MODULE 11 : KINETIC THEOREM OF GAS

KINETIC THEORY OF GASES

A molecules is smallest indivisible part of matter which occupies space molecule of matter are in motion. The theory developed on the basis of molecular motion is called **Kinetic theory of matter**. This theory is used to explain the macroscopic properties of matter. Kinetic theory of matters when applied to understand the behavior of gases called **'Kinetic theory of gases'**.

Kinetic theory of gases is the theory which explains the behavior of gases in terms of the motion of its molecules.

Kinetic theory of gases deals with the behavior of gases in terms of molecular (atomic) motion. Example of phenomena which can be explained by the kinetic theory of gases:-

(i) Microscopic variables of gases like pressure, volume and temperature
(ii) A gas occupies all the space provided to it.
(iii) The temperature of gas decreases in an adiabatic expansion.
(iv) The volume of fixed mass of a gas kept a constant temperature decreases as the pressure increased.
(v) A gas exerts pressure on the wall of its container.

A GAS

A gas is a compressible fluid which is confirmed by the shape of the container but ever fills it. Gas is the substance which has neither rigidity nor surface.

TYPES OF THE GAS

There are two types of gas molecules:

(i) Ideal gas (ii) Real gas

IDEAL GAS

Is the gas which strictly obeys gas laws i.e Boyle's Charles law and Gay Lussac's law. Ideal gas is the gas which obeys ideal gas equation.

Equation of Ideal gas

$PV = nRT$

P = Pressure of the gas
V = Volume of the gas
R = Universal gas constant
T = Absolute temperature
n = Number of moles

$$n = \frac{N}{N_A} = \frac{m}{Mr} = \frac{V}{V_{stp}}$$

N = Number of molecules of gas
NA = Avogadro's constant
m = Mass of the gas molecules
Mr = Molar mass of the gas.

REAL GAS

Is the gas which obeys Van der Waals gas equation.

Note that

An ideal gas has the following two characteristics.

(i) There is no intermolecular force of attraction or repulsion among the molecules of the gas.
(ii) Molecule of and ideal gas is a point which no geometrical dimensions.

PRESSURE OF A GAS

Qualitative account

How a gas exerts the pressure on the walls of the container?

Pressure of a gas is due to the random motion of the gas molecules and the collisions between the gas molecules with the wall of the container.

- When a molecule of gas collides with the walls of the container. Its momentum at right angles to that wall is reversed.
- Force exerted is equal to the average rate of change of the linear momentum. This is according to the Newton's second law of motion.
- Pressure exerted by the gas is equal to the force per unit area.

Qualitative account or Mathematical treatment of a pressure exerted by a gas.

Derivation of expression for the pressure exerted by a gas by using kinetic theory of gases.

POSTULATES OF KINETIC THEORY OF GASES

A number of assumptions are made to develop the kinetic theory of gases, regarding the nature of gas molecules:-

(i) The gas molecules are identical

(ii) A gas consists of particles called molecules which are in state of continuous random motion, moving in all directions with all possible velocities .

(iii) The volume of the molecules is negligible as compared to the volume occupied the gas.

(iv) The intermolecular forces of attraction or repulsion can be neglected.

(v) The collision between gas molecules with the wall of container is a perfect elastic collision (i.e No loss in kinetic energy in these collisions)

(vi) The duration of a collision is negligible compared with the time between the collision i.e the time spent in a collision is negligible as compared with that during which the molecules are moving independently.

(vii) The laws of Newtonian mechanic apply

(viii) Between collisions the molecules moves in a straight line with uniform velocity.

- The distance between two collisions is called the **'Free path of the molecules'** the average distance travelled by a molecule between successive collision is called **'mean free path'**.

Definition Duration collision

Is the time the gas molecules spend in contact with the wall of the container.

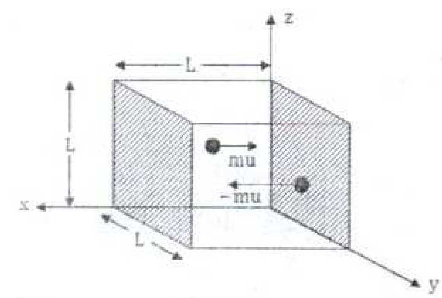
The time between collisions

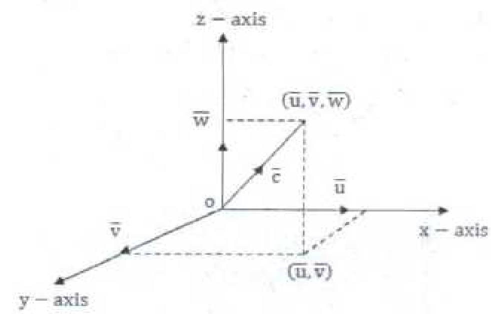
Is the time the molecules take to move to the opposite side of wall and back.

EXPRESSION OF A PRESSURE OF A GAS

Consider a gas enclosed in a cubical container of side L. The volume V of the gas is equal to L^3. Let N be the number of the molecules in the gas, m be the mass molecule and M be mass of the gas

Let U, V and W be the velocities of gas molecules in x, y and z – axes respectively





Assume that the motion of the gas molecule is under three dimensional motion. Let us consider the motion of a gas molecule in x – direction momentum of a gas molecule before the collision with the wall = mu

Momentum of a gas molecule after collision with a wall = -mu

(Since momentum = mu(--mu) = 2mu

Time interval between two successive collisions on face, $t = \frac{2L}{U}$

Rate of change of momentum $= \frac{2mu}{2L/U} = \frac{MU^2}{L}$

According to the Newton's second law of motion

$$P = \frac{mu^2}{L}$$

If there are N molecules in the container and their x – components of velocities are u_1 , u_2..........u_N respectively.

Total pressure in x – direction

$$P = P_1 + P_2 + ... + P_N$$

$$\text{Pressure} = \frac{mu^2}{L} / L^2 = \frac{mu^2}{L^3}$$

$$P = \frac{mu_1^2}{L^3} + \frac{mu_2^2}{L^3} + ... + \frac{mu_N^2}{L^3}$$

$$P = \frac{m}{L^3}\left[u_1^2 + u_2^2 + ... + u_N^2\right]$$

But $\bar{u}^2 = \dfrac{u_1^2 + u_2^2 + \dots\dots\dots u_N^2}{N}$

$N\bar{u}^2 = u_1^2 + u_2^2 + \dots\dots + u_N^2$

$P = \dfrac{Nm\bar{u}^2}{L^3} = \dfrac{Nm\bar{u}^2}{V}$

Since the motion of gas molecules are random
By using Pythagoras theorem

$\bar{c}^2 = \bar{u}^2 + \bar{v}^2 + \bar{w}^2$

But $\bar{u}^2 = \bar{v}^2 = \bar{w}^2$

$\bar{c}^2 = 3\bar{u}^2 = 3\bar{v}^2 = 3\bar{w}^2$

$\bar{u}^2 = \dfrac{1}{3}\bar{c}^2$

Now $P = \dfrac{Nm}{V}\left[\dfrac{1}{3}\bar{C}^2\right]$

$P = \dfrac{1}{3}\dfrac{Nm\bar{c}^2}{V}$

Let M = Nm = total mass of the gas

$P = \dfrac{1}{3}\dfrac{Nm\bar{c}^2}{V} = \dfrac{1}{3}\rho\bar{c}^2$

$P = \dfrac{1}{3}\rho\bar{c}^2$

DIFFERENT FORMS OF EXPRESSION OF PRESSURE OF A GAS

1. $P = \dfrac{1}{3}\dfrac{Nm\bar{c}^2}{V}$

2. Let

   $n = \dfrac{N}{V}$ = Number of the molecules per unit volume $P = \dfrac{1}{3}nm\bar{c}^2$

3. Let nm = ρ

   $P = \dfrac{1}{3}\rho\bar{c}^2$

4. $P = \dfrac{2}{3}\dfrac{N}{V}\left(\dfrac{1}{2}m\bar{c}^2\right) = \dfrac{2}{3}\dfrac{NK.E}{V}$

5. Let

   $E = \dfrac{NK.E}{V}$ = Average kinetic energy per unit volume $P = \dfrac{2}{3}E$

Note that

1. Cubical container used for the derivation of pressure of an ideal gas. Why?

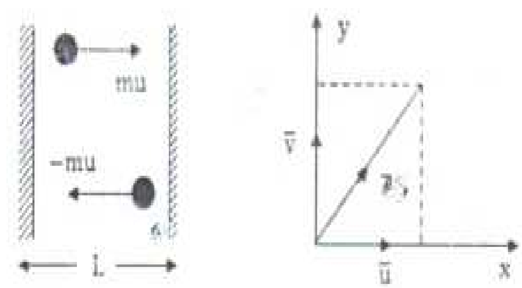
Reason

Since the gas of molecules travelled in equal distance in all direction at the same time interval. Then the velocities i.e mean square velocity possessed by the gas molecules is same in all direction, so that forces and pressure exerted by the gas in all direction are the same i.e $P_X = P_y = P_Z$

2. For the two dimensional motion of the gas molecules, pressure exerted by the gas molecules is given by $P = \dfrac{1}{2}\rho\bar{c}^2$

DERIVATION OF $P = \dfrac{1}{2}\rho\bar{c}^2$

Consider a motion of gas molecules as shown in the figure below.



Change of the momentum of gas molecules = mu −(mu) = 2mu

Rate of change of momentum

$= \dfrac{2mu}{t}$ but $t = \dfrac{2L}{u}$

$= \dfrac{2mu}{2L/u} = \dfrac{mu^2}{L}$

$\text{Pressure} = \dfrac{mu^2}{L^3}$

Total pressure in x – direction

$P = \dfrac{mu^2}{L^3}\left[u_1^2 + u_2^2 + \dots\dots\dots + u_N^2\right]$

But $N\bar{u}^2 = u_1^2 + u_2^2 + \dots\dots\dots + u_N^2$

Since $\bar{c}^2 = \bar{u}^2 + \bar{v}^2 = 2\bar{u}^2 = 2\bar{v}^2$

$\bar{u}^2 = \dfrac{1}{2}\bar{c}^2$

$P = \dfrac{1}{2}\rho\bar{c}^2$

DEFINITION MEAN VELOCITY OF GAS MOLECULES

Is defined as the arithmetical mean of the velocity of the gas molecules.

Let $C_1, C_2, C_3 \ldots\ldots C_N$ be the velocities of N gas molecules.

$$C = \frac{C_1 + C_2 + C_3 ------ C_N}{N}$$

From Maxwell Boltzmann statistics it can be obtained that

$$\bar{C} = \sqrt{\frac{8KT}{\pi m}} = \sqrt{\frac{8RT}{Mr}}$$

When m = mass of each gas molecules

K = Boltzman constant

T = Absolute temperature

R = Universal gas constant

$$m = \frac{\text{molecular weight}}{\text{Avogadro's number}} = \frac{Mr}{N_A}$$

$$K = 1.38\times10^{-23}\,\text{Jmolecule}^{-1}K^{-1}$$

Mean square velocity of gas molecule

Is defined as the average of the square of the velocities gas molecules

$$\bar{C}^2 = \frac{C_1^2 + C_2^2 + C_3 ------ C_N}{N}$$

ROOT MEAN SQUARE VELOCITY OF GAS MOLECULES

Is defined as the square root of the mean of the squares of the velocities of the gas molecules it is denoted by $C_{r.m.s}$ or C_r mathematical

$$C_{r.m.s} = \sqrt{\bar{C}^2} = \sqrt{\frac{C_1^2 + C_2^2 + ------ \mp C_N^2}{N}}$$

DIFFERENT FORMS OF AN EXPRESSION OF THE ROOT MEAN SQUARE VELOCITY.

(i) We know that $P = \dfrac{M\bar{C}^2}{3V}$

$$C_{r.m.s} = \sqrt{\frac{3PV}{M}} \quad M = Nm$$

(ii) Again $P = \dfrac{1}{2}\rho\bar{c}^2$

$$C_{r.m.s} = \sqrt{\frac{3P}{\rho}}$$

(iii) For an ideal gas

$$PV = nRT = \frac{N}{N_A}RT$$

$$PV = NKT \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

From the basis of kinetic theory of gases

$$PV = \frac{1}{3}Nm\bar{c}^2$$

$$PV = \frac{2}{3}N\left(\frac{1}{2}m\bar{c}^2\right)\ldots\ldots\ldots(2)$$

(1) = (2)

$$NKT = \frac{2}{3}N\left(\frac{1}{2}m\bar{c}^2\right)$$

$$\bar{C}^2 = \frac{3KT}{m}$$

$$C_{r.m.s} = \sqrt{\bar{C}^2} = \sqrt{\frac{3KT}{m}}$$

(iv) Also for an ideal gas

$$PV = nRT = \frac{m}{Mr}RT$$

$$PV = \left(\frac{m}{v}\right)\frac{RT}{Mr} = \frac{\rho RT}{Mr}$$

$$\frac{P}{\rho} = \frac{RT}{Mr}$$

Since $C_{r.m.s} = \sqrt{\dfrac{3P}{\rho}} = \sqrt{\dfrac{3RT}{Mr}}$

$$C_{r.m.s} = \sqrt{\frac{3RT}{Mr}}$$

- The r.m.s velocity of the molecule of a gas is directly proportional to the square root of the absolute temperature of the gas.

  Let V_1 and V_2 be the root mean square velocities of the gas at temperature θ_1 and θ_2 respectively

$$V \propto \sqrt{\theta} \;,\; V = K\sqrt{\theta_2}$$

$$V_1 = K\sqrt{\theta_1} \;,\; V_2 = K\sqrt{\theta_2}$$

$$\frac{V_2}{V_1} = \frac{K\sqrt{\theta_2}}{K\sqrt{\theta_1}} = \sqrt{\frac{\theta_2}{\theta_1}}$$

$$\frac{V_2}{V_1} = \frac{\sqrt{\theta_2}}{\sqrt{\theta_1}}$$

- The r.m.s speed of the molecules of different gases at the same temperature is inversely proportions to the square root of the molecular mass. So gases of higher molecular mass have smaller root mean square speed. As an illustration the r.m.s speed of hydrogen molecules is four time the r.m.s speed of oxygen molecules at the same temperature. Due to large values of r.m.s velocities, the effect of gravity on the molecular motion is not considered.

MOST PROBABLE VELOCITY OF GAS MOLECULES

Is the velocity which is possessed by maximum number of molecules in a gas sample it is denoted by C_P or C_{mp}.

Three different formulae of most probable speed are as under.

1. $C_p = \sqrt{\frac{2KT}{m}}$ (For 1 molecules of gas)
2. $C_p = \sqrt{\frac{2RT}{Mr}}$ (For 1 mole of gas)
3. $C_p = \sqrt{\frac{2PV}{Mr}}$ (For 1 mole of gas)

TYPES OF GAS CONSTANT

There are two types of gas constant

1. Molar gas constant (universal gas constant)
2. Gas constant for a unit fixed mass of a gas, r (specific gas constant).

UNIVERSAL GAS CONSTANT

$$R = \frac{\text{Pressure} \times \text{Volume}}{\text{Number of moles} \times \text{Temperature}}$$

For n moles of a perfect gas

$$R = \frac{PV}{nT} = \frac{PV}{\mu T}$$

For 1 mole of a perfect gas

$$PV = RT$$

$$R = \frac{PV}{T}$$

Unit of R

$$R = \frac{PV}{\mu T} = \frac{Nm^{-2} \times m^3}{mol \times K} = Jmol^{-1}K^{-1} \quad \text{OR}$$

$$R = \frac{1atm \times Litre}{mol \times K} = atm\ Lmol^{-1}K^{-1}$$

$\therefore$ S.I Unit of R is $Jmol^{-1}K^{-1}$ or $atmLmol^{-1}K^{-1}$

Numerical value of R

$$R = \frac{1.013 \times 10^5 \times 22.4 \times 10^3}{273}$$

$$R = 831Jmol^{-1}K^{-1} \ \textbf{or}$$

$$R = \frac{1atm \times 22.4L}{1mol \times 273}$$

$$R = 0.082\ atm\ Lmol^{-1}K^{-1}$$

Note that

1. Here R is called the universal gas constant because of the value of R is same for all gases.
2. $R = \frac{PV}{nT} = \frac{\text{Work done}}{\text{Number of moles} \times \text{Temperature}}$

   Physical significance: it's signifies the work done by a gas or on a gas per Kelvin.

SPECIFIC GAS CONSTANT

Let m and Mr be the mass and molar mass of the gas respectively.

Since $PV = nRT = \frac{m}{Mr} \cdot RT$

$$P = \left(\frac{m}{v}\right)\left(\frac{R}{Mr}\right)T$$

But $\frac{m}{v} = \rho$, $\frac{R}{Mr} = r$ or $P = \rho rT$

$$r = \frac{R}{Mr} = \frac{P}{\rho T}$$

SPECIFIC GAS CONSTANT

Is defined as universal gas constant per unit molar mass of the gas. Since the value of mass is different for the different gases. Therefore the value of specific gas constant is different for the different gases.

$\therefore$ S.I. Unit of r is JKg-1K-1

For 1gm of a gas $PV = rT$ or $r = \frac{PV}{T}$

Examples of specific gas constant

Gas	Molar mass (kg)	Molecular mass	$r = JKg^{-1}K^{-1}$
Hydrogen	2×10^{-3}	2	4156
Helium	4×10^{-3}	4	2077
Nitrogen	28×10^{-3}	28	297
Oxygen	32×10^{-3}	32	259.7
Argon	40×10^{-3}	40	207.9

BOLTZMANN'S CONSTANT K

Is defined as the ratio of universal gas constant to the Avogadro's constant

$$K = \frac{R}{N_A} = \frac{8.3LJmol^{-1}K^{-1}}{6.02\times10^{-23}mol^{-1}}$$

$$K = 1.38\times10^{-23}JK^{-1}$$

Expression of the number of the molecules of gas

For an ideal gas of n – moles

$$PV = nRT = \frac{N}{N_A}RT$$

$$PV = NKT$$

$$N = \frac{PV}{KT}$$

SIGNIFICANCE OF AVOGADRO'S NUMBER

$$N_A = \frac{\text{Universal gas constant}}{\text{Boltzman's constant}}$$

$$N_A = \frac{R}{K}$$

ONE MOLE of any substance is the amount of substance equal to the atomic mass or molecular mass in grams and contains N_A (6.02×10^{-23}) atoms or molecules.

The importance of Avogadro's number (mole concept) is clear from the following applications:-

1. It helps to calculate the actual mass of one atom of an element mass of one atom of an element $= \dfrac{\text{Gram atomic mass}}{\text{Avogadro's number}}$
2. It help to calculate the mass of one molecules of a substance.
   Mass of one molecule of substance $= \dfrac{\text{Gram atomic mass}}{\text{Avogadro's number}}$
3. It help to calculate the number O atoms in a given mass of an element.
   Suppose an element has a mass of X gram. Then the number molecules in X grams of the substance $= \dfrac{6.02\times10^{23}}{\text{Gram molecular mass}}\times X$
4. It helps to calculate the number of molecule present in a given volume of a gas at S.T.P
   Number of molecules in X litres of a gas at S.T.P $= \dfrac{6.02\times10^{23}}{22.4}\times X$

KINETIC ENERGY AND TEMPERATURE

(i) **Expression of average kinetic energy of a gas molecules.**

For an ideal gas equation

$$PV = nRT$$

$$PV = \frac{N}{N_A}\cdot RT \text{ ....(i)}$$

From the basis of kinetic theory of gases

$$PV = \frac{1}{3}Nm\bar{c}^2$$

(i) = (ii)

$$\frac{1}{3}Nm\bar{c^2} = \frac{N}{N_A}RT$$

$$\frac{2}{3}\left(\frac{1}{2}Nm\bar{c^2}\right) = \left(\frac{R}{N_A}\right)T$$

$$K.\bar{e} = \frac{1}{3}Nm\bar{c^2} = \frac{3}{2}\left(\frac{R}{N_A}\right)T = \frac{3}{2}KT$$

∴ Average kinetic energy of a gas molecule is directly proportional to the absolute temperature of the gas i.e $K.\bar{e} \propto T$

(ii) **Expression of average kinetic energy of one mole of a gas (molar kinetic energy of gas molecules).**

The number of molecules contained in one mole of a gas is equal to the Avogadro's number.

Average k.e of one mole of a gas

$$K.E = N_A \times K.\bar{e}$$

But $\quad k.\bar{e} = \dfrac{3}{2}\left(\dfrac{R}{N_A}\right)T$

$$K.E = \frac{1}{2}M r\bar{c}^2 = N_A \times \frac{3}{2}\left(\frac{R}{N_A}\right)T$$

$$K.E = \frac{1}{2}M r\bar{c}^2 = \frac{3}{2}RT$$

Kinetic interpretation of temperature

Since $\frac{1}{2}m\bar{c}^2 = \frac{3}{2}KT$

$$k.\bar{e} = \frac{3}{2}KT$$

$$T = \frac{3k.\bar{e}}{3k} = \frac{2k.\bar{e}}{3\left(R/N_A\right)}$$

Also $C_{r.m.s} \propto \sqrt{T}$

When heat is supplied to a gas its temperature increases and the root mean square velocity of the molecules also increases. The interpretation of the temperature in terms of the velocities of the gas molecules is called **'kinetic interpretation of the temperature'.**

NUMERICAL EXAMPLES

1. (a) Define the temperature of an ideal gas as a consequence of the kinetic theory.
   (b) A flexible container of oxygen has a volume of 10cm³. Find the mass of gas enclosed. Given that at S.T.P, $P = 1.01 \times 10^5 \times Nm^{-2}$, $T = 273K$, $V = 22.4 \times 10^{-3}m^3$.

