleration , g	Points toward –y, hence -

The motion of a projectile is a two dimensional motion because its motion can be described in xy – plane, therefore , projectile motion involves the combination of the two motions.

(i) Horizontal motion with constant velocity
(ii) Vertical motion with constant acceleration
These two motions takes place independent of each other this is called 'principle of physical independence of motions'.

HORIZONTAL MOTION

Initially , there is no horizontal component of the force acting on the projectile since the body is projected vertically initially.

According to the newton's second law

$$F_x = Ma_x \text{ but } F_x = 0$$

$$0 = Ma_x$$

$$a_x = 0$$

This shows that under horizontal motion no changes of the velocity sin ax = 0.

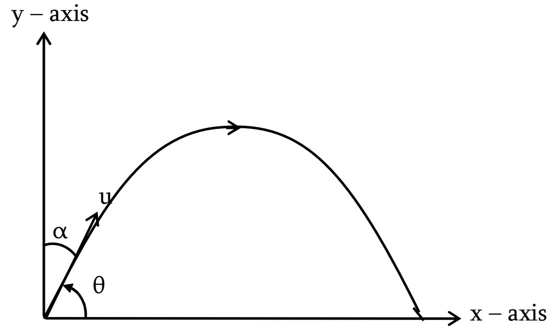
VERTICAL MOTION

$$Fy = May \text{ but } Fy = -mg$$

$$-mg = may$$

$$ay = -g$$

4. **ANGLE OF DEPARTURE (PROJECTILE)**
   Is defined as the angle between the line of the projectile with the horizontal line (plane) at the time , t = 0 i.e angle of projectile – is the angle between the direction of projectile makes with a horizontal plane at the origin.



θ = angle of departure

α = angle between the line of projectile with the vertical.

From the figure above

$$\alpha + \theta = 90^o$$

$$\theta = 90^o - \alpha$$

5. INITIAL VELOCITY OF A PROJECTILE

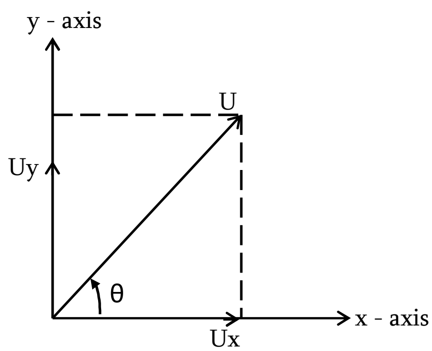
Is defined as the velocity of projectile at the origin i.e at time , t = 0 it can be denoted by using symbol of U or V_o.

Component of initial velocity of projectile

There are two components of the initial velocity of the projectile:-

(i) Initial horizontal components of the velocity , U_x.

(ii) Initial vertical component of the velocity , U_y



$$\cos\theta = \frac{U_x}{U} \quad , \quad U_x = U\cos\theta$$

$$\sin\theta = \frac{U_y}{U} \quad , \quad U_y = U\sin\theta$$

$$U_x = U\cos\theta$$

$$U_y = U\sin\theta$$

Expression of U in terms of U_x and U_y.

By using Pythagoras theorem.

$$U^2 = U_x^2 + U_y^2$$

$$U = \sqrt{U_x^2 + U_y^2}$$

Expression of angle of departure

$$\tan\theta = \frac{U_y}{U_x}$$

6. EQUATION OF UNIFORM MOTION. APPLIED TO THE PROJECTILE MOTION.

Equation of uniform motions are :-

$$V = u + at \text{ ....(i)}$$

$$S = ut + \frac{1}{2}at^2 \text{ ....(ii)}$$

$$V^2 = u^2 + 2as \text{ ....(iii)}$$

Each symbol have usual meaning

Now,

$$V = U + at$$

Horizontal motion (i.e ln x – direction)

$$v_x = u_x + a_x t$$

$$v_x = u\cos\theta + 0_x t$$

$$v_x = u\cos\theta$$

Vertical motion (i.e ln y – directions)

$$v_y = u_y + a_y t$$

$$v_y = u\sin\theta - gt$$

$$s = ut + \frac{1}{2}at^2$$

Horizontal motion (i.e ln x – direction)

$$s_x = u_x t + \frac{1}{2}a_x t^2$$

$$x = \left(u\cos\theta\right)t + \frac{1}{2} \times 0 \times t^2$$

$$x = \left(u\cos\theta\right)t$$

Vertical motion (ln y – direction)

$$s_y = u_y t + \frac{1}{2}a_y t^2$$

$$y = \left(u\sin\theta\right)t + \frac{1}{2}\left(-g\right)t^2$$

$$y = \left(u\sin\theta\right)t - \frac{1}{2}gt^2$$

$$V^2 = U^2 + 2as$$

In horizontal motion

$$v_x^2 = u_x^2 + 2a_x s_x$$

$$v_x^2 = u^2 \cos^2 \theta$$

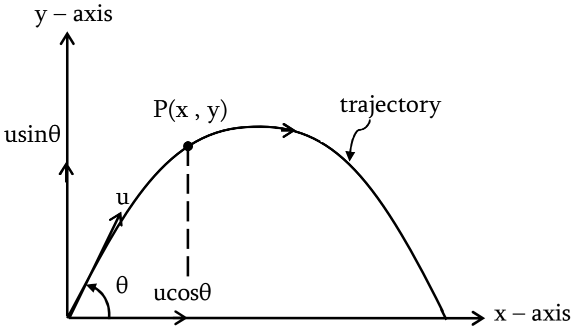
In vertical motion

$$v_y^2 = u_y^2 + 2a_y y$$

$$v_y^2 = u^2 \sin^2 \theta - 2gy$$

7. EQUATION OF THE TRAJECTORY FOR THE CASE OF OBLIQUE PROJECTILE.

Consider a projectile being projected from origin with initial velocity U at an angle θ from the horizontal as shown on the figure below.



Horizontal displacement

$$t = \frac{x}{u\cos\theta}$$

Vertical displacement

$$y = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$y = (u\sin\theta)\left[\frac{x}{u\cos\theta}\right] - \frac{g}{2}\left[\frac{x}{u\cos\theta}\right]^2$$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Let : $a = \tan\theta$, $b = \dfrac{g}{2u^2\cos\theta}$

$$y = ax - bx^2$$

This shows that the path followed by the projectile is the parabola.

Note that:

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$y = x\tan\theta - \frac{gx^2}{2u^2}\sec^2\theta$$

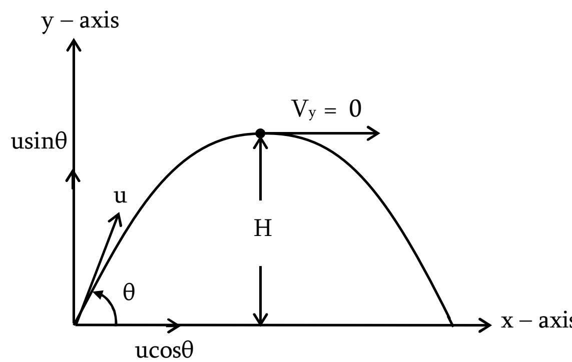
But : $\sec^2\theta = 1 + \tan^2\theta$

$$y = x\tan\theta - \frac{gx^2\left(1+\tan^2\theta\right)}{2u^2}$$

8. VERTICAL RANGE (H) (MAXIMUM HEIGHT ATTAINED)

Is the maximum vertical distance a projectile covers is the maximum height to which the projectile rises above the horizontal of plane of projectile is called vertical range.

EXPRESSION OF THE VERTICAL RANGE.



At the highest point , no further vertical motion , $V_y = 0$.

From the equation.

When $V_y = 0$, $y = H$

$$0^2 = U^2\sin^2\theta - 2gH$$

$$2gH = u2\sin2\theta$$

$$H = \frac{u^2\sin^2\theta}{2g}$$

EXPRESSION OF TIME TO REACH THE MAXIMUM HEIGHT (t_m)

This is the time taken by the projectile to reach on the maximum height and can be obtained by setting $V_y = 0$

Since $V_y = u\sin\theta - gt$

$$0 = u\sin\theta - gt_m$$

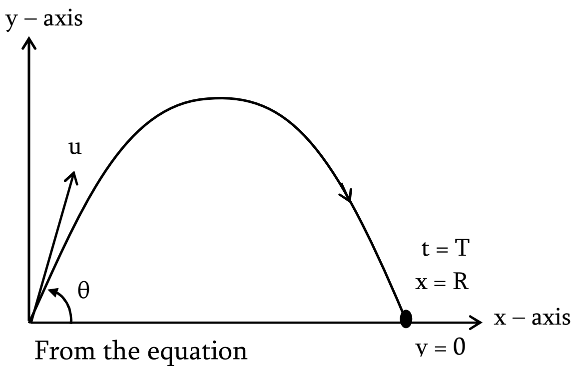
$$t_m = \frac{u\sin\theta}{g}$$

TIME OF ASCENT is the time taken by the projectile to reach the maximum height (the highest points) from the origin.

9. TIME OF FLIGHT (FLIGHT TIME)

Is the total time taken by the projection to return to the same level from where it was thrown. Time of flight , T is the total time taken by a projectile to complete its motion. i.e flight time is the time elapsing from the launching to the time the projectile returns to the ground again.

EXPRESSION OF A FLIGHT TIME.



From the equation

$$y = (u\sin\theta)t - \frac{1}{2}gt^2$$

When : $y = 0$, $t = T$

$$0 = (u\sin\theta)T - \frac{1}{2}gT^2$$

$$\frac{gT^2}{2} = (u\sin\theta)T$$

$$gT = 2u\sin\theta$$

$$T = \frac{2u\sin\theta}{g}$$

Relationship between T and t_m

$$T = \frac{2u\sin\theta}{g} = 2\left[\frac{u\sin\theta}{g}\right]$$

$$T = 2t_m$$

Time of flight is equal to the twice of the time taken by projectile to reach the maximum height. This is because the time of Ascent is equal to the time of descent. This fact is also clear from the symmetry of the curve.

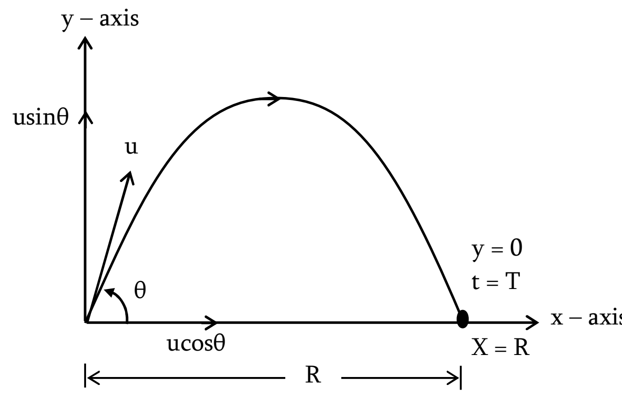
DESCENT TIME is the time taken by the projectile from the highest point to return back to the horizontal level (ground).

Ascent time = descent time.

10. HORIZONTAL RANGE , R

Is the total horizontal distance from the point of projectile to the point where the projectile comes back to the plane of projection. The horizontal distance travelled by the projectile during the flight time is known as horizontal range ,R

EXPRESSION OF THE HORIZONTAL RANGE.



Horizontal displacement

$$x = (u\cos\theta)t$$

When : $x = R$, $t = T = \frac{2u\sin\theta}{g}$

$$R = (u\cos\theta)T = \frac{2u^2\sin\theta\cos\theta}{g}$$

But :

$$2\sin\theta\cos\theta = \sin 2\theta$$

$$R = \frac{u^2\sin 2\theta}{g}$$

$$R = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{u^2\sin 2\theta}{g}$$

Alternatively

The equation of the horizontal range can be obtained from the equation of the trajectory.

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

When : $y = 0$, $x = R$

$$0 = R\tan\theta - \frac{gR^2}{2u^2\cos^2\theta}$$

$$\frac{gR^2}{2u^2\cos^2\theta} = R\tan\theta$$

$$\frac{gR}{2u^2\cos^2\theta} = \frac{\sin\theta}{\cos\theta}$$

$$R = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{u^2\sin 2\theta}{g}$$

Expression of the maximum horizontal range
Since,

$$R = \frac{u^2\sin 2\theta}{g}$$

When : $R = R_{max}$, $\sin 2\theta = 1$

$$R_{max} = \frac{u^2}{g}$$

Also,

$$u = \sqrt{R_{max}g}$$

Condition for the maximum
Horizontal range $\sin 2\theta = 1$

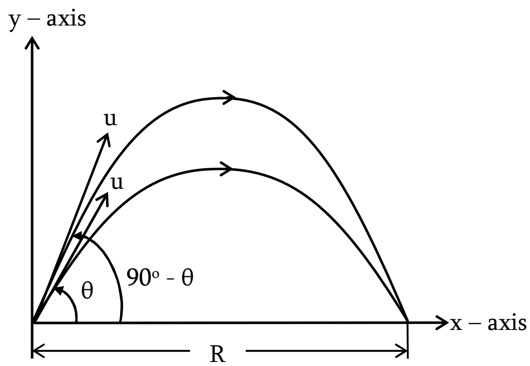
$$2\theta = \sin^{-1}(1)$$

$$2\theta = 90^\circ \ , \ \theta = 45^\circ$$

The maximum horizontal range of projectile is obtained when $2\theta = 90^\circ$ or $\theta = 45^\circ$.

11. POSSIBLE ANGLES WHICH GIVES THE SAME HORIZONTAL RANGE.

(horizontal range is a same for angle of projection θ and $90^\circ - \theta$).



From the equation of the horizontal range , R

$$R = \frac{u^2\sin 2\theta}{g}$$

$$\sin 2\theta = \frac{Rg}{u^2}$$

$$\theta = \frac{1}{2}\sin^{-1}\left[\frac{Rg}{u^2}\right]$$

The possible angles which give the same horizontal range are θ and $90^\circ - \theta$.

Note that:

A projectile possesses the same horizontal range when is projected at

(i) Angle θ with horizontal or angle θ with the vertical.

(ii) Angle θ and $90^\circ - \theta$ with the horizontal

(iii) Angle $45^\circ + \theta$ and $45^\circ - \theta$ with horizontal

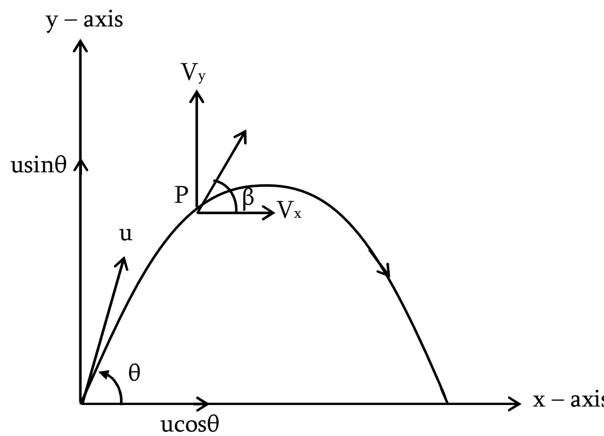
12. VELOCITY OF PROJECTILE AT ANY TIME, T

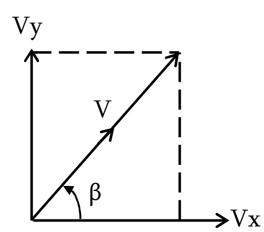
During the flight of the projectile , it is under the action of two velocities:-

(i) A constant horizontal velocity , Ux

(ii) Uniformly changing velocity along a vertical or y – axis.

Consider the position of the particle at P after a time , t as shown on the figure below.





At time t = 0

$u_x = u\cos\theta$, $u_y = u\sin\theta$

At any time , t

$$v_x = v\cos\beta = u\cos\theta$$

$$v_y = v\sin\beta = u\sin\theta - gt$$

By using Pythagoras theorem

$$V^2 = V_x^2 + V_y^2$$

$$V = \sqrt{V_x^2 + V_y^2}$$

$$= \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$$

$$= \sqrt{u^2\cos^2\theta + u^2\sin^2\theta - 2gtu\sin\theta + g^2t^2}$$

$$V = \sqrt{u^2 - 2gtu\sin\theta + g^2t^2}$$

Direction of the velocity of projectile at any instant of time.

$$\tan\beta = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta}$$

$$\tan\beta = \tan\theta - \frac{gt}{u\cos\theta}$$

$\therefore$ The angle β goes on changing with time.

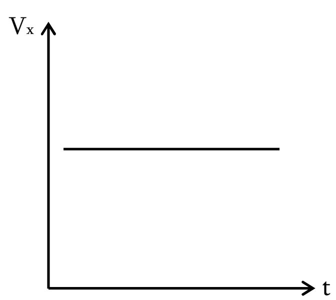
13. GRAPH OF PROJECTILE MOTION.

(i) Graph of velocity against time

(a) FOR HORIZONTAL MOTION

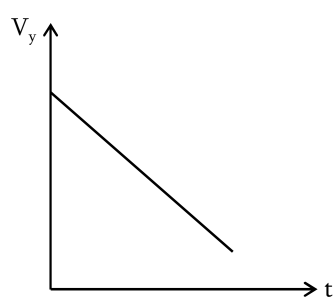
For horizontal motion

$$v_x = u_x = u\cos\theta$$



(b) FOR VERTICAL MOTION

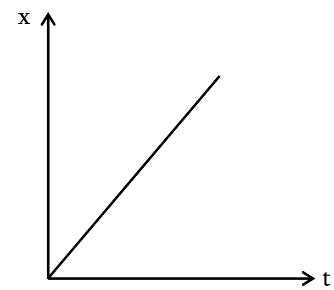
$$v_y = u\sin\theta - gt$$



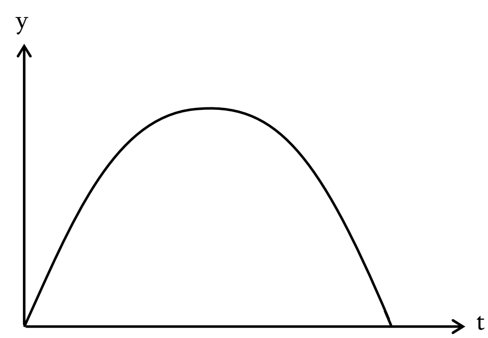
(ii) Graph of displacement against time

(a) FOR THE HORIZONTAL MOTION

$$x = (u\cos\theta)t$$

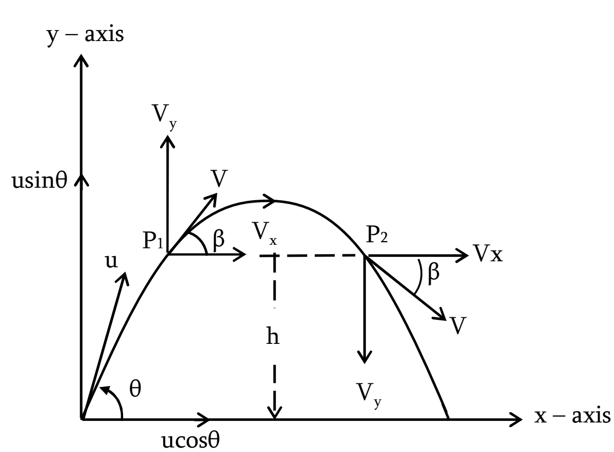


(b) FOR THE VERTICAL MOTION



14. VELOCITY OF PROJECTILE AT A GIVEN HEIGHT

The vertical component of velocity depends on height , h above the ground and since velocity along the horizontal direction is constant , then the speed of projectile is the same at all points at the same vertical height , h above the ground.



Apply the principle of conservation of mechanical energy.

$$\frac{1}{2}mu^2 = mgh + \frac{1}{2}mv^2$$

$$v^2 = u^2 - 2gh$$

$$v = \sqrt{u^2 - 2gh}$$

…………(27)

Direction of V

$$\tan\beta = \frac{v_y}{v_x}$$

$$\beta = \tan^{-1}\left[\frac{v_y}{v_x}\right] \quad \text{or} \quad \cos\beta = \frac{v_x}{v}$$

$$\beta = \cos^{-1}\left[\frac{v_x}{v}\right]$$

15. RELATIONSHIP BETWEEN R AND H

Given that : $R = \dfrac{2u^2\sin\theta\cos\theta}{g}$

$$H = \frac{u^2\sin^2\theta}{2g}$$

Take

$$\frac{R}{H} = \frac{2u^2\sin\theta\cos\theta}{g} \Big/ \frac{u^2\sin^2\theta}{2g}$$

$$\frac{R}{H} = 4\cot\theta$$

$$R = 4H\cot\theta$$

$$\cot\theta = \frac{R}{4H}$$

$$H = \frac{R}{4\cot\theta} = \frac{R}{4}\tan\theta$$

$$\tan\theta = \frac{4H}{R}$$

$$\theta = \tan^{-1}\left[\frac{4H}{R}\right]$$

16. RELATIONSHIP BETWEEN U , R AND H

Since $H = \dfrac{U^2\sin^2\theta}{2g}$

$$\frac{1}{\sin^2\theta} = \text{cosec}^2\theta = \frac{U^2}{2gH}$$

Since $1 + \cot^2\theta = \text{cosec}^2\theta$

$$\frac{U^2}{2gH} = 1 + \frac{R^2}{16H^2}$$

$$U = \left[2g\left(H + \frac{R^2}{16H}\right)\right]^{\frac{1}{2}}$$

17. RELATIONSHIP BETWEEN R, Y, X AND θ

From the equation of the trajectory

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

But $R = 2U^2\dfrac{\sin\theta\cos\theta}{g}$

$$2U^2 = \frac{Rg}{\sin\theta\cos\theta}$$

$$y = x\tan\theta - \frac{gx^2}{\dfrac{Rg}{\sin\theta\cos\theta}\cdot\cos^2\theta}$$

$$= x\tan\theta - \frac{gx^2\sin\theta\cos\theta}{Rg\cos^2\theta}$$

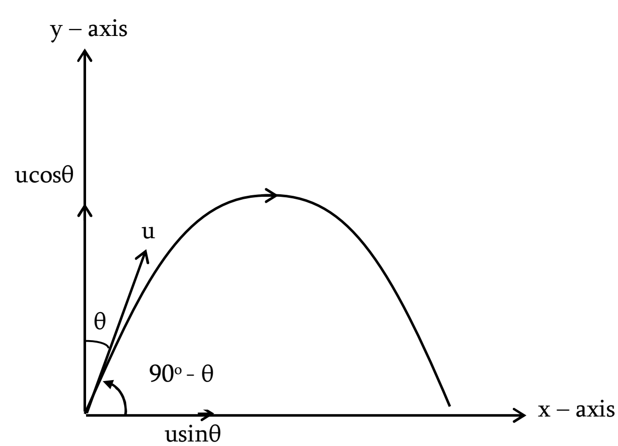
$$y = x\tan\theta - \frac{x^2}{R}\tan\theta$$

$$y = \left(1 - \frac{x}{R}\right)\tan\theta$$

$$\tan\theta = \frac{y}{x}\cdot\frac{R}{(R - X)}$$

SPECIAL CASE:

PROJECTILE FIRED AT AN ANGLE WITH THE VERTICAL.



(i) Trajectory of projectile

$$y = x\tan(90^\circ - \theta) - \frac{gx^2}{2u^2\cos^2(90^\circ - \theta)}$$

$$y = x\cot\theta - \frac{gx^2}{2u^2\sin^2\theta}$$

It is clear that the trajectory of the projectile is the parabolic.

(ii) Time of flight , T

$$T = \frac{2U\sin(90^\circ - \theta)}{g} = \frac{2u\cos\theta}{g}$$

Time to reach the maximum height ,

$$t_m = \frac{u\cos\theta}{g}$$

(iii) Vertical range

$$H = \frac{u^2\sin^2(90^\circ - \theta)}{g} = \frac{u^2\cos^2\theta}{g}$$

(iv) Horizontal range.

$$R = \frac{2u^2\sin(90^\circ - \theta)\cos(90^\circ - \theta)}{g} = \frac{u^2\sin 2\theta}{g}$$

(v) Velocity of the projectile at any time , t

$$V = \sqrt{u^2 + g^2t^2 - 2ugt\sin(90^\circ - \theta)}$$

$$V = \sqrt{u^2 + g^2t^2 - 2ugt\cos\theta}$$

And

$$\tan\beta = \tan(90^\circ - \theta) - \frac{gt}{u\cos(90^\circ - \theta)}$$

$$\tan\beta = \cot\theta - \frac{gt}{u\sin\theta}$$

NUMERICAL EXAMPLES

1. (a) (i) Explain why the horizontal motion of projectile is a constant or explain why the horizontal velocity in a projectile motion is a constant.

(ii) Mention two motions that add up to make projectile motion.

(b) A shell is fired at an angle of 30° to the horizontal with a velocity of 196m/s. Find the total of time of flight, horizontal range and maximum height attained by projectile ($g = 9.8m/s^2$)

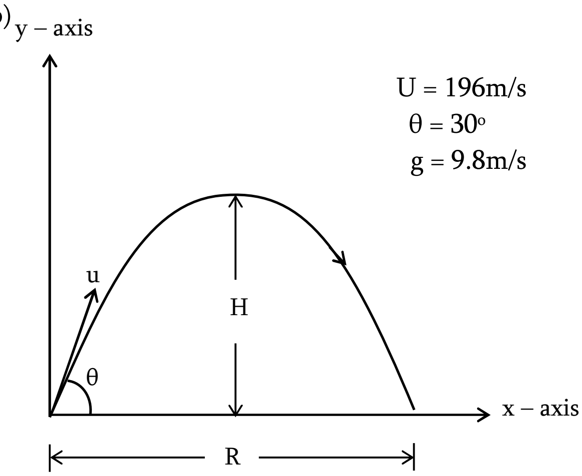
Solution

(a) (i) Because acceleration in the horizontal direction is equal to zero ($a_x = 0$). This show that under horizontal motion , no changes of the velocity therefore horizontal motion of projectile is a constant.

(ii) • Horizontal motion with a constant velocity

• Vertical motion with a constant acceleration.

(b)



Flight time , $T = \frac{2u\sin\theta}{g}$

$$T = \frac{2\times 196\sin 30^\circ}{9.8} = 20\,\text{sec}$$

$$T = 20\,\text{sec}$$

Horizontal range , $R = \frac{u^2\sin 2\theta}{g}$

$$R = \frac{196^2\times\sin 60^\circ}{g}$$

$$R = 3394.7\text{m}$$

Maximum height , $H = \frac{u^2\sin^2\theta}{2g}$

$$H = \frac{[196\sin 30^\circ]^2}{2\times 9.8}$$

$$H = 490\text{m}$$

2. A ball is thrown through up with a velocity of 40m/s at an angle of 30° above the horizontal. Calculate the maximum height reached by the ball ($g = 9.8m/s^2$)

Solution

The maximum height attained by the projectile

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$H = \frac{(40 \sin 30^\circ)^2}{2 \times 9.81}$$

$$H = 20.39m$$

3. (a) (i) What assumptions are made in the treatment of projectile motions?
   (ii) Give three examples of the projectile motion.
   (b) Find the velocity of projection of missile which has horizontal range of 120m if its time of flight is 4 second.

   Solution

   (a) Refer to your notes
   (b) From the equation

$$gT^2 = 2R\tan\theta$$

$$\theta = \tan^{-1}\left[\frac{gT^2}{2R}\right] = \tan^{-1}\left[\frac{10 \times 4^2}{2 \times 120}\right]$$

$$\theta = 33.69^\circ$$

Also

$$R = (u\cos\theta)T$$

$$U = \frac{R}{T\cos\theta} = \frac{120}{4\cos(33.69^\circ)}$$

$$U = 36.06m/s$$

4. (a) Show that the trajectory of a boy projected with an initial velocity , U at an angle θ to the horizontal is a parabolic.
   (b) A ball is kicked from the ground and made to move with initial velocity 8m/s at angle of 30° to the ground. Calculate the distance of the ball from the ground after 0.3second ($g = 9.8m/s^2$)

   Solution

   (a) Refer to your notes
   (b) From the equation

$$y = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$= (8\sin 30^\circ) \times 0.3 - \frac{1}{2} \times 9.8 \times (0.3)^2$$

$$y = 0.76m$$

5. (a) Define the following terms:-
   (i) Projectile (ii) Trajectory
   (iii) Flight time (iv) Horizontal range
   (v) Vertical range (v) Ascent time
   (b) At which point of the projectile path the speed is (i) minimum (ii) maximum?
   (c) What is the angle of projection of an oblique projectile if its range is $\frac{\sqrt{3}u^2}{2g}$?

   Solution

   (a) See your notes.
   (b) (i) At the highest point
   (ii) It is maximum at two points, one from where the projectile is thrown and another where it returns to the plane of projection.
   (c) Since

$$R = \frac{u^2 \sin 2\theta}{g} \quad \text{..........(i)}$$

$$R = \frac{\sqrt{3}U^2}{2g} \quad \text{..............(ii)}$$

$$\text{(i)} = \text{(ii)}$$

$$\frac{u^2 \sin 2\theta}{g} = \frac{\sqrt{3}u^2}{2g}$$

$$\sin 2\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{1}{2}\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\theta = 30^\circ$$

6. (a) A ball is projected with a speed of 10m/s. what are the two angle of projection for which the range is 5.0m?
   (b) Two seconds after projection a projectile is moving at 30° above the horizontal , after one more second it is moving horizontal. Find the magnitude and direction of its initial velocity ($g = 10m/s$)

Solution

(a) Since $R = \dfrac{u^2 \sin 2\theta}{g}$

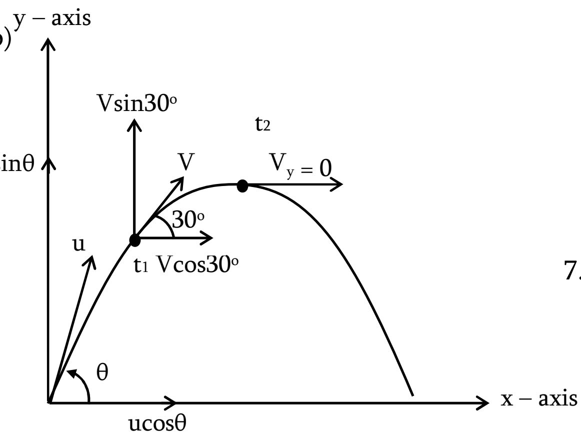
$$\theta = \frac{1}{2}\sin^{-1}\left[\frac{Rg}{u^2}\right]$$

$$\theta = \frac{1}{2}\sin^{-1}\left[\frac{5\times 10}{10\times 10}\right]$$

$\theta = 15^\circ$ or $90^\circ - \theta = 75$

$\therefore$ The two possible angle are 15°,75°

(b)



Let u be the initial speed and θ be the angle of projection. Then since after 3seconds , the projectile is moving horizontally , it must be at the height point.

Since: $t_m = t_2 = \dfrac{u\sin\theta}{g}$

$$3 = \frac{u\sin\theta}{g}$$

$u\sin\theta = 3g = 30$ ……………(i)

Let v be the speed after 2sec , then

$V\cos 30^\circ = u\cos\theta$ ………………(ii)

Also $V_y = u\sin\theta - gt$

$= u\sin\theta - 10\times 2 = 30 - 20$

$V\sin 30^\circ = 10$ …………………..(iii)

From equation (iii)

$$V = \frac{10}{\sin 30^\circ} = 20\text{m/s}$$

Equation (ii) becomes

$$u\cos\theta = 2\cos 30^\circ = 10\sqrt{3}$$

Takes

$$(u\cos\theta)^2 + (u\sin\theta)^2 = \left(10\sqrt{3}\right)^2 + 30^2$$

$$u^2\left[\cos^2\theta + \sin^2\theta\right] = 1200$$

$$u = \sqrt{1200} = 20\sqrt{3}\text{m/s}$$

Also $\dfrac{u\sin\theta}{u\cos\theta} = \dfrac{30}{10\sqrt{3}} = \sqrt{3}$

$$\tan\theta = \sqrt{3}$$

$$\theta = \tan^{-1}\left(\sqrt{3}\right)$$

$$\theta = 60^\circ$$

7. (a) A projectile is fired at an angle θ with the horizontal with a velocity U. Derive the expression for the maximum height by the projectile.

(b) The equation of trajectory of an oblique is

$$y = \sqrt{3}x - \frac{gx^2}{2}$$

What is the initial velocity and the angle of projection of projectile?

Solution

(a) See your notes

(b) Given that

$$y = \sqrt{3}x - \frac{gx^2}{2} \text{ ……………..(i)}$$

Also $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$ ……(ii)

On comparing equation (i) and (ii)

$\tan\theta = \sqrt{3}$, $\theta = 60°$

Also $u^2\cos^2\theta = 1$

$$u^2 = \frac{1}{\cos^2\theta}, \quad u = \frac{1}{\cos\theta}$$

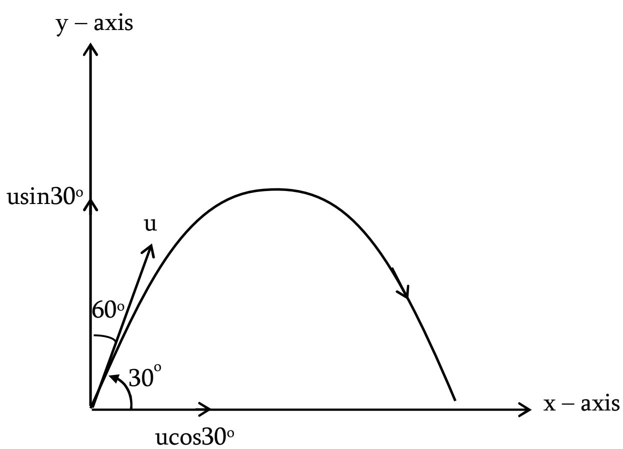
$$u = \frac{1}{\cos 60^\circ}$$

$$u = 2\text{m/s}$$

8. A particle is projected with velocity 50m/s at an angle 60° to the vertical calculate:-
   (i) It vertical height after 1.0sec
   (ii) It horizontal displacement after 1.0sec
   (iii) Its vertical component of velocity after 1.0second
   (iv) Its horizontal component of velocity after 1.0seconds ($g = 10m/s^2$)

Solution

$g = 10m/s^2$
$u = 50m/s$



(i) $$y = \left(u\sin 30^\circ\right)t - \frac{1}{2}gt^2$$

$$= \left(50\sin 30^\circ\right)\times 1 - \frac{1}{2}\times 10\times 1^2$$

$$y = 20m$$

(ii) $$x = \left(u\cos 30^\circ\right)t = \left(50\cos 30^\circ\right)\times 1$$

$$x = 43.30m$$

(iii) $$V_y = u\sin 30^\circ - gt$$

$$= 50\sin 30^\circ - 10\times 1$$

$$V_y = 15m/s$$

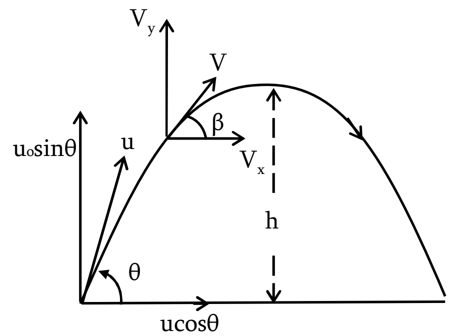
(iv) $$V_x = u\cos 30^\circ = 50\cos 30^\circ$$

$$V_x = 43.30m/s$$

9. Show that the y – component of the velocity of a projected particle with velocity U_o at an angle θ to the horizontal at a distance half way to the highest point is given by the equation.

$$V_y = \frac{U_o\sin\theta}{\sqrt{2}}$$

Solution



Since $$V_y^2 = U_0^2\sin^2\theta - 2gy$$

When $$y = \frac{H}{2}, \quad H = \frac{U_0^2\sin^2\theta}{2g}$$

$$V_y^2 = U_0^2\sin^2\theta - 2g\cdot\frac{H}{2}$$

$$= U_0^2\sin^2\theta - gH$$

$$= U_0^2\sin^2\theta - g\cdot\frac{U_0^2\sin^2\theta}{2g}$$

$$= \left(1-\frac{1}{2}\right)U_0^2\sin^2\theta = \frac{U_0^2\sin^2\theta}{2}$$

$$V_y = \frac{U_o\sin\theta}{\sqrt{2}}$$

10. If R be the horizontal range of a projectile and H its greatest height , prove that the initial speed is given by

$$U = \left[2g\left(H + \frac{R^2}{16H}\right)\right]^{\frac{1}{2}}$$

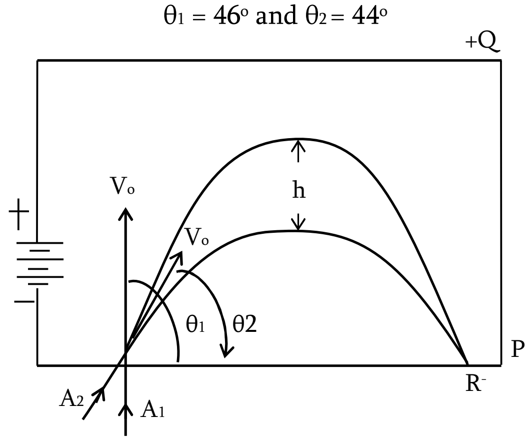
11. (a) (i) What is meant by the range of a projectile?
   (ii) Show that the maximum range of a projectile having an initial speed U is obtained when projected at an angle of 45° to the horizontal.

(b) Two α – particles enter through a shits (figure 1 below) from two sources A_1 and A_2 at angles θ_1 and θ_2 to the horizontal respectively. The initial velocity Vo and acceleration a are the same for both particles .

Given that $V_o = 6.0 \times 106\text{m/s}$,

$a = 4.0 \times 10^{13}\text{m/s}^2$,

$\theta_1 = 46^o$ and $\theta_2 = 44^o$



(i) Show that all particles are focused at the same point R.

(ii) Find the difference in the maximum height reached by the particles.

Solution

(a) (i) Refer to your notes

(iii) From the equation of horizontal range

$$R = \frac{u^2 \sin 2\theta}{g}$$

When $R = R_{max}$, $\sin 2\theta = 1$

$$2\theta = \sin^{-1}(1) = 90^\circ$$

$$\theta = 45^\circ$$ Hence shown.

