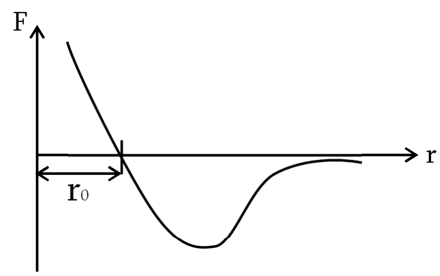
MODULE 9 : SURFACE TENSION

TERMS CONCERNING SURFACE TENSION

1. **INTERMOLECULAR FORCES** are the molecular forces of attraction between two molecules of substance. The variation of intermolecular force F with intermolecular distance r has been shown in the figure below.



$$F \propto \frac{1}{r^7} \quad \text{or} \quad F = \frac{-a}{r^7}$$

The intermolecular force of attraction decreases and become equal to zero for the value of $r = r_0 = 3.5\dot{A}$. This distance is called **normal or equilibrium distance** between the molecules.

ORIGIN OF INTERMOLECULAR FORCES.

The intermolecular forces are of electrical origin and these forces are known as Vander Waal's forces. When the distance between two molecule is greater than 35nm, the distribution of charges is such that mean distance between opposite charges in the molecules is slightly less than that between their like charges. So a force of attraction develops. When the intermolecular distance is less than 35nm, there is overlapping of the electron clouds of the molecules. This results in a strong force of repulsion. Attraction forces are usually taken as negative. Repulsive forces are usually taken as positive.

TYPES OF INTERMOLECULAR FORCES.

There are two types of intermolecular forces:-

(i) Force of cohesion or cohesive force

(ii) Force of adhesion or adhesive force.

(i) FORCE OF COHESION OR COHESIVE FORCE

Is the force of attraction amongst the molecules of the same substance. The solid have definite shape and size. It is due to strong forces of cohesive amongst their molecules. Liquid have a definite volume, but no definite shape. Thus, cohesive forces in case of liquids must be weaker than cohesive forces in case of solids. Therefore cohesive forces are very strong in solid, weak in liquids and extremely weak in gases.

Examples:

1. Definite shape of solids in due to the strong cohesive forces among their molecules.
2. Due to cohesive forces, two liquid drops brought in contact, coalesce into one.
3. We can suspend a heavy weight by a thin steel wire because of the strong forces of cohesion between the molecules of steel wire.

(ii) FORCE OF ADHESION OR ADHESIVE FORCE

Is the force of attraction acting between the molecules of different substances.

ADHESION is the property of a force of attraction between different kinds of molecules.

Example

(i) It is due to adhesive force that ink sticks on paper while writing.

(ii) Water wets the surface of a glass container . this is due to force of adhesion between water and glass molecules.

(iii) Cement , gum e.t.c are useful in glucing two surfaces together again on account of adhesive forces.

DAILY EXAMPLES OF COHESIVE AND ADHESIVE FORCES.

1. Two drops of water when brought into contact coalesces into one.
   Reason:
   This is due to the greater cohesive force than adhesive force.

2. The ink sticks on paper
   Reason:
   It is because the adhesive force between the ink and paper is greater than the cohesive force of the ink molecules.

3. Water wet the glass
   Reason:
   it is because the adhesive force between water molecules and glass molecules is greater than the force of cohesive between water molecules.

4. Mercury does not wet the glass
   Reason:
   It is because the adhesive force between mercury molecules and glass molecules is less than the force of cohesive between mercury molecules.

5. We able to write on the black board with a piece of chalk.
   Reason:
   it is because the adhesive force between chalk and wood molecules is much greater than force of cohesion between the chalk molecules and wood molecules.

6. Paint sticks to wood (and other surfaces) due to large force of adhesion between the surface of wood and paint.

2. INTER - MOLECULAR BINDING ENERGY OF LIQUID.

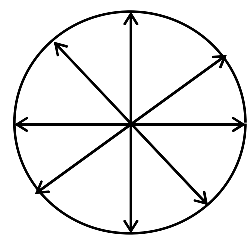
Is the minimum energy required to separate two molecules of a liquid from each others influence due to cohesive forces , the molecules of the liquid have some intermolecular binding energy. To remove a molecule from the liquid , some energy is to spent by outside agency which is equal to intermolecular binding energy. The intermolecular binding energy is less for water and more for mercury.

3. MOLECULAR RANGE

Is the maximum distance up to which a molecule can exert some measureable attraction on other molecules. It is different for different substances. The other of molecular range is 10^{-9}m in solids and liquids.

4. SPHERE OF INFLUENCE

It is an imaginary sphere drawn with a molecule as centre and molecular range as radius.



5. SURFACE FILIM

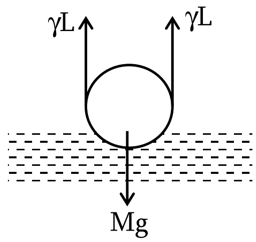
Is the top most layer of liquid at rest with thickness equal to the molecular range i.e it is the thin film of liquid near its surface and having thickness equal to the molecular range of that liquid. All the molecules in the surface film have additional potential energy as compared to the negative potential energy of the molecules in the interior of the liquid due to cohesive forces between molecules. The phenomenon of surface tension is intermately linked with thin film.

QUALITATIVELY DEFINITION OF SURFACE TENSION

Surface tension is the property of a liquid at rest by virtual of which its free surface behave like an elastic stretched skin covering the liquid. It has a tendency of contracting so as to occupy a small area as much as possible.

Observation from day to day life about surface tension.

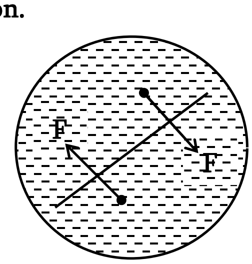
(i) A steel needle placed on water surface may float even through its density is higher than that of water. This is because the needle produces a depression on water surface at the point of contact. This means the water surface behaves as a stretched elastic membrane as a result surface tension force will balance with the weight of the needle.



(ii) Insect which are not wet by water can walk on the surface of water. But some are wet by water , hence they are gripped by water film and cannot escape all observation above shown that the water surface acts as stretched elastic membrane and hence the surface is under tension. This property of the liquid is called 'surface tension' i.e the surface of a liquid becomes like an elastic skin in a state of tension. This is known as **surface tension**.

MEASUREMENT OF SURFACE TENSION

Surface tension is due to the intermolecular attractions on the liquid surface and these forces produce a skin effect on the surface. Imagine a line AB drawn tangentially anywhere on the liquid surface at rest. The force of surface tension acts at right angles to this line on its side and also along the tangent to the liquid surface as shown on the figure below. The force acting per unit length of such a line gives the **quantitative measure of surface tension.**

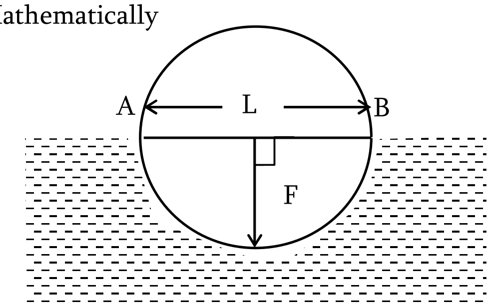


SURFACE TENSION

(Coefficient of surface tension , γ)

Is defined as the force per unit length acting in the liquid surface at right angles to a side of imaginary line drawn on the liquid surface.

Mathematically



$$\text{Surface tension} = \frac{\text{Force}}{\text{Length}}$$

$$\gamma = \frac{F}{L}$$

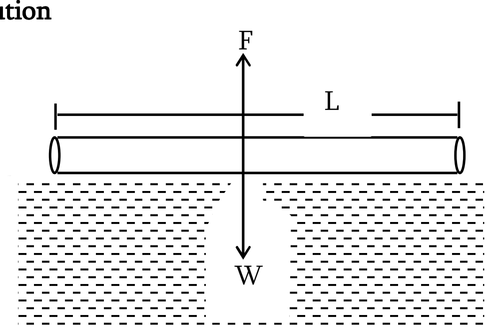
Surface tension is denotes by using symbol of γ or σ or T. The surface tension acts perpendicular to the line AB and tangential to the liquid surface. S.I unit of surface tension is Nm^{-1} and in cgs system is dyne/cm. the dimensional formula of surface tension is $[ML^0T^{-2}]$. Surface tension is a scalar because it has no specific direction for a given liquid. Surface tension force is given by $F = \gamma L$

SOLVED EXAMPLE TYPE A:

Example – 01

The maximum force in addition to the weight required to pull a wire 5cm long from surface of water at $20^{O}C$ is 728N; Calculate the surface tension of the water.

Solution



The film of the liquid covered the wire on the either side. The length which covered by surface tension force on the wire is 2L

At the equilibrium

$$F = W \quad \text{but} \quad F = 2\gamma L$$

$$2\gamma L = 728$$

$$\gamma = \frac{728}{2L} = \frac{728}{2\times0.05}$$

$$\gamma = 7280Nm^{-1}$$

$\therefore$ Surface tension of water , $\gamma = 7280Nm^{-1}$

Example – 02

A wire ring of 3cm radius resting flat on the surface of a liquid is raised. The pull required is 3.03gf more before the film breaks that its afterwards. Find the surface tension of the liquid.

Solution

Surface tension

$$F = 3.03\times10^{-3}\times9.8$$

$$F = 0.029694N$$

At the liquid touches the ring both along the inner and outer circumference , so force on the ring due to the surface tension.

$$F = 2\times2\pi r\gamma = 4\pi r\gamma$$

$$\gamma = \frac{F}{4\pi r} = \frac{0.029694N}{4\times3.14\times0.03m}$$

$$\gamma = 7.87\times10^{-2}Nm^{-1}$$

Example – 03

A disc of radius 0.03m rested on the liquid surface is raised . the pull required is 1.5 × 10-3kgwt more before the film breaks , that its after. Calculate the surface tension.

Solution

$$\gamma = \frac{F}{L} = \frac{F}{2\pi r}$$

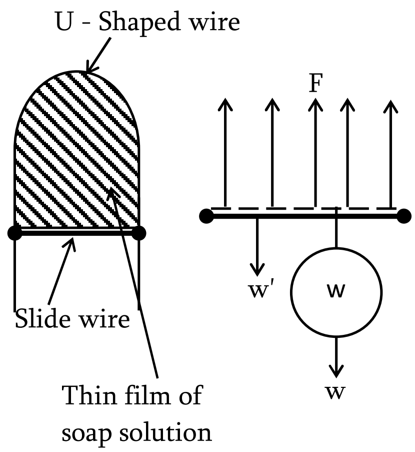
$$\gamma = \frac{1.5\times10^{-3}\times9.8}{2\times3.14\times0.03} = 77.98\times10^{-3}Nm^{-1}$$

$\therefore$ Surface tension of liquid , $\gamma = 77.98\times10^{-3}Nm^{-1}$

Example – 04

A U – shaped wire is dipped in a soap solution and then removed. The thin film formed between the wire and slide supports a weight , w if the surface tension of the soap water is 0.025N/m and the slid wire weigh is 10^{-3} and is 0.03m long. What is the magnitude of the weight w?

Solution



The available force are :

(i) Surface tension force f , acts vertically upward.

(ii) Weight of slide wire (w') acts downward

(iii) Weight of the hanging mass acts vertically downward , w.

At the equilibrium of the system

$$F = W + W'$$

$$W = F - W' = 28L - W'$$

$$= 2\times0.03\times0.025 - 10^{-3}$$

$$W = 0.5\times10^{-3}N$$

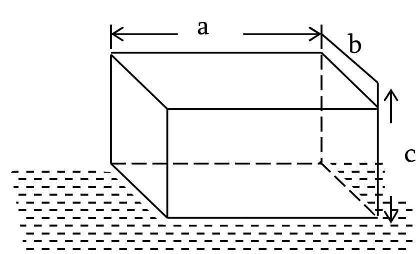
$\therefore$ The magnitude of weight , $W = 0.5\times10^{-3}N$

Example – 05

A rectangular plate of dimensions of 6cm by 4cm and thickness 2mm is placed with a largest face flat on the surface of the water. Calculate the force due to surface tension on the plate. What is the down ward force due to surface tension if the plate is placed vertically and its longest side just touches the water?

Solution

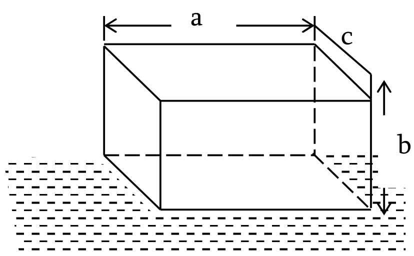
Case 1: When a largest face flat on water surface



Force due to the surface tension

$$F = 2\gamma(a+b)$$
$$= 2\times 0.07\times(6+4)\times 10^{-2}$$
$$F = 0.014N$$

Case 2: when plate is placed vertically.



$$F = 2\gamma(a+c)$$
$$= 2\times 0.07\times(6+0.2)\times 10^{-2}$$
$$F = 0.00868N$$

Example – 06

A circular wire ring of mass 1.85g and diameter 5cm is supported with its plane horizontal from one arm of a sensitive balance and beaker of water raised underneath it until the ring just lies on the surface. What additional mass must be placed on the other pan of the balance in order to lift off the ring clear of the water surface?

Solution

For the ring to be lifted.

Additional weight = surface tension force

$$Mg = 2\gamma(2\pi r)$$
$$Mg = 4\gamma\pi r$$
$$M = \frac{4\pi\gamma r}{g} = \frac{4\times 3.14\times 7.5\times 10^{-2}\times 0.05}{9.8}$$
$$M = 4.8\times 10^{-3}\text{kg} = 4.8\text{gm}$$

$\therefore$Additional mass , M = 4.8gm

Example – 07

A metal wire of density d floats , in a liquid of surface tension γ , in the horizontal position . calculate the maximum radius of the wire so that it may not sink.

Solution

At the equilibrium

Surface tension = weight of metal wire

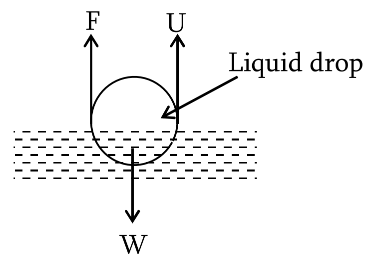
$$2\gamma L = L\times \pi r^2\times d\times g$$
$$r^2 = \frac{2\gamma}{\pi dg}$$
$$r = \sqrt{\frac{2\gamma}{\pi dg}}$$

$\therefore$ The maximum radius of the wire $r = \sqrt{\frac{2\gamma}{\pi dg}}$

Example – 08

A drop of liquid of density ρ is floating half – immersed in a liquid of density d. if γ is the surface tension, then what is the diameter of the drop of the liquid?

Solution



At the equilibrium of the liquid drop.

F + U = W

F = Surface tension force

U = up thrust , W = Weight of drop

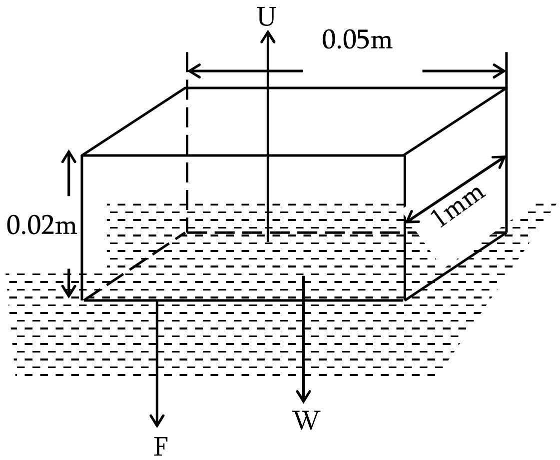
$$2\pi r\gamma + \frac{1}{2}\times\frac{4}{3}\pi r^3 dg = \frac{4}{3}\pi r^3\rho g$$
$$2\pi r\gamma = \frac{\pi r^3 g}{3}\left[4\rho - 2d\right]$$
$$r^2 = \frac{3\gamma}{g(2\rho - d)}$$
$$r = \sqrt{\frac{3\gamma}{g(2\rho-d)}}$$

$\therefore$ The diameter of drop $= 2r = \sqrt{\frac{12\gamma}{g(2\rho-d)}}$

Example – 09

A glass plate of length 0.05m, breath 0.02m and thickness 1mm, weighs 0.015kg in air. Calculate the apparent weight, when it is held vertically with its long ride horizontal and its lower half immersed in water , when the surface tension of water is 7.2×10^{-2}N/m.

Solution



Forces available acting on the plates

(i) Weight of the plate W = 0.015kgwt act in downward direction

(ii) Force due to the surface tension F = 2γ(L + d)

$$F = 7.2\times10^{-2}\times2(0.05+0.001)$$

$$F = \frac{7.2\times10^{-2}\times0.102}{9.8}\text{kgwt}$$

$F = 74.9\times10^{-5}\text{kgwt}$ in the downward direction.

(iii) Upthrust , U = Weight of the fluid displaced

$$U = L\times t\times\frac{b}{2}\times\rho\times g$$

$$= 0.05\times0.001\times\frac{0.02}{2}\times1000\times9.8$$

$U = 50\times10^{-5}\text{kgwt}$ in upward direction

Apparent weight = F + W – U

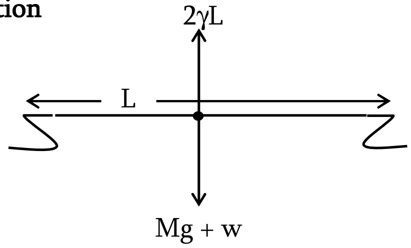
= 0.015 + 74.9 × 10^{-5} – 50 × 10^{-5}

Apparent weight is 0.01525kgwt

Example – 10

A U – shape wire is dipped in a soap solution and removed. The film formed between the wire and a light slider supports a weight of 2.5 × 10^{-3}N, which includes the small weight of the slider is 0.05m , what is the surface tension of the liquid?

Solution



At the equilibrium

$$2\gamma L = Mg + W$$

$$\gamma = \frac{Mg+\gamma}{2L} = \frac{2.5\times10^{-3}}{2\times0.05}$$

$$\gamma = 2.5\times10^{-2}\text{N/m}$$

Example – 11

A glass plate of length 10cm, breath 1.54cm and thickness 0.20cm has a mass of 8.2gm in air its held vertically with its long side horizontal and the lower half under water. Find the apparent weight of the plate surface tension of the water is 7.3 × 10^{-3}N/m.

Solution

Weight of the block in air

$$W = Mg = 8.2\times10^{-3}\times9.8$$

$$W = 80.36\times10^{-3}\text{N}$$

Force due to the surface tension

$$F = 2\gamma(L+t)$$

$$= 2\times7.3\times10^{-2}\times(10+0.2)\times10^{-2}$$

$$F = 14.89\times10^{-3}\text{N}$$

$$\text{Upthrust , } U = L\times t\times\frac{b}{2}\times\rho\times g$$

$$U = \frac{10\times10^{-2}\times1.54\times0.2\times10^{-4}\times1000\times9.8}{2}$$

$$U = 15.092\times10^{-3}\text{N}$$

Apparent weight = total downward force – U

= F+ W – U

= (80.36 + 14.89 – 15.092) × 10^{-3}

∴ Apparent weight = 80.158 × 10^{-3}N

Example – 12

A toy boat of white pine (specific gravity , 0.45) is floating on the surface of a calm lake. The wood is 8cm wide , 30cm long and 2cm thick.

(a) If to the water near the right hand 8cm side added a detergent that make the coefficient of surface tension 40% of its normal value, what is the direction of the net force on the boat?

(b) Ignoring viscous drag force of the water what will be the velocity of the boat 5seconds after detergent is added to the weight?

Solution

(a) The direction of the net force is towards the left hand side . the detergent lowers the surface tension to the right thus making left ward force due to surface tension greater than the rightward force and according to the Newton's law, the boat moves to the left.

(b) The net force towards to the left

$$F = W(\gamma_1 - \gamma_2) = W(\gamma_1 - 0.4\gamma_1)$$

$$F = 0.6\gamma_1 W$$

$$Ma = 0.6\gamma_1 W$$

$$a = \frac{0.6\gamma_1 W}{M} = \frac{0.6\gamma_1 W}{\rho \times L \times w \times t}$$

$$a = \frac{0.6 \times 0.072}{450 \times 0.3 \times 0.02} = 0.016 m/s^2$$

Now $V = u + at = 0 + 0.016 \times 5$

$$V = 0.08 m/s$$

EXERCISE 9.1

1. A U – shape wire is dropped in a soap solution and removed. The thin soap film formed between the wire and a light slider supports a weight of 4.2×10^{-3}N (which includes the small weight of the slider). The length of the slider is 0.05m. What is the surface tension of the liquid? **Answer:** 4.2×10^{-2}N/m.

2. A thin circular wire of 1.12cm radius is supported horizontally in a liquid and withdraw slowly by an upward force of 5.53×10^{-3}N. If the surface tension of the liquid is 0.032N/m. What is the mass of the wire? **Answer:** 1.05×10^{-4}kg.

3. A soap film is formed on a rectangular frame of length 0.07m dipping in soap solution , the frame hangs from the arm of a balance an extra weight of 0.38×10^{-3}kg is to be placed in the opposite pan to balance the pull on the frame. Calculate the surface tension of the soap solution given g = 9.81m/s^2. **Answer:** 0.027Nm^{-1}.

4. A wire 0.1m long is placed horizontally on the surface of water and gently pulled up with a force 1.456×10^{-2}N to keep the wire in equilibrium. Calculate the surface tension of water. Answer 0.07Nm^{-1}.

5. A square frame of side L is dipped in a liquid soap. When it is taken out of the liquid , a soap film is formed on it. If the surface tension of soap solution is γ, then what is the force acting on the frame? **Answer:** $8\gamma L$.

6. A ring of internal and external diameters 8.5×10^{-2}m and 8.7×10^{-2}m is supported horizontally from the pan of a physical balance such that it come in contact with a liquid. An extra force of 40N is required to pull it away from the liquid. Determine the surface tension of the liquid. Answer 74Nm^{-1}.

7. The material of a wire has a density of 1.4g/cm^3. If it is not wetted by a liquid of surface tension 44dyne/cm , find the maximum radius of the wire which can float on the surface of the liquid. **Answer**: 1.43mm.

8. A square wire frame of side 10cm is dipped in a liquid of surface tension 28×10^{-3}Nm^{-1} on taking out , a membrane is formed. What is the force acting on the surface of the wire frame? **Answer:** 0.0224N.

SOME SURFACE TENSION PHENOMENA

The following are phenomena which illustrate surface tension:-

1. Surface tension explain why a steel needle can be made to float on water even through steel is more denser than water. This is because steel needle create a depression in the liquid on the surface so that the surface tension force F , have acted upward direction which is capable to support the weight of the needle.

   Note that:

   If we add oil in the water the needle sinks. It is because oil lowers the surface tension of water and the weight of needle exceed , the upward direction of component of surface tension force.

2. Surface tension of oil is less than that of water when it sprayed it spreads and forms a thin film layer above water surface. This prevent mosquito to bread on the surface of oil – water.

3. Rain drops are spherical in shape , because each drop tends to acquire minimum surface area due to surface tension and for a given volume , the surface area of sphere is minimum.

4. Surface tension explains why water rises up on a capillary tube.

5. Elimination of the effect of gravity soap bubbles are almost perfect spheres because they have large surface areas and negligible masses i.e effect of gravity is almost nil.

6. When a shaving brush is dipped in water , its hair spread out on taking out the brush from water , the water film formed between the hairs while tending to make its surface area minimum due to surface tension will bring the hairs closer to each other. That is why , the hairs of shaving brush when taken out of water are pressed together.

DEMONSTRATION OF SURFACE TENSION

A loop of thread tied to a metal ring is dipped into soap solution and then withdrawn from the solution. A soap film is formed figure (a); surface tension forces on both sides of the thread counter balances and thread ha the shape shown

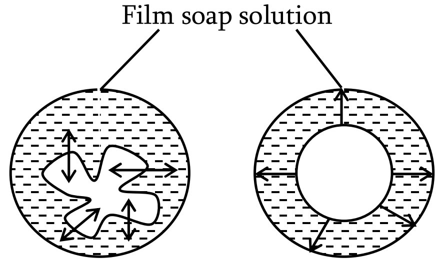


Figure (a) Figure (b)

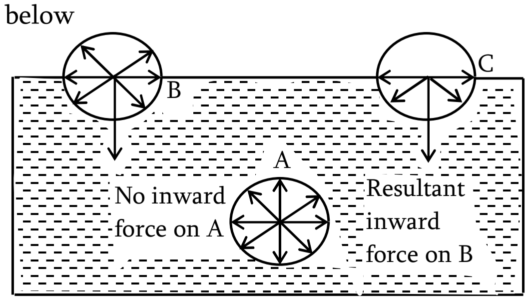
When the film inside the thread is punctured by using sharp edge of a needle as shown on figure (b) the thread is pulled into a circle by surface tension forces which now acts on one side of the thread only. Since a circle has maximum area for a given perimeter , the remaining film has a minimum area due to the surface tension.

