HEAT TRANSFER

2. HEAT TRANSFER

Heat transfer is the propagation of heat from a body at high temperature to a body at lower temperature i.e Heat transfer means the movement of heat energy from one point towards to the another point due to the temperature difference. Normally heat travels from hot region to cold region.

Example: Why does heat flows from a body at higher temperature to a body at lower temperature:

Reason: A fast moving marble flows when it hits slower moving marbles. It gives up some of its kinetic energy to the slower ones. The same situation occurs in heat flow. Molecules with move kinetic energy that are in contact with energetic molecules give up by some of their kinetic energy to the less energetic ones. Thus the direction of heat flow is from hotter body to the colder body. The total heat energy before and after contact are the same.

METHODS (WAYS) OF HEAT TRANSFER

There are three ways or principle means of heat transfer:

(i) Thermal conduction
(ii) Thermal convection
(iii) Thermal radiation.

1. THERMAL CONDUCTION

Thermal conduction is the process of heat transfer due to the vibration of particle about their mean position i.e conduction is the process in which heat flows from the hotter regions of a material to the colder region without their being any net movement of the material itself. Thermal conduction occurs in metals and the movement of heat takes place without any visible sign of movement of the particles in solid.

MECHANISM OF HEAT CONDUCTION IN METALS

Metals contain free electrons which are in thermal equilibrium with their atoms. In metals, there are electrons which are free to move about the whole of the lattice and collides with other particles which have the heat transfer from one point towards to the another point along the metals. Two mechanisms explain how heat is transferred by conduction.

(i) Lattice vibration i.e interatomic vibration
(ii) Motion of free electrons i.e collision between the particles

The conduction through solids occurs by a combination of the tow mechanism mentioned above.

- **Conduction by interatomic vibration**

  Atoms at a higher temperature vibrate move vigorously about their mean positions in the lattice that their colder neighbours. The vibration atoms transfer the energy to nearby atoms and cause them to vibrate move energetically as well. These in turns affect other atoms and thermal conduction occurs.

  Note that:

  1. PHONONS are particle like entities that carry the elastic wave during the propagation of heat in solids.
  2. Conduction process by vibrations of atoms in very slow since atoms are move massive compared to the electrons; thus the increases in vibrations of atoms in fairly small.

- **Conduction by movement of electrons**

  Metals have free electrons which are not bound to any particular atoms and thus can freely move about the solid. If the temperature increases, the electrons on the hot side of solid move faster than those on the cooler side. As the electrons undergo series of collisions, the faster electrons give off some of their energy to the slower electrons. So heat energy is transmitted through the whole conductor.

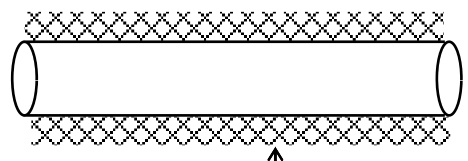
PARAMETER OF THERMAL CONDUCTION

1. **Conductor** – is the substance which allows heat energy to pass through it. Examples: All metals such as copper, Iron, Zinc, Aluminum, Lead, e.t.c.

2. **Bad conductor (insulator)** – Is the substance which does not allow heat energy to passing through it. Example: Rubber, wood, glass, waxy, cotton, e.t.c.

3. **Steady state condition** – Is an equilibrium point in which energy point in the bar maintains its temperature and that the rate of flow of heat through the bar is the same at all points. Under steady state hhconditions, the following points may be noted:-
   (i) The temperature at any point will remain constant with time.
   (ii) The temperature at two different points will be different.
   (iii) The rate of heat flows each section of the bar remain constant
   i.e $\frac{dQ}{dt}$ = constant

4. **Lagging** – is the process of covering outer surface of conductor by lagged material (insulator) in order to prevent heat loss from the conductor to the surrounding.

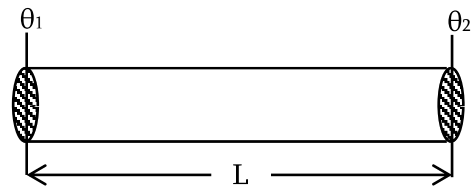


A conductor whose sides are well wrapped with a good heat insulator is called a **Lagged bar**.

5. **Unlagged bar** – is the conductor which does not covered by using insulating materials. For unlagged bar, the rate of heat flows is not constant since some of amount of heat energy is given out through the side of the conductor.

6. **Temperature difference** – is the difference in temperature between two opposite faces of cube or conductor.
   $\Delta\theta = \theta_1 - \theta_2 (\theta_1 > \theta_2)$

7. **Temperature gradient** – Is the temperature difference per unit length of the conductor.
   $$g = \frac{d\theta}{dx} = \frac{\text{temperature difference}}{\text{length of conductor}}$$



$$g = \frac{d\theta}{dx} = \frac{\theta_1 - \theta_2}{L}$$

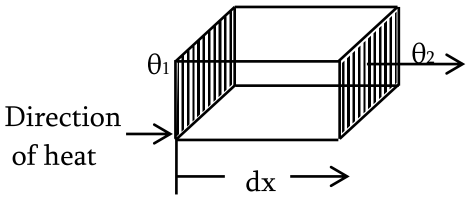
S.I. unit of the temperature gradient is Km^{-1} or $^{O}Cm^{-1}$.

8. **Cross – sectional area, A** – is the cross – sectional area of the conductor which allows heat to pass through it.

9. **Rate of heat flows** – is the amount of heat energy passing through the conductor per unit time through the given cross – sectional area of the conductor.

Expression of the rate of heat flows through a conductor.

Consider a slab of materials of cross – section area, A and thickness, dx subjected to a high temperature θ_1 on one side and lower temperature θ_2 on the other side.



Experimentally, it can been shown that, the quantity of heat energy passing through the conductor can be depends on the following factors:

(i) Cross – sectional area, A of a conductor (i.e $Q \propto A$), the larger the area, the more thermal energy is transmitted.

(ii) Time taken (i.e $Q \propto dt$). The longer the period of time, the move thermal energy is transmitted.

(iii) Temperature difference between the faces of slab ($Q \propto \theta_1 - \theta_2$). If there is large temperature difference, the large amount of thermal energy flows.

(iv) Length or thickness of the conductor $\left(Q \propto \frac{1}{dx}\right)$ on

combining the factors above

$$dQ \propto \frac{A(\theta_1 - \theta_2)dx}{dx}$$

$$dQ = \frac{-KA(\theta_1 - \theta_2)dx}{dx}$$

The rate of heat flows.

$$\frac{dQ}{dt} = \frac{-KA(\theta_1 - \theta_2)}{X} \text{ OR}$$

$$\frac{dQ}{dt} = KA\left(-\frac{d\theta}{dx}\right)$$

(This is called Fourier's law) in magnitude rate of heat flow is given by

$$\frac{dQ}{dt} = \frac{-KA(\theta_1 - \theta_2)}{X} = KA\frac{d\theta}{dx}$$

Where

$\frac{dQ}{dt}$ = Rate of heat flows

K = coefficient of thermal conductivity.

A = Cross – sectional area of a conductor.

$\frac{d\theta}{dx}$ = temperature gradient

Negative sign shows that temperature decreases as the length of conductor increases from the hot end.

The equation $\frac{dQ}{dt} = KA\frac{d\theta}{dx}$ is applicable under the following conditions:

(i) The steady state condition must be reached.

(ii) The conductor should be well lagged.

COEFFICIENT OF THERMAL CONDUCTIVITY

Qualitatively, thermal conductivity of a solid is a measure of the ability of the solid conductor heat through it.

Quantitatively, coefficient of thermal conductivity is defined as the rate of heat flows per unit cross – sectional area per unit temperature gradient.

$$K = \frac{dQ/dt}{A\, d\theta/dx}$$

S.I. unit of K is $Wm^{-1}K^{-1}$ or $Js^{-1}m^{-1}k^{-1}$ or $Cals^{-1}cm^{-1o}c^{-1}$.

Dimensional formula of

$$K = \frac{dQ/dt}{A\, d\theta/dx} = \frac{(ML^2T^{-2})/T}{L^2(K/L)}$$

$$[K] = [MLT^{-3}K^{-1}]$$

THERMAL CONDUCTIVITY OF DIFFERENT MATERIALS

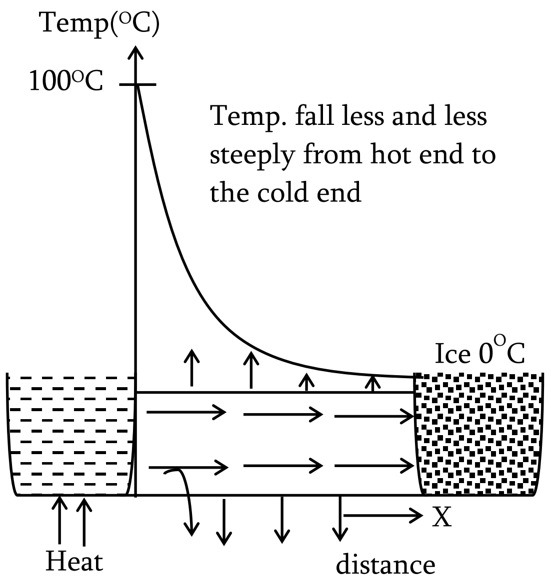
Material	K($Wm^{-1}k^{-1}$)	Material	K($Wm^{-1}k^{-1}$)
Silver	420	Mercury	8.0
Copper	380	Rubber	0.2
Aluminium	200	Iron	80
Steel	40	Lead	38
Glass	0.84	Ice	1.6
Brick	0.60	Felt	0.04
Concrete	0.84	Brass	109
Air	0.03	Wood	0.12 – 0.04

Note

1. The value of K is large in case of good conductors and it is in the case of insulators, as it clear from the table above.
2. Thermal conductivity of a material depends on the following factors:-
   (i) Temperature of material
   (ii) Nature of the material itself.

TEMPERATURE DISTRIBUTION ALONG THE CONDUCTOR (BAR).

1. Temperature distribution along the unlagged bar.



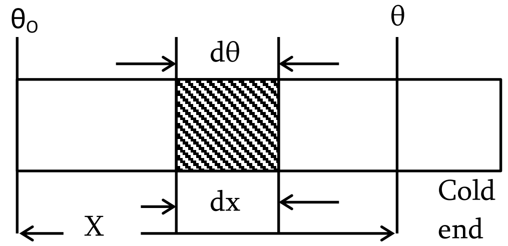
For the unlagged bar, the following points may be noted:-

(i) Some of amount of heat energy is given out from the bar to the surroundings.
(ii) Radiant heat lines are non – uniform
(iii) Rate of heat flow is not constant

Expression of temperature, θ with the length (thickness) X of the conductor

Expression of temperature θ at any point X along the unlagged bar is given by $\theta = \theta_o e^{-\mu x}$

Derivation $\theta = \theta_o e^{-\mu x}$



Temperature gradient, $\frac{d\theta}{dx} \alpha - \theta$

$$\frac{d\theta}{dx} = -\mu\theta$$

Negative sign shows that as dx increases, dθ decreases

Now, $\frac{d\theta}{\theta} = -\mu dx$

$$\int_{\theta_O}^{\theta} \frac{d\theta}{\theta} = -\mu \int_{O}^{X} dx$$

$$\log_e\left(\theta/\theta_O\right) = -\mu x$$

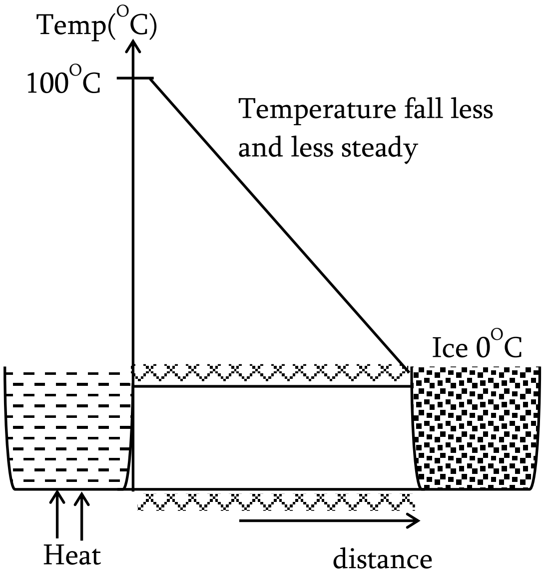
$$\theta = \theta_o e^{-\mu x}$$

Graph of temperature against distance, X is an exponential decay curve i.e there is a loss of heat to the surroundings since the metal bar is unlagged and the graph of fall of temperature against length of the bar is not a straight line.

- Expression of rate of heat flows through unlagged bar

$$\frac{dQ}{dt} = -KA\frac{d\theta}{dx} = -\beta\frac{d\theta}{dx}$$

2. Temperature distribution along the lagged bar.



For the lagged bar, the following points may be noted:

(i) There is no loss of heat from the bar to the surrounding.
(ii) Radial heat lines are uniformly
(iii) The rate of heat flows is constant.

Expression of temperature θ at any point X along the lagged bar is given by

$\theta = -mx + \theta_o$

Derivation

Since, the rate of heat flows

$$-H = KA\frac{d\theta}{dx}$$

$$\frac{-H}{KA} = \frac{d\theta}{dx}, \text{ Let } \frac{H}{KA} = m$$

$$-m = \frac{d\theta}{dx} \Rightarrow d\theta = -mdx$$

$$\int_{\theta_o}^{\theta} = -m\int_{o}^{x} dx$$

$$\theta - \theta_o = -mx$$

$$\theta = -mx + \theta_o$$

A graph of temperature against length of the bar is shown on the figure above. Since the metal bar is well lagged no heat is lost to the surrounding and a graph of fall of temperature against length of the bar is a straight line with negative gradient.

Expression of the rate of heat flows through the logged bar.

$$\frac{dQ}{dt} = KA\frac{d\theta}{dx} = \frac{KA(\theta_1 - \theta_2)}{L}$$

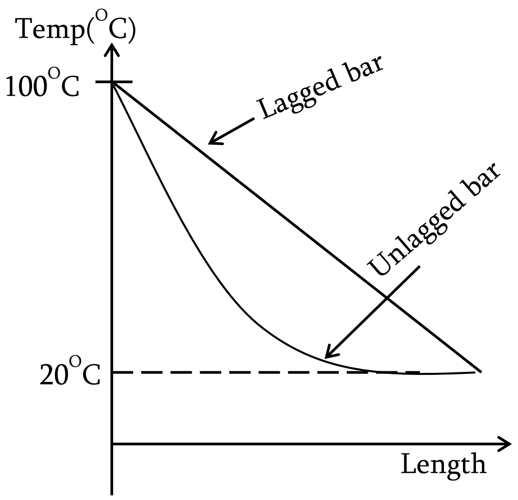
K = Thermal conductivity

A = cross – sectional area

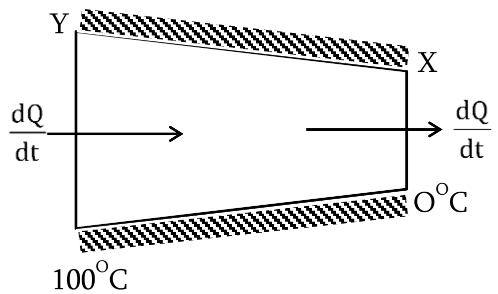
$\frac{d\theta}{dx}$ = temperature gradient

Note that

1. Graph of temperature against length for the given bar



2. Variation of g, k and A consider the figure below which shows a lagged bar of non – uniform cross – sectional area, A



The rate of heat flows

$$\frac{dQ}{dt} = KA\frac{d\theta}{dx} = KAg$$

$$g = \frac{d\theta}{dx} = \frac{1}{KA}\cdot\frac{d\theta}{dt}$$

Since the bar is lagged, then K and dQ/dt are constants $g \,\alpha\, \frac{1}{A}$, $g \,\alpha\, \frac{1}{K}$

Therefore the cross – sectional area of Y is lagger than that of X then temperature gradient at X is greater than that of Y. Also $g \,\alpha\, (\theta_1 - \theta_2)$ and $g \,\alpha\, 1/x$ if other factors are kept constant.

SOLVE EXAMPLES TYPE A

Example 1

Find the amount of thermal energy that flows per day through a solid oak wall 10cm thick, 3.00m long and 2.44m high, if the temperature of the inside wall is 21.1°C while the temperature of the outside wall is -6.67°C. Thermal conductivity of oak is $0.147 Jm^{-1}{}^{o}C^{-1}$

Solution

From the relation

$$Q = \frac{KA(\theta_1 - \theta_2)t}{X}$$

$$= \frac{0.147\times3\times2.44\left[21.1-^{-}6.67\right]\times24\times60\times60}{0.1}$$

$$Q = 2.58\times10^{7}\,J$$

∴ Thermal energy that flow per day through a solid oak is 2.58×10^7J.

Example 2

When steam at 100°C is passed into a metal cylinder, water at 100°C is collected at the rate of 200g/min. if the area and thickness of cylinder are $300cm^2$ and 20mm, respectively; determine the temperature of the outer surface. Take thermal conductivity of metal = $0.424\ cals^{-1}cm$. latent heat of steam = $540calg^{-1}$.

Solution

Heat abstracted from steam

$$Q = ML = 200\times540$$

Since $$Q = \frac{KA(T_1 - T_2)t}{X}$$

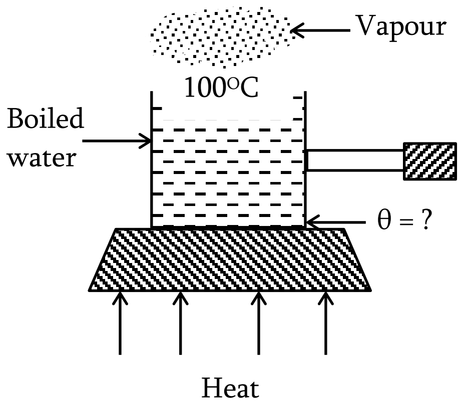
$$\therefore\quad 200\times540 = \frac{0.424\times300\times(100-T_2)\times60}{2}$$

On solving $T_2 = 71.7^{o}C$

Example 3

A brass boiler has a base area of $0.15m^2$ and thickness 1.0cm. If boils water at the rate of 6.0kg/min when placed on gas stove. Estimate the temperature of the part of the flame in contact with the boiler. Thermal conductivity of brass = $109Wm^{-1}K^{-1}$. Heat of vaporization = $2256 \times 103Jkg^{-1}$.

Solution



Heat absorbed by water

$$Q = \frac{KA(\theta - 100)t}{X}$$

$$Q = \frac{109\times0.15\times(\theta-100)\times60}{0.01} \quad ......(i)$$

Mass of water boiled per min = 6kg = 6000g

Heat used to boil water

$$Q = mL = 6000\times2256 \quad ......(ii)$$

(i) = (ii)

$$\frac{109\times0.15\times(\theta-100)\times60}{0.01} = 6000\times2256$$

$$\theta = 238^{o}C$$

Example 4

A thin – walled hot water tank, having a total surface are $5m^2$, contains $0.8m^3$ of water at a temperature of 350K. It is lagged with a 50mm thick layer of material of thermal conductivity $4 \times 10^{-2}Wm^{-1}K^{-1}$. The temperature of the outside surface of the lagging is 290K

(a) What electrical power must be supplied to an immersion heater to maint the temperature of the hot water at 350K? (Assume the thickness of the copper walls of the tank to be negligible).

(b) (i) What is the justification for the assumption that the thickness of the copper wall of the tank may be neglected? (thermal conductivity of copper = 400Wm^{-1}K^{-1})

(ii) If the heater were switched off, how long would it take for the temperature of hot water to fall 1K?

(Density of water = 1000Kgm^{-2}, specific heat capacity of water = 4170Jkg^{-1}K^{-1}).

Solution

(a) The electrical power required must be equal to the rate of conduction of heat through the lagged material.

$$P = KA\frac{d\theta}{dx} = \frac{KA\left[\theta_1 - \theta_2\right]}{X}$$

$$= \frac{4\times10^{-2}\times5\times(350-290)}{0.05}$$

$$P = 240W$$

(b) (i) The assumption justify that the temperature of the water inside the pipe is equal to that throughout the pipe thickness i.e the temperature of the outer surface of the pipe is equal to that of water inside of the pipe.

(ii) The initial rate of fall of heat

$$P = MC\frac{d\theta}{dt} = \rho VC\frac{d\theta}{dt}$$

$$dt = \frac{\rho VCd\theta}{P} \quad \text{but } d\theta = 1K$$

$$dt = \frac{1000\times0.8\times4170\times1}{240}$$

$$= 13{,}900s = 231.67\,\text{min}$$

$\therefore$ Time taken for temperature of hot water to fall 1K, dt = 232min (approx.)

Example 5 NECTA 2012/P1/5(C) (i)

An Aluminium sauce pan is in contact with a hot plate has a base of diameter 20cm and the thickness of 0.5cm. If the sauce contains water - boiling away at the rate of 0.15g/sec. Estimate the temperature at the base of the sauce pan. Given that latent heat of vapourization of water is 2.27 × 106Jkg^{-1} coefficient of thermal conductivity of aluminium is 210Wm^{-1}K^{-1}.

Solution

Rate of heat flows through the hot plate of sauce pan.

$$\frac{dQ}{dt} = \frac{KA(\theta-100)}{X} \ldots\ldots(1)$$

Rate of heat required to boiled water away into vapour.

$$\frac{dQ}{dt} = L\frac{dM}{dt} \ldots\ldots\ldots\ldots(2)$$

$$(1) = (2)$$

$$\frac{KA(\theta-100)}{X} = L\frac{dM}{dt}$$

$$\theta = 100 + \frac{4\times L}{\pi d^2 K}\cdot\frac{dM}{dt}$$

$$= 100 + \frac{4\times0.5\times10^{-2}\times2.27\times10^{6}\times0.15\times10^{-3}}{3.14\times210\times\left(20\times10^{-2}\right)^2}$$

$$\theta = 100.26^{0}C$$

Example 6

A cubical ice box of side 30cm has a thickness of 5.0cm. If 4.0kg of ice is put in the box, estimate the amount of ice remaining after 6 hours. The outside temperature is 45^{O}C and coefficient of thermal conductivity of the material of the box = 0.01Qm^{-1}K^{-1}, Latent heat of fussion of ice = 335 × 103Jkg^{-1}.

Solution

Area through which heat is lost = 6 × Area of one surface

= 6 × 0.3^2 = 0.54m^2.

Thickness, X = 0.05m

Heat absorbed by the ice

Q = ML …….(1)

Quantity of heat through the ice box

$$Q = \frac{KA(\theta_1 - \theta_2)}{X}$$

$$(1) = (2)$$

$$ML = \frac{KA(\theta_1 - \theta_2)t}{X}$$

$$M = \frac{KA(\theta_1 - \theta_2)t}{XL}$$

$$= \frac{0.01 \times 0.54 \times (45-0) \times 6 \times 3600}{0.05 \times 335 \times 10^3}$$

$M = 0.313Kg$

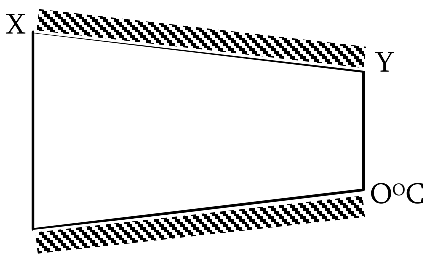
Mass of ice remaining = total mass of ice – mass of ice melted.

$Mr = 4 - 0.313$

$Mr = 3.687Kg$

Example 7

The figure below shows the lagged bar XY of non – uniform cross – sectional area, one end X is kept at 100°C? And the other end Y at 0°C. Describe and explain how temperature varies from X and Y in the steady state.



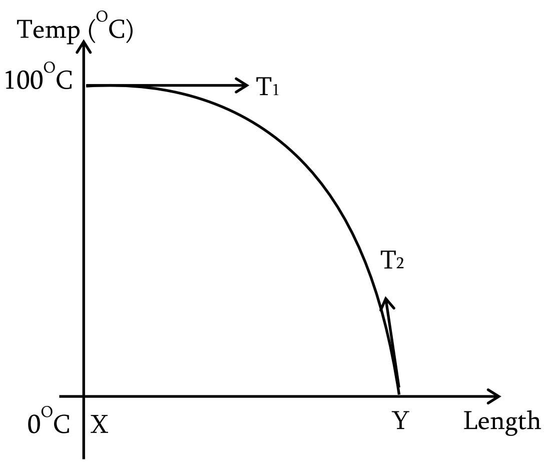
Solution

The rate of heat flows for lagged bar

$$\frac{dQ}{dt} = KA\frac{d\theta}{dx} = KAg$$

$$g = \frac{1}{KA} \cdot \frac{dQ}{dt}, \quad g \propto \frac{1}{A}$$

Graph of temperature against length



Since at X where A is greater, g is the smallest as shown by the slope of tangent T_1 and the temperature curve above g is greater as shown by the tangent T_2 on the curve above. Thus temperature gradient at Y is greater than that of X.

Example 8

A cylindrical rod with one end in a steam chamber and the other end in ice results melting 0.1×10^{-3}kg of ice per second. If the rod is replaced by another with half the length and double the radius of first and if thermal conductivity of the second rod is one fourth of the first, find the rate at which ice result melting.

Solution

Rate of heat conducted on the rod is equal to the rate of heat for melting the ice.

$$L_f\frac{dM}{dt} = \frac{KA(\theta_1 - \theta_2)}{L}$$

But $A = \pi r^2$

$$L_f\frac{dM}{dt} = \frac{\pi r^2 K(\theta_1 - \theta_2)}{L}$$

- For the first rod

$$L_f \cdot \frac{dM_1}{dt} = \frac{\pi r_1^2 K_1(\theta_1 - \theta_2)}{L_1} \text{.......(i)}$$

- For the second rod

$$L_f \cdot \frac{dM_2}{dt} = \frac{\pi r_2^2 K_2(\theta_1 - \theta_2)}{L_2}$$

(ii)/(i)

$$\frac{dM_2}{dt} = \frac{dM_1}{dt}\left(\frac{K_2}{K_1}\right)\left(\frac{r_2}{r_1}\right)^2\left(\frac{L_1}{L_2}\right)$$

$$= 0.1\left[\frac{K/4}{K}\right]\left[\frac{2r}{r}\right]^2\left[\frac{L}{L/2}\right]$$

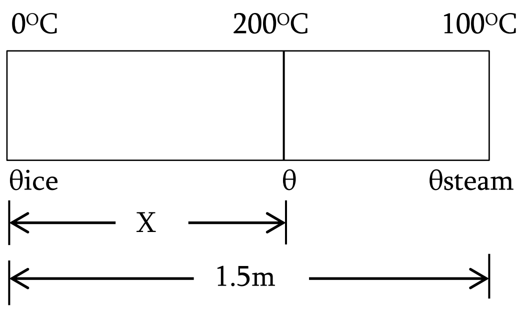
$$\frac{dM_2}{dt} = 0.2g/sec$$

Example 9

On end of a copper rod of uniform cross – section and of length 1.5m is kept in contact with ice and the other end with water at 100°C. At what point along its length should a temperature of 200°C be maintained so that in the steady state the mass of ice melted be equal to that of the steam produced in the same interval of time? Latent heat of fusion of ice = 80cal/g and latent heat of vaporization = 540cal/g

Solution

Let temperature θ = 200°C be maintained at a distance X from the end at 0°C.



Amount of heat flowing towards ice in time, t

$$Q = \frac{K(\theta - \theta_{ice})t}{X} = mL_{ice} \text{ ....(1)}$$

Amount of heat flowing toward water at 100°C in time, t

$$Q = \frac{K(\theta - \theta_{Steam})t}{1.5 - X} = mL_{Steam}$$

(1)/(2)

$$\frac{L_{ice}}{L_{Steam}} = \frac{\theta - \theta_{ice}}{\theta - \theta_{Steam}} \times \frac{(1.5 - X)}{X}$$

$$\frac{80}{540} = \left(\frac{200 - 0}{200 - 100}\right)\left(\frac{1.5 - X}{X}\right)$$

On solving X = 1.396m from cold end.

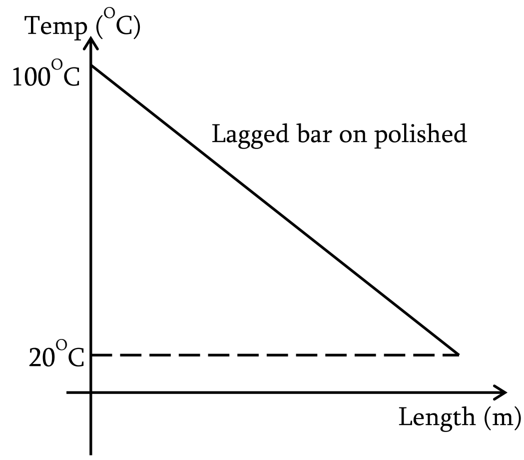
Example 10

(i) The ends of a straight uniform rod are maintained at temperatures of 100°C and 20°c, the room temperature being below 20°C. Draw sketch – graphs of the variations of the temperature of the long its length when the surface of the rod is
(a) Lagged
(b) Coated with soot
(c) Polished
Give a qualitative explaination of the forms of the graphs

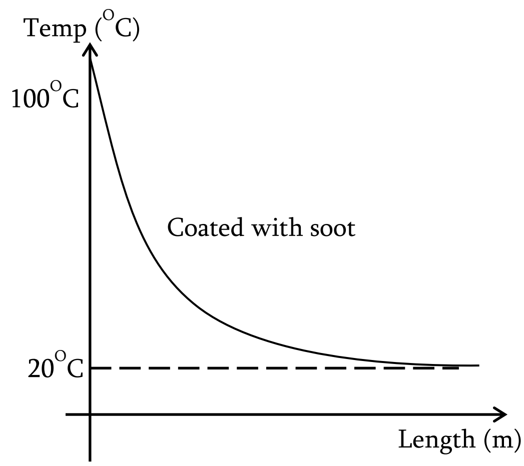
(ii) A liquid in a glass vessel of wall area 595cm^2 and thickness 2mm is agitated by a stirrer driven at a uniform rate by an electric motor rated at 100W. The efficiency of the conversion of electrical to mechanical energy in the motor is 75%. The temperature of the outer surface of the glass is maintained at 15°C. Estimate the equilibrium temperature of the liquid, stating any assumption you make thermal conductivity of glass = 0.84Wm^{-1}K^{-1}.

Solution

(i) (a) and (c)
Graph of temperature against length
(a)



(b)



(a) The rate of transfer of heat through a lagged bar is a uniform and so its temperature gradient. Therefore the graph of temperature against length is a straight line with negative gradient.

(b) The bar coated with soot has good radiating power this causes the loss of heat to its sides as it travels through it. Thus its temperature gradient will be decreasing with distance as shown by the decreasing slope of its curve i.e graph of temperature against length varies exponentially decayed curve.

(c) A pulish surface has very poor radiating power, so the rate of conduction of heat through it is approximately constant as very little heat is lost through its sides by radiation. Hence its graph is similar to that of a lagged bar

(ii) Assumption made

- All mechanical power is used to raise the internal energy (temperature) of the liquid in the glass.
- The temperature of the liquid is equal to temperature of the inside of the glass.
- There is no loss of heat to the surrounding and steady state has been reached.

