

TYPE: A SOLVED EXAMPLES

Example 1

- (a) During uniform circular motion name of the quantities which remain constant.
- (b) A motor car travelling at 30 m/s on a circular road of radius 500 m. it is increasing its speed at the rate of 2 m/s². What is its acceleration?

Solution

- (a) Speed, kinetic energy and angular velocity.
- (b) Here $V = 30$ m/s, $r = 500$ m. $a_T = 2$ m/s².

Centripetal acceleration

$$A_c = \frac{V^2}{r} = \frac{30 \times 30}{500} = 1.8 \text{ m/s}^2.$$

The magnitude of acceleration

$$a = \sqrt{a_c^2 + a_T^2} = \sqrt{(1.8)^2 + 2^2}$$

$$a = 2.7 \text{ m/s}^2.$$

Example 2.

- (a) Can a body move on a curved path without having acceleration?
- (b) A stone tied to the end of a string 2 m long is whirled in a horizontal circle with constant speed. If the stone makes 10 revolution in 20s. calculate the magnitude and direction of acceleration.

Solution

- (a) No. it is so because while moving on a curved path, the velocity of the body (which is represented by tangent to the curved path at a point) changes with time and the body is in accelerated state.

$$(b) a_c = \omega^2 r = (2\pi f)^2 r \bullet \left(\frac{f = 10 \text{ S}^{-1}}{20} \right)$$

$$a_c = \left(2 \times \frac{22}{7} \times \frac{1}{2} \right)^2 \times 2$$

$$a_c = 19.74 \text{ m/s}^2.$$

Example 3

Calculate the magnitude of linear acceleration of a particle moving in a circle of radius 0.5 m at the instant when its angular velocity is 2.5 rad S⁻² and its angular acceleration is 6 rad S⁻².

Solution

Tangential acceleration $a_T = \theta r = 0.5 \times 6 = 3 \text{ m/s}^2$.

Centripetal acceleration

$$a_c = \omega^2 r = 0.5 \times (2.5)^2 = 3.125 \text{ m/s}^2.$$

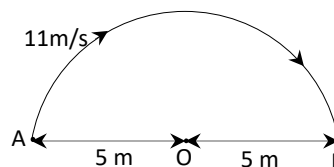
Magnitude of the total linear acceleration

$$a = \sqrt{a_T^2 + a_c^2} = \sqrt{3^2 + (3.125)^2}$$

$$a = 4.33 \text{ m/s}^2.$$

Example 4

- (a) A particle moves in a circle of radius 20 cm. its linear speed is given by $V = 2t$, where t is in second and V in m/s. Find the radial and tangential acceleration at $t = 3$ seconds.
- (b) A particle moves in a semi-circular path of radius 5.0 m with constant speed 11 m/s as shown on the figure below. Calculate:-



- (i) The time taken to travel from A to B
- $$\left(\pi = \frac{22}{7} \right).$$
- (ii) The average velocity.
- (iii) The average acceleration.

Solution

- (a) Linear speed at $t = 3$ sec. $V = 2t$
 $= 2 \times 3 = 6 \text{ m/s}^2$. Radial acceleration at this instant

$$a_c = \frac{V^2}{r} = \frac{6^2}{0.2}$$

$$a_c = 180 \text{ m/s}^2.$$

Tangential acceleration

$$A_T = \frac{dV}{dt} = \frac{d}{dt}(2t) = 2 \text{ m/s}^2$$

$$A_T = 2 \text{ m/s}^2.$$

- (b) (i) Time = $\frac{\text{arc AB}}{\text{speed}} = \frac{\pi \times 5}{11}$

$$T = \frac{10}{7} \text{ sec.}$$

- (ii) Average velocity = $\frac{\text{displacement}}{\text{time}}$

The displacement is diameter AB to the right

$$\bar{V} = \frac{10}{10/7} = 7 \text{ m/s}^2 \text{ to the right}$$

$$\bar{V} = 7 \text{ m/s}^2$$

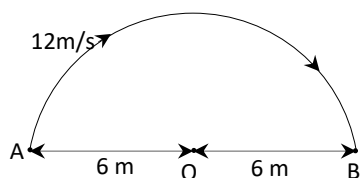
$$(iii) \text{ Average acceleration} = \frac{\text{change of velocity}}{\text{time}}$$

$$= \frac{11 - (-11)}{10/7} = \frac{22}{10/7}$$

$$a = \frac{154}{10} \text{ m/s}^2 = 15.4 \text{ m/s}^2$$

Example 5. NECTA 2019/P1/5

- (a) (i) In which aspect does circular motion differ from linear motion?
- (ii) Why there must be a force acting on a particle moving with uniform speed in a circular path?
- (b) (i) Figure 1. below shows a particle moving in a semi-circular path AB of radius 6 m with constant speed of 12 m/s. Calculate the average velocity.



- (ii) A stone tied to the end of string 80 cm long, is whirled in a horizontal circle with constant speed making 25 revolutions in 14 seconds. Determine the magnitude of its acceleration.

Solution

- (a) (i) For uniform circular motion the direction of centripetal acceleration is perpendicular to the direction of the velocity while for linear motion the direction of acceleration is on the same as the direction of the velocity.
- For circular motion the magnitude of accelerations remains constant but direction of acceleration continuously with time while for the linear motion, magnitude of acceleration changes with time.
- (ii) There must be a centripetal force for the particle moving in a circular path with

constant speed to maintain it to move in a circular path and always centripetal force is directed towards to the centre of the path.

$$(b) (i) \text{ Time, } t = \frac{\text{arc AB}}{\text{speed}} = \frac{\pi \times 6}{12}$$

$$t = \frac{\pi}{2} \text{ sec} = 1.57 \text{ sec}$$

$$\text{Average velocity} = \frac{\text{displacement}}{\text{time}}$$

$$\bar{V} = \frac{12}{1.57}$$

$$\bar{V} = 7.643 \text{ m/s}$$

$$(ii) r = 80 \text{ cm, } T = \frac{25}{14} \text{ sec}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{25/14} = \frac{28\pi}{25} \text{ rad s}^{-1}.$$

The acceleration of uniform circular motion is given by

$$a = \omega^2 r = \left(\frac{28\pi}{25} \right)^2 \times 80$$

$$a = 990.4 \text{ cm/s}.$$

The acceleration is directed along the radius of the circular path and towards the centre of the circle.

Example 6

A cyclist is riding with a speed of 27 kmhr⁻¹. As he approached a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.5 m/s. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?

Solution

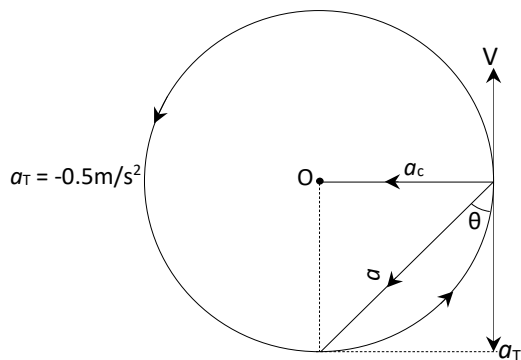
$$\text{Here } V = 27 \text{ kmh}^{-1} = 7.5 \text{ m/s.}$$

$$r = 80 \text{ m.}$$

Centripetal acceleration

$$a_c = \frac{V^2}{r} = \left(\frac{7.5}{80} \right)^2 = 0.7 \text{ m/s}^2.$$

Suppose that the cyclist applies brakes at the point A of the circular turn. Then retardation produced due to the brakes, say a_T will acts opposite to the velocity, V .



Total acceleration is given by

$$a = \sqrt{a_c^2 + a_T^2} = \sqrt{(0.7)^2 + (-0.5)^2}$$

$$a = 0.86 \text{ m/s}^2.$$

Direction of a

$$\tan \theta = \frac{a_c}{a_T} = \frac{0.7}{0.5} = 1.4$$

$$\theta = 54^\circ 28'$$

Example 7

A race car negotiates a curve of radius 50 m on a flat road. What is its angular displacement when its linear distance covered by the car is 78.5 m?

Solution

$$\theta = \frac{s}{r} = \frac{78.5}{50} = \frac{25}{14}$$

$$\theta = 1.54 \text{ rad} = 90^\circ.$$

Example 8

A satellite moving in a circular orbit at an altitude of 1000 km complete one revolution round the earth is 105 minutes. What is

- its angular velocity and
- speed? Radius of the earth, $R = 6400 \text{ km}$.

Solution

$$(i) \quad \omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{105 \times 60}$$

$$\omega = 9.97 \times 10^{-4} \text{ rad s}^{-1}$$

$$(ii) \quad V = \omega r = \omega(R+h) \\ = 9.9 \times 10^{-4} \times (64000 + 1000) \times 10^3 \\ V = 7,326 \text{ m/s}.$$

Example 9

- Is any work done by the centripetal force?
- Why centripetal force cannot do work?
- What is the power needed to maintain object in circular motion?

Solution

- No work done by the centripetal force. $W = FScos90^\circ = 0$
- The centripetal force cannot do work on the object because it is always perpendicular to the direction of the velocity.
- No power is needed to maintain uniform circular motion. $P = FVcos\theta = FScos90^\circ = 0$

Example 10

- Why do we express the angular displacement in radian?
- The angular displacement of a body is given by $\theta = 2t^2 + 5t - 3$. Find the value of the angular velocity and angular acceleration when $t = 2$ second.

Solution

- The length of an arc $L = r\theta$, in this equation θ is in radian and not in degree. Using this equation angular displacement can be converted into a linear displacement.

$$(b) \quad \theta = 2t^2 + 5t - 3$$

$$\omega = \frac{d\theta}{dt} = 4t + 5 \text{ when } t = 2$$

$$\omega = 13 \text{ rad/s}.$$

Angular acceleration

$$\alpha = \frac{d\omega}{dt} = \frac{d(4t + 5)}{dt}$$

$$\alpha = 4 \text{ rad/s}^2.$$

Example 11

A wheel starting from rest rotates with an angular acceleration $\alpha = (10 + 6t) \text{ rad/s}^2$. Where t is time in seconds. Determine the angle in radian through which the wheel has turned in first four seconds.

Solution

Since,

$$\alpha = \frac{d\omega}{dt} = 10 + 6t$$

$$d\omega = (10 + 6t)dt$$

$$\int d\omega = \int (10 + 6t)dt$$

$$\omega = 10t + 3t^2 + C$$

When $t = 0$, $\omega = 0$ $C = 0$

$$\omega = 10t + 3t^2$$

$$\omega = \frac{d\theta}{dt} = (10t + 3t^2)$$

$$d\theta = (10t + 3t^2)dt$$

$$\theta = \int_0^4 (10t + 3t^2)dt$$

$$\theta = 144 \text{ rad.}$$

Example 12

A motor revolving at 1800 r.p.m slows down uniformly to 120 r.p.m in 2 seconds. Calculate: -

- Angular acceleration of the motor.
- Number of revolutions it makes in this time.

Solution

Initial frequency, $f_1 = \frac{1800}{60} = 30 \text{ Hz}$

Final frequency, $f_2 = \frac{1200}{60} = 20 \text{ Hz}$

- Let α = Angular acceleration

$$\alpha = \frac{2\pi(f_2 - f_1)}{t} = \frac{2\pi(20 - 30)}{2}$$

$$\alpha = -31.43 \text{ rad s}^{-2}$$

- Let θ total angle turned in 2 sec.

Since $\omega_2^2 - \omega_1^2 = 2\alpha\theta$

$$\theta = \frac{4\pi^2(f_2^2 - f_1^2)}{2\alpha} = \frac{2 \times (3.14)^2 (20^2 - 30^2)}{-2 \times 31.43}$$

$$\theta = 314.28 \text{ rad.}$$

Number of revolutions

$$N = \frac{\theta}{2\pi} = \frac{314.28}{2 \times 3.14}$$

$$N = 50 \text{ revolutions.}$$

Example 13

Angular frequency of a particle is increased by 100 rev min⁻¹ in one minute when it moves in a circular path of radius 200 cm. find its

- Angular acceleration
- Tangential acceleration

Solution

$$(i) \quad \alpha = \frac{\Delta\omega}{t} = \frac{2\pi\Delta f}{t}$$

$$= \frac{2 \times 3.14}{60} \times \frac{100}{60}$$

$$\alpha = 0.1746 \text{ rad s}^{-2}$$

$$(ii) \quad a = \alpha r = 2 \times 0.1746$$

$$a = 0.3492 \text{ m/s}^2.$$

Example 14

The angular velocity of a fly wheel decreases uniformly from 1200 r.p.m to 900 r.p.m in 5 seconds. Find:-

- the angular acceleration.
- the angular displacement in 5 seconds.
- the number of rotations made by the wheel in 5 second.
- further time taken by the wheel to come to rest.

Solution

$$F_o = 1200 \text{ r.p.m} = \frac{1200}{60} = 20 \text{ HZ}$$

$$f = 900 \text{ r.p.m} = \frac{900}{60} = 15 \text{ HZ}$$

$$(i) \quad \alpha = \frac{2\pi(f - f_o)}{t} = \frac{2\pi(15 - 20)}{5}$$

$$\alpha = -6.284 \text{ rad s}^{-2}.$$

$$(ii) \quad \theta = \frac{2\pi(f + f_o)}{2} = \frac{2\pi(20 + 15) \times 5}{2}$$

$$\theta = 175\pi \text{ rad.}$$

- Let N be the number of rotations made by the wheel

$$N = \frac{\theta}{2\pi} = \frac{175\pi}{2\pi}$$

$$N = 87.5 \text{ rotations.}$$

$$(iv) \quad t = \frac{\omega - \omega_o}{\alpha} = \frac{0 - 30\pi}{-2\pi}$$

$$t = 15 \text{ sec.}$$

Example 15

The centripetal force acting on a body moving with speed in circular path is 10N. Without changing the path if the speed of the body is doubled, what will be the centripetal force?

Solution

$$\text{Let } F_1 = \frac{MV_1^2}{r}, F_2 = \frac{MV_2^2}{r}$$

$$\frac{F_2}{F_1} = \left(\frac{V_2}{V_1}\right)^2 = \left(\frac{2V}{V}\right)^2 = 4$$

$$F_2 = 4F_1 = 4 \times 10$$

$$F_2 = 40\text{N.}$$

Example 16

A metal bob of mass 0.6 kg is attached to the end of a string of length 1 m and the bob is whirled in a horizontal circle with a uniform speed of 12 m/s. what is the centripetal force acting on the bob? If the speed of the bob is 5 m/s. calculate the tension in the string.

Solution

$$\bullet F = \frac{MV^2}{r} = \frac{0.6 \times 12^2}{1}$$

$$F = 86.4\text{N.}$$

- Tension in the string = centripetal force

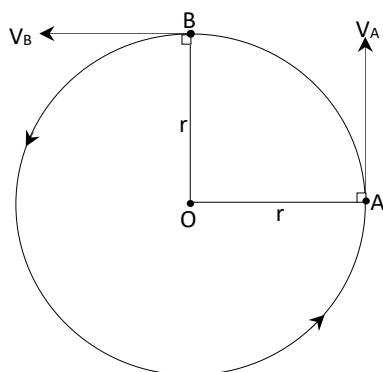
$$T = \frac{MV^2}{r} = \frac{0.6 \times 5^2}{1}$$

$$T = 15\text{N.}$$

Example 17

Consider the object of mass, M moving in a circular path with a constant speed.

- What is the direction of the velocity at some instant and quarter of revolution later.
- Explain why although the speed is constant, we say the velocity is changing?
- What would happen if there were no force acting on the object?

Solution

- The velocity is tangential to the circular path and thus the two velocities are at right angle to each other after revolution later.
- Speed can be constant if the direction is changing velocity can only be constant if the direction does not change i.e speed is the scalar quantity while the velocity is the vector quantity. Therefore in the circular motion velocity changes with the change when object moves in the circular path.
- If there is no centripetal force acts on the object then object continue to move in the linear motion i.e straight line with a constant speed.

Example 18

What provides the centripetal force in the following cases:

- Electron revolving around the nucleus.
- Earth revolving around the sun.
- Car taking a curve.
- A coin placed on a horizontal turn table.
- A train travelling round a horizontal circular arc.
- A stone tied to a string and whirled in a horizontal circle.
- Motion of charged particle in uniform magnetic field.

Solution

- Electrostatic force.
- Gravitational force.
- A component of the reaction force of the road.
- Friction force between the coin and the plane.
- The reaction force between the flange of the outer rail and the inner rail.
- Tension in the string.
- Magnetic force.

Example 19

What is the effect on the direction of the centripetal force when the revolving body reverses its direction of the motion?

Solution

The centripetal force will be directed towards the centre of the circle. This fact does not depend upon the sense of rotation of the particle.

Example 20

Why are passengers sitting in a vehicle thrown outwards, when the vehicle rounds a curve suddenly? OR Passengers are thrown outwards, when the bus takes a circular turn. Why?

Solution

When the bus takes a turn, the centripetal acceleration of the bus acts towards the centre of the circular turn and along its radius. Due to inertia, the passengers are thrown outwards. The force experienced by the passengers is called the centrifugal force.

Example 21

A car is taking a sudden turn to the left. A passenger in the front seat finds himself sliding towards the door. Explain, indicating the forces acting on the passenger and on the car at this instant.

Solution

The passenger in the front seat slides towards door i.e. away from the centre of the circular turn. It is due to the centrifugal force acting on the passenger.

Example 22

Why does a child in a merry-go-round press the side of his seat radially outward?

Solution

When the child presses the side of his seat radially outward in a merry-go-round, the side of the seat presses the child radially inwards in accordance with Newton's third law of motion. This force, which comes as reaction, provides the child the necessary centripetal force to move along a circular path.

TYPE A: TYPICAL PROBLEM

1. The wheel of an automobile is rotating with 4 rotations per second. Find its angular velocity.

If the radius of the flywheel is 50 cm. find the linear velocity of a point on its circumference. (Ans. $8\pi \text{ rads}^{-1}$, $400\pi \text{ cms}^{-1}$)

2. A body of mass 5 kg is revolving in a circle of diameter 0.30 m making 2000 revolution in 2 minutes. Calculate the linear velocity and centripetal acceleration. (Ans. 15.71 m/s , 1646.3 m/s^2)
3. Find the magnitude of the centripetal acceleration of a particle on the tip of a blade, 0.30 m in diameter rotating at 1200 rev/minute. (Ans. 2370.6 m/s^2)
4. Calculate the linear acceleration of a particle moving in a circle of radius 0.5 m at the instant when its angular velocity is 2 rads^{-1} and its angular acceleration is 6 rads^{-2} . (Ans. 3.6 m/s^2 , $33^\circ 42'$ with tangential acceleration)
5. Give explanation for each of the following observations:-
 - (a) If there is a net force on a particle in uniform circular motion, why does the particle's linear speed not change?
 - (b) As a car rounds a banked circular curve at constant speed, several forces are acting on it, for example, air resistance towards the rear, friction from the pavement in the forward direction, gravity, and the normal force from the tilted road surface. In what direction does the net force point?
6. What does the word "uniform" indicate in uniform circular motion?

TYPES B: SOLVED EXAMPLES**Example 1**

A 500 g stone attached to a string is whirled in a horizontal circle at a constant speed of 10 m/s . the length of the string is 1.0 m. Neglecting the effect of gravity, find:

- (a) The centripetal acceleration of the stone and
- (b) The centripetal force acting on the stone.

Solution

- (a) The centripetal acceleration

$$a_c = \frac{V^2}{r} = \frac{10^2}{1}$$

$$a_c = 100 \text{ m/s}^2.$$

- (b) The centripetal force,
- $F_c = Ma_c$

$$F_c = 0.5 \times 100 = 50\text{N}$$

$$F_c = 50\text{N}.$$

Example 2

A rubber stopper 13g is attached to a 0.93 m string. The stopper is swung in a horizontal circle, making 10 revolutions in 31.4 seconds. Find the tension in a string.

Solution

Using the relation $T = M\omega^2 r$

$$T = M(2\pi f)^2 r = \frac{0.013 \times 4\pi^2 \times 10^2 \times 0.93}{(31.4)^2}$$

$$T = 0.043\text{N}.$$

Example 3

A small mass of 1 kg is attached to the lower end of a string 1 m long whose upper end is fixed. The mass is made to rotate in a horizontal circle of radius 0.6 m. if the circular speed of the mass is constant. Find:

- Tension, F in the string.
- The period of the motion.

Solution

- (a) Consider vertical forces

$$F \cos \theta = Mg, \quad F = \frac{Mg}{\cos \theta}$$

$$F = \frac{MgL}{\sqrt{L^2 - r^2}} = \frac{1 \times 9.8 \times 1}{\sqrt{1^2 - (0.6)^2}}$$

$$F = 12.25\text{N}.$$

$$(b) \quad T = 2\pi \sqrt{\frac{L \cos \theta}{g}}$$

$$\cos \theta = \sqrt{\frac{L^2 - r^2}{L}}$$

$$T = 2\pi \sqrt{\frac{1 \times 0.8}{9.8}}$$

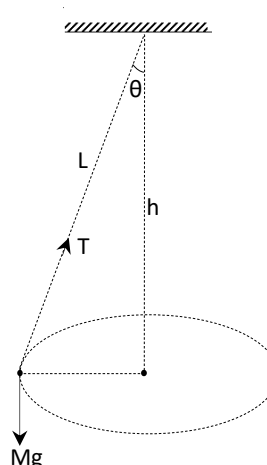
$$T = 1.8\text{sec}.$$

Example 4

A ball of mass 200 gm attached to the end of a cord of length 1 m and is whirled in a horizontal circle

as shown in the figure below, with $\theta = 10^\circ$. What is:

- The speed of the ball and
- The time period of revolution of the ball?