MOLECULAR THEORY OF THE SURFACE TENSION

This is the theory which explain the surface tension in terms of the intermolecular forces. The cohesive force among liquid molecules is responsible for the phenomena of the surface tension. The molecules inside the liquid are attracted equally in all directions by the other molecules. In contrast , the molecules on the surface experience an inward pull , because they are partially attracted by the other molecules. As the result , a network of force is then formed against inward pull in order to move molecules in the liquid surface. This results into a large potential energy on surface molecules. In order to attain minimum potential energy and hence the stable equilibrium , the free surface of the liquid tends to have the minimum surface area and it behaves like a stretched membrane.

- **Energy of liquid surface**

  This can be explained as shown on the figure below



In the bulk of the liquid , which begins only a few molecules diameters downward from the surface , a particular molecule such as A is surrounded by an equal number of molecules on all sides. This can be seen by drawing a sphere round A. The average distance apart of the molecule is such that the attractive forces balanced with the repulsive force. Thus the average intermolecular force between A and the surrounding molecules is zero. Since the sphere of influence of molecule B lies partly outside the liquid , the number of molecules attracting molecule B downwards is more than the number of molecules attracting it upwards.

Therefore there is a resultant downward force of cohesive acting on molecule B. For the case of sphere C , the number of molecules in the lower half of sphere of influence of C, attracting it downwards is very large as compared to number of molecules (only gas or vapour molecules) in the upper half of its sphere of influence attracting it upward. Therefore, there is maximum resultant downward force acting on molecule C. Due to this net in ward force on the molecules lying on the surface , then the surface experiences a tension called surface tension. For bringing a molecule to the surface of the liquid, the work has to be done against downward force. Hence, the molecules on the liquid surface have greater potential energy for the system to be stable its potential energy must be minimum. In order the potential energy associated with intermolecular force (resurface tension) can be minimum, the number of molecules which reside on the surface has to be minimum

Note that

The following are the ways in which minimum number of surface molecules of the liquid is obtained:-

1. A liquid molecules will have the smallest possible surface area.

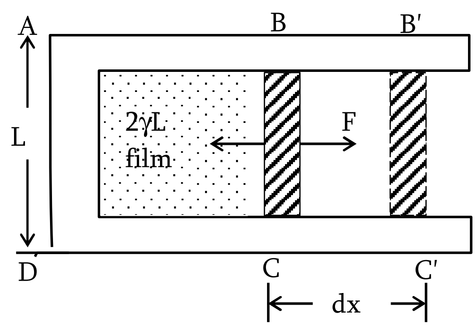
   For example:

   The liquid drops are spherical because for any given volume the shape which given volume the shape which gives the minimum surface area is a sphere. Also soap bubbles are almost perfect sphere because of their negligible masses and the presence of surface tension force tend to reduce the surface area of the bubble to a minimum surface area hence are spherical.

2. The area of liquid surface have least number of molecules i.e the average separation of the molecules on the surface of a liquid is greater than that of the molecules in the interior.

SURFACE TENSION AND SURFACE ENERGY

Consider a film of liquid stretched across a horizontal frame ABCD as shown on the figure below. If we dip the frame in soap solution a thin film is formed which pulls the wire AB towards left due to surface tension. We can increase the surface tension. We can increase the surface area of soap film by displacing the sliding wire AB outwards.



The surface tension on the sliding were BC of length , L is given F = 2γL (surface film has two surfaces) the work done by enlarge the surface area

$$W = Fdx = (2\gamma L)dx$$

$$\text{But } 2LdX = A$$

$$W = \gamma A \quad \text{or} \quad \gamma = \frac{W}{A}$$

Surface tension – is defined as work done per unit area in increasing the surface area of a liquid under isothermal condition.

Mathematically

$$\gamma = \frac{W}{A}$$

Free surface energy

$$\delta = \frac{\text{work done}}{\text{increase in surface area}}$$

$$\delta = \frac{W}{A} = \frac{\gamma A}{A} = \gamma$$

$$\delta = \gamma$$

The potential energy of increased area is called surface energy.

Surface energy of given liquid surface is defined as the amount of work done against the force of surface tension in forming the liquid surface of given area at a constant temperature. Another unit of surface tension is Jm^{-2}.

Note that.

1. When the surface area of a film increases , the temperature of the film is usually decreases. This is also happening if the film surface is increased under adiabatic condition when no heat enters or leave the system.

2. Under isothermal condition heat is added to restore the original temperature. This heat comes from mechanical work in increasing the surface.

- **WORK DONE IN BLOWING LIQUID DROP**

Case 1: expression of work done of blowing liquid drop. When a liquid drop is formed, its area increses from zero value to a certain value. If the radius of the resulting drop is R, then the increase in are is $4\pi r^2$. Assume that the drop is spherical in shape. Work done in blowing liquid drop

$$W = \gamma A = 4\pi r R^2\gamma$$

Case 2: expression of work done in blowing liquid from radius , R_1 to the radius R_2 (i.e $R_1 < R_2$)

$$W = W_2 - W_1 = 4\pi R_2^2\gamma - 4\pi R_1^2\gamma$$

$$W = 4\pi\gamma\left(R_2^2 - R_1^2\right)$$

- **WORK DONE IN BLOWING SOAP BUBBLE**

Case 1: When a soap bubble is formed, its area increases from zero value to a certain value. If the radius of the resulting soap bubble is R, then increase in area is $\left(4\pi R^2\right)\times 2$

Increase in area, $A = 8\pi R^2\gamma$

Work done in blowing soap bubble

$$W = \gamma A = 8\pi R^2\gamma.$$

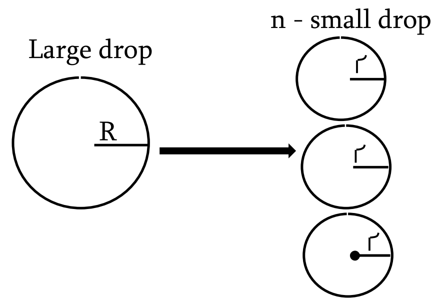
Case 2: expression of work done in blowing soap bubble from radius R_1 to radius R_2 ($R_1 < R_2$)

$$W = W_2 - W_1 = 8\pi\gamma\left(R_2^2 - R_1^2\right)$$

COALESCING AND BREAKING OF LIQUID DROPS AND SOAP BUBBLES.

Case 1: work done in breaking a large drop into small drop.

Suppose a large liquid drop of radius R breaks up into a tiny small drops each of radius r. this breaking must be done very slowly so that temperature remain constant so that it avoid the change of surface tension.



Apply the law of conservation of volume

Total volume of n – small drops = volume of the large drop

$$n\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$$

$$nr^3 = R^3 \text{ or } n = \left(\frac{R}{r}\right)^3$$

The radius of each small drop formed

$$r = n^{1/3}R$$

Increase in surface area when the drop formed

$$\Delta A = A_2 - A_1$$

$$= 4\pi r^2 n - 4\pi R^2 = 4\pi\left[nr^2 - R^2\right]$$

$$= 4\pi\left[n\left(n^{-1/3}R\right)^2 - R^2\right]$$

$$\Delta A = 4\pi R^2\left[n^{1/3} - 1\right]$$

Work done required to break the large drop into small drops.

$$W = \gamma\Delta A$$

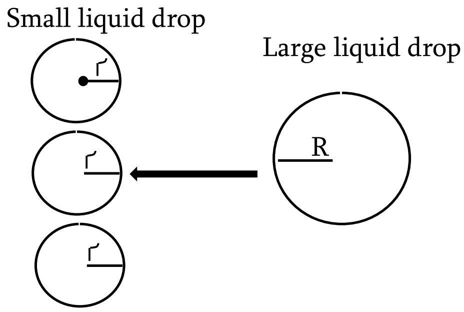
$$W = 4\pi R^2\gamma\left(n^{1/3} - 1\right) \quad \text{Or } n = \left(\frac{R}{r}\right)^3$$

$$W = 4\pi R^2\gamma\left[\frac{R}{r} - 1\right]$$

Since $\frac{R}{r} > 1$, there is an increase in energy when a large drop breaks into small ones.

Case 2: work done in coalescing liquid drop

When liquid drops each of radius r are joined, the energy is liberated energy of the large drop formed is less than total energy of small drops.



Applying the law of conservation of volume

$$\frac{4}{3}\pi r^3 n = \frac{4}{3}\pi R^3$$

$$R = rn^{1/3} \quad \text{or} \quad n = \frac{R^3}{r^3}$$

Decrease in surface area

$$\Delta A = A_2 - A_1$$

$$= 4\pi r^2 n - 4\pi R^2 = 4\pi\left(nr^2 - R^2\right)$$

$$\Delta A = 4\pi r^2\left(n - n^{2/3}\right)$$

$$W = \gamma\Delta A$$

$$= 4\pi r^2\left[n - n^{2/3}\right]\gamma$$

$$W = 4\pi\gamma r^2\left[n - n^{2/3}\right]$$

Also: $n = \dfrac{R^3}{r^3}$

Now : $W = 4\pi\gamma r^2\left[n - n^{2/3}\right]$

$$= 4\pi\gamma r^2\left[\frac{R^3}{r^3} - \left(\frac{R^3}{r^3}\right)^{2/3}\right]$$

$$= 4\pi\gamma r^2\left[\frac{R^3}{r^3} - \frac{R^2}{r^2}\right]$$

$$= 4\pi\gamma\left[\frac{R^3}{r} - R^2\right]$$

$$W = 4\pi\gamma R^2\left[\frac{R}{r} - 1\right]$$

SOLVED EXAMPLE: TYPE B

Example – 13

(a) Molecules on the free surface of a liquid have more potential energy than in the interior why?

(b) Calculate energy the change of surface area of a soap bubble when its radius decreases from 5cm to 1cm. give that the surface tension of soap solution is $2.0 \times 10^{-2}Nm^{-1}$.

Solution

(a) In the interior ,no net force acts on a molecule but as it approaches the surface , the attractive force pulling it back dominate therefore , work must be done on the molecule to bring it to the surface so molecules on the free surface of a liquid have more potential energy than those in the interior.

(b) $W = 2\gamma\Delta A = 8\pi\gamma\left[R_2^2 - R_1^2\right]$

$$= 8\times3.14\times2\times10^{-2}\left[\left(5\times10^{-2}\right)^2 - \left(1\times10^{-2}\right)^2\right]$$

$$W = 1.21\times10^{-3}J$$

Example – 14

(a) The surface tension of the soap solution is $0.03Nm^{-1}$. Calculate work required to produce a bubble of radius 0.5m.

(b) A spherical drop of mercury of radius 3mm falls to the ground and breaks into 10 small drops of equal size. Calculate the amount of work to be done in process. Surface tension of mercury $\gamma = 4.72\times10^{-1}Nm^{-1}$

Solution

(a) $W = \gamma\Delta A$

Since a soap bubble has two surfaces in contact with air , then

$$\Delta A = 2\times4\pi R^2 = 8\pi R^2$$

$$W = 8\pi R^2\gamma = 8\pi\times\left(0.5\right)^2\times0.03$$

$$W = 0.19J$$

(b) Applying the law of conservation of volume

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r^3 n$$

$$r = \frac{R}{n^{1/3}} = \frac{2\times10^{-3}m}{10^{1/3}}$$

Now $\Delta A = 4\pi nr^2 = 4\pi R^2$

$$W = \gamma\Delta A = 4\pi\gamma\left[nr^2 - R^2\right]$$

$$= 4\times3.14\times4.72\times10^{-1}\left[10\times\left(9.28\times10^{-4}\right)^2 - \left(2\times10^{-3}\right)^2\right]$$

$$W = 2.74\times10^{-5}J$$

Example – 15

(a) Find the work done in spraying a drop of water of 2mm diameter into 1000 droplets all of the same size surface tension of water is 72×10^{-3}N/m.

(b) If a number of small droplets of water each of radius r coalesce to form a single drop of radius, R , shown that the rise in the temperature is given by $\Delta\theta = \frac{3\gamma}{\rho C}\left[\frac{1}{r} - \frac{1}{R}\right]$

Solution

(a) Applying the law of conservation of volume.

$$\frac{4}{3}\pi nr^3 = \frac{4}{3}\pi R^3$$

$$n = \left(\frac{R}{r}\right) \quad \text{or} \quad r = n^{1/3}R$$

Change in energy

$$W = W_2 - W_1$$

$$= 4\pi r^2\gamma n - 4\pi R^2\gamma$$

$$= 4\pi\gamma\left(nr^2 - R^2\right)$$

$$= 4\pi\gamma R^2\left[n^{1/3} - 1\right]$$

$$W = 4\pi\times72\times10^{-3}\times\left(1\times10^{-3}\right)^2\left[1000^{1/3} - 1\right]$$

$$W = 8.139\times10^{-6}J$$

(b) Let n be number of water droplets each of radius r combine to form bigger drop of radius, R.

Area of n drops , $A_1 = 4\pi nr^2$.

Area of bigger drop $A_2 = 4\pi R^2$.

Now , $\Delta A = A_1 - A_2$

$$W = \gamma\Delta A = 4\pi\gamma\left(nr^2 - R^2\right)$$

Apply the law of conservation of volume

$$\frac{4}{3}\pi nr^3 = \frac{4}{3}\pi R^3$$

$$n = \frac{R^3}{r^3}$$

$$W = 4\pi\gamma\left[\frac{R^3}{r^3}r^2 - R^2\right]$$

$$W = 4\pi\gamma R^2\left[\frac{R}{r} - 1\right]$$

Amount of heat energy produced

$$W = MC\Delta\theta$$

$$W = \frac{4}{3}\pi R^3\rho C\Delta\theta$$

Example – 16

If the energy required to blow a soap bubble of radius r is E , show that the extra energy needed to double the radius of the bubble is given by $E = 24\pi\gamma r^2$ where γ is the surface tension of the soap solution.

Solution

Soap bubble have two surface increase in area.

$$\Delta A = 2\left(A_2 - A_1\right)$$

$$= 2\left[4\pi\left(2r\right)^2 - 4\pi r^2\right]$$

$$\Delta A = 8\pi\left[4r^2 - r^2\right] = 24\pi r^2$$

$$\text{Now } E = \gamma\Delta A = 24\pi\gamma r^2$$

Example – 17

Find the amount of work done required to break up a drop of water of radius 0.5cm into drops of water each of radii 1mm , surface tension of water is 7.0×10^{-2}N/m.

Solution

The number of drops formed

$$n = \left(\frac{R}{r}\right)^3 = \left(\frac{0.5}{0.1}\right)^2 = 125$$

Work done required to break water drop.

$$W = 4\pi R^2\gamma\left[n^{1/3} - 1\right]$$

$$= 4\times3.14\left(0.5\times10^{-3}\right)^2\left[125^{1/3} - 1\right]$$

$$W = 8.8\times10^{-5}J$$

Example – 18

A spherical drop of mercury of radius 2mm falls to the ground and breaks into 10smaller drops of equal size. Calculate the amount of work done that has to be done in the process. Surface tension of mercury is 0.472Nm^{-1}.

Solution

$$W = 4\pi R^2\gamma\left[n^{1/3} - 1\right]$$

$$= 4\times 3.14\left(0.2\times 10^{-2}\right)^2\times 0.472\times\left[10^{1/3} - 1\right]$$

$$W = 2.74\times 10^{-5}\,J$$

Example – 19

The work done in increasing the size of film from 10cm × 6cm to 10cm ×v11cm is 3.0 × 10^{-4}J. What is the surface tension of the film?

Solution

Since $W = \gamma \times$ increase in surface area

$$3.0\times 10^{-4} = 2\gamma\times 10\times 10^{-4}\left(11 - 6\right)$$

$$\gamma = 3.0\times 10^{-2}\,N/m$$

Example – 20

A soap film is a rectangular wire ring of size 3cm × 3cm. If the size of the film is changed to 3cm × 4cm. The calculate the work done in the process. The surface tension of soap is 3 × 10^{-2}Nm.

Solution

$W = \gamma \times$ Increase in surface area

$$= 2\times 3\times 10^{-2}\times 3\times\left(4 - 3\right)\times 10^{-4}$$

$$W = 1.8\times 10^{-5}\,J$$

Example – 21

How much energy is expanded in blowing a soap bubble of radius 10mm from a soap solution of surface tension 25 × 10^{-3}N/m. what becomes of this energy? Indicate any assumption made at arriving at your answer

Solution

$$W = 8\pi\gamma r^2$$

$$= 8\times 3.14\times 25\times 10^{-3}\times\left(10\times 10^{-3}\right)^2$$

$$W = 6.28\times 10^{-5}\,J$$

The expanded energy appears as the molecular potential energy associated with molecules in the new surface i.e free surface energy.

Assumptions:

1. The process takes place under isothermal condition.
2. No work done in pushing back the atmosphere.

Example – 22

Find the work down in blowing a spherical soap bubble of radius 3cm in an atmosphere at a pressure of 10Nm^{-2}. Surface tension of a soap solution is 25 × 10^{-3}Nm^{-1}.

Solution

Work done against surface tension force

$$W_1 = 8\pi\gamma r^2$$

$$= 8\pi\left(0.03\right)^2\times 25\times 10^{-3}$$

$$W_1 = 5.655\times 10^{-4}\,J$$

Work done against external pressure

$$W_2 = PV = \frac{4}{3}\pi r^3 p$$

$$= 10\times\frac{4}{3}\times 3.14\times\left(0.03\right)^3$$

$$W_2 = 1.1309\times 10^{-3}\,J$$

Total work done

$$W = W_1 + W_2$$

$$W = 1.7\times 10^{-3}\,J$$

Example – 23

Calculate work done against surface tension in blowing a soap bubble from a radius 10cm to 20cm. the surface tension of soap solution is 25 × 10^{-3}Nm^{-1}.

Solution

The soap bubble has two surfaces , external and internal.

Original total surface area, $A_1 = 2\times 4\pi R_1^2$

Final surface area, $A_2 = 2\times 4\pi R_2^2$

Increase in surface area

$$\Delta A = A_2 - A_1 = 8\pi\left[R_2^2 - R_1^2\right]$$

$$= 8\times 3.14\times 25\times 10^{-3}\left[\left(0.2\right)^2 - \left(0.1\right)^2\right]$$

$$W = 18.84\times 10^{-3}\,J$$

Example – 24

Several spherical drops of a liquid of radius r coalesce to form a single spherical drop of radius R. If γ is the surface tension. Calculate the energy release . if all the energy released converted into the kinetic energy , show that the velocity acquired by the drop is given by

$$V = \sqrt{\frac{6\gamma}{\rho}\left(\frac{1}{r} - \frac{1}{R}\right)}$$

Solution

Case 1: Energy released.

Let n be number of liquid droplets each of radius r combine to form a bigger drop of radius ,R.

Apply the law of conservation of volume

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r^3 n$$

$$n = \frac{R^3}{r^3}$$

Energy released is given by

$$E = \gamma\Delta A = \gamma\left(A_2 - A_1\right)$$

$$= \gamma\left[4\pi n r^2 - 4\pi R^2\right]$$

$$= 4\pi\gamma\left[nr^2 - R^2\right]$$

$$= 4\pi\gamma\left[\frac{R^3}{r^3}\cdot r^2 - R^2\right]$$

$$E = 4\pi\gamma R^3\left[\frac{1}{r} - \frac{1}{R}\right]$$

Case 2:

Let V be the velocity of the drop K.E of drop = ½ MV^2.

Apply the law of conservation of energy

k.e of drop = energy released

$$\frac{1}{2}MV^2 = 4\pi\gamma R^3\left[\frac{1}{r} - \frac{1}{R}\right]$$

$$\frac{1}{2}\left(\frac{4}{3}\pi R^3\rho\right)V^2 = 4\pi\gamma R^3\left(\frac{1}{r} - \frac{1}{R}\right)$$

$$V^2 = \frac{6\gamma}{\rho}\left(\frac{1}{r} - \frac{1}{R}\right)$$

$$V = \sqrt{\frac{6\gamma}{\rho}\left(\frac{1}{r} - \frac{1}{R}\right)}$$

Example – 25

A spherical drop of mercury of radius 2mm falls to the ground and breaks into 10 smaller drops of equal size. Calculate the amount of work that has to be done. (Surface tension of mercury = $4.72 \times 10^{-1}Nm^{-1}$) what is the minimum speed with which the original drop could have hit the ground? (Density of mercury = $13600kgm^{-3}$).

Solution

Case 1: the amount of work done

$$W = W_1 - W_2$$

$$= 4\pi\gamma R^2\left[n^{1/3} - 1\right]$$

$$= 4\pi \times 0.472 \times \left(2\times10^{-3}\right)^2\left[10^{1/3} - 1\right]$$

$$W = 2.74\times10^{-5}J$$

Case 2:

According to the principle of conservation of energy assuming that no loss of energy on this process

k.e on impact with ground = work done in break of drop

$$\frac{1}{2}MV^2 = W$$

$$\frac{1}{2}\left(\frac{4}{3}\pi R^3\rho\right)V^2 = W$$

$$V^2 = \frac{6W}{4\pi R^3\rho}$$

$$V = \sqrt{\frac{6\times2.74\times10^{-5}}{4\times3.14\left(2\times10^{-3}\right)^3\times13600}}$$

$$V = 0.35m/s$$

EXERCISE 9.2

1. (a) Define surface tension show how it is related to surface energy in a liquid.
   (b) The surface tension of a soap solution is $0.03Nm^{-1}$. What amount of work is required to produce a bubble of radius 0.5m?
   Answer: (b) $1.884 \times 10^{-3}J$

2. (a) Explain surface tension on the basis of molecular theory.
   (b) A spherical drop of mercury of radius 2mm falls to the ground and breaks into 10 smaller drops of equal size. Calculate the amount of work that has to be done in the process. Surface tension of mercury is $4.72 \times 10^{-1}Nm^{-1}$. **Answer** (b) $2.74 \times 10^{-5}J$

3. (a) Derive the relation between surface tension and surface energy.
   (b) A film of water is formed between two straight – parallel wires each 10cm long and at separation 0.5cm. Calculate the work required to increase 1mm distance between the wires surface tension of water $= 72 \times 10^{-3}Nm^{-1}$. **Answer** (b) $1.44 \times 10^{-5}J$

4. (a) Show that when a larger number of drops of a liquid (same size) coalesce to form a big drop, there is always liberation of energy.
   (b) A liquid drop of diameter D breaks up into 27tiny drops. Find the resulting change in energy. Take surface tension of the liquid as γ. **Answer:** (b) $2\pi D^2\gamma$

5. (a) A liquid drop of diameter 4mm breaks into 1000 droplets of equal size. Calculate the resultant change in surface energy, the surface tension of the liquid is $0.07Nm^{-1}$.
   (b) What amount of energy will be liberated if 1000 droplets of water each of diameter 10^{-6}cm coalesce to form a bigger drop. Surface tension of water is $75 \times 10^{-3}Nm^{-1}$.
   Answer: (a) $3.168 \times 10^{-5}J$ (b) $2.12 \times 10^{-14}J$

SHAPE OF THE LIQUID MENISCUS

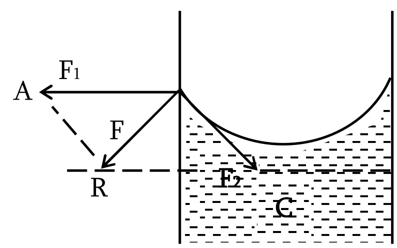
(a) Shape of liquid drops. Small liquid drops are spherical where as large drops are elliptical a liquid drop has potential energy due to (i) gravity and (ii) surface tension.
   - For a small drop , its penitential energy is small and may be neglected small drops may be stable if its surface tension is least. This happen when its surface area is least for a given volume , sphere has the least surface area and hence small drops are spherical.
   - For the large drop , energy is due to gravity is dominates to make its potential energy minimum it tries to bring the centre of gravity as low as possible and the drops are flattens..

(b) The shape of the liquid especially inside of the container can be determine by adhesive and cohesive force and also take the shape of that container.

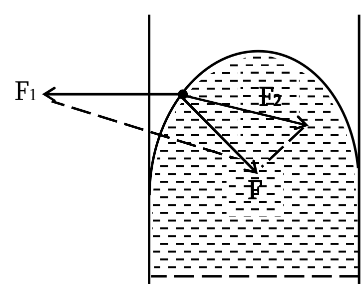
LIQUID MENISCUS is the curved surface of the liquid.

Different cases of the liquid meniscus.

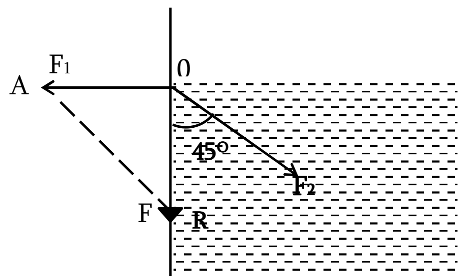
1. If the adhesive force (F_1) or BA is less than the cohesive force (F_2) or BC then the resultant force F or BR will acts to the left of the wall of the container and the shape of liquid is downward meniscus eg water.