(b) (i) Horizontal range

$$R = \frac{V_0^2 \sin 2\theta}{|a|}$$

For source A_1

$$R_1 = \frac{V_0^2 \sin 2\theta_1}{|a|} = \frac{(6\times10^6)^2 \sin(2\times46)}{4\times10^{13}}$$

$$R_1 = 0.90\text{m}$$

For source A_2

$$R_2 = \frac{V_0^2 \sin 2\theta_2}{|a|} = \frac{(6\times10^6)^2 \sin(2\times44)}{4\times10^{13}}$$

$$R_2 = 0.90\text{m}$$

Since $R_1 = R_2 = 0.90\text{m}$ this shows that all particles are focused at the same point R.

Alternatively

We seen that the two angles are complimentary angles then

$\sin\theta_1 \cos\theta_1 = \sin\theta_2 \cos\theta_2$ and therefore the ranges are the same.

(ii) $h = h_1 - h_2$

$$= \frac{V_0^2 \sin^2\theta_1}{2|a|} - \frac{V_0^2 \sin^2\theta}{2|a|}$$

$$= \frac{V_0^2}{2|a|}\left[\sin^2\theta_1 - \sin^2\theta_2\right]$$

$$= \frac{(6\times10^6)^2}{2\times4\times10^{13}}\left[(\sin 46^\circ)^2 - (\sin 44^\circ)^2\right]$$

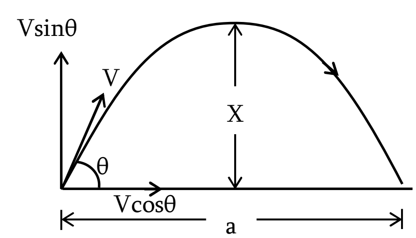
$$h = 0.233\text{m}$$

12. If the horizontal range of a particle projected with velocity V is 'a' show that the greatest height X attained is given by the equation

$$16gx^2 - 8v^2x + ga^2 = 0$$

Explain why the values of x are to be expected?

Solution



Horizontal range $a = \dfrac{2v^2 \sin\theta\cos\theta}{g}$

Greatest height $x = \dfrac{v^2 \sin^2\theta}{2g}$

$$\frac{1}{\sin^2\theta} = \text{cosec}^2\theta = \frac{v^2}{2gx} \ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

Takes

$$\frac{a}{x} = \frac{2v^2 \sin\theta \cos\theta}{g} \Big/ \frac{v^2 \sin^2\theta}{2g}$$

$$\frac{a}{x} = 4\cot\theta$$

$$\cot\theta = \frac{a}{4x} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

Since

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

$$1 + \left(\frac{a}{4x}\right)^2 = \frac{v^2}{2gx}$$

$$1 + \frac{a}{16x^2} = \frac{v^2}{2gx}$$

[Multiply by $16x^2g$ both sides]

$$16gx^2 + a^2g = 8v^2x$$

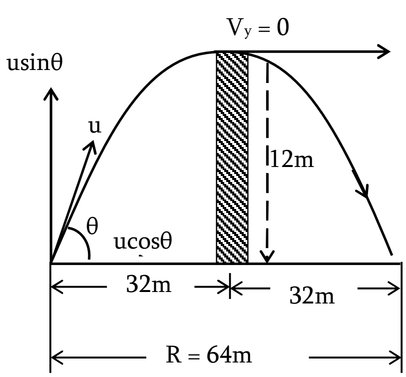
$$16x^2g - 8v^2x + a^2g = 0$$

Hence shown.

Since the obtained equation is the quadratic equation w.r.t x .Thus the two values of x are to be expected.

13. Find the velocity and direction of projection of particle which is passes in a horizontal direction just over the top of a wall which is 32m distant and 12m high.

Solution



$$\text{Since, } R = \frac{2u^2 \sin\theta\cos\theta}{g} \;,\; H = \frac{u^2\sin^2\theta}{2g}$$

$$\frac{R}{4} = 4\cot\theta$$

$$\cot\theta = \frac{R}{4H}$$

$$\frac{1}{\sin^2\theta} = \operatorname{cosec}^2\theta = \frac{u^2}{2gH}$$

Since $\quad 1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$$1 + \left(\frac{R}{4H}\right)^2 = \frac{U^2}{2gH}$$

$$u = \left[2gH\left(1 + \frac{R^2}{16H^2}\right)\right]^{\frac{1}{2}}$$

$$u = \left[2.9.8\times 12\left(1 + \frac{64^2}{16\times 12^2}\right)\right]^{\frac{1}{2}}$$

$$u = 25.56\text{m}/\text{s}$$

Angle of departure

Since $\quad \tan\theta = \frac{4H}{R}$

$$\theta = \tan^{-1}\left[\frac{4H}{R}\right] = \tan^{-1}\left[\frac{4\times 12}{64}\right]$$

$$\theta = 36.87^\circ$$

14. (a) (i) Mention two examples of projectile motion.
   (ii) Define the trajectory.
   (b) (i) Mention two uses of projectile motion
   (ii) Find the velocity and angle of projection of a particle which passes in a horizontal direction just over the top of a wall which is 12m high and 32m away.

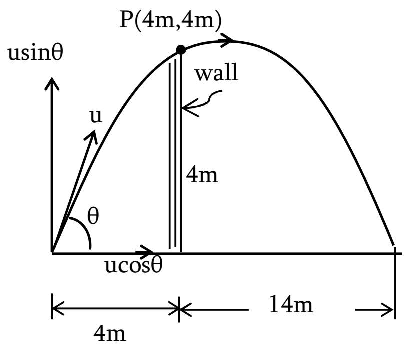
Answer : (b) (ii) 25.6m/s , 36.87°

15. A motor is at the same horizontal level as the bottom of a wall which is 20m from it. A shell from the motor just passes horizontally over the top of the wall , which is 9m high. Find velocity of projection both in magnitude and direction

Answer: 22.36m/s , 63.43°.

16. A ball is thrown from ground level so as to just clear a wall 4m high at a distance of 4m and fall at a distance of 14metres from the wall. Find the magnitude and direction of the velocity of the ball.

Solution



R = 14 + 4 = 18m

From the equation of trajectory

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$y = x\tan\theta\left[1 - \frac{x}{\frac{2}{g}u^2\sin\theta\cos\theta}\right]$$

$$y = x\tan\theta\left[1 - \frac{x}{R}\right]$$

$$\frac{y}{x} = \tan\theta\left[\frac{R-X}{R}\right]$$

$$\tan\theta = \frac{y}{x}\cdot\frac{R}{R-X} = \frac{4}{4}\cdot\frac{18}{18-4}$$

$$\tan\theta = \frac{18}{14} = \frac{9}{7}$$

$$\theta = \tan^{-1}\left(\frac{9}{7}\right) = 52.13^\circ$$

$$\theta = 52.13^\circ$$

Since $\tan\theta = \frac{9}{7}$, $\sin\theta = \frac{9}{\sqrt{130}}$

And $\cos = \frac{7}{\sqrt{130}}$

Since $R = \frac{2u^2\sin\theta\cos\theta}{g}$

$$u = \sqrt{\frac{Rg}{2\sin\theta\cos\theta}}$$

$$u = \sqrt{\frac{18\times9.8\times130}{2\times9\times7}}$$

$$u = \sqrt{182}\,m/s = 13.49\,m/s$$

17. A body is projected so that on its upward path it passes through a point X in horizontally and y in vertically from the point of projection show that if R is the range on a horizontal plane through the point of projection the angle of elevation , θ is

$$\theta = \tan^{-1}\left[\frac{y}{x}\cdot\frac{R}{R-X}\right] \text{ OR}$$

$$\tan\theta = \frac{y}{x}\cdot\frac{R}{R-X}$$

18. In a football match , a player stands 30m away from a goal keeper. The player kicks the ball toward the goalkeeper at an angle of elevation on of 40º with a speed of 20m/s the keeper has a time reaction of 0.2 seconds. Find
    (i) In which direction should the keeper run to catch the ball?
    (ii) The keeper minimum average speed is running to catch the ball just before reaching the ground.

    Solution

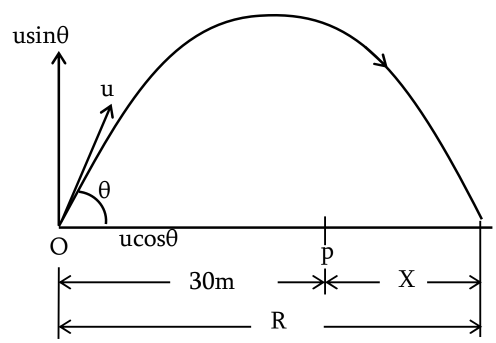
    (i) Horizontal range of the ball

    $$R = \frac{u^2\sin2\theta}{g} = \frac{(20)^2\sin(2\times40^\circ)}{9.8}$$

    $R = 40.2m$

    Therefore, the keeper has to run away from the kicker in order to catch the ball.

    (ii) Let x be the distance the keeper run



Distance the keeper should run

X = R – 30 = 40.2 – 30 = 10.2m

Flight time of the ball

$$T = \frac{2u\sin\theta}{g} = \frac{2\times 20\sin 40^\circ}{9.8}$$

$$T = 2.6236\,\text{sec}$$

Time taken by the keeper to run in order to catch the ball

t = T – reaction time of the keeper

t = 2.6236 – 0.2

t = 2.4236sec

The keeper average speed is given by

$$V = \frac{x}{t} = \frac{10.2}{2.4236}$$

$$V = 4.2\text{m}/\text{s}$$

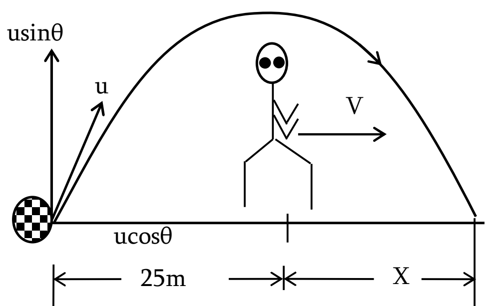
19. A football is kicked with a velocity of 20m/s at a projection angle of 45°. A receiver on the goal line 25m away in the direction of the kicks runs the same instant to meet the ball. What must be his speed if he has to catch the ball before hits the ground?

Solution

U = 20m/s

$\theta = 45^\circ$

g = 9.8m/s²



Horizontal range of the ball

$$R = \frac{u^2\sin 2\theta}{g} = \frac{(20)^2\sin(2\times 45^\circ)}{9.8}$$

$$R = 40.8163\text{m}$$

Distance covered by the receiver

$$R = x + 25$$

$$X = R - 25 = 40.8163 - 25$$

$$X = 15.8163\text{m}$$

Flight time of the ball

$$T = \frac{2u\sin\theta}{g} = \frac{2\times 20\sin 45^\circ}{9.8}$$

$$T = 2.8862\,\text{sec}$$

Speed of the receiver

$$V = \frac{X}{T} = \frac{15.8163}{2.8863}$$

$$V \approx 5.5\text{m}/\text{s (approx.)}$$

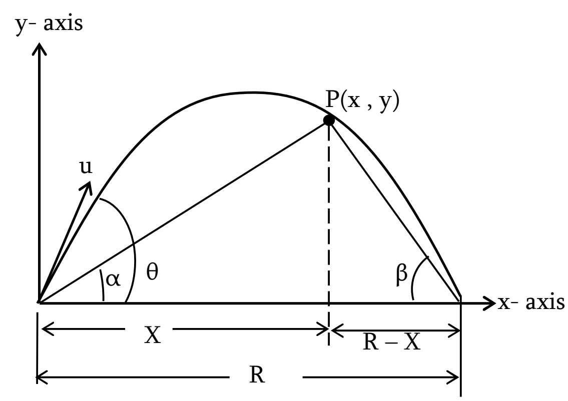
20. A player kicks a football at an angle of 30° with the horizontal and with an initial velocity of 19.6m/s. A second player standing at a distance of 20m from the first player and in the direction of the kick , starts running to meet the ball at the instant the ball is kicked. How far and how fast must he run in order to catch the ball before it hits the ground.

Answer : 13.9m , 6.975m/s

21. A particle is thrown over a triangle from one end of a horizontal base and grazing the vertex falls on the other end of the base. If α and β be the base angle and θ the angle of projection , prove that

$$\tan\theta = \tan\alpha + \tan\beta$$

Solution



From the figure above

$$\tan\alpha + \tan\beta = \frac{y}{x} + \frac{y}{R-x}$$

$$= \frac{y(R-X) + XY}{X(R-X)}$$

$$\tan\alpha + \tan\beta = \frac{y}{x}\cdot\frac{R}{R-x}\ \ldots\ldots\ldots\ldots\ldots\text{(i)}$$

From the equation

$$y = x\tan\theta\left[1 - \frac{x}{R}\right]$$

$$\tan\theta = \frac{y}{x}\cdot\frac{R}{R-x}\ \ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

(i) = (ii)

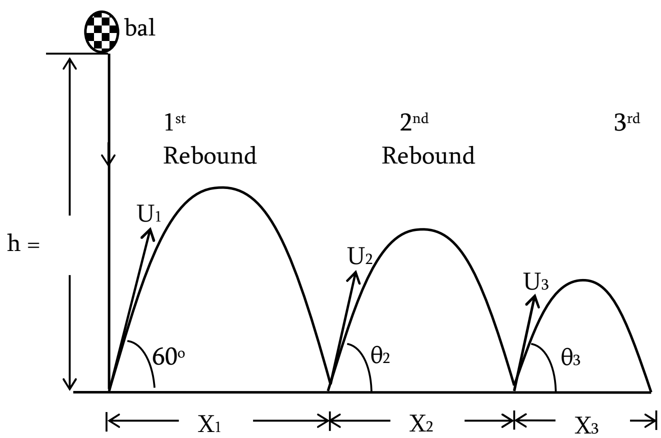
$$\tan\alpha + \tan\beta = \tan\theta$$
$$\tan\alpha + \tan\beta = \tan\theta \text{ Hence shown.}$$

22. (a) What (i) trajectory (ii) angle of projection
(b) A ball is dropped from a height of 20m and rebound obliquely at an angle of 60º ,with a velocity which is $\frac{3}{4}$ of the velocity with which hits the ground. This repeats to hit the ground and rebounds at an angle of $\frac{3}{4}$ and velocity of $\frac{3}{4}$ of the letter angle and velocity of rebound
(i) What is the time of flight just after the third rebound?
(ii) Calculate the horizontal distance travelled at that time.

Solution

(a) Refer to your notes

(b) Illustration



Apply the principle of conservation of energy

p.e = gain in k.e

$$mgh = \frac{1}{2}mv^2$$
$$v = \sqrt{2gh} = \sqrt{2\times 9.8\times 20}$$
$$v = 19.79899\text{m/s}$$

After 1st rebound

$$u_1 = \frac{3}{4}v = \frac{3}{4}\times 19.79899$$
$$u_1 = 14.84925\text{m/s}$$
$$\theta_1 = 60^\circ$$

Horizontal range

$$x_1 = \frac{u_1^2\sin 2\theta_1}{g}$$
$$x_1 = \frac{(14.84925)^2\sin(2\times 60^\circ)}{9.8}$$
$$x_1 = 19.49\text{m}$$

Flight time ,

$$t_1 = \frac{2u_1\sin\theta_1}{g}$$
$$t_1 = \frac{2\times 14.84925\sin 60^\circ}{9.8}$$
$$t_1 = 2.624\text{ sec}$$

After 2nd rebound

$$u_2 = \frac{3}{4}u_1 = 0.75\times 14.84925$$
$$u_2 = 11.13694\text{m/s}$$

$$\text{Also } \theta_2 = \frac{3}{4}\theta_1 = 0.75\times 60^\circ = 45^\circ$$

Horizontal range

$$x_2 = \frac{u_2^2\sin 2\theta}{g}$$
$$x_2 = \frac{(11.13694)^2\sin(2\times 60^\circ)}{9.8}$$
$$x_2 = 10.96\text{m}$$

Flight time

$$t_2 = \frac{2u_2\sin\theta_2}{g}$$
$$t_2 = \frac{2\times 11.13694\sin 45}{9.8}$$
$$t_2 = 1.607\text{ sec}$$

After 3rd Rebound

$$u_3 = \frac{3}{4}u_2 = 0.75\times 11.13694$$
$$u_3 = 8.3527\text{m/s}$$

$$\text{Also } \theta_3 = \frac{3}{4}\theta_2 = 0.75\times 45 = 33.75$$

Horizontal range

$$x_3 = \frac{(8.3527)^2\sin(2\times 33.75)}{9.8}$$
$$x_3 = 6.58\text{m}$$

Flight time

$$t_3 = \frac{2\times 9.3527\sin(33.75)}{9.8}$$

$$t_3 = 0.947\text{ sec}$$

(i) Total flight after third rebound

$$T = t_1 + t_2 + t_3$$

$$= 2.624 + 1.607 + 0.947$$

$$T = 5.178\text{ sec}$$

(ii) Horizontal range after this time

$$x = x_1 + x_2 + x_3$$

$$x = 37.03\text{m}$$

23. (a) A ball is projected with an initial velocity V_o at an angle θ to the horizontal, show that

(i) $x = (v_o\cos\theta)t$

(ii) $y = (v_o\sin\theta)t - \frac{1}{2}gt^2$

(iii) $H = \frac{V_0^2\sin^2\theta}{2g}$

Where x and y are the horizontal and vertical displacement after time , t respectively and H is the maximum height.

(b) A ball is projected upward with the initial velocity Vo at an angle θ from the horizontal. Draw sketch graphs showing the variables with time of the acceleration , velocity and displacement of the ball.

(i) For its horizontal motion

(ii) For its vertical motion

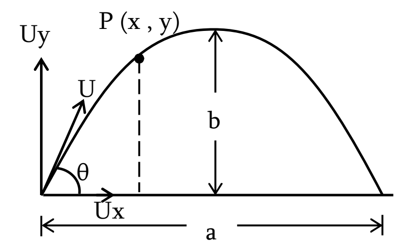
Neglect air resistance.

24. A particle projected from a point meets the horizontal plane through the points of projection after describing a horizontal distance , a and in the course of its trajectory attains a greatest height b above the point of projection. Find the horizontal and vertical components of velocity of projection in terms of a and b. show that when it has described a horizontal distance x is has attained a height of

$$\frac{4bx(a-x)}{a^2}$$

Solution

Case 1 : illustration



$U_x = U\cos\theta$, $U_y = U\sin\theta$

Now: $V_y^2 = U_y^2 - 2gy$ when $y = b$, $V_y = 0$

$$0^2 = U_y^2 - 2gb$$

$$U_y = \sqrt{2gb}$$

Flight time , $T = \frac{2U_y}{g}$

Horizontal range a = UxT

$$a = \frac{2U_yU_x}{g}$$

$$U_x = \frac{ag}{2u_y} = \frac{ag}{2\sqrt{2gb}}$$

Case 2:

proof : $y = \frac{4bx(a-x)}{a^2}$

From the equation

$y = U_yt - \frac{1}{2}gt^2$ but

$$U_y = \sqrt{2gb}$$

$$y = (\sqrt{2gb})t - \frac{1}{2}gt^2$$

Also

$$x = U_xt = \frac{agt}{2\sqrt{2gb}}$$

$$t = \frac{2\times\sqrt{2gb}}{ag}$$

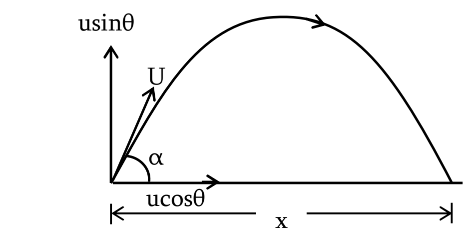
$$y = (\sqrt{2gb})\left[\frac{2x\sqrt{2gb}}{ag}\right] - \frac{1}{2}g\left[\frac{2x\sqrt{2gb}}{ag}\right]^2$$

On simplifying , we get

$$y = \frac{4bx(a-x)}{a^2}$$

25. Prove that the time of flight T and the horizontal range x of a projectile are connected by the equation $gT^2 = 2x\tan\alpha$ where α is the angle of elevation. Show that when the maximum horizontal range is 160km. the time of flight is about 3minutes and determine the muzzle velocity and the height of the trajectory.

Solution



$$\text{Horizontal range , } x = (u\cos\alpha)T$$

$$\text{Flight time , } T = \frac{2u\sin\alpha}{g}$$

$$\frac{T}{X} = \frac{2u\sin\alpha}{g} \Big/ (u\cos\alpha)T$$

$$\frac{T}{X} = \frac{2u\sin\alpha}{(gu\cos\alpha)T} = \frac{2\tan\alpha}{gT}$$

$$gT^2 = 2x\tan\alpha$$

Hence shown

$$T = \sqrt{\frac{2x\tan\alpha}{g}}$$

$$T = \sqrt{\frac{2\times 160,000\tan 45^\circ}{9.8}}$$

$$T = 180.70\,\text{sec} \approx 3 \text{ minutes}$$

Let : U = Muzzle velocity

$$U = \frac{X}{T\cos\alpha} = \frac{160,000}{18070\cos 45^\circ}$$

$$U = 12.5\text{m}/\text{s}$$

The height of trajectory

$$H = \frac{U^2\sin^2\theta}{2g} = \frac{(12.5\sin 45^\circ)^2}{2\times 9.81}$$

H = 3.982m

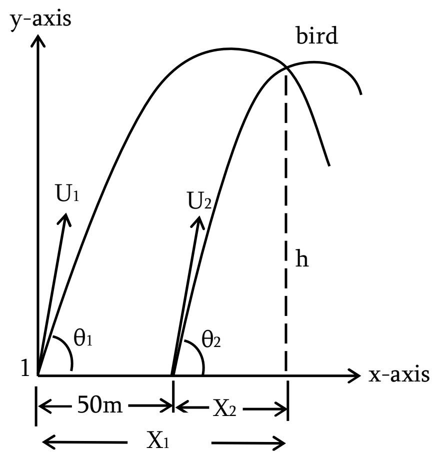
26. Two boys simultaneously aim their guns at a bird sitting on a tower. The first boy releases his shot with a speed of 100m/s at an angle of projection of 30°. The second boy is ahead of the first by a distance of 50m and releases his shot with a speed of 80m/s.

(i) How must be aim his gun so that both the shots hits the bird simultaneously?

(ii) What is the distance of the foot of the tower from the two boys and height of the tower?

(iii) With what velocities and when do the two shot hit the bird?

Solution



$U_1 = 100$m/s , $\theta_1 = 30^\circ$

Let x_1 and x_2 be horizontal displacement for the first and second shot respectively

h = vertical displacement

from the figure above

$$X_1 = X_2 + 50$$

(i) $\theta_2 = ?$ Let $\theta_2 = \theta$

The two shots will hit the bird simultaneously at a particular time t , if the horizontal and vertical displacements of the two shots are as shown above

$$\text{for first shot,} \quad h = (100\sin 30)t - \frac{1}{2}gt^2$$

$$\text{for second shot,} \quad h = (80\sin\theta)t - \frac{1}{2}gt^2$$

$$(100\sin 30)t - \frac{1}{2}gt^2 = (80\sin\theta)t - \frac{1}{2}gt^2$$

$$\sin\theta = \frac{100\sin 30}{80} = 0.625$$

$$\theta = \sin^{-1}(0.625) = 38^\circ 68'$$

$$\theta = 38^\circ 68' = 38.68^\circ$$

(ii) now

$$x_1 = (100\cos 30)t$$

$$x_2 = (80\cos 38.68)t$$

But: $x_1 = x_2 + 50$

$$x_1 - x_2 = 50$$

$$\left[100\cos(30) - 80\cos(38.68^\circ)\right]t = 50$$

$$t\left[86.6025 - 62.4499\right] = 50$$

$$t = \frac{50}{24.1526} = 2.070\,\text{sec}$$

Further, $x_1 = (100\cos 30)\times 2.070$

$$x_1 = 179.27\text{m}$$

$$x_2 = x_1 - 50 = 179.27 - 50$$

$$x_2 = 129.27\text{m}$$

Height of the tower

$$h = (100\sin 30)t - \frac{1}{2}gt^2$$

$$= (100\sin 30^\circ)\times 2.07 - \frac{1}{2}\times 9.8(2.07)^2$$

$$h = 82.504\text{m}$$

(iii) The two shots hit the bird after t = 2.07sec

Velocity at any instant of time

$$V = \sqrt{u^2 + g^2t^2 - 2utg\sin\theta}$$

For the first shot

$$V_1 = \sqrt{(100)^2 + (9.8\times 2.07)^2 - 2\times 100\times 2.07\times 9.8\sin 30^\circ}$$

$$V_1 = 91.56\text{m/s}$$

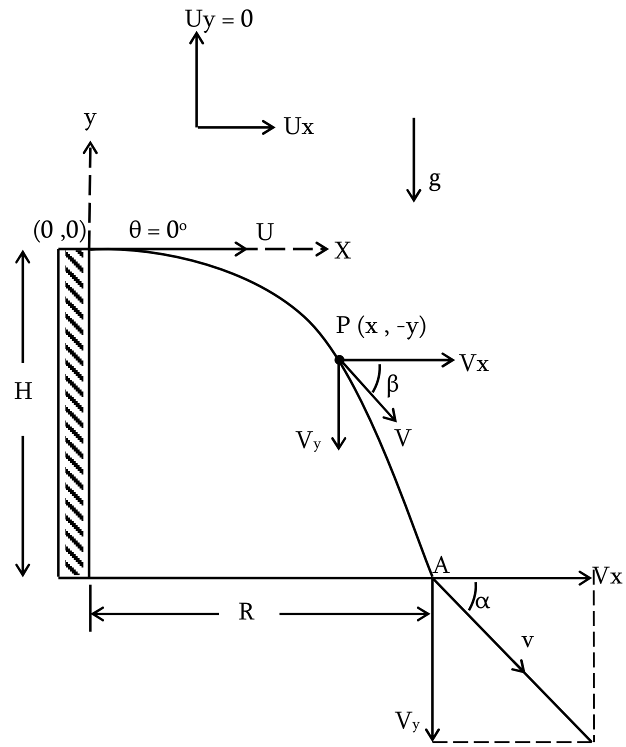
For the second shot

$$V_2 = \sqrt{80^2 + (9.8\times 2.07)^2 - 2\times 80\times 2.07\times 9.8\sin 38.68}$$

$$V_2 = 69.16\text{m/s}$$

CASE 2: HORIZONTAL PROJECTILE

Consider a projectile fired horizontal from the top of cliff as shown on the figure below .



Vector quantity	Sign to the vector quantity
Initial velocity Uy = 0	Points toward +y ,hence +
Vertical distance Sy	Point toward –y , hence -
Acceleration , g	Points towards – y, hence -

(i) Motion along the horizontal direction

$$x = (u\cos 0^\circ)t = ut$$

(ii) Motion along the vertical direction

Vertical displacement.

$$y = (u\sin 0^\circ)t - \frac{1}{2}gt^2$$

$$y = \frac{-1}{2}gt^2 \quad \text{but } t = \frac{x}{u}$$

$$y = \frac{-1}{2}g\left[\frac{x}{u}\right]^2 = \frac{1}{2}g\frac{x^2}{u^2}$$

$$y = \frac{-gx^2}{2u^2} \quad \text{Let } k = \frac{g}{2u^2}$$

$$y = -kx^2$$

This shows that the equation of the trajectory is the parabola

Cartesian coordinate P = [X , -y]

P = [ut , -½ gt²]

P = [ut , -½ gt²]

(iii) Expression of the flight time

$$-H = \frac{-1}{2}gT^2$$

$$T = \sqrt{\frac{2H}{g}}$$

(iv) Expression of the horizontal range

$$R = UT$$

$$R = U\cdot\sqrt{\frac{2H}{g}}$$

(v) Velocity of projectile at time , t

Since $\theta = 0$

$$V = \sqrt{u^2 + g^2t^2 - 2ugt\sin 0^\circ}$$

$$V = \sqrt{u^2 + g^2t^2}$$

Also

$$\tan\beta = \frac{-V_y}{V_x} - \frac{(u\sin 0^\circ - gt)}{u\cos\theta}$$

$$\tan\beta = \frac{gt}{u}$$

(vi) Velocity of projectile after striking on the ground

$$V = \sqrt{V_x^2 + V_y^2}$$

$$V_x = u\ ,\ V_y = u\sin 0^\circ - gT$$

$$V = \sqrt{u^2 + g^2T^2}$$

Direction of V

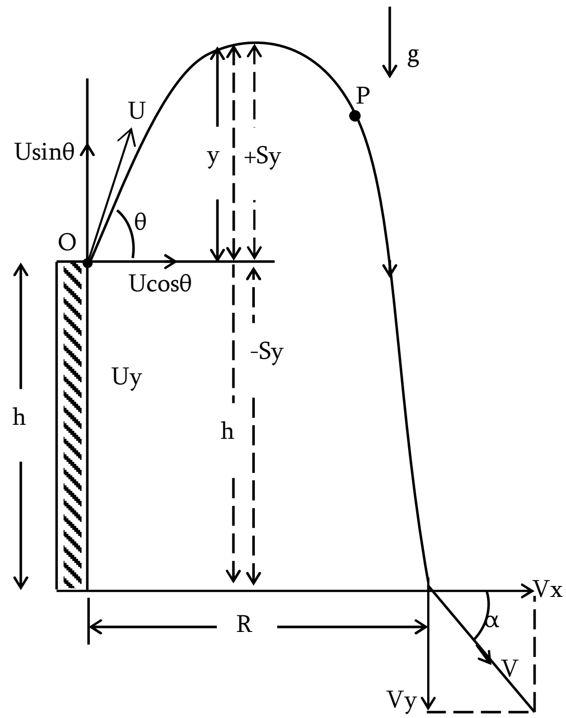
$$\tan\alpha = \frac{v_y}{v_x}\ ,\ \alpha = \tan^{-1}\left[\frac{v_y}{v_x}\right] \text{ or}$$

$$\cos\alpha = \frac{v_x}{v} = \frac{u}{\sqrt{u^2 + g^2T^2}}$$

$$\alpha = \cos^{-1}\left[\frac{u}{\sqrt{u^2 + g^2T^2}}\right]$$

CASE 3: OBLIQUE PROJECTILE FIRED FROM TOP OF THE CLIFF OR TOWER.

Consider the particle of mass , m projected from the top of cliff at an angle θ from the horizontal as shown on the figure below.



Vector quantity	Sign of the quantity
Initial velocity Uy = 0	Points toward +y ,hence +
Vertical distance Sy	Depend on the position of projectile
Acceleration , g	Points towards – y, hence -

(i) Expression for time to reach the maximum height

$$t_m = \frac{u\sin\theta}{g}$$

(ii) Expression of time taken by projectile to reach on the ground level (flight time).

From the equation

$$y - H = (u\sin\theta)t - \frac{1}{2}gt^2$$

When $y = 0$, $t = T$

$gT^2 - (2u\sin\theta)T - 2H = 0$

On solving quadraticall for

$$T = \frac{u\sin\theta \pm \sqrt{u^2\sin^2\theta + 2gH}}{g}$$

Omitting negative time

$$T = \frac{u\sin\theta + \sqrt{u^2\sin^2\theta + 2gH}}{g}$$

(iii) Expression of the horizontal

$$R = (u\cos\theta)T$$

$$R = \frac{u\cos\theta}{g}\left[u\sin\theta\sqrt{u^2\sin^2\theta + 2gH}\right]$$

(iv) Expression of maximum velocity of the projectile after strike the ground.

$$V^2 = V_x^2 + V_y^2$$

$$V = \sqrt{u^2 - gTu\sin\theta + g^2T^2}$$

Direction of V

$$\tan\beta = \tan\theta - \frac{gT^2}{u\cos\theta}$$

(v) Parabolic equation in terms of vertical height , H above the ground.
From the equation

$$y = (u\sin\theta)t - \frac{1}{2}gt^2$$

Let : $y = H - h$

$$H - h = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$H = h + (u\sin\theta)t - \frac{1}{2}gt^2$$

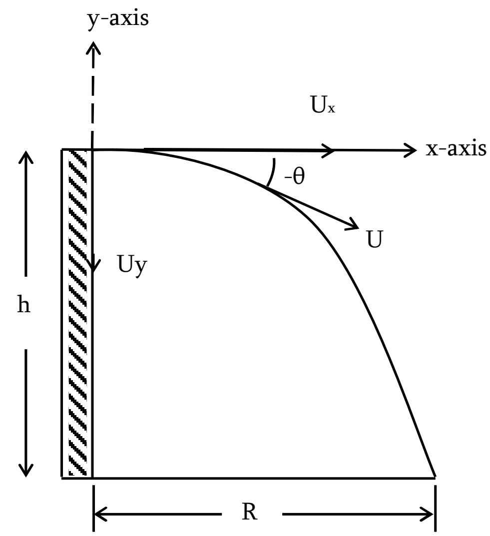
But: $t = \dfrac{x}{u\cos\theta}$

$$H = h + (u\sin\theta)\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$H = h + x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

CASE 4: PROJECTILE FIRED AT AN ANGLE θ BELOW THE HORIZONTAL LEVEL FROM THE TOP OF CLIFF.

Consider the figure below as illustrated.

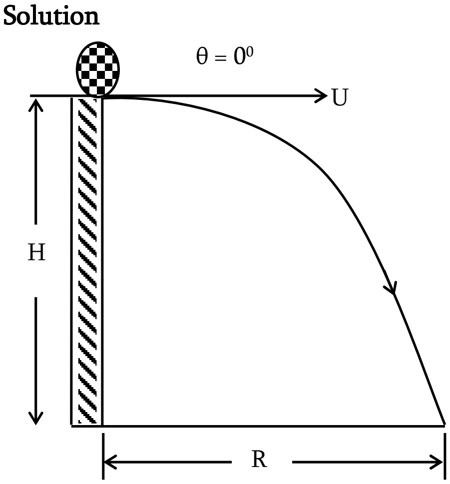


The mathematical treatment for this case is similar to the case 3 above , but the value θ and y are negative because are obtained below the point of the projections

NUMERICAL EXAMPLES

27. A ball projected horizontally with speed V from the top of a tower of height h , reaches the ground at a distance R from the top of the tower. Another ball , projected horizontally from the top of a tower of height 2h , reaches 2R from the foot of the tower. What is the initial speed of the second ball?

Solution



Horizontal range is given by

$$R = U\sqrt{\frac{2H}{g}}$$

For the first case

$R_1 = R$, $H_1 = h$, $U_1 = V$

$$R_1 = U_1\sqrt{\frac{2H_1}{g}}$$

$$R = V\sqrt{\frac{2h}{g}} \ldots\ldots\ldots\ldots\ldots\ldots(i)$$

For the second case

$$R_2 = U_2\sqrt{\frac{2H_2}{g}} \quad , H_2 = 2h$$

$$2R = V'\sqrt{\frac{2(2h)}{g}}$$

$$2R = V'\sqrt{\frac{4h}{g}}$$

$$2R = V'\sqrt{\frac{2h}{g}}\cdot\sqrt{2} \ldots\ldots\ldots\ldots(ii)$$

Insert equation (i) into (ii)

$$2V\sqrt{\frac{2h}{g}} = V'\sqrt{\frac{2h}{g}}\cdot\sqrt{2}$$

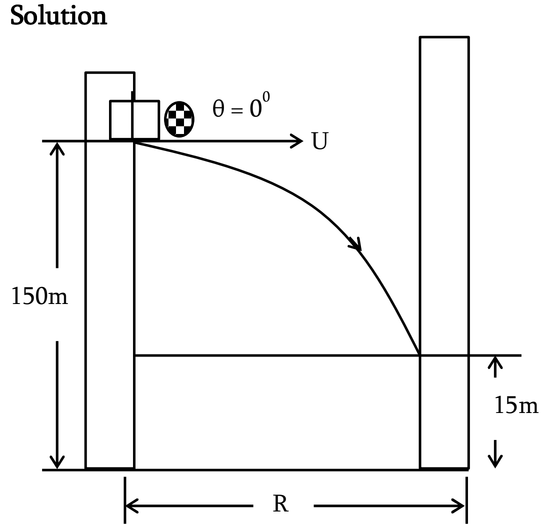
$$2V = V'\sqrt{2}$$

$$V' = \frac{2V}{\sqrt{2}} = \frac{2V}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$V' = \sqrt{2}V$$

28. Two tall buildings are 60m apart with what speed must a ball through horizontal from a small window 150m above the ground in one building so that it will enter a window 15m from the ground in the other building.

Solution



Solution

Time taken by the ball to reaches on the second building from the first building

$$-h = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$-135 = (u\sin 0^\circ)t - \frac{1}{2}\times 9.8t^2$$

$$t = \sqrt{\frac{2\times 135}{9.8}}$$

$$t = 5.20\,\text{sec}$$

Horizontal range

$$R = (u\cos\theta)t$$

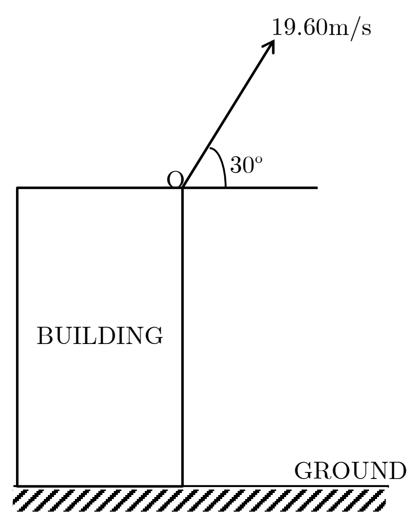
$$R = (u\cos\theta)t = ut$$

$$u = \frac{R}{t} = \frac{R}{t}$$

$$u = \frac{60}{5.20} = 11.54\text{m/s}$$

$$u = 11.54\text{m/s}$$

29. (a) Show that the trajectory of a body projected with an initial velocity Vo at an angle θ° to the horizontal is a parabola.
(b) State the two practical applications of the projectile motion.
(c) The top of a certain flat roofed building is 39.20m above the ground a small solid ball is projected from point O at the roof of the building with an initial speed of 19.60m/s as illustrated in the figure below



Calculate

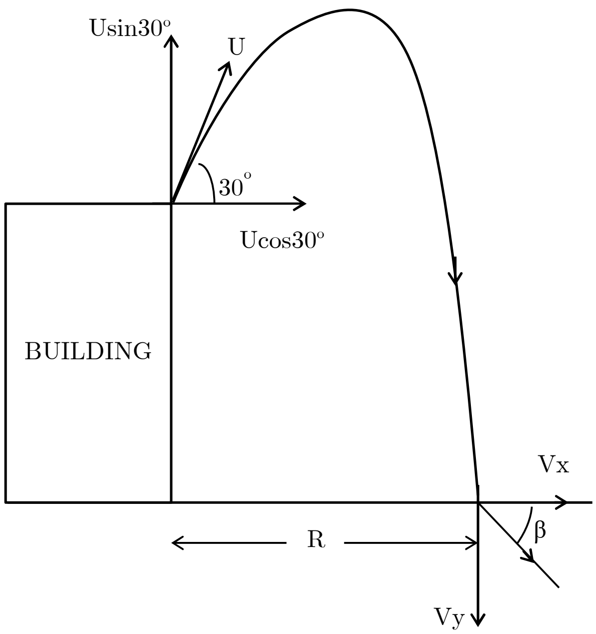
(i) The time by the ball to reach on the ground.