Solution

(a) From the basis of kinetic theory of the gas

$$\frac{1}{2}m\bar{c}^2 = \frac{3}{2}KT$$

$$T = \frac{m\bar{c}^2}{3k} = \frac{2k.\bar{e}}{3k}$$

$k.\bar{e}$ = Average k.e of gas molecule
k = Boltzaman's constant
$\bar{c}^2$ = Mean square velocity
m = mass of each gas molecules

(b) Number of moles of gas

$$\frac{M}{Mr} = \frac{V}{V_{S.T.P}}$$

$$M = Mr\left[\frac{V}{V_{S.T.P}}\right] = 0.032\left[\frac{10}{22.4\times10^{-3}}\right]$$

$$m = 14.3kg$$

2. (a) Find the average kinetic energy of gas molecules at a temperature of 27°C.
   (c) Find the root mean square speed of hydrogen molecules at 27°C. (Mass of the proton = 1.674×10^{-27}kg,) K = 1.38 × 10-23JK⁻¹.

Solution

(a) $$k.\bar{e} = \frac{3}{2}KT$$

$$= \frac{3}{2}\times1.38\times10^{-23}\times300$$

$$k.\bar{e} = 6.21\times10^{-23}J$$

(b) $$G_{r.m.s} = \sqrt{\frac{3KT}{m}} = \sqrt{\frac{3\times300\times1.38\times10^{-23}}{2\times1.674\times10^{-27}}}$$

$$G_{r.m.s} = 1926m/s$$

Alternatively

$$G_{r.m.s} = \sqrt{\frac{3KT}{m}} = \sqrt{\frac{3\times8.31\times300}{2\times1,008\times10^{-3}}}$$

3. Helium gas occupies a volume of 0.04m³ at pressure of 2.0×10^5Pa and temperature of 300K calculate.
   (i) The mass of helium
   (ii) The r.m.s speed of its molecules
   (iii) The r.m.s speed at 432K when the gas is heated at constant pressure to this temperature.
   (iv) r.m.s speed hydrogen molecules at 432K (Relative molecular masses at helium and hydrogen are 4 and 2 respectively and molar gas constant, R = 8.3Jmol⁻¹ K⁻¹).

Solution

(i) Let M = Mass of Helium

Assume that Helium is an ideal gas

$$PV = nRT \text{ but } n = \frac{M}{Mr}$$

$$PV = \frac{M}{Mr}RT$$

$$m = \frac{PVMr}{RT}$$

$$= \frac{2.0\times10^5\times0.04\times0.04}{8.3\times300}$$

$$m = 12.8\times10^{-3}Kg = 12.8gm$$

(ii) $G_{r.m.s} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3PV}{M}} \left(\rho = \frac{m}{v}\right)$

$= \sqrt{\frac{3\times2\times10^{5}\times0.04}{12.8\times10^{-3}}}$

(iii) Since $G_{r.m.s} \ \alpha \ \sqrt{T}$

$\frac{C_{r1}}{C_{r}} = \sqrt{\frac{T}{T_{1}}}$

$C_r = 1369m/s$

$T = 300$

$T_1 = 432$

$C_{r1} = C_r\sqrt{\frac{T}{T_1}}$

$= 1369\sqrt{\frac{432}{300}}$

$C_{r1} = 1643m/s$

(iv) $C_r \ \alpha \ \frac{1}{\sqrt{Mr}}$

$C_r \text{ of } H_2 = C_r \text{ of } He\sqrt{\frac{MrHe}{MrH_2}}$

$= 1643\sqrt{\frac{4}{3}}$

$C_r \text{ of } H_2 = 2324m/s$

4. A cylinder of volume $2 \times 10^{-3}m^3$ contains a gas at a pressure of $1.5MNm^{-2}$ and at temperature of 300K Calculate.
   (i) The number of moles of the gas
   (ii) The number of molecules of the gas
   (iii) The mass of the gas if its molar mass is $32 \times 10^{-3}kg$.
   (iv) The mass of one molecule of the gas
   ($R = 8.31Jmol^{-1}K^{-1}$, $NA = 6.02 \times 10^{23}mol^{-1}$)

Solution

(i) Assume that the gas is an ideal

$PV = nRT$

$n = \frac{PV}{RT} = \frac{1.5\times10^{6}\times10^{-3}}{8.31\times300}$

$n = 1.2moles$

(ii) $1mole \rightarrow 6.02 \times 10^{23}$ molecules

$1.2 \text{ moles} \rightarrow N$

$N = 1.2 \times 6.02 \times 10^{23}$

$N = 7.2 \times 10^{23}$ molecules.

(iii) $N = \frac{M}{Mr}$

$M = nMr = 1.2\times1.2\times32\times10^{-31}$

$M = 38.4\times10^{-3}Kg = 38.4gm$

(iv) $32\times10^{-3}kg \rightarrow 6.02\times10^{23}$ molecules

$x \rightarrow$ molecules

$x = \frac{32\times10^{-3}}{6.02\times10^{23}}$

$x = 5.3\times10^{-26}kg$

5. (a) Write down four assumptions about properties of molecules that are made in the kinetic theory in order to define on ideal gas. On the basis of this theory derive an expression for the pressure by an ideal gas.
   (b) Air at 273K and $1.01 \times 10^{25}Nm^{-2}$ pressure contains 2.70×10^{25} molecules per cubic metre. How many molecules per cubic metre will there be at a place where the temperature is 223K and the pressure $1.33 \times 10^{-4}Nm^{-2}$ (Nelkon 5th edition qn. 7).

Solution

(a) See your notes

(b) $T_1 = 273k$, $P_1 = 1.01\times10^{5}Nm^{-2}$

$n_1 = 2.70\times10^{25}$ molecules

$T_2 = 223K$, $P_2 = 1.33\times10^{-4}Nm^{-2}$

$n_2 = ?$

From the basis of kinetic theory of gas

$PV = \frac{1}{3}Nm\overline{c}^2$

$P = \frac{1}{3}\left(\frac{N}{V}\right)m\overline{c}^2 = \frac{1}{3}Nm\overline{c}^2$

$P = \frac{2}{3}n\left(\frac{1}{2}m\overline{c}^2\right)$

But $\frac{1}{2}m\overline{c}^2 = \frac{3}{2}KT$

$P = \frac{2}{3}n\left(\frac{3}{2}KT\right) = nKT$

$P_1 = n_1 \ , \ KT \ , \ P_2 = n_2KT_2$

$$\frac{P_2}{P_1} = \frac{n_2KT_2}{n_1KT_1}$$

$$n_2 = n_1\left[\frac{P_2}{P_1}\right]\left[\frac{T_1}{T_2}\right]$$

$$= 2.70\times10^{25}\left[\frac{1.33\times10^{-4}}{1.01\times10^{5}}\right]\left[\frac{273}{223}\right]$$

$$n_2 = 4.35263\times10^{16}\,m^{-3}$$

6. Nine particle have speed of 5, 8, 12, 12, 12, 14, 14, 17 and 20m/s. Find
   (i) The average speed
   (ii) The r.m.s speed
   (iii) The most probable speed of the particle.

Solution

(i) Average

$$\bar{C} = \frac{C_1+C_2+\text{--------}+C_n}{n}$$

$$\bar{C} = \frac{5+8+12+12+12+14+14+17+20}{9}$$

$$\bar{C} = 12.7m/s$$

(ii) $$C_{r.m.s} = \sqrt{\frac{C_1^2+C_2^2\text{--------}+C_n^2}{n}}$$

$$= \sqrt{\frac{5^2+8^2+12^2+12^2+12^2+14^2+14^2+17^2+20^2}{9}}$$

(iii) Three particle have speed of 12m/s and two particle have speed of 14m/s

∴ The most probable speed = 12m/s

7. At certain time the speed of seven particles are as follows:
   Calculate the root mean square of the particles

Solution

$$C_{r.m.s} = \sqrt{\frac{2^2+3+3^2+4^2+5^2+6^2}{7}}$$

$$C_{r.m.s} = 3.927922m/s$$

8. (a) What is the absolute temperature of a gas moving at speed (r.m.s) of 500m/s, if the average mass of the molecules is 8.0×10^{-26}kg.

(c) A flask of $10^{-3}m^3$ contain hydrogen gas at a pressure of 10^{-3} mmHg and a temperature of 27^{O}C. Calculate the
   (i) Root mean square speed of the molecules.
   (ii) Number of molecules present in the flask.
   (iii) Number of impacts per second per unit area on the wall of flask ($R = 8.31Jmol^{-1}K^{-1}$, $N_A = 6.02\times10^{23}mol^{-1}$).

Solution

(a) $$\frac{1}{2}m\bar{c}^2 = \frac{3}{2}\left(\frac{R}{N_A}\right)T$$

$$T = \frac{mcr^2}{3\left(\frac{R}{N_A}\right)} \quad \text{But } C_{r.m.s} = \sqrt{\bar{c}^2} \text{ or } Cr^2 = \bar{c}^2$$

$$T = \frac{8\times10^{-26}\times500^2}{3\times1.38\times10^{-23}}$$

$$T = 483K$$

(b) $$760mmHg \rightarrow 1.01\times10^5Nm^{-2}$$

$$10^{-3}mmHg \rightarrow P$$

$$P = \frac{1.01\times10^5\times10^{-3}Nm^{-2}}{760}$$

(i) Since $$C_{r.m.s} = \sqrt{\frac{3P}{\rho}}$$

Given the density of hydrogen = $0.09kgm^{-3}$

$$C_{r.m.s} = \sqrt{\frac{3\times1.01\times10^5\times10^3}{760\times0.09}}$$

$$C_{r.m.s} = 2.10m/s$$

(ii) Number of molecules present, N

Since $$n = \frac{N}{N_A} = \frac{PV}{RT}$$

$$N = \frac{N_APV}{RT}$$

$$= \frac{6.02\times10^{23}\times0.1329\times10^{-3}}{8.31\times300}$$

$$N = 3.215\times10^{16}\text{ molecules}$$

(iii) Number of impact per second per unit area.

$$= \frac{\text{change in momentum}}{\text{time} \times \text{area}}$$

$$= \frac{2mc}{\left(\frac{2L}{C}\right)A} = \frac{mc^2}{AL} = \frac{mc^2}{V}$$

But $\bar{c}^2 = \frac{c_1^2 + c_2^2 + ------ c_N^2}{N}$

Now: no impact per second per unit area.

$$= \frac{Nmc^2}{NV}$$

$$= \frac{Nm}{V}\left[\frac{c_1^2 + c_2^2 + -------- + c_N^2}{N}\right]$$

$$= \frac{Nm\bar{c}^2}{V} = N\rho\bar{c}^2$$

$$= 3.215\times10^{16}\times0.09\times(2.1)^2$$

$$= 1.2862\times10^{16}\,Nm^{-2}$$

9. (a) On the basis of the kinetic theory of gases, show that the different gases at the same temperature has the same average value of kinetic average of the molecules.

(b) Determine the r.m.s speed of air molecules at S.T.P, given that the density of mercury is $1.29kgm^{-3}$ density of mercury is $13600kgm^{-3}$ and barometer height at S.T.P is 760mmHg.

Solution

(a) Since $PV = \frac{1}{3}Nm\bar{c}^2$

$$PV = nRT = \frac{N}{N_A}RT$$

$$\frac{N}{N_A}RT = \frac{2}{3}N\left(\frac{1}{2}m\bar{c}^2\right)$$

$$\frac{3}{2}KT = k.\bar{e}$$

Gas 1: $k.\bar{e} = \frac{3}{2}KT_1$

Gas 2: $k.\bar{e} = \frac{3}{2}KT_2$

$\therefore$ At the same temperature, two different gases will have the same average of kinetic energy of molecules.

(b) $P = \rho mgh$

Since $C_{r.m.s} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3\rho mgh}{\rho}}$

$$= \sqrt{\frac{3\times13600\times9.8\times0.76}{1.29}}$$

$$C_{r.m.s} = 489.4m/s$$

10. A cylinder containing 19g of compressed air at a pressure 9.5 times that of the atmosphere is kept in a store at 7ºC. When it is moved to a workshop where the temperature is 27ºC a safety valve on the cylinder operates. Releasing some of air if the valve allows air to escape when its pressure exceeds 10times that of atmosphere. Calculate the mass of air that escapes.

Solution

Let P_0 = atmospheric pressure

V = Volume of the cylinder

For an ideal gas: $PV = nRT$, $n = \frac{PV}{RT}$

Initial number of moles at temperature (70ºC)

$P_1 = 9.5P_0 T_1 = 7+273 = 280$

$$n_1 = \frac{P_1V}{RT_1} = \frac{9.5P_0V}{280R} ....(i)$$

Final number of moles of air at temperature of (27ºC).

$P_2 = 10P_0$, $T_2 = 27+273 = 300K$

$$n_2 = \frac{P_2V}{RT_2} = \frac{10P_0}{300R} ....(ii)$$

Dividing equation (2) by (1)

$$\frac{n_2}{n_1} = \frac{\frac{10P_0V}{300R}}{\frac{10P_0V}{280R}}$$

$$\frac{n_1}{n_2} = \frac{10\times280}{9.5\times300} = 0.982456$$

Let M = mass of air left in the cylinder.

Since $n = \frac{M}{Mr}(n \;\alpha\; m)$

$$\frac{M}{19} = \frac{n_2}{n_1} = 0.982456$$

$$M = 19\times0.982456$$

$$M = 18.67Kg(approx)$$

Mass of air escaped

$$Me = 19-18.67$$

$$Me = 0.33Kg$$

11. A mole of an Ideal gas at 300K is subjected to pressure of 10^5P_2 and its volume is $0.025m^3$. Calculate.
(a) Molar gases constant R
(b) The Boltzmann's translational
(c) The average transilational kinetic energy of a molecule of the gas ($N_A = 6.0 \times 10^{23}mol^{-1}$).

Solution

(a) For an ideal gas equation

$$PV = nRT$$

$$R = \frac{PV}{nT} = \frac{10^5\times0.025}{1\times300}$$

$$R = 8.33Jmol^{-1}K^{-1}$$

(b) $$K = \frac{R}{N_A} = \frac{8.33}{6.0\times10^{23}}$$

$$K = 1.388\times10^{-23}JK^{-1}$$

(c) $$k.\bar{e} = \frac{3}{2}KT$$

$$= \frac{3}{2}\times1.388\times10^{-23}\times300$$

$$k.\bar{e} = 6.25\times10^{-21}J$$

12. (a) Equation relating pressure P volume V and thermodynamics temperature T of an ideal gas is PV = nRT. Identify the term n and R

(b) Nitrogen gas under an initial pressure of 5.0×10^6Pa at 15^OC is contained in a cylinder of volume $0.040m^3$. After a period of three years the pressure has fallen to 2.0×10^4Pa at the same temperature because of leakage. (Assume molar mass of nitrogen = $0.028Kgmol^{-1}$, R = $8.31Jmol^{-1}K^{-1}$ Avogadro's constant = $6.0 \times 10^{23}mol^{-1}$).
Calculate
(i) The mass of gas originally pressure in the cylinder.
(ii) The mass of gas which escaped from the cylinder in three years.
(iii) The average number of nitrogen molecules which escape from the cylinder per second.
(Take 1 year = 3.2×10^7sec).

Solution

(a) Refer to your notes

(b) Given that

$P0 = 5 \times 10^6P_a$, $T_0 = 298K$, $V = 0.04m^3$

$t = 3year$, $P_1 = 2\times 10^6P_a$

(i) Let M_0 = Mass of gas

For an Ideal gas

$$PV = nRT = \frac{M}{Mr}RT$$

$$M_0 = \frac{P_0VMr}{RT_0}$$

$$= \frac{5\times10^6\times0.04\times0.028}{8.3\times298}$$

$$M_0 = 2.3kg(approx)$$

(ii) Obtained first the mass of gas present after 3 years

$$M_1 = \frac{P_1VMr}{RT}$$

$$= \frac{2\times10^6\times0.04\times0.028}{8.3\times298}$$

$$M_1 = 0.904Kg$$

The mass of air escaped

$$\Delta M = M_0 - M_1$$

$$= 2.3-0.904$$

$$\Delta M = 1.396Kg \simeq 1.4Kg$$

(iii) $$n = \frac{\Delta M}{Mr}N_A$$

Let β = Number of molecules escaped per unit time.

$$\beta = \frac{n}{t} = \frac{\Delta MN_A}{Mrt}$$

$$\beta = \frac{1.4\times6\times10^{23}}{0.028\times3\times3.2\times10^7}$$

$$\beta = 3.2\times10^{17}s^{-1}$$

13. A flasks with a volume of 1.5 provided with a stop – cork contains ethane gas at 300K and atmospheric pressure. The molar mass of ethane is $30.1gmol^{-1}$ and the system is warmed to a temperature of 490k, with the stop cork open to the atmosphere. The stop cork is then closed, and the flask is cooled to its original temperature.
   (a) What is the final temperature of the ethane in the flask?
   (b) How many grams of ethane remain in the flask?

Solution

$V_1 = 1.5L$, $T_1 = 300K$, $P_1 = 1.01 \times 10^5Pa$

$Mr = 30.1gmol^{-1} = 30.1 \times 10^{-3}kgmol^{-1}$

$T_2 = 490K$

(a) The volume of the container remained constant, and eh pressure of the container changes but its initial value is equal to the atmospheric pressure. So when the temperature is raised some of the molecules escaped into air, then let us find the number of moles which remains in the flasks.

$$PV = nRT$$

$$n = PV/RT$$

$$\text{Let: } \frac{PV}{R} = \text{Constant}, n = \frac{K}{T}$$

$$\frac{n_2}{n_1} = \frac{T_1}{T_2}, \quad n_2 = n_1\left(\frac{T_1}{T_2}\right)$$

Then the gas is cooled to its original temperature.

$$P_0V_0 = n_1RT$$

$$P_1V_0 = n_2RT$$

$$\text{Take } \frac{P_1V_0}{P_0V_0} = \frac{n_2RT}{n_1RT}$$

$$\frac{P_1}{P_0} = \frac{n_2}{n_1} = \frac{T_1}{T_2}$$

$$P_1 = P_0\left[\frac{T_1}{T_2}\right] = 1.01\times10^5\left[\frac{300}{490}\right]$$

$$P_1 = 61.84\times10^3P_a = 61.84kPa$$

(b) The mass which remain in the flask

$$n_2 = n_1\left(T_1/T_2\right)$$

Since $m \alpha n$

$$\frac{m_2}{m_1} = \frac{n_2}{n_1}$$

$$\frac{m_2}{m_1} = \frac{T_2}{T_1}, \quad m_2 = m_1\left[\frac{T_1}{T_2}\right]$$

$$\text{Since } PV = nRT = \frac{m}{Mr}RT$$

$$P_0V_0 = \frac{m_1}{Mr}\cdot RT$$

$$M_1 = \frac{P_0V_0Mr}{RT_1}$$

$$M_2 = \frac{P_0V_0Mr}{RT_1}\left[\frac{T_1}{T_2}\right] = \frac{P_0V_0Mr}{RT_2}$$

$$M_2 = \frac{1.01\times10^5\times1.5\times10^{-3}\times30.1\times10^{-3}}{8.314\times490}$$

$$M_2 = 1.12\times10^{-3}Kg = 1.12gm$$

14. (a) (i) Define an ideal gas
   (ii) State four (4) assumption necessary for an ideal gas that are used develop the expression $P = \frac{1}{2}\delta\bar{C^2}$
   (iii) How is pressure explained in terms of the kinetic theory?
   (b) (i) without a detailed mathematical analysis argue the steps to follows in the deriving the relation $P = \frac{1}{2}\rho\bar{c}^2$
   (c) A mole of an ideal gas at 300K is subjected to a pressure $105Nm^{-2}$ and its volume is $2.5 \times 10^{-2}m^3$. Calculate the
   (i) Molar gas constant, R
   (ii) Boltzmann constant, K
   (iii) Average translational kinetic energy of molecules of the gas.

Solution

(a) (i) and
   (ii) See your notes
   (iii) Pressure of a gas is due to the random motion of the gas molecules and the collisions between the gas molecules with the wall of the container.

(b) (i) see your notes

(ii) Temperature of an ideal gas as consequence of kinetic theory of gas is defined by the equation

$$T = \frac{2k.\bar{e}}{3\left(R/N_A\right)} = \frac{2k.\bar{e}}{3k}$$

If A is the rate of the absorber then, initially the mass of molecules striking it per second $\frac{M}{t} = R_1 = \frac{nm\bar{c}A}{V}$

n = number of diatomic molecules each of velocity

$\bar{c}$ = Mean velocity

Doubling absolute of the vessel. In this case the number of diatomic molecules in the vessel becomes $n/2$ and their mean velocity change to $\bar{c}_1$

∴ Mass per second reading A

$$\frac{M}{t} = R_2 = \frac{nm\bar{c}_2A}{2V} \;.......(1)$$

The number of atoms produced in n and each has a mass $m/2$

$$\frac{M}{t} = R_2 = \frac{nm\bar{c}_2A}{2V} \;.......(2)$$

$\bar{c}_2$ = Mean velocity of the atoms. Now each atom must have the same energy as the diatomic molecule after dissociation occurs, otherwise there would be an interchange of energy in the mixture.