2. If the cohesive force (F_2) is greater than adhesive force (F_1), the resultant force (F) is acts towards the right of the liquid and the shape of the liquid in the container is the **upward meniscus**.



3. For the plane liquid surface pressure on the liquid side is equal to the pressure on the vapour side so that the resultant force acting on it due to the surface tension is equal to zero.



ANGLE OF CONTACT

Is defined as the angle between the solid surface and the tangent plane to the liquid surface measured through the liquid i.e the angle between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid is called angle of contact for pair of solid and liquid. It is represented by θ.

FACTORS AFFECTING ANGLE OF CONTACT

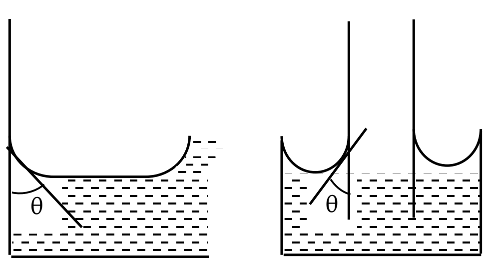
The value of angle of contact depends on the following factors:

1. Nature of the liquid
2. The nature of the solid and liquid in contact
3. The medium that exists above the free surface of the liquid.
4. Temperature ; angle of contact increases with increase in temperature.
5. Impurities present in the liquid angle of contact θ decreases on adding impurities to the liquid.

MEASUREMENT OF ANGLE OF CONTACT

1. **When angle of contact is a cute.**

   A liquid makes an a cute angle of contact with a solid surface if adhesive force between the liquid molecules themselves.



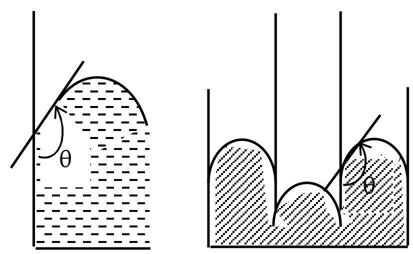
$0^{\circ} < \theta < 90^{\circ}$

For the liquid which wet the glass , the angle of contact is an acute angle eg. Pure water on the clear glass for an acute angle , the following points may be noted:

(i) Liquid will wet the glass
(ii) Meniscus of the liquid will be clearing agents.
(iii) Liquid will size in the capillary tube made of such a solid.

2. **When the angle of contact is obtuse.**

   A liquid makes an obtuse angle of contact with solid of cohesive themselves in greater than the adhesive forces between the solid surface and liquid molecules $90^{\circ} < \theta < 180^{\circ}$



For an obtuse angle of contact , the following points may be noted;

(i) Liquid will not wet the glass
(ii) Meniscus of the liquid will be convex
(iii) These liquids are used as water proofing agent
(iv) Liquid will get depressed in the capillary tube made of such a solid.

3. **When angle contact is zero**

   The angle of contact for pure water and clean glass is zero in this case , the resultant adhesive force is equal to the resultant cohesive force. Thus , the glass tube , the meniscus of will be exactly hemispherical

   Note that

   Contamination of the solid surface has a considerably effect on the angle of contact. Addition of detergent to the liquid (eg. Soap to water) will reduce the angle of contact and therefore help for washing.

CAPILLARITY.

The word 'capilla' mean hair in latin **capillary action or capillarity** is the action of rise or fall of a liquid. A tube having very small diameter called 'capillary tube' a narrow tube.

TYPES OF CAPILLARITY

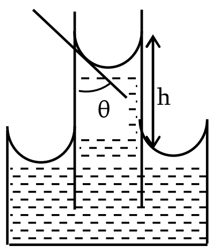
There are two types of capillarity

(i) Capillarity rise (ii) capillarity fall (depression)

CAPILLARITY RISE

If the capillarity tube is dipped in a liquid which wets the glass , the liquid rises up above the level outside. This is called **' capillarity rise'**.

This happen when the angle of contact is an acute angle eg. Water.

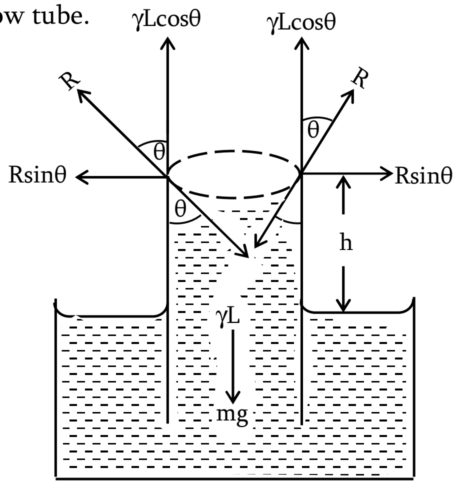


This happen when the angle of contact is an acute anlge. Eg. Water.

RISE OF LIQUID IN A CAPILLARY TUBE (ASCENT FORMULA)

Balance of forces method

Why liquid rises in the tube. When the tube is placed in a container containing liquid, round the boundary of the tube, where the liquid surface meets the tube, surface tension force exerts a down ward pull on the tube. From the Newton's third law action and reaction are equal and opposite. Therefore the tube exerts an equal but upward force on the liquid and this causes the liquid to rise up. The rise of the liquid stop to rise up until the component of the surface tension vertically is equal to the weight of the liquid inside of the narrow tube.



At the equilibrium, weight of the weight of the liquid which has been lifted up by height , h is equal to the vertical component of the force exerted by the tube.

$Mg = \gamma L\cos\theta$ but $L = 2\pi r$.

$\pi r^2\rho h = 2\pi r\gamma\cos\theta$

$$h = \frac{2\gamma\cos\theta}{\rho g r}$$

This relation is called Ascent formula where

γ = surface tension of the liquid

r = radius of capillary tube

ρ = density of the liquid

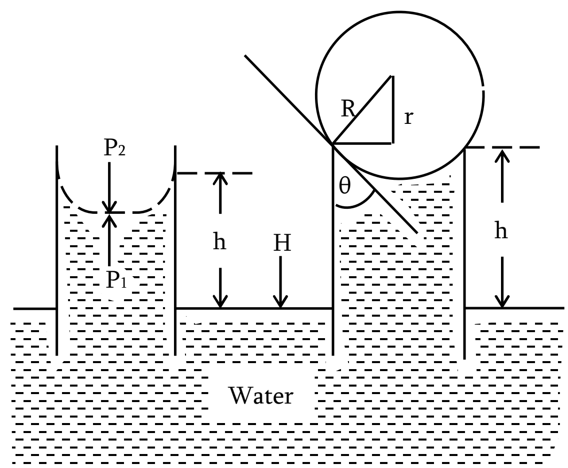
g = acceleration due to gravity

θ = angle of contact

h = height to which the liquid rises above liquid surface.

R = reaction acting tangentially (outwards) on the liquid meniscus.

Alternative method , capillary rise can be derived by using the pressure method



We shall now calculate the capillarity rise of water by using the excess pressure formulae.

If P_2 = H atmospheric pressure and P_1 is the pressure in the liquid.

$$\text{Now } P_2 - P_1 = \frac{2\gamma}{R}$$

$$\text{But } P_1 = H - \rho g h$$

Equation (1) becomes

$$H - (H - \rho g h) = \frac{2\gamma}{R}$$

$$\rho g h = \frac{2\gamma}{R} \text{ but } R = \frac{r}{\cos\theta}$$

$$\rho gh = \frac{2\gamma}{r/\cos\theta} = \frac{2\gamma\cos\theta}{r}$$

$$h = \frac{2\gamma\cos\theta}{\rho gr}$$

It is clears from this formula that the height which a liquid raises in a capillary tube is

(i) Inversely proportional to the radius r of the tube.
(ii) Inversely proportional to the density ρ of the liquid.
(iii) Directly proportional to the surface tension γ of the liquid.
(iv) Directly proportional to the cosine of an angle of contact.

- For those liquid which wet the glass , the angle of contact is acute, then cos θ is positive and h is positive. Therefore the level of liquid in a capillary tube will rise when the angle of contact is less than 90°.
- For those liquids which do not wet the glass, the angle of contact θ is obtuse, when cosθ is negative and h is negative. Therefore the level of liquid in a capillary tube will fall when angle of contact is more than 90°.
- The level of the liquid in a capillary tube will remain unchanged when angle of contact is 90°.

Additional concepts.

1. From the equation

$$h = \frac{2\gamma\cos\theta}{\rho gr}$$

Since γ, θ , ρ and g are constants, then $h \propto 1/r$. Capillary effects are more pronounced in the tube of smaller radii like capillary tube

$$hr = \frac{2\gamma\cos\theta}{\rho g} = \text{constant}$$

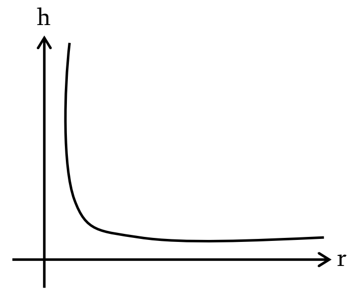
This is known as **JURIN'S LAW**

JURIN'S LAW state that 'The smaller the radius of the capillary tube the greater is the rise or fall of the liquid in it'.

Let h_1 and h_2 be height rises by the liquid in the capillary tube whose radii are r_1 and r_2 respectively.

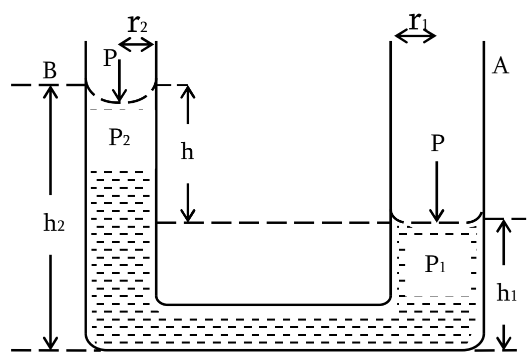
Now; $h_1r_1 = h_2r_2$ or $h_2 = h_1\left(\frac{r_1}{r_2}\right)$

Graph of h against r



2. **Excess pressure due to two limbs of U – tube.**

Let P_1 and P_2 be pressure inside of U – tube at the point A and B respectively as shown on the figure below. If h_1 and h_2 are levels of the heights of liquid in the two limbs whose radii are r_1 and r_2 respectively.



(i) Expression of the difference in height of levels of liquid in a U – tube.

At the tube A: $h_1 = \frac{2\gamma\cos\theta}{\rho gr_1}$

At the tube B: $h_2 = \frac{2\gamma\cos\theta}{\rho gr_2}$

The difference in height

$$h = h_2 - h_1 = \frac{2\gamma\cos\theta}{\rho gr_2} - \frac{2\gamma\cos\theta}{\rho gr_1}$$

$$h = \frac{2\gamma\cos\theta}{\rho g}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{2\gamma\cos\theta}{\rho g}\left(\frac{r_1 - r_2}{r_1r_2}\right)$$

If the angle of contact is zero.

$$\cos\theta = \cos 0^{O} = 1$$

(ii) Expression of the expression

$$\text{Since } h = \frac{2\gamma\cos\theta}{\rho g}\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$$

$$P_2 - P_1 = \rho g h = 2\gamma\cos\theta\left(\frac{1}{r_2}-\frac{1}{r_2}\right)$$

$$P_2 - P_1 = 2\gamma\left(\frac{1}{r_2}-\frac{1}{r_1}\right)\cos\theta$$

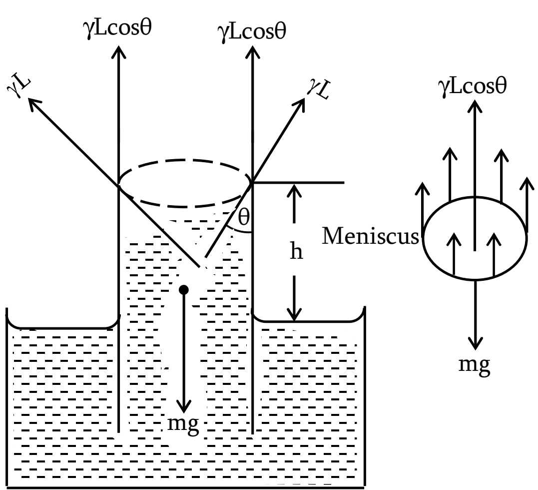
Comment

The height of liquid column in the two limbs differs because the excess pressure below the meniscus depends on the radius of the limb. So the limb with smaller radius is expected to have high pressure as compared with the limb with large radius i.e $h \; \alpha \; {}^{1}\!/_{r}$.

3. The expression of h on the capillary tube, if the effect of liquid meniscus is taken into account.

$$h = \frac{2\gamma\cos\theta}{\rho g r} - \frac{r}{3}$$

$$\text{Derivation: } h = \frac{2\gamma\cos\theta}{\rho g r} - \frac{r}{3}$$



Volume of the liquid in the tube above the free surface of liquid

= volume of cylinder + volume of of height , h and radius, r height r and radius of volume of hemisphere of radius r.

$$V = \pi r^2 h + \pi r^2 r - \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$V = \pi r^2\left(h + \frac{r}{3}\right)$$

Weight of the liquid

$$Mg = \rho g V = \pi r^2 \rho g\left(h + \frac{r}{3}\right)$$

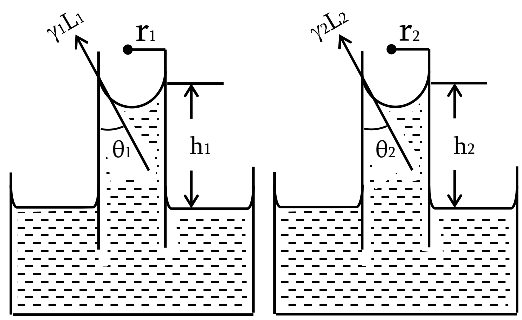
At the equilibrium of the liquid column inside of the tube.

$$\pi r^2 \rho g\left(h = \frac{r}{3}\right) = 2\pi\gamma r\cos\theta$$

If the tube is very narrow $\frac{r}{3}$ can be neglected as compared to h.

$$h = \frac{2\gamma\cos\theta}{\rho g r}$$

4. Comparisons of height raised of the liquids column on the capillary tubes of different radii.



$$\text{Since; } \quad h = \frac{2\gamma\cos\theta}{\rho g r}$$

$$\text{For liquid 1: } h_1 = \frac{2\gamma_1\cos\theta_1}{\rho_1 g r_1}$$

$$\text{For liquid 2: } h_2 = \frac{2\gamma_2\cos\theta_2}{\rho_2 g r_2}$$

$$\text{Takes } \quad \frac{h_2}{h_1} = \left(\frac{\gamma_2}{\gamma_1}\right)\left(\frac{r_1}{r_2}\right)\left(\frac{\rho_1}{\rho_2}\right)\frac{\cos\theta_2}{\cos\theta_1}$$

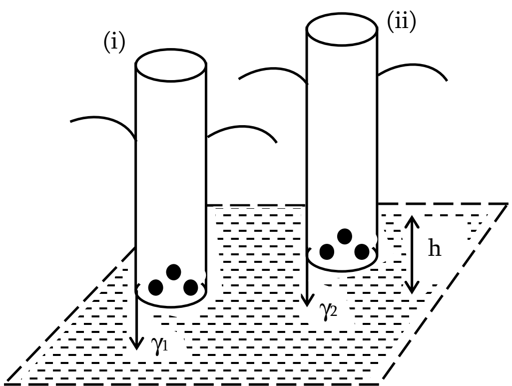
Note that:

For the pure water and clean glass, the angle of contact θ is approximately 0°

$\cos\theta = 1$. In this case $h = \dfrac{2\gamma}{\rho g r}$ and the height of liquid rises is corresponding to the greatest height.

5. **Determination of density of a liquid by a floating the tube.**

Suppose a test tube floats on the surface of water whose surface tension is γ_1. If the small drop of liquid detergent is added on a surface of a liquid is lowered, hence the tubes change in surface tension force is equal to the weight of the liquid displaced by the portion of the tube height, h



F = Up thrurst = weight of fluid displaced

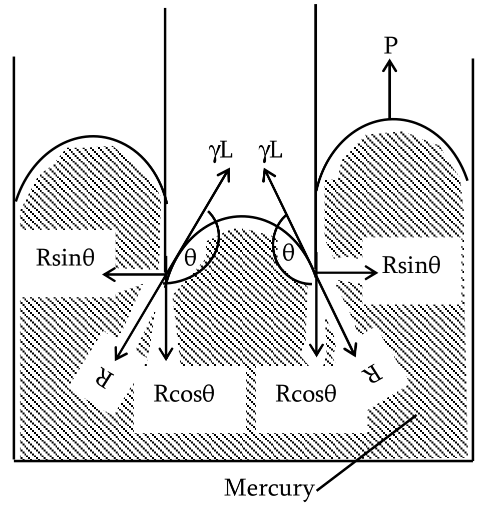
$$2\pi r(\gamma_1 - \gamma_2) = Mg$$

$$2\pi r(\gamma_1 - \gamma_2) = \pi r^2 h\rho g$$

$$\rho = \frac{2(\gamma_1 - \gamma_2)}{hrg}$$

6. **Capillary depression**

When a glass capillary tube is dipped in mercury, the mercury shows depression as shown in the figure below. In such a case, the formula for descent of mercury can be derived as follows.



Total downward force $= 2\pi\gamma r\cos\theta$

Total downward pressure $= \dfrac{2\pi\gamma r\cos\theta}{\pi r^2}$

$$= \frac{2\gamma\cos\theta}{r}$$

Mercury outside the capillary exerts on upward pressure hρg on the mercury in the tube. At the equilibrium

$$h\rho g = \frac{2\gamma\cos\theta}{r}$$

$$h = \frac{2\gamma\cos\theta}{\rho g r}$$

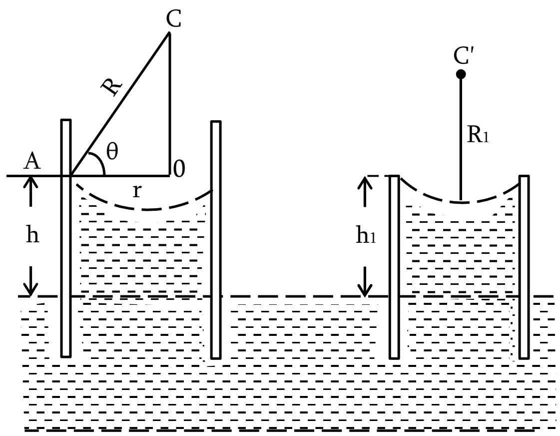
This is known as Descent formula.

RISE OF LIQUID IN A CAPILLARY TUBE OF INSUFFICIENT LENGTH

Insufficient length means that the liquid can rise to a greater height than length of the tube. The rise of liquid in a capillary tube is given by

$$h = \frac{2\gamma\cos\theta}{\rho g r} \text{..........(i)}$$

This equation has been derived on the assumption that the capillary tube is so narrow that the radius of the liquid meniscus is equal to the radius of the capillary tube. But this is not correct.



Let C be the centre of curvature of liquid meniscus. Let R be the radius of curvature of the meniscus in the triangle AOC

$$\cos\theta = \frac{\overline{AO}}{\overline{AC}} = \frac{r}{R}$$

$$r = R\cos\theta$$

From the equation , $h = \dfrac{2\gamma\cos\theta}{(R\cos\theta)\rho g}$

$$h = \frac{2\gamma}{R\rho g}$$

$$Rh = \frac{2\gamma}{\rho g} = \text{constant} \;......\text{(ii)}$$

If the length of capillary tube is less than h , then the liquid rises to the top of the capillary tube and starts spreading until the liquid are in equilibrium. The liquid will be in equilibrium when equation (ii) is satisfied. Let h_1 be insufficient length of the capillary tube and R_1 be the new radius of curvature of the meniscus.

$$R_1h_1 = Rh$$

Since $h_1 < R_1 > R$, so there is an increase in the radius of curvature of the liquid meniscus. But the angle of contact remain constant. This is because the angle of contact depends only on the nature of solid or liquid. It is independent of the manner in which the two are brought in contact with each other.

APPLICATIONS OF CAPILLARITY

1. A blotting paper soaks ink by capillary action. The pores of the blotting paper act as capillaries.
2. A towel soaks water on account of capillarity action
3. The oil in an oil wick rises up through the narrow spaces between the threads of the wick which acts as fine capillary tubes.
4. Water retained in a piece of sponge on account of capillarity.
5. Walls get damped in rainy season due to the absorption of water by bricks by the capillarity action.
6. A pen rib is split at the tip to provide a narrow capillary tube and the ink is drawn up to the point continuously.
7. Water comes out of an earthen pot through small pores by capillary action.
8. Ploughing of fields is essential for preventing moisture in the soil by ploughing, the fine capillaries in the soil are broken water from within the soil shall not rise and evaporate off.
9. Sand is drier than clay. This is because holes between the sand particles are not so fine as compared to that of clay, as to draw up water by capillarity action.

SOLVED EXAMPLES

Example – 26

(a) What is meant by the term ‘surface tension of liquid’ and ‘angle of contact’.

(b) Account for the following :

(i) Small needle may be placed on the surface of water in a beaker so that it floats

(ii) If a small quantity of detergent is added to the water, the needle sinks.

(c) (i) A uniform capillary tube of radius r is held vertically and lowered in a liquid of density ρ and surface tension γ. Show that the liquid rises a height , h is given by

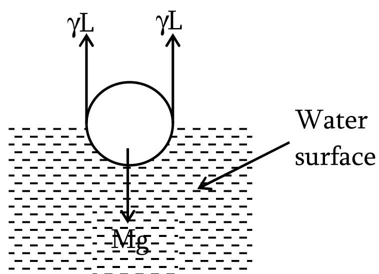
$$h = \frac{2\gamma\cos\theta}{\rho g r}$$

Each symbol have usual meaning.

(ii) A capillary tube is immersed in water surface tension $7 \times 10^{-2}Nm^{-1}$ and the water rises 6.0m in the capillary tube. What will be the difference in the mercury levels if the same capillary tube is immersed in mercury?
Surface tension of mercury = $0.84Nm^{-1}$.
Angle of contact between the mercury and glass = 140^O.
Density of mercury = $136000kgm^{-3}$
Density of water = $1000kgm^{-3}$.

Solution

(b) (i) When the needle is careful placed on the water surface it may float



If it will float the surface is not broken. The surface tension force will counter balance with the weight of the needle. Therefore the formation of thin elastic skin in the form of state of tension develops the surface tension force.

(ii) If a small detergent is added to the water , the surface tension of water reduces the resultant surface tension force becomes less than the weight of the needle, therefore the needle will sinks.

(c) (i) $h = \dfrac{2\gamma\cos\theta}{\rho g r}$ (see your notes)

(ii) For water: $h_1 = \dfrac{2\gamma_1 \cos\theta_1}{\rho_1 g r}$

For mercury: $h_2 = \dfrac{2\gamma_2 \cos\theta_2}{\rho_2 g r}$

Takes $\dfrac{h_2}{h_2} = \dfrac{2\gamma_2 \cos\theta_2}{\rho_2 g r} \times \dfrac{\rho_1 g r}{2\gamma_1 \cos\theta_1}$

$$h_2 = h_1\left(\frac{\gamma_2}{\gamma_1}\right)\left(\frac{\rho_1}{\rho_2}\right)\frac{\cos\theta_2}{\cos\theta_1}$$

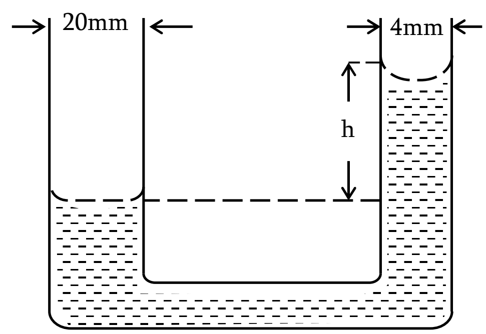
$$= 6.2\left(\frac{0.84}{7\times10^{-2}}\right)\left(\frac{1000}{13600}\right)\frac{\cos 140^O}{\cos 0^O}$$

$$h_2 = -4.2\times10^{-2}\,m$$

The minus sign show that the height h is below the mercury surface i.e h is depression.

