MODULE 6 : GRAVITATION

Gravitation – is the force of attraction exist between the Earth and a body in the universe i.e The force of attraction between any two bodies in the universe is called gravitational force or gravitation.

GRAVITY OR EARTH'S GRAVITATIONAL PULL – Is the force of attraction exerted by earth on a body lying on or near the surface of the Earth. Force due to gravity is given by

F = Mg
M = mass of the body
g = Acceleration due to gravity

The acceleration produced in a body when it falls freely under effect of gravity alone is called **'acceleration due to gravity'**. The value of g on or near to the Earth surface, g = $9.8 m/s^2$.

Note that

- The value of g varies slightly from one space to place on the Earth's surface.
- W = mg or g = w/m. Therefore unit of g can be expressed as m/s^2 or N/kg.

DIFFERENCES BETWEEN GRAVITATION AND GRAVITY.

GRAVITATION	GRAVITY
This is the force of attraction between any two bodies in the universe.	This is the force of attraction exerted by the earth towards its centre on a body lying on or near the Earth's surface
This occurs between two bodies separated by distance between them	Exists within a single body
It cannot be zero unless $r \rightarrow \alpha$	It can be zero at the centre of the Earth.

- When the heavy bodies are revolve around (the Earth) to each other, there're are existence of the force of attraction between them. This force is known as 'Gravitational force'. examples of heavy bodies:- moon, planets, sun, Asterooids, Metoroids.etc.

- There are several laws that guide the study movement of bodies in the universe called laws of planetary motion: vix:
  1. Kepler's laws planetary motion
  2. Newton's universal law of gravitation.

1. **KEPLER'S LAWS**

A great advance was made by Johannes Kepler's about 1609. He studies for many years the record of observation of the planets made by **TYCHO – BRAHE** and he discovered three laws known by his name and called **Kepler's law**. Sometimes Kepler's laws are known as **'LAWS OF PLANETORY MOTION'** because the laws made on detailed by study of motion of the planets about the sun. There are three laws of Kepler's laws of planetary motion:-

- Kepler's first law (law of orbit)
- Kepler's second law (law of Areas)
- Kepler's third law (law of period)

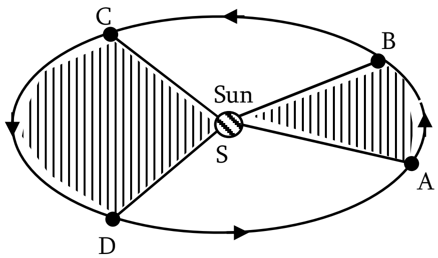
(i) **KEPLER'S FIRST LAW**

State that 'The path of each planet about the sun is an ellipse with sun at the one focus of the ellipse'. In the other words, each planet moves around the sun in a elliptical path with the sun at one focus by using law of universal gravitation, Newton's showed that the general path of a planet under the influence of an inverse – square law of force is an ellipse with the centre of force at one focus if the path is closed. Kepler's first law is also known as orbital rule (law of orbit).

(ii) **KEPLE'S SECOND LAW**

State that ''Each planet moves in such a way that an imaginary line drawn from the sun to the planet sweeps out equal areas in equal periods of time'.

- The second law of Keplers can be used to predict the speed of a planet in one part of its orbit if we know its speed in the other part.



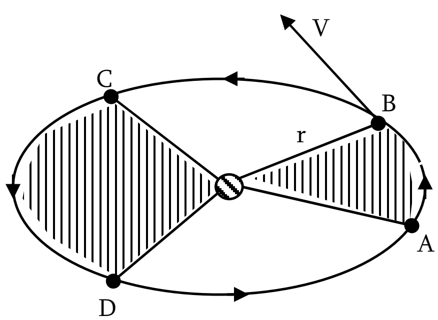
$$\frac{dA}{dt} = \text{constant i.e } A \propto t$$

$$\frac{A}{t} = \frac{dA}{dt} = \text{constant}$$

$$\frac{A_1}{t_1} = \frac{A_2}{t_2} = \text{-----} \frac{A_n}{t_n} = \text{constant}$$

Proof:

Relationship between Kepler's second law and angular momentum. Consider the motion of the planet around the sun as shown on the figure below



Angular momentum L = mvr. The time rate of area dA/dt sweept out by a planet is directly proportional to the angular momentum of planet about the sun.

$$\frac{dA}{dt} \propto L$$

The dominant force on each planet is the gravitational attraction force of the sun which is radially directed force that exerts no torque on the planet. Thus torque on the planet in their orbit is equal to zero and the angular momentum of any planet about the sun is constant. This is accordance to the principle of conservation of angular momentum.

$$\frac{dA}{dt} = KL$$

$$\frac{dA}{dt} = \text{Constant}$$

This means that the time rate at which the planet sweeps out on area is constant. This is the Kepler's second law. Sometime second Kepler's law is known as **'Area rule'**.

(iii) **KEPLER'S THIRD LAW**

State that ' The square of the period of any planet (time needed for one revolution about the sun) is directly proportional to the cube of the planet's average distance from the sun'. If T is the period of the planet and R is its average distance from the sun.

$T^2 \propto R^3$

$T^2 = KR^3$

$$\frac{T^2}{R^3} = \text{Constant}$$

Kepler's third law is also known as period rule law of period)

VALIDITY OF KEPLER'S LAW

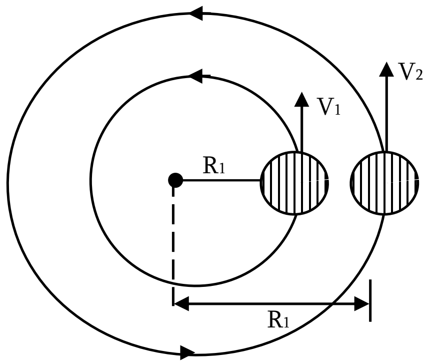
(i) It is approximately true that the orbit speed is constant.

(ii) The orbits of the planets are circles.

Note that

Since $\frac{T^2}{R^3} = \text{Constant}$ i.e T^2/R^3 is the same for all planets. Hence smaller the orbits of the planet around the sun, the shorter the time it takes to complete one revolution.

- If T_1 and T_2 are the periods of two planets and R_1 and R_2 are their respectively average distances from the sun.



$$\frac{T^2}{R^3} = \text{Constant}$$

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^2}$$

Periodic time of each planet revolving around the sun is given by.

$$P_1:\ T_1 = \frac{2\pi R_1}{V_1}$$

$$P_2:\ T_2 = \frac{2\pi R_2}{V_2}$$

$$\frac{T_2}{T_1} = \frac{2\pi R_2}{V_2} \Big/ \frac{2\pi R_1}{V_1}$$

$$\frac{T_2}{T_1} = \left(\frac{R_2}{R_1}\right)\left(\frac{V_1}{V_2}\right)$$

INVERSE SQUARE LAW

State that ' The force of attraction between the planet and the sun is inversely proportional to the square of the distances of the separation between them'.

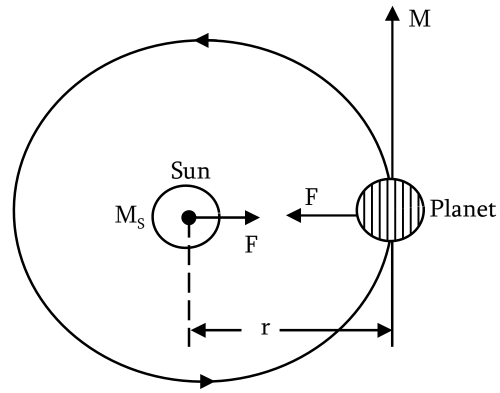
$$F \ \alpha \ \frac{1}{r^2}$$

$$F = \frac{K}{r^2}$$

K = constant of proportionality

- **Derivation of Kepler's third law by using inverse square law.**

  Consider the motion of the planet around the sun as shown on the figure below



Centripetal force on the planet

$$F = M\omega^2 r \ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

According to the inverse square law

$$F = \frac{K}{r^2} \ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

$$\text{(i)} = \text{(ii)}$$

$$M\omega^2 r = K/r^2$$

$$\omega^2 = \frac{K}{Mr^3} \quad \text{but } \omega = \frac{2\pi}{T}$$

$$\frac{4\pi^2}{T^2} = \frac{K}{Mr^3}$$

$$T^2 = \left(\frac{4\pi^2 M}{K}\right) r^3$$

$$\text{Let } \frac{4\pi^2 M}{K} = \text{Constant}$$

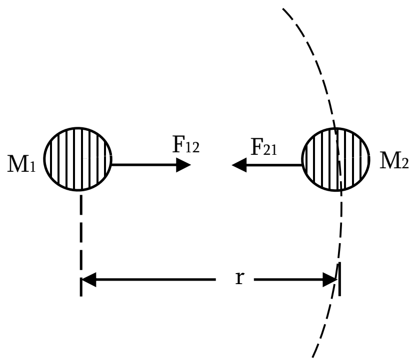
$$T^2 = r^3 \times \text{constant}$$

$$T^2 \ \alpha \ r^3$$

This is Kepler's third law of motion

NEWTON'S UNIVERSAL LAW OF GRAVITATION

State that 'Everybody in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of distance between their centres i.e The maginitude of force of attraction between the planet and the sun is directly proportional to the product of their masses and inversely proportional to the square of their distance apart. This force acts along the line joining the centres of the bodies mathematically.



$$F \propto \frac{M_1M_2}{r^2}$$

$$F = \frac{GM_1M_2}{r^2}$$

Where G is a constant of proportionality known as universal gravitational constant.

r = centre to centre distance between two heavy bodies.

Sometimes, the expression of Newton's universal law of gravitation can be written as

$$F = \frac{-GM_1M_2}{r^2}$$

Minus sign show that the force is the force of attraction i.e Minus sign shows that M_2 is attracted to M_1 and that the force is directed towards to M1. The force on M_1 by M_2 is F_{12} and is equal in magnitude, but opposite in direction i.e $F_{12} = -F_{21}$

NOTE THAT

The following points may be noted:

1. The gravitational force between two bodies is always attractive.
2. Newton's universal law of gravitation is known as universal law because it appears to be true for masses all over the world.
3. It's obey the inverse square law $\left(F \propto 1/r^2\right)$.
4. The gravitational force is central force i.e it acts towards the line joining the two centres of the particle, so the force is said to have spherical symmetry.
5. The gravitational force is an example of conservative force.
6. The value of $G = 6.67 \times 10^{-11}Nm^2kg^{-2}$ and does not depend upon the nature and size of the masses it also does not depends upon the nature of the medium between the two bodies.

$$G = \frac{Ir^2}{M_1M_2}$$

7. The gravitational force of attraction between two bodies is not altered to the presence of the other bodies. Thus one mass will experience the same attractive force from a second body independently of whether or not a third mass is placed between them.

VALIDITY OF THE NEWTON'S UNIVERSAL LAW OF GRAVITATION.

1. The law is valid for point masses.
2. It valid for the two bodies of any size provided they each have spherical symmetry (eg sun and Earth is a good approximately)
3. It is valid when one body has spherical symmetrical and the other is small compared with the separation of their centre.
4. This law fails if the distance between the objects is less than 10-9 i.e of the order of intermolecular distance.

RESULTANT GRAVITATIONAL FORCES BY PRINCIPLE OF SUPERPOSITION.

The resultant gravitational force on point mass M due to the numbers of masses i.e M_1, M_2 , M_3.....Mn can be obtained by using principle of the superposition.

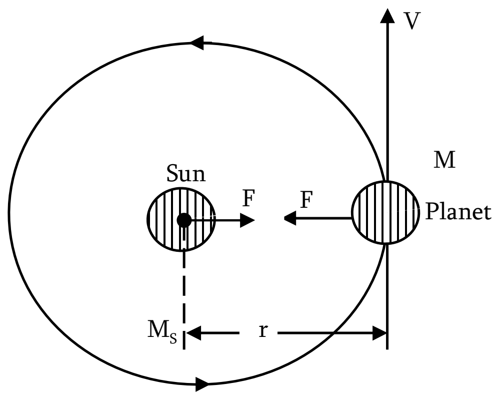
PRINCIPLE OF SUPERPOSITION

State that ' The resultant gravitational force on the point of mass is equal to the vector sum of all individual forces acting on that point of mass, M due to M_1, M_2.........Mn i.e

$$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_n}$$

- Derivation of Kepler's Third law by using Newton's universal law of gravitation.

  Consider the planet of mass M which is revolving around the sun as shown on the figure below



According to the Newton's universal law of gravitation.

$$F = \frac{GMM_s}{r^2} \text{........................(i)}$$

Centripetal force on the planet

$$F = M\omega^2 r = \frac{4\pi^2 Mr}{T^2} \text{...........(ii)}$$

$$\text{(i)} = \text{(ii)}$$

$$\frac{4\pi^2 Mr}{T^2} = \frac{GMM_S}{r^2}$$

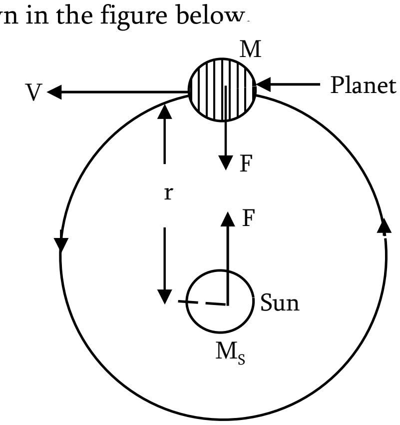
$$T^2 = \left(\frac{4\pi^2}{GMs}\right) r^3$$

$$T^2 \; \alpha \; r^3$$

This implies the kepler's third law we have derived kepler's third law only for uniform circular motion of the planet but the result is true for elliptical orbits if we use the mean distance from the sun, r.

DERIVATION OF NEWTON'S UNIVERSAL LAW OF GRAVITATION FROM KEPLER'S THIRD LAW.

Consider a planet of mass M revolving around the sun of mass Ms in a circular orbit of radius r as shown in the figure below.



For the one complete revolution of the planet

$$V = \frac{2\pi r}{T} \; , \; T = \frac{2\pi r}{V}$$

Centripetal force on the planet

$$F = M\omega^2 r = \frac{4\pi^2 Mr}{T^2} \text{.........(i)}$$

This centripetal force is provided by sun gravitational force on the planet.

According to the Kepler's third law

$$T^2 = Kr^3 \text{.............(ii)}$$

Putting equation (ii) into (i)

$$F = \frac{4\pi^2 Mr}{Kr^3} = \frac{4\pi^2 M}{Kr^2}$$

$$F = \left(\frac{4\pi^2}{K}\right)\left(\frac{M}{r^2}\right) \text{...........(iii)}$$

Since force F on the planet is directly proportional to the mass of the planet and inversely proportional to the square distance from the sun. by Newton's third law of motion, the force exerted by the planet on the sun F α M. The symmetry suggest that

F α Ms

$$\frac{4\pi^2}{K} \; \alpha \; Ms$$

$$\frac{4\pi^2}{K} = GMs$$

$$\text{Now, } F = \frac{GM_s M}{r^2}$$

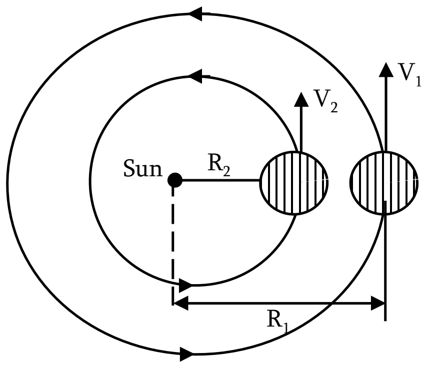
NUMERICAL EXAMPLES.

1. The distance of the two planets from the sun are 10^{13}m and 10^{12}m respectively. Find the ratio of the time periods and the speeds of the two planets.

   Solution

   $R_1 = 10^{13}$M, $R_2 = 10^{12}$m

   Let T_1 and T_2 be periodic times of first and second planet. Respectively and their corresponding velocities are V_1 and V_2



According to the Kepler's third law

$$\frac{T^2}{R^3} = \text{Constant}$$

$$\frac{T_1^2}{R_1^3} = \frac{T_2^2}{R_2^3}$$

$$\frac{T_1}{T_2} = \left[\frac{R_1}{R_2}\right]^{3/2} = \left[\frac{10^{13}}{10^{12}}\right]^{3/2}$$

$$\frac{T_1}{T_2} = 10\sqrt{10}$$

Again $\frac{V_1}{V_2} = ?$

$$V_1 = \frac{2\pi R_1}{T_1}, \quad V_2 = \frac{2\pi R_2}{T_3}$$

$$\frac{V_1}{V_2} = \left(\frac{R_1}{R_2}\right)\left(\frac{T_2}{T_1}\right) = \frac{10^{13}}{10^{12}} \times \frac{1}{10\sqrt{10}}$$

$$\frac{V_1}{V_2} = \frac{1}{\sqrt{10}}$$

2. (a) A Saturn year is 29.5 times the Earth's year. How far is the Saturn from the sun if the Earth is 1.5×10^8km away from the sun.
   (b) calculate the period of Neptune around the sun. Given that diameter of its orbit is 30times the diameter of the Earth's orbits around the sun, both orbits assume circular.

Solution

(a) Given that $T_S = 29.5T_E$

$R_E = 1.5 \times 10^8$Km, $R_S = ?$

According to the Kepler's third law

$T^2 \alpha R^3$

$$\therefore \frac{T_S^2}{T_E^2} = \frac{R_S^3}{R_E^3}$$

$$R_S = R_E\left[\frac{T_S}{T_E}\right]^{2/3} = 1.5\times10^8\left[29.5\right]^{2/3}$$

$$R_S = 14.32\times10^{11}\,\text{Km}$$

(b) Let T_2 = Periodic time of Neptune

T_1 = Periodic time of Earth

$$\frac{T_2^2}{R_2^3} = \frac{T_1^2}{R_1^3}$$

$$T_2 = T_1\left[\frac{R_2}{R_1}\right]^{2/3} = 1\text{yr}\left[\frac{30R_1}{R_1}\right]^{3/2}$$

$$T_2 = 164.3\text{years}$$

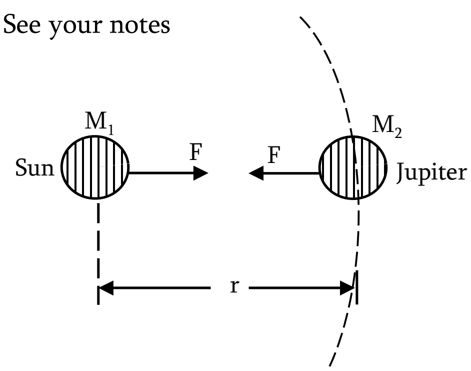
3. The moon has a period of 28days and an orbital radius 3.8×10^8km. What is orbital radius of a satellite that has a period of one day? Answer 4.1×10^4km.

4. (a) State the Newton's universal law of gravitation.
   (b) The mass of planet Jupiter is 1.9×10^{27}kg and that of the sun is 1.99×10^{30}kg. The mean distance of the sun from the Jupiter is 7.8×10^{11}m, calculate the gravitational force in which the sun exerts on Jupiter. Take $G = 6.67 \times 10^{-11}\text{Nm}^{-2}\text{kg}^{-2}$.

Solution

(a) See your notes

(b)



Apply Newton's universal law

$$F = \frac{GM_1M_2}{r^2}$$

$$F = \frac{6.67\times10^{-11}\times1.99\times10^{30}\times1.9\times10^{27}}{\left(7.8\times10^{11}\right)^2}$$

$$F = 4.15\times10^{23}\,\text{N}$$

5. How fast (in m2/s) is area swept out
   (i) The radius from sun to Earth?
   (ii) The radius from Earth to moon? Sun to Earth distance = 1.496 × 10^{11}m, Earth distance = 3.845 × 10^8m and period of revolution of moon $= 271/3$ days (27 1/3 days).

Solution

(i) T = 365 days = 365 × 24v× 60 × 60

$$\frac{dA}{dt} = \frac{\pi R^2}{T} = \frac{\pi\left(1.496\times10^{11}\right)^2}{365\times24\times60\times60}$$

$$\frac{dA}{dt} = 2.23\times10^{15}\,m^2/s$$

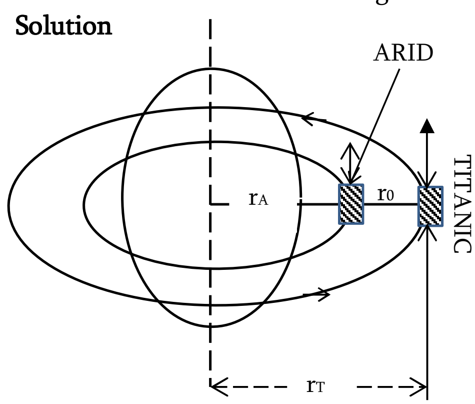
(ii) $T = \frac{82}{3}\times24\times60\times60$

$$\frac{dA}{dt} = \frac{\pi\left(3.845\times10^{8}\right)^2}{\frac{82}{3}\times24\times60\times60}$$

$$\frac{dA}{dt} = 1.97\times10^{11}\,m^2/s$$

6. The satellite ARID describe a very nearly circular orbit of radius 9.0 × 10^8m round the planet Uranus with period of 2.16 × 10^5sec, calculate.
   (i) The mass of Uranus
   (ii) The radius of the orbits TITANIC (another satellite of Uranus) if its period of revolution is 7.49 × 10^5sec.
   (iii) The distance of closest approach between the two satellite during their motion
   G = 6.67 × 10-11Nm-2kg-2

Solution



r_A = orbital radius of ARID satellite

r_T = orbital radius of TITANIC satellite

Given that;

$r_A = 1.9\times10^8\,m$, $T_A = 2.16\times10^5\,Sec$

$T_T = 7.49\times10^5\,Sec$

$G = 6.67\times10^{-11}\,Nm^{-2}kg^{-2}$

Let M = Mass of Uranus

M_1 = Mass of ARID satellite

Consider the motion of ARID around the Uranus.

Gravitation force = Centripetal force

$$\frac{GMM_1}{r_A^2} = M_1\omega^2 r_A$$

$$M = \frac{\omega^2 r_A^3}{G}$$

$$M = \left(\frac{2\pi}{T_A}\right)^2\cdot\frac{r_A^3}{G}$$

$$= \left(\frac{2\pi}{2.16\times10^5}\right)^2\cdot\left(\frac{1.9\times10^8}{6.67\times10^{-11}}\right)^2$$

$$M = 8.69\times10^{25}\,Kg$$

(ii) r_T = ?

Apply Kepler's third law

$$\frac{T^2}{T^3} = \text{a constant}$$

$$\frac{T_A^2}{r_A^3} = \frac{T_T^2}{r_T^3}$$

$$r_T = r_A\left[\frac{T_T}{T_A}\right]^{2/3}$$

$$= 1.9\times10^8\left[\frac{7.49\times10^5}{2.16\times10^5}\right]^{2/3}$$

$$r_T = 4.35\times10^8\,m$$

(iii) r_O = closest distance of approach between TITANIC and ARID satellite.

$$r_0 = r_T - r_A$$

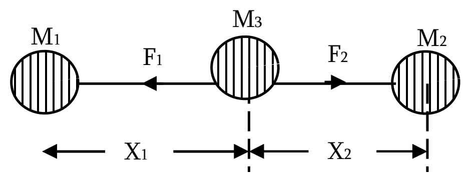
$$= (4.35-1.9)\times10^8$$

$$r_0 = 2.45\times10^8\,m$$

7. Two particles of mass 0.20kg and 0.30kg are placed 0.15m apart. A third particle of mass 0.050kg is placed between them on the line joining the first two particles calculate.
(a) The gravitational force acting on the third particle if it is placed 0.05m from 0.30kg and
(b) Where along the line it should be placed for no gravitational force be exerted on it.