K.E is proportional to the absolute temperature [2K.E → 2T]

$$\frac{1}{2}\left(\frac{m}{2}\right)\bar{c}_2^2 = \frac{1}{2}m\bar{c}_1^2$$

Doubling the temperature double the kinetic energy of the diatomic molecules.

$$\bar{c}_1^2 = 2\bar{c}^2 \text{ or } \bar{c}_1 = \sqrt{2}\bar{c}$$

$$\bar{c}_2 = \sqrt{2}\bar{c}_1 = \sqrt{2}\cdot\sqrt{2}\bar{c} = 2\bar{c}$$

From (1) and (2) the rate at which A is gaining mass.

$$R = R_2 + R_3$$

$$= \frac{nmA}{2V}\left(\bar{C}_1 + \bar{C}_2\right)$$

$$= \frac{nmA}{2V}\left[2\bar{c} + \sqrt{2}\bar{c}\right]$$

$$R = \frac{nm\bar{C}A}{2V}\left[2+\sqrt{2}\right]$$

Relative increase in rate

$$\frac{R}{R_1} = \frac{\frac{nm\bar{C}A}{2V}\left[2+\sqrt{2}\right]}{nm\bar{C}A/V}$$

$$= \frac{1}{2} = \left[2+\sqrt{2}\right]$$

$$= 1+\frac{\sqrt{2}}{2} = 1+\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{\sqrt{2}}$$

$$\frac{R}{R_1} = 1+\frac{1}{\sqrt{2}}$$

(c) See the solution of example.

15. Calculate the r.m.s velocity of oxygen molecules at 57°C. (Density of oxygen at S.T.P 1.424Kgm^{-3}) P = 1.01 × 10^{5}Nm^{-2}.

Solution

Root means square velocity at S.T.P

$$C_r = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3\times1.01\times10^5}{1.424}}$$

$$C_r = 461.28\text{m/s}$$

Since $C_r \propto \sqrt{T}$, $C_r = K\sqrt{T}$

$$C_{r1} = K\sqrt{T_1}$$

$$\frac{C_{r1}}{C_r} = \frac{K\sqrt{T_1}}{K\sqrt{T}} = \sqrt{\frac{T_1}{T}}$$

$$C_{r1} = C_r\sqrt{\frac{T_1}{T}} = 461.282\sqrt{\frac{330}{273}}$$

$$C_r = 507.16\text{m/s}$$

∴ R.M.S velocity of oxygen at 57°C , Cr1 = 507.16m/s.

GAS LAWS

The gas laws involving the following laws;

(i) Boyles's law
(ii) Charles's law
(iii) Pressure law
(iv) Avogadro's law
(v) Dalton law of partial pressure
(vi) Graham's law of diffusion

I. BOYLE'S LAW

State that 'for a given fixed mass of a gas, volume of a gas is inversely proportional to pressure applied on it, if temperature of the gas is kept constant.

Mathematically

$$V \,\alpha\, \frac{1}{P} \;\text{ or }\; P \,\alpha\, \frac{1}{V}$$

At a constant temperature

$$P = \frac{\text{Constant}}{V}$$

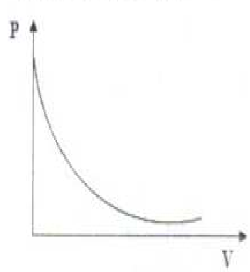
$$PV = \text{Constant}$$

The law means that if the pressure of a given mass of a gas is double, the volume of a as is reduced to half of its original value when the temperature is kept constant.

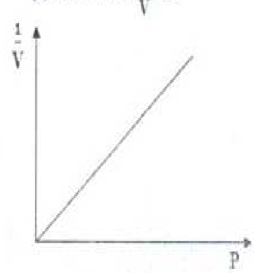
Let P_1 and P_2 be the pressure of the gas when the volume of the gas are V_1 and V_2 respectively.

$$P_1V_1 = P_2V_2 = \text{Constant}$$

(i) GRAPH OF P Vs V



(ii) GRAPH OF $\frac{1}{V}$ Vs P



The graph shows that with the increase in pressure, there is a decreases in the volume of a gas at constant temperature.

NOTE

(a) Boyle's law is perfectly obeyed at high temperature and low pressure
(b) Derivation of Boyle's law from basis of kinetic theory of gases.

From the basis of the kinetic theory of gases.

$$PV = \frac{1}{3}Nm\bar{c}^2$$

$$Nm = M = \text{total mass of the gas}$$

$$PV = \frac{1}{3}M\bar{c}^2$$

$$PV = \frac{2}{3}\left(\frac{1}{2}M\bar{c}^2\right) \text{ but } \frac{1}{2}M\bar{c}^2 = \frac{3}{2}KT$$

$$PV = KT \quad T = \text{Constant}$$

$$PV = \text{Constant} \text{ or } P \,\alpha\, \frac{1}{V}$$

This is statements of Boyle's law.

II. CHARLE'S LAW

State that 'for the given fixed mass of the gas volume is directly proportional to its absolute temperature, if the pressure of the gas remain constant' i.e $V \,\alpha\, T$

$$\frac{V}{T} = \text{Constant}$$

If a given mass of a gas has a volume V_1 at temperature θ_1 °C constant pressure to the temperature θ_2 °C, its new volume is V_2.

$$\frac{V_2}{T_2} = \frac{V_1}{T_2}$$

$$\frac{V_2}{V_1} = \frac{273.15+\theta_2}{273.15+\theta_1} = \frac{T_2}{T_1}$$

$$\frac{V_2}{V_1} = \frac{T_2}{T_1} \text{ or } V_2 = V_1\left[\frac{T_2}{T_1}\right]$$

Let

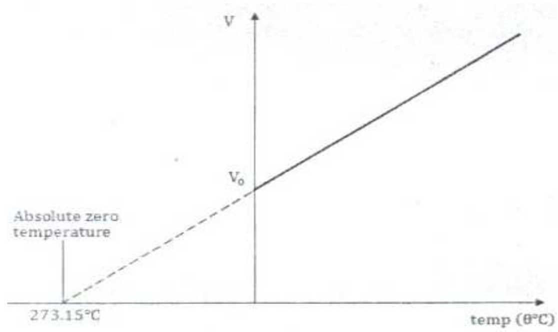
$$V_2 = V = \text{Volume of the gas at temperature } \theta$$

$$V_1 = V_0 = \text{Volume of the gas at } 0^0\text{C}$$

$$\frac{V}{V_0} = \frac{\theta+273.15}{273.15}$$

$$V = V_0\left[1+\frac{\theta}{273.15}\right]$$

GRAPH OF VOLUME AGAINST TEMPERATURE



Since the volume of the gas at the temperature θ is given by $V = \frac{V_0}{273.15}\theta + V_0$

Thus Charle's law can also be defined as 'Pressure remaining constant, the volume of a given mass of the gas increases or decreases by $\frac{1}{273.15}$ of its volume at 0°C for the each 1°C rise or fall in temperature.

Derivation of Charles's law from the kinetic theory gases.

$$PV = \frac{1}{3}Nm\bar{c}^2$$

$$PV = \frac{2}{3}N\left(\frac{1}{2}m\bar{c}^2\right) \quad \text{But} \quad \frac{1}{2}m\bar{c}^2 = \frac{3}{2}KT$$

$$PV = \frac{2}{3}N\left(\frac{3}{2}KT\right)$$

$$PV = NKT$$

$$V = \left(\frac{NK}{P}\right)T$$

$V \alpha T$

This implies the statements of the Charles's law

III. PRESSURE LAW

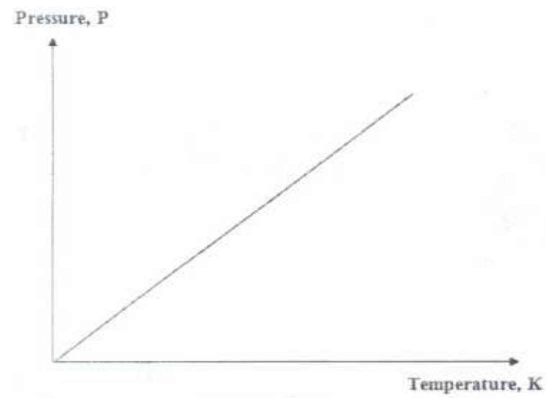
State that 'If the volume of the given fixed mass of the gas remains constant pressure of the gas is directly proportional to its absolute temperature i.e $P \alpha T$

$$\frac{P}{T} = \text{Constant}$$

For the given mass of the gas that has a pressure P_1 at temperature θ_1 °C and is heated at constant volume to the temperature its new pressure is given by.

$$\frac{P_2}{P_1} = \frac{273.15+\theta_2}{273.15+\theta_1} = \frac{T_2}{T_1}$$

GRAPH OF PRESSURE AGAINST TEMPERATURE.



Derivation of charl's law rom the kinetic theory of gases.

Since

$$PV = \frac{1}{3}M\bar{c}^2 \quad \text{and} \quad \bar{c}^2 \alpha T$$

$PV \alpha T$

It V is kept constant $P \alpha T$

This implies the pressure law

Note

From the following Gas laws

$$PV = \text{constant (Boyle's law)}$$

$$\frac{V}{T} = \text{Constant (Charle's law)}$$

$$\frac{P}{T} = \text{Constant (Pressure law)}$$

On combining the three equations above, we have.

$$\frac{PV}{T} = \text{Constant}$$

Now $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$$\frac{PV}{T} = \text{Constant}$$

This is known as Gas condition for S.TP

$$P = 1.01\times10^5 Nm^{-2} = 760mmHg = 1atm$$

$$V = 22.4dm^3 \quad , \quad T = 273K\left(0^0C\right)$$

IV. PERFECT GAS EQUATION

In practice, the gas do not obey the gas laws at all values of pressure and temperature. It is because of the intermolecular forces between the gas molecules.

Definition A PERFECT GAS

Is the gas whose molecules are free from intermolecular attraction and obeys the gas laws at all values temperature and pressure. As gas which strictly obeys all the gas law is called **Ideal gas or perfect gas.**

PERFECT GAS EQUATION

Is the equation which shows the relationship between the pressure, volume and temperature of a given mass of the gas.

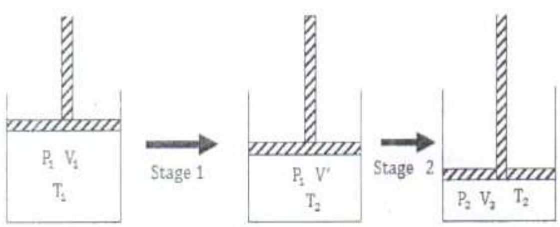
- For an ideal gas of n = 1 mole
  PV = RT
- For an ideal gas of n number of moles
  PV = Nrt

DERIVATION OF PERFECT GAS EQUATION

This can be derived by using Boyle's and Charles's laws.

Consider the ideal gas occupying a volume V_1 at pressure P_1 and absolute temperature T_1.

Let use



Apply Charles's law

$$\frac{V_1}{V_1} = \frac{T_2}{T_1}$$

$$V' = V_1\frac{T_2}{T_1}$$

Now apply Boyle's law for the stage 2

$$\frac{V_2}{V'} = \frac{P_1}{P_2}$$

$$V_2 = \frac{P_1}{P_2}V'$$

$$V_2 = \left(\frac{P_1}{P_2}\right)\left(\frac{V_1T_2}{T_1}\right)$$

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_2} = \text{Constant}$$

$$\frac{PV}{T} = \text{Constant}$$

The constant is known as Gas constant (Universal constant)

$$\frac{PV}{T} = R$$

$$PV = RT$$

This is known as perfect gas equation.

Derivation of perfect gas equation from basis of the kinetic theory of gases.

$$\text{Since } PV = \frac{1}{3}Nm\overline{c}^2$$

$$PV = \frac{2}{3}N_A\left(\frac{1}{2}m\overline{c}^2\right)$$

$$PV = RT$$

This is the perfect gas equation

V. AVOGADRO'S LAW

State that 'Equal volume of all gases at the same temperature and pressure contains equal number of molecules. i.e $N_1 = N_2$.

Derivation of Avogadro's law from the basis of kinetic theory gases

Consider two gases having the same temperature, pressure and volume. Let one gas contains N_1 molecules each of mass M_1. Let second gas contains N_2 molecules each of mass M_2. Let C_1 and C_2 be r.m.s velocity of the gases.

$$\text{Gas 1: } PV = \frac{1}{3}N_1m_1C_1^2$$

$$\text{Gas 2: } PV = \frac{1}{3}N_2m_2C_2^2$$

$$\frac{1}{3}N_1m_1C_1^2 = \frac{1}{3}N_2m_2C_2^2$$

$$\frac{2}{3}N_1\left(\frac{1}{2}m_1C_1^2\right) = \frac{2}{3}N_2\left(\frac{1}{2}m_2C_2^2\right)$$

$$\text{But: } \frac{1}{2}m_1C_1^2 = \frac{1}{2}m_2C_2^2 = \frac{3}{2}KT$$

$$N_1 = N_2$$

This implies the statement of the Avogadro's law.

VI. GRAHAM'S LAW DIFFUSION

State that 'For any specified temperature and pressure the relative rates of diffusion of two gases are inversely proportional to the square root of their densities.

$$R \propto \frac{1}{\sqrt{\rho}} \; , \; R = \frac{\text{Constant}}{\sqrt{\rho}}$$

Gas 1: $R_1 = \frac{K}{\sqrt{\rho_1}}$

Gas 2: $R_2 = \frac{K}{\sqrt{\rho_2}}$

$$\frac{R_1}{R_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

Derivation of graham's law from basis of the kinetic theory of gases.

Consider two gases at the same pressure P.

Let C_1 and C_2 be the r.m.s velocities of their molecules.

Let ρ_1 and ρ_2 be their respectively densities.

According to the kinetic theory of gases

$$P = \frac{1}{3}\rho_1 C_1^2 = \frac{1}{3}\rho_2 C_2^2$$

$$\frac{C_1^2}{C_2^2} = \frac{\rho_2}{\rho_1} \text{ or } \frac{C_1}{C_2} \text{ or } = \sqrt{\frac{\rho_2}{\rho_1}}$$

Let R_1 and R_2 be the rates of diffusion of two gases. These depend upon the velocity of the gas molecules.

$R \propto C$

$R_1 \propto C1$ and $R_2 \propto C_2$

$$\frac{R_1}{R_2} = \frac{C_1}{C_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

VII. DALTON'S LAW OF PARTIAL PRESSURE

State that ' The resultant pressure exeted by a mixture of gases or vapour which do not interact in any way is equal to the sum of their individual pressure'.

Consider a number of gases or vapour mixed together in a vessel. It is assumed that they do not interact with on another in any way.

Let ρ_1, ρ_2, ρ_3.........be their densities.

Let C_1, C_2..........be their respective r.m.s velocities.

Let P = total pressure exerted by the mixture.

$$P = \frac{1}{3}\rho_1 C_1^2 + \frac{1}{3}\rho_2 C_2^2$$

But $P_1 = \frac{1}{3}\rho_1 C_1^2$, $P_2 = \frac{1}{3}\rho_2 C_2^2$

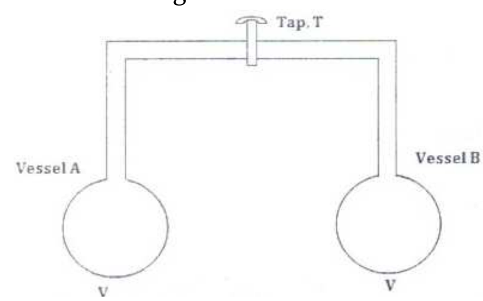
$$P = P_1 + P_2 + P_3 + --------$$

Which proves Dalton's law of partial pressure.

APPLICATIONS OF THE GAS LAW'S

A. SYSTEM OF THE CONNECTED BULBS OR VESSELS.

Consider two vessels A and B of equal volume are connected by a tube of negligible volume as shown on the figure below.



Before the tap is open the initial pressure and temperature on the vessels A and B and B are P_1, P_2 and T_1, T_2 respectively. If the tap T is open, then the gases moves from the bulb of higher pressure to the bulb of the lower pressure until the pressure on the both bulb's or vessels are the same. Such pressure is known as a common pressure, obtain expression of the common pressure P.

Assumptions made in our derivation

(i) No escaping of gases from the system to the surrounding since the system in closed system.

(ii) Assume that the gases in the vessel or bulb are ideal gas i.e PV = nRT.

(iii) Initial total number of moles of gas is equal to the final number of moles of gas ($n_i = n_f$). Assume that $P_1 > P_2$ and after tap open, the temperature on vessel A rises to T. Therefore gases moves from the vessel A to B until the pressure on the both vessels are the same and temperature of the gases on the vessel B is remains constant initial number of moles.

$$n = \frac{RV}{RT}$$

$$n_i = n_A + n_B$$

$$= \frac{P_1V}{RT_1} + \frac{P_2V}{RT_2}$$

$$n_i = \frac{V}{R}\left[\frac{P_1}{T_1} + \frac{P_2}{T_2}\right] \text{..........(i)}$$

Final total number of moles of gases.

$$n_f = \frac{PV}{RT} + \frac{PV}{RT_2}$$

$$n_f = \frac{PV}{R}\left[\frac{1}{T} + \frac{1}{12}\right] \text{........(ii)}$$

$$n_f = n_i$$

$$\frac{PV}{R}\left[\frac{1}{T} + \frac{1}{12}\right] = \frac{V}{R}\left[\frac{P}{T_1} + \frac{P_2}{T_2}\right]$$

$$P = \frac{\frac{P_1}{T_1} + \frac{P_2}{T_2}}{\frac{1}{T} + \frac{1}{12}}$$

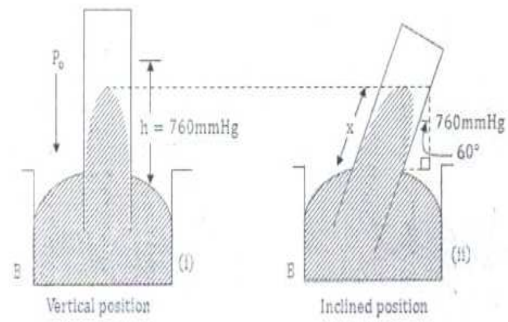
B. ATMOSPHERIC PRESSURE AND VARIATION OF ATMOSPHERIC PRESSURE WITH THE HEIGHT, h

ATMOSPHERIC PRESSURE

Is the pressure exerted due to the atmosphere.

A BAROMETER

Is an instrument used for measuring the pressure of the atmosphere which is required for weather – forecasting. An accurate form of barometer consists basically of a vertical barometer tube about a meter long containing mercury with a vacuum at the closed top as shown on the figure below. The other end of the tube us below the surface of mercury containing in a vessel.



Po = Atmospheric pressure

The atmospheric pressure supports the column of mercury in the tube. Suppose the mercury is at a vertical height, h above the level of mercury in vessel B.

$$P = \rho gh$$
$$= 0.76 \times 9.8 \times 13600$$
$$P_O = P = 1.013 \times 10^5 Pa$$

If the barometer is individual of an angle $\theta = 60^O$ to the vertical. The length of the mercury along the slanted side of the tube is X mm say. If the atmospheric pressure here is the same as in figure (i). This means that the vertical height of the mercury surface is still 760mm.

$$\cos 60^\circ = \frac{760}{X}$$

$$760mm = X\cos 60$$

$$X = \frac{760mm}{\cos 60^\circ}$$

$$X = 1520mm$$

VARIATION OF ATMOSPHERIC PRESSURE WITH HEIGHT.

Atmospheric pressure is equal to the pressure at the base of column of mercury of about 0.76m high. The whole mass of the atmosphere can therefore be considered to be equal to a mass of mercury of this height and having a cross section equal to that of the Earth is surface area.

Earth radius = 6.4×10^6m

Mass of atmosphere

= $0.76 \times 40 \times (6.4 \times 10^6)^2 \times 13{,}600$

= 5.3×1018Kg

If the density of air is constant $\delta = \rho_a = 1.29 kgm^{-3}$ the height h of the atmosphere would be given by
h × 1.29 = 0.76 × 13,600
h = 8 × 10^3m = 8km (approx.)

Atmospheric pressure varies with the height above sea – level. If δ is the density of air at a height h, them for a height dh.
dh = -gρdh……….(i)

Minus sign shows that the pressure diminishes as h increases.
For one mole of the gas

$$\rho = \frac{M}{V}$$

V = Volume of the mass of a one mole

$$dh = -\frac{Mg}{V}dh$$

If the gas obey Boyle's law i.e under isothermal condition.

$$PV = RT = \text{Constant}$$

$$V = \frac{RT}{P}$$

$$dP = \frac{Mg}{V}dh = -\frac{Mg}{RT/P}dh$$

$$dP = \frac{-MgP}{RT}dh$$

$$\frac{dP}{P} = -\frac{Mg}{RT}dh$$

$$\int_{P0}^{P}\frac{dP}{P} = \frac{Mg}{RT}\int_{0}^{h}dh$$

$$\left[\log_e^P\right]_{P0}^{P} = -\frac{Mg}{RT}\left[h\right]_0^h$$

$$\log_e^P - \log_e^{P_0} = \frac{-Mg}{RT}(h-0)$$

$$\log_e^{\left(P/_{po}\right)} = \frac{-Mg}{RT}$$

In exponential form

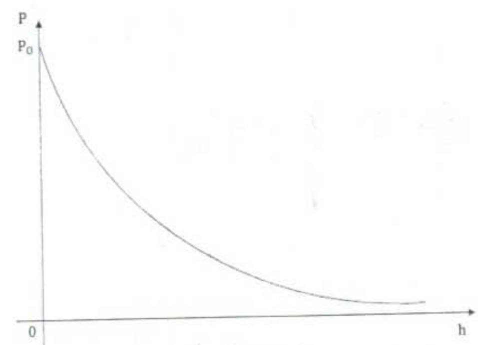
$$P/_{P_0} = e^{\frac{-Mg}{RT}}$$

$$P = P_{0e}^{\frac{-Mg}{RT}}$$

Po = Atmospheric pressure at the sea level

Therefore, atmospheric decreases with increase of the height from the sea level.