**Solution**

$$(i) \quad V = \sqrt{gL \sin \theta \tan \theta}$$

$$= \sqrt{9.8 \times 1 \times \sin 10^\circ \times \tan 10^\circ}$$

$$V = 0.55\text{m/s}.$$

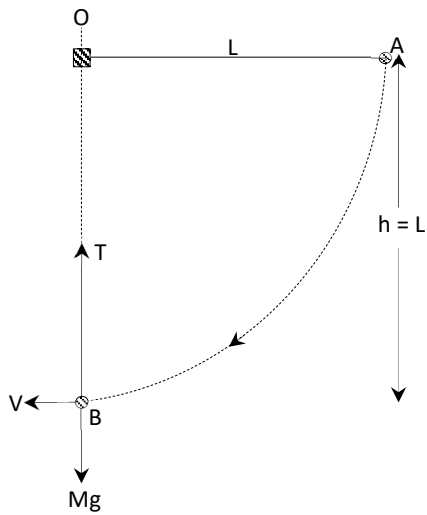
$$(ii) \quad T_1 = 2\pi \sqrt{\frac{L \cos \theta}{g}} = 2\pi \sqrt{\frac{1 \times \cos 10^\circ}{9.8}}$$

$$T_1 = 1.9\text{sec}.$$

Example 5

A pendulum bob of mass, M held out in the horizontal position and then released. If the length of the string is L . Calculate

- The velocity of the bob and
 - The force on the string when the bob reaches the lowest position.
-

**Solution**

Apply the Law of conservation of energy.
Decrease in P.E of the bob in falling through height L = Increase in K.e

$$MgL = \frac{1}{2}MV^2$$

$$V = \sqrt{2gL}$$

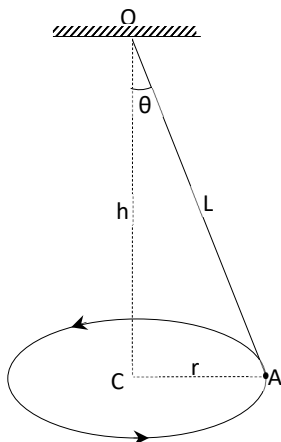
(ii) At the lowest position.

$$\frac{MV^2}{L} = Mg + \frac{2MgL}{L}$$

$$T = 3Mg.$$

Example 6

A ball of mass 0.1 kg is suspended by a string 30 cm long. Keeping the strings always taut, the ball describes horizontal circle of radius 15 cm . find the angular speed

Solution

$$\omega = \frac{g}{h}$$

$$h = \sqrt{OA^2 - CA^2}$$

$$= \sqrt{0.3^2 - 0.15^2}$$

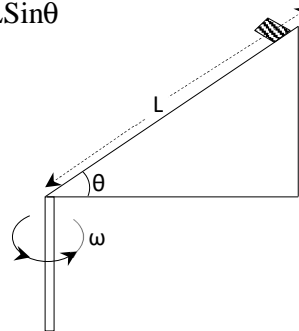
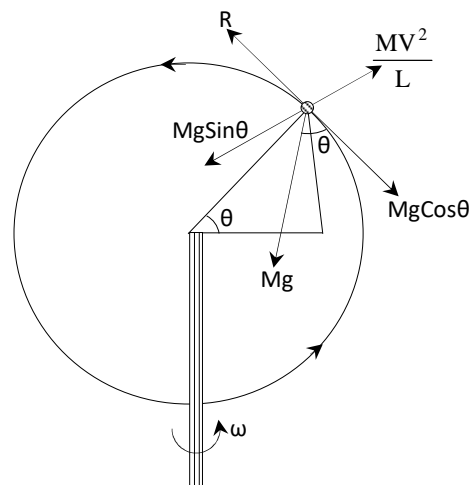
$$h = 0.26 \text{ m.}$$

$$\omega = \sqrt{\frac{9.8}{0.26}}$$

$$\omega = 6.14 \text{ rad/s}$$

Example 7

In the figure below shows a child's toy made of a small wedge which has an acute angle, θ . The slopping side is frictionless. The wedge is attached to a rod. The rod is rotated and the wedge is spun at a constant speed. Show that when the mass, M rises up the wedge at a distance L the speed of mass is $V = \sqrt{gL \sin \theta}$

**Solution**

When the mass M is at the top of the wedge, the component of the weight parallel to the sloping side provide necessary centripetal force.

$$Mg \sin \theta = \frac{MV^2}{L}$$

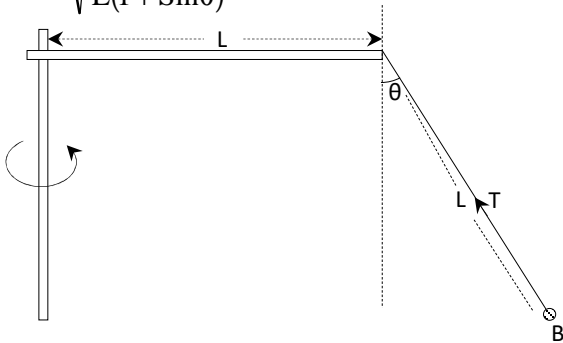
$$V^2 = gL \sin \theta$$

$$V = \sqrt{gL \sin \theta} \text{ Hence shown.}$$

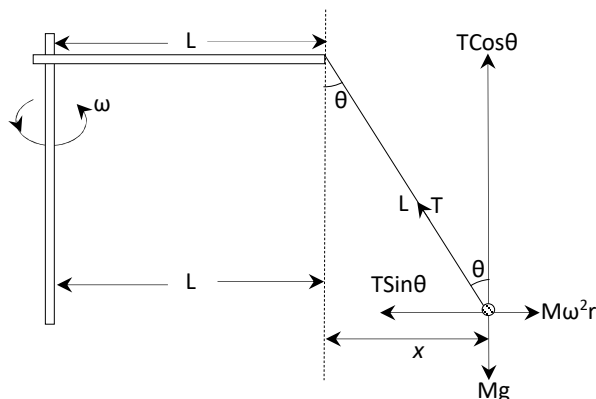
Example 8

Instead of a wedge in the figure below, a uniform rod of length L is attached to the central rod. The free end of the rod of length L a simple pendulum of length is suspended. The mass of the bob is M . When the system is rotated with an angular velocity ω , the angle made by the string with the vertical is θ . Show that the angular velocity of rotation is

$$\omega = \sqrt{\frac{g \tan \theta}{L(1 + \sin \theta)}}$$



Solution



$T \sin \theta$ provides the centripetal force.

$$\text{Since } \sin \theta = \frac{x}{L}, x = L \sin \theta$$

$$r = x + L = L(1 + \sin \theta)$$

At the equilibrium

$$T \sin \theta = M \omega^2 r$$

$$T \sin \theta = M \omega^2 L (1 + \sin \theta)$$

$$T \cos \theta = Mg \quad (2)$$

Dividing equation (1) and (2)

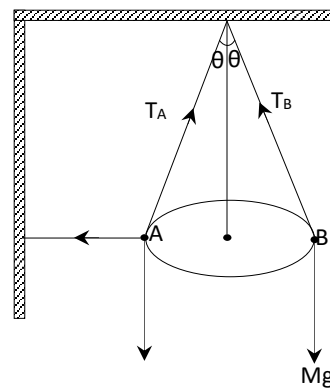
$$\frac{T \sin \theta}{T \cos \theta} = \frac{M \omega^2 (1 + \sin \theta) L}{Mg}$$

$$\tan \theta = \frac{\omega^2 L (1 + \sin \theta)}{g}$$

$$\omega = \sqrt{\frac{g \tan \theta}{L(1 + \sin \theta)}}$$

hence shown

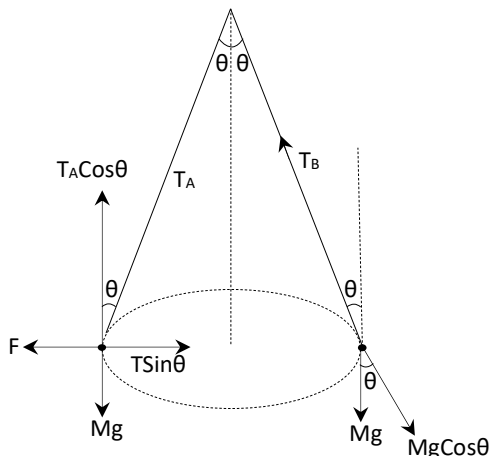
Example 9



Using two light cords, a bob is held in equilibrium at A. The horizontal cord is cut and the bob is allowed to swing as a pendulum. Calculate the ratio of the tension in the supporting cord in position B to that in position A

Solution

Consider the FBD as shown below



At the equilibrium

$$T_A \cos \theta = Mg$$

$$T_B = Mg \cos \theta$$

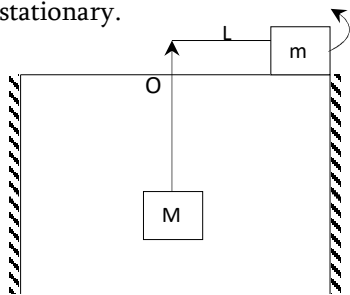
$$T_B = (T_A \cos \theta)(\cos \theta)$$

$$T_B = T_A \cos^2 \theta$$

$$\frac{T_B}{T_A} = \cos^2 \theta$$

Example 10

Two masses m and M are connected by a light string that passes through a smooth hole O at the centre of a table. Mass m lies on the table and M hangs vertically. m is moved round in a horizontal circle with O as the centre. If L is the length of the string from O to m then, find the frequency with which m should revolve so that M remains stationary.



Solution

Let T = tension in the string. At the equilibrium.

$$T = Mg \quad \text{(i)}$$

T provides the necessary centripetal force for the revolution of m .

$$T = m\omega^2 L \quad \text{(ii)}$$

$$(i) = (ii)$$

$$m\omega^2 L = Mg$$

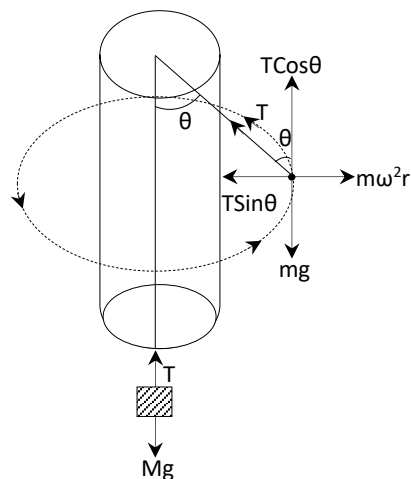
$$\omega = \sqrt{\frac{Mg}{mL}} \quad \text{but} \quad \omega = 2\pi f$$

$$2\pi f = \sqrt{\frac{Mg}{mL}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{Mg}{mL}}$$

Example 11

A large mass M and small mass m hang at the two ends of a string that passes through a smooth tube as shown in figure below.



The mass m moves round in a circular path which lies in the horizontal plane. The length of the string from the mass m to the top of the tube is L and θ is the angle this length makes with the vertical. What should be the frequency of the mass m so that M remains stationary?

Solution

At the equilibrium

$$T \sin \theta = m\omega^2 L = m\omega^2 L \sin \theta$$

$$T = m\omega^2 L \quad \text{(i)}$$

$$\text{Also } T = Mg \quad \text{(ii)}$$

$$(i) = (ii)$$

$$m\omega^2 L = Mg, \quad \omega = \sqrt{\frac{Mg}{mL}}$$

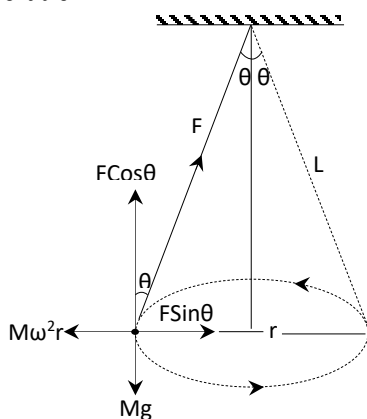
$$2\pi f = \sqrt{\frac{Mg}{mL}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{Mg}{mL}}$$

Example 12

A sphere of mass 200 gm is attached to an inextensible string of length 130 cm whose upper end is fixed to the ceiling the sphere is made to describe a horizontal circle radius 50 cm.

- Calculate the time period of revolution.
- What is the tension in the string.

Solution

$$(i) T = 2\pi \sqrt{\frac{L \cos \theta}{g}} = 2\pi \sqrt{\frac{130 \times 120}{130 \times 980}}$$

$$T = 2.2 \text{ sec.}$$

$$(ii) F \cos \theta = Mg, F = \frac{Mg}{\cos \theta}$$

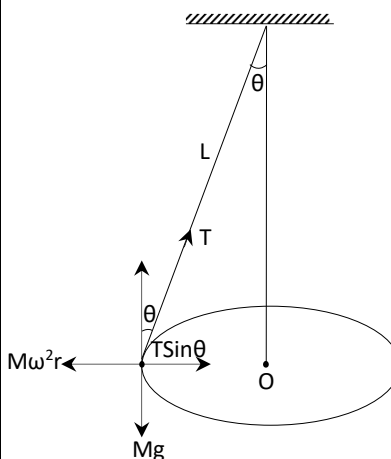
$$F = \frac{0.2 \times 9.8}{\frac{120}{130}}$$

$$F = 2.12 \text{ N.}$$

Example 13

A string of length 1 m is fixed at one end and carries a mass of 100 gm at the other end. The string makes $\frac{2}{\pi}$ revolution per second around a vertical axis passing through its fixed end. Calculate:-

- The angle of inclination of the string with the vertical.
- The tension in the string.
- The linear velocity of the mass.

Solution

- At the equilibrium

$$T \cos \theta = Mg$$

$$\cos \theta = \frac{Mg}{T} = \frac{0.1 \times 9.8}{1.6}$$

$$\cos \theta = 0.6125.$$

$$\theta = 52^\circ 14'$$

$$(b) T = M\omega^2 L = 0.1 \times 4\pi^2 \times \frac{4 \times 1}{\pi^2}$$

$$T = 1.6 \text{ N}$$

$$(c) V = \omega r = 2\pi f L \sin \theta$$

$$= 2\pi \times \frac{2}{\pi} \times 1 \times \sin 52^\circ 14'$$

$$V = 3.6 \text{ m/s.}$$

Example 14

A special prototype model aeroplane of mass 400 gm long attached to its body. The other end of the control line is attached to a fixed point. When the aeroplane flies with its wings horizontal in a horizontal circle, making one revolution in every 4S, the control wire is elevated 30° above the horizontal. Draw a diagram showing the forces exerted on the plane and determined:-

- The tension in the control wire.
- The lift on the plane.

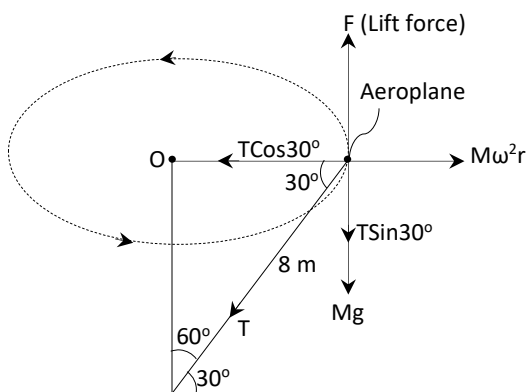
Solution

Diagram of the forces

- (a) Let T = tension in the control wire.

At the equilibrium

$$T \cos 30^\circ = M\omega^2 r$$

$$T = \frac{M\omega^2 r}{\cos 30^\circ} = \frac{0.4 \times \left(\frac{\pi}{2}\right)^2 \times 8 \sin 60^\circ}{\cos 30^\circ}$$

$$T = 8.0 \text{ N.}$$

- (b) At the equilibrium of Aeroplane.

$$F = Mg + T \sin 30^\circ$$

$$= 0.4 \times 10 + 8 \sin 30^\circ$$

$$F = 8.0 \text{ N.}$$

Example 15

A particle of mass 80 g rests at 16 cm from the centre of a turn table. If the maximum frictional force between the particle and the turn table is 0.72 N. What is the maximum angular velocity at which the turn table could rotate without the particle slipping?

Solution

$$\text{Since } f = \mu Mg, \mu = \frac{F}{Mg}$$

$$\mu = \frac{0.72}{0.08 \times 9.8} = 0.91837$$

Maximum angular speed

$$\omega = \sqrt{\frac{\mu g}{r}} = \sqrt{\frac{0.91837 \times 9.8}{0.16}}$$

$$\omega = 7.5 \text{ rad/s.}$$

Example 16

A small button placed on a horizontal rotating platform with diameter 0.320 m will revolve with the platform when it is brought up to a speed of 40 rev/min, provided. The button is more than 0.150 m from the axis.

- (a) What is the coefficient of static friction between the button and the platform?
 (b) How far from the axis can the button be placed, without slipping, if the platform rotates at 60 rev/min?

Solution

$$\begin{aligned} \text{(a)} \quad \mu &= \frac{\omega^2 r}{g} = \frac{(2\pi f)^2 r}{g} \\ &= \frac{(2\pi \times \frac{40}{60})^2 \times 0.15}{9.8} \end{aligned}$$

$$\mu = 0.27$$

$$\begin{aligned} \text{(b)} \quad r &= \frac{\mu g}{(2\pi f)^2} = \frac{0.2686 \times 9.8}{(2\pi \times 1)^2} \\ r &= 0.067 \text{ m.} \end{aligned}$$

Example 17

A string 0.6 m long can just sustain a weight of 18 kg, without breaking. A mass of 1.8 kg is attached to one end of the string and revolves uniformly on a smooth table. Find the greatest number of complete revolutions the mass can make, in a minute without breaking the string

Solution

$$\text{Since } T_{\max} = M\omega^2 r$$

$$2\pi f = \omega = \sqrt{\frac{T_{\max}}{rM}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{Mg}{rM}} = \frac{1}{2\pi} \sqrt{\frac{18 \times 9.8}{1.8 \times 0.6}}$$

$$f = 2.03 \text{ rev/s} = 122 \text{ rev/min.}$$

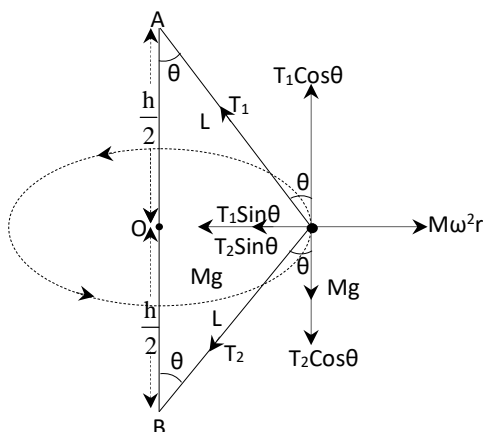
Example 18

A particle is attached by means of two equal strings to two points A and B in the same vertical line and describes a horizontal circle with uniform angular speed.

- (a) Prove that, in order that both strings may remain stretched, the angular speed must exceed $2\sqrt{\frac{2g}{h}}$ where $h = AB$.

- (b) If the angular speed is $2\sqrt{\frac{2g}{h}}$, prove that the ratio of the tensions of the strings is 5:3

Solution



- (a) At the equilibrium
Vertical components of the forces.
 $(T_1 - T_2)\cos\theta = Mg$
Horizontal component of forces
 $(T_1 + T_2)\sin\theta = M\omega^2 r$

But $\cos\theta = \frac{h}{2L}$, $\sin\theta = \frac{r}{L}$

Now $(T_1 - T_2) \frac{h}{2L} = Mg$

$T_1 - T_2 = 2 \frac{MgL}{h}$ (i)

Also

$(T_1 + T_2)\sin\theta = M\omega^2 L\sin\theta$

$T_1 + T_2 = M\omega^2 L$ (ii)

On solving simultaneously equation (i) and (ii)

$T_1 = \frac{1}{2} ML \left(\omega^2 + \frac{2g}{h} \right)$

$T_2 = \frac{1}{2} ML \left(\omega^2 - \frac{2g}{h} \right)$

For both string to remain stretched, $T_2 > 0$

$\frac{1}{2} ML \left(\omega^2 - \frac{2g}{h} \right) > 0$

$\omega > \sqrt{\frac{2g}{h}}$ proved

- (b) Given that $\omega = 2\sqrt{\frac{2g}{h}}$

$\omega^2 = \frac{8g}{h}$

$T_1 = \frac{ML}{2} \left(\omega^2 + \frac{2g}{h} \right) = \frac{ML}{2} \left(\frac{8g}{h} + \frac{2g}{h} \right)$

$T_1 = \frac{5MgL}{h}$

Also

$T_2 = \frac{ML}{2} \left(\omega^2 - \frac{2g}{h} \right) = \frac{ML}{2} \left(\frac{8g}{h} - \frac{2g}{h} \right)$

$T_2 = 2 \frac{MgL}{h}$

Takes $\frac{T_1}{T_2} = \frac{5MgL/h}{2MgL/h} = \frac{5}{2}$

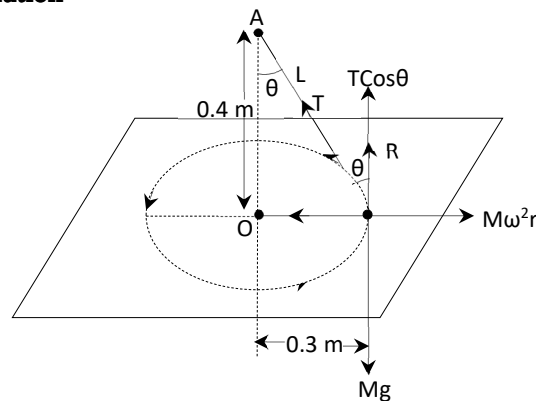
$\frac{T_1}{T_2} = \frac{5}{2} = 5:2$

Example 19

A particle is attached by means of a light inextensible string to a point 0.40 m above a smooth table in a circle of radius 0.30 m with angular velocity ω .

- (a) Find the reaction on the table in terms of ω .
(b) Hence, find the maximum angular velocity for which the particle can remain on the table.

Solution



By using Pythagoras theorem

$$L = \sqrt{0.4^2 + 0.3^2} = 0.5 \text{ M}$$

From the figure above

$$\sin\theta = \frac{0.3}{0.5} = \frac{3}{5}, \cos\theta = \frac{0.4}{0.5} = \frac{4}{5}$$

$$\tan\theta = \frac{3}{4}$$

- (a) At the equilibrium vertical component of the forces.

$$T\cos\theta + R = Mg$$

$$R = Mg - T\cos\theta$$

$$= Mg - T \cdot \frac{4}{5}$$

$$R = Mg - \frac{4}{5}T \quad \text{---(i)}$$

Horizontal component of force

$$T\sin\theta = M\omega^2 r$$

$$T = \frac{M\omega^2 r}{\sin\theta} = \frac{0.3M\omega^2}{3/5}$$

$$T = 0.5 M\omega^2 \quad \text{---(ii)}$$

Putting equation (ii) into (i)

$$R = Mg - \frac{4}{5}(0.5M\omega^2)$$

$$R = M(g - 0.4\omega^2).$$

- (b) For the maximum speed of particle to remain on the table, $R = 0$.

$$0 = m(g - 0.4\omega^2)$$

$$0 = g - 0.4\omega^2$$

$$\omega = \sqrt{\frac{g}{0.4}} = \sqrt{\frac{10}{0.4}}$$

$$\omega = 5 \text{ rad/s.}$$

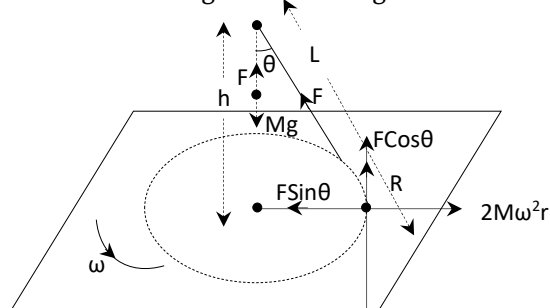
Example 20

A mass 2 m rests on a smooth horizontal table and is connected to a mass m by a light inextensible string passing through a small ring fixed at a height h above the table. If the mass 2 m is made to describe a circle of radius $\frac{1}{2}h$ having its centre on the table vertically below the ring show that the time it takes to describe the circle once.

$$T = 2\pi\sqrt{\frac{h\sqrt{5}}{g}}$$

Solution

Let R be normal reaction force on the table and F is the tension throughout the string.



At the equilibrium

$$F\cos\theta + R = 2mg$$

$$F = mg$$

$$R = 2mg - F\cos\theta$$

$$= 2mg - mgF\cos\theta \quad \text{---(i)}$$

$$F\sin\theta = 2M\omega^2 r$$

$$\omega = \sqrt{\frac{F\sin\theta}{2Mr}} = \sqrt{\frac{Mg}{2M} \frac{\frac{h/2}{\sqrt{h^2 + (h/2)^2}}}{(h/2)}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{g}{2\sqrt{5/4}h^2}} = \sqrt{\frac{g}{h\sqrt{5}}}$$

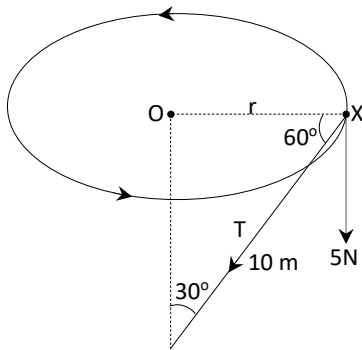
$$T = 2\pi\sqrt{\frac{h\sqrt{5}}{g}} \text{ Hence shown}$$

Example 21

A model aeroplane X has a mass of 0.5 kg and has a control wire OX of length 10 m attached to it when it flies in a horizontal circle with its wings horizontal, figure below. The wire OX is then inclined at 60° to the horizontal and fixed to a point O and X takes 2 seconds to fly once round its circular path.