Solution

(a) M_1 = Mass of the 1st particle
M_2 = Mass of the 2nd particle
M_3 = Mass of the 3rd particle



Apply the Newton's universal law of gravitation

$$F_1 = \frac{GM_1M_3}{X_1^2}, \quad F_2 = \frac{GM_2M_3}{X_2^2}$$

Let F = Resultant gravitational force on M3

$$F = F_2 - F_1$$

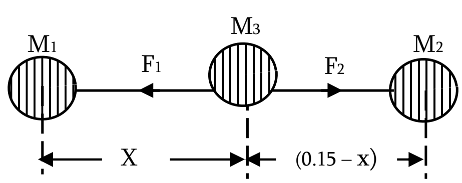
$$= \frac{GM_2M_3}{X_2^2} - \frac{GM_1M_3}{X_1^2}$$

$$= GM_3\left[\frac{M_2}{X_2^2} - \frac{M_1}{X_1^2}\right]$$

$$= 6.67\times10^{-11}\times0.05\left[\frac{0.3}{(0.05)^2} - \frac{0.2}{(0.1)^2}\right]$$

$$F = 33.5\times10^{-11}\text{N towards to 0.30kg}$$

(b) Let X be the position of mass M_3 from M_1 where no gravitational force



For no resultant gravitational force on M_3

$$F_1 = F_2$$

$$\frac{GM_1M_3}{X^2} = \frac{GM_2M_3}{(0.15-X)^2}$$

$$\frac{M_1}{X^2} = \frac{M_2}{(0.15-X)^2}$$

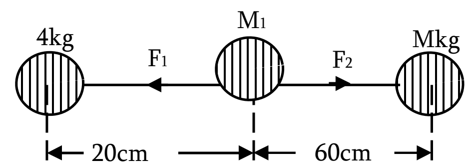
$$\frac{0.2}{X^2} = \frac{0.3}{(0.15-X)^2}$$

On solving X = 0.067m

∴ The particle of mass M_3 must be placed at a distance of 0.067m from M_1.

8. Two small spheres of mass 4.0kg and M kg are placed 80cm apart. If the gravitational force is zero at a point 20cm from 4kg mass along the line between the two masses, calculate the value of M.

Solution



For no resultant gravitational force

$$F_1 = F_2$$

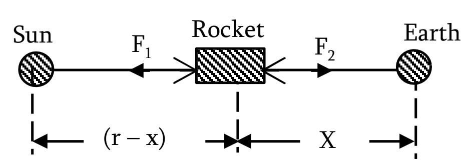
$$\frac{GM_1\times4}{(20)^2} = \frac{GM_1M}{(60)^2}$$

$$\frac{4}{(20)^2} = \frac{M}{(60)^2}$$

$$M = 36\text{kg}$$

9. A rocket is fired from the Earth towards the sun. at what point on its path is the gravitational force on the rocket is zero? Mass of sun $M_s = 2\times10^{30}$kg ; mass of Earth = M_E = 6.0×10^{24}kg. Neglecting the effect of other planets orbital radius of Earth $r = 1.5\times10^{11}$m.

Solution



For no gravitational force on the rocket

$$F_1 = F_2$$

$$\frac{GM_E M}{X^2} = \frac{GM_S M}{(r-x)^2}$$

$$\left[\frac{r-x}{x}\right]^2 = \frac{M_S}{M_E}$$

$$\frac{r-x}{x} = \sqrt{\frac{M_S}{M_E}}$$

$$r = x\left[1+\sqrt{\frac{M_S}{M_E}}\right]$$

$$7.8\times10^{11} = x\left[1+\sqrt{\frac{2\times10^{30}}{6\times10^{24}}}\right]$$

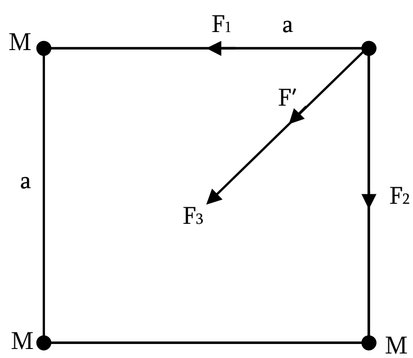
$$x = 2.59\times10^{8}\,m$$

10. (a) What is meant by gravitational force?
    (b) Three masses, each equal to M are placed at the three corners of a square of side 'a'. calculate the force of attraction on a unit mass at the fourth corner.

Solution

(a) Gravitation force – is force of attraction between any two material bodies in the universe.

(b)



$$F_1 = F_2 = \frac{GM}{a^2}$$

Let F' = Resultant force between F_1 and F_2

$$F' = \sqrt{F_1^2+F_2^2} \quad \text{(Pythagoras theorem)}$$

$$= \sqrt{2}\,F_1 = \sqrt{2}\,\frac{GM}{a^2}$$

Again $r^2 = a^2+a^2 = 2a^2$ (Pythagoras theorem).

Since F_3 and F' acts in the same direction.

The resultant force

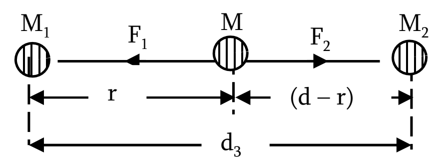
$$F = F_3 + F'$$

$$= \frac{\sqrt{2}GM}{a^2}+\frac{GM}{2a^2}$$

$$F = \frac{GM}{a^2}\left[\sqrt{2}+\frac{1}{2}\right]$$

11. Two stationary particles of masses M_1 and M_2 are a distance d apart. A third particle, lying on the line joining the particles, experiences no resultant gravitational force. What is the distance of this particle from M_1?

Solution



According to the Newton's universal law of gravitation.

$$F_1 = \frac{GMM_1}{r^2}$$

$$F_2 = \frac{GMM_2}{(d-r)^2}$$

For no resultant gravitational force

$$F_1 = F_2$$

$$\frac{GMM_1}{r^2} = \frac{GMM_2}{(d-r)^2}$$

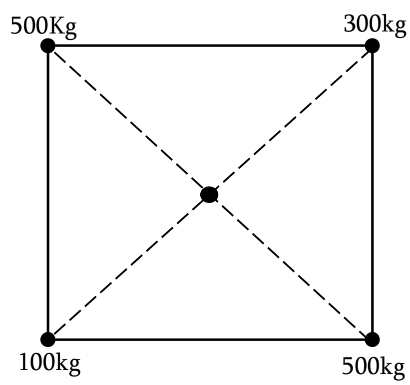
$$\left(\frac{d-r}{r}\right)^2 = \frac{M_2}{M_1} \Rightarrow \frac{d}{r}-1 = \frac{\sqrt{M_2}}{\sqrt{M_1}}$$

$$\frac{d}{r} = \frac{\sqrt{M_2}+\sqrt{M_1}}{\sqrt{M_1}}$$

$$r = d\left[\frac{\sqrt{M_1}}{\sqrt{M_1}+\sqrt{M_2}}\right]$$

EXERCISE NO 1

12. Four sphere forms the corner of a square whose side is 2.0cm long. What are the magnitude and direction of the net gravitational force from them on a central sphere with mass =, M_S = 250kg?



Answer : 0.017N towards the 300kg sphere.

13. Consider two solid uniform spherical objects of the same density ρ. One has a radius R and other a radius 2R. They are in outer space where the gravitational fields from other objects are negligible. If they are at rest with their surfaces touching, what is the contact force between the objects due to their gravitational attraction?

Answer : $\frac{128}{81}G\pi^2R^4\rho^2$

14. (a) What are gravitation and gravity?
(b) The mass of the Earth is 6×10^{24}kg and that of moon 7.4×10^{22}kg. If the distance between their centre is 3.8×10^8m. Calculate at what point on the line joining their centres there is no gravitational force. Neglecting the effect of the planets and the sun. Answer 3.4×10^8m from the Earth.

15. The gravitational force on a mass of 1kg at the Earth's surface is 10N. Assuming the Earth is a sphere of radius R, Calculate the gravitational force on a satellite of mass 100kg in a circular orbit of radius 2R from the centre of the Earth. ($g = 10Nkg^{-1}$). Answer 250N.

- **UNIVERSAL GRAVITATION CONSTANT G.**

Is defined as the force of attraction between two bodies of a unit mass which is separated at a unit distance

Expression of G

According to the Newton's universal law of gravitation

$$F = \frac{GM_1M_2}{r^2}$$

$$G = \frac{Fr^2}{M_1M_2}$$

Unit of G

$$G = \frac{Nm^2}{Kg^2} = Nm^2kg^{-2}$$

∴ S.I unit of G is Nm^2kg^{-2}.

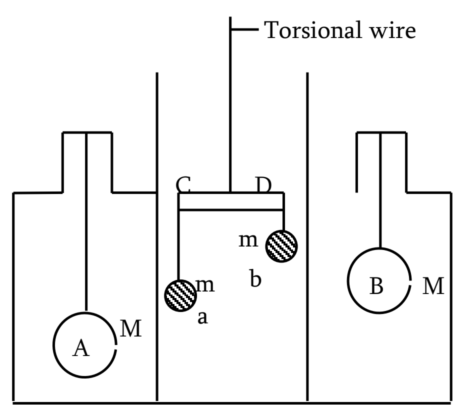
Dimensional of G

$$[G] = \frac{[F][r]^2}{[mass]^2} = M^{-1}L^3T^{-2}$$

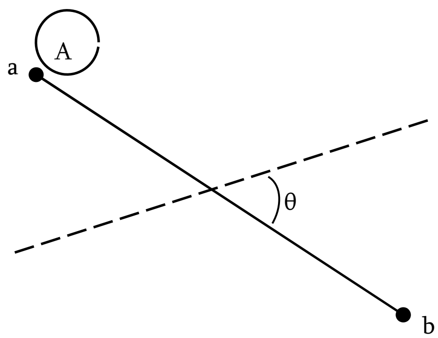
$$[G] = [M^{-1}L^3T^{-2}] = M^{-1}L^3T^{-2}$$

DETERMINATION OF G

The value of gravitation constant, G was first determined exprerimentally by an English physicist HENRY CAVENDISH in 1708. He accomplished this with an extremely sensitive torsional balance. Arrangement of the apparatus can be shown in the figure below.



Plane view



Two identical mass balls of masses in at the ends of light rod were suspended by a very delicated fibre. Two large balls of masses M were brought close to the small masses m. the attractive force, F between m and M causes the fibre to twist. The equilibrium is reached when the restoring torque due to the torque fibre is equal to the torque due to the gravitational attractive.

Let L = Length of C to D

r = distance of separation between M and m.

M = Mass of large sphere

m = mass of small sphere

at the equilibrium

gravitational torque = restoring torque

gravitation force $F = \frac{GMm}{r^2}$

gravitational torque = $\frac{G.MmL}{r^2}$

restoring torque $\tau \alpha \theta$, $\tau = C\theta$

Now, $\frac{GMmL}{r^2} = C\theta$

$$G = \frac{C\theta r^2}{MmL}$$

C = Torsional constant

θ = Angular displacement by knowing the value of M,m, θ, r, c and L, then G can be calculated from the information given.

- **TORSIONAL CONSTANT** – Is defined as the restoring torque per unit deflection, $C = \frac{\tau}{\theta}$

  S.I. Unit of C is Nmrad-1. The tensional constant (c) is determined by allowing bar CD to oscillate through a small angle θ and then noting its periodic time, T

Periodic time for Torsional pendulum

$$T = 2\pi\sqrt{\frac{I}{C}}$$

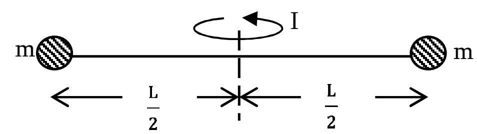
I = Moment of inertia of system

$$T^2 = \frac{4\pi^2 I}{C}$$

$$C = \frac{4\pi^2 I}{T^2}$$

Since $G = \frac{C\theta r^2}{MmL} = \left(\frac{4\pi^2 I}{T^2}\right)\left(\frac{\theta r^2}{MmL}\right)$

Moment of inertia of the system



$$I = m\left[\frac{L}{2}\right]^2 + m\left[\frac{L}{2}\right]^2 = \frac{mL^2}{2}$$

Now, $G = \frac{4\pi^2\theta r^2}{T^2 MmL}\cdot\left(\frac{ML^2}{2}\right)$

$$G = \frac{2\pi^2\theta r^2 L}{T^2 M}$$

NUMERICAL EXAMPLES

16. In an experiment using covendish balance, the smaller sphere have a mass of 5.0×10^{-3}kg each , the larger sphere have a mass of 12kg each, the length of the bar is 100cm, the torsional constant of the fibre is 3.56×10^{-8}Nm per radian, the angle of twist is 4.86×10^{-3} radian and the distance between the centre of each pair of heavy and light sphere is 15cm. compute the value of gravitational constant G from this data.

Solution

$$C\theta = \frac{GM_M L}{r^2}$$

$$G = \frac{C\theta r^2}{MmL}$$

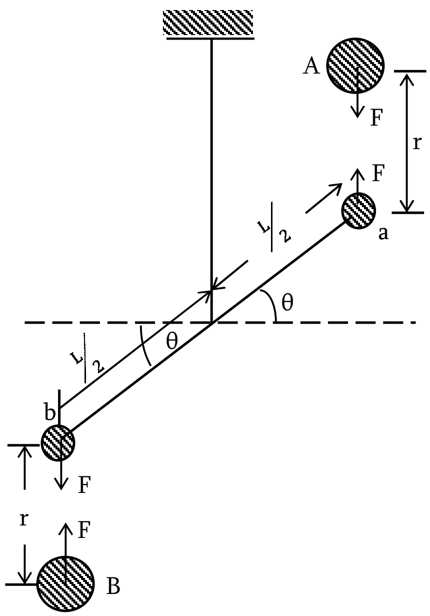
$$G = \frac{\left(3.56\times10^{-8}\right)\left(4.86\times10^{-3}\right)\times\left(0.15\right)^2}{12\times5\times10^{-3}\times1}$$

$$G = 6.488\times10^{-11}\,Nm^2kg^{-2}$$

17. NECTA 2007/P2/1(b)

Two small sphere each of mass 10gm are attached to a light rod 50cm long. The system is set into oscillation and the period of torsion oscillation is found to be 7700seconds. To produce maximum Torsion to the system two large sphere each of mass 10kg are placed near each suspended sphere; if the angular is 3.96×10^{-3}rad and the distance between the centre of the large spheres and small sphere is 10cm. determine the value of the universal gravtitaional constant, G from the given information. (06 marks)

Solution



According to the Newton's universal law of gravitation.

$$F = \frac{GMm}{r^2}$$

Gravitation torque, $\tau_g = \frac{GMmL}{r^2}$ …..(i)

Restoring torque $\tau = c\theta$ ……….(ii)

(i) = (ii)

$$c\theta = \frac{GMmL}{r^2}$$

$$G = \frac{c\theta r^2}{MmL}$$

Periodic time of oscillation of the Torsional pendulum.

$$T = 2\pi\sqrt{\frac{I}{C}}$$

$$C = \frac{4\pi^2 I}{T^2}$$

Since $I = m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{2}$

$$C = \frac{2\pi^2 mL^2}{T^2}$$

Now

$$G = \left(\frac{2\pi^2 mL^2}{T^2}\right)\left(\frac{\theta r^2}{MmL}\right)$$

$$G = \frac{2\pi^2\theta r^2 L}{T^2 M}$$

$$G = \frac{2\times(3.14)^2\times 3.96\times 10^{-3}\times(0.5)^2\times 0.1}{(7700)^2\times 10}$$

$$G = 6.67\times 10^{-11}\,Nm^2kg^{-2}$$

18. A sphere of mass 40kg is attracted by another sphere of mass 15kg with a force of $1/10$ mgwt. Find the value of constant of gravitation if centes of sphere are 0.2m apart.

Solution

M_1 = 40kg , M_2 15kg

$$F = \frac{1}{10}mgwt = \frac{1}{10}\times 10^{-3}gwt$$

$$= \frac{1}{10}\times 10^{-6}kgwt$$

(1kgwt = 9.8 Newton's)

$$F = \frac{GM_1M_2}{r^2}$$

$$G = \frac{Fr^2}{M_1M_2} = \frac{9.8\times 10^{-7}\times(0.2)^2}{40\times 15}$$

$$G = 6.533\times 10^{-11}\,Nm^2kg^{-2}$$

19. (a) G is called the universal gravitational constant. Why?

(b) Two bodies on the surface of the Earth do not move towards each other due to force of attraction given by Newton's law of gravitation. Why?

(c) Why Earth does not fall towards sun due to its attraction?

Solution

(a) G has the same value for microscopic as well as macroscopic bodies found anywhere in the universe it does not depend on the nature of medium between bodies and also on the nature of the bodies.

(b) The masses involved are small so the force of attraction is very small to cause any acceleration. The force of attraction between 1kg masses separated by 1m is only 6.67×10^{-11}N.

(c) The force of attraction provides the centripetal force and Earth moves in a stable orbit with a velocity perpendicular to the force so it does not fall.

20. A mass M is broken into two parts m and (M – m). How m and M related so that gravitational force between the two parts is maximum?

Solution

According to the Newton's universal law of gravitation.

$$F = \frac{Gm(M-m)}{r^2}$$

$$F = \frac{GMm}{r^2} - \frac{Gm^2}{r^2}$$

Differentiate F w.r.t m

$$\frac{dF}{dm} = \frac{d}{dm}\left[\frac{GMm}{r^2} - \frac{Gm^2}{r^2}\right]$$

$$\frac{dF}{dm} = \frac{GM}{r^2} - \frac{2Gm}{r^2}$$

When $F = F_{max}$; $\frac{dF}{dm} = 0$

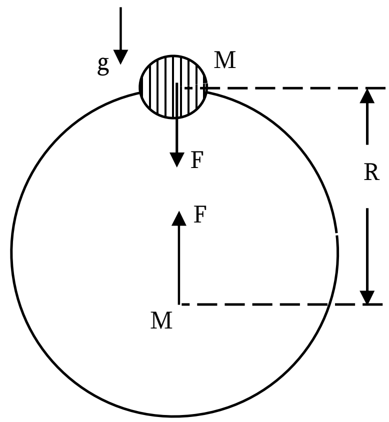
$$0 = \frac{GM}{r^2} - 2\frac{Gm}{r^2}$$

$$\frac{2Gm}{r^2} = \frac{GM}{r^2}$$

$$2M = m \;,\; m = \frac{M}{2}$$

RELATION BETWEEN G and g

Consider the figure below which shows the body of mass m which lies on the Earth surface.



According to the Newton's universal law of gravitation.

The gravitational force between the body of mass m and Earth.

$$F = \frac{GMm}{R^2} \ldots\ldots.(i)$$

Weight of an object on Earth surface

$$F = mg \ldots\ldots(ii)$$

(i) = (ii)

$$mg = \frac{GMm}{R^2}$$

$$g = \frac{GM}{R^2} \text{ or } gR^2 = GM$$

M = Mass of the Earth

R = Earth radius

g = acceleration due to gravity

G = Universal gravitation constant

This equation shows that g depends on the mass of the planet and its radius.

MASS OF THE EARTH (M)

From the relationship between g and G

$$GM = gR^2$$

$$M = \frac{gR^2}{G}$$

Given that; $g = 9.8 m/s^2$, $R = 6.4 \times 10^6 m$

$$G = 6.67 \times 10^{-11} Nm^2kg^2$$

$$M = \frac{9.8 \times (6.4 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$M = 6.02\times10^{24}\,Kg$$

DENSITY OF THE EARTH

$$Density = \frac{mass}{volume} = \frac{M}{V}$$

$$M = \frac{gR^2}{G}$$

Assuming Earth to be a homogeneous sphere its volume.

$$Volume, \quad V = \frac{4\pi R^3}{3}$$

$$\rho = \frac{gR^2}{G}\times\frac{3}{4\pi R^3} = \frac{3g}{4\pi RG}$$

$$\rho = \frac{3g}{4\pi RG}$$

Given that; $g = 9.8 m/s^2$,

$G = 6.67 \times 10^{-11} Nm2kg^{-2}$

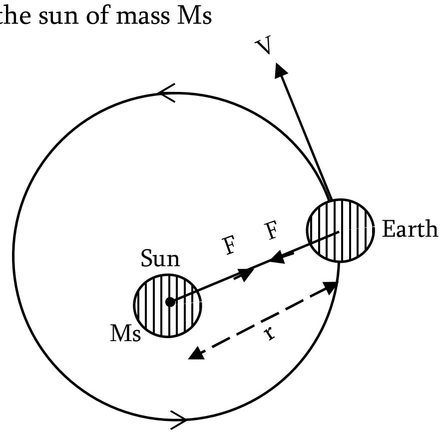
$$R = 6.4\times10^6\,m$$

$$\rho = \frac{3\times9.8}{4\times3.14\times6.4\times10^6\times6.67\times10^{-11}}$$

$$\rho = 5.51\times10^3\,kg/m^3$$

MASS OF THE SUN

Consider the motion of the Earth of mass M moving in the circular orbit of radius, r about the sun of mass Ms



Centripetal force on the Earth

$F = M\omega^2 r$........(i)

Gravitation force of attraction between the sun and Earth

$$F = \frac{GMms}{r^2} \text{......(ii)}$$

$$(i) = (ii)$$

$$\frac{GMMs}{r^2} = M\omega^2 r$$

$$Ms = \frac{\omega^2 r^3}{G} \text{ but } \omega = \frac{2\pi}{T}$$

$$Ms = \left(\frac{2\pi}{T}\right)^2\cdot\frac{r^3}{G}$$

Given that; $r = 1.5 \times 10^{11} m$, $T = 365 days$

$G = 6.67 \times 10^{-11} Nm^2kg^{-2}$

$$Ms = \left(\frac{2\times3.14}{365\times24\times3600}\right)^2\times\frac{\left(1.5\times10^{11}\right)^3}{6.67\times10^{-11}}$$

$$Ms = 2.0\times10^{30}\,kg$$

NUMERICAL EXAMPLES

21. The acceleration due to gravity at the moon's surface is $1.67 m/s^2$. If the radius of the moon is $1.74 \times 10^6 m$, calculate the mass of the moon, $G = 6.67 \times 10^{-11} Nm2kg^{-2}$

Solution

Since $GM = Gr^2$

$$M = \frac{gR^2}{G} = \frac{1.67\times\left(1.74\times10^6\right)}{6.67\times10^{-11}}$$

$$M = 7.58\times10^{22}\,Kg$$

22. If the radius of the Earth were increased by a factor of 3, by what factor would its density have to be changed to keep 'g' the same?