GRAPH OF ATMOSPHERIC PRESSURE P AGAINST HEIGHT, H ABOVE SEA LEVEL.

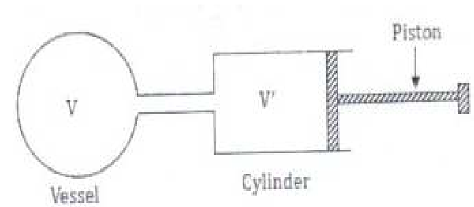


C. VACUUM PUMP

This is pump used to evacuate a system by reducing pressure of the system. Generally and system with low pressure is called a vacuum, why? It is a simple fact that pressure is caused by collision between the molecules for the gas and the walls of the container (from the basis of kinetic theory of gases) so reducing the pressure means reducing the number of collisions which intently removes the gas molecule in a system. It works a little but different from a compression pump.

Work principle

Firstly a pump is connected to a vessel of volume V of V′ is the volume of the cylinder of the pump and Po is the initial pressure in the vessel.



When a piston is pushed out wards valve 1 connecting pump cylinder and the vessel opens allowing air in the vessel to mix up with that in the pump cylinder. Valve 1 is a valve that connects the pipe and pump cylinder while valve is the one that connect cylinder and the environment (the two valves are not shown in the figure above). When the gas enters the pump cylinder the volume it occupies increases from V (Volume occupied by the vessel alone) to V + V′ (Volume occupied by vessel barrel of the pump (cylinder)). When volume increases, pressure of the whole interconnected system (vessel and pump cylinder) become P_1.

Now when the piston compressed, valve 1 closes, valve 2 opens to allow the gas molecules in the cylinder to get expelled out of the whole system.

After one toke (first compression and release of the piston)

Pressure in the vessel changes from P_O to P_1 and volume change from V_O to V_1.

Apply Boyle's law

$$P_0V_0 = P_1V_1$$

$$V_0 = V \ , \ V_1 = V + V'$$

$$P_0V = P_1(V+V')$$

$$P_1 = \frac{P_0V}{V+V'} = P_0\left(\frac{V}{V+V'}\right)^1$$

After two strokes (second compression and release of the piston)

Pressure in the system (pump cylinder and vessel) becomes, P_2 again

Apply Boyles's law.

$$P_1V_1 = P_2V_2$$

$$V_1 = V \ , \ V_2 = V + V'$$

$$P_1V = P_2(V+V')$$

$$P_2 = P_1\left[\frac{V}{V+V'}\right]$$

$$= P_0\left[\frac{V}{V+V'}\right]\left[\frac{V}{V+V'}\right]$$

$$P_2 = P_0\left[\frac{V}{V+V'}\right]^2$$

Generally, for n – strokes

$$P_n = P_0\left[\frac{V}{V+V'}\right]^n$$

NUMERICAL EXAMPLES

Calculate the root mean square speed of air molecules at a temperature it 27°C. One mole of air has a mass of 29gm. How $V_{r.m.s}$ of air molecules does compare to the speed of sound in air (340m/s).

Solution

Since one mole (6.023×10^{23} molecules) of air has a mass of 29gm. The average mass of air molecules.

$$m = \frac{29\times10^{-3}}{6.02\times10^{23}}$$

$$m = 4.8\times10^{-26}\,Kg$$

$$V_{r.m.s} = \sqrt{\frac{3KT}{m}}$$

$$V_{r.m.s} = \sqrt{\frac{3\times1.38\times10^{-23}\times300}{4.8\times10^{-26}}}$$

$$V_{r.m.s} = 509m/s$$

Comparison:

sound waves are caused by air molecules oscillating in and out of regions of varying are density. Since the molecules move into or out of the regions faster than their random speed, the speed with which sound wave are formed and travel is limited by this random speed.

$$\frac{V}{V_{r.m.s}} = \frac{340}{509} = 0.667 = \frac{2}{3}$$

$$V = \frac{2}{3}V_{r.m.s}$$

∴ The speed of sound in a gas is about two – third of the root mean square speed of its molecules.

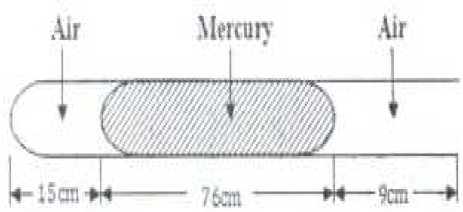
16. (a) On reducing the volume of a gas at constant temperature the pressure of a gas increases. Why?

(b) A metre long narrow tube (crossed at one end) held horizontally contains 76cm long of mercury thread which traps 15cm column of air. What happens if the tube is held vertically with open end at the bottom?

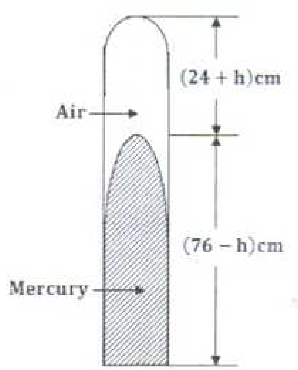
Solution

(a) On reducing the volume, the number of molecules per unit volume increase. Therefore more number of molecules collides with the walls per second and hence a large momentum is transferred to the walls per second. if the volume is halved, them the number of molecules per m^3 will be double. This according to the kinetic theory of gases, the pressure will be double. This is the explanation of Boyle's law on the basis of kinetic theory.

(b) Let A cm^3 = cross sectional area of the tube
When tube is held horizontally.



Pressure of air enclosed $P_1 = 76cmHg$
Volume of air enclosed $V_1 = 15Acm^3$
When the tube is held vertically



When the tube is held vertically the mercury first reaches the open end and the length of air becomes 15 + 9 = 24cm

Since pressure exerted by 24cm of air and 76cm of Hg is greater than atmospheric pressure, mercury will flow but of the tube until pressure becomes equal

Let $h = P_2 = 76 - (76 - h) = h$ cm of Hg

$V_2 = A(24 + h)\ cm^3$

Apply Boyles's law

$P_1V_1 = P_2V_2$

$76 \times 15A = h \times (24 + h)A$

$1140 = h(24 + h)$

$h^2 + 24h - 1140 = 0$

Solve for h quadratically

$$h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$h = \frac{-24 \pm \sqrt{24^2 - 4 \times 1 \times (-1140)}}{2}$$

$$h = \frac{-24 \pm 71.6}{2}$$

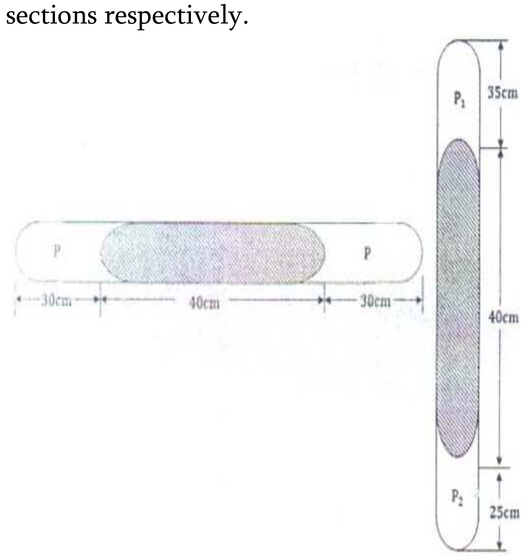
$h = 23.8cm$ or $-41.8cm$

Since h cannot be negative. Therefore, when the tube is held vertically, the mercury thread will decrease in length by 23.8cm.

17. A column of mercury with length of 40cm be in the middle of a horizontal capillary tube evacuated partially and soldered at both ends. If the tube is now placed vertically, the mercury column shifts through 5cm. Find the initial pressure when the ends are sealed. Given the length of the tube is 100cm.

Solution

Let P be the pressure when the tube is held horizontally. In the vertical position, let P_1 and P_2 be the pressure at the upper and lower sections respectively.



Apply Boyle's law for each section.

$P \times 30A = P_1 \times 35A$

$30P = 35P_1$

$6P = 7P_1$……….(i)

Also

$P \times 30A = 25P_2A$

$30P = 25P_2$

$6P = 5P_2$…….(ii)

$6P = 7P_1 = 5P_2$

Also

$30P = 25\ (40 + P1)$

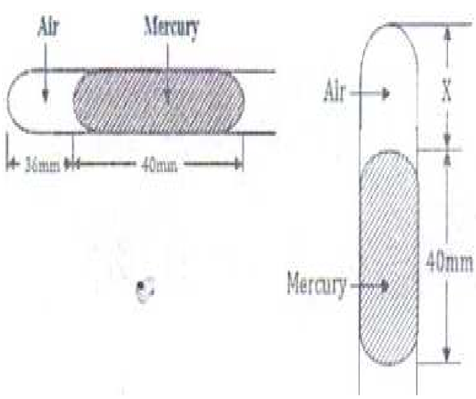
$P_1 = 100,\ P_2 = 140$

$$P = 25\frac{(40+100)}{30}$$

$P = 1167.7\text{mmHg}$

18. (a) (i) State Boyle's law

(ii) Uniform capillary tube contains air trapped by a mercury thread 40mm long. When the tube is placed vertically with the open end of the tube, downwards the length of the air column is no X. calculate X if the atmospheric pressure is 760mmHg. State any assumption you have used.



(b) (i) List down the four assumptions of the kinetic theory of gases.

(ii) Determine the absolute temperature of a gas in which the average molecule of mass 8.0 × 10-26kg is moving with (r.m.s) speed of 600m/s.

(c) The Doppler broadening of a spectral line is proportional to the r.m.s speed of the atoms emitting light. Which source of light has greater Doppler broadening than the other source? Mercury lamp at 250K or Krypton lamp at 88K. Mathematical treatment of your answer is required. Mass of mercury = 200gm and mass of Krypton 840gm.

Solution

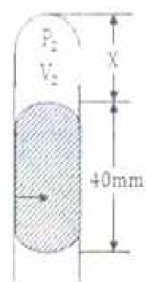
(a) (i) Refer to your notes

(ii) When the tube is in horizontal

Let A = Uniform cross – sectional area of the tube

$V_1 = 36\text{Amm}^3$, $P_1 = Pa = 760\text{mmHg}$

When the tube is the vertical



$Pa = P_2 + 40\text{mmHg}$

$P_2 = Pa - 40$

$= 760 - 40$

$P_2 = 720\text{mmHg}$

$V_2 = AX\text{mm}^3$

Assumption

Assume that the Boyle's law is hold

$$P_1V_1 = P_2V_2$$

$$760 \times 36A = 720 \times Ax$$

$$X = \frac{760 \times 36}{720}$$

$$X = 38\text{mm}$$

(b) (i) Refer to you notes

(ii) Since $\frac{1}{2}m\bar{c}^2 = \frac{3}{2}\left(\frac{R}{N_A}\right)T$

$$T = \frac{m\bar{c}^2}{3\left(R/N_A\right)} = \frac{8\times10^{-6}\times(500)^2}{3\left[\frac{8.31}{6.02\times10^{23}}\right]}$$

$$T = 483K$$

(c) Molecules moving away $\lambda_1 = \left(1+\frac{V}{C}\right)\lambda$

Molecules moving towards $\lambda_2 = \left(1-\frac{V}{C}\right)\lambda$

Broadening spectral lines

$$\Delta\lambda = \lambda_1 - \lambda_2$$

$$= \left(1+\frac{V}{C}\right)\lambda - \left(1-\frac{V}{C}\right)\lambda$$

$$\Delta\lambda = \frac{2\Delta\lambda}{C} \text{ But } \Delta\lambda \ \alpha \ \text{Vr.m.s}$$

$$V_{r.m.s} = \sqrt{\frac{3RT}{Mr}}$$

For mercury lamp

$$V_1 = \sqrt{\frac{3\times8.31\times250}{0.2}}$$

$$V_{r.m.s} = 176.5\text{m}/\text{s}$$

For Krypton lamp

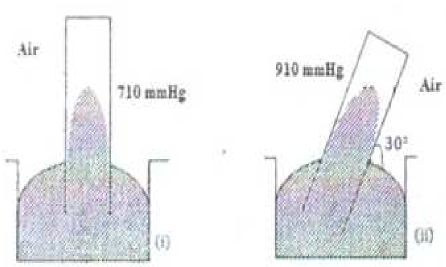
$$V_{r.m.s_2} = \sqrt{\frac{3\times8.31\times88}{0.84}}$$

$$V_{r.m.s} = 51.1\text{m}/\text{s}$$

Since $\Delta\lambda \ \alpha \ \text{Vr.m.s} \ \alpha \sqrt{\frac{3RT}{Mr}}$

∴ $V_{r.m.s}$ of mercury is greater than $V_{r.m.s}$ of Krypton. Therefore mercury lamp has greater Doppler broadening lines.

19. A barometer tube 190mm long above the mercury in the reservoir, contain a little air above the mercury column inside it when vertical (see figure (i) below) the mercury column is 710mm above the mercury in the reservoir. When inclined at 30° to the horizontal figure (ii) the mercury 910mmHg long the barometer tube. Assume the air obeys Boyle's law PV = Constant. Calculate the atmospheric pressure.



Vertical position Inclined position

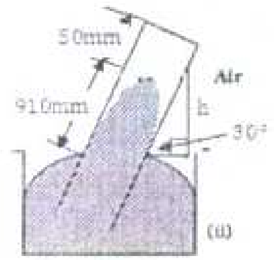
Solution

Let A = Uniform cross – sectional area of the tube when the tube is in vertical position

$V_1 = A(960 - 710) = 250\text{Amm}^3$

$P_1 = (P_a - 710)\text{mmHg}$

When the tube is inclined at an angle of 30°



Inclined position

$$\sin 30^0 = \frac{h}{910\text{mm}}$$

$$h = 910\text{mm} \sin 30^0$$

$$h = 455\text{mm}$$

$$P_2 = (P_a - 455)\text{mmHg}$$

Apply Boyle's law

$$P_1V_1 = P_2V_2$$

$$(P_a - 710)250A = (P_a - 455)50A$$

$$5(P_a - 710) = P_a - 455$$

$$5P_a - P_a = 3550 - 455$$

$$4P_a = 3095$$

$$P_a = 773.75\text{mmHg}$$

∴ Atmospheric pressure, 773.8mmHg.

(a) (i) List basic assuming underlying the derivation of the expression $P = \frac{1}{3}\rho\bar{c}^2$

(ii) A vessel of volume 50cm³ contains hydrogen at pressure of 1.0Pa and temperature of 27°C. Estimate the number of molecules in the vessels and their not mean square speed. (Mass of 1 mole of hydrogen molecules = 2.0×10^{-3}kg/mol).

(b) (i) State Boyle's law

(ii) A barometer tube 960mm long above the mercury in the reservoir contains a little air above the mercury column inside it. 710mm above the mercury in the reservoir, but when inclined at an agel of 30^{O} to horizontal the column is 910mm along the barometer tube. Calculate the atmospheric pressure.

(iii) A flask containing air is corked when the atmospheric pressure is 750mmHg and the temperature is 17^{O}C. The temperature of the flask is now raised gradually. The cork blows out when the pressure in the flask exceeds atmospheric pressure by 150mmHg. Calculate the temperature of the flask when the blow occurs.

Solution

(a) (i) See your notes

(ii) $V = 50cm^3 = 50 \times 10^{-6}m^3$

$P = Pa$, $T = 27^{O}C = 300K$

N = Number of molecules in the vessel

For an ideal gas

$$PV = NKT$$

$$N = \frac{PV}{KT} = \frac{1\times50\times10^{-6}}{1.38\times10^{-2}\times300}$$

Let Cr.m.s = mean square speed

$$m = \frac{Mr}{N_A} = \frac{2\times10^{-3}}{6\times10^{23}}$$

$$\text{Since } \frac{1}{2}m\bar{c}^2 = \frac{3}{2}KT$$

$$\bar{c}^2 = \frac{3KT}{m}$$

$$C_{r.m.s} = \sqrt{\bar{c}^2} = \sqrt{\frac{3KT}{m}}$$

$$= \sqrt{3\left(\frac{R}{N_A}\right)\frac{K}{m}}$$

$$C_{r.m.s} = \sqrt{\frac{3\times8.31\times300}{6.0\times10^{23}\times3.33\times10^{-27}}}$$

$$C_{r.m.s} = 1.934.75m/s$$

(b) (i) Refer to your notes

(ii) See solution example 19

(iii) Given that

$P_1 = 750mmHg$, $T_1 = 17^{O}C = 290K$

$P_2 = 760 + 150 = 910mmHg$

$T_2 = ?$

Apply pressure law

$$P/T = \text{Constant}$$

$$\frac{P_1}{T_1} = P_2/T_2$$

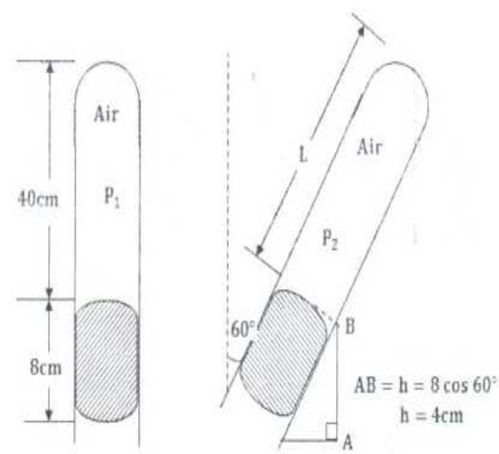
$$T_2 = \left(\frac{P_2}{P_1}\right)T_1 = 290\left[\frac{910}{750}\right]$$

$$T_2 = 351.87K$$

20. A column of air 40cm in a glass tube with a cross sectional area 0.5cm^3 and arranged vertically with a sealed end upward is isolated by a common of mercury 8cm long, the temperature 27^{O}C. How will the length of air column changes, if the tube is inclined at 60^{O}C from the vertical and the temperature is simultaneously raised by 30^{O}C? Assume the atmospheric pressure to be 76cm of mercury. Find the mass of air enclosed in the tube. (Density of air at S.T.P = $2993 \times 10^{-3}gmcm^3$).

Solution

Casa: 1



When the tube is in the vertical position

At 27^{O}C (300K) T = 300K,

$P_1 = Pa - 8 = 76 - 8 = 68cmHg$.

Volume of air $V_1 = 40 \times 05 = 20cm^3$.

When the tube inclined at an angle 60^{O} with vertical and temperature is raised to 30^{O}C (330K).

$T_2 = 330K$

$P_2 = 76 - 4 = 72$cmHg

Volume of air, $V_2 = 0.5L cm^3$.

Now $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$$\frac{68\times20}{300} = \frac{72\times0.5L}{330}$$

$$L = \frac{68\times20\times330}{300\times72\times0.5}$$

$$L = 41.5cm$$

So the length of air column increased by

$$\Delta L = 41.5-40$$

$$\Delta L = 1.5cm$$

Case 2: $\frac{P_0V_0}{T_0} = \frac{P_1V_1}{T_2}$

$$V_0 = V_1\left(\frac{P_1}{P_0}\right)\left(\frac{T_0}{T_1}\right)$$

$$= 20\times\frac{68}{76}\times\frac{273}{300}$$

$$V_0 = 16.284cm^3$$

$$\text{Mass} = \rho V_0$$

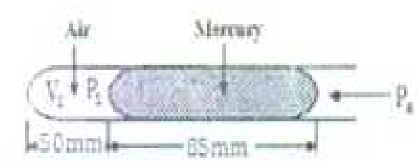
$$= 16.284\times10^{-3}\times L.293$$

$$M = 21.05\times10^{-3}gm$$

21. A uniform capillary tube closed at one end, contained air trapped by a thread of mercury 85mm long. When the tube was held horizontally the length of the air column was 50mm, when it was held vertically with the closed end downward, the length was 45mm. Find the atmospheric pressure (Take $g = 10m/s^2$ density of mercury $= 14 \times 10^3 kgm^{-3}$).

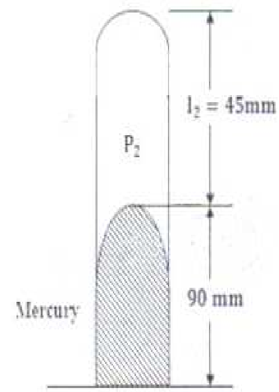
Solution

Case 1: When the tube is held horizontally



$P_1 = P_a$, $V_1 = AL_1 = 50A\ mm^3$

Case 2: when the tube is held vertically



$V_2 = Al_2$

$V_2 = 45A mm^3$

$P_2 = Pa + \rho gh$

Apply Boyle's law

$P_1V_1 = P_2V_2$

$50APa = (Pa + \rho gh)\ 45A$

$50APa = 45Pa + 45\rho gh$

$5Pa = 45\rho gh$

$Pa = 9\rho gh$

$= 9 \times 14 \times 10^3 \times 10 \times 90 \times 10^{-3}$

$Pa = 1.1 \times 10^5 Nm^2 (Pa)$ (approx.)