Calculate

- (a) The tension T in the control wire.
(b) The upward force on X due to air.

**Solution**

$$(a) \omega = \frac{2\pi}{T} = \pi \text{ rad s}^{-1}$$

$$F = M\omega^2 r = T \cos 60^\circ$$

$$\text{but } r = L \sin 30^\circ = 10 \sin 30^\circ = 5 \text{ m}$$

$$T \cos 60^\circ = 0.5 \times 5 \pi^2$$

$$T = \frac{2.5 \pi^2}{0.5}$$

$$T = 50 \text{ N (approx)}$$

$$(b) \text{ Upward force due to the air} = \text{weight of X} + T \cos 30^\circ$$

$$\mu = 48 \text{ N (approx)}$$

Example 22

A long playing record revolves with speed of $33\frac{1}{3}$ rev/min and has a radius of 15 cm. Two coins are placed at 4 cm and 14 cm always from the centre of the record. If the coefficient of friction between the coins and the record is 0.15, which of the two coins, if any will revolve with record?

Solution

For a coin to revolve with the record the force of friction must be sufficient to provide the necessary centripetal force.

$$M\omega^2 r \leq \mu Mg \text{ or } r \leq \frac{\mu g}{\omega^2}$$

$$r \leq \frac{0.15 \times 9.8 \times 60^2}{4\pi^2 (100/3)^2}$$

$$r \leq 0.12 \text{ m}$$

$$r \leq 12 \text{ cm.}$$

Therefore, the coin at 4 cm distance from the centre will revolve with the record since it fills condition above.

Example 23 NECTA 1995

What is the speed of an aircraft moving in a horizontal circle with radius 300 m. if the acceleration experienced by the path is $4g$.

Solution

The centripetal acceleration

$$a = \frac{V^2}{r} = 4g$$

$$V^2 = 4gr, V = \sqrt{4gr}$$

$$V = \sqrt{4 \times 9.8 \times 300}$$

$$V = 108.44 \text{ m/s.}$$

Example 24

A man stands on the Earth at latitude 60°N . Calculate:-

(a) Its angular velocity.

(b) Its linear velocity.

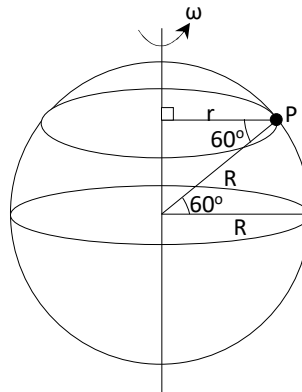
(c) Its acceleration due to the rotation of the Earth about its axis. ($g = 9.8 \text{ m/s}^2$, $R = 6400 \text{ km}$)

Solution

$$(a) \omega = \frac{2\pi}{T} = \frac{2\pi}{24 \times 3600}$$

$$\omega = 7.3 \times 10^{-5} \text{ rad s}^{-1}$$

(b)



$$\cos 60^\circ = \frac{r}{R}$$

$$r = R \cos 60^\circ$$

$$V = \omega r$$

$$= \omega R \cos 60^\circ$$

$$= 7.3 \times 10^{-5} \times 6.4 \times 10^6 \times \frac{1}{2}$$

$$V = 233.6 \text{ m/s}$$

$$(c) \ a = \omega^2 r = (7.3 \times 10^{-5})^2 6.4 \times 10^6 \times \frac{1}{2}$$

$$a = 0.017 \text{ m/s}^2.$$

Example 25

One end of an elastic string 0.6 m long, is attached to a fixed point on a smooth table and the other end to a mass of 1.8 kg resting on the table. If the 1.8 kg mass were suspended vertically by the string the extension would be 10 cm. The mass is made to describe a circle round the fixed point at 40 r.p.m. calculate the extension of the string.

Solution

Elastic constant of the string

$$K = \frac{Mg}{e} = \frac{1.8 \times 9.8}{0.1}$$

$$K = 176.4 \text{ Nm}^{-1}.$$

Tension in the string when the mass describes the circle

$$T = M\omega^2 r = M(2\pi f)^2 r$$

$$= 1.8 (2\pi \times 40/60)^2 0.6$$

$$T = 18.95 \text{ N}.$$

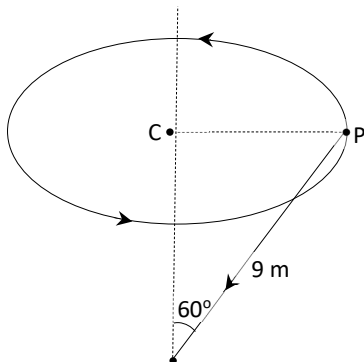
Extension produced is given by

$$e = \frac{T}{K} = \frac{18.95}{176.4}$$

$$e = 0.107 \text{ m}.$$

Example 26

A toy aircraft of mass 0.5 kg is attached to one end of a light inextensible string of length 9 m. The other end of the string is attached to a fixed point O. The aircraft moves with constant speed in a horizontal circle. The string is taut, and makes an angle of 60° with the upward vertical at O as shown in figure below.

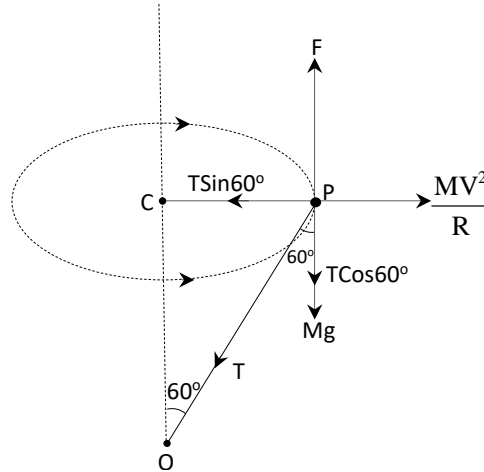


In a simplified model of the motion, the aircraft is treated as a particle and the force of the air on the aircraft is taken to act vertically upward with magnitude 8 N. Find

- The tension in the string.
- The speed of the aircraft

Solution

Consider the FBD as shown below.



- At the equilibrium of an aircraft.

$$T \cos 60^\circ + Mg = F$$

$$0.5T + 0.5 \times 10 = 8$$

$$T = 6 \text{ N}.$$

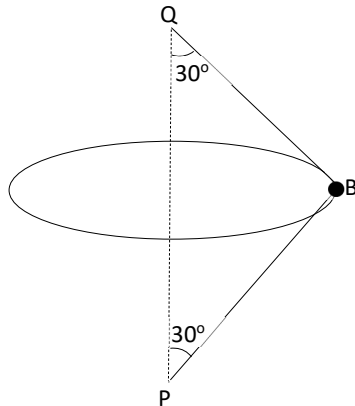
-
- $$\frac{MV^2}{R} = T \sin 60^\circ$$

$$\frac{0.5V^2}{9 \sin 60^\circ} = 6 \sin 60^\circ$$

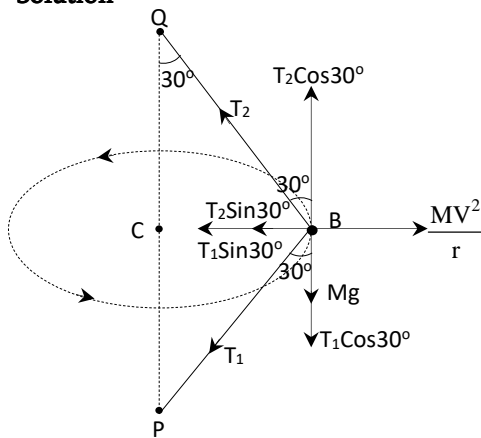
$$V = 9 \text{ m/s}.$$

Example 27

A small ball B of mass 0.4 kg is attached to fixed points P and Q on a vertical axis by two light inextensible of equal length both strings are taut and each is inclined at 30° to the vertical. The ball moves in a horizontal circle as shown in figure below.



- (a) It is given that when the ball moves with a speed 6m/s, the tension in the string QB is three times the tension in the string PB. Calculate the radius of the circle. The ball now moves along this circular path with the minimum possible speed.
- (b) State the tension in the string PB in this case and find the speed of the ball.

Solution

- (a) At the equilibrium for the ball.

$$T_2 \cos 30^\circ - T_1 \cos 30^\circ = Mg$$

$$3T_1 \cos 30^\circ - T_1 \cos 30^\circ = Mg$$

$$2T_1 \cos 30^\circ = 0.4 \times 10$$

$$T_1 = 2.31 \text{ N}$$

$$T_2 = 3T_1 = 6.93 \text{ N}$$

Also

$$\frac{MV^2}{r} = T_1 \sin 30^\circ + T_2 \sin 30^\circ$$

$$\frac{0.4 \times 6^2}{r} = (2.31 + 6.93) \sin 30^\circ$$

$$r = 3.12 \text{ m.}$$

- (b) When the ball moves along this circular path with the minimum possible speed, the tension T_1 in the string is zero

$$\text{Then } T_2 \cos 30^\circ = Mg$$

$$T_2 = \frac{0.4 \times 10}{0.866} = 4.62 \text{ N}$$

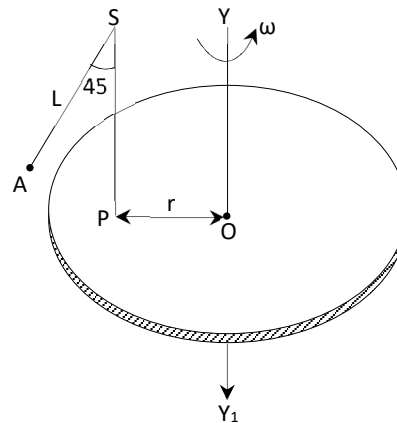
$$\text{Also } \frac{MV^2}{r} = T_2 \sin 30^\circ$$

$$\frac{0.4 V^2}{3.12} = 4.62 \times 0.866$$

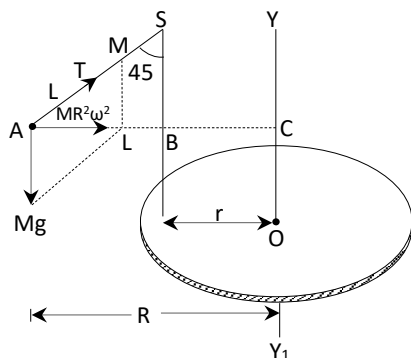
$$V = 4.25 \text{ m/s.}$$

Example 28

A ball of mass, m is attached to the end of a thread fastened to the top of a vertical rod which is secured to the horizontally revolving round the table shown below. With what angular velocity will the table rotate, if the thread forms an angle of 45° with the vertical? Given that the length of the thread is 6 cm and the distance of the rod from the axis of rotation is 10 cm.

**Solution**

Consider the FBD as shown on the figure below.



$$\text{Now } R = AC = AB + BC \\ = L \sin 45^\circ + r$$

$$R = \frac{L}{\sqrt{2}} + r$$

$$L = 6 \text{ cm} = 0.06 \text{ m}, r = 10 \text{ cm} = 0.1 \text{ m}$$

$$R = \frac{0.06}{\sqrt{2}} + 0.1$$

$$R = 0.1424 \text{ m.}$$

The tension T_1 , weight Mg and the centripetal force $F = M\omega^2 R$ can be represented respectively by the sides AM , ML and AL of the right angles $\triangle ALM$

$$\text{Now, } \tan 45^\circ = \frac{AL}{ML} = \frac{M\omega^2 R}{Mg}$$

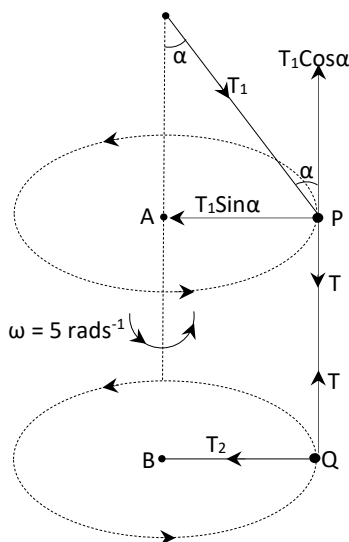
$$\tan 45^\circ = \frac{R\omega^2}{g}$$

$$\omega = \sqrt{\frac{g \tan 45^\circ}{R}} = \sqrt{\frac{9.8}{0.1424}}$$

$$\omega = 8.3 \text{ rad s}^{-1}.$$

Example 29

The particles P and Q have masses 0.8 kg and 0.4 kg respectively. P is attached to a fixed point A by a light inextensible string, which is inclined at an angle α to the vertical. Q is attached to a fixed point B , which is vertically below A , by a light inextensible string of length 0.3 m . The string BQ is horizontal. P and Q are joined to each other by a light inextensible string which is vertical. The particles rotate in horizontal circle of radius 0.3 m about the axis through A and B with constant angular speed 5 rad/s (See figure below)



- By consider the motion of Q , find the tension in the string PQ and BQ .
- Find the tension in the string AP and the value of α .

Solution

$$M_P = 0.8 \text{ kg } M_Q = 0.4 \text{ kg}$$

$$\omega = 5 \text{ rad s}^{-1} \quad R = 0.3 \text{ m}$$

- The tension in the string PQ

$$T = M_Q g = 0.4 \times 10$$

$$T = 4 \text{ N.}$$

The tension in the string BQ

$$T_2 = M_Q R \omega^2 = 0.4 \times 0.3 \times 5^2$$

$$T_2 = 3 \text{ N.}$$

- At the equilibrium at P

$$T_1 \sin \alpha = M_P \omega^2 R$$

$$T_1 \sin \alpha = 0.8 \times 0.3 \times 5^2$$

$$T_1 \sin \alpha = 6 \text{ N.}$$

Also

$$T_1 \cos \alpha = (M_P + M_Q)g$$

$$= (0.8 + 0.4) \times 10$$

$$T_1 \cos \alpha = 12 \text{ N.}$$

$$\text{Takes } \frac{T_1 \sin \alpha}{T_1 \cos \alpha} = \frac{6}{12} = \frac{1}{2}$$

$$\alpha = \tan^{-1} \frac{1}{2}$$

$$\alpha = 26.56^\circ.$$

Again

$$T_1 = \sqrt{6^2 \times 12^2}$$

$$T_1 = 13.4\text{N}.$$

Example 30

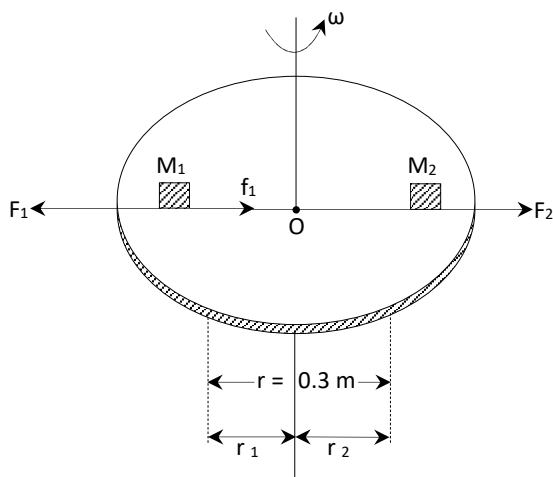
Two blocks of mass $M_1 = 10\text{ kg}$ and $M_2 = 5\text{ kg}$ connected to each other by a mass less inextensible string of length 0.3 m are placed along a diameter of a turn table. The coefficient of friction between the table and M_1 is 0.5 , while there is no friction between M_2 and the table. The table is rotating with an angular velocity of 10 rad s^{-1} about a vertical axis passing through its centre O . The masses are placed along the diameter of the table on the either side of the centre O , such that the mass M_1 is at a distance of 0.124 m from O . The masses are observed to be at rest w.r.t to an observer on the turn table.

- Calculate the frictional force on M_1 .
- What should be the minimum angular speed of the turn table so that the masses will slip from this position.

Solution

- Here $M_1 = 10\text{ kg}$, $M_2 = 5\text{ kg}$
 $\omega = 10\text{ rad s}^{-1}$, $\mu = 0.5$
 $r = 0.3\text{ m}$, $r_1 = 0.124\text{ m}$
 $r_2 = r - r_1 = 0.3 - 0.124 = 0.176\text{ m}$.

As the turn table rotates, the centrifugal forces F_1 and F_2 act on the masses M_1 and M_2 in the directions away from the centre O of the turn table.



Force f due to friction acts on mass M_1 towards the centre O i.e. in a direction opposite to that of F_1 .

Now

$$F_1 = M_1 r_1 \omega^2; F_2 = M_2 r_2 \omega^2$$

$$f = \mu M_1 g.$$

As the two masses are in equilibrium

$$F_1 - f = F_2$$

$$f = F_1 - F_2 = (M_1 r_1 - M_2 r_2) \omega^2$$

$$f = (10 \times 0.124 - 5 \times 0.176) \times 10^2$$

$$f = 36\text{N}.$$

- The masses will slip from their positions on the turn table, if $f \leq F_1 - F_2$. If ω_{\min} is the angular velocity of the turn table which f is just equal to $F_1 - F_2$.

$$f = (M_1 r_1 - M_2 r_2) \omega_{\min}^2$$

$$\omega_{\min} = \sqrt{\frac{f}{M_1 r_1 - M_2 r_2}}$$

$$= \sqrt{\frac{\mu M_1 g}{M_1 r_1 - M_2 r_2}}$$

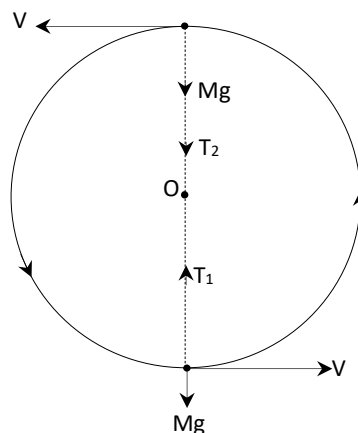
$$= \sqrt{\frac{0.5 \times 10 \times 9.8}{10 \times 0.124 - 5 \times 0.176}}$$

$$\omega_{\min} = 11.67\text{ rad s}^{-1}$$

TYPE C: SOLVED EXAMPLES**Example 1.**

A string 0.5 m in length is used to whirl a 1.0 kg stone in a vertical circle at uniform speed of 5 m/s . determine the tension in the string. When the stone is

- At the top of the circle
- At the bottom of the circle.

Solution**Diagram**

(i) At the top

$$T_2 = \left(\frac{V^2}{r} - g \right) = 1 \left(\frac{5^2}{0.5} - 9.8 \right)$$

$$T_2 = 40.2 \text{ N.}$$

(ii) At the bottom

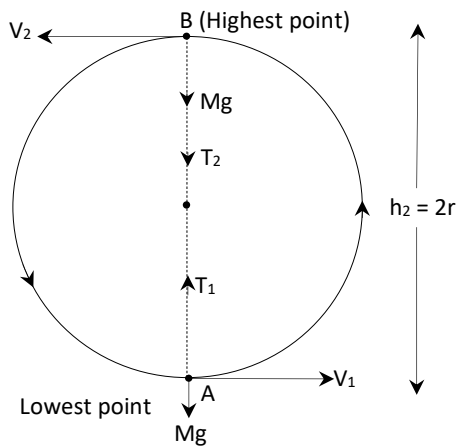
$$T_1 = M \left(\frac{V^2}{r} + g \right)$$

$$= 1 \times \left(\frac{5^2}{0.5} + 9.8 \right)$$

$$T_1 = 59.8 \text{ N.}$$

Example 2

One end of a string of length 1.5 m is tied to a stone of mass 0.4 kg and the other end to a small pivot on a smooth vertical board. What is the minimum speed of the stone required at its lowest most point that the string does not slack at any point in its motion along vertical circle?

Solution

At the point B: $V_2 = \sqrt{gr}$

Apply the law of conservation of mechanical energy.

$$(P.e + K.e)_A = (P.e + K.e)_B$$

$$\frac{1}{2}MV^2 = Mgh_2 + \frac{1}{2}MV_2^2$$

$$V_1^2 = V_2^2 + 2gh_2$$

$$= gr + 2g(2r)$$

$$V_1^2 = 5gr$$

$$V_1 = \sqrt{5gr} = \sqrt{5 \times 9.8 \times 1.5}$$

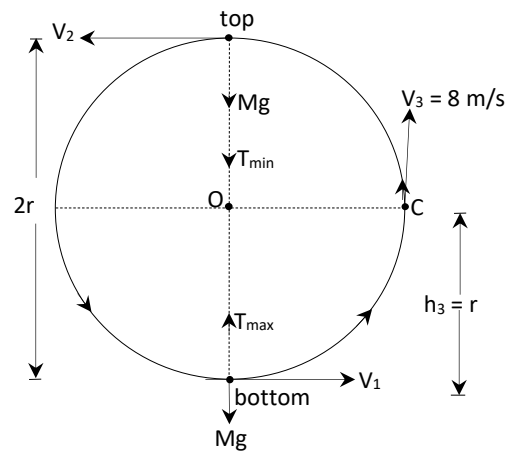
$$V_{\min} = 8.573 \text{ m/s.}$$

Example 3

A body of 50 g is whirled in a vertical circle of radius 60 cm. determine the maximum and minimum tensions in the string when its velocity at horizontal position is 8 m/s.

Solution

Consider the motion of a body in a vertical circle as show below:



Apply the principle of the conservation of mechanical energy.

$$P.e_1 + K.e_1 = P.e_3 + K.e_3$$

$$Mgh_1 + \frac{1}{2}MV_1^2 = Mgh_3 + \frac{1}{2}MV_3^2$$

$$2gh_1 + V_1^2 = 2gh_3 + V_3^2$$

$$h_1 = 0, V_3 = 8 \text{ m.s, } h_3 = r$$

$$V_1 = \sqrt{V_3^2 + 2gr}$$

$$= \sqrt{8^2 + 2 \times 9.8 \times 0.6}$$

$$V_1 = 8.7 \text{ m.s.}$$

$$\text{Since } T_{\max} = \frac{MV_1^2}{r} + Mg$$

$$= \frac{50 \times 10^{-3} \times 8.7^2 + 50 \times 10^{-3} \times 9.8}{0.6}$$

$$T_{\max} = 6.8 \text{ N.}$$

Again

$$\frac{1}{2}MV_2^2 + Mgh_2 = \frac{1}{2}MV_3^2 + Mgh_3$$

$$V_2 = \sqrt{V_3^2 - 2gr}$$

$$= \sqrt{82 - 2 \times 9.8 \times 0.6}$$

$$V_2 = 7.2 \text{ m/s.}$$

Now

$$T_{\min} = \frac{MV_2^2}{r} - Mg$$

$$= 50 \times 10^{-3} \left(\frac{(7.2)^2}{0.6} - 9.8 \right)$$

$$T_{\min} = 3.8 \text{ N.}$$

Example 4

A stone is tied to a weightless string and revolved in a vertical circle of radius 5 m. what should be the minimum speed of the stone at the highest point of the circle so that the string does not slack? What should be the speed of the stone at the lowest point of vertical circle?

Take $g = 98 \text{ m/s}^2$.