Example – 27

A clean open ended glass U – tube has vertical limbs one of which has a uniform diameter of 4.0mm and other 20mm, clean water is introduced in the tube is observed that the height of water meniscus is different for the two limbs as shown on the figure below. Calculate the difference h in the height level of water into two limbs of U – tube. (angle of contact = 0^O , Surface tension of water = $7 \times 10^{-2}Nm^{-1}$, density of water = $1000kgm^{-3}$, acceleration due to gravity = $9.81m/s^2$)



Solution

$$d_1 = 20mm\ ,\ r_1 = \frac{d_1}{2} = 100mm = 10\times10^{-3}m$$

$$d_2 = 4mm\ ,\ r_2 = \frac{d_2}{2} = 2mm = 2\times10^{-3}m$$

$$h = \frac{2\gamma}{\rho g}\left(\frac{r_1 - r_2}{r_1 r_2}\right)\cos\theta$$

$$= \frac{2\times7\times10^{-2}}{1000\times9.81}\left[\frac{10\times10^{-3} - 2\times10^{-3}}{10\times2\times10^{-6}}\right]\cos 0^O$$

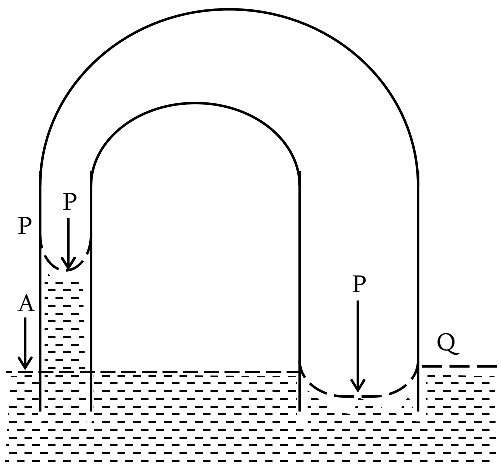
$$h = 5.708\times10^{-3}m = 5.708mm$$

Example – 28

(a) The surface tension of water is $7.0 \times 10^{-2}Nm^{-1}$ and angle of contact of water with glass is zero. Explain what these statement mean.

(b) A glass U – tube is inverted with open ends of the straight limbs of diameters respectively 0.5mm and 1mm , below the surface of water in a beakers. The air pressure in the upper part is increased until the meniscus in one level is level with water outside. Find the height of water in the other limb.

(Density of water = 1000kgm^{-3})



Solution

(a) This means that the force of about 7.0×10^{-2}N per metre acting in the surface of water at right angles to one side on line drawn in the surface. The angle between the tangent to the water surface at the point of contact and the solid surface inside the water is equal to zero.

(b) Excess pressure on limb P

$$P-\left(A-\rho gh\right) = \frac{2\gamma}{r_1}$$

$$P - A + \rho gh = \frac{2\gamma}{r_1} \text{……..….(i)}$$

Excess pressure on limb Q

$$P - A = \frac{2\gamma}{r_2} \text{……..(ii)}$$

Putting equation (ii) into (i)

$$\frac{2\gamma}{r_2} + \rho gh = \frac{2\gamma}{r_1}$$

$$h = \frac{2\gamma}{\rho g}\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

$$h = \frac{2\times 0.075}{1000\times 9.8}\left(\frac{0.05-0.025}{0.05\times 0.025}\right)\frac{\times 10^{-3}}{\times 10^{-6}}$$

$$h = 3.1\times 10^{-2}\text{m}\left(\text{approx}\right)$$

Example – 29

Liquid rises to a height of 5.0cm in a capillary tube and mercury falls to a depth of 2.0cm in the same capillary tube. If the density of liquid is 1.2g/cm^3 , of mercury is 13.6g/cm^3 and angle of contact of liquid and mercury with capillary tube 0^O and 35^O respectively. Find the ratio of the surface tension for mercury and liquid.

Solution

For liquid , h_1 = 5.0cm , $\theta_1 = 0^O$, ρ_1 = 1.2g/cc

For mercury , h_2 = -2.0cm , θ_2 = 135O, ρ_2 = 13.6g/cc

Since $h = \frac{2\gamma\cos\theta}{\rho gr}$ or $\gamma = \frac{hrg\rho}{2\cos\theta}$

Or $\gamma \ \alpha \ \frac{h\rho}{\cos\theta}$

$$\frac{\gamma_2}{\gamma_1} = \left(\frac{h_2}{h_1}\right)\left(\frac{\rho_2}{\rho_1}\right)\frac{\cos\theta_1}{\cos\theta_2}$$

$$= \frac{-2\times 13.6\times\cos 0^O}{5\times 1.2\times\cos 135^O}$$

$$\frac{\gamma_2}{\gamma_1} = 6.41$$

Example – 30

The tube of mercury barometer is 4mm in diameter. How much error does the surface tension cause in the reading? Surface tension of mercury = 540×10^{-3}Nm^{-1} angle of contact = 135^O, density of mercury = 13600kgm^{-3}.

Solution

Here r = 2mm = 2×10^{-3}m , $\gamma = 540 \times 10^{-3}$Nm^{-1}, $\theta = 135^O$.

Error in barometer reading = depression of mercury level due to S.T

$$= \frac{2\gamma\cos\theta}{r\rho g} = \frac{2\times 540\times 10^{-3}\cos 135^O}{2\times 10^{-3}\times 13.6\times 10^{-3}\times 9.8}$$

$$= -2.86\times 10^{-3}\text{m}$$

∴ Error in barometer reading = -2.86mm

Example – 31

The radii of two capillary tubes A and B are in the ratio of 3:1. If water rises to a height of 3cm in tube A, how much it will rise in tube B. If the tube A is inclined at an angle of 30° with the vertical, then what will the position of water in the tube.

Solution

Case 1:

In a capillary tube , the liquid will rise or fall through a height of $h = \frac{2\gamma\cos\theta}{r\rho g}$

For the given liquid , γ , ρ , θ and g are constant.

$$\text{So ;} \quad hr = \frac{2\gamma\cos\theta}{\rho g} = \text{constant}$$

$$h_A r_A = h_B r_B \quad \text{but} \quad \frac{r_A}{r_B} = \frac{3}{1}$$

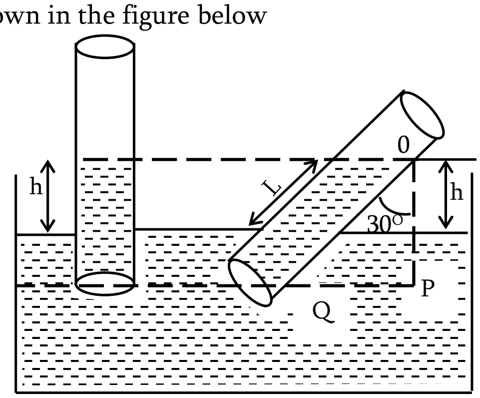
$$h_B = h_A \frac{r_A}{r_B}$$

$$h_B = 3\times 3 = 9\text{cm}$$

$$h_B = 9\text{cm}$$

Case 2:

Now , it is given that the capillary tube A is inclined at an angle 30° to the vertical. The vertical height (h = 3cm) of the liquid will remain same as shown in the figure below



Here , L is the length of water in the capillary tube

Then , in ΔOPQ

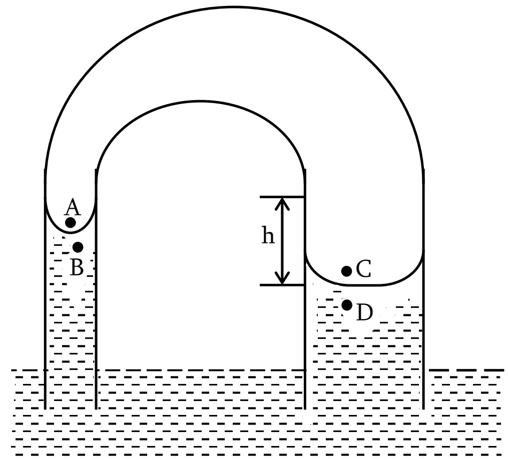
$$\cos 30^O = h/L$$

$$L = \frac{h}{\cos 30^O} = \frac{3\text{cm}}{\cos 30^O}$$

$$L = 3.464\text{cm}$$

Example – 32

A glass U – tube is such that the diameter of one limb is 3.0mm and that of the other is 6.0mm. The tube is inverted vertically with the open ends below the surface of water in a beaker. What is the difference between the heights to which water rises in the two limbs? Surface tension of water is 0.07Nm^{-1}. Assume that the angle of contact is 0°.



Solution

Let P_A , P_B , P_C and P_D be the pressure at the points A , B, C, and D respectively

$$\text{Now :}\ P_A = P_B + \frac{2\gamma}{r_1}$$

$$P_C = P_D + \frac{2\gamma}{r_2}$$

When r_1 and r_2 are radii of the two limbs

$$\text{But} \quad P_A = P_C$$

$$P_B + \frac{2\gamma}{r_1} = P_D + \frac{2\gamma}{r_2}$$

$$P_D - P_B = 2\gamma\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$h\rho g = 2\gamma\left[\frac{r_2 - r_1}{r_1 r_2}\right]$$

$$h = \frac{2\gamma}{\rho g}\left[\frac{r_2 - r_1}{r_1 r_2}\right]$$

$$= \frac{2\times 0.07}{1000\times 9.8}\left[\frac{3.0-1.5}{3\times 1.5}\right]\frac{\times 10^{-3}}{\times 10^{-6}}$$

$$h = 4.76\times 10^{-3}\text{m} = 4.76\text{mm}$$

Example – 33

(a) What will happen if the length of the capillary tube is smaller than the height to which the liquid rises?

(b) Water rises in a capillary tube to a height of 2.0cm. In another capillary tube whose radius is one – third of it, how much the water will rise?

Solution

(a) The liquid will rise up to the top of the capillary tube but will not overflow. The radius of curvature of the meniscus increases and the meniscus becomes more and more flat. The liquid cannot emerge as of fountain from the upper end of the tube.

(b) Since $hr = \text{constant}$

$$h_1 r_1 = hr$$

$$h_1\left(\frac{r}{3}\right) = hr$$

$$h_1 = 3h = 3\times 2 = 6.0\text{cm}$$

$$h_1 = 6.0\text{cm}$$

Example – 34

(i) Water rises to a height h inside a clean glass capillary tube of radius 0.2mm when the tube is placed vertically inside a beaker of water. Calculate h if the surface tension of water $7 \times 10^{-2}\text{Nm}^{-1}$ and the angle of contact is zero.

(ii) The tube is now pushed into water until 4.0cm of its length is above the surface. Describe and explain what happens.
Density of water = 1000kgm^{-3}, $g = 10\text{m/s}^2$.

Solution

(i) Since $h = \dfrac{2\gamma\cos\theta}{\rho g r}$

$$= \frac{2\times 7\times 10^{2}\cos 0^{O}}{1000\times 10\times 0.2\times 10^{-3}}$$

$$h = 0.07\text{m}$$

(ii) When the tube is pushed down so that height h1 in figure below is 4cm , the angle of contact at the top of the tube changes from 0^{O} to θ

Now ; $\cos\theta = \dfrac{r}{R}$

R = radius of curvature

Excess pressure $P_2 - P_1 = \dfrac{2\gamma}{R}$

P_2 = Atmospheric pressure , Pa

$$P_1 = Pa - h_1\rho g$$

$$h_1\rho g = \frac{2\gamma}{R}\text{........(i)}$$

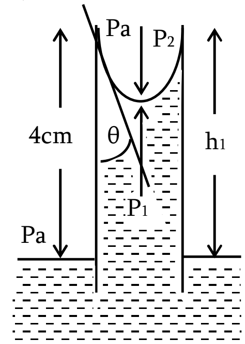
$$h\rho g = \frac{2\gamma}{r}\text{........(ii)}$$

$$\text{(i)} = \text{(ii)}$$

$$\frac{h_1}{h} = \frac{r}{R} = \cos\theta$$

$$\cos\theta = \frac{4}{7} \text{ or } \theta = 55^{O}$$

$$\theta = 55^{O}$$

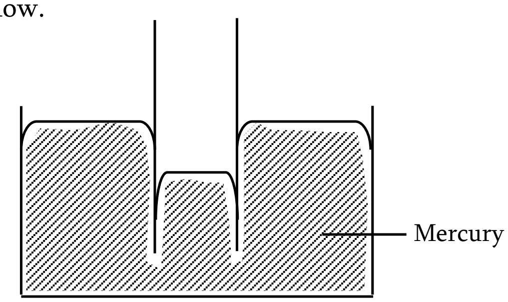


Example – 35

If water rises in the capillary tube 5.8cm above the free surface of the outer liquid. What happen to the mercury level in the same tube when it is placed in a dish of mercury? Illustrate this by the aid of diagram. Calculate the difference in level between the mercury surfaces inside and outside (surface tension of water is $75 \times 10^{-3}\text{Nm}^{-1}$, surface tension of mercury is $547 \times 10^{-3}\text{Nm}^{-1}$, angle of contact of mercury with clean glass = 130^{O}, density of mercury is 13600kgm^{-3} and density of water is 1000kgm^{-3}.

Solution

The mercury is depressed a distance h below the outside level and convex as shown in the figure below.



Let r be the radius of the capillary tube.

For water $h_1 = \dfrac{2\gamma_1}{\rho g r_1} = \dfrac{2\gamma_1}{\rho g r_1}$

For mercury $h_2 = \dfrac{2\gamma_2\cos\theta_2}{\rho_2 g r}$

Takes $\dfrac{h_2}{h_1} = \dfrac{2\gamma_2\cos\theta_2}{\rho_2 g r} \times \dfrac{\rho_1 g r}{2\gamma_1}$

$$h_2 = \left(\frac{\gamma_2}{\gamma_1}\right)\left(\frac{\rho_1}{\rho_2}\right)h_1 \cos\theta_2$$

$$= \left(\frac{547\times10^{-3}}{75\times10^{-3}}\right)\left(\frac{1000}{13600}\right)\times 5.8\cos 50^{O}$$

$$h_2 = 0.02\text{m} = 2\text{cm}$$

Example – 36

A capillary tube of inside diameter 1mm is dipped vertically in a liquid of surface tension $63 \times 10^{-3}\text{Nm}^{-1}$ and density 1262kgm^{-3}. Find the height of capillary rise if the angle of contact is 10^{O}.

Solution

According to the Ascent formula

$$h = \frac{2\gamma\cos\theta}{\rho g r} = \frac{2\times63\times10^{-3}\cos 10^{O}}{1269\times9.8\times9.8\times0.5\times10^{-3}}$$

$$h = 0.0197\text{m}$$

Example – 37

A capillary tube of inside diameter 1mm is dipped vertically into water so that the length of its part protruding over the water surface is 20mm. What is the radius of the meniscus? Surface tension of water is $72 \times 10^{-3}\text{Nm}^{-1}$.

Solution

The height to which water rises in the tube

$$h = \frac{2\gamma}{\rho g r} = \frac{2\times72\times10^{-3}}{1000\times9.8\times0.5\times10^{-3}}$$

$$h = 29.4\times10^{-3}\text{m}$$

The length of the capillary tube protruding over the water surface.

$$h_2 = 2.0\times10^{-3}\text{m}$$

Let r_2 be radius of curvature of the meniscus

By Jurin's law

$$hr = h_2 r_2$$

$$r_2 = \frac{hr}{h_2} = \frac{29.4\times10^{-3}\times0.5\times10^{-3}}{2.0\times10^{-3}}$$

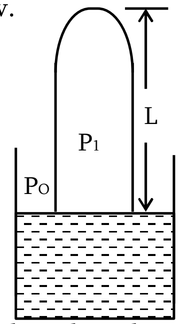
$$r_2 = 0.735\times10^{-3}\text{m}$$

Example – 37

A capillary tube of length 12cm and inside diameter 0.03mm is dipped vertically into water, the top end of the capillary tube is sealed. If the outside pressure is $1.01 \times 10^{5}\text{Nm}^{-2}$. Find the length to which the capillary tube should be submerged in water so that the levels inside and outside coincided ($\gamma = 72 \times 10^{-3}\text{Nm}^{-1}$).

Solution

Let L be initial length of air column and A cross – section area of the tube. Initially the tube is held on the water surface as shown on the figure below.

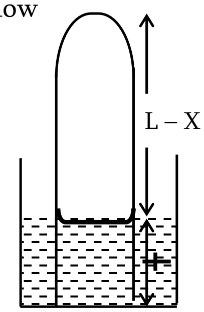


P_1 = initial pressure of air inside of the tube.

Po = Atmospheric pressure

P_1 = Po, V_1 = AL

Now the tube is lowered in the water, till the level of water inside and outside the tube becomes the same. Let X be the length of the tube inside of the water will be concave as shown on the figure below



$$V_2 = A(L - X)$$

$$P_2 = Po + \frac{2\gamma}{r}$$

P_2 = final pressure of air inside of the tube

Apply Boyle's law

$$P_1V_1 = P_2V_2$$

$$PoAL = \left(Po + \frac{2\gamma}{r}\right)A(L - X)$$

$$\left(\frac{2\gamma}{r} + Po\right)X = \frac{2\gamma L}{r}$$

$$X = \frac{L}{1 + \frac{Por}{2\gamma}}$$

$$X = \frac{12\times10^{-2}}{1 + \frac{1.01\times10^{5}\times0.015\times10^{-3}}{2\times72\times10^{-3}}}$$

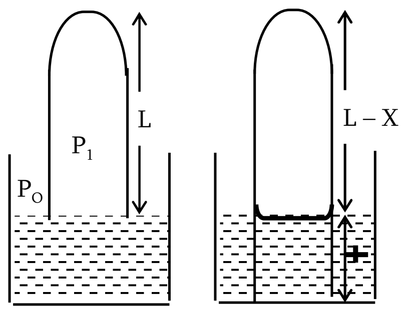
$$X = 1.041\times10^{-2}\text{m}$$

Example – 39

A glass capillary sealed at the upper end of length 0.11m and internal diameter 2×10^{-5}m. The tube is immersed vertically into a liquid of surface tension 5.06×10^{-2}Nm^{-1}. To what length, the capillary has to be immersed so that the liquid level inside and the outside the capillary becomes the same. What will happen to the water level inside the capillary, if the seal is now broken?

Solution

Let A and L be the uniform cross – sectional area and length of the tube respectively.



According to the Boyle's law

$$P_1V_1 = P_2V_2$$

$$PoAL = \left(Po + \frac{2\gamma}{r}\right)A\left(L - X\right)$$

$$PoL = \left(Po + \frac{2\gamma}{r}\right)\left(L - X\right)$$

$$X = \frac{L}{1 + \frac{Por}{2\gamma}}$$

$$= \frac{0.11}{1 + \frac{1.01 \times 10^5 \times 1.0 \times 10^{-5}}{2 \times 5.06 \times 10^{-2}}}$$

$$X = 0.01\text{m}$$

If the seal is now broken, the water will rise in the tube due to the capillary action.

Example – 40

A hydrometer with glass stem 5.0mm in diameter floats in water and due to a slightly greasiness of the stem the angle of contact is 25^{O}. A small quantity of detergent is added to the water and this reduces the surface tension from 7.0×10^{-2} to 2.4×10^{-2}Nm^{-1} and angle of contact becomes zero. Because of the detergent, does the hydrometer float higher or lower? How much?

Solution

When the detergent is added the surface tension is reduced. This is turn to reduced. This is turn to reduces the downward pull due to surface tension and hence the hydrometer will float higher. The change in height is given by

$$h = h_1 - h_2$$

$$= \frac{2\gamma_1 \cos\theta_1}{\rho gr} - \frac{2\gamma_2 \cos\theta_2}{\rho gr}$$

$$h = \frac{2}{\rho gr}\left(\gamma_1 \cos\theta_1 - \gamma_2 \cos\theta_2\right)$$

$$= \frac{2}{9.8 \times 1000 \times 2.5 \times 10^{-3}}\left[0.07\cos 25^{O} - 0.024\cos 0^{O}\right]$$

$$h = 3.22 \times 10^{-3}\text{m} = 3.2\text{mm}$$

Example – 41

A glass tube having walls 2.5mm thick is suspended vertical from a balance and counter poised. It is lowered into water of surface tension 7×10^{-2}Nm^{-1}. What is the length of the tube is immersed when balance pointer is back to zero?

Solution

Let r_1 and r_2 be the inside and outside radii of the tube respectively and h is the length of the tube immersed. The thickness of the glass tube is given by $t = r_1 - r_1 = 2.5$mm.

The total downward force due to surface tension is

$$F = 2\pi\gamma\left(r_1 + r_2\right) \ldots\ldots\text{(i)}$$

This force balances with the up thrust (i.e weight of the fluid displaced)

$$U = \pi\left(r_2^2 - r_1^2\right)h\rho g$$

$$U = \pi\left[\left(r_2 - r_1\right)\left(r_1 + r_2\right)\right]\rho gh \ldots\ldots\text{(ii)}$$

(i) = (ii)

$$2\pi\gamma\left(r_1 + r_2\right) = \pi\left[\left(r_2 - r_1\right)\left(r_1 + r_2\right)\right]\rho gh$$

$$h = \frac{2\gamma}{\left(r_2 - r_1\right)\rho g}$$

$$= \frac{2 \times 0.071}{2.5 \times 10^{-3} \times 1000 \times 9.8}$$

$$h = 5.8 \times 10^{-3}\text{m} = 5.8\text{mm}$$

Example – 42

A glass U – tube has legs of diameters 4.00mm and 2.00mm and contains mercury. Find the difference in mercury levels in the two tubes , given that the surface tension of mercury is 0.465Nm^{-1}, the angle of contact is 127^{O} and the density of mercury is 13600kgm^{-3}.

Solution

The difference in liquid level in the U – tube is given by

$$h = \frac{2\gamma}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\cos\theta$$

$$= \frac{2\times 0.465}{13600\times 9.8}\left[\frac{1}{1} - \frac{1}{2}\right]\frac{\cos 127^{O}}{10^{-3}}$$

$$h = -2.1\times 10^{-3}\text{m} = -2.1\text{mm}$$

Example – 43

A rectangular vessel with open top has a hole of diameter 0.40mm in its base, each side of which is 5cm, the vessel is of 150g and floats vertically in water of surface tension 0.075Nm^{-1}. What is the greatest additional load that can be put in the vessel before water enters the hole?

Solution

Let the additional load be M and the height that the vessel sinks is h. excess pressure on the liquid curved at the hole.

$$F = A\Delta P = \frac{2\gamma}{r}L^2$$

$$= \frac{2\times 0.075\times\left(5\times 10^{-2}\right)^2}{2\times 10^{-4}}$$

$$F = 1.875\text{N}$$

This upward force balances with the weight

$$1.875 = \left(0.15 + M\right)\times 9.8$$

$$M = 0.0413\text{kg} = 41\text{gm}$$

Example – 44

A glass of rod of radius r_2 is inserted symmetrically into a vertical capillary tube of radius r_2 such that their lower ends are at the same level. The arrangement is now dipped in water. Calculate the height to which water will rise into the tube (γ = surface tension of water, ρ = density of water)

Solution

Total upward force due to surface tension

$$= \gamma\left(2\pi r_1 + 2\pi r_2\right) = 2\pi\gamma\left(r_1 + r_2\right)$$

This supports the weight of the liquid column of height, h.

Weight of liquid column

$$W = h\rho g\left(\pi r_2^2 - \pi r_1^2\right)$$

$$W = \pi r\rho g\left(r_2 - r_1\right)\left(r_2 + r_1\right)$$

$$\text{Now; } 2\pi\gamma\left(r_1 + r_2\right) = \pi\rho g h\left(r_2 - r_1\right)\left(r_2 + r_1\right)$$

$$h = \frac{2\gamma}{\left(r_2 - r_1\right)\rho g}$$

Example – 45

A capillary tube of inside radius 5 × 10^{-4}m is dipped in water of surface tension 0.075Nm^{-1}. To what height is the water raised by the capillary action above the normal water level? Calculate the wright of water raised. Given that angle of contact = 0^{O}.

Solution

$$\text{Since } h = \frac{2\gamma\cos\theta}{\rho g r} - \frac{r}{3}$$

$$= \frac{2\times 0.075\cos 0^{O}}{1000\times 9.8\times 5\times 10^{-4}} - \frac{5\times 10^{-4}}{3}$$

$$h = 3.04\times 10^{-2}\text{m} = 3.04\text{cm}$$

Weight of water raised

$$w = mg = \rho v g$$

$$= \pi r^2\left(h + r/3\right)\rho g$$

$$= 3.142\times\left(5\times 10^{-4}\right)^2\left[3.04\times 10^{-2} + \frac{5\times 10^{-4}}{3}\right]\rho g$$

$$w = 2.4\times 10^{-5}\times 9.8\text{N}$$

$$w = 2.4\times 10^{-5}\text{kgf}$$

Example – 46

A clean glass capillary tube is held vertically in water rises to a height of 7cm the tube is now depressed and only 5cm of its length is above water. What will be the angle of contact?

Solution

$$\text{Since } h = \frac{2\gamma\cos\theta}{\rho g r}, \quad h \;\alpha\; \cos\theta$$

$$\frac{h_2}{h_1} = \frac{\cos\theta_2}{\cos\theta_1}$$

$$h_1 = 7\text{cm}\ ,\ \theta_1 = 0^O\ ,\ h_2 = 5\text{cm}$$

$$\frac{\cos\theta_2}{\cos 0^O} = \frac{5}{7}$$

$$\theta_2 = \cos^{-1}\left(\frac{5}{7}\right) = 44.4^O$$

$$\theta_2 = 44.4^O$$

Example – 47

If a 5×10^{-2}m long capillary tube with 0.1×10^{-2}m internal diameter open at both ends is slightly dipped in water having surface tension 0.075Nm^{-1} state whether

(i) Water will rise half way in the capillary

(ii) Water will rise up to upper end of capillary.