Mechanical Power = rate of conduction of heat through the glass

$$0.75\times100 = \frac{0.84\times595\times10^{-4}\left(\theta-15\right)}{2\times10^{-3}}$$

$$\theta = 18^{\circ}C$$

Example 11

(a) State two factors upon which the thermal conductivity of a material depends.

(b) Two points are identical except that one case the flat bottom is aluminium and in the other it is copper each pot contains the same amount of boiling water and sits on a heating element that has a temperature of 155°C. In the aluminium bottom pot, the water boils away completely in 360seconds. How long does it take the water in copper – bottom pot to boil away completely? Thermal conductivity of aluminium $K_1 = 240Wm^{-1}K^{-1}$, Copper, $K_2 = 390Wm^{-1}K^{-1}$.

Solution

(a) • Temperature of the material
• Nature of the material itself

(b) Rate of heat flows Through the Conductor = Rate of heat gained by ice

$$KA\frac{d\theta}{dx} = \frac{mL}{t}$$

Since A, $\frac{d\theta}{dx}$, M and L are kept constant.

$$K\ \alpha\ \frac{1}{t} \text{ or } Kt = \text{constant}$$

$$t_2 = \frac{K_1t_1}{K_2} = \frac{240\times360}{390}$$

$$t_2 = 224\text{seconds}$$

ARRANGEMENT OF BARS

Bars can be arranged into two ways;-

(i) Bars in series connection

(ii) Bars in parallel connection

1. **BARS IN SERIES CONNECTION**

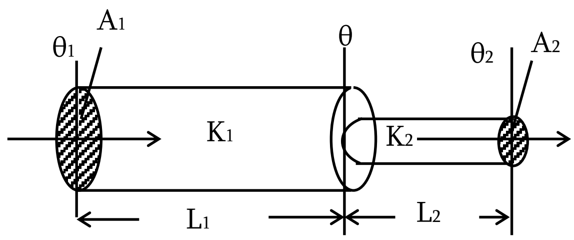
Composite bar – is the bar which can be formed when two or more bars are joined end to end. Different cases for composite bar.

Case 1: lagged composite bar.

For the lagged composite bar, the rate of heat flows on each bar are the same i.e

$$\frac{dQ}{dt} = \text{constant}$$

Consider the composite bar as shown on the figure below



Assume that $\theta_1 > \theta > \theta_2$

Rate of heat flows in

Bar1: $$\frac{dQ}{dt} = \frac{K_1A_1\left(\theta_1-\theta\right)}{L_1}$$

Bar 2: $$\frac{dQ}{dt} = \frac{K_2A_2\left(\theta-\theta_2\right)}{L_2}$$

Since $\frac{dQ}{dt}$ = constant

$$\left(\frac{dQ}{dt}\right)_1 = \left(\frac{dQ}{dt}\right)_2$$

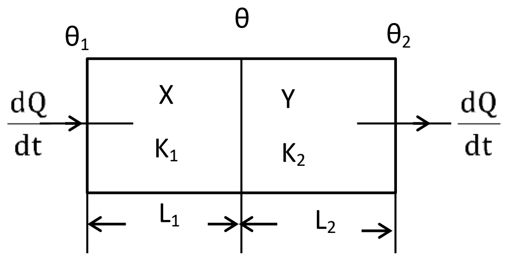
$$\frac{K_1A_1\left(\theta_1-\theta\right)}{L_1} = \frac{K_2A_2\left(\theta-\theta_2\right)}{L_2}$$

Generally

$$\left(\frac{dQ}{dt}\right)_1 = \left(\frac{dQ}{dt}\right)_2 = ------\left(\frac{dQ}{dt}\right)_n$$

Special case

Consider bar X and Y which are connected in series as shown in the figure below.



Assume that $\theta_1 > \theta > \theta_2$

Bar X: $$\frac{dQ}{dt} = \frac{K_1A_1\left(\theta_1-\theta\right)}{L_1}$$

$$\frac{L_1}{K_1A_1}\cdot\frac{dQ}{dt} = \theta_1-\theta \text{.......(i)}$$

Bar Y: $$\frac{dQ}{dt} = \frac{K_2A_2\left(\theta-\theta_2\right)}{L_2}$$

$$\frac{L_2}{K_2A_2}\cdot\frac{dQ}{dt} = \theta-\theta_2 \text{......(ii)}$$

(i) + (ii)

$$\left(\frac{L_1}{K_1A_1}+\frac{L_2}{K_2A_2}\right)\frac{dQ}{dt} = \theta_1-\theta_2$$

$$\frac{dQ}{dt} = \frac{\theta_1-\theta_2}{\frac{L_1}{K_1A_1}+\frac{L_2}{K_2A_2}}$$

If the bars have the same physical conditions

$L_1 = L_2 = L$, $A_1 = A_2 = A$

$$\frac{dQ}{dt} = \frac{\theta_1-\theta_2}{\frac{L}{A}\left[\frac{1}{K_1}+\frac{1}{K_2}\right]}$$

2. Expression of the temperature at the junction θ. At the steady state condition

$$\left(\frac{dQ}{dt}\right)_1 = \left(\frac{dQ}{dt}\right)_2$$

$$\frac{K_1A_1\left(\theta_1-\theta\right)}{L_1} = \frac{K_2A_2\left(\theta-\theta_2\right)}{L_2}$$

On solving

$$\theta = \frac{K_1A_1L_2\theta_1+K_2A_2L_1\theta_2}{K_1A_1L_2+K_2A_2L_1}$$

- If the bars have the same physical conditions.

$$\theta = \frac{K_1\theta_1+K_2\theta_2}{K_1+K_2}$$

3. Expression of total effectively thermal conductivity for bars in series.

Assume that the bars have the same physical condition

$$\frac{dQ}{dt} = \frac{A\left(\theta_1-\theta_2\right)}{L\left[\frac{1}{K_1}+\frac{1}{K_2}\right]} \text{..........(i)}$$

Also

$$\frac{dQ}{dt} = \frac{KA\left(\theta_1-\theta_2\right)}{2L}$$

(i) = (ii)

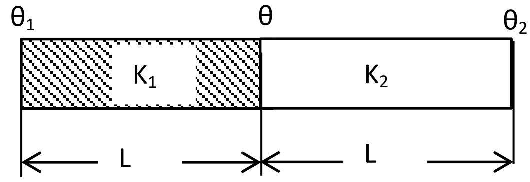
$$\frac{KA\left(\theta_1-\theta_2\right)}{L} = \frac{A\left(\theta_1-\theta_2\right)}{L\left(\frac{1}{K_1}+\frac{1}{K_2}\right)}$$

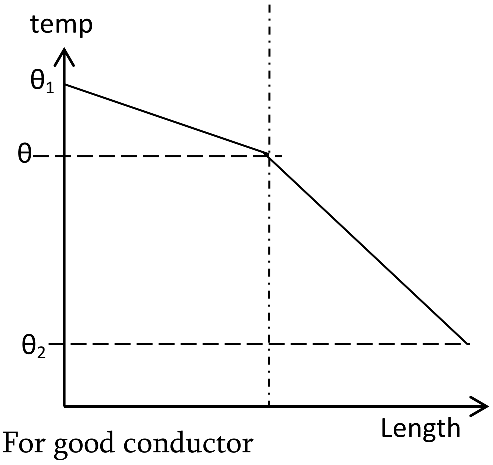
Now $\frac{1}{K} = \frac{1}{2}\left[\frac{1}{K_1}+\frac{1}{K_2}\right]$ or

$$K = \frac{2K_1K_2}{K_1+K_2}$$

Case 2: composite bar of good conductor and poor conductor. For the lagged composite bar.

$$\frac{dQ}{dt} = \text{constant}$$





For good conductor

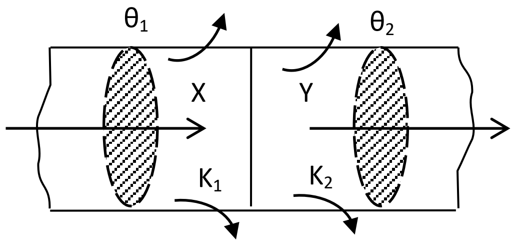
$$\frac{dQ}{dt} = \frac{K_1 A(\theta_1 - \theta)}{L}$$

For poor conductor

$$\frac{dQ}{dt} = \frac{K_2 A(\theta - \theta_2)}{L}$$

So if the lengths are equal, the temperature drop for the good conductor will be less than that of the poor conductor as shown on the graph above.

Case 3: for unlagged composite

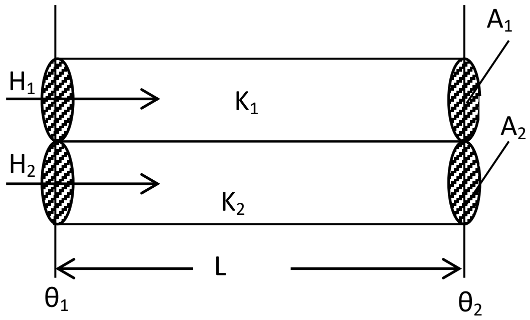


Assume the $\theta_1 > \theta > \theta_2$

$$\left(\frac{dQ}{dt}\right)_1 > \left(\frac{dQ}{dt}\right)_2$$

2. BARS IN THE PARALLEL CONNECTION

Case 1: for the parallel conductors, the total rate of heat flows is equal to the rate of heat flows on each bar.



Now

$$\frac{dQ}{dt} = H_1 + H_2$$

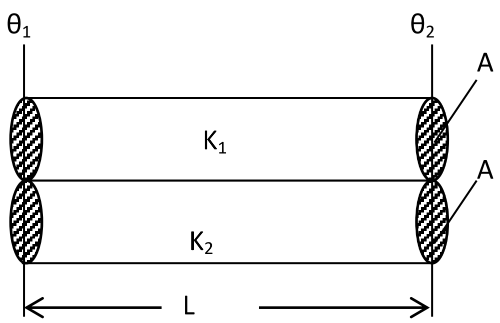
$$\frac{dQ}{dt} = \frac{K_1 A_1(\theta_1 - \theta_2)}{L} + \frac{K_2 A_2(\theta_1 - \theta_2)}{L}$$

$$\frac{dQ}{dt} = (K_1 A_1 + K_2 A_2)\left[\frac{\theta_1 - \theta_2}{L}\right]$$

General, for n – conductors being connected in parallel connection

$$\frac{dQ}{dt} = \left(\frac{dQ}{dt}\right)_1 + \left(\frac{dQ}{dt}\right)_2 + \text{------}$$

Case 2: expression of the total effectively thermal conductivity for the two bars in parallel connection having the same physical condition



The rate of heat flows

$$\frac{dQ}{dt} = (K_1 + K_2)\frac{A(\theta_1 - \theta_2)}{L}$$

Total effectively area = 2A

Also $\frac{dQ}{dt} = \frac{2KA(\theta_1 - \theta_2)}{L}$ ......(ii)

(i) = (ii)

$$\frac{2KA(\theta_1 - \theta_2)}{L} = \frac{(K_1 + K_2)A(\theta_1 - \theta_2)}{L}$$

$$K = \frac{K_1 + K_2}{2}$$

THERMAL RESISTANCE AND U – VALUES OF THE MATERIAL.

Thermal resistance – Is defined as the temperature difference per unit rate of heat flows along the conductor.

Since. $\frac{dQ}{dt} = \frac{\theta_1 - \theta_2}{L/KA} = \frac{\theta_1 - \theta_2}{R_H}$

$$R_H = \frac{L}{KA} = \frac{\theta_1 - \theta_2}{dQ/dt}$$

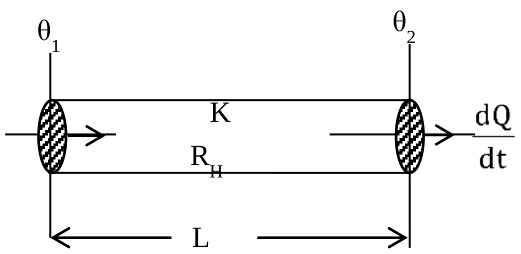
This is analogous to Ohm's law equation. $I = \frac{V}{R}$

The quantity $R_H = \frac{L}{KA}$ is thermal resistance in analogy to electrical resistance. $\frac{dQ}{dt}$ is called heat current.

S.I. Unit of thermal resistance is KW^{-1}

Dimension of RH = $[M^{-1}L^{-2}T^3K]$

Relationship between R_H and K



Assume that $\theta_1 > \theta_2$

Rate of heat flows in terms of R

$\frac{dQ}{dt} = \frac{\theta_1 - \theta_2}{R_H}$ ....(i)

Rate of heat flows in terms of K

$\frac{dQ}{dt} = \frac{KA(\theta_1 - \theta_2)}{L}$ ....(ii)

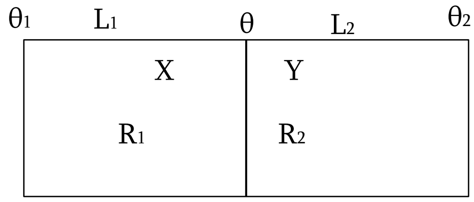
(i) = (ii)

$$\frac{KA(\theta_1 - \theta_2)}{L} = \frac{\theta_1 - \theta_2}{R_H}$$

$$R_H = \frac{L}{KA}$$

CONDUCTORS

Consider the figure below which shows bar X and Y being connected in series connection



Assume that $\theta_1 > \theta > \theta_2$

Rate of heat flows in conductor

$$X : H = \frac{\theta_1 - \theta}{R_1} \text{ or } HR_1 = \theta_1 - \theta$$

$$Y : H = \frac{\theta - \theta_2}{R_2} \text{ or } HR_2 = \theta - \theta_2$$

$$H(R_1 + R_2) = \theta_1 - \theta_2$$

$$H = \frac{dQ}{dt} = \frac{\theta_1 - \theta_2}{R_1 + R_2}$$

Expression of total thermal resistance in series

$$R_H = R_1 + R_2$$

$$\frac{L_1 + L_2}{KA} = \frac{L_1}{K_1A} + \frac{L_2}{K_2A}$$

$$K = \frac{L_1 + L_2}{L_1/K_1 + L_2/K_2}$$

Expression of the temperature at the junction

$$H_1 = H_2$$

$$\frac{\theta_1 - \theta}{R_1} = \frac{\theta - \theta_2}{R_2}$$

On solving, $\theta = \frac{R_1\theta_2 + R_2\theta_1}{R_1 + R_2}$

THERMAL RESISTANCE IN THE PARALLEL CONDUCTORS.

$$\frac{1}{R_H} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

THERMAL RESISTANCE COEFFICIENT (XC)

Is defined as the thickness or length of material per unit thermal conductivity of the conductor

$$X_C = \frac{L}{K} = AR_H$$

S.I. Unit of thermal resistance coefficient is Km^2W^{-1}.

Relationship between thermal conductivity and temperature gradient for two bars in series connection.

Assume that the bar is lagged and have same physical condition.

$$\frac{dQ}{dt} = KAg$$

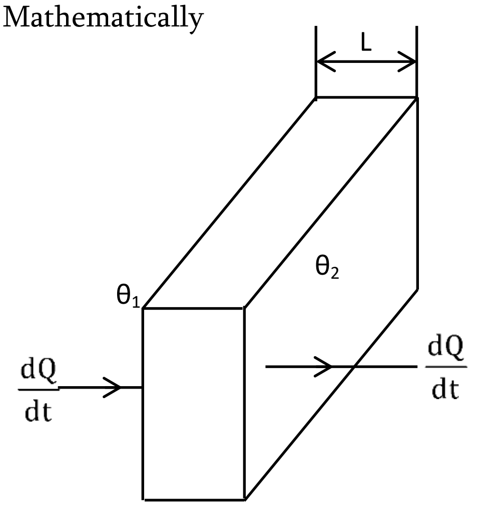
$$K = \frac{1}{Ag} \cdot \frac{dQ}{dt} \quad , \quad K \alpha \frac{1}{g}$$

$$K_1 g_1 = K_2 g_2$$

U – VALUE OF THE MATERIAL

Is defined as the heat conducted per unit time per unit area per unit temperature difference of its opposite side

Mathematically



$$U = \frac{\text{Rate of heat flows}}{\text{Area} \times \text{temperature difference}}$$

$$U = \frac{dQ/dt}{A(\theta_1 - \theta_2)}$$

Rate of heat flows in terms of U – value

$$\frac{dQ}{dt} = AU(\theta_1 - \theta_2)$$

S.I Unit of U – value is $Wm^{-2}K^{-1}$

Dimensional formula of U – value is $[U] = [ML^2T^{-2}K^{-1}]$

Relationship between U – value and K

$$\text{Since } \frac{dQ}{dt} = \frac{KA(\theta_1 - \theta_2)}{L} \text{.......(1)}$$

$$\text{Also } \frac{dQ}{dt} = AU(\theta_1 - \theta_2) \text{.......(ii)}$$

$$\text{(i)} = \text{(ii)}$$

$$\frac{KA(\theta_1 - \theta_2)}{L} = AU(\theta_1 - \theta_1)$$

$$U = \frac{K}{L} \text{ or } K = LU$$

$$\text{Since } X_C = \frac{L}{K} = \frac{1}{U}$$

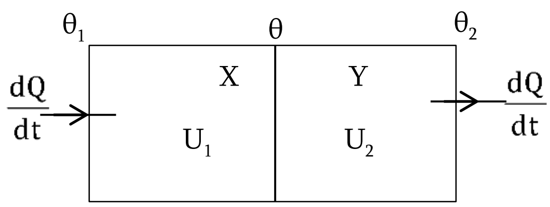
$$U = \frac{1}{X_C}$$

The U – values of the material depends on the following factors:-

(i) Thermal conductivity of material
(ii) Thickness of the material
(iii) Temperature difference

U – VALUE IN SERIES CONDUCTORS

Consider the conductor X and Y which are connected in series as shown in the figure below



Assume that $\theta_1 > \theta > \theta_2$

Rate of heat flows through the conductor

$$X : \frac{dQ}{dt} = AU_1(\theta_1 - \theta)$$

$$\frac{1}{AU_1}\cdot\frac{dQ}{dt} = \theta_1 - \theta \ldots\ldots(i)$$

$$Y: \frac{dQ}{dt} = AU_2(\theta - \theta_2)$$

$$\frac{1}{AU_2}\cdot\frac{dQ}{dt} = \theta - \theta_2$$

(i) + (ii)

$$\frac{1}{A}\left[\frac{1}{U_1}+\frac{1}{U_2}\right]\frac{dQ}{dt} = \theta_1 - \theta_2$$

$$\frac{dQ}{dt} = \frac{A(\theta_1-\theta_2)}{\frac{1}{U_1}+\frac{1}{U_2}} \ldots\ldots(i)$$

Expression of total U – values for the series conductors.

$$\text{Now } \frac{dQ}{dt} = AU(\theta_1-\theta_2)\ldots\ldots.(2)$$

(1) = (2)

$$U = \frac{1}{\frac{1}{U_1}+\frac{1}{U_2}}$$

$$\frac{1}{U} = \frac{1}{U_1}+\frac{1}{U_2} \quad \text{or} \quad U = \frac{U_1U_2}{U_1+U_2}$$

U – VALUE FOR PARALLEL CONDUCTOR S

$$U_P = U_1 + U_2 + ------ U_n$$

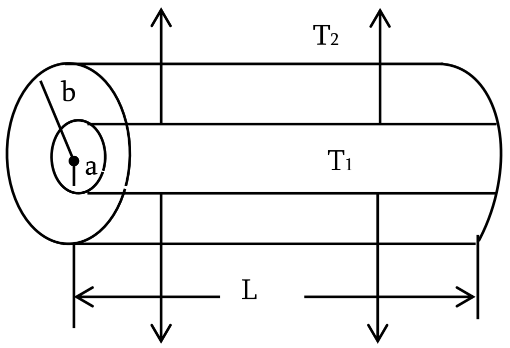
APPLILCATIONS OF U – VALUES OF MATERIAL.

1. Used in the construction of building
2. Used in the construction of glass material, bridge, plastic etc.

Engineers are based on their calculation on the U – value of material rather than their thermal conductivity because they are concerned with the air temperature inside and outside of the room and not the temperature within the material.

RADIAL HEAT LINES

(a) Radial heat lines on the concentric cylinder



The rate of heat flows for the small portion.

$$H = -KA\frac{d\theta}{dr} \quad \text{but} \quad A = 2\pi rL$$

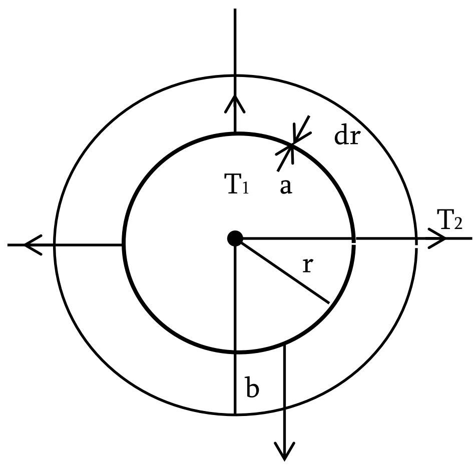
$$H = -2\pi rLK\frac{d\theta}{dr}$$

$$H\frac{dr}{r} = -2\pi KLd\theta$$

$$\int_a^b \frac{dr}{r} = -2\pi KL\int_{T_1}^{T_2} d\theta$$

$$H = \frac{dQ}{dt} = \frac{2\pi KL(T_1-T_2)}{\log_e\left(b/a\right)}$$

(b) Radial lines on the spherical conductor



Rate of heat flows through the small portion of the sphere.

$$H = -KA\frac{d\theta}{dr}, \quad A = 4\pi r^2$$

$$H = -4\pi r^2K\frac{d\theta}{dr}$$

$$Hr^{-2}dr = -4\pi Kd\theta$$

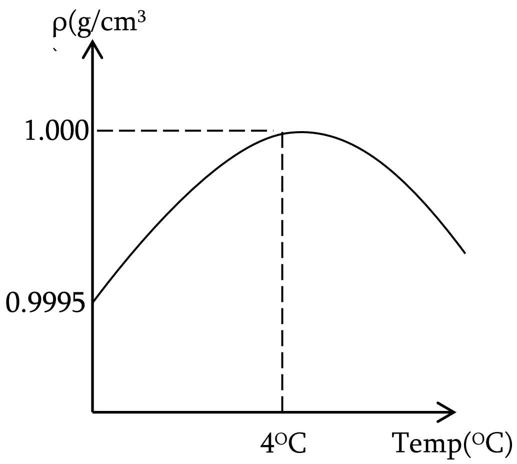
$$H\int_a^b r^{-2}dr = -4\pi K\int_{T_1}^{T_2} d\theta$$

$$H = \frac{dQ}{dt} = \frac{4\pi kab(T_1 - T_2)}{b-a}$$

FREEZING OF POND

Anomalous expansion of water.

When water is at 0°C is heated, it first contracts as the temperature rose to 4°C and then expands as other liquids as the temperature rises from 4°C to 100°C. As the mass remain constant, while the volume decreases and reaches the minimum value at 4°C; the density increases and reaches the maximum value at 4°C. This behavior of water is said to be **anomalous**



Density of water near the surface becomes maximum when the temperature reaches 4°C and it starts moving down. Less dense water from below rises up and as soon as its temperature becomes 4°C, then it foes down and similarly whole water of lake attains a temperature of 4°C first. Now, when temperature of water near the surface becomes less than 4°C it remains at the top because density of water decreases when the temperature goes beyond 4°C. Due to this reason, freezing starts from top and aquatic animals and plant life can survive at bottom of a frozen lake.

THE GROWTH OF ICE THICKNESS ON THE LAKE/ POND

The water in a lake/pond freezes when the temperature of air in contact with it falls below 0°C. The density of ice is less than water and so the ice formed floats on the surface of water. Let X be the thickness of layer of ice formed. Let the temperature of air just above the ice be – 0°C and the temperature of water below ice in the pond or lake be 0°C.

Assumption made in our calculation.

- Heat is transferred by conduction through the ice layer.
- There is no absorption of heat from the bulky of the water beneath the ice

  Let A be cross – sectional area of the ice. Suppose the thickness of ice increases by dx in a time, dt. Mass of ice formed

  $M = \rho A dx$

  Rate of heat energy needed to convert water into ice.

  $$\frac{dQ}{dt} = L\frac{dM}{dt}$$

  $$\frac{dQ}{dt} = \rho AL\frac{dx}{dt} \ldots\ldots.(i)$$

  This rate of heat is conducted through the layer of ice of thickness, X let K = Thermal conductivity of ice

  $$\frac{dQ}{dt} = \frac{KA(\theta_1 - \theta_2)}{X} = \frac{KA\theta}{X} \ldots\ldots(ii)$$

  (i) = (ii)

  $$\rho AL\frac{dx}{dt} = \frac{KA(\theta_1 - \theta_2)}{X} = \frac{KA\theta}{X}$$

- Expression of increase in the thickness of the ice in small time dt

  $$dx = \frac{K(\theta_1 - \theta_2)dt}{L\rho X} = \left(\frac{K\theta}{L\rho X}\right)dt$$

- Expression of rate of the increases of thickness of ice

  $$\frac{dx}{dt} = \frac{K(\theta_1 - \theta_2)}{L\rho X} = \frac{K\theta}{L\rho X}$$

- Expression of time taken by the ice to growth from thickness X_1 to X_2

$$\text{Now } dt = \frac{\rho LX}{K\theta}dx$$

$$\int_0^t dt = \frac{\rho L}{K\theta}\int_{X_1}^{X_2} xdx$$

$$t = \frac{\rho L}{2K\theta}\left[X_2^2 - X_1^2\right] \text{ OR}$$

$$t = \frac{\rho L}{2K(\theta_1 - \theta_2)}\left[X_2^2 - X_1^2\right]$$

The thickness of the ice at a later time, t

$$X_2 = X_t = \sqrt{X_1^2 + 2\left(\frac{K\theta}{\rho L}\right)t}$$

- Expression of the rate of mass of ice increases on the lake surface.

$$L\frac{dm}{dt} = \frac{KA\theta}{X}$$

$$\frac{dm}{dt} = \frac{KA\theta}{X} = \frac{KA(\theta_1 - \theta_2)}{LX}$$

L = Specific latent heat of fusion of ice.

A GOOD CONDUCTOR OF HEAT IS ALSO A GOOD CONDUCTOR OF ELECTRICITY.

The conduction of both heat and electricity is due to the movement of free electrons. This suggested that free electrons are the charges carries which carries electrical and thermal energy in metals (conductors). So when metal is heated free electrons gain thermal energy and distribute this energy by collisions. With the fixed positive ions in the solid lattice. So it is no surprise that there must be some relationship between thermal conduction and electrical conduction. In the year 1853, Wiedman and Franz established a relationship between thermal conductivity, K and electrical conductivity, δ.

WIEDMANN – FRANZ LAW

State that 'The ratio of thermal and electrical conductivities is proportional to the absolute temperature T of the material.

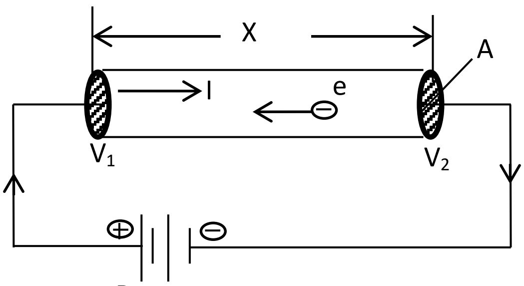
$$\frac{K}{\delta} \alpha\, T \text{ OR } \frac{K}{\delta T} = \text{Constant}$$

ANALOGUE BETWEEN ELECTRICAL CONDUCTIVITY AND THERMAL CONDUCTIVITY.

The rate of heat flows along the conductor under the steady state condition

$$\frac{dQ}{dt} = \frac{KA(\theta_1 - \theta_2)}{X} \ldots\ldots(i)$$

When the same conductor is connected to the source of electric current e.g battery as shown on the figure below.



Electric current passing through a conductor of resistance, R given by

$$I = \frac{V_1 - V_2}{R} = \frac{V}{R}$$

$$\text{But } R = \frac{\rho X}{A} = \frac{X}{\delta A}$$

ρ = Electrical resistivity

δ = electrical conductivity.

Electrical conductivity is the reciprocal of electrical resistivity

$$\delta = \frac{1}{\rho}$$

$$\text{Now } I = \frac{V}{R} = \frac{V\delta A}{X}$$

$$I = \delta \times A \times \frac{V}{X}$$

∴ Rate of heat flows

$$\frac{dQ}{dt} = K \times A \times \text{Potential gradient}$$

The following points may be noted:-

1. The rate of heat flow is similar to the electric charges flow rate. Therefore for any substance which can be considered as good electrical conductor is also good thermal conductor and this is accordance to the Wiedemann Franz law.

2. In general, the conduction of heat involves two processes:-
   (i) Due to the lattice vibrations of the atoms about their equilibrium position or we say it is due to the presence of photons. This process is slow.
   (ii) This is due to presence of free elections. This process is quick. In metals, the free the hotter region into the colder region. This is achieved region. This is achieved through the increased velocity of free electrons caused by the rise in temperature of the material. Due to the increased velocity of the electrons more from hotter to the colder parts faster so the process is quick.
3. Comparisons of the law governing to the conduction of heat and electricity.

Thermal quantity	**Electrical quantity**
Rate of heat flows	Rate of electric charges flows
Thermal conductivity	Electrical conductivity
Temperature difference	Potential difference
Temperature gradient	Potential gradient
Thermal resistance	Electrical resistance

THERMAL CONDUCTORS AND THERMAL INSULATORS.

Thermal conductors – are those materials which have high value of the thermal conductivity and conduct heat well.

Examples: all metals thermal conductors have the following characteristics:-

(i) Have high value of thermal conductivity in order to allow fast rate of heat flows through the material.
(ii) Have low specific heat capacity, this ensures that little heat is required to rise its temperature.

THERMAL INSULATORS – are those material which have low value of thermal conductivity and conduct heat poorly example; glass, air, plastic. Thermal insulators have the following characteristics

(i) Low thermal conductivity
(ii) High specific heat capacity.

APPLICATION OF THERMAL CONDUCTIVITY ON THE DOMESTIC COOKING UTENSILS.

The following requirements are needed in the case of a cooking utensils.

1. **Have high value of coefficient of thermal conductivity.**
   This is in order to conduct heat quickly to the utensils. Also this enable high sufficient rate of heat flows from the source to the utensil or cooking pot.
2. **Have low value of specific heat capacity.**
   To ensure that the amount of heat given required to raise the temperature of a unit mass of vessel is maximum. So little amount of heat is used to raised high temperature of the vessel
3. **Low coefficient of expansion**
   To ensure that there is no considerable expansion of the vessel during the cooking process.
4. **High melting point**
   Since the cooking process required high temperature, thus the material should have haigh melting point to ensure that the vessel will stand with high temperature without damaged during cooking process.

APPLICATIONS OF THERMAL CONDUCTIVITY TO EVERYDAY LIFE.

1. **Cooking utensils are provided with wooden handles** - This is because wood is a bad conductor of heat. So the wooden handle would not permit heat to be conducted from hot utensil to the hand. Therefore, the hot cooking utensils can be easily held in hand through the wooden handle.
2. **Ice packed in saw dust** – this is because saw dust and air trapped inside are poor conductors of heat. So ice will got get heat from the surrounding and it will not melt.