Solution

$$r = 5 \text{ m } V_H = ?$$

$$V_L = ? \quad g = 9.8 \text{ m/s}^2.$$

The string will not slack at the highest point when

$$V_H = \sqrt{gr} = \sqrt{9.8 \times 5}$$

$$V_H = 7 \text{ m/s}$$

At the bottom of vertical circle

$$V_L = \sqrt{5gr} = \sqrt{5 \times 9.8 \times 5}$$

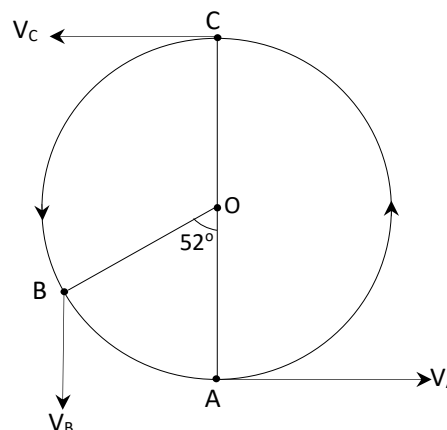
$$V_L = 15.65 \text{ m/s}$$

Example 5

Consider the figure below which shows the particle of mass $M = 5 \text{ kg}$ moving in a circle as shown in the figure below. If the (radius) length of the string is 1.5 m and the body is then pulled to the position B from C such that the string makes an angle of 52° with the vertical

- With what velocity should the body be realised from B so that it reaches at C with a velocity of 2.20 m/s ?
- With what velocity will the body pass the lowest point A?

- Find the tension in the string at the lowest point, A.



Solution

- $V_B = ?$

Apply the principle of conservation of the mechanical energy.

$$(P.e + K.e)_B = (P.e + K.e)_C$$

$$Mgh + \frac{1}{2}MV_B^2 = Mg(2r) + \frac{1}{2}MV_C^2$$

$$V_B^2 = V_C^2 + 4gr - 2gh$$

$$\text{but } h = r(1 - \cos\theta)$$

$$V_B = \sqrt{V_C^2 + 4gr - 2gr(1 - \cos\theta)} =$$

$$\sqrt{2.2^2 + 4 \times 9.8 \times 1.5 - 2 \times 9.8 \times 1.5(1 - \cos 52^\circ)} \quad V_B$$

$$= 7.23 \text{ m/s.}$$

- $V_A = ?$

Apply principle of conservation of mechanical energy.

$$\frac{1}{2}MV_A^2 = \frac{1}{2}MV_C^2 + 2Mgr$$

$$V_A = \sqrt{V_C^2 + 4gr}$$

$$= \sqrt{(2.2)^2 + 4 \times 9.8 \times 1.5}$$

$$V_A = 7.97 \text{ m/s.}$$

- Tension on the string at the lowest position.

$$T_A = M \left(\frac{V_A^2}{r} + g \right)$$

$$= 5 \left(\frac{(7.97)^2}{1.5} + 9.8 \right)$$

$$T_A = 260.736 \text{ N.}$$

Example 6. NECTA 2007/P1/4

(a) (i) What is meant by centripetal force.

(ii) Derive the expression of $a = \frac{V^2}{r}$ where a ,

V and r stand for the centripetal acceleration, linear velocity and radius of a circular path respectively.

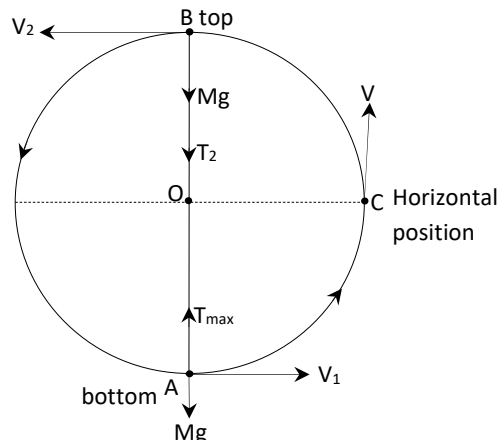
(b) A ball of mass 0.5 kg attached to a light inextensible string rotates in a vertical circle of radius 0.75 m such that it has a speed of 5 m/s. When the string is in horizontal. Calculate:-

- (i) The speed of the ball and the tension in the string at the lowest point of its circular path.
- (ii) Evaluate the work done by Earth's gravitational force and by the tension in the string as the ball moves from its highest to its lowest point?

Solution

(a) Refer to your notes.

(b) Consider the figure below.



(i) Apply the law of conservation of energy.

$$(P.e + K.e)_A = (P.e + K.e)_C$$

$$\frac{1}{2} M V_1^2 = \frac{1}{2} M V^2 + Mgh \text{ but } h = r$$

$$V_1 = \sqrt{V^2 + 2gr}$$

$$V_1 = \sqrt{5^2 + 2 \times 9.8 \times 0.75}$$

$$V_1 = 6.30 \text{ m/s.}$$

Tension of the string at the bottom.

$$T_{\max} = M \left(\frac{V_1^2}{r} + g \right)$$

$$= 0.5 \left(\frac{6.3^2}{0.75} + 9.8 \right)$$

$$T_{\max} = 31.36 \text{ N.}$$

(ii) The work done by the gravitational force as the ball moves from its highest point to its lowest point is the increase in the kinetic energy of the body. By the law of the conservation of energy increase in K.e = decreases in P.e

$$\Delta K.E = 2Mgr$$

By the work energy theorem.

$$W = \Delta K.E = 2Mgr$$

$$= 2 \times 0.5 \times 9.8 \times 0.75 \text{ J}$$

The tension in the string does not do any work done on the ball as the tension in the string acts perpendicular to the ball.

$$W = 0$$

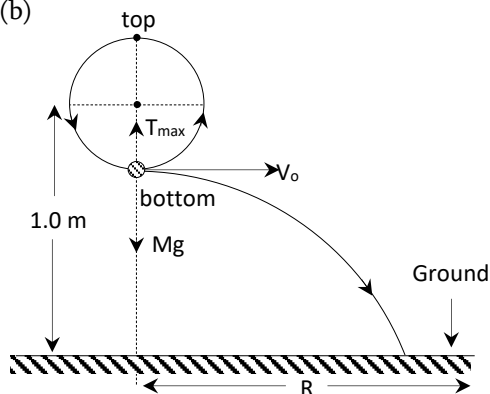
Example 7

- (a) Explain what is meant by angular velocity. Derive an expression for the force required to keep a particle of mass M move in a circle of radius r with uniform angular velocity, ω .
- (b) A stone of mass 500 gm is attached to a string of length 50 cm which will break if the tension in it exceeds 20N. The stone is whirled in a vertical circle, the axis of rotation being at a height of 100 cm above the ground. The angular speed is very slowly increased until the string breaks. In what position is this break most likely to occur and at what angular speed? Where will the stone hit the ground?

Solution

(a) Refer to your notes.

(b)



The position in which this likely to occur is at the LOWEST POSITION since the tension in the string is maximum at this point.

$$T_{\max} = M \left(\frac{V_1^2}{r} + g \right) \text{ but } V = \omega r$$

$$\omega = \sqrt{\frac{1}{r} \left(\frac{T_{\max}}{M} - g \right)}$$

$$= \sqrt{\frac{1}{0.5} \left(\frac{20}{0.5} - 9.8 \right)}$$

$$\omega = 7.77 \text{ rad s}^{-1}.$$

Let R = Horizontal range

Flight time

$$h = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$$

$$t = \sqrt{\frac{2 \times 0.5}{9.8}} = 0.319 \text{ sec}$$

Now

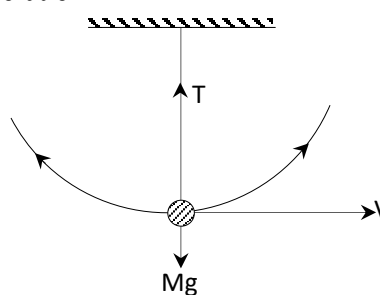
$$R = V_{0t} = \omega r t$$

$$= 0.5 \times 7.77 \times 0.319$$

$$R = 1.2410 \text{ m}.$$

Example 8

A boy weighing 55 kg swings on a rope 4.5 m long. If he passes through the lowest position with a velocity of 4 m/s. What is the corresponding tension in the string?

Solution

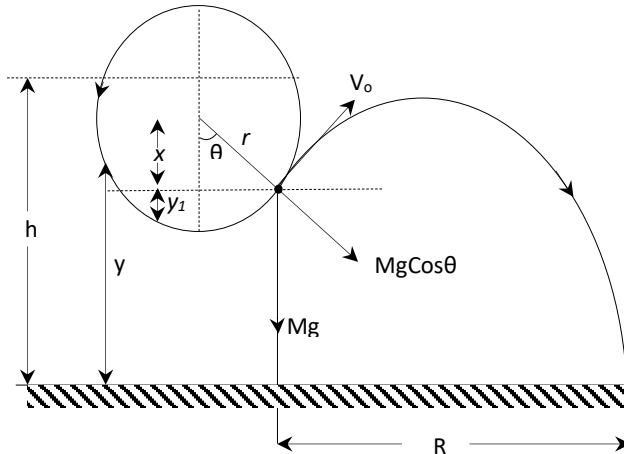
$$T = M \left(\frac{V_1^2}{L} + g \right)$$

$$= 55 \left(\frac{4^2}{4.5} + 9.8 \right)$$

$$T = 734.6 \text{ N}.$$

Example 9

A stone of mass 2.0 kg and attached to the end of a string 1.0 m long is whirled round vertical circle with constant speed. The horizontal axis of rotation is 10 m above the ground. As the stone moves upwards from its lowest position the string snaps just when it makes an angle of 60° to the vertical. Calculate the horizontal distance travelled by the stone before striking the ground. (Assume tension T is 810N before its snaps).

Solution

$$\cos\theta = x/r \Rightarrow x = r\cos\theta$$

$$y_1 = r - x = r - r\cos\theta$$

$$X = 1 \times \cos 60^\circ = 0.5 \text{ m.}$$

Now

$$y = h - x = 10 - 0.5 = 9.5 \text{ m.}$$

Let V_o = velocity of a projectile.

$$\frac{MV_o^2}{r} = T - Mg\cos\theta$$

$$V_o = \left(\frac{r}{M} (T - Mg\cos\theta) \right)^{\frac{1}{2}}$$

$$= \left(\frac{1}{2} (810 - 2 \times 9.8 \cos 60^\circ) \right)^{\frac{1}{2}}$$

$$V_o = 20 \text{ m/s.}$$

Flight time, t

$$-y = (V_o \sin\theta)t - \frac{1}{2}gt^2$$

$$-9.5 = (20 \sin 60^\circ)t - 4.9t^2.$$

On solving t quadratically and omitting negative time.

$$t = 4.01 \text{ sec.}$$

horizontal range

$$R = (V_o \cos\theta)t$$

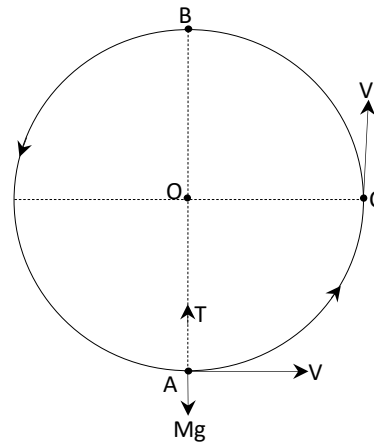
$$= (20 \cos 60^\circ) \times 4.01$$

$$R = 40.17 \text{ m.}$$

Example 10

A stone of mass 0.5 kg tied to a rope of length 0.5 m revolves along a circular path in a vertical plane. The tension in the rope at the bottom point of the

circle is 35N. To what height the stone will rise if the rope breaks the moment the velocity is directed upwards. (Take $g = 10 \text{ m/s}^2$).

Solution

At the point A

$$\frac{MV^2}{L} = T - Mg$$

$$V^2 = \frac{L}{M} (T - Mg)$$

$$= \frac{0.5}{0.5} (45 - 0.5 \times 10)$$

$$V^2 = 40 \text{ m}^2/\text{s}^2.$$

When the rope breaks, the stone rises to height h until its entire kinetic energy is converted into potential energy.

$$\frac{1}{2}MV^2 = Mgh$$

$$h = \frac{V^2}{2g} = \frac{40}{2 \times 10}$$

$$h = 2 \text{ m.}$$

Example 11

A bucket containing water is tied to one end of a rope of length 2.5m and rotated about the other end in a vertical circle so that water does not spill even when bucket is upside down. What is the minimum velocity of the bucket at which this happens? How many revolution per minute is it making? ($g = 10 \text{ m/s}^2$)

Solution

Water in the bucket will not spill when weight of water is balanced by centrifugal force.

$$Mg = \frac{MV^2}{r}, V = \sqrt{rg}$$

$$V = \sqrt{2.5 \times 10} = 5 \text{ m/s.}$$

This is the velocity at the highest point.

$$\text{Angular velocity, } \omega = \frac{V}{r} = \frac{5}{2.5}$$

$$\omega = 2 \text{ rads}^{-1}.$$

Frequency of rotation

$$f = \frac{\omega}{2\pi} \text{ r.p.s} = \frac{60 \text{ r.p.m}}{\pi}$$

$$f = \frac{60}{\pi} \text{ r.p.m}$$

Example 12

In circus the diameter of globe of death is 20 m. from what minimum height must a motor cyclist start in order to go around the globe successfully?

Solution

$$\text{Here } r = \frac{20}{2} = 10 \text{ m, } h = ?$$

On rolling down the incline loss in P.e = gain in K.e

$$Mgh = \frac{1}{2} MV^2$$

$$V = \sqrt{2gh}$$

For looping the loop minimum velocity at the lowest point.

$$V_{\min} = \sqrt{5gr}$$

$$\sqrt{5gr} = \sqrt{2gh}$$

$$h = \frac{5}{2} r = \frac{5}{2} \times 10$$

$$h = 25 \text{ m.}$$

Example 13

A small stone of mass 200 g is tied to one end of a string of length 80 cm. holding the other end in hand, the stone is whirled into a vertical circle. What is the minimum speed that needs to be imparted at the lowest point of the circular path, so that the stone is just able to complete the vertical

circle? What would be the tension at the lowest point of the circular path? (Take $g = 10 \text{ m/s}^2$).

Solution

Given that: $M = 200 \text{ g} = 0.2 \text{ kg}$

$$L = r = 80 \text{ cm} = 0.8 \text{ m, } V_L = ?$$

$$T_L = ?$$

$$\text{Since } V_L = \sqrt{5gr} = \sqrt{5 \times 10 \times 0.8}$$

$$V_L = 6.32 \text{ m/s.}$$

$$T_L = 6 Mg = 6 \times 0.2 \times 10 = 12 \text{ N.}$$

Example 14

A mass less string of length 1.2 m has a breaking strength of 2 kgwt. A stone of mass 0.4 kg tied to one end of the string is made to move in a vertical circle by holding the other end in hand. Can the particle describe the vertical circle? (Take $g = 10 \text{ m/s}^2$).

Solution

Breaking strength = 2 kgwt

$$F = 2 \text{ kgwt} = 20 \text{ N.}$$

$$\text{Now } T_{\max} = T_L = 6 Mg$$

$T_{\max} = 6 \times 0.4 \times 10 = 24 \text{ N}$, which is more than breaking strength of string. Therefore, the string will break and the particle cannot describe the vertical circle.

Example 15

A small stone of mass 0.2 kg tied to a massless, inextensible string rotated in a vertical circle of radius 2 m. If the particle is just able to compete the vertical circle, what is its speed at the highest point of the circular path? How would the speed get affected if the mass of the stone is increased by 50%? (Take $g = 10 \text{ m/s}^2$)

Solution

When the particle is just able to complete the vertical circle,

$$V_L = \sqrt{5gr} \text{ and}$$

$$V_H = \sqrt{gr} = \sqrt{10 \times 2} = 4.47 \text{ m/s.}$$

As V_H does not depend upon mass, M of the stone, therefore, the value of V_H will not be affected by any change in mass of the stone.

Example 16

A particle of mass 150 g is attached to one end of a massless inextensible string. It made to describe a vertical circle of radius 1 m. when the string is making an angle of 48.2° with the vertical, its instantaneous speed is 2 m/s. what is the tension in the string in this position? Would this particle be able to complete its circular path?

Solution

From the equation

$$T = \frac{MV^2}{L} + Mg \cos \theta$$

$$= \frac{0.15 \times 2^2}{1} + 0.15 \times 10 \cos 48.2^\circ$$

$$T = 1.6 \text{ N.}$$

To be able to complete the vertical circle

$$V_H = \sqrt{gr} = \sqrt{10 \times 1} = 3.16 \text{ m/s.}$$

As instantaneous velocity is 2 m/s, it cannot increase (to 3.16 m/s) on reaching the highest point as gravity is opposing the motion. Therefore, the particle cannot complete its circular path.

Example 17

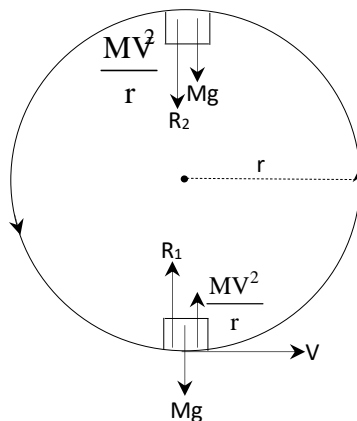
An aeroplane flying in the sky dives with a speed of 360 km/hr in a vertical circle of radius 200 m. the weight of pilot sitting in its is 75 kg. Calculate the force with which the pilot presses his seat when the aeroplane is (i) at the lowest position (ii) at the highest position. ($g = 10 \text{ m/s}^2$)

Solution

R_1 is normal reaction at the lowest position and R_2 is normal reaction at the highest position of vertical circle.

$$r = 200 \text{ cm, } M = 75 \text{ kg}$$

$$V = 360 \text{ km/hr} = 100 \text{ m/s}$$



- (i) At the lowest position

$$R_1 - Mg = \frac{MV^2}{r}$$

$$R_1 = M \left(\frac{V^2}{r} + g \right) = 75 \left(\frac{100^2}{20} + 10 \right)$$

$$R_1 = 4500 \text{ N} = 450 \text{ kgwt.}$$

- (ii) At the highest position

$$R_2 + Mg = \frac{MV^2}{r}$$

$$R_2 = M \left(\frac{V^2}{r} - g \right) = 75 \left(\frac{100^2}{20} - 10 \right)$$

$$R_2 = 3000 \text{ N} = 300 \text{ kgwt.}$$

TYPE D: SOLVED EXAMPLES**Example 1. NECTA 2014/P1/3(C)**

- (i) Define the term radial acceleration.
 (ii) An insert is realised from rest at the top of the smooth bowling ball such that it slides over the ball. Prove that it will lose its footing with the ball at an angle of about 48° with the vertical.

Example 2

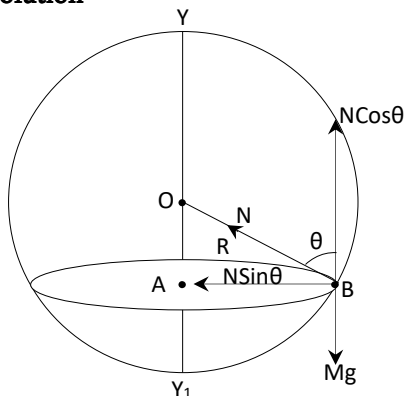
A thin circular wire of radius R rotates about its vertical diameter with an angular frequency, ω . Show that a small bead on the wire remains at its lower most point for

$\omega \geq \sqrt{\frac{g}{R}}$. What is the angle made by the radius

vector joining the centre to the bead with the vertical downward direction for

$\omega = \sqrt{\frac{2g}{R}}$ Neglect friction.

Solution



A bead is placed at the point B, $r = R \sin \theta$

At the equilibrium

$$N \cos \theta = Mg \quad \text{(i)}$$

$$N \sin \theta = M \omega^2 r \quad \text{(ii)}$$

Also

$$N \sin \theta = M \omega^2 R \sin \theta$$

$$N = M \omega^2 R \quad \text{(iii)}$$

Putting equation (iii) into (ii)

$$M \omega^2 R \cos \theta = Mg$$

$$\cos \theta = \frac{g}{R \omega^2}$$

Now $(1 \cos \theta) \leq 1$

$$\frac{g}{R \omega^2} \leq 1$$

$$\omega^2 \geq \frac{g}{R}, \omega \geq \sqrt{\frac{g}{R}}$$

Hence shown

$$\text{If } \omega^2 \geq \frac{2g}{R}, \cos \theta \geq \frac{r}{R \omega^2}$$

$$\cos \theta = \frac{g}{\frac{2g}{R} R} = \frac{1}{2}$$

$$\theta = \cos^{-1} \left(\frac{1}{2} \right) = 60^\circ$$

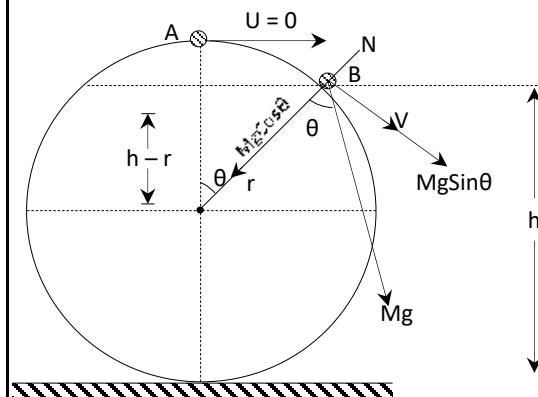
Example 3

A small mass M kg is balanced at the top of the large spherical smooth globe of radius r placed on the ground. It now given a very small push so that it starts to slide down a frictionless globe. The mass is observed to lose contact with globe at a height that is greater than the radius of the globe. Calculate:-

- The initial velocity just when the mass lose contact.
- At what height from the ground will the mass lose contact with the globe?

Solution

Let h be the height from the ground which the mass lose contact and V be its initial velocity at that instant.



At the point B

$$\frac{MV^2}{r} = Mg \cos \theta - N$$

For the particle loses contact

$$N = 0, \frac{MV^2}{r} = Mg \cos \theta$$

$$V^2 = rg \cos \theta \quad \text{(i)}$$

Apply the principle of the conservation of mechanical energy.

$$(P.e + K.e)_A = (P.e + K.e)_B$$

$$2Mgr = \frac{1}{2} MV^2 + Mgh$$

$$4gr = V^2 + 2gh$$

$$\cos\theta = \frac{h-r}{r}, \quad r\cos\theta = h-r$$

$$h = r(1 + \cos\theta)$$

Now

$$4gr = V^2 + 2gh(1 + \cos\theta)$$

$$V^2 = 2gr(1 - \cos\theta) \quad \text{---(ii)}$$

$$(i) = (ii)$$

$$rg\cos\theta = 2gr(1 - \cos\theta)$$

$$3gr\cos\theta = 2gr$$

$$\text{but } r\cos\theta = h - r$$

$$3g(h - r) = 2gr$$

$$3h = 5r$$

$$h = \frac{5}{3}r$$

From the equation

$$4gr = V^2 + 2gh$$

$$4gr = V^2 + 2g \cdot \frac{5}{3}r$$

$$V = \sqrt{\frac{2}{3}rg}$$

$$(i) \quad V = \sqrt{\frac{2}{3}rg}$$

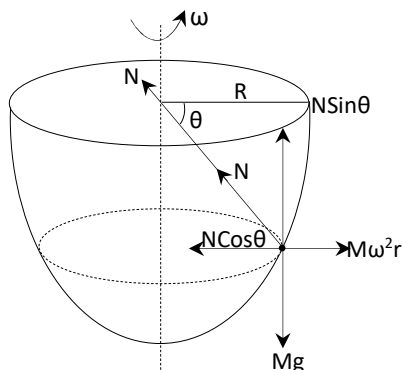
$$(ii) \quad h = \frac{5}{3}r$$

Example 4

A particle of mass 0.3 kg moves with angular velocity of 10 rad s^{-1} in a horizontal circle of radius 20 cm inside a smooth hemispherical bowl. Find the reactions of the bowl on the particle and the radius of the bowl.

