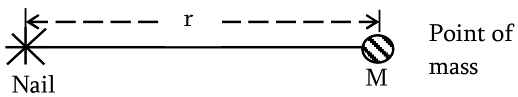
Example – 01

A point mass 200gm is attached to one end of string and another end is attached to a nail. The mass is made to rotate in a circle of radius 20cm. What is the moment of inertia of the particle about the axis of the nail?

Solution

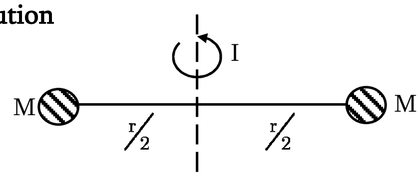


$$\begin{aligned} M.I &= Mr^2 \\ &= 0.20\times(0.2)^2 \\ &= 0.4kgm^2 \end{aligned}$$

Example – 02

The distance between the two atoms in a diatomic molecules is 1.21×10^{-10}m. The mass of each oxygen atom is 2.66×10^{-26}kg. Treating the atoms as point masses. Find the moment of inertia of the molecule about an axis passing perpendicular to the line joining the centre of the atoms.

Solution



M.I of the system

$$I = M\left(\frac{r}{2}\right)^2 + M\left(\frac{r}{2}\right)^2$$

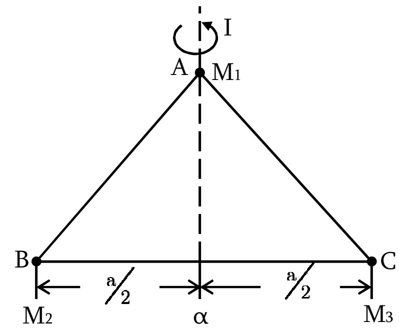
$$I = \frac{1}{2}Mr^2$$

$$I = \frac{1}{2}\times 2.66\times10^{-26}\times\left(1.21\times10^{-10}\right)^2$$

$$I = 1.95\times10^{-46}Kgm^2$$

Example – 03

Three mass points M_1 , M_2 , M_3 are located at the vertices of an equilateral triangle of length 'a' as shown in the figure below. What is the moment of inertia of the system about an axis along the altitude of the triangle passing through M_1?



Solution

M.I of the system about the altitude AQ

$$I = M_1r_1^2 + M_2r_2^2 + M_3r_3^2$$

$$= M_1(0)^2 + M_2\left(\frac{a}{2}\right)^2 + M_3\left(\frac{a}{2}\right)^2$$

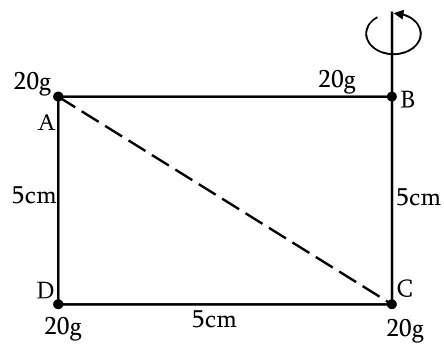
$$I = \frac{a^2}{4}\left(M_2 + M_3\right)$$

Example – 04

Four small bodies A , B , C and D which can be considered as particles connected by rods of negligible masses as shown in figure below. Find the M.I of the system:

(i) About an axis coinciding with rod BC and

(ii) About an axis passing through A and perpendicular to the plane of diagram.



Solution

(i) M.I about an axis coinciding with BC

$$I = 20(AB)^2 + 20(BB)^2 + 20(CC)^2 + 20(DC)^2$$

$$BB = CC = 0 \;,\; AB = DC = 5cm$$

$$I = 20(5)^2 + 20(0)^2 + 20(0)^2 + 20(5)^2$$

$$I = 1000gcm^2$$

(ii) M.I about an axis through the point A and perpendicular to the plane of the diagram.

$$I = 20(AA)^2 + 20(BA)^2 + 20(CA)^2 + 20(DA)^2$$

Now

$$CA = \sqrt{(AB)^2 + (BC)^2} = \sqrt{5^2 + 5^2}$$

$$\overline{CA} = 5\sqrt{2}\text{cm}$$

$$I = 20(0)^2 + 20(5)^2 + 20(5\sqrt{2})^2 + 20(5)^2$$

$$I = 2000\text{gcm}^2$$

Example – 05

Calculate the moment of inertia about a transverse axis through the centre of a disc , whose radius is 20cm. its density is 9gcm^{-3} and its thickness is 7cm.

Solution

Mass of the disc

$$M = \pi R^2 t\rho$$

$$= \frac{22}{7} \times (20)^2 \times 7 \times 9$$

$$M = 79200\text{g}$$

M.I of the disc about the centre , $I = \frac{1}{2}MR^2$

$$= \frac{1}{2} \times 79200 \times (20)^2$$

$$I = 1.584 \times 10^7 \text{gcm}^2$$

Example – 06

(a) (i) How is moment of inertia of a rigid body measured?

(ii) Why is the moment of inertia also called rotational inertia?

(b) What are the factors on which moment of inertia of a body depends?

(c) Derive an expression for the moment of inertia of a hollow sphere (thick spherical shell) about a diameter. Given that the M.I of a solid sphere of mass M and radius R about a diameter is $2/5\ MR^2$

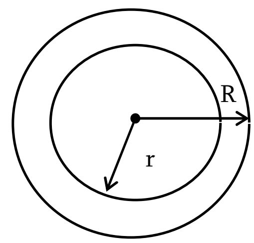
Solution

(a) (i) The moment of inertia of a rigid body about a given axis of rotation is the sum of the products of the masses of its particles and the squares of their respective perpendicular distances from the axis of rotation.

(ii) The moment of inertia is also called rotational inertia because it is a measure of inertia of a rotating body

(b) Refer to your notes

(c) Consider a solid sphere. Suppose a smaller concentric sphere is removed from the solid sphere we get a hallow or a thick spherical shell as shown below.



r = internal radius of the hallow sphere

R = External radius

Volume of the shell $= \frac{4}{3}\pi(R^3 - r^3)$

Shell $\rho = \dfrac{M}{\frac{4}{3}\pi(R^3 - r^3)} = \dfrac{3M}{4\pi(R^3 - r^3)}$

M.I of the solid sphere of radius about a diameter

$$= \frac{2}{5}(\text{mass})(\text{radius})^2$$

$$I_S = \frac{2}{5} \times \frac{4}{3}\pi R^3 \rho R^2 = \frac{8}{15}\pi R^5 \rho$$

M.I of the solid sphere of radius r about a diameter.

$$I_r = \frac{8}{15}\pi r^5 \rho$$

M.I of the hollow sphere about the diameter, I

$$I = I_S - I_r$$

$$= \frac{8}{15}\pi\rho R^5 - \frac{8}{15}\pi\rho r^5$$

$$= \frac{8}{15}\pi\rho(R^5 - r^5)$$

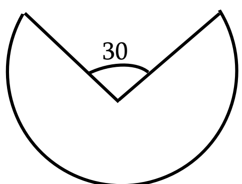
$$= \frac{8\pi}{15} \cdot \frac{3M}{4\pi\left(R^3 - r^3\right)} \cdot \left(R^5 - r^5\right)$$

$$I = \frac{2}{5} M \frac{\left(R^5 - r^5\right)}{\left(R^3 - r^3\right)}$$

Example – 07

From a complete ring of mass , M and radius R a 30^{O} sector is removed. What is the moment of inertia of the complete ring about an axis passing through the centre of the ring and perpendicular to the plane of the ring?

Solution



Mass of incomplete ring

$$= M - \frac{M}{2\pi} \times \frac{\pi}{6} = \frac{11M}{12}$$

M.I of incomplete ring

$$I = \frac{11}{12} MR^2$$

EXAMPLES

1. Expression of radius of gyration of uniform rod about an axis passing through the center and perpendicular to its length.

$$I_G = MK^2 = \frac{ML^2}{12}$$

$$K = \frac{L}{\sqrt{12}}$$

2. Expression of radius of gyration of uniform circular disc about an axis passing through the centre and perpendicular to the plane of disc.

$$I_G = \frac{1}{2} MR^2 = MK^2$$

$$K = \frac{R}{\sqrt{2}}$$

3. Expression of radius of gyration of uniform solid sphere about an axis through the centre.

$$I_G = \frac{2}{5} MR^2 = MK^2$$

$$K = \sqrt{\frac{2}{5}} R$$

NUMERICAL EXAMPLES

Example – 08

If the radius of a sphere is 5cm , calculate the radius of gyration.

(i) About its diameter

(ii) About its tangent

Solution

(i) M.I of the sphere about its diameter

$$I_G = \frac{2}{5} MR^2 = MK^2$$

$$K = \sqrt{\frac{2}{5}}\, R = \sqrt{\frac{2}{5}} \times 5cm$$

$$K = 3.162cm$$

(ii) Apply parallel axess theorem

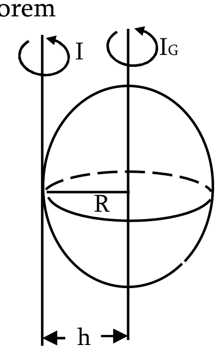
$$I = I_G + Mh^2$$

$$= \frac{2}{5} MR^2 + MR^2$$

$$I = \frac{7}{5} MR^2$$

But $I = MK^2$

$$MK^2 = \frac{7}{5} MR^2$$



$$K = \sqrt{\frac{7}{5}}\, R = \sqrt{\frac{7}{5}} \times 5cm$$

$$K = 5.92cm$$

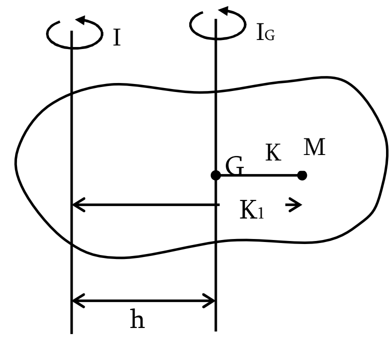
Example – 09

(a) Radius of gyration of a body about an axis at a perpendicular distance of 6cm from its centre of mass is 10cm. Find its radius of gyration about a parallel axis through its centre of mass.

(b) The diameter of flywheel increases by 1%. What will be the percentage increase in moment of inertia about axis of symmetry?

Solution

(a) K_1 = 10cm , h = 6cm , K = ?, K_1 = radius of gyration of the body about the given axis.



Apply parallel axes theorem

$$I = I_G + Mh^2$$
$$MK_1^2 = MK^2 + Mh^2$$
$$K_1^2 = K^2 + h^2$$
$$K = \sqrt{K_1^2 - h^2} = \sqrt{10^2 - 6^2}$$
$$K = 8cm$$

(b) $I = MR^2$

$$\log_e^I = \log_e\left(MR^2\right) = \log_e^M + \log_e^{R^2}$$
$$\log_e^I = \log_e^M + 2\log_e^R$$

On differentiating

$$\frac{dI}{I} = 2\frac{dR}{R}$$
$$\frac{dI}{I}\times 100\% = 2\left[\frac{dR}{R}\times 100\%\right]$$
$$= 2\times 1\%$$
$$\frac{dI}{I}\times 100\% = 2\%$$

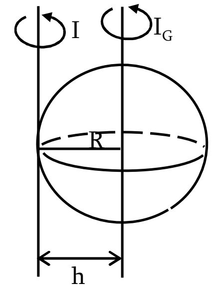
Example – 10

(a) Calculate the moment of inertia of a uniform disc of mass 0.4kg and radius 10cm about an axis through its edge and perpendicular to the plane of the disc.

(b) Find the radius of gyration of sphere of radius 15cm without an axis 5cm from the centre ($I_G = 0.4MR^2$)

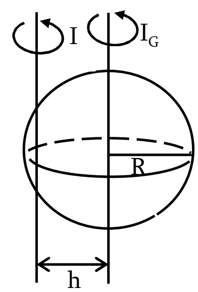
Solution

(a) Apply parallel axes theorem



$$I = I_G + Mh^2$$
$$= \frac{1}{2}MR^2 + MR^2$$
$$I = \frac{3}{2}MR^2$$
$$= \frac{3}{2}\times 0.4\times\left(0.1\right)^2$$
$$I = 6.0\times 10^{-3}\,Kgm^2$$

(b) Apply parallel axes theorem



$$I = I_G + Mh^2$$
$$MK^2 = 0.4MR^2 + Mh^2$$
$$K = \sqrt{0.4R^2 + h^2}$$
$$= \sqrt{0.4\left(0.15\right)^2 + \left(0.05\right)^2}$$
$$K = 0.11m$$

Example – 11

(a) Define angular acceleration and moment of inertia of rigid body.

(b) Some rivers flows towards the equator and hence transport more sediment from higher to lower latitudes. How does the process affect the earth rotation?

Solution

(b) This process tends to show down the earth rotation, due to the deposition of sediment and particles which can be carried by the river tends to increase the moment of inertia.

Example – 12

(a) The mass of a disc is 700gm and its radius of gyration is 20cm. What is its moment of inertia if it rotates about an axis passing through its centre and perpendicular to the face of the disc?

(b) The mass of a flywheel is concentrated on the rim why?

Solution

(a) $I = MK^2$

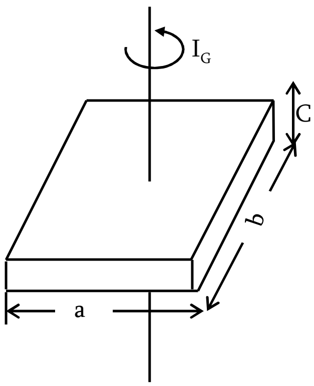
$$= 0.7\times\left(0.2\right)^2$$

$$I = 0.028Kgm^2$$

(b) This is to increase the moment of inertia. Hence its opposition to any change in uniform rotatory motion is large. So when a fly wheel of large M.I is used the engine runs smoother and steadier.

Example – 13

Derive an expression of moment of inertia of slab about perpendicular axis through the centre as shown on the figure below.



Solution

Consider the mass of small portion

$$\frac{dM}{M} = \frac{adx}{ab} = \frac{dx}{b}$$

$$dM = \frac{Mdx}{b}$$

M.I of the strip of mass ,dM

$$dI_G = \frac{1}{2}a^2dm$$

Apply the parallel axes theorem

$$dI = dI_G + h^2dM \qquad h = x$$

$$dI = \frac{a^2}{12}dM + x^2dM$$

$$I = \frac{a^2}{12}\int_0^M dM + \frac{M}{b}\int_{-b/2}^{b/x} x^2dx$$

$$I = \frac{M}{12}\left(a^2 + b^2\right)$$

NUMERICAL EXAMPLES

Example – 14

(a) (i) Why is moment of inertia called rotational inertia?

(ii) Give the physical significance of moment of inertia.

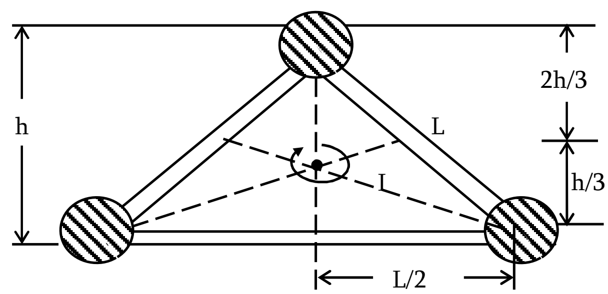
(b) Calculate moment of inertia of system made by connecting three spherical balls of mass, M using rods of length , L if the system rotates about an axis through the centre of gravity and perpendicular to the plane of the system.

Solution

(a) (i) The moment of inertia is called rotational inertia for the reason that it gives the measure of inertia of a body during its rotational motion.

(ii) The moment of inertia of body plays the same role in rotational motion as its mass does in linear motion.

(b) Let I = Moment of inertia of the system.



By using Pythagoras theorem

$$h = \sqrt{L^2 - \frac{L^2}{4}} = \sqrt{L^2 - \left(L/2\right)^2}$$

$$h = \frac{L\sqrt{3}}{2}$$

Since $r_1 = r_2 = r_3 = r = \frac{2h}{3} = \frac{2}{3}\cdot\frac{L\sqrt{3}}{2}$

$$r = \frac{L\sqrt{3}}{3}$$

Now $I = M_1r_1^2 + M_2r_2^2 + M_3r_3^2$

$$= Mr^2 + Mr^2 + Mr^2$$

$$= 3Mr^2 = 3M\left(\frac{L\sqrt{3}}{3}\right)^2$$

$$I = ML^2$$

Example – 15

If the radius of a sphere is 5cm. calculate the radius of gyration.

(i) About its diameter

(ii) About its tangent.

Solution

(i) M.I of the sphere about its diameter.

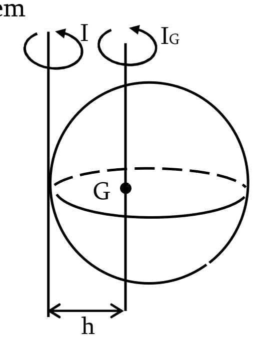
$$I_G = \frac{2}{5}MR^2 \quad \text{but} \quad I_G = MK^2$$

$$MK^2 = \frac{2}{5}MR^2$$

$$K = R\cdot\sqrt{\frac{2}{5}} = 5\times\sqrt{\frac{2}{5}}$$

$$K = 3.162\text{cm}$$

(ii) Apply parallel axes theorem



$$I = I_G + Mh^2$$

$$= \frac{2}{5}MR^2 + MR^2$$

$$I = \frac{7}{5}MR^2$$

$$\text{But } I = MK^2$$

$$MK^2 = \frac{7}{5}MR^2$$

$$K = R\cdot\sqrt{\frac{7}{5}} = 5\times\sqrt{\frac{7}{5}}$$

$$K = 5.92\text{cm}$$

Example – 16

(a) If no external torque acts on a body , will its angular velocity remain conserved?

(b) A wheel is rotating at 90revmin^{-1}, What is the torque required to bring it to rest in 5 revolutions if the moment of inertia of the wheel is 0.80kgm^2.

Solution

(a) If no external torque acts ($\tau = 0$) then angular momentum of the body will remain conserved i.e $I\omega$ = constant as in the rotational motion , the moment of inertia (I) of the body can change due to the change in position of the axis of rotation, the angular speed may not remain conserved. However , if the position of the axis of rotation also remains fixed , the angular speed will remain conserved.

(b) $\omega_0 = 2\pi f_0 = \dfrac{2\pi\times 90}{60} = 3\pi\text{rads}^{-}$

$$1\text{ rev} \longrightarrow 2\pi$$

$$5\text{ rev} \longrightarrow \theta$$

$$\theta = \frac{5\text{rev}\times 2\pi}{1\text{rev}}$$

$$\theta = 10\pi\text{rad}$$

$$\text{Now } \omega^2 = \omega_0^2 + 2\alpha\theta$$

The wheel will comes to rest , $\omega = 0$

$$0^2 = \omega_0^2 + 2\alpha\theta$$

$$\alpha = \frac{-\omega_0^2}{2\theta} = \frac{-(3\pi)^2}{2\times 10\pi}$$

$$\alpha = -0.45\pi\text{rads}^{-2}$$

Negative sign show that the flywheel is showing down.

$$\tau = I\alpha$$

$$= 0.80\times(-0.45\pi)$$

$$\tau = -0.36\pi\text{Nm}$$

Negative sign show that the torque is applied in the opposite direction to that of the original rotation.

Example – 17

(a) If no external torque acts on a body , will its angular velocity remain constant? Explain.

(b) A flywheel rotates on a bearing which exerts a constant frictional torque of 12Nm an external torque of 36Nm acts on the flywheel for a time of 15sec, after which the time is removed. If the angular velocity of the flywheel increases from zero to 60rads^{-1} in 15second period.

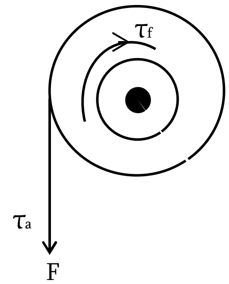
(i) Calculate the moment of inertia of the flywheel.

(ii) Find at what time the flywheel will come to rest.

Solution

(a) If no external torque acts on body, angular momentum (L) of the body will remain constant. Now L = $I\omega$ therefore , angular velocity will remain constant , only if the moment of inertia of the body remains constant.

(b) (i)



Resultant torque

$\tau = \tau_a - \tau_f$

τ_a = applied or external torque

τ_f = frictional torque

$\tau = 36 - 12 = 24Nm$

$\tau = 24Nm$

Since $\omega_0 = 0$

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{60-0}{15}$$

$$\alpha = 4rads^{-2}$$

Again $\tau = I\alpha$, $I = \frac{\tau}{\alpha}$

$$I = \frac{24}{4}$$

$$I = 6.0kgm^2$$

(ii) When the external torque is removed the flywheel shows down and net torque τ_1 of -12Nm now acts on it due to friction.

$$\alpha_1 = \frac{\tau_1}{I} = \frac{-12}{6} = -2rads^{-2}$$

$\omega_0 = 60rads^{-2}$, $\omega = 0$

Finally the flywheel comes to rest($\omega = 0$)

$$t_1 = \frac{\omega - \omega_1}{t} = \frac{0-60}{-2}$$

$$t_1 = 30\,sec$$

$\therefore$ After $t_1 = 30$ second the external torque is removed.