3. **Birds swell their feathers in winter –** by doing so, the birds enclose air between their bodies and the feathers so it prevent loss of heat from the body of the bird and it maintains its body temperature.

4. **Cooking utensils are made of aluminium an brass –** This is because aluminium and brass are good conductors of heat. They rapidly absorbed heat from the fire and supply it to the food to be cooked.

5. **In winter metals appears colder than the wood** - In winter human body is at high temperature than the surrounding objects. So heat flows from our body to the metal when we touch it. But no heat flows from the body to the wood, as it poor conductor of heat. So metals appears colder.

6. **Tow thin blankets are warmer than a single blanket** – because the still layer air is between them being a bad conductor of heat does not allow the conduction of heat.

7. **The refrigerator body is made of thick walls of fibre glass** – which is an insulator and prevents the flow of heat into refrigerator.

8. **Eskimos make double walled houses of using ice blocks –** the air enclosed between the double walls the conduction of heat from inside the house to surroundings.

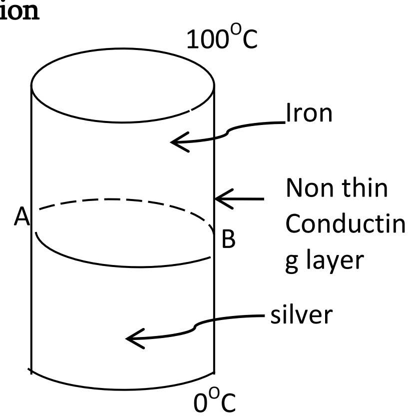
9. **A hot liquid remains hot and a cold liquid remains cold in a thermo flask –** this is due to the fact that a vacuum is created between the two walls of the thermos flask. Heat can neither flow from inside the flask to outside nor from outside air to the liquid inside the flask. Note that, in this way, loss of heat by conduction and convection has been minimized. To minimize loss of heat by radiation, the surface is made shining.

SOLVE EXAMPLES TYPE B

Example 1

Two cylinders of equal physical conditions are placed on the top of other as illustrated in the figure below. The lower part of the cylinder is kept at $0^{O}C$ and the upper part at $100^{O}C$. Given that the thermal conductivity of silver is eleven times that of iron. Find the temperature of surface AB.

Solution



Let θ be the temperature on the surface AB.

For the composite lagged bar, $\frac{dQ}{dt}$ = constant

$$\left(\frac{dQ}{dt}\right)_{Iron} = \left(\frac{dQ}{dt}\right)_{Silver}$$

Under the same physical conditions.

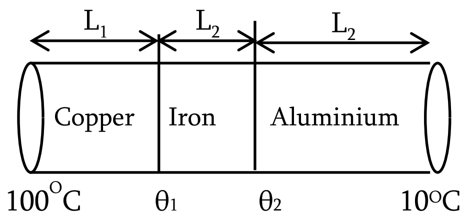
$$A_I = A_S = A$$
$$L_I = L_S = L$$
$$\frac{K_I A(100-\theta)}{L} = \frac{11K_I A(\theta-0)}{L}$$
$$100-\theta = 11\theta$$
$$100^{\circ}C = 12\theta$$
$$\theta = \frac{100^{\circ}C}{12} = 8.33^{\circ}C$$

∴ Temperature of surface AB. θ = $8.33^{O}C$.

Example 2

A composite bar is made up of copper 10cm long, 8cm long of iron and aluminium 12cm long each having the same cross – sectional area. If the extreme ends of the bars are maintained at $100^{O}C$ and $10^{O}C$ respectively. Find the temperature at two junctions. Thermal conductivity of copper, iron and aluminium are 400, 40 and 20$Wm^{-1}K^{-1}$. Respectively.

Solution



Assume that the composite bar is under steady state condition.

$$\left(\frac{dQ}{dt}\right)_{Copper} = \left(\frac{dQ}{dt}\right)_{AL} = \left(\frac{dQ}{dt}\right)_{Iron}$$

Now $\left(\frac{dQ}{dt}\right)_{Copper} = \left(\frac{dQ}{dt}\right)_{Iron}$

$$\frac{K_1A(100-\theta_1)}{L_1} = \frac{K_2A(\theta_1-\theta_2)}{L_2}$$

$$\frac{400A(100-\theta_1)}{10} = \frac{40A(\theta_1-\theta_2)}{8}$$

$$9\theta_1-\theta_2 = 800\,..........(1)$$

Also $\left(\frac{dQ}{dt}\right)_{Copper} = \left(\frac{dQ}{dt}\right)_{AL}$

$$\frac{400A(100-\theta_1)}{10} = \frac{20(\theta_2-10)}{12}$$

$$24\theta_1+\theta_2 = 2410\,......(2)$$

On solving equation (1) and (2)

$$\theta_1 = 97.3^{\circ}C$$

$$\theta_2 = 75.4^{0}C$$

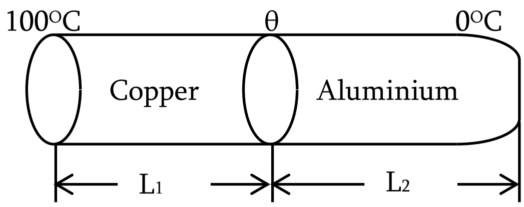
Example 3

An Ideally lagged compound bar 25cm long consists of a copper bar 15cm long joined to an aluminium bar 10cm long and of equal cross – sectional area. The free end of the copper is maintained at 100°C and the free end of the aluminimu at 0°C. Calculate:-

(i) The temperature at the junction

(ii) The temperature gradient in each bar when the steady state conditions have been reached. Thermal conductivity of copper and aluminium are 3.9$Wcm^{-1}K^{-1}$ and 2.1$Wcm^{-1}K^{-1}$ respectively.

Solution



(i) Let θ be junction temperature under steady state condition.

$$\frac{dQ}{dt} = \text{constant}$$

$$\frac{K_1A(100-\theta)}{L_1} = \frac{K_2A(\theta-0)}{L_2}$$

$$\frac{3.9(100-\theta)}{15} = \frac{2.1}{10}$$

On solving, θ = 55.3°c.

(ii) ● Temperature gradient in aluminium bar

$$g_L = \frac{\theta_1-\theta_2}{L_1}$$

$$g_{AL} = \frac{55.3-0}{10}$$

$$g_{AL} = 5.53^{0}C/cm$$

- Temperature gradient in copper bar

$$g_{CU} = \frac{100-55.3}{15}$$

$$g_{CU} = 2.98^{0}C/cm$$

Example 4: NECTA 1996/P2/6(a)

State two important thermal characteristics of an ideal cooking pot.

Example 5: NECTA 1997/P2

Assuming that you are managing a metal box company. What required for 'thermal conductivity" specific heat capacity" coefficient of expansion" and melting point" would you want to the material to be used as cooking utensil to satisfy.

Example 6: NECTA 2005/P1/5(a)

(i) What is the difference between ice point and triple point of water.

(ii) Several cooking utensils for sale are rated at 'HIGH' or 'LOW' in terms of their thermal efficiency for the following properties:-
- Thermal conductivity
- Specific heat capacity
- Coefficient of expansion and
- Melting point

Explain briefly the thermal ratings you would observe with respect to each property in purchasing a cooking utensils.

Example 7

(a) (i) Define the coefficient of thermal conductivity.

(ii) Explain why good thermal conductors are also good electrical conductors.

(b) (i) Describe the difference in conduction of heat through a well lagged bar and a non – lagged bar.

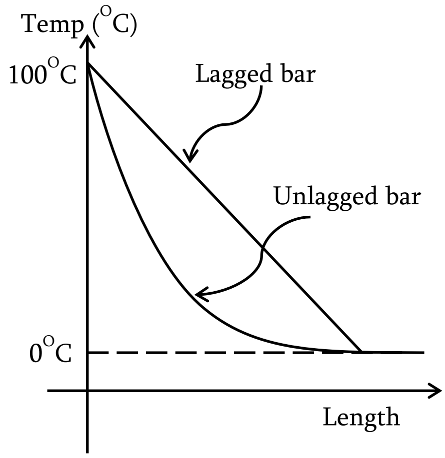
(ii) Two perfectly lagged metal bar X and Y are then in parallel. When the bars are in series the hot end X is maintained at 90°C and the cold end of Y maintained at 30°C. When the bar are in parallel the hot end of each is maintained at 90°C and cold end of each is maintained at 30°C. Calculate the ratio of the total rate of flow of heat in the parallel arrangement. The length of each bar is L and the cross – sectional area of each is A. [Thermal conductivity of X is $400 Wm^{-1}K^{-1}$ and that of Y is $200 Wm^{-1}k^{-1}$].

Solution

(a) (i) Refer to your notes.

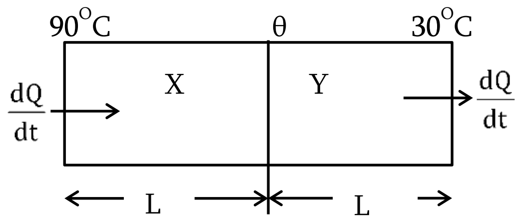
(ii) In both cases, the conduction is due to the movement of the free electrons. If there are enough electrons on the conduction band, then material will be good thermal conductor and good electrical conductor and this is accordance to the Wiedemann and Franz law i.e $\frac{K}{\delta T}$ = Constant

(b) (i)



Lagged bar	Unlagged bar
Rate of heat flows is constant i.e $\frac{dQ}{dt} = KA\frac{d\theta}{dx}$ a	Rate of heat flows is not constant i.e $\frac{dQ}{dt} = -KA\frac{d\theta}{dx}$
Radial heat lines are uniformly a cross the bar	Radial heat lines are non – uniformly across the bar
No amount of heat energy given out from the bar to the surrounding	Amount of heat energy is given out to the surrounding.

(ii) For the lagged bars in series connection



Since $\frac{dQ}{dt}$ = Constant

$$\left(\frac{dQ}{dt}\right)_X = \left(\frac{dQ}{dt}\right)_Y$$

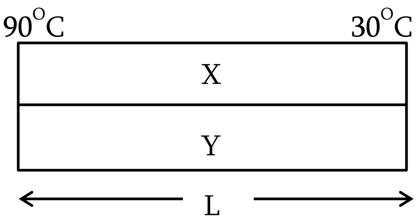
$$\frac{400A(90-\theta)}{L} = \frac{200A(\theta-30)}{L}$$

On solving, $\theta = 70^{\circ}C$

Rate of heat flows on bars in series connection.

$$\left(\frac{dQ}{dt}\right)_S = \frac{400A(90-70)}{L} = \frac{8000A}{L}$$

For lagged bar in parallel



$$\left(\frac{dQ}{dt}\right)_P = \left(\frac{dQ}{dt}\right)_X + \left(\frac{dQ}{dt}\right)_Y$$

$$\left(\frac{dQ}{dt}\right)_P = \frac{400A(90-30)}{L} + \frac{200A(90-30)}{L}$$

$$\left(\frac{dQ}{dt}\right)_P = \frac{36000A}{L} \ldots\ldots(ii)$$

(ii)/(i)

$$\frac{\left(\frac{dQ}{dt}\right)_P}{\left(\frac{dQ}{dt}\right)_S} = \frac{\frac{3600A}{L}}{\frac{8000A}{L}}$$

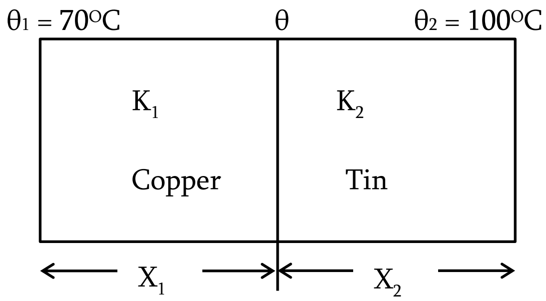
$$\frac{\left(\frac{dQ}{dt}\right)_P}{\left(\frac{dQ}{dt}\right)_S} = 4.5$$

Example 8

A boiler is made of a copper plate 3mm thick. The inner surface of the copper plate is coated with a layer of tin of thickness 0.3mm. The surface are of the plate is 150cm^2. The outside temperature of copper plate is 700^{O}C. Find the maximum amount of steam that could be produced per hour at atmospheric pressure. The coefficient of thermal conductivities of copper and tin are 397Wm^{-1}k^{-1} and 63Wm^{-1}k^{-1} respectively ($L = 2268 \times 10^3$Jkg^{-1})

Solution

Let θ be the temperature of the interface



At the steady state condition

$$\left(\frac{dQ}{dt}\right)_{copper} = \left(\frac{dQ}{dt}\right)_{tin}$$

$$K_1A\frac{(\theta_1-\theta)}{X_1} = K_2A\frac{(\theta-\theta_2)}{X_2}$$

$$\frac{397(700-\theta)}{3\times10^{-3}} = \frac{63(\theta-100)}{0.3\times10^{-3}}$$

On solving $\theta = 331.7°C$

Heat flows through copper in one hour

$$Q = \frac{K_1A(\theta_1-\theta)t}{X_1}$$

$$= \frac{397\times150\times10^{-4}(700-331.7)\times3600}{3\times10^{-3}}$$

$$Q = 2.63\times10^{10}J$$

Let M = Mass of steam produced in one hour.

$$M = \frac{Q}{L} = \frac{2.63\times10^{10}}{2268\times10^{3}} \quad [Q = mL]$$

$$M = 1.16\times10^{4}Kg$$

Example 9: NECTA 2013/P1/9

(a) Compare the law governing to the conduction of heat and electricity pointing out corresponding quantities in each case.

(b) (i) Write three laws governing the black body radiation.

(ii) A cup of tea kept in a room with temperature of 22^{O}C cools from 66^{O}C to 63^{O}c in 1 minute. How long will the same cup of tea take to cool from the temperature of 43^{O}C to 40^{O}C under the same condition?

(c) A lagged copper rod is uniformly heated by a passage of an electric current. Show by considering a small section dx that the temperature θ varies with distance X long a rod in a way that $K\left[\frac{d^2T}{dx^2}\right] = -H$ where K is a thermal conductivity and H is the rate of heat generation per unit volume.

Solution

(a) • Heat flow is due to the temperature gradient while current flow is due to the potential gradient.

• Heat flow is proportional to the temperature difference or temperature gradient) while current flow is proportional to the potential difference (potential gradient)

• Thermal conductivity determine heat flows while electrical conductivity determine the current flow.

(b) (i) • Stefan's law

• Plank's law

• Wieri's displacement law

(ii) Applying Newton's law of cooling

$$\frac{d\theta}{dt} = \frac{-KS}{MC}(\theta - \theta_s)$$

$$\frac{(66-63)^{o}C}{1\,min} = \frac{-KS}{MC}[66-22] \ldots..(1)$$

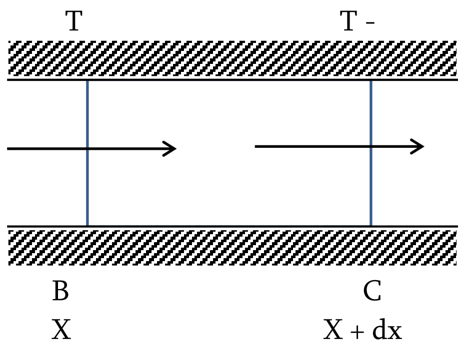
Also

$$\frac{(43-40)^{0}C}{t} = \frac{-KS}{MC}[43-22] \ldots\ldots(2)$$

Dividing equation (1) and (2)

t = 2minutes

(c) Consider a small portion BC of lagged copper rod



If A is a cross – sectional area, then the rate of heat flows through point B

$$H_1 = -KA\frac{dT}{dx}$$

Rate of heat flows through point C

$$H_2 = -KA\frac{dT}{dx} + \frac{d}{dx}\left[-KA\frac{dT}{dx}\right]dx$$

$$H_2 = -KA\frac{dT}{dx} - KA\frac{d^2T}{dx^2}\cdot dx$$

Net heat per second lost by section i.e Rate of heat generated in BC.

$$H_0 = H_2 - H_1$$

$$= -KA\frac{dT}{dx} - KA\frac{d^2T}{dx^2}\cdot dx + KA\frac{dT}{dx}$$

$$H_0 = -K(Adx)\cdot\frac{d^2T}{dx^2}$$

$$\frac{H_0}{Adx} = \frac{H_0}{V} = H$$

$$H = -K\frac{d^2T}{dx^2}$$

$$-H = K\frac{d^2T}{dx^2} \text{ hence shown}$$

Example 10

A composite slab is made of two parallel layers of two different materials of thermal conductivities K_1 and K_2 are of the same thickness. Show that the equivalent thermal conductivity of the slab is

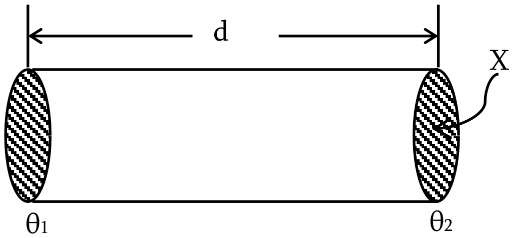
$$K = \frac{2K_1K_2}{K_1 + K_2}$$

Example 11

The rate of flow of energy through a perfectly lagged metal bar of area of cross – sectional X length, d and thermal conductivity K may be considered to be analogous to the rate of charge through an electrical conductor of cross – sectional area, A length L and resistivity ρ. Show that the thermal resistance of the metal bar which corresponds to the electrical resistance of the conductor is $\frac{d}{kx}$ extending analogy to the thermal conductors in series, show that the effectively thermal conductivity K of composite wall consisting of the two parallel sided layer of material of thickness d_1 and d_2 and thermal conductively is given by $K = \dfrac{d_1 + d_2}{\dfrac{d_1}{K_1} + \dfrac{d_2}{K_2}}$

Solution

Case 1:



Rate of heat flows along the conductor

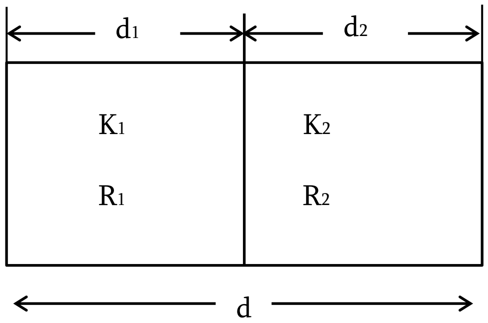
$$\frac{dQ}{dt} = \frac{KX(\theta_1 - \theta_2)}{d}$$

$$\frac{d}{kx} = \frac{\theta_1 - \theta_2}{\frac{dQ}{dt}} = R$$

Thermal resistance

$$R = \frac{d}{kx} \text{ Hence shown}$$

Case 2: For the conductors in series connection



Thermal resistance for each bar

$$R_1 = \frac{d_1}{K_1X}, \quad R_2 = \frac{d_2}{K_2X}$$

Total effectively thermal resistance in series connection

$$R = R_1 + R_2$$

$$= \frac{d_1}{K_1X} + \frac{d_2}{K_2X}$$

$$\frac{d}{KX} = \left(\frac{d_1}{K_1} + \frac{d_2}{K_2}\right)\frac{1}{X}$$

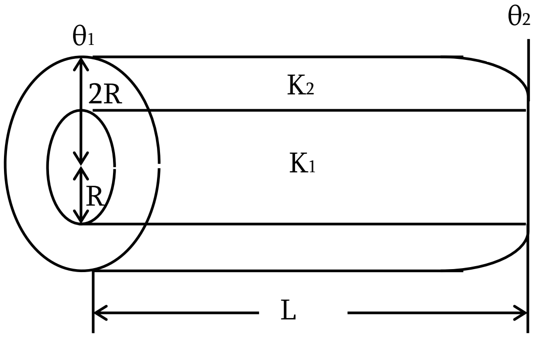
$$\frac{d_1 + d_2}{K} = \frac{d_1}{K_1} + \frac{d_2}{K_2}$$

$$K = \frac{d_1 + d_2}{\frac{d_1}{K_1} + \frac{d_2}{K_2}} \text{ Hence shown}$$

Example 12

A cylinder of radius R made of a material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius R and the outer radius 2R made of material of thermal conductivity K_2. The two ends of combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is steady state. Calculate the effectively thermal conductivity of the system.

Solution



Total thermal resistance in parallel connection

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

But $R_1 = \dfrac{L}{K_1A_1}$, $R_2 = \dfrac{L}{K_2A_2}$

Now $\dfrac{KA}{L} = \dfrac{K_1A_1}{L} + \dfrac{K_2A_2}{L}$

$$A = \pi(2R)^2 = 4\pi R^2$$

$$A_1 = \pi R^2, \quad A_2 = \pi\left[(2R)^2 - R^2\right]$$

$$A_2 = 3\pi R^2$$

$$4\pi R^2 K = K_1\pi R^2 + 3\pi R^2 K_2$$

$$K = \frac{K_1 + 3K_2}{4}$$

Example 13

A is a compound slab made up of two layers, one of thickness d1 and thermal conductivity K_1, the other of thickness d_2 and thermal conductivity K_2. B is slab of thickness $d_1 + d_2$ and thermal conductivity, K, one of face of each slab is maintained at θ_1 and other at θ_2 until a steady state is reached. If the rate of conduction of heat per unit area through the two slabs is the same; find the relation between K_1 and K_2.

Solution

Rate of conduction of heat through the first slab, A per unit area

$$\frac{1}{A}\cdot\frac{dQ}{dt} = \frac{\theta_1-\theta_2}{\frac{d_1}{K_1}+\frac{d_2}{K_2}} \text{........(1)}$$

Rate of heat flows through slab, B per unit area.

$$\frac{1}{A}\cdot\frac{dQ}{dt} = \frac{\theta_1-\theta_2}{\frac{d_1+d_2}{K}} \text{........(2)}$$

$$(1) = (2)$$

$$\frac{d_1}{k_1}+\frac{d_2}{k_2} = \frac{d_1+d_2}{k}$$

$$k_1 = \frac{kk_2d_1}{d_1k_2+(k_2-k)d_2}$$

Example 14

(a) (i) What is meant by the statement that thermal conductivity of brick is $0.48Wm^{-1}k^{-1}$?

(ii) A brick wall is 0.25m thick and the temperature difference between the exposed surfaces is 30°C. Calculate the heat passing through it per square metre per hour. If the brick is covered on, one side with plaster 5cm thick, calculate the heat per square metre per hour now flowing, the temperature difference of the exposed surfaces being the same as before. Thermal conductivity of plaster is $0.004Wm^{-1}k^{-1}$.

(b) One end of uniform metal bar is maintained at 100°C and the other end at 20°C, when the bar is (i) lagged (ii) unlagged. Draw sketches illustrating the temperature variation along the bar in each case and explain the variation.

Solution

(a) (i) This mean that the brick of unit cross – sectional area will transfer heat by conduction at the rate of 0.048W when its opposite faces are maintained at a unit temperature gradient.

(ii) Case 1: the rate of heat flow per unit area

$$\frac{1}{A}\frac{dQ}{dt} = \frac{K(\theta_2-\theta_1)}{X}\times 3600$$

$$= \frac{0.048\times 30\times 3600}{0.25}$$

$$\frac{1}{A}\cdot\frac{dQ}{dt} = 20736Jm^{-2}$$

Case 2: when the brick is covered with plaster then,

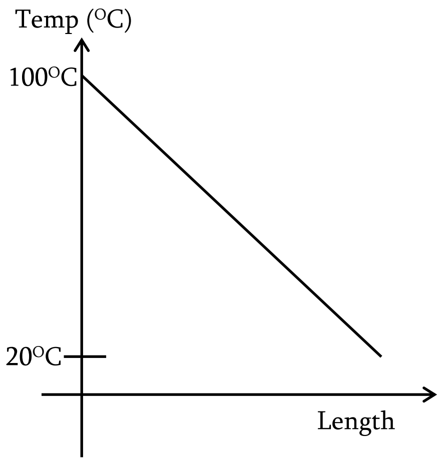
$$\frac{1}{A}\cdot\frac{dQ}{dt} = \frac{\theta_1-\theta_2}{\frac{X_1}{K_1}+\frac{X_2}{K_2}}$$

$$= \frac{30}{\frac{0.25}{0.048}+\frac{0.05}{0.004}}\times 3600$$

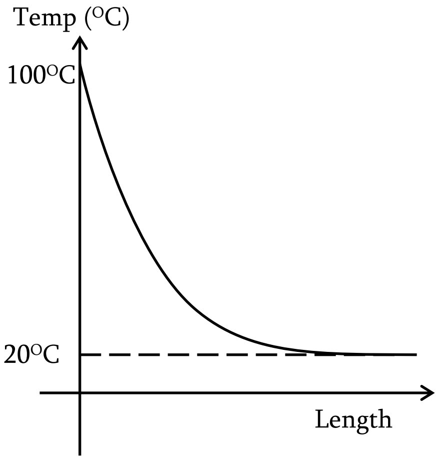
$$\frac{1}{A}\cdot\frac{dQ}{dt} = 6099Jm^{-2}$$

(b) Graph of temperature against length

(i) For the lagged bar



(ii) For unlagged bar



Example 15

(a) Ice is forming on the surface of the pond. When it is 4.6cm thick, the temperature of the surface of the ice in contact with air is 260K whilst the surface in contact with water is at temperature 273 (a) calculate the rate of heat per unit area from the water.

(b) Hence determine the rate at which the thickness of ice is increasing, given that thermal conductivity of ice = $2.3Wm^{-1}k^{-1}$ density of water = 1000kgm-3, specific latent heat of fussion = $3.25 \times 10^5 Jkg^{-1}$.

Solution

(a) Rate of heat flows through the ice per unit area.

$$\frac{1}{A}\cdot\frac{dQ}{dt} = \frac{K\left[\theta_2-\theta_1\right]}{X}$$

$$= \frac{2.3\left[273-260\right]}{4.6\times10^{-2}}$$

$$\frac{1}{A}\cdot\frac{dQ}{dt} = 6.5\times10^{2}Wm^{-2}$$

(b) Let $\frac{dx}{dt}$ = Rate of increases of the thickness of ice.

$$\frac{dx}{dt} = \frac{1}{A}\cdot\frac{dQ}{dt}\cdot\frac{1}{\rho L}$$

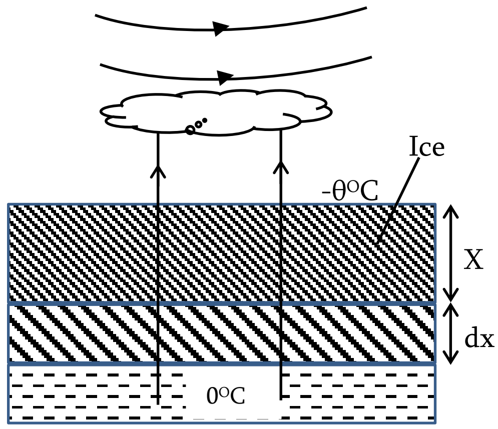
$$= \frac{6.5\times10^{2}}{3.25\times10^{5}\times1000}$$

$$\frac{dx}{dt} = 0.2m/s$$

Example 16

The ice on the pond is 30mm thick. If the air temperature above the ice is -8°C, how long will it take for the thickness of ice to increase to 40mm. given that thermal conductivity of ice is $2.0Wm^{-1}k^{-1}$, specific latent heat fussion is $3.3 \times 105Jkg^{-1}$, density of ice = $0.9 \times 10^3 kgm^{-3}$.

Solution



Rate of heats flows through the ice

$$\frac{dQ}{dt} = \frac{KA\left(\theta_2-\theta_1\right)}{X}\;.........(i)$$

Rate of heat energy needed to convert water into ice.

$$\frac{dQ}{dt} = \frac{L\rho Adx}{dt}\;.........(ii)$$

(i) = (ii)

$$L\rho A\frac{dx}{dt} = \frac{KA\left(\theta_2-\theta_1\right)}{X}$$

$$\int_0^t dt = \frac{\rho L}{K\left(\theta_2-\theta_1\right)}\int_{X_1}^{X_2} xdx$$

$$t = \frac{\rho L}{2K\left(\theta_2-\theta_1\right)}\left[X_2^2-X_1^2\right]$$

$$= \frac{3.3\times10^{5}\times0.9\times10^{3}}{2\times2\left[0-^{-}8\right]}\left[\left(40\times10^{-3}\right)^2-\left(30\times10^{-3}\right)^2\right]$$

$$t = 6.5\times10^{3}\,sec$$

Example 17

A layer of ice 20cm thick has formed on a pond. The temperature of air is -10^{O}C. Find how long will it take for another 0.1cm layer of water to freeze? Given that thermal conductivity of ice = 2.1Wm^{-1}k^{-1}. Latent heat capacity of ice = 3.36 × 10^{5}Jkg^{-1} and density of ice = 1000kgm^{-3}.

Solution

Let A (in m2) be the surface area of the layer of ice formed. Thickness of the layer of ice formed = 0.1cm = 0.1 × 10^{-2}m

∴ Volume of the ice to be formed V = A × 10^{-3}m^{3}

Mass of ice formed M = ρV = A × 10^{-3} × 1000 = Akg

For the mass, M of water in this layer to freeze into ice, the amount of heat that should be removed from the water

Q = mL = A × 3.36 × 10^{5}J

During the formation of ice, heat has to traverse the varying thickness of the ice.

Initial thickness of ice

$d_1 = 20\text{cm}$

Final thickness of ice

$d_2 = d_1 + \text{thickness of ice formed}$

$d_2 = 20+0.1 = 20.1\text{cm}$

Average thickness of ice

$$d = \frac{d_1+d_2}{2} = \frac{20+20.1}{2}$$

$d = 20.05\text{cm} = 20.05\times10^{-2}\text{m}$

Temperature difference on the two side of ice layer

$T_1 - T_2 = 0-(-10) = 10^{0}\text{C}$

Quantity of heat passing through the ice

$$Q = \frac{KA(T_1-T_2)t}{d}$$

$$\text{Now } A\times3.36\times10^{5} = \frac{2.1\times A\times10\times t}{20.05\times10^{-2}}$$

On solving t = 3,208sec

Example 18

On a hot day, the surface water of a pond is warmer than the water below, but on a day when it is nearly freezing, the surface water is colder. Why?

Solution

The density of water is maximum at 4^{O}C. On a hot day, the temperature of whole of water in the pond is more than 4^{O}C. The warmer water, being rarer, reaches the top and hence on a hot day, water of the pond is warmer at the top. On the other hand, on an extremely cold day, when temperature is below 4^{O}C, the nearly freezing water (having temperature just above 0oc but below 4^{O}C) being rarer appears at the top but water having temperature close to 4^{O}C being heavier goes down. Therefore, on a day, when it is nearly freezing, water of the pond is colder at the top than at the bottom.

Example 19: NECTA 2020/P1/6

(a) (i) Why is it preferred to purchase a cooking utensil of low specific heat capacity? (03 marks)

(ii) How does a fish survive in a pond during an extreme winter season even if the pond is deep frozen on the surface? (03 marks)

(b) The ice on a pond is 10mm thick. If the temperature above and below its surface are 263K and 273K respectively, calculate the rate of heat transfer through the ice (04 marks)

- Thermal conductivity of ice K = 2.3Wm^{1}K^{-1}
- Density of water = 1000kgm^{-3}
- Specific latent heat of fusion of water = 3.25 × 015Jkg^{-1}

Solution

(a) (i) Refer to your notes.