(ii) The maximum speed reached by the ball on the ground.

Solution

(a) , (b) refer to your notes

(c)



(i) t = time taken by the ball to reach on the ground

$$-h = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$-392 = (19.60\sin 30^\circ)t - \frac{1}{2}\times 9.8t^2$$

$$4.9t^2 - 9.8t - 39.2 = 0$$

On solving t quadratically

t = 4.0sec

(ii) $V_x = u\cos\theta = 19.6\cos 30^\circ$

$V_x = 5.90m/s$

$V_y = u\sin\theta - gt$

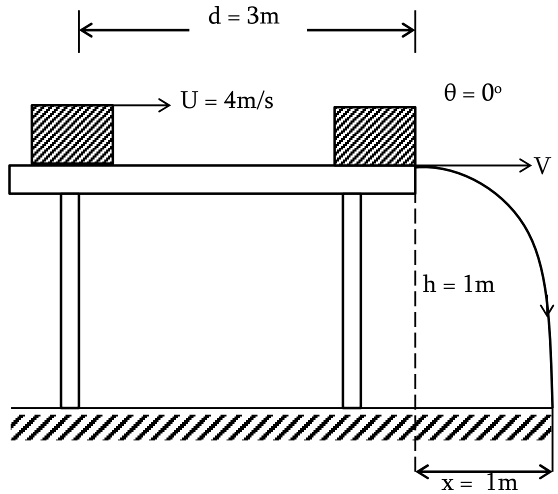
$= 19.6\sin 30^\circ - 9.8\times 4$ = -29.4m/s

The magnitude of velocity of particle strike on the ground

$$V = \sqrt{V_x^2 + V_y^2}$$

30. A block passes at point 3m from the edge of the table with a velocity of 4m/s. it slides toward another edge which is 1m high and strike the floor 1m from the edge of the table. What is the kinetic friction between the block and the table.

Solution



Apply work – energy theory

$$K.E_1 - K.E_2 = \text{work done}$$

$$\frac{1}{2}MU_1^2 - \frac{1}{2}MU_2^2 = f.d \text{ but } f = \mu mg$$

$$\frac{1}{2}MU^2 - \frac{1}{2}MV^2 = \mu mgd$$

$$U^2 - V^2 = 2\mu gd$$

$$\mu = \frac{U^2 - V^2}{2gd} \ldots\ldots\ldots\ldots\ldots\text{(i)}$$

Since $-h = \frac{-1}{2}gt^2$

$t^2 = \frac{2h}{g}$ but $x = vt$

$$\left(\frac{x}{v}\right)^2 = \frac{2h}{g}, \quad v^2 = \frac{gx^2}{2} \ldots\ldots\ldots\text{(ii)}$$

Putting equation (ii) into (i)

$$\mu = \frac{u^2 - \frac{gx^2}{2h}}{2gd}$$

$$= \frac{2hu^2 - gx^2}{4ghd}$$

$$= \frac{2\times 1\times 4^2 - 10\times 1^2}{4\times 10\times 1\times 3}$$

$$\mu = 0.18$$

31. A gun on the top of a vertical cliff 90m high , fires a shell horizontally with a velocity of 350m/s
   (a) How long it takes the shell to strike on the ground?
   (b) How far is the shell from the bottom the cliff when it strikes the ground?
   (c) With what speed the shell strikes the ground?

Solution

(a) Flight time

$$T = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 90}{9.8}}$$

$$T = 4.29\,\text{sec}$$

(b) $R = UT = 30\times 4.29$

$$R = 1500\text{m}$$

(c) $V = \sqrt{U^2 + 2gh}$

$$V = \sqrt{(350)^2 + 2\times 9.8\times 90}$$

$$V = 352.5\text{m/s} = 352.5\text{m/s}$$

Direction of V

$$\alpha = \cos^{-1}\left[\frac{U}{V}\right] = \cos^{-1}\left[\frac{350}{352.5}\right]$$

$\alpha = 6.8^\circ$ below the horizontal.

32.(a) Why does a hunter raise the barret of his riffle when aiming at a distance target?
   (b) A hunter aims directly at a target 200m away. By how much will he miss the target if the muzzle velocity of the bullet is 500m/s.

Solution

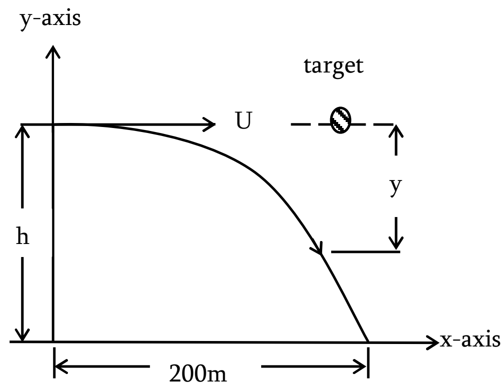
(a) A hunter raises the barralel so as to offset the effect of gravity which tend to pull down the bullet and thus cause to mas the target.

(b) Using the parabola equation

$$y = \frac{gx^2}{2u^2} = \frac{9.8\times(200)^2}{2\times(500)^2}$$

$$y = 0.784\text{m}$$

He will miss the target by 78.4cm



33. A jet flying horizontally at a uniform speed of 60m/s at an altitude of 204m above ground drops a bomb to hit a moving truck that is 250m from point A on a level ground , when the plane is vertically above point A. The truck is 4m high and moving the direction away from A. If the bomb it the top of the truck , determine the average speed of the truck.

Solution

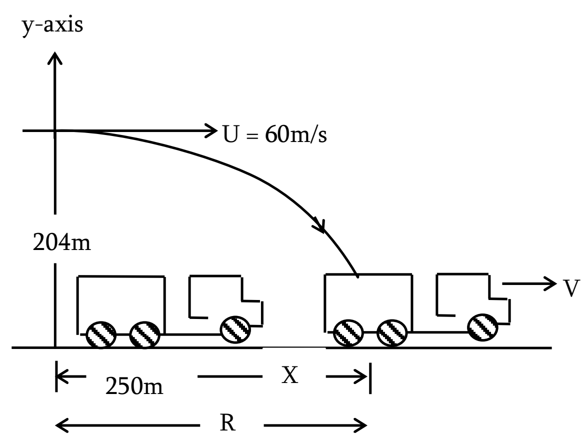
The flight time of the bomb

$$T = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 200}{9.8}}$$

$$T = 6.3888\,\text{sec}$$

Horizontal range of bomb

$$R = UT = 6.3888\times 60$$



$$R = 383.33\text{m}$$

But : $R = 250 + X$

$$X = R - 250 = 383.33 - 250$$

$$X = 133.33\text{m}$$

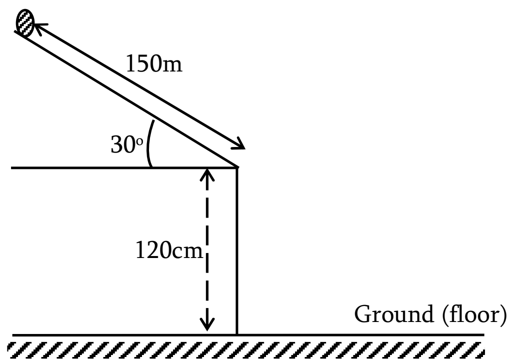
Average speed of the truck

$$V = \frac{X}{T} = \frac{133.33}{6.3888}$$

$$V = 20.87\text{m/s}$$

34. (a) Show that the trajectory of a body projected with an initial velocity V_o at an angle θ to horizontal is a parabola.

(b) A particle is released from rest 150cm up a smooth inclined plane making an angle of 30^o with horizontal. The lower end of the plane is resting on the edge or table 120cm above a horizontal floor as illustrated below.



Calculate :-

(i) Speed of the particle it reaches edge of the table.

(ii) The speed of the particle when strike the floor.

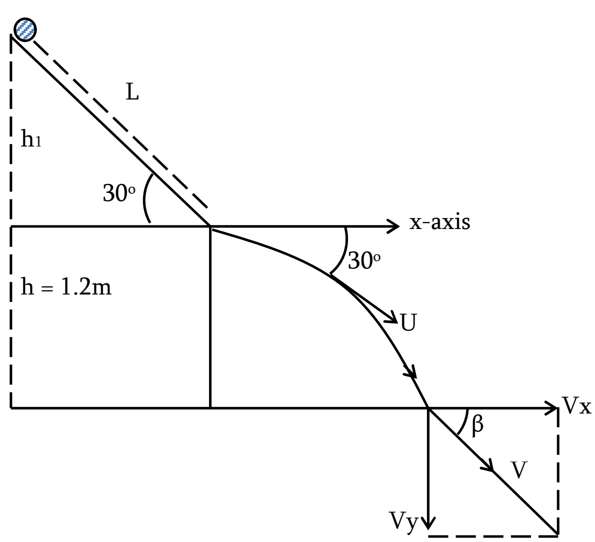
(iii) The horizontal distance from the table to the point at which the particle strike the floor.

Solution

(a) Refer to your notes

(b) (i) U = speed of the particle when reaches at the edge of the table

Apply the law of conservation of energy



Less in P.E = gain in K.E

$$Mgh = \frac{1}{2}mu^2$$

$$u = \sqrt{2gh} = \sqrt{2\times9.8\times u}$$

But $u = L\sin 30^\circ$

$$u = \sqrt{2\times9.8\times1.5\sin 30^\circ}$$

$$u = 3.8m/s$$

Alternatively from the equation of uniform motion.

$$u^2 = V_0^2 + 2aL \text{ But } V_o = 0\text{ , } a = g\sin30^o$$

$$u^2 = 2gL\sin 30^\circ$$

$$u = \sqrt{2gL\sin 30^\circ}$$

$$= \sqrt{2\times9.8\times1.5\sin 30^\circ}$$

$$u = 3.83m/s$$

(ii) The time taken by the ball (particle) from the edge of the table to the ground.

$$-h = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$-1.2 = (3.83\sin 30^\circ)t - \frac{1}{2}\times9.8t^2$$

$$4.9t^2 + 1.9t^z - 1.2 = 0$$

$$4.9t^2 + 1.9t - 1.2 = 0$$

On solving quadratically and omitting negative time.

$$t = 0.3365\text{ sec} \approx 0.34\text{ sec}$$

$$V = \sqrt{u^2 + g^2t^2 + 2gtu\sin\theta}$$

$$= \sqrt{(3.83)^2 + (9.8\times0.34)^2 + 2\times9.8\times0.34\times383\times30^\circ}$$

$$V = \sqrt{(3.83)^2 + (2.8\times0.34)^2 + 12.762}$$

$$V = 6.2m/s$$

Alternatively apply principle of conservation of mechanical energy

P.E + K.E = constant

$$Mgh_1 + \frac{1}{2}Mu^2 = \frac{1}{2}mv^2$$

$$v = \sqrt{u^2 + 2gh} \text{ but } u_2 = 2gh$$

$$v = \sqrt{2g(h+u)}$$

$$v = \sqrt{2\times9.8(15\sin 30^\circ + 1.2)}$$

$$v = 6.2m/s$$

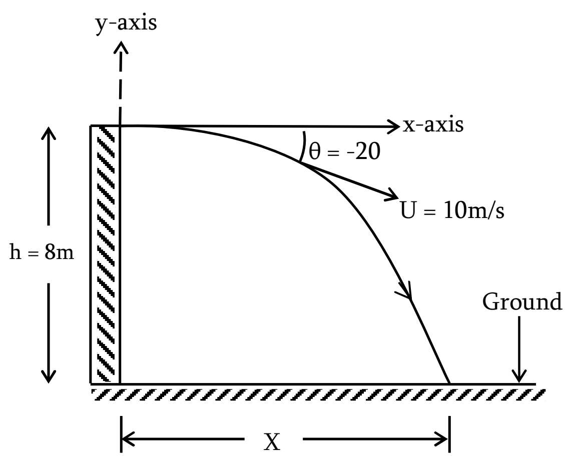
(iii) Horizontal distance

$$R = (u\cos\theta)t$$
$$= (3.83\cos 30^\circ)\times 0.34$$
$$R = 1.128\text{m}$$

35. From the top of a building 45m high a stone is thrown at angle of 30° to the horizontal with an initial velocity of 20m/s find:-
   (i) The flight time
   (ii) The distance from the foot of the building where it strikes the ground.
   Answer (i) 4.22sec (ii) 73m

36. Two tall building are 200m apart with what speed must a ball be thrown horizontal from a window of one building 2km above the ground in one building so that it will enter a window 40m from the ground in the other?
   Answer : 10m/s

37. (a) What is a projectile . Give two application of projectile motion.
   (b) What is time of flight of a projectile?
   (c) A ball was tossed from the window 8.0m above the ground. The ball left it with a velocity of 10m/s at an angle 20° below the horizontal. How far horizontal from the window , will the ball hit the ground?
   Solution



Time taken by the ball to reach on the ground

$$-h = (u\sin\theta)t - \frac{1}{2}gt^2$$
$$-8 = (10\sin(-20))t - \frac{1}{2}\times 9.8t^2$$
$$-8 = -(10\sin 20)t - 4.8t^2$$

On solving quandratically for t and omitting negative time

$$t = 0.98\,\text{sec}.$$

Horizontal displacement

$$x = (u\cos\theta)t$$
$$= [10\cos(-20)]\times 0.98$$
$$x = 9.2\text{m}$$

38. A cricketer can throw a ball to a maximum horizontal distance of 100m. How much high above the ground can the cricketer thrown the same ball?
   Solution
   Maximum horizontal range

$$R_{max} = \frac{u^2}{g}, \quad u = \sqrt{R_{max}g}$$
$$u^2 = 100g$$

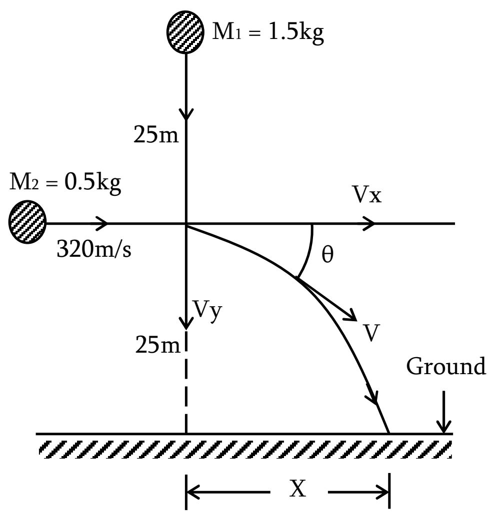
For upward motion of the ball

$$v^2 - u^2 = -2gh$$
$$0^2 - 100g = -2gh$$
$$h = \frac{100g}{2g}$$
$$h = 50\text{m}$$

39. An object of mass 1.5kg is dropped from a point 50m above the ground. After falling through half that height it is hit by a shell travelling horizontally with a velocity of 320m/s. If the shell has a mass of 0.5kg and the bodies coalesce after impact.
   (i) Draw the complete path of the motion
   (ii) Calculate the horizontal displacement of the system on hitting the ground.
   (iii) Calculate the velocity with which the coalesced mass hits the ground.

Solution

(i) Diagram for the complete path of the motion.



(ii) Apply the principle of conservation of linear momentum along the y – direction.

$$M_1U_{1y} + M_2U_{2y} = (M_1 + M_2)V_y$$

But

$$U_{2y} = 0$$

$$V_y = \frac{M_1U_{1y}}{M_1 + M_2}$$

Apply the law of conservation of energy.

$$mgh = \frac{1}{2}mu_{1y}^2$$

$$u_{1y} = \sqrt{2gh} = \sqrt{2\times9.8\times25}$$

$$u_{1y} = 22.1m/s$$

$$v_y = \frac{22.1\times1.5}{(1.5+0.5)}$$

$$v_y = 16.6m/s$$

Along x – direction

$$m_1u_{1x} + m_2u_{2x} = (m_1 + m_2)v_x$$

But $u_{1x} = 0$

$$v_x = \frac{m_2u_{2x}}{m_1 + m_2} = \frac{0.5\times320}{1.5+0.5}$$

$$v_x = 80m/s$$

$$\tan\theta = \frac{v_y}{v_x}, \quad \theta = \tan^{-1}\left[\frac{v_y}{v_x}\right]$$

$$\theta = \tan^{-1}\left[\frac{16.6}{80}\right] = 11.7^O$$

By using Pythagoras theorem

$$v^2 = v_x^2 + v_y^2 = (80)^2 + (16.6)$$

$$v = 81.7m/s$$

Time taken to strike on the ground

$$-25 = [81.7\sin(-11.7)]t - 4.9t^2$$

$$4.9t^2 + (81.7\sin117)t - 25 = 0$$

$$4.9t^2 + 16.6t - 25 = 0$$

Solve for t quadratically , t = 1.13sec

Horizontal displacement

$$x = v_x \cdot t$$

$$x = 80\times1.13 = 90.4m$$

(iii) V = 81.7m/s

40. (a) State two practical applications of a projectile motion.

(b) A shell of mass 30kg is released from height 600m above the ground on half way its journey it collided with another shell of 25kg moving horizontally 300m/s . They coalesce together after collision.

(i) Draw path for the whole motion

(ii) Calculate:-

- The time taken for both to return on the ground.
- Horizontal distance covered after collision.

41. (a) Prove the maximum horizontal range is four times of maximum height attained by projectile fired along the required oblique direction.

(b) The horizontal range of a projectile is $4\sqrt{3}$ times of its maximum height. calculate the angle of projection.

Solution

(a) Since $R = \frac{u^2\sin2\theta}{g}$

For the maximum range

$$R = R_{max},\ \theta = 45^o$$

$$R_{max} = \frac{u^2}{g}$$

Also

$$H = \frac{u^2\sin^2\theta}{2g} = \frac{u^2 2}{g}\left[\sin 45^\circ\right]^2$$

$$H = \frac{u^2}{2g}\left[\frac{\sqrt{2}}{2}\right] = \frac{u^2}{4g}$$

$$H = R_{max}/4$$

(b) $R = \dfrac{2u^2\sin\theta\cos\theta}{g}$, $H = \dfrac{u^2\sin^2\theta}{2g}$

Given that : $R = 4\sqrt{3}H$

$$\frac{2u^2\sin\theta\cos\theta}{g} = 4\sqrt{3}\,\frac{u^2\sin^2\theta}{2g}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = 30^O$$

42. A particle of mass M_o kg is projected at an angle θ with horizontal at a speed of V_o m/s show that the potential energy possessed by the particle half – way up is given by.

$$P.E = \frac{M_o u^2\sin^2\theta}{4}$$

Solution

$P.E = M_o gh$ but $h = \dfrac{H}{2}$

$$= M_o gh = \frac{MgH}{2}$$

$$= \frac{Mg}{2}\left[\frac{u^2\sin^2\theta}{2g}\right]$$

$P.E = \dfrac{M_o u^2\sin^2\theta}{4}$ Hence shown

43. A gun is fired from the top of a cliff of height h and short attains a maximum height of (h + b) above the sea level and strikes the sea at 'a' distance from the foot of the cliff. Prove that the angle of elevation of a gun is given by the equation.

$$a^2\tan^2\theta - 4ab\tan\theta - 4bh = 0$$

Solution

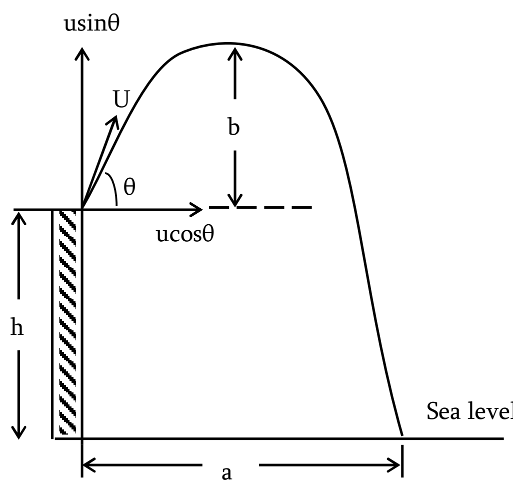
Horizontal motion

$$a = (u\cos\theta)t$$

$$t = \frac{a}{u\cos\theta} \quad \text{.................................(i)}$$

Vertical motion

$$-h = (u\sin\theta)t - \frac{1}{2}gt^2$$



$$h = \frac{1}{2}gt^2 - (u\sin\theta)t \quad \text{.......................(ii)}$$

Putting equation (i) into (ii)

$$h = \frac{1}{2}g\left(\frac{a}{u\cos\theta}\right)^2 - u\sin\theta\left(\frac{a}{u\cos\theta}\right)$$

$$h = \frac{ga^2}{2u^2\cos^2\theta} - a\tan\theta$$

At the maximum height , $V_y = 0$

$$V_y^2 = U^2\sin^2\theta - 2gy$$

$$0^2 = U^2\sin^2\theta - 2gb$$

$$U^2 = \frac{2gb}{\sin^2\theta}$$

Now ;

$$h = \left(\frac{ga^2}{2\cos^2\theta}\right)\cdot\frac{\sin^2\theta}{2gb} - a\tan\theta$$

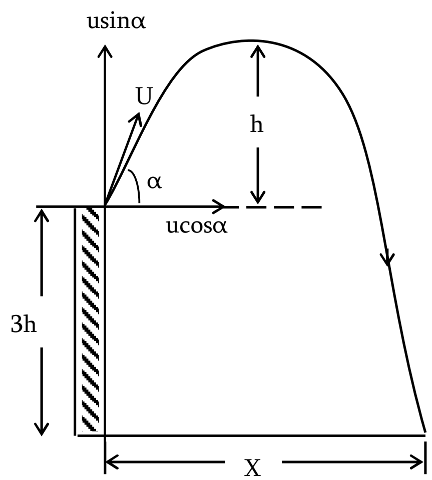
$$h = \frac{a^2}{4b}\tan^2\theta - a\tan\theta$$

$$a^2\tan^2\theta - 4ba\tan\theta - 4bh = 0,$$

Hence shown.

44. A particle is projected from a point at a height 3h above a horizontal plane the direction of projection making an angle α with the horizontal . Show that if the greatest height above the point of projection is h , the horizontal distance travelled after striking the plane is X = 6hcotα.

Solution



Horizontal motion $X = (u\cos\alpha)t$

$$t = \frac{X}{u\cos\alpha}$$

For the vertical motion

$$-3h = (u\sin\alpha)t - \frac{1}{2}gt^2$$

$$-3h = (u\sin\alpha)\left[\frac{X}{u\cos\alpha}\right] - \frac{g}{2}\left[\frac{X}{u\cos\alpha}\right]^2$$

$$-3h = x\tan\alpha - \frac{gX^2}{2u^2\cos^2\alpha} \text{ ....(i)}$$

Since $V_y^2 = U^2\sin^2\alpha - 2gy$

When Vy = 0 , y = h

$$0 = U^2\sin^2\alpha - 2gh$$

$$U^2 = \frac{2gh}{\sin^2\alpha} \text{ ....(ii)}$$

Putting equation (ii) into (i)

$$-3h = x\tan\alpha - \frac{gx^2}{2\cos^2\alpha}\left[\frac{\sin^2\alpha}{2gh}\right]$$

$$-3h = x\tan\alpha - \frac{x^2\tan^2\alpha}{4h}$$

$$X^2\tan^2\alpha - (4h\tan\alpha)x - 12h^2 = 0$$

Solve for x quadratically

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$X = \frac{4h\tan\alpha \pm \sqrt{(4h\tan\alpha)^2 - 4\tan^2\alpha(-12h^2)}}{2\tan^2\alpha}$$

$$X = \frac{4h\tan\alpha + 8h\tan\alpha}{2\tan^2\alpha}$$

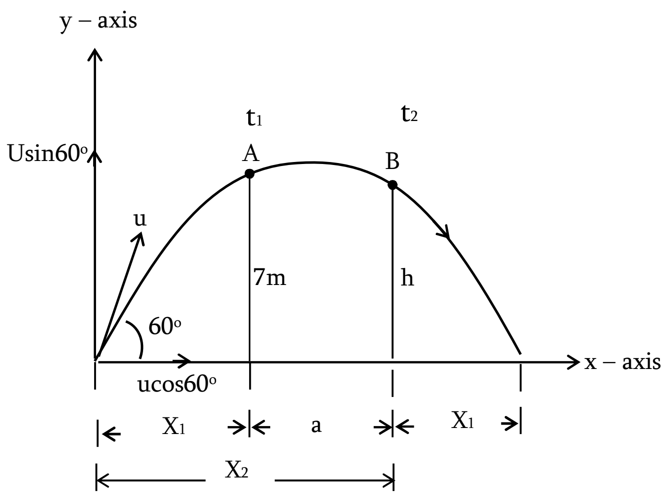
$$X = \frac{12h\tan\alpha}{2\tan^2\alpha} = 6h\cot\alpha$$

X = 6hcotα hence shown.

45. (a) (i) Define the term trajectory
(ii) Briefly explain why the horizontal component of the initial velocity of a projectile always remains constant
(b) (i) List down two limitations of projectile motion
(ii) A body projected from the ground at the angle 60° is required to pass just above the two vertical wall each of height 7m. If the velocity of the projection is 100m/s, calculate the distance between two walls
(c) Fireman stand at horizontal distance of 38m from edge of the burning storey building aimed to raise streams of water at an angle of 60° into the first floor through an open window which is at 20m high from the ground level. If water strikes on this floor 2m away from the outer edge.
(i) Sketch the diagram of the trajectory
(ii) What speed will the water leave the muzzle of the fire hose?

Solution

(a) (i) Refer to your notes
(ii) Because acceleration in the horizontal motion i.e in x – direction is equal to zero ($a_x = 0$)
(b) (i) see your notes



By using the equation of trajectory for oblique projectile

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

At point A or B

$$7 = x\tan 60^\circ - \frac{9.8x^2}{2\times 100^2\cos^2 60^\circ}$$

$$7 = 1.732X - 1.96\times 10^{-3}X^2$$

Solve quadratically for x

$X_1 = 4.06\text{m}$ or $X_2 = 879.63\text{m}$

Distance between the two vertical walls.

a = 879.63 – 4.06m

a = 875.57m

Alternative method

From the equation

$$y = (u\sin\theta)t - \frac{1}{2}gt^2 \qquad [g = 9.8\text{m/s}^2]$$

$$7 = (100\sin 60)t - 4.9t^2$$

On solving quadratically for t

$t_1 = 0.081$sec , $t_2 = 17.59$sec

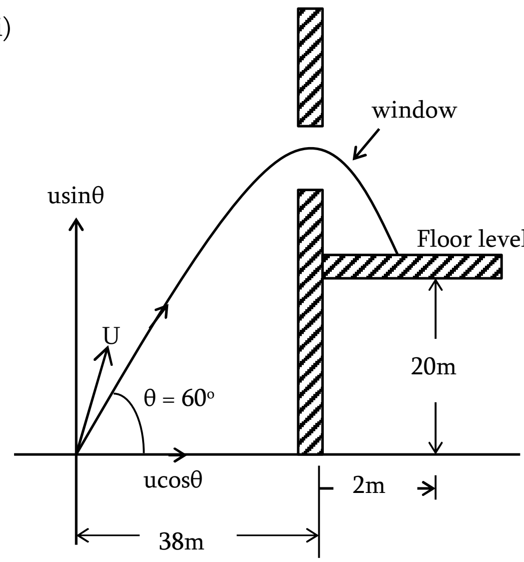
The distance between two walls

$$a = (t_1 - t_2)u\cos\theta$$

$$= (17.59 - 0.081)\times 100\cos 60^\circ$$

$$a = 875.55\text{m}$$

(d) (i)



$$X = 38 + 2 = 40$$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$20 = 40\tan 60^\circ - \frac{9.8(40)^2}{2(u\cos 60^\circ)^2}$$

(ii) By using equation of the trajectory

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$20 = 40\tan 60^O - \frac{9.8(40)^2}{2(u\cos 60^O)}$$

On solving for u , u = 25.2m/s

46. (a) (i) List down two main assumptions in deriving the equation of projectile motion

(ii) Why the horizontal motion of a projectile is constant ?

(b) A ball is thrown horizontally with a speed of 14m/s from a point 6.4m above the ground.

Calculate

(i) The horizontal distance travel in that time

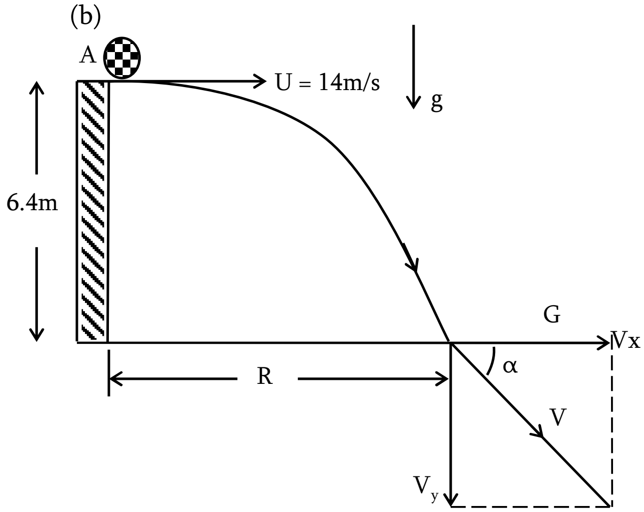
(ii) It's velocity when it reaches the ground?

(c) A man stands in a lift which is being accelerated upward at 3.2m/s². If the man has a mass of 65kg, what is the net force exerted on the man by the floor of the lift?

Solution

(a) refer to your notes

(b)



(i) Flight time

$$T = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 6.4}{9.8}}$$

$T = 1.143\,\text{sec}$

The horizontal distance travelled

$R = UT = 1.143\times 14$

$R = 16\text{m}$

(ii) V = Velocity of the ball reached on the ground

$$V = \sqrt{V_x^2 + V_y^2}$$

$V_x = U = 14\text{m/s}$

$V_y = (u\sin 0^\circ) - gt = -gt$

$V_y = -9.8\times 1.143$

$V_y = -11.2\text{m/s}$

$$V = \sqrt{(14)^2 + (-11.2)^2}$$

$V = 17.93\text{m/s}$

$$\tan\alpha = \frac{V_y}{V_x}, \quad \alpha = \tan^{-1}\left(V_y/V_x\right)$$

$$\alpha = \tan^{-1}\left[\frac{-11.2}{14}\right] = -38.7^\circ$$

Direction of V is 38.7° below the ground level.

Alternative

Apply the principle of conservation of mechanical energy.

$$(\text{p.e} + \text{k.e})_A = (\text{p.e} + \text{k.e})_G$$

$$mgh + \frac{1}{2}mu^2 = 0 + \frac{1}{2}mv^2$$

$$v = \sqrt{u^2 + 2gh}$$

$$= \sqrt{14^2 + 2\times 9.8\times 6.4}$$

$$v = 17.9\text{m/s}$$

47. (a) Distinguish the term projectile and trajectory as applied to projectile motion.

(b) A meteorite is traced by radar as it falls through the earth's atmosphere when its altitude is 3.0×10^4m. The screen shows that the meteorite is travelling with a velocity of 58.3m/s at an angle of 28.3° below the horizontal. In the absence of air and at re – entrance to the earth's atmosphere.

(i) How much time elapses before the meteorite strike the earth?

(ii) What is the magnitude and direction of the velocity of the meteorite just before the impact with the earth?

(c) (i) What are assumption made in the treatment of projectile motion?

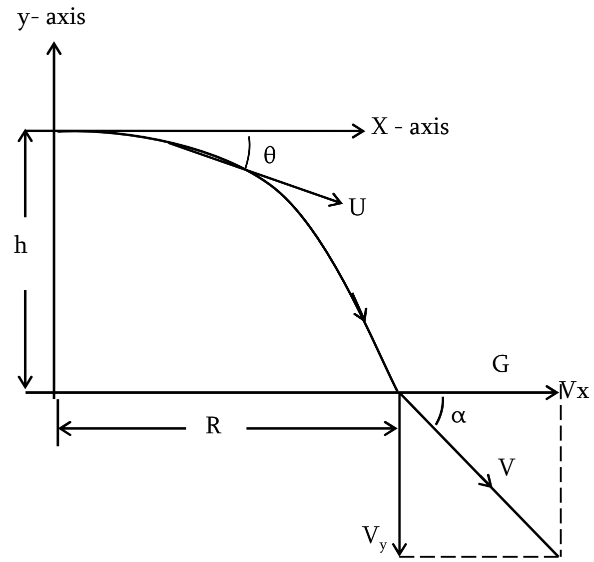
(ii) Find the angle of projection of a projectile at which the horizontal range and the maximum height have equal values.

Solution

(a) See your notes

(b) (i) Flight time

$$-h = (u\sin\theta)t - \frac{1}{2}gt^2$$



$$-30000 = \left(58.3\sin\left(-28.3\right)\right)t - 4.9t^2$$

$$4.9t^2 + 27.64t - 30000 = 0$$

On solving quadratically and omitting negative time

t = 75.48seconds

(ii) Apply the principle of conservation of mechanical energy

$$\frac{1}{2}mu^2 + mgh = \frac{1}{2}mv^2 + 0$$

$$v = \sqrt{u^2 + 2gh} = \sqrt{\left(5.3\right)^2 + 2g \times 30000}$$

V = 769m/s

Direction of V

$$\cos\alpha = \frac{V_x}{V}, \quad \alpha\cos^{-1}\left[\frac{V_x}{V}\right]$$

$$\alpha = \cos^{-1}\left[\frac{58.3\cos 28.3}{769}\right] = 86.2^{\circ}$$

(c) (i) see your notes

(ii) $\tan\theta = {}^{4H}\!/_{R}$

$$\theta = \tan^{-1}\left[\frac{4H}{R}\right] = \tan^{-1}\left[4\right]$$

θ = 75.96°.

48. (a) (i) Define the term range of a projectile.

(ii) Show that the range of a projectile R is given by the equation

$$R = \frac{u^2 \sin 2\theta}{g}$$

All symbols carry their usual meaning .

49. (a) (i) What is meant by term 'projectile' as applied to the projectile motion.

(ii) Give two (2) practical applications of projectile motion at your locality?

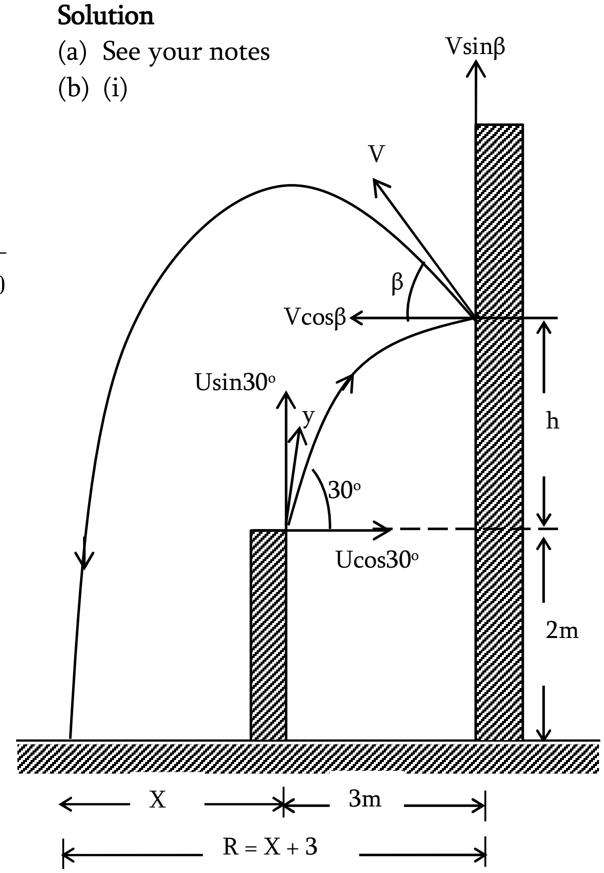
(b) (i) A ball is thrown on a vertical wall from a point 2m above the ground and 3m from the wall. The initial velocity of the ball is 20m/s at an angle of 30° above the horizontal. If the collision of the ball with the wall is perfectly elastic how far behind the thrower does the ball hit the ground?

(ii) The ceiling of a long hall is 25m high. Determine the maximum horizontal distance that a ball throw with a speed of 40m/s can go without hitting the ceiling of the wall.