Example – 18

(a) (i) Define radius of gyration of a body.

(ii) Find the radius of gyration of a solid sphere of a diameter 2.0m rotating about a diameter as an axis.

(b) A wheel of radius 0.72m and moment of inertia 4.8kgm² has a constant force of 10N applied tangentially at the rim calculate.

(i) Angular acceleration

(ii) The angular speed after 4.0seconds from rest

(iii) The number of revolutions made in 4.0 seconds.

(c) The maximum and minimum distance of a comment from the sun are 1.40×10^{12}m and 7.0×10^{10}m. If its velocity nearest to the sun is 6.0×10^4m/s. Find its velocity in the furthest position from the sun. State assumptions made in your calculations.

Solution

(a) (i) Refer to your notes

(ii) M.I of a sphere

$$I_G = MK^2 = \frac{2}{5}MR^2$$

$$K = \sqrt{2/5}\,R = 1.0m \times \sqrt{2/5}$$

$$K = 0.63m$$

(b) (i) Torque on the wheel

$$\tau = I\alpha = FR$$

$$\alpha = \frac{FR}{I} = \frac{10 \times 0.72}{4.8}$$

$$\alpha = 1.5rads^{-2}$$

(ii) $\omega = \omega_0 + \alpha t = 0 + 1.5 \times 4$

$$\omega = 6.0rads^{-1}$$

(iii) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$$= 0 \times 4 + \frac{1}{2} \times 1.5 \times 4^2$$

$$\theta = 12rad$$

$$N = \frac{\theta}{2\pi} = \frac{12}{2\pi}$$

$$N = 1.91 \simeq 2 \text{ revolution}$$

(c) Apply the principle of conservation of angular momentum.

$$MV_1r_1 = MV_2r_2$$

$$V_1 = \frac{V_2r_2}{r_1} = \frac{6 \times 10^4 \times 7 \times 10^{10}}{1.4 \times 10^{12}}$$

$$V_1 = 3.0 \times 10^3\,m/s$$

Example – 19

(a) What do you understand by the term moment of inertia of a rigid body?

(b) (i) State the perpendicular axes theorem of moment of inertia for a body in the form of lamina.

(ii) Calculate the moment of inertia of a thin circular disc of radius 50cm and mass 2kg about an axis along a diameter of the disc.

(c) A wheel mounted on an axle that is not frictionless is initially at rest. A constant external torque of 50Nm is applied to the wheel for 20second. At the end of the 20sec, the wheel has an angular velocity of 600revmim-1. The external torque is the n removed and the wheel comes to rest after 120sec more.

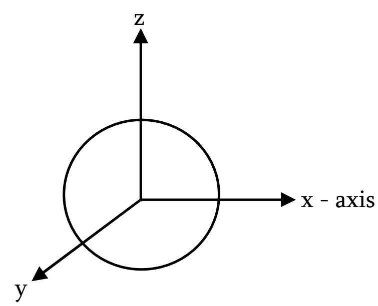
(i) Determine the moment of inertia of the wheel

(ii) Calculate the frictional torque which is assumed to be constant.

Solution

(a) Refer to your notes

(b) (ii)



Apply perpendicular axes theorem

$$I_Z = I_X + I_y$$

Due to the symmetry property.

$$I_X = I_y$$

$$2I_X = 2I_y = I_Z$$

$$I_X = I_y = \frac{1}{2}I_Z = \frac{1}{2}I_G$$

$$I_X = I_y = \frac{1}{2}\left(\frac{1}{2}MR^2\right) = \frac{1}{4}MR^2$$

$$= 0.25\times2\times(0.5)^2$$

$$I_X = I_y = 0.125\text{kgm}^2$$

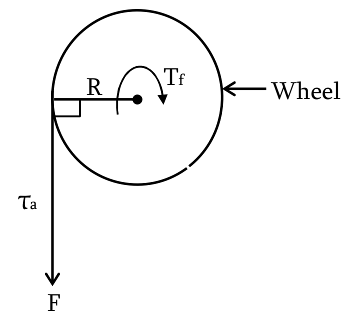
(c) $\tau_a = 50\text{Nm}$, $t_1 = 20\,\text{sec}$

$$\omega = \frac{6000\times2\pi}{60} = 20\pi\text{rads}^{-1}$$

$$t_2 = 120\,\text{sec}$$

τ_f = frictional torque

(i) Before the external torque is removed.



Net to torque $\tau = \tau_a - \tau_f = I\alpha$

$$\alpha = \frac{\tau_a - \tau_f}{I}$$

$$\frac{\omega}{t_1} = \frac{50-\tau_f}{I}$$

$$\frac{20\pi}{20} = \frac{50-\tau_f}{I}$$

$$\pi = \frac{50-\tau_f}{I}\ \ldots\ldots\ldots\ldots\text{(i)}$$

When the external torque is removed

$$\omega = 0,\ \tau_a = 0\ ,\ \omega_0 = 20\pi\text{rads}^{-1}$$

$$\alpha = \frac{\omega-\omega_0}{t_2} = \frac{-\omega_0}{t_2}$$

$$\frac{-I_f}{I} = \frac{-20\pi}{120}$$

$$\frac{\pi}{6} = \frac{\tau_f}{I}\ \ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

On solving equations (1) and (2)

(ii) $\tau_f = \dfrac{\pi I}{6} = 7.14$

$$\tau_f = 7.14\text{Nm}$$

Example – 20

(a) Why are spokes fitted in the cycle wheel?

(b) If two circular disc of different radii and the same weight and thickness are made from metals of different densities , which disc will have the larger moment of inertia about the central axis?

(c) The moment of inertia of two rotating bodies A and B about the same axis are I_A and I_B ($I_A > I_B$) and their angular momenta are equal which has greater rotational kinetic energy?

Solution

(a) The spokes of cycle wheel increase its moment of inertia , the greater is the opposition to any change in uniform rotational motion. As a result , the cycle runs smoother and steadier. If the cycle wheels had no spokes , the cycle would be driven in jerks and hence unsafe.

(b) $I_1 = \frac{1}{2}MR_1^2$, $I_2 = \frac{1}{2}MR_2^2$

$$\frac{I_1}{I_2} = \frac{R_1^2}{R_2^2}$$

Now; $M = \pi R_1^2 t\rho_1$; $R_1^2 = \frac{M}{\pi t\rho_1}$

$$R_2^2 = \frac{M}{\pi t\rho_2}$$

$$\frac{R_1^2}{R_2^2} = \frac{\rho_2}{\rho_1}$$

Now; $\frac{I_1}{I_2} = \frac{\rho_2}{\rho_1}$, $I \alpha \frac{1}{\rho}$

Therefore the disc which have greater density have smaller rotational inertia.

(c) Rotational K.E $= \frac{1}{2}I\omega^2$

$$K.E = \frac{1}{2}I\frac{L^2}{I^2} = \frac{L^2}{2I}$$

Let K_A and K_B are the rotational k.e of A and B respectively. Since the angular momentum of the two bodies are the same.

$$\frac{K.A}{K_B} = \frac{L^2}{2I_A} \Big/ \frac{L^2}{2I_B} = \frac{I_B}{I_A}$$

Since $I_A > I_B$, then $K_B > K_A$

Example – 21

(a) Explain each of the following in terms of rotational dynamics:-

(i) It is easier to loosen a nut on the bolt using a long spanner than using a short one.

(ii) An ice – skater spins more easily while her arms folded than when her arms stretched.

(iii) In hand – driven grinding machine , handle is put near the circumference of the wheel or stone.

(b) The radius of gyration of a body about an axis at a distance 8cm from its centre of mass is 12cm. Find its radius of gyration about a parallel axis through its centre of mass.

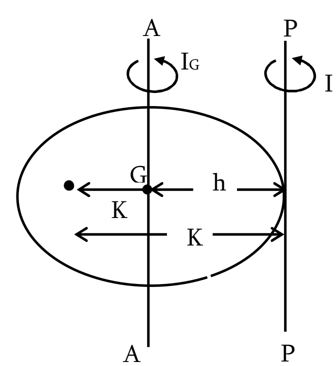
Solution

(a) (i) It is easier to loosen a nut on the bolt using a long spanner than using a short one because with a long spanner more torque is exerted to rotate the nut by applying a small force , while with a short spanner move force is required to produce the needed torque to rotate the nut.

(ii) With airms folded more mass is brought close to the axis of rotation , hence the moment of inertia is small but the angular velocity is large on the other hand , if her arms are outstretched some mass is displaced away from the axis of rotation and the moment of inertia is large but angular velocity is small. The angular momentum is conserved in each case.

(iii) For a given force , torqued can be increased if the perpendicular distance of the point of application of force from the axis of rotation is increased. This explain as to why handle is put near the circumference of the wheel or stone.

(b)



M.I about an axis $PP^1 = I = MK^2$
M.I of the body through a parallel axis through its centre of mass $I_G = MK_1^2$
Apply parallel axes theorem

$$I = I_G + Mh^2$$
$$MK^2 = MK_1^2 + Mh^2$$
$$K_1 = \sqrt{K^2 - h^2} = \sqrt{12^2 - 8^2}$$
$$K_1 = 4\sqrt{5}\text{cm}$$

Example – 22

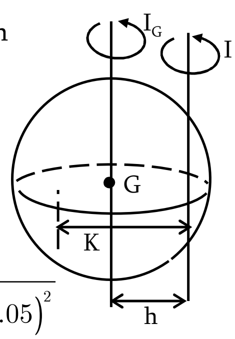
(a) How a swimmer jumping from a height is able to increase the number of loops mad in the air?

(b) Find the radius of gyration for a sphere of radius 15cm without an axis 5cm from the centre. (Given that $I_G = 0.4Mr^2$).

Solution

(a) The swimmer can increase the number of loops in air by pulling his arms and legs in wards i.e by decreasing the moment of inertia by doing so the angular velocity ω increases because angular momentum ($L = I\omega$) remain constant.

(b) Apply parallel axes theorem



$$I = I_G + Mh^2$$
$$I = 0.4Mr^2 + Mh^2$$
$$I = MK^2$$
$$MK^2 = M\left(0.4r^2 + h^2\right)$$
$$K = \sqrt{0.4r^2 + h^2}$$
$$= \sqrt{0.4(0.15)^2 + (0.05)^2}$$
$$K = 0.11\text{m}$$

Example – 23

Two rings of equal mass and thickness but of different materials are acted upon by the same torque about an axis passing through their centre and perpendicular to the plane of the rings. If the radii of the rings are in ratio 1:4, find the ratio of their angular accelerations.

Solution

Let R_1 and R_2 be the radii of the two rings , each having mass , M.

$$I_1 = MR_1^2 \ , \ I_2 = MR_2^2$$

$$\frac{I_1}{I_2} = \frac{MR_1^2}{MR_2^2} = \left(\frac{R_1}{R_2}\right)^2 = \left(\frac{1}{4}\right)^2$$
$$\frac{I_1}{I_2} = \frac{1}{16}$$

If α_1 and α_2 are the angular accelerations produced in them, due to the torque τ acting on them.

$$\tau = I_1\alpha_1 = I_2\alpha_2$$
$$\frac{\alpha_1}{\alpha_2} = \frac{I_2}{I_1} = 16$$
$$\frac{\alpha_1}{\alpha_2} = 16:1$$

Example – 24

A grind stone has a moment of inertia of $1.6 \times 10^{-3}\text{kgm}^{-2}$ when a constant torque is applied, the flywheel reaches an angular velocity of 1200revmin^{-1} in 15seconds. Assuming its started from rest, find:-

(i) The angular acceleration
(ii) The torque applied
(iii) The angular turned
(iv) The work done on the flywheel by the torque.

Solution

$$\omega_0 = 0 \ , \ \omega = \frac{2\pi \times 1200}{60} = 40\pi\text{rads}$$

(i) $$\alpha = \frac{\omega - \omega_0}{t} = \frac{40\pi}{15}$$
$$\alpha = 8.38\text{rads}^{-2}$$

(ii) $$\tau = I\alpha = 1.6\times10^{-3}\times8.38$$
$$\tau = 0.0134\text{Nm}$$

(iii) $$\omega = \left(\frac{\omega_0 + \omega}{2}\right)$$
$$\theta = \left(\frac{\omega_0 + \omega}{2}\right)t = \left(\frac{0 + 40\pi}{2}\right)\times15$$
$$\theta = 300\pi\text{rad}$$

(iv) $$w = \tau\theta = 0.0134\times300\pi$$
$$w = 12.6\text{J}$$

Example – 25

A flywheel on a motor increase its rate of rotation uniformly from 120revmin^{-1} to 300revmin^{-1} in 10seconds. Calculate:

(a) It angular acceleration

(b) Its angular displacement in this time.

Solution

$$\omega_0 = 2\pi f_0 = \frac{2\pi\times 120}{60} = 4\pi rads^{-1}$$

$$\omega = 2\pi f = \frac{2\pi\times 300}{60} = 10\pi rads^{-1}$$

(a) $\alpha = \frac{\omega-\omega_0}{t} = \frac{10\pi-4\pi}{10}$

$\alpha = 0.6\pi rads^{-2}$

(b) $\theta = \left(\frac{\omega+\omega_0}{2}\right)t = \left(\frac{4\pi+10\pi}{2}\right)\times 10$

$\theta = 70rad$

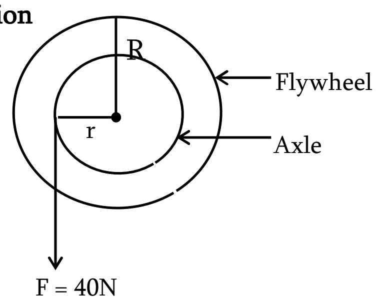
Example – 26

A heavy flywheel of mass 15kg and radius 0.2m is mounted on a horizontal axle of radius 0.01m and negligible mass compaired with flywheel neglecting friction, find:-

(i) The angular acceleration if a force of 40N is applied tangentially to the axle.

(ii) The angular velocity of the flywheel after 10second

Solution



(i) M.I of flywheel $I = \frac{1}{2}MR^2$

$$\tau = I\alpha = \frac{MR^2\alpha}{2} = Fr$$

$$\alpha = \frac{2Fr}{MR^2} = \frac{2\times 40\times 0.01}{15\times 0.2^2}$$

$\alpha = 1.3rads^{-2}$

(ii) $\omega = \omega_0 + \alpha t$

$= 0 + 1.3\times 10$

$\omega = 13rads^{-1}$

Example – 27

The moment of inertia of a solid flywheel about its axis is 0.1kgm2. It is set in rotation by applying a tangential force of 20N with a rope wound round the circumference of the radius of the wheel being 0.1m

(i) Calculate the angular acceleration of the flywheel.

(ii) What would be the acceleration if a mass of 2kg were hung from the end of the rope.

Solution

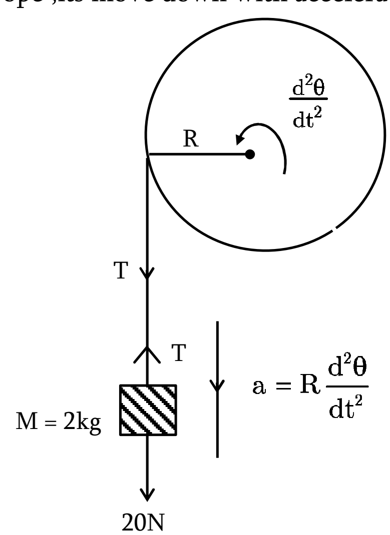
(i) Torque on the flywheel.

$$\tau = FR = I\frac{d^2\theta}{dt^2}$$

$$\alpha = \frac{d^2\theta}{dt^2} = \frac{FR}{I} = \frac{20\times 0.1}{0.1}$$

$\alpha = 20rads^{-2}$

(ii) If a mass M of 2kg hung from the end of the rope ,its move down with acceleration , a.



Resultant force on the mass

$Mg - T = Ma$ ………….…….(1)

Torque on the flywheel

$\tau = TR = I\frac{d^2\theta}{dt^2}$ ……………..….(2)

Linear acceleration

$$a = \alpha R = R\frac{d^2\theta}{dt^2}$$

Now , equation (i) becomes

$$Mg - T = MR\frac{d^2\theta}{dt^2}$$

(Multiply by R both side)

$$MgR - TR = MR^2\frac{d^2\theta}{dt^2} \text{.........(3)}$$

Putting equation (2) into (3)

$$MgR - I\frac{d^2\theta}{dt^2} = MR^2\frac{d^2\theta}{dt^2}$$

$$MgR = \left(I + MR^2\right)\frac{d^2\theta}{dt^2}$$

$$\alpha = \frac{d^2\theta}{dt^2} = \frac{MgR}{I + MR^2}$$

$$= \frac{2\times10\times0.1}{0.1 + 2\left(0.1\right)^2}$$

$$\alpha = 1.6.7\text{rads}^{-2}$$

Since $a = \alpha R$

$$= 16.7\times0.1$$

$$a = 1.67\text{m/s}^2$$

Example – 28

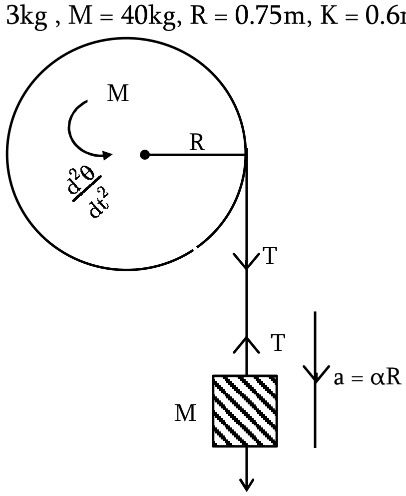
A 3kg block hang from a string wound on 40kg wheel. The wheel has a radius of 0.75m and radius of gyration of 0.6m. Find:-

(i) The angular acceleration of the wheel

(ii) The distance the blocks falls in first 10seconds.

Solution

m = 3kg , M = 40kg, R = 0.75m, K = 0.6m



(i) Let α = angular acceleration

M.I of the flywheel , $I = MK^2$

Resultant force on the block

$$ma = mg - T$$

Torque $\tau = TR = I\alpha$

$$T = \frac{I\alpha}{R} \quad \text{but} \quad \alpha = {}^{a}\!/_{R}$$

$$T = \frac{Ia}{R^2}$$

$$ma = mg - \frac{Ia}{R^2}$$

$$a\left(m + {}^{I}\!/_{R^2}\right) = mg$$

$$a = \frac{mg}{m + \frac{I}{R^2}} = \frac{mg}{m + \frac{MK^2}{R^2}}$$

$$a = \frac{3\times9.8}{3 + \frac{40\left(0.6\right)^2}{\left(0.75\right)^2}} = 1.03\text{m/s}^2$$

$$\alpha = \frac{a}{R} = \frac{1.03}{0.75}$$

$$\alpha = 1.37\text{rads}^{-2}$$

(ii) $S = ut + \frac{1}{2}at^2 \qquad u = 0$

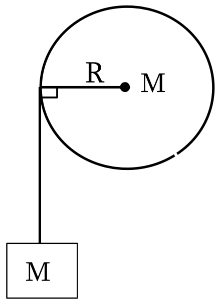
$$= \frac{1}{2}\times1.03\times10^2$$

$$S = 51.5\text{m}$$

Example – 29

(a) Define 'Angular momentum' and state a mathematical relation between angular momentum (L) of a body and its moment of inertia (I)

(b) A pulley of mass M and radius R mounted on an axle is free rotate about an axis through its centre and perpendicular to its plane (see figure below). A light cord is wrapped around the rim of the wheel and mass, M is suspended from the end of the cord.



Find an expression in terms of M, m and R for

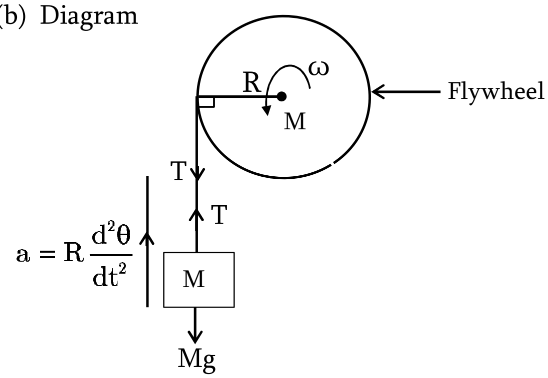
(i) The angular acceleration of the disc.