Solution

$$g = \frac{GM}{R^2}$$

Let: ρ = density of the Earth

$$\rho = \frac{M}{4/3\,\pi R^3} \text{ or } M = \frac{4}{3}\pi R^3\rho$$

$$\text{Now: } g = \frac{G}{R^2}\times\frac{4}{3}\pi R^3\rho$$

$$g = \frac{4}{3}\pi G\rho R$$

$4/3$, π , G are constant

For no change in value of g

$$R \,\alpha\, \frac{1}{\rho}$$

$\therefore$ If R is made 3R, ρ must become $\rho/3$

23. (a) What will be the acceleration due to gravity on the surface of moon if its radius is ¼ th the radius of the Earth and its mass $\frac{1}{80}$ th of the mass of the Earth?

(b) A man can jump 1.5m on the Earth. calculate the maximum approximate height he might be able to jump on a planet whose density is one quarter that of the Earth and whose radius is one – third that of the Earth.

Solution

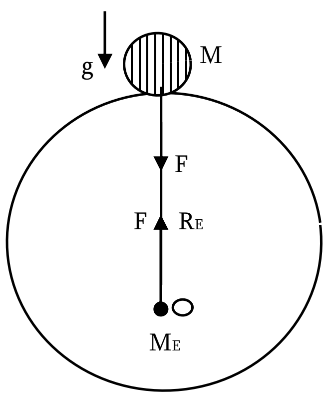
(a) Let M_E = Mass of the Earth

R_E = Earth radius

M_m = Mass of the moon

R_m = Moon radius

Consider the body of mass M placed on the Earth – surface.



Gravitation force = weight of the body

$$\frac{GM_EM}{R_E^2} = Mg$$

$$g = \frac{GM_E}{R_E^2} \quad ....(i)$$

Similarly, when the body lies on the moon surface.

$$g_m = \frac{GM_m}{R_m^2}$$

$$\frac{(ii)}{(i)}$$

$$\frac{g}{g_m} = \frac{GM_E}{R_E^2} \Big/ \frac{GM_m}{R_m^2}$$

$$\frac{g_m}{g} = \left(\frac{M_m}{M_E}\right)\left(\frac{R_E}{R_m}\right)^2$$

$$g_m = g\left(\frac{M_m}{M_E}\right)\left(\frac{R_E}{R_m}\right)^2$$

$$R_m = \frac{R_E}{4}, \quad R_E = 4R_m$$

$$M_m = \frac{M_E}{80}, \quad M_E = 80M_m$$

$$g_m = 9.8\left[\frac{M_m}{80M_m}\right]\left[\frac{4R_m}{R_m}\right]^2$$

$$g_m = 1.96\text{m}/\text{s}^2$$

(b) $h = 1.5\text{m}$, $h_p = ?$

$\rho_P = \frac{1}{4}\rho_E$, $\rho_E = 4\rho_P$

$R_P = \frac{1}{3}R_E$, $R_E = 3R_P$

Let R_P and R_E are radii of planet and Earth respectively. Assumption: initial kinetic energy of the man when start to jump on the Earth and given planet are the same.

p.e of man on = p.e of man on the planet the Earth.

$$Mg_ph_p = Mg_Eh$$

$$h_p = \left(\frac{g_E}{g_p}\right)h$$

Acceleration due to gravity on the Earth – surface. $g_E = \frac{GM_E}{R_E^2}$

Planet surface: $g_p = \frac{GM_p}{R_p^2}$

$$\frac{g_E}{g_P} = \frac{GM_E}{R_E^2} \Big/ \frac{GM_P}{R_P^2}$$

$$\frac{g_E}{g_P} = \left(\frac{M_E}{M_P}\right)\left(\frac{R_P}{R_E}\right)^2$$

Also $M_E = \frac{4}{3}\pi R_E^3\rho_E$

$$M_P = \frac{4}{3}\pi R_P^3\rho_P$$

$$\frac{M_E}{M_P} = \frac{\frac{4}{3}\pi R_E^3\rho_E}{\frac{4}{3}\pi R_P^3\rho_P} = \left(\frac{\rho_E}{\rho_P}\right)\left(\frac{R_E}{R_P}\right)^3$$

Now $$\frac{g_E}{g_P} = \left(\frac{M_E}{M_P}\right)\left(\frac{R_P}{R_E}\right)^2$$

$$= \left(\frac{\rho_E}{\rho_P}\right)\left(\frac{R_E}{R_P}\right)^3\cdot\left(\frac{R_P}{R_E}\right)^2$$

$$h_p = \left(\frac{\rho_E}{\rho_P}\right)\left(\frac{R_E}{R_P}\right)h$$

$$= \left(\frac{4\rho_p}{\rho_p}\right)\left(\frac{3R_P}{R_P}\right)\times 1.5m$$

$$h_p = 1.5\times 12 = 18m$$

$$h_p = 18m$$

24. If the radius of the Earth shrinks by 1.5% (mass remaining the same) then how would the value of acceleration due to gravity changes?

Solution

Since $g = \frac{GM}{R^2} = GMR^{-2}$

$$\log_e^g = \log_e\left[GMR^{-2}\right]$$

$$= \log G + \log_e^M \mp 2\log_e^R$$

(On differentiate both side)

$$\frac{dg}{g} = -2\frac{dR}{R}$$

Percentage error

$$\frac{dg}{g}\times 100\% = -2\left[\frac{dR}{R}\times 100\%\right]$$

$$\frac{dg}{g}\times 100\% = 3\%$$

25. (a) What is the acceleration due to gravity on the surface of a planet that has a radius half that of Earth and the same average density as the Earth?

(b) The mass of the planet Jupiter is 1.9×10^{27}kg and that of the sun is 1.99×10^{30}kg the mean distance of the Jupiter from the sun is 7.8×10^{11}m. Calculate the gravitational force which the sun exert on the Jupiter. Assume that the Jupiter moves in circular orbit around the sun. Calculate the speed of Jupiter.

Answer

(a) $4.9 m/s^2$

(b) 4.14×10^{23}N, 1.304×10^4m/s

VARIATION OF g

The value of acceleration due gravity g is not constant at all points.

The value of g changing due to the following factors:

(i) Variation of g with altitude (i.e height)

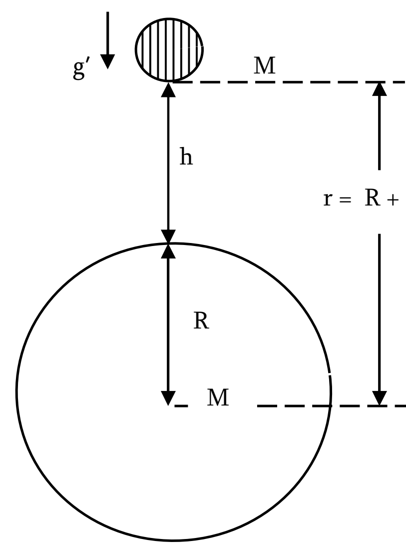
(ii) Variation of g with depth

(iii) Variation of g due to shape of Earth

(iv) Variation of g due to rotation of Earth

VARIATION OF g WITH ALTITUDE HEIGHT

Consider a body of mass m which is located at a distance h above the Earth – surface



1st case; Assume that a body of m rest on the earth – surface and the value of g can be calculated as follows $mg = \frac{GMm}{R^2}$

$$g = \frac{GM}{R^2} \quad\ldots\ldots(i)$$

2nd case; A body of mass m is a height h above the earth – surface

$$g' = \frac{GM}{r^2} \quad\ldots\ldots(ii)$$

Dividing equation (ii) by (i)

$$\frac{g'}{g} = \frac{GM}{r^2}\times\frac{R^2}{GM}$$

$$g' = \frac{gR^2}{r^2} = g\left[\frac{R}{R+h}\right]$$

$$g' = g\left[\frac{R}{R+h}\right]^2$$

Also $g' \alpha \frac{1}{r^2}$

Again $g' = g\left[\frac{R}{R+h}\right]^2 = g\left[\frac{R+h}{R}\right]^{-2}$

$$g' = g\left[1+\frac{h}{R}\right]^{-2}$$

Expanding this equation by using Binomial expansion and neglecting higher powers of h/R

$$(1+x)^n = 1+nx+\frac{n(n-1)x^2}{2!}+----$$

$$\left[1+\frac{h}{R}\right]^{-2} = 1-\frac{2h}{R}+---$$

$$\left[1+\frac{h}{R}\right]^{-2} \simeq 1-\frac{2h}{R}$$

$$g' = g\left(1-\frac{2h}{R}\right) \quad (g'<g)$$

Therefore, the value of g decreases as we go above the Earth surface

- Percentage decrease in g

  $$g-g' = g-g\left(1-\frac{2h}{R}\right)$$

  $$g-g' = \frac{2gh}{R}$$

  Fractional decrease in g

  $$\frac{g-g'}{g} = \frac{2h}{R}$$

  Percentage decrease in g

  $$\left(\frac{g-g'}{g}\right)\times 100\% = \frac{2h}{R}\times 100\%$$

- Loss in weight

  $$\Delta w = M\Delta g = \frac{2Mgh}{R}$$

Note that

(i) $g' = \frac{gR^2}{(R+h)^2} = \frac{GM}{(R+h)^2}$

(ii) If $R = h$

$$g' = \frac{gR^2}{(2R)^2} = \frac{g}{4}$$

$$g' = \frac{g}{4}$$

Variation of g with depth

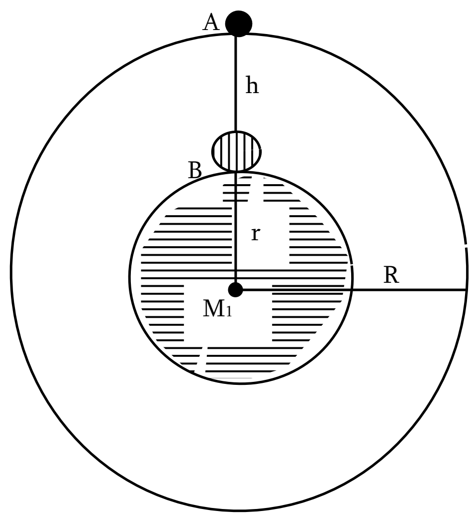
Consider the Earth to be a sphere radius R and mass M. at the Earth surface

$$g = \frac{GM}{R^2}$$

Since $M = \frac{4}{3}\pi R^3\rho$

$$g = \frac{4}{3}\pi R^3\rho\cdot\frac{G}{R^2}$$

$$g = \frac{4}{3}\pi R\rho G \,.........(i)$$



Consider a point B which is inside the Earth at a depth h below the Earth – surface. The value of acceleration due to gravity.

$$g' = \frac{GM_1}{r^2}$$

$$M_1 = \frac{4}{3}\pi r^3\rho$$

$$g' = \frac{4}{3}\pi r\rho G \,.........(ii)$$

$(ii)/(i)$

$$\frac{g'}{g} = \frac{4/3\,\pi r\rho G}{4/3\,\pi R\rho G} = \frac{r}{R}$$

$$g' = \frac{gr}{R} \qquad g' \alpha r$$

Since $r = R - h$

$$g' = g\left[\frac{R-h}{R}\right] = g\left[1-\frac{h}{R}\right]$$

$$g' = g\left[1-\frac{h}{R}\right] \qquad (g' < g)$$

- Percentage decrease in g

$$\Delta g = g - g' = g - g\left(1 - h/R\right)$$

$$\Delta g = \frac{gh}{R}$$

Fractional decrease in g

$$\frac{\Delta g}{g} = \frac{g-g'}{g} = \frac{h}{R}$$

Percentage decrease in g

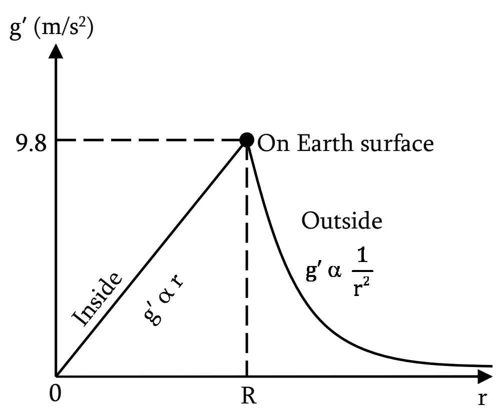
$$\left(\frac{g-g'}{g}\right)\times 100\% = \frac{h}{R}\times 100\%$$

Loss in weight $\Delta W = M\Delta g = \dfrac{Mgh}{R}$

Graph of g′ against r

The graph below shows that g′ is a function of r where r is the distance from the centre of the Earth. outised of the Earth: $g' \alpha\ 1/r^2$

Inside of the Earth : $g' \alpha\ r$



- Weight of a body at the centre at the centre of Earth.

At a depth h below the free surface of Earth

$$g' = g\left(1 - h/R\right)$$

At the centre of the Earth, h = R g′ = 0

If M is the mass of a body lying at the centre of the Earth, then its weight is equal to zero.

- Comparison of height and depth for the same change in g. as we have seen above, the value of g decrease as we go above the surface of Earthh or when we go below the Earth – surface this can be taken to mean that value of g is maximum on the surface of Earth.

Now $g' = \left(1-\frac{2h}{R}\right)$ and $g' = g\left(1-\frac{h_1}{R}\right)$

$$g\left(1-\frac{2h}{R}\right) = g\left(1-\frac{h_1}{R}\right)$$

$2h = h_1$ or $h_1 = 2h$

∴ The value of acceleration due to gravity at a height h is same as the value of acceleration due to gravity at a depth $(h_1 = 2h)$. But this is true only of h is very small.

- **Variation of g with latitude**

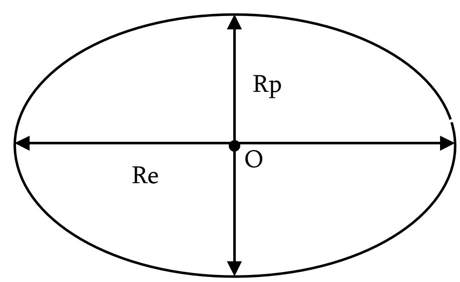
The value of acceleration due to gravity changes with latitude due to

(a) Shape of the Earth

(b) Rotation of the Earth about its own axis.

SHAPE OF EARTH

The Earth is not a perfect sphere it is flattened at the poles (where latitude is 90°) and bulges out at the equator (where latitude is 0°)



The equatorial radius Re is grater than the polar radius Rp by nearly 21km

since $g = \frac{GM}{R^2}$, $g\ \alpha\ \frac{1}{R^2}$

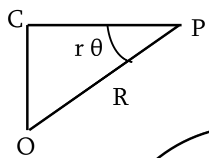
i.e $Re > Rp$ $\quad g_p > g_e$

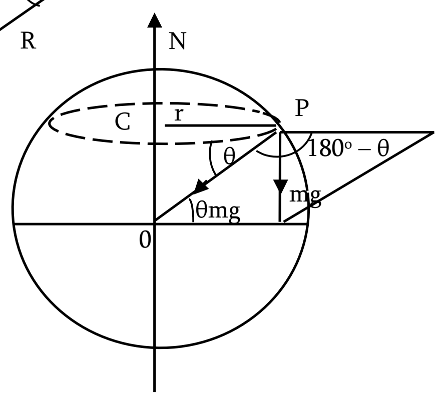
At poles: The value of g is maximum i.e $9.83 m/s^2$.

At the equator: The value of g is least i.e $9.78 m/s^2$.

ROTATION OF THE EARTH

As the Earth rotates about the axis through north and south poles the value of acceleration due to gravity varies from one place to the another place depending on the angle made from the centre of the Earth relatives to the equator as shown on the figure below.





The apparent weight Mg′ of the particle is the resultant of the true weight mg and the centrifugal force. Fe = Mω²r due to rotation of Earth, the particle at P is attracted towards a point O′ which is close to O.

Apply parallelogram law of forces

$$(mg')^2 = (mg)^2 + (m\omega^2 r)^2 + 2(mg)(m\omega^2 r)\cos(180^\circ - \theta)$$

$$(mg')^2 = (mg)^2 + (m\omega^2 r)^2 - 2(mg)(m\omega^2 r)\cos\theta$$

$$(g')^2 = g^2 + \omega^4 R^2\cos^2\theta - 2R\omega^2\cos^2\theta$$

$$(g') = g^2\left[1 + \frac{\omega^4 R^2\cos^2\theta}{g^2} - \frac{2R\omega^2\cos^2\theta}{g}\right]$$

$$g' = g\left[1 - \frac{R^2\omega^4\cos^2\theta}{g^2} - \frac{2R\omega^2\cos^2\theta}{g}\right]^{\frac{1}{2}}$$

Now $R = 6.4\times10^6\,m$

$$\frac{R\omega^2}{g} = \frac{6.4\times10^6}{9.8}\left(\frac{2\pi}{24\times3600}\right)^2$$

(Neglecting the higher powers of $\frac{R\omega^2}{g}$)

$$g' = g\left(1 - \frac{2R\omega^2\cos^2\theta}{g}\right)^{1/2}$$

By using Binomial expansion

$$g' = g\left[1 - \frac{1}{2}\times\frac{2R\omega^2\cos^2\theta}{g} + \text{higher terms}\right]$$

Neglecting the terms containing higher powers

$$g' = g\left[1 - \frac{R\omega^2\cos^2\theta}{g}\right]$$

$$g' = g\left(-R\omega^2\cos^2\theta\right)$$

- At the equator θ = 0°
- At the poles θ = 90o

  ∴ The value of g is maximum at the poles. This is expected because the particle at the poles moves in a circle of zero radius. Thus, no centrifugal force acts on the particle.

  Note that:

  The difference in the values of acceleration due to gravity at the poles and equator is given by

  $$g_p - g_e = g - (g - \omega^2 R)$$

  $$g_p - g_e = R\omega^2$$

- When a body of mass M is moved from equator to either pole, the weight of the body increases by $\Delta w = m(g_p - g_e) = M\omega^2 R$

NUMERICAL EXAMPLES

26. Assuming the Earth to be a sphere of uniform mass density how much would a body weight half way down to the centre of the Earth. If its weighted 250N on the surface?

Solution

$Mg = 250N$, $h = \frac{R}{2}$

Acceleration due to gravity below the Earth – surface

$$g' = g\left(1 - \frac{h}{R}\right)$$

$$Mg' = Mg\left(1 - \frac{h}{R}\right)$$

$$= 250\left(1 - \frac{1}{2}\right)$$

$$Mg' = 125N$$

27. (a) Calculate the height above the Earth's surface at which the value of acceleration due to gravity reduces to half its value of the Earth's surface. Assume the Earth to be sphere of radius 6400km.
(b) If the radius of the Earth were increased by a factor of 3, by what factor would its density have to be changed to keep 'g' the same?

Solution

(a) $$g' = g\left[\frac{R}{R+h}\right]^2 \quad , \quad g' = \frac{g}{2}$$

$$\frac{1}{2} = \left(\frac{R}{R+h}\right)^2 \quad \text{or} \quad \frac{R}{R+h} = \frac{1}{\sqrt{2}}$$

$$\frac{R+h}{R} = \sqrt{2}$$

$$\frac{h}{2} = \sqrt{2}-1 = 0.414$$

$$h = 0.414R = 0.414\times 6400$$

$$h = 2649.6\text{Km}$$

(b) $$g = \frac{G}{R^2}\cdot\frac{4}{3}\pi R^3\rho = \frac{4}{3}\pi G\rho R$$

$$g = \frac{G}{R^2}\cdot\frac{4}{3}\pi R^3\rho = \frac{4}{3}\pi G\rho R$$

$^4/_3$, π , G are constant

For no change in value of g

$$R \,\alpha\, \frac{1}{\rho}$$

∴ If R is made 3R, ρ must become $^{\rho}/_{3}$

28. (a) Calculate the imaginary angular velocity of the Earth for which the effective acceleration due to gravity at the equator becomes zero. In this condition what will be the length (in hours) of the day?
(b) How far above the Earth – surface does the value of g becomes 16% value on the surface ($g = 10m/s^2$).

Solution

(a) Since $g_e = g - \omega^2 R$ but $g_e = 0$

$$0 = g - \omega^2 R$$

$$\omega^2 R = g \quad , \quad \omega = \sqrt{\frac{g}{R}}$$

$$\omega = \sqrt{\frac{10}{6.4\times 10^6}} = 1.25\times 10^{-3}$$

$$\omega = 1.25\times 10^{-3}\,\text{rad/s}$$

Length of the day

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{1.25\times 10^{-3}}$$

$$T = 5024\,\text{sec} = 1.4\text{hr}$$

(b) $$\frac{g'}{g}\times 100\% = 16\% \quad , \quad \frac{g'}{g} = 0.16$$

Since $$g' = g\left(\frac{R}{R+h}\right)^2$$

$$\frac{g'}{g} = \left(\frac{R}{R+h}\right)^2 = \frac{16}{100}$$

$$h = \frac{3}{2}R = \frac{3}{2}\times 6400\text{km}$$

$$h = 9600\text{km}$$

29. (a) Calculate the effect of rotation of Earth on the weight of the body at a place at latitude 45°. Take radius of Earth = 6.37×10^6m.
(b) Find the percentage decrease in the weight of a body when taken to a depth of 32km below the surface of Earth. Radius of Earth is 6400km.

Solution

(a) $$g' = g - R\omega^2\cos^2\theta$$

$$\omega = \frac{2\pi}{24\times 3600} = 7.2\times 10^{-5}\,\text{rads}^{-1}$$

$$g - g' = R\omega^2\cos^2\theta$$

$$= 6.37\times 10^6 \times \left(7.27\times 10^{-5}\right)^2\left(\cos 45^\circ\right)^2$$

$$g - g' = 0.0168\,\text{m/s}^2$$

It shows that the value of acceleration due to gravity at the pole is greater than at latitude 45° by $0.0168m/s^2$. Hence the weight of a body of 1kg mass at the poles will be greater than at 45° latitude by 0.0168N.

(b) Since $$g' = g\left(1 - \frac{d}{R}\right)$$

$$Mg' = Mg\left(1 - \frac{d}{R}\right)$$

$$\frac{Mg'-Mg}{Mg} = \frac{d}{R}$$

$$\left(\frac{Mg'-Mg}{Mg}\right)\times 100\% = \frac{d}{R}\times 100\%$$

$$= \frac{32}{6400}\times 100\%$$

% age decreases in weight = 0.5%

30. (a) How much faster than its present rate should the Earth rotate about its axis so that the weight of a body at equator becomes zero?
(b) Also calculate the new length of the day?
(c) What should happen if the rotation becomes still faster?

Solution

(a) Since $g' = g\left(1-\frac{R\omega^2}{g}\cos^2\theta\right)$

At the equator $\theta = 0^\circ$, $\cos 0^\circ = 1$

$$g_e = g-\omega^2 R = g\left(1-\frac{\omega^2 R}{g}\right)$$

It can be shown that

$$\frac{R\omega^2}{g} = \frac{1}{288}$$

In order that the weight of a body at the equator to be zero $g_e = 0$. This means that

$\frac{R\omega^2}{g} = 1$ so, $R\omega^2$ should be 288times greater. Since R is constant, therefore ω^2 should be 288times more than its present value thus ω should be $\sqrt{288}$ times i.e 16.97 times greater. So the Earth should rotate 17times faster than its present value so that the weight of a body at the equator becomes zero.