(iii) Water will overflow out of the upper end of capillary?

Solution

Let h be the height to which water rises in a capillary tube.

$$h = \frac{2\gamma\cos\theta}{\rho gr} = \frac{2\times 0.075\cos 0^O}{1000\times 9.8\times 0.05}$$

$$h = 0.306\text{m}$$

But length of capillary tube be $h' = 5\times 10^{-2}\,\text{m}$

(i) Since $h > h'/2$, therefore the first possibility is ruled out.

(ii) Since the tube is insufficient length, therefore water will rise up to the upper end of the tube.

(iii) The water will not over flow out of the upper end of the capillary. The water will rise up to the upper end of the capillary. The liquid meniscus will adjust its radius of curvature R'h' = Rg, where R is the radius of curvature that the liquid meniscus would posses if the capillary tube were of sufficient length.

$$R' = \frac{Rh}{h'} = \frac{rh}{h'}\quad\left[R = \frac{r}{\cos\theta} = \frac{r}{\cos\infty}\right]$$

$$R' = \frac{0.05\times 10^{-2}\times 0.306}{5\times 10^{-2}}$$

$$R' = 3.06\times 10^{-4}\,\text{m}$$

EXERCISE 9.3

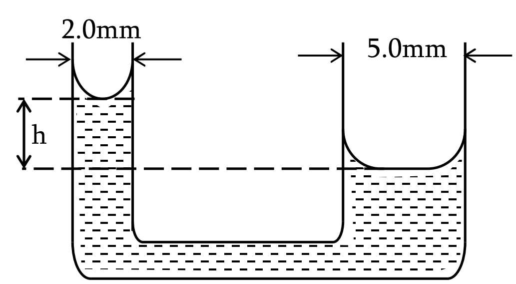
1. Water rises up in a glass capillary up to a height of 9.0cm while mercury falls down by 3.4cm in the same capillary. Assume angles of contact for water – glass and mercury glass as 0^O and 135^O respectively. Determine the ratio of surface tension of mercury and water.
   Density of mercury = 13600kgm^{-3}.
   Density of water = 1000kgm^{-3} ; **Answer** 7.2:1

2. The radius of a capillary tube is 0.025mm. It is held vertically in a liquid whose density is $0.8 \times 10^3\text{kgm}^{-3}$, surface tension is $3.0 \times 10^{-2}\text{Nm}^{-1}$ and for which the cosine of angle of contact is 0.3. Determine the height to which the liquid will rise in the tube relative to the liquid surface outside. Take g = 10m/s^2 ; **answer.** 9cm.

3. A capillary tube of radius 0.4mm is dipped vertically in water. Find up to what height the water will rise in the capillary. If the capillary is inclined at angle of 60^O with the vertical, how much length of the capillary is occupied by water? Surface tension of water = $7.0 \times 10^{-2}\text{Nm}^{-1}$, density of water = 1000kgm^{-3}.
   Answer 3.57, 7.14cm.

4. Liquid rises up in a glass capillary up to a height of 7.0cm. The radius of the tube is 0.1mm and density of liquid is $0.8 \times 10^3\text{kgm}^{-3}$. If the angle of contact between the liquid and the wall of the tube is zero; determine the surface tension of the liquid.
   Answer $27.4 \times 10^{-3}\text{Nm}^{-1}$.

5. A U – tube with two upright limbs of diameter 5.0mm and 2.0mm (figure below shown) containing water of surface tension $5.0 \times 10^{-2}\text{Nm}^{-1}$, angle of contact is zero and density of water is 1000kgm^{-3}. Find the difference in height.



6. A U – tube is supported with its limbs vertical and partly filled with water. If the inner diameter of the limbs are 1cm and 0.01cm respectively, what will be the difference in

heights of water in the two limbs? S.T of water is $70 \times 10^{-3}Nm^{-1}$, angle of contact $\theta = 0^{O}$.

7. Mercury in capillary tube suffers a depression of 13.2mm. Find the diameter of the tube, if angle of contact of mercury is 140^{O} and density $13600kgm^{-3}$, surface tension of mercury is $540 \times 10^{-3}Nm^{-1}$; **answer** $9.406 \times 10^{-4}m$.

8. Water rises in a capillary tube to a height 2.0cm. in another capillary tube whose radius is one third of it, how much the water will rise? If the first capillary tube is inclined at an angle of 60^{O} with the vertical then what will be the position of water in the tube. **Answer** 6.0cm , 4.0cm.

9. One end of a capillary tube of radius r is immersed vertically in water and the mass of water rises in the capillary tube is 5g. If one end of another capillary tube of radius 2r is immersed vertically in water. What will be the mass of water that will rise in it? **Answer** 10gm

10. A capillary tube of inner diameter $4 \times 10^{-3}m$ stands vertically in a bowl of mercury. The density of mercury is $13500kgm^{-3}$ and its surface tension is $0.544Nm^{-1}$. The level of mercury inside the tube is $3 \times 10^{-3}m$ below the level outside. Calculate the angle of contact of mercury with glass. **Answer** $124^{O}33'$.

11. A clean glass capillary tube of internal diameter $0.60 \times 10^{-3}m$ is held vertically with its lower end in water and with $80 \times 10^{-3}m$ of the tube above the surface. How much high does he water rise? If the tube is now lowered until $30 \times 10^{-2}m$ of its length is above the surface, what happens? Surface tension of water is 0.072Nm-1. **Answer** $4.898 \times 10^{-2}m$, water will rise up to the upper end with it meniscus of curvature, $R' = 4.898 \times 10^{-5}m$.

12. A tube of diameter $0.5 \times 10^{-3}m$ and length $10 \times 10^{-2}m$ is dipping with its lower end just inside water. The surface tension of water is $0.072Nm^{-1}$. Calculate the height to which water will rise in the tube if half the length of the tube is submerged below the water surface, what will happen to the water column in the tube? **Answer** $5.88 \times 10^{-2}m$, water will not overflow.

PRESSURE DIFFERENCE ACROSS A SPHERICAL SURFACE

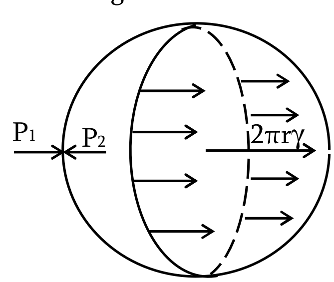
The force of surface tension is related to the magnitude of the curvature of a liquid surface or a bubble formed in a liquid. Every molecule on the liquid surface experience a force of surface tension that acts tangentially to the liquid surface at rest. The resultant force normal to the surface acts on curved surface of liquid. For convex surfaces the resultant force is directed towards the centre of the curvature while for concave surface is directed outwards for the centre of curvature for the equilibrium of the curvature liquid surface, there must be an excess pressure for that balances the resultant force due to the surface tension.

- The pressure inside a soap bubble is greater than the pressure of the air outside the bubble otherwise the combined effect of the external pressure and the surface tension force in the film would cause the bubble to collapse.
- The pressure inside an air bubble in liquid exceeds the pressure in the liquid.
- The pressure inside the mercury drop is greater than that of the outside. Therefore there is always a pressure difference across any curved liquid surface **excess pressure** is the difference in internal pressure and external pressure acting on the bubble.

Different cases for the excess pressure.

Case 1: Excess pressure in air bubble or curved liquid surface.

An air bubble is found in the liquid and have only one surface. Consider the bubble of curved liquid as show in the figure below.



The forces available acting on the bubble.

(i) The surface tension force $F = 2\pi r\gamma$ acting to the right.

(ii) The resultant force due to the excess pressure acting perpendicular to an area $A = \pi r^2$ the area of flat face of the hemisphere to the left. For the equilibrium of the bubble

$$P_2\pi r^2 = P_1\pi r^2 + 2\pi r\gamma$$

$$(P_2 - P_1)\pi r^2 = 2\pi r\gamma$$

$$P_2 - P_1 = P = \frac{2\gamma}{r}$$

Where

P_1 = External pressure act on the bubble.

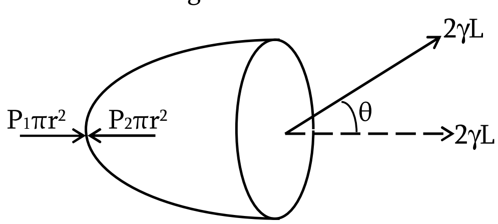
P_2 = Internal pressure act on the bubble

γ = surface tension of air or liquid

r = radius of air bubble or liquid drop.

Note that.

1. If the surface tension forces acts at the certain angle , θ on the liquid surface as shown in the figure below .



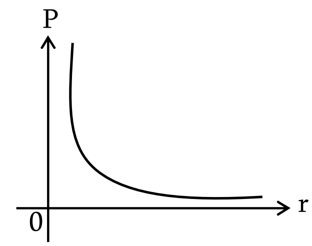
At the equilibrium of half bubble

$$P_2\pi r^2 = P_1\pi r^2 + 2\pi\gamma r\cos\theta$$

$$(P_2 - P_1)\pi r^2 = 2\pi r\gamma\cos\theta$$

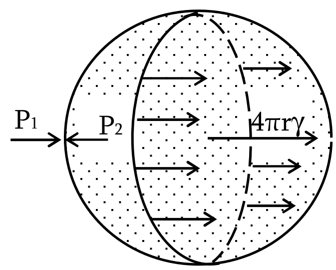
$$P = P_2 - P_1 = \frac{2\gamma\cos\theta}{r}$$

2. Applications.
   (i) Tiny fog drops behave like solids due to excess pressure inside them.
   (ii) When ice – skates slide over the surface of ice, the ice melts slightly as a result of this , tiny drops are formed. Due to the large internal pressure, these drops behave as ball bearings these explains the ease with which ice – skates slide over the surface of ice.
3. Graph of excess pressure against radius of bubble $P = \frac{2\gamma}{r}$, $P\,\alpha\,\frac{1}{r}$ since $P\alpha\frac{1}{r}$ i.e smaller the bubble the greater the excess pressure. This explain why one needs to blow hard to start a balloon growing once the balloon has grown less energy is needed to make it expand more.

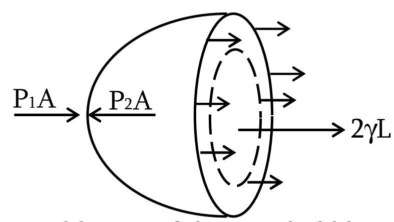


Case 2: excess pressure of a soap bubble

A soap bubble has two liquid surfaces in contact with air , one being inside and other outside of the bubble. Consider the soap bubble or half soap bubble as shown on the figure below.



OR



At the equilibrium of the soap bubble

$$P_2\pi r^2 = P_1\pi r^2 + 4\pi r\gamma$$

$$(P_2 - P_1)\pi r^2 = 4\pi r\gamma$$

$$P_2 - P_1 = \frac{4\gamma}{r}$$

Excess pressure on the soap bubble

$$P = P_2 - P_1 = \frac{4\gamma}{r}$$

Let $P_1 = P_o$ = Atmospheric pressure (external pressure)

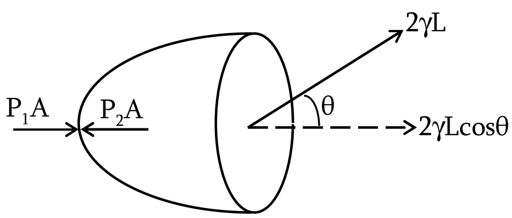
$P_2 = P$ = Internal pressure of soap bubble

Expression of the internal pressure of the soap bubble.

$$P_2 = P = P_O + \frac{4\gamma}{r}$$

Note that

1. If the surface tension forces acts at the certain angle to the direction of the soap bubble as shown on the figure below.



At the equilibrium half soap bubble

$$P_2A = P_1A + 2\gamma L\cos\theta$$

$$\left(P_2 - P_1\right)A = 2\gamma L\cos\theta$$

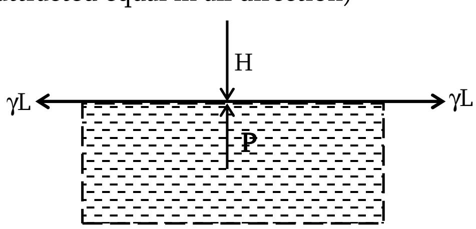
$$P_2 - P_1 = \frac{2\gamma\left(2\pi r\right)\cos\theta}{\pi r^2}$$

$$P_2 - P_1 = \frac{4\gamma\cos\theta}{r}$$

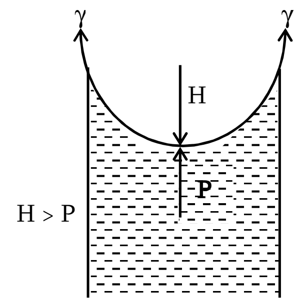
2. The excess pressure depends on the following factors:-
   (i) The surface tension of the liquid
   (ii) The radius of curvature of the liquid meniscus.

Case 3. Pressure difference across a liquid surface.

1. **For a plane liquid surface**
   Pressure on the liquid side is equal to the pressure on the vapour side. This is because the resultant force on any molecules on the surface due to surface tension is zero. (i.e molecules are attracted equal in all direction)



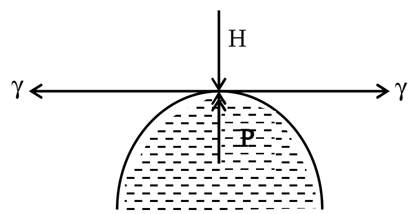
2. **For a concave liquid surface (meniscus)**
   The resultant force due to surface tension on any molecule on the surface is directed outward from the centre of curvature. This results to the greater pressure on the concave (outside) surface than on the convex side (inside)



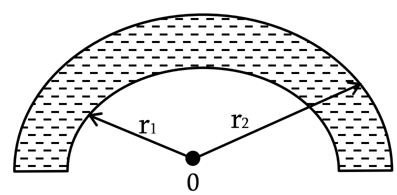
$$H - P = \frac{2\gamma}{r}$$

$$H = P + \frac{2\gamma}{r}$$

3. **For a convex liquid surface (meniscus)**
   The resultant force due to surface tension on any molecule on the surface is directed in ward towards the centre of curvature this results to a greater pressure on the convex (inside) surface than on the convex side (surface)

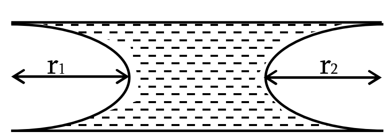


4. If r_1 and r_2 are the radii of the curved liquid surfaces:
   (a) The curvatures are in the same direction as shown on the figure below.



Excess pressure inside of the liquid is given by. $P = P_1 - P_2 = \gamma\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

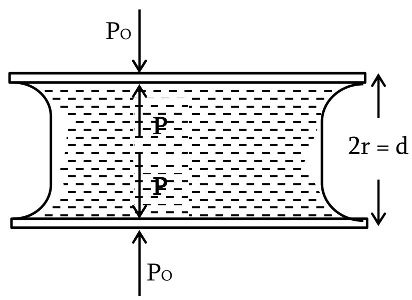
   (b) If the curvatures are in mutually opposite directions as shown in the figure below



Excess pressure is given by

$$P = P_1 - P_2 = \gamma\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

5. Force of attraction between two glass plates separated by a thin layer of liquid.
When two glass plates are separated by a very thin layer of a liquid, it becomes very difficult to separate the plates by applying a force perpendicular to the plates. This is because the pressure between the plates is less than the external pressure (i.e atmospheric pressure). Thus work must be done to overcome the force due to the pressure difference.



The force F pushing the plates together is given by work done in creating the two surfaces.

$$W = Fd = 2\gamma A$$

$$F = \frac{2\gamma A}{d} = \frac{2\gamma A}{2r}$$

$$F = \frac{2\gamma A}{d} = \frac{\gamma A}{r} = A\Delta P$$

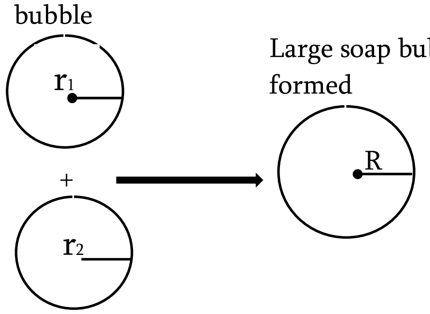
RADIUS OF THE NEW BUBBLE FORMED WHEN TWO BUBBLES COALESCE (FUSION OF A SOAP BUBBLES)

Fusion of a soap bubble is the joining of two or more bubbles to form a large bubble.

Assumptions made on the derivation.

(i) The process of coalescing of bubbles is an isothermal process.

(ii) The effect of atmospheric pressure is negligible. If the two soap bubbles of radii r_1 and r_2 are joined together to form a large bubble of radius , R under isothermal condition



Expression of the radius of large soap bubble formed can be obtained by using different methods:

Method 1: buy using law of conservation of energy

Energy of large Soap bubble = total energy of the two soap bubble

$$W = W_1 + W_2$$

$$\gamma A = \gamma A_1 + \gamma A_2$$

$$A = A_1 + A_2$$

$$8\pi R^2 = 8\pi r_1^2 + 8\pi r_2^2$$

$$R^2 = r_1^2 + r_2^2$$

$$R = \sqrt{r_1^2 + r_2^2}$$

Method 2: by using Boyle's law

Let V_1 and V_2 be the volume of the soap bubbles and V be volume of large soap bubble formed.

$$V_1 = \frac{4}{3}\pi r_1^3 \quad , \quad V_2 = \frac{4}{3}\pi r_2^3$$

$$V = \frac{4}{3}\pi R^3$$

Excess pressure inside of the soap bubble

$$P_1 = \frac{4\gamma}{r_1} \quad , \quad P_2 = \frac{4\gamma}{r_2}$$

$$P = \frac{4\gamma}{R}$$

Assume that Boyle's law hold

$$PV = P_1V_1 + P_2V_2$$

$$\frac{4\gamma}{R}\left(\frac{4}{3}\pi R^3\right) = \frac{4\gamma}{r_1}\left(\frac{4}{3}\pi r_1^3\right) + \frac{4\gamma}{r_2}\left(\frac{4}{3}\pi r_2^3\right)$$

$$R^2 = r_1^2 + r_2^2$$

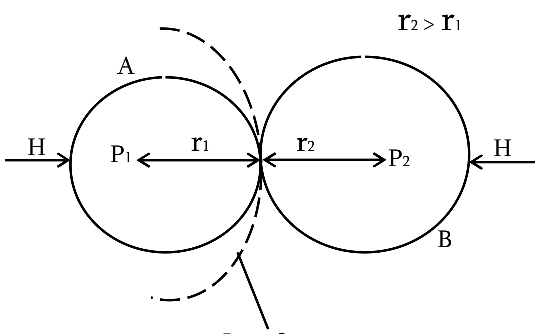
$$R = \sqrt{r_1^2 + r_2^2}$$

Generally, for n – soap bubble combined together to form large soap bubble

$$R = \sqrt{r_1^2 + r_2^2 + \cdots\cdots\cdots + r_n^2}$$

RADIUS OF INTERFACE WHEN TWO SOAP BUBBLES OF DIFFERENT RADII ARE IN CONTACT

Common interface of the soap bubble is the common face between the two bubbles when are joined together. When the small soap bubbles are joined with a large bubble, common interface is towards to the large bubble because the pressure on the large bubble is less than that of the small bubble. Consider two soap bubbles of radii r_1 and r_2 in contact with each other in figure below shown. Let R be the radius of common boundary.



Let H be atmospheric pressure
Excess pressure on small bubble (A)

$$P_1 - H = \frac{4\gamma}{r_1} \text{.........(i)}$$

Excess pressure on large bubble (B)

$$(P_1 - H) - (P_2 - H) = \frac{4\gamma}{r_1} - \frac{4\gamma}{r_2}$$

$$P_1 - P_2 = 4\gamma\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\frac{4\gamma}{R} = 4\gamma\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$\frac{1}{R} = \frac{1}{r_1} - \frac{1}{r_2} \quad \text{or} \quad R = \frac{r_1 r_2}{r_1 - r_2}$$

This equation is hold only under isothermal condition.

SHAPE OF COMMON INTERFACE

The shape of common interface may be either convex or concave shape. This can be depends on the pressure on the small and large soap bubble since, $P = \frac{4\gamma}{r}$, $P \propto \frac{1}{r}$

(i) The shape of common interface is convex towards to the large bubble.
(ii) The shape of common interface is concave towards to the small bubbles.

Note that:

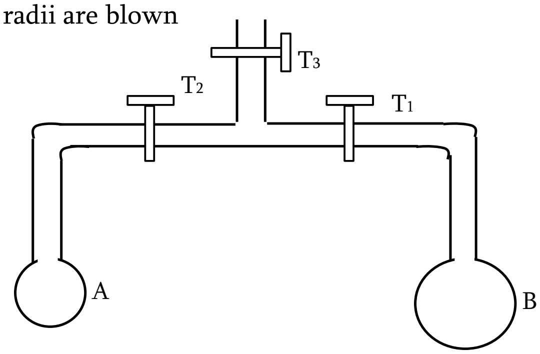
When two soap bubbles of radii r_1 and r_2 combine to form a single bubble of radius R at constant temperature and pressure, then the surface tension of soap solution is given by

$$\gamma = \frac{Po\left(R^3 + r_1^2 - r_2^3\right)}{4\left(r_1^2 + r_2^2 - R^2\right)}$$

Where Po = atmospheric pressure.

EXPERIMENT TO DEMONSTRATE THE EXCESS PRESSURE

The following experiment beautifully illustrate the facts of excess pressure by using the apparatus in figure below, two soap bubbles A and B of different radii are blown



The tap T_3 is kept closed A and B re connected by the taps T_1 and T_2 respectively. The air from smaller bubble passes into the capillary tube to the large soap bubble A gradually shrinks while the big soap bubble B is seen to expand. This is because the excess pressure inside of the smaller bubble is greater than the large soap bubble $\left[P = \frac{4\gamma}{r}\ ,\ p\alpha\frac{1}{r}\right]$

FACTORS AFFECTING SURFACE TENSION

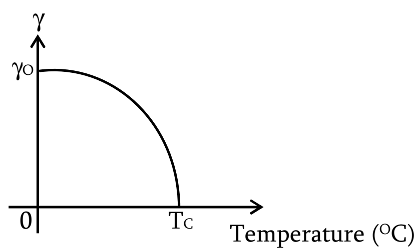
1. **Temperature**

   The surface tension of a liquid decreases with rise in temperature and for some liquid become zero at the critical temperature. The surface tension decreases because with increase in the temperature, the kinetic energy of the molecules of the liquid increase which reduce the effect of molecular force of attraction.

   Critical temperature (Tc)

   Is the temperature at which the surface tension of liquid becomes equal to zero.

   Graph of variation of surface tension with temperature.



   The variation of surface tension with temperature is given by

   Where $\gamma = \gamma_O\left[1-\alpha\theta\right]$

   γ_O = Surface tension at 0^OC.

   γ = Surface tension at 0^OC.

   α = temperature coefficient of surface tension for water $\gamma = 0.0756Nm^{-1}$ (0^OC) and $\gamma = 0.072Nm^{-1}(20^OC)$

   Applications:

   (i) The surface tension of hot soup is less than that of cold soup. Consequently the hot soup will spread over a large area of the tongue. For this reasons, hot soup is tastier than the cold soup.

   (ii) If you float two matchsticks on the surface of water and touch the surface of water between the match sticks by a hot needle, the matchsticks will fly apart. It is because hot needle lowers the surface tension of water between the matchsticks and the pull of the water molecules on the outside of matchsticks becomes greater than the pull on the inside as a result, the matchsticks fly apart.

2. **Nature of liquid**

   Different liquids have different surface tension.

3. **Contamination (i.e the presence of impurities or detergents).** In general impurities in a liquid lower its surface tension. It is because addition of impurities results in liquid spreading out as a thin film which means surface tension is lowered.

   Applications:

   (i) The cleaning action of soap is due to its ability to lower the surface tension of water, making it possible for the water and soap to penetrate more redly into the poles of the cloth being washed.

   (ii) Mosquitoes hang their eggs from the surface of water. When a small amount of oil is poured on water, it surface tension of water is reduced. This breaks the elastic film of water surface and mosquitoes eggs are killed by drowing.

4. **Nature of media**

   The surface tension of a liquid is affected by the nature of the media in contact with the surface tension of the liquid.

5. The electrification of a liquid decrease its surface tension. This is because the liquid surface experiences an outward normal pressure due to electrification. This increases the surface area and hence the surface tension decreases.

6. Presence of any dissolved substances in the liquid. A highly soluble substance like sodium chloride when dissolved in water, increases the surface tension of water but the sparingly soluble substances like phenol when dissolved

in water, decreases the surface tension of water.