(ii) Tension in the cord.

Solution

(a) Refer to your notes

(b) Diagram



(i) Resultant force on the block

$$mg - T = ma \text{ ....(1)}$$

Torque on flywheel

$$\tau = FR = I\alpha \quad \text{but} \quad I = \frac{1}{2}MR^2$$

$$FR = \frac{1}{2}MR^2\alpha \;, \quad F = T$$

$$TR = \frac{1}{2}MR^2\alpha$$

$$2T = MR\alpha$$

$$T = \frac{1}{2}MR\alpha$$

$$a = \alpha R$$

From equation (1)

$$mg - T = mR\alpha$$

$$mg - \frac{1}{2}MR\alpha = mR\alpha$$

$$mg = \left(mR + \frac{1}{2}MR\right)\alpha$$

$$\alpha = \frac{2mg}{(2m + M)R}$$

(ii) Tension on the cord

$$T = \frac{1}{2}MR\alpha$$

$$T = \frac{Mmg}{M + 2m}$$

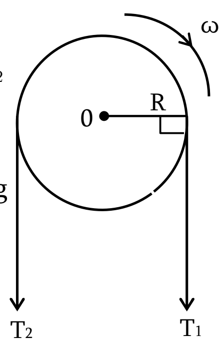
Example – 30

A flywheel of mass 50kg is made in form of a circular disc of radius 10cm and is driven by the belt whose tensions at the points where runs on and off the rim of the wheel are 20N and 49N respectively. If the wheel is rotating at a certain speed of 180revolutions per minute, find how long it will be before the speed has reached 720revolutions per minute while the flywheel is rotating at this later speed , the belt slip off and break applied. Find the constant working couple required to stop the wheel in 12second.

Solution

Case 1:

The flywheel is of the form of the disc , so as its M.I about an axis through its centre of mass and perpendicular to the plane.



$$I = \frac{1}{2}MR^2 = \frac{1}{2}\times 50\times 0.1^2$$

$$I = 0.25\text{kgm}^2$$

Resultant tension on the string

$$T = T_2 - T_1 = 49 - 20$$

$$T = 29\text{N}$$

The resultant tension in the string produces torque in the wheel.

Torque $\tau = TR = I\alpha$

$$\alpha = \frac{IR}{I} = \frac{29\times 0.1}{0.25}$$

$$\alpha = 11.6\text{rads}^{-2}$$

$$\omega = 2\pi f = 2\pi\times\frac{720}{60} = 24\pi\text{rads}^{-1}$$

Since $\alpha = \dfrac{\omega - \omega_0}{t}$, $t = \dfrac{\omega - \omega_0}{\alpha}$

$$t = \frac{24\pi - 6\pi}{11.6} = 4.87\text{ sec}$$

∴ It will take 4.87sec before the speed reached 720revolutions per minute.

Case 2: during deceleration

$$\omega_0 = 2\pi f = 2\pi \times \frac{720}{60} = 24\pi rads^{-1}$$

$$\omega = 0$$

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{0 - 24\pi}{12}$$

$$\alpha = -2\pi rads^{-2}$$

Opposing torque $\tau = -I\alpha$

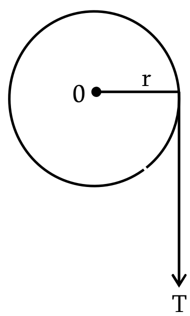
$$\tau = -(-2\pi \times 0.25)$$

$$\tau = 1.57Nm$$

Example – 31

A solid cylinder of mass 50kg and radius 0.5m is free to rotate about its axis which is horizontal. A string is wound round the cylinder with one end attached to its and the hanging freely. Find the tension in the string required to produce an angular acceleration of 2rev/s².

Solution



Torque , $\tau = Tr = I\alpha$

$$T = \frac{I\alpha}{r}$$

But $I = \frac{1}{2}Mr^2$

$$T = \frac{1}{2}Mr^2 \cdot \frac{\alpha}{r}$$

$$= \frac{1}{2}Mr\alpha = \frac{1}{2} \times 50 \times 0.5 \times 2 \times 3.14 \times 2$$

$$T = 157N$$

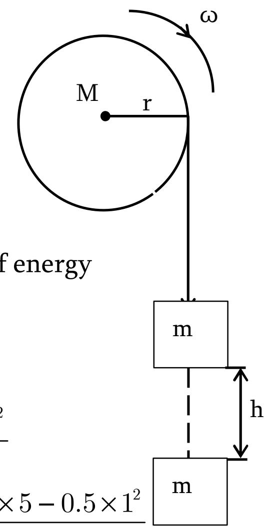
Example – 32

A mass of 0.5kg hangs from the rim of a wheel of radius 0.2m by a light string which is wound round the wheel. When released from rest the mass falls through a distance of 5m in 10seconds , rotating the wheel. Find the moment of inertia of the wheel. (g = 10m/s²).

Solution

Let V be the velocity after falling through 5m

$$h = \left(\frac{v+u}{2}\right)t \quad \text{but} \quad u = 0$$



$$5 = \left(\frac{0+v}{2}\right) \times 10$$

$$V = 1m/s$$

Angular velocity of flywheel

$$\omega = \frac{V}{r} = \frac{1}{0.2} = 5rads^{-1}$$

Apply the law of conservation of energy

Gain in K.e = loss in p.e

$$\frac{1}{2}I\omega^2 + \frac{1}{2}MV^2 = mgh$$

$$I = \frac{2mgh - mv^2}{\omega^2}$$

$$= \frac{2 \times 0.5 \times 10 \times 5 - 0.5 \times 1^2}{5^2}$$

$$I = 1.98kgm^2$$

Example – 33

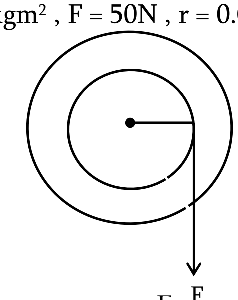
A flywheel is mounted on a horizontal axle which has a radius of 0.06m. A constant force of 50N is applied tangentially to the axle. If the moment of inertia of system (flywheel and axle) is 4kgm². Calculate:

(a) The angular acceleration of the flywheel

(b) The number of revolutions that the flywheel makes in 16seconds assuming that its starts from rest.

Solution

I = 4kgm² , F = 50N , r = 0.06m



(a) Torque $\tau = F \quad I\alpha$

$$\alpha = \frac{Fr}{I} = \frac{50 \times 0.06}{4}$$

$$\alpha = 0.75rads^{-2}$$

(b) If θ is the angle turned through in the time , t = 16sec.

$$\omega_0 = 0$$

$$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\times 0.75\times 16^2$$

$$\theta = 96\text{rad}$$

Number of revolution

$$N = \frac{\theta}{2\pi} = \frac{96}{2\pi}$$

$$N = 15\text{rev (approx.)}$$

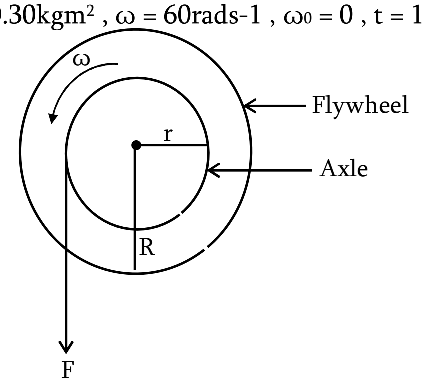
Example – 34

A flywheel of moment of inertia $0.30kgm^2$ mounted on a fixed axle accelerates uniformly from rest to an angular velocity of $6rads^{-1}$ in 12sec. find:-

(a) The angular acceleration
(b) The torque causing the wheel to accelerates.
(c) The number of revolution in this 12seconds period.

Solution

$I = 0.30kgm^2$, $\omega = 60rads-1$, $\omega_0 = 0$, $t = 12sec$



(a) $$\alpha = \frac{\omega - \omega_0}{t} = \frac{60-0}{12}$$

$$\alpha = 5rads^{-2}$$

(b) $$\tau = I\alpha = 0.3\times 5$$

$$\tau = 1.5Nm$$

(c) $$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\times 5\times 12^2$$

$$\theta = 360\text{rad}$$

$$N = \frac{\theta}{2\pi} = \frac{360}{2\pi} \simeq 57\text{rev}$$

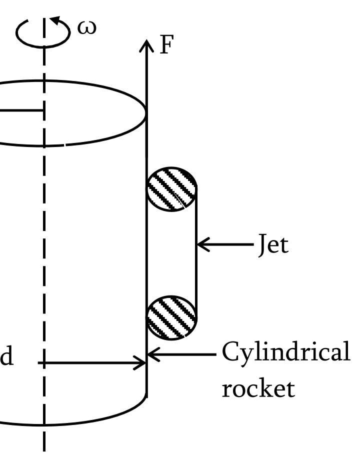
Example – 35

A cylindrical rocket of diameter 2.0m develops a spinning motion in space of period 2.0sec about an axis of these cylinder to eliminate this spin two jets motors which are attached to the rocket at the opposite ends of a diameter are fired until the spinning motion cease each motor turns the rocket in the same direction and provides a constant thrust of 4.0×10^3N in the direction tangentially to the surface of the rocket and in the plane perpendicular to its axis. If the moment of inertia of the rocket about its cylindrical axis is $6.0\times 10^5kgm^2$. Calculate the number of revolutions made by the rocket during the firing and the time for which the motors are fired.

Solution

$F = 4\times 10^3N$, $I = 6.0\times 10^5kgm^2$, $d = 20cm$

$$r = \frac{d}{2} = 1.0m$$



Torque $\tau = -Fd = I\alpha$

F = applied force

d = distance between the coplanar force

$$\alpha = \frac{-Fd}{I} = \frac{-4\times 10^3\times 2}{6\times 10^5}$$

$$\alpha = \frac{-8}{600}rads^{-2}$$

(Negative sign means retardation)

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi rads^{-1}$$

Since $$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$0^2 = \pi^2 + 2\left(\frac{-8}{600}\right)\theta$$

$$\theta = 37.5\pi^2\text{rad}$$

Number of revolution

$$N = \frac{\theta}{2\pi} = \frac{37.5\pi^2}{2\pi}$$

$$N = 58.9 \text{ revolutions.}$$

Time taken

$$\omega = \omega_0 + \alpha t$$

$$0 = \pi + \frac{-8}{600}t$$

$$t = 235.6\,\text{sec}$$

Example – 36

Two masses M_1 = 15kg and M_2 = 10kg are attached to the ends of a cord which passed over the pulley on an Atwood's machine. The mass of the pulley is M = 10kg and its radius is R = 0.1m. Calculate the tensions in the cord, the acceleration and the number of revolution made by the pulley at the end of 2seconds from the starts.

Solution

As the pulley has a finite mass, the two tensions T_1 and T_2 are not equal.

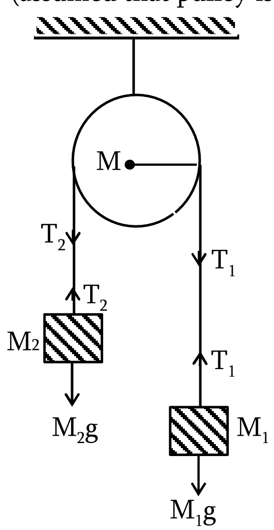
Let: a = linear acceleration

Resultant forces

M_1: $M_1g - T_1 = M_1a$……….(i)

M_2: $T_2 - M_2g = M_2a$……….(ii)

Here the tensions in the cords exert torque on the pulley (assumed that pulley is a solid disc)



Torque on the pulley.

$$\tau = (T_1 - T_2)R$$

But $\tau = I\alpha = \frac{1}{2}MR^2\alpha \quad \left[\alpha = \frac{a}{R}\right]$

$$\tau = \frac{1}{2}MR^2 \cdot \frac{a}{R} = \frac{1}{2}MRa$$

Now; $(T_1 - T_2)R = \frac{1}{2}MRa$

$$T_1 - T_2 = \frac{1}{2}Ma \text{……..(iii)}$$

Adding equation (i) and (ii)

$$(M_1 - M_2)g = (M_1 + M_2)a + (T_1 - T_2) \text{…(iv)}$$

Putting equation (iii) into (iv)

$$(M_1 - M_2)g = (M_1 + M_2)a + \frac{Ma}{2}$$

$$a = \frac{(M_1 - M_2)g}{M_1 + M_2 + M/2}$$

$$= \frac{(15 - 10)\times 9.8}{15 + 10 + 10/2}$$

$$a = 1.63\text{m/s}^2$$

Tension , T_1

$$T_1 = M_1(g - a) = 15(9.8 - 1.63)$$

$$T_1 = 114.3\text{N}$$

Also $T_2 = M_2(g + a) = 10(9.8 + 1.63)$

$$T_2 = 114.3\text{N}$$

Angular acceleration

$$\alpha = \frac{a}{R} = \frac{1.63}{0.1}$$

$$\alpha = 16.3\text{rads}^{-2}$$

Since $\theta = \frac{1}{2}\alpha t^2 \quad (\omega_0 = 0)$

$$= \frac{1}{2}\times 16.3 \times 2^2$$

$$\theta = 32.6\text{rad}$$

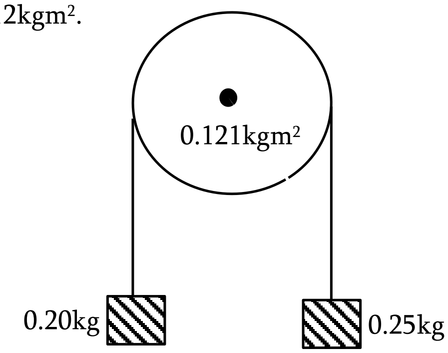
Number of revolution , $N = \frac{\theta}{2\pi}$

$$N = 32.6$$

$$N = 5.2 \text{ Revolutions.}$$

Example – 37

Masses 0.20kg and 0.25kg suspended as in the figure below , from a light cord which passes over a wheel of radius 0.15m and moment of inertia $0.12kgm^2$.



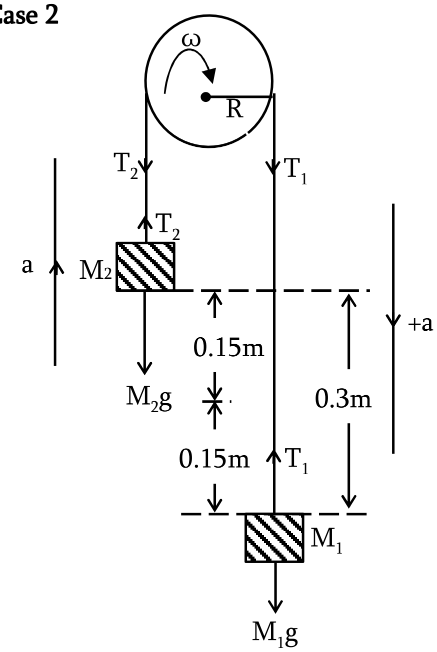
Initially , the two masses are held at the same horizontal level. Explain what happens when they are released from rest if the cord does not slip on the wheel. Assume that the wheel rotates freely about its axis , calculate the angular velocity of the wheel and the speed of the each mass , when the vertical distance between the masses is 0.3m.

Solution

Case 1:

When they are released from the rest the block of mass M_1 = 0.25kg moves downward while the block of mass M_2 = 0.2kg moves upward with the same linear acceleration.

Case 2



Resultant forces on each block

M_1: $M_1 - T_1 = M_1a$

$2.5 - T_1 = 0.25a$……..(i)

M_2: $T_2 - M_2g = M_2a$

$T_2 - 2 = 0.2a$………..(ii)

Adding equation (i) and (ii)

$$T_2 - T_1 + 2.5 - 2 = 0.45a$$

$$T_1 - T_2 = 0.5 - 0.45a \text{ ……….(iii)}$$

Torque on the flywheel

$$\tau = (T_1 - T_2)R = I\alpha$$

$$T_1 - T_2 = \frac{I\alpha}{R} = \frac{Ia}{R^2} \text{ …….(iv)}$$

Putting equation (iv) into (iii)

$$\frac{Ia}{R^2} = 0.5 - 0.45a$$

$$\frac{0.12a}{(0.15)^2} = 0.5 - 0.45a$$

$$a = 0.865m/s^2$$

$$\alpha = \frac{a}{R} = \frac{0.865}{0.15} = 0.577rads^{-2}$$

Since $V^2 = u^2 + 2as \quad (u = 0)$

$$V = \sqrt{2as} = \sqrt{2 \times 0.0865 \times 0.15}$$

$$V = 0.161m/s$$

Angular velocity , ω

$$\omega = \frac{V}{R} = \frac{0.161}{0.15}$$

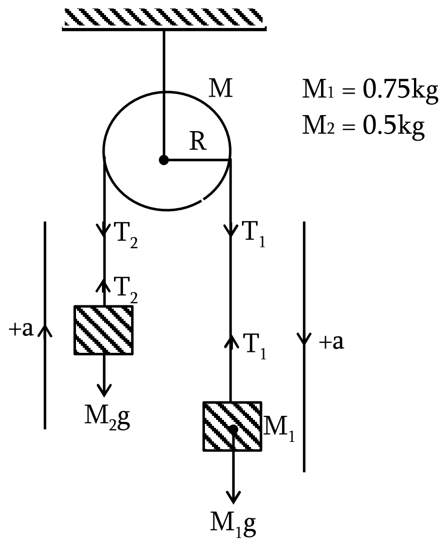
$$\omega = 1.074rads^{-1}$$

Example – 38

In an at woods , machine the pulley mounted in horizontal frictionless bearing has radius R = 0.05m. the cord passing over the pulley carries a block of mass M_1 = 0.75kg at one end and block of mass 0.5kg on other end when set free from rest , the heavier block is observed to fall a distance of 1metre in 10seconds. Compute the moment of inertial of the pulley.

Solution

M_1 = 0.75kg , M_2 = 0.5kg



Since the heavier blocks falls through distance S in t = 10sec.

$$u = 0$$

$$S = \frac{1}{2}at^2 \quad , \quad a = \frac{2s}{t^2}$$

$$a = \frac{2\times1}{10^2} = 0.02m/s^2$$

Linear velocity at the end of 10sec

$$V = u + at = 0 + 0.02\times10$$

$$V = 2m/s$$

The loss in p.e

$$\Delta p.e_1 = M_1gh = 0.75\times9.81\times1$$

$$\Delta p.e_1 = 7.35J$$

The lighter mass ascends 1m.

$$\Delta p.e_2 = M_2gh = 0.5\times9.81\times1$$

$$\Delta p.e_2 = 4.9J$$

Since no friction on the pulley.

Apply the law of conservation of energy

Loss in p.e = gain of k.e of two blocks + rotational k.e of the pulley.

$$\Delta p.e = \frac{1}{2}(M_1 + M_2)V^2 + \frac{1}{2}I\omega^2$$

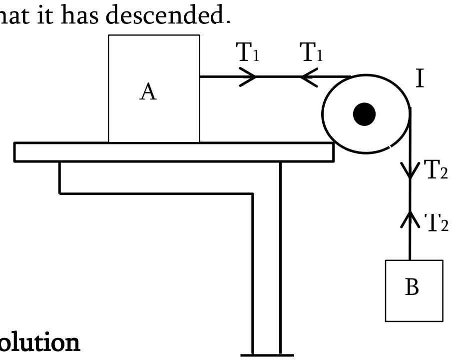
$$= \frac{1}{2}(M_1 + M_2)V^2 + \frac{1}{2}I\left(\frac{V}{R}\right)^2$$

$$2.45 = \frac{1}{2}\times1.25\times(0.2)^2 + \frac{I}{2}\left(\frac{0.2}{0.05}\right)^2$$

$$I = 0.3031kgm^2$$

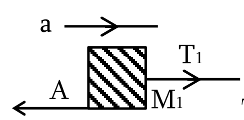
Example – 39

The pulley in the figure below has radius R and a moment of inertia , I. The rope does not ship over the pulley and the pulley spins on a frictionless axle. The coefficient of kinetic friction between block A and the table top is μ_k. The system is released from rest , and block B descends. Block A has a mass M_1 and block B has mass, M_2. Calculate the speed of block B as a function of the distance d that it has descended.