Alternative

Since $g' = g\left(1-\frac{R\omega^2}{g}\cos^2\theta\right)$

At the equator , $\theta = 0^\circ$, $\cos 0^\circ = 1$

$$g_e = g-R\omega^2$$

Suppose when ω is changed to ω', ge (and hence mg_e) becomes zero.

$$0 = g-R\omega'^2 \text{ OR } \omega' = \sqrt{\frac{g}{R}}$$

$$\frac{\omega'}{\omega} = \frac{1}{\omega}\sqrt{\frac{g}{R}} = \frac{1}{\omega}\sqrt{\frac{981}{6.4\times 10^8}}$$

(b) Since the Earth then rotates 17times faster, therefore, the Earth makes 17rotations in 24hours and hence the length of the day would be $\frac{24}{17}\text{hr} = 1.412\text{hours}$.

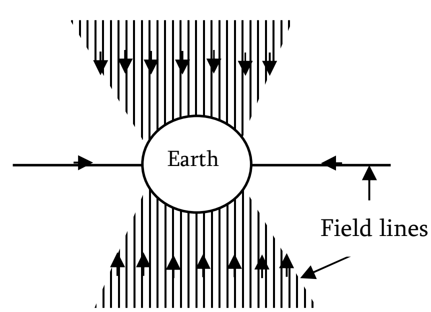
(c) If the rotation becomes still faster i.e faster than 17times its present value, the increased centripetal acceleration on the bodies will be greater than acceleration due to gravity. Therefore, there will now be a resultant force actin on them outwards and all objects kept loose on the equator will start leaving the Earth's surface.

GRAVITATION FIELD AND GRAVITATIONAL FIELD STRENGTH OR INTENSITY.

The gravitational field due to a material body is the space around the body in which any other mass experiences of force of attraction.

Definition Gravitational field - is the region in which space where the force of attraction can be exerted from one body towards to the another body i.e

- The gravitational field is the space around a mass or an assembly of masses over which it can exert gravitational forces on other masses.
- The gravitational field can be specified by the direction of gravitational lines of the force i.e we respect the gravitational field of a mass by fields lines. Suppose a particle of mass M is located above the Earth surface as shown in the figure below.

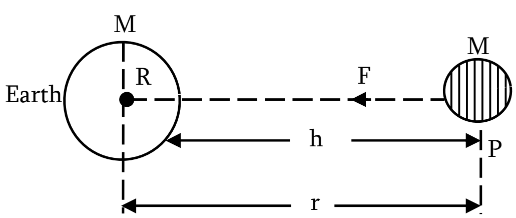


GRAVITATIONAL FIELD INTENSITY

Gravitational field strength at a point in the gravitational field is the force experience by a unit mass placed at that point i.e gravitational field strength – is defined as the gravitational force per unit mass acting on a test mass placed at that point. Gravitational field strength is a vector quantity. It is denoted by I or E or g′.

Mathematically:

consider gravitational field of an isolated body of mass M. in order to determine the gravitational field strength at a point P in a gravitational field, a test mass m is placed at the point P. The test mass is supposed to be small that it does not alter gravitational field in any manner.



$$I = \frac{\text{Gravitational force}}{\text{mass}}$$

$$I = g'\frac{fg}{m}$$

Apply the Newton's universal law of gravitation.

$$F = \frac{GMm}{r^2}$$

$$I = \frac{GMm}{r^2} \Big/ m$$

$$I = \frac{GM}{r^2}$$

Since $GM = gR^2$, $r = R+h$

$$I = \frac{GM}{(R+h)^2} = \frac{gR^2}{(R+h)^2}$$

S.I Unit of gravitational field intensity is Nkg^{-1} or m/s^2.

Dimensional formula of I

$[I] = [M^0LT^{-2}]$

Note that

1. Intensity of gravitational field at a point duet to the point of mass, m is given by

$$I = g' = \frac{GM}{r^2}$$

I is directed towards the centre of mass M. Suppose that the test mass is free to move. Then it will move towards M with an acceleration a

$$I = \frac{ma}{m} = a$$

Hence the intensity of gravitational field at a point in a gravitational field is equal to the free acceleration experienced by test mass kept at that point.

2. Gravitational field intensity on the Earth – surface.

$$g' = \frac{GM}{R^2} = \frac{gR^2}{R^2}$$

$$g' = g$$

Hence the gravitational field intensity at any point near the surface of the Earth is equal to the acceleration due to gravity at that point.

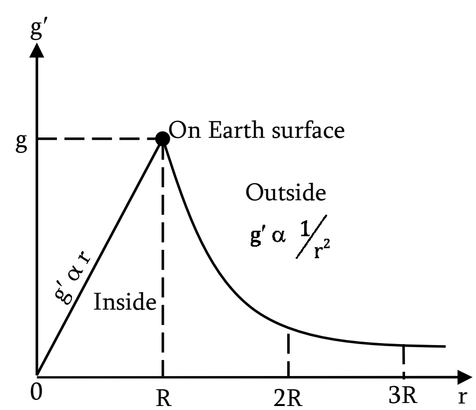
3. Gravitational field intensity at the infinite.

$$g' = \frac{GM}{r^2} \ , \ r \longrightarrow \alpha$$

$$g'_{\alpha} = \frac{GM}{(\alpha)^2} = 0$$

$$g'_{\alpha} = 0$$

GRAPH OF GRAVITATIONAL FIELD INTENSITY G′ AGAINST R.



Gravitational potential gravitational potential difference and gravitational potential energy.

The gravitational potential determines the direction of motion of a body in the gravitational field of another body.

Definition gravitational potential at a point is defined as the amount of work done in bringing a unit mass from infinite to that point. Since gravitational potential represents the work done, therefore it is a scalar quantity it is represented by V, The gravitational field intensity due to a certain mass is zero at infinity so gravitational potential at the indefinite distance also is equal to zero. As a given mass always moves from higher to lower gravitational potential in a gravitational field, it follows the gravitational potential goes approached so gravitational potential is of such gravitational potential is of such a nature that its maximum value is zero and it decreases as the given mass is approached. Hence gravitational potential at any point in the gravitational field is NEGATIVE.

Mathematically

$$\text{Gravitational potential} = \frac{\text{Work done}}{\text{mass}}$$

$$V = \frac{W}{M}$$

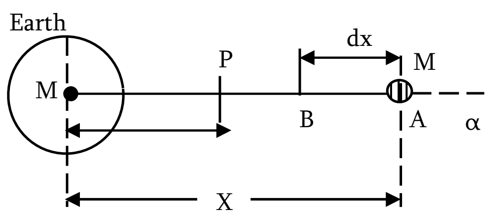
S.I Unit of gravitational potential is JKg^{-1}.

Dimensional formula of V

$[V] = [M^0L^2T^{-2}]$

EXPRESSION OF GRAVITATINAL POTENTIAL AT A POINT

Consider a point P at a distance r from a point mass M.



According to the Newton's universal law of gravitation,

$$F = \frac{GMm}{X^2}$$

If a mass at A moved to B through a small distance , dx

$$dw = Fdx = \frac{GMmdx}{X^2}$$

Total work done by the gravitational field to bring a mass m from infinity to point P.

$$w = \int_{\alpha}^{r} \frac{GMm}{X^2} dx$$

$$w = GMm \int_{\alpha}^{r} X^{-2} dx$$

$$= GMm \left[\frac{X^{-2+1}}{-2+1} \right]_{\alpha}^{r}$$

$$= -GMm \left[\frac{1}{X} \right]_{\alpha}^{r}$$

$$w = -GMm \left[\frac{1}{r} - \frac{1}{\alpha} \right]$$

$$w = \frac{-GMm}{r}$$

Gravitational potential

$$V = \frac{w}{m} = \frac{-GMm}{r} \Big/ m$$

$$V = \frac{-GM}{r}$$

$$R+h = r \;,\; GM = gR^2$$

$$V = \frac{-gR^2}{(R+h)}$$

Gravitational potential on te Earth surface, h = 0

$$V = \frac{-gR^2}{R+h} = -gR$$

$$V_E = -gR$$

Gravitational potential at the infinite $r \rightarrow \alpha$

$$V_\alpha = -GM\left[\frac{1}{\alpha}\right] = 0$$

$$V_\alpha = 0$$

Note that

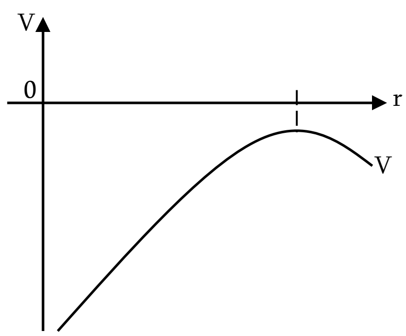
1. Gravitational potential at a point is given by

$$V = -\frac{GM}{r}$$

The gravitational potential is negative, since the work is done by the gravitational field and not against field in bringing a unit mass from infinity to the point under consideration i.e the negative sign indicates that the gravitational potential at infinity is zero i.e higher than the potential close to the Earth.

2. Graph of gravitational potential against distance r.

$$V = -\frac{GM}{r} \quad \left(V \propto \frac{-1}{r}\right)$$



Gradient at any point on the curve

$$\text{Slope} = \frac{dv}{dr} \quad \text{but } V = -\frac{GM}{r}$$

$$\frac{dv}{dr} = \frac{d}{dr}\left[-\frac{GM}{r}\right]$$

$$\frac{dv}{dr} = -GM\frac{d}{dr}\left[r^{-1}\right]$$

$$\frac{dv}{dr} = \frac{GM}{r^2} = I$$

Therefore the gradient at any point on the gravitational potential curve represent **gravitational field intensity**.

3. **GRAVITATIONAL POTENTIAL DIFFERENCE** – is defined as the work done in moving a unit mass from one point to another point in a gravitational field.

4. Graph of a force (F) and gravitational potential (V) for moon satellite.

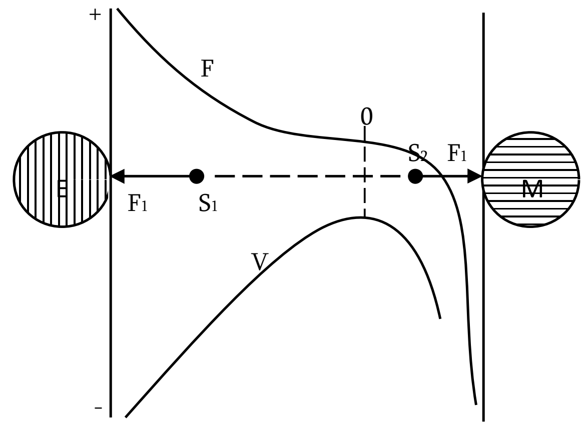


Figure above shows roughly how the resultant force F on a satellite varies after launched from the Earth E towards to the moon, M. The direction of force (F) changes from F_1 and S_1, where Earth's gravitational pull is greater than that of the moon, to F_2 at S_2 near the moon where the pull of this body is now stronger than that of the Earth at point O, the gravitational pull of the Earth is balanced by that of the moon. The maximum value of V occurs just below V. Here the resultant force F is zero.

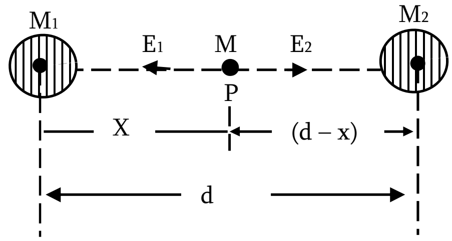
Since $E = -\frac{dv}{dr}$, the gradient dv/dr of the potential curve is zero.

- At one point on a line between the Earth and the moon, the gravitational field caused by the two bodies is zero.

Reasons:

This is because at that point, the force of attraction between the Earth and the body is equal in magnitude to the force of attraction between the moon and the body. Since they acts in opposite direction, they cancel each other therefore the result to zero gravitational field.

Expression of gravitational potential between two bodies of masses M_1 and M_2 where by gravitational field due to them at that point is equal to zero.



At the point P, the resultant electric field is equal to zero.

$$E_1 = E_2$$

$$\frac{GM_1}{X^2} = \frac{GM_2}{(d-X)^2}$$

$$\left[\frac{d-X}{X}\right]^2 = \frac{M_2}{M_1}$$

$$\frac{d}{X} - 1 = \sqrt{\frac{M_2}{M_1}}$$

$$\frac{d}{X} = \frac{\sqrt{M_2}}{\sqrt{M_1}} + 1 = \frac{\sqrt{M_2}+\sqrt{M_1}}{\sqrt{M_1}}$$

$$X = d\left[\frac{\sqrt{M_1}}{\sqrt{M_1}+\sqrt{M_2}}\right]$$

Let V = Gravitational potential at P

$$V = V_1 + V_2$$

$$= \frac{-GM_1}{X} + \frac{-GM_2}{d-X}$$

$$= -G\left[\frac{M_1}{X} + \frac{M_2}{d-X}\right]$$

$$V = -G\left[\frac{M_1}{\frac{d\sqrt{M_1}}{\sqrt{M_1}+\sqrt{M_2}}} + \frac{M_2}{d - \frac{d\sqrt{M_1}}{\sqrt{M_1}+\sqrt{M_2}}}\right]$$

On simplify , we get

$$V = \frac{-G}{d}\left[M_1 + M_2 + 2\sqrt{M_1M_2}\right] \quad \text{OR}$$

$$V = \frac{-G}{d}\left[\sqrt{M_1}+\sqrt{M_2}\right]^2$$

GRAVITATIONAL POTENTIAL ENERGY (U)

The potential energy of a body near to the Earth – surface is given by

g.p.e = Mgh

M = Mass of the body

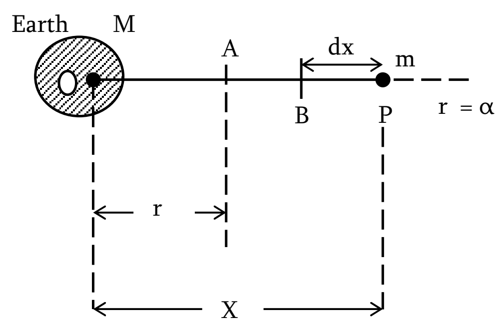
g = Acceleration due to gravity

h = height above the Earth – surface

Definition gravitational potential energy of a body at a point is defined as the amount of work done in bringing the given body from infinity to that point against t the gravitational force.

EXPRESSION OF GRAVITATIONAL POTENTIAL ENERGY.

Suppose the mass m is kept at point P as shown in figure below i.e the body of mass m moves from the infinity to the point, P



Gravitational force between the body and the Earth when it's at the point P.

$$F = \frac{GMm}{X^2}$$

The corresponding change in work done by the body to moves from P to B

$$dw = Fdx$$

$$dw = \frac{GMmdx}{X^2}$$

Total work done by the gravitational field when a body of mass, m is moved from infinity to a distance r from O.

$$w = GMm\int_{\alpha}^{r} \frac{dx}{X^2}$$

$$= GMm\int_{\alpha}^{r} X^2 dx$$

$$= -GMm\left[\frac{1}{X}\right]_{\alpha}^{r}$$

$$= -GMm\left[\frac{1}{r}-\frac{1}{\alpha}\right]$$

$$w = \frac{-GMm}{r}$$

This work done is equal to the gravitational potential energy U of mass, m.

$$U = \frac{-GMm}{r}$$

$$\text{Now } U = \frac{-GMm}{r} = \left(-\frac{GM}{r}\right)\times m$$

U = gravitational potential × mass

Note that

1. Generally the gravitational potential energy is given by $U = \frac{-GMm}{r}$

   Gravitational potential energy at the infinite ($r = \alpha$)

   $$U_{\alpha} = -GMm\left[\frac{1}{\alpha}\right]$$

   $$U_{\alpha} = 0$$

   The minus sign show that all gravitational potential energy is below that at the infinite.

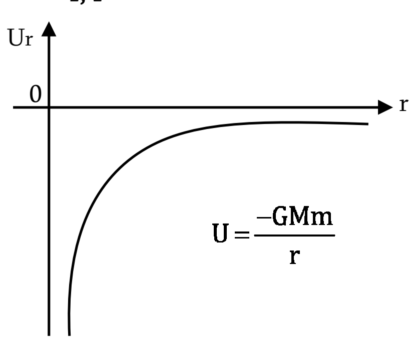
2. Gravitational potential energy at the point above the Earth – surface is given by

   $$U_r = \frac{-GMm}{r} = \frac{-GMm}{(R+h)}$$

   Also $GM = gR^2$

   $$U_r = \frac{-GMm}{(R+h)} = \frac{-gR^2m}{(R+h)}$$

3. **GRAPH OF GRAVITATIONAL POTENTIAL ENERGY AGAINST DISTANCE, r**



From the graph above, gradient or slope of the curve.

$$\text{Slope} = \frac{du}{dr} = \frac{d}{dr}\left[\frac{-GMm}{r}\right]$$

$$\frac{du}{dr} = \frac{+GMm}{r^2}$$

Therefore, the gradient at any point on the gravitational potential energy curve represents the force pulling the body towards the planet i.e gravitational force of attraction.

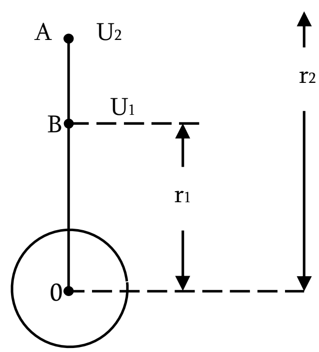
4. The gravitational potential energy of the body when the body is at the point r_1 and r_2 above the Earth surface as illustration in the figure below is obtain as follows:-

At A

$$U_2 = \frac{-GMm}{r_2}$$

At B

$$U_1 = \frac{-GMm}{r_1}$$



Since $r_2 > r_1$ the change of the gravitational potential energy, ΔU is given by.

$$\Delta U = U_2 - U_1$$

$$= \frac{-GMm}{r_2} - \left(\frac{-GMm}{r_1}\right)$$

$$\Delta U = GMm\left[\frac{1}{r_2}-\frac{1}{r_1}\right] \quad \text{OR}$$

$$\Delta U = GMm\left[\frac{r_2 - r_1}{r_1 r_2}\right]$$

Since $r_1 < r_2$, the change in potential energy is positive. It means that if the body is moved away from the Earth, the gravitational potential energy is increases.

GRAVITATIONAL POTENTIAL ENERGY OF A MASS M AT A HEIGHT, H ABOVE THE EARTH – SURFACE.

The gravitational potential energy of mass m at a height h above the surface of Earth is given by.

$$U = \frac{-GMm}{R+h}$$

$$U = \frac{-GMm}{R\left(1+h/R\right)} = \frac{-GMm}{R}\left(1+h/R\right)^{-1}$$

By using Binomial expression.

$$(1+x)^n = 1+nx+n(n-1)\frac{x^2}{2!}+---$$

$$\left[1+\frac{h}{R}\right]^{-1} = 1-\frac{h}{R}+\frac{(-1)(-1-1)}{2!}\left(\frac{h}{R}\right)^2+--$$

$$\left(1+\frac{h}{R}\right)^{-1} = 1-\frac{h}{R}+\left(\frac{h}{R}\right)^2+---$$

Since $h/R \ll 1$, neglecting all terms contain the highest power.

$$\left[1+\frac{h}{R}\right]^{-1} \simeq 1-\frac{h}{R}$$

Now $U = \frac{-GMm}{R}\left(1-\frac{h}{R}\right)$

$$= \frac{-GMm}{R}+\frac{GMmh}{R^2}$$

But $\frac{GM}{R^2} = g$

$$U = \frac{-GMm}{R}+mgh$$

Since $-\frac{GMm}{R}$ = Gravitational potential energy of mass, m at the surface of the Earth. according to convection, the gravitational potential energy at the surface of Earth is taken as zero, since $h = 0$

$-\frac{GMm}{R} = 0$, $U = p.e$

$p.e = mgh$

WORK DONE IN RAISING A BODY FROM THE SURFACE OF EARTH TO A LARGE HEIGHT, h

Potential energy of the body on the surface of Earth $= \frac{-GMm}{R}$

Work done $W = \frac{-GMm}{R+h}-\left[\frac{-GMm}{R}\right]$

$$= \frac{GMm}{R}-\frac{GMm}{R+h}$$

$$GMm\left[\frac{1}{R}-\frac{1}{R+h}\right] = \frac{GMmh}{R(R+h)}$$

$$= \frac{mgR^2h}{R(R+h)} \qquad \left[\because g = \frac{GM}{R^2}\right]$$

$$= \frac{mgRh}{R+h} = \frac{mgh}{1+h/R}$$

$$W = \frac{mgh}{1+\frac{h}{R}}$$

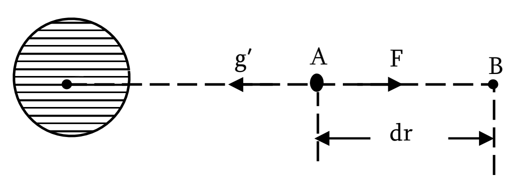
A special case: when h = R, then

$$W = \frac{mgR}{1+\frac{R}{R}} = \frac{1}{2}mgR$$

$$W = \frac{1}{2}mgR$$

RELATIONSHIP BETWEEN GRAVITATIONAL FIELD STRENGTH AND GRAVITATIONAL POTENTIAL GRADIENT.

Suppose that a particle of mass, m is moved by force F from A to B in gravitational field strength, g′



The change in work done by the particle in going from A to B

$$dw = Fdr$$

But $g' = \frac{-F}{M}$

(Minus sign shows that g′ and F are in opposite direction)

$$F = -mg'$$

Now: $dw = -mg'dr$

$$dw = -mg'dr$$

$$dw = mdv$$

$$mdv = -mg'dr$$

$$g' = I = \frac{dv}{dr}$$

Note that:

Gravitational field intensity

$$g' = \frac{GM}{r^2}$$

Gravitational potential

$$V = \frac{-GM}{r}$$

Now $\quad -V = \frac{GM}{r}$

$$g' = \frac{1}{r}\left[\frac{GM}{r}\right] = \frac{-V}{r}$$

$$g' = \frac{V}{r}$$

NUMERICAL EXAMPLES

31. (a) (i) What is meant by Gravitational field?
(ii) Outline five (5) properties of gravitation field lines.
(b) With what velocity must a body be thrown upward from the surface of the Earth so that it reaches height 10R? Radius of the Earth = 6.4×10^6m, mass of the Earth $M = 6 \times 10^{24}$kg, $G = 6.67 \times 10^{-11}Nm^2Kg^{-2}$.

Solution

(a) (i) Refer to your notes

(ii)
- They are parallel
- They are perpendicular to the surface when they enter or leave.
- They are closer together near the Earth and far distant points.
- They are field strength determinant.
- They determine the direction of force of attraction

(b) From the equation of

$$\Delta P.e = GMm\left[\frac{1}{R} - \frac{1}{R+h}\right]$$

But h = 10R

$$= GMm\left[\frac{1}{R} - \frac{1}{R+10R}\right]$$

$$\Delta P.e = \frac{GMm\left[11R - R\right]}{11R^2} = \frac{10GMmR}{11R^2}$$

$$\Delta P.e = \frac{10GMm}{11R} \quad \text{.........(i)}$$

Also change in K.E

$$\Delta K.e = \frac{1}{2}M\left(V^2 - U^2\right)$$

$$\Delta K.e = \frac{1}{2}M\left[V^2 - 0^2\right] = \frac{1}{2}MV^2$$

Apply the law of conservation of energy

$$\Delta k.e = \Delta p.e$$

$$\frac{1}{2}MV^2 = \frac{10GMm}{11R}$$

$$V = \sqrt{\frac{20GM}{11R}}$$

$$V = \sqrt{\frac{20 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{11 \times 6.4 \times 10^6}}$$

$$V = 1.0663 \times 10^4 m/s$$

32. Calculate the gravitational field intensity on the surface of Mars assuming it to be uniform sphere. Given that the mass of Mars is 6.420×10^{23}kg and its radius is 3.375×10^6m. $G = 6.67 \times 10^{-11}Nm^2kg^{-2}$.