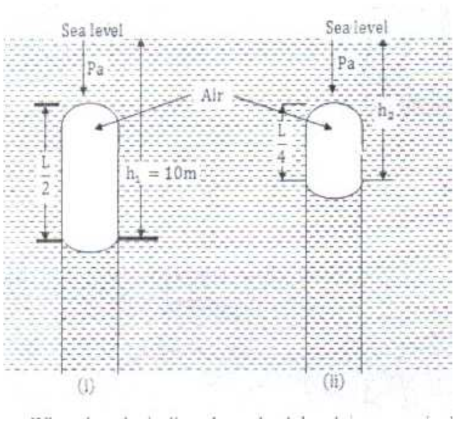
22. A uniform vertical glass tube at the lower end and sealed at the upper end is lowered into sea water thus trapping the air in the tube. It is observed that when the tube is submerged to a depth of 10m the sea water has entered the lower half of the tube. To what depth must the tube be lowered so that the sea – water fills three quarters of the tube?

Solution

When the tube is dipped to a depth of 10m the pressure on air gap

$$P_1 = P_a + \rho_w gh_1$$

$$V_1 = \left(\frac{1}{2}L\right)A$$



When the tube is dipped to a depth h_2, the pressure in the air gap.

$$P_2 = P_a + \rho_w g h_2$$

$$V_2 = \left(\frac{1}{4}L\right)A$$

Assume that the process takes place under isothermal condition temperature remain constant.

Apply that Boyle's law

$$P_1V_2 = P_2V_2$$

$$\left(P_a + \rho_w g h_1\right)\frac{L}{2}A = \left(P_a + \rho_w g h_2\right)\frac{L}{4}A$$

$$2\left(P_a + \rho_w g h_1\right) = P_a + \rho_w g h_2$$

$$P_a + 2\rho_w g h_1 = \rho_w g h_2$$

$$h_2 = \frac{P_a + 2\rho_w g h_1}{\rho_w g}$$

Given that

$$\rho_w = 1000kgm^{-3} \quad g = 10mls^2$$

$$P_a = 1\times10^5 Nm^{-2}$$

$$h_2 = \frac{1\times10^5 + 2\times1000\times10\times10}{1000\times10}$$

$$h_2 = 30m$$

∴ The tube must be lowered 30m from the sea level so as to get filled with water three quarter and leaning a quarter space of air gap.

Assignment

23. A mercury barometer tube with a scale attacked has a little air above the mercury. The top of the tube is 1.0m above the level of the mercury in the reservoir when the tube is held vertically the length of the mercury column is 700mm when the tube is inclined at 60^O to the vertical the reading of the mercury level on the scale is 950mm. To what height would be mercury have risen in the vertical tube had there not been air in it? Answer : 745mmHg

24. A faulty mercury barometer consists of an inverted vertical tube of length 800mm with its open end in a mercury reservoir. On a certain day, when the atmospheric pressure was known to be 102.5KPa and the temperature was 20^OC, the height of the mercury in the tube was 760mm above the lever of mercury in the reservoir.
    (a) Calculate the pressure of gas in the tube in space above the mercury. The density of mercury is $13600Km^{-3}$ and $g = 9.8NKg^{-1}$.
    (b) On the next day the temperature changes to 15^OC and the height of the mercury column falls to 755mm. calculate the pressure of the gas in the top of the tube and the atmospheric pressure (UND 103)

Solution

(a) At temperature of 20^OC

$T_1 = 293K$, $V_1 = A(800 - 760) = 40mm^3A$

Pa = 102.5KPa

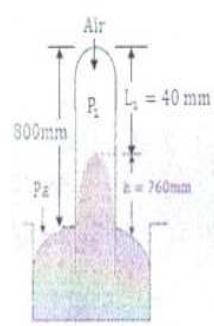
Let P_1 = Pressure of the gas in the tube

$Pa = P_L + \rho gh$

$P_1 = P_a - \rho gh1$

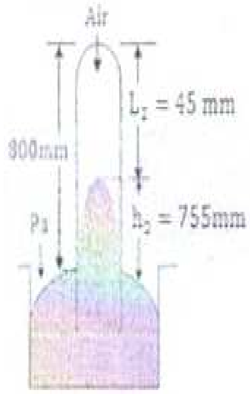
$= 102.5 \times 10^3 - 13600 \times 98 \times 0.76$

$P_1 = 1.21 \times 10^3 P_a$



On the 1^{st} day

(b) On the 2nd day



$V_2 = (800 - 755)\,A = 45Amm^3$

$T_2 = 15 + 273 = 288K$

By using the equation

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

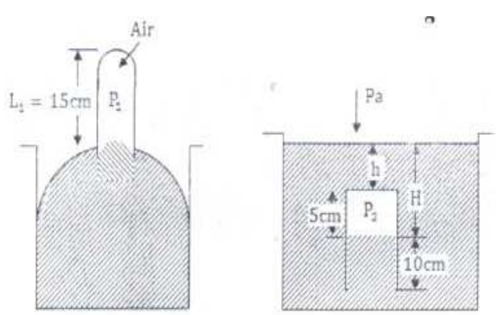
$$P_2 = P_1\left(\frac{T_2}{T_1}\right)\left(\frac{V_1}{V_2}\right)$$

$$= 1.21KPa\left[\frac{288}{293}\right]\left[\frac{40A}{45A}\right]$$

$$P_2 = 1.06KPa$$

25. A test tube 15cm long is pushed down into mercury with the closed end upper most, until. The mercury rises 5cm up the tube. If the barometer height is 76cm how far is the closed end of the tube below the surface of mercury?

Solution



(i) Before pushing (ii) After pushing

When we push the test tube down on the liquid some of the liquid tries to fill the air space in the test and this cause increase in pressure of the air inside of the tube. From figure (i) the level of mercury inside and outside of the test tube is conceded after pushing the test tube downward.

$$P_2 = P_a + H\ ,\ V_2 = AL_2 = 10Acm^3$$

Apply Boyle's law

$$P_LV_L = P_2V_2$$

$$76\times 15A = (76+H)10A$$

$$H = 38cm$$

The height of the closed end from the mercury level in the reservoir as shown in the figure (ii)

$$H = h + 10$$

$$h = H - 10 = 38cm - 10cm$$

$$h = 28cm$$

26. A diving bell of uniform cross section is 3m high and contains air at 67°C. The atmospheric pressure is 0.75m of mercury. The bell is now lowered into water at 17°C and the water rises 1m inside the bell. How far is the top of the diving bell below the surface of the water?

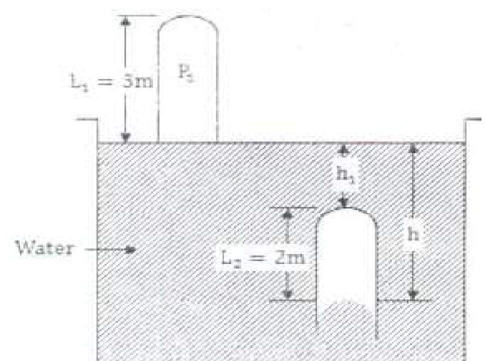
Solution

At 67°C , $P_1 = Pa T_1 = 67 + 273 = 340$

$V_1 = 3AM^3 P_1 = Pa = 750mmHg$

At 17°C $P_2 = Pa + h$, $V_2 = 2Am^3$

$T_2 = 273 + 17 = 290K$



Apply the equation

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$P_2 = \frac{P_1V_1T_2}{T_1V_2} = \frac{0.75\times 3A\times 290}{2A\times 340}$$

$$P_2 = 0.96m$$

$$P_2 = P_a + h$$

$$h = P_2 - P_a = 0.96 - 0.75$$

$$h = 0.21m$$

∴ The weight of the top of the diving bell from the water surface.

27. (a) (i) Define the temperature of an ideal gas as a consequence of the kinetic theory.
(ii) Without a detailed mathematical analysis argues the steps to follow in deriving the relation $\rho = \frac{3P}{\overline{C}^2}$ where all symbol carry their usual meaning.
(b) (i) Show that average kinetic energy of an ideal gas is directly proportional to the absolute temperature of the gas.
(c) It is found that molecules of a certain gas travel in a tube at a distance of 40cm without collision. If the molecules is 2×10^{-8}m, estimate the
(i) Number of molecules per unit volume in the tube
(ii) Pressure of the gas at a temperature of 27°C.

28. (a) Briefly give comments on the following observations.
(i) Polyatomic and diatomic gases have large molar heat capacities than mono atomic gases.
(ii) Cubical container is used for the derivation of pressure of an ideal gas
(b) (i) What is meant by a gas constant?
(ii) Helium gas occupies a volume of 4×10^{-2}m³ at a pressure of 2×10^5Pa and temperature of 300K. Calculate the mass of helium and the r.m.s of its molecules.

Solution

(a) (i) The degree of freedom of diatomic and polyatomic gases are larger than that of mono atomic gas.
- The molecules of diatomic and polyatomic gases have translational as well as translational kinetic energy. While for mono atomic gas internal energy is due to the only translational kinetic energy is equal to zero. Thus its rotational kinetic energy is equal to zero. These show that polyatomic and diatomic gases have large molar heat capacities than mono atomic gases.

(b) (i) see your notes

(ii) $V = 4\times10^{-2}m^3$, $P = 2\times10^5 Pa$
T = 300K
For an ideal gas

$$PV = nRT = \frac{m}{Mr}RT$$

$$m = \frac{PVMr}{RT} = \frac{2\times10^5\times4\times10^{-2}\times4\times10^{-3}}{8.31\times300}$$

$$m = 12.8\times10^{-3}Kg = 12.8gm$$

Root mean square velocity

$$C_{r.m.n} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3PV}{M}}$$

$$= \sqrt{\frac{3\times2\times10^5\times0.04}{12.8\times10^{-3}}}$$

$$C_{r.m.n} = 1369 m/s$$

29. (a) Explain the meaning of the following
(i) Root mean square velocity
(ii) Ideal gas
(b) (i) Outline the main assumption of kinetic theory of gases.
(ii) From the assumption stated in (b) (i) above differentiate ideal and real gases.
(c) (i) Calculate the pressure in mmHg exerted by hydrogen gas if the number of molecules per cm3 is 6.8×10^5 and the root mean square of the molecules is 1.9×10^3m/s.
(ii) Give comment on the effect of pressure obtained in (c) (i) above on mercury in the fortin barometer an in cathode ray tube.
(d) (i) Derive the relation between the pressure of the atmosphere and the height above the ground assuming that the temperature of the atmospheric is uniform.
(ii) From the relationship derived in (d) (i) above sketch a group of pressure against the height above the sea level.

Solution

(a) See your notes
(b) (i) see your notes

(ii) Column chart

Ideal gas	Real gas
The volume of molecules is negligible compared to the volume occupied by the gas	The volume of molecules is not negligible in relation to the volume occupied by the gas especially at high pressure
The force of attraction between molecules is negligible	The force attraction between the molecules is not negligible
Internal energy is a function of temperature alone	Internal energy is a function of temperature and other factors
At 0K an ideal gas occupies 0 volume	At 0K, Real gas still occupies some volume.

(c) (i) $P = \frac{1}{3}\rho\bar{c}^2$ but $\rho = \frac{Nm}{V}$

$M = \frac{Mr}{N_A}$, $\rho = \frac{N}{V}\times\frac{Mr}{N_A}$

$P = \frac{1}{3}\frac{N}{V}\frac{Mr}{N_A}\bar{C}^2$

$= \frac{1}{3}\times 6.8\times10^{15}\times\frac{2.16\times10^{-3}}{6.02\times10^{23}}\times\left(1.9\times10^{3}\right)^2$

$P = 29.36\text{Nm}^{-2}$

But $\rho = \rho hg$

$h = \frac{P}{\rho g} = \frac{29.36}{13600\times9.8} = 0.22\times10^{5}\text{m}$

$h = 0.22\text{mmHg}$

$\therefore P = 0.22\text{mmHg}$

(ii) Comment:

Above a mercury column this pressure leads to incorrect readings of the atmospheric pressure. This is because above the column there should be a vacuum such that the height of the column is only a result of the atmospheric pressure exerted on the mercury. With hydrogen in the tube, the reading of the barometer will be:

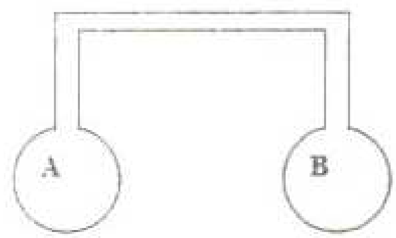
Atmospheric pressure – 0.22mmHg

In a cathode ray tube this pressure is low enough to allow current conduction. At this pressure the illumination of air disappears although current conduction takes place.

(d) See your notes.

SYSTEM OF CONNECTED BULBS

30. Two vessels A and B of equal volume are connected by a tube of negligible volume as shown in the figure below.



The vessels contain a total mass of 2.50×10^{-3}kg of air and initially both vessels are at 27^{0}C when the pressure is $1.01 \times 10^{5}\text{Nm}^{-2}$, vessel A is cooled to 0^{0}C and vessel B is heated to 100^{0}C. Calculate

(a) The mass of the gas now in each vessel

(b) The pressure in the vessels.

Solution

(a) Let M_A = Mass of the gas in the vessels A

M_B = Mass of the gas in vessel B. Since the pressure is equal on the two vessels.

Let the final pressure be P.

Now $PV = nRT = \frac{M}{Mr}\times RT$

Vessel A: Mass of the gas (M_A) at temperature of 273K

$PV = M_A\left(\frac{R}{Mr}\right)\times273$ ....(i)

Vessel B: Contains MB of gas at 373K

$PV = M_B\left(\frac{R}{Mr}\right)\times373$

(i) = (ii)

$M_A\left(\frac{R}{Mr}\right)273 = M_B\left(\frac{R}{Mr}\right)373$

$$273M_A = 373M_B$$

$$M_A = \left(\frac{373}{273}\right)M_B$$

Total mass of the gas on the system

$$M_T = M_A + M_B$$

$$2.5\times10^{-3} = M_A + M_B$$

$$2.5\times10^{-3} = \left(\frac{373}{273}\right)M_B + M_B$$

$$2.5\times10^{-3} = \left(\frac{373+273}{273}\right)M_B$$

$$M_B = 2.5\times10^{-3}\left[\frac{273}{373+273}\right]$$

$$M_B = 1.06\times10^{-3}\,\text{kg}$$

Also $M_A = \left(\frac{373}{273}\right)M_B$

$$M_A = 1.44\times10^{-3}\,\text{kg}$$

$$\therefore \quad M_A = 1.44\times10^{-3}\,\text{kg and}$$

$$M_B = 1.06\times10^{-3}\,\text{kg}$$

(b) Initial conditions for the whole system

$$T_0 = 273+27 = 300\text{K}$$

$$P_0 = 1.01\times10^5\,\text{Nm}^{-2}$$

$$M_T = 2.5\times10^{-3}\,\text{kg}$$

Now $P_0V_0 = M_T\left(\frac{R}{M_T}\right)T_0$

$$1.01\times10^5\times2V = 2.5\times10^{-3}\left(\frac{R}{Mr}\right)\times300 \ldots\text{(iii)}$$

To find the final pressure P, we can make the use of equation (i) or (ii)

Now from equation (i)

$$PV = 1.44\times10^{-3}\left(\frac{R}{Mr}\right)\times273$$

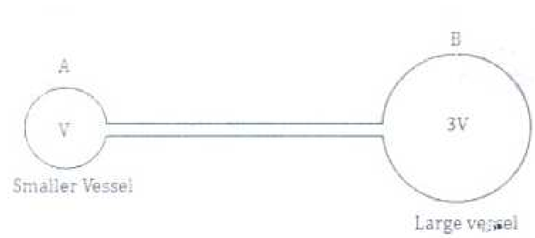
Dividing equation (i) by (ii)

$$\frac{PV}{1.01\times10^5\times2V} = \frac{1.44\times10^{-3}\left(\frac{R}{Mr}\right)\times273}{2.5\times10^{-3}\left(\frac{R}{Mr}\right)\times300}$$

$$P = 1.06\times10^5\,\text{Nm}^{-2}$$

31. Two vessels are having three times the volume of the other, are connected by a narrow tube of negligible volume. Initially the whole system is filled with a gas at a pressure of 105 × 10^5Pa and a temperature of 290K. The smaller vessel is now cooled to 250K and larger heated to 400. Find the final pressure in the system.

Solution



Total initial number of moles

$$n_i = \frac{P_1V}{RT_1} + \frac{3P_1V}{RT_1} = \frac{4P_1V}{RT_1} \ldots\ldots\text{(i)}$$

Total final number of moles

$$n_f = \frac{PV}{RT_A} + \frac{3PV}{RT_B} = \frac{PV}{R}\left[\frac{1}{T_A} + \frac{3}{T_B}\right] \ldots\text{(ii)}$$

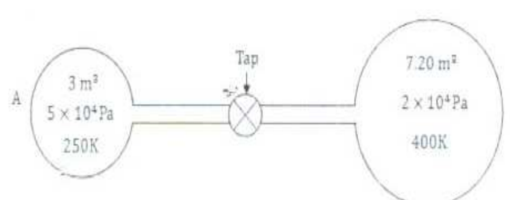
$$n_f = n_i$$

$$\frac{PV}{R}\left[\frac{1}{T_A} + \frac{3}{T_B}\right] = \frac{4P_1V}{RT_1}$$

$$P = \frac{4P_L}{T_1\left[\frac{1}{T_A} + \frac{3}{T_B}\right]} = \frac{4\times1.05\times10^5}{290\left[\frac{1}{250} + \frac{3}{400}\right]}$$

32. Initial vessels A contains 3.00cm³ of an ideal gas at a temperature of 250K and pressure of 5.00 × 10^4Pa while vessels B contains 7.20m³ of the same gas at 400 and 2.00 × 10^4Pa (see figure below). Find the pressure after the connecting tap has been opened and the system has reached equilibrium assuming that A is kept at 250K and B at 400K connecting?

Solution



When the tip is opened, some gas moves from A to B thereby reducing the pressure in A and increasing it in bulb B. This process continue until equilibrium is reached and pressure in bulb A become equal to pressure in B.

Let P = Final pressure of the gas

Since : PV = nRT

$$n = \frac{PV}{RT}$$

Initial total number of moles

$$n_i = \frac{5\times10^4\times3.00}{250}+\frac{2\times10^4\times270}{400R}$$

$$n_i = \frac{960}{R} \ldots\ldots\ldots\ldots\ldots(i)$$

Final total number of moles

$$n_f = \frac{3P}{250R}+\frac{7.2P}{400R}$$

$$n_f = \frac{3.0\times10^{-2}P}{R} \ldots\ldots\ldots(ii)$$

$$n_f = n_i$$

$$\frac{3.0\times10^{-2}P}{R} = \frac{960}{R}$$

$$P = 3.20\times10^4\,\text{Pa}$$

33. Two perfect gases at absolute temperature T_1 and T_2 are mixed. There is no loss of energy. Find the temperature of mixture if masses of molecules are M_1 and M_2 and the number of molecules in the gases are n_1 and n_2 respectively.

Solution

Since the gases are perfect, they do not interact mutually average kinetic energy of molecules.

Average kinetic energy of a molecule

$$E = \frac{3}{2}KT$$

Kinetic energy of one perfect gas

$$K.E_1 = \frac{3}{2}n_1KT_1$$

Kinetic energy of second perfect gas

$$K.E_2 = \frac{3}{2}n_2KT_2$$

Total initial kinetic energy

$$E_0 = \frac{3}{2}n_1KT_1+\frac{3}{2}Kn_2T_2$$

$$E_0 = \frac{3}{2}K\left(n_1T_1+n_2T_2\right)$$

Total kinetic energy of mixture

$$E = \frac{3}{2}KT\left(n_1+n_2\right)$$

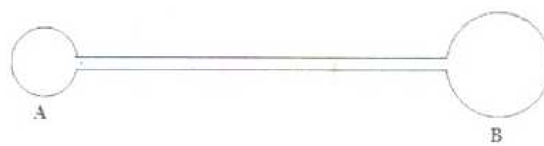
Since there is no loss of energy

$$E = E_0$$

$$\frac{3}{2}KT\left(n_1+n_2\right) = \frac{3}{2}K\left(n_1T_1+n_2T_2\right)$$

$$T = \frac{n_1T_1+n_2T_2}{n_1+n_2}$$

34. Two vessels A, B of equal volume are connected by a narrow tube of negligible internal volume. Initially the whole system is filled with 3g of dry air at pressure of 105Pa and temperature 300K. The temperature of the vessels B is now raised to 600K, the temperature of A remaining 300K.