Solution

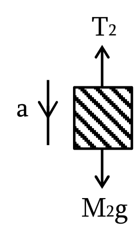
Consider FBD of body A



$$T_1 - f = M_1a$$

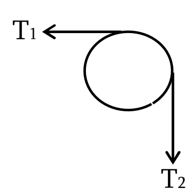
$$T_1 - \mu M_1g = M_1a \text{ ....(i)}$$

Body B



$$M_2g - T_2 = M_2a \text{ ....(ii)}$$

For the pulley



Torque on the pulley

$$(T_2 - T_1)r = I\alpha$$

$$T_2 - T_1 = \frac{Ia}{r} \text{ ....(iii)}$$

Adding equation (i) , (ii) and (iii)

$$M_2g - \mu M_1g = \left(M_1 + M_2 + \frac{I}{r^2}\right)a$$

$$a = \frac{\left(M_2 - \mu M_1\right)g}{M_1 + M_2 + \frac{I}{r^2}}$$

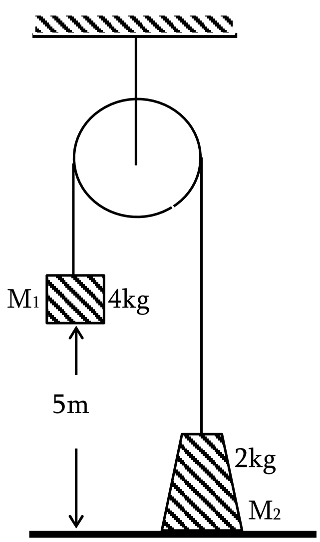
Since $V^2 = U^2 + 2ad \quad (u = 0)$

$$V = \sqrt{2ad}$$

$$V = \sqrt{2gd\left[\frac{M_2 - \mu M_1}{M_1 + M_2 + I/r^2}\right]}$$

Example – 40

The pulley as shown on the figure below has radius 0.16m and moment of inertia 0.38kgm². The rope does not ship on the pulley rim use energy methods to calculate the speed of the 4kg block just before it strikes the floor.



Solution

Apply the law of conservation of energy

$$p.e_1 = k.e \text{ of pulley} + p.e_2 + k.e_1 + k.e_2$$

$$M_1gh = \frac{1}{2}I\omega^2 + M_2gh + \frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2$$

$$V_1 = V_2 = V \quad , \quad \omega = \frac{V}{r}$$

$$2\left(M_1 - M_2\right)gh = \frac{IV^2}{r^2} + \left(M_1 + M_2\right)V^2$$

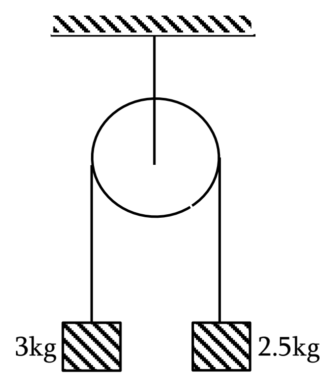
$$V = \sqrt{\frac{2\left(M_1 - M_2\right)gh}{I/r^2 + M_1 + M_2}}$$

$$= \sqrt{\frac{2\left(4-2\right)\times 9.8\times 5}{0.38/0.16^2 + 4 + 2}}$$

$$V = 3m/s$$

Example – 41

Two bodies of mass 2.5kg and 3kg are hanging on the pulley by mean of light inextensible string as shown in the figure below. Radius of the wheel is 0.2m and its mass is 0.5kg. Calculate time taken for distance separation between bodies to be 3m assuming that the bodies were initially at the same height.



Solution

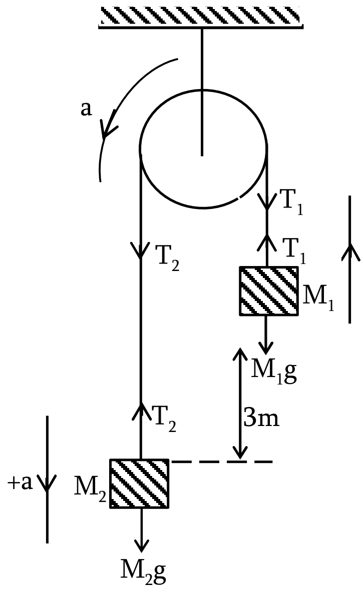
$M_1 = 2.5kg$, $M_2 = 3.0kg$

Resultant force on

M_1: $T_1 - M_1g = M_1a$ ……(i)

M_2: $M_2g - T_2 = M_2a$ …..(ii)

Torque on the pulley



$$\left(T_2 - T_1\right)r = I\alpha \qquad \left[\alpha = \frac{a}{r}\right]$$

$$T_2 - T_1 = \frac{Ia}{r^2} \text{.....(iii)}$$

(i) + (ii) + (iii)

$$\left(M_2 - M_1\right)g = \left(M_1 + M_2 + \frac{I}{r^2}\right)a$$

$$a = \frac{\left(M_2 - M_1\right)}{M_1 + M_2 + \frac{I}{r^2}} \qquad \left[I = \frac{1}{2}Mr^2\right]$$

$$= \frac{\left(3 - 2.5\right)\times 9.8}{3 + 2.5 + \frac{0.5}{2}}$$

Since $S = Ut + \frac{1}{2}at^2$

$$1.5 = \frac{1}{2}\times 0.852t^2$$

$$t = 1.88 \text{ sec}$$

Example – 42

A flywheel with an axle 1cm in diameter is in frictionless bearing and set in motion by applying a steady tension of 2N to a thin thread wound tightly round the axle. The moment of inertia of the system about the axis of rotation is 5×10^{-4}kgm^2 calculate.

(a) Angular acceleration of the flywheel when 1m has been pulley off the axle.

(b) The constant retarding couple which must then be applied to stop the flywheel in one turn with tension on the rope completely removed.

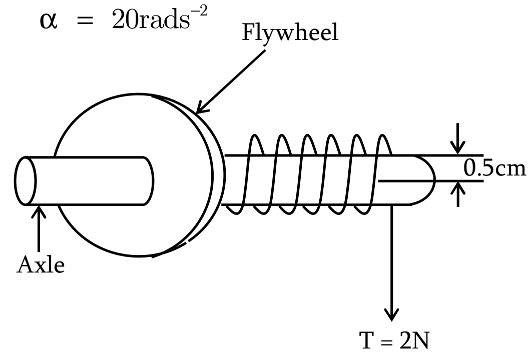
Solution

(a) $\tau = Tr = I\alpha \qquad \left(\alpha = \frac{a}{r}\right)$

$$\alpha = \frac{Ir}{I}$$

$$\alpha = \frac{2\times 5\times 10^{-3}}{5\times 10^{-4}}$$

$$\alpha = 20\text{rads}^{-2}$$



(b) Retarding couple. When 1m has been pulled off the axle , it means that the axle has rotated by n – turns. When these n – turns are multiplied by the circumference , we get total distance that the axle has rotated (that is 1m)

$$2\pi rn = 1$$

$$n = \frac{1}{2\pi r}$$

Total angular displacement for the axle after n – revolutions.

$$\theta = 2\pi n = 2\pi\times\frac{1}{2\pi r} = \frac{1}{r}$$

$$\theta = \frac{1}{0.5\times 10^{-2}} = 200\text{rad}$$

Since $\omega^2 = \omega_0^2 + 2\alpha \qquad \left(\omega_0 = 0\right)$

$$\omega^2 = 8000\text{rad}^2\text{s}^{-2}$$

Now to stop the flywheel in one complete turn

$$\theta_2 = 2\pi\text{rad}$$

$$\omega_2^2 = \omega^2 + 2\alpha_2\theta_2$$

$$0 = 8000 + 2\alpha_2 \times 2\pi$$

$$\alpha_2 = \frac{2000}{\pi}\,\text{rads}^{-2}$$

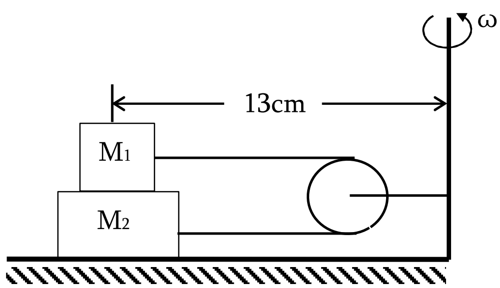
$$\tau = I\alpha_2$$

$$= 5\times10^{-4} \times \frac{2000}{\pi}$$

$$\tau = 0.318\text{Nm}$$

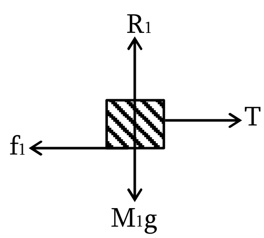
Example – 43

In a turn table arrangement as shown in the figure below. A block of mass $M_1 = 0.9$kg and $M_2 = 1.7$kg and they are 13cm from the axis of rotation. The coefficient of static friction between the blocks is 0.1 , the pulley being smooth. Find the angular speed of rotation of the turn table for which the blocks just to slide.



Solution

Consider the FBD for M_1



Resultant force on M_1

$$T - f_1 = M_1\omega^2 r$$

$$T - \mu M_1 g = M_1\omega^2 r$$

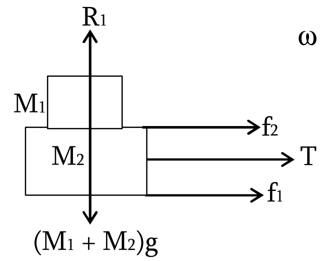
Consider the FBD for M_2

Resultant force on M_2

$$T + \mu M_1 g + \mu\left(M_1 + M_2\right)g = M_2\omega^2 r \,\ldots.\text{(ii)}$$

Takes (ii) – (i)

$$\left(M_2 - M_1\right)\omega^2 r = \left(3M_1 + M_2\right)\mu g$$



$$\omega = \sqrt{\frac{\mu g\left(3M_1 + M\right)}{\left(M_2 - M_1\right)r}}$$

$$= \sqrt{\frac{0.1\times9.8\left(3\times0.9 + 1.7\right)}{\left(1.7 - 0.9\right)\times13\times10^{-2}}}$$

$$\omega = 6.4\text{rads}^{-1}$$

Example – 44

(a) A flywheel starts rotating from rest because of an external torque 1.5Nm. The torque is removed after the flywheel rotated by 48 revolutions it continues to rotate by 145 revolutions more before coming to rest. If the total time taken is 50 seconds , calculate the moment of inertia of the flywheel and the maximum angular velocity by it.

(b) A wheel of radius 50cm having 30spokes each of mass 100gm is travelling forward at 5m.s. If the mass of the rim of the wheel is 1000g, calculate the moment of inertia , kinetic energy and angular momentum of a wheel.

Solution

(a) Torque acting on the flywheel $\tau = 1.5$Nm

Number of revolutions completed N = 48

Angular displacement covered

$$\theta = 2\pi N = 2\pi\times48 = 96\pi\text{rad}$$

Since $\theta = \left(\frac{\omega_0 + \omega}{2}\right)t = \left(\frac{0+\omega}{2}\right)t_1$

$$96\pi = \frac{\omega t_1}{2}$$

$$\omega t_1 = 192\pi \,\ldots\ldots\text{(i)}$$

When the torque is removed , let α_2 be the angular retardation and t_2 be the time taken by the flywheel before coming to rest.

Angular displacement covered.

$$\theta = 2\pi N = 2\pi\times145 = 290\pi\text{rad}$$

Now $\theta = \left(\frac{\omega + \omega'}{2}\right)t$

$$\theta = \left(\frac{\omega + 0}{2}\right)t_2 = \frac{\omega t_2}{2}$$

$$290\pi = \frac{\omega t_2}{2}$$

$$\omega t_2 = 580\pi$$

(i) = (ii)

$$(\omega t_2 + \omega t_2) = 580\pi + 192\pi$$

$$\omega(t_2 + t_1) = 772\pi \text{ but } t_1 + t_2 = 50\,\text{sec}$$

$$50\omega = 772\pi$$

$$\omega = 15.44\text{rads}^{-1}$$

Let α be the angular acceleration during the time when the torque was acting

Since $\omega^2 = \omega_0^2 + 2\alpha\theta$

$$(15.44\pi)^2 = 0^2 + 2\times 96\pi\alpha$$

$$\alpha = 3.899\,\text{rad}/\text{s}^2$$

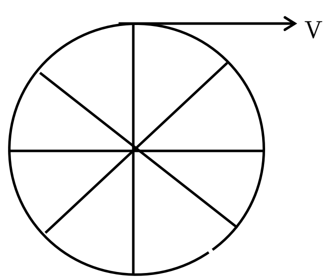
Torque $\tau = I\alpha$

I = M.I of the flywheel

$$I = \frac{I}{\alpha} = \frac{1.5}{3.899}$$

$$I = 0.3847\text{kgm}^2$$

(b)



Total M.I of the system

I = Ispokes + Irim

Spokes are similar to rods

Rotating about one end.

$$\text{Ispokes} = \frac{1}{3}ML^2 \times \text{number of spokes}$$

$$= \frac{1}{3}\times 0.1\times(0.5)^2\times 30$$

$$\text{Ispoke} = 0.25\text{kgm}^2$$

M.I of the wheel

$$\text{Irim} = MR^2 = 1\times(0.5)^2$$

$$\text{Irim} = 0.25\text{kgm}^2$$

$$I = 0.25 + 0.25 = 0.5\text{kgm}^2$$

Total energy

$$E = \frac{1}{2}I\omega^2 + \frac{1}{2}MV^2$$

$$= \frac{1}{2}I\frac{V^2}{R^2} + \frac{1}{2}MV^2$$

$$= \frac{V^2}{2}\left[\frac{I}{R^2} + M\right] = \frac{5^2}{2}\left[\frac{0.5+4}{(0.5)^2}\right]$$

$$E = 75\text{J}$$

Angular momentum

$$L = I\omega = \frac{IV}{R} = \frac{0.5\times 5}{0.5}$$

$$L = 5\text{kgm}^2\text{s}^{-1}$$

Example – 45

(a) (i) State the principle of conservation of angular momentum.

(ii) How does an ice – skater , a ballet dancer or an acrobat take advantage of principle of conservation of angular momentum?

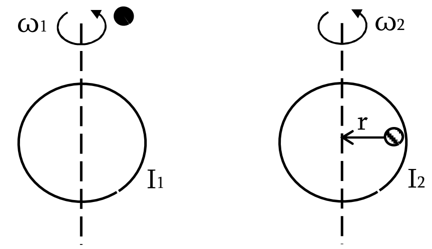
(b) A horizontal wooden disc is rotated about a vertical axis through its centre at 70 r.p.m. A dirt of 300gm falls vertically on the disc and embeds its sharp point in the wood at a distance of 10cm from the axis. If this reduced the rate of rotation of the disc to 50 r.p.m. Determine the moment of inertia of the wooden disc about the vertical axis.

Solution

(a) (i) Refer to your notes

(ii) An ice – skater , a ballet dancer or an acrobat is able to change his angular speed during the course of his performance when the performer stretches out his hand and legs , his moment of inertia increase and the angular speed decrease on the other hand , when he folds his hands and legs near his body , the moment of inertia decreases and he is able to increase his angular speed.

(b) Before the dirty falls After the dirty falls



Apply the principle of conservation of angular momentum.

$$I_1\omega_1 = I_2\omega_2$$

$$I_1(2\pi f_1) = I_2(2\pi f_2)$$

Let I = M.I of the disc

I_0 = M.I of dirty

$$If_1 = (I + I_0)f_2$$

$$I = I_0\left[\frac{f_2}{f_1 + f_2}\right] \quad \text{but } I_0 = Mr^2$$

$$= Mr^2\left[\frac{f_2}{f_1 + f_2}\right]$$

$$= 0.3\times(0.1)^2\left[\frac{50}{70-50}\right]$$

$$I = 7.5\times10^{-3}\,kgm^2$$

Example – 46

(a) (i) Define the angular velocity of a rotating body and give its S.I unit. A car wheel has its angular velocity changing from 2rads^{-1} to 30rads^{-1} in 20seconds. If the radius of the wheel is 400mm. Calculate:-

(ii) The angular acceleration

(iii) The tangential linear acceleration of a point on the rim of the wheel.

(b) A large wheel of radius 40cm having 10spokes on its is made to spin about an axle at 3rev per second. A 25cm long arrow is shot parallel to the axle but perpendicular to the surface of the rotating wheel without hitting any of the spokes and enters at a point where one of the spokes has just passed.

(i) What minimum speed should the arrow have?

(ii) Does it matter where (between the axle and the rim you aim?)

(c) (i) A recording disc rotates steadily at 45rev per minute on a turntable. When a small mass of 0.02kg is dropped gently onto the disc at a distance of 0.04m from its axis of rotation and sticks the rate of revolution falls to 36revmin^{-1}. Calculate the moment of inertia of the disc about its centre.

(ii) State and write down the principle used in your calculation in (i) above.

Solution.

(a) (i) Refer to your notes.

(ii) $\alpha = \dfrac{\omega - \omega_0}{t} = \left(\dfrac{30-2}{20}\right)$

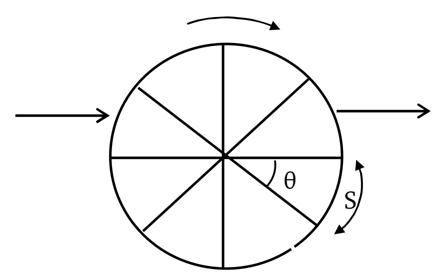
$\alpha = 1.4\,rads^{-2}$

(iii) $V = \omega R$

$a = \alpha R = 1.4\times0.4$

$a = 0.56\,m/s^2$

(b)



Circumference of the wheel

$C = 2\pi r = 2\pi\times0.4 = 0.8\pi$

The distance between the spokes

$$S = \frac{C}{N} = \frac{0.8\pi}{10}$$

$S = 0.08\pi m$

Speed of the wheel , $V = \omega r = 2\pi fr$

$V = 2\pi\times2\times0.4$

$V = 2.4\pi m/s$

The time taken by an arrow to travel that distance is the same as time taken by the wheel to turn through an angle , θ.

$$v = \frac{s}{t}\ ,\ t = \frac{s}{v}$$

$$t = \frac{0.08\pi}{2.4\pi}$$

$$t = \frac{1}{30}\text{ second}$$

Then arrow should travel a distance equal to its length through the wheel without hits the wheel.

$$V_{min} = \frac{L}{t} = \frac{0.25}{\frac{1}{30}}$$

$$V_{min} = 7.5\text{m/s}$$

(ii) Yes, it matter if the arrow is shot near the axle it may collide with the spokes since the relative angular speed at the centre is very low compaired to the point of min

(c) (i) Apply principle of conservation of angular momentum.

$$I_d\omega_0 = I_1\omega_1 \quad , \quad I_1 = I_d + Mr^2$$

$$I_d\left(2\pi f_0\right) = \left(I_d + Mr^2\right)2\pi f_1$$

$$I_d = Mr^2\left[\frac{f_0}{f_1 - f_0}\right]$$

$$= 0.02\times\left(0.04\right)^2\left[\frac{36}{45-36}\right]$$

$$I_d = 1.28\times10^{-4}\text{kgm}^2$$

(ii) Principle of conservation of angular momentum.

Example – 47

Two boys each of mass 25kg on the opposite ends of a horizontal beam of mass 10kg and length 2.6m. The beam is rotating about a vertical axis through its centre at 5revolutions per minute. Find the initial angular momentum. What would be the angular velocity if each body moves 0.6m toward the centre of the beam without to uching the floor. Calculate also the change in kinetic energy of the system.

Solution

Mass of each boy , M = 25kg

Mass of the beam , m = 10kg

Length of the beam L = 2.6m

Distance of each boy from the axis of rotation

$r_1 = 1.3$.

Initial angular velocity

$$\omega_1 = 5\text{r.p.m} = \frac{2\pi\times5}{60} = \frac{\pi}{6}\text{rads}^{-1}$$

M.I of the system

I_1 = M.I of the rod + M.I of the boy

$$= \frac{ML^2}{12} + 2Mr_1^2$$

$$= \frac{10\times\left(2.6\right)^2}{12} + 2\times25\times\left(1.3\right)^2$$

$$I_1 = 90.13\text{kgm}^2$$

Initial angular momentum

$$L_1 = I_1\omega_1 = 90.13\times\frac{\pi}{6}\ \dots\dots\dots\text{(i)}$$

$$L_1 = 47.168\text{kgm}^2\text{s}^{-1}$$

When each boy moves 0.6m towards the centre , the distance of each boy from the centre.

$$r_2 = 1.3 - 0.6 = 0.7\text{m}$$

New M.I of the system

$$I_2 = \frac{ML^2}{12} + 2mr_2^2$$

$$= \frac{10\times\left(2.6\right)^2}{12} + 2\times25\times\left(0.7\right)^2$$

$$I_2 = 30.13\text{kgm}^2$$

Let ω_2 = final angular velocity

Final angular momentum

$$L_2 = I_2\omega_2$$

Apply the principle of conservation of angular momentum

(i) = (ii)

$$I_1\omega_1 = I_2\omega_2$$

$$90.13\times\frac{\pi}{6} = 30.13\times2\pi f_2$$

$$f_2 = 0.25\text{r.p.s} = 15\text{r.p.m}$$

Initial kinetic energy

$$E_1 = \frac{L_1^2}{2I_1} = \frac{1}{2}\frac{\left(I_1\omega_1\right)}{I_1} = \frac{1}{2}I_1\omega_1^2$$

$$= \frac{1}{2}\times90.13\times\left(\frac{\pi}{6}\right)^2$$

$$E_1 = 12.35\text{J}$$

Final kinetic energy

$$E_2 = \frac{1}{2I_2}\left(I_2\omega_2\right)^2$$

$$= \frac{\left[30.13\times0.5\pi\right]^2}{2\times30.13}$$

$$E_2 = 37.13J$$

Change in K.E , $\Delta E = E_2 - E_1$

$\Delta E = 37.13 - 12.35$

$\Delta E = 24.78J$

Example – 48

Two discs of moment of inertia I1 and I2 about their respective axes (normal to the disc and passing through the centre) and rotating with angular speed and brought into contact face to face with their axis of rotation coincident.