Solution

Gravitational field strength

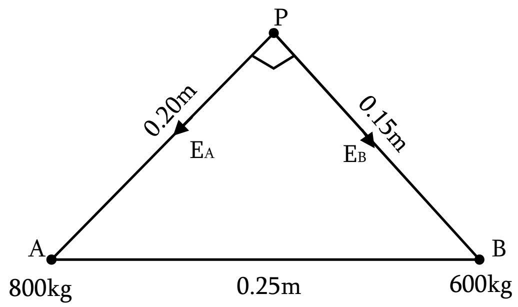
$$g'_m = \frac{GM}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times 6.420 \times 10^{23}}{\left(3.375 \times 10^6\right)^2}$$

$$g'_m = 3.76Nkg^{-1}$$

33. Two masses 800kg and 600kg are at a distance of 0.25m apart. Compute the magnitude of intensity of gravitational field at a point distant 0.2m from 800kg mass and 0.15m from the 600kg mass.

Solution



Gravitational field intensity at P due to the mass at A.

$$E_A = \frac{GM_1}{r_1^2} = \frac{G\times800}{(0.2)^2}$$

$$E_A = 20{,}000\ G \text{ along } PA$$

Also

$$E_B = \frac{GM_2}{r_2^2} = \frac{G\times600}{(0.15)^2}$$

$$E_B = \frac{80{,}000G}{3} \text{ along } PB$$

In ΔAPB , $\overline{PA}^2 + \overline{PB}^2 = \overline{AB}^2$

$\therefore < APB = 90^{O}$

Magnitude of resultant gravitational field at P.

$$E = \sqrt{E_A^2 + E_B^2}$$

$$E = G\sqrt{(20000)^2 + \left(\frac{80{,}000}{3}\right)^2}$$

$$= 6.67\times10^{-11}\times\frac{10{,}000}{3}$$

$$E = 6.67\times10^{-11}\times\frac{10{,}000}{3}$$

$$E = 2.22\times10^{-6}\,NKg^{-1}$$

34. Calculate the gravitational potential due a body of a mass 10kg at a distance of
(i) 10m and
(ii) 20m from the body. Given that $G = 6.67 \times 10^{-11}Nm^2Kg^{-2}$.

Solution

Gravitational potential is given by $V = \frac{-GM}{r}$

(i) V = ? , r = 10m

$$V = \frac{-6.67\times10^{-11}\times10}{10}$$

$$V = -6.67\times10 JKg^{-1}$$

(ii) $$V = \frac{-6.67\times10^{-11}\times10}{20}$$

$$V = -3.33\times10^{-12}\,Jkg^{-1}$$

35. (a) Two bodies of masses M_1 and M_2 are placed distant d apart. Show that the position where the gravitational field due to them is zero, the gravitational potential is given by

$$V = \frac{-G}{d}\left[M_1 + M_2 + 2\sqrt{M_1M_2}\right]$$

(b) Assuming that the Earth is a uniform sphere of radius 6.4×10^6m and mass 6.0×10^{24}kg, find the gravitational field strength at a point.
(i) On the surface
(ii) On height 0.50times its radius above the Earth's surface.

Solution

(a) Refer to your notes

(b) (i) $$g_E' = \frac{GM}{R^2}$$

$$= \frac{6.67\times10^{-11}\times6.0\times10^{24}}{(6.4\times10^6)^2}$$

$$g_E' = 9.8\,Nkg^{-1}$$

(ii) $R_1 = 1.5R$

$$g' \alpha \frac{1}{r^2}\ ,\ g' = \frac{K}{R^2}$$

$$g_1' = \frac{K}{R_1^2}\ ,\ g_E = \frac{K}{R^2}$$

$$g_1' = g_E'\left[\frac{R}{R_1}\right]^2 = 9.8\left[\frac{R}{1.5R}\right]^2$$

$$g_1' = 4.36\,Nkg^{-1}$$

36. (a) (i) State the Newton's law of gravitational and show that the speed V of a particle in an orbit of radius r round a planet of mass M is given by

$$V = \left[\frac{GM}{r}\right]^{\frac{1}{2}}$$

(ii) Define gravitational field strength and gravitational potential at a point on the Earth's gravitational field. How are they related?

(iii) Given a graph of the variation of gravitational potential with distance away from the Earth; how could the graph of gravitational field strength with distance be derived?

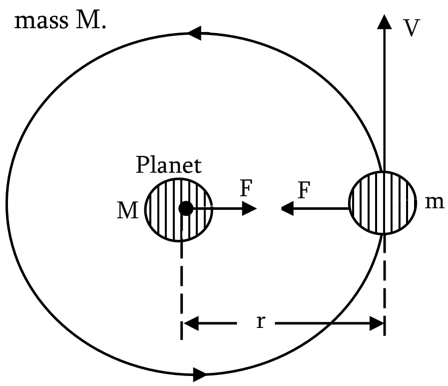
(b) (i) At one point on a line between the Earth and the moon the gravitational field caused by the two bodies is zero. Explain briefly why this so?

(ii) The mass of moon is $1/81$ the mass of Earth and its radius $1/4^{th}$ that of the Earth. if the acceleration due to gravity at the surface of the Earth is $9.8 m/s^2$. What is its value at the surface of the moon.

Solution

(a) (i) Refer to your notes

Consider the motion of the particle of mass m revolves around the planet of mass M.



Centripetal force On the particle = Gravitation force

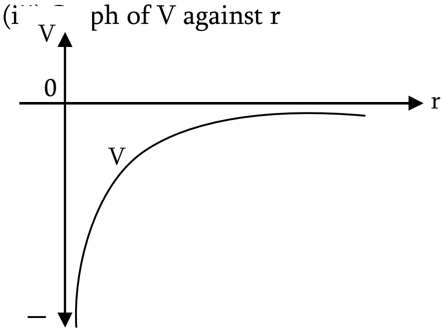
$$\frac{MV^2}{r} = \frac{GMm}{r^2}$$

$$V^2 = \frac{GM}{r}$$

$$V = \left[\frac{GM}{r}\right]^{\frac{1}{2}}$$

(ii) See your notes

(i ph of V against r



The negative of the gradient at any point of the graph of V against r is the gravitational field strength at that distance.

$$V = \frac{-GM}{r}$$

$$\frac{dv}{dr} = \frac{d}{dr}\left[\frac{-GM}{r}\right] = \frac{GM}{r^2}$$

$$E = \frac{-dv}{dr} = \frac{GM}{r^2}$$

Therefore the graph of gravitational field strength can be derived from the graph of potential against distance at any point , negative the value of those slopes and plotting the graph if the obtained slope versus distance. The resulting graph will be gravitational field strength versus distance.

(b) (i) see your notes

(ii) On the Earth – surface

$$g_e = \frac{GM}{R^2}$$

On the moon

$$g_m = \frac{GMm}{R_m^2}$$

$$\frac{g_m}{g_e} = \frac{GMm}{R_m^2} \Big/ \frac{GM}{R^2}$$

$$g_m = g_e\left[\frac{Mm}{M}\right]\left[\frac{R}{R_m}\right]^2$$

But $\frac{Mm}{M} = \frac{1}{81}$, $M = 81Mm$

$\frac{R_m}{R} = \frac{1}{4}$, $R = 4R_m$

$$g_m = 9.8\left[\frac{Mm}{81Mm}\right]\left[\frac{4Rm}{R_m}\right]$$

$$g_m = 1.93m/s^2$$

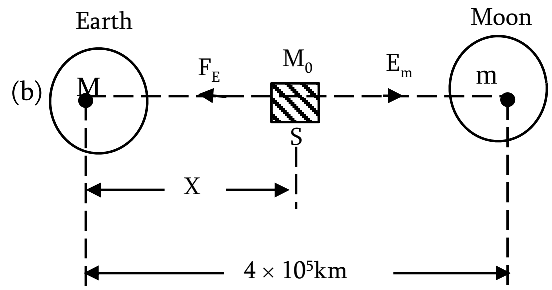
37. NECTA 2009/P2/4
   (a) (i) State Kepler's laws of planetary motion.
   (iii) Explain the variation of acceleration due to gravity, g inside and outside of the Earth.
   (b) Derive the formulae of mass and density of the Earth.

38. (a) (i) The gravitational potential energy of a body on the Earth surface is -6.5×10^6J. What do you mean by this statement.
   (ii) The gravitational potential energy of a body at the surface of Earth is negative. What does it mean?
   (b) The mass of the Earth is 81times that of the moon and the distance from the centre of the Earth to that of moon is about 4.0×10^5km. calculates the distance from the centre of the Earth where the resultant gravitational force becomes zero when a space craft is launched from the Earth to the moon. Draw a sketch showing roughly how the gravitational force on the space craft varies in its journey.

Solution

(a) (i) it means that 6.5×10^6J of energy is required on the surface of Earth to send the body outside the gravitational field of the Earth.
   (ii) It means that the gravitational potential energy of the body at the surface of the Earth is less than at infinity (p.e of the body at infinity is zero). It follows that a body at infinity would fall towards the Earth; a body on the Earth does not fall to infinity.

(b)



Gravitational force on the space craft s due to the Earth is in opposite direction to that of the moon.

$$F_E = F_M$$

$$\frac{GMm}{X^2} = \frac{GmM}{(4\times10^5 - X)^2}$$

$$\frac{M}{m} = \frac{X^2}{(4\times10^5 - X)^2}$$

$$\text{But: } \frac{M}{m} = \frac{81}{1} = 81$$

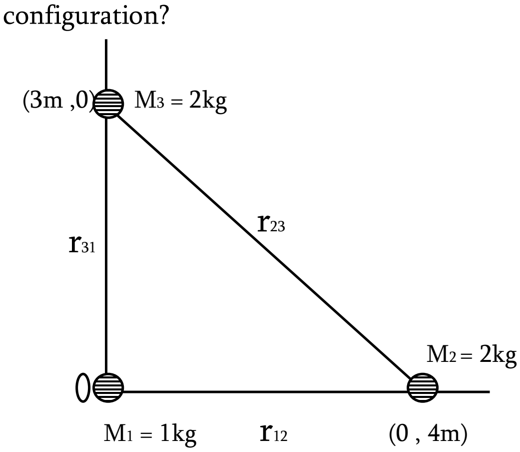
$$81 = \left[\frac{X}{4\times10^5 - X}\right]^2$$

$$9 = \frac{X}{4\times10^5 - X}$$

On solving $X = 3.6 \times 10^5$Km

The resultant F on Mo due to the Earth acts towards the Earth until S is reached. It then acts towards the moon. So F changes in direction after S is passed.

39. Three masses are in the configuration as shown in the figure below. What is the total gravitational potential energy of the configuration?



Solution

By using principle of superposition, the total gravitational potential energy of the system is

$$U = \left(\frac{-GM_1M_2}{r_{12}}\right) + \left(\frac{-GM_2M_3}{r_{23}}\right) + \left(\frac{-GM_1M_3}{r_{31}}\right)$$

$$= -G\left[\frac{M_1M_2}{r_{12}} + \frac{M_2M_3}{r_{23}} + \frac{M_1M_3}{r_{31}}\right]$$

$$U = -1.31\times10^{-10}\,J$$

$$= -6.67\times10^{-11}\left[\frac{1\times2}{4} + \frac{2\times2}{5} + \frac{2\times1}{3}\right]$$

$$U = -1.31\times10^{-10}\,J$$

40. At a point above the surface of Earth, the gravitational potential is -5.12 × 10^7Jkg $^{-1}$ and the acceleration due to gravity is 6.4m/s^2. Assuming the mean radius of Earth to be 6400km; calculate the height of this point from the surface of the Earth?

Solution

Gravitational potential at this point

$$V = \frac{-GM}{r} = -5.12\times10^7 \;.......(i)$$

Acceleration due to gravity at this point

$$g = \frac{GM}{r^2} = 6.4 \;...............(ii)$$

(i) / (ii)

$$\frac{5.12\times10^7}{6.4} = \frac{GM}{r} \Big/ \frac{GM}{r^2}$$

$$r = 8\times10^6\,m = 8000km$$

Since $R + h = r$

$$h = r - R$$
$$= 8000 - 6400$$
$$h = 1600km$$

MOTION OF THE SATELLITE

Definition Satellite – is an object moving constantly around a planet in a closed and stable orbit.

TYPES OF SATELLITE

(i) Artificial satellite

(ii) Natural satellite

ARTIFICIAL SATELLITE

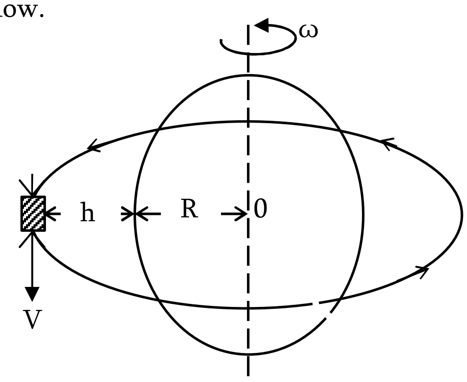
Is the man made satellite that orbits around the Earth or some other heavenly body. Artificial satellite cirching the Earth are now quite common they are called Earth – satellites for example; communication satellite.

NATURAL SATELLITE

A heavenly body that revolves around the planet in a close and stable orbit is called a Natural satellite' i.e this is the satellite which is created by natural (GOD) example; Moon is the natural satellite of the Earth.

EARTH – SATELLITE

An Earth satellite is an object launched in space by man to an orbit of the Earth for scientific investigation above the Earth – surface. There are kept in their orbit by the gravitation force of attraction of the Earth. suppose a satellite of mass m is to be put into circular orbit around the Earth at a height h above its surface as shown in the figure below.



ORBITAL VELOCITY

Is the velocity with which a satellite revolves in its closed orbit. i.e The velocity required to put a satellite into a give orbit around the earth is called orbital velocity of the satellite in that orbit. This velocity is imparted in a direction parallel to the tangent of the orbit.

- Expression for orbital velocity

  The centripetal force $\frac{mv^2}{r}$ required by the satellite to keep moving in a circular orbit is produced by the gravitation force $\frac{GMm}{r^2}$ between the satellite and the Earth.

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v^2 = \frac{GM}{r}$$

$$V = \left[\frac{GM}{r}\right]^{\frac{1}{2}} = \sqrt{\frac{GM}{r}}$$

Since r = R + h , GM = gR²

$$V = \sqrt{\frac{GM}{R+h}} = R\sqrt{\frac{g}{R+h}}$$

If g' be the value of acceleration due to gravity at a height h

$$GM = g'(R+h)^2$$

$$V = \sqrt{\frac{g'(R+h)^2}{(R+h)}} = \sqrt{g'(R+h)}$$

$$V = \sqrt{g'(R+h)}$$

Special case:

Since the satellite is launched very close to the Earth – surface, then neglecting the value of h (i.e h = 0).

$$V = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{gR^2}{R}}$$

$$V = \sqrt{gR} = \sqrt{6.4\times10^6\times9.81}$$

$$V = 7.92\times10^3 m/s \simeq 8km/s \text{ (approx.)}$$

∴ Orbital velocity of the satellite is 8km/s (approximately)

- In practice the satellite is carried by a rocket to the height of the orbit and then given an impulse by firing jets, to deflect it in a direction parallel to the tangent of the orbit. Its velocity is boosted to 8km/s so that it stays in the orbit since this motion may continue indefinitely we may say that the orbit is stable.

Note that

The orbital velocity of a satellite revolving around any planet is given by

$$V_P = \sqrt{\frac{GM_p}{R_p+h}}$$

M_P = mass of the planet

R_P = radius of the planet

h = height of the satellite above planet's surface.

TIME PERIOD OF SATELLITE

Is the time taken by the satellite to complete one revolution around the Earth.

It is denoted by

$$T = \frac{\text{Circumference of circular orbit}}{\text{orbital speed}}$$

$$T = \frac{2\pi r}{V} = \frac{2\pi(R+h)}{V}$$

But $V = \left[\frac{GM}{R+h}\right]^{\frac{1}{2}}$

$$T = \frac{2\pi(R+h)}{\sqrt{\frac{GM}{R+h}}}$$

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

Also $GM = gR^2$

$$T = 2\pi\sqrt{\frac{(R+h)}{gR^2}} = \frac{2\pi}{R}\cdot\sqrt{\frac{(R+h)^3}{g}}$$

Again $GM = g'(R+h)^2$

Now $T = 2\pi\sqrt{\frac{(R+h)^3}{g'(R+h)^2}}$

$$T = 2\pi\sqrt{\frac{R+h}{g'}}$$

g' = Acceleration due to gravity at height, h.

Time period of the satellite in terms of the density of the Earth.

$$M = \frac{4}{3}\pi R^3\rho$$

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

$$= 2\pi\sqrt{\frac{(R+h)^3}{G}\times\frac{1}{4/3\,\pi R^3\rho}}$$

$$T = \sqrt{\frac{3\pi(R+h)^3}{G\rho R^3}}$$

Special case

When the satellite is very close to the surface of the Earth (h = 0)

$$T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R}{g}} = \sqrt{\frac{3\pi}{G\rho}}$$

Given that : R = 6.4 × 106m

$$g = 9.8m/s^2$$

$$T = 2\pi\sqrt{\frac{6.4\times10^6}{9.8}}$$

$$T = 5075\sec \simeq 84\,\text{min utes}$$

Thus, the orbital speed of satellite revolving very near to the Earth surface is about 8km/s and its period of revolution is nearly 84minutes.

HEIGHT OF SATELLITE ABOVE THE EARTH – SURFACE

The periodic time of the satellite is given by

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

$$T^2 = \frac{4\pi^2(R+h)^3}{gR^2}$$

$$(R+h)^3 = \frac{T^2gR^2}{4\pi^2}$$

$$R+h = \left[\frac{T^2gR^2}{4\pi^2}\right]^{\frac{1}{3}}$$

$$h = \left[\frac{gT^2R^2}{4\pi^2}\right]^{\frac{1}{3}} - R$$

SALIENT FEATURE OF AN EARTH SATELLITE

(i) The orbital speed of an Earth satellite is independent of the mass of the satellite and depends only upon its height above the Earth's surface. The greater the height of the satellite above the Earth's surface the smaller orbital speed of satellite and vice – versa.

(ii) The time period of Earth satellite depends only upon its height above the Earth's surface. The greater the height of the satellite above Earth's surface, the greater is the time period of the satellite.

(iii) The orbital speed of a satellite close to Earth's surface is about 8km/s.

(iv) The relation between escape velocity Ve from the Earth's surface and orbital velocity (V) close to the Earth's surface

$$Ve = \sqrt{2}V$$

Expression Of Angular Momentum Of The Earth Satellite.

$$L = mVr$$

But $V = \sqrt{\frac{GM}{r}}$

$$L = mr.\sqrt{\frac{GM}{r}} = \sqrt{\frac{m^2r^2GM}{r}}$$

$$L = \sqrt{GMm^2r}\cdot \quad [r = R+h]$$

ORBIT

Orbit is the curved path in space that is followed by an object going around and round planet moon or start.

TYPES OF ORBIT

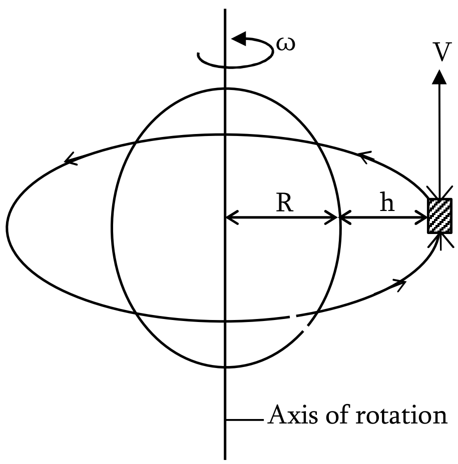
There are two types of orbit

1. Parking orbit
2. Free orbit

PARKING ORBIT

Is the orbit on which a body such as a satellite remains over the position above the Earth on it own motion. i.e parking orbit – is the orbit which a body such as satellite revolve around the Earth and its time taken is equivalent to the periodic time taken by the Earth to rotates about its own axis (24hours). Sometimes parking orbit is known as Geostationary orbit (synchronous orbit). The satellite which can be found on parking orbit is known as Geostationary satellite.

Geostationary satellite is the satellite in a circular orbit around the Earth in the equatorial plane which appears stationary to an observer on the Earth. A geostationary satellite is so named **because it appears to be stationary to an observer on the earth**. This satellite is also named as geosynchrous satellite because the angular speed of the satellite is synchromised with the angular speed of Earth about its axis i.e synchronous satellite rotates in the same direction as the Earth about its axis from west to East with a period of 24houors as that of the Earth. These satellite is used in the communication satellite (SYNCOMS) and their orbit is called **Geosynchronous orbit**.



- **HEIGHT OF PARKING ORBIT ABOVE EARTH'S SURFACE.**

Since $h = \left[\frac{gR^2T^2}{4\pi^2}\right]^{\frac{1}{3}} - R$

$$= \left[\frac{9.8\times\left(6.4\times10^6\right)^2\times\left(24\times3600\right)^2}{4\pi^2}\right]^{\frac{1}{3}}$$

$- 6.4\times10^6$

$h = 3600\times10^3 = 36000\text{km}$

SPEED OF SATELLITE IN PARKING ORBIT

$$V = \frac{2\pi\left(R+h\right)}{T}$$

$$= \frac{2\times3.4\times\left(6.4\times10^6+36000\times10^3\right)}{24\times3600}$$

$V = 3.1\times10^3 \text{m/s} = 3.1\text{km/s}$

Therefore a geostationary satellite revolves around the Earth at a height of 36000km above Earth's surface with an orbital speed of 3.1km/s.

CONDITION FOR PLACING SATELLITE IN THE PARKING ORBIT.

1. It should revolve in an orbit concentric and coplanar with the equatorial plane.
2. Its motion should be synchrous with the axial rotation of the Earth i.e time period of the satellite should be 24 hours.
3. It should rotate in the same direction as the Earth is rotating.
4. The height of the parking orbit should be 36000km above the Earth surface

USES OF ARTIFICIAL SATELLITE

1. They are used to lean about the atmosphere near the Earth.
2. They are used to forecast weather.
3. They are used to study radiation from the sun and the outer space.
4. They are used to receive and transmit various radio and television signals.
5. They are sued to know the exact shape and dimensions of the Earth.
6. Space flights are possible due to artificial satellite.
7. They are used in military purpose.

FREE ORBIT - Is the orbit in which a body such as a satellite changes its position relative to the Earth with time. Free orbit is the kind of the orbit in which satellite is free to revolves around the Earth and its time taken revolves around the Earth is quietly different from the periodic time taken by the Earth to rotates about its own axis.

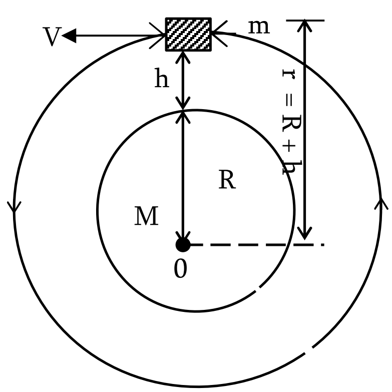
ENERGY OF SATELLITE.