Solution

(a) See your notes

(b) (i)



By using equation of trajectory

$$h = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$h = 3\tan 30^\circ - \frac{9.81\times 3^2}{2\times\left(20\cos 30^\circ\right)^2}$$

$h = 1.585m$

Vertical component of velocity V

$$V_y^2 = \left(u\sin\theta\right)^2 - 2gh$$

$$V_y^2 = \left(20\sin 30^\circ\right)^2 - 2\times 9.81\times 1.585$$

$$V_y = 8.303m/s$$

Also

$$V_x = 20\cos 30^\circ = 17.32m/s$$

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{\left(8.303\right)^2 + \left(17.321\right)^2}$$

$$V = 19.21m/s$$

$$\tan\beta = \frac{V_y}{V_x} = \frac{8.303}{17.321}$$

$$\beta = 25.61^\circ$$

Also

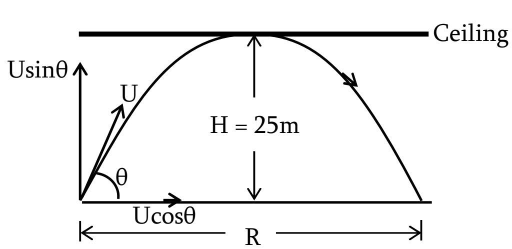
$$-\left(h+2\right) = \left(x+3\right)\tan\left(25.61\right) - \frac{9.81\left(x+3\right)^2}{2\times 9.81\times\left(19.21\right)^2}$$

$$-\left(1.585+2\right) = \left(x+3\right)\tan\left(25.61\right) - \frac{9.81\left(x+3\right)^2}{2\times 9.81\left(19.2\cos 25.61\right)^2}$$

on solving for $x = 32.533m$

(ii) We should adjust the angle of projection θ so that $H = 25m$

Since $H = \dfrac{U^2\sin^2\theta}{2g}$



$$\sin^2\theta = \frac{2gH}{U^2}, \quad \theta = \sin^{-1}\left[\sqrt{\frac{2gH}{U^2}}\right]$$

$$\theta = \sin^{-1}\left[\sqrt{\frac{2\times 9.81\times 25}{40\times 40}}\right]$$

$$\theta = 33.6^\circ$$

Horizontal range

$$R = \frac{U^2\sin 2\theta}{g} = \frac{40^2\sin\left(2\times 33.6\right)}{9.81}$$

$$R = 150.50m$$

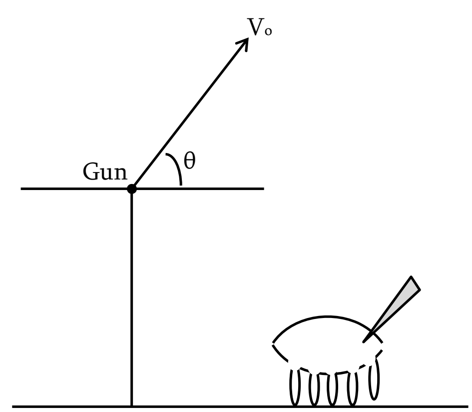
50. (a) What is projectile?

(b) A body of 2.5gm mass is pushed horizontally and rolls off from the top of a stair way with the velocity of 4.5m/s.

(i) Which step will this body hit first if the steps are 30cm high and 30cm wide

(ii) Determine the time taken to hit the above obtained step

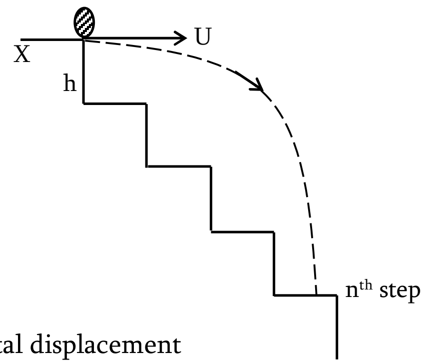
(c) An antitank gun is located on the edge of plateau that is 60m above the surrounding plain. The gun screw sights on the plain on a horizontal distance of 2.2km from the gun. At the same moment the tank crew see the gun , he starts to move directly away from it with an acceleration of 0.9m/s2. If the autitank fires a shell with the muzzle velocity of 240m/s at an elevation of 10o above the horizontal , how long should the gun screw wait before firing if , the shell has to hit the tank



Solution

(a) Refer to your notes

(b) (i) Diagram



Horizontal displacement

$$X = ut \quad , \quad t = x/u$$

Vertical displacement

$$h = \frac{1}{2}gt^2 = \frac{1}{2}g\left[\frac{u}{x}\right]^2$$

$$h = \frac{9.8x^2}{2(4.5)^2} = 0.241975x^2$$

But $h = 0.3n$

$$0.3n = 0.241975(0.3n)^2$$

On solving $n \simeq 14$steps(approx.)

$\therefore$ the steps which the body will hits first is 14steps.

(ii) From the equation

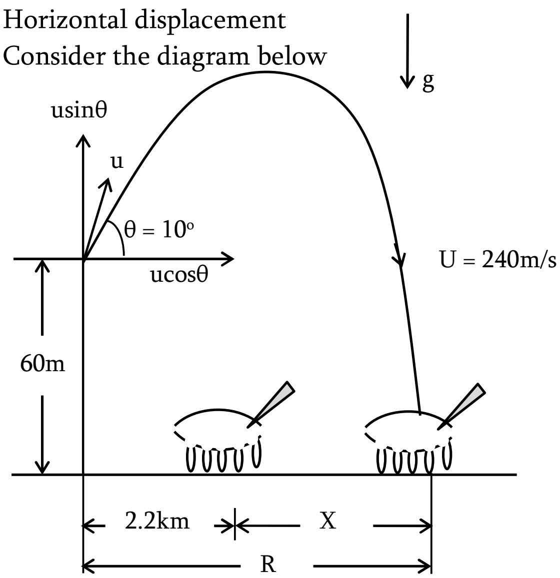
$$t = \frac{u}{x} = \frac{0.3n}{u}$$

$$t = \frac{0.3 \times 14}{4.5}$$

$$t = 0.9184\,\text{sec}$$

Horizontal displacement

(c) Consider the diagram below



Time taken by the projectile to reach on the ground.

$$-h = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$-60 = (240\sin 10^O)t - 4.9t^2$$

$$4.9t^2 - 41.676t - 60 = 0$$

On solving quadratically and omitting negative sign $t = 9.7599$sec.

Horizontal displacement travelled by the projectile.

$$R = (240\cos 10^O) \times 9.7599$$

$$R = 2,306.790045\text{m}$$

But $R = X + 2200$

$$X = R - 2200$$

$$X = 106.7900453\text{m}$$

The time taken by the tank to more this distance as it accelerates with acceleration, $a = 0.9\text{m/s}^2$.

$$S = X = \frac{1}{2}at_1^2$$

$$t_1 = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2 \times 106.7900453}{0.9}}$$

$$t_1 = 15.40\,\text{sec}$$

So the time to wait before firing to the tank is

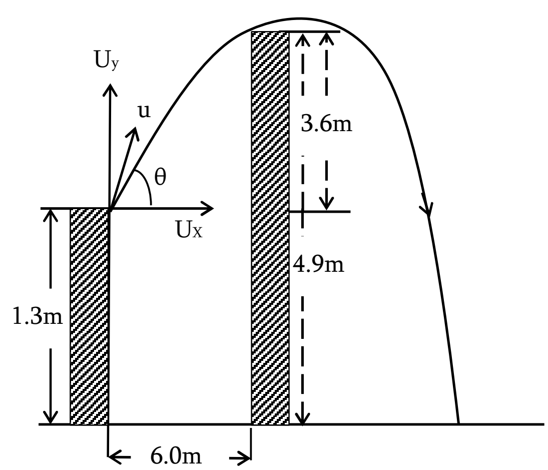
$$\Delta t = t_1 - t$$

$$= 15.40 - 9.7599$$

$$\Delta t = 5.65\,\text{sec}$$

51. A ball is thrown with a velocity whose horizontal component is 12m/s from a point1.3m above the ground and 6m away from a vertical wall4.9m high is such away as just clear the wall. At what time will reach the ground?

Solution



From the equation of the trajectory

$$y = x\tan\theta - \frac{gx^2}{2\left(u\cos\theta\right)^2}$$

But: $u\cos\theta = 12m/s$, $x = 6$, $y = 3.6m$

$$6\tan\theta = \frac{9.8\times 36}{2\times 12^2} + 3.6$$

$$\tan\theta = 4.825$$

$$\theta = 78.3^{O}$$

Also : $u\cos = u_x$

$$u = \frac{u_x}{\cos\theta} = \frac{12}{\cos\left(78.3^{O}\right)}$$

$$u = 59.18m/s$$

From the equation of the vertical displacement

$$y = \left(u\sin\theta\right)t - \frac{1}{2}gt^2$$

$$-1.3 = \left(59.18\sin 78.3\right)t - 4.9t^2$$

On solving quadratically and omitting negative time $t = 2.1sec$

52.(a) (i) What is meant by the terms projectile and trajectory?

(ii) A package of medical supplies is released from a small plane flying over an isolated jungle settlement. The plane flies horizontal with a speed of 20m/s at an altitude of 20m. Where will the package strike the ground?

(b) A ball is thrown with an initial velocity Vo of 48m/s directed at angle θ of 37° with the vertical find:-

(i) The x – and y – components of V_o

(ii) The position of the ball and the magnitude and direction of its velocity when t = 2sec

(iii) The highest point of the ball and time taken to reach there

(iv) Range of the ball

Solution

(a) (i) See your notes

(ii) The time taken by the package to reach on the ground

$$h = \frac{1}{2}gt^2$$

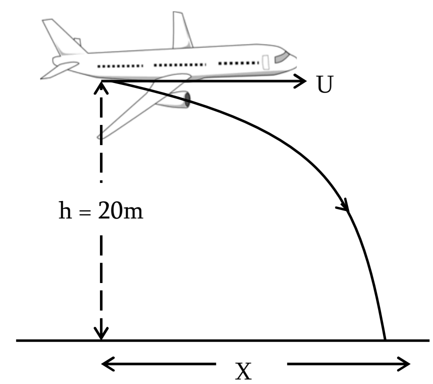
$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 20}{9.8}}$$

$$t = 2.0203\,sec$$

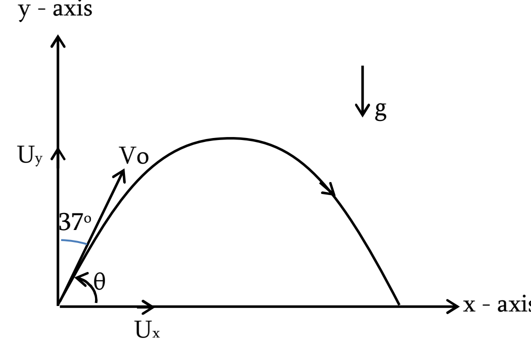
The horizontal distance

$$x = ut = 20\times 2.0203$$

$$x = 40.41m$$



(b)



$$\theta = 90^{\circ} - 37^{\circ} = 53^{\circ}$$

(i)

$$U_x = V_o\cos\theta = 48\cos 53^{\circ} = 28.89m/s$$

$$U_y = V_o\sin\theta = 48\cos 53^{\circ} = 38.33m/s$$

(ii) after t = 2sec

$$y = \left(V_o\sin\theta\right)t - \frac{1}{2}gt^2$$

$$= 38.33\times 2 - \frac{1}{2}\times 9.8\times 2$$

$$y = 57.06m$$

$$x = u_x t = 28.89\times 2 = 57.78m$$

Position of particle after 2sec (5778m , 57.06m)

$$V = \sqrt{V_0^2 + g^2t^2 - 2gtV_o \sin\theta}$$

$$= \sqrt{(48)^2 + (9.8\times2)^2 - 2\times9.8\times48\times2\sin53^\circ}$$

$V = 34.43m/s$

Direction of V

$$\tan\beta = \tan\theta - \frac{gt}{V_o\cos\theta}$$

$$= \tan 53^\circ - \frac{9.8\times2}{48\cos 53^\circ}$$

$$\beta = \tan^{-1}\left[\tan 53^\circ - \frac{9.8\times2}{48\cos53^\circ}\right]$$

$$\beta = 32^\circ 57'$$

(iii) maximum height attained

$$H = \frac{(V_o\sin\theta)^2}{2g} = \frac{U_y^2}{2g}$$

$$H = \frac{(38.33)^2}{2\times9.8}$$

$H = 74.96m$

Time to reach the maximum height

$$t_m = \frac{V_o\sin\theta}{g} = \frac{48\sin53^\circ}{9.8}$$

$t_m = 3.9\,sec$

(iv) horizontal range

$$R = \frac{V_o^2\sin2\theta}{g} = \frac{48^2\sin(2\times53^\circ)}{9.8}$$

$R = 226m$

53. (a) (i) Give two examples of projectiles and describe their trajectory in a certain coordinate system.

(ii) Show that the maximum range of a projectile of fixed initial speed U is obtained when it is launched at an angle of 45° to the horizontal. (Ignore the effect of air resistance)

(b) A stone is projected horizontally with velocity 3m/s from the top vertical cliff 200m high. Calculate:

(i) The time it takes to reach on the ground

(ii) Its distance from the foot of the cliff

(iii) Its vertical and horizontal components of velocity when it hits the ground.

Answer : (b)(i) 6.4sec (ii) 19.2m (iii) $V_x = 3m/s$, $v_y = 62.6m/s$

54. (a) Show that the trajectory of a body projected with an initial velocity V at an angle θ to the horizontal is a parabola.

(b) Which standing on an open truck moving at a velocity of 35m/s a man sees a duck flying directly overhead. The man shoots an arrow at the truck and misses it. The arrow leaves the bow with a vertical velocity of 98m/s the truck acceleration to a constant speed of 40m/s in the same direction just after the mane has shot at the duck if the truck open board at which the man is standing 2.0m above the ground

(i) How long will the arrow remain in air before hitting the ground.

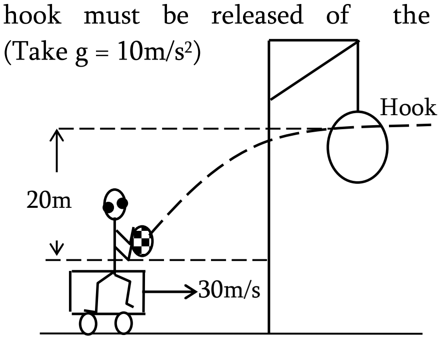
(ii) When will the arrow land in relation to the position of the truck?

55. A man riding on a small flat car travelling with a constant velocity of 30m/s. He wishes to thrown a ball through a stationary hook 20m above his hand such that the ball moving horizontally as it passes just through the hook he through a ball with velocity of 40m/s with respect to himself as shown on the diagram below.

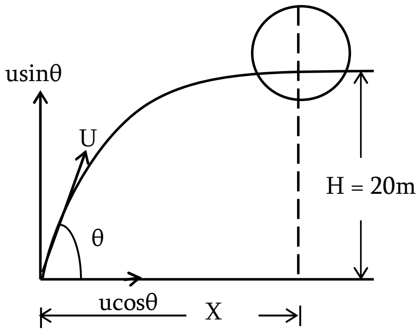
(a) what must be the vertical component of the initial velocity of the ball

(b) Find the time of flight

(c) At what horizontal distance in front of the hook must be released of the ball? (Take $g = 10m/s^2$)



Solution



(a) From the equation

$$V_y^2 = U_y^2 - 2gy$$

When the ball passes the hook horizontally, $V_y = 0$, $y = H$

$$0^2 = U_y^2 - 2gH$$

$$2gH = U_y^2$$

$$U_y = \sqrt{2gH}$$

$$= \sqrt{2\times10\times20} = 20m/s$$

∴ The initial velocity component of the velocity Uy = 20m/s

(b) Flight time , T

$$T = \frac{2u\sin\theta}{g} = \frac{2u_y}{g}$$

$$= \frac{2\times20}{10} = 40\,\text{sec}$$

$$T = 4.0\,\text{sec}$$

(c) The horizontal distance

$$X = \left(V_o + U\cos\theta\right)\frac{T}{2}$$

Since $U_y = U\sin\theta$, $\sin\theta = U_y/U$

$$\theta = \sin^{-1}\left[\frac{U_y}{U}\right] = \sin^{-1}\left[\frac{20}{40}\right]$$

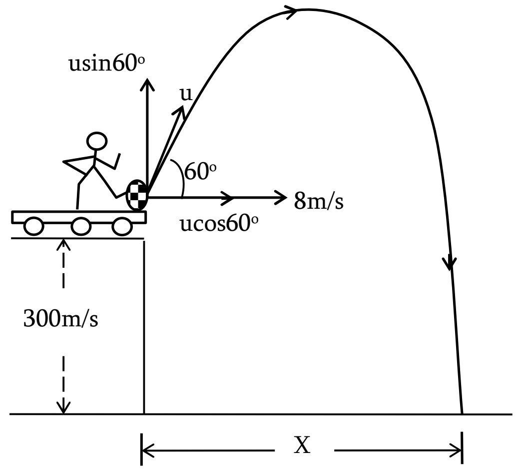
$$\theta = 30^\circ$$

$$X = \left[30 + 40\cos30^\circ\right]\times2$$

$$X = 129.30m$$

56. A man is standing on horizontal convey belt moving at 8m/s which is on the top of building 300m high. Then he throwing a ball at 40m/s at an angle of 60° with horizontal. Calculate:-
   (i) Time taken to hits the ground
   (ii) The horizontal distance covered by the ball

Solution



(i) From the equation

$$h = \left(u\sin\theta\right)t - \frac{1}{2}gt^2$$

$$-300 = \left(40\sin60^\circ\right)t - \frac{1}{2}\times10t^2$$

$$5t^2 - \left(20\sqrt{3}\right)t - 300 = 0$$

On solving quadratically and omitting negative time t = 11.95sec.

(ii) Horizontal displacement

$$X = \left(V_o + u\cos\theta\right)t$$

$$X = \left(8 + 40\cos60^\circ\right)\times11.95$$

$$X = 334.6m$$

57. The range of a rifle bullet is 1000m where θ is the angle of projection. If the bullet is fired with the same angle from a car travelling at 36km/hr towards the target , show that the range will be increased by $\frac{1000}{7}\sqrt{\tan\theta}, m$

Solution

The horizontal range is given by

$$R = \frac{2U^2 \sin\theta\cos\theta}{g} \quad \text{but} \quad R = 1000$$

$$1000 = \frac{2u^2 \cos\theta\sin\theta}{g}$$

$$1000 = \frac{2}{g}(u\sin\theta)(u\cos\theta) \text{........(i)}$$

When the bullet is fired from a car travelling at 36km/hr (= 10m/s) towards the target , the horizontal component of initial velocity relatively to the car becomes (10 + ucosθ)m/s , while the vertical component remain unchanged

$$R_1 = \frac{2}{g}u\sin\theta(10 + u\cos\theta)$$

$$R_1 = \frac{2u^2\sin\theta\cos\theta}{g} + \frac{20u\sin\theta}{g}$$

$$R_1 = R + \frac{20u\sin\theta}{g}$$

$$R_1 - R = \frac{20u\sin\theta}{g} \text{..........(ii)}$$

From the equation (i)

$$1000 = \frac{2u^2\sin\theta\cos\theta}{g}$$

$$u = \sqrt{\frac{1000}{2\sin\theta\cos\theta}}$$

Now, equation (ii) becomes

Let ;

$$R_1 - R = \Delta R$$

$$\Delta R = \frac{20\sin\theta}{g}\sqrt{\frac{1000}{2\sin\theta\cos\theta}}$$

$$= \sqrt{1000} \times 20\sqrt{\frac{g\sin^2\theta}{g^2\sin\theta\cos\theta}}$$

$$= \sqrt{1000} \times \sqrt{\frac{400}{g}} \cdot \sqrt{\tan\theta}$$

$$= \sqrt{\frac{1000 \times 1000 \times 0.4}{g}} \cdot \sqrt{\tan\theta}$$

$$= 1000\sqrt{\frac{0.4}{g}} \cdot \sqrt{\tan\theta}$$

$$\Delta R = \frac{1000}{7} \cdot \sqrt{\tan\theta}$$

58. (a) State whether the following statement is TRUE or FALSE 'A projectile fired from the ground follows a parabolic path. The speed of the projectile is minimum at the top of the path'. Give reasons in support of your answer.

(b) In the long jumping , does matter how high you jump? Why factors determine the span of the jump?

Solution

(a) The statement is true. The velocity of the projectile fired with a velocity u making angle θ with the horizontal has two components. The horizontal component is the ucosθ and the vertical component is usinθ. The horizontal component remain constant throughout the motion whereas the vertical component goes on decreasing and ultimate because zero at the top of the path at this position , the net velocity of the projectile is minimum.

(b) If can athlete takes jump with velocity u and an angle θ horizontal

$$H = \frac{u^2\sin^2\theta}{2g} \quad \text{and}$$

$$R = \frac{2u^2\sin\theta\cos\theta}{g}$$

$$\frac{R}{H} = \frac{2u^2\sin\theta\cos\theta}{g} \Big/ \frac{u^2\sin^2\theta}{2g}$$

$$R = 4H\cot\theta$$

Yes. In long jumping , it does matter how high you jump. Its follows that the length of the jump (R) depends on the following factors:-

- Height of jump
- Angle of projection , θ
- Since H it depends on U , one can say that length of jump depends on the velocity U of the jump and angle , θ.

59. (a) Mention two motions that add up to make up projectile motion.

(b) (i) In long jump does it matter how high you jump? State the factors which determine the span of the jump?

(ii) Derive an expression that relates the span of the jump and the factors you have mentioned.

(c) A bullet is fired from a gun on the top of a cliff 140m high with a velocity of 150m/s at an angle or elevation of 30^{0} to the horizontal. Find the horizontal distance from the roof of a cliff to the point where the bullet lands on the ground.

Answer : (c) 2206.804m

60. A ball is thrown from a point distant 'a' from a smooth vertical wall against the wall , and return to the point of projection. Prove that the velocity , U of projection and the elevation α of projection are connected by the equation.

$$u^2 \sin 2\alpha = ag\left[\frac{1+e}{e}\right]$$

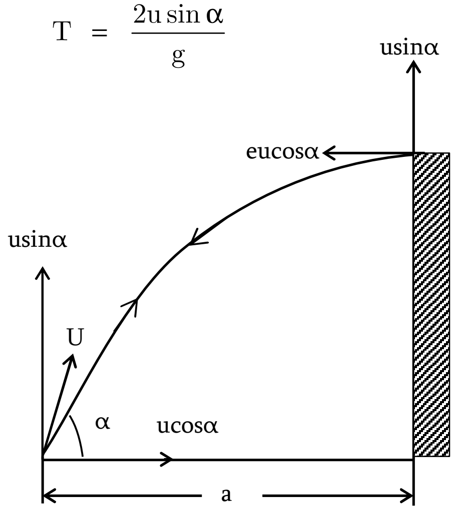
Where e is the coefficient of restitution between the ball and the wall.

Solution

Since the wall is smooth , the vertical component of the velocity is unaffected by the impact.

The flight time is given by

$$T = \frac{2u\sin\alpha}{g}$$



The ball approaches the wall with horizontal velocity ucosα and rebounds with horizontal velocity , eucosα

$$e = \frac{\text{velocity of separation}}{\text{velocity pf approach}}$$

$$e = \frac{V_{sep}}{U\cos\alpha}$$

$$V_{sep} = eu\cos\alpha$$

Let t_1 and t_2 be time taken to reach on the wall and rebound a horizontal , distance , a respectively.

$$t_1 = \frac{a}{u\cos\alpha}, \quad t_2 = \frac{a}{eu\cos\alpha}$$

$$T = t_1 + t_2$$

$$T = \frac{a}{u\cos\alpha} + \frac{a}{eu\cos\alpha}$$

$$T = a\left[\frac{1+e}{eu\cos\alpha}\right]$$

But : $$T = \frac{2u\sin\alpha}{g}$$

$$\frac{2u\sin\alpha}{g} = a\left[\frac{1+e}{eu\cos\alpha}\right]$$

$$2u^2\sin\alpha\cos\alpha = ag\left[\frac{1+e}{e}\right]$$

$$u^2\sin 2\alpha = ag\left[\frac{1+e}{e}\right]$$

Hence shown.

61. A ball is thrown with a speed of 19.6m/s at an angle of elevation of 45^{0}. It strike a vertical wall 9.8m away and returns to the point of projection. Find the coefficient of the restriction between the ball and the wall.

Solution

From the equation

$$u^2\sin 2\alpha = ag\left[\frac{1+e}{e}\right]$$

$$eu^2\sin 2\alpha = ag + age$$

$$eu^2\sin 2\alpha - age = ag$$

$$e\left[u^2\sin 2\alpha - ag\right] = ag$$

$$e = \frac{ag}{u^2\sin 2\alpha - ag}$$

$$e = \frac{9.8\times 9.8}{(19.6)^2\sin(2\times 45) - 9.8\times 9.8}$$

$$e = \frac{1}{3} = 0.33$$

62.(a) (i) Define the term range of projectile

(ii) Show that the horizontal motion of a projectile is not affected by time

(iii) Prove that , the time of flight T and the horizontal range X a projectile are connected by the equation.

$$gT^2 = 2x\tan\theta$$

(b) A rifle is aimed horizontally at a target 130m away. The bullet hits the target 0.75m below the aiming point. What is the muzzle speed of the bullet?

63. If T_1 and T_2 are the times of flight of a projectile for two angle of projection θ_1 and θ_2 such that $\theta_1 + \theta_2 = 90^\circ$ show that

$$T_1T_2 = \frac{2R}{g}$$

where R is the horizontal range of the projectile.

Solution

Flight time , $T = \frac{2u\sin\theta}{g}$

Given that $\theta_1 + \theta_2 = 90^\circ$

$\theta_2 = 90^\circ - \theta_1$

For the angle of projection

θ_1 , the flight time , $T_1 = \frac{2u\sin\theta_1}{g}$

for the angle of projection θ_2

$$T_2 = \frac{2u\sin\theta_2}{g} = \frac{2u\sin(90-\theta_1)}{g}$$

$$T_2 = \frac{2u\cos\theta_2}{g}$$

$$T_1T_2 = \left(\frac{2u\sin\theta_1}{g}\right)\left(\frac{2u\cos\theta_1}{g}\right)$$

$$T_1 \cdot T_2 = \frac{2}{g}\left[\frac{2u^2\sin\theta_1\cos\theta_1}{g}\right]$$

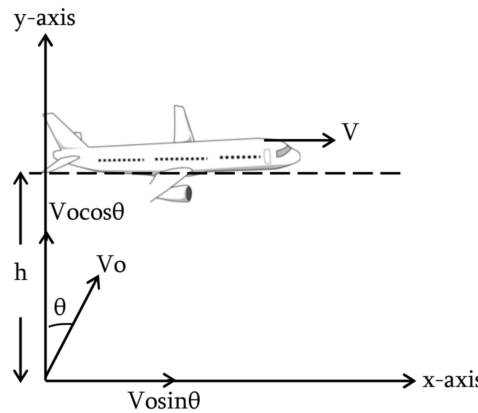
$$T_1T_2 = \frac{2R}{g}$$

64. A fighter plane flying horizontally at an altitude of 1.5km with speed 720km/hr passes directly overhead on anti – air craft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed 600m/s to hit the plane? At what minimum altitude should the pilot fly the plane to avoid being hit? ($g = 10m/s^2$).

Solution

$$V = 720kmhr^{-1} = \frac{720\times1000}{3600} = 200m/s$$

$$V_o = 600m/s \quad , \quad h = 1.5km = 1500m$$



When the plane is just above the gun let the shell be fired at an angle θ with the vertical so as to hit the plane in time t seconds. The horizontal distance travelled by the plane (= vt) will be equal to the horizontal distance covered by the shell $\left[= (V_o\sin\theta)t\right]$

$$Vt = (V_o\sin\theta)t$$

$$\sin\theta = \frac{V}{V_o} = \frac{200}{600} = \frac{1}{3}$$

$$\theta = \sin^{-1}\left(\frac{1}{3}\right) = 19.5^O$$

The maximum height to which rises

$$H = \frac{V_o\sin^2(90-\theta)}{2g} = \frac{V_0^2\cos^2\theta}{}$$

$$= \frac{(600)^2}{2\times9.8}\left[\cos19.5\right]^2$$

$$H = 16300m = 16.3km$$

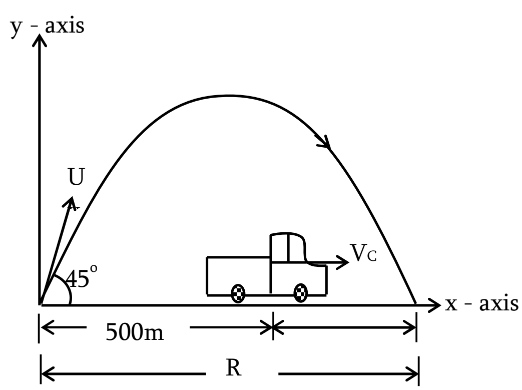
∴ The pilot should fly the plane at a minimum height of 16.3km to avoid being hit by the shell.

65. A gun kept on a horizontal straight road is used to hit a car travelling along the same road away from the gun with a uniform speed of 72km/hr. The car is at a distance of 500m from the gun when the gun is fired at an angle of 45⁰, with the horizontal find:-
(i) The distance of the car form the gun when the shell hit it.
(ii) The speed of projection of the shell from the gun.

Solution

The car is travelling at a constant speed $V_c = 72\text{kmhr}^{-1} = 20\text{m}/\text{s}$ when the car is at a distance of 500m from the gun, the shell is fired with initial velocity , U making an angle of 45° with horizontal

Let: t = time taken by the shell to hit the car



The car travelled a horizontal distance x in time, t

Horizontal range of the shell

R = 500 + x = 500 + 20t ....(i)

Flight time of the shell

$$T = \frac{2u\sin\theta}{g} = \frac{2u\sin 45^\circ}{g} = \frac{\sqrt{2}u}{g}$$

$$t = T = \frac{\sqrt{2}U}{g} \text{ ....(ii)}$$

Putting equation (ii) into (i)

$$R = 500 + \frac{20\sqrt{2}U}{g} \text{ ....(iii)}$$

Range of the projectile

$$R = \frac{U^2\sin(2\times45^\circ)}{g} = \frac{U^2}{g} \text{ ....(iv)}$$

(iv) = (iii)

(i) $500 + \frac{20\sqrt{2}U}{9.8} = \frac{U^2}{9.8}$

On solving U = 85.56m/s

(ii) $R = \frac{U^2}{g} = \frac{(85.56)^2}{9.8}$

$R = 746.9\text{m}$

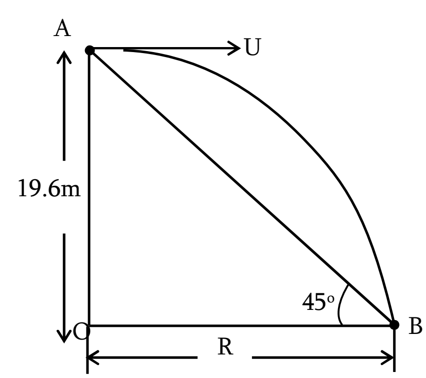
PROJECTILE MOTION INVOLVED TWO OR MORE COLLIDES BODIES WHEN ARE THROWN SIMULTANEOUSLY.

The following are the problems solving techniques for projectile motion of two bodies thrown simultaneously.

(i) Identify a common parameter for both projectile eg. Distance , time
(ii) Formulate equations for the common parameter for both bodies.
(iii) Equate the two equations and solve for unknown.
(iv) If the bodies collide their initial vertical velocities are the same horizontal plane.
(v) If the bodies collide their initial horizontal velocities are the same , if they are projected from the same vertical plane.

66. From the top of a building 19.6m high a ball is projected horizontal after how long does it strike ground if the line joining the point of projection to the point where it hits the ground makes an angle of 45° with the horizontal what is initial velocity of the ball? of 60°. How far from A is the point of collision and after what time will this happen?

Solution



From the figure above

$$\tan 45^\circ = \frac{\overline{OA}}{\overline{OB}} = \frac{19.6}{R}$$

$$R = 19.6\text{m}$$

Let t = time taken by the ball to hits on the ground.

$$h = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 19.6}{9.8}}$$

$$t = 2\,\text{sec}$$

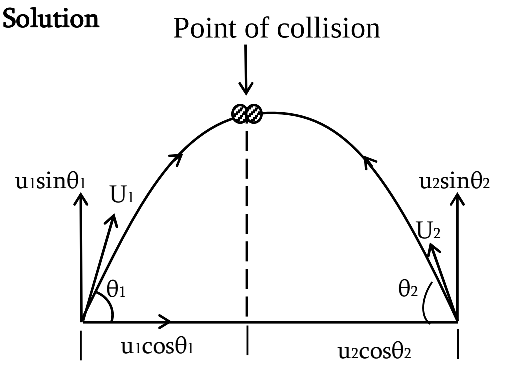
Since $R = ut \;,\; u = R/t$

$$u = \frac{19.6}{2}$$

$$u = 9.8\text{m}/\text{s}$$

67. A and B are two points on the ground 30m apart. A TOY SCUD missle is fired from A towards B with a speed of 30m/s at an elevation of 45° to the horizontal while simultaneously a TOY PATRIOT missle fired from B of 60° how far from A is a point of collision after what time will this happen?

Solution



$\theta_1 = 45^\circ, \theta$ … $^\circ, U_1 =$ … $= 20\text{m}/\text{s}$

Time taken by the two projectiles from the point of projections to the point of the collision are the same.

The horizontal distance covered by the SCUD

$$x = \left(U_1 \cos\theta_1\right)t$$

Horizontal distance covered by PATROIT

$$30 - x = \left(u_2 \cos\theta_2\right)t \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

Dividing equation (i) by (ii)

$$\frac{x}{30-x} = \frac{\left(u_1\cos\theta_1\right)t}{\left(u_2\cos\theta_2\right)t}$$

$$\frac{x}{30-x} = \frac{u_1\cos\theta_1}{u_2\cos\theta_2} = \frac{30\cos 45^\circ}{20\cos 60^\circ}$$

$$\frac{x}{30-x} = 2.12$$

On solving $x = 20.38\text{m}$

Again $x = \left(u_1\cos\theta_1\right)t$

$$t = \frac{x}{u_1\cos\theta_1} = \frac{20.38}{30\cos 45^\circ}$$

$$t = 0.96\,\text{sec}$$

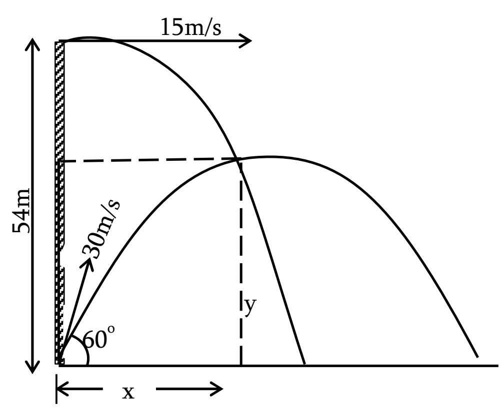
68. Stone A is projected horizontally from the top of the tower 54m at a velocity of 15m/s. At the same instant stone B is projected from the bottom of the tower with a velocity of 30m/s at an angle of 60° to the horizontal find:-

(i) The height above the ground where the stone collide.

(ii) The horizontal distance from the tower where the collision occurs.

Solution

The common quantities described by both stones are x , y and t



(i) for the stone A

vertical motion at the point of collision

$$-\left(54 - y\right) = \left(u\sin 0^\circ\right)t - \frac{1}{2}gt^2$$

$$54 - y = \frac{1}{2}gt^2$$

$$y = 54 - \frac{1}{2}gt^2 \;\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

For the stone B at the point of collision

$$y = \left(u_2 \sin 60^\circ\right)t - \frac{1}{2}gt^2$$

$$y = \left(30 \sin 60^\circ\right)t - \frac{1}{2}gt^2 \;....(ii)$$

(i) = (ii)

$$\left(30 \sin 60\right)t - \frac{1}{2}gt^2 = 54 - \frac{1}{2}gt^2$$

$$t = \frac{54}{30 \sin 60^\circ}$$

$$t = 2.078\,\text{sec}$$

From equation (i)

$$y = 54 - \frac{1}{2} \times 9.8 \times \left(2.078\right)^2$$

$$y = 32.83\text{m}$$

(ii) The horizontal distance , x

$$X = \left(U_2 \cos 60^O\right)t$$

$$X = \left(30 \cos 60^O\right) \times 2.078$$

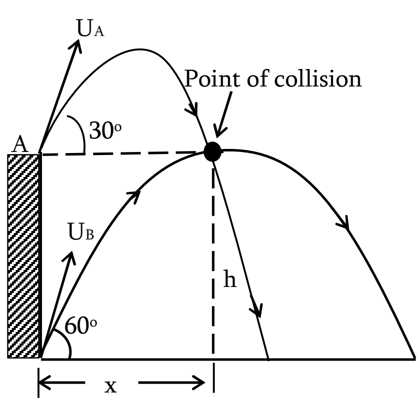
$$X = 31.17\text{m}$$

69. Two particles A and B projected at the same time in the same vertical plane. A is projected at the height 2m above the ground making an angle 30^O with the horizontal. B is projected from the ground with a velocity of 20m/s making an angle of 60^O with the horizontal find:-

(i) The initial velocity of A

(ii) The horizontal distance travelling before A and B collides.

Solution



The common parameters are X, h and t

(i) For the horizontal motion

For particle A

$$X = \left(U_A \cos 30^\circ\right)t \;.......................(i)$$

For particle B

$$X = \left(U_B \cos 60^O\right)t \;......................(ii)$$

(i) = (ii)

$$\left(U_A \cos 30^O\right)t = \left(U_B \cos 60^O\right)t$$

$$U_A = \frac{U_B \cos 60^O}{\cos 30^O} = \frac{20 \cos 60^O}{\cos 30^O}$$

$$U_A = 11.55\text{m / s}$$

(b) For the vertical motion

For particle A

$$y = \left(U_A \sin 30^O\right)t - \frac{1}{2}gt^2$$

$$y = \left(11.55 \sin 30^O\right)t - 4.9t^2 \;........(i)$$

For the particle B

$$h + y = \left(20 \sin 60^O\right)t - \frac{1}{2}gt^2$$

$$2 + y = 17.32t - 4.9t^2$$

$$y = 17.3t - 2 - 4.9t^2$$

$$17.3t - 2 - 4.9t^2 = 5.775t - 4.9t^2$$

$$t = \frac{2}{17.3 - 5.775} = 0.173\,\text{sec}$$

Horizontal distance

$$x = \left(20 \cos 60^O\right) \times 0.173$$

$$x = 1.73\text{m}$$

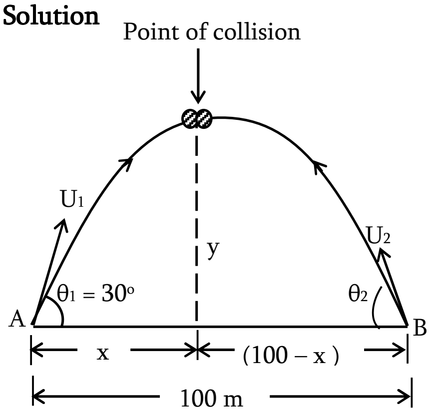
70. A particle is projected from point A with an initial velocity of 60m/s at 30^O to the horizontal. At the same time a second particle is projected in the opposite direction with an initial velocity of 50m/s from point B level with A and 100m form A.