(i) What is the angular speed of the two discs system?

(ii) Show that kinetic energy of the combined system is less than the sum of initial kinetic energies of two discs. How do you account of this.

Solution

(i) Initial angular momentum of the system

$$Li = I_1\omega_1 + I_2\omega_2$$

Let ω = angular velocity as two discs combine together.

$$L_f = \left(I_1 + I_2\right)\omega$$

Before the conservation of angular momentum

$$\left(I_1 + I_2\right)\omega = I_1\omega_1 + I_2\omega_2$$

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

(ii) Kinetic energy of the system before the discs combined together.

$$E_1 = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$$

Final kinetic energy of the disc combined together.

$$E_2 = \frac{1}{2}\left(I_1 + I_2\right)\omega^2$$

$$= \frac{1}{2}\left(I_1 + I_2\right)\left(\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}\right)^2$$

$$E_2 = \frac{1}{2}\frac{\left(I_1\omega_1 + I_2\omega_2\right)^2}{\left(I_1 + I_2\right)}$$

Change in kinetic energy

$$\Delta E = E_1 - E_2$$

$$\Delta E = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 - \frac{\left(I_1\omega_1 + I_2\omega_2\right)^2}{2\left(I_1 + I_2\right)}$$

$$\Delta E = \frac{I_1I_2\left(\omega_1 - \omega_2\right)^2}{2\left(I_1 + I_2\right)}$$

Example – 49

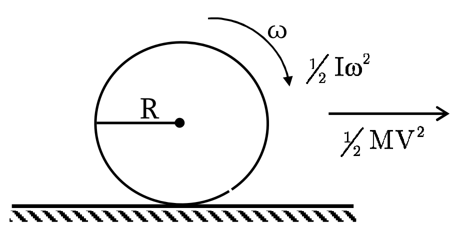
(i) State the principle of conservation of angular momentum and show how the principle can be demonstrated in the life

(ii) A uniform disc of radius R is rotating in its own plane with angular velocity , ω_0 when it is placed flat on a rough table. If μ is the coefficient of sliding friction is independent of velocity show that the time for the disc to come to rest is $\frac{3}{4}\frac{R\omega_0}{\mu g}$ how does the kinetic energy of rotation of the disc vary with time?

Solution

(i) Refer to your notes

(ii) Consider the disc rotating with angular velocity , ω as shown on the figure below.



Total kinetic energy

$$E = \frac{1}{2}MV^2 + \frac{1}{2}I\omega_0^2$$

But $I = \frac{1}{2}MR^2$, $V = \omega_0R$

Now; $E = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_0^2 + \frac{1}{2}M\left(\omega_0R\right)^2$

$$E = \frac{3}{4}MR^2\omega_0^2 \text{......(i)}$$

Work done by the disc against friction ,

$W = fs = \mu mgs$

$S = R\theta$

$W = \mu MgR\theta$……(ii)

Apply work – energy theorem

(i) = (ii)

$$\mu mgR\theta = \frac{3}{4}MR^2\omega_0^2$$

$$\theta = \frac{3}{4}\frac{R\omega_0^2}{\mu g}$$

Since $\theta = \omega_0 t$

$$\omega_0 t = \frac{3}{4}\frac{R\omega_0^2}{\mu g}$$

$$t = \frac{3}{4}\frac{R\omega_0}{\mu g}$$ hence shown.

Kinetic energy

After a time , t the velocity angular velocity

$$\omega = \omega_0 - \alpha t$$

Angular velocity is decreases and becomes equal to zero. Therefore angular acceleration is negative ($\omega = 0$)

$$0 = \omega_0 - \alpha t$$

$$\alpha = \frac{\omega_0}{t} = \frac{\omega_0}{\frac{3}{4}\frac{R\omega_0}{\mu g}}$$

$$\alpha = \frac{4\mu g}{3R}$$

$$k.e = \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left[\omega_0 - \alpha t\right]^2$$

$$= \frac{1}{4}MR^2\left[\omega_0 - \frac{4\mu gt}{3R}\right]^2$$

$$k.e = \frac{4\mu^2 g^2 m}{9}\left(t - \frac{3R\omega_0}{4\mu g}\right)^{\frac{1}{2}}$$

Hence the kinetic energy decreases to zero according to this square law (parabolic) variation with time , t

Example – 50

(a) A solid disc and a ring both of the radius 10.0cm are placed on a horizontal table simultaneously, with initial angular speed equal to $10\pi s^{-1}$. Which of the two start to roll earlier? The coefficient of kinetic friction $\mu_k = 0.2$.

(b) How will you distinguish between a hardboiled egg and a raw egg by spinning each on a table top?

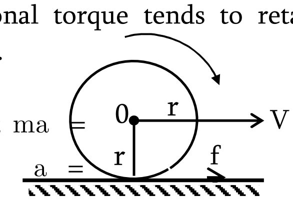
Solution

(a) $r = 0.1m$, $\omega_0 = 10\pi rads^{-1}$, $\mu_k = 0.2$.

from the figure , it is clear that frictional force makes the centre of mass to accelerate and the frictional torque tends to retard the angular speed.

Now ; ma =

a =



Since $v = u_0 + at = 0 + \mu_k gt$

$v = \mu_k gt$……(i)

Also $\tau = \mu_k mgr = I\alpha$

$$\alpha = \frac{\mu_k mgr}{I}$$

Since $\omega = \omega_0 - \alpha t$

$$\omega = \omega_0 - \frac{\mu_k mgrt}{I} \text{……(2)}$$

The rolling begins ,when $V = \omega r$

From equation (1) and (2)

$$\mu_k gt = r\left[\omega_0 - \frac{\mu_k mgrt}{I}\right]$$

For the ring , $I = Mr^2$

$$r\omega_0 - \frac{\mu_k mgr^2 t}{mr^2} = \mu_k gt$$

$$r\omega_0 = 2\mu_k gt$$

$$t = \frac{r\omega_0}{2\mu_k g} = \frac{0.1\times 10\pi}{2\times 0.2\times 9.8}$$

$$t = 0.255\text{second}$$

For the disc , $I = \frac{1}{2}Mr^2$

$$r\omega_0 - \mu_k \frac{mgr^2t}{\frac{1}{2}Mr^2} = \mu_k gt$$

$$r\omega_0 = 3\mu_k gt$$

$$t = \frac{r\omega_0}{3\mu_k g} = \frac{0.1\times 10\pi}{3\times 0.2\times 9.8}$$

$$t = 0.170\,\text{sec}$$

Example – 51

(a) If the radius of the earth assumed to be a perfect sphere, suddenly shrinks to half of its present value, the mass of the Earth remaining unchanged, what will be the duration decrease of the day.

(b) A uniform cylinder of radius 20cm is given an initial angular speed 35rad/s about an axis parallel to its length which passes through its centre. The cylinder is gently lowered onto a horizontal frictional surface and released. The coefficient of friction of the surface is μ = 0.5. How long does it take before the cylinder starts to roll without slipping? What distance does the cylinder travel between its release point and the point at which it commences to roll without slipping?

Solution

(a) Apply the law of conservation of angular momentum.

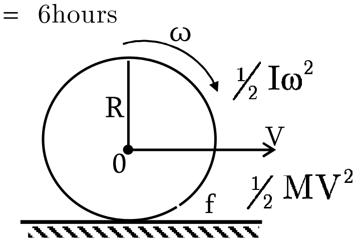
$$I_1\omega_1 = I_2\omega_2$$

$$\frac{2}{5}MR_1^2\left(\frac{2\pi}{T_1}\right) = \frac{2}{5}MR_2^2\left(\frac{2\pi}{T_2}\right)$$

$$T_2 = \left(\frac{R_2}{R_1}\right)^2 T_1 = \left(\frac{R_1}{2R_1}\right)^2 \times 24$$

$$T_2 = 6\text{hours}$$

(b)



The initial velocity of centre of mass is zero i.e u = 0.

The frictional force , f causes the centre of mass of the cylinder to accelerate

$$f = \mu mg = ma$$

$$a = \mu g$$

Since $v = u + at$

$$v = \mu gt$$

Frictional torque causes retardation in angular speed. The torque τ about centre of mass of the cylinder if placed on table is given by

$$\tau = -\alpha I = fR$$

$$\mu mgR = -\alpha I$$

$$\alpha = \frac{-\mu MgR}{I}$$

Applying , $\omega = \omega_0 + \alpha t$

$$\omega = \omega_0 - \frac{\mu mgRt}{I}$$

Now, the cylinder stops slipping as soon as the noslip conditions

$V = \omega R$ is satisfied.

$$V = R\left(\omega_0 - \frac{\mu MgRt}{I}\right)$$

$$\mu gt = R\left(\omega_0 - \frac{\mu MgRt}{I}\right)$$

$$\mu gt = \omega_0 R - \frac{\mu MgR^2t}{I}$$

But $I = \frac{1}{2}MR^2$

$$\mu gt = \omega_0 R - \frac{\omega MgR^2t}{\frac{1}{2}MR^2}$$

$$t = \frac{R\omega_0}{3\mu g}$$

$$= \frac{0.2\times 35}{3\times 0.15\times 9.8}$$

$$t = 1.59\,\text{sec}$$

Whist it is slipping the cylinder travels a distance.

$$S = \frac{1}{2}at^2 = \frac{1}{2}\mu gt^2$$

$$= \frac{1}{2}\times 0.15\times 9.8(1.59)^2$$

$$S = 1.86\text{m}$$

Example – 52

(a) (i) Define the moment of inertia

(ii) State the parallel axes theorem .

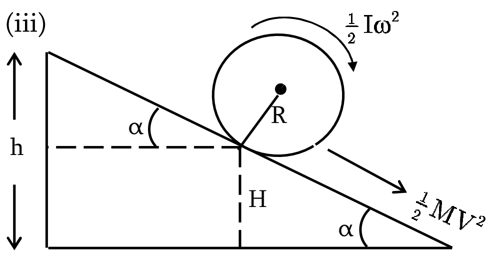
(iii) A solid cylinder of mass 'M' and radius 'R' rolls without slipping down a plane inclined at an angle (α) to the horizontal. Write down the total energy of the cylinder in terms of the mass of cylinder M and velocity V of the centre of mass of cylinder and any other quantities which you defined.

(b) Calculate the maximum speed of a solid cylinder rolling without slopping at the bottom of inclined plane of length 1.6m inclined at 60^{O} to the horizontal. Assume that the cylinder started from the rest at the top of the inclined plane.

Solution

(a) (i) , (ii) Refer to your notes

(iii)



Total k.e of rolling body

$$K.E = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$

Total energy of rolling body

$$E = p.e + k.e$$

$$E = MgH + \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$

(b) Apply the principle of conservation of energy

$$Mgh = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$

$$\omega = V/R$$

$$Mgh = \frac{1}{2}\frac{IV^2}{R^2} + \frac{1}{2}MV^2$$

$$Mgh = \frac{V^2}{2}\left[M + \frac{I}{R^2}\right]$$

But $h = L\sin\alpha$

$$MgL\sin\alpha = \frac{V^2}{2}\left[\frac{I}{R^2} + M\right]$$

$$V = \left[\frac{2MgL\sin\alpha}{M + I/R^2}\right]^{\frac{1}{2}}$$

But $I = \frac{1}{2}MR^2$

$$\frac{I}{R^2} = \frac{M}{2}$$

$$V = \left[\frac{2MgL\sin\alpha}{\frac{M}{2} + M}\right]^{\frac{1}{2}} = \left[\frac{4}{3}gL\sin\alpha\right]^{\frac{1}{2}}$$

$$= \left[\frac{4\times 9.8\times 1.6\sin 60^{O}}{3}\right]^{\frac{1}{2}}$$

$$V = 3.2\text{m/s}$$

Example – 53

(a) (i) State the parallel axes theorem

(ii) What is the radius of gyration of a body?

(iii) Find the moment of inertia of a solid sphere of mass M and radius 'a' about any axis which passes through any point on its surface given that its moment about its centre of gravity is $2/5Ma^2$.

(b) A sphere and a cylinder of the same mass and radius starts from rest at the same point rolls down at the same plane inclined to an angle 30^{O} to the horizontal:

(i) Which body gets to the bottom first and with what acceleration?

(ii) If the later body has reach the bottom with the acceleration of the former body , what would be the angle of inclination to the horizontal for the body?

(c) A body like hoop , solid cylinder or sphere of mass M and radius 'a' whose moment of inertia about its centre of gravity is MK^2 (Where K is the radius of gyration is made to roll down a slope which is inclined at an angle θ to the horizontal having a total length , h . find the final velocity ,V possessed by the body when it reaches to the bottom of the plane.

(i) In terms of h , k and g

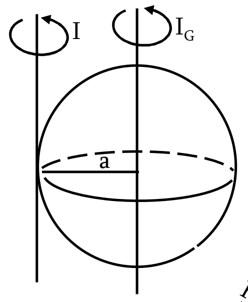
(ii) In terms of θ , k , g and a.

(d) Find also the uniform acceleration of the body down the plane in terms of g, θ , k and a.

Solution

(a) (i) , (ii) Refer to your notes

(ii) apply the parallel axes theorem

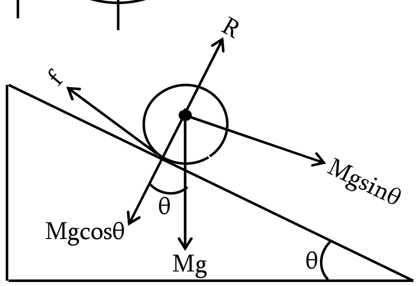


$$I = I_G + Mh^2$$

$$= \frac{2}{5}Ma^2 + Ma^2$$

$$I = \frac{7}{5}Ma^2$$

(b)



From the newton's second law of motion

Mgsinθ – f = Ma………..(1)

Torque produced by the frictional force

$$\tau = FR = I\alpha$$

$$f = \frac{I\alpha}{R} = \frac{Ia}{R^2} \;\ldots\ldots\ldots(2)$$

Putting equation (2) into (1)

$$Mg\sin\theta - \frac{Ia}{R^2} = Ma$$

$$Mg\sin\theta = a\left[\frac{I}{R^2} + M\right]$$

$$a = \frac{Mg\sin\theta}{M + \frac{I}{R^2}}$$

(i) Let a_1 = acceleration of sphere

$$a_1 = \frac{Mg\sin\theta}{M + \frac{2M}{5}} = \frac{5Mg\sin\theta}{7M}$$

$$= \frac{5}{7}g\sin\theta = \frac{5}{7}\times 9.8\sin 30^O$$

$$a_1 = 3.5m/s^2$$

a_2 = acceleration of the cylinder

$$I = \frac{1}{2}MR^2 \;,\; I = \frac{M}{2}$$

$$a_2 = \frac{Mg\sin\theta}{M + \frac{1}{2}} = \frac{2}{3}g\sin\theta$$

$$= \frac{2}{3}\times 9.8\sin 30^O$$

$$a_2 = 3.27m/s^2$$

Since $a_1 > a_2$, therefore the solid sphere will reach at the bottom first than the solid cylinder.

(ii) $\theta = 30^O$

$$a_1 = \frac{5}{7}g\sin 30^O = 3.5m/s^2$$

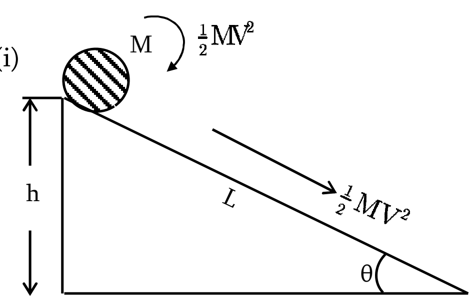
$$a_1 = a_2$$

Let θ_1 = New angle

$$3.5 = \frac{2}{3}g\sin\theta_1$$

$$\theta_1 = 32.39^O$$

(c) (i)



Apply the principle of conservation of energy

$$Mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}MV^2 \qquad \left[\omega = \frac{V}{a}\right]$$

$$Mgh = \frac{1}{2}I\frac{V^2}{a^2} + \frac{1}{2}MV^2 \quad \text{But } I = MK^2$$

$$Mgh = \frac{MV^2}{2}\left[\frac{K^2}{a^2} + 1\right]$$

$$V = \left[\frac{2gh}{1+\left(\frac{k}{a}\right)^2}\right]^{\frac{1}{2}}$$

(ii) Since $h = L\sin\theta$

$$V = \left[\frac{2gL\sin\theta}{1+\left(\frac{k}{a}\right)^2}\right]^{\frac{1}{2}}$$

(d) Let a_O = Uniform acceleration down the slope , $U = 0$

$V^2 = U^2 + 2aL = 2a_OL$

$V^2 = 2a_OL$ ....(i)

Apply the law of conservation of energy

$$MgL\sin\theta = \frac{V^2}{2}\left[M+\frac{I}{a^2}\right]$$

$$V^2 = \frac{2gML\sin\theta}{M+\frac{I}{a^2}}$$

$$2a_OL = \frac{2MgL\sin\theta}{M+\frac{I}{a^2}}$$

$$a_O = \frac{Mg\sin\theta}{M+\frac{I}{a^2}}$$

For the solid sphere , $I = \frac{2}{5}Ma^2$

$$\frac{I}{a^2} = \frac{2}{5}M$$

$$a_O = \frac{Mg\sin\theta}{M+\frac{2}{5}M}$$

$$a_O = \frac{5}{7}g\sin\theta$$

In terms of g , θ k and a

$I = MK^2$

$$a_O = \frac{Mg\sin\theta}{M+\frac{MK^2}{a^2}}$$

$$a_O = \frac{g\sin\theta}{1+\left(\frac{k}{a}\right)^2}$$

Example – 54

(a) Define the moment of inertia angular velocity , and angular momentum.

(b) A ballet dancer can spin faster by folding her arms than with arms outstretched. Explain why?

(c) A solid sphere and a cylinder both of the same radius are released simultaneously from the top of a 10m plane inclined at an angle of 30^O with horizontal. Determine which of the body will be first to reach the bottom of the plane? Which of the two objects which have greater angular momentum at the bottom of the plane?

Solution

(a) Refer to your notes

(b) When the arms of the ballet dancer are folded her moment of inertia I decreases but since angular momentum is conserved , her angular velocity increases. $\left(I \propto \frac{1}{\omega}\right)$

(c) For sphere , $I_G = \frac{2}{5}MR^2$

For the cylinder , $I_G = \frac{1}{2}MR^2$

Total energy at the top of plane.

$E_1 = Mgh = MgL\sin\theta$

$= M\times 9.8\times 10\sin 30^O$

$E_1 = 49M$ joule

After bottom of the plane.

$$E_2 = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$

- For the cylinder

$$E_2 = \frac{1}{2}MV_C^2 + \frac{1}{2}\frac{1}{2}MR^2\left(\frac{V_C}{R}\right)^2$$

$$E_2 = \frac{3}{4}MV_C^2$$

By the law of conservation of energy

$E_1 = E_2$

$$49M = \frac{3}{4}MV_C^2$$

$$V_C = 8.083m/s$$

- For the solid sphere

$$E_2 = \frac{1}{2}MV_S^2 + \frac{1}{2}\frac{2}{5}MR^2\left(\frac{V_S}{R}\right)^2$$

$$E_2 = \frac{7}{10}MV_S^2$$

Then $E_1 = E_2$

$$49M = \frac{7}{10}MV_S^2$$

$$V_S = 8.37m/s$$

Hence the sphere will reach at at the bottom first sine $V_S > V_C$. Therefore sphere will have a greater angular momentum that the cylinder at the bottom of the plane.

Example – 55

(a) How will you distinguish between a hardboiled egg and a row egg by spinning each on the top?

(b) Show that the rotational kinetic energy of a ball rolling over a horizontal plane is 2/7 of its total kinetic energy.