APPLICATIONS OF SURFACE TENSION

1. The surface tension of soap solution is low, it can spread over large area. Hence it can wash clothes more effectively hot soap solution proves still better as surface tension decreases further on heating.
2. The lubricant oils and paints have low surface tension. So they can spread properly.
3. Since the surface tension of oil is smaller than that of water, therefore oil spreads on water, this property is used to calm down the stormy waves at sea.
4. Mosquito larvae (eggs) are killed by pouring small of water is reduced and its elastic film is broken the larvae (eggs) are killed by drowing.
5. In soldering, addition of 'flux' reduces the surface tension of molten tin , hence , it spreads.
6. Antiseptic like Detol have low surface tension , so they spread faster.

SOLVED EXAMPLES

Example –48

(a) A soap bubble has a diameter of 4mm. calculate the pressure inside it if the atmospheric pressure is $10^5 Nm^{-2}$. Surface tension of soap solution is $2.8 \times 10^{-2} Nm^{-1}$.

(b) Estimate the total surface energy of a million drops of water each of radius 0.1mm, if the surface tension of water is $7.0 \times 10^{-2} Nm^{-1}$. State any assumption made.

Solution

(a) $P = Po + \frac{4\gamma}{r}$

$$= 10^5 + \frac{4 \times 4.8 \times 10^{-2}}{2 \times 10^{-3}}$$

$$P = 100.056 \times 10^3 Nm^{-2}$$

$\therefore$ Pressure inside of soap bubble

$$P = 100.056 \times 10^3 Nm^{-2}$$

(b) Total surface energy

$$W = 4\pi R^2 N\gamma$$

$$= 10^6 \times 4\pi\left(10^{-4}\right)^2 \times 7 \times 10^{-2}$$

$$W = 8.8 \times 10^{-3} J$$

Assumptions

- The drops are perfectly spherical in shape.
- The temperature is kept constant.

Example – 49

Find the gauge pressure inside a soap bubble of radius 2cm surface tension of a soap solution is $0.025 Nm^{-1}$.

Solution

Gauge pressure is the pressure difference inside of a soap bubble and atmospheric pressure.

$$\Delta P = P - Po = \frac{4\gamma}{r}$$

$$= \frac{4 \times 0.025}{0.02}$$

$$\Delta P = 5 Nm^{-2}$$

Example – 50

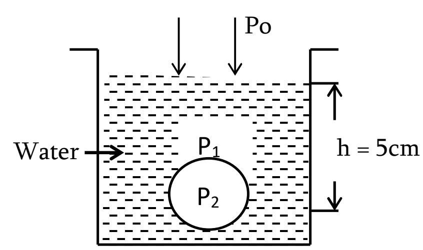
An air bubble of radius 1mm is formed into a disc of water at a depth of 5cm below the surface. If the atmospheric pressure is 10^5Pa, calculate

(i) The absolute pressure at that depth

(ii) The pressure inside the air bubble

Solution

Po = atmospheric pressure



(i) Absolute (actual) pressure is the atmospheric pressure plus pressure due to liquid column.

$$P_1 = Po + \rho gh$$

$$= 10^5 + 1000 \times 10 \times 5 \times 10^{-2}$$

$$P_1 = 100500 Pa$$

(ii) The pressure inside of the bubble

$$P_2 = Po + \rho gh + \frac{2\gamma}{r} = P_1 + \frac{2\gamma}{r}$$

$$= 100500 + 2 \times \frac{0.072}{10^{-3}}$$

$$P_2 = 100644\text{Pa}$$

$$\left[\text{Here } \cos 0^{\circ} = 1\right]$$

$$P_2 - P_1 = 1.86 \times 10^3\,\text{Nm}^{-2}$$

Example – 51

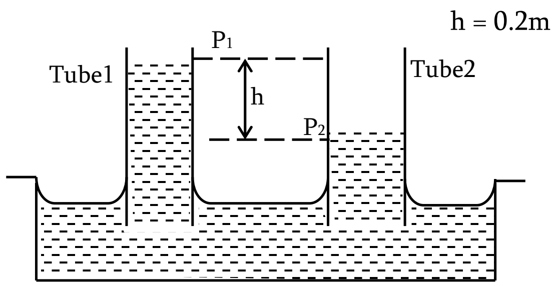
A barometer contains two uniform capillaries of radii 7.2×10^{-4}m and 1.44×10^{-3}m. If the height of liquid in narrow tube is 0.2m more than in the wide tube, calculate the true pressure difference.

Density of liquid = 1000kgm^{-3}

Surface tension = $72 \times 10^{-3}\text{Nm}^{-1}$

g = 9.8m/s^2.

Solution



Let P_1 and P_2 be pressure of tube whose radii are r_1 and r_2 i.e tube 1 and tube 2 respectively. Pressure just below the meniscus of tubes

$$P_1' = P_1 - \frac{2\gamma\cos\theta}{r_1}$$

For tube 2: $$P_2' = P_2 - \frac{2\gamma\cos\theta}{r_2}$$

Takes

$$P_2' - P_1' = \left(P_2 - \frac{2\gamma\cos\theta}{r_2}\right) - \left(P_1 - \frac{2\gamma\cos\theta}{r_1}\right)$$

But : $P_2' - P_1'$ = Pressure exerted by height h of the water column.

Now : $$P_2 - P_1 + 2\gamma\cos\theta\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \rho g h$$

$$P_2 - P_1 = \rho g h - 2\gamma\cos\theta\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Let : $P_2 - P_1$ = Pressure difference

$$P_2 - P_1 = 1000 \times 9.8 \times 0.2 - 2 \times 72 \times 10^{-3}\left[\frac{1}{7.2 \times 10^{-3}} - \frac{1}{1.44 \times 10^{-3}}\right]$$

Example – 52

The limbs of a manometer consists of uniform capillary tubes of radii 1.44×10^{-3} and 7.2×10^{-4}m. If the height of the liquid in the narrow tube is 0.2m more than in wide tube, calculate the true pressure difference.

Solution

Let the pressure in the wide and narrow limbs be P_1 and P_2 respectively. If R_1 and R_2 are the radii of meniscus in the wide and narrow limb, then pressure just below the meniscus of wide limb

$$= P_1 - \frac{2\gamma}{R_1}$$

Pressure just below the meniscus of narrow limb

$$= P_2 - \frac{2\gamma}{R_2}$$

Pressure difference is given by

$$\left(P_1 - \frac{2\gamma}{R_1}\right) - \left(P_2 - \frac{2\gamma}{R_2}\right) = \rho g h$$

$$P_1 - P_2 = h\rho g - 2\gamma\left(\frac{1}{R_2} - \frac{1}{R_2}\right)$$

$$= 0.2 \times 10^3 \times 9.8 - 2 \times 72 \times 10^{-3}\left[\frac{1}{7.2 \times 10^{-4}} - \frac{1}{1.44 \times 10^{-3}}\right]$$

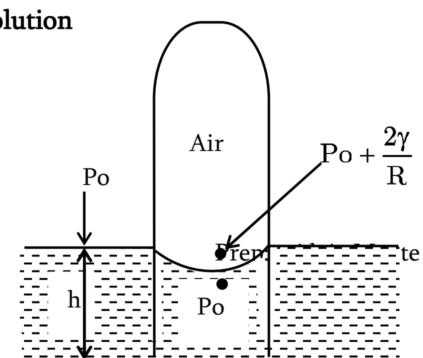
$$P_1 - P_2 = 1860\text{Nm}^{-2}$$

Example – 53

A glass tube of internal radius 5×10^{-4}m is dipped vertically into a vessel containing mercury such that the lower end of the tube is 10^{-2}m below the surface of mercury. Calculate the gauge pressure of air inside the tube to blow a hemispherical bubble at the lower end of the tube. Surface tension of mercury = $3.5 \times 10^{-2}\text{Nm}^{-1}$.

Density of mercury = 1300kgm^{-3}.

Solution



Pressure just below the meniscus Po (same as outside) pressure just above meniscus is $Po+\frac{2\gamma}{R}$

Gauge pressure required

$$P_G = \frac{2\gamma}{R}+h\rho g$$

$$= \frac{2\times3.5\times10^{-2}}{5\times10^{-4}}+10^{-2}\times13600\times9.8$$

$$P_G = 1472Nm^{-2}$$

Example – 54

(a) Explain the following phenomena:

(i) A drop of oil placed on the surface of water spreads out, but a drop of water placed on oil contracts to a spherical shape.

(ii) A drop of liquid under no external force is always spherical in shape.

(iii) When two soap bubbles of different diameters are connected by a tube, the smaller bubble gradually collapse and larger bubble grow bigger.

(iv) Water in a clean glass surface tends to spread out, while mercury on the same surface tends to form drop.

(b) A spherical air bubble 1.0mm in diameter is formed at a depth of 10cm below the surface of some water in beaker. If the atmospheric pressure is 1.01 × 10^5Nm^{-2}, what must be the pressure inside the bubble? Coefficient of surface tension of water is $0.073Nm^{-1}$ and density of water is $1000kgm^{-3}$.

Solution

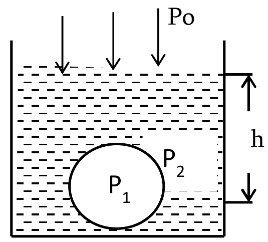
(a) (i) The adhesive force among oil water molecules is greater than the cohesive force among oil molecules, in the second case the cohesive force among molecules is greater than the adhesive force.

(ii) When there is no external force, the only force on the drop is the surface tension force, which tries to make the drop to have least surface area , hence the drop becomes spherical.

(iii) Because the pressure inside the smaller bubble is greater than that inside the large bubble and the pressure tends to equalize.

(iv) The adhesive force between water molecules and glass molecules is greater than cohesive force between water molecules so it spreads for mercury cohesive force between molecules is greater so they form drops.

(b)



Excess pressure in air bubble is given by

$$P_1-P_2 = \frac{2\gamma}{r}$$

$$P_1 = P_2+\frac{2\gamma}{r} \quad \text{but} \quad P_2 = Po+\rho gh$$

$$P_1 = Po+\rho gh+\frac{2\gamma}{r}$$

$$= 1.01\times10^5+1000\times9.8\times0.1+\frac{2\times0.073}{0.5\times10^{-3}}$$

$$P_1 = 1.02\times10^5 N/m^2$$

Example – 55

A vessel containing mercury has a hole at its bottom. The diameter of a hole is 60 × 10^{-6}m. Find the maximum height of mercury in vessel so that it does not flow out through the hole. Surface tension of mercury is 54 × $10^{-2}Nm^{-1}$.

Solution

Radius of the hole , r = 30 × 10^{-6}m

Let h be the maximum height

$$\rho gh = \frac{2\gamma}{r}$$

$$h = \frac{2\gamma}{\rho gr} = \frac{2\times54\times10^{-6}}{13600\times9.8\times30\times10^{-6}}$$

$$h = 0.27m$$

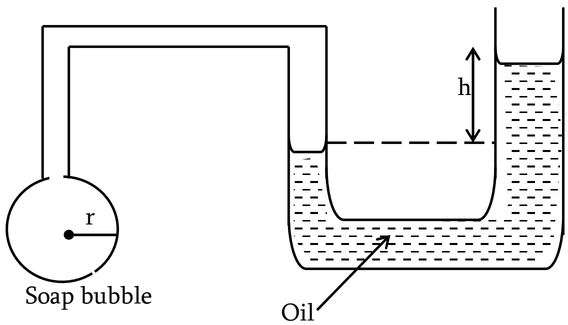
Example – 56

(a) The excess pressure inside a soap bubble of radius 6mm is balanced by 2mm column of oil of density 800kgm^{-3}. Find the surface tension of soap solution.

(b) Two soap bubbles one of radius 3cm and the other of radius 4cm each coalesce in vacuum under isothermal conditions. Calculate the radius of the bubble formed.

Solution

(a) Diagram



Excess pressure inside Soap bubble = pressure exerted by oil column

$$\frac{4\gamma}{r} = \rho gh$$

$$\gamma = \frac{\rho ghr}{4}$$

$$= \frac{800\times9.8\times2\times10^{-3}\times6\times10^{-3}}{4}$$

$$\gamma = 2.35\times10^{-2}\text{Nm}^{-1}$$

(b) Let r_1 and r_2 be the radius of two spherical bubbles and P_1 and P_2 be corresponding pressure inside of the soap bubbles.
Let V_1 and V_2 be the volumes of the bubble before they coalesce and V be the volume of bubble formed.

Now : $P_1 = \frac{4\gamma}{r_1}$, $P_2 = \frac{4\gamma}{r_2}$

$V_1 = \frac{4}{3}\pi r_1^3$, $V_2 = \frac{4}{3}\pi r^3$

Apply Boyle's law

$$PV = P_1V_1 + P_2V_2$$

$$\frac{4\gamma}{R}\left(\frac{4}{3}\pi R^3\right) = \frac{4\gamma}{r_1}\left(\frac{4}{3}\pi r_1^3\right) + \frac{4\gamma}{r_2}\left(\frac{4}{3}\pi r_2^3\right)$$

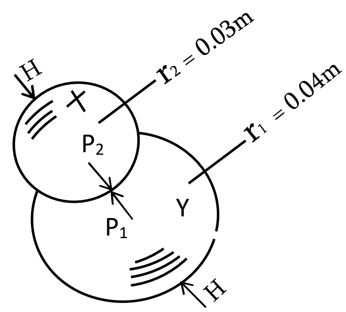
$$R^2 = r_1^2 + r_2^2$$

$$R = \sqrt{r_1^2 + r_2^2} = \sqrt{3^2 + 4^2}$$

$$R = 5\text{cm}$$

Example – 57

A soap bubble X of radius 0.03m and another bubble Y of radius 0.04m are brought together so that the combined bubble has a common interface s of radius r, figure below shown calculate, r.



Solution

Excess pressure on soap bubble Y

$$P_1 - H = \frac{4\gamma}{r_1}$$

Excess pressure on soap bubble X

$$P_2 - H = \frac{4\gamma}{r_2}$$

Takes $(P_2 - H) - (P_1 - H) = 4\gamma\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

$$P_2 - P_1 = 4\gamma\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

But $P_2 - P_1 = \frac{4\gamma}{r}$

$$\frac{4\gamma}{r} = 4\gamma\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$r = \frac{r_1r_2}{r_1 - r_2} = \frac{0.04\times0.03}{0.04 - 0.03}$$

$$r = 0.12\text{m}$$

Example – 58 NECTA 2008/P1/3(B)

Two soap bubbles have radii ratio 2:3

(i) Compare excess pressure inside these bubbles.

(ii) Show that the ratio of the work done in blowing these bubbles is $4/9$.

Solution

Let r_1 and r_2 be the radius of the first and second bubble respectively.

$$\frac{r_1}{r_2} = 2:3 = \frac{2}{3}$$

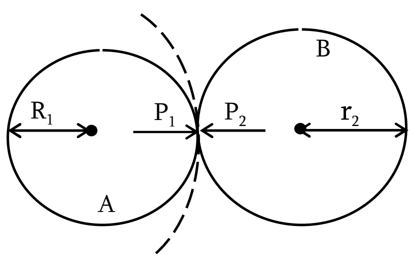
(i) Excess pressure on soap bubble

$$P = \frac{4\gamma}{r}$$

$$P_1 = \frac{4\gamma}{r_1}, \quad P_2 = \frac{4\gamma}{r_2}$$

$$\frac{P_1}{P_2} = \frac{4\gamma}{r_1}\Big/\frac{4\gamma}{r_2} = \frac{r_2}{r_1}$$

$$\frac{P_1}{P_2} = \frac{2}{3} = 3:2$$



Excess pressure in A, $P_1 = \dfrac{4\gamma}{R_1}$

Excess pressure in B, $P_2 = \dfrac{4\gamma}{R_2}$

Now

$$P_1 - P_2 = \frac{4\gamma}{R_1} - \frac{4\gamma}{R_2} = 4\gamma\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{4\gamma}{R} = 4\times 0.07\left[\frac{1}{0.002} - \frac{1}{0.004}\right]$$

$$\frac{4\gamma}{R} = 1000\gamma$$

$$R = 0.004\text{m}$$

The pressure inside the smaller bubble is greater than inside the bigger bubble. Since excess pressure is always towards concave surface, the common surface is concave towards the centre of the smaller bubble.

(ii) Work done, $W = 4\pi\gamma r^2 \times 2$

$$W_1 = 8\pi\gamma r_1^2, \quad W = 8\pi\gamma r_2^2$$

$$\frac{W_1}{W_2} = \frac{8\pi\gamma r_1^2}{8\pi\gamma r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$

$$\frac{W_1}{W_2} = \left(\frac{2}{3}\right)^2 = \frac{4}{9} = 4:9$$

$$\frac{W_2}{W_2} = \frac{4}{9} = 4:9$$

Example – 59

Two separate air bubbles (radii 0.002m and 0.004m) formed of the same liquid (surface tension is 0.07Nm^{-1}) come together to form a double bubble. Find the radius and sense of curvature of the internal film of surface common to both the bubbles.

Solution

Let R_1 and R_2 be radii of the two bubbles A and B and R be the radius of curvature of the common internal film of the double bubble of the two bubbles.

Example – 60 NECTA 2021/P2/3

(b) (i) Determine the height at which water will rise in a capillary tube of radius 5.0×10^{-5}m if the angle of contact between water and the material of the tube is approximately zero.

(c) If the surface tension of mercury at room temperature is 4.72×10^{-1}N/m; determine the excess pressure inside a drop of mercury of radius 0.2cm.
Surface tension of water = 0.073N/m
Density of water = 103kg/m^3, g = 9.8m/s^2.

Solution

(b) (i) The height of water rises in the capillary tube.

$$h = \frac{2\gamma\cos\theta}{\rho g r}$$

$$= \frac{2\times 0.073\times\cos 0^{\circ}}{1000\times 9.8\times 5\times 10^{-5}}$$

$$h = 0.298\,(\text{approx})$$

(c) Excess pressure on the mercury drop

$$P - Po = \frac{2\gamma}{r}$$

Let P = excess pressure in side of drop

$$P = Po + \frac{2\gamma}{r}$$

$$= 1.01\times10^{5} + \frac{2\times4.72\times10^{-1}}{0.2\times10^{-2}}$$

$$P = 1.01472\times10^{5}\,Nm^{-2}$$

Example – 61. NECTA 2020/P2/3

(a) Briefly explain the following observations:-

(i) The rise of the liquid is affected if the top of capillary tube is closed.

(ii) Rain drops are spherical in shape.

(b) (i) Why brick walls are plastered with cement?

(ii) A barometer contains two uniform capillary tubes of radii 6.5×10^{-4}m and 1.24×10^{-3}m. If the height of water in narrow tube is 0.2m more than that in the wide tube, calculate the true pressure difference.

(c) (i) What is meant by surface tension? Give its S.I units.

(ii) During the rain, 64 rain drops combined into a single drop. Calculate the ration of the total surface energy of the 64 drops to that of a single drop.

Solution

(a) (i) If the drop of the capillary tube is closed, there will be a small rise in the capillary tube. The rise of the liquid in the capillary tube due to surface tension is opposite by the downward force exerted by the compressed air above the liquid in the tube. The liquid will rise in the tube till the two forces balance each other. Therefore , there will be small rise of liquid column if the top of the capillary tube is closed.

(ii) The free surface of a rain drop tries to acquire minimum surface area. Since for a given volume, the surface area of a sphere is minimum, the rain drops acquired spherical shape.

(b) (i) Bricks have pores. During rainy season, the water will be sucked in due to capillary action. To prevent it, brick walls are plastered.

(ii)

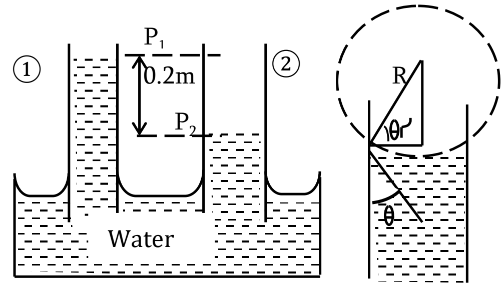


Figure (a) Figure (b)

Let P_1 and P_2 be pressure in the tubes 1 and 2 just above the meniscus. Then , at the point just below the meniscus of the liquid in the two tubes, the pressure will be

$$P_1' = P_1 - \frac{2\gamma}{R_1} \quad \text{and} \quad P_2' = P_2 - \frac{2\gamma}{R_2}$$

Where R_1 and R_2 are radii of the meniscus of the liquid in the two tubes.

From the figure (b) $\frac{r}{R} = \cos\theta$

$$R = \frac{r}{\cos\theta}$$

Now;

$$P_1' = P_1 - \frac{2\gamma\cos\theta}{r_1} \quad , \quad P_2' = P_2 - \frac{2\gamma\cos\theta}{r_2}$$

Then

$P_2' - P_1'$ = Pressure exerted by height h of the water column.

$$\left(P_2 - \frac{2\gamma\cos\theta}{r_2}\right) - \left(P_1 - \frac{2\gamma\cos\theta}{r_1}\right) = h\rho g$$

$$P_2 - P_1 = h\rho g - 2\gamma\cos\theta\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$P_2 - P_1 = 0.2\times1000\times9.8$$

$$-2\times0.072\cos0^{O}\left[\frac{1}{6.5\times10^{-4}} - \frac{1}{1.24\times10^{-3}}\right]$$

$$P_2 - P_1 = 1,854.591\,Nm^{-2}$$

$\therefore$ True pressure difference is $1854.591\,Nm^{-2}$

(c) (i) Refer to your notes

(ii) Let r radius of each small rain drop and R be radius of a single drop formed.

Apply the law of conservation of volume

$$64\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$$

$$R = 4r$$

Total surface energy of 64 drops.

$$W_1 = 4\pi\gamma r^2 \times 64 = 256\pi\gamma r^2$$

Surface energy of single large drop

$$W_2 = 4\pi\gamma R^2 = 4\pi\gamma(4r)^2$$

$$W_2 = 64\pi\gamma r^2$$

Now ; $\frac{W_1}{W_2} = \frac{256\pi\gamma r^2}{64\pi\gamma r^2}$

$$\frac{W_1}{W_2} = 4 = 4:1$$

Example – 62. NECTA 2018/P2/4

(a) Mention any two factors which affect the surface tension of the liquid and in each case explain two typical examples.

(b) Why molecules on the surface of a liquid have more potential energy than those within the liquid? Briefly explain.

(c) (i) Derive an expression for excess pressure inside a soap bubble of radius R and surface tension γ where the pressure inside and outside the bubble are P_2 and P_1 respectively.

(ii) A soap bubble has a diameter of 5mm. calculate the pressure inside if the atmospheric pressure is 10^5Pa and the surface tension of a soap solution is $2.8 \times 10^{-2}Nm^{-1}$.

(d) Water rises up in a glass capillary tube up to a height of 9.0cm while mercury falls down by 3.4cm in the same capillary. Assume angles of contact for water glass and mercury – glass as 0^{O} and 135^{O} respectively. Determine the ratio of surface tension of mercury and water.

Example – 63 NECTA 2005/P1/4

(a) (i) Define 'Free surface energy' in relation to the liquid surface.

(ii) Explain what happen if two bubbles of unequal radii are joined by a tube without bursting.

(iii) A spherical drop of mercury of radius 5.0mm falls on the ground and breaks into 1000 equal droplets, calculate the amount of work done in breaking the drop.

(b) (i) Two capillary tube of radii r and R are placed in a beaker containing a liquid of density, ρ. Show from the first principle in which tube the liquid will rise highest given that $r < R$

(ii) Suppose the Xylem tubes in the actively growing outer layer of a tree are uniform cylinder and that the rising form cylinders and that the rising of a sap is due entirely to capillarity with a contact angle of 45^{O} and surface tension 5×10^{-2}N/m. what will the maximum radius of the tubes be for a tree 20m tall?

Solution

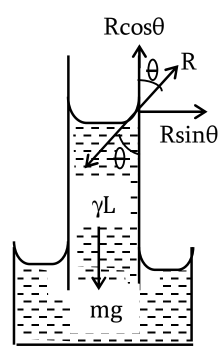
(a) (ii) The excess pressure , P inside of soap bubble is given by $P = \frac{4\gamma}{r}$ it follows that the pressure inside of the small bubble is greater than the inside of large bubble since $P \alpha \frac{1}{r}$. Therefore , when the two bubble into the larger bubble, so that the smaller bubble will collapse, whereas the large bubble will increases in size.

(iii) $W = 4\pi\gamma R^2\left[n^{1/3} - 1\right]$

$$= 4 \times 3.14 \times 0.482 \times \left(5 \times 10^{-3}\right)\left[(1000)^{1/3} - 1\right]$$

$$W = 1.335 \times 10^{-3} J$$

(b) (i) consider capillary tube of radius r immersed on liquid as shown below



At equilibrium

$$R\cos\theta = Mg$$

But $R = \gamma L$

$$\gamma L\cos\theta = Mg$$

$$2\pi r\gamma\cos\theta = \rho\pi r^2 hg$$

$$h = \frac{2\gamma\cos\theta}{r}$$

$$h \ \alpha \ \frac{1}{r}$$

Therefore ,the tube whose radius , r liquid will raise higher because its radius is small compared to the tube whose its radius is R(i.e r < R)

(ii) Since $\Delta P = \frac{2\gamma\cos\theta}{r}$

$$\rho gh = \frac{2\gamma\cos\theta}{r}$$

$$r = \frac{2\gamma\cos\theta}{\rho gh} = \frac{2\times 5\times 10^{-2}\times\cos 45^{O}}{1000\times 9.8\times 2}$$

$$r = 3.6\times 10^{-7}\,m$$

∴ The maximum radius, $r = 3.6\times 10^{-7}\,m$

Example – 64 NECTA 2003/P1/4

(a) (i) What is surface tension.