(a) The new pressure in the system

(b) The mass of air in A and B

Solution

(a) Initial total number of moles before raising of temperature

$$n = \frac{PV}{RT}$$

$$n_i = \frac{P_iV}{RT_i}+\frac{P_iV}{RT_i}$$

$$n_i = \frac{2P_iV}{RT_i}\ ,\ P_i = 10^5\,\text{Pa}\ ,V = V\ ,\ T_i = 300\text{K}$$

$$n_i = \frac{2\times10^5\,V}{300R}$$

$$n_I = \frac{2000V}{3R} \ldots\ldots\ldots(i)$$

After temperature rises in the vessels B

$$n_f = \frac{PV}{RT_A} = \frac{PV}{RT_B}$$

$$= \frac{PV}{R}\left[\frac{1}{T_A}+\frac{1}{T_B}\right] = \frac{PV}{R}\left[\frac{1}{300}+\frac{1}{6000}\right]$$

$$n_f = \frac{PV}{200R}$$
$$n_f = n_i$$
$$\frac{PV}{200R} = \frac{2000V}{3R}$$
$$P = 1.33\times10^5\,Pa$$

(b) Since $n = \frac{PV}{RT}$

$$\frac{M}{Mr} = \frac{PV}{RT}$$
$$M = \frac{PVMr}{RT}$$

Before temperature rises, mass of gas in bulb $A\left[T_A = 300K\right]$

$$M_A = Mr\frac{PV}{RT_A}\ \ldots\ldots..(i)$$

After temperature rises of bulb A

$$M_A = \frac{MrPV}{RT'_A}\ \ldots\ldots\ldots(ii)$$
$$\frac{M'_A}{M_A} = \frac{MrPV}{RT'_A}\Big/\frac{MrP_iV}{RT_A}$$
$$\frac{M'_A}{M_A} = \left(\frac{P}{P_i}\right)\left(\frac{T_A}{T'_A}\right)$$

Initially $M'_A = M_A = M_B = 1.5gm$

$$T_A = T'_A$$
$$M'_A = 1.5gm\left(\frac{1.33\times10^5}{10^5}\right)$$
$$M'_A = 2.0gm$$

But $M'_A + M'_B = 3gm$

$$M'_B = 3-2 = 1gm$$

$\therefore$ New mass of A, $M'_A = 2.0gm$

New mass of B, $M'_B = 1.0gm$

35. Two gas container with volume of 100cm³ and 1000cm³ respectively are connected by a tube of negligible are volume and contain air at a pressure of 1000mm of mercury. If the temperature of both vessels is originally 0ºC, how much air will pass through the connecting the tube when the temperature of the smaller tube is raised to 100ºC? Give your answer is cm³ measured at 0ºC and 760mmHg.

Solution

Since $n = \frac{PV}{RT}$

Initial total number of moles of air

$$n_I = \frac{P\left(V_1+V_2\right)}{RT} = \frac{1000\left(1000+1000\right)}{R+300}$$
$$n_i = \frac{1000\times1100}{300R}\ \ldots\ldots\ldots\ldots..(i)$$

Total final number of moles of air

$$n_f = \frac{P\times100}{R\times373}+\frac{P\times1000}{R\times273}\ \ldots\ldots(ii)$$

(i) = (ii)

$$\frac{100P}{373R}+\frac{1000R}{273R} = \frac{1000\times1100}{300R}$$
$$P = \frac{1100\times373}{4003}$$
$$P = 1025mmHg$$

As the question, we now have to connect the initial volume of 100cm³ of air in the smaller container at 0ºC and 1000mmHg to 0ºC and 760mmHg and the final volume of 100cm³ at 100ºC and 1025mmHg to 0ºC and 760mmHg.

Since PV = Constant

Initial volume at 0ºC and 760mmHg.

$$P_iV_i = P_fV_f$$
$$V_i = \left(\frac{P_f}{P_i}\right)V_f$$
$$= \left(\frac{1000}{760}\right)\times100cm^3$$
$$V_i = 131.6cm^3$$

Final volume at 0ºC and 760mmHg

$$\frac{P_iV_i}{T_i} = \frac{P_fV_f}{T_f}$$
$$V_f = \left(\frac{P_i}{P_f}\right)\frac{T_f}{T_i}$$
$$V_f = 100\times\frac{1025}{760}\times\frac{273}{373}$$
$$V_f = 98.7cm^3$$

Volume of air at 0ºC and 760mmHg flows out

V = 131.6 – 98.7 = 33cm3 (approx.)

36. A vacuum pump has a cylinder of volume V and is connected to a vessel of volume v to pump out air from the vessels. The initial pressure in the vessel in P. Calculate the reduced pressure after n strokes of the pump. What number of strokes is needed the pressure from $2.0 \times 10^5 P_a$ to $1.0 \times 10^5 P_a$ if the closed vessel has a volume of $200cm^3$ and the pump cylinder is $20cm^3$.

Solution

Case : 1 on the first part of the stroke when the pump piston goes down, the air in the vessels expands from V to V + v

Let P_1 = Reduced pressure

Apply Boyle's law

$$P_1(V+v) = PV$$

$$P_1 = P\left[\frac{V}{V+v}\right] \ldots\ldots(i)$$

On the next part of the stroke, when the piston rises the air drown is pushed out into the atmospheric by the pump leaving a pressure P_1 in the vessel. After the second stroke, the reduced pressure P_2 is given by

$$P_2 = P_1\left(\frac{V}{V+v}\right) = P\left(\frac{V}{V+v}\right)^2$$

After in strokes the reduced pressure Pn

$$P_n = P\left[\frac{V}{V+v}\right]^n$$

Case : 2 $V = 200cm^3$, $V = 20cm^3$,

$P = 2.0 \times 10^5 P_a P_n = 1 \times 10^3 Pa$

$$1.0\times10^3 = 2\times10^5\left[\frac{200}{200+20}\right]^n$$

$$\frac{1}{200} = \left(\frac{200}{220}\right)^n = \left(\frac{10}{11}\right)^n$$

$$\log\left(\frac{1}{200}\right) = n\left[\log 10 - \log 11\right]$$

$$n = \frac{\log\left(\frac{1}{200}\right)}{\left[\log 10 - \log 11\right]}$$

$$n = 56\text{strokes}$$

37. The barrel of an exhaust pump has an effective volume of $100cm^3$ and is being used to extract air from a $1000cm^3$ flask. Ignoring the volume of the connecting tube and assuming the temperature of the air to remain constant throughout, calculate the number of complete strokes of the pump needed to reduce the pressure on an air in the flask to one hundredth of initial value.

Solution

$V = 1000cm^3$, $V = 100cm^3$

$$P_n = \frac{P}{100}$$

By using equation

$$P_n = P\left[\frac{V}{V+V}\right]^n$$

$$\frac{P_n}{P} = \left[\frac{1000}{1000+100}\right]^n$$

$$\frac{1}{100} = \left(\frac{1000}{1100}\right)^n$$

$$\log\left(\frac{1}{100}\right) = n\log\left(\frac{1000}{1100}\right)$$

$$n = \frac{\log\left(\frac{1}{100}\right)}{\log\left(\frac{1000}{1100}\right)}$$

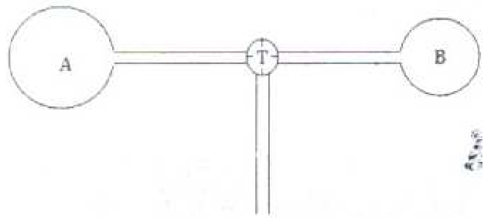
$$n = 48$$

So 48 strokes will cause the pressure to $\frac{1}{100}$.

Thus in order for the pressure to below this, the number of strokes should be 49.

38. What is the internal energy of the gas? Starting from the expression $P=\frac{1}{3}\rho\bar{c}^2$, show that the internal energy of an ideal monatomic gas is $\frac{3}{2}PV$ and discus the interpretation of temperature in the kinetic theory. Explain the following observations.
    (a) When pumping up a bicycle tyre the pump barrel gets warm and

(b) When a gas at high pressure is a container is suddenly released, the container cools.



Two bulbs A of volume 100cm3 and B of volume 50cm^3, are connected to a three way up T which enables them to be filled with gas or evacuated. The volume of the tubes may be neglected.

(c) Initially bulb A is filled with an ideal gas at 10^{o}C to pressure of 3.0 × 10^{5}P$_a$. bulb B is filled with an ideal gas at 100^{o}C to pressure of 1.0 × 10^{5}P$_a$. the two bulbs are connected with A maintained at 10^{o}C at and B at 100 calculate pressure at equilibrium.

(d) Bulb A filled at 10^{o}C to a pressure of 3.0 × 10^{5}Pa is connected to a vacuum pump with a cylinder of volume 20cm^3. Calculate the pressure in A air after one inlet stroke of the pump. The air in the pump is now expelled into the atmosphere. Calculate the pressure in A after the second stroke. Calculate the number of strokes of the pump to reduce the pressure in A to 1.0 × 10^{3}P$_a$. The whole system is maintained at 10^{o}C throughout the process.

Solution

Case 1: Internal energy – is defined as the sum of the potential energy and kinetic energy of the gas. The total amount of energy within the body is called the internal energy

Proof: for idea gas of mono atomic gas

$$U = \frac{3}{2}PV$$

For the expression

$$P = \frac{1}{3}\rho\bar{c}^2$$

But $\rho = \frac{Nm}{V}$

$$P = \frac{1}{3}\frac{Nm\bar{c}^2}{V}$$

$$PV = \frac{1}{3}Nm\bar{c}^2$$ (from n = 1 moles of ideal mono atomic gas) N = NA

$$= \frac{2}{3}\left(\frac{1}{2}N_A m\bar{c}^2\right) \text{ but } u = \frac{1}{2}N_A m\bar{c}^2$$

$$PV = \frac{3}{3}U$$

$$U = \frac{3}{2}PV \text{ Hence shown}$$

Case 2:

(a) When the tyre is being inflated quickly or pumped no heat exchange allowed between the gas inside of the tyre and the surrounding. Simply the process is adiabatic. The work done in pumping the tyre appears as the rise in internal energy of the gas. An increase in internal energy is corresponding to the temperature rises and therefore the pump barrel get warm.

(b) This also an example adiabatic process. The work of done by the gas escapes appears as the decrease in internal energy of the gas and so the container becomes cools.

(c) V_A = 100cm^3, V_B = 50cm^3, T_A = 283K, T_B = 373K, P_A = 3 × 10^{5}Pa, P_B = 10^{5}Pa. Initial total number of moles before connecting the bulbs.

$$n_i = n_A + n_B$$

$$= \frac{P_A V_A}{RT_A} + \frac{P_B V_B}{RT_B}$$

$$n_i = \frac{1}{R}\left[\frac{P_A V_A}{RT_A} + \frac{P_B V_B}{RT_B}\right]$$

$$n_f = n_i$$

$$\frac{P}{R}\left[\frac{V_A}{T_A} + \frac{V_B}{T_B}\right] = \frac{1}{R}\left[\frac{P_A V_A}{RT_A} + \frac{P_B V_B}{RT_B}\right]$$

$$P\left[\frac{100\times10^{-6}}{283} + \frac{50\times10^{-6}}{373}\right]$$

$$= \left[\frac{3\times10^5\times100\times10^{-6}}{283} + \frac{10^5\times50\times10^{-6}}{373}\right]$$

$$P = 2.45\times10^5 \text{Pa}$$

(d) For the vacuum pump

$$P_n = P\left[\frac{V}{V+V}\right]^n$$

$P = 3 \times 10^5$Pa, $V = 100$cm^3, $V = 20$cm^3

(i) For one stroke

$$P_1 = 3\times10^5\left[\frac{100}{100+20}\right]^1$$

$$P_1 = 2.5\times10^5\,\text{Pa}$$

(ii) For second stroke

$$P_2 = 3\times10^5\left[\frac{100}{100+20}\right]^n$$

$$P_2 = 2.1\times10^5\,\text{Pa}$$

(iii) The number of n strokes to reduce the pressure to 10^3Pa

$$10^3 = 3\times10^5\left[\frac{100}{100+20}\right]^n \text{ on solving}$$

$$n = 31 \text{ stroke (approx.)}$$

REAL GASES

Definition of Real gases

This is the gas which obeys Van Waals equation at the higher pressure and low temperature and deviate from ideal behavior i.e **real gas** is the gas which there are intermolecular force of attraction and volume of gas is not negligible compared to the volume of the container.

WHY REAL GASES DEVIATE FROM IDEAL BEHAVIOUR?

At higher pressure low temperature and low volume, the two assumptions of the kinetic theory of an ideal gas should be neglected namely:

(i) Intermolecular force of attraction should be neglected.

(ii) Volume of gas molecules is neglected compared to the volume of container.

Therefore a gas normally is real gas and thus for real gas:-

1. The intermolecular force of attraction may not be neglected are put into significant consideration. So in a real gas, the molecules attract each other, hence there is a pressure factor due to attraction of considered.
2. Volume occupied by a gas molecule is neglected compared to the volume of container i.e the volume of gas molecules is also significantly considered.

Therefore these two basis criterions are usefully in dealing with real gases and for ideal gas these statement are put into negation.

Van – der Waals observation of real gases

Correction of the pressure.

In view of real gases at higher pressure, low pressure and low volume, the molecules are closer and transaction forces play part and in fact repulsive force which give an extra pressure P1 on the walls say a/v^2 were a being constant and v being volume of an ideal gas which is the part of the pressure.

Total pressure = P + P_1

Since $P_1 \alpha \rho^2$ and $\rho^2 \alpha 1/v^2$ for the mole of gas

$$P_1 \alpha \frac{1}{V^2}, \quad P_1 = \frac{a}{V^2}$$

$$\text{Total pressure (bulk pressure)} = P+\frac{a}{V^2}$$

In actual practice, at high pressure the size of molecules of the gas becomes significant and cannot be neglected in the comparison to the volume of the container. Therefore the intermolecular force should not be neglected.

Correction of the volume

If b is the volume occupied by molecules of real gases at higher pressure, then the free volume of molecules of the Van der Waals equation is given by n by v – b

Generally the equation of the state is given by

(Total pressure) (free volume of gas molecule) = RT

For n = 1 mole of a gas

$$\left(P+\frac{an^2}{V^2}\right)(V-nb) = nRT$$

Where P = Pressure of the gas

V = Volume of the gas

n = Number of the moles

R = Universal gas constant

T = Absolute temperature

Note that

1. The correction of a pressure of a gas depends
   (a) The number of molecules striking unit area of the wall of the container per second.
   (b) The number of molecules present in given volume.
2. If attractive forces, between molecules are not negligible. The molecules approaching the container walls are attracted by the molecules behind them and experience resultant force inward (i.e away from the wall). Due to this reason and reduces momentum of the approaching molecules and hence the pressure. The observed pressure P is less than ideal gases pressure P_1.

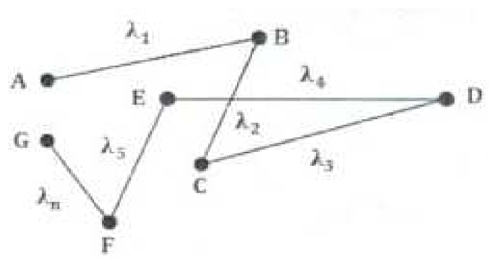
MEAN FREE PATH OF THE GAS (λ)

The molecules of a gas hence a finite size and behave like hard sphere. Even at ordinary temperature, the molecules posses large velocities; since due to the molecular motion is random therefore the molecules frequently collide against one another. In each collision, there is a change in the magnitude and direction of the velocity of molecules between two successive collision a molecule moves along a straight path with uniform velocity. This path is known as **'free path'**.

DEFINITION MEAN FREE PATH (λ) OF THE GAS. Is the average distance travelled by the molecule between two successive collisions.

Mathematically

Imagine a molecules starting from A and suffering collisions at B, C, D, E, F and G.



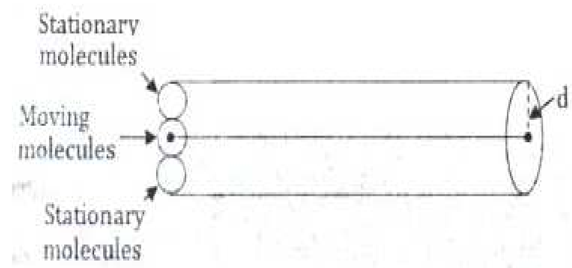
$$\lambda = \frac{\text{Total distances travelled}}{\text{Number of successive collisions of gas}}$$

Let $\lambda_1, \lambda_2, \lambda_3, \lambda_4 \ldots\ldots\lambda_n$ to be the free paths travelled by the molecules in n successive collisions. Then the men free path of a gas is given by.

$$\lambda = \frac{\lambda_1 + \lambda_2 + \lambda_3 + \lambda 4 + \ldots\ldots + \lambda_n}{n}$$

DERIVATION OF EXPRESSION FOR MEAN FREE PATH

(i) The gas molecules inside of the container are rigid sphere of diameter, d
(ii) Assume that only one molecule of a gas is in motion while the other molecules of the gas remaining stationary.



Let N = Number of collusions (molecules)
n = Number of molecules per unit volume

$$n = \frac{N}{V} = \frac{N}{\pi d^2 L}$$

d = Inter molecular diameter

$$\text{Now } \lambda = \frac{\text{Distance travelled}}{\text{Number of collusions}} = \frac{L}{N}$$

But N = πd2Ln

$$\lambda = \frac{L}{\pi d^2 Ln} = \frac{L}{\pi n d^2}$$

In the above derivation, we have assumed that all, but molecule is in motion. but this assumption is not corret. All the molecules are in random motion . so we chance of collision of the molecules are grater. Taking this into account, the mean free path can be shown to be $\sqrt{2}$ time less than that given equation above.

∴ The correct expression of the mean free path is given by $\lambda = \dfrac{1}{\sqrt{2}\pi n d^2}$

DIFFERENT FORMS OF THE EXPRESSION OF THE MEAN FREE PATH.

1. $\lambda = \dfrac{1}{\sqrt{2}\pi nd^2} = \dfrac{1}{4\pi\sqrt{2}nr^2}$

   r = radius of the gas molecules
2. From the basis of kinetic theory of the gas

   $PV = NKT$

   $n = \dfrac{N}{V} = \dfrac{P}{KT}$

   Now $\lambda = \dfrac{1}{\sqrt{2}\pi\left(\dfrac{N}{V}\right)d^2} = \dfrac{1}{\sqrt{2}\pi\left(\dfrac{P}{KT}\right)d^2}$

   $\lambda = \dfrac{KT}{\sqrt{2}\pi Pd^2} = \dfrac{KT}{4\sqrt{2}\pi Pd^2}$

   T = Absolute temperature

   K = Boltzmann's constant

3. mn = mass of per unit volume of the gas and represent the density of the gas ρ.

   $\lambda = \dfrac{1}{\sqrt{2}\pi d^2} = \dfrac{m}{\sqrt{2}\pi mnd^2}$

   $\lambda = \dfrac{m}{\sqrt{2}\pi\rho d^2}$

4. $\lambda = \dfrac{V}{\sqrt{2}\pi d^2}$

5. Mean free path of the gas at S.T.P = VSTP, $N = N_A$ for one mole of a gas.

   $n = \dfrac{N}{V} = \dfrac{N_A}{V_{S.T.P}}$

   Now : $\lambda = \dfrac{1}{\sqrt{2}\pi\left(\dfrac{N_A}{V_{S.T.P}}\right)d^2}$

ADDITION CONCEPTS

The following points may be noted under the mean free path:

1. $\lambda \,\alpha\, 1/n$. The smaller the number of molecules per unit volume of the gas the large is the mean free path.
2. The smaller the diameter, the large is mean free path $\lambda \,\alpha\, 1/d^2$.
3. The smaller density, the large is mean free path, in this case for the vacuum $\rho = O$, $\lambda = \infty$
4. The smaller the pressure of a gas the larger the mean free path $\left(\lambda \,\alpha\, 1/P\right)$
5. The higher the temperature of a gas, the larger is the mean free path of the gas ($\lambda \,\alpha\, T$)

MEAN FREE PATH AND VISCOSITY OF THE GAS

Using the basic definition of viscous drag force

$$F = -nA\frac{dv}{dx}$$

n_A = coefficient of viscous

A = Area

The pressure exerted by the gas

$$P = \frac{F}{A} = \frac{n_A \frac{dv}{dx}}{A}$$

$$P = n\frac{dv}{dx} \quad \text{.........(i)}$$

From the basic of the kinetic theory of the gas

$$P = \frac{1}{3}nm\bar{c}^2 = \frac{1}{3}\rho\bar{c}^2$$

(i) = (ii)

$$n\frac{dv}{dx} = \frac{1}{3}\rho\bar{c}^2$$

Let : $\dfrac{dv}{dx} = \dfrac{C_{r.m.s}}{\lambda}$

$$\eta = \frac{C_{r.m.s}}{\lambda} = \frac{1}{3}\rho\bar{c}^2$$

$$\eta = \frac{\rho\lambda\bar{c}^2}{3C_{r.m.s}}$$

But $C_{r.m.s} = \sqrt{\bar{c}^2}$

$$C_r^2 = \bar{c}^2$$

$$\eta = \frac{\delta\lambda C_{r.m.s}^2}{3C_{r.m.s}}$$

$$\eta = \frac{1}{3}\rho\lambda C_{r.m.s}$$

DEFINITION COLLISION FREQUENCY (F)

Is defined as the number of collision made by the molecules per second

$$f = \frac{\bar{c}}{\lambda}$$

$\bar{c}$ = Mean speed (velocity) of gas

λ = mean free path

$$\text{Now } \frac{1}{\lambda} = \sqrt{2}\pi n d^2 = \frac{\sqrt{2}\pi p d^2}{KT}$$

$$f = \sqrt{2}\pi \bar{c} n d^2 = \frac{\sqrt{2}\pi p \bar{c} d^2}{KT}$$

FACTORS AFFECTING MEAN FREE PATH AND COLLISION FREQUENCY

1. **EFFECT OF TEMPERATURE**

   The mean free path is directly proportional to the absolute temperature of the gas. If other factors are kept constant

$$\lambda \alpha T$$

$$\frac{\lambda_1}{\lambda_2} = \frac{T_1}{T_2}$$

   On the other side, collision frequency is inversely proportional to the absolute temperature of the gas i.e $f \alpha \frac{1}{T}$.