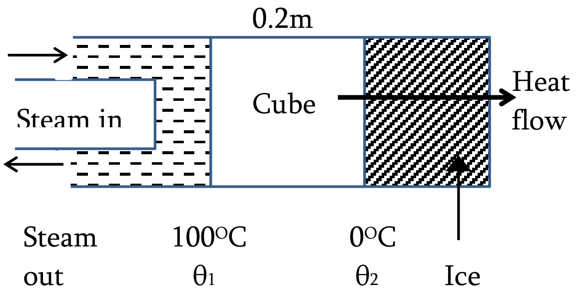
(ii) When it is extreme cold, the temperature of water in ponds starts falling. On getting colder, water becomes denser and it goes down. To replace it, the warmer water from below rises up. However, it happens so, till the temperature of water at the bottom of the pond becomes 4°C. It is becomes, the density of water is maximum at 4°C. Now, as the temperature lowers, further, ice is formed at the surface of the pond with water below. In this manner, the fishes can survive in the extreme winter, when ponds are frozen.

(b) Refer solution of example 15.

Example 20

One face of the cube of side 0.2m is in contact with ice and the opposite face is in contact with steam. If all other sides are well lagged. Calculate the mass of the ice that melts during one hour. Given that the thermal conductivity of the metal = $40Wm^{-1}k^{-1}$. Latent heat of fusion of ice = $336KJkg^{-1}$.

Solution



Rate of heat energy needed to melt the ice

$$\frac{dQ}{dt} = \frac{ML}{t} \ldots\ldots\ldots\ldots\ldots(i)$$

Rate of heat flows through the cube

$$\frac{dQ}{dt} = \frac{KA(\theta_1 - \theta_2)}{X} \ldots..(ii)$$

(i) = (ii)

$$\frac{ML}{t} = \frac{KA(\theta_1 - \theta_2)}{X}$$

$$M = \frac{KA(\theta_1 - \theta_2)t}{XL}$$

But $A = (\text{Side})^2 = (0.2m)^2$

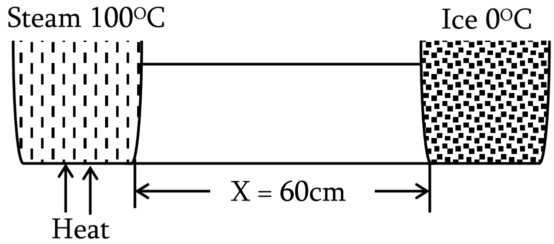
$$M = \frac{40 \times (0.2)^2 [100 - 0] \times 3600}{0.2 \times 336 \times 10^3}$$

$$M = 8.571kg$$

Example 21

A copper rod of length 60cm and 8mm in radius is taken and its one end is kept in boiling water and the other end is ice at 0°C. If 72gm of ice melts in one hour, what is the thermal conductivity of copper. (Latent heat of fusion of ice = $336KJKg^{-1}$)

Solution



Since $mL = \dfrac{KA(\theta_1 - \theta_2)t}{X}$

$$K = \frac{mLX}{\pi r^2 (\theta_1 - \theta_2)t}$$

$$= \frac{72 \times 10^{-3} \times 336 \times 10^3 \times 0.6}{3.14 \times (8 \times 10^{-3})^2 \times (100 - 0) \times 3600}$$

$$K = 200.64Wm^{-1}K^{-1}$$

Example 22 : NECTA 2011/P1/5

(a) (i) Briefly explain what is meant by thermal conduction and define the coefficient of thermal conductivity.

(ii) Ice cubes of mass 5.0g at 0°C are placed inside a spherical container having an outside diameter of 40cm, 2mm thick and of thermal conductivity $5 \times 10^{-4}Wm^{-1}K^{-1}$. How long will it take for all the ice cubes to melt if the room temperature is 30°C?

(b) Two metal rods A and B of lengths 40cm and 80cm respectively having the same cross – sectional area of $10cm^3$ are joined end to end. If the composition is perfectly lagged and the free end of A is fixed at 100°C while that of B is passed to a point at 0°C. Determine.

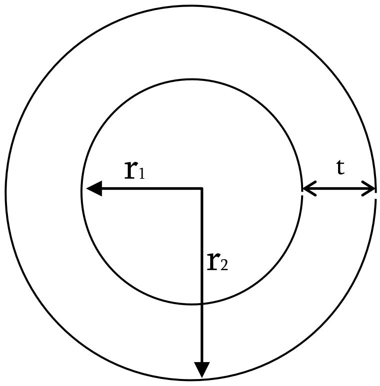
(i) The junction temperature of the two metal rods.

(ii) The quantity of heat that flows per minute in steady state. (Thermal conductivities of rods A and B are 360 and $80 Wm^{-1}K^{-1}$ respectively, specific latent heat of fusion, $L = 3.33 \times 10^5 Jkg^{-1}$)

Solution

(a) (i) Refer to your notes

(ii) Fig



$r_2 = 20cm = 0.2m$

$r_1 = r_2 - t$

$r_1 = 19.8cm$ or $0.198m$

The rate of heat flows across a spherical shell.

$$\frac{dQ}{dt} = \frac{4\pi K r_1 r_2 (\theta_2 - \theta_1)}{r_2 - r_1}$$

$$= \frac{4\pi \times 5 \times 10^{-4} \times 0.2 \times 0.198 (30 - 0)}{0.2 - 0.198}$$

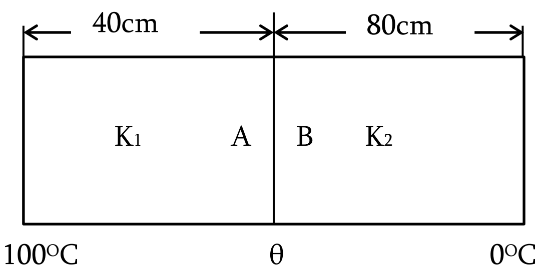
$$\frac{dQ}{dt} = 3.73W$$

But $\frac{dQ}{dt} = \frac{ML}{t}$

$$t = \frac{ML}{\frac{dQ}{dt}} = \frac{5 \times 10^{-3} \times 3.33 \times 10^5}{3.73}$$

$$t = 446.116 sec = 7.43 min$$

(b) (i)



Let θ = temperature at the junction

Under the steady condition

$$\left(\frac{dQ}{dt}\right)_A = \left(\frac{dQ}{dt}\right)_B$$

$$\frac{K_1 A (100 - \theta)}{L_1} = \frac{K_2 A (\theta - 0)}{L_2}$$

$$\frac{3600(100 - \theta)}{0.4} = \frac{80\theta}{0.8}$$

On solving, $\theta = 90°C$

(ii) $$Q = \frac{K_1 A (100 - \theta) t}{L_1}$$

$$= \frac{360 \times 10 \times 10^{-4} (100 - 90) \times 60}{0.4}$$

$$Q = 540J$$

Example 23

Determine the rate of heat flow on concentric cylinder of length 10cm being maintained at the temperature of 40°C. The inner and outer radii are 5cm and 20cm respectively and the thermal conductivity of the cylinder is $54 Wm^{-1}K^{-1}$.

Solution

Rate of heat flows through the concentric cylinder

$$\frac{dQ}{dt} = \frac{2\pi K L \Delta\theta}{\log_e \left(b/a \right)}$$

$$= \frac{2 \times 3.14 \times 54 \times 0.1 \times 40}{\log_e \left(20/5 \right)}$$

$$\frac{dQ}{dt} = 978.9W$$

Example 24

Determine the rate of heat flows on the concentric spherical conductor whose inner and outer radii are 20cm and 40cm respectively. Given that $K = 20 Wm^{-1}K^{-1}$ and the temperature in the outer surface is 48°C while the temperature on the inner part of the conductor is 92°C.

Solution

The rate of heat flows along concentric spherical conductor.

$$\frac{dQ}{dt} = \frac{4\pi Kab(\theta_2 - \theta_1)}{b-a}$$

$$= \frac{4\times3.14\times20\times0.2\times0.4(92-38)}{(0.4-0.2)}$$

$$\frac{dQ}{dt} =$$

Example 25

(a) What is the relationship between U – value and its thermal conductivity?

(b) A uninsulated roof has u – value of 1.9. Its U – value when lagged is 0.4. Calculate the U – value of lagging material and its thermal conductivity if its 2cm thick?

Solution

(a) $U = \frac{K}{L}$ or $K = LU$

(b) $U = 0.4$, $U_1 = 1.9$, $U_2 = ?$

For the series conductors

$$\frac{1}{U} = \frac{1}{U_1} + \frac{1}{U_2}$$

$$\frac{1}{U_2} = \frac{1}{U} - \frac{1}{U_1} = \frac{1}{0.4} - \frac{1}{1.9}$$

$$U_2 = 0.506Wm^{-2}K^{-1}$$

Let K = thermal conductivity of lagged material.

$$K = U_2L = 0.506\times2.0\times10^{-2}$$

$$K = 0.01Wm^{-1}K^{-1}$$

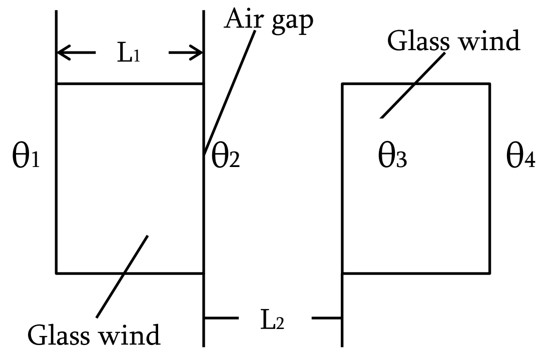
Example 25

A double glazed window consists of two planes of glass each 4mm thick separated by 10mm layer of air. Assume the thermal conductivity of glass to be 50 times greater than that of air. Calculate the ratio of:-

(i) Temperature gradient in the glass to the temperature gradient in air gap.

(ii) Temperature difference across one plane of the glass to the temperature difference across the air gap.

Solution



$L_1 = 40mm$, $L_2 = 10mm$

(i) Rate of heat flows

$$\frac{dQ}{dt} = KAg$$

$$g = \frac{1}{KA}\cdot\frac{dQ}{dt} \quad \left[g \propto \frac{1}{K}\right]$$

$$\frac{g_1}{g_2} = \frac{K_2}{K_1} = \frac{K_2}{50K_2}$$

$$\frac{g_1}{g_2} = \frac{1}{50} = 0.02$$

(ii) Let $\Delta\theta_1$ = temperature difference across the plane $\Delta\theta_2$ = temperature difference across the layer of air.

Rate of heat flows

$$H = \frac{KA\Delta\theta}{L} \Rightarrow \Delta\theta = \frac{LH}{KA}$$

Now $\Delta\theta \propto \frac{L}{K}$

$$\frac{\Delta\theta_1}{\Delta\theta_2} = \left(\frac{L_1}{L_2}\right)\left(\frac{K_2}{K_1}\right)$$

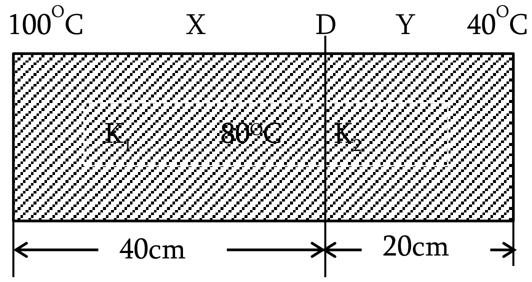
$$= \frac{40}{10} \times \frac{1}{50}$$

$$\frac{\Delta\theta_1}{\Delta\theta_2} = 0.008$$

Example 26

(a) The temperature of the water side of a metal boiler is 140^O and 3000kg of water are evaporated per square meter of the boiler in one hour. Calculate the temperature of the other side of the boiler if the thermal conductivity of the metal is 84W/mk, the boiler is 12mm thick and the specific latent heat of steam is 2.27×10^6J/kg.

(b) In the figure below, X and Y are two metals bars of respectively thermal conductivities K_1 and K_2 in series.



The outsides are at 100^OC and 40^OC respectively and their junction D is 80^OC in the steady state. If the lengths of the bars are 40cm and 20cm as shown, calculate the ratio of K_1/K_2.

Solution

(a) Let $\theta =$ temperature of the other sides

Quantity of heat required to convert water into steam is equal to the heat conducted through boiler plate.

$$mL = \frac{KA(\theta - 140)t}{X}$$

$$\theta = \frac{\left(M/A\right)LX}{Kt} + 140$$

$$= \frac{3000 \times 2.27 \times 10^6 \times 12 \times 10^{-3} + 140}{84 \times 3600}$$

$$\theta = 410°C$$

(b) Under steady state condition

$$\frac{K_1A(100-80)}{40} = \frac{K_2A(80-40)}{20}$$

$$\frac{K_1}{K_2} = 4:1$$

Example 27

A room ha 4m × 4m × 10cm concrete roof (K_1 = $1.26Wm^{-1}\ ^OC^{-1}$). At some instant the temperature outside is 46^OC and inside is 32^OC.

(i) Neglecting convection, calculate the amount of heat flowing per second into the room through the roof.

(ii) If bricks (K_2 = $0.65Wm^{-1}\ ^OC^{-1}$) of thickness 7.5cm roof. Calculate the new rate of heat flow under the same conditions.

Solution

(i) Dimension of the room

= 4m × 4m × 0.1m

X_1 = 0.1m, A_1 = 4m × 4m = $16m^2$

Thermal resistance

$$R_1 = 4.96 \times 10^{-3} W^{-1}\ ^0C$$

Rate of heat flows

$$H_1 = \frac{\theta_1 - \theta_2}{R_1} = \frac{(46-32)^0 C}{4.96 \times 10^{-3} W^{-1}\ ^0C}$$

$$H_1 = 2822W$$

(ii) Thermal resistance of the bricks

$$R_2 = \frac{X_2}{K_2A_2} = \frac{7.5 \times 10^{-2}}{0.65 \times 16}$$

$$R_2 = 7.2 \times 10^{-3} W^{-1}\ ^0C$$

Rate of heat flows through the brick

$$H_2 = \frac{(46-32)^0 C}{7.2 \times 10^{-3} W^{-1}\ ^0C}$$

$$H_2 = 1152W$$

Example 28

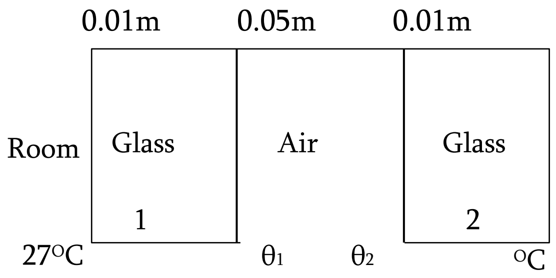
(a) The tile feels colder than the wooden floor even through both floor materials are at the same temperature. Why?

(b) A double plane window used for insulating a room thermally from outside consists of two glass sheets each are of area $1m^2$ and thickness 0.01m separated by a 0.05m thick stagnant air space. In the steady state, the room glass interface are at constant temperature of 27^OC and 0^OC respectively. Calculate the rate of heat flow through the window plane. Also find the temperature of the interface. Given that thermal conductivity of glass and air are $0.8Wm^{-1}K^{-1}$ and $0.08Wm^{-1}K^{-1}$.

Solution

(a) It is because tile is a better heat conductor than the wooden floor, the heat transferred from your foot to the wood is not conducted away rapidly. So the wood quickly heat up on its surface to the temperature of your foot but the tile conducts the heat away rapidly and thus can take away from your foot, so its surface temperature drops.

(b)



Let K_1 = thermal conductivity of glass = $0.8W^{-1}K^{-1}$.

K_2 = thermal conductivity of air = $0.08Wm^{-1}K^{-1}$.

Rate of heat flows through

Glass 1: $H = \dfrac{K_1A(27-\theta_1)}{X_1}$

$$H = \frac{0.8A(27-\theta_1)}{0.01} \text{.........(1)}$$

Air space $H = \dfrac{K_2A(\theta_1-\theta_2)}{X_2}$

$$H = \frac{0.08A(\theta_1-\theta_2)}{0.05} \text{........(2)}$$

Glass 2: $H = \dfrac{0.8A(\theta_2-0)}{0.01}$ ....(3)

Under steady state condition

$$\frac{dQ}{dt} = \text{constant}$$

(1) = (2) = (3)

$$\frac{0.08A(27-\theta_1)}{0.01} = \frac{0.08A(\theta_1-\theta_2)}{0.05}$$

$$1350 = 51\theta_1 - \theta_2 \text{.....(1)}$$

Again $\dfrac{0.8A(27-\theta_1)}{0.01} = \dfrac{0.8A\theta_2}{0.01}$

$$27 = \theta_1 + \theta_2$$

On solving equation (1) and (2) simultaneously

$\theta_1 = 26.48^0C$

$\theta_2 = 0.52^0C$

Rate of heat flows

$$H = \frac{dQ}{dt} = \frac{0.8\times1\times(27-26.48)}{0.01}$$

$H = 48W$

Example 29: NECTA 2021/P1/5

(a) (i) State the law applied when a body is cooling under forced convection (02 marks)

(ii) Write the mathematical expression of the how stated in 5(a)(i) and briefly give the physical meaning of each term (03 marks)

(b) If ends of a straight uniform metal rod are maintained at temperature of 100°C and 20°C while the room, temperature being below 20°C

(i) Sketch the graph of variation of temperature of the rod versus its length when its surface is unlagged (03 marks)

(ii) Comment on the nature of the graph draw in 5(b)(i) (02 marks)

Solution

(a) (i) State that 'The rate of heat loss by a body is directly proportional to the temperature difference between the body and the surrounding under forced convection.

(ii) $\dfrac{dQ}{dt} = -KS\,(\theta-\theta_S)$

$\dfrac{dQ}{dt}$ = rate of heat loss

θ = temperature of the hot body

θ_S = temperature of the surrounding

S = Surface area

K = Convectional coefficient

Negative sign indicates that the body is losing heat

(b) Refer to your notes.

Example 30

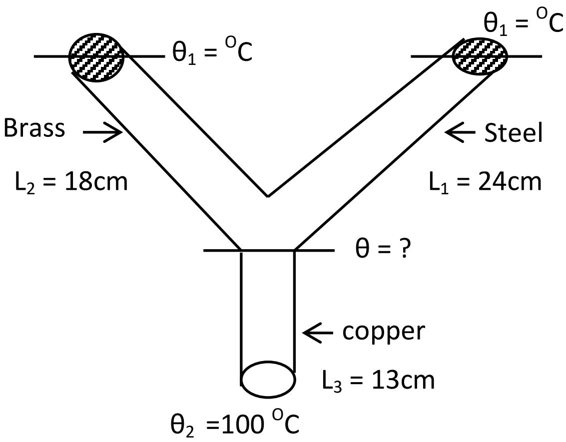
Rods of copper, brass and steel are welded together to form a Y shaped figure each of cross – section area of each rod is 2.00cm^2. The free end of the copper rod is maintained at 100^{O}C and free ends of the brass and steel stands at 0^{O}C. The length of the rods are copper is 13cm, brass is 18cm and steel is 24cm. Assume there is no heat loss from the surfaces of the rods.

(a) What is the temperature of the junction point?

(b) What is the rate of heat flows in each of three rods? Thermal conductivities of copper, steel and brass rods are 385, 50.2 and 109Wm^{-1}K^{-1} respectively.

Solution

(a) The steel rod and brass rod are parallel connection. The parallel connection is varies with copper



$$\text{Now } \left(\frac{dQ}{dt}\right)_3 = \left(\frac{dQ}{dt}\right)_1 + \left(\frac{dQ}{dt}\right)_2$$

$$\frac{K_3A(\theta_2-\theta)}{L_3} = \frac{K_1A(\theta-0)}{L_1} + \frac{K_2A(\theta-0)}{L_2}$$

$$\theta = \theta_3\left[\frac{\frac{K_3}{L_3}}{\frac{K_1}{L_1}+\frac{K_2}{L_2}+\frac{K_3}{L_3}}\right]$$

$$= 100^0C\left[\frac{\frac{385}{0.13}}{\frac{50.2}{0.24}+\frac{109}{0.18}+\frac{385}{0.13}}\right]$$

$$\theta = 78.4^0C$$

(b) Rate of heat flows through brass

$$\left(\frac{dQ}{dt}\right)_{brass} = \frac{K_2A\theta}{L_2}$$

$$= \frac{109\times2\times10^{-4}\times78.4}{0.18}$$

$$\left(\frac{dQ}{dt}\right)_{brass} = 9.4951W$$

$$\text{Again } \left(\frac{dQ}{dt}\right)_{steel} = \frac{K_1A\theta}{L_1}$$

$$= \frac{50.2\times2\times10^{-4}\times78.4}{0.24}$$

$$\left(\frac{dQ}{dt}\right)_{steel} = 3.27973W$$

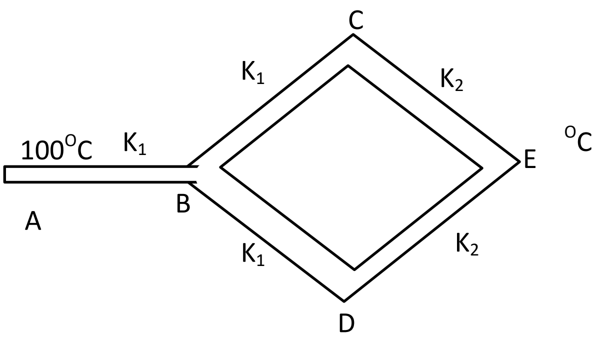
Rate of heat flows through the copper

$$\left(\frac{dQ}{dt}\right)_{copper} = 9.4951+3.27973$$

$$\left(\frac{dQ}{dt}\right)_{copper} = 12.77483W$$

Example 31

In the figure below, the bars AB, BC and BD are of the same dimensions and are made of the same material of coefficient of thermal conductivity K$_1$, the bars CE and DE are made of the same material of coefficient of thermal conductivity K$_2$. The dimensions of these two bars are same as that of the first three bars. If the end A is at 100oc and the end E at 0^{O}C. What is the temperature of the junction B? Given that K$_1$:K$_2$ = 2:1



Solution

Thermal resistance, $R = \dfrac{L}{KA}$

Let thermal resistance of the bar AB, BC and BD are R_1 each and that of CE and DE are R_2 each.

Total thermal resistance of bars BC and CE are in series

$$Rs_1 = R_1 + R_2$$

Also BD and DE are in series

$$Rs_2 = R_1 + R_2$$

Since $Rs_1 // Rs_2$

$$R_P = \frac{R_{S_1} + R_{S_1}}{R_{S_1} + R_{S_2}}$$

$$= \frac{(R_1+R_2)(R_1+R_2)}{(R_1+R_2)+(R_1+R_2)}$$

$$R_P = \frac{R_1+R_2}{2}$$

The net thermal resistance

$$R_T = R_1 + R_P$$

$$= R_1 + \frac{R_1+R_2}{2}$$

$$R_T = \frac{3R_1+R_2}{2}$$

The total rate of heat flows between point A and E

$$\frac{dQ}{dt} = \frac{100-0}{\frac{3R_1+R_2}{2}} = \frac{200}{3R_1+R_2} \ldots\ldots(1)$$

Rate of heat flows in bar AB

$$\frac{dQ}{dt} = \frac{100-\theta}{R_1} \ldots\ldots.(2)$$

$$(1) = (2)$$

$$\frac{100-\theta}{R_1} = \frac{200}{3R_1+R_2}$$

But $R_1 = \dfrac{L}{K_1A}$, $R_2 = \dfrac{L}{K_2A}$

$$\frac{200}{\frac{3}{K_1}+\frac{1}{K_2}} = \frac{100-\theta}{\frac{1}{K_1}}$$

Given that $\dfrac{K_1}{K_2} = \dfrac{2}{1}$, $K_1 = 2K_2$

$$\frac{200}{\frac{3}{2K_2}+\frac{1}{K_2}} = \frac{100-\theta}{\frac{1}{2K_2}}$$

On solving, $\theta = 60^\circ C$

$\therefore$ Temperature at the junction B, $\theta = 60^\circ C$

EXPERIMENT TO DETERMINE THERMAL CONDUCTIVITY OF A GOOD CONDUCTOR (e.g. metals).

The thermal conductivity of a good conductor can be determined by using SEARLE'S METHOD. The experiment is based on the equation $Q = \dfrac{KA(\theta_2-\theta_1)t}{L}$ for this the following conditions must be satisfied:-

(i) Measurable amount of heat must flow through the bar per second

(ii) The temperature gradient along the conductor must be steep enough so that it can be accurately measured.

Description of apparatus of searle's method

The searle's apparatus consists of:-

1. Long bar compared to its diameter in order to insure that sufficient temperature difference exist between its end in order to be accurately measured.
2. The bar is long and have small value of cross – sectional area, A in order to make the temperature gradient big at the same time giving satisfactory rate of heat flows.
3. The bar is lagged with felt material in order to prevent heat lost from the bar to the surrounding.

DESCRIPTION

The specimen bar is heated from one end using a steam chamber (jacket) and cooled at the other end by circulating water. The water leaving at B is warmer than that coming in at A, so that the temperature fall continuously along the bar; if water come in at B and out at A it would tend to reverse the temperature gradient at the end of the bar and might upset it as far as D or C. When the apparatus has been running for some time a steady state condition is attained when:-

(i) The rate of heat flows along the bar is constant for all section since the bar is lagged.

Theory

In the steady state condition, the rate of heat flows along the bar

$$\frac{Q}{t} = \frac{KA(\theta_2 - \theta_1)}{L}$$

$$Q = \frac{KA(\theta_2 - \theta_1)}{L} \text{.........(1)}$$

The amount of heat absorbed by circulating water after time, t

$$Q = MC(\theta_4 - \theta_3) \text{..........(2)}$$

C = Specific heat capacity of water

M = Mass of water collected in time, t

(1) = (2)

$$\frac{KA(\theta_2 - \theta_1)t}{L} = MC(\theta_4 - \theta_3)$$

$$K = \frac{MCL(\theta_4 - \theta_3)}{A(\theta_2 - \theta_1)t}$$

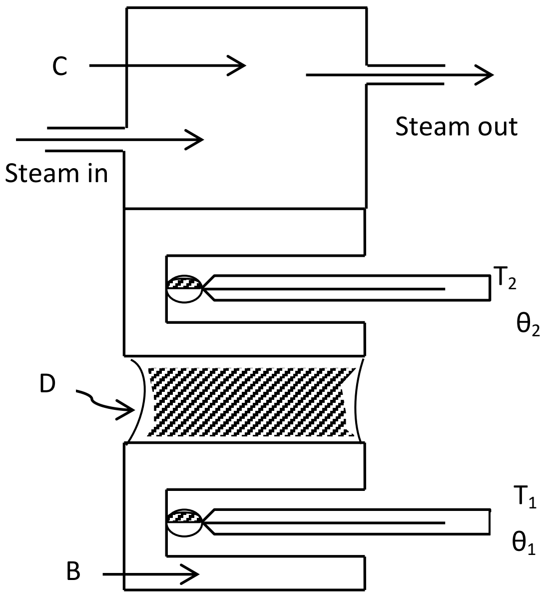
Source of Error Precautions

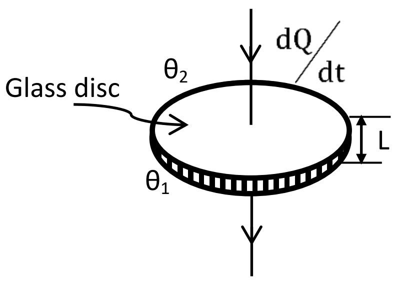
1. The readings of the thermometer should b taken only after they indicate steady temperature
2. The rate of water should be uniform and slow.
3. The rod must be perfectly lagged.
4. The difference between the initial readings of the various thermometers must be noted.

DETERMINATION OF THERMAL CONDUCTIVITY OF POOR CONDUCTOR eg. Glass.

The thermal conductivity of poor conductor such as a glass can be determined by using **LEE'S DISC METHOD** i.e by using thin disc material.

1. The disc is thin to ensure sufficient heat flows through it to be measurable.
2. The area A of cross – section and thickness L is small this shape help to reduce heat loss to the surrounding from the specimen.
3. No lagging is needed since little heat lost through the sides of the disc in comparison with its face.**Description**





The specimen is made in form of a disc. The disc D is placed on a thick brass slab B containing a thermometer T_1 and is heated from above by a steam chest C whose thick base also carries thermometers T_2. The experiment has two main parts:-

PART 1:

Steam is passed until the temperature θ_1 and θ_2 are under steady state. At steady state, the rate of heat flowing through D and B will be the same. The conducted through D is equal to the heat radiated from the side and bottom surface of the disc B. Rate of heat flows through the disc

$$\frac{dQ}{dt} = \frac{KA(\theta_2 - \theta_1)}{L} = \frac{K\pi R^2(\theta_2 - \theta_1)}{L}$$

R = radius of the disc

L = thickness of the disc

K = thermal conductivity of the sisc.

PART 2

The specimen disc D is removed and steam chest C is kept directly on the disc B and heated, till its temperature rises by 5^{O}C or 10^{O}C above θ_1. The steam chest is removed and disc B is suspended separately and allowed to cool. Its temperature is noted at regular interval say half a minute until it falls to about 5^{O}C or 10^{O}C below.

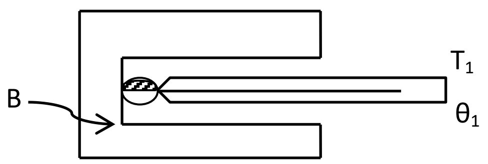
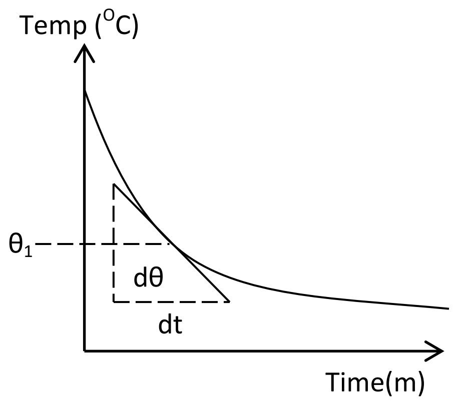


Table of result

Time e(min)	Temp (^{O}C)



The rate of temperature fall of B at θ_1 is equal to the slope of tangent at $\theta 1$

$$\text{Slope} = \frac{dQ}{dt} = \frac{b}{a}$$

The rate of heat loss by radiation from B

$$\frac{dQ}{dt} = MC\frac{d\theta}{dt} \ldots\ldots(2)$$

M = Mass of slab or disc B

C = Specific heat capacity

(1) = (2)

$$\frac{KA(\theta_2 - \theta_1)}{L} = MC\frac{d\theta}{dt}$$

$$K = \frac{MCL\,{}^{d\theta}\!/_{dt}}{A(\theta_2 - \theta_1)} = \frac{MCL\,{}^{d\theta}\!/_{dt}}{\pi R^2(\theta_2 - \theta_1)}$$

SOLVE EXAMPLES

Example 1 : NECTA 2019/P1/7

(a) (i) Why water is preferred as a cooling agent in many automobile engines? (02 marks)

(ii) A thermometer has wrong calibrations it reads the melting point of ice as -10^{o}C. If it reads 40^{o}C in a place where the temperature reads 30^{o}C, determine the boiling point of water on this scale (03 marks)

(b) (i) Analyse three practice applications of solids in day life situations. (03 marks)

(ii) A closed metal vessel containing water at 750^{o}C, has a surface area of 0.5m^2 and uniform thickness of 4.0mm. If its outside temperature is 15^{o}C, calculate the heat loss per minute by conduction (02 marks)

Solution

(a) (i) Water has a high value of specific heat capacity. It means a relatively small amount of water will absorb a large amount of heat for a correspondingly (Q = MCDT) small temperature rise because of this, water is a very useful cooling agent and is used in the cooling system of automobiles and other engines. If a liquid of low specific heat were used in cooling system, its temperature would rise higher for a comparable absorption of heat.