(ii) Derive an expression for the excess pressure inside a spherical bubble in a liquid.

(iii) A soap bubble of radius 3.0cm and another soap bubble of radius 4.0cm are brought together so that the combined bubble has a common interface of radius, r find the value of r and comment on the shape of the interface. (surface tension of soap solution is $2.5\times 10^{-2}Nm^{-1}$)

(b) A clear glass tube of internal diameter 0.6mmi held vertically with its lower end in water and with 80mm of the tube above the surface. How high does the water rise in the tube. What happen if the tube is lowered until only 30mm of its length is above the surface?

Solution

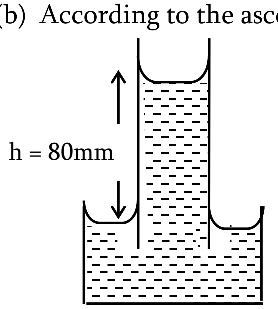
(a) (iii) Let R = Radius of common interface

$$\frac{1}{R} = \frac{1}{r_1} - \frac{1}{r_2} \quad \text{or} \quad R = \frac{r_2 r_1}{r_2 - r_1}$$

$$R = \frac{3\times 4}{4-1} = 12cm$$

∴ Radius of common interface , R is 12cm. The pressure inside the smaller bubble is greater than inside the bigger bubble. Since excess pressure is always towards concave surface, the common surface is concave towards the centre of the smaller bubble.

(b) According to the ascent formula



$$h = \frac{2\gamma\cos\theta}{\rho gr}$$

$$= \frac{2\times 7.2\times 10^{-2}\cos 0^{O}}{0.3\times 10^{-3}\times 1000\times 9.8}$$

$$h = 0.04898m$$

∴ The height rises by water is 0.04898m

When the tube is only 30mm above the surface, water in the capillary tube will form a new angle of contact , θ.

Now : $h_1 = \frac{2\gamma\cos\theta}{\rho gr}$

$$\cos\theta = \frac{h_1\rho gr}{2\gamma}$$

$$= \frac{30\times 10^{-3}\times 0.3\times 10^{-3}\times 9.8\times 1000}{2\times 7.2\times 10^{-2}}$$

$$\theta = 52.2^{O}$$

Example – 65 NECTA 2001/P2/2

(a) (i) How does surface tension varies with temperature.

(ii) Define surface tension in terms of surface energy and calculate the change in surface energy of a soap bubble when its radius decreases from 5cm to 1cm.

(b) A soap solution of surface tension γ is used to form a film between a horizontal rod of length L and a length of weightless in extensible thread attached to each end of the rod. A weight , W is attached at the mid pont of the thread. Show that

(i) The shape of each half of the thread is circular.

(ii) The tension in the thread is

$$T = \frac{\gamma L}{\cos\theta - \sin\beta}$$

Where β is the angle made at the ends of the rod and θ is the angle at the centre due to the weight.

Solution

(a) (i) The surface tension of the liquid decreases with the rise of temperature because the rise of temperature tends to increases of kinetic energy of molecules so reduces the intermolecular force of attraction.

Therefore the surface tension of the liquid decreases with the rise of temperature.

$$\gamma = \gamma_o\left[1-\alpha\theta\right]$$

(ii) Surface tension is defined as the amount of the work done needed to create the new surface under the isothermal condition , i.e $w = \gamma A$

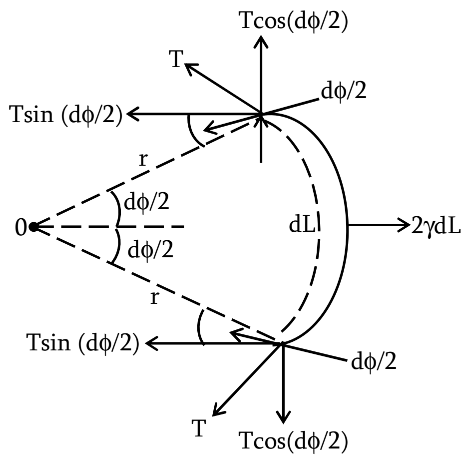
$$\gamma = \frac{W}{A}$$

$$W = 8\pi\gamma\left[R_1^2 - R_2^2\right]$$

$$= 8\times 3.14\times 2\times 10^{-2}\left[(0.05)^2-(0.01)^2\right]$$

$$W = 1.21\times 10^{-3}\,\text{J}$$

(b) (i) To show that thread is the circular. Consider the small element dL shown on the figure below.



The tension at each thread is T acts along the tangent to the element forming an angle $d\phi$ as shown on the figure above, which is also angle subtended where the normal meets.

At the equilibrium

$$T\sin\left(\frac{d\phi}{2}\right) + T\sin\left(\frac{d\phi}{2}\right) = 2\gamma dL$$

$$2T\sin\left(\frac{d\phi}{2}\right) = 2\gamma dL$$

If $d\phi$ is very small angle measured in radian

$$\sin\left(\frac{d\theta}{2}\right) = \frac{d\phi}{2}$$

$$2T\cdot\frac{d\phi}{2} = 2\gamma dL$$

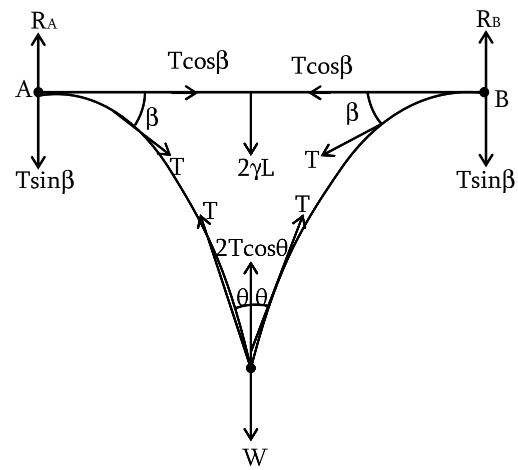
$$\frac{dL}{d\phi} = \frac{T}{2\gamma}$$

Let r = radius of curvature

$$r = \frac{dL}{d\phi} = \frac{T}{2\gamma} = \text{constant}$$

$\therefore$The shape of each half thread is a circular.

(ii) Consider the free body diagram below



For the rod when is in equilibrium

$$2\gamma L + 2T\sin\beta = R_A + R_B \;.......(i)$$

For the whole system in equilibrium

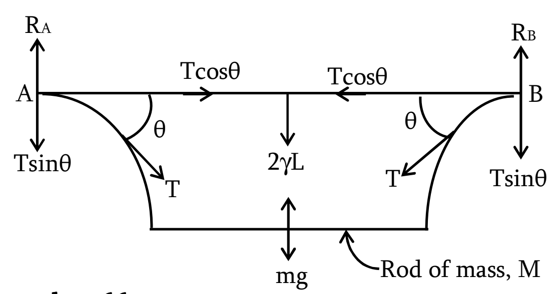
$$W = 2T\cos\theta = R_A + R_B \;......(ii)$$

$$2T\cos\theta = 2\gamma L + 2T\sin\beta$$

$$T\left(\cos - \sin\beta\right) = \gamma L$$

$$T = \frac{\gamma L}{\cos\theta - \sin\beta}$$

Hence shown



Example – 66

What is surface tension? A liquid of surface tension, γ is used to form a film between a horizontal rod of length L and another shorter rod of mass M, ends of each rod, the film is films the vertical plane with rods and strings.

(i) What is the shape of the string?

(ii) Show that the tension in each string is given by

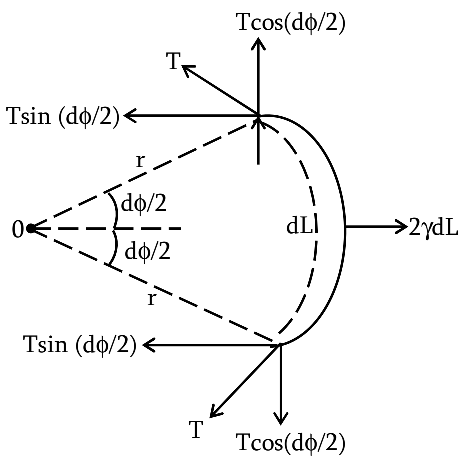
$$T = \frac{Mg - 2\gamma L}{2\sin\theta}$$

Where θ is the angle which the tangent to each string makes with the upper rod.

Solution

(i) The shape of the string

Consider the figure below



At the equilibrium of the film

$$T\sin\left(\frac{d\phi}{2}\right) + T\sin\left(\frac{d\phi}{2}\right) = 2\gamma dL$$

$$2T\sin\left(\frac{d\phi}{2}\right) = 2\gamma dL$$

If $\frac{d\phi}{2}$ is very small angle measured in radian

$$\sin\left(\frac{d\phi}{2}\right) \simeq \frac{d\phi}{2}$$

$$2T\frac{d\phi}{2} = 2\gamma dL$$

$$\frac{dL}{d\phi} = \frac{T}{2\gamma} = r = \text{constant}$$

r = radius of curvature.

∴ Each shape makes by the string is a circular in shape.

(ii) At the equilibrium of upper rod.

$$2\gamma L + 2T\sin\theta = R_A + R_B \ldots\ldots.(1)$$

But $R_A + R_B = Mg \ldots\ldots\ldots\ldots(2)$

(1) = (2)

$$2\gamma L + 2T\sin\theta = Mg$$

$$2T\sin\theta = Mg - 2\gamma L$$

$$T = \frac{Mg - 2\gamma L}{2\sin\theta}$$

Hence shown.

Example – 67

A plane soap film is formed over a wire frame and a fine rubber band is dropped onto the film, the rubber band when unextend forms a circle of diameter 5.0cm. The film inside the rubber band broken upon the band is stretched to form a circle of diameter 5.4cm. It's also found that the rubber band when cut and is used as an elastics, it stretched to double of its original length when suspended a weight of 2 × 10-2N. Evaluate the surface tension of the soap solution.

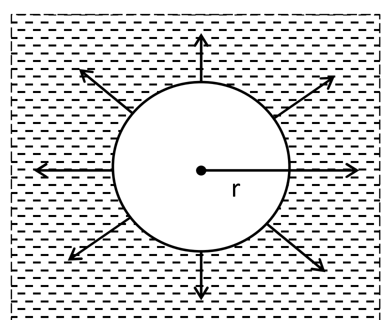
Solution

Length of the unstretched rubber band

$$L = 2\pi r = 2\pi \times 2.5 \times 10^{-2}$$

$$L = 5\pi \times 10^{-2}\,m$$

According to the Hooke's law, tension T developed in the stretched rubber band when dipped in the solution.



$$T = KX \;,\; X = \text{extension}$$

$$T = K\left[2\pi R - 2\pi r\right]$$

$$= K\left[2\pi \times 2.7 \times 10^{-2} - 2\pi \times 2.5 \times 10\;\right]$$

$$T = 4\pi K \times 10^{-3} \ldots\ldots(i)$$

Doubling the rubber band from its original length gives the extension

$$e = 2L - L = L$$

$$L = 5\pi \times 10^{-2}\,m$$

Since $F = KL$

$$2\times10^{-2} = K\times5\pi\times10^{-2}$$

$$K = \frac{2\times10^{-2}}{5\times10^{-2}} = \frac{2}{5\pi}\text{Nm}^{-1}$$

For equation (1)

$$T = 4\pi\times\frac{2}{5\pi}\times10^{-3} = \frac{8}{5}\times10^{-3}\text{N}$$

For string to be circular in shape

$$r = \frac{T}{2\gamma}, \quad \gamma = \frac{T}{2r}$$

$$\gamma = \frac{\frac{8}{5}\times10^{-3}}{2\times2.5\times10^{-2}} = 0.032\text{Nm}^{-1}$$

∴ Surface tension of the soap solution is $\gamma = 0.032\text{Nm}^{-1}$.

Example – 68

(a) Give definition of the coefficient of surface tension in terms of energy.

(b) Explain the following phenomena with the reference to the soap bubble:

(i) If formed freely in air, it has a spherical in shape.

(ii) The pressure inside it is greater than the external pressure

(c) (i) What is the effect of temperature (θ) on coefficient of surface tension (γ).

(ii) Sketch the graph of γ against temperature θ for water.

(d) A spherical soap bubble of radius, R is flowing in a gas inside a cylinder fitted with frictionless piston. The piston is allows with drawn with no rise in temperature of gas until the radius of the bubble is doubled. Show that the final pressure P of the gas in the cylinder is given by

$$P = \frac{Po}{8} - \frac{3\gamma}{2R}$$

Where Po is the original pressure of the gas in cylinder and γ is the coefficient of surface tension of the soap solution.

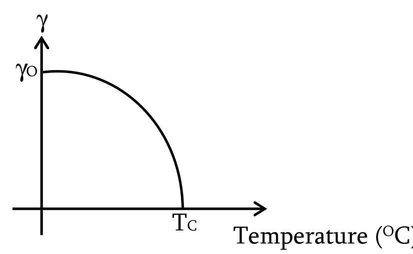
Solution

(b) (i) The curvature of a soap bubble is determined by the surface tension forces of the liquid. The forces act in wards in all direction in a liquid so that the area of the surface of the bubble will be minimum, hence spherical in shape.

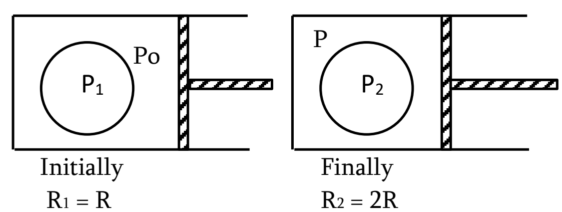
(ii) The surface tension forces due to the surface tension of the solution plus the external pressure on the bubble touch to cause the collapse of the bubble. Hence to avoid this pressure inside of the bubble. Hence to avoid this pressure inside of the bubble must be greater than the external to the surface tension forces which will balance with it.

(c) (i) surface tension of the liquid decreases with rise of temperature.

(ii) Graph of surface tension against temperature



(d) Consider the figure below



Applying Boyle's law

$$P_1V_1 = P_2V_2$$

$$\left(Po + \frac{4\gamma}{R}\right)\frac{4}{3}\pi R^3 = \left(P + \frac{2\gamma}{R}\right)\frac{32}{3}\pi R^3$$

$$Po + \frac{4\gamma}{R} = \left(P + \frac{2\gamma}{R}\right)8$$

On simplification

$$P = \frac{Po}{8} - \frac{3\gamma}{2R}$$

Example – 69

There is a soap bubble of radius 2.4×10^{-4}m in an air cylinder which is originally at a pressure of 105Nm^{-2}. The air in the cylinder is now compressed isothermally until the radius of the bubble is halved. Calculate now the pressure of air in the cylinder. The surface tension of soap film is $0.08 Nm^{-1}$.

Solution

Initial pressure of air in the cylinder

$$Po = 10^5 Nm^{-2}$$

Initial volume of soap bubble

$$V_1 = \frac{4}{3}\pi R^3$$

Pressure inside of a soap bubble

$$P_1 = Po + \frac{4\gamma}{R}$$

Let P be final pressure of air in the cylinder when radius is halved. Pressure inside of soap bubble.

$$P_2 = P + \frac{4\gamma}{R/2} = P + \frac{8\gamma}{R}$$

$$V_2 = \frac{4}{3}\pi\left(R/2\right)^3 = \frac{1}{6}\pi R^3$$

Apply the Boyle's law

$$P_1V_1 = P_2V_2$$

$$\left(Po + \frac{4\gamma}{R}\right)\frac{4}{3}\pi R^3 = \left(P + \frac{8\gamma}{R}\right)\frac{\pi R^3}{6}$$

$$P = 8Po + \frac{24\gamma}{R}$$

$$= 8\times10^5 + \frac{24\times0.08}{2.4\times10^{-4}}$$

$$P = 8.08\times10^5 Nm^{-2}$$

Example – 70

Two spherical soap bubbles combine if V is the change in volume of the contained air, A is the change in total surface area, then show that $3PaV + 4A\gamma = 0$ where γ is the surface tension and Pa is the atmospheric pressure.

Solution

Let r_1 and r_2 be radii of the two bubbles and r be the radius of resulting bubble.

Total pressure inside of the bubbles

$$P_1 = Pa + \frac{4\gamma}{r_1} \quad , \quad P_2 = Pa + \frac{4\gamma}{r_2}$$

$$P = Pa + \frac{4\gamma}{r} \quad , \quad V_1 = \frac{4}{3}\pi r_1^3$$

$$V_2 = \frac{4}{3}\pi r_2^3 \quad , \quad V = \frac{4}{3}\pi r^3$$

Applying Boyle's law

$$PV = P_1V_1 + P_2V_2$$

$$\left(Pa + \frac{4\gamma}{r}\right)\frac{4}{3}\pi r^3 = \left(Pa + \frac{4\gamma}{r_1}\right)\frac{4}{3}\pi r_1^3 + \left(Pa + \frac{4\gamma}{r_2}\right)\frac{4}{3}\pi r_2^3$$

On arrangement

$$Pa\left[\frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 - \frac{4}{3}\pi r^3\right] + \frac{4\gamma}{3}\left[4\pi r_1^2 + 4\pi r_2^2 - 4\pi r^2\right] = 0$$

Let

$$A = 4\pi r_1^2 + 4\pi r_2^2 - 4\pi r^2$$

$$V = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 - \frac{4}{3}\pi r^3$$

$$PaV + \frac{4\gamma A}{3} = 0$$

$$3PaV + 4\gamma A = 0 \text{ Hence shown.}$$

Example – 71

Two soap bubbles of radii r_1 and r_2 coalesces to form a single bubble of radius R, show that the surface tension is given by

$$\gamma = \frac{P\left(R^3 - r_1^2 - r_2^3\right)}{4\left(r_1^2 + r_2^2 - R^2\right)}$$

Where P is the external pressure and γ is the surface tension of soap solution.

Solution

Pressure inside of bubble of radius , r_1

$$P_1 = P + \frac{4\gamma}{r_1} \quad , \quad V_1 = \frac{4}{3}\pi r_1^3$$

Pressure inside of bubble of radius , r_2

$$P_2 = P + \frac{4\gamma}{r_2} \quad , \quad V_2 = \frac{4}{3}\pi r_2^3$$

Pressure inside of bubble of radius , R

$$P_3 = P + \frac{4\gamma}{R} \quad , \quad V_3 = \frac{4}{3}\pi R^3$$

Since temperature remains the same during the change , from Boyle law we have

$$P_1V_1 + P_2V_2 = P_3V_3$$

$$\left(P + \frac{4\gamma}{r_1}\right)\frac{4}{3}\pi r_1^3 + \left(P + \frac{4\gamma}{r_2}\right)\frac{4}{3}\pi r_2^3 = \left(P + \frac{4\gamma}{R}\right)\frac{4}{3}\pi R^3$$

$$P\left(r_1^3 + r_2^3 - R^3\right) + 4\gamma\left(r_1^2 + r_2^2 - R^2\right) = 0$$

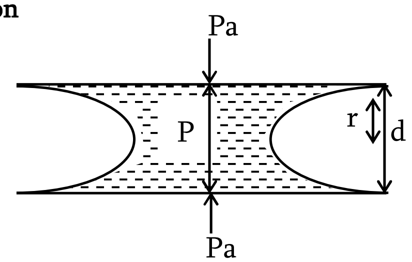
$$4\gamma\left(r_1^2 + r_2^2 - R^2\right) = P\left(R^3 - r_1^3 - r_2^3\right)$$

$$\gamma = \frac{P\left(R^3 - r_1^2 - r_2^3\right)}{4\left(r_1^2 + r_2^2 - R^2\right)}$$

Example – 72

A small drop of clean water is squeezed between two glass plates so that a very thin layer of large are is formed. If the thickness of water film is 10^{-6}m and surface area of it is 40cm^2, calculate the force required to separate the two glass plates. Surface tension of water is 7×10^{-2}N/m.

Solution



Excess pressure is given by

$$Pa - P = \frac{T}{r} = \frac{2T}{d}$$

T = Surface tension

$$\text{Force , } F = \left(Pa - P\right)A = \frac{2TA}{d}$$

$$= \frac{2 \times 7 \times 10^{-2}}{10^{-6}} \times 40 \times 10^{-4}$$

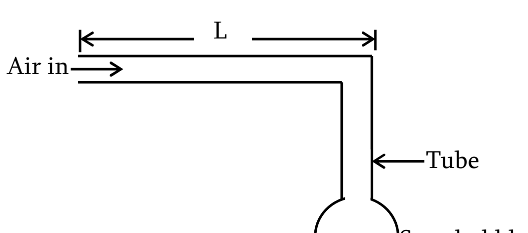
$$F = 560N$$

Example – 73

A soap bubble of radius 4cm and surface tension 0.03N/m is blown at the end of a tube of length 10cm and internal radius 0.2cm. If the coefficient of viscosity of air is 1.85×10^{-5}Nm^{-2}, find the time taken by the bubble to be reduced to a radius of 2cm.

Solution

Let R be the radius of the bubble and r be the radius of the tube.



The volume of the bubble

$$V = \frac{4}{3}\pi R^3$$

Differentiate with respect to the time

$$\frac{dv}{dt} = 4\pi R^2 \frac{dR}{dt} \text{ ....(i)}$$

According to the Poiseulli's formula

$$\frac{dv}{dt} = \frac{\pi P r^4}{8\eta L} \quad \text{but } P = \frac{4\gamma}{R}$$

$$\frac{dv}{dt} = \frac{\pi r^4}{8\eta L} \cdot \frac{4\gamma}{R} = \frac{\pi r^4 \gamma}{2\eta RL} \text{ ....(ii)}$$

(i) = (ii)

$$4\pi R^2 \frac{dR}{dt} = \frac{\pi r^4 \gamma}{2\eta RL}$$

$$dt = \frac{8\eta L R^3}{\gamma r^4} dR$$

$$\int dt = \frac{8\eta L}{\gamma r^4} \int_{R_2}^{R_1} R^3 dR$$

$$t = \frac{8\eta L}{\gamma r^4}\left[\frac{R_1^4 - R_2^4}{4}\right]$$

$$= \frac{8 \times 1.85 \times 10^{-5} \times 0.1}{0.03 \times \left(0.002\right)^4}\left[\frac{\left(0.04\right)^4 - \left(0.02\right)^4}{4}\right]$$

$$t = 18.4 \text{ sec}$$

Example – 74

(a) Explain the following phenomena with reference to the surface tension.
 (i) Soldering becomes easy due to the addition of flux . Why?
 (ii) A drop of oil placed on the surface of water spreads out. But a drop of water placed oil contracts to spherical shape. Why?

(b) At one end of capillary tube of radius r and length , L there is a soap bubble of radius, R. the bubble is connected to the atmosphere through capillary tube. After how long the radius of the bubble will reduced to zero? Coefficient of viscosity of air is η.

Solution

(a) (i) Flux reduces the surface tension of molten tin and this causes tin to spread easily.

(ii) The adhesive force among oil – water molecules is great than cohesive force among oil molecules. In the second case, the cohesive force among molecules is greater than adhesive force.

(b) The excess pressure inside of the soap bubble,

$$P = \frac{4\gamma}{R}$$

Volume of the bubble , $V = \frac{4}{3}\pi R^3$

Rate of decrease of volume of soap bubble

$$\frac{dV}{dt} = -4\pi R^2 \frac{dR}{dt} \ldots\ldots(1)$$

The volume flow rate through a pipe can be obtained by using Poiseulli's formula.

$$\frac{dV}{dt} = \frac{\pi \Delta p r^4}{8\eta L} \quad \text{but} \quad \Delta P = \frac{4\gamma}{R}$$

$$\frac{dV}{dt} = \frac{\pi r^4 \gamma}{2\eta RL} \ldots\ldots.(2)$$

(1) = (2)

$$-4\pi R^2 \frac{dR}{dt} = \frac{\pi r^4 \gamma}{2\eta RL}$$

$$\frac{\gamma r^4}{8\eta L}\int_0^t dt = -\int_R^0 R^3 dR$$

$$t = \frac{2\eta L R^4}{\gamma r^4}$$

Example – 75

(a) Excess pressure of one soap bubble is four times the other soap bubble. Determine the ratio of the surface energy of the bubbles.

(b) A soap bubble is made of 8.0mg of a soap solution and is filled with hydrogen of density 0.09kgm^{-3} it just floats in air of density 1.29kgm^{-3}. What is the excess pressure inside of the bubble? The surface tension of soap solution is $2.5 \times 10^{-2}\text{Nm}^{-1}$.