2. **EFFECT OF THE PRESSURE**

   The higher pressure exerted by the gas molecules implies the smaller the volume of the molecules. Then the more and more closer to the molecules of the gas, the number of collision per second becomes high which is in turn of reduce the mean free path of the gas molecules.

$$P = \frac{1}{\lambda}\left|\frac{P_1}{P_2} = \frac{\lambda_2}{\lambda_1}\right|$$

$$\text{Since } P = \frac{1}{3}\frac{N}{V}m\bar{c}^2 \left(P \alpha N\right)$$

$$\text{Then } N \alpha \frac{1}{\lambda}$$

$$\text{Also } f = \frac{\sqrt{2}\pi p \bar{c} d^2}{KT}\left(f \alpha P\right)$$

3. EFFECT OF VISCOSITY OF THE GAS

$$\text{Since } \lambda = \frac{3\eta}{\rho C_{r.m.s}} \text{ or } \lambda = \frac{1}{3}\lambda\rho\bar{c}^2$$

$$\eta \alpha \lambda$$

THE DEGREE OF FREEDOM (F)

The term degree of freedom 'may be defined in the following three ways.

(i) Degree of freedom – is defined as the total number of independent way the gas molecules can possessing energy.

(ii) Degree of freedom – is the total number of possible independent ways in which the position and configuration of a mechanical system may change.

(iii) Degree freedom – is the total number of coordinates or independent quantities required to completely specify the position and configuration (arrangement of constitute particles in space) of a dynamical system.

The degree of freedom of the gas (system) is given by

$f = 3N - K$

f = Degree of freedom

N = Number of particle in the system

K = Number of independent relations among the particles.

DEFINITION THERMAL AGITATION

Is the random motion of gas molecules due to the kinetic energy of the gas molecules which depends on the temperature.

INTERNAL ENERGY, U

Is the total amount of the energy within the body i.e internal energy of any gas is defined as the sum of the potential energy and kinetic energy of the gas. The internal energy of the gas molecules can be depends on the kind of the molecules which the gas consists and absolute temperature.

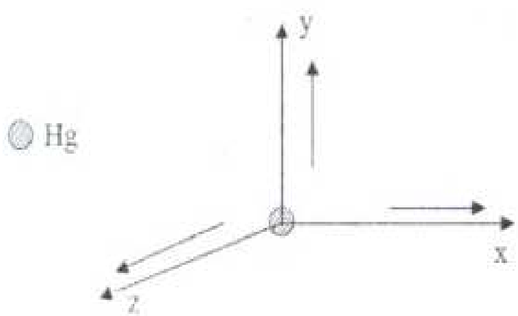
- The thermal energy of the gas is the 'differential' since it can be depends only on the absolute temperature of the gas. sometimes, the internal energy of the gas is defined as the kinetic energy of gas molecules due to the thermal agitation.

TYPES OF GAS MOLECULES

1. Monatomic gas
2. Diatomic gas
3. Polyatomic gas

1. MONATOMIC GAS

Is the gas molecules which consists of a single atom example. Chemical inert gas and metallic vapour mercury sodium (Na) Helium (He) Neon (Ne) gas etc. structure of Monatomic gas



The internal energy of a gas is due to the translational kinetic energy and rotational kinetic energy.

$$K.\overline{E} = K.e_T + K.e_R$$

The moment of the inertia of a monatomic gas is very negligible thus the rotational kinetic energy can be neglected i.e approximately equal to zero. In other words the internal energy of monatomic gas is due to only translational kinetic energy.

- The monatomic gas has three (3) degree of freedom due to the translational only.

Total degree of freedom = degree of freedom of translation + degree of freedom of rotation

$F = 3 + 0 = 3$

Alternative: By using the equation $f = 3N - K$

$N = 1, K = 0$

$F = 3 \times 1 - 0$

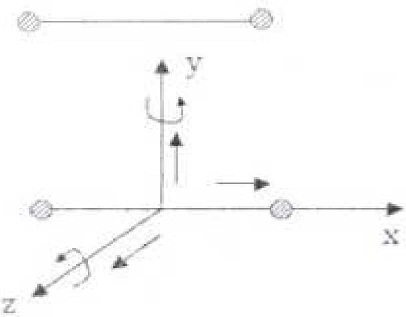
$F = 3$

2. DIATOMIC GAS

Is the gas which consists of two atoms only

Example : Oxygen gas (O_2), Hydrogen gas (N_2), Chloride gas (Cl_2), Carbon monatomic (CO) etc.

Structure of diatomic gas



- The molecules of a diatomic gas have translational as well as rotational kinetic energy.
- The degree of freedom due to the rotational kinetic energy of two (2) while due to the translational kinetic energy is three (3)
- Total degree of freedom

$f = 3 + 2$

$f = 5$

Alternative:

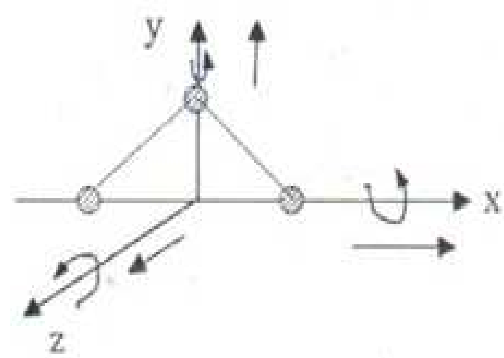
$N = 2, K = 1$

$f = 3N - K = 3 \times 2 - 1$

$f = 5$

3. POLYATOMIC GAS

Is the gas molecules which consists of more than two atomic. Examples: water vapor (H_2O), Ozone (O_3), Sulphur dioxide (H_2S), Carbon dioxide (CO_2) , Methane (CH_4) e.t.c.



A poly atomic molecules, unless, it happens to consists of molecules all in a straight line has no axis about which moment of inetia is negligible. Thus have kinetic energy of rotation about the three mutually perpendicular axes.

- It has three degree of freedom due to the rotational kinetic energy plus three degree of freedom due to the translational kinetic energy. Therefore, the total degree of freedom for polyatomic gas is six (6).
  Alternative
  For Tri atomic gas
  N = 3 , K = 3
  $f = 3N - K = 3 \times 3 - 3$
  $f = 6$

Note that

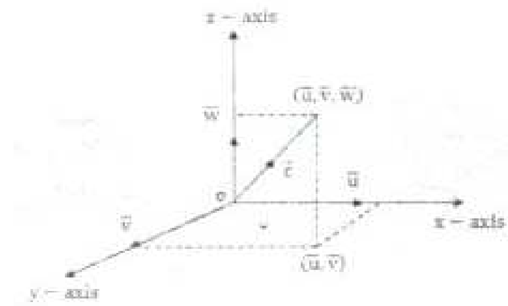
1. The molecules of a diatomic or polyatomic gas have appreciable amount of moment of inertia because their structure appreciably exerted in space. Thus the molecules of diatomic and polyatomic gas have translational as well as rotational kinetic energy due to the rotational energy as well as translational kinetic energy.
2. At very high temperature (above 500K), a gas molecules possesses vibratory motion also in addition to translator and rotatory motion. at ordinary temperature possesses translational and rotational motion only

PRINCIPLE (LAW) OF EQUIPARTION OF ENERGY

State that ‘for a dynamical system in the thermal equilibrium, the energy of the system is equally distributed amongst the various degree of freedom and the energy associated with each degree of freedom per molecule is ½ KT, where K is the Boltzmann’s constant’ i.e The average kinetic energy of molecules in each degree of freedom (translational as well as rotational) is the same and is equal to ½ KT where K is Boltzmann’s constant and T absolute temperature’.

Derivation: $\frac{1}{2}m\bar{u}^2 = \frac{1}{2}m\bar{v}^2 = \frac{1}{2}m\bar{w}^2 = \frac{1}{2}KT$

Consider the rotational as well as translational motion of a gas molecules in the directions x, y and z axes whose velocities are $\bar{u}$, $\bar{v}$ and $\bar{w}$ respectively.



By using Pythagoras theorem

$$\bar{c}^2 = \bar{u}^2 + \bar{v}^2 + \bar{w}^2$$

Since the molecules of a gas do not pile up any corner of the vessel containing the gas, their average velocities in all directions must be the same.

$$\bar{u}^2 = \bar{v}^2 = \bar{w}$$

$$\bar{c}^2 = 3\bar{u}^2 = 3\bar{v}^2 = 3\bar{w}^2$$

From the basis of kinetic theory of gases

$$\frac{1}{2}m\bar{c}^2 = \frac{3}{2}KT$$

$$\frac{1}{2}m\left(3\bar{u}^2\right) = \frac{3}{2}KT$$

$$\frac{1}{2}mu^2 = \frac{1}{2}KT$$

$$\therefore \frac{1}{2}m\bar{u}^2 = \frac{1}{2}m\bar{v}^2 = \frac{1}{2}m\bar{w}^2 = \frac{1}{2}KT$$

$\therefore$ Average kinetic energy of gas molecules in each of freedom $= \frac{1}{2}KT$

Average kinetic energy of gas molecules for the f – degree of freedom $K.\bar{E} = \frac{1}{2}fKT$

VALIDITY OF PRINCIPLE OF EQUIPARTITION OF ENERGY

The law is valid at the room temperature and above. At very low temperature the gas is near liquification its fail.

EXPRESSION OF INTERNAL ENERGY OF GAS MOLECULES

U = Internal energy
N_A = Avogadro’s number

$$U = N_A K.\bar{E}$$

But $K.\bar{E} = K\bar{e}_R + K\bar{e}_R$

1. **FOR MONATOMIC GAS MOLECULE**

$$K.\bar{E}_R = 0$$

$$K.\bar{E}_T = \frac{3}{2}KT$$

$$K.\bar{E} = K\bar{e}_R + K\bar{e}_R$$

$$K.\bar{E} = 0 + \frac{3}{2}KT = \frac{3}{2}KT$$

Now $U = N_A K.\bar{E} = N_A\left(\frac{3}{2}KT\right)$

But $N_A K = R$

$$U = \frac{3}{2}RT \text{ or } du = \frac{3}{2}RdT$$

2. **FOR DIATOMIC GAS MOLECULES**

$$K.\bar{E} = K\bar{e}_R + K\bar{e}_R$$

$$= \frac{3}{2}KT + \frac{3}{2}KT$$

$$K.\bar{E} = \frac{5}{2}KT$$

$K.\bar{E}$ = Average k.e molecules

Now $U = N_A K.\bar{E}$

$$= N_A\left[\frac{5}{2}KT\right] \text{ OR } U = \frac{5}{2}KT$$

$$dU = \frac{5}{2}RdT$$

3. **FOR POLYATOMIC GAS**

$$K.E = \frac{3}{2}KT + \frac{3}{2}KT = \frac{6}{2}KT$$

Now : $U = N_A K.\bar{E}$

$$U = N_A\left[\frac{6}{2}KT\right] = \frac{6}{2}RT$$

$$U = \frac{6}{2}RT = 3RT$$

$U = 3RT$ OR $dU = 3RT$

R = Universal gas constant

dT = Temperature

NUMERICAL EXAMPLES

39. (a) Equal volume of Argon and zone initially are at the same temperature θ_0 and then heated to temperature θ. What will you note about the internal energies. Explain why?.

(b) Define the term collision frequency and explain how it varies with temperature of the gas.

Solution

(a) At temperature θ_0 the ozone and Argon will have equal internal energy since ozone and Argon have the different number of atoms. Argon has less internal energy rather than ozone because Argon is monatomic gas while ozone is the polyatomic gas.

40. (a) Estimate the mean heat free path of a molecules of air at 27°C and 1 atm. Model the molecules as spheres with radius 2×10^{-10}m.

(b) Estimate the mean free time of oxygen molecule with V = Cr.m.s at 27°C and 1 atmosphere.

Solution

T = 300k, P = 1atm = 1.01 × 105Pa, V = Vr.m.s , r = 2 × 10-10m.

(a) $$\lambda = \frac{KT}{4\pi\sqrt{2}pr^2}$$

$$\lambda = \frac{1.38\times10^{-23}\times300}{4\pi\sqrt{2}\times1.01\times10^5\times\left(2\times10^{-10}\right)^2}$$

(b) $$V_r = \bar{C} = \sqrt{\frac{3RT}{Mr}}$$

$$\bar{c} = f\lambda$$

$$\bar{c} = \frac{\lambda}{t}$$

$$t = \frac{\lambda}{\sqrt{\frac{3RT}{Mr}}} = \lambda\sqrt{\frac{Mr}{3RT}}$$

$$t = 5.8\times10^{-8}\sqrt{\frac{32\times10^{-3}}{3\times8.314\times300}}$$

$$t = 1.2\times10^{-10}\text{ sec}$$

41. (a) Cooking gas containers are kept in a lorry moving with uniform speed. What will be the effect on the temperature of the gas molecules inside?

(b) Estimate the mean free path and collision frequency of nitrogen molecules in a cylinder containing nitrogen at 2 atm and temperature 17^oC. Take the radius of nitrogen molecules to be 1A. Molecular mass of nitrogen = 28.

Solution

(a) As the long is moving at a uniform speed, the translational motion of the gas molecules will not be effected. Hence the temperature of the gas molecules will remain the same.

(b) P = 2atm = $2.026 \times 10^5 Nm^{-2}$

T = 17 + 273 = 290K

Mr = 28×10^{-3}kg, d = $1 \times 2\,\mathring{A}$, d = 2×10^{-10}m

Now, mass of nitrogen molecules

$$m = \frac{Mr}{N_A} = \frac{28\times10^{-3}}{6.02\times10^{23}}$$

$$m = 4.65\times10^{-26}$$

Also volume occupied by the nitrogen gas

$$V = \frac{RT}{P} = \frac{8.31\times290}{6.02\times10^{23}}$$

$$V = 1.19\times10^{-2}m^3$$

Density of nitrogen gas

$$\rho = \frac{M}{V} = \frac{28\times10^{-3}}{1.19\times10^{-2}}$$

$$\rho = 2.353kgm^{-3}$$

$$\text{Now } \lambda = \frac{m}{\sqrt{2}\pi d^2\rho}$$

$$= \frac{4.65\times10^{-26}}{\sqrt{2}\times3.14\times\left(2\times10^{-10}\right)^2\times2.353}$$

$$\lambda = 1.11\times10^{-7}m$$

$$\text{Now } V_{r.m.s} = \sqrt{\frac{3p}{s}} = \sqrt{\frac{3\times2.02\times10^5}{2.353}}$$

$$V_{r.m.s} = 508.24m/s$$

Collision frequency

$$f = \frac{V_{r.m.s}}{\lambda} = \frac{508.24}{1.11\times10^{-7}}$$

$$f = 4.58\times10^9 s^{-1}$$

42. Find
(i) The mean free path and
(ii) Collision frequency of nitrogen molecules at temperature of 20^oC and pressure of 1atm. Assume a molecular diameter of 2.0 × 10^{-10}m. Given that the average speed of nitrogen molecule at 20^oC is 511m/s.

Solution

$$P = 1atm = 1.01\times10^5 Nm^{-2}$$

$$K = 1.38\times10^{-23}JK^{-1} \;,\; T = 293K$$

$$d = 2\times10^{-10}m$$

(i) Mean free path

$$\lambda = \frac{KT}{\sqrt{2}\pi p d^2}$$

$$= \frac{1.38\times10^{-23}\times293}{\sqrt{2}\pi\times1.01\times10^5\times\left(2\times10^{-10}\right)^2}$$

$$\lambda = 2.25\times10^{-7}m$$

(ii) $$f = \frac{\bar{c}}{\lambda} = \frac{511}{2.25\times10^{-7}}$$

$$f = 2.27\times10^9 \text{collisions per sec}$$

43. Estate the mean free path of air molecules at S.T.P the diameter of O_2 and N_2 molecules is about 3.0 × 1010

Solution

At S.T.P 1 mole of ideal gas occupies a volume of 22.4 × 10^{-3}m

$$n = \frac{N_A}{V} = \frac{6.02\times10^{23}}{22.4\times10^{-3}}$$

$$n = 2.69\times10^{25}\text{moleculles}/m^2$$

Since

$$\lambda = \frac{1}{\sqrt{2}\pi d^2}$$

$$= \frac{1}{\sqrt{2}\times3.24\times2.69\times10^{25}\times\left(3\times10^{-16}\right)^2}$$

$$\lambda = 9.0\times10^{-8}m$$

44. (a) What is meant by the internal energy
(c) Define the mean free path of a molecule
(d) Calculate the mean free path of a gas molecule having a collision diameter of 2A^O at a standard temperature and pressure [Na = 6.02 × $10^{23}mol^{-1}$]

Solution

(a) And (b) refer to your notes

(c) $\lambda = \frac{1}{\sqrt{2}\pi d^2} = \frac{1}{\sqrt{2}\pi\left(\frac{N_A}{V}\right)d^2}$

$$\lambda = \frac{V}{\sqrt{2}\pi N_A d^2}$$

$$= \frac{22.4\times10^{-3}}{\sqrt{2}\times3.14\times6.02\times10^{25}\times\left(2\times10^{-16}\right)^2}$$

$$\lambda = 2.09\times10^{-7}\,m$$

45. The pressure and temperature of top of Mount Kilimanjaro are 3.26×10^4Pa and 250K respectively while at DSM are 1.01×10^5Pa and 300K. The mean free path of Nitrogen in DSM is 1.6×10^{-7}m. What is the mean free path at the mountain Kilimanjaro?

Solution

At mountain Kilimanjaro

$P_1 = 3.26\times10^4 P_a P_2 = 1.01\times10^5 P_a$

$T_1 = 250K$, $T_2 = 300K$

$\lambda_1 = ?$, $\lambda_2 = ?$

Since $\lambda = \frac{KT}{\sqrt{2}\pi P_1 d^2}$, $\lambda_2 = \frac{KT}{\sqrt{2}\pi P_2 d^2}$

$$\frac{\lambda_1}{\lambda_2} = \frac{\frac{KT_1}{\sqrt{2}\pi P_1 d^2}}{\frac{KT_2}{\sqrt{2}\pi P_2 d^2}}$$

$$\frac{\lambda_1}{\lambda_2} = \left(\frac{T_1}{T_2}\right)\left(\frac{P_2}{P_1}\right)$$

$$\lambda_1 = \lambda_2\left(\frac{T_1}{T_2}\right)\left(\frac{P_2}{P_1}\right)$$

$$= 1.6\times10^{-7}\left[\frac{250}{300}\right]\left[\frac{1.01\times10^5}{3.26\times10^4}\right]$$

$$\lambda_1 = \text{-------}\,m$$

46. Calculate the diameter of a molecule if $n = 2.79\times10^{25}$ molecules per m3 and mean free path is $2.2\times10^{-8}\,m$ (Answer d = 0.606nm)

47. (i) Define the mean free path for the molecule of the gas
    (ii) How the mean free path for the molecule of a gas is affected by the temperature?

48. (a) (i) What is meant by the mean free path of a gas molecule .
    (ii) Show that the mean free path of a gas and absolute temperature T is given by $\lambda = \frac{K_B T}{\sqrt{2}\pi P d^2}$ where KB is the Boltzman's constant and d the molecular diameter.

    (b) (i) Derive an expression for the average kinetic energy of one molecule of a gas, assuming the formula for the pressure of an ideal gas
    (ii) Explain how a gas exerts pressure in any case.

    (c) A cylinder of volume $2 \times 10^{-3}m^3$ containing a gas at pressure $1.5 \times 10^6Nm^{-2}$ and at temperature of 300K calculate.
    (i) The number of mole of the gas
    (ii) The number of molecules of the gas contains.
    (iii) The mass of the gas if its molar mass is 32×10^{-3}Kg
    (iv) The mass of the one molecule of the gas.

    Answer

    (c) (i) 1.2mole
    (ii) 7.24×10^{23} molecules
    (iii) 38.4×10^3kg
    (iv) 5.5×10^{-16}kg

49. (a) (i) Write down the Van der Waals equation and define each term in its usual meaning.

    (b) (i) On the gases, show that two theory of gases, show that two different gases at the same temperature will have the same average value of the kinetic energy of the molecules.

(ii) Determine the r.m.s speed of air at S.T.P given that the density of air is $1.29kgm^{-3}$ density of mercury is $13600kgm^{-3}$ and the barometer height is 760mmHg

(c) Define the mean free path λ of the gas molecule and state how it is affected by the temperature.

(d) If the mean free path of molecules of air at $0^{o}C$ and 1.0 atmospheric pressure and $27^{o}C$?

50. NECTA 2008

(a) (i) State the two (2) assumptions necessary for a real gas that we used to develop the expression $\left(P+\frac{a}{v^2}\right)(v-b) = RT$

(ii) What is the difference between vapour and gas

(iii) Helium gas occupies volume of $0.04m^3$ at a pressure of $2.0 \times 10^5Nm^{-2}$ and temperature of 300K. Calculate the mass of helium and the root mean square speed of its molecules.

(b) (i) Calculate the mass of air containing in $50cm^3$ flask at $9.33 \times 104Nm^{-2}$ and $20^{o}C$. Assume that air composition is approximately 80% of nitrogen and 20% of oxygen by mass and molar masses of nitrogen and oxygen are 28gm and 32gm respectively.

(ii) Two vessels A and B of equal volume are connected by tube of negligible volume. The vessel contain a total mass of $2.5 \times 10^{-3}kg$ of air and initially both vessels are at $1.01 \times 10^5Nm^{-2}$ vessels A is cooled to $0^{o}C$ and vessels B is heated to $100^{o}C$. Calculate the new masses of the gas in each vessel.