(i) Find the angle of projection of the second particle if they collide.

(ii) The time at which collision occurs

(iii) The horizontal distance from A to where collisions occurs.

Solution



(i) For the vertical motion
Vertical displacement

For A: $y = \left(60\sin 30\right)t - \frac{1}{2}gt^2$

$y = 30t - 4.9t^2$ ....(i)

For B: $y = \left(50\sin\theta_2\right)t - \frac{1}{2}gt^2$

$y = \left(50\sin\theta_2\right)t - 4.9t^2$ ......(ii)

(i) = (ii)

$\left(50\sin\theta_2\right)t - 4.9t^2 = 30t - 4.9t^2$

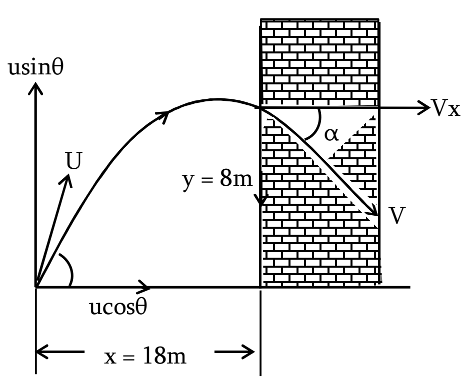
$\left(50\sin\theta_2\right) = 30$

$\theta_2 = \sin^{-1}\left[\frac{30}{50}\right] = 36.87^O$

$\theta_2 = 36.87^\circ$

71. A ball thrown at an angle of 60° above the horizontal strikes a building 18m away at a point 8m above the point from which it is thrown.
(a) Find the magnitude of the initial velocity of the ball
(b) Find the magnitude and direction of the ball just before it strikes the building.

Solution



(a) horizontal displacement

$x = \left(u\cos\theta\right)t$

$t = \frac{x}{u\cos\theta} = \frac{18}{u\cos 60^O} = \frac{36}{u}$

$t = \frac{36}{u}$

For the vertical motion

$y = \left(u\sin\theta\right)t - \frac{1}{2}gt^2$

$8 = \left(u\sin 60^\circ\right)\frac{36}{u} - 4.9t^2$

$8 = \left(u\sin 60^O\right)\frac{36}{u} - 4.9\left(\frac{36}{u}\right)^2$

$8 = 31.177 - \frac{6350.4}{u^2}$

On solving u = 16.55m/s

(b) horizontal velocity just before striking the wall

$V_x = u\cos\theta = 16.55\cos 60^O$

$V_x = 8.28m/s$

Vertical velocity just before striking the wall

$V_y^2 = \left(u\sin\theta\right)^2 - 2gy$

$= \left(16.55\sin 60^O\right) - 2\times 9.8\times 8$

$V_y = 6.978m/s$

Magnitude of V

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{\left(8.28\right)^2 + \left(6.978\right)^2}$

$V = 10.5m/s$

Direction of V

$\alpha = \tan^{-1}\left[\frac{V_y}{V_x}\right] = 38.3^O$

$\alpha = 38.3^O$

72. A ball, 26.9m from the goal post is kicked with an initial velocity of 19.8m/s at an angle θ above the ground. Between what two angle θ_1 and θ_2 will the ball clear the 2.74m high crossbar?

Solution

From the equation

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$2.74 = 26.9\tan\theta - \frac{9.8(26.9)^2}{2(19.8)^2\cos^2\theta}$$

$$2.74 = 26.9\tan\theta - 9.044\sec^2\theta$$

But : $\sec^2\theta = 1+\tan^2\theta$

$$2.74 = 26.9\tan\theta - 9.044(1+\tan^2\theta)$$

$$9.044\tan^2\theta - 26.9\tan\theta + 11.78 = 0$$

On solving for θ

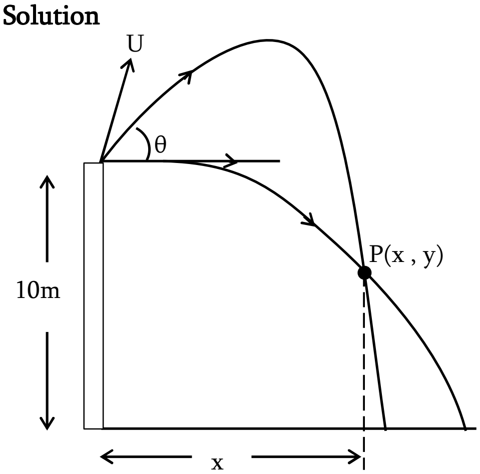
θ = 67.7° or θ = 28°.

73. Two guns, situated on the top of a hill of height 10m, fire one shot each with the same speed $5\sqrt{3}$ m/s at the some interval of time one gun fires horizontally and the other fires up wards at an angle 60 horizontal the shoot collides in air at point p. Find

(i) the time interval between the firings

(ii) the coordinate of point p. Take origin of the coordinate system at the foot of hill right below the muzzle and trajectories in xy-plane.

Solution



(i) Let the time taken by the shot which is projected at 60° and by the what which is projected horizontal to reach the point P be t_1 and t_2 respectively

$$x = (u\cos\theta)t$$

$$x = (5\sqrt{3}\cos 60^\circ)t_1 = 5\sqrt{3}t_2$$

$$y = 10 + (5\sqrt{3}\sin 60^\circ)t_1 - \frac{1}{2}gt_1^2$$

$$y = 10 - \frac{1}{2}gt_2^2$$

$$t_1 = 2\,\text{sec}$$

Let : Δt = time interval

$$\Delta t = t_1 - t_2 = 2-1$$

$$\Delta t = 1\,\text{sec}$$

(ii) The coordinates at point , P

$$x = 5s\sqrt{3}t_1 = 5\sqrt{3}\times 1 = 5\sqrt{3}$$

$$y = 10 - \frac{1}{2}\times 10\times 1$$

$$P(x,y) = (5\sqrt{3}\text{m}, 5\text{m})$$

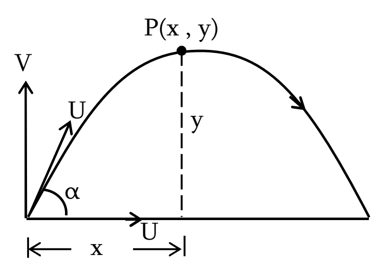
74. A bullet is fired with a velocity whose horizontal and vertical components of the velocities are U, V. Find its position at time t if the horizontal velocity is 600m/s , find elevation at which it must be fired if it to hit a mark 2m above the muzzle (initial) at a distance of 500m (g = 9.8m/s²).

Solution

Since $x = ut$

$$y = vt - \frac{1}{2}gt^2$$

Position of particle $= \left[ut, vt - \frac{1}{2}gt^2\right]$



$$\text{Now}: \quad t = \frac{u}{x}$$

$$y = vt - \frac{1}{2}gt^2 = v\left(\frac{x}{u}\right) - \frac{1}{2}g\left(\frac{x}{u}\right)^2$$

$$y + \frac{gx^2}{2u^2} = \left(\frac{v}{u}\right)x$$

$$\frac{v}{u} = \frac{y}{x} + \frac{gx}{2u^2} \quad \text{but } \tan\theta = \frac{v}{u}$$

$$\tan\theta = \frac{y}{x} + \frac{gx}{2u^2}$$

$$\theta = \tan^{-1}\left[\frac{y}{x} + \frac{gx}{2u^2}\right]$$

$$= \tan^{-1}\left[\frac{2}{500} + \frac{9.8 \times 500}{2(600)^2}\right]$$

$$\theta = 0.62^{o}$$

75. A shell bursts on contact with the ground and fragments fly in directions with speed up to 39.2m/s. show that a man 78.4m away is in danger for $4\sqrt{2}$ seconds.

Solution

$$\text{Horizontal range}, \ R = \frac{u^2 \sin 2\theta}{g}$$

$$\sin 2\theta = \frac{Rg}{u^2}$$

$$\sin 2\theta = \frac{78.4 \times 9.8}{(39.2)^2} = \frac{1}{2}$$

$$2\theta = 30^\circ, 150^\circ \quad , \quad \theta = 15^{o}, 75^{o}$$

Thus , there will be two times of flight

$$t_1 = \frac{2u \sin 15^\circ}{g} \quad , \quad t_2 = \frac{2u \sin 75^\circ}{g}$$

The man will be ≠ in danger for time $t_2 - t_1$

$$t_2 - t_1 = \frac{2u}{g}\left(\sin 75^\circ - \sin 15^\circ\right)$$

$$= \frac{2 \times 39.2}{9.8}\left[\sin 75^\circ - \sin 15^\circ\right]$$

$t_2 - t_1 = 4\sqrt{2}$ second Here shown.

76. With the same velocity of projection , the horizontal range of a projectile is half the greatest range , find the two possible angles of projection. Prove that the ratio of the greatest height reached in these two possible path is given by

$$\frac{H_1}{H_2} = \frac{2-\sqrt{3}}{2+\sqrt{3}}$$

Solution

Case 1: Horizontal range , $R = \dfrac{u^2 \sin 2\theta}{g}$

Greatest range , $R_{max} = \dfrac{u^2}{g}$

Given that $\quad R = \dfrac{1}{2}R_{max}$

$$\frac{u^2 \sin 2\theta}{g} = \frac{u^2}{2g}$$

$$\sin 2\theta = \frac{1}{2}$$

$$2\theta = \sin^{-1}\left[\frac{1}{2}\right] = 30^{o}$$

$$\theta_1 = \theta = 15^{o} \quad , \quad \theta_2 = 90^{o} - \theta$$

$$\theta_1 = 15^{o} \quad , \quad \theta_2 = 75^{o}$$

∴The two possible angles are 15^{o} , 75^{o}

Case 2: Proof : $\dfrac{H_1}{H_2} = \dfrac{2-\sqrt{3}}{2+\sqrt{3}}$

Greatest height reached by the projectile.

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$H_1 = \frac{u^2 \sin^2\theta_1}{2g} = \frac{u^2 \sin^2 15^\circ}{2g}$$

$$H_2 = \frac{u^2 \sin^2\theta_2}{2g} = \frac{u^2 \sin^2 75^\circ}{2g}$$

$$\frac{H_1}{H_2} = \frac{u^2 \sin^2 15^\circ}{2g} \Big/ \frac{u^2 \sin^2 75^\circ}{2g}$$

$$\frac{H_1}{H_2} = \frac{\sin^2 15^{o}}{\sin^2 75^\circ} = \frac{\left[\sin 15^\circ\right]^2}{\left[\sin 75^{o}\right]^2}$$

$$= \frac{\left[\sin\left(45^\circ - 30^\circ\right)\right]^2}{\left[\sin\left(45^\circ + 30^\circ\right)\right]^2}$$

$$= \frac{\left[\sin 45^{\circ}\cos 30^{o} - \sin 30^{o}\cos 45^{o}\right]^2}{\left[\sin 45^{o}\cos 30^{o} + \sin 30^{o}\cos 45^{o}\right]^2}$$

$$= \frac{\left[\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2}\right]^2}{\left[\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\frac{\sqrt{2}}{2}\right]^2}$$

$$= \frac{\left[\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)\right]^2}{\left[\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)\right]^2}$$

$$= \frac{\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)^2}{\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)^2}$$

$$= \frac{\left(\frac{\sqrt{3}}{2}\right)^2 - 2\frac{1}{2}\frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)^2}{\left(\frac{\sqrt{3}}{2}\right)^2 + 2\frac{1}{2}\frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)^2}$$

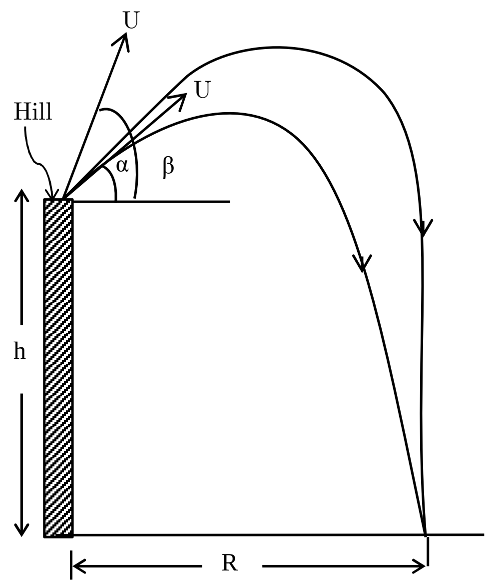
$$= \frac{\frac{3}{4} - \frac{\sqrt{3}}{2} + \frac{1}{4}}{\frac{3}{4} + \frac{\sqrt{3}}{2} + \frac{1}{4}} = \frac{1 - \frac{\sqrt{3}}{2}}{1 + \frac{\sqrt{3}}{2}}$$

$$\frac{H_1}{H_2} = \frac{2-\sqrt{3}}{2+\sqrt{3}} \text{ Hence shown}$$

77. Two shots are projected from a gun at the top of a hill with the same velocity u at angle of projection α and β respectively. If the shoots strike the horizontal ground through the foot of the hill at the same point , show that the height h of the hill above the plane is given by

$$h = \frac{2u^2\left(1 - \tan\alpha\tan\beta\right)}{g\left(\tan\alpha + \tan\beta\right)^2}$$

Solution



From the equation of the trajectory

$$-h = R\tan\alpha - \frac{gR^2}{2u^2\cos^2\alpha} \text{............(i)}$$

$$\text{And } -h = R\tan\beta - \frac{gR^2}{2U^2\cos^2\beta} \text{.......}(ii)$$

(i) = (ii)

$$R\tan\alpha - \frac{gR^2}{2u^2\cos^2\alpha} = R\tan\beta - \frac{gR^2}{2u^2\cos^2\beta}$$

$$R\left(\tan\alpha - \tan\beta\right) = \frac{gR^2}{2u^2\cos^2\alpha} - \frac{gR^2}{2u^2\cos^2\beta}$$

$$\frac{2u^2}{g}\left(\tan\alpha - \tan\beta\right) = R\left(\sec^2\alpha - \sec^2\beta\right)$$

But: $\sec^2\alpha = 1 + \tan^2\alpha$

$\sec^2\beta = 1 + \tan^2\beta$

$$\frac{2u^2}{g}\left(\tan\alpha - \tan\beta\right) = R\left[1 + \tan^2\alpha - 1 - \tan^2\beta\right]$$

$$\frac{2u^2}{g}\left(\tan\alpha - \tan\beta\right) = R\left(\tan^2\alpha - \tan^2\beta\right)$$

$$\frac{2u^2}{g}\left(\tan\alpha - \tan\beta\right) = R\left[\left(\tan\alpha - \tan\beta\right)\left(\tan\alpha + \tan\beta\right)\right]$$

$$\frac{2u^2}{g} = R\left(\tan\alpha + \tan\beta\right)$$

$$R = \frac{2u^2}{g(\tan\alpha + \tan\beta)}$$

Putting the value of R in equation (i)

$$h = R\left[\frac{gR}{2u^2\cos^2\alpha} - \tan\alpha\right]$$

$$= \frac{2u^2}{g(\tan\alpha+\tan\beta)}\left[\frac{\sec^2\alpha}{\tan\alpha+\tan\beta} - \tan\alpha\right]$$

$$= \frac{2u^2}{g(\tan\alpha+\tan\beta)^2}\left[\sec^2\alpha - \tan^2\alpha - \tan\alpha\tan\beta\right]$$

$$h = \frac{2u^2}{g(\tan\alpha+\tan\beta)}(1-\tan\alpha\tan\beta)$$

Hence shown.

78. A heavy particle is projected from a point O at an angle of elevation α and describes a parabola under gravity. If coordinates axes are taken horizontally and vertically through O prove that the equation of the parabola is

$$y = x\left[1 - \frac{x}{R}\right]\tan\alpha$$

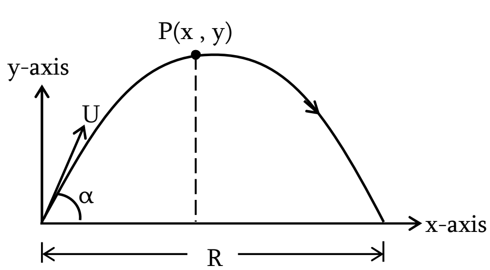
Where R is the horizontally range . If the distance between two points on the parabola which are at the same height h above the horizontal is 2a, show that

$$R(R - 4h\cot\alpha) = 4a^2$$

Solution

Case : 1

$$y = x\left[1 - \frac{x}{R}\right]\tan\alpha$$



From the equation of trajectory

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

But

$$R = \frac{2u^2\sin\alpha\cos\alpha}{g}$$

$$\frac{g}{2u^2} = \frac{\sin\alpha\cos\alpha}{R}$$

$$y = x\tan\alpha - \frac{x^2\sin\alpha\cos\alpha}{R\cos^2\alpha}$$

$$= x\tan\alpha - \frac{x}{R}\tan\alpha$$

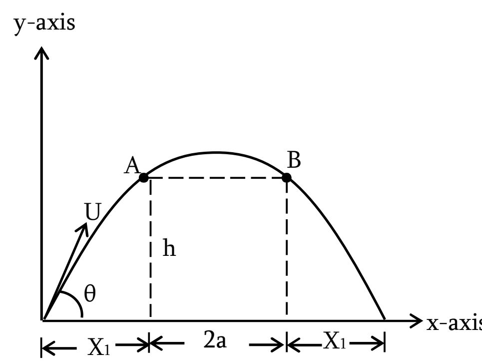
$$y = x\tan\alpha\left[\frac{R-x}{R}\right]$$

$$y = x\left[1-\frac{x}{R}\right]\tan\alpha$$

Hence shown

Case : 2

$$R(R-4h\cot\alpha) = 4a^2$$



The range R is given by

$$R = 2(a + x_1)$$

$$x_1 = \frac{R}{2} - a$$

Using the second equation of motion

$$h = (u\sin\theta)t - \frac{1}{2}gt^2$$

Solving as quadratic in t , we get

$$t = \frac{u\sin\alpha - \sqrt{u^2\sin^2\alpha - 2gh}}{g}$$

$$x_1 = (u\cos\alpha)t = \frac{R}{2} - a$$

$$(u\cos\alpha)\left[\frac{u\sin\alpha - \sqrt{u^2\sin^2\alpha-2gh}}{g}\right] = \frac{R}{2} - a$$

$$\frac{2u^2\sin\alpha\cos\alpha}{g} - \frac{2u\cos\alpha\sqrt{u^2\sin^2\alpha - 2gh}}{g} = R - 2a$$

$$R - \frac{2u\cos\alpha\sqrt{u^2\sin^2\alpha - 2gh}}{g} = R - 2a$$

$$\frac{2u\cos\alpha\sqrt{u^2\sin^2\alpha - 2gh}}{g} = 2a$$

$$u\cos\alpha\sqrt{u^2\sin^2\alpha - 2gh} = ag$$

(Square both side)

$$u^2\cos^2\alpha\left(u^2\sin^2\alpha - 2gh\right) = a^2g^2$$

$$\frac{1}{4}\left(\frac{2u^2\sin\alpha\cos\alpha}{g}\right)^2 - \frac{2hu^2\cos^2\alpha}{g} = a^2$$

$$\frac{1}{4}R^2 - \frac{2hu^2\cos^2\alpha}{g} = a^2$$

From $R = \dfrac{2u^2\sin\alpha\cos\alpha}{g}$

$$u^2 = \frac{Rg}{\sin\alpha\cos\alpha}$$

$$\frac{1}{4}R^2 - \frac{2h\cos^2\alpha}{g}\times\frac{gR}{2\sin\alpha\cos\alpha} = a^2$$

$$\frac{1}{4}R^2 - Rh\cot\alpha = a^2$$

$$4a^2 = R\left(R - 4h\cot\alpha\right)\text{ Shown.}$$

79. A particle is projected from level ground in such a way that its horizontal and vertical component of velocities are 20m/s and 10m/s respectively find:-
(a) maximum height of the particle
(b) its horizontal distance from the projection when it returns to the ground
(c) The magnitude and direction of its velocity on landing
Answer : (a) 5m (b) 40m (c) 22.3m/s

80. (a) Define the following terms:-
(i) Projectile (ii) range (iii) Trajectory.
(b) (i) Show that the maximum range is obtained when projected at an angle of 45° to the horizontal.
(ii) a hunter is intending to kill a lion is estimated to be 180m from him by striking his poisoned arrow if the initial velocity of the hunters arrow is 48m/s and the hunter is projected his arrow at an angle of 30° to the horizontal , please advise the following to the hunter:-
- Is the hunter going to kill the lion? Elaborate why you think so.
- If your answer is no , advice the hunter what he should do in order to succeed if the initial velocity of the arrow and the distance of the lion are fixed.

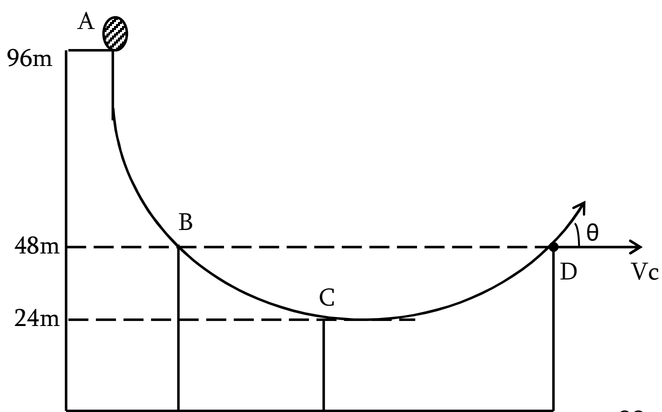
81. (a) (i) What do you understand by the term projectile motion.
(ii) What are to be considered for a projectile to attain maximum range?
(b) A 60gm block starts from rest and slides 30m down the roof with a slope of 2 horizontal to 1 vertical and the coefficient of friction of 0.1. If the vertical distance from the edge of the roof to the ground is 40m, find how far from the wall , the block strikes the ground?

82. (a) (i) What is a projectile?
(ii) What is a trajectory?
(b) Give two assumptions made in a projectile motion.
(c) A motor is at the same horizontal level as the bottom of a shell which is 20m from it. A shell from the motor just passes horizontally over the top of the wall which is 9.0m high. Find the velocity of projection both in magnitude and direction.

83. Particles A and B are projected simultaneously on the same vertical plane. Particle A is projected from highest h , vertically and B at point 2h horizontally from the origin respectively in the rectangular Cartesian coordinate they are projected at angle α and β with the horizontal respectively. If they collides , show that the ratio of their initial speed of A to B.

$$\frac{U_A}{U_B} = \frac{2\sin\beta + \cos\beta}{2\sin\alpha + \cos\alpha}$$

84. (a) State three factors on which the range of a projectile fired from gun depends.
(b) A small ball is released from the top of a parabolic less run away ABCD. Refer to the figure below . Find the tangential velocities attained at points C and D. Hence determine the range of the ball will cover from point E of the figure below.



85. A ball thrown from a point P with velocity V at an angle α to the horizontal reaches point Q after t second. Find the horizontal and vertical distance of Q from P and show that if PQ is inclined at θ to the horizontal the distance of motion of the ball when at Q is inclined to the horizontal at an angle

$$\tan^{-1}\left[2\tan\theta - \tan\alpha\right]$$

Answer : $x = (v\cos\alpha)t, y = (v\sin\alpha)t\frac{-1}{2}gt^2$

86. A particle is projected with a velocity whose horizontal and vertical components are U, V so as to pass through a point whose horizontal and vertical distances from the point of projection are h , k prove that.

$$2u^2k + gh^2 = 2uvh$$

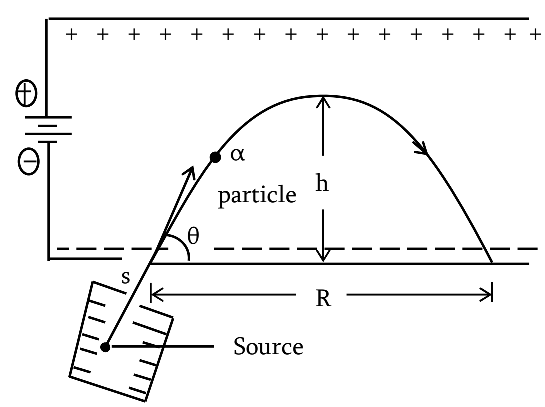
A particle is projected so as to pass through two points whose horizontal and vertical distances from the point of projection are (36 ,11) and (72 , 14)m. find the velocity and direction of projection.
Answer : 16.8m/s, $\theta = \tan^{-1}(0.42)$.

87. (a) A projectile is launched with speed V at an angle θ above the horizontal. The launch point is at a height H above the ground is given by

$$x = \frac{v\cos\theta}{g}\left[v\sin\theta + \left(v^2\sin^2\theta + 2gH\right)^{\frac{1}{2}}\right]$$

Where g is the acceleration due to the gravity.
(b) A bullet is fired out to sea in a horizontal from the gun situated on the top of a cliff 78.4m.
(i) Calculate the distance out to sea at which the bullet will strike the surface of the water given that the initial velocity of bullet is 240m/s
(ii) What will be the inclination to the horizontal at which the bullet strikes the surface of the water?

88. Figure below , α – particles from a bit of radioactive material enter through slit s into the space between two large parallel metal plates A and B connected to a source of voltage. As a result of the uniform electric field between the plates, each particle has a constant acceleration $a = 4 \times 10^{13} m/s^2$ normal to and toward B. If $V_o = 6 \times 10^{13}$m/s and $\theta = 45^o$. Determine h and R.



Answer: h = 0.225m , R = 0.90m.

89. From a point at a height h above the horizontal ground a particle A is projected with a velocity V_o in an upward direction making an angle θ with the horizontal. Another particle B is also projected with the same velocity V_o from the same point but downward direction directed opposite to A. Show the two particles will strike the ground at a distance.

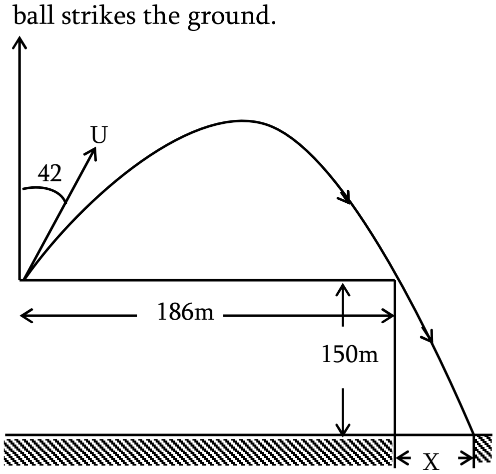
$$\frac{2V_o \cos\theta}{g} \cdot \sqrt{V_o^2 \sin^2\theta + 2gh} \text{ apart.}$$

90. A ball rolls from the top of stair way with a horizontal velocity Um/s. If the steps are 'h' metre and 'w' metre hit the edge of the nth step if

$$n = \frac{2hu^2}{gw^2}$$

91. A body is projected downward an angle of 30° with the horizontal from the top of a building 196m high. Its initial speed is 60m/s. How long will it take before striking the ground? Also find how far from the foot of the building the body will strike and at what angle with the horizontal?
Answer: t = 3.96m/s , θ = 52.96 = 53°.

92. (a) A ball is thrown upward with an initial velocity U at angle of 42 to the vertical from a point 186 from edge of a vertical cliff 150m high as shown below. The ball just clears the edge of the cliff find:-
(i) The initial velocity , U
(ii) The distance x beyond the cliff where the ball strikes the ground.



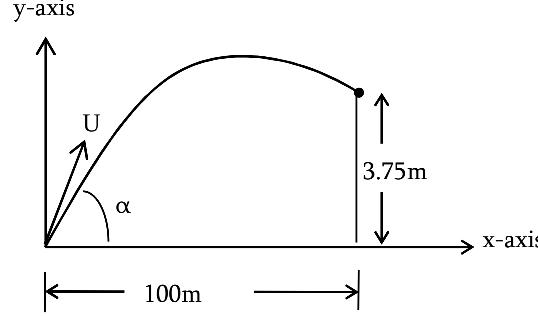
(b) A hunter aims his gun and fire bullet directly at a monkey on a tree. At the instant , the bullet leaves the barrel , the monkey drops will the bullet hit the monkey?

93. From the top of tower 20m high , a stone A is thrown with a velocity of 20m/s in an upward direction making an angle of 30° with the horizontal. Another stone is projected from the same point with the same velocity but in the downward direction exactly opposite to A. Find the distance between the stones , when they strike the ground ($g = 10m/s^2$)
Answer : 77.46m.

94. (a) A particle is projected inside of tunnel which is 2m high. If the initial speed is V_o. Show that the maximum range inside the tunnel is given by.

$$R = 4\sqrt{\frac{V_o^2}{g} - 4}$$

(b) If the target is a strip which is 3.75m high , find the values between which the angle projection must lies so that the arrow hits the targets.



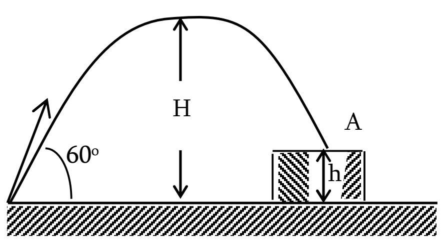
95. A particle is projected with the velocity $2\sqrt{hg}$ so that it just clears tow walls of equal height h which are a distance 2h from each other. Show that the time of passing between the walls is

$$2\sqrt{\frac{h}{g}}.$$

96. A particle is aimed at a mark which is in the same horizontal plane is the point of projection , it falls 10m short of the target when it is projected with an elevation of 75^{o} and falls 10m ahead of the tangent when its projected with an elevation of 45^{o}. Find the correct elevation so that it exactly hit the target. Given that the initial velocity of projection is the same in each case.
Answer :24.3^{o}.

97. (a) Prove that the motion of one projectile as viewed from another projectile will be a straight line motion.
(b) At what angles a ball should be thrown with a velocity of 24m/s just to a wall 16m high at a distance of 32m ($g = 10m/s^2$).
Answer (b) 67.9^{o} or 48.7^{o}

98. A stone is projected at a speed of 36.576m/s directly 60^{o} above the horizontal , at a cliff of height h as shown in the figure below.



The stone strikes the ground at A. 5.5sec after launching. Calculate the height h of the cliff and the speed of the stone just before impact at A. Also calculate the maximum height reached above the ground ($g = 9.75m/s^2$)
Answer: 26.77m , 28.56m/s , 51.45m.

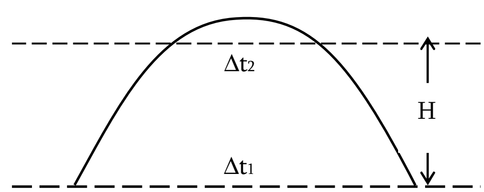
99. If R is the horizontal range for θ inclination and h is the maximum height reached by the projectile, show that the maximum range is given by

$$R_{max} = \frac{R^2}{8h} + 2h$$

100.(a) Prove that the range of a projectile for two angles α and β is the same where $\alpha + \beta = 90^{o}$.
(b) From the top of a tower 156.8m high , a projectile is thrown up with a velocity of 39.2m/s making an angle of 30^{o} with the horizontal. Calculate the distance from the foot of the tower where it strikes the ground and time taken by it to do so.
Answer: (b) 271.6m , 8seconds.

101.Particle P and Q of masses 20g and 40g respectively projected from points A and B on the ground. The initial velocities of P and Q makes 45^{o} and 135^{o} angle respectively with the horizontal as shown in the figure below. Each particle has an initial speed of 49m/s. the separation AB is 245m both particles travel in the same vertical plane and undergo a collision after the collision P retrace its path. Determine the position of Q when it hits the ground. How much time after the collision , the particle Q reaches the ground.
Answer: 122.5m from A or B , 3.54sec.

102.At the national physical laboratory in Tanzania , MR. MGOTE do experiment of a measument of the value of acceleration due to gravity was made by throwing a glass ball straight up in a evacuated tube and letting it returns as shown in the figure below.



If the time interval between the two passages across the lower level is Δt_1 and the time interval the two passes across the upper level is Δt_2, then show that

$$g = \frac{8H}{\Delta t_1^2 - \Delta t_2^2}$$

H represent the distance between the two levels.

103.(a) Explain what a projectile is giving at least two of its applications.

(b) (i) State Newton's first law of motion

(ii) A balloon is rising steadily and vertically upwards with a velocity of 20m/s. when it is at a height of 80m above the ground , a ball is released from it to fall to the ground. Draw a diagram to show the by ball the ball and calculate the time taken to reach on the ground.

(c) A footballer standing on top of a flat cliff , a height h above the horizontal ground , kicks two identical balls horizontally. The first with a velocity of V_o and the second with a velocity of $2V_o$ calculate:-

(i) The time of flight of each ball.

(ii) The horizontal displacement of the second ball in terms of h and V_o.

104. (a) (i) Define projectile

(ii) A heavy object at the top of a tall building is thrown horizontally with a velocity of the show that the path it described is a parabola.

(b) A ball is projected horizontal with a velocity V_o of magnitude 5m/s. find its position and velocity after 0.5seconds.

(c) Using equation of trajectory of projectile , derive an expression for range of that projectile.

105.What is the effect of air resistance on the time of flight and horizontal range of the projectile.

Answer: The effect of the air resistance is to increase the flight time and decrease the horizontal range.

106. Is the acceleration of a projectile equal to zero when it reaches the top of its trajectory? If not why not?

Answer: No , it is not equal to zero at the top , because acceleration on is the result of the earth gravitational field.

107.A projectile of mass M is projected with velocity , V at an angle θ with the horizontal. What is the magnitude of change in momentum of projectile after time , t?

Answer:

change in momentum = impulse
= force × time
= mgt

108.An object is thrown upward at an angle θ above the ground eventually returning to earth.

(a) is there any place along the trajectory where velocity and acceleration are perpendicular? If so where?

(b) Is there any place where the velocity and acceleration are parallel? If so where? In each case , explain.

Answer:

(a) Yes , velocity and acceleration are perpendicular at the highest point of trajectory where the velocity component along the vertical becomes equal to zero. The net velocity is the constant horizontal velocity which is perpendicular to acceleration.

(b) No there is no place where velocity and acceleration are parallel. This is because throughout the trajectory , the horizontal velocity is constant and perpendicular to acceleration , g.

109.A rifle, at the height H above the ground fires a bullet parallel to the ground. At the same instant and the same height , a second bullet is dropped from rest. In the absence of air resistance.

(a) Which bullet strikes the ground first? Explain.

(b) Which bullet strikes the ground with a greater speed? Justify your answer.

Answer

(a) Both bullets will strikes the ground at the same time. This is because both bullets start with zero initial velocity along the vertical at the same vertical height , h the time taken is the function of the height , h

$$t = \sqrt{\frac{2h}{g}}$$

(b) The first bullet will strike the ground with a greater speed. This is because on falling the given height it acquires a vertical speed equals to that of second bullet on striking the ground. Since the first bullet starts with a horizontal speed which is unchanging the resultant speed on ground is greater than that of bullet 2.

110. Why does a tennis ball bounces higher on hills than in plains?

Answer: The maximum height of a projectile is inversely proportional to the value of acceleration due to gravity. So the smaller the acceleration due to gravity , greater is the maximum height since the value of g is less on the hill than on the plains, therefore a tennis ball will bounce higher on the hills than in plans.

PROJECTILE – MOTION HIGHER ORDER THINKING SKILLS

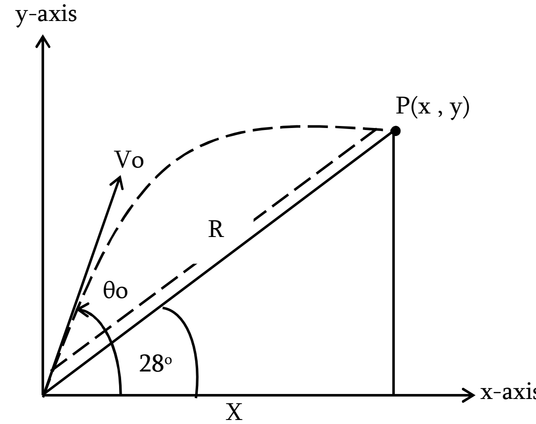
EXAMPLES

1. (a) Outline the motions that add up to make projectile motion.
   (c) A ball is thrown upwards with an initial velocity of 33m/s from a point 65° on the side of a hill which slopes upward uniformly at an angle of 28°.
      (i) Ata what distance up the slope does the ball strike?
      (ii) Calculate the time of flight of the ball

Solution

(a) • Horizontal motion with constant velocity.
   • Vertical motion with constant acceleration.

(c) (i)



$\theta_o = 65^\circ$, $V_o = 33$m/s , $g = 9.8$m/s²

The trajectory equation of the ball

$$y = x\tan\theta_o - \frac{gx^2}{2V_o^2\cos^2\theta_o}$$

$$y = x\tan 65^\circ - \frac{9.8x^2}{2(33)^2\cos^2 65^\circ}$$

$$y = 2.14x - 0.025x^2 \ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

From the figure above

$$\tan 28 = \frac{y}{x}$$

$$y = x\tan 28^\circ = 0.53x \ldots\ldots\ldots\text{(ii)}$$

(i) = (ii)

$$2.14x - 0.025x^2 = 0.53x$$

On solving x = 64.4m

The distance along the inclined plane

$$\cos 28^\circ = x/R$$

$$R = \frac{x}{\cos 28^\circ} = \frac{64.4\text{m}}{\cos 28^\circ}$$

$$R = 64.4\text{m}$$

(ii) t = time of flight

$$x = \left(V_o\cos 68^\circ\right)t$$

$$t = \frac{x}{V_o\cos 65^\circ} = \frac{64.4}{33\cos 65^\circ}$$

$$t = 4.34\,\text{sec}$$

2. (a) A 75kg hunter fires a bullet of mass 10g with a velocity of 400m/s from a gun of mass 5kg. calculate the
    (i) Recoil velocity of the gun
    (ii) Velocity acquired by the hunter during firing. (03 marks)
   (b) A jumbo jet travelling horizontally at 50m/s at a height of 500m from sea level drops a luggage of food to a disaster area.
    (i) At what horizontal distance from the target should the luggage be dropped? (03 marks)
    (ii) Find the velocity of the luggage as it hit the ground. (02 marks)

3. (a) (i) How does projectile motion differ from uniform circular motion?
    (ii) A rifle shoots a bullet with muzzle velocity of 1000m/s at a small target 200m away. How high above the target must the rifle be aimed so that the bullet hit the target?
   (b) (i) Where does the object strike the ground when thrown horizontally with a velocity of 15m/s from the top of a 40m high building?
    (ii) Find the speed of travel when a man jumps a maximum horizontal distance of 1m spending a minimum time on the ground.