Solution

(a) The egg which spins at a lower rate will be raw egg. In a raw egg , the liquid matter tires to get away from the axis of rotation , thereby increasing the moment of inertia. Since $\tau = I\alpha$, α decreases i.e the row egg will spin at a smaller rate. On the other hand, the hardboiled egg will rotate faster like a rigid body.

(b) For the ball

$$I = \frac{2}{5}MR^2$$

$$K.E_R = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{V}{R}\right)^2$$

$$K.E_R = \frac{1}{5}MV^2$$

$$\text{Total K.E} = \frac{1}{2}I\omega^2 + \frac{1}{2}MV^2$$

$$= \frac{1}{5}MV^2 + \frac{1}{2}MV^2$$

$$K.E = \frac{7}{10}MV^2$$

$$\frac{K.E_R}{K.E} = \frac{MV^2}{5}\Big/\frac{7MV^2}{10} = \frac{2}{7}$$

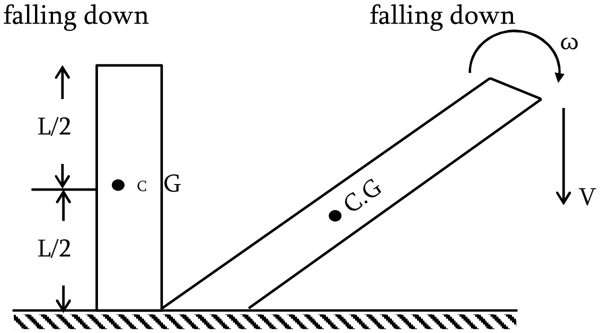
$$K.E_R = \frac{2}{7}K.E$$

Example – 56

A rod of length 30cm , held vertically with one end on the ground is allowed to fall on the ground. Find the angular velocity of the rod when its upper end to uches the ground. Also find the liner velocity of the middle point of the rod at that instant (g = 10m/s²).

Solution

Before the rod falling down when the rod falling down



Apply the law of conservation of energy

Here the p.e of the rod is converted into rotational k.e

$$\frac{MgL}{2} = \frac{1}{2}I\omega^2$$

$$\omega^2 = \frac{MgL}{I} \quad \text{but } I = \frac{1}{3}ML^2$$

$$\omega^2 = \frac{MgL}{\frac{1}{3}ML^2} = \frac{3g}{L}$$

$$\omega = \sqrt{\frac{3g}{L}} = \sqrt{\frac{3\times10}{0.3}}$$

$$\omega = 10rads^{-1}$$

Linear velocity of the middle point of the rod.

$$V = \frac{L\omega}{2} = 0.15\times10$$

$$V = 1.5m/s$$

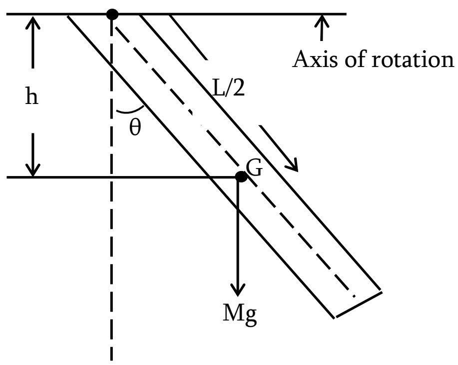
Example – 57

A uniform rod of length 3.0m is suspended at one end so that it can move about an axis perpendicular to its length and it held inclined at 60° to the vertical and then released. Calculate the angular velocity of the rod when

(i) It is inclined at 30O to the vertical

(ii) Reaches the vertical (moment of inertia of rod about end $= \text{mass} \times \frac{(\text{lenght})^2}{3}$)

Solution



From the figure above

$$\cos\theta = \frac{h}{L/2} \quad , \quad h = \frac{1}{2}\cos\theta$$

$$\text{Rotational K.E} = \frac{1}{2}I\omega^2 \,.........(1)$$

$$\text{p.e} = Mgh = \frac{MgL\cos\theta}{2}$$

$$\text{Change in p.e} = \frac{MgL}{2}\left[\cos\theta_2 - \cos\theta_1\right]$$

Apply the law of conservation of energy

(1) = (2)

$$\frac{1}{2}I\omega^2 = \frac{MgL}{2}\left(\cos\theta_2 - \cos\theta_1\right)$$

$$\omega = \left[\frac{MgL}{I}\left(\cos\theta_2 - \cos\theta_1\right)\right]^{\frac{1}{2}}$$

$$\text{But } I = \frac{ML^2}{3}$$

$$\omega = \left[\frac{MgL}{ML^2/3}\left(\cos\theta_2 - \cos\theta_1\right)\right]^{\frac{1}{2}}$$

$$\omega = \left[\frac{3g}{L}\left(\cos\theta_2 - \cos\theta_1\right)\right]^{\frac{1}{2}}$$

(i) $\theta_1 = 60^O \quad , \quad \theta_2 = 30^O$

$$\omega = \left[\frac{3\times 9.8}{3}\left(\cos 30^O - \cos 60^O\right)\right]^{\frac{1}{2}}$$

$$\omega = 1.89\text{rads}^{-1}$$

$$\omega \simeq 1.9\text{rads}^{-1}\,(\text{approx})$$

(ii) $\theta_2 = 0^O \quad , \quad \theta_1 = 60^O$

$$\omega = \left[\frac{3\times 9.8}{3}\left(\cos 0^O - \cos 60^O\right)\right]^{\frac{1}{3}}$$

$$\omega = 2.2\text{rads}^{-1}$$

Example – 58

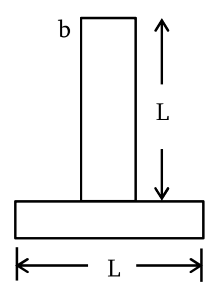
A pendulum is constructed from two identical uniform rods a and b each of length L and mass M connected at right angles to form a 'T' by joining the centre of rod a to one end of rod b. The 'T' is then suspended from the free end of the rod b and the pendulum swings in the plane of the 'T'.

(a) Calculate the moment of inertia of the 'T' about the axis of rotation.

(b) Give an expression for the kinetic and potential energies in terms of the angle θ of the inclination to the vertical of the pendulum.

(c) Derive the equation of the motion of the pendulum.

(d) Show that the period of small oscillation is given by $T = 2\pi\sqrt{\frac{17L}{18g}}$



Solution

(a) The M.I of a thin rod of length L and mass M about its centre is $\frac{ML^2}{12}$

Apply parallel axis theorem givens M.I of rod b

$$I_b = \frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{3}$$

Apply the principle of parallel axes theorem to a rod 'a' gives its M.I about the point of suspension.

$$I_a = \frac{ML^2}{12} + ML^2 = \frac{13}{12}ML^2$$

Total M.I of the system

$$I = I_a + I_b$$

$$= \frac{ML^2}{3} + \frac{13}{12}ML^2$$

$$I = \frac{17}{12}ML^2$$

(b)

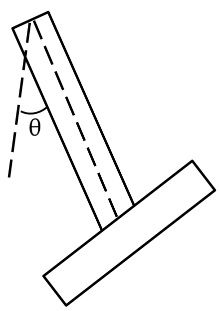


Figure on left hand shows the 'T' at an angle θ to the vertical. The k.e of the system k.e $= \frac{1}{2}I\left(\frac{d\theta}{dt}\right)^2$

At an angle θ the centre of mass of rod b has been raised through a distance $L\frac{(1-\cos\theta)}{2}$ and the centre of mass of rod a has been raised by L (1 – cos θ)

$$P.E = \frac{MgL(1-\cos\theta)}{2} + MgL(1-\cos\theta)$$

$$P.E = \frac{3MgL}{2}(1-\cos\theta)$$

With respect to the value of θ = 0°

(c) The equation of motion of the pendulum can be derived by using the fact that the total energy of system is constant.

P.E + K.E = Constant

$$\frac{17}{24}ML^2\left(\frac{d\theta}{dt}\right)^2 + \frac{3}{2}MgL(1-\cos\theta) = \text{constant}$$

differentiating this expression w.r.t time gives

$$\frac{17}{24}2.ML^2\left(\frac{d\theta}{dt}\right)\cdot\frac{d^2\theta}{dt^2} + \frac{3}{2}MgL\sin\frac{d\theta}{dt} = 0$$

Which can be rearranged to gives

$$\frac{d^2\theta}{dt^2} = \frac{-18}{17}g\frac{\sin\theta}{L}$$

(d) When the oscillation is small sinθ ≃ θ

$$\frac{d^2\theta}{dt^2} = \frac{-18}{17}\left(\frac{g}{L}\right)\theta$$

For an angular S.H.M

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta$$

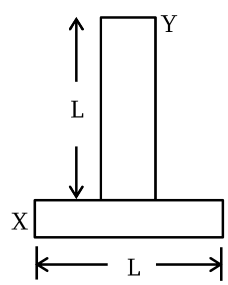
$$-\omega^2\theta = \frac{-18}{17}\left(\frac{g}{L}\right)\theta$$

$$\omega = \sqrt{\frac{18g}{17L}}, \quad \omega = \frac{2\pi}{T}$$

$$T = 2\pi\sqrt{\frac{17L}{18g}} \text{ hence shown}$$

Example – 59

A pendulum is constructed from two identical uniform rods X and Y , each of length L and mass M connected at right angles to form a T by joining the centre of rod X to one end of rod Y.



The T is then suspended from the free end of the rod Y and the perpendicular swings in the plane of T about the axis of rotation.

(i) Calculate the moment of inertia I of the T about the axis of rotation.

(ii) Obtain the expression of the k.e and p.e in terms of the angle θ of inclination to the vertical oscillation of the pendulum.

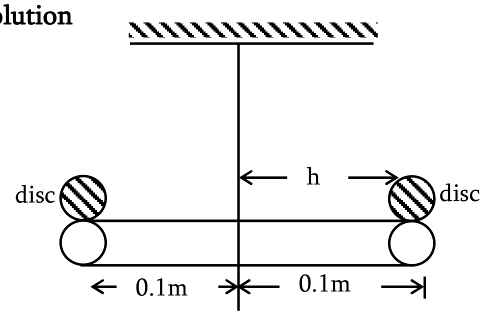
(iii) Show that the period of oscillation is given by

$$T = 2\pi\sqrt{\frac{17L}{18g}}$$

Example – 60

A uniform cylinder 20cm long suspended by a steel wire attached to its mid – point , so that its long axis is horizontally . it found to oscillate with a period of 2sec. when the wire is twisted and released when a small thin disc of mass 10gm is attached to each end , the period is found to be 2.3sec. Calculate the moment of inertia of the cylinder about axis of rotation.

Solution



$$T = 2\pi\sqrt{\frac{I}{Mgh}}$$

For the case of cylinder only

$$T_1 = 2\,\text{sec} \quad , \quad I = I_G$$

$$2 = 2\pi\sqrt{\frac{I_G}{Mgh}} \quad \text{........(1)}$$

For the case of the small disc attached at the end of the cylinder.

$$T_2 = 2.3\,\text{sec}$$

Apply parallel axes theorem

$$I = I_G + 2Mh^2$$

$$T_2 = 2\pi\sqrt{\frac{I_G + 2Mh^2}{Mgh}}$$

$$2.3 = 2\pi\sqrt{\frac{I_G + 2Mh^2}{Mgh}} \quad \text{........(2)}$$

Dividing equation (1) by (2)

$$\frac{2}{2.3} = \sqrt{\frac{I_G}{I_G + 2Mh^2}}$$

$$\left(\frac{2.0}{2.3}\right)^2 = \frac{I_G}{I_G + 2\times10\times10^{-3}\times(0.1)^2}$$

On solving

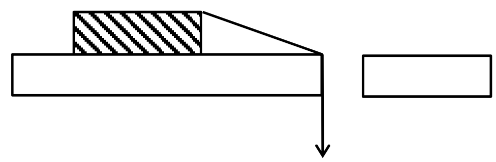
$$I_G = 6.2\times10^{-4}\,\text{kgm}^2$$

Example – 61

A block of mass 0.05kg is attached to a cord passing through a hole in a horizontal frictionless surface as shown in the figure below. The block is originally revolving at the distance of 0.2m from hole with an angular velocity of 3rads^{-1}. The rod is then pulled from below shorten the radius of the circle in which the block revolving to 0.1m, the block may be considered point mass.

(a) What is the angular velocity.

(b) Find the change in kinetic energy of the blocks.



Solution

(a) Since $I_1\omega_1 = I_2\omega_2$

$$M_1r_1^2\omega_1 = M_1r_2^2\omega_2 \quad [M_1 = M_2]$$

$$\frac{\omega_2}{\omega_1} = \left(\frac{r_1}{r_2}\right)^2$$

$$\omega_2 = \omega_1\left[\frac{r_1}{r_2}\right]^2$$

$$= 3.0\left[\frac{0.2}{0.1}\right]^2$$

$$\omega_2 = 12\text{rads}^{-1}$$

(b) Initial kinetic energy

$$E_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}M_1r_1^2\omega_1^2$$

$$= \frac{1}{2}\times0.05\times(0.2)^2(3)^2$$

$$E_1 = 0.0095\text{J}$$

Final kinetic energy

$$E_2 = \frac{1}{2}I_2\omega_2^2 = \frac{1}{2}M_1r_2^2\omega_2^2$$

$$= \frac{1}{2}\times 0.05\times(0.1)^2\times 12^2$$

$$E_2 = 0.036\text{J}$$

Now; $\Delta E = E_2 - E_1 \quad (E_2 > E_1)$

$$= 0.036 - 0.0095$$

$$\Delta E = 0.027\text{J}$$

Example – 62

(a) If angular momentum is conserved in a system, whose moment of inertia is decreased , will its rotational kinetic energy be also conserved? Explain.

(b) The initial angular velocity of a circular disc of mass M is ω_1. The two small spheres of mass m are attached gently to two diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc?

Solution

(a) Suppose that the moment of inertia of the system decreases from I to I'. Then , angular speed will increase from ω to ω', such that

$$I\omega = I'\omega'$$

$$\omega' = \frac{I\omega}{I'}$$

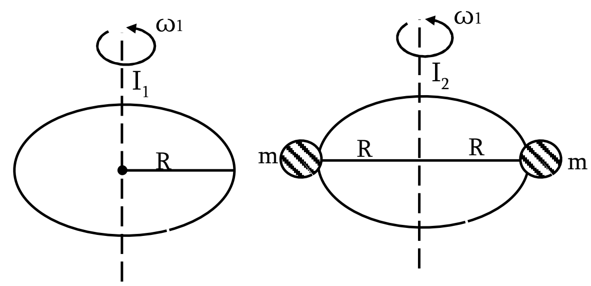
The kinetic energy of rotation of the system.

$$= \frac{1}{2}I'(\omega')^2 = \frac{1}{2}I'\left[\frac{I\omega}{I'}\right]^2$$

$$= \frac{1}{2}\frac{I^2}{I'}\omega^2$$

As I' < I , it follows that the kinetic energy of rotation of the system will increase , when its moment of inertia decreases. Hence , the kinetic energy of rotation will not be conserved.

(b) Before the sphere Attached on the disc   After the two sphere attached to the disc



Initial angular momentum

$$L_1 = I_1\omega_1 = \frac{1}{2}MR^2\omega_1$$

Final angular momentum

$$L_2 = \left[\frac{1}{2}MR^2 + MR^2 + MR^2\right]\omega^2$$

$$L_2 = \left[\frac{1}{2}MR^2 + 2MR^2\right]\omega_2$$

Apply the principle of conservation of angular momentum.

$$L_2 = L_1$$

$$\left[\frac{M}{2} + 2m\right]R^2\omega_2 = \frac{1}{2}MR^2\omega_1$$

$$\omega_2 = \left(\frac{M}{M+4m}\right)\omega_1$$

Example – 63

What will be the duration of the day, if the earth suddenly shrinks to 1/64 of its original volume , mass remaining unchanged? M.I of solid sphere = 2/5 MR^2.

Solution

Let T_1 and ω_1 be the period of revolution and angular velocity respectively of the earth before contraction. Let T_2 and ω_2 be the corresponding quantities after contraction. Let I_1 and I_2 be the moments of inertia of the earth before and after contraction respectively.

Apply the law of conservation of angular momentum.

$$I_1\omega_1 = I_2\omega_2$$

$$\left(\frac{2}{5}MR_1^2\right)\left(\frac{2\pi}{T_1}\right) = \left(\frac{2}{5}MR_2^2\right)\left(\frac{2\pi}{T_2}\right)$$

$$T_2 = \left(\frac{R_2}{R_1}\right)^2 T_1 \text{.......(1)}$$

Volume after contraction

$$= \frac{1}{64} \text{ volume before contraction}$$

$$\frac{4}{3}\pi R_2^3 = \frac{1}{64}\left(\frac{4}{3}\pi R_1^3\right)$$

$$\frac{R_2}{R_1} = \frac{1}{4} \text{................(2)}$$

Putting equation (2) into (1)

$$T_2 = \left(\frac{1}{4}\right)^2 \times T_1 = \frac{1}{16} \times 24\text{hrs}$$

$$T_2 = 1.5\text{hrs}$$

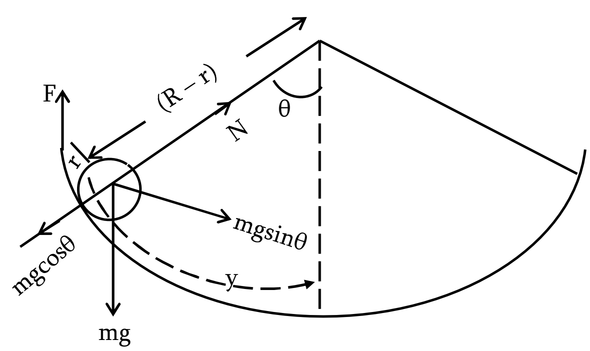
Example – 64

A sphere of radius r rolls without splipping on a concave surface of large radius of curvature R. show that the motion of the centre of gravity of the sphere is approximately simple harmonic motion with a period

$$T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$$

Solution

Let F be the force of static friction between the sphere and the concave mirror and N be normal reaction force



There is no acceleration in a direction normal to the plane.

$$N = Mg\cos\theta \text{.........(i)}$$

Resultant force along the plane mirror

$$Mg\sin\theta - F = Ma \text{..............(ii)}$$

The torque produced by the force of friction about the centre of mass.

$$\tau = I\alpha = Fr$$

$$F = \frac{I\alpha}{r} = \frac{I}{r}\cdot\frac{a}{r} = \frac{Ia}{r^2} \text{.......(iii)}$$

Putting equation (iii) into (i)

$$Mg\sin\theta - \frac{Ia}{r^2} = Ma$$

$$Mg\sin\theta = a\left[M + \frac{I}{r^2}\right]$$

$$a = \frac{Mg\sin\theta}{M + \frac{I}{r^2}}$$

But $I = \frac{2}{5}Mr^2$, $\frac{I}{r^2} = \frac{2}{5}M$

$$a = \frac{Mg\sin\theta}{M + \frac{2}{5}M} = \frac{5}{7}g\sin\theta$$

Restoring force $F = -Ma$

$$F = \frac{-5}{7}Mg\sin\theta$$

From the figure above

$$\sin\theta \simeq \theta = \frac{y}{R-r}$$

If θ is very small angle measured in radian.

$$F = -\frac{5}{7}Mg\left(\frac{y}{R-r}\right)$$

$$Ma = \frac{-5}{7}Mg\left(\frac{y}{R-r}\right)$$

$$a = \frac{-5}{7}g\left(\frac{y}{R-r}\right)$$

$$a = -\left(\frac{5g}{7(R-r)}\right)y \text{.......(iv)}$$

For S.H.M

$$a = -\omega^2 y$$

$$\text{(iv)} = \text{(iv)}$$

$$-\omega^2 y = -\left[\frac{5g}{7(R-r)}\right]y$$

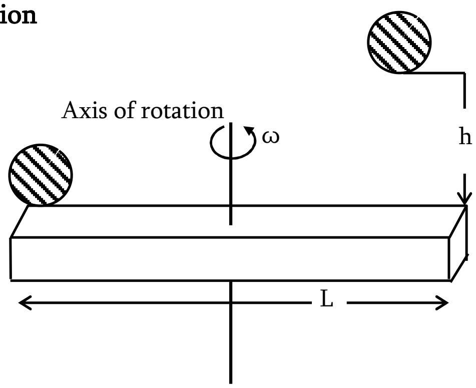
$$\omega = \sqrt{\frac{5g}{7(R-r)}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{7(R-r)}{5g}}$$ Hence shown

Example – 65

A 5.0kg ball is dropped from a height of 12.0m above one end of a uniform bar that pivots as its centre. The bar has mass 8.0kg and is 4.0m in length at the other end of the bar sits another 5.0kg ball , unattached to the bar? The dropped ball sticks to the bar after the collision. How high will the other ball go after the collision?