A satellite revolving around the Earth has both kinetic energy and potential energy. The kinetic energy is due to motion of the satellite. The gravitational potential energy is because of the gravitational force of attraction by Earth on the satellite.

The total energy of satellite

$E = E_K + E_P$……..(i)

Consider the figure below which shows the satellite revolves around the Earth – surface at the height h above the Earth – surface



- **Expression of k.e of the satellite**

Since the path followed by the satellite to revolve around the Earth is the circular path. Centripetal force on the satellite = gravitation force.

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$mv^2 = \frac{GMm}{r}$$

(Multiply by ½ both side)

$$\frac{1}{2}mv^2 = \frac{GMm}{2r}$$

$$E_K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

Also $GM = gR^2$, $r = R+h$

$$E_K = \frac{GMm}{2(R+h)} = \frac{gR^2m}{2(R+h)}$$

- **Expression of p.e of satellite**

The change in work done due to the small displacement of satellite

$$dW = Fdx$$

$$F = \frac{GMm}{x^2}$$

$$\int dw = GMm\int_{\alpha}^{r} x^{-2}dx$$

$$p.e = E_p = GMm\left[\frac{-1}{x}\right]_{\alpha}^{r}$$

$$E_p = GMm\left[\frac{-1}{r} - \frac{-1}{\alpha}\right]$$

$$E_p = \frac{-GMm}{r} = \frac{-GMm}{R+h}$$

Also $$E_p = \frac{-gR^2m}{R+h}$$

The gravitational potential energy at the infinite is zero. Minus sing shows that all gravitational potential energy is below that at the infinite.

- **Expression of the total energy of satellite**

$$E = E_p + E_K$$

$$= \frac{-GMm}{r} + \frac{GMm}{2r}$$

$$E = \frac{-GMm}{2r} = \frac{-GMm}{2(R+h)}$$

Or $GM = gR^2$

$$E = \frac{-gR^2m}{2(R+h)}$$

Additional concept

1. Relationship between E, EP and EK

Since $E = \frac{-GMm}{2r}$, $E_p = \frac{-GMm}{r}$

$$E_K = \frac{GMm}{2r}$$

Now $E = -\left[\frac{GMm}{2r}\right] = -E_K$

$$E = -E_K$$

Also $E = \frac{1}{2}\left[\frac{-GMm}{r}\right] = \frac{E_P}{2}$

$$E = \frac{E_P}{2} \text{ or } E_p = 2E$$

2. The total energy of the satellite is negative. At the infinity (r = α), p.e as well as a k.e of the satellite is zero. Thus E = 0. The K.e can never be negative. Therefore negative sign on the total energy of the satellite means that in order to send the satellite to infinity, we have to give energy to the satellite i.e positive work must be done to free the satellite from the gravitational force.

BINDING ENERGY OF SATELLITE

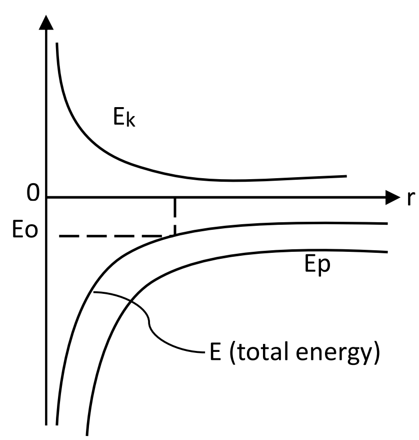
Is the energy required to remove the satellite from tis orbit around the Earth to infinity.

$$B.E = E_{\alpha} - E$$

$$= 0 - \frac{-GMm}{2r}$$

$$B.E = \frac{GMm}{2r} = \frac{GMm}{2(R+h)} = \frac{gR^2m}{2(R+h)}$$

3. Graph of E_K, E_P and E of the satellite with distance, r



$$E_K = \frac{GMm}{2r}, \quad E_P = \frac{-GMm}{r}$$

$$E = \frac{-GMm}{2r}$$

Suppose the total energy of satellite is Eo when it is in an orbit of radius, ro due to friction the satellite losses part of its energy. So its total enrgy becomes less than Eo and its moves closer to Earth (or planet). When its comes closer, its k.e increases. This means when a satellite loses energy its comes closer and closer to Earth with increased speed. When it enters the denser atmosphere its get burnt due to the heat generated.

PRINCIPLE OF LAUNCHING EARTH SATELLITE

When a satellite is to be placed in the orbit is if first carried to the desired height by the rocket at lift – off the exhaust gases built up an up thrust which exceeds rocket weight. Now the rocket accelerated upwards initially the rocket goes vertically so that it passes through the denser atmosphere with least time. The first stage burns off and falls back to the Earth. Now the rocket is tilted gradually, the second stage burns and attain high velocity. Using the final state of the rocket, the satellite is turned in the horizontal direction and given proper velocity in a direction parallel to the tangent of the orbit. The satellite is kept in its orbit by the gravitational attraction of the Earth.

Note that

(i) Principle of launching satellite in space, the following principle must be applied:-
- A satellite must be raised to the desired height.
- At this height a satellite has to be given a necessary speed and direction.

This velocity must be large enough to make a satellite to follow the curvature of the Earth. in this stage the satellite in circular orbit is regarded as falling object in the concentric to the Earth's spherical surface

Fig 126 (SCANN)

(ii) Why are space rockets usually launched from West to East?

Reason;

We know that Earth rotates about its axis from west to east. Therefore any point on the Earth's surface has linear velocity from west to East. When a rocket I launched from west to East, the linear velocity of Earth is added to the launching velocity of the rocket.

ENERGY REQUIRED TO LIFT A SATELLITE TO A CIRCULAR ORBIT.

Suppose an artificial satellite of mass m lifted from Earth – surface (mass M and radius, R) and put in a circular orbit with a radius, r. initial total energy when satellite is on Earth surface.

$$E_0 = P.E_1 + K.E_1$$

But $P.E_1 = \frac{-GMm}{R}$, $K.E_1 = 0 \ (V = 0)$

$$E_0 = \frac{-GMm}{R} \ldots\ldots.(i)$$

Final total energy of satellite on the given orbit

$$E = K.E_2 + P.E_2$$

$$= \frac{+GMm}{2r} + \frac{-GMm}{r}$$

$$E = \frac{-GMm}{2r} \ldots\ldots(ii)$$

Energy required to lift satellite is given by

$$\Delta E = E - E_0$$

$$= \frac{-GMm}{2r} - \frac{-GMm}{R}$$

$$= \frac{GMm}{R} - \frac{GMm}{2r}$$

$$\Delta E = GMm\left[\frac{1}{R} - \frac{1}{2r}\right]$$

But $r = R + h$

$$\Delta E = GMm\left[\frac{1}{R} - \frac{1}{2(R+h)}\right]$$

$$= GMm\left[\frac{2R + 2h - R}{2R(R+h)}\right]$$

$$= GMm\left[\frac{R + 2h}{2R(R+h)}\right]$$

$$\Delta E = \frac{GMm(R+2h)}{2R(R+h)}$$

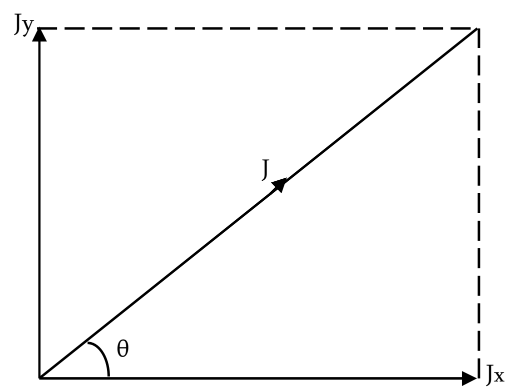
Also $GM = gR^2$

$$\Delta E = \frac{gR^2m(R+2h)}{2R(R+h)}$$

$$\Delta E = \frac{gRm(R+2h)}{2(R+h)}$$

MAGNITUDE AND DIRECTION OF IMPULSE OF THE SATELLITE

Suppose a satellite is taken by a rocket to the required height and then is given a certain horizontal velocity. The total impulse that is needed by a satellite to move it's in the Earth's curvature must be well known



J_y = Vertical impulse (Rocket impulse)

J_x = Horizontal impulse

J = Satellite impulse

$J_x = mV_o$, $J_y = mV_r$

V_o = orbital velocity

V_r = rocket velocity

m = mass of satellite

By using Pythagoras theorem

$$J^2 = J_X^2 + J_y^2$$

$$J = m\sqrt{V_0^2 + V_r^2}$$

Direction of impulse

$$\tan\theta = \frac{J_y}{J_x} = \frac{mV_r}{mV_0} = \frac{V_r}{V_0}$$

$$\tan\theta = \frac{V_r}{V_0}$$

ESCAPE VELOCITY (Ve)

When we throw a body vertically upwards with certain velocity, the body returns to the Earth's surface after some time. However, when the body is thrown with a velocity to the escape velocity , the body overcomes the Earth's gravitational pull and also the resistance of Earth's atmosphere. This body never returns to the surface of Earth again. This velocity is known as Escape velocity.

DEFINITION ESCAPE VELOCITY – is defined as minimum velocity with which a body may be projected vertically upward such that it escapes from the earth's gravitational pull permanently.

EXPRESSION FOR ESCAPE VELOCITY

Consider a boy of mass, m lying at a distance X from the centre of the Earth. Let M be the mass of Earth. according to Newton's law of gravitation, the gravitational force F of attraction between the body and the Earth is given by

$$F = \frac{GMm}{X^2}$$

If dw be the work done in raising a body through a small distance, dx

$$dw = fdx$$

$$dw = \frac{GMmdx}{x^2}$$

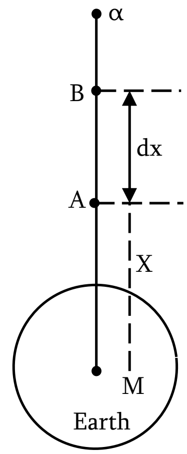
Total work done in rising the body from earth – surface to the infinity

$$w = \int_R^{\alpha} \frac{GMmdx}{x^2}$$

$$w = GMm\left[\frac{-1}{x}\right]_R^{\alpha}$$

$$w = GMm\left[\frac{-1}{\alpha} - \frac{-1}{R}\right] = \frac{GMm}{R} \text{.......(i)}$$

If Ve be the escape velocity, then k.e imported to the body



$$k.e = \frac{1}{2}mV_e^2 \text{.......(ii)}$$

$$(i) = (ii)$$

$$\frac{1}{2}mV_e^2 = \frac{GMm}{R}$$

$$V_e^2 = \frac{2GM}{R}$$

$$Ve = \sqrt{\frac{2GM}{R}} \text{.......(1)}$$

Since $GM = gR^2$

$$Ve = \sqrt{\frac{2gR^2}{R}} = \sqrt{2gR}$$

$2R = D =$ diameter of the Earth

$$Ve = \sqrt{2gR} = \sqrt{gD} \text{.......(2)}$$

Escape velocity in terms of the density of the Earth

$$M = \frac{4}{3}\pi R^3\rho$$

$$Ve = \sqrt{\frac{2G}{R} \times \frac{4}{3}\pi R^3\rho}$$

$$Ve = \sqrt{\frac{8\pi G\rho R^2}{3}} = R\sqrt{\frac{8\pi G\rho}{3}} \text{......(3)}$$

$$Ve = D \cdot \sqrt{\frac{2\pi\rho G}{3}} \text{.......(4)}$$

Equation (1) , (2), (3) and (4) give different expressions for the escape velocity of a body. value of Ve.

$$g = 9.8m/s^2 \quad , \quad R = 6.4 \times 10^6 m$$

$$Ve = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$

$$Ve = 11200m/s = 11.2km/s$$

Discussion

1. In deriving expression for escape velocity, we have neglected the atmospheric resistance to the body, so in actual practice, the value of escape velocity is slightly greater than the value calculate from these expression.
2. If a body is to be projected from the surface of any plate rather than Earth, then the escape velocity will be different. This is because the escape velocity depends upon the mass and the

radius of the planet from which the body is projected.

3. Escape velocity does not depend upon the mass of the body.
4. If we ignore the Earth's rotation, then the escape velocity does not , depend upon the direction in which the body is projected.
5. If the body is projected from a point above the Earth - surface, then

$$Ve = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2GM}{R+h}}$$

Note that

(i) $\sqrt{\frac{2GM}{r}} < \sqrt{\frac{2GM}{R}}$

(ii) The orbital velocity of a satellite close to Earth – surface $V = \sqrt{gR}$

Escape velocity

$$Ve = \sqrt{2gR} = \sqrt{2}\cdot\sqrt{gR}$$

$$Ve = \sqrt{2}V = 1.41V$$

(iii) If the velocity of projection

(iv) Of the body from the surface of a planet is greater than escape velocity (Ve) of the planet, the body will escape out from the gravitational field of that planet and will move in the interstellar space with velocity V_1. We can find the value of V_1 by applying law of conservation of energy i.e

$$\frac{1}{2}MV_1^2 + \frac{1}{2}MV_e^2 = \frac{1}{2}MV^2$$

$$V_1^2 = V^2 - V_e^2$$

$$V_1 = \sqrt{V^2 - V_e^2}$$

DERIVATION FORMULA FOR ESCAPE VELOCITY FROM ENERGY CONSIDERATION.

Consider a projectile mass, m leaving the Earth's surface with escape velocity Ve.

Total energy of the body on the Earth – surface

$$E_1 = p.e_1 + k.e_1$$

$$E_1 = \frac{-GMm}{R} + \frac{1}{2}mV_e^2 \;........(i)$$

When the projectile reaches at the infinity, it stops so it has no k.e it also has zero g.p.e because it is out of the gravitational field of the Earth.

Total energy at the infinity

$$E_2 = 0 \;......(i)$$

Apply the principle of conservation of energy

$$E_1 = E_2$$

$$\frac{-GMm}{R} + \frac{1}{2}mV_e^2 = 0$$

$$V_e^2 = \frac{2GM}{R}$$

$$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

EXPRESSION OF ESCAPE VELOCITY

1. Escape velocity on the Earth surface of the body.

$$V_e = \sqrt{2gR} = \sqrt{\frac{2GM}{R}}$$

$$V_e = 11.2km/sec$$

2. Escape velocity on the moon surface

$$V_m = \sqrt{\frac{2GMm}{R_m}} = 2km/sec$$

3. Escape velocity in terms of density of Earth

$$M = \frac{4}{3}\pi R^3\rho$$

$$V_e = \sqrt{\frac{2GM}{R}}$$

$$= \sqrt{\frac{2G}{R}\times\frac{4}{3}\pi R^3\rho}$$

$$V_e = R\cdot\sqrt{\frac{8\pi G\rho}{3}}$$

4. Escape velocity of a body at a certain height h above the Earth – surface. $V_e = \sqrt{\frac{2GM}{R+h}}$

THE EARTH ATMOSPHERE

The molecules of air at normal temperature and pressure have average velocity of the order of the which is much less than the escape velocity. Therefore the gravitational force of attraction of Earth keeps the atmosphere round the Earth.

Explanation of the probable nature of the atmosphere of planets, sun, moon e.t.c base d on the escape velocity.

Escape velocity of the body from the planet is given by $Ve = \sqrt{\frac{2GM}{R}}$

By substituting their respectively masses and radii.

1. Ve of the earth = 11.2km/s
2. Ve of the moon = 2km/s
3. Ve of the mercury = 4.2km/s
4. Ve of the jupter = 61km/s
5. For the sun, Ve = 618km/s.

If the average velocity of a gas molecules is near the velocity of escape, then there is greater chances of these molecules escaping away from the regions of the atmosphere. The root mean velocity of hydrogen is nearly 2km/s oxygen gas and carbon dioxide gas is about 0.5km/s. This is much less than the escape velocity for the Earth, so these gases are retained on the Earth surface. Therefore there is existence of the Earth's atmosphere. For the moon, escape velocity is 2km/s so the molecules of hydrogen can easily escape from the moon surface. Similar other gases with higher r.m.s velocity will also escape from the moon atmosphere also the gravitational attraction on the moon is much less than of the Earth and accounts for the lack of atmosphere round the moon.

Note that

Since escape velocity depends upon mass and radius of the planet, it is different for the different planets

OCEAN TIDES

Ocean tides – is the periodic rise and fall of oceanic water due to the gravitational attraction of the moon and the sun on the Earth – surface. There is a tendency of Moon to pool Earth's water (ocean) and area facing the moon experience high tides (floods) where as other area experience low tides (ebbs) due to the less gravitational force on or sun towards to the surface of the Earth.

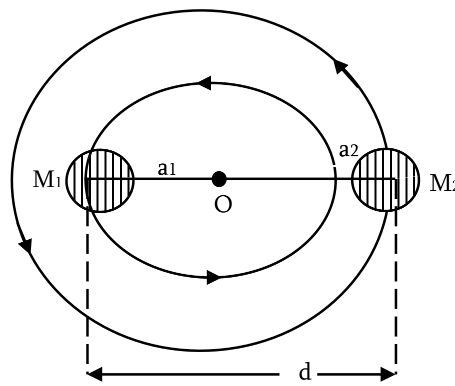
BINARY SYSTEM (STARS)

A binary system is one in which two heavy bodies revolve about a common centre of mass (the word binary means two).

Estimation of mass of sun using a binary system method.

It has been found that the sun of mass M_1 and the Earth of mass M_2 both revolve around a common centre of mass M located at O.

Let a_1 and a_2 be the radii of revolution of sun and Earth respectively d be distance between centre – centre of Earth to the sun.



$$a_1 + a_2 = d$$
$$a_2 = d - a_1$$

From the definition of centre of mass

$$M_1a_1 = M_2a_2$$
$$M_1a_1 = M_2(d - a_1)$$
$$M_1a_1 = M_2d - M_2a_1$$
$$a_1(M_1 + M_2) = M_2d$$
$$a_1 = \frac{M_2d}{M_1 + M_2}$$

Also $a_2 = d - a_1 = d - \frac{M_2d}{M_1 + M_2}$

$$a_2 = \frac{M_1d}{M_1 + M_2}$$

The centripetal force acquired by each mass is provided by gravitational force.

Centripetal force on M_1

$$M_1\omega^2a_1 = \frac{GM_1M_2}{d^2}$$
$$\omega^2 = \frac{GM_2}{d^2a_1}$$

Common angular velocity

$$\omega = \sqrt{\frac{GM_2}{d^2a_1}} \text{ but } \omega = \frac{2\pi}{T}$$

$$\frac{2\pi}{T} = \sqrt{\frac{GM_2}{d^2a_1}}$$

$$T = 2\pi\sqrt{\frac{d^2a_1}{GM_2}}$$

Centripetal force on M_2

$$M_2\omega^2a_2 = \frac{GM_1M_2}{d^2}$$

$$\omega^2a_2 = \frac{GM_1}{d^2}$$

$$\omega = \sqrt{\frac{GM_1}{d^2a_2}} \quad \Big| w = \frac{2\pi}{T}$$

$$T = 2\pi\sqrt{\frac{d^2a_2}{GM_1}} = 2\pi\sqrt{\frac{d^2\left(d-a_1\right)}{GM_1}}$$

Mass of the sun

$$T^2 = \frac{4\pi^2d^2a_2}{GM_1}$$

$$M_1 = \frac{4\pi^2d^2a_2}{GT^2}$$

Also $T = 2\pi\sqrt{\frac{d^2a_1}{GM_2}}$

But $a_1 = \frac{M_2d}{M_1+M_2}$

$$T = 2\pi\sqrt{\frac{d^2}{GM_2}\left(\frac{M_2d}{M_1+M_2}\right)}$$

$$T = 2\pi\sqrt{\frac{d^3}{G\left(M_1+M_2\right)}}$$

WEIGHTLESSNESS

This is the physical feelings occurs when the effective weight of a body equals to zero i.e g = 0; then w = mg = 0. The statement of weightlessness can be observed in eth following situations:-

(i) Body at the center of Earth. at the centre of Earth, g = 0. Thus, if a body is taken to the centre of the Earth, its effective weight becomes zero.

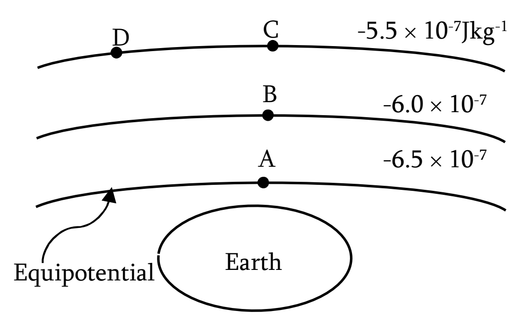
(ii) Body at will point in space null point is the point where by two gravitational pull are equal and opposite i.e as the body moves from the Earth's surface the gravitational pull of Earth decreasing but gravitational pull of the moon increasing, at the equilibrium point where Eart''s and moon gravitational pull equal to the value of acceleration due to gravity, g = 0. Therefore at this point a body feels weightlessness (w = 0).

(iii) Body in a freely falling lift when a body is lying in a lift and falls freely, the effective acceleration, $g' = g - a = g - g = 0$ therefore, the effective weight of a body in the freely falling lift is zero.

(iv) Body in a satellite orbiting Earth. when a satellite is circulating in its orbit along the curvature of Earth, there is no contact force on it or other objet inside it, therefore a satellite is said to be in free falling a satellite feels weightlessness (because the only acceleration due to gravity is acting on satellite.)

EQUIPOTENTIAL

Are lines of constant potential equipotential surface – is the surface which have the same potential.



The gravitational potential

$$V = -\frac{GM}{r}$$

The work done of mass m to moves from A to B

$$W = \left(V_B - V_A\right)M$$

The work done of mass m to moves from D to C is zero. Since $V_B = V_C$ this is known as an Equipotential.

NUMERICAL EXAPLES

41. NECTA 2013/P1/3(C)

(i) With the aid of a labeled diagram, sketch the possible orbits for a satellite launched from the Earth (3 marks)

(ii) From the diagram in (c) (i) above, write down an expression for the velocity of a satellite corresponding to each orbit.

42. (a) The gravitational potential energy of a body on the surface of Earth is -6.5 × 10^6J. What do you mean by this statement.

(b) A spaceship is launched in a circular orbit closed to Earth's surface. What additional velocity ha now to be imported to the space ship in the orbit to overcome the gravitational pull?
($R = 6400Km$, $g = 9.8m/s^2$)

Solution

(a) It means that 6.5 × 10^6J of energy is required on the surface of Earth to send the body outside the gravitational field of the Earth.

(b) Orbital velocity of spaceship in the circular orbit.

$$V = \sqrt{\frac{gR^2}{R+h}}$$

When the satellite is closed to the Earth surface , $h \simeq 0$

$$V = \sqrt{gR} = \sqrt{9.8\times6.4\times10^6}$$

$$V = 7.9195\times10^3 m/s$$

$$V = 7.9195 km/s$$

Escape velocity

$$V_e = \sqrt{2gR} = \sqrt{2}V = \sqrt{2}\times7.9195$$

$$V_e = 11.200 km/s$$

Additional velocity required

$$V_a = V_e - V$$

$$= 11.2 - 7.9195$$

$$V_a = 3.2805 km/s$$

43. (a) On a planet whose size is the same and mass 4 times as that of Earth, find the amount of work done to lift 6kg of mass vertically upward though a distance of 3m on the planet. The value of g on the planet. The value of g on the surface of Earth is $10m/s^2$.