($g = 10m/s^2$)

Solution

(a) (i) In projectile motion , the acceleration (= g) is constant in both magnitude and direction but velocity changes in both magnitude in circular motion , the velocity and acceleration are constant in magnitude but change their direction continuous.

(ii) Time taken by bullet to reach the target.

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{200}{1000}$$

$$t = \frac{2}{10} = \frac{1}{5}\text{sec}$$

Vertical fall of bullet due to the gravity

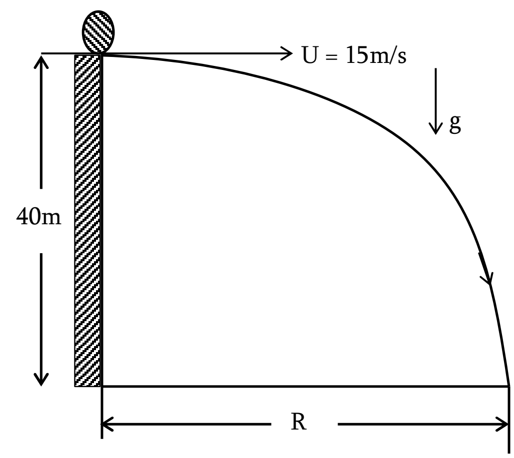
$$y = \frac{1}{2}gt^2 \quad (u = 0)$$

$$= \frac{1}{2}\times 10\times\left(0.2\right)^2$$

$y =$

The rifle must be aimed above the target for the bullet to hit the target.

(b) (i)



The flight time , $h = \frac{1}{2}gt^2$

$$t = \sqrt{\frac{2h}{g}}$$

Horizontal range , $R = ut$

$$R = U\sqrt{\frac{2h}{g}}$$

$$= 15\times\sqrt{\frac{2\times 40}{10}}$$

$R = 42.43m$

(ii) Since $R = \frac{u^2\sin 2\theta}{g}$, $R = R_{max}, \theta = 45^\circ$

$$R_{max} = \frac{u^2}{g}, u = \sqrt{R_{max}}$$

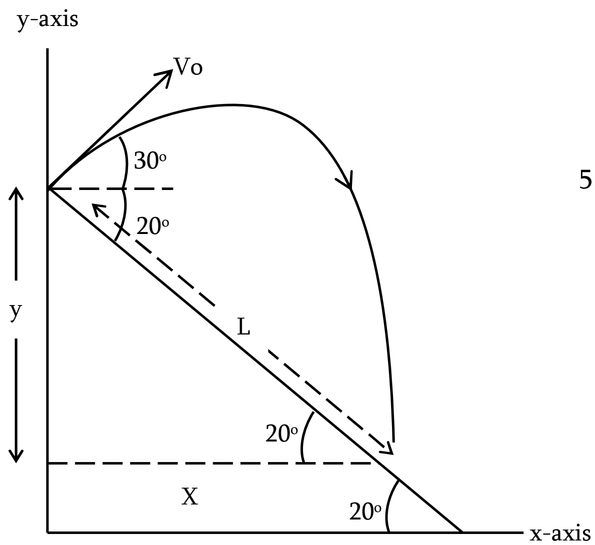
Velocity along the road

$$V_x = u\cos\theta = \sqrt{1\times 10}\cos 45^\circ$$

$V_x = 2.236m/s$

4. A ball is thrown upward with initial velocity V_o = 15m/s at an angle of 30° with the horizontal. The thrower stands near the top of a long hill which slopes downward at an angle of 20°
(a) When does the ball strike the slope?
(b) How far down the slope the ball strikes?
(c) Indicate with what velocity the ball hits?

Solution



(a) The equation of motion of the balls

$$x = \left(v_o \cos 30^\circ\right)t = \left(15\cos 30^\circ\right)t$$

$$x = 13t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

$$-y = \left(v_o \sin 30^\circ\right)t - \frac{1}{2}gt^2$$

$$-y = \left(15\sin 30^\circ\right)t - \frac{1}{2}\times 9.8t^2$$

$$-y = 7.5t - 4.9t^2 \ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

$$y = 4.9t^2 - 7.5t$$

When the ball strikes the hill its coordinates satisfy the equation

$$\frac{y}{x} = \tan 20$$

$$y = x\tan 20^\circ \ldots\ldots\ldots\ldots\ldots\text{(iii)}$$

(i) = (ii)

$$x\tan 20^\circ = 4.9t^2 - 7.5t$$

$$(13t)\tan 20 = 4.9t^2 - 7.5t$$

On solving , t = 2.49sec

(b) Now

$$x = 13t = 13\times 2.49$$

$$x = 31.4\text{m}$$

$$y = 4.9(2.49)^2 - 7.5(2.49)$$

$$y = -11.8\text{m}$$

By using Pythagoras theorem

$$L = \sqrt{x^2 + y^2}$$

$$L = 34.5\text{m}$$

Or $\cos 20^\circ = \frac{x}{L}, L = \frac{x}{\cos 20^\circ} = \frac{31.4}{\cos 20^\circ}$

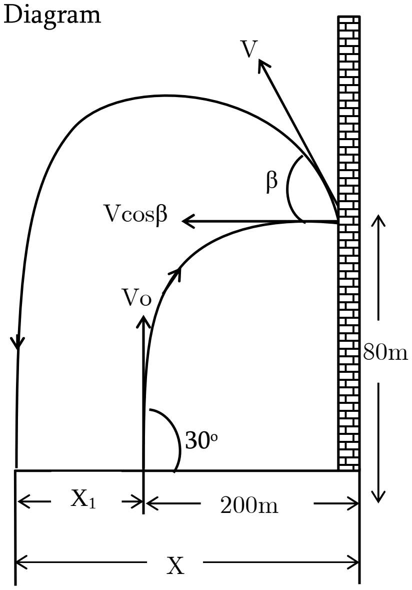
$$L = 34.5\text{m}$$

(c) For the velocity of the ball just before impact

5. (a) A ball is thrown at a point 200m from the foot edge of a vertical cliff 80m high with velocity of Vo at an angle 30° above the horizontal . If the ball will makes a perfectly elastic collision with the edge of the cliff.
(i) Find the value of Vo
(ii) How far behind the thrower does the ball strike the ground?
(b) A shell is fired from a gun from a point on the hill which slopes , upward uniformly at an angle of 30° to the horizontal. The edge of the barrel to the horizontal is 67° at what distance up the slope does the shell strike and in what time? Take the velocity of the shell leaving the gun as Vo = 40m/s

Solution

(a) Diagram



(i) Since

$$y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$$

$$\frac{gx^2}{2v_0^2\cos^2\theta} = x\tan\theta - y$$

$$V_0^2 = \frac{gx^2}{2\cos^2\theta\left(x\tan\theta - y\right)}$$

$$= \frac{9.8\left(200\right)^2}{2\left(\cos 30^\circ\right)\left[200\tan 30^\circ - 80\right]}$$

$$Vo = \sqrt{\frac{9.8\left(200\right)^2}{1.5\left(35.47\right)}}$$

$$Vo = 85.84\text{m/s}$$

(ii) Also $X = \left(Vo\cos\theta\right)t$

$$200 = \left(85.84\right)t$$

$$t = 2.69\,\text{sec}$$

The horizontal and vertical components of velocity at any time , t

$$V_X = Vo\cos 30^\circ = 85.84\cos 30^\circ$$

$$V_X = 74.34\text{m / s}$$

$$V_y = V_O\sin\theta - gt$$

$$= 85.84\sin 30^\circ - 9.8\times 2.69$$

$$V_y = 16.56\text{m / s}$$

Apply Pythagoras theorem

$$V = \sqrt{V_x^2 + V_y^2}$$

$$V = \sqrt{\left(74.34\right)^2 + \left(16.56\right)^2}$$

$$V = 76.16\text{m / s}$$

$$\beta = \tan^{-1}\left(V_y / V_x\right) = \tan^{-1}\left(\frac{16.56}{74.34}\right)$$

$$\beta = 12.6^\circ$$

After impact , trajectory equation is gives

$$-y = x\tan\beta - \frac{gx^2}{2V^2\cos^2\beta}$$

$$-80 = x\tan\left(12.6^\circ\right) - \frac{9.8x^2}{2\left(76.16\right)^2\left(\cos 12.6\right)^2}$$

$$9.8x^2 - 2469.37x - 8838924 = 0$$

On solving

X = 451.67m or -199.69m

On omitting negative sign

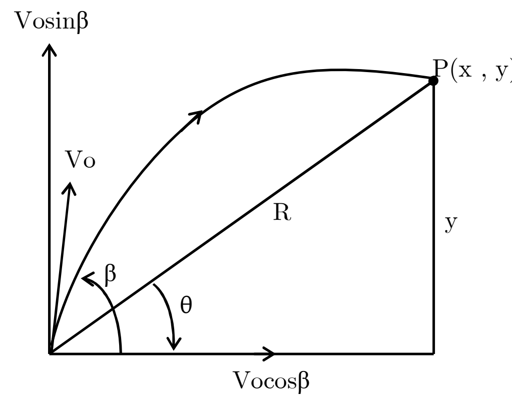
X = 451.67m

But $X_1 + 200 = 451.67M$

$X_1 = 251.67m$

∴ The ball will strike the ground 251.67m behind the thrower.

(b)



The coordinates of the shell

$$X = \left(V_o\cos\beta\right)t \ldots\ldots\ldots\ldots\text{(i)}$$

$$y = \left(V_o\sin\beta\right)t - \frac{1}{2}gt^2 \ldots\ldots\ldots\ldots\text{(ii)}$$

Also

$$y = x\tan\theta \ldots\ldots\ldots\ldots\text{(iii)}$$

(iii) = (ii)

$$x\tan\theta = \left(V_o\sin\beta\right)t - gt^2$$

$$\left(V_o\cos\beta\right)t\tan\theta = \left(V_o\sin\beta\right)t - \frac{1}{2}gt^2$$

$$t = \frac{2V_o}{g}\left(\sin\beta - \cos\beta\tan\theta\right)$$

$$= \frac{2\times 40}{9.8}\left[\sin 67^\circ - \cos 67^\circ\tan 30^\circ\right]$$

$$t = 5.67\text{seconds}$$

Now $X = \left(40\cos 67^\circ\right)\times 5.67$

$$X = 88.62\text{m}$$

But

$$R\cos 30^\circ = X$$

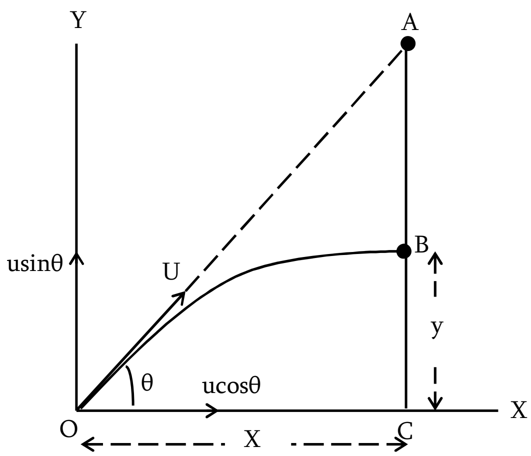
$$R = \frac{X}{\cos 30^\circ} = \frac{88.62}{\cos 30^\circ}$$

$$R = 102.3\text{m}$$

∴ The shell strikes 102.3m up the slope in 5.67seconds

6. A hunter aims his gun and fires a bullet directly at a monkey in a tree. At the instant the bullet leaves the barrel of the gun the monkey drops. Will the bullet hit the monkey. Substantiate your answer with proper reasoning.

Solution



Let the monkey stationed at A be fired with a gun from O with a velocity U at an angle θ with horizontal direction OX. Draw AC perpendicular to OX. Let the bullet across the vertical line AC at B after time, t and coordinates of B be (x , y) w.r.t origin , O.

$$t = \frac{OC}{U\cos\theta} = \frac{X}{U\cos\theta} \text{..........(i)}$$

In ΔOAC,

$$AC = OC\tan\theta = X\tan\theta \text{.........(ii)}$$

Travelled by the bullet in time , t

$$y = (u\sin\theta)t - \frac{1}{2}gt^2 \text{.............(iii)}$$

$$AB = AC - BC = X\tan\theta - y$$

$$AB = X\tan\theta\left[(u\sin\theta)t - \frac{1}{2}gt^2\right]$$

$$= X\tan\theta - \left[(u\sin\theta)\cdot\frac{X}{U\cos\theta} - \frac{g}{2}\frac{X^2}{U^2\cos^2\theta}\right]$$

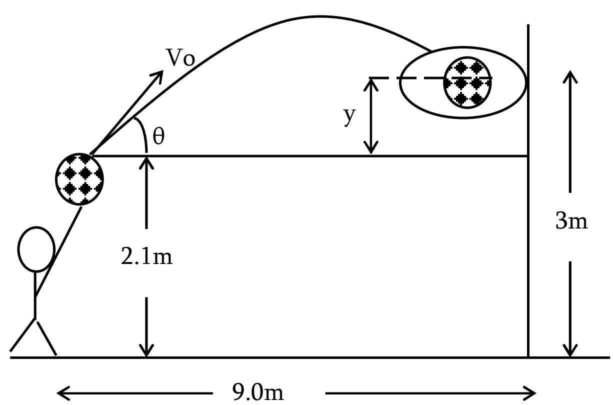
$$= X\tan\theta - X\tan\theta + \frac{1}{2}gt^2$$

$$AB = \frac{1}{2}gt^2$$

It means that the bullet will pass through the point B on the vertical line AC at a distance $\frac{1}{2}gt^2$ below point , A the distance through which the monkey falls vertically in time , $t = \frac{1}{2}gt^2 = AB$. It means the bullet and monkey will pass through the point B simultaneously. Therefore the bullet will hit the monkey.

7. A basketball player releases the ball 2.10m above the floor when he is 9.0m from the basket.

Solution



For vertical motion

$$y = (V_o\sin\theta)t - \frac{1}{2}gt^2$$

$$3 - 2.1 = (V_o\sin\theta)\times 1.5 - \frac{1}{2}9.8\times 1.5^2$$

$$1.5V_o\sin\theta = 11.925 \text{........................(i)}$$

For horizontal motion

$$X = (V_o\cos\theta)t$$

$$1.5V_o\cos\theta = 90 \text{............................(ii)}$$

$$(i) = (ii)$$

$$\frac{1.5V_o\sin\theta}{1.5V_o\cos\theta} = \frac{11.925}{9}$$

$$\theta = \tan^{-1}\left(\frac{11.925}{9}\right)$$

$$\theta = 53^\circ$$

$$V_o = \frac{11.925}{1.5\sin 53^\circ}$$

$$V_o = 9.96 m/s$$

Maximum height

$$h = \frac{V_0^2 \sin^2\theta}{2g} = \frac{(9.96\sin 53^\circ)^2}{2\times 9.8}$$

$$h = 3.23m$$

Total maximum height above the floor reached by the ball

Hmax = 323 + 210

Hmax = 5.33m

8. Three seconds after projection , a projectile is moving at 30_o above the horizontal , after two more seconds it is moving horizontally. Find the magnitude and direction of its initial velocity. Answer :55.3° , 59.6m/s.

9. (a) Define the term trajectory
   (b) A point O is vertically above a fixed point M of a horizontal plane. A particle P is projected from O with speed 5V at an angle $\cos^{-1}\left(\frac{3}{5}\right)$ above the horizontal and hits the plane at point K, a distance $\frac{48V^2}{g}$ from M. show that the height of O above M is of magnitude $\frac{64V^2}{g}$

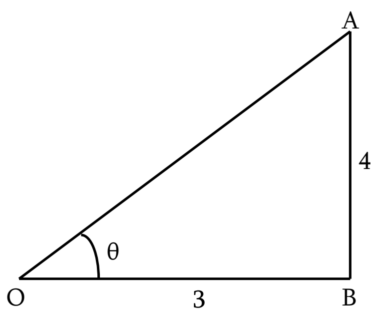
Solution

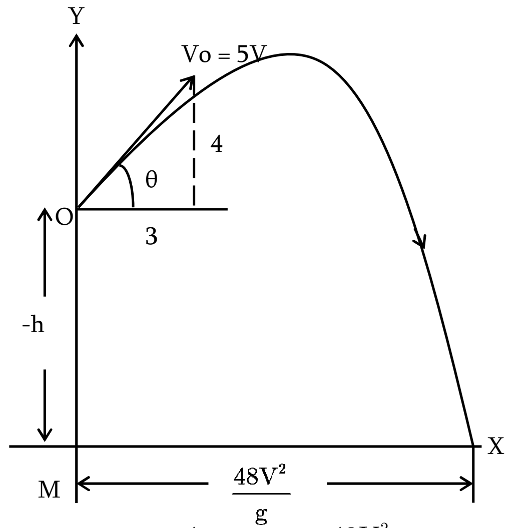
$$\cos^{-1}\left(\frac{3}{5}\right) = \theta$$

$$\cos\theta = \frac{3}{5}$$

$$3^2 + (AB)^2 = 5^2$$

$$AB = 4$$





$$\tan\theta = \frac{4}{3}, \quad x = \frac{48V^2}{g}$$

From the equation of trajectory

$$y = x\tan\theta - \frac{gx^2}{2V_0^2\cos^2\theta}$$

$$-h = \frac{48V^2}{g}\cdot\frac{4}{3} - \frac{g\left(48\,V^2/g\right)^2}{2(5V)\left(3/5\right)^2}$$

On simplifying g

$$h = \frac{64V^2}{g} \quad \text{Hence shown}$$

10. A stone is projected from a point O on horizontal ground with speed Um/s ata angle θ above the horizontal , where $\sin\theta = \frac{3}{5}$. The stone is at its highest point where it has travelled a horizontal distance of 19.2m.
   (a) find the value of U after passing through its highest point , the stone strike a vertical wall at a point 4m above the ground.
   (b) Find the horizontal distance between O and the wall. At the instant , when the stone hits the wall , the horizontal component of the stone's velocity component of the stone's velocity doesnot change as a result the stone hitting the wall.
   (c) Find the distacne from the wall of the point , where the stone reaches the ground.

Solution

(a) The stone is at its highest point , when it has travelled a horioznto distance of 19.2m The horizontal range of the stone

$R = 19.2\times 2 = 38.4m$

Now

$$R = \frac{U^2\sin^2\theta}{g}$$

$$\frac{U^2\sin^2\theta}{g} = 38.4$$

$$\frac{2u^2\sin\theta\cos\theta}{10} = 38.4$$

$$u^2\sin\theta\cos\theta = 192$$

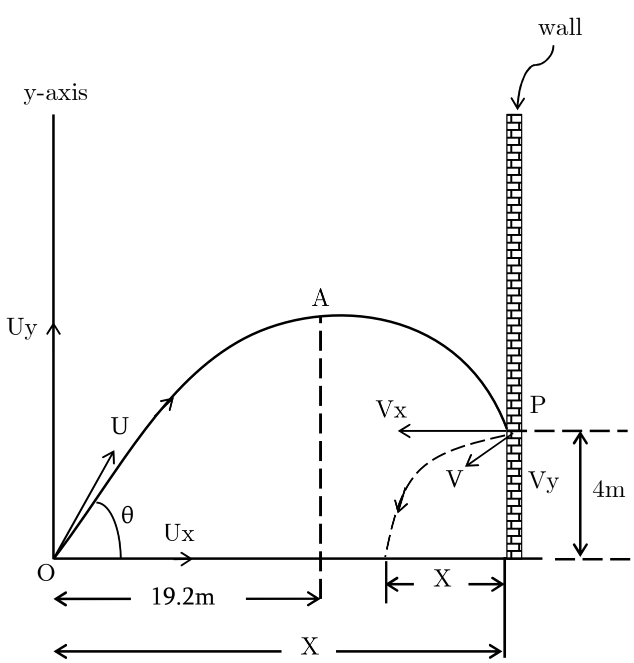
$$\sin\theta = \frac{3}{5} \quad , \quad \cos\theta = \frac{4}{5}$$

$$u^2\cdot\frac{3}{5}\times\frac{4}{5} = 192$$

$$u^2 = 400$$

U = 200m/s

(b) Let t be the time taken by the stone to hit the wall at the point P, 4m above the ground as shown in figure below.



For motion of stone along vertical

$$U_y = U\sin\theta = \frac{20\times 3}{5} = 12m/s$$

$U_y = 12m/s \quad , \quad y = 4m$

Now

$$y = U_y t - \frac{1}{2}gt^2$$

$$4 = 12t - \frac{1}{2}\times 10t^2$$

$$5t^2 - 12t + 4 = 0$$

(c) $V_X = \frac{1}{2}U_X = \frac{1}{2}U\cos\theta$

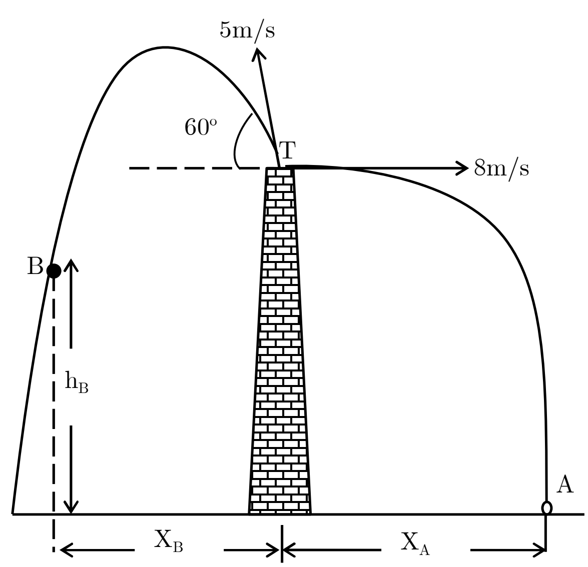
$$V_X = \frac{1}{2}\times 20\times\frac{4}{5} = 8m/s$$

Here , t = 0.4sec the other value of t obtained above corresponding to the time taken by the stone to rise through a height of 4m above the point , O

$$X' = V_X t = 8\times 0.4$$

$$X' = 3.2m$$

11. Particle A and B are projected simultaneously from the top T of a vertical tower and move in the same vertical plane T is 7.2m above horizontla ground A is projected horizontal with speed 8m/s and B is projected ata n angle60o above the horizontal with speed 5m/s A and B move away from each other as shown in the figure below.



(a) Find the time taken for A to reach the ground
(b) At athe instant , when A hits the ground,
    (i) Show that B is approximetly 5.2m above the ground and
    (ii) Find the distance AB

Solution

(a) For particel A : UA = 0
If the particel A reaches the ground after time , t

$$y_A = U_A t + \frac{1}{2}gt^2$$

$$7.5 = \frac{1}{2} \times 10 \times t^2$$

$$t = 1.2\text{second}$$

(b) (i) For particle B
If Y_B is distance travelled by the particle in 1.2sec , then

$$-y_B = \left(U_{By} \sin\theta\right)t - \frac{1}{2}gt^2$$

$$-y_B = \left(5\sin 60^\circ\right) \times 1.5 - \frac{1}{2} \times 10\left(1.2\right)^2$$

$$y_B = 2\text{m}$$

Height of B above the ground after 1.2sec

$$h_B = 7.2 - 2$$

$$h_B = 5.2\text{m}$$

(ii) Horizontal distance travelled by the particle A is in 1.2sec

$$X_A = 8 \times 1.2$$

$$X_A = 9.6\text{m}$$

Horizontal distacne travelled by the particle B is 1.2sec

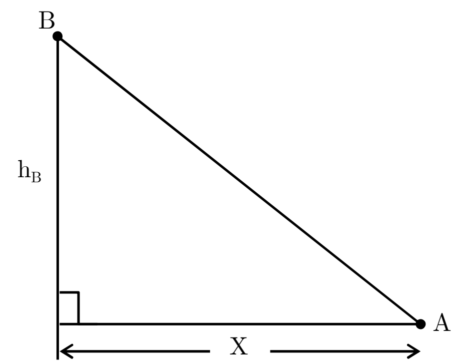
$$X_B = 5\cos 60^\circ \times 1.2$$

$$X_B = 3\text{m}$$

Horizontal distance between tow particles

$$X = X_A + X_B$$

$$X = 9.6 + 3 = 12.6\text{m}$$



∴ The required d ce between the tow particles

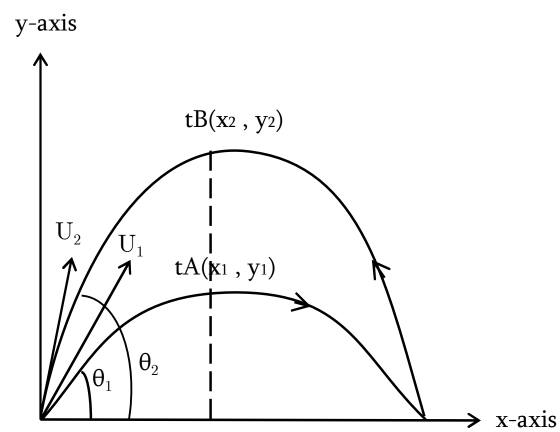
$$AB = \sqrt{X^2 + h_B^2}$$ (pythagorous theorem)

$$= \sqrt{\left(12.6\right)^2 + \left(5.2\right)^2}$$

$$AB = 13.63\text{m}$$

12. Show that the motion of one projectile as seen from another projectile will always be straight line motion.

Solution



$$x_1 = \left(u_1\cos\theta_1\right)t \quad , \quad y_1 = \left(u_1\sin\theta\right)t - \frac{1}{2}gt^2$$

$$x_2 = \left(u_2\cos\theta_2\right)t \quad , \quad y_2 = \left(u_2\sin\theta\right)t - \frac{1}{2}gt^2$$

Now

$$x_2 - x_1 = \left(u_2 \cos\theta_2 - u_1 \cos\theta_1\right) t$$

$$y_2 - y_1 = \left(u_2 \sin\theta_2 - u_1 \sin\theta_1\right) t$$

Let

$$y_2 - y_1 = y \quad , \quad x_2 - x_1 = x$$

$$\frac{y}{x} = k \text{ (a constant)}$$

Which is an equation of a straight line. Hence the motion of one projectile as seen from another projectile is a straight line motion.

13. The maximum height attained by a projectile is increased by 10% by increasing its speed of projection , without changing the angle of projection. What will the percentage increase in the horizontal range.

Solution

The maximum height

$$H = \frac{u^2 \sin^2\theta}{2g}$$

Let ΔH be the increase in H when U changes by Δu

$$\frac{dH}{du} = \frac{2u \sin^2\theta}{2g}$$

$$\Delta H = \frac{2u\Delta u \sin^2\theta}{2g}$$

$$\frac{\Delta H}{H} = \frac{2u\Delta u \sin^2\theta}{2g} \Big/ \frac{u^2 \sin^2\theta}{2g}$$

$$\frac{\Delta H}{H} = 2\frac{\Delta u}{u}$$

But

$$\frac{\Delta H}{H} \times 100\% = 10\% \quad \text{or} \quad \frac{\Delta H}{H} = 0.1$$

$$2\frac{\Delta u}{u} = \frac{\Delta H}{H} = 0.1$$

Also

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$\Delta R = \frac{2u\Delta u}{g} \sin 2\theta$$

$$\frac{\Delta R}{R} = \frac{2u\Delta u}{g} \sin 2\theta \Big/ \frac{u^2 \sin 2\theta}{g}$$

$$\frac{\Delta R}{R} = \frac{2\Delta u}{u} = 0.1$$

% increase in horizontal range

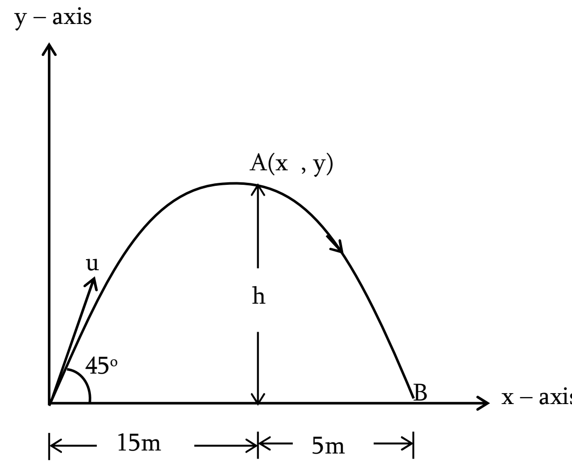
$$\frac{\Delta R}{R} \times 100\% = 0.1 \times 100\%$$

$$\frac{\Delta R}{R} \times 100\% = 10\%$$

14. From a point on the ground at a distance 15m from the foot of a vertical wall a ball is thrown at an angle of 45° which just clears the top of the wall afterwards strikes the ground at a distance 5m on the other side. Find the height of the wall.

Solution

Let U be the velocity of projection from O at an angle 45° which just clear the top A of wall of height , h as shown on the figure below.



Horizontal range ,

$$R = 15 + 5 = 20\text{m}$$

$$\frac{u^2 \sin 2\theta}{g} = 20$$

$$\frac{u^2}{g} = 20 \ldots\ldots\ldots\ldots\text{(i)}$$

Let (x ,y) be the coordinate of A then $x = 15\text{m}$, $y = h$.

From the equation of trajectory

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$h = 15\tan 45^\circ - \frac{1}{2} \times \frac{15^2}{20 \times \frac{1}{2}}$$

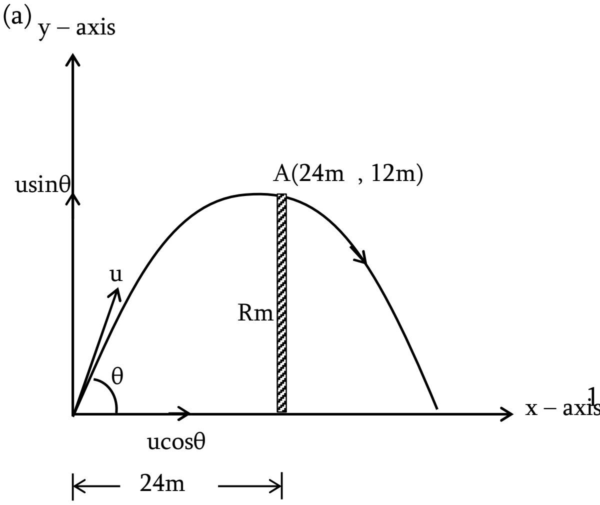
$$h = 3.75\text{m}$$

15. (a) At what angle should a body be projected with a velocity 20m/s just pass over the obstacle 12m high at a horizontal distance of 24m? (take $g = 10m/s^2$)

(b) A body is projected with a velocity of 40m/s after 2sec it crosses a vertical pole of height 20.40m. Find the angle of projection and horizontal range of projectile ($g = 9.8m/s^2$)

Solution

(a)



Horizontal displacement

$$x = (u\cos\theta)t$$

$$24 = (20\cos\theta)t$$

$$t = \frac{24}{20\cos\theta} = \frac{6}{5\cos\theta}$$

Horizontal displacement

$$y = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$12 = (20\sin\theta)t - \frac{10t^2}{2}$$

$$= (20\sin\theta)\frac{6}{5\cos\theta} - 5\left(\frac{6}{5\cos\theta}\right)^2$$

$$12 = 24\tan\theta - \frac{5\times 36}{25\cos^2\theta}$$

$$= 24\tan\theta - \frac{36}{5}\sec^2\theta$$

$$12 = 24\tan\theta - \frac{36}{5}(1+\tan^2\theta)$$

$$60 = 120\tan\theta - 36 - 36\tan^2\theta$$

$$36\tan^2\theta - 120\tan\theta + 96 = 0$$

$$3\tan^2\theta - 120\tan\theta + 8 = 0$$

$$(\tan\theta - 2)(3\tan\theta - 4) = 0$$

$\tan\theta = 2$ or $4/3$

(b) Since

$$y = u_y t - \frac{1}{2}gt^2$$

$$20.4 = 40\sin\theta\times 2 - \frac{1}{2}\times 9.8(2)^2$$

$$20.4 = 80\sin\theta - 19.6$$

$$\sin\theta = \frac{20.4+19.6}{8} = \frac{1}{2}$$

$$\theta = 30^\circ$$

Horizontal range

$$R = \frac{u^2\sin 2\theta}{g}$$

$$R = \frac{40^2}{9.8}\sin(2\times 30^\circ)$$

$$R = 141.4m$$

16. From the top of a tower 156.8m high a projectile is thrown up with velocity of 39.2m , making an angle 30^o with horizontal direction. Find the distance from the foot of tower where it strikes the ground and time taken by it to do so.

Answer : t = 8.0sec , R = 271.57m

17. (a) Find the angle of projection at which the horizontal range and maximum height are equal.

(b) A projectile has a range of 60m and reaches a maximum height of 12m. Calculate the angle at which the projectile is fired and initial velocity of projectile (take g = 10m/s)

Answer: (a) $75^o58'$ (b) 38^o248m/s

18. Two tall building face each other and are at a distance of 180m from each other with what velocity must a ball be thrown horizontally from a window 55m above the ground in one building, so that it enters a window 10.9m above the ground in second window $g = 9.8m/s^2$.

Answer : 60m/s.

19. The height y and x along the horizontal plane of a projectile on a certain plane of a projectile on a certain planet (with no surrounding atmosphere) are given by

$$x = 6tm \; ; \; y = (8t - 5t^2)$$

where t is in seconds find

(i) The velocity with which the projectile was projected

(ii) The angle with the horizontal at which projectile was projected.

(iii) Acceleration due to gravity.

Solution

$$x = 6tm \quad , \quad y = \left(8t - 5t^2\right)$$

The x and y component of velocity are

$$V_x = \frac{dx}{dt} = 6m/s$$

$$V_y = \frac{dx}{dt} = \left(8 - 10t\right)m/s$$

(i) The initial x and y component of velocity are obtained by putting t = 0

$$V_{ox} = 6m/s \quad , \quad V_{oy} = 8 - 10 \times 0 = 8m/s$$

Initial velocity

$$V_o = \sqrt{V_{ox}^2 + V_{oy}^2} = \sqrt{6^2 + 8^2}$$

$$V_o = 10m/s$$

(ii) Angle of projection with the horizontal is given by

$$\theta = \tan^{-1}\left[\frac{V_{oy}}{V_{ox}}\right] = \tan^{-1}\left[\frac{8}{6}\right]$$

$$\theta = 53.1^\circ$$

(iii) Acceleration due to gravity

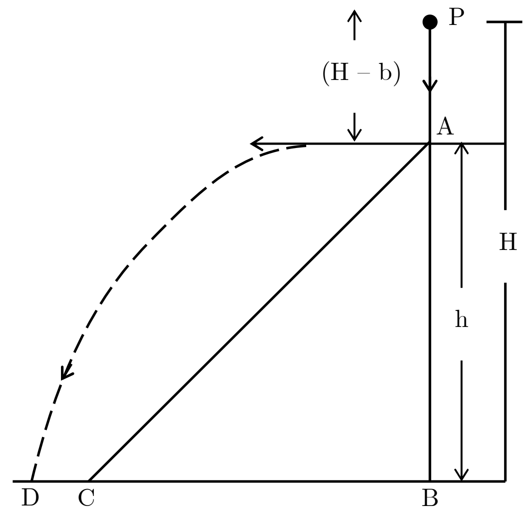
$$a_y = \frac{d}{dt}\left(V_y\right) = \frac{d}{dt}\left(8 - 10t\right)$$

$$a_y = -10m/s^2$$

$\therefore$ Acceleration due to gravity on the planet is $10m/s^2$ downward.

20. A ball falling freely from a given height H hits an inclined plane in its path at a height h as a result of this impact , the direction of velocity of the ball becomes horizontal for what value of h/H , the ball will take maximum time to reach the ground.

Solution



Let t_1 be time taken by the particle from point P to A.

$$H - h = \frac{1}{2}gt_1^2$$

$$t_1 = \sqrt{\frac{2\left(H - h\right)}{g}}$$

Motion from A to D at point A the initial vertical velocity is again zero. Let t_2 be time taken by the ball in reaching from A to D.

$$h = \frac{1}{2}gt_2^2 \quad \text{or} \quad t_2 = \sqrt{\frac{2h}{g}}$$

Total time t taken by the ball is given by

$$t = t_1 + t_2$$

$$= \sqrt{\frac{2\left(H - h\right)}{g}} + \sqrt{\frac{2h}{g}}$$

$$t = \sqrt{\frac{2}{g}}\left[\left(H - h\right)^{\frac{1}{2}} + h^{1/2}\right]$$

$$\frac{dt}{dh} = \sqrt{\frac{2}{g}}\left[-\frac{1}{2}\left(H - h\right)^{-1/2} + \frac{1}{2}h^{-1/2}\right]$$

For the maximum time taken

$$\frac{dt}{dh} = 0$$

$$0 = \sqrt{\frac{2}{g}}\left[\frac{-1}{2}\left(H - h\right)^{\frac{-1}{2}} + \frac{1}{2}h^{\frac{-1}{2}}\right]$$

$$-\left(H - h\right)^{\frac{-1}{2}} + h^{\frac{-1}{2}} = 0$$

$$\left(H - h\right)^{\frac{-1}{2}} = h^{\frac{-1}{2}}$$

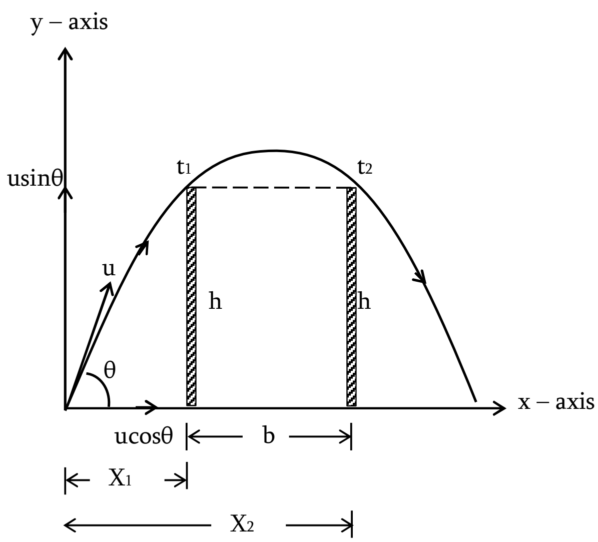
$$H - h = h$$

$$\frac{h}{H} = \frac{1}{2}$$

CLEARING POSTS OF EQUAL HEIGHT

A projectile can clear posts of equal height , as projectile retraces vertical displacement attained during upward flight while going down.