Solution

Consider the figure below.



Inclination of the normal to the horizontal is given by

$$\tan\theta = \frac{g}{\omega^2 r} = \frac{9.8}{10^2 \times 0.2}$$

$$\theta = \tan^{-1}(0.49) = 26.1^\circ$$

Normal reaction force

$$N = \frac{Mg}{\cos\theta} = \frac{0.3 \times 9.8}{\sin(26.1^\circ)}$$

$$N = 6.68 \text{ N.}$$

Radius of the bowl.

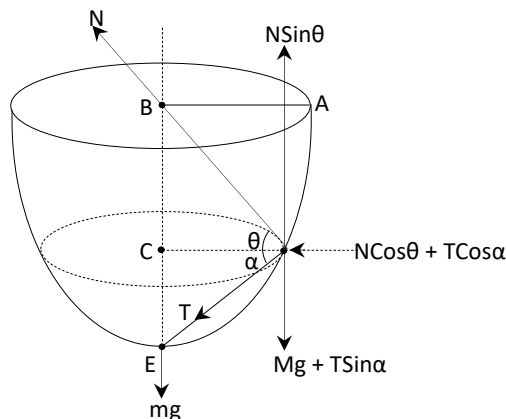
$$R = \frac{r}{\cos\theta} = \frac{20}{\cos(26.1^\circ)}$$

$$R = 22.3 \text{ cm.}$$

Example 5

A particle of mass M describes a horizontal circle of radius 24 cm inside a smooth hemispherical bowl of internal radius 25 cm which is held with its rim horizontal. A fine weightless thread tied to the particle passes through a small smooth hole at the bottom of the bowl and supports another particle of mass m which hangs at rest. Calculate the speed of the first particle.

Solution



At the equilibrium by balancing the forces.

$$T = Mg, \text{ and } Mg + T\sin\alpha = N\sin\theta$$

$$Mg + Mg\sin\alpha = N\sin\theta \quad \text{---(i)}$$

The centripetal force is given by

$$N\cos\theta + T\sin\alpha = M\omega^2 r$$

$$N\cos\theta + Mg\cos\alpha = M\omega^2 r \quad \text{---(ii)}$$

From the figure above

$$\overline{CE} = \overline{BE} - \overline{BC} = 25 - \sqrt{25^2 - 24^2},$$

$$\overline{CE} =$$

$$18$$

$$\cos\theta = \frac{24}{25}, \sin\theta = \frac{7}{25}$$

$$\sin\alpha = \frac{3}{5}, \cos\alpha = \frac{4}{5}$$

$$\tan\alpha = \frac{18}{24} = \frac{3}{4}$$

Now putting these values into equation (i) and (ii)

$$Mg + \frac{3}{5}Mg = \frac{7}{25}N$$

$$N = \frac{40}{7}Mg$$

$$\text{Also } \frac{40}{7} \times \frac{24}{25}Mg + \frac{4}{5}Mg = 0.24M\omega^2$$

$$\frac{44g}{7} = 0.24\omega^2$$

$$\omega = 16.02 \text{ rads}^{-1}$$

Speed of the first particle

$$V = \omega r$$

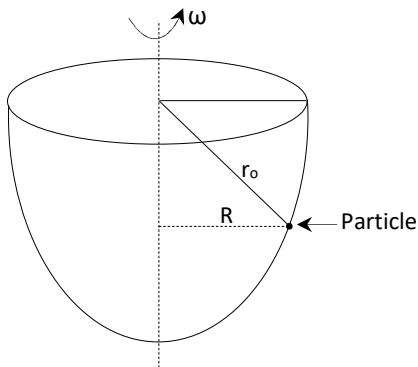
$$= 16.02 \times 0.24$$

$$V = 3.84 \text{ m/s}$$

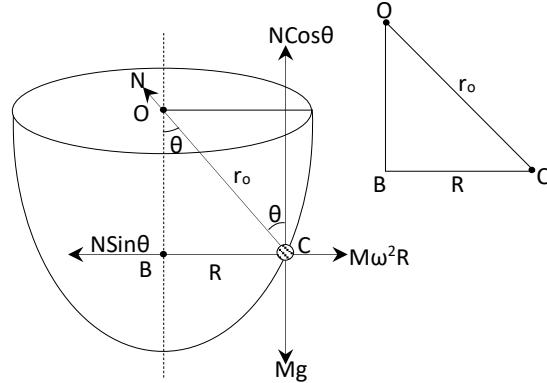
Example 6

A hemispherical bowl of radius r_0 is rotating with constant angular velocity ω about its vertical axis of symmetry. If a particle placed on the inside surface of the bowl is at rest relative to the surface at a distance R from the axis, show that

$$R = \sqrt{r_0^2 - \frac{g^2}{\omega^4}}$$



Solution



At the equilibrium of the particle

$$N \sin \theta = M\omega^2 r \quad \text{--- (i)}$$

$$N \cos \theta = Mg \quad \text{--- (ii)}$$

(i) (ii)

$$\frac{N \sin \theta}{N \cos \theta} = \frac{M\omega^2 R}{g}$$

$$\tan \theta = \frac{\omega^2 R}{g}$$

$$\text{But } \tan \theta = \frac{R}{\sqrt{r_0^2 - R^2}}$$

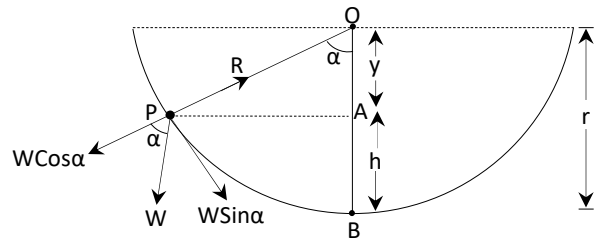
$$\frac{\omega^2 R}{g} = \frac{R}{\sqrt{r_0^2 - R^2}}$$

On simplifying we get

$$R = \sqrt{r_0^2 - \frac{g^2}{\omega^4}}$$

Example 7

Calculate the height up to which an insect can crawl up a fixed bowl in the form of a hemisphere of radius r . given coefficient of friction $= \frac{1}{\sqrt{3}}$.



Solution

The insect can crawl up the bowl from B to P through a height $BA = h$.

At the equilibrium

$$R = \omega \cos \alpha, F = \omega \sin \alpha$$

F = friction force

$$\mu = \frac{F}{R} = \frac{\omega \sin \alpha}{\omega \cos \alpha} = \tan \alpha$$

$$\text{In } \triangle OPA, \tan \alpha = \frac{PA}{OA}$$

$$\tan \alpha = \sqrt{\frac{r^2 - y^2}{y^2}} = \frac{1}{\sqrt{3}}$$

$$\frac{r^2 - y^2}{y^2} = \frac{1}{3}$$

$$y^2 = \frac{3r^2}{4}, y = \frac{\sqrt{3}r}{2}$$

$$h = BA = OB - OA = r - y$$

$$h = r - \frac{\sqrt{3}}{2}r = 0.134r$$

$$h = 13.4\% \text{ of } r.$$

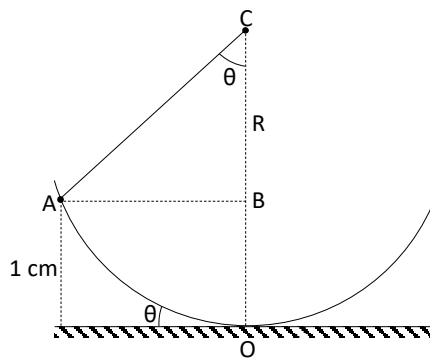
Example 8

A particle of mass 1 g executes an oscillatory motion on the concave surface of a spherical dish of radius 2m placed on a horizontal plane, shown on the figure below. If the motion of the particle begins from a point on the dist at a height of 1 cm; from the horizontal plane and coefficient of friction is 0.01, find the total distance covered by the particle before coming to rest

Solution

In the figure

$$CO = CA = 200 \text{ cm}, OB = 1 \text{ cm } BC = CO - OB = 200 - 1 = 199 \text{ cm}$$



$$\text{In } \triangle ABC, \cos \theta = \frac{BC}{AC} = \frac{199}{200}$$

Treating the dist as an inclined plane and using principle of conservation of energy.

Loss in P.e = work done against friction.

$$Mgh = \mu R \times S = \mu Mg \cos \theta \times S$$

$$S = \frac{Mgh}{\mu Mg \cos \theta} = \frac{h}{\mu \cos \theta}$$

$$= \frac{1}{0.01 \times 199/200} = \frac{200}{199}$$

$$S = 1.005 \text{ m.}$$

Example 9

A hemispherical bowl of radius R is rotating about its axis of symmetry which is kept vertical. A small block is kept in the bowl and rotates with it without slipping. If the surface of the bowl is smooth and the angle made by the radius through, the block with the vertical is θ , show that the angular speed of the bowl is given by

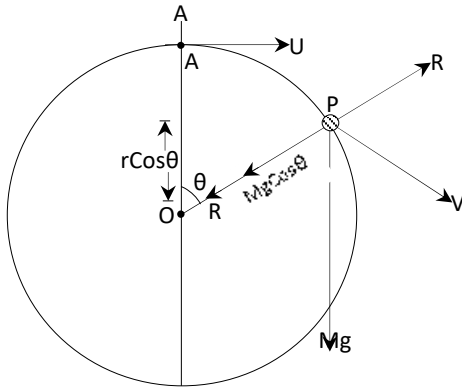
$$\omega = \sqrt{\frac{g \cos \theta}{R}}$$

Example 10

An object of mass M is resting on top of a hemispherical mound of ice whose radius of curvature is R . The object is given a small push and start sliding down the mound. Show that the object will lose contact with the surface of ice at a vertical height of $\frac{2R}{3}$

Example 11

A small bead of mass M is threaded on a smooth circular of radius, r and centre O , and which is fixed in a vertical plane. The bead is projected with speed U from the highest point A of the wire. Find the reaction on the bead due to the wire when the bead is at P , in terms of M , g , r , U and θ where $\theta = \hat{AOP}$

Solution

Apply the principle of conservation of energy
increase in K.e = decrease in P.e

$$\frac{1}{2}MV^2 - \frac{1}{2}MU^2 = Mgh$$

At the equilibrium when the particle is at OP

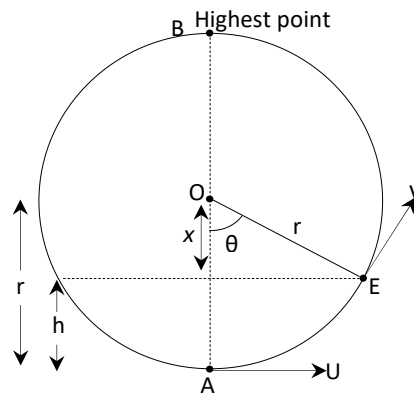
$$Mg \cos \theta - R = \frac{M}{r}[U^2 + 2gh(1 - \cos \theta)]$$

$$R = Mg(3 \cos \theta - 2) - \frac{MU^2}{r}$$

Example 12

A small bead is thread on a smooth circular wire of radius r which is fixed in a vertical plane. The bead is projected from the lowest point of the wire with speed $\sqrt{6gr}$. Find the speed of the bead when it turned through

- (a) 60° (b) 90° (c) 180° (d) 300°

Solution

From the figure above

$$X = r \cos \theta, h = r - x = r - r \cos \theta$$

Apply the principle of the conservation of mechanical energy.

$$(P.e + K.e)_E = (P.e + K.e)_A$$

$$Mgh + \frac{1}{2}MV^2 = \frac{1}{2}MU^2$$

$$V^2 = U^2 - 2gh$$

$$V^2 = U^2 - 2gh(r - r \cos \theta)$$

$$\text{but } U = \sqrt{6gr}, U^2 = 6gr$$

$$U^2 = 6gr + 2gr \cos \theta - 2gr$$

$$V = \sqrt{2gr(2 + \cos \theta)}$$

(a) At $\theta = 60^\circ$

$$V = \sqrt{2gr(2 + \cos 60^\circ)}$$

$$V = \sqrt{5gr}$$

(b) $\theta = 90^\circ, V = \sqrt{2gr(2 + \cos 90^\circ)}$

$$V = \sqrt{4gr}$$

(c) $\theta = 180^\circ, V = \sqrt{2gr(2 + \cos 180^\circ)}$

$$V = \sqrt{2gr}$$

(d) $\theta = 300^\circ, V = \sqrt{2gr(2 + \cos 300^\circ)}$

$$V = \sqrt{5gr}$$

Example 13

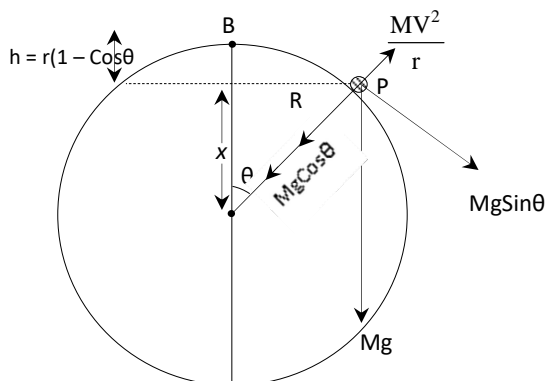
A smooth circular tube is held fixed in a vertical plane. A particle of mass, M which can slide inside the tube is slightly displaced from rest at the highest point of the tube. Find the reaction between the particle and the tube when it is at an

angular distance θ from the highest point of the tube. Also find the vertical component of the acceleration of the particle when $\theta = 120^\circ$.

Solution

Consider the figure below:

$$h = r(1 - \cos\theta)$$



Apply the principle of conservation of energy.

Gain in K.e = Loss in P.e

$$\frac{1}{2}MU^2 = Mgh$$

$$U^2 = 2gh$$

At the point, P $h = r(1 - \cos\theta)$

$$U^2 = 2gr(1 - \cos\theta)$$

Reaction on the particle at P.

$$Mg\cos\theta + R = \frac{MU^2}{r}$$

$$R = \frac{MU^2}{r} - Mg\cos\theta$$

$$R = \frac{M}{r}[2gr(1 - \cos\theta)] - Mg\cos\theta$$

$$R = Mg(2 - 3\cos\theta) \text{ in ward.}$$

Acceleration of the particle at $\theta = 120^\circ$

$$Ma = Mg\cos\theta + R$$

$$= Mg\cos\theta + Mg(2 - 3\cos\theta)$$

$$Ma = Mg\cos 120^\circ + Mg(2 - 3\cos 120^\circ)$$

$$Ma = 3Mg$$

$$a = 3g.$$

Vertical component of the acceleration.

$$a_y = a\cos\theta = 3g\cos 120^\circ.$$

$$a_y = \frac{3g}{2}$$

Example 14

Prove that the velocity V with which a body must be projected at the lowest part of a loop apparatus of radius R in the vertical plane so that it passes at the highest position with minimum velocity, is given by

$$V = \sqrt{5gr}$$

TYPE E: SOLVED EXAMPLES

EXAMPLE 01.

A small toy car moves round a circular track of radius 4m. the car makes one revolution in 10 second calculate:-

- The speed of the car.
- Its centripetal acceleration
- The centripetal force exerted by the track mass of the car is 200gm.
- The safe speed with which the car can move around without toppling, if the distance between wheels is 4cm and the height of the centre of gravity of the car from the horizontal is 2cm.

Solution

$$(a) \quad V = \frac{2\pi r}{T} = \frac{2 \times 3.14 \times 4}{10}$$

$$V = 2.51 \text{ mfs}$$

$$(b) \quad a = \frac{v^2}{r} = \frac{(2.51)^2}{4}$$

$$a = 1.57 \text{ mfs}^2$$

$$(c) \quad F = Ma = 0.2 \times 1.57$$

$$F = 0.314 \text{ N}$$

$$(d) \quad V_{\max} = \sqrt{\frac{rag}{h}} = \sqrt{\frac{9.8 \times 0.02 \times 4}{0.02}}$$

$$V_{\max} = 6.26 \text{ mfs}$$

Example 02.

A car whose wheels are 1.4m apart laterally and whose centre of gravity is 0.5m above the ground moves round a curve of radius 60m. assuming no slipping of the wheels on the road find the highest speed at which the car can round the curve without overturning.

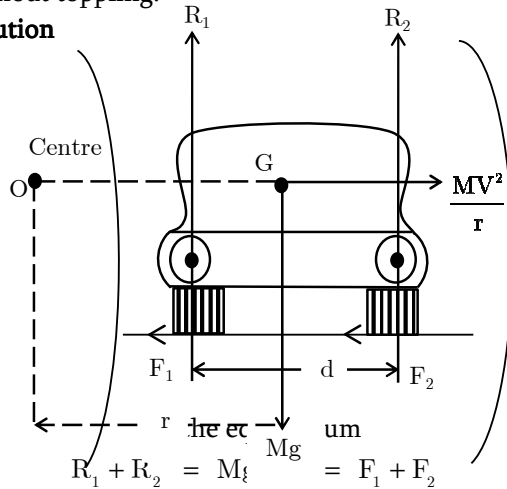
Solution

$$V_{\max} = \sqrt{\frac{rdg}{2h}} = \sqrt{\frac{1.4 \times 60 \times 9.8}{2 \times 0.5}}$$

$$V_{\max} = 2.69 \text{ m/s}$$

Example 03.

A car whose wheels are 1.5m apart laterally and whose centre of the gravity is 1.5m above the ground round a curve of radius 250m. find the maximum speed at which the car can travel without toppling.

Solution

At the maximum speed when the car is moving without toppling, $R_1 = 0$, $R_2 = Mg$.

Take the moment of force about G.

$$Fh = \frac{R_2 d}{2}$$

$$\frac{MV_{\max}^2}{r} h = \frac{Mgd}{2}$$

$$V_{\max} = \sqrt{\frac{rgd}{2h}} = \sqrt{\frac{9.8 \times 1.5 \times 250}{2 \times 1.5}}$$

$$V_{\max} = 35 \text{ m/s}$$

Example 04.

Find the maximum speed at which car can take turn round a curve of 30m radius on a level road if the coefficient of friction between the tyres and the road is 0.4.

Solution

$$r = 30 \text{ m}, \mu = 0.4, g = 10 \text{ m/s}^2$$

$$V_{\max} = \sqrt{\mu rg} = \sqrt{0.4 \times 30 \times 10}$$

$$V_{\max} = 11 \text{ m/s}$$

Example 05

A car travels on a flat, circular track of radius 200m at 30m/s and has a centripetal acceleration = 4.5 m/s^2 .

- If the mass of the car is 1000kg, what frictional force is required to provide the acceleration.
- If the coefficient of static friction is 0.8, what is the maximum speed at which the car can circle the track?

Solutions.

- Frictional force required

$$F = Ma = 1000 \times 4.5$$

$$F = 4,500 \text{ N}$$

- $V = \sqrt{\mu rg} = \sqrt{0.8 \times 200 \times 9.8}$
 $V = 39.6 \text{ m/s}$

Example 06.

A car is moving at 30km/hr in a circle of radius. Find the minimum value of μ for the car to make the turn without skidding.

Solution

$$\text{Since } \mu = \frac{v^2}{rg} = \frac{(8.3)^2}{60 \times 9.8}$$

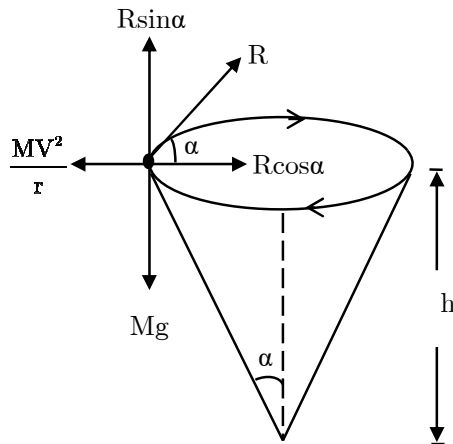
$$\mu = 0.12$$

Example 07

A particle describes a horizontal circle on the smooth surface of an inverted cone. The height of the plane of the circle above the vertex is 9.8cm. Find the speed of the particle take $g = 9.8 \text{ m/s}^2$.

Solution.

$$h = 9.8 \text{ cm}, V = ?$$



$$R \cos \alpha = \frac{mv^2}{r}, \quad R \sin \alpha = mg$$

Dividing, we get

$$\tan \alpha = \frac{rg}{v^2} \text{ or } \frac{r}{h} = \frac{rg}{v^2}$$

$$V = \sqrt{gh} = \sqrt{9.8 \times 9.8 \times 10^{-2}}$$

$$V = 0.98 \text{ m/s}$$

Example 08

What is the maximum speed with which a cyclist can turn around curved path of radius 10m, if the coefficient of friction force between the tyres and road is 0.5? If the cyclist wants to avoid overturning by what angle the cyclist must lean from the vertical?

Solution.

$$V_{\max} = \sqrt{\mu rg} = \sqrt{0.5 \times 10 \times 9.8}$$

$$V_{\max} = 7 \text{ m/s}$$

For no overturning

$$\tan \theta = \mu, \quad \theta = \tan^{-1}(\mu)$$

$$\theta = \tan^{-1}(0.5) = 26.8^\circ$$

Example 09.

A cyclist speeding at 18km/hr on a level road takes a sharp circular turn of radius 3m without reducing the speed. The coefficient of static friction between

the tyres and the road is 0.1, will the cyclist slip while taking the turn?

Solution

The condition for the cyclist not to slip

$$V^2 \leq \mu rg$$

$$\leq 0.1 \times 3 \times 9.8$$

$$V^2 \leq 2.94 \text{ m}^2/\text{s}^2$$

Thus, for the cyclist not to slip, V^2 should be less than $2.94 \text{ m}^2/\text{s}^2$.

$$V = 18 \text{ km/hr} = 5 \text{ m/s}$$

$$V^2 = 25 \text{ m}^2/\text{s}^2$$

It's clear that the condition above is not obeyed. Hence the cyclist will slip.

Example 10.

A highway road designed for an average speed of 72km/hr has a turn of radius 50m. to what angle must the road be banked so that cars travelling at 72km/hr may not overturn.

$$V = \frac{72 \times 10^3}{3600} = 20 \text{ m/s}$$

$$\text{Since } \tan \theta = \frac{v^2}{rg}, \quad \theta = \tan^{-1} \left[\frac{v^2}{rg} \right]$$

$$\theta = \tan^{-1} \left(\frac{20^2}{50 \times 9.8} \right)$$

$$\theta = 39^\circ$$

Example 11

A circular race track of radius 400m is banked at an angle of 10° . If the coefficient of friction between the wheels of a race car and the road is 0.2, what is the (i) optimum speed of the car to avoid wear and tear on its tyres. (ii) maximum permissible speed to avoid slipping.

Solution

Here $r = 400 \text{ m}$, $\theta = 10^\circ$, $\mu = 0.2$

(i) Optimum speed of the car to avoid wear and tear

$$V = \sqrt{rg \tan \theta} = \sqrt{400 \times 9.8 \tan 10^\circ}$$

$$V = 26.29 \text{ m/s}$$

(ii) Maximum permissible speed

$$V_{\max} = \sqrt{\frac{rg(\mu + \tan \theta)}{1 - \mu \tan \theta}}$$

$$= \sqrt{\frac{400 \times 9.8 (0.2 + 1763)}{1 - 0.2 \times 0.1763}}$$

$$V_{\max} = 39.10 \text{ m/s}$$

Example 12

A cyclist speeding at 6m/s in a circle of 36m diameter makes an angle θ with the vertical. What is the value of θ ? Also determine the maximum possible value of the coefficient of friction between the tyres and the road.