Solution



The speed of the first ball before it hits the bar is given by

$$V = \sqrt{2gh} = \sqrt{2\times 9.8\times 12}$$

$$V = 15.34m/s$$

Let ω be the angular velocity of the bar just after collision. Take the axis at the centre of the bar. Initial angular momentum of the ball

$$L_1 = MVr = 5.0\times 15.34\times 2$$

$$L_1 = 1533.4Kgm^2s^{-1}$$

Immediately after the collision the bar and both balls are rotating together.

Final angular momentum

$$L_2 = \left(\frac{ML^2}{12} + 2mr^2\right)\omega$$

Apply the law of conservation of angular momentum

$$L_2 = L_1$$

$$\left(\frac{1}{12}\times 8\times 4^2\times 2\times 5\times 2^2\right)\omega = 153.4$$

$$\omega = 3.03rads^{-1}$$

Just after the collision the second ball has linear velocity.

$$V = \omega r = 2\times 3.03$$

$$V = 6.06m/s$$

Apply the principle of conservation energy

As the second ball moving upward its k.e is converted into potential energy.

$$\frac{1}{2}MV^2 = mgH$$

$$H = \frac{V^2}{2g} = \frac{(6.06)^2}{2\times 9.8}$$

$$H = 1.87m$$

Example – 66

(a) State and explain the following.

(i) Theorem of parallel axes

(ii) Theorem of perpendicular axes.

(b) A metre stick is held vertically with one end on the floor and is then allowed to fall. Find the velocity of the other end , when it hits the floor , assuming that the end on the floor does not ship.

Solution

(a) Refer to your notes

(b) When the sticks hits the ground p.e of the stick in the vertical

Position = k.e of rotation , when it hits the ground.

$$\frac{MgL}{2} = \frac{1}{2}I\omega^2$$

$$MgL = I\omega^2$$

But $I = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3}ML^2$

$$MgL = \frac{1}{3}ML^2\omega^2$$

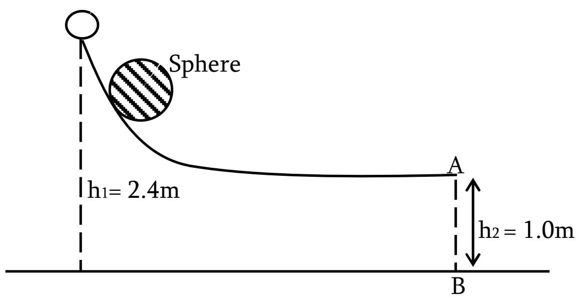
$$\omega = \sqrt{\frac{3g}{L}}$$

Now , the linear velocity with which stick hits the floor.

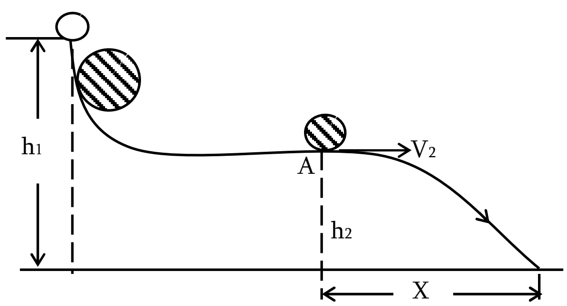
$$V = \omega L = L\cdot\sqrt{\frac{3g}{L}} = \sqrt{3gL}$$
$$= \sqrt{3\times 9.8\times 1}$$
$$V = 5.4 m/s$$

Example – 67

A small sphere rolls down without shipping from the top of a track in a vertical plane. The track has an elevated section and a horizontal part. The horizontal part is 1.0m above the ground and the top of the track is 2.4m above the ground. Find the distance on the ground w.r.t the point B (which is vertically below the end A of the track as shown below , where the sphere lands) During the flight as a projectile , does the sphere continues to rotate about its centre of mass?



Solution



Let M be the mass and R ,the radius of the sphere.
When the sphere is at point O
Total energy of sphere at O

$$E_1 = Mgh_1 \ldots\ldots\ldots\ldots\ldots\ldots(i)$$

When the sphere is at point A
Total energy of sphere at A

$$E_2 = p.e + k.e$$
$$= Mgh_2 + \frac{1}{2}MV_2^2 + \frac{1}{2}I\omega^2$$
$$= Mgh_2 + \frac{1}{2}MV_2^2 + \left(\frac{2}{5}MR^2\right)\left(\frac{V_2}{R}\right)^2$$
$$E_2 = Mgh_2 + \frac{7}{10}MV_2^2 \ldots\ldots(ii)$$

Apply the principle of conservation of energy

$$E_2 = E_1$$
$$Mgh_2 + \frac{7}{10}MV_2^2 = Mgh_1$$
$$\frac{7}{10}MV_2^2 = g(h_1 - h_2)$$
$$V_2 = \sqrt{\frac{10g}{7}(h_1 - h_2)}$$
$$V_2 = \sqrt{\frac{10\times 9.8}{7}(2.4-1)}$$
$$V_2 = 4.427 m/s$$

If t is the time taken by the sphere to reach on the ground

$$h_2 = \frac{1}{2}gt^2 \quad (u = 0)$$
$$t = \sqrt{\frac{2h_2}{g}} = \sqrt{\frac{2\times 1}{9.8}}$$
$$t = 0.452\,\text{sec}$$

Horizontal range

$$X = V_2 t = 4.427\times 0.452$$
$$X = 1.9999m \simeq 2m$$
$$X = 2m$$

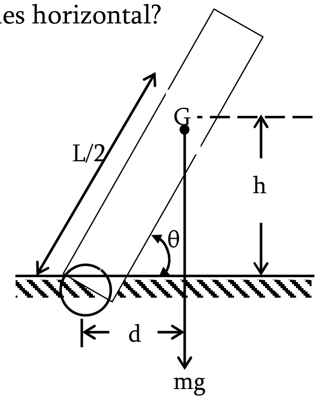
Example – 68

A uniform draw bridge 8m long is attached to the road way by a frictionless hinge at one end , and it can be raised by a cable attached to the other end. The bridge is at rest , suspended at 60° above the horizontal , when the cable suddenly break;

(a) Find the angular acceleration of the bridge just after the cable breaks.
(b) Could you use the equation $\omega = \omega_0 + \alpha t$, to calculate angular speed of the drawbridge a later time? Explain why?

(c) What is the angular speed of the bridge as it becomes horizontal?

Solution



(a) Torque $= I\alpha = Mgd$

$$\left(\frac{1}{3}ML^2\right)\alpha = Mgd$$

$$\alpha = \frac{3gd}{L^2}$$

$$\cos\theta = \frac{d}{L/2}, \quad d = \frac{L}{2}\cos\theta$$

$$\alpha = \frac{3gL\cos\theta}{2L^2} = \frac{3g\cos\theta}{2L}$$

$$\alpha = \frac{3\times 9.8\cos 60^O}{2\times 8}$$

$$\alpha = 0.92\text{rad/s}^2$$

(b) The angular acceleration , α depends on the angle the bridge makes with the horizontal, therefore α is constant during the motion and $\omega = \omega_0 + \alpha t$ cannot be used.

(c) Apply the principle of conservation of energy

$$Mgh = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2$$

$$Mgh = \frac{1}{6}ML^2\omega^2$$

$$\omega^2 = \frac{6g}{L^2}\cdot\frac{L}{2}\sin 60^O = \frac{3g\sin 60^O}{L}$$

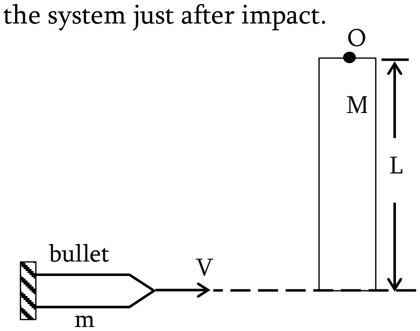
$$\omega = \sqrt{\frac{3g\sin 60^O}{L}} = \sqrt{\frac{3\times 9.8\sin 60^O}{8}}$$

$$\omega = 1.78\text{rads}^{-1}$$

Example – 69

(a) A small ball at the end of a string that passes through a tube is swang in a horizontal circle of radius 0.3m at a speed of 2.8m/s. What will be tangential speed down so as to reduce the radius of the circle to 0.2m?

(b) A rod of length L and mass M is hinged at point O. A small bullet of mass , m hits the rod as shown in the figure below. The bullet gets embedded in the rod. Find the angular velocity of the system just after impact.



Solution

(a) $r_1 = 0.3$m, $r_2 = 0.2$m , $V_1 = 2.8$m/s , $V_2 = ?$

Since there is no external torque acting on the ball , angular momentum can be conserve.

$$MV_1r_1 = MV_2r_2$$

$$V_2 = \frac{V_1r_1}{r_2} = \frac{2.8\times 0.3}{0.2}$$

$$V_2 = 4.2\text{m/s}$$

(b) Initial angular momentum of the system.

Li = Mvl

After the bullet gets embedded , then the system acquires angular velocity , ω.

Final angular momentum of the system.

$$L_f = I\omega$$

I = M.I of the bullet about an axis through O + M.I of rod about the axis through O.

$$I = mL^2 + \frac{1}{3}ML^2$$

$$I = \left(\frac{M+3m}{3}\right)L^2$$

$$L_f = \left(\frac{M+3m}{3}\right)L^2\omega$$

Apply the law of conservation of angular momentum

$$\left(\frac{M+3m}{3}\right)L^2\omega = MvL$$

$$\omega = \frac{3mV}{(M+3m)L}$$

Example – 70

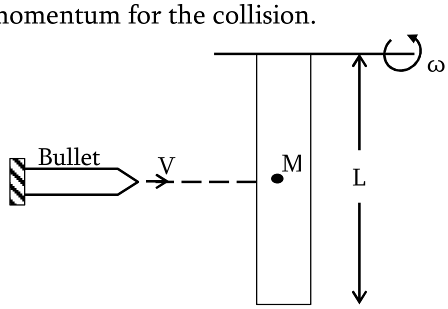
A uniform rod of length 2.5m rests on a frictionless surface. The rod pivots about fixed frictionless axis at one end. The rod is initially at rest a bullet travelling parallel to the horizontal surface and perpendicular to the rod with speed 460m/s strikes the rod at its centre and becomes embedded in it. The mass of the bullet is one – fourth the mass of the rod.

(a) What is the final angular speed of the rod just after the bullets impact? During the collision , why is the angular momentum conserved , but not the linear momentum?

(b) What is the ratio of the kinetic energy of the system after the collision to the kinetic before the collision.

Solution

(a) Apply the law of conservation of angular momentum for the collision.



$$mvr = (Irod + Ibullet)\omega$$

$$mvr = \left(\frac{1}{3}ML^2 + mr^2\right)\omega$$

$$m = \frac{M}{4}, \quad r = \frac{L}{2}$$

$$\frac{V}{8} = \frac{1gL\omega}{48}$$

$$\omega = \frac{6V}{19L} = \frac{6\times 460}{19\times 2.5}$$

$$\omega = 58.1\text{rad/s}$$

Linear momentum is not conserved during collision because of the force applied to the rod at the axis. But since external force acts at the axis it produces no torque and angular momentum is conserved.

(b) Initial kinetic energy of the bullet before collision.

$$K_1 = \frac{1}{2}mV^2 = \frac{1}{2}\cdot\frac{1}{4}MV^2$$

$$K_1 = \frac{1}{8}MV^2$$

Kinetic energy of the system after collision

$$K_2 = \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}\left[Irod + Ibullet\right]\omega^2$$

$$= \frac{1}{2}\left[\frac{1}{3}ML^2 + mr^2\right]\omega^2$$

$$= \frac{1}{2}\left[\frac{1}{3}ML^2 + \frac{M}{4}\cdot\left(\frac{L}{2}\right)^2\right]\omega^2$$

$$= \frac{1}{2}\left[\frac{19}{48}ML^2\right]\omega^2$$

$$= \frac{1}{2}\left[\frac{19}{48}ML^2\right]\left[\frac{6V}{19L}\right]^2$$

$$K_2 = \frac{3}{152}MV^2$$

$$\therefore \quad \frac{K_2}{K_1} = \frac{3}{19}$$

Example – 71

A uniform circular disc of mass 20kg and radius 0.15m is mounted on a horizontal cylindrical axle of radius 0.015m and negligible mass. Neglecting frictional losses in the bearing. Calculate:

(a) The angular velocity acquired from rest by the application for 12seconds of a force of 20N tangentially to the axle.

(b) The kinetic energy of the disc at the end of this period.

(c) The time required to bring to rest if a breaking force of 1N were applied tangentially to its rim.

Solution

(a) M.I of the disc , $I = \frac{1}{2}Ma^2$

Torque due to 20N tangential to the axle

$\tau = Fr = I\alpha$

$$\alpha = \frac{Fr}{I} = \frac{Fr}{\frac{1}{2}Ma^2}$$

$$= \frac{20\times0.015}{\frac{1}{2}\times20\times(0.15)^2}$$

$\alpha = 1.333rads^{-2}$

Since $\omega = \omega_0 + \alpha t$

$\omega = \omega_0 + \alpha t$ but $\omega_0 = 0$

$\omega = 1.333\times12$

$\omega = 16rads^{-1}$

(b) $k.e = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}Ma^2\right)\omega^2$

$$= \frac{1}{4}Ma^2\omega^2$$

$$= \frac{1}{4}\times20\times(0.15)^2\times16^2$$

$k.e = 28.8J$

(c) Decelerating torque , $\tau = Fa$

$$\alpha = \frac{\tau}{I} = \frac{Fa}{\frac{1}{2}Ma^2} = \frac{2F}{Ma}$$

$$\alpha = \frac{2\times1}{20\times0.15}$$

$$\alpha = \frac{2}{3}rads^{-2}$$

Time to bring disc to rest

$$t = \frac{\text{initial angular velocity}}{\text{angular deceleration}}$$

$$= \frac{16}{2/3}$$

$t = 24\,sec$

Example – 72

(a) A grind stone has a moment of inertia of 600kgm2. A constant couple is applied and the grindstone is found to have a speed of 150revolution per minutes , 10seconds after starting from rest.j calculate the couple applied.

(b) By applying torqued of 980Nm to a flywheel, its angular velocity is increased from 10 to 20 revolutions per second in two minutes? Determine the moment of inertia.

Solution

(a) $\omega = \omega_0 + \alpha t \quad [\omega_0 = 0]$

$\omega = \alpha t$ but $\omega = 2\pi f$

$$\alpha = \frac{2\pi f}{t} = \frac{2\pi}{10}\left[\frac{150}{60}\right]$$

$\alpha = 1.571rads^{-2}$

Torque $\tau = I\alpha = 600\times1.571$

(b) $\omega_0 = 2\pi f_0 = 2\pi\times10 = 20\pi rads^{-1}$

$\omega = 2\pi f = 2\pi\times20 = 40\pi rads^{-1}$

$$\alpha = \frac{\omega-\omega_0}{t} = \frac{40\pi-20\pi}{120}$$

$$\alpha = \frac{\pi}{6}rads^{-2}$$

$$I = \frac{\tau}{\alpha} = \frac{980\times6}{\pi}$$

$I = 1871.42kgm^2$

Example – 73

A flywheel of mass 65.4kg is made in the form a circular disc of radius 18.0cm and is driven by the belt whose tension at the point where it runs on and off the rim of the wheel are 2kgwt and 5kgwt respectively if the wheel is rotating at a certain at 60r.p.m , find how long will it be before the speed has reached 210 r.p.m. While the flywheel is rotating at this later speed, the belts ships off and a break applied. Find the constant working couple required to stop the wheel is 7 revolutions.

Solution

The flywheel is in the form of disc

$$I = \frac{1}{2}MR^2 = \frac{1}{2}\times65.4\times(0.18)^2$$

$I = 1.05948kgm^2$

Tension on the belt on one side

$T_1 = 2kgwt$

Net tension for the rotation of the wheel

$$T = T_2 - T_1$$
$$T = 5 - 2 = 3\text{kgwt}$$
$$T = 3 \times 9.8 = 29.4\text{N}$$

Torque on the flywheel

$$\tau = Tr = I\alpha$$

Angular acceleration

$$\alpha = \frac{Tr}{I} = \frac{29.4 \times 0.18}{1.05948}$$
$$\alpha = 4.99\text{rads}^{-2}$$
$$\omega_0 = 2\pi f_0 = 2\pi\left(\frac{60}{60}\right) = 2\pi\text{rads}^{-1}$$
$$\omega = 2\pi f = 2\pi\left(\frac{210}{60}\right) = 7\text{rads}^{-1}$$

(i) Time required

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{7\pi - 2\pi}{4.99}$$
$$t = 3.14\,\text{sec}$$

(ii) Now , in this case $\omega_0 = 2\pi f$

$$\omega_0 = 2\pi\left(\frac{210}{60}\right) = 7\pi\text{rads}^{-1}$$
$$\omega = 0$$
$$N = 7$$

Angle traced, $\theta = 2\pi N$

$$\theta = 2\pi \times 7 = 14\pi\text{rad}$$

Let α = angular retardation

$$\omega^2 - \omega_0^2 = -2\alpha\theta$$
$$\alpha = \frac{49\pi^2}{2 \times 14\pi} = \frac{7\pi}{4}\text{rads}^{-2}$$
$$\tau = I\alpha = 1.05948 \times \frac{7}{4} \times \frac{22}{7}$$
$$\tau = 5.82\text{Nm}$$

Example – 74

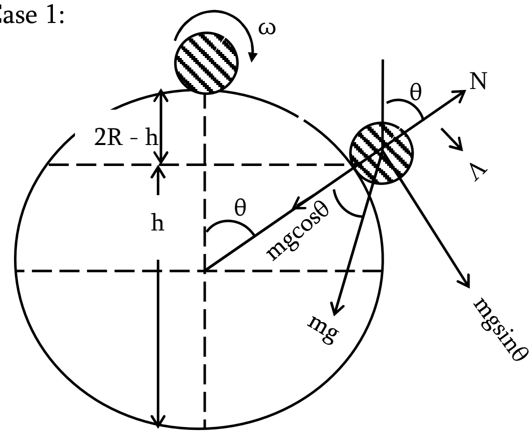
A small sphere is rolling down a large sphere of radius , R. Show that the small sphere will lose contact with the large sphere at a height

$$h = \frac{27}{17}R$$

Show that if a block of ice is used to slide down the large sphere , show also that the block of ice will lose contact with large sphere when $h = \frac{5}{3}R$

Solution

Case 1:



By the law of conservation of energy

Loss in p.e = gain in rolling k.e

$$Mg(2R - h) = \frac{1}{2}I\omega^2 + MV^2$$

But $I = \frac{2}{5}M^2r^2$, $\omega = \frac{V}{r}$

$$Mg(2R - h) = \frac{1}{2} \times \frac{2}{5}Mr^2\left(\frac{V}{r}\right)^2 + \frac{1}{2}MV^2$$
$$V^2 = \frac{10g}{7}(2R - h)\,..........(1)$$

Also $Mg\cos\theta - N = \frac{MV^2}{R}$

When the body loss contact , N = 0

$$Mg\cos\theta = \frac{MV^2}{R}$$
$$V^2 = gR\cos\theta\,.............(2)$$
$$(1) = (2)$$
$$gR\cos\theta = \frac{10g}{7}(2R - h)$$
$$R\cos\theta = \frac{10}{7}(2R - h)$$

But $\cos\theta = \frac{h - R}{R}$

$$\left(\frac{h - R}{R}\right)R = \frac{10}{7}(2R - h)$$
$$h - R = \frac{10}{7}(2R - h)$$

$$h = \frac{27}{17}R \text{ Hence shown}$$

Case 2:

If block of ice is used instead of a sphere , the potential energy loss is converted into sliding kinetic energy.

$$\text{Now}: \quad Mg(2R-h) = \frac{1}{2}MV^2$$

$$V^2 = 2g(2R-h) \ldots\ldots.(1)$$

$$\text{Again}: Mg\cos\theta - N = \frac{MV^2}{R}$$

For the body to lose contact

$$N = 0$$

$$Mg\cos\theta = \frac{MV^2}{R}$$

$$V^2 = gR\cos\theta = gR\left(\frac{h-R}{R}\right)$$

$$V^2 = g(h-R) \ldots\ldots..(2)$$

$$(1) = (2)$$

$$g(h-R) = 2g(2R-h)$$

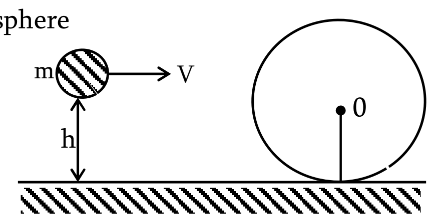
$$h = \frac{5}{3}R \text{ Hence shown.}$$

Example – 75

The sphere of mass M and radius R shown in the figure below , lies on a rough plane when a particle of mass m travelling at a speed V collide and stick with it. The line of motion of the particle is at a height h above the plane. Find

(a) The angular speed of the system about the centre of mass of the sphere just after the collision.