(b) Two metal sphere of the same material and equal radius R are touching each other. Show that force of attraction between them is directly proportional to R^4.

Solution

(a) On the planet, $g_p = \frac{GM_p}{R_p^2}$

On the Earth, $g_E = \frac{GM_E}{R_E^2}$

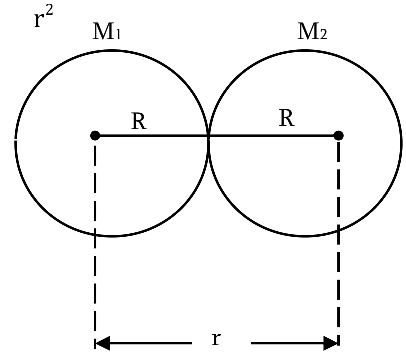
$$\frac{g_p}{g_E} = \left(\frac{M_p}{M_E}\right)\left(\frac{R_E}{R_p}\right)^2 = 4\times1^2 = 4$$

$$g_p = 4g_E = 4\times10 = 40m/s^2$$

Work done $W = Mg_ph = 6\times40\times3$

$w = 720J$

(b) $F = \frac{GM_1M_2}{r^2}$



If d is the density of the material of the sphere.

$$M_1 = \frac{4}{3}\pi R^3 d \ , \ M_2 = \frac{4}{3}\pi R^3 d$$

$$r = 2R$$

$$\text{Now, } F = \frac{G\left(\frac{4}{3}\pi R^3 d\right)\left(\frac{4}{3}\pi R^3 d\right)}{(2R)^2}$$

$$F = \frac{4}{9}G\pi^2 d^2 R^4$$

$F \propto R^4$ Hence shown.

44. NECTA 2016/P1/5

(a) (i) Mention one application of parking orbit. (01 mark)

(ii) Briefly explain how parking orbit of a satellite is achieved? (1.5 marks)

(b) The Earth satellite revolves in a circular orbit at a height of 300km above the Earth's surface. Find the

(i) Velocity of the satellite (2 marks)

(ii) Period of the satellite (1.5 marks)

(c) (i) Why are space rockets usually launched from west to east? (1.5 marks)

(ii) A spaceship is launched into a circular orbit close to the Earth's surface. What additional velocity has to be imparted to the spaceship in order to overcome the gravitational pull? (2.5 marks)

Given that $g = 9.8m/s^2$

$G = 6.67 \times 10^{-11}Nm2kg^{-2}$

$R = 6.4 \times 10^6m$

Solution

(a) Refer to your notes

(b) (i) since $\frac{mv^2}{r} = \frac{GMm}{r^2}$

$$V = \left[\frac{GM}{r}\right]^{\frac{1}{2}} \quad \text{but} \quad r = R+h$$

$$Gm = gR^2$$

$$V = \left[\frac{gR^2}{R+h}\right]^{\frac{1}{2}}$$

$$V = \left[\frac{9.8\left(6.4\times10^6\right)^2}{6.4\times10^6\times300\times10^3}\right]^{\frac{1}{2}}$$

V =

(ii) $T = \frac{2\pi(R+h)}{V}$

$= 2\times3.14\left(6.4\times10^6+300\times10^3\right)$

T =

(c) (i) We know that Earth rotates about its axis from west to east. Therefore, any point on the Earth's surface has linear velocity from west to east. When a rocket is launched from west to East, the linear velocity of Earth is added to the launching velocity of the rocket

(ii) See solution example 42 (b)

45. NECTA 2015/P1/6

(a) (i) State Newton's law of gravitation (01 mark)

(ii) Use the law started in (a) (i) to derive Keppler's law third law (1.5 marks)

(b) (i) Briefly explain why Newton's equation of universal gravitation does not hold for bodies falling near the surface of the Earth? (1.5 marks)

(ii) Show that the total energy of a satellite in a circular orbit equal half its potential energy? (1.5 marks)

(c) (i) What would be the length of a day if the rate of rotation of the Earth were such that the acceleration due to gravity g = 0 at the equator? (2.5 marks)

(ii) Calculate the height above the Earth's surface for a satellite in a parking orbit (02 marks)

$g = 9.8m/s^2$, $G = 6.67 \times 10^{-11}Nm^2kg^2$

Radius of the Earth, $Re = 6.5 \times 10^6m$.

46. (a) The escape velocity of a body from Earth is 11.2km/s. if the radius of a planet be half the radius of Earth and its mass be one – fourth of Earth. What will be the escape velocity for the planet?

(b) With what velocity should a body be projected horizontally at a height of 30km from the ground so that it becomes the satellite of the Earth? Neglecting friction. Also calculate the time period of revolution of the satellite radius of the Earth = 6370km.

Solution

(a) Escape velocity for Earth

$$V_e = \sqrt{\frac{2GM}{R}}$$

Escape velocity for planet

$$V_P = \sqrt{\frac{2GM_P}{R_P}}$$

$$\frac{V_P}{V_e} = \sqrt{\left(\frac{M_P}{M}\right)\left(\frac{R}{R_P}\right)}$$

$$\frac{V_P}{V_e} = \sqrt{\left(\frac{m/4}{m}\right)\left(\frac{R}{R/2}\right)} = \frac{1}{\sqrt{2}}$$

$$V_p = \frac{V_e}{\sqrt{2}} = \frac{11.2km/s}{\sqrt{2}}$$

$$V_p = 8km/s$$

(b) Suppose the mass of the body is m. Let radius of Earth be R and the projection velocity to be V.

Centripetal force = weight of a body

$$\frac{mV^2}{R+h} = mg'$$

$$V^2 = g'(R+h)$$

$$\text{But } g' = g\left[1-\frac{2h}{R}\right]$$

$$V^2 = g(R+h)\left(1-\frac{2h}{R}\right)$$

$$V = \sqrt{g(R+h)\left(1-\frac{2h}{R}\right)}$$

$$V = \sqrt{9.8(6370+30)\times10^3\times\left(1-\frac{2\times30}{6370}\right)}$$

$$V = 7882m/s = 7.882km/s$$

$$\text{Time period} \quad T = \frac{2\pi(R+h)}{V}$$

$$T = \frac{2\times3.14(6370+30)}{7.882}$$

$$T = 5099sec = 1.416h$$

47. (a) What is the difference between Geostationary and Ordinary satellite?

(b) A rocket is launched vertically upwards from the surface of the Earth with initial velocity, Vo. Show that is velocity V at a height h is given by

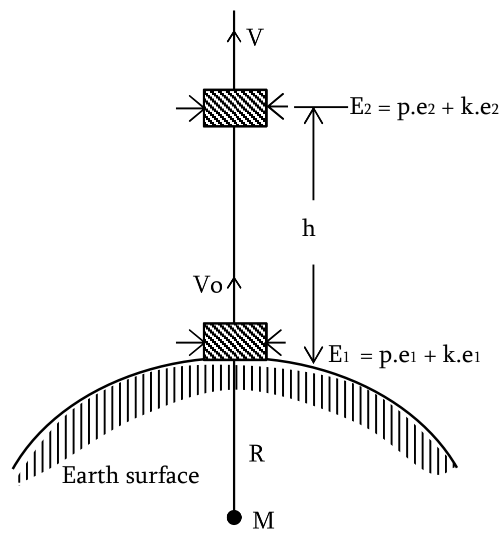
$$V_0^2 - V^2 = \frac{2gh}{1+\frac{h}{2}}$$

Where R is the radius of the Earth and g is the acceleration due to gravity at Earth's surface. Hence find the maximum height reached by a rocket fired with 90% escape velocity.

Solution

(a) A geostationary satellite orbits around the Earth at a height of about 3600km above Earth's surface and its motion is synchronous with Earth rotation about its axis. Therefore, it has a time period of 24 hours and stays over the same place relative to the observer on the Earth. An ordinary satellite has a time period other than 24 hours and hence appears to be moving w.r.t Earth.

(b)



According to the principle of conservation of mechanical energy.

$$E_1 = E_2$$

$$p.e_1 + k.e_1 = p.e_2 + k.e_2$$

$$\frac{-GMm}{R} + \frac{1}{2}mV_0^2 = \frac{-GMm}{R+h} + \frac{1}{2}mV^2$$

$$V_0^2 - V^2 = \frac{2GM}{R} - \frac{2GM}{R+h}$$

$$= 2GM\left[\frac{1}{R} - \frac{1}{R+h}\right]$$

$$= 2gR^2\left[\frac{R+h-h}{R(R+h)}\right]$$

$$= \frac{2gRh}{R+h} = \frac{2gRh/R}{\frac{R+h}{R}}$$

$$V_0^2 - V^2 = \frac{2gh}{1+\frac{h}{2}} \quad \text{Hence shown}$$

Now Vo = 90% of Ve but $V_e = \sqrt{2gR}$

$$V_0 = \frac{90}{100}.\sqrt{2gR} \;,\; V = 0$$

$$V_0^2 = \left(\frac{90}{100}\right)^2 \cdot 2gR$$

$$\text{Then } \left(\frac{90}{100}\right)^2 \cdot 2gR - 0^2 = \frac{2gh}{1+h/R}$$

$$\text{On solving } \quad h = \frac{81R}{19}$$

48. NECTA 2005/P2/2

(b) (i) List two (2) ways of describe 'g' as applied to gravitation. Give its appropriate units in each case. Assuming the Earth to be a uniform sphere of radius 6.4 × 10^6m and Me = 6 × 10^{24}kg, calculate the.

(ii) Gravitational potential at a point 6 × 10^5m above the Earth's surface.

(iii) Work done in taking a 5.0kg mass from the Earth's surface to a point where the gravitational field of the Earth is negligible.

(c) What is the binding energy of neglecting the presence of the other planets or satellites, calculating the binding energy of this system. (Take mass of the sun Ms = 3.3 × 10^5Me, Mass of the Earth, Me = 6 × 10^{24}kg, and radius of Earth – sun orbit r = 1.5 × 10^{11}m.

Solution

(b) (i) Acceleration due to gravity is the acceleration that is experienced by the free falling body. It is S.I. Unit is m/s².
Gravitational force per unit mass on the region of gravitational field. Its S.I. Unit is N/kg.

(ii) Since $V = \frac{-GM}{R+h}$

$$= \frac{-6.67\times10^{-11}\times6\times10^{24}}{\left(6.4\times10^6+6\times10^5\right)}$$

$$V = -5.7428\times10^7\,\text{Jkg}^{-1}$$

(iii) $$dw = \frac{GMmdr}{r^2}$$

$$\int_0^w dw = GMm\int_a^b r^{-2}dr$$

$$w = GMm\left[\frac{1}{a}-\frac{1}{b}\right]$$

$$= 6.67\times10^{-11}\times6\times10^{24}\times5\left[\frac{1}{6.4\times10^6}-\frac{1}{\alpha}\right]$$

w =

(c) In this case the Binding energy of the Earth – sun system is the amount of energy required in order to set the Earth free from the sun's gravitational field that is the energy to put the Earth out of control of sun's gravitational field.

$$M_1 = 3.3\times10^5\,\text{Me} \;,\; M_2 = 6\times10^{24}\,\text{kg}$$

$$b = \alpha \qquad a = 1.5\times10^{11}\,\text{m}$$

$$B.E = GM_1M_2\left[\frac{1}{a}-\frac{1}{b}\right]$$

$$= 6.67\times10^{-11}\times3.3\times10^5\times\left(6.0\times10^{24}\right)^2\left[\frac{1}{1.5\times10^{11}}-\frac{1}{\alpha}\right]$$

$$B.E = 5.3064\times10^{33}\,\text{J}$$

49. NECTA 2002/P2/1

(a) (i) State Newton's law of gravitational. Use the law to derive Kepler's third law.

(ii) Explain why Newton's equation of universal gravitation does not hold for bodies falling near the surface of the Earth?

(b) (i) With regard to the Earth – moon system discuss the formation of tides.

(ii) A satellite of mass 600kg is in a circular orbit at a height of 2 × 10^3km above the Earth's surface. Calculate the orbit speed, the kinetic energy and its gravitation potential energy.

(c) Jupiter has a mass 318times that of the Earth, its radius is 11.2 times the Earth's radius. Using this information to estimate the escape velocity of a body from Jupiter's surface, if the escape velocity from the Earth's surface is 11.2km/s

Solution

(a) (i) Refer to your notes

(ii) Reason:

- Universal gravitational constant, G is very small.
- The gravitational forces between the bodies on Earth's surface is small. These are reasons which makes why this law does not hold for the bodies falling near to the Earth – surface.

(b) (i) Refer to your notes

(ii) Orbital speed, $V = \left[\dfrac{gR^2}{R+h}\right]^{\frac{1}{2}}$

$$V = \left[\frac{9.8\times\left(6\times10^6\right)^2}{\left(6.4+2\right)\times10^6}\right]^{\frac{1}{2}}$$

$$V = 6912.79\text{m/s}$$

- K.e of the satellite

$$\text{k.e} = \frac{GMm}{2(R+h)} = \frac{gR^2m}{2(R+h)}$$

$$= \frac{9.8\left(6.4\times10^6\right)^2\times600}{2\left(6.4+2\right)\times10^6}$$

$$\text{k.e} = 1.4336\times10^{10}\text{J}$$

- P.e of the satellite

$$\text{p.e} = \frac{-gR^2m}{R+h}$$

$$= \frac{-9.8\left(6.4\times10^6\right)^2\times600}{\left(6..4+2\right)\times10^6}$$

$$\text{p.e} = -2.8672\times10^{10}\text{J}$$

(c) Escape velocity on the surface of the Earth

$$V_e = \sqrt{\frac{2GM}{R}}$$

Escape velocity on the Jupiter

$$V_J = \sqrt{\frac{2GM_J}{R_J}}$$

$$\frac{V_J}{V_e} = \sqrt{\left(\frac{M_J}{M}\right)\left(\frac{R}{R_J}\right)}$$

$$\frac{R_J}{R} = 11.2\ ,\ R_J = 11.2R$$

$$\frac{M_J}{M} = 318\ ,\ M_J = 318M$$

$$V_J = V_e\cdot\sqrt{\left(\frac{M_J}{M}\right)\left(\frac{R}{R_J}\right)}$$

$$= 11.2\sqrt{\left(\frac{318M}{M}\right)\left(\frac{R}{11.2R}\right)}$$

$$V_J = 59.68\text{km/s}$$

50. NECTA 2001/P2/1

(b) Taking the Earth to be a uniform sphere of radius 6,400km and the value of g at the surface to be 9.8m/s2. Calculate the total energy needed to raise a satellite of mass 2000kg into an orbit at an altitude of 8,000km.

(c) (i) Explain the term parking orbit of a satellite.

(ii) Explain briefly how the satellite is sent into orbit when the intended altitude has been reached. What would happen if this procedure of a putting satellite in an orbit failed to overcome into effect?

Solution

(b) Total energy of the satellite ate the Earth surface.

$$E_1 = \text{p.e} = \frac{-GMm}{R} = \frac{-gR^2m}{R}$$

$$E_1 = -gRm$$

Total energy of the satellite at the altitude, h

$$E_2 = \text{p.e}+\text{k.e} = \frac{-GMm}{2(R+h)}$$

$$E_2 = \frac{-gR^2m}{2(R+h)}$$

$$\Delta E = E_1 - E_2 = -gRm - \frac{-gR^2m}{2(R+h)}$$

$$\Delta E = gRm\left[\frac{R+2h}{2(R+h)}\right]$$

$$= 9.8\times6.4\times10^6\times2000\left[\frac{6.4\times10^6+2\times8000\times10^3}{2\left(6.4\times10^6+8000\times10^3\right)}\right]$$

$$\Delta E = 9.76\times10^{10}\text{J}$$

(c) Refer to your notes.

51. A satellite orbits the Earth at height 500km from its surface compute its;
(a) Kinetic energy
(b) Potential energy
(c) Total energy if mass of the satellite = 300kg, mass of the Earth = 6.0 × 106m, G = 6.67 × 10-11Nm2kg-2.

Solution

(a) $$K.E = \frac{GMm}{2(R+h)}$$
$$= \frac{6.67\times10^{-11}\times6\times10^{24}\times300}{2(6.4\times10^{6}+500\times10^{3})}$$
$$K.E = 8.7\times10^{9}J$$

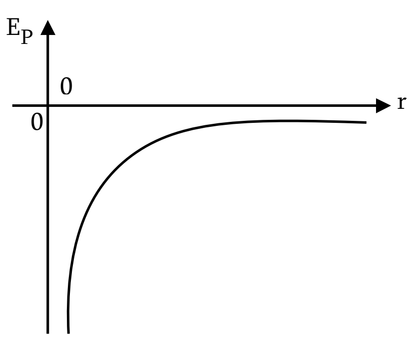
(b) $$P.E = \frac{-GMm}{R+h}$$
$$= \frac{-6.67\times10^{-11}\times6\times10^{24}\times300}{6.4\times10^{6}+500\times10^{3}}$$
$$P.E = -17.4\times10^{9}J$$

(c) $$E = P.E + K.E$$
$$= 8.7\times10^{9} - 17.4\times10^{9}$$
$$E = -8.7\times10^{9}J$$

Note: Will your answer alter if the Earth were to shrink suddenly to half its size?
Hints : When the Earth shrinks to half the size

R will become $\frac{6.4\times10^{6}}{2} = 3.2\times10^{6}m$

52. (a) The gravitational potential energy Eᴘ of a body varies with its distance r from the centre of a planet as shown in the figure below. What does the gradient at any point on the curve represent?



(b) A satellite of mass 100kg moves in a circular orbit of radius 8000km round the Earth, assumed to be a sphere of radius 6400km. Calculate the total energy to place the satellite in orbit from the Earth, assuming that g = 9.8Nkg^{-1} at the Earth – surface.

Solution

(a) Gravitational potential energy

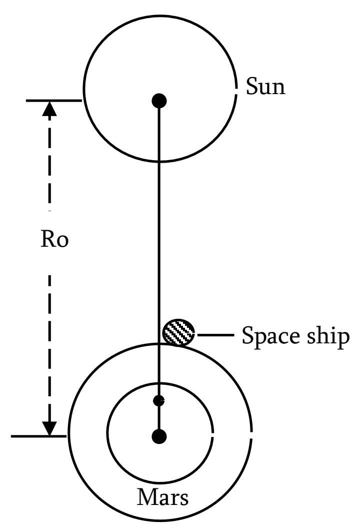
$$U = E_P = \frac{-GMm}{r}$$
$$\frac{dU}{dr} = \frac{d}{dr}\left[\frac{-GMm}{r}\right] = \frac{GMM}{r^2}$$
$$\text{Force of attraction} = \frac{dU}{dr} = \frac{+GMm}{r^2}$$

Thus, the gradient at any point on the gravitational potential energy curve represent the force pulling the body towards the planet i.e gravitational force of attraction.

(b) E = 3.76 × 10^{10}J

53. A spaceship is stationed on Mars. How much energy must be expended on the spaceship to rocket it out of the solar system?
Mars of the spaceship = 1000kg
Mass of the sun = 2 × 10^{30}kg
Mass of the Mars = 6.4 × 10^{23}kg
Radius of mars = 3395km
Radius of the orbit of mars = 2.28 × 10^{8}km
G = 6.67 × 10^{-11}Nm^{2}kg^{-2}.

Solution



Energy required to pull out the spaceship = total energy .

Total energy of space ship = p.e due to attraction of mars + p.e due to attraction of sun + k.e

p.e due to attraction of mars $= \frac{-GM_m M}{R_m}$

k.e of spaceship is zero

p.e due to attraction of sun $= \frac{-GM_s M}{R_0}$

$$E = \frac{-GM_m M}{R_m} + \frac{-GM_s M}{R_0}$$

Energy required to pull out the spaceship'

$$\Delta E = E\alpha - E = GM\left[\frac{M_m}{R_m} + \frac{M_s}{R_0}\right]$$

$$\Delta E = 6.67\times10^{-11}\times1000\left[\frac{6.4\times10^{23}}{3395\times10^{3}} + \frac{2\times10^{30}}{2.28\times10^{11}}\right]$$

$$\Delta E = 5.976\times10^{11}\,J$$

54. (a) (i) Explain the weightless of a man in a satellite.

(ii) Explain why moon has no atmosphere?

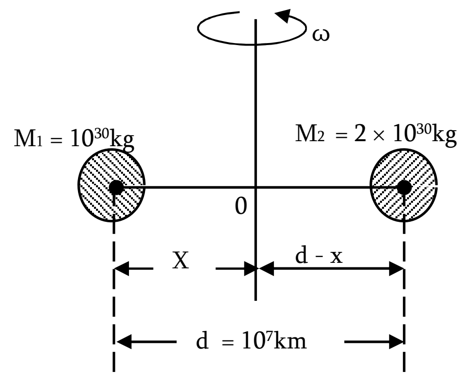
(b) A binary stars consists of two dense spherical masses of 10^{30}kg and 2×10^{30}kg whose centres are 10^7km apart and which rotate together with a uniform angular velocity ω about an axis intersects the line joining their centres. Assuming that only forces acting on the stars arise from their mutual gravitational attraction and that each mass may be taken to acts at its centre. Show that the axis of rotation passes through the centre of the system and find the value of ω.

Solution

(a) (i) The weight of the person provides the necessary centripetal force. so he feels weightless weightlessness does not mean absence of gravity. It is a situation where the person feels that he is not attracted by any force.

(ii) The escape velocity of moon is 2km/s. the root mean square velocity (r.m.s) for hydrogen is 2km/s. Thus hydrogen can easily escape from the moon atmosphere. Further , the molecule of other gases having high velocity will also leak from its atmosphere. Hence the moon has no atmosphere similarly for other smaller planets

(b)



Taking the moment about O

$$M_1X = M_2(d-x)$$

$$(M_1+M_2)X = M_2d$$

$$X = \frac{M_2d}{M_1+M_2}, \quad \frac{1}{X} = \frac{M_1+M_2}{M_2d}$$

Centripetal force On M_1 = Gravitational force between M_1 and M_2

$$M_1\omega^2 = \frac{GM_1M_2}{d^2}$$

$$\omega = \sqrt{\frac{GM_2}{d^2X}}$$

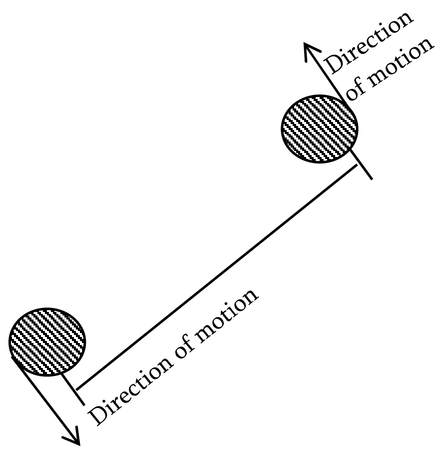
$$\omega = \sqrt{\frac{GM_2}{d^2}\cdot\frac{(M_1+M_2)}{M_2d}}$$

$$\omega = \sqrt{\frac{G(M_1+M_2)}{d^3}}$$

$$\omega = \left[\frac{6.7\times10^{-11}(10^{30}+2\times10^{30})}{(7\times10^3)^3}\right]^{1/3}$$

$$\omega = 1.4\times10^{-5}\,rad/sec(approx)$$

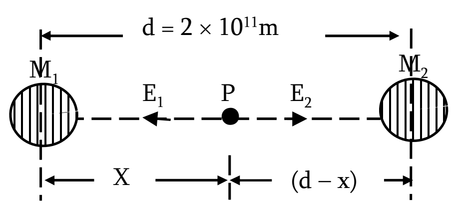
55. Two stars each of mass 4×10^{30}kg separated by 2×10^{11}m. The stars rotates about the centre of mass of the system.