(ii) Refer to your notes

(b) (i)
- While laying the railways tracks, a small gap is left between the successive lengths of the rails. This gap provide to allow for the expansion rails during summer.
- In bridge, one end is rigidly fastered to its abutment while the other rests on the rollers. This provision allows the expansion and contraction to take place during changes in temperature.
- The fact that a solid expands on heating and contracts on cooling in revelting.
- The concrete roads and floors are always made in sections and enough space is provided between the sections. This provision allows expansion and contraction to take place due to the change in temperature.

(ii) The quantity of heat flow per minute

$$Q = \frac{KA(\theta_1 - \theta_2)t}{X}$$

$$Q = \frac{400 \times 0.5 \times (75 - 15) \times 60}{4.0 \times 10^{-3}}$$

$$Q = 1.80 \times 10^8 \text{J}$$

Example 2

(a) Animal in the forest find shelter from cold holes in the snow. Why?

(b) A person walking during a dry atmosphere at 40°C will sweat about 1litre per hour. Assume that all water in sweat evaporates during that time. Calculate the rate of heat loss from body of the person $L = 540 \times 4.2 \times 10^3 \text{Jkg}^{-1}$.

Solution

(a) Snow has trapped air (in the ice there is no air) which acts as a heat insulator. Therefore the snow prevents the transmission of heat from the animal to the outside.

(b) $$\frac{dQ}{dt} = L\frac{dm}{dt} = \rho L\frac{dv}{dt}$$

$$= \frac{1000 \times 540 \times 42 \times 10^3 \times 10^{-3}}{3600}$$

$$\frac{dQ}{dt} = 630\text{W}$$

Example 3: NECTA 2002/P1/6

(a) (i) The thermal conductivity β of a substance may be defined by the equation $\frac{dQ}{dt} = -\beta A\frac{d\theta}{dx}$ identify briefly each term in this equation and explain minus sign.

(ii) Describe briefly one method of measuring the thermal conductivity of a bad conductor in the form of a disc.

(b) On end of a lagged bar copper rod is placed in a steam chest and 0.6kg mass of copper is attached to the other end of the rod which has an area of 2cm². When the steam is at 100°C is passed into the chest and a steady state is reached the temperature of mass of copper is rises by 4oc per minute. If the temperature of the surrounding is 15°C. Calculate the length of the rod. Given that the thermal conductivity of copper = $3.6\text{Wcm}^{-1}{}^{o}\text{c}^{-1}$. Specific heat capacity of copper rod is $4.0 \times 10^2 \text{Jkg}^{-1}\text{k}^{-1}$

Solution

(a) (i) Given that

$$\frac{dQ}{dt} = -\beta A\frac{d\theta}{dx}$$

Where β = thermal conductivity

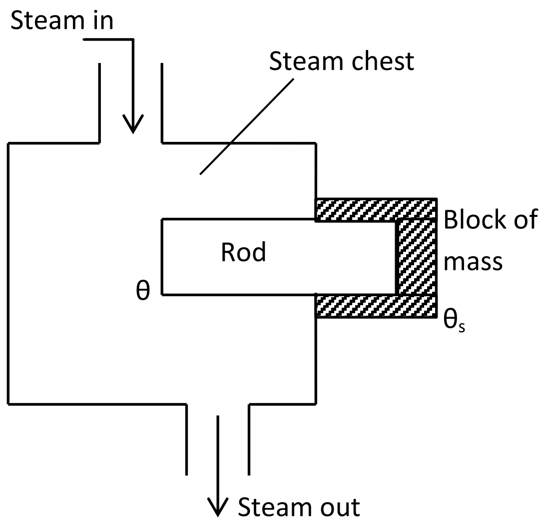
A = Cross – sectional area

$\frac{d\theta}{dx}$ = temperature gradient

$\frac{dQ}{dt}$ = Rate of heat flows minus sign show that temperature decreases as the length of bar increases from hot end

(ii) Refer to your notes

(b) Diagram



Rate of heat flows Through copper rod = Rate of heat gained by block of mass M attached at the end of rod.

$$\frac{KA(\theta-\theta_S)}{L} = MC\frac{d\theta}{dt}$$

$$L = \frac{KA(\theta-\theta_S)}{MC\,{}^{d\theta}\!/_{dt}}$$

$$\frac{d\theta}{dt} = 4^\circ C_{min} = 0.0667^\circ Cs^{-1}$$

$$L = \frac{360\times2\times10^{-4}(100-15)}{0.6\times400\times0.0667}$$

$$L = 0.3825m = 38.25cm$$

Example 4

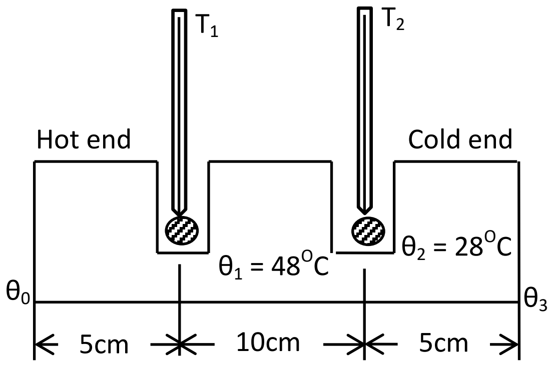
In an experiment for measuring the thermal conductivity of copper, heat is supplied at a rate of 80W to one end of well lagged uniform copper bar of cross – sectional area 10cm² and total length 20cm. The heat is removed by water cooling at the other end of the bar. Two thermometers T_1 and T_2 are used to record the temperatures within the bar at distances of 5cm and 15cm from the hot end of the bars. These are 48°C and 28°C respectively.

(a) Calculate the value of thermal conductivity of copper.

(b) Estimate the rate of flow (in gramm per minute) of cooling water sufficient of the temperature rise not exceed 5°C.

(c) Estimate the temperature at the end of the bar. Specific heat capacity of water = $4200Jkg^{-1}k^{-1}$.

Solution



(a) Rate of heat flows through copper

$$\frac{dQ}{dt} = \frac{KA(\theta_1-\theta_2)}{L}$$

$$80 = \frac{K\times10\times10^{-4}(48-28)}{10\times10^{-2}}$$

$$K = 400Wm^{-1}K^{-1}$$

(b) Since $\frac{dQ}{dt} = C\theta\frac{dm}{dt}$

$$\frac{dm}{dt} = \frac{{}^{dQ}\!/_{dt}}{C\theta}$$

$$= \frac{80}{4200\times5}kg/sec$$

$$= \frac{80\times10^{-3}}{4200\times5\times{}^{1}\!/_{60}}g/min$$

$$\frac{dm}{dt} = 228.57g\,min^{-1}$$

(c) **Method 1**

$$\frac{dQ}{dt} = \frac{KA(\theta_2-\theta_3)}{L}$$

$$80 = \frac{400\times10\times10^{-4}(28-\theta_3)}{5\times10^{-2}}$$

$$\theta_3 = 18^\circ C$$

Method 2:

We can use the concept of temperature gradient.

$$\frac{\theta_1-\theta_2}{10} = \frac{\theta_2-\theta_3}{5}$$
$$\frac{48-28}{10} = \frac{28-\theta_3}{5}$$
$$\theta_3 = 18^\circ C$$

Example 5

(a) In the determination of thermal conductivity of a poor conductor such as cardboard or ebonite, the substance is made thin and faily of large surface area. Explain why this is so?

(b) A circular disc of glass 3mm thick and 110mm in diameter is placed between two metal disc A and B. The temperature of lower disc B becomes constant at 93°C and the temperature of A at 96.5°C when the steam is passed. When B is warmed about 93°C with the glass disc on it and cooling curve obtained the rate of cooling at 93°C is found to be 0.042Ks^{-1}. Calculate the thermal conductivity of glass if the mass of B is 0.94Kg and specific heat capacity is 400JKg^{-1}K^{-1}.

Solution

(a) This shape helps to reduce the heat to the surroundings from the specimen and is made thin in order to insure sufficient heat flow through to be measurable.

(b) Since rate of heat flows through the disc is equal to the rate of heat loss to the surrounding by B at temperature $\theta_1 = 93^\circ$C.

$$\frac{KA(\theta_2-\theta_1)}{L} = MC\frac{d\theta}{dt}$$
$$K = \frac{4MCL\,{}^{d\theta}/_{dt}}{\pi d^2(\theta_2-\theta_1)}$$
$$= \frac{4\times0.94\times400\times3\times10^{-3}\times0.042}{3.14\times(110\times10^{-3})^2\times(96.5-93)}$$
$$K = 1.42\text{Wm}^{-1}\text{k}^{-1}$$

Example 6

In Lee's disc experiment two discs are separated by gap of thickness 5mm. the space between the discs contains a gas of thermal conductivity 3.88 × 10^{-5}Wm^{-1}K^{-1}. At the steady state the temperature, of the two sides of the discs are 368K and 333K. If the area of cross – section of the slab is 25cm^2, calculate the quantity of heat crossing the gas per second.

Solution

Since $Q = \dfrac{KA(\theta_1-\theta_2)t}{X}$

But $\theta_1-\theta_2 = 368-333 = 35\text{k}$

$$Q = \frac{3.88\times10^{-5}\times25\times10^{-4}\times35\times1}{5\times10^{-3}}$$
$$Q = 6.79\times10^{-4}\text{J}$$

Example 7

In an experiment to determine the thermal conductivity of copper with searles's apparatus, 100gm of water flowed in 4.2minutes past the cold end of the copper bar, its initial and final temperatures being 25°c and 52°C respectively. If the temperature difference between two points in the bar 10cm apart is 23°C and the area of cross – section of the bar is 5cm^2, calculate the coefficient of thermal conductivity of copper. Specific heat capacity of water 4,200Jkg^{-1}K^{-1}.

Solution

Heat gained by water

Q = MCdT

= 100 × 10-3 × 4200 × (52 – 25)

Q = 11340J

Thermal conductivity of copper

$$K = \frac{Qd}{A(\theta_1-\theta_2)t}$$
$$= \frac{11340\times0.10}{5\times10^{-4}\times23\times252}$$
$$K = 292.3\text{Wm}^{-1}\text{k}^{-1}$$

Example 8

(a) Draw a well labeled diagram of a searle's method of measuring thermal conductivity of a good conductor. Why is the metal bar made thick and long? In the experiment the temperature at two points 0.1m apart of the bar are 72°C and 60°C respectively. If the bar diameter is 0.08m, the mass of water collected is 30csec is 0.24kg and its temperature rise is 6.9°C, find thermal conductivity of bar. Specific heat capacity of water = $4200JKg^{-1}K^{-1}$.

(b) A cylindrical bar of iron of length 10cm is joined to a cylindrical bar of copper of length 15cm and the same cross – sectional area, thus making a composite bar. If the ends of the iron and copper bars are maintained at 100°C and 10°C respectively. Calculate the temperature of junction of the bars. Thermal conductivity of iron and copper = 50and $380Wm^{-1}K^{-1}$ respectively.

Solution

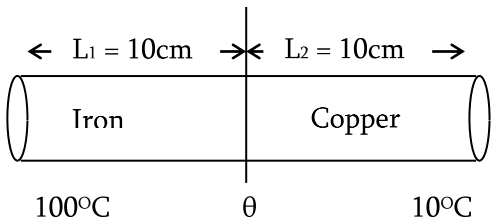
(a) The metal bar is made thick so as allow a measurable quantity of heat to flow through the bar and it is long to achieve sufficient temperature different across its ends.

$$K = \frac{MCL(\theta_3 - \theta_4)}{At(\theta_1 - \theta_2)}$$

$$= \frac{0.24 \times 4200 \times 0.1 \times 6.4}{\frac{\pi \times (0.08)^2}{4} \times (72 - 60)}$$

$$K = 384.4Wm^{-1}K^{-1}$$

(b) Let θ = temperature of the junction.



Under steady state condition

$$\left(\frac{dQ}{dt}\right)_{Iron} = \left(\frac{dQ}{dt}\right)_{Copper}$$

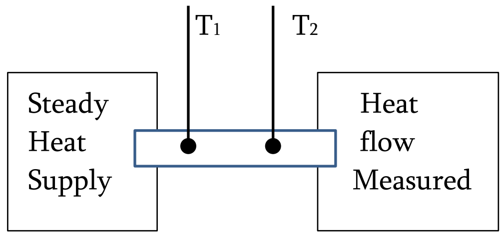
$$\frac{50A(100-\theta)}{10} = \frac{380A(\theta-10)}{15}$$

On solving θ = 25°C

Example 9

(i) Define thermal conductivity

(ii) Figure below represents, in outline, the apparatus used in searle's bar method for determining the thermal conductivity of copper.



(a) Why is a thick bar used in this determination?

(b) Why must it be insulated except at its tow ends?

(c) Why does one wait for some time before taking readings?

(d) Does it matter where thermometer T_1 and T_2 are placed along the bar?

(e) One end of the insulated copper bar, which is of 0.2m and cross – sectional are $1.2 \times 10^{-3}m^2$, is maintained at a steady temperature by an electric heater which is supplying heat to the bar at T_1 is 0.06m from the hot end and thermometer T_2 is cool end water flows into a circulating coil at 15.3°C and leaves at 16.7°C. Taking thermo conductivity of copper to be $400Wm^{-1}K^{-1}$ and specific heat capacity of water $4200JKg^{-1}K^{-1}$ estimate the rate at which water is flowing through the circulating coil and also the reading of each of the thermometers' T_1 and T_2.

Solution

(i) Refer to your notes

(ii) (a) In order to allow a measurable quantity of heat to flow through it.

(b) This is in order to prevent heat lost from the bar to the surrounding. At the ends the heat is allowed to escape so that steady state can be attained. If heat were prevented to escape at the ends, the heat of the bar would accumulate and temperature at each end point on the bar would be increasing with time.

(c) A time must be elapse to achieve steady state conditions so one must wait for steady state to be achieved.

(d) Yes it matter. Thermometers are placed at a large separation so that a large temperature difference is recorded this minimuize the errors of the thermometers and also enable a measureable temperature gradient to be obtained.

(e) Rate of heat gained by water

$$P = \frac{dQ}{dt} = C(\theta_3 - \theta_4)\frac{dm}{dt}$$

$$\frac{dm}{dt} = \frac{P}{C(\theta_3 - \theta_4)}$$

$$= \frac{100}{4200(16.7 - 15.3)}$$

$$\frac{dm}{dt} = 0.017kgs^{-1} = 17gs^{-1}$$

The rate of heat flows along the bar

$$P = \frac{KA(\theta_1 - \theta_2)}{0.14 - 0.06} = \frac{KA(\theta_2 - 16.7)}{0.2 - 0.14}$$

$$\theta_2 = \frac{P(0.2 - 0.14) + 16.7}{KA}$$

$$\theta_2 = 29.2^0C$$

Again

$$\theta_1 = \frac{P(0.14 - 0.06)}{KA} + \theta_2$$

$$\theta_1 = 45.9^oC$$

Example 10

(a) Define thermal conductivity and state a unit in which is expressed.

(b) Explain why in an experiment to determine thermal conductivity of copper using a Searle's arrangement, it is necessary.
 (i) That the bar should be thick, of uniform cross – section and have its sides well lagged.
 (ii) That the temperatures used in the calculation should be steady values finally registered by the thermometers.

(c) Straight metal bars X and Y of circular section and equal in length are joined end to end. The thermal conductivity of the material of X is twice that of the material Y and the uniform diameter of X is twice of Y. The exposed ends of X and Y are maintained at 100°C and 0°C respectively and the sides of the bars are ideally lagged. Ignoring the distortion of the heat flow at the junction, sketch a graph to illustrate how the temperature varies between the ends of the composite bar when conditions are steady. Explain and calculate the steady temperature of the junction.

Solution

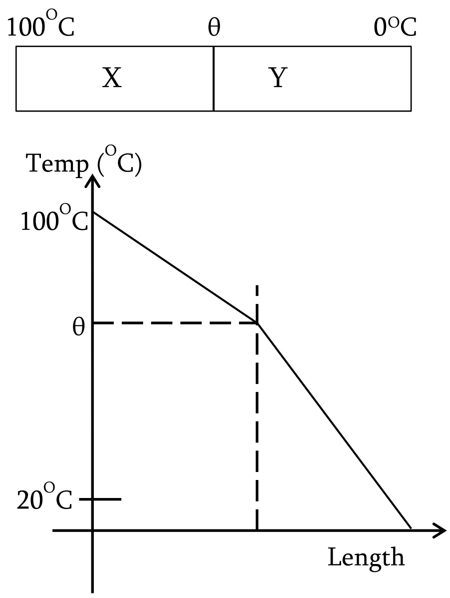
(a) Refer to your notes

(b) (i) The bar is lagged to prevent the loss of heat through its sides and it is made thick so that enough quantity of heat flows through it.
 (ii) This is to ensure that the rate flow of heat through every section of the bars is constant i.e steady state have been reached.

(c) Since temperature gradient

$$g \; \alpha \; \frac{1}{KA} \; , \; A = \frac{\pi d^2}{4}$$

Therefore, g is inversely proportional to the product of the thermal conductivity and square of diameter of the bar, so the temperature gradient of X will be lower than that of Y due to its high thermal conductivity. The graph can be shown on the figure below.



Let θ be temperature at the junction.

$$K_X A_X \left[\frac{100-\theta}{L}\right] = K_Y A_Y \left(\frac{\theta - 0}{L}\right)$$

$$2K_Y (2dy)^2 (100-\theta) = Kyd_y^2\theta$$

$$\theta = \frac{800}{9} = 89^\circ C \text{ (approx)}$$

Example 11

(a) Given major similarity and one major difference between heat conduction and wave propagation.

(b) The wall of container used for keeping objects cool consists of two thickness of wool 0.5cm thick separated by a space 1cm wide packed with a poorly conducting material. Calculate the rate of flow of heat per unit area into the container if the temperature difference between the internal and external surface is 20°C thermal conductivity of wood = $0.0024 Wcm^{-1}K^{-1}$; of the poorly conducting material = $0.00024 Wcm^{-1}K^{-1}$.

Solution

(a) Similarity heat conduction and wave propagation.

- Both heat conduction and wave propagation involve the transfer of energy.
- Both are due to vibrations of molecules (atoms) of the media.

Difference between heat conduction and wave propagation.

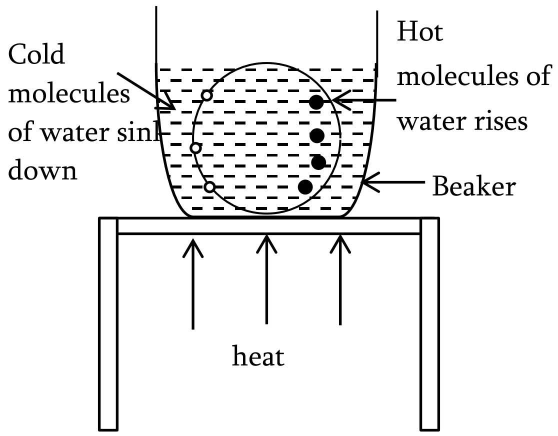
- Heat conduction is affected by temperature gradient across the sides of the media whereas wave propagation is affected by temperature of the medium (i.e elasticity and density of the medium)

(b)

2. THERMAL CONVECTION

Thermal convection is the process of heat flow or transfer by the actual deposition of hot particles through the fluid forming convectional current i.e convection is the process by which heat is transferred through a fluid from one point to another due to the movement of fluid particles (molecules). The movement of the material in thermal convection is due to the difference in densities of the hotter and colder parts.

DEMONSTRATION OF CONVECTION



A beaker of water is heated from underneath. When the water above the flame heated up and expand its density decreases and so density of hot molecules of water is less than the density of cold molecules of water tend to rise up while to sink down. The movement of water molecules forming the convection currents. This phenomena also can be explained by the kinetic theory of gases.

TYPES OF THERMAL CONVECTION

There are two types of thermal convection:

1. Natural (free) convection
2. Forced convection.

THERMINOLOGIES APPLIED TO THE THERMAL CONVECTION.

1. **Natural/free convection** – Is the kind of thermal convection in which the heated material flow due to the difference in the density or pressure. This takes place without any forced agent applied on the heated materials.
   Examples:
   (i) Movement of air molecules around a fire.
   (ii) Land breeze and sea breeze.
   (iii) Air flow at a beach is an example of natural convection, as is the mixing that occurs as surface water in an ocean cools and sinks.
   (iv) Convection currents in a heated water i.e boiling of water.

2. **Force convection** – is the kind of thermal convection in which the heated materials flows due to the applied forced due to the applied forced agent on it i.e are forced to move by means of pump or blower.
   Examples
   (i) Boiling of the wind on a cap of a tea
   (ii) In some hot air and hot water boiling systems forced convection is much quicker ways of loosing heat than that of the natural convection.

3. **Rate of loss of heat** – is the amount of heat loss per unit time from a body to the surrounding. It can be denoted as dQ/dt. S.I. Unit of rate of heat loss is Watt(Js^{-1}).

4. **Excess temperature** – is the difference between the temperature of hot body and the temperature of the surrounding. Excess temperature, $d\theta = \theta - \theta s$

5. **Laws of cooling (law of thermal convection)** there are two laws of cooling.
   (i) Newton's law of cooling
   (ii) Five – fourth law of cooling or Dulong – petit law.

NEWTON'S LAW OF COOLING

State that 'Under the condition of forced convection, the rate of heat lost by a body is directly proportional to the excess temperature over the surrounding.

Analytical treatment of Newton's law of cooling

Let M be the mass of hot body whose its temperature θ at any time, t, θs is the temperature of the surrounding and C is the specific heat capacity of the body. Let dθ be decrease in temperature of body in the small interval of time, dt i.e $-\frac{dQ}{dt} \propto \theta - \theta_s$

$$\frac{dQ}{dt} = -K(\theta - \theta_S)$$

Negative sign indicates that the body is losing heat K is a constant of proportionality depends on the area and nature of the surface of the body.

K = es

e = conventional coefficient

s = surface area

$$-\frac{dQ}{dt} = es(\theta - \theta_s)$$

Convectional coefficient or emissive power of the surface of the body is the rate of loss of heat by the body per unit surface area per unit excess temperature. S.I.Unit of e is $Wm^{-2}K^{-1}$. The rate of heat loss by the body is given by

$$\frac{dQ}{dt} = MC\frac{d\theta}{dt} \text{.........(ii)}$$

(i) = (ii)

$$-es(\theta - \theta_s) = MC\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = -\frac{es}{MC}(\theta - \theta_S)$$

Let $\lambda = \frac{es}{MC} = \frac{K}{MC}$

λ is a constant

$$\frac{d\theta}{dt} = -\lambda(\theta - \theta_s)$$

An alternative statement of Newton's law of cooling state that 'The rate of fall of temperature (cooling) of an object is directly proportional to the excess temperature over the surrounding under forced convection.

Expression of the temperature of body at any time, t

$$\frac{d\theta}{\theta-\theta_s} = -\lambda dt$$

$$\int \frac{d\theta}{\theta-\theta_s} = -\lambda \int dt$$

$$\log_e(\theta-\theta_s) = -\lambda t + c$$

In exponential form

$$\theta-\theta_s = e^{-\lambda t+c}$$

$$\theta-\theta_s = Ae^{-\lambda t}$$

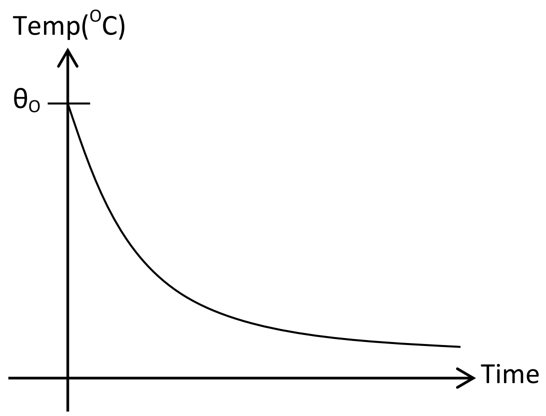
$$\theta = \theta_s + Ae^{-\lambda t}$$

When t = 0, $\theta = \theta o$

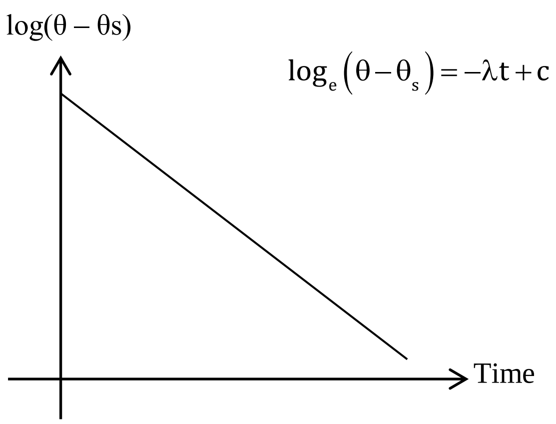
$$\theta_o - \theta_s = A$$

$$\theta = \theta_S + (\theta_0 - \theta_S)e^{-\lambda t} \text{ or } \theta = \theta_s(1-e^{-\lambda t}) + \theta_0 e^{-\lambda t}$$

GRAPH OF TEMPERATURE AGAINST TIME, t



GRAPH OF $\log_e(\theta-\theta_s)$ AGAINST TIME, t



VALIDITY OF NEWTON'S LAW OF COOLING

1. Under forced convection, the law is valid under all excess temperature over the surrounding.
2. Under natural or free convection, the law is valid under small temperature range of the excess temperature
   i.e $20^0C \le \theta-\theta_S \le 30°C$ or $20K \le \theta-\theta_S \le 30K$

2. DULONG PETIT OR FIVE FOURTH POWER LAW.

State that 'Under the condition of natural convection, the rate of heat loss by a body is directly proportional to the five fourth power of the excess temperature over the surrounding provide that the excess temperature is not less due to the modification of the Newton's law of cooling to be valid under all excess temperature and this experiment have been done by the **DULONG** and **PETIT**.

Validity of Dulong – Petit law

This law appears to be true for higher excess temperature such as from 50K to 300K

Expression of rate of heat loss under forced convection

$$\frac{dQ}{dt} = -K(\theta-\theta_S)$$

Expression of rate of heat loss under natural convection.

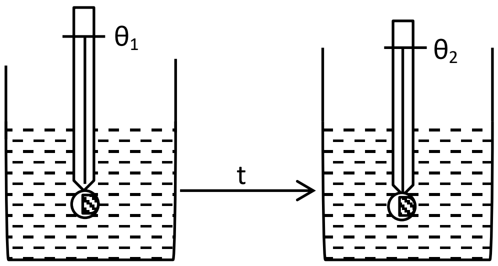
$$\frac{dQ}{dt} = -K(\theta-\theta_S)^{5/4}$$

FACTORS AFFECTING THERMAL CONVECTION.

1. Excess temperature
2. Medium in which convection takes place.
3. Surface area of the body
4. The volume of cooling body
5. Nature of the surface enclosure.

Additional concepts

1. Let us now calculate the time taken t by the body to cool from the temperature θ_1 to θ_2.



According to the Newton's law of cooling

$$\frac{d\theta}{\theta-\theta_s} = -Kdt$$

$$\int_{\theta_1}^{\theta_2}\frac{d\theta}{\theta-\theta_s} = -K\int_0^t dt$$

$$\left[\log_e(\theta-\theta_s)\right]_{\theta_1}^{\theta_2} = -kt$$

$$\log_e(\theta_2-\theta_s)-\log_e(\theta_1-\theta_s) = -kt$$

$$kt = \log_e\left(\frac{\theta_1-\theta_s}{\theta_2-\theta_s}\right)$$

$$t = \frac{1}{k}\log_e\left(\frac{\theta_1-\theta_s}{\theta_2-\theta_s}\right)$$

2. **HEAT LOSS AND TEMPERATURE FALLS**

The rate of heat loss from the body depends on the following factors:-

(i) Excess temperature i.e $\frac{dQ}{dt}\alpha-(\theta-\theta_s)$

(ii) The exposed surface area, S of the calorimeter (body) i.e $\frac{dQ}{dt}\alpha\ S$

(iii) Nature of the surface of the body (e). for example a dull surface losses heat a little faster than a shiny body because it is a better radiator Generally, the rate of heat loss is given by

$$\frac{dQ}{dt} = -ks(\theta-\theta_s)\text{........(i)}$$

Also $\frac{dQ}{dt} = MC\frac{d\theta}{dt}$.......(ii)

(i) = (ii)

$$MC\frac{d\theta}{dt} = -ks(\theta-\theta_s)$$

$$\frac{d\theta}{dt} = \frac{-ks}{MC}(\theta-\theta_S) = \frac{-KS}{\rho CV}(\theta-\theta_S)$$

From the equation above the following points may be noted:

I. The rate of heat loss is directly proportional to the rate of temperature falls i.e $\frac{dQ}{dt}\ \alpha\ \frac{d\theta}{dt}$

II. The rate of heat loss is directly proportional to the surface area of the body i.e $\frac{dQ}{dt}\ \alpha\ S$

III. The rate of temperature fall is directly proportional to the excess temperature over the surrounding i.e $\frac{d\theta}{dt}\ \alpha\ (\theta-\theta_s)$

comparison of the rate of temperature falls of the same body at the different temperature i.e θ_1 and θ_2.

$$\left(\frac{d\theta}{dt}\right)_2 = \left(\frac{d\theta}{dt}\right)_1\left[\frac{\theta_2-\theta_s}{\theta_1-\theta_s}\right]$$

IV. The rate of temperature falls is directly proportional to the ratio of surface area to the volume of the body i.e $\frac{d\theta}{dt}\ \alpha\ \frac{S}{V}$

For the spherical body

$$S = 4\pi R^2\ ,\ V = \frac{4}{3}\pi R^3$$

$$\frac{S}{V} = \frac{4\pi R^2}{\frac{4}{3}\pi R^3} = \frac{3}{R}$$

$$\frac{d\theta}{dt}\ \alpha\ \frac{S}{V}\ \alpha\ \frac{1}{R}$$

On comparing the rate of temperature falls for the two spheres having different radii

$$\left(\frac{d\theta}{dt}\right)_2 \Big/ \left(\frac{d\theta}{dt}\right)_1 = \frac{R_1}{R_2}$$

If the bodies have the same nature of the surface, then the rate of temperature falls is inversely proportional to the linear dimensions.

Daily examples

(i) Small body cool faster than the large body.

(ii) A tiny body should be more the roughly wrapped up than a grown man.