Consider the figure below



Expression of separation between two vertical walls can be obtained by using two methods:

Method 1:

We can use equation of trajectory and we can solve for the values of x_1 and x_2 quadratically

$$y = h = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$\frac{gx^2}{2u^2\cos^2\theta} - x\tan\theta - h = 0$$

$$\text{Let}: a = \frac{g}{2u^2\cos^2\theta} \quad , \quad b = -\tan\theta$$

$$c = -h$$

On solving quadratically

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Separation between two vertical walls

$$b = x_2 - x_1$$

Method 2:

The equation of motion for displacement yield two values of time for a given height , one of which corresponding to the time for upward flight and other for the downward flight time.

$$h = \left(u\sin\theta\right)t - \frac{1}{2}gt^2$$

$$2h = \left(2u\sin\theta\right)t - gt^2$$

$$gt^2 - \left(2u\sin\theta\right)t + 2h = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{2u\sin\theta \pm \sqrt{4u^2\sin^2\theta - 8gh}}{2g}$$

$$t_1 = \frac{u\sin\theta - \sqrt{u^2\sin^2\theta - 2gh}}{g}$$

$$t_2 = \frac{u\sin\theta + \sqrt{u^2\sin^2\theta - 2gh}}{g}$$

Now

$$x_1 = \left(u\cos\theta\right)t_1 \quad , \quad x_2 = \left(u\cos\theta\right)t_2$$

$$b = x_2 - x_1$$

$$b = u\left(t_2 - t_1\right)\cos\theta$$

Note that

If there are two values for which a projectile given angular projection is at the same height , then the sum of these two times is equal to the time of flight.

$$T = t_1 + t_2$$

$$= \frac{u\sin\theta - \sqrt{u^2\sin^2\theta - 2gh}}{g} + \frac{u\sin\theta + \sqrt{u^2\sin^2\theta - 2gh}}{g}$$

$$T = t_1 + t_2 = \frac{2u\sin\theta}{g}$$

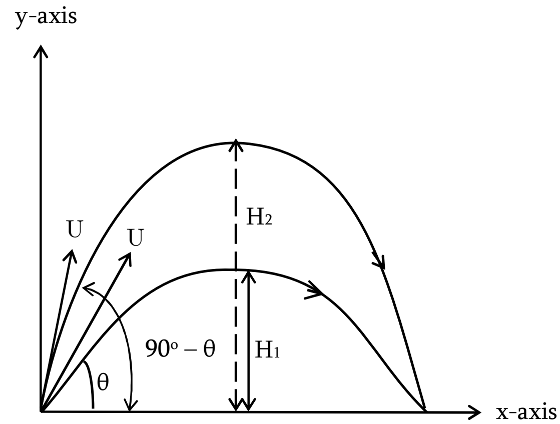
T = time of flight

21. (a) If there two values of time for which a projectile given angular projection is at the same height , then show the sum of these two times is equal to the time of flight.
(b) There are two angles of projection for which the horizontal range is the same . prove that the sum of the maximum heights for these two angles does not depend upon the angle of projection

Solution

(a) refer to your notes

(b)



The maximum height

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

Now

$$H_1 = \frac{u^2 \sin^2 \theta}{2g}$$

$$H_2 = \frac{u^2 \sin^2 (90^\circ - \theta)}{2g} = \frac{u^2 \cos^2 \theta}{2g}$$

Takes

$$H_1 + H_2 = \frac{u^2 \sin^2 \theta}{2g} + \frac{u^2 \sin^2 (90^\circ - \theta)}{2g}$$

$$= \frac{u^2 \sin^2 \theta}{2g} + \frac{u^2 \cos^2 \theta}{2g}$$

$$H_1 + H_2 = \frac{u^2}{2g}$$

Which is independent of the angle of projection , θ.
Hence proved.

22. A projectile is thrown with a velocity of 50m/s at an angle 60° with the horizontal. The projectile just clears two post of height 30m each find;
(i) The position of throw on the ground from the post
(ii) Separation between the post ($g = 10m/s^2$)

Solution

(i) by using the equation

$$h = (u \sin \theta) t - \frac{1}{2} g t^2$$

$$30 = (50 \sin 60^\circ) t - \frac{1}{2} \times 10 t^2$$

$$30 = 43.3t - 5t^2$$

On solving quadratically

$$t_1 = 07594 \text{ sec} \quad \text{or}$$

$$t_2 = 7.9005 \text{ sec}$$

Position of throw

$$x_1 = (u \cos \theta) t_1$$

$$x_1 = 18.98\text{m}$$

Also

$$x_2 = (50 \cos 60^\circ) t_2$$

$$x_2 = 197.52\text{m}$$

(ii) separation between post

$$b = x_2 - x_1$$

$$b = 178.5325\text{m}$$

23. A body is projected with initial velocity of 20m/s at an angle of 30° to the horizontal and it just clear two vertical walls of the same height of 2m. Find the separation between the two vertical wall ($g = 9.8m/s^2$)

Answer: 26.8m

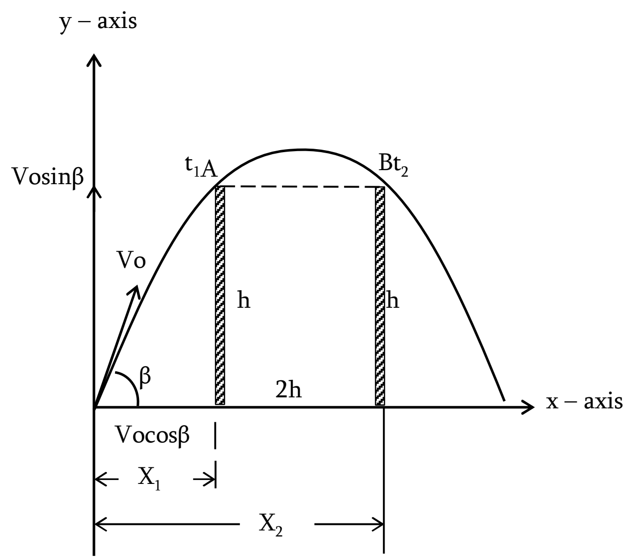
24. A particle is projected with a velocity $2\sqrt{gh}$ so that it clears two walls of equal heights 'h' in time t_1 and t_2 respectively. The two walls are at a distance of 2h from each other if the time of passing between the two walls is $2\sqrt{\frac{g}{h}}$

(a) Find the angle of projection

(b) Show that $t_1 + t_2 = 2\sqrt{\frac{3h}{g}}$

Solution

(a) suppose a particle is projected with initial velocity Vo at a an angle of elevation β with horizontal



Let t_1 and t_2 be times taken by the particle to passes through point A and B respectively.

Motion of a particle along x – direction

$$X_1 = (V_o \cos\beta)t_1 = (2\sqrt{gh}\cos\beta)t_1 \ldots\ldots\text{(i)}$$

$$X_2 = (V_o \cos\beta)t_2 = (2\sqrt{gh}\cos\beta)t_2 \ldots\ldots\text{(ii)}$$

Motion of particle along y – axis

$$h = (V_o \sin\beta)t_1 - \frac{1}{2}gt_1^2 = (V_o \sin\beta)t_2 - \frac{1}{2}gt_2^2$$

$$(2\sqrt{gh}\sin\beta)t_1 - \frac{1}{2}gt_1^2 = (2\sqrt{gh}\sin\beta)t_2 - \frac{1}{2}gt_2^2$$

But

$$X_2 - X_1 = 2h$$

$$2h = (2\sqrt{gh}\cos\beta)t_2 - (2\sqrt{gh}\cos\beta)t_1$$

$$2\sqrt{gh}(t_2 - t_1)\cos\beta = 2h$$

$$\cos\beta = \frac{2h}{2\sqrt{gh}(t_2 - t_1)}$$

$$\cos\beta = \frac{2h}{2\sqrt{gh} \times 2\sqrt{g/h}} = \frac{1}{2}$$

$$\beta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\beta = 60^\circ$$

(b) From equation (iii)

$$(2\sqrt{gh}\sin\beta)t_1 - \frac{1}{2}gt_1^2 = (2\sqrt{gh}\sin\beta)t_2 - \frac{1}{2}gt_2^2$$

$$2\sqrt{gh}\sin\beta(t_2 - t_1) = \frac{g}{2}(t_2^2 - t_1^2)$$

$$2\sqrt{gh}\sin\beta(t_2 - t_1) = \frac{1}{2}g(t_2 - t_1)(t_1 + t_2)$$

$$t_1 + t_2 = \frac{4\sqrt{gh}\sin\beta}{g}$$

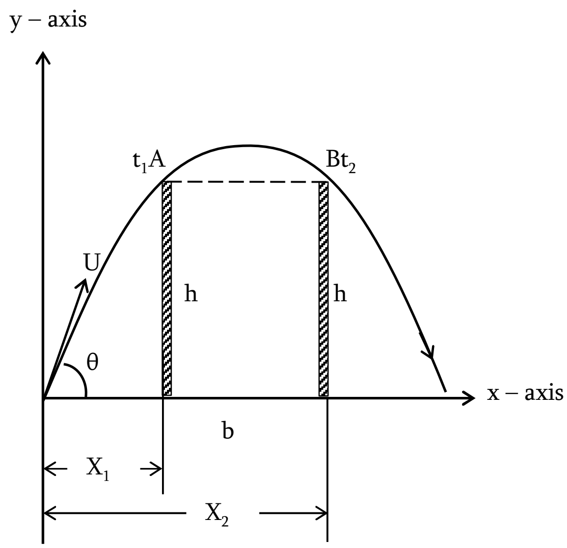
$$= \frac{4\sqrt{gh}}{g} \cdot \sin 60^\circ$$

$$t_1 + t_2 = 2\sqrt{\frac{3h}{g}}$$

Hence shown.

25. At time t = 0 sec a particle is projected from point O with speed of 50m/s and direction which makes an acute angle θ with the horizontal plane through O. Find in terms of θ an expression for the range of the particle from O. the particle also reaches a height of 10m above the horizontal plane through O at a time t1sec and t2sec. Find in terms of θ , an expression of t_1 and t_2. Given that $t_2 - t_1 = \sqrt{17}\ \text{sec}$. Find θ and also show that $R = \frac{250 \cdot \sqrt{3}}{2}\text{m}$

Solution



Case 1: horizontal range

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{50^2 \sin 20}{10}$$

$$R = 250 \sin 2\theta$$

Case 2: since $h = (u \sin\theta)t - \frac{1}{2}gt^2$

$$10 = (50\sin\theta)t - 5t^2$$

$$5t^2 - (50\sin\theta)t + 10 = 0$$

On solving quadraticaaly

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_1 = \frac{50\sin\theta - \sqrt{2500\sin^2\theta - 200}}{10}$$

$$t_2 = \frac{50\sin\theta + \sqrt{2500\sin^2\theta - 200}}{10}$$

Case 3:

$$t_2 - t_1 = \sqrt{17}$$

$$\frac{50\sin\theta + \sqrt{2500\sin^2\theta - 200}}{10} - \frac{\left(50\sin\theta - \sqrt{2500\sin^2\theta - 200}\right) = \sqrt{17}}{10}$$

On solving θ = 30°

Now

$$R = 250\sin 2\theta$$

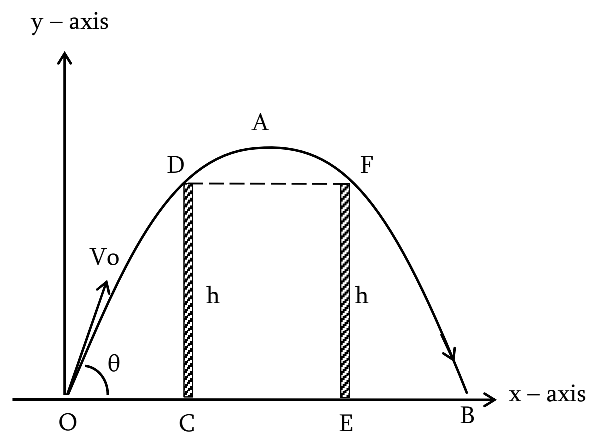
$$= 250\sin(2\times 30^\circ)$$

$$R = 250\frac{\sqrt{3}}{2}\text{m}$$

Hence shown

26. A particle is projected with the velocity $2\sqrt{hg}$ so that it clear two walls of equal height 'h' which are a distacne 2h from each other. Show that the time of passing between the walls is $2\sqrt{\frac{h}{g}}$

Solution



A particle is projected with the velocity $2\sqrt{hg}$ so that it clear two walls of equal height 'h' which are a distacne 2h from each other. Show that the time of passing between the walls is $2\sqrt{\frac{h}{g}}$

Let t_1 and t_2 be times after which the projectile is at D and F respectivel distance between two wall

$2h = (V_o \cos\theta)(t_2 - t_1)$ ....(i)

The height of the wall

$$h = (V_o \sin\theta)t - \frac{1}{2}gt^2$$

$$gt^2 - (2V_o \sin\theta)t + 2h = 0$$

By equation is quadratic in t and will have two values of t_1 and t_2 on the same value of y = h

$$t_1 + t_2 = \frac{2V_o \sin\theta}{g} \qquad \left[\alpha + \beta = \frac{-b}{a}\right]$$

Also

$$t_1 t_2 = \frac{2h}{g} \qquad \left[\alpha\beta = \frac{c}{a}\right]$$

Now

$$(t_1 - t_2)^2 = (t_1 + t_2)^2 - 4t_1t_2$$

$$= \frac{4V_0^2 \sin^2\theta}{g^2} - \frac{8h}{g}$$

$$= \frac{4\times\left(2\sqrt{hg}\right)^2 \sin^2\theta}{g^2} - \frac{8h}{g}$$

$$= 16\frac{h}{g}\sin^2\theta - \frac{8h}{g}$$

$$= \frac{8h}{g}\left[2\sin^2\theta - 1\right]$$

$$t_1 - t_2 = \sqrt{\frac{8h}{g}\left(2\sin^2\theta - 1\right)}$$

Putting equation (2) into (1)

$$2h = 2\sqrt{hg}\cos\theta \times \sqrt{\frac{8h}{g}\left(2\sin^2\theta - 1\right)}$$

[square both side]

$$4h^2 = 4gh\cos^2\theta \cdot \frac{8h}{g}\left(2\sin^2\theta - 1\right)$$

$$1 = 8\cos^2\theta\left(1 - 2\cos^2\theta\right)$$

$$16\cos^4\theta - 8\cos^2\theta + 1 = 0$$

$$\left(4\cos^2\theta - 1\right)^2 = 0$$

$$4\cos^2\theta = 1 \quad , \quad \cos\theta = \frac{1}{2}$$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

Now

$$t_2 - t_1 = \sqrt{\frac{8h}{g}\left[2\times\frac{3}{4} - 1\right]}$$

$$t_2 - t_1 = 2\sqrt{\frac{h}{g}}$$

Hence shown.

27. A particle is projected from a point A at ground level at an angle of elevation of 60°, it just clears the top of each of the two walls that are 4.0m high. If the first of the walls is at a distance of 3.0m from A find.
    (i) The speed of projection
    (ii) The distance of the second wall from A
    Answer: (i) 12.1m/s (ii) 10m

28. An object is projected so that it just clears two obstacles , each 25m high which are situated 160m from each other. If the time of passing between the obstacles is 2.5s , calculate the full range of projection and initial velocity of the object (g = 9.8m/s2)
    Answer : 277.1m , 82.04m/s.

CHARACTERISTICS OF PROJECTILE MOTION

1. Projectile motion is two dimensional motion.
2. A projectile motion consists of two perpendicular motion i.e
   (i) A horizontal motion with constant velocity.
   (ii) Motion along vertical under gravity i.e with constant acceleration g(= 9.8m/s²).
3. Projectile motion is confirmed in a vertical plane determined by the direction of the initial velocity.

PROJECTILE MOTION VERSUS UNIFORM CIRCULAR MOTION

1. Both are two dimensional motions.
2. In projectile motion , the acceleration is constant in both magnitude and direction but the velocity changes in both magnitude and direction. However , in uniform circular motion , both velocity and acceleration are constant in magnitude but both changes their directions continuously.
3. In projectile motion $\vec{a}\left(=\vec{g}\right)$, make various angle with $\vec{v}$ during the course of the motion. However , in the case of uniform circular motion , $\vec{a}_c$ is perpendicular to $\vec{v}$ at every instant
4. For projectile motion , $\vec{a}\left(=\vec{g}\right)$ is constant in both magnitude and direction. Therefore , kinematics equations for constant acceleration can be used for this type of motion. However , this equation cannot be used for uniform circular motion because the acceleration is not constant in direction.

EXAMPLES

28. For the angular projection θ, the velocity of projectile is U. let H be the maximum height reached be the projectile and R be its horizontal range , show that the maximum horizontal range is given by

$$R_{max} = \frac{R^2}{8H} + 2H$$

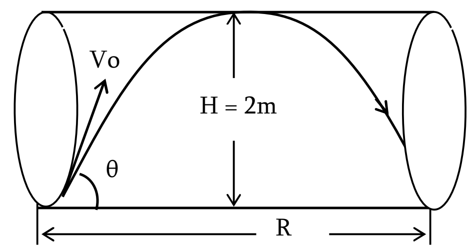
30. (a) A projectile is projected inside of tunnel which is 2m high. If the initial speed is V_o show that the maximum range inside the tunnel is given by

$$R = 4\sqrt{\frac{V_0^2}{g} - 4}$$

(b) A projectile has the same range R when the maximum height attained by it is either H_1 or H_2 . find the relation between R_1 H_1 and H_2

Solution

(a)



Horizontal range : $R = \frac{2u^2 \sin\theta\cos\theta}{g}$

Max. height : $H = \frac{u^2\sin^2\theta}{2g}$

$$\frac{R}{H} = 4\cot\theta$$

$$\cot\theta = \frac{R}{4H}$$

Also $\frac{1}{\sin^2\theta} = \frac{u^2}{2g} = \text{cosec}^2\theta$

Since $\text{cosec}^2\theta = 1 + \cot^2\theta$

$$\frac{u^2}{2gH} = 1 + \frac{R^2}{16H^2}$$

$$\frac{u^2}{g} = 2H\left(1 + \frac{R^2}{16H^2}\right)$$

But : $R_{max} = \frac{u^2}{g}$

$$R_{max} = 2\left[H + \frac{R^2}{16H}\right]$$

Let : $U = V_o$

$$R_{max} = \frac{u^2}{g} = \frac{v_0^2}{g}$$

$$\frac{v_0^2}{g} = 2\left[1 + \frac{R}{16H}\right]$$

$$V_0^2 = 2g\left[1 + \frac{R^2}{16H}\right]$$

But : $H = 2m$

$$V_0^2 = 2g\left[1 + \frac{R^2}{32}\right]$$

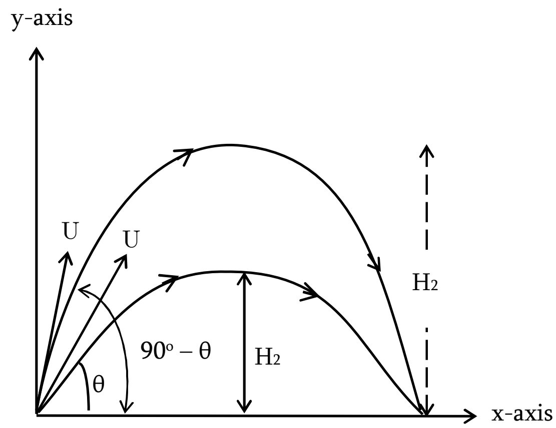
$$V_0^2 = 2g + \frac{2gR^2}{32} = 2g + \frac{gR^2}{16}$$

$$R^2 = \left(V_0^2 - 2g\right)\frac{16}{g}$$

$$R = 4\sqrt{\frac{V_0^2}{g} - 2} \quad \text{hence}$$

shown.

(b) Consider the figure below as shown.



The maximum height

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

When H = H_1 , $\theta_1 = \theta$

$$H_1 = \frac{u^2 \sin^2 \theta_1}{2g} = \frac{u^2 \sin^2 \theta}{2g}$$

$$H_2 = \frac{u^2 \sin^2 \theta_2}{2g}, \quad \theta_2 = 90^\circ - \theta$$

$$H_2 = \frac{u^2 \sin^2 (90^\circ - \theta)}{2g} = \frac{u^2 \cos^2 \theta}{2g}$$

Now:

$$\frac{2gH_1}{u^2} = \sin^2 \theta, \quad \frac{2gH_2}{u^2} = \cos^2 \theta$$

Takes

$$H_1 \cdot H_2 = \frac{u^2 \sin^2 \theta}{2g} \cdot \frac{u^2 \cos^2 \theta}{2g}$$

$$= \frac{[u \sin \theta \cos \theta]^2}{4g^2}$$

$$= \frac{1}{16}\left[\frac{2u^2}{g} \sin \theta \cos \theta\right]^2$$

$$H_1 \cdot H_2 = \frac{R^2}{16}$$

$$R^2 = 16H_1H_2$$

$$R = \sqrt{16H_1H_2} = 4\sqrt{H_1H_2}$$

$$R = 4\sqrt{H_1H_2}$$

31. (a) A particle is projected inside a tinnel which is 4.0m high. If the initial speed is Vo , show that the maximum range inside the tunnel is given by.

$$R = 4\sqrt{2}\left[\sqrt{\frac{V_0^2}{g} - 8}\right]$$

(b) A golf ball is projected with speed 49m/s at an elevation β from a point A on the first floor which is $3\frac{4}{15}$ m above the horizontal ground. The golf ball hits the ground at point Q which is at a horizontal distance 98m from point A

(i) Show that $6\tan_\beta^2 - 30\tan\beta + 5 = 0$

(ii) Hence find to the nearest degree , the two possible angles of elevation.

(iii) Find the nearest second , the smallest possible time of direct flight.

Answer : (b) (ii) 78°,10° (iii) 2second

32. Show that when a body is projected from two different angle θ_1 and θ_2 with the same initial velocity such that they attain the same horizontal range , then the ratio of the maximum heights attained is given by

$$\frac{H_1}{H_2} = \tan^2 \theta_1$$

Solution

The maximum height attained by the projectile.

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

$$H_1 = \frac{u^2 \sin^2 \theta_1}{2g}$$

$$H_2 = \frac{u^2 \sin^2 \theta_2}{2g}$$

Takes : $\dfrac{H_1}{H_2} = \dfrac{u^2 \sin^2 \theta_1}{2g} \Big/ \dfrac{u^2 \sin^2 \theta_2}{2g}$

$$\frac{H_1}{H_2} = \frac{\sin^2 \theta_1}{\sin^2 \theta_2}$$

Since θ_1 and θ_2 should be complimentary angles , then

$$\theta_1 + \theta_2 = 90^\circ$$

$$\frac{H_1}{H_2} = \frac{\sin^2\theta_1}{\sin^2(90^\circ - \theta_1)} = \frac{\sin^2\theta_1}{\cos^2\theta_1}$$

$$= \tan^2\theta_1$$

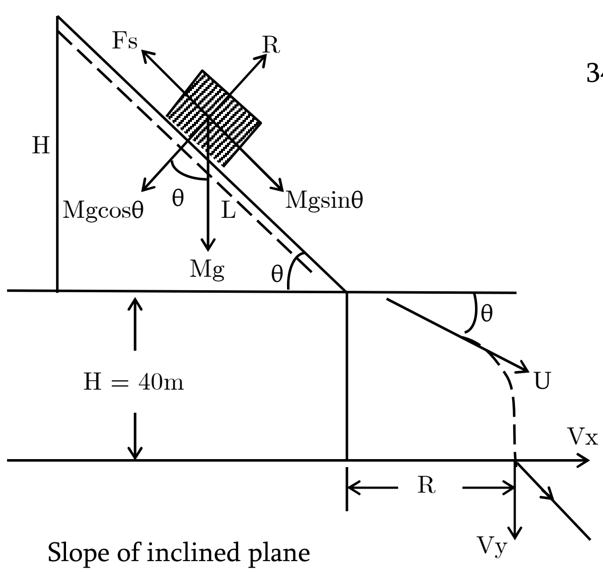
$\frac{H_1}{H_2} = \tan^2\theta_1$. Hence shown.

33. (a) (i) What do you understand by the term projectile motion.
 (ii) What are to be considered for a projectile to attain maximum range.
 (b) A 60gm block starts from rest and slides 30m down the roof with a slope of 2 horizontal to 1 vertical and the coefficient of friction of 0.1. if the vertical distance from the edge of the roof to the ground is 40m , find how far from the wall the block strikes the ground?

Solution

(a) Refer to your notes

(b) Diagram



Slope of inclined plane

$$\tan\theta = \frac{1}{2}, \quad \sin\theta = \frac{1}{\sqrt{2}}, \quad \cos\theta = \frac{2}{\sqrt{5}}$$

Apply the principle of conservation of energy

Loss in P.E = gun in K.E + workdone gain Friction

$$\sin\theta = \frac{H}{L}, \quad H = L\sin\theta$$

$$mgH = FsL + \frac{1}{2}MU^2$$

$$mgL\sin\theta = \mu mgL\cos\theta + \frac{1}{2}MU^2$$

$$U = \sqrt{2gL(\sin\theta - \mu\cos\theta)}$$

$$= \sqrt{2 \times 9.8 \times 30\left[\frac{1}{\sqrt{5}} - \frac{2\times 0.1}{\sqrt{5}}\right]}$$

$$U = 14.5m/s$$

Flight time

$$t = \frac{-u\sin\theta + \sqrt{u^2\sin^2\theta + 2gh}}{g}$$

$$= \frac{-14.5 \times \frac{1}{\sqrt{5}}\sqrt{(14.5)^2\left(\frac{1}{\sqrt{5}}\right)^2 + 2\times 9.8\times 40}}{9.8}$$

$$t = 2.2711\,\text{sec}$$

Horizontal range

$$R = (u\cos\theta)t = 14.5 \times \frac{2}{\sqrt{5}} \times 2.2711$$

$$R = 29.45m$$

34. (a) (i) In the projectile motion , what is the relationship between the directions of acceleration and velocity?
 (ii) What assumptions made in the treatment of projectile motion?
 (iii) What is the nature of the projectile motion?
 (b) (i) Is projectile motion two dimensional motion?
 (ii) What is the characteristics of projectile motion?

Solution

(a) (i) in projectile motion (non – vertical), $\vec{a}(=g)$ is constant in both magnitude and direction (downward) and makes various angle with $\vec{v}$ during the course of motion.
 (ii) Refer to your notes
 (iii) It is a case of two dimensional motion (i.e motion in a plane) with constant acceleration.

(b) (i) Yes , since the projectile moves horizontal as well as vertically it is a case of two dimensional motion i.e two coordinates are required to specify the position of the projectile at any time

(ii) A projectile motion consists of two perpendicular motions viz

- A horizontal motion with constant velocity.
- Motion along vertical under acceleration ($g = 9.8m/s^2$)

35. (a) (i) Is there any point on the projectile path at which the acceleration is perpendicular to the velocity.

(ii) In the projectile motion , when is the kinetic energy of projectile maximum and minimum?

(b) At which point of projectile motion.

(i) K.E is maximum

(ii) P.E is maximum

(iii) K.E is minimum

(iv) Velocity is minimum

(v) Velocity is maximum.

(c) Prove that the maximum horizontal range is four times the maximum height attained by the projectile.

Solution

(a) (i) Yes at the highest point of the parabolic path the projectile has only horizontal velocity because the vertical component of the velocity is zero. However acceleration due to gravity is acting vertically downward.

(ii) The K.E of the projectile is maximum at the point of projection or point of reaching the ground and is minimum at the highest point.

(b) (i) Plane of projection

(ii) At the highest point

(iii) At the highest point

(iv) At the highest point

(v) Plane of projection or at a point where it hits the ground.

(c) $R_{max} = \frac{V_0^2 \sin^2\theta}{g}$, $\theta = 45^\circ$

$$R_{max} = \frac{V_0^2}{g}$$

$$H_{max} = \frac{V_0^2 \sin^2\theta}{2g} = \frac{V_0^2\left(\sin 45^\circ\right)^2}{2g}$$

$$H_{max} = \frac{V_0^2}{4g} = \frac{R_{max}}{4}$$

$R_{max} = 4H\,max$. Shown

And angle of projection (2½ marks)

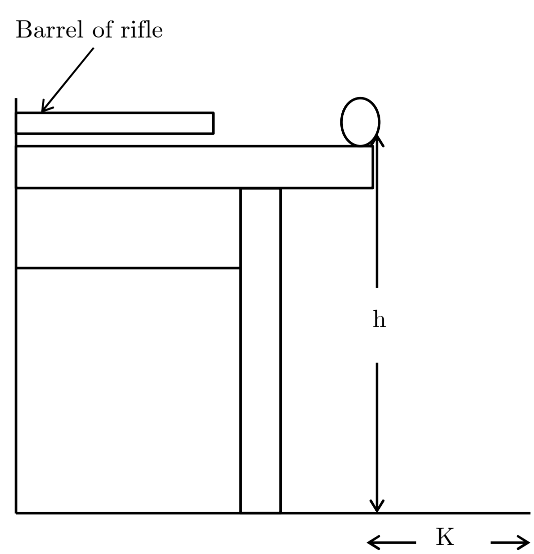
(ii) Show that the trajectory is always a parabolic path (02 marks)

(c) A ball is thrown with an initial velocity of 48m/s at an angle $\theta = 37^\circ$ with the vertical find

(i) The x and y components of Vo (01 mark)

(ii) The position of the ball , including magnitude of its velocity t = 2sec (03 marks)

42. A student devised the following experiment to determine the velocity of a pullet from an air rifle



A piece of plasticine of mass, m is balanced on the edge of a table such that it just fails to fall off. A pellet of mass m is fired horizontally into the plasticine and remain embedeed in it. As a result the plasticine reaches the floor a horizontal distance k away. The height of the table is h.

(i) Show that the horizontal velocity of the plasticine with pallet embedded is $k\left[\frac{g}{2h}\right]^{1/2}$

(ii) Obtain an expression for the velocity of the pellet before impact with the plasticine.

43. A golf ball is projected with speed 49m/s at an elevation β from a point A on the first floor which is $3\frac{4}{15}$ m above the horizontal ground. The golf ball hits the ground at a point Q which is at a horizontal distance 98m from point A.
(i) Show that $6\tan^2\beta - 30\tan\beta + 5 = 0$
(ii) hence find to the nearest degree the two possible angles of elevation.
(iii) find to the nearest second , the smallest possible time of direct flight
Answer: (a) 78°, 40° (iii) 2 second.

44. A fireman is attempting to direct water through the open window of a top room in a tall building. The window is 25m above the ground. The water leaves the hose at an angle 40° to the horizontal at a height of 1.5m above the ground. The fireman finds that if he stands 35m away from the base of the wall the jet of the water travelling in an upward derivation just goes through the open window. Determine the speed of water as it leaves the hose.
Answer :41.75m/s.

45. (a) During the wall of a soldier of one part was ordered to bomb a certain enemy's town situated about the same horizontal level with the point of projection so the soldier had to find the horizontal range first. He was told that the trajectory for the bomb is given by the expression

$$y = x\tan\beta = \frac{gx^2}{2V_0^2\cos^2\beta}$$

Show that the range of the soldier got is given by the expression.

$$R = \frac{V_0^2\sin 2\beta}{g}$$

(b) A player kicks a ball at an angle of elevation of 60° it just clears the top of each two walls that are 4.0m high if the first of the walls is at a distance of 3.0m from point A find:-
(i) Speed of projection
(ii) The distance of the second wall from A
Answer: (b) (i) 12.1m/s (ii) 10m.

46. A projectile fired with speed V at an angle θ from point A on the ground , reaches the ground at a horizontal distance d metres , from point A.
(i) prove that $V = \sqrt{gd\operatorname{cosec}\theta}$ and that greatest height attained by the projectile is $\frac{1}{4}d\tan\theta$
(ii) If the projectile just clears an obstacle of height , h metres at a horizontal distance , a metre from A, show that $\tan\theta = \frac{hd}{a(d-a)}$

47. (a) Find the velocity and direction of the projection of a particle which passes in horizontal direction just cover the top of a wall which is 36m distant and 26.6m/s
(b) A body is projected with speed V_1 at an angle of 45° to the horizontal from a point P at the same instant , another body is projected vertically upward from a point O with speed V_2 the point O is vertically below the highest point of the path of the first boy. If the bodies collide at the highest point , find the value of

$$\frac{V_1}{V_2}$$

Answer: (a) 26.6m/s , 48° (b) $\sqrt{2}$

48. (a) A and B are two points $60\sqrt{3}$ m a part on level ground. A particle P is projected from a towards B with speed 45m/s at 30° to the horizontal at the same instant a particle Q is projected from B towards A with speed $15\sqrt{3}\text{m/s}$ at 60° to the horizontal.

(i) Prove that P and Q are always at the same height.
(ii) Find the time and height at which collision occur.

(b) An aeroplane flies at a height h with a constant speed U_o in a straight horizontal line so as to pass vertically over a certain gun. At the instant when the aeroplane is directly over it , the gun fires a shell which hits the plane. Find the minimum muzzle velocity V_o of the shell and correct angle of elevation α of the gun at this velocity in terms of U_o , g and h

Answer :
(a) (ii) 2.0sec , 25.4m
(b) $V_o = \sqrt{U_0^2 + 2gh}$, $\alpha = \tan^{-1}\left[\sqrt{\frac{2gh}{U_0^2}}\right]$

49. An aeroplane flies with a uniform speed U_o in the horizontal course at an altitude 2.25km the pilot drops a parcel as the plane passes directly above the gun on the ground and at the same instant a bullet is fired from the gun with velocity of V_o at an angle 30° to the horizontal . Find;-
(a) The muzzle velocity V_o of the bullet if it is to be hits by the parcel at the highest possible altitude.
(b) Time of flight of the parcel from the instant it is released to the time it hit the bullet.
(c) The value of U_o.
(d) The positions of the air craft and the altitude of the parcel when the bullet hit the parcel.

Answer :

(a) 297m/s	(b) 15sec
(c) 257m/s	(d) 3855m,1125m

50. (a) Show that the vertical displacement of the ball after being projected with initial velocity U is given by

$$y = x\tan\theta - \frac{gx^2}{2u^2}\left(1+\tan^2\theta\right)$$

Where x is the horizontal displacement.

(b) If the horizontal range of a particle projected with velocity U is R , show that the greatest height it attained is given by the equation

$$16gH^2 - 8u^2H + gR^2 = 0$$

51. Two projectiles are fired simultaneously from point P and Q on a horizontal ground and collides head on when travelling horizontally. The first projectile is fired with a speed V_o(m/s) at an elevation α and the second fired with a speed ½ Vo(m/s) at an elevation β. If

$$PQ = \frac{V_0^2 \sin\beta}{2g}$$

Show that:
(i) $2\sin\alpha = \sin\beta$
(ii) $2 = 2\cos\alpha + \cos\beta$
(iii) $\cos\alpha = \frac{7}{8}$

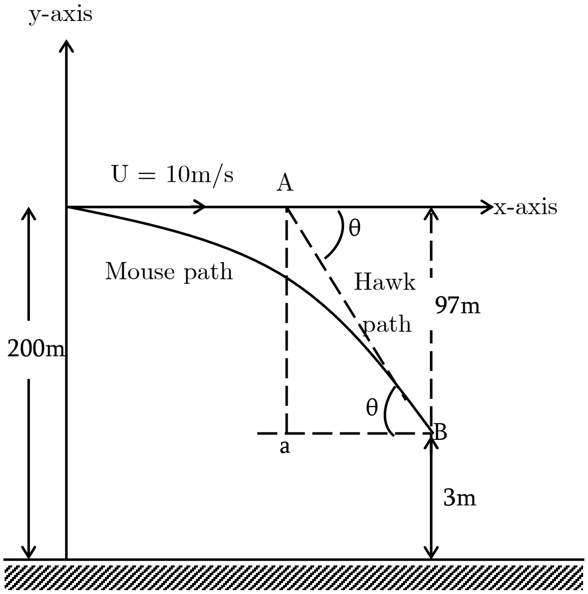
52. A particle is projected with an initial speed V_o to pass through a point which is 5h horizontally and h vertically from the point of projection. Show that if there are two angles of projection for which this is possible , then

$$V_0^2 > 20\left(h + 125\right)$$

53. A hawk is flying horizontally at 10m/s in a straight line above the ground. A mouse it was carrying is released from it grasp. The hawk continues on its path at the same speed for two seconds before attempting to retrieve its prey. To accomplish the retrieval it dives in a straight line at constant speed and recapture the mouse 3.0m above the ground.
(i) Find the diving speed of the hawk
(ii) What angle did the hawk make with the horizontal during its descent?