(i) Determine the gravitational potential at a point where gravitational field strength is zero ($G = 6.67 \times 10^{-11} Nm^2kg^{-2}$)
(ii) Calculate the linear speed of each star in the system.
(iii) Determine the time period of rotation
(iv) Calculate the force on each star in the system.

Solution

(i) Let X be a distance from one of star at which gravitational field strength be equal to zero.



$$M_1 = M_2 = 4\times10^{30}\,kg$$

At the neutral point

$$E_1 = E_2$$

$$\frac{GM_1}{X^2} = \frac{GM_2}{(d-X)^2} \qquad [GM_1 = GM_2]$$

$$X^2 = (d-X)^2$$

$$X = d - X$$

$$X = \frac{d}{2} = \frac{2\times10^{11}\,m}{2}$$

$$X = 1.0\times10^{11}\,m$$

Gravitational potential at the neutral point

$$U = \frac{-GM_1}{X} + \frac{-GM_2}{d-X}$$

$$= -GM_1\left[\frac{1}{X} + \frac{1}{d-X}\right]$$

$$= -6.67\times10^{-11}\times4\times10^{30}\left[\frac{1}{1.0\times10^{11}} + \frac{1}{(2-1)\times10^{11}}\right]$$

$$U = -5.36\times10^{9}\,JKg^{-1}$$

(ii) Centripetal force = Gravitation force

$$\frac{M_1V^2}{X} = \frac{GM_1M_2}{d^2}$$

$$V = \sqrt{\frac{GM_2X}{d^2}}$$

$$= \sqrt{\frac{6.7\times10^{-11}\times4\times10^{30}\times10^{11}}{(2\times10^{11})^2}}$$

$$V = 2.5\times10^{4}\,m/s$$

(iii) $$\frac{GM_1M_2}{d^2} = M_1\omega^2X \qquad \left[\omega = \frac{2\pi}{T}\right]$$

$$T = 2\pi\sqrt{\frac{d^2X}{GM_2}}$$

$$T = 2\times3.14\sqrt{\frac{(2\times10^{11})^2\times1\times10^{11}}{6.7\times10^{-11}\times4\times10^{3}}}$$

$$T = 2.43\times10^{7}\,sec$$

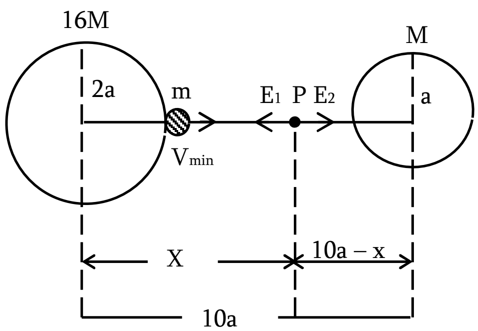
(iv) $$F = \frac{GM_1M_2}{d^2}$$

$$= \frac{6.7\times10^{-11}\times(4\times10^{30})^2}{(2\times10^{11})^2}$$

$$F = 2.68\times10^{28}\,N$$

56. Distance between the centres of the two stars is 10a. The masses of these stars are M and 16M and their radii a and 2a respectively. A body of mass m is fired straight from the surface of the larger star towards the smaller star. What would be its minimum initial speed to reach the surface of the smaller star? Obtain the expression in terms of G, M and a.

Solution



Initial total energy

$$E = p.e + k.e$$

$$= \frac{1}{2}mM_{mm}^2 + \frac{-G(16)m}{2a} + \frac{-GM_m}{8a}$$

$$E = \frac{1}{2}mV_{min}^2 - \frac{8GM_m}{a} - \frac{GM_m}{8a} \ ....(1)$$

At zero gravitational field (i.e at point P)

$$E_1 = E_2$$

$$\frac{G(16M)}{X^2} = \frac{GM}{(10a - X)^2}$$

$$\frac{16}{X^2} = \frac{1}{(10a - X)^2}$$

$$\frac{10a - X}{X} = \sqrt{\frac{1}{16}} = \frac{1}{4}$$

$$40a - 4X = X$$

$$40a = 5X$$

$$X = 8a$$

But at point P, the gravitational force cancels each other, so the particle will be at rest and the velocity will be zero. The particle will gain minimum speed when it starts moving again. Total energy at P.

$$E_P = \frac{-G(16M)m}{8a} + \frac{-GM_m}{2a}$$

Apply the law of conservation of energy (1) = (2)

$$\frac{1}{2}mV_{min}^2 + \frac{-8GM_m}{a} + \frac{-GM_m}{8a} = \frac{-G(16M)m}{8a} + \frac{-GM_m}{2a}$$

$$V_{min}^2 = \frac{45GM}{4a} = \frac{9 \times 5GM}{4a}$$

$$V_{min} = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

57. Explain what is meant by the 'constant of gravitation'. A proposed communication satellite would revolve round Earth in a circular orbit in the equatorial plane at a height of 35880km above the Earth's surface. Find the period of revolution of the satellite in hours and comment on the result. Take mean radius of earth = 6370km, mass of the Earth = 5.98 × 10^{24}kg, constant of gravitation = 6.67 × 10^{-11}Nm^{2}kg^{-2}.

Solution

$$T = \frac{2\pi(R+h)}{V} \text{ but } V = \sqrt{\frac{GM}{R+h}}$$

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

$$T = 2 \times 3.14\sqrt{\frac{(6.37 \times 10^6 + 3.588 \times 10^7)^3}{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}}$$

$$T = 86398.5sec = 24hours$$

Since the time period of the satellite is 24hours, it is clear that the satellite is geostationary.

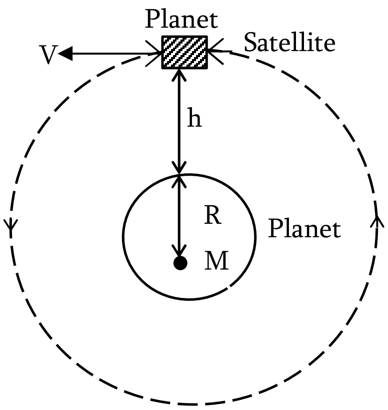
58. (a) A satellite is in a circular orbit about a planet of radius R. If the altitude of the satellite is h and its period is T, show that the density of the planet is

$$\rho = \frac{3\pi}{GT^2}\left[1 + \frac{h}{R}\right]^3$$

(b) Two stars each of mass 2 × 10^{30}kg are approaching each other for a head – on collision when they are at a distance 109km apart their speeds are negligible. What is the speed with which they collide? The radius of each star is 10^4km. assume the stars to collide,
G = 6.67 × 10^{-11}Nm2kg^{-2}.

Solution

(a) Consider the motion of satellite around the planet.



Centripetal force On satellite = gravitational force

$$m\omega^2(R+h) = \frac{GMm}{(R+h)^2}$$

$$\omega^2(R+h)^3 = GM$$

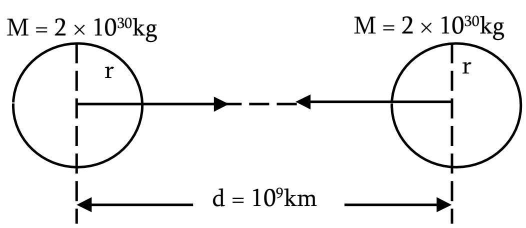
$$M = \frac{\omega^2}{G}(R+h)^3$$

But $M = \frac{4}{3}\pi R^3\rho$, $\omega = \frac{2\pi}{T}$

$$\frac{4}{3}\pi R^3\rho = \frac{4\pi^2}{GT^2}(R+h)^3$$

$$\rho = \frac{3\pi}{GT^2}\left[1+\frac{h}{R}\right]^3 \text{ Hence shown.}$$

(b) Before the collisions, stars approaching each other.



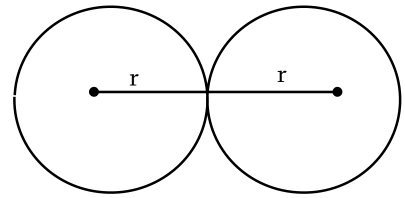
Total energy before collision

$$E_0 = p.e + k.e$$

$$= \frac{-GMM}{d} + 0$$

$$E_0 = \frac{-GM^2}{d} \quad ....(1)$$

During collision



Total energy of the system during collision

$$E = p.e + k.e$$

$$= \frac{-GM^2}{2r} + \frac{1}{2}MV^2 + \frac{1}{2}MV^2$$

$$E = \frac{-GM^2}{2r} + MV^2 \quad ......(2)$$

Apply the law of conservation of energy

$$E = E_0$$

$$\frac{-GM^2}{2r} + MV^2 = \frac{-GM^2}{d}$$

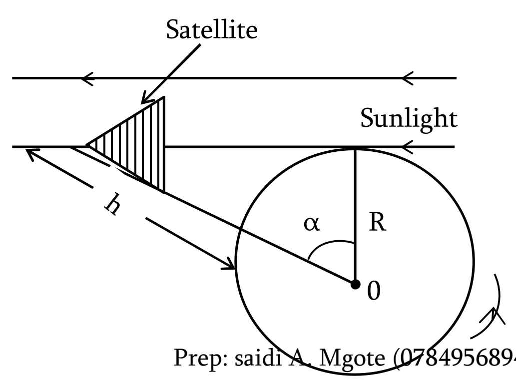
$$V^2 = \frac{GM}{2r} - \frac{GM}{d}$$

$$V = \left[GM\left(\frac{1}{2r} - \frac{1}{d}\right)\right]^{\frac{1}{2}}$$

$$V = \left[6.67\times10^{-11}\times2\times10^{30}\left(\frac{1}{2\times10^7} - \frac{1}{10^{12}}\right)\right]^{\frac{1}{2}}$$

$$V = 2.58\times10^6\, m/s$$

59. An artificial satellite can often be seen as bright high in the sky long after sun set shown in the figure below. What must be the minimum altitude in metres if satellite moving above the Earth's equator for it to appear visible directly overhead two hours after sunset? Satellite.



Given that R = 6400Km

Solution

h = ?

Assume that the satellite is on the parking orbit

t = 2hr

T = 24hrs

$$\alpha = \left(\frac{t}{T}\right)\times 360^0$$

$$= \left(\frac{2}{24}\right)\times 360^\circ$$

$$\alpha = 30^\circ$$

From the figure above

$$\cos\alpha = \frac{R}{R+h}, \quad R+h = \frac{R}{\cos\alpha}$$

$$h = R\left[\frac{1}{\cos\alpha}-1\right] = 6400\left[\frac{1}{\cos 30^0 - 1}\right]$$

$$h = 990.30\text{km}$$

60. (a) (i) State Kepler's laws of planetary motion

(ii) Suppose that the radius of the Earth was to shrink by 1%, its mass remaining the same would the acceleration due to gravity (g) on the Earth's surface increase or decrease and by what percentage? (03 marks)

(b) (i) Define the term Escape velocity (½ mark)

(ii) The escape velocity of projectile on the Earth's surface is 11.2km/s thrice this speed. What is the speed of the body far away from the Earth? ignore the presence of the sun and other planets (3½ marks)

(iii) A satellite orbits the Earth at height of 500km from its surface. Calculate its kinetic energy, given that the mass of satellite is 300kg.

Solution

(a) (i) Refer to your notes

(ii) Given that $\frac{dR}{R}\times 100\% = -1\%$

Since $G = \frac{GM}{R^2} = GMR^{-2}$

Apply natural logarithm both side

$$\log_e^g = \log_e\left[GMR^{-2}\right]$$

$$\log_e^g = \log_e^G + \log_e^M \mp 2\log_e^R$$

On differentiating both side

$$\frac{dg}{g} = 0+0-2\frac{dR}{R}$$

$$\frac{dg}{g}\times 100\% = -2\left[\frac{dR}{R}\times 100\%\right]$$

$$= -2X-1\%$$

$$\frac{dg}{g}\times 100\% = 2\%$$

(b) (i) Refer to your notes

(ii) Ve = 11.2km/s

Velocity of projection V = 3Ve

Let Vo = Velocity of projectile after escaping the gravitational pull.

Apply the law of conservation of energy

$$\frac{1}{2}MV^2 = \frac{1}{2}MV_0^2 + \frac{1}{2}MV_e^2$$

$$V_0^2 = V^2 - V_e^2$$

$$V_0^2 = \left(3V_e\right)^2 - V_e^2 = 8V_e^2$$

$$V_0 = \sqrt{8}V_e = \sqrt{8}\times 11.2$$

$$V_0 = 31.68\text{km/s}$$

$$V_0 = 31.68\text{km/s}$$

61. (a) Explain why any resistance to the forward motion of an artificial satellite result in an increase in its speed?

(b) Show that the free fall acceleration of a body at the Earth's surface, g can be expressed as

$$g = \frac{4\pi\rho GR}{3}$$

Where ρ and R are the average density and radius of the Earth respectively.

(c) The moon moves around the Earth in orbit which is approximately circular and of radius 60R, g = 9.8m/s².

(i) Calculate the moon acceleration towards the Earth.

(ii) Estimate the speed of the moon relative to the Earth.

(iii) Explain how the value of g, G and R can be used to determine the mean density of the Earth

Solution

(a) If the satellites forward motion is reduced it will go into a lower orbit thus losing g.p.e and gaining k.e. its speeds up as result despite some energy turning into heat.

(b) Since $mg = \frac{GMm}{R^2}$

$$g = \frac{GM}{R^2} = \frac{G}{R^2}\left[\frac{4}{3}\pi R^3\rho\right]$$

$$g = \frac{4}{3}\pi\rho GR$$

(c) (i) since $g = \frac{GM}{R^2} = \frac{K}{R^2}$ (On Earth)

$$g_m = \frac{K}{R_m^2}$$

$$\frac{g_m}{g} = \left(\frac{R}{R_m}\right)^2, \quad g_m = g\left[\frac{R}{R_m}\right]^2$$

$$= 9.81\left[R/\right]^2$$

$$g_m = 2.7\times10^{-3}\,m/s^2$$

(ii) Weight of an Object = Centripetal force of an object

$$mg_m = \frac{MV^2}{R_m}$$

$$V = \sqrt{g_m R_m}$$

$$= \sqrt{60\times6.4\times10^6\times2\times10^{-3}}$$

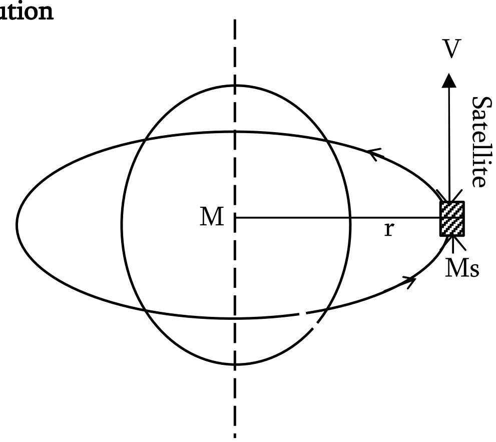
$$V = 1018.23\,m/s$$

62. (a) Prove that the angular momentum of a satellite of mass Ms revolving round the Earth of mass M in an orbit of radius r is equal to $\left[GMM_S^2 r\right]^{1/2}$

(b) The escape velocity of a projectile on the Earth's surface is 11.2km/s. A body is projected out with 4 times this speed. What is the speed of the body for away from the Earth?

Solution

(a)



$$F_C = F_g$$

$$\frac{M_S V^2}{r} = \frac{GMMs}{r^2}$$

$$V = \sqrt{\frac{GM}{r}}$$

Angular momentum

$$L = MsVr$$

$$= Msr\sqrt{\frac{GM}{r}}$$

$$L = \sqrt{GMM_S^2 r}$$

(b) Apply the principle of conservation of energy

$$\frac{1}{2}MV^2 = \frac{1}{2}MV_e^2 + \frac{1}{2}MV_0^2$$

$$V_0^2 = V^2 - V_e^2$$

$$= (4V_e)^2 - V_e^2 = 15V_e^2$$

$$V_0 = V_e\sqrt{15} = 11.2\sqrt{15}$$

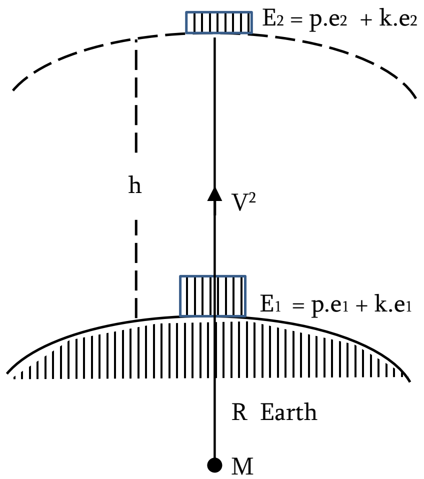
$$V_0 = 43.4\,km/s$$

63. (a) a projectile is fired vertically from the Earth surface with an initial velocity of 10km/s. Neglecting atmospheric retardation how far above the surface of the Earth would it go? Take the Earth's radius as 6400km.

(b) The escape velocity of a body on the surface of the Earth is 11.2km/s. A body is projected away with twice this speed. What is the speed if the body at infinity. Ignore the presence of other heavy bodies.

Solution

(a)



Apply the law of conservation of energy

$$p.e_1 + k.e_1 = p.e_2 + k.e_2$$

$$\frac{1}{2}MV^2 + \frac{-GMm}{R} = \frac{-GMm}{R+h} + 0$$

$$\frac{1}{2}MV^2 = \frac{GMmh}{R(R+h)} \qquad \left[gm = gR^2\right]$$

$$h = \frac{V^2R^2}{2GM - V^2R} = \frac{V^2R^2}{2gR^2 - V^2R}$$

$$h = \frac{V^2R}{2gR - V^2}$$

$$= \frac{\left(10^4\right)^2 \times 6.4 \times 10^6}{2 \times 9.8 \times 6.4 \times 10^6 - \left(10^4\right)^2}$$

$$h = 2.5 \times 10^7\,m = 2.5 \times 10^4\,km$$

(b) If V is the velocity of projection and Vo is the velocity at infinity.

Apply the principle of conservation of energy.

$$\frac{1}{2}mV^2 + \frac{-GMm}{R} = \frac{1}{2}mV_0^2 + 0$$

But $V = 2V_e$, $V_e = \sqrt{\frac{2GM}{R}}$

$$V_e^2 = \frac{2GM}{R}$$

Now $\frac{1}{2}mV^2 - \frac{GMm}{R} = \frac{1}{2}mV_0^2$

$$V^2 - \frac{2GM}{R} = V_0^2$$

$$V^2 - V_e^2 = V_0^2$$

$$(2V_e)^2 - V_e^2 = V_0^2$$

$$3V_e^2 = V_0^2$$

$$V_0 = \sqrt{3}V_e$$

$$= \sqrt{3} \times 11.2$$

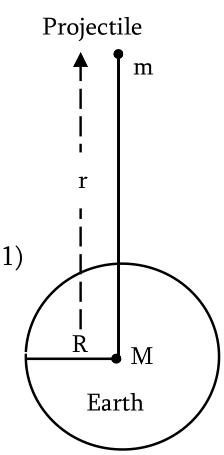
$$V_0 = 19.4km/s$$

64. A projectile is fired upwards from the surface of the Earth with a velocity KVe. Where Ve is the escape velocity and K < 1. Neglecting air resistance, show that the maximum height to which it will rise, measured from the centre of the Earth, is $\frac{R}{1-K^2}$ where R is the radius of the Earth.

Solution

Let r be maximum distance from the centre of the Earth to which the projectile rises

Initial total energy of projectile on Earth surface.



$$E_0 = p.e + k.e$$

$$= \frac{-GM_m}{R} + \frac{1}{2}mV^2$$

$$= \frac{-GMm}{R} + \frac{1}{2}m(kV_e)^2$$

$$E_0 = \frac{-GMm}{R} + mK^2V_e^2 \,.......(1)$$

Total final energy

$$E = p.e + k.e$$

$$= \frac{-GM_m}{r} + 0$$

$$E = \frac{-GM_m}{r} \,........(2)$$

Apply the law of conservation of Energy

$$E = E_0$$

$$\frac{-GM_m}{R} + \frac{1}{2} mK^2V_e^2 = \frac{-GM_m}{r}$$

But $\quad V_e^2 = \frac{2GM}{R}$

$$\frac{-GM_m}{R} + \frac{1}{2} mK^2 \frac{2GM}{R} = \frac{-GM_m}{r}$$

$$\frac{-1}{R} + \frac{K^2}{R} = \frac{-1}{r}$$

$$\frac{1}{r} = \frac{1}{R} - \frac{K^2}{R}$$

$$\frac{1}{r} = \frac{1-K^2}{R}$$

$$r = \frac{R}{1-K^2}$$

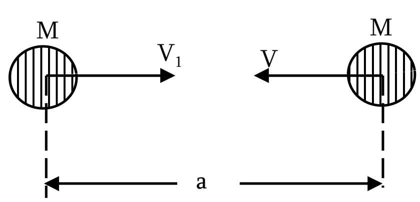
65. Two particles having masses m and M attract each other according to the law of gravitation initially they are at rest at an infinite distance apart. Show that their velocity of approach is

$$\sqrt{\frac{2G(M+m)}{a}}$$

Where a is their separation.

Solution

If V_1 and V are velocities of m and M at separation, a



Apply the principle of conservation of linear momentum.

$$0 = mV_1 - MV$$

$$mV_1 = MV$$

$$V_1 = \frac{MV}{m} \;\ldots\ldots\ldots\ldots(1)$$

Apply the law of conservation of energy

$$\frac{1}{2}mV_1^2 + \frac{1}{2}MV^2 - \frac{GMm}{a} = 0$$

$$\frac{1}{2}mV_1^2 + \frac{1}{2}MV^2 = \frac{GMm}{a}$$

$$mV_1^2 + MV^2 = \frac{2GMm}{a} \;\ldots\ldots.(2)$$

Putting equation (1) into (2)

$$m\left[\frac{MV}{m}\right]^2 + MV^2 = \frac{2GMm}{a}$$

$$\frac{M^2V^2}{m} + MV^2 = \frac{2GMm}{a}$$

$$V = m\sqrt{\frac{2G}{a(M+m)}}$$

Also $V_1 = M\sqrt{\frac{2G}{a(M+m)}}$

Relative velocity of approach

$$V_0 = V_1 + V$$

$$= M\sqrt{\frac{2G}{a(M+m)}} + m\sqrt{\frac{2G}{a(M+m)}}$$

$$= (M+m)\sqrt{\frac{2G}{a(M+m)}}$$

$$V_0 = \sqrt{\frac{2G(M+m)}{a(M+m)}}$$

$$V_0 = \sqrt{\frac{2G(M+m)}{a}}$$

66. Sun and the Earth revolve around a common centre of mass with common period of revolution T. If M_1 and M_2 are the masses of the sun and Earth respectively and 'a' is the separation between them, then prove that

$$M_1 + M_2 = \frac{4\pi^2}{G} \cdot \frac{a^3}{T^2}$$

Where G is the universal constant of gravitation.

Solution