(iii) A small coal which falls out of the fire can picked up sooner than the one

V. From the equation

$$\frac{d\theta}{dt} = -\frac{KS}{MC}(\theta-\theta_S)$$

Then $\frac{d\theta}{dt} \alpha \frac{1}{M}$, $\frac{d\theta}{dt} \alpha \frac{1}{C}$

VI. Average method under Newton's law of cooling. If the temperature of body changes from θ_1 and θ_2 in time interval dt and θs is the temperature of the surrounding. Mean temperature of the body

$$\theta = \frac{\theta_1+\theta_2}{2}$$

According to the Newton's law of cooling

$$\frac{d\theta}{dt} = -\frac{KS}{MC}(\theta-\theta_S)$$

$$\frac{d\theta}{dt} = -\frac{KS}{MC}\left[\frac{\theta_1+\theta_2}{2}-\theta_S\right] \text{ or}$$

$$\frac{\theta_1-\theta_2}{t} = -\frac{KS}{MC}\left[\frac{\theta_1+\theta_2}{2}-\theta_S\right]$$

This relation can be used for problem on Newton's law of cooling provided the temperature difference between the body and surrounding is small.

FACTORS AFFECTING CONVECTIONAL COEFFICIENT.

Unlike thermal conductivity, the convectional coefficient K or e is not a property of a solid, fluid but depends upon on many parameters:-

1. Varies with geometry of solid and its surface area.
2. The velocity of fluid
3. Thermal conductivity
4. Density of the fluid
5. Difference in temperature and pressure of the fluid.

SOLVED EXAMPLE

Example 1

(a) Write down an equation which represents the rate of the cooling of the body through
    (i) Forced convection
    (ii) Natural convection
    Define the symbols used in your equation.

(b) A body cools from 80°C to 50°C in 5minutes. Calculate the time it takes to cool from 60°C to 30°C. The temperature of the surrounding is 20°C.

Solution

(a) Refer to your notes.

(b) By using the equation

$$\log_e\left(\frac{\theta_2-\theta_s}{\theta_1-\theta_s}\right) = -kt$$

When a body cools for the first 5minutes

$$\log_e\left[\frac{50-20}{80-20}\right] = -5k\,\text{min}$$

$$\log_e\left(\tfrac{3}{6}\right) = -5\text{min}\times k \;\ldots\ldots\text{(i)}$$

When a body cools for next time, t

$$\log_e\left(\frac{30-20}{60-20}\right) = -kt$$

$$\log_e\left(\tfrac{1}{4}\right) = -kt \;\ldots\ldots\ldots\text{(ii)}$$

(ii)/(i)

$$\frac{-kt}{5\text{min}\times k} = \frac{\log_e\left(\tfrac{1}{4}\right)}{\log_e\left(\tfrac{3}{6}\right)}$$

$$t = 10\,\text{minutes}$$

∴ The time taken by the body to cool from 60°C to 30°C is 10minutes.

Example 2

A body cools in 7minutes from 60°C to 40°C. What will be its temperature after the next 7minutes? The temperature of the surrounding is 10°C. Assume Newton's law of cooling holds throughout the process.

Solution

Using the relation

$$\log_e\left(\frac{\theta_2-\theta_s}{\theta_1-\theta_s}\right) = -kt$$

When a body cools for the first 7 minutes

$$\log_e\left[\frac{40-10}{60-10}\right] = -7\min\times k$$

$$\log_e\left(3/4\right) = -7\min\times k\,........(i)$$

When a body cools for the next 7 minute

$$\log_e\left(\frac{\theta-10}{40-10}\right) = -7\min\times k\,.......(ii)$$

On solving equation (i) and (ii) $\theta = 28^{O}C$

$\therefore$ The temperature of the body after the next 7 minutes is $28^{O}C$.

Example 3

A liquid cools from $70^{O}C$ to $60^{O}C$ in 5minutes. Calculate the time taken by the liquid to cool from $60^{O}C$ to $50^{O}C$, If the temperature of the surrounding is constant at $30^{O}C$.

Solution

Applying Newton's law of cooling under average method.

$$\frac{d\theta}{dt} = \frac{\theta_1-\theta_2}{t} = -k\left[\frac{\theta_1+\theta_2-\theta_s}{2}\right]$$

For the first stage of cooling

$$\frac{70-60}{2} = -k\left[\frac{70+60-30}{2}\right]$$

$$2^{\circ}C\min^{-1} = -35^{0}C\times K\,........(i)$$

For the second stage of cooling

$$\frac{60-50}{t} = -k\left[\frac{60+50-30}{2}\right]$$

$$\frac{10^{\circ}C}{t} = -25^{\circ}C\times t\,......(ii)$$

Take (i) / (ii)

$$\frac{2}{10/t} = \frac{35}{25}$$

$$t = 7\text{minutes}$$

$\therefore$ The time taken by the body to cool from $60^{O}C$ to $50^{O}C$ is 7minutes.

Example 4

A body initially at $80^{O}C$ cools to $64^{O}C$ in 5minutes and to $52^{O}C$ in 10minutes. Calculate the temperature of the surroundings. Also find the temperature after 15minutes.

Solution

In the first stage of cooling

$$\frac{\theta_1-\theta_2}{t} = -K\left[\frac{\theta_1+\theta_2}{2}-\theta_S\right]$$

$$\frac{80-64}{5} = -K\left[\frac{80+64}{2}-\theta_S\right]$$

$$\frac{16}{5} = -K\left[72-\theta_S\right].......(ii)$$

In the second stage of cooling

$$\frac{64-52}{5} = -K\left[\frac{64+52}{2}-\theta_S\right]$$

On solving equation (i) and (ii)

$$\theta_s = 16^{\circ}C$$

$\therefore$ Temperature of the surrounding, $\theta_s = 16^{\circ}C$.

In the third stage of cooling.

$$\frac{52-\theta}{15} = -K\left[\frac{52+\theta}{2}-16\right]......(iii)$$

On dividing equation (iii) by (i)

$\theta = 43^{O}C$.

Example 5

A liquid takes 5minutes to cools from $80^{O}C$ to $50^{O}C$. How much time will take to cools from $60^{O}C$ to $30^{O}C$? The temperature of the surrounding is $20^{O}C$.

Solution

According to the Newton's law of cooling under average method.

$$\frac{d\theta}{dt} = \frac{\theta_1-\theta_2}{t} = -K\left[\frac{\theta_1+\theta_2}{2}-\theta_S\right]$$

In the first stage of cooling

$$\frac{80-50}{5} = -K\left[\frac{80+50}{2}-20\right]$$

$$6^{0}C\min^{-1} = K\times45^{0}C\,......(i)$$

In the second stage of cooling

$$\frac{60-30}{t} = -K\left[\frac{60+30}{2}-20\right]$$

$$\frac{30}{t} = -K \times 25^0 C \ldots\ldots(ii)$$

On dividing equation (i) by (ii)

t = 9minutes

Example 6

A body in constant temperature enclosure cools from 90°C to 80°C in 10minutes and from 80°C to 70°C in 12minutes. What will be its temperature after another 15minutes.

Solution

Let θs be the temperature of the surrounding.

According to the Newton's law of cooling.

$$\frac{d\theta}{dt} = \frac{\theta_1 - \theta_2}{2} = -K\left(\frac{\theta_1 + \theta_2}{2} - \theta_S\right)$$

$$1^0 C\min^{-1} = -K\left[85 - \theta_S\right] \ldots\ldots\ldots(i)$$

In the second stage of cooling

$$\frac{80-70}{12} = -K\left[\frac{80+70}{2} - \theta_S\right]$$

$$\frac{10}{20}\,{}^0 C\min^{-1} = -K\left[75 - \theta_S\right] \ldots\ldots\ldots(ii)$$

On dividing equation (i) by (ii) and solving

$$\theta_s = 25^{\circ}C$$

In the third stage of cooling

$$\frac{70-\theta}{t} = -K\left[\frac{70+\theta}{2} - 25\right] \ldots\ldots(iii)$$

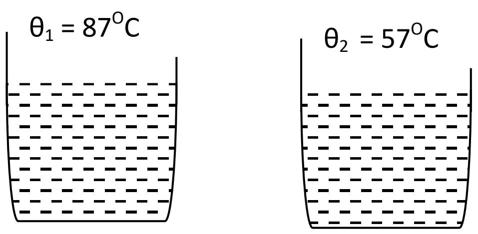
Dividing equation (i) by (iii) and on solving θ = 60°C

Example 7

Wind blows over hot liquid placed in a beaker in a laboratory whose average room temperature is 27°C. The liquid state of cooling is 5°C min^{-1}, when it is at temperature of 87°C. Calculate the liquid rate when it's at a temperature of 57°C.

Solution

θs = 27°C



$$\frac{d\theta_1}{dt} = 5^0 C\min^{-1} \qquad \frac{d\theta_2}{dt} = ?$$

According to the Newton's law of cooling

$$\frac{d\theta}{dt} = -K\left(\theta - \theta_S\right)$$

Now $\frac{d\theta_1}{dt} = -K\left(\theta_1 - \theta_S\right) \ldots\ldots\ldots(i)$

$$\frac{d\theta_2}{dt} = -K\left(\theta_2 - \theta_S\right) \ldots\ldots\ldots(ii)$$

Dividing equation (ii) by (i) we get

$$\frac{d\theta_2}{dt} = \frac{d\theta_1}{dt}\left[\frac{\theta_2 - \theta_s}{\theta_1 - \theta_s}\right]$$

$$= 5^0 C\min^{-1}\left[\frac{57-27}{87-27}\right]$$

$$\frac{d\theta_2}{dt} = 2.5^0 C\min^{-1}$$

∴ The rate of cooling when the temperature of the liquid is at 57°C is 2.5°Cmin^{-1}.

Example 8

(a) Why a body at a higher temperature than the body at lower temperature?

(b) A hot body is placed in cooling surroundings. Its rate of cooling is 3°C min^{-1} when its temperature is 70°C and 1.5°C min^{-1} when its temperature is 50°C.

  (i) Determine the temperature of the surrounding

  (ii) Determine the rate of cooling when the body of body is 40°C.

Solution

(a) According to the Newton's law of cooling, the rate of heat loss on the body is directly proportional to the excess temperature over the surrounding under forced convection. So the body at higher temperature have large excess temperature than the body at the lower temperature. Therefore the body at higher temperature cools faster.

(b) (i) Since $\frac{d\theta}{dt} = -K\left(\theta - \theta_S\right)$

When $\theta_1 = 70^{\circ}C$, $\frac{d\theta_1}{dt} = 3^{\circ}C\min^{-1}$

$$3°C\,min^{-1} = -K[70-\theta_s] \ldots\ldots(1)$$

When $\theta_2 = 50°C$, $\frac{d\theta_2}{dt} = 1.5°C\,min^{-1}$

$$1.5°C\,min^{-1} = -K[50-\theta_s]$$

On dividing equation (1) by (2)

$$\theta_s = 30°C$$

$\therefore$ Temperature of the surrounding is 30°C

(ii) $\frac{d\theta}{dt} = -K[40-30] \ldots\ldots(3)$

On comparing equation (1) and (3)

$$\frac{d\theta}{dt} = 3°C\,min^{-1}\left[\frac{40-30}{70-30}\right]$$

$$\frac{d\theta}{dt} = 0.75°C\,min^{-1}$$

Example 10

Two similar spherical bodies of radii R and 2R are temperature are at the same temperature. If they are kept to cool under the same condition. Show quantitatively which of the two spherical body will cool faster?

Solution

According to the Newton's law of cooling.

$$\frac{d\theta}{dt} = -\frac{-KS}{\rho VC}(\theta-\theta_s)$$

$\frac{d\theta}{dt}\ \alpha\ \frac{s}{v}$ If other factors are kept constant

$$\frac{S}{V} = \frac{4\pi R}{{}^4\!/_3\,\pi R^3} = \frac{3}{R} \ldots\ldots(1)$$

For the second sphere

$$\frac{d\theta_2}{dt} = \frac{K}{2R}$$

Putting equation (1) into (2)

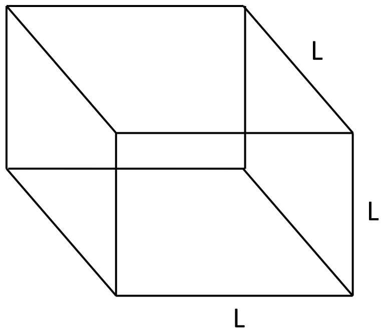
$$\frac{d\theta_2}{dt} = \frac{1}{2}\frac{d\theta_1}{dt} \text{ or } \frac{d\theta_1}{dt} = 2\frac{d\theta_2}{dt}$$

$\therefore$ The rate of cooling for the smaller sphere is twice that of large sphere. Thus the smaller sphere will cool faster.

Example 11

You are given two hot cubes at the same temperature and both are cooled by forced convections. The cubes are made from the same material but one has a side of L meters and other 2L meters. Which cube is at quicker rate of cooling?

Solution



Area of cube = L × L × L × L × L × L

$S = 6L^2$

Volume of cube, V = L3

$$\frac{S}{V} = \frac{6L^2}{L^3} = \frac{6}{L}$$

Rate of cooling of cube of side L

$$\frac{d\theta_1}{dt} = \frac{K}{L} \ldots\ldots\ldots(1)$$

Rate of cooling of cube of side, 2L

$$\frac{d\theta_2}{dt} = \frac{K}{2L} \ldots\ldots\ldots(2)$$

Putting equation (1) into (2)

$$\frac{d\theta_2}{dt} = \frac{1}{2}\cdot\frac{d\theta_1}{dt}$$

$\therefore$ The smaller cube will cool faster than the large cube.

Example 12

Two solid copper spheres of diameter 10cm and 5cm are at temperature which are respectively 10°C and 5°C above that of the surroundings. Assuming Newton's law and conditions to apply, compare the rate of fall of temperature of the two spheres. Indicate any further assumptions made in your calculations.

Solution.

According to the Newton's law of cooling the rate of heat loss by the spheres.

$$MC\frac{d\theta}{dt} = -KS(\theta-\theta_s)$$

$$\frac{4}{3}\pi r^3\rho C\frac{d\theta}{dt} = -K4\pi r^2(\theta-\theta_s)$$

$$\frac{d\theta}{dt} = \frac{-3K}{\rho Cr}(\theta-\theta_s)$$

For the first sphere

$$\frac{d\theta_1}{dt} = \frac{-3K}{\rho cr_1}(\theta_1-\theta_s)\ldots\ldots(1)$$

For the second sphere

$$\frac{d\theta_2}{dt} = \frac{-3K}{\rho cr_2}(\theta_2-\theta_s)\ldots\ldots(2)$$

Dividing equation (1) by (2)

$$\frac{d\theta_2}{dt}\bigg/\frac{d\theta_2}{dt} = \left(\frac{r_2}{r_1}\right)\left(\frac{\theta_1-\theta_s}{\theta_2-\theta_s}\right)$$

$$= \frac{5}{10}\times\frac{10}{5} = 1$$

$$\frac{d\theta_1}{dt} = \frac{d\theta_2}{dt}$$

$\therefore$ The sphere will cools at the same rate.

Example 13

A sphere of radius r, density ρ and specific heat capacity C is heated to temperature θ and then cooled in an enclosure at temperature θo. How the rate of fall of temperature is related with r, ρ and C?

Solution

Since $\frac{dQ}{dt} = K\frac{d\theta}{dx}$

But $Q = MC\theta$

$$MC\frac{d\theta}{dt} = KA\frac{d\theta}{dx} \text{ or } \frac{d\theta}{dt} = \frac{KA}{MC}\cdot\frac{d\theta}{dx}$$

Since $K\frac{d\theta}{dx} = \text{constant}$

$$\frac{d\theta}{dt} \alpha \frac{A}{MC}$$

Also A α r^2 and M α $r^3\rho$

$$\frac{d\theta}{dt} \alpha \frac{r^2}{r^3\rho C}$$

$$\frac{d\theta}{dt} \alpha \frac{1}{\rho Cr}$$

Example 14

A calorimeter of heat capacity 30J K^{-1} contains 100cm^3 of glycerine (density = 1.2 X $10^3 kgm^{-3}$) and its cools from 80°C to 70°C in 3minutes, the room temperature being 20°C. When the glycerine is replaced by 100cm^3 of water, the water cools from 43°C to 33°C in 16.5 minutes. What is the specific heat capacity of glycerine?

Solution

According to the Newton's law of cooling.

Rate of heat loss by water and calorimeter

$$\frac{d\theta}{dt} = -\frac{KS}{\rho CV}\left(\frac{\theta_1+\theta_2}{2}-\theta_S\right)$$

For the glycerine

$$\frac{80-70}{3} = \frac{-KS}{\rho_1C_1V}\left(\frac{80+70}{2}-20\right)$$

$$\frac{10}{3}\,^{o}C\min^{-1} = \frac{-KS}{\rho_1C_1V}\times 55\ldots\ldots.(i)$$

For the water

$$\frac{43-33}{16.5} = \frac{-KS}{\rho_2C_2V}\left[\frac{43+33}{2}-20\right]$$

$$\frac{10}{16.5}\,^{o}C\min^{-1} = \frac{-KS}{\rho_2C_2V}\times 18$$

Dividing equation (ii) by (i) and make subject C1

$$C_1 = \left(\frac{3}{16.5}\right)\left(\frac{55}{18}\right)\left(\frac{\rho_2}{\rho_1}\right)C_2$$

$$= \left(\frac{3}{16.5}\right)\left(\frac{55}{18}\right)\left(\frac{1000}{1200}\right)\times 4200$$

$$C_1 = 2.2\times10^3 JKg^{-1}K^{-1}$$

$\therefore$ Specific heat capacity of glycerine is $2.2\times10^3 JKg^{-1}K^{-1}$

Example 15

A copper calorimeter of mass 50g containing 100cm^3 of water cools at the rate of $2^{O}Cmin^{-1}$ when the temperature of the water is $50^{O}C$. If the water is replaced by 100cm^3 of a liquid of specific heat capacity 2200$JKg^{-1}K^{-1}$ and relative density 0.8, what will be the rate of cooling when the temperature of the liquid is 40°C. Assume the cooling takes place according to the Newton's law of cooling and that the constant temperature of the surrounding is 15°C in each case. Specific heat capacity of water = 4200Jkg^{-1}K^{-1} specific heat capacity of copper = 400Jkg^{-1}K^{-1}.

Solution

According to the Newton's law of cooling. Rate of heat loss by water and calorimeter $\alpha \quad \theta-\theta_s$

$$\left(MC+M_\omega C_\omega\right)\frac{d\theta_1}{dt} = -KS\left(\theta_1-\theta_S\right)$$

$$\left[0.05\times400+0.1\times4200\right]\times2 = -KS\left[50-15\right]$$

$$880 = -35\times KS \,\ldots\ldots\text{(i)}$$

For the case of given liquid

$$\left(MC+M_LC_L\right)\frac{d\theta_2}{dt} = -KS\left(\theta_2-\theta_S\right)$$

$$\left(MC+\rho_LV_LC_L\right)\frac{d\theta_2}{dt} = -KS\left(\theta_2-\theta_S\right)$$

$$= -KS\left[40-15\right]$$

$$196\frac{d\theta}{dt} = -25\times KS \,\ldots\ldots\ldots\text{(ii)}$$

On solving equation (i) by (ii)

$$\frac{d\theta_2}{dt} = 3.21\,°C\,min^{-1}$$

Example 16

A calorimeter containing 40g of water cools from 60°C to 55°C in 1min 36sec. The same calorimeter, when containing 50g of a liquid of specific heat capacity 2140JKg^{-1}K^{-1} takes 1min 8sec to cool through the same temperature range under the same conditions of cooling. What is the specific heat capacity of calorimeter?

Solution

Let C′ = thermal capacity of calorimeter

According to the Newton's law of cooling

$$\left(MC+C'\right)\frac{d\theta}{t} = -K\left(\theta-\theta_S\right)$$

For water

$$\left(M_\omega C_\omega+C'\right)\frac{d\theta}{t_1} = -K\left(\theta-\theta_S\right)\ldots\ldots\text{(i)}$$

For the given liquid

$$\left(M_LC_L+C'\right)\frac{d\theta}{t_2} = -K\left(\theta-\theta_S\right)\ldots\ldots\text{(ii)}$$

(i) = (ii)

$$\frac{t_1}{t_2} = \frac{M_\omega C_\omega+C'}{M_LC_L+C'}$$

$$\frac{60+36}{60+8} = \frac{40\times10^{-3}\times42\times10^3+C'}{50\times2140+C'}$$

$$\frac{96}{68} = \frac{168+C'}{107+C'}$$

$$C' = 41.1429\,J\,^0C^{-1}$$

Example 17 NECTA 2008/P1/6(a)

(i) Distinguish between forced and natural convection and state the laws governing these process.

(ii) A piece of copper of mass 50g is heated to 100°C and then transferred to a well-insulated copper colorimeter of mass 25g containing 100g of water at 10°C. Neglecting heat loss, calculate the final steady temperature of water after it has been well stirred. Specific heat capacity of water = 4200Jkg^{-1}K^{-1}. Specific heat capacity of copper = 400Jkg^{-1}K^{-1}.

Solution

(i)

Force convection	Natural convection
Is the kind of thermal convection in which the heated materials flows due to the applied forced agent or pump blower.	Is the kind of thermal convection in which the heated materials flows due to the difference in density or pressure.
Law governing is the Newton's law of cooling.	Law governing is a dulong – petit law.

(ii) Apply the principle of calorimeter.

Heat lost by copper = heat gained by calorimeter + water

$$M_1C_1\left(100-\theta\right) = \left(M_2C_2+M_WC_W\right)\left(\theta-10\right)$$

$$50 \times 400(100 - \theta) = (25 \times 400 + 100 \times 4200)(\theta - 10)$$

On solving $\theta = 14^{o}C$

Example 18

(a) Can we boil water inside a satellite revolving earth?

(b) When hot water is poured into thick bottomed glasses, they get cracked many times why?

Solution

(a) No, water gets boiled with the process of convection and it is based on the fact a liquid becomes lighter on becoming hot and rises up. Now inside the satellite revolving around earth, everything is under the state of weightlessness, so this process cannot take place and hence, boiling of water is not possible.

(b) This is because glass is a bad conductor of heat. It does not allow the heat to pass quickly to the lower surface. As a result, different layers of the glass bottom attain different temperature and expand differently. As a result, glass may break at the bottom.

Example 19

In order to loosen a stopper from the neck of a glass bottle, boiling water is poured around the neck why?

Solution

Due to poor conductivity of the glass, the neck of the bottle expands but not the stopper. This helps in loosening the stopper from the neck of the bottle.

Example 20

(a) (i) Give two examples of natural convection.
(ii) Give two examples of forced convection.

(b) Can the rate of loss of heat be the same for two liquids? Comment.

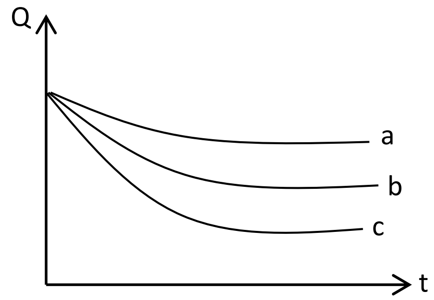
Solution

(a) (i) see breeze and trade wind are example of natural convection.
(ii) Forced air heating systems in home and the cooling system of an automobile engine are the examples of forced convection.

(b) The rate of loss of heat can be the same for the two liquids if their cooling takes place under identical conditions.

Example 21

Cooling curves are drawn for the three liquids a, b, c shown in the figure below. For which liquid specific heat is least?



Solution

When a body cools by radiation

$$\frac{d\theta}{dt} = \frac{-K}{MS}(\theta - \theta_0)$$

$$\frac{d\theta}{dt} \propto \frac{1}{S}$$

Where S is specific heat capacity. From graph we note that $d\theta/dt$ is more for liquid C, so specific heat capacity is least for liquid C.

Example 22

Water is heated from below but not from top. Why?

Solution

When water is heated from below, its density decreases, it becomes light and rises up. Cooler water from above comes down and takes the position vacated by hot water. So convectional currents are set up heated. When water is heated from top, it will conduct very small amount of heat to the bottom part of water because water is a poor conductor of heat. Hence water will not be heated properly.

Example 23

(a) What is the basic condition for Newton's law of cooling to be obeyed?

(b) Is the rate of cooling be the same thing as the rate of loss of heat? Explain

Solution

(a) Refer to your notes

(b) No, the rate of cooling of a body at a temperature is defined as the fall in temperature per second at the temperature, while the rate of loss of heat is the quality of heat lost per second from a body at a given temperature.

Example 24

Sometimes when we shake a bottle vigorously containing hot liquid, its cork blows off. Why?

Solution

On shaking a hot liquid, its temperature increases and some of the liquid is converted into vapours. The vapours pressure inside the bottle may become high enough to blow off the cork.

Example 25

(a) In an Earth – satellite, the transfer of heat by convection is not possible. Why?

(b) A solid sphere of copper of radius R and a hollow sphere of the same material of inner radius r and outer radius R are heated to the same temperature and allowed to cool in the same environment. Which of them starts cooling faster?

Solution

(a) The process of transfer of heat by convection is based upon the fact that a liquid becomes lighter on becoming hot and rises up. in condition of weightlessness this is not possible so, the transfer of heat by convection is not possible inside of a satellite.

(b) Since $\frac{dQ}{dt} = MC\frac{d\theta}{dt}$

$$\frac{d\theta}{dt} = \frac{1}{MC}\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} \alpha \frac{1}{M}$$

Since the surface area of hollow and solid sphere are equal. But mass of hollow sphere is less than that of the solid sphere. So the cooling of hollow sphere is faster than the solid sphere.

PRACTICE PROBLEMS

1. (a) Distinguish between natural and forced convection. Give one example of each.
   (b) A body cools from 40°C to 30°C in next 3 minutes. What is the temperature of surroundings? [ans. (b) 14°C]

2. Water placed in a container cools from 80°C to 70°C in 5 minutes. How long will it take to cools from 60°C to 40°C if temperature of surroundings is 12°C. [ans. 41.41min]

3. A body cools from 80oc to 50°C in 5 minutes. Calculate the time takes to cool from 60°C to 30°C. The temperature of the surroundings in 20°C. [ans. 10min]

4. A body cools from 80°C to 70°C in 5 minutes and further to 60°C in 11 minutes. What will be its temperature after 15 minutes from the starts? Also calculate the temperature of the surroundings. [ans. 54.4°C, 15°C]

5. A body cools from 70°C to 50°C in 6 minutes when the temperature of the surroundings is 30°C. What will be the temperature of the body after further 12 minutes if cooling proceeds according to Newton's law of cooling? [ans. 34°C]

6. (a) State Newton's law of cooling and give one limitation of the law.
   (b) In a cooling experiment it is found that the rate of the temperature fall of a liquid when its temperature is 75°C is 2.4°C/min. calculate the rate of cooling of a liquid of the liquid at

65°C if the room temperature is 29°C. [ans.(b) 1.88°C/min]

7. EZEB 2006/P1/5
   (a) (i) Can water heated to temperature higher than 100°C?
   (ii) Why steam burns more severely than boiling water?
   (b) Why a body at higher temperature losses heat faster than one at lower temperature?

8. A cooper colorimeter of mass 120gm contains 100gm of paraffin at 15°C. If 48gm of aluminum at 100°C is transferred to the liquid and final temperature is 27°C, calculate the specific heat capacity of paraffin neglecting heat losses. (Specific heat capacities) of aluminium and copper are $1000Jkg^{-1}k^{-1}$ and $400Jkg^{-1}k^{-1}$ respectively. [ans. $2440Jkg^{-1}k^{-1}$]

9. 200g of water in a copper vessel of mass 300g cool from 60°c to 40°c in 12 minutes. The water is replaced by an equal volume of alcohol which cools through the same range under the same conditions in 6 minutes. If the density of alcohol is $800kgm^{-3}$, what is the specific heat capacity of alcohol? Density of water = $1000kgm^{-2}$ specific heat capacity of water = $4200Jkg^{-1}k^{-1}$, heat capacity of copper = $30Jk^{-1}$. [ans. $2500Jkg^{-1}k^{-1}$]

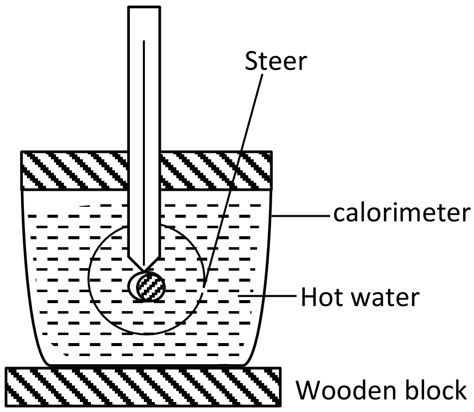
10. Into a vessel containing 500gm of water at 10°C (the room temperature) is placed a 60w electric immersion heater. The temperature of the water is observed to rise to 20°C in 7 minutes after switching the heater. What is the highest temperature to which the water can be raised by the heater under the conditions of the experiment and what time elapses before this temperature is reached? Assume the heat losses confirm to Newton's law throughout the temperature range involved. (specific heat capacity of water = $4200Jkg^{-1}k^{-1}$)

11. A copper block of mass 2.5kg is heated in a furnace to a temperature of 500°C and then placed on a large ice block. What is the maxiumum amount of ice that can melt. (specific heat capacity of copper = $0.39Jg^{-1}k^{-1}$, latent heat of fussion of water = $335Jg^{-1}$) [ans. 1.5kg].

12. A pan filled with hot food cools from 94°C to 86°C in 2 minutes when the room temperature is at 20°C. How long will it take to cool from 71°C to 69°C? [ans.0.7min = 42sec]

EXPERIMENT TO VERIFY THE NEWTON'S LAW OF COOLING

Apparatus colorimeter, thermometer, steer, piece of wood, stop, watch, cardboard, source of hot water. Apparatus can be arranged as shown in figure below.



Data

Measurement of room temperature

Initial room temperature, θ_i = °C

Final room temperature, θ_f = °C

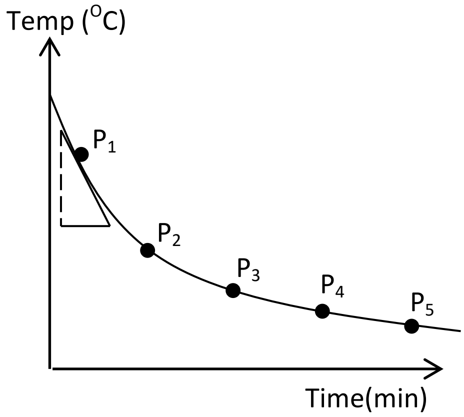
Average room temperature

$$\theta_R = \frac{\theta_i + \theta_f}{2}$$

Table of result

Temp (°C)				
Time (min)				

GRAPH OF TEMPERATURE AGAINST TIME



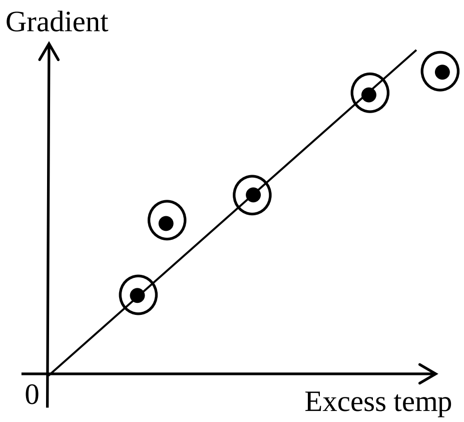
- Choose five (5) points on your curve, then draw the tangent line on each point.
- Determine the slope or gradient of each point in which the tangent line is drawn.