Solution

Here, $V = 6 \text{ m/s}$, $r = \frac{36}{2} = 18 \text{ m}$

$$\tan \theta = \frac{v^2}{rg} = \frac{6 \times 6}{18 \times 9.8} = 0.2040$$

$$\theta = \tan^{-1}(0.2040) = 11^\circ 32'$$

$$\theta = 11^\circ 32'$$

Also $\mu = \tan \theta = 0.2040$

Example 13

A car is speeding on a horizontal road curving round with a radius 60m. The coefficient of friction between the wheels and the road is 0.5 the height of centre of gravity of the car from the road level is 0.3m and the distance between the wheels is 0.8m. Calculate the maximum safe velocity for negotiating the curve will the car skid or topple if this velocity is exceeded?

Solution

$r = 60 \text{ m}$, $\mu = 0.5$, $h = 0.3 \text{ m}$, $d = 0.8 \text{ m}$

For no skidding

$$\tan \theta = \mu = \frac{V^2}{rg}$$

$$V = \sqrt{\mu rg} = \sqrt{0.5 \times 60 \times 9.8}$$

$$V = 17.15 \text{ m/s}$$

$$\text{For toppling, } \frac{mv^2 h}{r} = mga \left\{ a = \frac{d}{2} \right\}$$

$$V = \sqrt{\frac{rdg}{2h}} = \sqrt{\frac{9.8 \times 60 \times 0.8}{2 \times 0.3}}$$

$$V = 28 \text{ m/s}$$

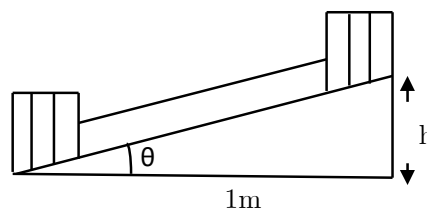
Hence the maximum safe velocity for negotiating the curve is 17.15m/s. Beyond this speed, skidding starts until the car topples at $V = 28 \text{ m/s}$.

Example 14.

A train has to take a circular turn of radius 500m with a speed of 36km/hr. By how much the outer rail should be raised above the inner rail so that there is no side pressure on the rails. The distance between the rails is 1m ($g = 10 \text{ m/s}^2$).

Solution

Let the outer rail be raised by h metre.



$$\tan \theta = \frac{h}{1} = \frac{V^2}{rg}$$

$$h = \frac{V^2}{rg} = \left(\frac{36 \times 1000}{3600} \right)^2 \times \frac{1}{500 \times 10}$$

$$h = 0.02 \text{ m} = 2 \text{ cm}$$

Example 15

A racing car of 1000kg moves round a banked track at a constant speed of 108km/hr. Assuming the total reaction on at the wheels is normal to the track and the horizontal radius of the track is 100m calculate the angle of inclination of the track to the horizontal and the reaction at the wheels ($g = 10 \text{ m/s}^2$)

Solution

$$V = \frac{108 \times 1000}{3600} = 30 \text{ m/s}$$

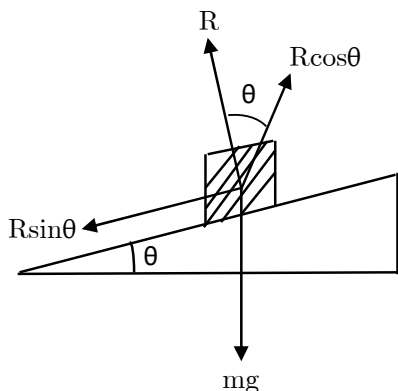
$$\text{Since } \tan \theta = \frac{V^2}{rg}, \quad \theta = \tan^{-1} \left(\frac{V^2}{rg} \right)$$

$$\theta = \tan^{-1} \left[\frac{30 \times 30}{100 \times 10} \right]$$

$$\theta = 42^\circ \text{ (approx.)}$$

Let $R_1 + R_2 = R = \text{total reactions}$

Forces on the wheels



At the equilibrium of the car

$$R \cos \theta = mg, \quad R = \frac{mg}{\cos \theta}$$

$$R = \frac{1000 \times 10}{\cos 42^\circ} = 13456.33 \text{ N}$$

$$R = 13456.33 \text{ N}$$

Example 16

A hollow cylinder of radius 10cm rotates about its axis which is vertical. A small body remains in constant with the inner wall if the frequency of rotation is 200rev per minutes but falls at lower frequency. Find the coefficient of friction between the body and the cylinder.

Solution

Since the body is moving in circle and N is directed towards to the centre.

$$N = m\omega^2 r$$

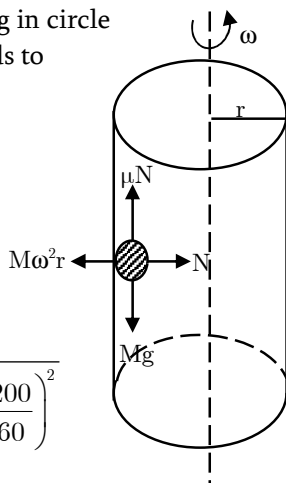
$$\mu N = mg$$

$$\mu m\omega^2 r = mg$$

$$\mu = \frac{g}{\omega^2 r}$$

$$\mu = \frac{9.8}{0.1 \times 4\pi^2 \left(\frac{200}{60}\right)^2}$$

$$\mu = 0.225$$



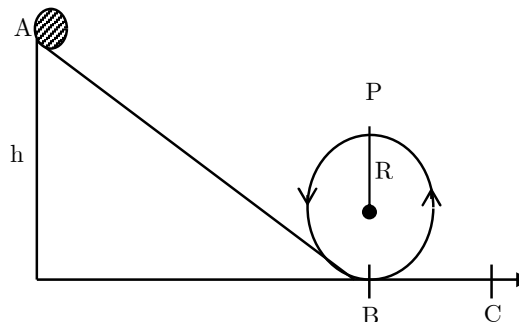
Example 17

NECTA 2001/P1/3

- (a) (i) Why are road beds and high ways banked on curve.

- (ii) What is the smallest radius of a circle at which a cyclist can travel, if his speed is 30km/hr and the coefficient of friction is 0.32? ($g = 10\text{m/s}^2$).

- (b) Figure below shows a toy runway after release from a point A a small model car runs down the slope loops the loop and travels on towards c. The radius of the loop is 0.25m.



- (i) Outline the energy changes as the model car moves from A to C ignore frictional effect.
- (ii) What is the minimum speed with which the car must pass point P at the top of the loop if it is to remain in constant with the runway?

Solution

- (a) (i) Refer to your notes

$$(ii) \text{ since } \frac{mv^2}{r} = \mu mg$$

$$r = \frac{v^2}{\mu g} = \frac{8.33 \times 8.33}{0.32 \times 10}$$

$$r = 21.7 \text{ m}$$

- (b) (i) initially, the car is at the rest when is at the point A, then its possessing only gravitational potential energy from point A to point B the car loses gravitational potential energy (g.p.e) and gains kinetic energy (k.e). From point B to point P its loses some of its kinetic energy and gain gravitational potential energy. From point B to P its loses some of its kinetic energy and gain gravitational potential energy, it

gained and regains its original kinetic energy at point B from B to C kinetic energy is constant.

- (ii) At the highest point P

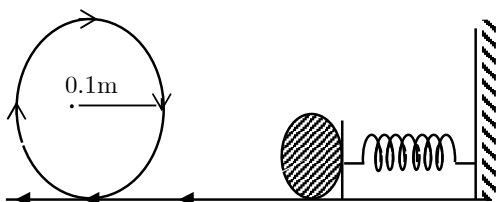
$$Mg = \frac{MV^2}{r}$$

$$V = \sqrt{rg} = \sqrt{10 \times 0.25}$$

$$V = 1.581 \text{ m/s}$$

Example 18.

A compressed spring is used to propel a ball bearing along a track which contains a circular loop of radius 0.1m in a vertical plane below.



The spring obeys Hooke's law (i.e $F = KX$). It requires a force of 0.2N to compress it by 1mm. the spring is compressed by 30mm. calculate:-

- The energy stored in the spring ($E = \frac{1}{2} Fx$)
- A ball bearing of mass 25g is placed at the end of the spring which is then released, calculate the:-
 - Speed with which the ball bearing leave the spring.
 - Speed of the ball at the top of the loop.
 - Force exerted by the track on the ball at the top of the loop. (Ignore friction).

Solution

- (a) Force constant on the spring, $\frac{F}{x} = K$

$$K = \frac{0.2}{1 \times 10^{-3}} = 200 \text{ Nm}^{-1}$$

Energy stored in the spring

$$E = \frac{1}{2} KX^2$$

$$= \frac{1}{2} \times 200 \times (30 \times 10^{-3})^2$$

$$E = 9.0 \times 10^{-2} \text{ J}$$

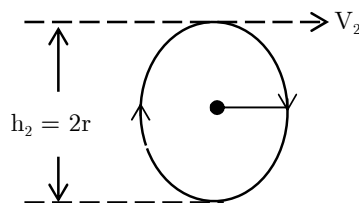
- (b) (i) Apply the law of conservation of energy
Loss in elastic = gain in k.e
p.e on the spring of the ball

$$9.0 \times 10^{-2} = \frac{1}{2} MV^2$$

$$V = \left[\frac{2 \times 9 \times 10^{-2}}{0.025} \right]^{1/2}$$

$$V = 2.7 \text{ m/s}$$

- (ii) Let V_2 be speed of the ball reached at the top of loop



Apply the law of conservation of mechanical energy

p.e + k.e = Elastic p.e on spring

$$Mgh_2 + \frac{1}{2} MV_2^2 = \frac{1}{2} KX^2$$

$$Mgh_2 + \frac{1}{2} MV_2^2 = 9 \times 10^{-2}$$

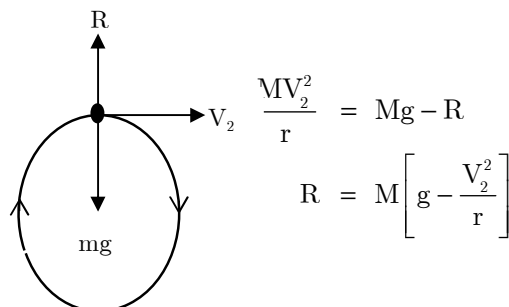
$$V_2^2 = \frac{2 \times 9 \times 10^{-2}}{M} - 2gh_2$$

$$h_2 = 2r$$

$$V_2 = \left[\frac{2 \times 9 \times 10^{-2}}{0.02} - 4 \times 10 \times 0.1 \right]^{1/2}$$

$$V_2 = 1.8 \text{ m/s}$$

- (iii) Let R = force exerted on the ball at the top of the loop



$$\frac{MV_2^2}{r} = Mg - R$$

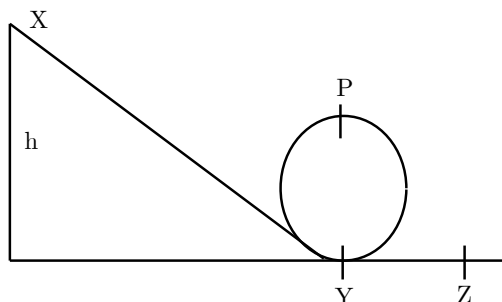
$$R = M \left[g - \frac{V_2^2}{r} \right]$$

$$R = 0.025 \left[10 - \frac{(1.8)^2}{0.1} \right]$$

$$R = 0.55\text{N}$$

Example 19

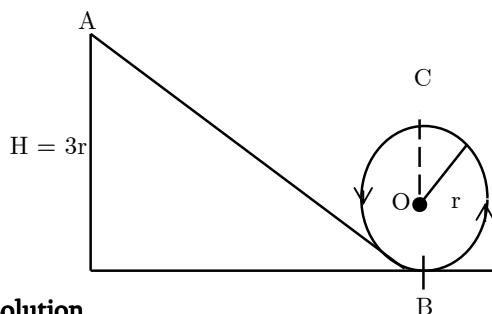
- (a) Write down an expression for the force required to maintain the motion of a body of mass M moving with constant speed V in a circle of radius R . In which direction does the force act?
- (b) Figure below shows a toy runway after release from a point such as X , a small model car runs down the slope, loops the loop, and travels on towards Z . The radius of the loop is 0.25m .



- (i) Ignoring the effect of friction outline the energy changes as the model moves from X to Z .
- (ii) What is the minimum speed with which the car must pass point P at the top of the loop if it is to remain in contact with the runway?
- (iii) What is the minimum value of h which allows the speed calculated in (ii) to be achieved? The effect of friction can be ignored (Assume that $g = 10\text{m/s}^2$)
- Ans: (b) (ii) 1.58m/s (iii) 0.625m .

Example 20

Figure below shows a smooth looping the loop track. A particle of mass M is released from point A as shown. If $H = 3r$, would the particle loop the loop? What is the force on the circular track when the particle is at point (i) B (ii) C ?

**Solution**

Here $H = 3r$

When the particle of mass M is released from A , the velocity acquired by it on reaching B is $V_B = \sqrt{2gH} = \sqrt{6gr}$ which is greater than $\sqrt{5gr}$, the minimum velocity required at B for looping the loop. Hence the particle will loop the loop.

- (i) Force exerted by the circular track on the particle at B .

$$\begin{aligned} N_1 &= Mg + \frac{MV_B^2}{r} \\ &= Mg + \frac{M6gr}{2} \\ N_1 &= 7Mg \end{aligned}$$

- (ii) Force exerted by circular track on the particle at C .

$$N_2 = \frac{MV_C^2}{r} - Mg$$

$$\text{Where } V_C^2 = V_B^2 - 4gr = 6gr - 4gr$$

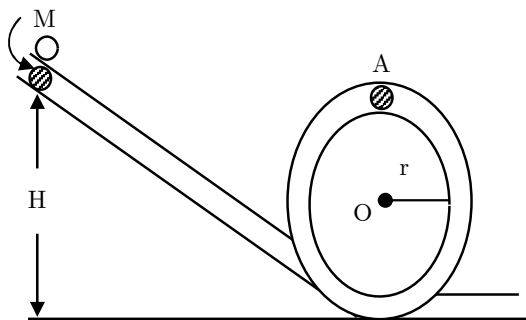
$$V_C^2 = 2gr$$

$$N_2 = \frac{M \cdot 2gr}{r} - Mg$$

$$N_2 = Mg$$

Example 21

The figure below shows a frictionless track. A block of mass M starts from rest at O . The block on reaching A pushes a track with a force equal to twice its weight. Obtain the value of H in terms of r .

**Solution**

Total energy at point , O

$$E_1 = Mgh$$

At the point A

$$E_2 = \text{k.e} + \text{p.e} = \frac{1}{2}MV^2 + 2Mgr$$

Apply the law of conservation of mechanical energy

$$E_1 = E_2$$

$$Mgh = \frac{1}{2}MV^2 + 2Mgr$$

$$V^2 = 2g(H - 2r) \dots\dots\dots(i)$$

At the point A

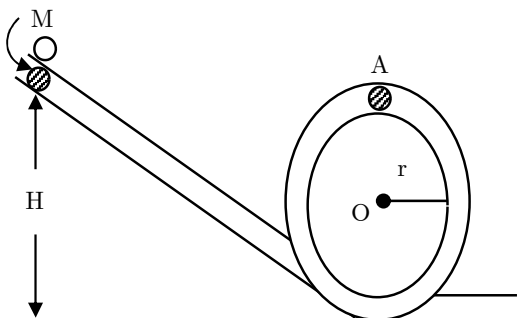
$$\frac{MV^2}{r} = Mg = 2Mg$$

$$\frac{MV^2}{r} = 3Mg$$

$$V^2 = 3gr \dots\dots\dots(ii)$$

$$(i) = (ii)$$

Its weight. Obtain the value of H in terms of r.

**Solution**

Total energy at point , O

$$E_1 = Mgh$$

At the point A

$$E_2 = \text{k.e} + \text{p.e} = \frac{1}{2}MV^2 + 2Mgr$$

Apply the law of conservation of mechanical energy

$$E_1 = E_2$$

$$Mgh = \frac{1}{2}MV^2 + 2Mgr$$

$$V^2 = 2g(H - 2r) \dots\dots\dots(i)$$

At the point A

$$\frac{MV^2}{r} = Mg = 2Mg$$

$$\frac{MV^2}{r} = 3Mg$$

$$V^2 = 3gr \dots\dots\dots(ii)$$

$$(i) = (ii)$$

Which is greater than $\sqrt{5gr}$, the maximum velocity required at B for looping the loop. Hence the particle will loop the loop.

(i) Force exerted by the circular track on the particle at B.

$$N_1 = Mg + \frac{MV_B^2}{r}$$

$$= Mg + \frac{M6gr}{r}$$

$$N_1 = 7Mg$$

(ii) Force exerted by circular track on the particle at C.

$$N_2 = \frac{MV_C^2}{r} - Mg$$

$$\text{Where } V_C^2 = V_B^2 - 4gr = 6gr - 4gr$$

$$V_C^2 = 2gr$$

$$N_2 = \frac{M \cdot 2gr}{r} - Mg$$

$$N_2 = Mg$$

Example 22

The figure below show a frictionless track. A block of mass M starts from rest on at O. The block on reaching A pushes a track with a force equal to twice.

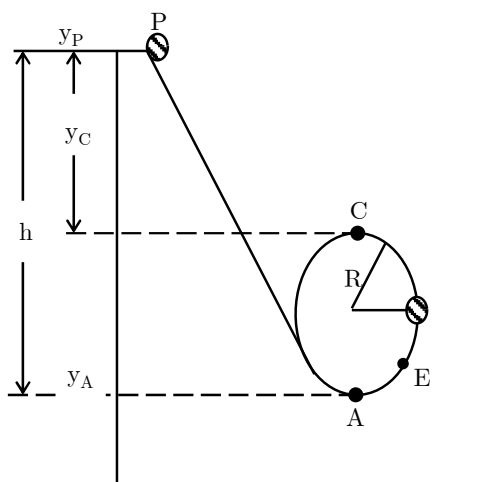
$$3gr = 2g(H - 2r)$$

$$3gr = 2gH - 4gr$$

$$H = \frac{7}{2}r$$

Example 23

The figure below shows a toy run away after being released from a point X a small model car run down the slope loops the loop as illustrated on the figure below.



(i) Show that $V_C^2 = 2g(h - 2R)$ and to complete loops of the loop track $V_C^2 > 0$, $h > 2R$

(ii) Show that $V_A^2 = 2gh$ and reaction at the point A is $R_A = Mg \left[1 + \frac{2h}{R} \right]$

(iii) Show that $R_C = Mg \left[\frac{2h}{R} - 5 \right]$ and

$$b \geq 2.5R \text{ and } V_C \geq \sqrt{gR}.$$

Solution

(i) Apply the principle of conservation of mechanical energy.

$$(k.e + p.e)_p = (k.e + p.e)_c$$

$$V_p = 0 \text{ Since the body starts from the rest}$$

$$Mgh = \frac{1}{2}MV_C^2 + 2MgR$$

$$V_C^2 = 2g(h - 2R)$$

Hence shown

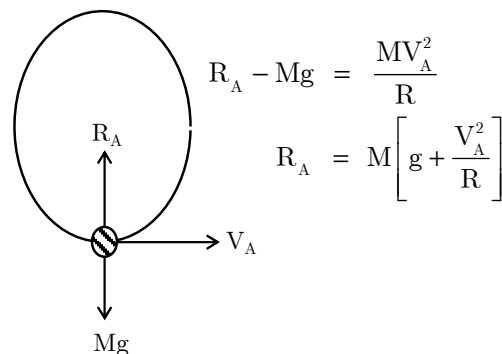
$$V_C = \sqrt{2g(h - 2R)}, \text{ for } V_C^2 > 0$$

$$2g(h - 2R) > 0$$

$$h > 2R$$

Hence shown.

(ii) At the point, A



Apply the principle of conservation of mechanical energy.

$$(p.e + k.e)_A = (p.e + k.e)_P$$

$$Y_A = 0, V_P = 0$$

$$\frac{1}{2}MV_A^2 = \frac{1}{2}MV_P^2 + Mgh$$

$$V_A^2 = 2gh$$

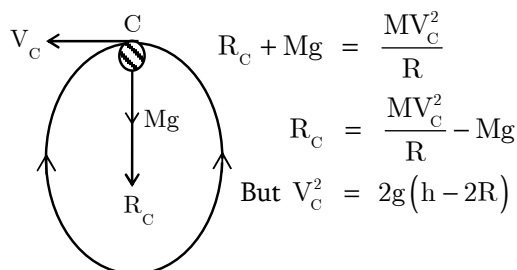
Now,

$$R_A = Mg + \frac{2Mgh}{R}$$

$$R_A = Mg \left[1 + \frac{2h}{R} \right]$$

Hence shown.

(iii) At the point C



$$R_C + Mg = \frac{MV_C^2}{R}$$

$$R_C = \frac{MV_C^2}{R} - Mg$$

$$\text{But } V_C^2 = 2g(h - 2R)$$

$$R_c = 2g(h - 2R)\frac{M}{R} - Mg \quad \frac{1}{2}MV^2 = Mgh, V^2 = 2gh$$

$$R_c = Mg\left[\frac{2h}{R} - 5\right]$$

For the body to be contact with the track ,
 $R_c \geq 0$

$$Mg\left[\frac{2h}{R} - 5\right] \geq 0, h \geq 2.5R$$

$$\text{Also } V_c^2 = 2g(h - 2R)$$

$$\frac{V_c^2}{2g} = h - 2R$$

$$h = \frac{V_c^2}{2g} + 2R$$

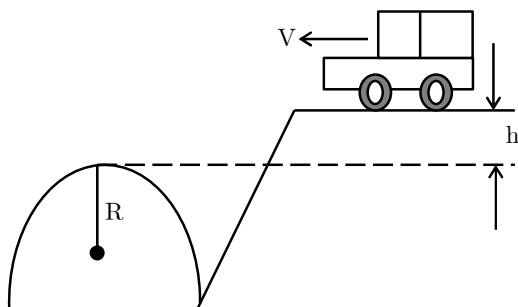
$$\frac{V_c^2}{2g} + 2R \geq 2.5R$$

$$V_c \geq \sqrt{gR}$$

Hence shown

Example 24

Figure below shows a loops the loop track radius , R a car (without engine) starts from a plat form at a distance h above the top of the loop and goes around the loop and goes around the loop without falling off the track. Find the minimum value of h for a successful looping (Neglecting friction)



Solution

Let the gravitational potential energy be zero at the platform and the car starts with negligible speed. Suppose the speed of the car at the top most of the loop is V.

Apply the law of conservation of energy.

At the top of circular path

$$Mg + N = \frac{MV^2}{R}$$

$$Mg + N = \frac{2Mgh}{R}$$

Where $h = h_{\min}$, $N = 0$

$$h_{\min} = \frac{R}{2}$$

Example 25

NECTA

2006/P1/3

- (a) (i) Explain what is meant by angular velocity and centripetal acceleration.
- (ii) Why does a motorbike rider bend while going around a corner?
- (b) (i) What is the maximum speed at which the car can safely go round a circular curve of radius 48m on a horizontal road if the coefficient of static friction between tyres and road is 0.8?
- (ii) Will the car overturn or skid if it just exceeds the speed started in (b) (i) above? Assume the width between the wheels is 1.5 metre and the centre of gravity of the car is 0.6m above the road.