Solution

(i)



Time taken by the mouse to travel the distance from O to B (caught point)

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 197}{9.8}}$$

$t = 6.3407$ sec

Time of descent of the hawk

td = 6.3407 – 2.00

td = 4.3407 sec

distance travelled by the hawk as it dives to catch the mouse (AB): CB =Utd = 43.407m

$$AB = \sqrt{(AC)^2 + (CD)^2} = \sqrt{197^2 + 43407^2}$$

$AB = 201.725m$

The diving speed of the hawk

$$V = \frac{AB}{td}$$

$$= \frac{201.725}{4.3407}$$

$V = 46.5m/s$

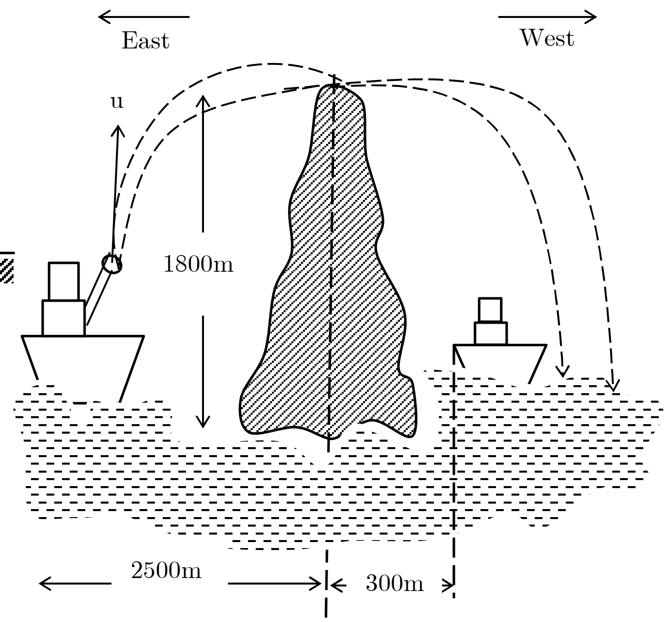
(ii) Angle made by the hawk in the descent

$$\tan\theta = \frac{AC}{CB}, \quad \theta = \tan^{-1}\left(\frac{AC}{CB}\right)$$

$$\theta = \tan^{-1}\left(\frac{197}{43.407}\right)$$

$\theta = 77.6^\circ$ below the horizontal

54. An enemy ship is on the East side of mountain Island as shown in the figure below. The energy ship can maneuver to within 2500m of the 1800m high mountain peak can shoot projectiles with an initial speed of 250m/s. if the western shoreline is horizontally 300m from the peak. What are the distances from the western shore at which a ship can be safe from the bombardment of the enemy ship?



Solution

By using equation of trajectory

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

For the projectile to clear the hill x = 2500m , y = 1800m

$$1800 = 2500\tan\theta - \frac{9.8(2500)^2}{2(250)^2\cos^2\theta}$$

$$490\tan^2\theta - 2500\tan\theta + 2290 = 0$$

On solving

$\theta = 75.64^o$ or 50.12^o.

The range of the projectiles

$$R = \frac{U^2\sin 2\theta}{g}$$

$\theta = 75.64^\circ$

$$R_1 = \frac{(250)^2 \sin(2\times 75.64^\circ)}{9.8}$$

$$R_1 = 3065\text{m}$$

$$\theta = 50.12^\circ$$

$$R_2 = \frac{(250)^2 \sin(2\times 50.12^\circ)}{9.8}$$

$$R_2 = 6276\text{m}$$

The ship will be safe if it stays at a distance less than 3065 – 2500 – 300 = 3476m from the store line.

60. (a) (i) Justify the statement that projectile motion is two dimensional motion (02 marks)

(ii) A rocket was launched with a velocity of 50m/s from the surface of the moon at an angle of 40° to the horizontal distance covered after half time of flight (03 marks)

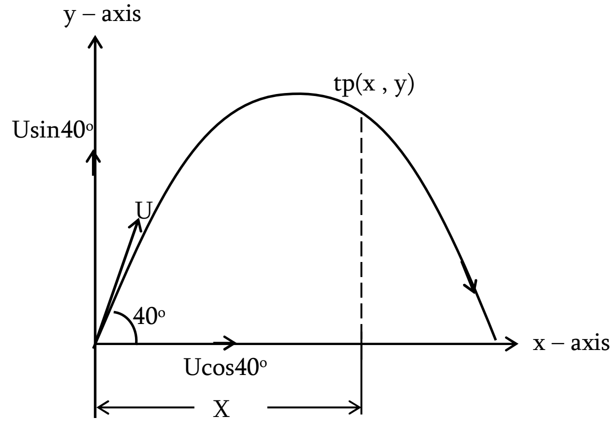
(b) (i) Show that the angle of projection θ_o for a projectile launched from the origin is given by $\theta^\circ = \tan^{-1}\left[\frac{4hm}{R}\right]$ where R stand for horizontal range and h_m is the maximum vertical height (02 marks)

(ii) Determine the angle of projection for which the horizontal range of a projectile is $4\sqrt{3}$ times its maximum height (03 marks)

Solution

(a) (i) Because two coordinates are required to describe projectile motion since it moves horizontally as well as vertically i.e two coordinates are required to specify the position of the projectile at any time.

(ii) Diagram



$$t = \frac{T}{2}, \text{ but}$$

$$T = \frac{2u\sin\theta}{g}, \frac{T}{2} = \frac{u\sin\theta}{g}$$

Horizontal distance

$$x = (u\cos 40^\circ)t$$

$$= (u\cos 40^\circ)\left(\frac{u\sin 40}{9}\right)$$

$$= \frac{u^2 \sin 40^\circ \cos 40^\circ}{g}$$

$$= \frac{(50)^2 \sin 40^\circ \cos 40^\circ}{9.8}$$

$$X =$$

(b) (i) since $R = \dfrac{2u^2 \sin\theta\cos\theta}{g}$

$$h_m = \frac{u^2\sin^2\theta}{2g}$$

Takes

$$\frac{h_m}{R} = \frac{u^2\sin^2\theta}{2g} \times \frac{g}{2u^2\cos\theta\sin\theta}$$

$$\frac{h_m}{R} = \frac{\tan\theta}{4}$$

$$\tan\theta = \frac{4h_m}{R} \text{ OR } \theta = \tan^{-1}\left[\frac{4h_m}{R}\right]$$

$$\theta = \tan^{-1}\left[\frac{4h_m}{R}\right]$$

(ii) $R = 4\sqrt{3}H$

$$\frac{2u^2 \sin\theta \cos\theta}{g} = 4\sqrt{3}\,\frac{u^2 \sin^2\theta}{2g}$$

$$\frac{1}{\sqrt{3}} = \tan\theta$$

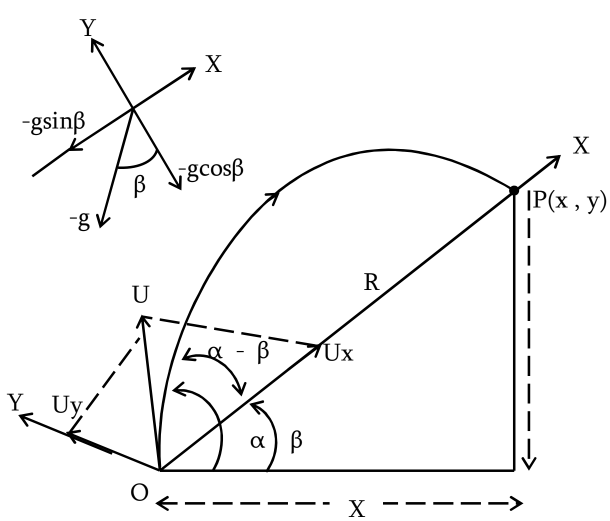
$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 60^\circ$$

$$\theta = 60^\circ$$

PROJECTILE ON AN INCLINED PLANE

Projectile motion on an inclined plane is the one of various types of projectile motion. This aspect is different from normal projectile since the point of projection and point of striking on the ground are not on the same plane here you have two different case:

CASE 1 : OBJECT PROJECTED UP THE PLANE



IMPORTANT CHARACTERISTICS OF ASPECTS OF PROJECTILE MOTION ON AN INCLINED PLANE.

1. Coordinate X along the incline plane and Y peperndicular to the inclined plane.
2. Angle of projection $\alpha - \beta$.
3. Range R is measured along the inclined plane from point O to p.
4. Components of initial velocity $U_x = U\cos(\alpha - \beta)$, $U_y = U\sin(\alpha - \beta)$.
5. Acceleration due to gravity is vertically dowards and its make an angle β with the negative Y – axis components of acceleration $a_x = -g\sin\beta$, $a_y = -g\cos\beta$

PARAMETERS OF THE MOTION

(i) Expression of the flight time , T

Since $S_y = U_y t + \frac{1}{2}a_y t^2$

When $S_y = 0$, $t = T$

$$0 = U_y T + \frac{1}{2}a_y T^2$$

$$T = \frac{-2U_y}{a_y} = \frac{-2U\sin(\alpha-\beta)}{-g\cos\beta}$$

$$T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

(ii) Expression of the horizontal range , R

Since $S_x = U_x t + \frac{1}{2}a_x t^2$

When $S_x = R$, $t = T$, $a_x = -g\sin\beta$

$U_x = u\cos(\alpha - \beta)$

$$R = u\cos(\alpha-\beta)T - \frac{1}{2}(g\sin\beta)T^2$$

$$= u\cos(\alpha-\beta)\left[\frac{2u\sin(\alpha-\beta)}{g\cos\beta}\right] - \frac{1}{2}g\sin\beta\left[\frac{2u\sin(\alpha-\beta)}{g\cos\beta}\right]$$

$$= \cos(\alpha-\beta)\cdot\frac{2u^2\sin(\alpha-\beta)}{g\cos\beta} - \sin\beta\,\frac{2u^2\sin^2(\alpha-\beta)}{g\cos^2\beta}$$

$$= \frac{2u^2\sin(\alpha-\beta)}{g\cos\beta}\left[\cos(\alpha-\beta) - \frac{\sin(\alpha-\beta)\sin\beta}{\cos\beta}\right]$$

$$= \frac{2u^2\sin(\alpha-\beta)}{g\cos\beta}\left[\frac{\cos(\alpha-\beta)\cos\beta - \sin(\alpha-\beta)\sin\beta}{\cos\beta}\right]$$

$$R = \frac{2u^2\sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta} \quad \ldots\ldots\ldots\ldots\ldots\text{(2)}$$

Again

$$R = \frac{U^2}{g\cos^2\beta}\left[\sin(2\alpha-\beta) - \sin\beta\right]$$

Expression of the maximum horizontal range

When $R = R_{max}$, $\sin(2\alpha - \beta) = 1$

$$R\max = \frac{U^2}{g\cos^2\beta}\left[1 - \sin\beta\right]$$

And for maximum range

$$\sin(2\alpha - \beta) = 1$$

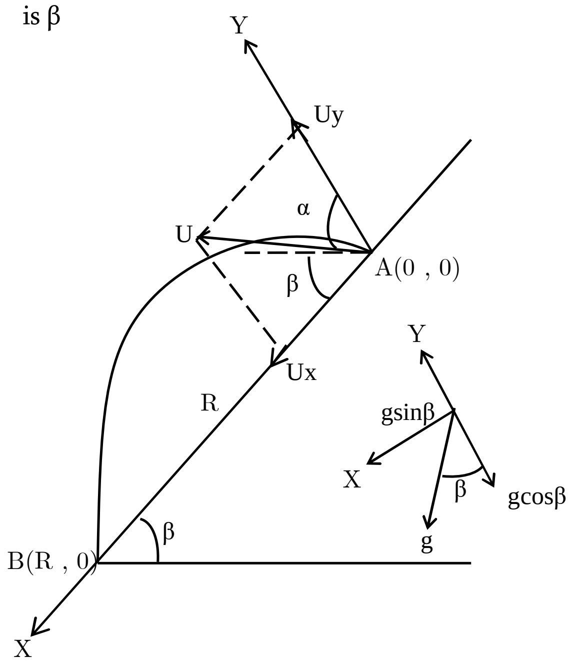
$$2\alpha - \beta = \frac{\pi}{2}$$

$$\alpha = \frac{\pi}{4} + \frac{\beta}{2} \quad \text{.............................(4)}$$

Equation 4 represent the angle of projection for a given speed to achieve maximum possible range which is given by equation (3)

Case 2: object projected down the plane

Let us consider a case shown in the following figure where an object thrown from top of the incline , with a velcoity U at an angle α with the horizontal here angle of plane with the horizontal is β



In this case angle between initial velocity and X – axis becomes α + β.

Components of initial velocity of projectile

$U_x = U\cos(\alpha + \beta)$, $U_y = U\sin(\alpha + \beta)$

Components of acceleratin due to gravity

$a_x = g\sin\beta$, $a_y = -g\cos\beta$

similarly components of displacement can be written as folllows:

$S_x = R$, $S_y = O$

Now we can proceed with the calculations same as the previous case. All the results obtained can be written by replacing β by –β hence we can write the following

$$T = \frac{2u\sin(\alpha - \beta)}{g\cos\beta}$$

$$R = \frac{2u^2\sin(\alpha - \beta)\cos\alpha}{g\cos^2\beta} \quad \text{or}$$

$$R = \frac{u^2}{g\cos^2\beta}\left[\sin(2\alpha + \beta) + \sin\beta\right]$$

$$R\max = \frac{u^2}{g\cos^2\beta}\left[1 + \sin\beta\right]$$

$$\alpha = \frac{\pi}{2} - \frac{\beta}{2}$$

SOLVED EXAMPLES

EXAMPLE

A projectile is thrown from the base of an incline of angle 30°. what should be the angle of projection as measured from the horizontal direction so that range on the incline is maximum?

Solution

From the equation

$$R = \frac{u^2}{g\cos^2\beta}\left[\sin(2\alpha - \beta) - \sin\beta\right]$$

The range is maximum when

$$\sin(2\alpha - \beta) = 1$$

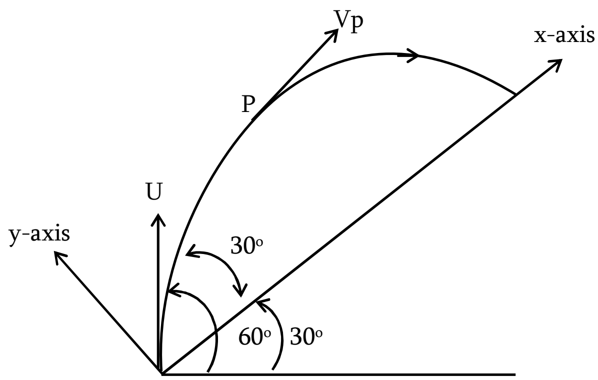
$$2\alpha = \beta + 90^\circ$$

$$\alpha = \frac{30^\circ + 90^\circ}{2}$$

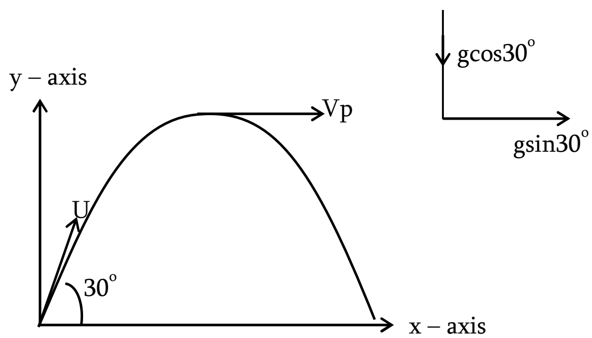
$$\alpha = 60^\circ$$

Example

A projectile is projected form the foot of an incline of angle 30° with the velocity 30m/s. the angle of projection as measured from the horizontal is 60°. find the speed wihen the projectile is paralled to the incline.



Equivalent diagram



Flight time

$$T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$

$$= \frac{2\times 30\sin(60^\circ - 30^\circ)}{9.8\cos 30^\circ}$$

$$T = \frac{60}{9.8}\tan 30^\circ = 3.5\,\text{sec}$$

The time taken by the projectile to reach at the highest position

$$t = \frac{T}{2} = \frac{3.53\,\text{sec}}{2} = 1.765\,\text{sec}$$

Velocity of the projectile at the point p parallel to the inclined plane

$$V_p = u\cos 30^\circ - (g\sin 30^\circ)t$$

$$= 30\cos 30^\circ - (9.8\sin 30^\circ)\times 1765$$

$$V_p = 17.3\text{m}/\text{s}$$

51. A particle is projected with a velocity of 245m/s at an elevation of 60° to the horizontal from the foot of a plane of inclination 30°. Find the range on the inclined plane and the time of flight.

Solution

The horizontal range

$$R = \frac{2u^2\sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}$$

$$= \frac{2\times 245^2\sin(60^\circ - 30)\cos 60^\circ}{9.8\cos^2 30^\circ}$$

$$R = 4083.33\text{m}$$

Also

$$T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$

$$= \frac{2\times 45\sin(60^\circ - 30^\circ)}{9.8\cos 30^\circ}$$

$$T = 28.87\,\text{sec}$$

52. A particle is projected with a velocity of 20m/s at an angle of 45° to the horizontal. Find its range on a plane inclined at 30° to the horizontal when projected (i) up (ii) down the plane.

Solution

(i) Up the inclined plane the horizontal range.

$$R = \frac{2u^2\sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}$$

$$= \frac{2\times 20^2\sin(45^\circ - 30)\cos 45^\circ}{9.8\cos^2 30^\circ}$$

$$R = 19.92\text{m}$$

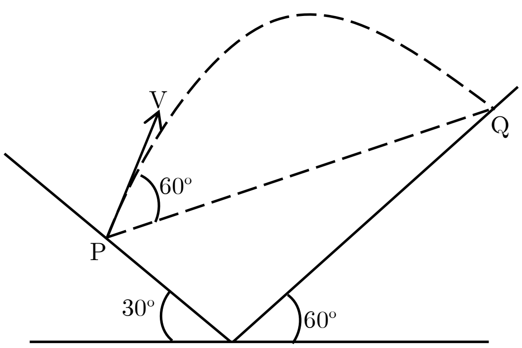
(ii) For the downward the plane

$$R = \frac{2u^2\sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}$$

$$= \frac{2\times 20^\circ\sin(45^\circ - 30^\circ)\cos 45^\circ}{9.8\cos^2 30^\circ}$$

$$R = 74.34\text{m}$$

53. Two inclined planes of angle 30° and 60° are placed to ** each other at the base as shown in the figure below. A projectile is projected at right angle with a speed of 103m/s from point P and hits the other incline at point Q normally. Calculate the time of flight.



Solution

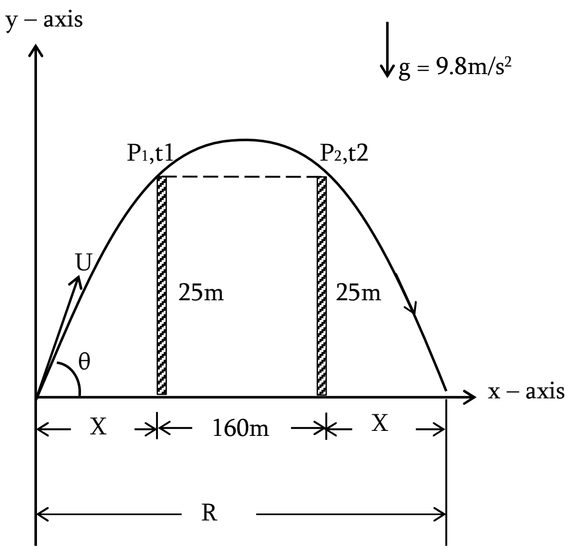
$$T = \frac{2u\sin\theta}{g}$$

$$= \frac{2\times 103\sin 60^{\circ}}{9.8}$$

$$T = 18.2\,\text{sec}$$

54. An object is projected so that it just clears two obstacles , each 25m high , which are situated 160m from each other if the time of passing between the obstacles is 2.5sec. Calculate the full range of projection and the initial velocity of the object.

Solution



From the figure above

$R = 2x + 160$

Points $P_1 = (X, 25)$ time , t

$P_2 = [X + 160, 25]$, time , t

Horizontal displacement

$$X = (u\cos\theta)t_1 \ldots\ldots\ldots\ldots\ldots(i)$$

$$X + 160 = (u\cos\theta)t_2 \ldots\ldots\ldots\ldots\ldots(ii)$$

Dividing equation (ii) by (i)

$$\frac{X+160}{X} = \frac{(u\cos\theta)t_2}{(u\cos\theta)t_1}$$

$$\frac{X+160}{X} = \frac{t_2}{t_1} \ldots\ldots\ldots\ldots\ldots(iii)$$

Vertical displacement

$$h = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$h = (u\sin\theta)t - 4.9t^2$$

$$h + 4.9t^2 = (u\sin\theta)t$$

At the point P_1

$$25 + 4.9t_1^2 = (u\sin\theta)t_1 \ldots\ldots\ldots\ldots\ldots(iv)$$

At the point P_2

$$25 + 4.9t_2^2 = (u\sin\theta)t_2 \ldots\ldots\ldots\ldots\ldots(v)$$

Dividing equation (iv) by (v)

$$\frac{25 + 4.9t_1^2}{25 + 4.9t_2^2} = \frac{t_1}{t_2}$$

$$t_2(25 + 4.9t_1^2) = t_1(25 + 4.9t_2^2)$$

$$25\left(t_2 - t_1\right) = 4.9\left(t_1t_2 - t_2t_1^2\right)$$

$$25\left(t_2 - t_1\right) = 4.9t_1t_2\left(t_2 - t_1\right)$$

$$25 = 4.9t_1t_2$$

$$t_1t_2 = \frac{25}{4.9} = 5.1 \text{ ....(vi)}$$

Since

$$t_2 - t_1 = 2.5$$

$$t_2 = 2.5 + t_1$$

Now equation (vi) becomes

$$t_1\left(2.5 + t_1\right) = 5.1$$

$$t_1^2 + 2.5t_1 - 5.1 = 0$$

On solving quadratically

$$t_1 = 1.33 \text{ sec}$$

$$t_2 = 3.83 \text{ sec}$$

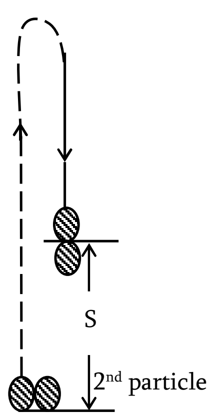
Now equation (iii) becomes

$$\frac{x + 160}{x} = \frac{t_2}{t_1} = \frac{3.83}{1.33}$$

55. A particle is projected vertically upwards with a speed U , after an interval of time t another particles is projected upward from the same point and with the same initial velocity. Prove that the particles will meet at the height $\left[\frac{4u^2 - g^2t^2}{8g}\right]$

Solution

Diagram



For the two bodies to meet

For the 1st particle.

$$S = ut_1 - \frac{1}{2}gt_1^2 \text{ ....(i)}$$

Where t_1 time taken for them to meet

For the 2nd particle

$$S = u\left(t_1 - t\right) - \frac{1}{2}g\left(t_1 - t\right)^2$$

Since the second particle lag behind by t

$$S = ut_1 - ut - \frac{1}{2}g\left(t_1^2 - 2t_1t + t^2\right)$$

$$S = ut_1 - \frac{1}{2}gt_1^2 - ut + gt_1t - \frac{1}{2}gt^2$$

$$S = s - ut + gt_1t - \frac{1}{2}gt^2$$

$$0 = -ut + gt_1t - \frac{1}{2}gt^2$$

$$ut + \frac{1}{2}gt^2 = gt_1t$$

$$t_1 = \frac{u + \frac{1}{2}gt}{g} \text{ ....(ii)}$$

Putting equation (ii) into (i)

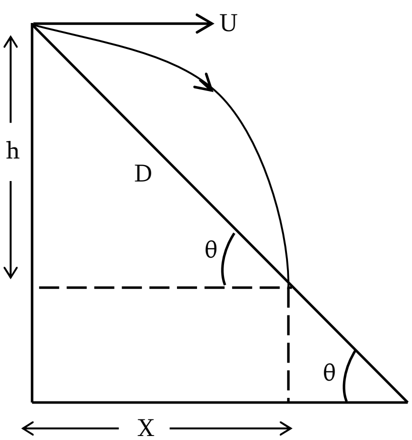
$$S = u\left[\frac{u + \frac{1}{2}gt}{g}\right] - \frac{1}{2}g\left[\frac{u + \frac{1}{2}gt}{g}\right]^2$$

$$S = \frac{u^2}{g} + \frac{ut}{2} - \frac{u^2}{2g} - \frac{ut}{2} - \frac{gt^2}{8}$$

$$S = \frac{u^2}{2g} - \frac{gt^2}{8}$$

$$S = \left[\frac{4u^2 - g^2t^2}{8g}\right] \text{ hence show}$$

56. A particle is thrown horizontally from the top of an inclined plane , with a speed of 5m/s. if the inclination of the plane is 30° with the horizontal , how far from the point of projection will the particle strike the plane?
Solution
The given situation is shown below



Horizontal distance covered by particle in time , t

$x = ut \quad , \quad t = \frac{x}{u}$

Vertical distance covered in time

$h = u_y t + \frac{1}{2}gt^2$ but uy = 0

$h = \frac{1}{2}gt^2 = \frac{gx^2}{2u^2}$

From the figure above

$\tan\theta = \frac{x}{h} \quad , \quad h = x\tan\theta$

Now $x\tan\theta = \frac{gx^2}{2u^2}$

$x\tan\theta - \frac{gx^2}{2u^2} = 0$

$x\left[\tan\theta - \frac{gx}{2u^2}\right] = 0$

x = 0 is the initial point , s

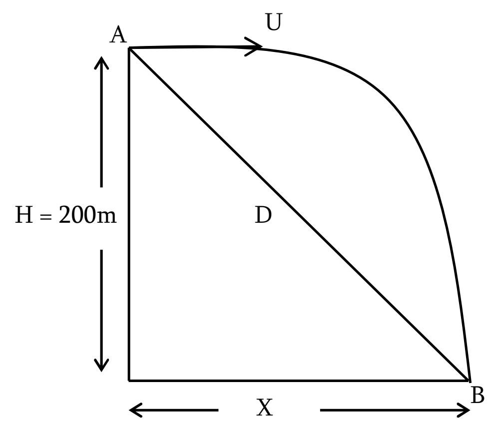
$\tan\theta - \frac{gx}{2u^2} = 0$

$$x = \frac{2u^2\tan\theta}{g}$$

Distance of the point of projection of the particle from the point where it strike is

$$D^2 = x^2 + h^2$$
$$D = \sqrt{x^2 + h^2}$$
$$= \sqrt{x^2 + (x\tan\theta)^2}$$
$$= x\sqrt{1 + \tan^2\theta} = x\sec\theta$$
$$D = x\sec\theta$$
$$= \frac{2u^2\tan\theta}{g}\cdot\sec\theta$$
$$D = \frac{2\times 5^2\tan 30^\circ\sec 30^\circ}{9.8}$$
$$D = 3.4\text{m}$$

57. A helicopter moving horizontally with a velocity of 360km/hr has to drop a food packet directly into a relief fund office. If the height of helicopter from the ground is 200m at what distance the helicopter must be from the fund office when the packet is dropped?
Solution



Here A represent the position of helicopter and B represents the position of relief fund office.
Now vertical displacement

$$h = uyt + \frac{1}{2}gt^2$$
$$200 = 0 + \frac{1}{2}\times 9.8t^2$$
$$t = \sqrt{\frac{400}{9.8}} \approx 6.4\,\text{sec}$$

Horizontal distance covered by the food packet in time , t x = ut = 100 × 6.4 = 640m
Distance between the helicopter and relief fund office.

$$D^2 = h^2 + x^2$$

$$D = \sqrt{h^2 + x^2}$$

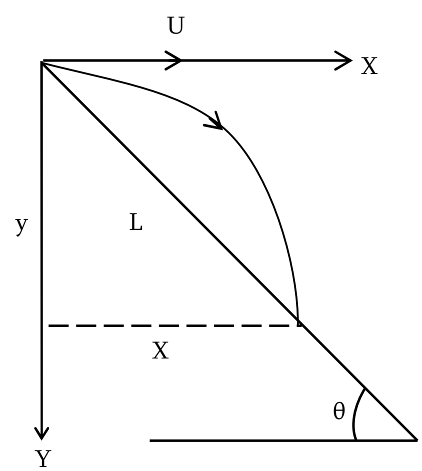
$$= \sqrt{200^2 + 640^2}$$

$$D = 670.52\text{m}$$

58. A particle is projected from the top of an inclined plane in the horizontal direction with a speed , U. the plane is inclined at an angle θ with the horizontal at what distance will the particle hit the plane?

Solution

Let us select point of projection as origin and X and Y – axis as shown in the figure below.



Horizontal displacement

$$x = ut \quad \Rightarrow \quad t = \frac{x}{u}$$

Vertical displacement

$$y = U_y t + \frac{1}{2}gt^2 \quad , \quad U_y = 0$$

$$y = \frac{1}{2}g\left(\frac{u}{x}\right)^2 = \frac{gx^2}{2u^2}$$

From the figure above

$$\tan\theta = \frac{y}{x} \quad , \quad y = x\tan\theta$$

$$x\tan\theta = \frac{gx^2}{2u^2}$$

$$x = \frac{2u^2\tan\theta}{g}$$

Now

$$y = \frac{gx^2}{2u^2} = \frac{g}{2u^2}\left[\frac{2u^2}{g}\tan\theta\right]^2$$

$$y = \frac{2u^2}{g}\tan^2\theta$$

Distance of the stone from point of projection can be written as

$$L = \sqrt{x^2 + y^2}$$

$$= 2u^2\tan\theta \cdot \sqrt{1 + \tan^2\theta}$$

$$L = \frac{2u^2}{g}\tan\theta\sec\theta$$

59. A particle is projected from origin at an angle θ with the horizontal. Horizontal direction is selected as X – axis and vertical as Y – axis. It is found that particle passes through the points (a ,b) and (b, a). Calculate range and angle of projection.

Solution

From the equation

$$y = x\left(1 - \frac{x}{R}\right)\tan\theta$$

Given coordinates must satisfy the above equation

Point A(a , b)

$$b = a\left(1 - \frac{a}{R}\right)\tan\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

Point B(b , a)

$$a = b\left(1 - \frac{b}{R}\right)\tan\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

Take (i) and (ii)

$$\frac{b}{a} = \frac{a\left(1 - \frac{a}{R}\right)\tan\theta}{b\left(1 - \frac{b}{R}\right)\tan\theta}$$

$$\frac{b^2}{a^2} = \frac{1 - \frac{a}{R}}{1 - \frac{b}{R}}$$

$$\frac{b^2}{a^2} = \frac{R - a}{R - b}$$

$$b^2R - b^3 = a^2R - a^3$$

$$R = \frac{a^3 - b^3}{a^2 - b^2} = \frac{(a - b)(a^2 + ab + b^2)}{(a - b)(a + b)}$$

$$R = \frac{a^2 + ab + b^2}{a + b}$$

From equation (i)

$$b = a\left[1 - \frac{b}{R}\right]\tan\theta$$

$$b = a\left[1 - \frac{a}{\frac{a^2+ab+b^2}{a+b}}\right]\tan\theta$$

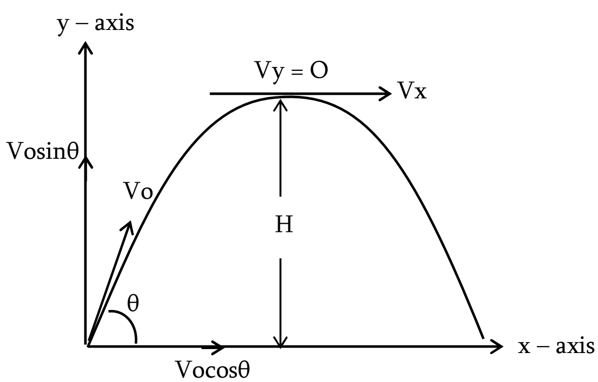
$$b = a\left[1 - \frac{\left(a^2+ab\right)}{a^2+ab+b^2}\right]\tan\theta$$

$$\frac{b}{a} = \left[\frac{a^2+ab+b^2-a^2-ab}{a^2+ab+b^2}\right]\tan\theta$$

$$\frac{b}{a} = \left[\frac{b^2}{a^2+ab+b^2}\right]\tan\theta$$

60. A particle is aimed at a mark which is in the same horizontal plane as the point of projection. It falls 10m short of the target when it is projected with an elevation of 75° and fall 10m ahead of the target when it is projected with an elevation of 45°. Find the correct elevation of projection so that it exactly hit the target. It is given that the initial velocity of projection is the same in each case.

Solution



The horizontal distance of the target is given by

$$R = \frac{V_0^2 \sin 2\theta}{g}$$

Where θ is the correct elevation of projection when θ = 70°, we have

$$R - 10 = \frac{V_0^2 \sin 150^\circ}{g} \text{ ....(i)}$$

When θ = 45°

$$R + 10 = \frac{V_0^2 \sin 90^\circ}{g} \text{ ....(ii)}$$

Adding equation (i) and (ii)

$$2R = \frac{V_0^2}{g}\left(\sin 150^\circ + \sin 90^\circ\right)$$

$$2R = \frac{3}{4}\cdot\frac{V_0^2}{g} \quad \text{but} \quad R = \frac{V_0^2 \sin 2\theta}{g}$$

$$\frac{V_0^2 \sin 2\theta}{g} = \frac{3}{4}V_0^2$$

$$\sin 2\theta = \frac{3}{4}$$

$$\theta = \frac{1}{2}\sin^{-1}\left(\frac{3}{4}\right) = 243^\circ$$

$$\theta = 24.3^\circ$$

61. (a) An object falling freely from a given height, H hit an inclined plane in its path at a height h. as a result of this impact , the direction of velocity of the object becomes horizontal for what value of h/H, the ball will take maximum time to reach the ground. (05 marks)

(b) A ball is kicked with an initial velocity of 8m/s such that it just passes over the barrier which is 2.2m high neglecting air resistance calculate:-

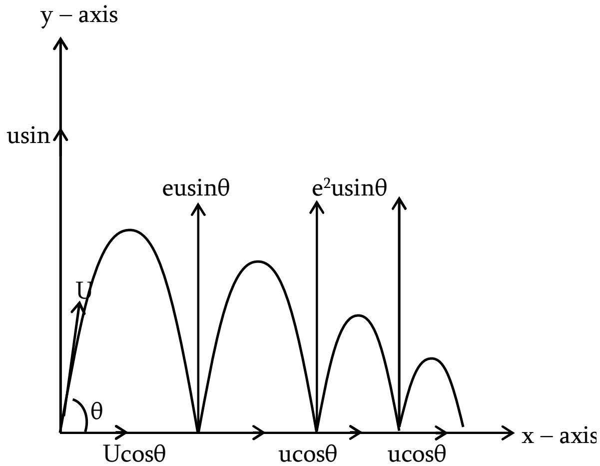
(i) The horizontal velocity of the ball (03 marks)

(ii) The total time of flight (02 marks)

62. A ball is projected from a point on a smooth horizontal floor at an angle θ with the horizontal speed of projection is u ball collides with the floor several times. Find the total time and horizontal distance covered by the ball.

Solution

Floor is smooth hence it applies no force in the horizontal direction and component of velocity of ball remains Ucosθ after every collision but due to collision vertical component of velocity becomes e times after every collision.



Flight time $T = \frac{2U_y}{g}$

Total flight time

$$T = \frac{2u\sin\theta}{g} + \frac{2eu\sin\theta}{g} + \frac{2e^2u\sin\theta}{g} + \ldots$$

$$= \frac{2u\sin\theta}{g}\left[1 + e + e^2 + e^3 + \cdots\right]$$

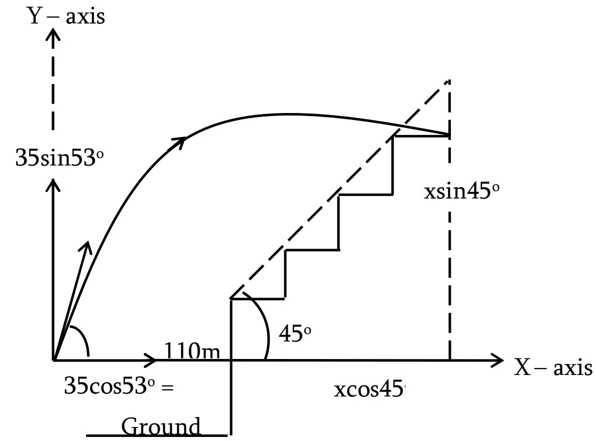
$$T = \frac{2u\sin\theta}{g(1-e)}$$

Horizontal range of projectile

$$R = (u\cos\theta)T = (u\cos\theta)\left[\frac{2u\sin\theta}{g(1-e)}\right]$$

$$R = \frac{u^2\sin 2\theta}{g(1-e)}$$

63. In a cricket match a batsman hit a six ball is hit 1m above the ground level from a point which is 110m away from the hence of stadium ball leaves the bat with a speed of 35m/s and at an angle 53° above the horizontal height and width of benches are 1m each. Assume that the benches are perpendicular to the plane of motion of the ball which bench will the ball hit?



Imaging that the length of each step is $\sqrt{2}$ m. Let the ball hit on this plane at a distance x from its bottom. Components of displacement of the ball

$$s_x = 110 + x\cos 45^\circ, \quad s_x = 110 + \frac{x}{\sqrt{2}}$$

Also $s_y = x\sin 45^\circ = \frac{x}{\sqrt{2}}$

Components of initial velocity

$$u_x = 21m/s \;,\; u_y = 28m/s$$

$$a_x = 0 \;,\; a_y = -9.8m/s^2$$

x – axis $s_x = u_x t + \frac{1}{2}a_x t^2$

$$s_x = 110 + \frac{x}{\sqrt{2}} = 21t \ldots\ldots\ldots\text{(i)}$$

$$\frac{x}{\sqrt{2}} = 28t - 4.9t^2$$

Putting the value of $x/\sqrt{2}$ from equation (ii) into (i)

$$110 + 28 - 4.9t^2 = 21t$$

$$4.9t^2 - 7t - 110 = 0$$

On solving $t = 5.5\,\text{sec}$

From equation (i)

$$110 + \frac{x}{\sqrt{2}} = 21 \times 5.5$$

$$x = 5.5\sqrt{2}$$

We have already discussed that each step is $\sqrt{2}$m, hence $\frac{x}{\sqrt{2}} = 5.5$

Hence the ball hits the 6th step.