  Slope $= \dfrac{\Delta\theta}{\Delta t} = \dfrac{\theta_1 - \theta_2}{t_1 - t_2}$

- Determine the excess temperature corresponding to each point and make a table of result as shown below.

Point	Temp (^OC)	$(\theta - \theta_R)$ (^OC)	Gradient $\dfrac{\Delta\theta}{\Delta t}\left({}^{\circ}c\,min^{-1}\right)$
P_1			
P_2			
P_3			
P_4			
P_5			

GRAPH OF GRADIENT AGAINST EXCESS TEMPERATURE



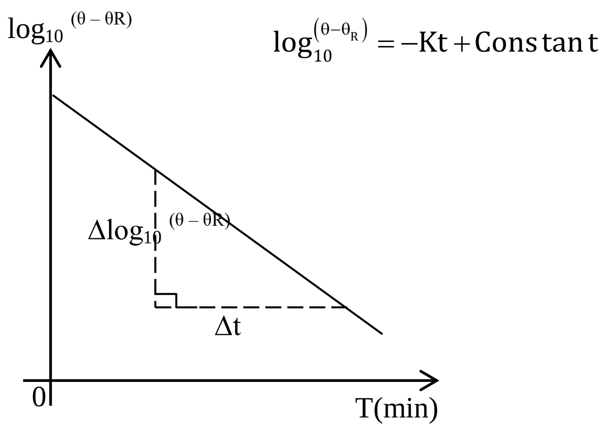
Comment

Since the graph of gradient against excess temperature is the straight line passing through the origin, then the given liquid obeys Newton's law of cooling which state that 'The rate of heat loss is directly proportional to the excess temperature over the surrounding under the forced convection'.

Alternative method

We can plot a graph of $\log_{10}\left(\theta - \theta_R\right)$ against time to verify the Newton's law of cooling.

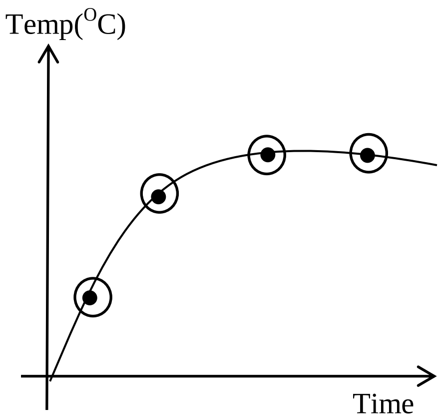
GRAPH OF $\log_{10}$ $(\theta - \theta_R)$ AGAINST TIME



Comment

Since the graph of $long_{10}$ $(\theta - \theta_R)$ against time is the straight line with negative gradient then the given liquid obey's newton's law of cooling.

GROWTH CURVE OF BOILING WATER



APPLICATIONS OF THERMAL CONVECTION IN DAILY LIFE.

1. **Heat of rooms** – the mechanics of heating a room by a heater is entirely based on convection. The air molecules in immediate contact with heater are heated up. these air molecules acquire sufficient energy and rise upward. The cool air at the top being denser moves down to take place. The cool air is in turn heated and move upward. In this way convection currents are set up in the room which transfer heat to different parts of the room.
2. **Cooling of transformer** – a transformer is always kept in a tank containing oil so as to remove the heat generated on it due to the flow of current. The heat is removed by convection.
3. **Hot water heating of a room –** hot water from a boiler is circulated in metallic spiral pipe in the room. The heat energy in the water is radiated into the room from the metal tube and cooled after returns to the boiler again. This heat keep the air in room warm.
4. Ventilation – ventilation or exhaust fan in a room help to remove impure and warm air from a room. The fresh air from outside blows into the room. This is all due to the convection current set up in the room.
5. **To regulate temperature of human body heat** transfer in the human body involves a combination of mechanisms. These to gether maintain remarkably uniform temperature in the human body in spite of large changes in environmental changes. The chief internal mechanism is forced convection. The heat serves as the pump and the blood as the circulating fluid.
6. **Chimney** – kitchen rooms are provided with chimney through which hot air in the room goes out. Fresh air containing oxygen enter through the window and doors into the kitchen to support burning of the fuel.
7. **Land and sea breezes the heat from the sun is** land rather than sea – water moreover, the specific heat of land is low comp aired to the sea – water. Consequently, the rise in temperature of land is higher as compared to that of sea – water. To sum – up, land is hotter than the sea during day time. As a result of this, the colder air over the sea blows towards the land. This is called **'sea – breeze'** at night air blows from land towards to the sea. This is called **'Land breeze'**
8. **Formation of trade winds** – The steady wind blowing from North – East, near the surface of the Earth is called trade – winds. Due to the difference of pressure at the poles and equator, the air at the pole moves up (get heated strongly) moves towards the poles and so on. In this way wind is formed in the atmosphere.
9. **Smoke rises from volcanic eruptions (industries).** Normally these smokes are hot and thus less dense therefore they rise upwards.

3. THERMAL RADIATION

It is the process by which heat is transmitted from one place to another without heating the intervening medium. .i.e thermal radiations the process of heat transfer from one point to wards to the another point by the invisible electromagnetic wave or radiation.

Radiation – is the total energy emitted by a body including heat, light, X – rays, δ – rays etc. the radiation emitted by a bod due to its temperature is called 'thermal radiation' for example, heat and light from the sun can be reached on the Earth surface by means of thermal radiation.

NATURE AND PROPERTIES OF THERMAL RADIATION.

According to Maxwell, thermal radiations are electromagnetic wave. Heat radiation have the wavelength range from 8×10^{-7}m to 4×10^{-4}m (infrared) radiant energy behave like light energy except it does not produce sensation of light. All objects above okemit thermal radiation continuously. The energy and wavelength of the thermal radiation depend on the temperature and nature of the hot radiating body.

Note that

At the law temperature, the bod emits mainly infrared and at high temperature its emits visible and ultraviolent radiation in addition to the infrared. Therefore 'Heat meant just an infrared'.

PROPERTIES OF THERMAL RADIATION

1. Thermal radiation travels in straight line like light.
2. Thermal radiation travels with the velocity of light in vacuum (i.e 3×10^8m/s).
3. Thermal radiation obey inverse square law.
4. Thermal radiation obeys the laws of reflections.
5. Thermal radiation obey the law of refraction
6. Thermal radiation suffer total internal reflection
7. Thermal radiation can be polarized like light
8. Thermal radiation shows the phenomena like interference and diffraction.
9. Thermal radiation do not affected by the medium in which they pass.

RADIATION INTENSITY, I

Is the amount of the radiation energy falling on the body per second per metre square.

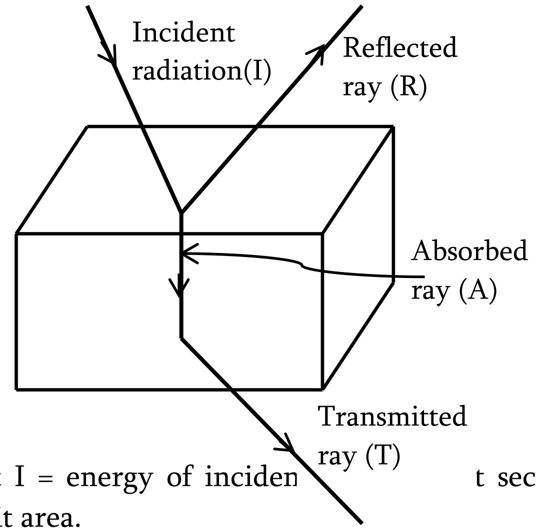
$$\text{Radiation intensity} = \frac{\text{radiation energy}}{\text{Area} \times \text{time}}$$

$$I = \frac{E}{A \times t} = \frac{P}{A}$$

S.I Unit of radiation intensity is Wm^{-2}.

REFLECTION, TRANSMISSION AND ABSORPTION

When the radiation energy fall on the given body, some of radiation are reflected (R), absorbed (A) and transmitted (T) and this depends on the nature of the body.



Let I = energy of inciden t sec per unit area.

Now $I = A + R + T$

$$I = \frac{A}{I} + \frac{R}{I} + \frac{T}{I}$$

$$1 = a + r + t \text{..........}(i)$$

Where a, r and t are the absorptive, reflectance and the body respectively. This equation show that

(a) If a body is a good absorber, $a \to 1$, $t \to O$ and $r \to o$. for perfect absorber eg black body $a = 1$, $r = 0$, $t = 0$.

(b) For the shine body (surface) the value of R is very large while A and T are very small. For good reflector, $t \to 0$, $a \to 0$, $r \to 0$. For perfect reflector $r = 1$, $a = 0$, $t = 0$.

(c) For the glass surface. The value of R will be average A is small and T is greater than A it can be shown that for a given wavelength; the equation (1) can be represented by $1 = t_\lambda + r_\lambda + a_\lambda$ where λ is a particular wavelength we should specify the wavelength because a body may absorb one wavelength and may reflect and transmit other wavelengths.

TERMINOLOGIES OF THERMAL RADIATION

1. **ABSORTANCE (Absorpting factor)** – is the ratio of the amount of energy of the thermal radiation absorbed in a certain time to the total amount of energy falling normally upon it in the same time.

$$a = \frac{\text{Radiat power obsorbed}}{\text{Radiant power incident}}$$

$$a = \frac{\Phi_a}{\Phi_i}$$

2. **REFLECTANCE (REFLECTION FACTOR)** – is the ratio of amount of energy of the thermal radiation reflected by a body in a certain time to the total amount of energy falling normally upon it in the same time.

$$r = \frac{\text{radiant power reflected}}{\text{radiant power incident}}$$

$$r = \frac{\Phi_r}{\Phi_I}$$

3. **TRANSMITTANCE (Transmission factor)** – is the ratio of amount of energy of thermal radiation transmitted by a body in a certain time to the total amount of energy falling normally upon it in the same time

$$t = \frac{\text{radiant power transmitted}}{\text{radiant power incident}}$$

$$t = \frac{\Phi_t}{\Phi_i}$$

4. **MONOCHROMATIC ABSORPTIVE POWER** Spectral absorption power of a body) is defined as the ratio of the heat energy absorbed in a certain time to the total incident heat energy upon it in the same time within a unit wavelength range around the wavelength λ i.e between (λ – ½) and (λ + ½).

5. **MONOCHROMATIC EMISSIVE** – The monochromatic emissive power of a body at a given temperature and for a given wavelength, λ is defined as the radiant energy of that wavelength emitted per second per unit area of its surface within a unit wavelength interval around λ i.e between (λ – ½) and (λ + ½). It is denoted by a_λ.

6. **EMISSIVE POWER** – Is the total energy of all wavelength radiated per second per unit area of the body

Emissive pow

$$\text{Emissive power} = \frac{\text{total power radiated by a body}}{\text{surface area of a body}}$$

$$\text{Emissive power} = \frac{P}{A}$$

Since the wavelength emitted range from O to infinity.

$$\text{Emissive power } E = \int_o^\alpha e_\lambda d\lambda$$

S.I Unit of emissive power is Jm-2S-1(Wm-2)

Thermal radiation emitted by any hot body depends upon.

(i) The temperature of the body
(ii) The nature of the surface
(iii) The size of the body
(iv) The nature of the surroundings..

7. **EMISSION** – is the process by which the body gives out heat.

8. **ENERGY DENSITY** – is the amount of energy of radiation per unit volume.

9. **EMISSIVITY (e, or ε)** – is defined as the ratio of the emissive power of a body to the emissive power blank body at the same temperature

$$e = \varepsilon = \frac{E}{E_b} = \frac{P}{P_b}$$

E = emissive power of a body

E_b = emissive power of a black body.

The emissivity is the number whose value depends on the nature of the surface of the body and its value range from zero to 1 i.e

$o \le e \le 1$

- For the perfectly black body e = 1, for shiny surface e → o, for pulished copper , e = 0.3

10. **SPECTRAL EMISSIVITY, e_λ** – is the ratio of a spectral emissive power of a body at a given temperature to spectral emissive power of a black body at the same temperature

$$\varepsilon_\lambda = \frac{e_\lambda}{E_\lambda}$$

Note that

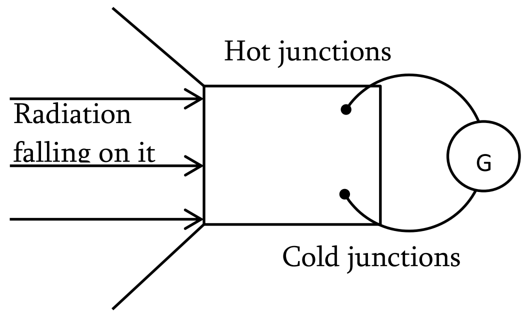
I. E_λ = O for a perfect reflector or perfect transmitter.

II. The instrument used to measure thermal radiations are:-

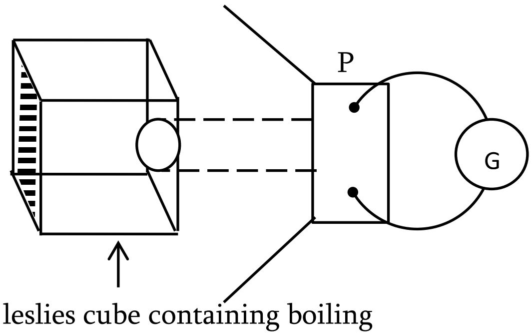
1. The thermopile
2. Boys radio – micrometer
3. The balometer

THERMOPILE

Thermopile is a device used for detecting and measuring the intensity of heat radiations. It works on the principle of thermo – electric effect. It is consist the number of thermocouple joined in series one junction is made to receive the heat radiation and become hot, the other is shielded, from the radiation to keep it cold. Due to the temperature difference, a small e.m.f is produced and current flows through the galvanometer. This indicate the detection of thermal radiation.



Experiment to determine which surfaces a good emitter and which are poor emitters. A cubical metal tank (leslies cube) whose sides have variety of finishes melt black dull white, highly polished slilvered dull black etc is used the tank containing boiling water which are kept at a constant temperature



G = Galvanometer

P = Thermopile fitted with a blackned conical mouth piece. The galvanometer deflection is facing on the malt black surface and least when it is facing on the highly polished surface is the worst radiator and the malt black surface is the best radiator.

DIA THERMANOUS AND DIA THERMANOUS BODY

A diathermanous body is the body that absorbs little of eh radiation passing through it. i.e a body translucent to visible light eg. Calcium fluoride. An adiathermaous body is the body that absorbs strongly the radiation passing through it. (i.e body apaque to visible light) an example is a mass of water.

THE BLACK BODY

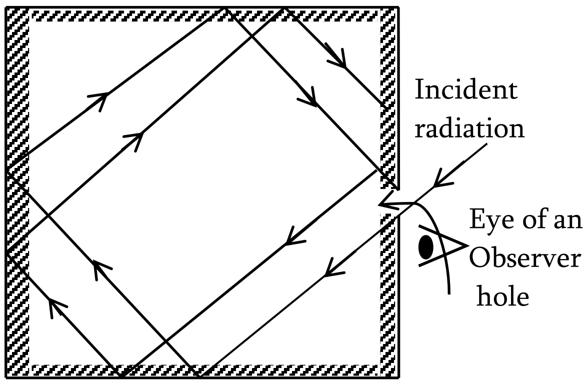
Black body – is the body which absorbs all radiation fall on it and reflects or transmits none. i.e R = O, T = O and A = 1. A perfect black body is body which absorbs completely all the radiation falling on it. Since a perfectly black body is a perfect absorber it also be a perfect radiator. When a perfect black body is heated to a high temperature it emits radiation of all possible wavelength. No anybody which is a perfect black body by 100%

PERFECT THERMAL SOURCE

Is a body that radiates all the heat generated in it and absorbs none heat falls on it.

Examples of black body
A surface coated with lamp black or platinum black, Wien's black body, frerys black body e.t.c

PREPARATION AND HOW TO REALIZE A BLACK BODY IN THE LABORATORY
A good black body can be made simply by punching a small hole in the lid of a closed empty tin as shown in the figure below



The hole looks almost black, although the shining tin is a good reflector. The holes looks black because the radiation that enters through it is reflected from the inside wall many times and is partly absorbed at each reflection until none remains. If you look inside the tin through the hole, it appears black thus realizes a black body.

BLACK BODY RADIATOR (CAVITY RADIATION) - Is a body which emits radiation in which its characteristics (i.e. intensity of each frequency or wavelength) can be determined by its own temperature and does not depend on the nature of the surface. Sometime black body radiator is called **temperature radiation because the amount of radiation emitted can be determined by its own temperature.**

BLACK BODY TEMPERATURE – is the temperature of a black body which emits radiation of the same description as the body under consider. The amount of radiation emitted by a body depends on the following factors:-

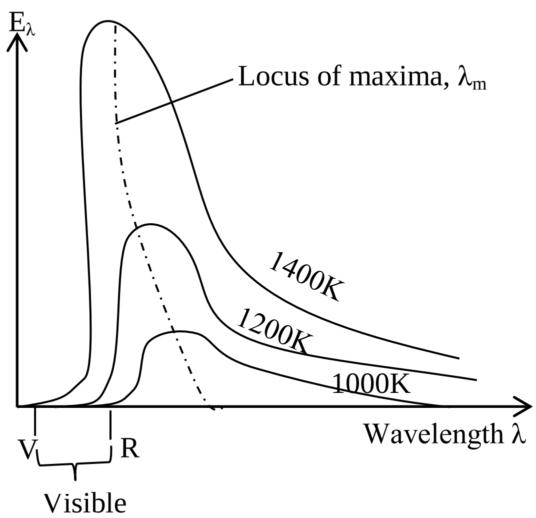
1. The surface area of a body
2. The nature of the surface of a body
3. The surface temperature of the body.

- **A dull black surface** emits more radiation that a polished surface under identical condition.
- A body at higher temperature emits more radiation. Than when it is at a lower temperature.
- The amount of radiation from a body is directly proportional to its surface area.

RELATIVE INTENSITY, E_λ – Is the amount of energy radiated by black body per second per unit metre square per unit wavelength.

QUALITY OF RADIATION OF PERFECT BLACK BODY – is the relative intensity of wavelength in that body on a given temperature. The quality of a radiation from a perfect black body can be depends only on its own temperature.

SPECTRAL DISTRIBUTION OF RADIANT ENERGY FOR BLACK BODY RADIATION. – is the graph which shows the variation of relative intensity E_λ emitted by a black body with wavelength at various temperatures.



Special feature
The following conclusions or points may be noted from the curve above:-

1. No curve enters to the visible region. Since the radiation corresponding to the

wavelength of visible region is absorbed by the fluorspar prism (i.e part of apparatus of experiment)

2. At a given temperature the energy is not uniformly distributed in the radiation spectrum of hot body.

3. At each temperature, the black body emits continuous heat radiation spectrum.

4. The energy associated with the radiation at a particular wavelength increases with increase in the temperature of the black body.
5. For all wavelength an increase in temperature cause an increase in the energy emission.

6. The wavelength of maximum emission shifts towards smaller wavelength as the temperature of black body increases i.e $\lambda_m \alpha \frac{1}{T}$ It is known as Wien's displacement law.
   Definition: wavelength of maximum emission – is the wavelength corresponding to which energy associated is maximum.

7. The energy (E_{max}) emitted corresponding to the wavelength of maximum emission (λ_m) increases with fifth power of absolute temperature of the black body i.e $E_{max} \alpha T^5$

8. The area under a curve is found to be proportional to the fourth power of the absolute temperature, T of the perfect black body. Area of the curve α T4 i.e the area enclosed by a particular curve represent the total radiant energy emitted per second per unit area by the black body at given temperature.

$$E = \int_0^\alpha E_\lambda d_\lambda = T^4 \text{ (Stefani's law)}$$

Note that:
Stars which are hotter than sun such as **'SIRIUS'** and **'VEGAS'** looks blue and not white because the peak of their radiation curve lie further towards the invisible blue than does the peak of the sun light.

LAW OF BLACK BODY RADIATION
There are three laws of the black body radiation:-
1. Wien's law of displacement
2. Stefan's law
3. Pank's law.

WIEN'S DISPLACEMENT LAW – State that 'The product of the wavelength, λ_m at which maximum energy is emitted and the absolute temperature (T) of the black body is always constant i.e 'The wavelength, λ_m is inversely proportional to the absolute temperature of the black body, T' $\lambda_m \alpha \, {}^{1}\!/_{T}$

$$\lambda_m = \frac{6}{T} \text{ or } \lambda_m T = b$$

b = constant of proportionality known as Wien's constant.
b = 2.9 × 10-3mk
If λ_1 and λ_2 be the wavelengths of black body radiation at temperatures T_1 and T_2 respectively

$$\lambda_1 T_1 = \lambda_2 T_2$$

This law is called 'Displacement law' because as temperature increases the maximum intensity of radiation emitted by a black body gets shifted or displacement towards the shorter wavelength side.

Applications of Wien's law of displacement.
1. This law can be used to determine the temperature of heavy bodies such as sun, moon and stars. The solar radiation having maximum intensity has a wavelength, $\lambda_m = 4753\overset{\circ}{A}$ and the corresponding

temperature at the sun surface is calculated

$$\text{as } T = \frac{6}{\lambda_m} = \frac{2.892\times10^{-3}\,mk}{4753\times10^{-10}\,m}$$

$$T = 6084K$$

2. When a piece of iron is heated, it first appears dull red, on further heating it appears as reddish yellow and then white. This can be explained using Wien's displacement law. More heat is given to iron, more heat it radiates and as its temperature increases, the maximum intensity radiation emitted from it has smaller wavelength.

STEFAN'S LAW OF BLACK BODY (Stefan – Boltzmann law) – state that 'The total amount of radiant energy emitted by perfect black body per second per unit area is directly proportional to the fourth power of the absolute temperature of surface of the body' E α T4 or E = T4

E = Total energy emitted per unit area per unit time(sec)

T = Absolute temperature.

σ = Stefan's constant its volume is $5.67 \times 10^{-8} Wm^{-2}K^{-4}$ power of radiation emitted by the black body.

$$P_b = E.A = \sigma A T^4$$

A = Surface area of black body

Expression of power of radiation emitted by the given body

Let e = emissivity of a body

$$e = \frac{P}{P_b}$$

$$P = eP_b = e\sigma AT^4$$

Limitations of Stefan's law

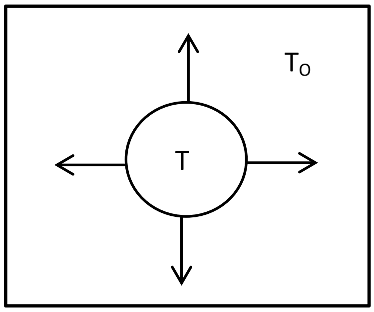
1. The law is hold for the perfect black body radiation only.
2. This law applies black body where there is a large excess temperature above 30oc.
3. The law hold when it receives no heat energy from the surroundings

Applications of Stefan's law

1. Used to determine the temperature of the sun.
2. Used to deduce the Newton's law of cooling.

DEDUCTION OF NEWTON'S LAW OF COOLING FROM THE STEFAN'S LAW.

Consider a hot body at a temperature, T(K) placed in a constant temperature enclosure at To(K). Let A be the surface area of the body and e is the emissivity of the body (relative emittance).



Net rate of heat loss due to the radiation is given by $\frac{dQ}{dt} = eA\sigma\left(T^4 - T_0^4\right)$

Let: $T = To + \Delta T$, $\Delta T = T - To$

$$\frac{dQ}{dt} = eA\sigma\left[\left(T_o + \Delta T\right)^4 - T_0^4\right]$$

$$= eA\sigma\left[T_0^4\left(1+\frac{\Delta T}{T_0}\right)^4 - T_0^4\right]$$

$$= eA\sigma T_0^4\left[\left(1-\frac{\Delta T}{T_0}\right)^4 - 1\right]$$

Expand $\left(1+\frac{\Delta T}{T_0}\right)^4$ by using binomial approximation.

$$\left(1+X\right)^n = 1 + n\times + -------$$

$$\left(1+\frac{\Delta T}{T_0}\right)^4 = 1 + 4\frac{\Delta T}{T_0} + ------$$

Since $\Delta T \ll T_0$, then neglecting all terms containing the highest power of $\Delta T / T_0$

Now $\left(1+\frac{\Delta T}{T_0}\right)^4 \simeq 1+4\frac{\Delta T}{T_0}$

$$\frac{dQ}{dt} = eA\sigma T_0^4\left[1+4\frac{\Delta T}{T_0}-1\right]$$

$$\frac{dQ}{dt} = \left(4eA\sigma T_0^3\right)\Delta T$$

$$= \left(4eA\sigma T_0^3\right)\left(T-T_0\right)$$

Now $\frac{dQ}{dt}\ \alpha\ \left(T-T_0\right)$

$$\frac{dQ}{dt}\ \alpha\ \left[\left(273+\theta\right)-\left(273+\theta_0\right)\right]$$

$$\frac{dQ}{dt}\ \alpha\ \theta-\theta_0$$

This is the statement of the Newton's law of cooling.

PLANK'S LAW (DISTRIBUTION FORMULA)

The curve showing the variation of E_λ with λ at constant, temperature T obey's Plank's formula which is given by

$$E_\lambda = \frac{C_1}{\lambda^5\left(e^{C_2/T\lambda}-1\right)}$$

Where C_1 and C_2 are constant

$C_1 = 4.99 \times 10^{-24}$Jm

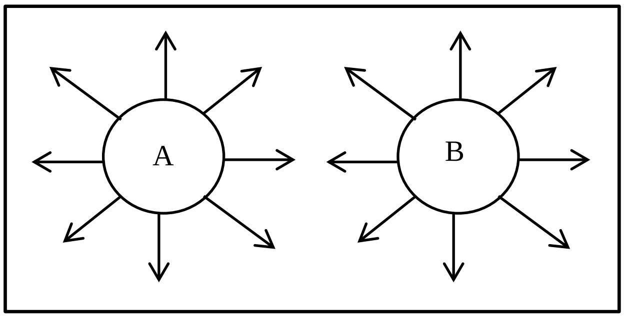
$C_2 = 1.44 \times 10^{-2}$mk

Plank's formula agrees with bath's Wien's displacement law and Stefan's law.

PREVOST'S THEORY OF HEAT EXCHANGE

State that 'When the temperature of the body is constant, the body is losing heat by radiation and gaining heat by absorption at equal rates'. If the body emits radiation at the rate which is determined only by the nature of its surface and absorbs radiation at a rate which is determined by the nature of its surface and the temperature of its surroundings.

Consider the system of two bodies inside of the enclosure.



Let T_1 and T_2 be the temperature of the body A and B respectively. Heat energy can be exchange between body A and B by radiation. If $T_1 > T_2$ heat is transferred from body A to B until $T_1 = T_2$. If the body A (being at higher temperature) emits more heat by radiation compaired to those emitted by B body A absorbs radiation from body B but the rate of absorption of energy is less than the rate of emission and so it loose energy. When $T_1 = T_2$, the rate at which it absorbs energy is equal to rate of emission i.e radioactive equilibrium is reached. Radioactive equilibrium is a situation in which a body receives (absorbs) heat at a rate that is equal to that it emits'. Now if $T_1 > T_2$, the net rate of heat radiated is given by $P = P_A - P_B = \sigma A\left(T_1^4 - T_2^4\right)$ in a dynamic equilibrium P = 0, $T_1 = T_2$.

PREVOSTS EXPLANATION

Everybody radiates heat depending upon its absolute temperature.

Examples

1. When we sit before a five, we feel warm. This is because our body is receiving more thermal energy per unit area from the five than it's is losing by its own radiation.
2. When we sit near a block ice we feel cold. This is because our body losses more heat energy by radiation than what it receives from the ice.

   Note that:

   a) Prevost argued that radiation of thermal energy is not a 'one – way traffic' with one , body emitting energy and another one receiving them. It should be considered as mutual exchange between two bodies.

b) Prevost's theory of heat exchange leads to the fact that good absorbers are good radiators and vice – versa is true.

SURFACE TEMPERATURE OF THE SUN

The surface temperature of the sun can be measured from solar constant and solar luminosity.

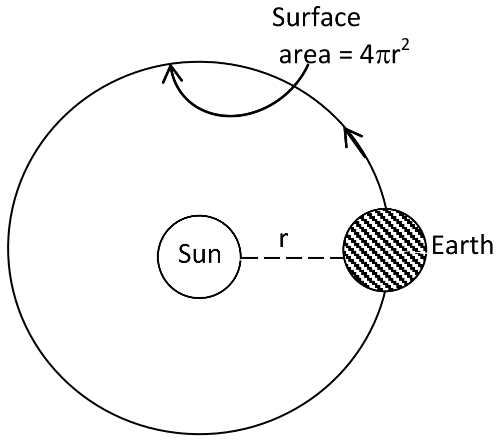
SOLAR CONSTANT (S OR Q)

Is the amount of radiant energy received per second per unit area by a perfect black body placed on the earth with its surface perpendicular to the direction of the radiation from the sun. The value of solar constant is $1.388 \times 10^3 Wm^{-2}$. Sometimes solar constant is **known as solar energy flux**.

SOLAR – LUMINOSITY (LS) – Is the amount of energy emitted by the sun per second in all direction.

LS = total radiation emitted by the sun per second.

Consider the motion of earth around the sun



The solar luminosity is

$LS = 4\pi r2S$

$= 4\pi \times (1.496 \times 10^{11})2 \times 1.388 \times 10^3$

$LS = 3.9 \times 10^{26} W$

Expression for surface temperature of the sun. assumption made:

1. The sun is a black body
2. There is no loss of energy of solar radiation as it travels from the sun to the earth.
3. The sun radius and earth radius is smaller compared to the mean distance from the centre of the sun to the centre of the earth.

Let R be the radius of the sun and Ts be sun temperature.

Solar luminosity is given by

$Ls = 4\pi r^2 s$…………………(i)

The total energy radiated by sun per second

$E = 4\pi R^2 \sigma T_s^4$…………….(ii)

(i) = (ii)

$4\pi R^2 \sigma T_s^4 = 4\pi r^2 S$

$$T_S = \left(\frac{r}{R}\right)^{\frac{1}{2}} \left(\frac{S}{\sigma}\right)^{\frac{1}{4}}$$

Given that

$R = 7.0 \times 10^8$, $R = 1.496 \times 10^{11} m$

$\sigma = 5.67 \times 10^{-8} Wm^{-2}K^{-4}$

$S = 1.388 \times 103 Wm^{-2}$

$$T_S = \left(\frac{1.496\times10^{11}}{7.0\times108}\right)^{\frac{1}{2}} \left(\frac{1.388\times10^{3}}{5.67\times10^{-8}}\right)^{\frac{1}{4}}$$

$$T_s = 5{,}800K \simeq 6000K$$

Solar constant – is defined as the maximum rate of the amount of energy per square metre reached at the earth surface from the sun since

$$4\pi r^2 s = 4\pi R^2 \sigma T_s^4$$

$$S = \left(\frac{R}{r}\right)^2 \cdot \sigma T_s^4$$

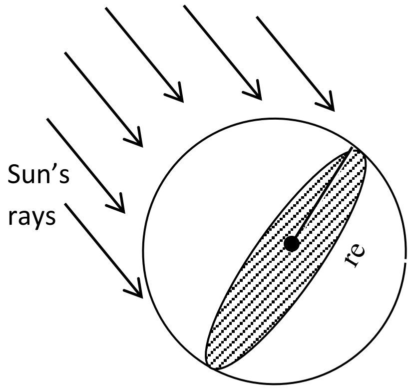
$$S = 1400 Wm^{-2} (\text{approx})$$

Estimation of the surface temperature of the earth.