(c) (i) Show that the mean free path of molecules in an ideal gas can be written in terms of the pressure as where all symbols carry their usual meaning. $\lambda = \frac{KT}{4\sqrt{2}\pi\rho P}$

(ii) Assuming the air to be composed of spherical nitrogen molecules with radius $1.7 \times 10^{-10}m$. Find the mean free path of air molecules under standard condition.

Solution

(a) (i) see your notes

(ii) Gas is the substance which is in the gaseous phase and is above its critical temperature while vapour is the substance which is in the gaseous phase and is below its critical temperature. A vapour can be liquefied simply by increasing the pressure, a gas cannot.

(iii) $m = \frac{PVMr}{RT} = \frac{2\times10^5\times0.04\times0.04}{8.3\times300}$

$m = 12.8\times10^5 kg = 12.8gm$

$C_{r.m.s} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3\times2\times10^4\times0.04}{12.8\times10^{-3}}}$

$C_{r.m.s} = 136m/s$

(b) (i) For a mixture of gas in a container the total pressure is equal to the sum of individual pressure of the constituent gases (Dalton's law of partial pressure).

Let: $P = P_N + P_0$ ……………….(i)

P_N = Pressure due to nitrogen

P_0 = Pressure due to oxygen

Now $P_N = \frac{nRT}{V} = \frac{M_N RT}{M_{rN} V}$ …….(ii)

M_N = Mass of nitrogen

M_{rN} = Molar mass of nitrogen

$P_0 = \frac{nRT}{V} = \frac{M_0 RT}{M_{r0} V}$ …………..(iii)

Putting equation (ii) and (iii) into (i)

$$P = \frac{M_N RT}{M_{rN} V} + \frac{M_0 RT}{M_{r0} V}$$

$$9.33\times10^4 = \frac{M_N RT}{M_{rN} V} + \frac{M_0 RT}{M_{r0} V}$$

$$9.33\times10^4 = \left(\frac{M_N}{M_{rN} V} + \frac{M_o}{M_{rO}}\right)\frac{RT}{V} \quad ….(iv)$$

But we are given that the mass composition of nitrogen and oxygen is 80% to 20% respectively

$$\frac{M_N}{M_0} = \frac{80}{20} \rightarrow M_N = 4M_0$$

$$\text{Now } \frac{RT}{V}\left[\frac{4M_0}{M_{rN}}+\frac{M_0}{M_{r0}}\right] = 9.33\times10^4$$

$$M_0 = \frac{9.33\times10^4 V}{RT\left[\frac{4}{M_{rN}}+\frac{1}{M_{r0}}\right]}$$

$$M_0 = \frac{9.33\times10^4 50\times10^{-3}}{8.31\times293\left[\frac{4}{28\times10^{-3}}+\frac{1}{32\times10^{-3}}\right]}$$

$$M_0 = 0.011\text{kg}$$

Also $M_N = 4M_0 = 0044\text{kg}$

Total mass of air $= M_N + M_0$

$M = 0.055\text{kg}$

(ii) See solution of example 30

(c) See your notes

51. A hot air balloon stays aloft because hot air at atmospheric pressure is less dense than cooler at air at the same pressure. If the volume of the balloon is 500m³ and the surrounding air is at 15°C, what must be the temperature of the air in the balloon be for it to lift a total load of 290Kg (in addition to the mass of the hot air)? The density of air at 15°C and atmospheric pressure is 1.2Kgm-3 and 1.02 × 10⁵Pa.

Solution

$V_0 = 500\text{cm}^3$, $T_1 = 288\text{k}$, $M_L = 290$,

$T_2 = ?$ $\rho = 1.23\text{Kgm}^{-3}$.

At the equilibrium

Total weight of air of the balloon + load = up thrust.

$M_{ag} + Mg = Up$

M_a = Mass of air inside of the balloon

M_L = Mass of the load

$M_a = \rho_2 V_0$

ρ_2 = Density of air outside of the balloon

$\rho_2 V_0 g + M_L g = \rho_1 V_b g$

$$\rho_2 = \rho_1 - \frac{M_L}{V_b}$$

Since $PV = nRT = \frac{M_0}{Mr}RT$

$$R = \left(\frac{m}{v}\right)\frac{RT}{Mr} = \frac{\rho RT}{Mr}$$

$$\rho \,\alpha\, \frac{1}{T}$$

$$\rho = \frac{K}{T}, \quad \frac{\rho_2}{\rho_1} = \frac{T_1}{T_2}$$

$$T_2 = T_1\left[\frac{\rho_1}{\rho_1 - \frac{M_L}{V_b}}\right]$$

$$= 288\left[\frac{1.23}{1.23-\frac{290}{500}}\right]$$

$T_2 = 549 = 272^0\text{C}$

52. A cylinder 1.00m tall with inside diameter 0.120m is used to hold propane gas (molar mass 44.1g/mol) for use in a barbecue. It is initially filled with gas until the gauge pressure is $1.3 \times 10^6 P_a$ and the temperature is 22°C. The temperature gas remains emptied out of the tank, until gauge pressure is 2.5 × 10⁵Pa. calculate the mass of propane that has been used.

Solution

$P_1 = 1.3\times10^6\text{Pa}$, $T_1 = 295\text{k}$

$P_2 = 2.5\times10^5\text{Pa}$, $\Delta M = ?$

Volume of the cylinder

$$V_1 = Ah = \frac{\pi d^2 h}{4}$$

$$V_1 = \frac{3.14(0.12)^2}{4}\times1 = 0.011\text{m3}$$

For ideal gas

$$PV = \frac{mRT}{Mr}$$

$$P = \left(\frac{RT}{VMr}\right)^m (P \,\alpha\, m)$$

$$\frac{P_2}{P_1} = \frac{M_2}{M_1}$$

$$M_2 = \left(\frac{P_2}{P_1}\right)M_1$$

Since $\Delta M = M_1 - M_2$

$$= M_1 - \frac{P_2}{P_1}M_1$$

$$\Delta M = M_1\left[1 - \frac{P_2}{P_1}\right]$$

Now $P_1V_1 = \frac{M_1RT}{Mr}$

$$M_1 = \frac{P_1V_1Mr}{RT}$$

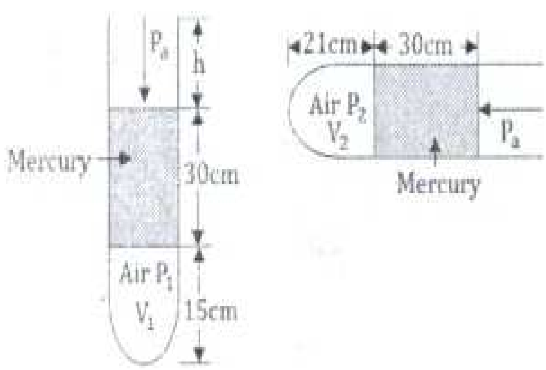
$$\Delta M = \frac{P_1V_1Mr}{RT}\left[1 - \frac{P_2}{P_1}\right]$$

$$= \frac{1.3\times0.011\times44.1\times10^{-3}}{8.314\times295}\left[1 - \frac{2\times10^5}{1.3\times10^6}\right]$$

$$\Delta M = 0.21Kg$$

53. A uniform narrow tube with one end closed has a 30cm long thread of mercury enclosing a column of air at the closed end. When the tube is held vertically with the open end up, the length of the air column is 15cm and when it held horizontally the length is 21cm. find the atmospheric pressure in cm of mercury.

Solution



Let be the height of atmospheric pressure in cm of mercury when the tube is held vertically.

$$P_1 = P_a + h_{Hg}\rho g = h\rho g + h_{Hg}\rho g$$

$$P_1 = \left(h + h_{Hg}\right)\rho g = \left(h+30\right)\rho g \text{ ....(1)}$$

Let A = Area of cross – section of the tube

$V_1 = 15A$(air)

$$P_2 = P_{atm} = \rho gh$$

$$V_2 = A\times21 = 21A$$

Apply Boyle's law

$$P_1V_1 = P_2V_2$$

$$\left(h+30\right)\rho g\times15A = h\rho g\times21A$$

$$15\left(h+30\right) = 21h$$

$$15h + 45 = 21h$$

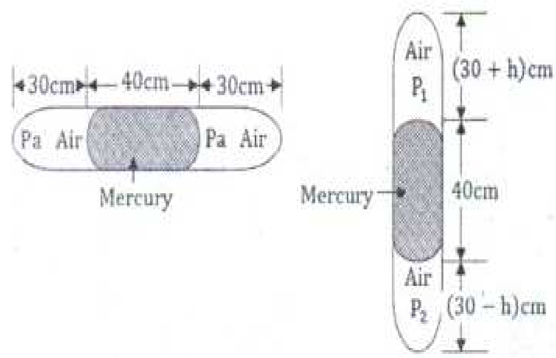
$$h = 75cm$$

$\therefore$ The atmospheric pressure is 75cm of mercury.

54. A thin tube sealed at both ends is 100cm long. It is kept horizontally, the middle 40cm containing mercury and the two equal ends containing air at standard atmospheric pressure. The tube is turned and kept in vertical position by what amount will the mercury displaced?

Solution

Let h be the displacement of mercury column when the tube is held in the vertical position.



A = Uniform cross – sectional area of the tube

Apply the Boyle's law

$$P_1V_1 = P_2V_2$$

$$P_1\left(30+h\right)A = P_2\left(30-h\right)A = Pa\times30A$$

$$P_1 = \frac{30Pa}{30+h}$$

$$P_2 = \frac{30Pa}{30-h}$$

Also $P_2 = P_1 + h_{Hg}$

$$\frac{30Pa}{30-h} = \frac{30Pa}{30+h} + h_{Hg}$$

Pa = 76cm of mercury

$$\frac{30\times76}{30-h} = \frac{30\times76}{30+h} + 40$$

Now $h^2 + 114h - 900 = 0$

On solving $h = 7.41cm$

$\therefore$ The mercury column is displaced by 7.41cm

ASSIGNMENT TO THE STUDENTS

55. (a) Explain why a chest type deep freezer (lid at the top) is thought to be more efficient than up right type (door size).

(b) Applying the kinetic theory of gases give explanations on each of the following.

(i) Ether placed in a dish by an open window is found to be at lower temperature than its surrounding.

(ii) If an empty aerosol can is left in strong sunlight it may explode.

(iii) When lighting a gas cooker an explosion is less likely if the match is struck and ten gas turned on than if he gas is turned on first and then the match is struck.

(c) A hand pump is being used to inflate a bicycle tyre in which the air is already 400KPa above atmospheric pressure. The initial volume of air trapped by the piston in the pump is $1 \times 10^{-4}m^3$ at atmospheric pressure and temperature of 27^0C. The volume of air trapped is $2.5 \times 10^{-5}m^3$ at the moment when the one – way valve between the pump and the tyre begins to open. Calculate the temperature of the air in the pump.

(d) A lump of ice of mass 25gm at 0^0C is added to glass containing $200cm^3$ of mineral water at 20^0C. If the thermal equilibrium is reached with the content of the glass at 90^0C.

(i) Calculate the specific latent heat of fusion of ice.

(ii) State any assumption made to arrive at your answer in (d) (i) above.

56. (a) (i) Define the Bulk modulus of a gas

(ii) Find the ratio of the adiabatic bulk molecules of a gas to that of its Isothermal bulk molecules in terms of the specific heat capacities of the gas

(b) (i) State the assumption that are made for the kinetic theory.

(ii) Given a balloon cube of side 10cm containing 10^{22} oxygen molecules at constant pressure having translational speed of 500m/s. calculate the pressure of the gas in mmHg if each molecules has a mass of 5×10^{-26}kg.

57. A bicycle tyres has a volume of $1.2 \times 10^{-3}m^3$ when fully inflicted. The barrel of bicycle pump has a working volume of $9 \times 10^{-3}m^3$. How many strokes of this pump are need to inflate a completely flat tyre (with zero in it) to a total pressure of 3.0×10^5Pa the atmospheric pressure being 10^5Pa.

Solution

$V_0 = 1.2\times10^{-3}m \quad \Delta V_b = 9\times10^{-5}m^3$

$P_T = 3\times10^5 Pa \;,\; Pa = 10^5 Pa$

In order to get the number of strokes need to inflated the tyre we must find he work done by one stroke and then we can get total work done for n strokes.

$$W_0 = Pa\Delta V$$

$$W_T = nW_0 = nPa\Delta V_b$$

But $W_T = P_T\Delta V_T = P_T V_0$

$$nPa\Delta V = P_T V_0$$

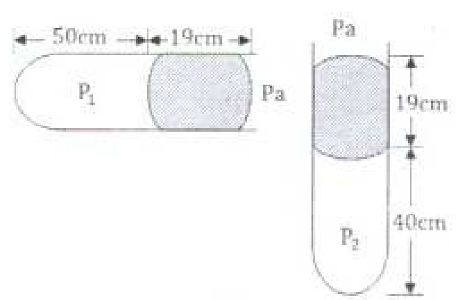
$$n = \frac{P_T V_0}{P_a \Delta V_b} = \frac{3\times10^5\times1.3\times10^{-3}}{10^5\times9\times10^{-5}}$$

$$n = 40 strokes$$

Explain why the barrel of bicycle pump becomes hot when the tyre is being inflated quickly.

58. A narrow glass tube of uniform bore and closed at one end contains air between closed end and mercury thread 19cm long. When the tube is horizontal the air column is 50cm, but when the tube is vertical with the open end uppermost, the column of air is 40cm long. What is the barometric height?

Solution



When the tube is in horizontal

$P_1 = Pa$, $V_1 = 50Acm^3$

When the tube is in vertical position

$P_2 = Pa+h$, $V_2 = 40Acm$

Apply Boyle's law

$P_1V_1 = N_2$

$50Pa = (Pa+19)40$

$Pa = 76$ cm of mercury

59. (a) Why does the cycle tube burst in the summer?
(b) Why is the temperature of air less on peak of hills than that at sea level?
(c) If perfume bottle opened in a room, the essence will spread rapidly but certainly not at a very high speed. Why ?
(d) What is the reason for the difference in temperature of air containing out of a balloon filled under high pressure and that of air outside the balloon?

Solution

(a) Due to increase in pressure of the atmosphere in the summer the temperature of the air molecules in the tube also increases. Therefore the pressure inside the tube increases in accordance with the kinetic theory. When the pressure increases beyond a limit, the tube gets burst.
(b) According to the kinetic theory, the pressure of a gas is directly proportional to tis absolute temperature hence due to increases in pressure the temperature of air also decreases.
(c) The molecules in air travel only a short distance (about 10 times their diameter) before colliding with another molecule. The average velocity of random motion is high but the velocity of one ground of molecules diffusing through a different kind is much lower.
(d) According to the kinetic theory of gases if pressure energy of gas molecules or temperature increases. Hence temperature of sir inside the balloon is more than air outside the balloon. (ABC – Modern Physics).

60. A narrow uniform glass tube 80cm long and open at both ends, the ends is half immersed in mercury. Then the top of the cube is closed and it is taken out of mercury. A column of mercury 22cm long then remains in the tube. What is the atmospheric pressure (answer: 70.7cmHg).

61. An open glass tube is immersed in mercury, so that a length of 8cm of the tube projects above the mercury. The tube is then closed and raise through 44cm. What is the fraction of the tube will be occupied by the air after it has been raised? The atmospheric pressure = 7cmHg (answer 0.2965)

62. A mercury thread of length 10cm is containing the middle of a narrow horizontal tube of 100cm and sealed at the both end. The air in both halves of the tube in under a pressure of 76cm of mercury. What distance will the mercury column move, if the tube in placed vertically?

63. On a day when the temperature was 15^{O}C and atmospheric pressure was 100KPa, a car tyre contained air at a pressure of 180KPa above atmospheric pressure. Assuming no change of the volume pressure in the car tyre as result of the temperature falling to 0^{O}C.

64. A cycle pump of volume 80cm^3 is connected via a valve to an air – tight steel vessel containing 400cm^3 of air at a pressure of 101KPa.
(a) Calculate the pressure of the air in the vessel after one stroke of the pump.
(b) How many strokes of the pump are needed to double the pressure of the air in the vessels.

Answer (a) 121KPa (b) n = 4 stroke

65. Two sealed identical glass bulbs are connected together by means of tubing and valve. With the valve open the two bulb are placed in melting ice and the gas pressure throughout equalizes to initial pressure P_0. The connecting valve is then closes and one bulb is transferred to boating water at 100°C. Calculate the new pressure in terms of P_0 in the hotter bulb. If the valve is opened to allow the pressure to equalize, calculate the final pressure in terms of P0 with one bulb at 100°C and the other one at 0°C. (UND.105) Answer $1.37P_0$,$1.15P_0$).

66. Two identical gas cylinder each contain 20kg of compressed air at 1000KPa pressure and 275K on of the cylinders is fitted with a safety valve which releases air from the cylinders into the atmosphere, if the pressure in the cylinder rises above 1100KPa. The cylinders are moved to a room where the temperature is 310K. Calculate.
    (a) The pressure in the cylinder which is not fitted with a safety valve.
    (b) The mass of gas lost from the other cylinder.
    Answer
    (a) 1127KPa (b) 0.48Kg.

67. A mercury barometer tube of length 1000mm above reservoir level contains as small quantity of dry are in the space above the mercury at the top of the cube. On a day when the atmospheric pressure in 101KPa and the temperature is 20°C, the barometer height 740mm. the next day when the temperature is 15°C, the barometer height is 750mm, calculate the atmospheric pressure on the second day. (A column of mercury of height 60mm, gives a pressure of 101KPa at its base) (UND.10.21 (Answer : 102KPa)

68. (a) Calculate the r.m.s speed of the molecules of oxygen at
    (i) 0°C
    (ii) 100°C the molar mass of oxygen molecules is 0.032kg.
    (b) Calculate the total kinetic energy of 1 mole of oxygen at
    (i) 0°C (ii) 100°C
    Answer
    (a) (i) 461m/s (ii) 539m/s
    (b) (i) 340KJ (ii) 4.64KJ

69. (a) Calculate the r.m.s speed of the molecules of nitrogen gas the molecules of nitrogen gas at 10°C. The molar mass of nitrogen molecules is 0.028kg.
    (b) Explain why a ammonia gas released at one end of a room speeds shows throughout the room even through the air molecules moves at very high speed.
    Answer
    (a) 510m/s

70. (a) A sealed flask of volume 80cm^3 contains argon gas at a pressure of 10KPa and a temperature of 27°C. Calculate the number of molecules of argon gas in the vessel.
    (b) Calculate the r.ms speed of the molecules in the bulb argon has a molar mass of 0.018kg.
    (c) Calculate the pressure and r.m.s speed of the argon molecules in then flask if its temperature is increased to 127°C. (UND. 10.10)
    Answer
    (a) 1.93×10^{20}
    (b) 645m/s
    (c) 744m/s , 13.3KPa

71. A faulty mercury barometer contains a little air and its glass tube is 80cm long when 1cm of the tube is below the mercury the barometer reads 76cm but when 5cm are below it reads 74cm. What is the true valve of the barometer height? (Answer 77cm).

72. At what depth in water would an air bubble just fail to raise? Take atmospheric pressure as 750mmHg and the density of air at that pressure as $1.25kgm^{-3}$. Ignore temperature changes, surface tension and vapour pressure.

Hint

The air bubble just fails to rise when the pressure due to the depth of water is equal to the pressure inside a bubble which is equal to atmospheric pressure.

$$\rho_{Hg}h_{Hg}g = \rho_a g h_W$$

$$h_W = \frac{\rho_{Hg}h_{Hg}}{\rho_a} = \frac{13600\times0.75}{1.25}$$

73. A faulty barometer contains some air above the mercury. On a day when the temperature is $12^{O}C$ and the atmospheric pressure is 755mmHg, this barometer reads 69.8cm and the length of the space above the mercury is 5.5cm. What is the atmospheric pressure on a day when this barometer reads 70.5cm the temperature being $7^{O}C$ and length of space above mercury is 7cm (Answer 750.6mmHg).

74. A long uniform horizontal capillary tube, sealted at one end and open at the other end contains air trapped behind a short column of water. A the length L of the trapped air column at pressure 300K and 360K is 10cm and 30cm respectively. Given that the vapour pressure of water at the same temperature is 4KPa and 63KPa respectively. Calculate the atmospheric pressure.

Hint:

$T_1 = 300$, $T_2 = 360K$, $L_1 = 10cm$, $L_2 = 300cm$, $V_{P1} = 4KPa$, $V_{P2} = 62KPa$

Atmospheric pressure = Pressure of air column + vapour pressure

For the tube in horizontal position

$P_1 = Pa + V_{P1} = Pa - V_{P1}$

New pressure of air trapped inside

$P_2 = Va - V_2$

Apply Boyle's law

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{(V_a - V_{P1})AL_1}{T_1} = \frac{(V_a - V_{P2})AL_2}{T_2}$$

$$\frac{(Pa-4)10}{300} = \frac{(Pa-62)30}{360}$$

$$Pa = 101KPa$$

75. An automobile tyre has a volume of $0.015m^3$ in a cold day when the temperature of the air in the tyres $5^{O}C$ and atmospheric pressure is 1.02atm under these conditions the gauge pressure is measured to be 1.70atm after the car is driven on the high way for 30mm, the temperature of the air in the tyres has risen to $45^{O}C$ and the volume has risen to, $0.0159m^3$ what then is the gauge pressure (UP 18.59) (Answer Pg = 1.92 atm)

Hint : $P = P_g + P_g \left(P_g = P - P_a\right)$