(b) The value of h in terms of R for which the sphere



Starts pure rolling on the plane. Assume that the mass M of the sphere is large compared to the mass of the particle so that the centre of mass of the combined system is not appreciably shifted form the centre of the sphere. Moment of inertia of the sphere about the centre of mass $= \frac{2}{5}MR^2$

Solution

(a) If V_1 is the linear speed of the combined system, conservation of linear momentum gives

$$mV = (M+m)V_1$$

$$V_1 = \frac{mV}{M+m} \ldots\ldots..(1)$$

Angular momentum of the particle before collision is Mv (h – R). If the system rotates with angular speed ω after collision, the angular momentum of the system becomes.

$$I\omega = \left(I_{\text{Sphere}} + I_{\text{Particle}}\right)\omega$$

$$I\omega = \left(\frac{2}{5}MR^2 + mR^2\right)\omega$$

Applying the law of conservation of angular momentum about the centre of mass of the sphere as M>>m,

$$mv(h-R) = \left(\frac{2}{5}MR^2 + mR^2\right)\omega$$

$$\omega = \frac{mv(h-R)}{\frac{2}{5}(M+m)R^2} = \frac{mv(h-R)}{\frac{2}{5}MR^2}$$

$$(M + m \simeq M)$$

$$\omega = \frac{5mv(h-R)}{2MR^2}$$

(b) The sphere will start rolling just after the collision

$$V_1 = \omega R$$

$$\frac{mv}{M+m} = \frac{5mv(h-R)}{2MR^2}\cdot R$$

$$\frac{1}{M+m} = \frac{5(h-R)}{2MR}$$

$$\frac{1}{M} = \frac{5(h-R)}{2MR}$$

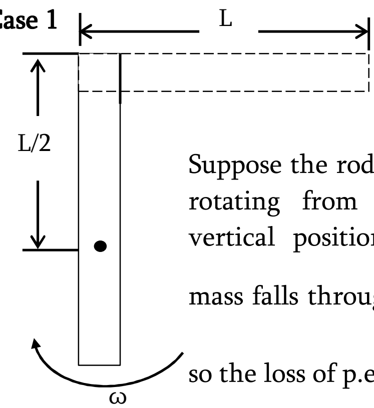
$$h = \frac{7}{5}R$$

Example – 76

uniform rod of length L is freely pivoted at one end. It is initially held horizontally and then released from rest. What is the angular velocity at the instant when the rod is vertical? When the rod is vertical it breaks as its midpoint. What is the largest angle from the vertical reached by the upper part of the rod in its subsequent motion? (assume that no impulsive forces are generated when the rod breaks).

Solution

Case 1



Suppose the rod has mass , m . In rotating from a horizontal to vertical position, the centre of mass falls through a distance $\frac{L}{2}$, so the loss of p.e is $\frac{mgL}{2}$

The M.I of a rod of length L and mass m about the end is $I = \frac{1}{3}mL^2$

K.e of the rod = $\frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mL^2\right)\omega^2 = \frac{1}{6}mL^2\omega^2$

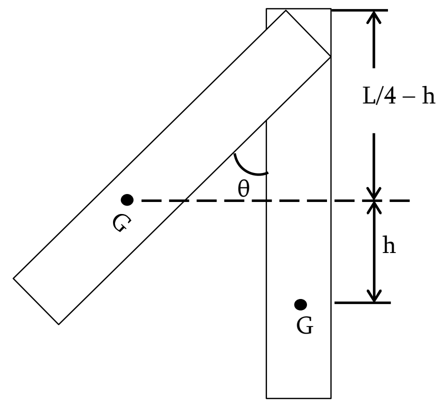
Apply the law of conservation of energy

$$\frac{mgL}{2} = \frac{1}{6}mL^2\omega^2$$

$$\omega = \sqrt{\frac{3g}{L}}$$

Case 2:

When the rod breaks , the mass and the length of the part of the rod are both half of the corresponding values for the unbroken rod.



Now, M.I of the rod

$$I = \frac{1}{3}\left(\frac{m}{2}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{24}mL^2$$

K.e of this part of the rod

$$k.e = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{24}mL^2\right)\times\frac{3g}{L}$$

$$k.e = \frac{1}{16}mgL$$

If the rod now rotates through an angle θ, then

$$\cos\theta = \frac{\frac{L}{4}-h}{L/4} = \frac{L-4h}{L}$$

$$h = \frac{L}{4}(1-\cos\theta)$$

∴ The centre of mass of the rod will rise through a distance $h = \frac{L}{4}(1-\cos\theta)$

Grain in p.e $= \left(\frac{m}{2}\right)gh = \frac{mgL}{8}(1-\cos\theta)$

Apply the law of conservation of energy

$$\frac{1}{16}mgL = \frac{mgL}{8}(1-\cos\theta)$$

$$1-\cos\theta = \frac{1}{2}$$

$$\theta = 60^O$$

Example – 77

(a) (i) Justify the statement that 'if no external torque acts on a body, its angular velocity will not conserved.

(iv) A car is moving with a speed of 30m/s on a circular track of radius 500m. If its speed is increasing at the rate of $2m/s^2$; find its resultant linear acceleration.

(b) An object of mass 1kg is attached to the lower end of a string 1m long whose upper end is fixed and made to rotate in a horizontal circle of radius 0.6m. If the circular speed of the mass is constant , find the ;-

(i) The tension in the string

(ii) Period of motion

Example – 78

(a) (i) What is meant by moment of inertia of a body.

(ii) List two factors on which the moment of inertia of a body depends.

(b) A thin sheet of aluminium of mass 0.032kg has the length of 0.25m and width of 0.1m. find its moment of inertia on the plane about an axis parallel to the:-

(i) Length and passing through its centre of mass , m.

(ii) Width and passing through the centre of mass , m in its own plane

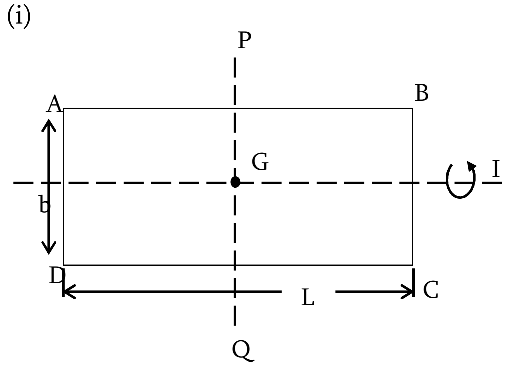
(c) (i) Define the term angular momentum.

(ii) A thin circular ring of mass M and radius r is rotating about its axis with constant angular velocity ω1. If two objects each of mass m are attached gently at the ring , what will be the angular velocity of the rotating wheel?

Solution

(a) Refer to your notes

(b) (i)



M.I of lamina about an axis PQ parallel to AD

Similarly M.I of the lamina about an axis RS parallel to AB or DC passing through the centre G

$$I_X = \frac{1}{12}mb^2$$

$$I_X = \frac{1}{12}\times 0.032\times(0.1)^2$$

$$I_X = 2.67\times10^{-5}\,kgm^2$$

(ii) $I = I_X + I_y$ (perpendicular axes theorem)

$$I = \frac{M}{12}\left(l^2 + b^2\right)$$

$$= \frac{0.032}{12}\left(0.25^2 + 0.1\right)^2$$

$$I = 1.933\times10^{-4}\,kgm^2$$

(c) Refer to your notes

Example – 79

(a) (i) Define torque and gives its S.I unit.

(ii) A disc of moment of inertia $2.5 \times 10^{-4}kgm^2$ is rotating freely about an axis through its center at 20rev/min. if some wax of mass 0.04kg is dropped gently on the disc 0.05m from its axis , what will be the new revolutions per minute of the disc?

(b) Explain briefly why a

(i) High diver can turn more somersaults , before striking the water?

(ii) Dancer on skates can spin faster by folding her arms?

(c) A flywheel of moment of inertia 0.4kgm2 is mounted on horizontal axle of radius 0.01m. if a force of 60N is applied tangentially to the axle:

(i) Calculate the angular velocity of the flywheel after 5seconds form rest.

(ii) List down two assumptions taken to arrive at your answer in 6(c) (i)

Solution

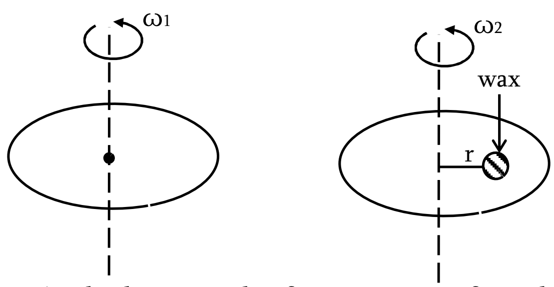
(a) (i) refer to your notes

(ii) $I_d = 2.5\times10^{-4}\,kgm^2$

$$f_1 = 20rev/min = \frac{20rev}{60\,sec} = 0.33rev/s$$

$$f_1 = 0.33rev/s \;,\; f_2 = ?$$

Before wax dropped On the disc | After wax dropped on the disc



Apply the principle of conservation of angular momentum.

$$I_d f_1 = \left(I_d + Mr^2\right) f_2$$

$$f_2 = \left(\frac{I_d}{Mr^2 + I_d}\right) f_1$$

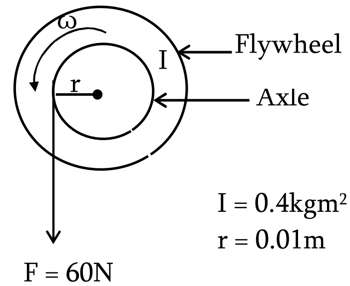
$$= \frac{2.5\times10^{-4}\times20}{2.5\times10^{-4} + 0.04\times(0.05)^2}$$

$$f_2 = 14.29rev\,min^{-1}$$

(b) (i) When a high diver jumps from spring board, he curls his body rolling in his arms and legs. This decreases M.I in which case his angular velocity increases here then performs more somersaults as the diver is about to touch the surface of water , he stretches out his limbs by so doing ,he increases his moment of inertia , thereby reducing his angular velocity.

(ii) By folding her arms , dancer skater reduces her moment of inertia , leading to an increase in her angular speed. This is because when she brings her arms and legs closer to the axis of rotation her M.I about its axis is reduced so angular speed ω increases to ensure that L = Iω is conserved.

(c)



(i) ω = Angular velocity of the flywheel , $t = 5sec$, $\omega_0 = 0$?

Torque on the flywheel

$$\tau = Fr = I\alpha$$

$$\alpha = \frac{Fr}{I} = \frac{60\times0.01}{0.4}$$

$$\alpha = 1.5rads^{-2}$$

Since $\omega = \omega_0 + \alpha t$

$$\omega = \alpha t = 1.5\times5$$

$$\omega = 7.5rads^{-1}$$

(ii) Assumptions made:-

- Moment of inertia of axles is very negligible compaired to the M.I fo the flywheel.
- No friction force between the flywheel and axle.

Example – 80

(a) (i) Define the term moment of inertia.

(ii) Briefly explain the meaning of the term radius of gyration.

(b) A uniform solid cylinder of mass M and radius r rolls without shipping about its centre from rest through a distance(s) along a plane inclined at an angle θ. Derive an expression for the linear acceleration *a* of the cylinder down the plane given that the moment of inertia of the cylinder about its centre $I = \frac{1}{2}Mr^2$ (its angular velocity ω and the linear velocity down the plane is V)

(c) A constant torque of 200Nm twists a wheel about its centre. The moment of inertia about its axis is $100kgm^2$ find:-
   (i) The angular velocity gained in 4second.
   (ii) The kinetic energy gained after 20 revolutions.

Example – 81

(a) Define the following terms
   (i) Torque (ii) angular acceleration (iii) radius of gyration.

(b) A cylindrical rocket of diameter 2.0m develops a spinning motion in space of period 2.0seconds about the axis of the cylinder. To stop the spin two jet motors which are attached to the rocket at opposite ends of the diameter are fired until the spinning motion ceases each motor turn the rocket in the same direction and provides a constant thrust of 4×10^3N in the direction of tangential to the surface of the rocket and in plane perpendicular to its axis. If the moment of inertia of the rocket about its cylindrical axis is $6.0 \times 10^5 kgm^2$, calculate :-
   (i) Angular acceleration of the spinning of the rocket and its angular speed.
   (ii) Number of revolutions made by the rocket during firing
   (iii) Time for which the motor are fired.

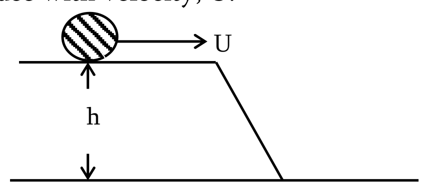
(c) A wheel of radius 50cm and mass 1000gm having 30spokes each of mass 1000gm is travelling forward at 5m/s. If the mass of the rim is 1000gm, determine the;-
   (i) Moment of inertia.
   (ii) Kinetic energy and angular momentum
   (iii) Linear momentum

Example – 82

(a) State the principle of angular momentum

(b) An ice skater spins at 4πrad/s with her arms extended.
   (i) If the moment of inertia her arms folded is 80% to that with arms extended , what is her angular velocity when she folds her arms.
   (ii) Find the fractional change in rotational K.E

(c) A disc rolls without slipping along a horizontal surface with velocity, U.



The disc is then encounters a smooth drop of height 'h' after which it continues with new velocity V at all times the disc remains in vertical plane as shown in the figure above show that $V = \sqrt{U^2 + \frac{4gh}{3}}$

Example – 83

Explain briefly the following phenomena;-

(a) Why is the handle in the flour grinding machine put near the circumference?

(b) A broad handle is use to resolve a screw why?

(c) Why is it more difficult to revolve a stone by tying it to longer string than by trying it to a shorter string?

(d) How is a swimmer jumping into water from a height able to make loop in air.

Solution

(a) A torque is required to revolve the machine. The torque is equal to the multiplication of applied force and its perpendicular distance from the axis of rotation. Clearly, more the distance the more torque can be applied by the same force on account of this , the handle is put near the circumference.

(b) By using broad handle the perpendicular distance of point of action of force from the axis of rotation increases and hence more torque can be applied with the same force. This facilitates to revolution of the screw.

(c) Let the moment of inertia of stone tied to shorter string be F1 and that of longer string I_2. If τ_1 and τ_2 respectively be torque in the first and second case to produce the same angular acceleration , α then $\tau_1 = I_1\alpha$ and $\tau_2 = I_2\alpha$. Since particle is more distant from the axis of rotation in second case then in first case, we shall have to apply more torque in second case. Evidently it will be easier to revolve stone tied to a shorter string.

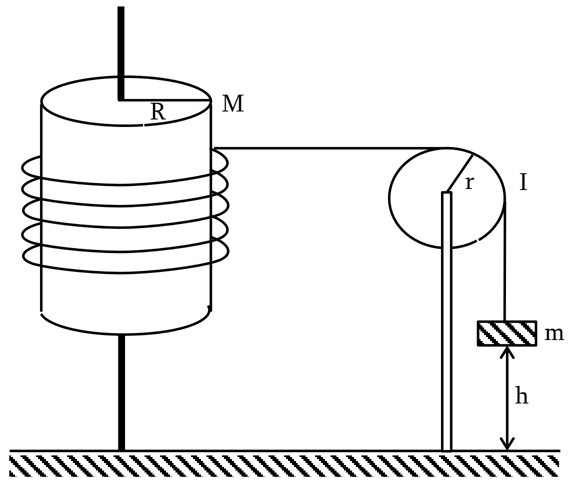
(d) This decreases the moment of inertia I of the swimmer since his angular momentum $I\omega$ remains constant , if I decreases his angular velocity increases and we easily forms a loop in air.

Example – 84

(a) Define angular momentum and give its dimensions.

(b) A grinding wheel in a form of solid cylinder of 0.2m diameter and 3kg mass is rotated at 3600rev/minute.
   (i) What is its kinetic energy?
   (ii) Find how far it would have to fall to acquire the same kinetic energy as in 2(b)(i) above.

(c) A uniform solid cylinder of mass M and radius R rotates about a vertical axis on a uniform bearing. A massless cord rapped with many turns round the cylinder passes over a pulley of rotational inertia I and radius r and then attached to a small mass , M that is otherwise free fall under the influence of gravity as shown in the figure below.



If there is no friction in the pulley axle and the cord does not slip, what is the speed of the small mass after it has fall a distance h from rest.

Example – 85

(a) (i) discuss how Newton's law of motion are related to angular motion.
   (ii) The moment of inertia of a circular disc of mass M and diameter d about an axis through its centre and perpendicular to its plane is given by Md2/8. Deduce an expression for the radius of gyration of the same disc about an axis through its rim and perpendicular to its plane.
   (iii) Apply the knowledge you learned in rotation of rigid bodies to explain why spokes are fitted in the cycle wheels.

(b) A grindstone has a moment of inertia of $1.6 \times 10^{-3}kgm^2$. When a constant torque is applied , the flywheel reaches an angular velocity of 1200 rev/min in 15seconds. Assuming it started from the rest , find the:-
   (i) Angular acceleration
   (ii) Unbalanced torque applied
   (iii) Angle turned through in 15sec
   (iv) Work done (w) on the flywheel by the torque.

Example – 86

(a) Show that the K.E of rotation of a rigid body about an axis with an angular velocity , ω is given by K.E $= \frac{1}{2}I\omega^2$ where I is the moment of inertia of body about the given axis.

(b) A solid sphere of mass , M and radius ‘a’ rolls without slipping down a plane inclined at an angle θ to the horizontal obtain an expression for the acceleration of the centre of the solid sphere in terms of g and θ.

Example – 87

(a) (i) Define the moment of inertia of a body.
(ii) State the parallel axes theorem for moment of inertia.

(b) (i) What is the torque (τ)?
(ii) A uniform disc of radius R and mass M is mounted on an axle supported in fixed frictionless bearings. A light cord is wrapped around the rim of the wheel and mass m is attached at the end of the cord. Find the angular acceleration of the disc using the relation $\tau = \frac{dL}{dt}$ and hence the tension in the string or cord.

(c) (i) State the principle of conservation of angular momentum.
(ii) A body stands on the plat form that can only rotate about a vertical axis holding an axle of a rim – loading bicycle , wheel with its axis vertical. The wheel is spinning about this vertical axis with angular velocity ω_0 of the wheel. What will happen?

Example – 88

Calculate the torque τ and angular momentum gained by a steel ball of mass 4.5kg and radius 50mm at the length 12m if the ball is released from the top of the inclined plane 9m above the ground
Ans.0.42Nm ; 1.01Nm.

Example – 89

(a) (i) State the principle of conservation of angular momentum.
(ii) If the earth suddenly contracted to half of its present radius without any external torque on it, by how much would be the day reduced?

(b) (i) Define the term torque?
(ii) A solid cylinder of mass m is placed on a rough inclined plane of inclination θ to the horizontal. Show that the minimum friction force applied (required) for rolling without slipping is $\frac{1}{3}mg\sin\theta$ and the minimum coefficient of friction is $\frac{1}{3}\tan\theta$
Ans.(a) (ii) 18hours

Example – 90

(a) What is mean by
(i) Radius of gyration
(ii) Moment of inertia

(b) (i) What can’t angular momentum and linear momentum be added.
(ii) A sphere rolls down an inclined plane of 40^{O} from the horizontal. Find its acceleration (M.I is $0.4MR^2$)

Example – 91

(a) (i) Define the term moment of inertia.
(ii) Why is a large torque required to bring about a large change in rotation , if the moment of inertia of the rigid body about that axis is large (02).
(iii) State the principle of conservation of angular momentum. Give two applications of this principle.

(b) (i) An ice – skater is spinning about a vertical axis through his body at a speed of 0.5revolutions per second. He extends his arms horizontally with a weight of mass 2kg in each hand. Assume that the moment of inertia of the skater himself remain constant is $0.8kgm^2$ and the distance of the weights from the axis is 0.9m. Find.
(ii) The total angular momentum of the skater and the weights about the vertical axis.

(c) If the skater in 1(b) pulls his hand to the sides so that the two weights are at distance of 0.2m from the axis of rotation.
   (i) Calculate the final rotation speed friction of ice can be neglected.
   (ii) What is the change in the total kinetic energy of the skater and the weights? Has this kinetic energy been increased or decreased? How can your account for this change?

Example – 92

(a) In problems involving linear motion the following equation are often used:-