Assumption made up:

1. Assume that Earth and the sun are black bodies.
2. There is no loss of energy of solar radiation as it travels from the sun to the Earth.
3. The Earth receives radiation from the sun only and no heat generated within the Earth.

Let r_e be mean radius of the earth and Te be the temperature of the earth.



Power radiated by the sun

$$P_s = 4\pi R^2\sigma T_S^4$$

Power received by the Earth

$$P_e = \frac{\pi r_e^2}{4\pi r^2}\times \text{power radiated by sun}$$

Power radiated by the Earth $P_e = 4\pi\sigma r_e^{r} T_e^4$

Assume that the radiation is under equilibrium

Power radiated by the Earth = Power received by the Earth

$$4\pi\sigma r_e^2 T_e^4 = \frac{\pi r_e^2}{4\pi r^2}\times 4\pi R^2\sigma T_S^4$$

$$T_e = \left(\frac{R}{2r}\right)^{\frac{1}{2}} T_S$$

$$T_e = \left(\frac{7.0\times 10^8}{2\times 1.5\times 10^{11}}\right)^{\frac{1}{2}}\times 6000\text{K}$$

$$T_e = 289.82\text{K}$$

ALBEDO (REFLECTIVE CAPACITY)

Is the ratio of sun's energy reflected by a planet to that energy absorbed by the planet.

$$\text{Albedo} = \frac{E_R}{E_a} = \frac{E_R}{E_a}\times 100\%$$

IMPORTANCE OF ALBEDO

1. Give us the information regarding to the presence of the atmosphere and clouds formation on the planet.
2. It is applied on the agriculture activities. Since albedo help for the formation of rain full.

APPLICATION OF THERMAL RADIATION

1. Kettle handle are painted black. This enable them to be handled as they cool by losing heat.
2. The cooling coils of refrigerator are painted black so that they loss heat more efficiently.
3. White clothing is more comfortable to wear in the summer than black clothing. White or light – colored clothing reflects away much of the incoming energy by radiation.
4. **Tea post** – have shiny surface and since shiny surface are bad emitters of radiation, shiny tea posts are able to keep the liquid in it warm for a longer period of time as compared to black – coloured tea pots in addition, shiny surfaces are bad absorbers, thus shiny teapots or container keep cold liquid for a longer time.
5. **Green houses** – are used in cold climates to help grow plants by trapping heat. Day time, infrared radiation from the sun passes through glass roof into the greenhouse. This warm up the plants and help them grow, however, the plants to emit infrared radiation but the infrared radiation emitted is different and cannot pass through the glass roof thus it gets trapped inside. Overtime, the temperature in the greenhouse will increase.

SOLVED EXAMPLES

Example 1

If the radiated power per manometer wavelength from the sun peaks at 490nm approximately. Estimate the temperature of the sun's surface, assuming the sun radiates as a black body ($b = 2.93\times 10^{-3}$mk).

Solution

Since $T = \dfrac{b}{\lambda_m} = \dfrac{2.93\times 10^{-3}}{490\times 10^{-9}}$

$$T = 5{,}979.592\text{K}$$

Example 2

The temperature of a furnace is 2324°C and the intensity is maximum in its radiation spectrum nearly at $12000\,\mathring{A}$. If the intensity in the spectrum of a star is maximum nearly at $4800\,\mathring{A}$, then calculate the surface temperature of the sun.

Solution

According to the Wien's law

$$\lambda_m T = \text{Constant}$$

$$\lambda_2 T_2 = \lambda_1 T_1$$

$$T_2 = T_1\left(\frac{\lambda_1}{\lambda_2}\right) = 2597K\left(\frac{12000}{4800}\right)$$

$$T_2 = 6492.5K = 6219.5^0C$$

Example 3

A black body at 2000K emits radiation with $\lambda_m = 1250nm$. Use this result to calculate the surface temperature of sirrius star if λ_m of sirrius is 71nm. Assume that the sirrius behave like a black body.

Solution

Since $\lambda_m T = \text{Constant}$

$$\lambda_1 T_1 = \lambda_2 T_2$$

$$T_2 = \frac{T_1\lambda_1}{\lambda_2} = \left(\frac{1250}{71}\right)\times 2000K$$

$$T_2 = 31211.3K$$

Example 4

Radiation from the moon gives maxima at $\lambda = 4700\,\mathring{A}$ and $\lambda = 14\times10^{-6}m$. What conclusions can you draw from this? Given $b = 2.9\times10^{-3}mk$.

Solution

According to the Wien's law

$$\lambda T = b$$

$$T_1 = \frac{b}{\lambda_1} = \frac{2.9\times10^{-3}}{4700\times10^{-10}}$$

$$T_1 = 6170K$$

Also $T_2 = \dfrac{b}{\lambda_2} = \dfrac{2.9\times10^{-3}}{14\times10^{-6}}$

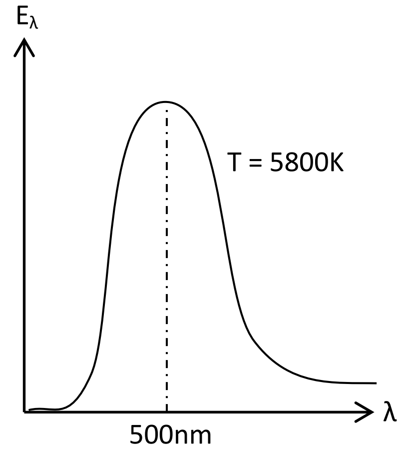
$$T_2 = 207K$$

Conclusion draw. The first one is the temperature of the sun because sun radiation are reflected from the moon's surface. The other is the moon own temperature.

Example 5

(a) Define a black body give one example of a body which approximates to a black body.

(b) The figure below show how Eλ, the energy radiated per unit area per second per unit wavelength for radiation from the sun's surface



Calculate the wavelength (λm) at which the corresponding curves have peak for:

(i) Radiation in the sun core where the temperature is approximately 15 × 10^6K.

(ii) Radiation in interstellar space which corresponds to a temperature approximately 2.7K.

(iii) Name the part of the electromagnetic spectrum to which the calculated wavelength belongs in each case.

Solution

(a) Refer to your notes.

(b) (i) Since λmT = Constant

$$\lambda_1 = \frac{b}{T_1} = \frac{2.9\times10^{-3}}{15\times10^{6}}$$

$$\lambda_1 = 1.93\times10^{-10}m$$

(ii) $\lambda_2 = \frac{b}{T_2} = \frac{2.9\times10^{-3}}{2.7}$

$\lambda_2 = 1.1\times10^{-3}\text{m}$

(iii) At T1 = 15 × 106K lies on X – Ray region

At T2 = 2.7K lies on infra-red region

Example 6

(a) Comment on the statement 'The radiation of a black body is white'

(b) A metal sphere with black surface and radius 30mm is cooled (-73°C) 200K and placed inside temperature of (27°C) 300K. Calculate the initial rate of temperature rise of the sphere assuming the sphere is black body. assume density of metal = 8000kgm^{-3}, specific heat capacity of metal = 400Jkg^{-1}k^{-1} and Stefan's constant = 5.67 × 10-8Wm^{-2}K^{-4}.

Solution

(a) A typical black body absorbs all the radiations incident on it but, when heated to very high temperature, its emits radiations of all wavelengths, which appear as white. For example, sun is considered as a black body although it emits white radiation.

(b) Heat energy gained per second from the surroundings

$$P = 4\pi r^2\sigma\left(T_0^4 - T^4\right) \ldots\ldots.(i)$$

Rate of heat energy gained by sphere

$$\frac{dQ}{dt} = MC\frac{d\theta}{dt} \ldots\ldots\ldots(ii)$$

$$(i) = (ii)$$

$$\frac{4}{3}\pi r^3\rho C\frac{d\theta}{dt} = 4\pi r^2\sigma\left(T_0^4 - T^4\right)$$

$$\frac{d\theta}{dt} = \frac{3\sigma\left(T_0^4 - T^4\right)}{\rho Cr}$$

$$= \frac{3\times5.67\times10^{-8}\times\left(300^4 - 200^4\right)}{0.03\times8000\times400}$$

$$\frac{d\theta}{dt} = 0.12\text{KS}^{-1}$$

Example 6

A spherical black body of radius 12cm radiates 450W power at 500K. If the radius were halved and the temperature doubled, what would be the power radiated?

Solution

Power radiated by the black body, $P = A\sigma T^4$

$$P = 4\pi R^2\sigma T^4$$

$$P_1 = 4\pi R_1^2\sigma T_1^4 \;,\; P_2 = 4\pi R_2^2\sigma T_2^4$$

$$\frac{P_2}{P_1} = \left(\frac{R_2}{R_1}\right)^2\left(\frac{T_2}{T_1}\right)^4$$

$$\frac{P_2}{P_1} = \left(\frac{R/2}{R}\right)^2\left(\frac{2T}{T}\right)^4 = 4$$

$$P_2 = 4P_1 = 4\times450\text{W}$$

$$P_2 = 1800\text{W}$$

Example 7

(a) A cube of side 1.0cm has a grey surface that gives 50% of emission of radiation of black body at the same temperature. The cube's temperature is 700°C. Calculate the power radiated by the cube.

(b) What radius would a black body sphere have so that it would radiated the power calculate in (a) when it is at 300°C? $\sigma = 5.7\times10^{-8}\text{Wm}^{-2}\text{K}^{-4}$

Solution

(a) Area of a cube, A = 6L^2

$$A = 6\times1\text{cm}^2 = 6\text{m}^2 = 6\times10^{-4}\text{m}^2$$

Now $P = e\sigma AT^4$

$$= 0.5\times5.7\times10^{-8}\times6\times10^{-4}\times\left(973\right)^4$$

$$P = 15.33\text{W}$$

(b) For sphere, e = 1

$A = 4\pi R^2$

$$P = 4\pi R^2\sigma T_1^4$$

$$R = \sqrt{\frac{P}{4\pi\sigma T^4}}$$

$$= \sqrt{\frac{15.33}{4\times3.14\times5.7\times10^{-8}\times\left(573\right)^4}}$$

$$R = 14.094\times10^{-3}\text{m}$$

Example 8

(a) Why is the pupil of the eye black?

(b) (i) A body which has a surface area $5.0cm^2$ and a temperature of 727^oC radiates 300J of energy each one minute. What is its emissivity? $\sigma = 5.67 \times 10^{-8}Wm^{-2}K^{-4}$.

Solution

(a) Any radiation that enters the pupil of the eye is reflected from inside many times and is partly absorbed at each reflection until one remains for this reason pupil of eye is black.

(b) (i) according to the Stefan's law, $E = \sigma T^4$

$\therefore$ The energy radiated by the body of emissivity, e surface area, A per second

$$Q = e\times EAt$$

$$Q = e\sigma AT^4t$$

$$e = \frac{Q}{\sigma AT^4t}$$

$$= \frac{300}{5.67\times10^{-8}\times5\times10^{-4}\times(1000)^4\times60}$$

$$e = 0.176$$

$\therefore$ Emissivity of a body e = 0.176.

Example 9

A tungsten filament of total surface area $0.45cm^2$ is maintained at a steady temperature of 2227^oC calculate the electrical energy is dissipated if all this energy is radiated to the surroundings. (Emissivity of tungsten at 2227^oC is 0.3).

Solution

$$\text{Since } P = eA\sigma T^4$$

$$= 0.3\times0.45\times10^{-4}\times5.7\times10^{-8}\times(2500)^4$$

$$P = 30.06W$$

Example 10

An electrical heating element has surface area of $6 \times 10^{-3}m^2$ and working temperature of 1300K. It is used to boil water. Estimate the rate at which heater losses heat by radiation to water when water starts to boils. Emissivity of heater is 0.5 and $\sigma = 5.7 \times 10^{-8}Wm^{-2}K^{-4}$.

Solution

Net power loss by heater is given by

$$\frac{dQ}{dt} = eA\sigma\left[T_1^4 - T_2^4\right]$$

$$= 0.5\times6\times10^{-3}\times\left[(1300)^4-(373)^4\right]\times5.7\times10^{-8}$$

$$\frac{dQ}{dt} = 485.08W$$

Example 11 NECTA 1998/P1

(a) Explain why a body at 1000K is red hot whereas the body at 2000K is white – hot.

(b) Determine the temperature of electrical bulb filament rated 100W whose diameter is 0.2mm and length is 40cm if 75% of power developed is lost through the radiation.

Solution

(a) A body at 1000K is red – hot since this is the ordinary temperature, the maximum emission of radiation is corresponds to the longer wavelength (i.e red – hot) and at 2000K is white – hot because at higher temperature the maximum emission of radiation is corresponds to the shorter wavelength i.e white-hot and this is according to the Wien's law of displacement

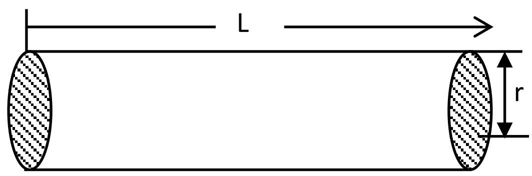
i.e $\lambda_m \; \alpha \; {}^1\!/_T$

(b) P_1 = 100W

Power lost as heat by radiation

$$P = P_1\times\frac{75}{100} = \frac{75}{100}\times100W$$

$$P = 75W$$



According to the Stefan's law

$$P = \sigma AT^4 = 2\pi rL\sigma T^4$$

$$T = \left[\frac{P}{2\pi r\sigma L}\right]^{1/4}$$

$$= \left[\frac{75}{2\pi\times0.1\times10^{-3}\times0.4\times5.7\times10^{-8}}\right]^{1/4}$$

$$T = 2690.8K$$

Example 12

A perfect black body sphere maintained at 273K radiates heat at a rate of 200W inside an enclosure maintained at OK. What rate will the sphere radiates energy in the following cases:-

(a) The radius is doubled
(b) The temperature is raised by 746K
(c) The temperature of enclosure is raised to 300K. What is the net rate of heat loss by the sphere is above.

Solution

For perfect black body, e = 1

$$P = \sigma A\left(T^4 - T_0^4\right)$$

(a) $P_1 = 4\pi R_1^2\sigma\left(T_1^4 - T_0^4\right)$.........(i)

$P_2 = 4\pi R_2^2\sigma\left(T^4 - T_0^4\right)$.........(ii)

(ii)/(i)

$$\frac{P_2}{P_1} = \left(\frac{R_2}{R_1}\right)^2 = \left[\frac{2R}{R}\right]^2$$

$$P_2 = 4P_1 = 4\times200W$$

$$P_2 = 800W$$

(b) $P_1 = \sigma A\left(T_1^4 - T_0^4\right)$........(i)

$P_2 = \sigma A\left(T_2^4 - T_0^4\right)$.........(ii)

Take (ii)/(i)

$$\frac{P_2}{P_1} = \frac{\left(T_2^4 - T_0^4\right)}{\left(T_1^4 - T_0^4\right)}$$

$$P_2 = 200W\left[\frac{(746)^4 - 0^4}{(373)^4 - 0^4}\right]$$

$$P_2 = 3200W$$

(c) $P_2 = \sigma A\left[T_1^4 - T_{01}^4\right]$

$$= \sigma A T_1^4\left[1 - \left(\frac{T_{01}}{T_1}\right)^4\right]$$

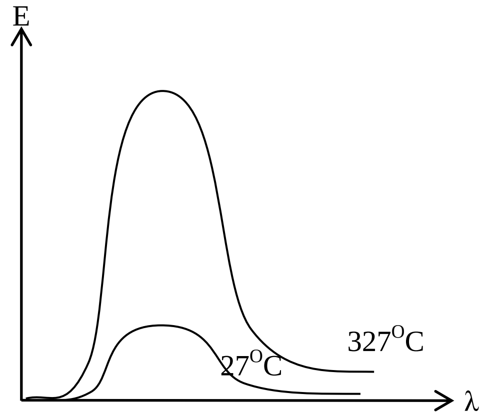
$$= P_1\left[1 - \left(T_{01}/T_1\right)^4\right]$$

$$= 200W\left[1 - \left(\frac{300}{373}\right)^4\right]$$

$$P_2 = 116.31W$$

Example 13

The spectrum of a black body at two temperatures 27ºC and 327ºC is shown in the figure below. Let A_1 and A_2 be respective areas under the two curves. Estimate the ratio A_2/A_1



Solution

T_1 = 27ºC = 27 + 273 = 300K

T_2 = 327ºC = 327 + 273 = 600K

Area (A) under intensity wavelength curve measures the amount of thermal energy (E) radiated per second per unit area from a black body.

$$\frac{A_2}{A_1} = \frac{E_2}{E_1} = \frac{\sigma T_2^4}{\sigma T_1^4}$$

$$\frac{A_2}{A_1} = \left(\frac{T_2}{T_1}\right)^4 = \left(\frac{600}{300}\right)^4$$

$$\frac{A_2}{A_1} = 2^4 = 16$$

Example 14

(a) In summer one should not wear black clothes. Why?
(b) An aluminium foil of relative emittance 0.2 is placed between two conceatric sphere at temperature 300k and 200k respectively.

Calculate the temperature of the foil after the steady state is reached. Also calculate the rate of energy transfer between one of the spheres and the foil.

Solution

(a) In summer one should not wear black clothes because black or coloured clothes absorb more heat than white clothes.

(b) Let the temperature of the sphere be T which is assumed to be less than 300k but greater than 200K. At steady state condition. Power gained by the foil from 1st sphere = power lost by foil to the 2nd sphere.

$$eA\sigma\left[(300)^4 - T^4\right] = eA\sigma\left[T^4 - 200^4\right]$$

$$T = \sqrt[4]{\frac{(300)^4 + (200)^4}{2}}$$

$$T = 263.9\text{K}$$

Rate of energy transfer between the two sphere

$$\frac{P}{A} = e\sigma\left[(300)^4 - T^4\right]$$

$$= 0.2 \times 5.67 \times 10^{-8}\left[(300)^4 - (263.9)^4\right]$$

$$\frac{P}{A} = 36.86\text{Wm}^{-2}$$

Example 15

(a) What is Prevost's theory of heat exchanges? Describe some phenomenon of theoretical or practical importance to which it applies.

(b) A metal sphere of 1cm diameter, whose surface acts as a black body is placed at the focus of a concave mirror with aperture of diameter 60cm directed towards the sun. If the solar radiation falling normally on the earth is at the rate of 0.14Wcm^{-2}. Stefan constant is taken as $6 \times 10^{-8}\text{Wm}^{-2}\text{K}^{-4}$ and the mean temperature of the surroundings is 27°C, calculate the maximum temperature which sphere could theoretically attain, starting any assumptions you make.

Solution

(a) Refer to your notes

(b) The solar power incident on the mirror which is focused to the sphere.

$P = 1400 \times$ normal area of mirror

$= 1400 \times \pi \times (0.30)^2$

$P = 126\pi$w

Net power absorbed = $P - e\sigma A(300)^4$

Let T be its maximum temperature. Then net absorbed = $\varepsilon\sigma AT^4$

$$\varepsilon A\sigma T^4 = P - \varepsilon A(300)^4$$

$$T = \sqrt{\frac{P}{\varepsilon A\sigma} - (300)^4}$$

$$= \sqrt{\frac{P}{\varepsilon\sigma 4\pi r^2} - (300)^4}$$

$$= \sqrt{\frac{126\pi}{1 \times (4\pi \times 0.005^2) \times 6 \times 10^{-}} - (300)^4}$$

$$T = 2140.5\text{K}$$

Example 16

The sun's rays are focused by a concave mirror of diameter 12cm fixed with its axis towards the sun onto a copper colorimeter, where they are absorbed. If the thermal capacity of the colorimeter and its contents is 248J/K and the temperature rises 8°C in 2 minutes. Calculate the heat received by a square meter of the earth's surface when the rays are incident normally.

Solution

Heat absorbed per second by the colorimeter and its contents $= \dfrac{248 \times 8}{2 \times 60}$

$= 16.533\text{Js}^{-1}$

Heat absorbed by the mirror per unit area per minute.

$= 1462 \times 60$

$= 87712\text{Jmin}^{-1}\text{m}^{-2}$

$\therefore$ Heat received in 1min by a square metre of the Earth's surface = $87712\text{Jmin}^{-1}\text{m}^{2}$

Example 17

A blackned metal sphere of diameter 10mm is placed at the focus of a concave mirror of diameter 0.5m directed towards the sun. if the solar power incident on the mirror is $1600Wm^{-2}$. Calculate the maximum temperature can attain.

Solution

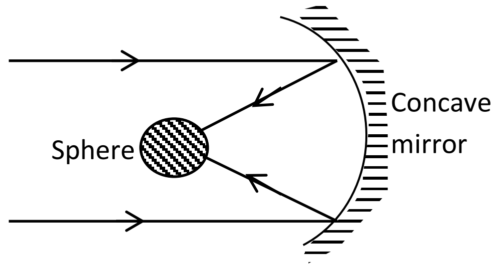
Power incident on the mirror

$P = \pi r^2 \times$ Intensity

$= 3.14 \times (0.25)^2 \times 1600$

$P = 314W$

Power absorbed by the sphere = 314W



Let T be the equilibrium temperature of the sphere. Power radiated

$P = \sigma AT^4 = 4\pi R^2\sigma T^4$

As the equilibrium power absorbed by the sphere is equal to the power radiated by sphere.

$$314 = 4\pi R^2\sigma T^4$$

$$T = \left[\frac{314}{4\pi R^2\sigma}\right]^{1/4}$$

$$= \left[\frac{314}{4\times3.14\times\left(5\times10^{-3}\right)^2\times5.7\times10^{-8}}\right]^{1/4}$$

$$T = 2047K$$

Example 18

The tungsten filament of an electric lamp is 60W has a length of 0.5m and diameter of 6.0×10^{-5}m. The radiation energy from the lamp is 80% of its power values. Calculate the average power per m^2 radiated from the filament surface area.

Solution

Let P = power of filament

P_1 = Power of radiation as the heat.

$$P_1 = \frac{80}{100}\times60W = 48W$$

Surface area of filament $A = 2\pi RL$

$$= \frac{P_1}{2\pi RL} = \frac{48}{2\pi\times3\times10^{-5}\times0.5}$$

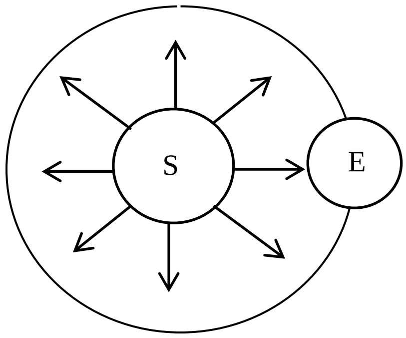
$$= 5.1\times10^3 Wm^{-2}$$

Note that:

The power radiated m^{-2} is high because the temperature of the filament is high approximate to 2000K.

Example 19

The solar constant, the energy per second arriving at the Earth's surface, E from the sun is about $1400Wm^{-2}$ as illustrated figure below.



Estimate the sun's temperature if its radius is 7.0×10^8m, the sun distance from the Earth is 1.5×10^{11}m and the sun radiation = KAT^4 where $K = 6.0 \times 10^8$, A = surface cross-sectional area of the sphere and T is Kelvin temperature.

Solution

Radiation from sun's surface

$P_s = KAT^4 = 4\pi R_s^2 K T_s^4$

Radiation of Earth's surface per m^2 is equal to the solar constant.

$$Q = \frac{P_S}{\text{Total area}} = \frac{P_S}{4\pi r^2}$$

$$Q = \frac{4\pi K R_S^2 T_S^4}{4\pi r^2}$$

$$T_S = \left[\frac{Q}{K}\left(\frac{r}{R_S}\right)^2\right]^{1/4}$$

$$= \left[\frac{1400}{6\times10^8}\left(\frac{1.5\times10^{11}}{70\times10^8}\right)^2\right]^{1/4}$$

$$T_S = 6000K$$

Example 20

The tungsten filament of an electric lamp has a length of 0.5m and diameter 6.0×10^{-5}m. The power rating of the lamp is 60W. Assuming the radiation from the filament is equivalent to 80% than of a perfect black body radiator at the same temperature. Estimate the steady temperature of the filament. (Stefan's constant = $5.7 \times 10^{-8} Wm^2K^{-4}$)

Solution

When the temperature is under steady state.
Power radiated from filament = power received

$0.8 \times 5.7 \times 10^{-8} \times 2\pi \times 10^{-5} \times 0.5 \times T^4 = 60$

T = 1933K

Example 21

(a) What is meant by black body? how black body can be realized?

(b) State the Stefan's law of radiation and indicate the conditions of its application.

(c) A solid metal sphere is found to cool at the rate of 1.2°C per minute when the its temperature is 127°C. At what rate will a sphere of three times the radius cool when is at a temperature of 327°C, in each case external temperature is 27°C. Assume Stefan's law applies in each case.

Solution

(c) Since $MC\left(\frac{d\theta}{dt}\right)_1 = 4\pi r^2\sigma\left(T_1^4 - T_0^4\right)$

$$\frac{4}{3}\pi r^3\rho c\left(\frac{d\theta}{dt}\right)_1 = 4\pi r^2\sigma\left(T_1^4 - T_0^4\right)\ldots.(1)$$

For a sphere of three times the radius

$$\frac{4}{3}\pi(3r)^3\rho c\left(\frac{d\theta}{dt}\right)_2 = 4\pi(3r)^2\sigma\left(T_2^4 - T_0^4\right)..(2)$$

On dividing equation (2) by (1), we get

$$\left(\frac{d\theta}{dt}\right)_2 = \frac{1}{3}\left[\frac{T_2^4 - T_0^4}{T_1^4 - T_0^4}\right]\cdot\left(\frac{d\theta}{dt}\right)_1$$

$$= \frac{1}{3}\left[\frac{600^4 - 300^4}{400 - 300^4}\right]\times 1.2$$

$$\left(\frac{d\theta}{dt}\right)_2 = 2.8^\circ C\,min^{-1}$$

Example 22

The intensity of the sun's radiation falling normally on the earth is about $1500Wm^{-2}$. If this radiation falls on the metal object of area $10cm^2$, mass 0.05kg and specific heat capacity $500Jkg^{-1}k^{-1}$ calculate the initial rate of temperature rise of the object. What happens to the temperature of the object as the sun's radiation. Continues to fall on its?

Solution

The initial rate of rise of temperature

$$\frac{d\theta}{dt} = \frac{P}{MC} = \frac{1500\times10\times10^{-4}}{0.05\times500}$$

$$\frac{d\theta}{dt} = 0.06KS^{-1}$$

The temperature of the object will continue rising until an equilibrium is established where the rate of heat its loss to the surroundings balances to that it gains from the surroundings.

Example 23

Luminocity of Rigel star in onion constellation is 17000 times that of sun. If the surface temperature of the sun is 6000K, Calculate the temperature of the star.

Solution.

The energy radiated by a black body per unit area per second is given by

$$E = \sigma\left(T^4 - T_0^4\right)$$

If $T_0 \ll T$

$$E = \sigma T^4$$

Let E_1 and E_2 are luminosities of the star and the sun respectively and T_1, T_2 their respectively temperature.

$$\frac{E_1}{E_2} = \frac{\sigma T_1^4}{\sigma T_2^4}$$

$$T_1 = T_2\left[\frac{E}{E_2}\right]^{1/4} = 6000\left[17000\right]^{1/4}$$

$$T_1 = 68520K$$

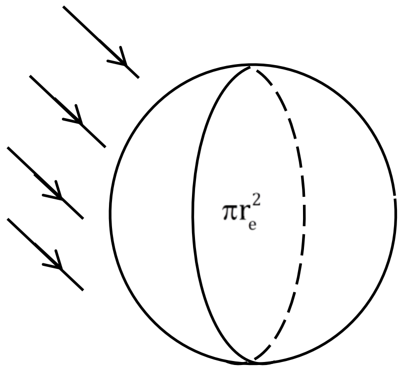
Example 24

Estimate the temperature Te of the Earth assuming that is radiactive equilibrium with the sun. Assuming the radius of the sun

$r_S = 7.0 \times 10^8$ m, temperature of the surface is 6000k distance of Earth from the sun,

$R = 1.5 \times 10^{11}$m.

Solution



Power radiated from the sun

$$P_S = 4\pi r_s^2 \sigma T_S^4$$

Power received by the Earth

$$P_e = \frac{\pi r_e^2}{4\pi R^2} \times \text{Power radiated by the sun}$$

Assuming the radioactive equilibrium

Power radiated By Earth = Power received by the Earth

$$4\pi r_e^2 \sigma T_e^4 = \frac{4\pi r_s^2 \sigma T_s^4}{4\pi R^2} \times \pi r_e^2$$

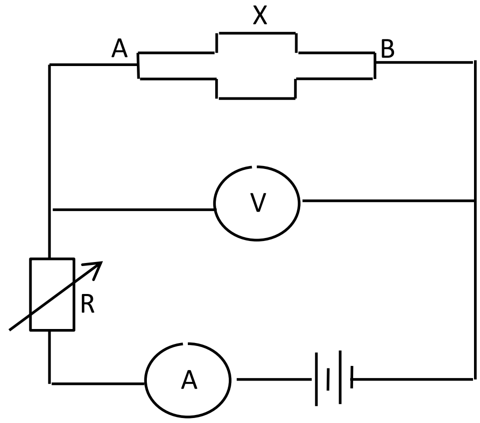
$$T_e = \left[\frac{r_s}{2R}\right]^{1/2} T_S$$

$$= \left[\frac{7.0\times10^8}{2\times1.5\times10^{11}}\right]^{1/2} \times 6000\text{K}$$

$$T_e = 209\text{K}$$

Example 25

In the figure below AB is the wire of length 1.3m and diameter 0.3mm placed in an evacuated glass tube X. The wire is coated black. The current in the wire is gradually increased to 20A. At this point the voltmeter reads 30V and



The wire is about to melt. Assuming that the wire radiates as a black body and the radiation from the glass is negligible.

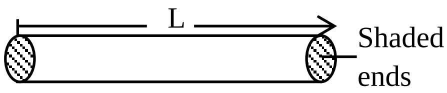
(i) Estimate the melting point of the metal wire.

(iv) Calculate the wavelength which is emitted with maximum intensity from the wire at this temperature.

Stefan's Constant = 5.7×10^{-8} Wm^{-2} K^{-4}.

Solution.

(i)



Radiating surface area

$A = 2\pi rL$.

Rate of heat generated = power radiated

$$IV = \sigma A T^4$$

$$T = \left[\frac{IV}{2\pi rL\sigma}\right]^{1/4}$$

$$= \left[\frac{2\times30}{5.7\times10^{-8}\times2\pi\times1.5\times10^{-4}\times1.3}\right]^{1/4}$$

$$T = 1700\text{K}$$

(ii) $$\lambda_m = \frac{b}{T} = \frac{2.9\times10^{-3}}{1700}$$

$$\lambda_m = 1.7\times10^{-6}\text{m}$$