Solution

(a) Refer to your notes

(b) (i) For the maximum safe speed.

$$\mu Mg = \frac{MV^2}{r}$$

$$V_{\max} = \sqrt{\mu rg} = \sqrt{0.8 \times 48 \times 9.8}$$

$$V_{\max} = 19.40 \text{ m/s}$$

(ii) For no overturning of the car

$$V \leq \sqrt{\frac{rag}{h}}$$

$$\leq \sqrt{\frac{0.75 \times 48 \times 9.8}{0.6}}$$

$$V \geq 24.24 \text{ m/s}$$

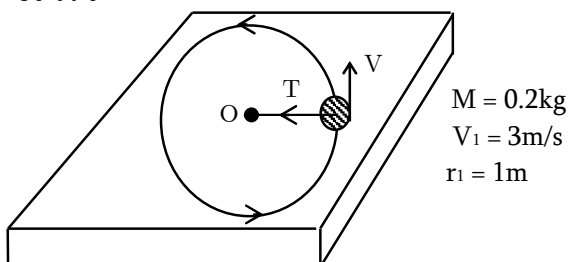
\therefore The car will not overturn , it will just skid.

Example 26

A 200gm mass lying on a smooth table is attached to 100cm length of string whose other end is fixed at the point O on the table calculate:-

- The initial tension in the string if the mass is pushed with an initial velocity of 3m/s in a direction perpendicular to the string.
- The new velocity of the mass if the string is shorted to 50cm while the mass rotates.

Solution



$$(i) \quad T = \frac{MV_1^2}{r_1} = \frac{0.2 \times 3 \times 3}{1}$$

$$T = 1.8\text{N}$$

- Apply the principle of conservation of angular momentum.

$$MVr = \text{Constant}$$

$$MV_1 r_1 = MV_2 r_2$$

$$V_2 = V_1 \left[\frac{r_1}{r_2} \right] = \frac{3 \times 100}{50}$$

$$V_2 = 6\text{m/s}$$

Example 27

A curve in a road has 60m radius. The angle of bank of the road is 47° . Find the maximum speed a car have without skidding if the coefficient of static friction between the tyres and road is 0.8.

Solution

Since

$$V_{\max} = \sqrt{\frac{gr(\mu + \tan \theta)}{1 - \mu \tan \theta}}$$

$$= \sqrt{\frac{9.8 \times 60 (0.8 + \tan 47^\circ)}{1 - 0.8 \tan 47^\circ}}$$

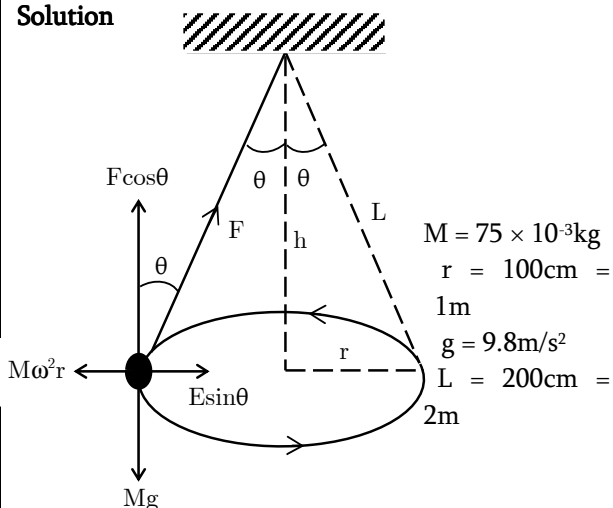
$$V_{\max} = 88\text{m/s}$$

Example 28

A thin inextensible string 200cm long has its upper end attached to a fixed point and a small lead sphere of mass 75gm is attached to the lower end of the string. If the sphere is made to describe a horizontal surface of radius 100cm with the uniform speed. Find the:

- Tension in the string
- Time taken to describe a complete circle.
- The angular velocity

Solution



- F = tension in the string

$$L^2 = h^2 + r^2$$

$$h = \sqrt{L^2 - r^2}$$

$$\cos \theta = \frac{\sqrt{L^2 - r^2}}{L}$$

At the equilibrium

$$F \cos \theta = Mg$$

$$F = \frac{Mg}{\cos \theta} = \frac{MgL}{\sqrt{L^2 - r^2}}$$

$$F = \frac{75 \times 10^{-3} \times 9.8 \times 2}{\sqrt{2^2 - 1^2}}$$

$$F = 0.84957\text{N}$$

- Periodic time of conical pendulum

$$T = 2\pi \sqrt{\frac{L \cos \theta}{g}} = 2\pi \sqrt{\frac{h}{g}}$$

$$T = 2 \times 3.14 \times \sqrt{\frac{\sqrt{3}}{9.8}}$$

$$T = 2.638 \text{ sec}$$

$$(iii) \omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{2.63}$$

$$\omega = 2.379 \text{ rad/sec}$$

Example 29

A car travels over a hump back bridge of radius of curvature 45m. Calculate the maximum speed of the car if the wheels are to remain in contact with the bridge.

Solution

$$V_{\max} = \sqrt{gR} = \sqrt{9.8 \times 45}$$

$$V_{\max} = 21 \text{ m/s}$$

Example 30

A road curve of 200m radius is banked at the correct angle for a speed of 15m/s. if a car rounds this curve at 30m/s, what is the minimum coefficient of friction between the tyres and the road so that the car will not skid?

Solution.

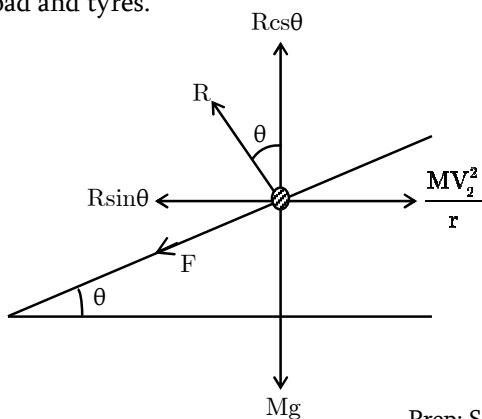
Let θ = correct angle of banking

$$\tan \theta = \frac{V_1^2}{rg}$$

$$\theta = \tan^{-1} \left[\frac{V_1^2}{rg} \right] = \tan^{-1} \left[\frac{15 \times 15}{9.8 \times 200} \right]$$

$$\theta = 6.5^\circ$$

For the car take the curve without skidding at a higher speed, there must be friction between the road and tyres.



Resolving horizontally.

$$F \cos 6.5^\circ + R \sin 6.5^\circ \geq \frac{MV_2^2}{r} \dots\dots(i)$$

Resolving vertically

$$R \cos 6.5^\circ = Mg \dots\dots(ii)$$

$$\frac{\mu R \cos 6.5^\circ + R \sin 6.5^\circ}{R \cos 6.5^\circ} \geq \frac{MV_2^2}{r} / Mg$$

$$\frac{\mu \cos 6.5^\circ + \sin 6.5^\circ}{\cos 6.5^\circ} \geq \frac{V_2^2}{rg}$$

$$\frac{\mu \cos 6.5^\circ + \sin 6.5^\circ}{\cos 6.5^\circ} = \frac{30 \times 30}{200 \times 9.8}$$

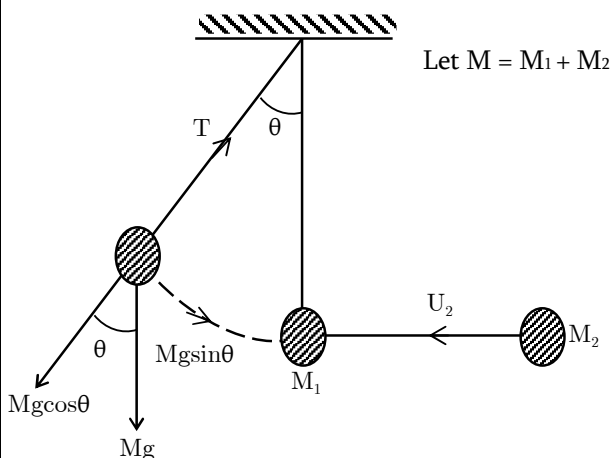
$$\mu = 0.345$$

Example 31.

A mass of 2.9kg is suspended from a string 50cm and is at rest another body of mass 100gm moving horizontally with a velocity of 150m/s, strikes and sticks to it.

(a) What is the tension in the string when it makes an angle of 60° with the vertical?

(b) Will it complete the circle?

Solution

(a) Apply the principle of conservation of linear momentum

$$M_1 U_1 + M_2 U_2 = (M_1 + M_2) V$$

$$V = \frac{M_1 U_1 + M_2 U_2}{M_1 + M_2} = \frac{0.1 \times 150}{2.9 + 0.1}$$

$$V = 5 \text{ m/s}$$

Tension on the string at that point

$$T = M \left[\frac{V^2}{r} - g \cos \theta \right]$$

$$= 3 \left[\frac{5^2}{0.5} - 9.8 \cos 60^\circ \right]$$

$$T = 135.3\text{N}$$

At the lowest point the velocity is 5m/s in order to complete the circle the minimum velocity at the bottom of the circle.

$$V_1 = \sqrt{5gr}$$

$$V_1 = \sqrt{5 \times 9.8 \times 0.5} = 4.95\text{m/s}$$

Since $V_1 < V$, hence the body will complete the circle.

Example 32**NECTA 2015/P1/3**

- (a) (i) Mention three effects of looping the loop.
 (ii) Why there must be a force acting on the particle moving with uniform speed in a circular path? Write down an expression for its magnitude.
- (b) A driver negotiating a sharp bend usually tend to reduce the speed of the car.
 (i) What provides the centripetal force on the car?
 (ii) Why is it necessary to reduce its speed?
- (c) A ball of mass 0.5kg is attached to the end of a cord whose length is 1.5m then whirled in the horizontal circle. If the cord can withstand a maximum tension of 50N, calculate the :-
 (i) Maximum speed the ball can have before the cord breaks
 (ii) Tension in the cord if the ball speed is 5m/s

Solution

- (a) (i) water stays in the bucket as the bucket is made to loops the loop.
- A pilot does not fall as an aeroplane execute a vertical loop.
 - Passenger of a roller coaster do not fall over as it loops the loop.
- (ii) A particle moving with uniform speed in the circular path has an acceleration. According to the Newton's second law, there must be a force associated with that acceleration

and is the force which maintain the particle moving in a circular path. The magnitude of the force, $F = MV^2/r$

- (b) (i) Friction force between the road and the tyres.
 (ii) It is necessary to reduce speed as to avoid overturning or skidding of the car.

$$(c) (i) V = \sqrt{\frac{T_{\max} r}{M}} = \sqrt{\frac{50 \times 1.5}{0.5}}$$

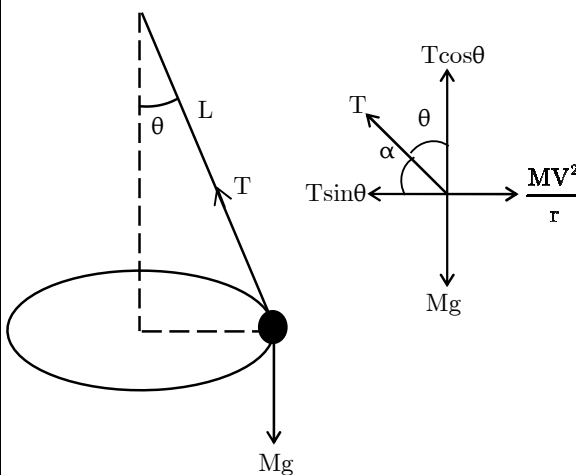
$$V = 12.25\text{m/s}$$

$$(ii) T = \frac{0.5 \times 5 \times 5}{1.5}$$

$$T = 8.33\text{N}$$

Example 33.

A man whirls a stone round his head on the end of a string 4.0m long. Can the string be in a horizontal circle? if the stone has a mass of 0.4kg and the string will break if the tension in it exceeds 8N, what is the smallest angle the string can make with the horizontal? What is the speed of the stone? (Take $g = 10\text{m/s}^2$).

Solution

At the equilibrium

$$T \cos \theta = Mg \dots\dots\dots(i)$$

$$T \sin \theta = \frac{MV^2}{r} \dots\dots\dots(ii)$$

From equation (i)

$$T = \frac{Mg}{\cos \theta}$$

When the string is in horizontal position

$$\theta = 90^\circ, \cos 90^\circ = 0$$

$$T = \frac{Mg}{0} = \infty$$

Thus, the tension must be infinite which it is impossible so the string cannot be in the horizontal plane. The maximum angle θ is given by the breaking tension of the string in the equation.

$$T \cos \theta = Mg$$

$$8 \cos \theta = 0.4 \times 10$$

$$\cos \theta = \frac{1}{2}, \theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\theta = 60^\circ$$

The angle with horizontal

$$\alpha = 90^\circ - \theta = 90^\circ - 60^\circ$$

$$\theta = 30^\circ$$

$$\text{Since } T \sin \theta = \frac{MV^2}{r}, \quad r = L \sin \theta$$

$$T \sin \theta = \frac{MV^2}{L \sin \theta}$$

$$V = \sqrt{\frac{TL \sin^2 \theta}{M}} = \sqrt{\frac{8 \times 4 (\sin 60^\circ)^2}{0.4}}$$

$$V = 7.7 \text{ m/s}$$

Example 34

A 0.6kg and a 1.2kg model Airplane fly in horizontal circles on guidelines that are parallel to the ground. The speeds of the plane are the same and the same type of cord is used for each guideline. The smallest circle on which the 0.6kg plane can fly without breaking its guideline has a radius of 3.5m. What is the radius of the smallest circle on which the 1.2kg plane can fly?

Solution

$$\text{For the 0.6kg plane: } T = \frac{M_1 V^2}{r_1} \dots\dots\dots(i)$$

$$\text{For the 1.2kg plane: } T = \frac{M_2 V^2}{r_2} \dots\dots\dots(ii)$$

$$(i) = (ii)$$

$$\frac{M_1 V^2}{r_1} = \frac{M_2 V^2}{r_2}$$

$$r_2 = \frac{M_2 r_1}{M_1} = \frac{1.2 \times 3.5}{0.6}$$

$$r_2 = 7 \text{ m}$$

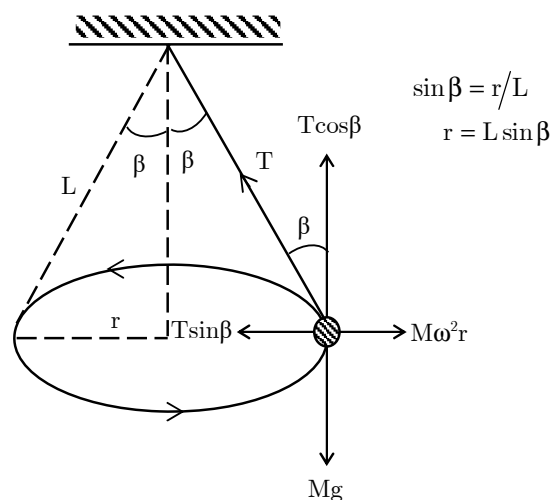
Example 35

A stone 0.5kg is tied to a string of length 0.25m and is rotated in a horizontal plane at a constant rate of 1.5 r.p.s.

(i) Find the tension in the string if the string makes an angle β with the vertical.

(ii) Calculate the value of β

Solution



(i) At the equilibrium resolves horizontally

$$T \sin \beta = M \omega^2 r = M \omega^2 L \sin \beta$$

$$T = M \omega^2 L = M (2\pi f)^2 L$$

$$= 0.5 [2 \times 3.14 \times 1.5]^2 \times 0.25$$

$$T = 11.09205 \text{ N}$$

(ii) $T \cos \beta = Mg$

$$\cos \beta = \frac{Mg}{T}$$

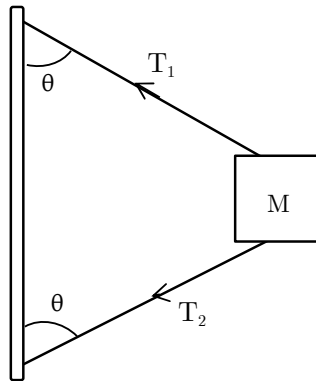
$$\beta = \cos^{-1} \left[\frac{Mg}{T} \right]$$

$$= \cos^{-1} \left[\frac{0.5 \times 9.8}{11.09205} \right]$$

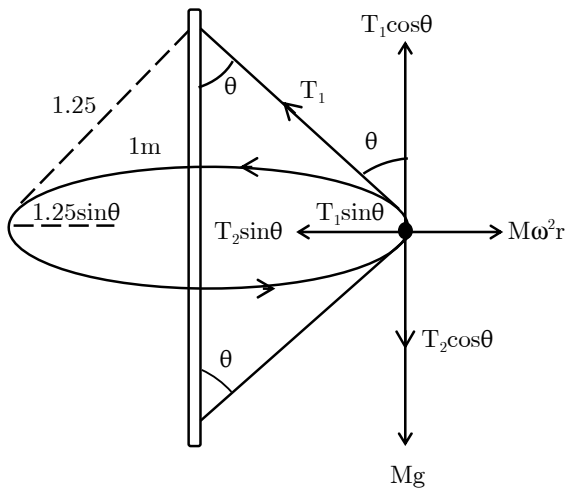
$$\beta = 63.8^\circ$$

Example 36

The 4kg block in the figure below is attached to a vertical rod by means of the two strings when the system rotates about the axis of the rod, the string are extended as shown in the figure below and the tension in the upper string is 80N.



- What is the tension in the lower cord ?
- How many revolutions per minute does the system make ?
- Find the number of revolutions per minute at which the lower cord just goes slack.

Solution

(a) At the equilibrium

$$T_1 \cos \theta = T_2 \cos \theta + Mg$$

$$T_2 = T_1 - \frac{Mg}{\cos \theta}$$

$$= 80 - \frac{4 \times 9.8}{1/1.25}$$

$$T_2 = 31\text{N}$$

(b) Also

$$T_1 \sin \theta + T_2 \sin \theta = M\omega^2 r$$

$$(T_1 + T_2) \sin \theta = M\omega^2 \times 1.25 \sin \theta$$

$$T_1 + T_2 = 1.25M\omega^2$$

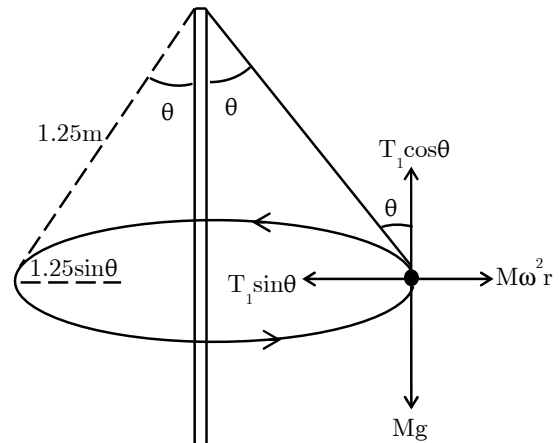
$$\text{But } \omega = 2\pi f$$

$$f = \frac{1}{2\pi} \sqrt{\frac{T_1 + T_2}{1.25M}}$$

$$= \frac{1}{2\pi} \sqrt{\frac{80 + 31}{1.25 \times 4}}$$

$$f = 0.75\text{Hz} = 45\text{r.p.s}$$

(c) If the lower cord just g slack



$$T_1 \sin \theta = M\omega^2 r$$

$$T_1 \cos \theta = Mg$$

$$\tan \theta = \frac{\omega^2 r}{g} = \frac{\omega^2 1.25}{g}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{1.25 \cos \theta}} = \frac{1}{2\pi} \sqrt{\frac{9}{1.25}}$$

$$f = 0.45\text{Hz} = 30\text{r}$$

Example 37

Two particles, each of mass M are attached to the two ends of a light string of length L which passes through a hole at the centre of a smooth table. One particle describes a circle on the table angular velocity ω_1 and the other describes a circle as a conical pendulum with angular velocity ω_2 below the table. If L_1 and L_2 are the lengths of the position of the string above and below the table, then show that.

$$(i) \quad \frac{L_1}{L_2} = \frac{\omega_2^2}{\omega_1^2} \text{ and}$$

$$(ii) \quad \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} < \frac{L}{g}$$

Solution

(i) Let T = tension on the string

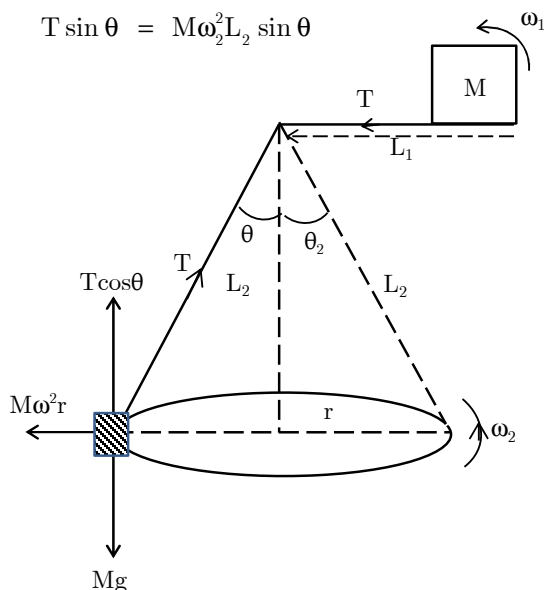
For the particle on the table

$$T = M\omega_1^2 L_1 \dots\dots\dots(i)$$

For the particle below the table

$$T \sin \theta = M\omega_2^2 r, \quad r = L_2 \sin \theta$$

$$T \sin \theta = M\omega_2^2 L_2 \sin \theta$$



$$T = M\omega_2^2 L_2 \dots\dots\dots(2)$$

$$\text{Also } T \cos \theta = Mg \dots\dots\dots(3)$$

$$(1) = (2)$$

$$ML_1\omega_1^2 = ML_2\omega_2^2$$

$$\frac{L_1}{L_2} = \frac{\omega_2^2}{\omega_1^2}$$

Hence shown.

(ii) Now.

$$L_1 = \frac{T}{M\omega_1^2}, \quad L_2 = \frac{T}{M\omega_2^2}$$

Total length of the string

$$\begin{aligned} L &= L_1 + L_2 \\ &= \frac{T}{M\omega_1^2} + \frac{T}{M\omega_2^2} \\ &= \frac{T}{M} \left[\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} \right] \end{aligned}$$

$$\frac{ML}{T} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2}$$

$$\text{But } T = \frac{Mg}{\cos \theta}, \quad \frac{1}{T} = \frac{\cos \theta}{Mg}$$

$$ML \cdot \frac{\cos \theta}{Mg} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2}$$

$$\cos \theta = \frac{g}{L} \left[\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} \right]$$

$$\text{Since } |\cos \theta| < 1$$

$$\frac{g}{L} \left[\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} \right] < 1$$

$$\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} < \frac{L}{g}$$

Hence shown.

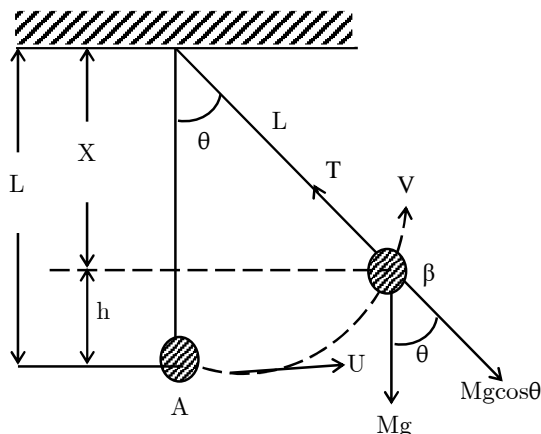
Example 38

A store of mass 0.1kg is suspended from the end weightless string of length 1.0m and is allowed to swing in the vertical. The speed of the mass is 2m/s when the string make an angle θ , with the vertical. calculate the tension in the string at $\theta = 60^\circ$. calculate the speed of the store when it is in the lowest position ($g = 9.8\text{m/s}^2$).

Solution

$$M = 0.1\text{kg}, \quad r = L = 1.0\text{m}$$

$$V = 2\text{m/s}, \quad \theta = 60^\circ, \quad g = 9.8\text{m/s}^2.$$



At the equilibrium when the bob is at point B

$$T - Mg \cos \theta = \frac{MV^2}{r}$$

$$T = Mg \cos \theta + \frac{MV^2}{r}$$

$$= 0.1 \times 9.8 \cos 60^\circ + \frac{0.1 \times 2 \times 2}{1}$$

$$T = 0.89\text{N}$$

From the figure above

$$h = L - X = L - L \cos \theta$$

$$h = 1 - \cos 60^\circ = 0.5\text{m}$$

Apply the principle of conservation of mechanical energy.

$$(p.e + k.e)_A = (p.e + k.e)_B$$

$$\frac{1}{2} MU^2 = \frac{1}{2} MV^2 + Mgh$$

$$U = \sqrt{V^2 + 2gh}$$

$$= \sqrt{2^2 + 2 \times 9.8 \times 0.5}$$

$$U = 3.715\text{m/s}$$

Example 39

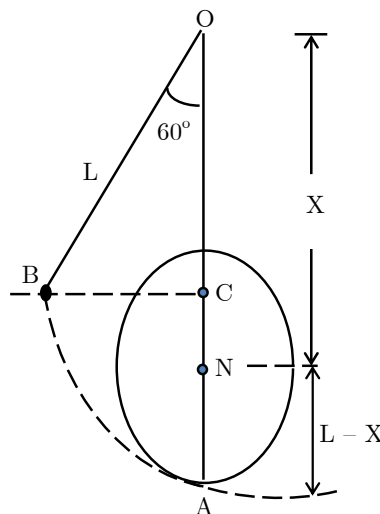
A nail is located at a certain distance vertically below the point of suspension of simple pendulum. The pendulum bob is released from the position where the string makes an angle of 60° with the vertical. Calculate the distance the nail from point

of suspension such that the bob will just perform revolution with the nail as centre. Assume the length of the pendulum to be 1m.

Solution

Let OB = length of pendulum

L, N be the nail L at a distance X from O



NA = radius of the vertical circle.

$$NA = L - X, \quad \theta = 60^\circ.$$

$$CO = OB \cos 60^\circ = \frac{1}{2}$$

Height through which the bob is raised

$$h = L - \frac{L}{2} = \frac{L}{2}$$

Apply the law of conservation of energy

Loss in k.e = gain in p.e

$$\frac{1}{2} MV^2 = Mgh$$

$$V^2 = 2gh = 2g \frac{L}{2}$$

$$V^2 = gL \dots\dots\dots(i)$$

For bob to perform revolution about the nail, the minimum speed of the bob at the point A is $\sqrt{5gr}$

$$V = \sqrt{5gr} = \sqrt{5g(L - X)}$$

$$V^2 = 5g(L - X)$$

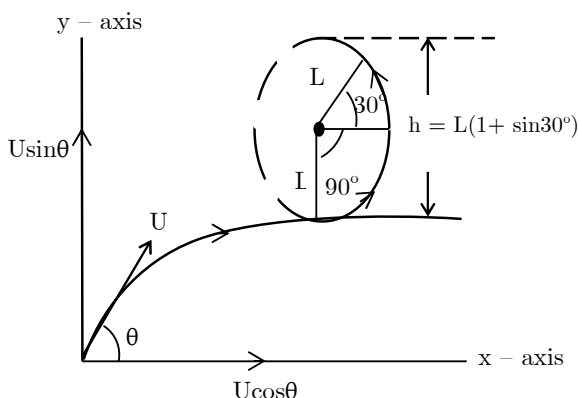
$$X = \frac{4}{5} L = 0.8L = 0.8 \times 1$$

$$X = 0.8\text{m}$$

Example 40.

A bullet of mass M is fired with a velocity of 50m/s at an angle θ with the horizontal at the highest point of its trajectory it collides lead on with a bob of mass $3M$ suspended by a massless string of length $10/3\text{ m}$ and gets embedded in the bob after collision of the string moves to an angle of 120° find.

- (a) The angle, θ
 (b) The length of the string so that the system could just complete a vertical circle.

Solution

- (a) At the highest point of the bullet motion, its velocity is only the horizontal component.

Apply the principle of conservation of linear momentum.

$$MV_x = (M + 3M)V$$

$$MU \cos \theta = 4MV$$

$$V = 12.5 \cos \theta$$

Apply the principle of conservation of energy.

$$\begin{array}{lcl} \text{k.e of bullet} & = & \text{p.e of the} \\ \text{bob system} & & \text{system at h} \end{array}$$

$$\frac{1}{2}(M + 3M)V^2 = (M + 3M)gh$$

$$V^2 = 2gh$$

$$V = \sqrt{2gL(1 + \sin 30^\circ)}$$

$$\text{Now, } 12.5 \cos \theta = \sqrt{3gL}$$

$$\cos \theta = \frac{\sqrt{3gL}}{12.5}$$

$$\theta = \cos^{-1} \left[\frac{\sqrt{3 \times 9.8 \times 10/3}}{12.5} \right]$$

$$\theta = 37.6^\circ$$

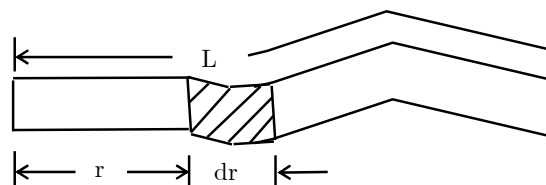
$$(b) V^2 = 5gr, \quad r = \frac{V^2}{5g}$$

$$r = \frac{(12.5 \cos 37.6^\circ)^2}{5 \times 9.8}$$

$$r = 2\text{m}$$

Example 41

A wire of mass 0.45kg and length 2.0m is spun in a horizontal circle. The breaking stress of the wire is $8 \times 10^9\text{Pa}$ and the cross-sectional area is $8 \times 10^{-6}\text{m}^2$. Calculate the maximum frequency of rotation before wire breaks and find at which point it will break

**Solution**

$$\text{Let, } \rho = \frac{M}{L}, \quad M = \rho \theta$$

L = Length of the wire

Consider the forces acting on the short length of the wire.

$$T + dT - T = -\omega^2 r dm$$

$$dT = -\omega^2 r dm$$

$$\text{But } \int = \frac{dm}{dr}, \quad dm = \int dr$$

$$dT = -\omega^2 r \int dr$$

$$\int dT = -\omega^2 \int r dr$$

$$T = \frac{-\omega^2 \int r^2}{2} + C$$

$$\text{When } r = L, T = 0, \quad C = \frac{\omega^2 L^2}{2}$$

$$\text{Now } T = \frac{1}{2} M \omega^2 \left(L - \frac{r^2}{L} \right)$$

$$\text{When } T = T_{\max}, r = 0$$

$$T_{\max} = \frac{1}{2} M \omega^2 L$$

$$\omega = \sqrt{\frac{2T_{\max}}{ML}}, \quad \omega = 2\pi f$$

$$f_{\max} = \frac{1}{2\pi} \sqrt{\frac{2T_{\max}}{ML}}$$

$$\text{Breaking stress} = \frac{T_{\max}}{A}$$

$$T_{\max} = A \times \text{breaking stress}$$

$$f_{\max} = \frac{1}{2\pi} \sqrt{\frac{2A \times \text{breaking stress}}{ML}}$$

$$= \frac{1}{2} \sqrt{\frac{2 \times 8 \times 10^{-6} \times 8 \times 10^9}{0.4 \times 2}}$$

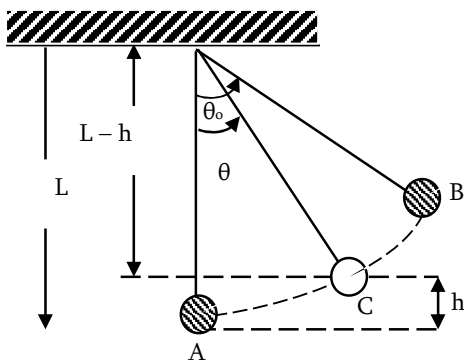
$$f_{\max} = 63.6 \text{ r.p.s}$$

∴ The wire will break at the point of attachment.

Example 42

A 40kg mass, hanging at the end of a rope of length L oscillates in a vertical plane with an angular amplitude of θ_0 . What is the tension in the rope when it makes an angle θ with the vertical? If the breaking strength of the rope is 80kg, what is the maximum angular amplitude with which the mass can oscillate without the rope breaking?

Solution



Let B be the position of the mass when the angular amplitude is θ_0 .

Let C be the position of the mass when the angular displacement is θ .

The height of C above A is given by

$$h = L - L \cos \theta = L(1 - \cos \theta)$$

The height of B above A is given by

$$h_0 = L(1 - \cos \theta_0)$$

Apply the principle of conservation of mechanical energy

$$\text{Total energy at point C} = \text{Total energy at point B}$$

$$(p.e + k.e)_C = (p.e + k.e)_B$$

$$\frac{1}{2} MV^2 + MgL(1 - \cos \theta) = MgL(1 - \cos \theta_0)$$

$$MV^2 = 2MgL(\cos \theta - \cos \theta_0)$$

At the point C

$$T - Mg \cos \theta = \frac{MV^2}{L}$$

$$\text{Now, } \frac{MV^2}{L} = 2Mg(\cos \theta - \cos \theta_0)$$

Then

$$T - Mg \cos \theta = 2Mg(\cos \theta - \cos \theta_0)$$

$$T = Mg(3 \cos \theta - \cos \theta_0)$$

$$T = M(3 \cos \theta - \cos \theta_0) \text{ kgwt}$$

$$T = 40[3 \cos \theta - \cos \theta_0] \text{ kgwt}$$

The tension will be maximum at A when $\theta = 0^\circ$ using the maximum value of tension in the above equation.

$$80 = 40[3 \cos 0^\circ - 2 \cos \theta_0]$$

$$2 = 3 - 2 \cos \theta_0$$

$$\cos \theta_0 = \frac{1}{2}, \quad \theta_0 = 60^\circ$$

CONCEPTUAL PROBLEMS

Example 43

- A pilot does not fall down when his aeroplane loops the loop why?
- A cyclist negotiates a curve at high speed and bends more than a cyclist negotiating the same curve at low speed why?

Solution

- This is because when the pilot is at highest point of the loop, the weight of the pilot is balanced by the centrifugal force.

- (b) Since $\tan \theta = V^2/rg$, each symbol have usual meaning greater the value of V , more will be the value of θ so, cyclist negotiating a curve at high speed shall have to bend more.

Example 44

Is it correct to say that the banking of road reduces the wear and tear of the tyres of automobile? If yes, explain.

Solution

If the road is not banked, then the necessary centripetal force will be provided by the force of friction between the tyres and the road. On the other hand, a component of the normal reaction provide the necessary centripetal force this reduces the wear and tear of the tyres.

Example 45

Explain the following observations:-

- (a) When the car takes turn round a curve, passenger sitting in the car tends to slide. To which side does the passenger slide?
- (b) Comment on the statement 'sharper the curve, more is the bending'

Solution

- (a) The passenger slides away from the centre of the curve due to the centrifugal force.
- (b) The angle θ which a cyclist should make with the vertical while taking circular turn of radius r with velocity, V is given by $\tan \theta = V^2/rg$. Sharper the curve, smaller is the radius and greater is the value of θ .

Example 46

- (a) A stone tied to the end of a string is whirled in a horizontal circle when the strings breaks the stone flies away tangentially. Why?
- (b) Passengers are thrown outwards, when the bus takes a circular turn why?

Solution

- (a) When a stone moves in a circular path, its instantaneous velocity is always tangential to the circle when the string breaks, the centripetal force ceases to act. According to

the inertia, stone continues its motion along the tangent to the circular path. Hence it flies away tangentially.

- (b) When the bus takes a turn, the centripetal acceleration of the bus acts towards the centre of the circular turn and along its radius due to inertia, the passengers are thrown outwards the force experienced by the passengers is called the centrifugal force.

Example 47

Why is the outer rail of a curved railway track generally raised over the inner?

Solution

When the outer rail of curved track is raised, weight of the train provides a force component along the radius of the curved track towards the centre. This component of the weight provides the necessary centripetal force to the train to enable it to move along the curved path.

Example 48

Why does an aeroplane tilt while making a curved flight?

Solution

The plane tilts to provide the necessary centripetal force. If θ is the angle of the tilt then $\tan \theta = V^2/rg$ where V is the speed of the plane and r is the radius of the circular path.

Example 49

Explain why it is desirable to bank a road at corners and calculate the optimum angle of banking for a curve of radius r , to be traversed at a speed, V .

Solution

Roads are banked along corners so as to reduce the dependency of friction on providing the necessary centripetal force to take it. This enables cars to take it with greater speed without overturning as well as reducing the wear and tear of tyres due to friction.

Example 50

Explain the following observation:-

- (a) When athletes run around a curve, they usually lean inward.
- (b) A cyclist leans inward while taking a turn.

Solution

They lean inward to provide the necessary centripetal force to take the curve.

Example 51

Explain why racing cars can travel very fast on banked round track than on flat track.

Solution

On flat track, the centripetal force is provided by friction between the tyres and track. If limiting friction is exceeded then the car will overturn. On banked track, the centripetal force can be provided wholly by the normal reaction on the car. This enables the car to take the curve with greater speed without overturning.

Example 52

Explain the following observations:-

- (a) What is the need for banking of roads?
- (b) Why are wheels of an automobile made circular.
- (c) Why does a child in a merry-go-round press the side of his seat radially outward?

Solution

- (a) When a curved road is banked, horizontal component of normal reaction of the road provides the necessary centripetal force to move the vehicle along the curved path. This reduces wear and tear of tyres.
- (b) Circular wheels roll on the road. Therefore motion of the vehicle is opposed by rolling friction which is much smaller than the sliding friction.
- (c) This is because of centrifugal force acting radially outwards on the child rotating actually in the merry-go-round.

Example 53

How does banking of roads reduce wear and tear of the tyres?

Solution

When a curved road is unbanked, force of friction between the tyres and the road provides the necessary centripetal force. Friction has to be increased suitably that will cause wear and tear. However, when the curved road is banked, a component of normal reaction of the ground provides the necessary centripetal force. The role of friction becomes secondary. Therefore, wear and tear of the tyres is reduced.

REVISION**EXERCISE 4. NECTA 2017/P1/4**

1. (a) (i) Justify the statement that if no external torque acts on a body, its angular velocity will not conserve.
- (ii) A car is moving with a speed of 30 m/s on a circular track of radius 500 m. If its speed is increasing at the rate of 2 m/s^2 , find its resultant linear acceleration.
- (b) An object of mass 1 kg is attached to the lower end of a string 1 m long whose upper end is fixed and made to rotate in a horizontal circle of radius 0.6 m. If circular speed of the mass is constant, find the:
 - (i) Tension in the string
 - (ii) Period of motion

2. JECAS 2017/P1/5

- (a) Briefly explain the following observations:-
 - (i) Why are passengers of a car rounding a curve thrown outwards?
 - (ii) Why are wheels made circular?
 - (iii) Why are fast moving vehicles given streamlines shape?
 - (iv) When does the body start moving?
- (b) A small box is placed on the surface of a horizontal disc at a point 5 cm from the centre of the disc. The box is on the point of slipping when the disc rotates at 1.4 rev/sec. Find the coefficient of friction between the box and the surface of the disc.
- (c) (i) Define the term 'Banking of road'

- (ii) At what angle must a truck with a bend of radius 1.5m banked for safe running of the trains at a speed of 48km/hr.

3. NECTA 2016/P1/3(C)

A boy ties a string around a stone of mass 0.15kg and then whirls it in a horizontal circle at constant speed. If the period of rotation of the stone is 0.4sec and the length between the stone and boy's hand is 0.5m

- (i) Calculate the tension in the string.
(ii) State one assumption taken to each the answer in 3(c)(i)

4. NECTA 2013/P1/3

- (a) Why is technically advised to bank a road at corners?
(b) A wheel rotates at a constant rate of 10revolution per second. Calculate the centripetal acceleration at a distance of 0.80m from the centre of the wheel.

5. NECTA 2012/P1/3

- (b) A child is whirling a 0.012kg ball on a string in a horizontal circle whose radius is 0.10cm. if the ball travels once round the circle in 0.5sec.
(i) Determine the centripetal force acting on the ball.
(ii) Why does such a force do not work in a circular motion
(c) If the speed of the ball in (b) is doubled , by what factor does the centripetal force increases ans.

- (a) (i) 0.00189N
(b) Factor of 4

6. NECTA 2011/P1/3

- (a) Mention the sources of centripetal force in each of the following cases:-
(i) An object at the end of a string is whirled in a horizontal circle.
(ii) A car travelling round a banked road.
(iii) The moon orbiting the earth

- (b) (i) An object of mass 4kg is whirled in a vertical circle of radius 2m with a speed of 5m/s calculate the maximum and minimum tension in the string connecting the object and the centre of the circle.

- (ii) Find the angle of banking necessary for a corner of radius 80m if cars are to travel round this corner at a speed of 120km/hr without relying on the frictional force ($g = 10\text{m/s}^2$).

Ans: (b) (i) 90N , 10N
(ii) 54.2°

7. NECTA 2010/P1/3

- (a) What is the origin of centripetal force for:-
(i) A satellite orbiting around the earth.
(ii) An electron in the hydrogen atom?
(b) A small mass of 0.15kg is suspended from a fixed point by a thread of fixed length. The mass is given a push so it moves along a circular path of radius 1.82m in a horizontal plane at a steady speed, taking 18sec to make 10 complete revolutions calculate:-

- (i) The speed of the small mass
(ii) The centripetal acceleration.
(iii) The tension in the thread

Ans (b) (i) 6.35m/s
(ii) 22.18m/s²
(iii) 3.64N

8. A particle of mass 5kg is slightly disturbed from rest on the top of a smooth hemisphere , radius 4m and centre O resting with its plane face on horizontal ground. Show that the particle leaves the surface of the hemisphere at the point B , where the angle between the radius BO and the upward vertical is $\cos^{-1}\left(\frac{2}{3}\right)$

9. NECTA 2000/P1/2

- (a) Show that the period of a body of mass M revolving in a horizontal circle with constant velocity V at the end of a string of length L is independent of mass of the object.
- (b) A ball of mass 100g is attached to the end of a string and is swung in a circle of radius 100cm at a constant velocity of 200cm/s while in motion the string is shortened to 50cm . calculate:-
- (c) A car travels over a humpback bridge of radius of curvature 45m . calculate the maximum speed of the car if the wheels are to remain in constant with the bridge.

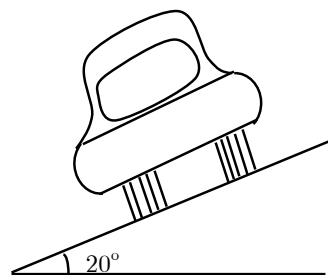
10. NECTA 2005/P2/2a

- (i) What is the centripetal force?
- (ii) Why does a centripetal force not do any work in a circular orbit?
- (iii) A conical pendulum is an example on which a force acts centripetal show that

the period T is given by $T = 2\pi\sqrt{\frac{L \cos \theta}{g}}$

11. NECTA 2008/P1/3(a)

- (i) Distinguish uniform circular motion from non- uniform circular motion
- (ii) A racing car (figure 1) goes around a circular curve as fast as it can without skidding. The radius of the curve is 50m and the road is banked at 20° to allow faster speed. If the coefficient of static friction between the road and the tyre is 0.80 resolve forces into horizontal and vertical component and the equation. Apply Newton's law of motion and the equation for maximum frictional force to determine maximum speed of the car.

**12. NECTA 1989/P2/1**

- (a) When is a corner said to be correctly banked? Explain briefly why a banked corner can be negotiated at a greater speed than the corner on a flat road?
- (b) On end of a light inelastic string of length L is attached to a fixed point and the other end supports a small bob of mass M . The bob set into oscillation so that it describes a horizontal circle with a constant angular velocity ω if the string makes an angle θ with the vertical show that:-

(i) The tension in the string $T = M\omega^2 L$

(ii) $\theta = \cos^{-1} \left[\frac{g}{L\omega^2} \right]$

- (iii) The period of revolution of the bob

$$T = 2\pi\sqrt{\frac{L \cos \theta}{g}}$$

- (c) An empty cylindrical tin of internal diameter 0.10m is placed on the centre of a turn – table so that the axis of symmetry of the tin coincided with the axis of rotation of turn – table is set into rotation it is found that a small object placed on the inner vertical surface of the tin remain impinged there if the speed of rotation of turn table is 28rad/s^{-1} (or more).
- (i) Draw a diagram to show all the forces which act on the object and
- (ii) Calculate the coefficient of friction between the object and the inner surface of the tin.

13. NECTA 1988/P1/2

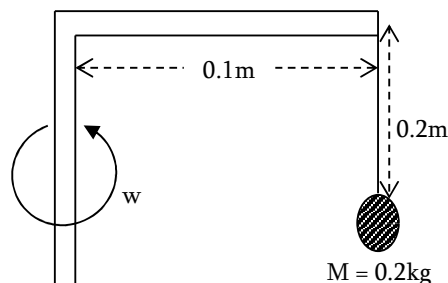
- (a) Stone tied to an inextensible string is whirled in a vertical circle at a constant speed, V . if the string snaps as a result of extension.
- At what position of motion of stone is this likely to occur?
 - Sketch the path followed by the string snaps to the time it hits the ground.
- (b) A body of weight, W slides without friction from the rest at the top of a sphere of radius, r if the body falls through a vertical distance, h ($h < r$) deduce an expression in terms of w , r and h for the normal reaction s of the sphere.

$$\text{Ans: } S = \left(\frac{r - 3h}{r} \right) W$$

14. (a) Why doesn't water contained in a bucket fall off when the bucket is whirled in a vertical plane?
- (b) A gramophone record rotates at $33\frac{1}{2}$ r.p.m and has a radius of 0.15m. determine:-
- Its angular velocity
 - The speed of a point on its circumference.
 - The speed of a point mid-way between its centre and its circumference.
15. A pendulum bob of mass 2kg is attached to one end of the string of length 1.2m. the bob moves in horizontal circle such away that the string is inclined at 30° to the vertical. Calculate :-
- The tension in the string.
 - Its velocity.
 - The period of the motion.
16. (a) How many revolution per second must apparatus figure below rotates about the vertical axis in order that the cord shall make an angle of 45° to the vertical and hence find the tension on the string?

- (b) Find the angle θ which makes with the vertical so that the system is rotate at 1.5rads^{-1} .

Ans: (a) 1.01Hz, $T = 2.77\text{N}$.



17. (a) What is banking of roads? Why is it necessary?
- (b) (i) Obtain an expression for angle of banking of a curved road. Show that it is independent of the mass of vehicle?
- (ii) On what factors angle of banking depends.
18. (a) Explain centripetal force and centrifugal force. Why centrifugal force is called a pseudo force?
- (b) Derive the expression for the maximum speed of a vehicle on the banked road state the factors on which the optimum speeds depends.
19. A stone of mass 0.4g is tied to a string and rotated in a vertical circle of radius 1.2m. Calculate the speed of the stone for which the tension in the string is zero at the highest point of the circle. What is the tension at the lowest point in this case?
- Ans: 3.43m/s, 23.52N.
20. A bucket of water tied to one end of a rope of length 3m is rotated in a vertical circle about the other end in such a way that water in it does not spill. Calculate the minimum velocity of the bucket at which this happens.
- Ans: 1.823rad/s