Solution

(a) Since $P = \frac{4\gamma}{r}$

$$P_1 = \frac{4\gamma}{r_1}, \quad P_2 = \frac{4\gamma}{r_2}$$

$$\frac{P_1}{P_2} = \frac{r_2}{r_1} = 4$$

The surface energy of soap bubble

$$W = 8\pi\gamma r^2$$

$$W_1 = 8\pi r\gamma_1^2, \quad W = 8\pi\gamma r_2^2$$

$$\text{Takes } \frac{W_1}{W_2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{1}{4}\right)^2$$

(b) When the soap bubble floats

Upthrust = total weight of the bubble

$$\rho_a\left(\tfrac{4}{3}\pi r^3\right)g = Mg + \rho\left(\tfrac{4}{3}\pi r^3\right)g$$

$$r^3 = \frac{M}{\frac{4}{3}\pi(\rho_a - \rho)} = \frac{8\times10^{-6}}{\frac{4}{3}\pi(1.29 - 0.09)}$$

$$r = 0.0117\text{m}$$

The excess pressure on the soap bubble

$$\Delta P = \frac{4\gamma}{r} = \frac{4\times2.5\times10^{-2}}{0.0117}$$

$$\Delta P = 8.6\text{Nm}^{-2}$$

Example – 76

An air bubble inside a liquid of surface tension $1.0 \times 10^{-3}\text{Nm}^{-1}$ grows from radius of 1.0×10^{-5}m to 1.0×10^{-4}m is 6μs (6×10^{-6}s). Calculate the average rate of change of pressure inside the bubble.

Solution

Average rate of change of pressure

$$\frac{dp}{dt} = \frac{2\gamma}{t}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{2\times1\times10^{-3}}{6\times10^{-6}}\left[\frac{1}{1\times10^{-5}} - \frac{1}{1\times10^{-4}}\right]$$

$$\frac{dp}{dt} = -3.0\times10^7\,\text{Pas}^{-1}$$

EXERCISE 9.4

1. A soap bubble in a vacuum has a radius of 3cm and another soap bubble in the vacuum has radius of 6cm. If two bubbles coalesce under isothermal conditions. Calculate the radius of the bubble formed. **Answer** $\sqrt{45}$cm

2. NECTA 2009/P1/3

   (a) Define the terms

   (i) Surface tension (ii) angle of contact

(b) (i) Show that the excess pressure inside a soap bubble floating in a soap solution of surface tension, γ is $\Delta P = \frac{4\gamma}{r}$ where r is radius of the soap bubble

(ii) Two spherical bubbles of radii 30cm and 10cm coalesce so that they are made from the same solution and the radii of the bubbles remain the same after they joined together. Calculate the radius of curvature of their common surface.

(c) A small oil drop of radius 2×10^{-6}m falls with terminal velocity of 4.0×10^{-4}m/s through the air. What is the new terminal velocity for an oil drop of half this radius? **Answer** (b) (ii) 15mm.

3. Two soap bubbles have radii in the ration1:3 compare the excess of pressure inside these bubbles. Also compare the work done in blowing these bubbles. **Answer** $^{3}/_{1}$, $^{1}/_{9}$

4. (a) Find the gauge pressure inside a soap bubble of diameter 5cm the surface tension of soap solution is 25×10^{-3}Nm^{-1}.

(b) A soap bubble has a diameter of 4mm. calculate the pressure inside it if the atmospheric pressure is 10^5Pa. surface tension of soap solution = 2.8×10^{-2}Nm^{-1}. **Answer** (a) 4Pa (b) 1.00056×10^5Pa

5. Calculate the depth of water at which an air bubble of radius 0.4mm may remain in equilibrium surface tension of water = 7 × 10^{-2}Nm^{-1} and g = 9.8m/s. **Answer** 3.57cm.

6. A small hollow sphere which has a small hole in it is immersed in water to a depth 40cm before any water penetrates into it. If the surface tension of water is 7.5×10^{-2}Nm^{-1}, find the radius of the hole. **Answer** 0.003827cm.

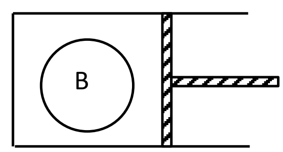
7. A glass tube of 1mm bore is dipped vertically into a container of mercury with its lower end 2cm below the mercury surface. What must be the gauge pressure of air in the tube in order to blow a hemispherical bubble at its end? Given density of mercury = 136000kgm^{-3} and surface tension of mercury = 35 × 10^{-3}Nm^{-1}. **Answer** 2805.6 Pa.

8. A liquid of surface tension γ is used to form a film between a horizontal rod of length L and another shorter rod of mass M suspended from its end by two light inextensible strings at the ends of each rod. The film fills the vertical plane within the rods and strings.

(i) Show that the shape of each string is circular.

(ii) Show that the surface tension of the film is given by $\gamma = \frac{Mg}{4r\sin\theta + 2L}$

Where θ is the angle in which the tangent to each string makes with the upper rod and r is the radius of curvature of each string.

9. 

In figure above, a spherical soap bubble B of radius r is suspended in a gas inside a cylinder which is fitted with a frictionless piston. The piston is slowly with drawn with no rise in temperature of the gas until the radius of the bubble is doubled. Show that the final pressure P of the gas in the cylinder is given by

$$P = \frac{1}{8}\left(Po - \frac{12\gamma}{R}\right)$$

Where Po is the original pressure of the gas in the cylinder and γ is the coefficient of surface tension.

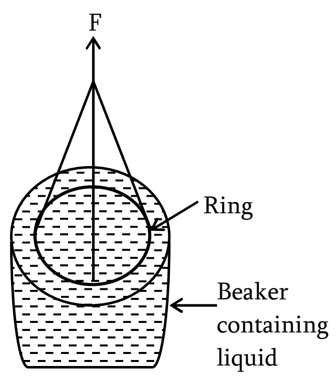
10. (i) Calculate the work done against tension surface forces in blowing a soap bubble of 2cm diameter if the surface tension of soap solution is 2.5×10^{-2}Nm^{-1}.

(ii) Find the work done required to break up a drop of water of radius 0.5cm into drops of water each of radii 1mm assuming

isothermal conditions. (Surface tension of water = 7 × 10-2Nm-1).
Answer (i) 1.57 × 10^{-5}J (ii) 8.8 × 10^{-5J}.

FORCE OF SURFACE TENSION ON A LIGHT RING RADIUS, R FLOATING ON A LIQUID.

Consider a light ring of radius, R floating on a liquid surface tension (γ) with its plane horizontal.



The ring is in contact with two surface films on each side. The total force acting on the ring to the surface tension is given by

$$F = 4\pi\gamma R$$

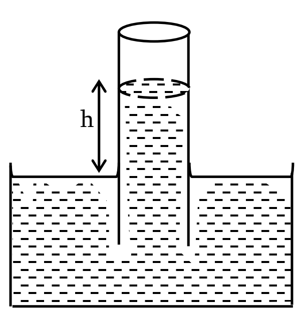
MEASUREMENT OF SURFACE TENSION OF LIQUID OR SOAP SOLUTION

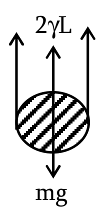
This can be measured by using the following method:-

1. Capillary method
2. Microscopic method
3. Jaeger's method

1. **CAPILLARY METHOD**

Suppose that the clean capillary tube is placed on the liquid of surface tension, γ and rises up to height, h with zero angle of contact, as shown on the figure below





The section of the meniscus is hemisphere in shape. The liquid does not rise up to the tube when surface tension forces is equal to the weight of the in the tube.

$$2\pi\gamma r = Mg$$

$$2\pi\gamma r = \pi r^2 h\rho g$$

$$\gamma = \frac{h\rho gr}{2}$$

Each symbol have usual meaning. Assumptions made on the derivation above:

(i) The glass tube to be a tangent of the liquid surface meeting it.

(ii) Neglecting the weight of the small liquid above the bottom of the meniscus.

2. **MICROSCOPIC METHOD**

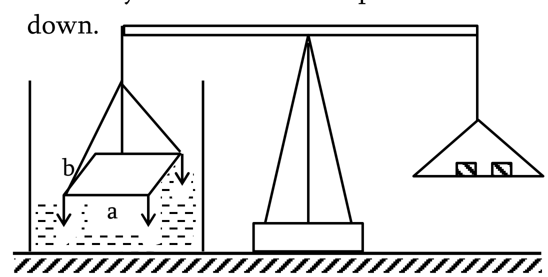
Microscopic slide is an instrument used for measurement of the surface tension of the liquid and soap bubble solution.

SURFACE TENSION OF LIQUID WHICH FORM UNSTABLE FILMS.

Liquid such as water cannot forms a stable film. The following methods may be used to determine surface tension of unstable films:

(i) **Surface tension of liquid by microscope slide (glass – block) method.**

The surface tension of liquid (water) can be measured by weighing a microscope slide in air and then lowering it until it just meet the surface of the liquid (water). The surface tension forces acting round the boundary of the slide and pulls the slide down.



If a and b are the length and thickness of the slide then surface tension forces acts over the total length 2(a + b) as shown above. If a mass M is required to counter balance the surface tension forces to pull the glass clear of the liquid, then at equilibrium.

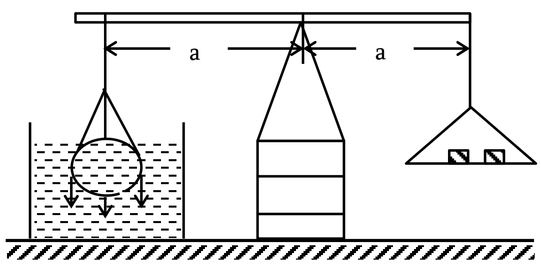
Weight added = surface tension forces

$$Mg = 2\gamma\,(a + b)$$

$$\gamma = \frac{Mg}{2(a+b)}$$

(ii) **Surface tension of liquid by a wire ring method.**

A wire ring of internal radius r_1 and external radius r_2 is laid flat on the surface of the liquid and counter poised (balanced) by a mass on the other side of pan. Surface tension forces acting on the ring points downwards (inside the liquid) an extra mass M placed on the pan will just detach the ring from the liquid if its weight is equal to the surface tension forces. Acts on both sides of the ring



At the equilibrium

Total force due to = extra weight

Surface tension

$$2\pi\gamma r_1 + 2\pi\gamma r_2 = Mg$$

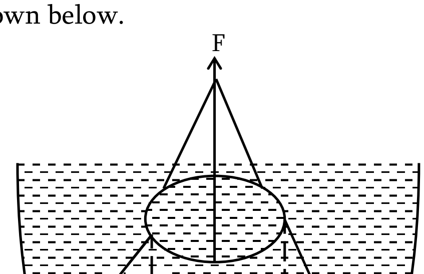
$$\gamma = \frac{Mg}{2\pi(r_1 + r_2)}$$

If the thickness of the ring is very small i.e negligible $r_1 + r_2 \simeq 2r$

$$\gamma = \frac{Mg}{4\pi r}$$

Special case

If the liquid forms an angle of contact θ with the ring, the surface tension forces are directed. Downwards at an angle θ as shown below.



At equilibrium

$$2\pi\gamma(r_1 + r_2)\cos\theta = Mg$$

$$\gamma = \frac{Mg}{2\pi(r_1 + r_2)\cos\theta}$$

If the thickness of the ring is very small i.e negligible $r_1 + r_2 \simeq 2r$

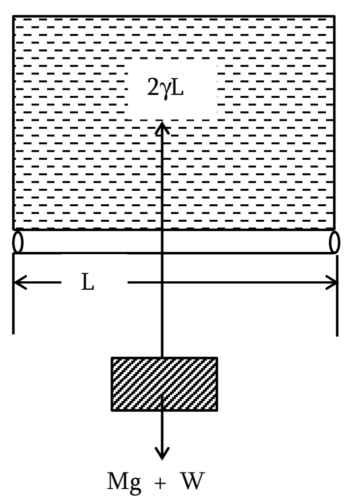
$$\gamma = \frac{Mg}{4\pi r\cos\theta}$$

SURFACE TENSION OF LIQUID WHICH FORM STABLE FILMS.

Liquid such as soap solution can form a stable film, which can be exists in air for sometime. The following methods may be used to determine the surface tension of a stable film.

(i) **Surface tension of soap solution by a U – shape wire frame and a sliding wire.**

Consider a U – shaped wire frame and a sliding wire of length dipped in a soap solution will be formed between the wires. Surface tension force will pull the sliding wire in an upward direction and a mass M may be required to balance the sliding wire.



The soap film has two thin layers separated by a liquid, hence surface tension forces acts on both sides of film at the equilibrium of slide wire

$$Mg + w = 2\gamma L$$

$$\gamma = \frac{Mg + w}{2L}$$

If the weight of slide wire is very small, W = 0

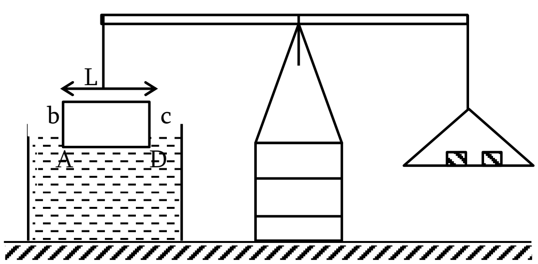
$$\gamma = \frac{Mg}{2L}$$

(ii) **Surface tension of soap solution by a squared wire frame.**

A soap film is formed in three sided metal (wire) frame ABCD and the apparent weight is found by balancing the level. When a mass M is placed on the other side of a pan and just breaks the film then we say that, the extra weight Mg introduced has balanced with the downward surface tension forces. At the equilibrium

Surface tension forces = extra weight introduced

$$2\gamma L = Mg$$



$$\gamma = \frac{Mg}{2L}$$

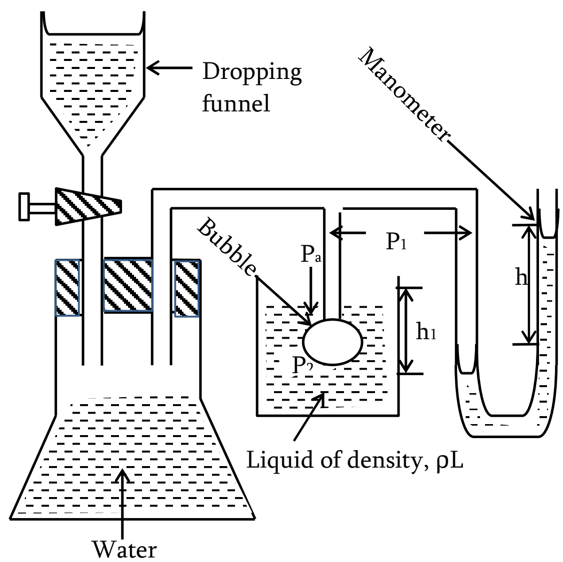
Example : Why a film of water cannot supported in a rectangular frame?

Reasons:

This is due to the fact that soap drams downward in a vertical film, so that the top of film has lower concentration of soap than the bottom. The surface tension at the top is greater than the bottom (soap diminishes surface tension of pure water). The upward pull on the film by the top bar is hence greater than the downward pull on the film by the lower bar. The net upward pull supports the weight of the film. In the case of pure weight, however, the surface tension would be the same at the top and bottom and hence these is net force in this case to support a water film in a rectangular frame.

EXPERIMENT FOR MEASUREMENT OF SURFACE TENSION OF SPHERICAL SOAP BUBBLE (LIQUID BUBBLE).

1. **JAEGER METHOD**.



By forming a bubble inside a liquid and measuring the excess pressure, jaeger was able to determine the variation of surface tension of a liquid with temperature as shown in the figure above. Assuming the bubble is hemispherical with the radius equal to that of the tube and let Pa be atmospheric pressure, P_2 the pressure in the liquid outside of the bubble. The manometer pressure (pressure inside of the bubble is $P_1 = Pa + \rho gh$.

Pressure in liquid outside of the bubble $P_2 = Pa + \rho_1 g h_1$.

Excess pressure in a bubble

$$P_1 - P_2 = \frac{2\gamma}{r}$$

$$(Pa + \rho gh) - (Pa + \rho_1 gh_1) = \frac{2\gamma}{r}$$

$$\gamma = \frac{gr}{2}(\rho h - \rho_1 h_1)$$

r = radius of capillary tube a bubble is formed.
ρ = density of the liquid in the manometer
ρ_1 = density of liquid.

Note that

1. The pressure inside the apparatus can be gradually increased by allowing water to enter the flask from the dropping funnel and the increase is recorded by the manometer.
2. h is read just as the bubble breaks away from the narrow tube. This is maximum value and corresponding to maximum excess pressure so r is minimum.
3. By adding warm liquid to the vessel, the variation of the surface tension with temperature can be determined. Experiments show that surface tension of water and liquid decreases with increasing temperature along fairly smooth curve.

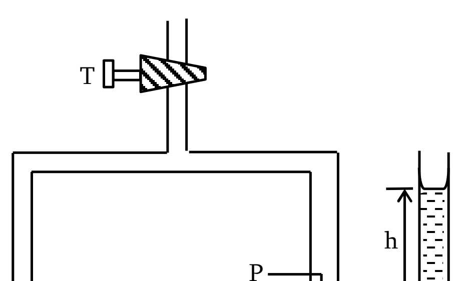
ADVANTAGES OF THE JAEGER'S METHOD

(i) It is easily adapted to investigate how surface tension varies with temperature.
(ii) Contamination is reduced because a fresh liquid surface (bubble) is formed continually.
(iii) Knowledge of the angle of contact is not required.

DISADVANTAGE

Absolute measurements are not reliable because the assumption that the minimum bubble radius equals to the radius of the tube is not quite true.

SURFACE TENSION OF SOAP BUBBLE BY USING MANOMETER METHOD.



The surface tension of soap solution can be obtained by blowing a small soap bubble at the end B of a tube connected by a manometer. The tap, T is then closed. The excess pressure P in the bubble.

$$P = \frac{4\gamma}{r} = \rho gh$$

If the diameter of the tube – bubble formed is d; then $r = d/2$

$$P = \rho gh = \frac{8\gamma}{d}$$

$$\gamma = \frac{\rho ghd}{8}$$

Where ρ is the density of the liquid in the manometer.

Molecular bonds and surface energy

Molecules which reach the surface form interior break and reform bonds with neighbours, all around them as they rise. At the surface, however half of the bonds have not reformed. The bond energy thus released results in greater in the surface than in the bulk liquid. Expression of surface tension, γ from the molecular theory is given by $\gamma = \frac{1}{4}NnE$

N = Number of molecules per unit area in the surface.
n = Number of nearest neighbours per molecule in the liquid.
E = Energy per molecular pair.
(The energy to break permanently the bonds between a pair of neighboring molecules in the liquid).

SURFACE ENERGY AND LATENT HEAT

Energy is needed to change a molecule in the liquid surface into a vapour outside. This amount of energy is needed permanently break bonds with it neighboring molecules of the liquid. Thus the energy needed to evaporate a liquid is related to its free surface energy or surface tension. The latent heat of evaporation is therefore related to the free surface energy.

Total surface energy

When the surface area of a liquid is increased, the surface energy is increased. The liquid cools while the surface, S increased and heat flows from the surroundings to restore the temperature. The increase in the total surface energy per unit area, E is given by $E = \gamma + H$

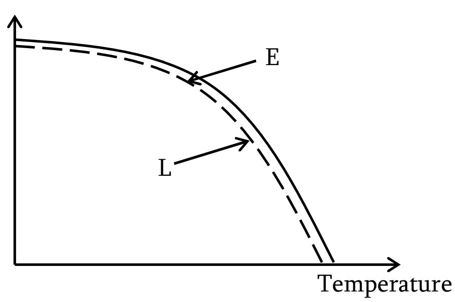
H = internal energy per unit area absorbed by the surface from the surroundings.

$$H = -\frac{\theta d\gamma}{d\theta}$$

Since γ decreases as temperature rises, then $\frac{d\gamma}{d\theta}$ is negative.

$$\text{Now } E = \gamma - \frac{d\gamma}{d\theta}$$

Graph of variation of E and L with temperature



SOLVED EXAMPLES

Example – 77 NECTA 2010/P1/4(b)

(a) (i) State the surface tension in terms of energy.

(ii) The surface tension of water at 20°C is $7.28 \times 10^{-2}Nm^{-1}$. The vapour pressure is $2.33 \times 10^{3}Pa$. determine the radius of smallest spherical water droplet which it can form without evaporating?

(b) A circular thin wire 3cm in radius is suspended with its plane horizontal by a thread passing through the 10cm mark of a metere rule pivoted at its centre and is balanced by 8gm weight, suspended at 80cm mark. When the ring is just brought in contact with the surface of a liquid, the 8gm weight has to be moved to 90cm mark to just detach the ring from the liquid. Find the surface tension of the liquid (Assume the angle of contact is zero).

Solution

(a) (ii) Let r be radius of smallest spherical water droplet.

$$\text{Since } P = \frac{2\gamma}{r}, \quad r = \frac{2\gamma}{P}$$

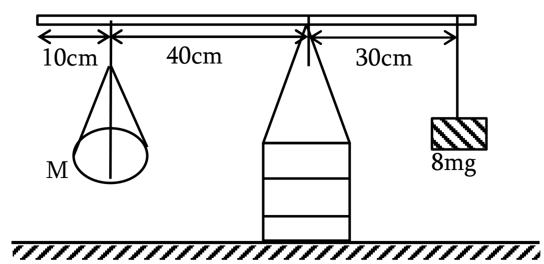
$$r = \frac{2 \times 7.28 \times 10^{-2}}{2.33 \times 10^{3}}$$

$$r = 6.25 \times 10^{-5}\,m$$

$\therefore$ Radius of smallest water drop

$$r = 6.25 \times 10^{-5}\,m$$

(b) First, let us to obtain mass of ring when are in air.



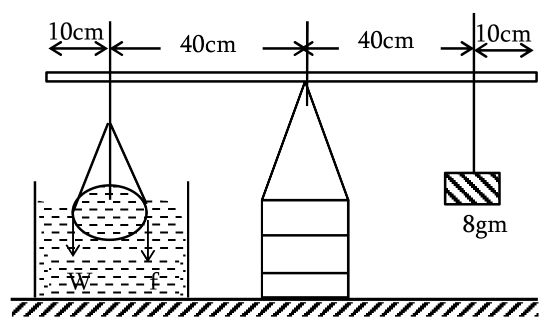
Apply the principle of moment of force

$$M \times 40 = 30 \times 8$$

$$M = \frac{30 \times 8}{40} = 6gm$$

Weight of ring, $W = 6 \times 10^{-3} \times 9.8 = 0.0588N$.

When the ring just touches liquid both weight of the ring and surface tension force acts down ward.



By principle of moments of force

$$\left(W+F\right)\times 40 = 8g\times 40$$

$$F = 8g - W$$

$$= 8\times 10^{-3}\times 9.8 - 0.0588$$

$$F = 0.0196N$$

But $F = 4\pi\gamma r$

$$\gamma = \frac{0.0196}{4\times 3.14\times 3\times 10^{-2}} = 5.2\times 10^{-2}Nm^{-1}$$

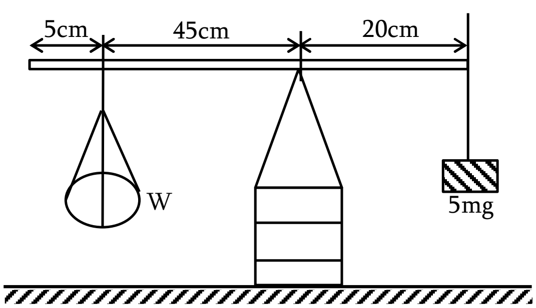
$\therefore$ Surface tension of liquid is $5.2\times 10^{-2}N/m$

Example – 78

A circular ring of thing wire of radius 2cm is suspended horizontally by a thread passing through the 5cm mark of the metre rule pivoted at its centre, and the ring is balanced by a 5gm mass suspended at the 70cm mark. A beaker of liquid is raised until the ring just touches the surface of the liquid. If the 5gm mass is moved to the 86cm mark, the ring just part from the liquid. Find the surface tension of the liquid.

Solution

Let W be the mass of the ring in air.

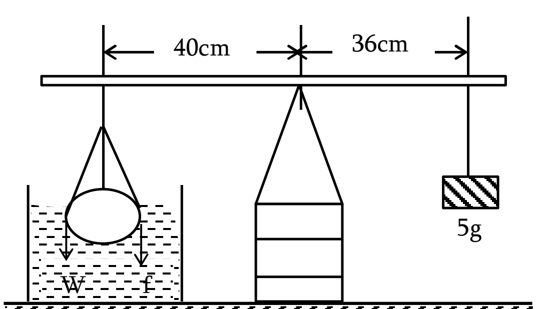


By the principle of moment of a force

$$W\times 45 = 20\times 5$$

$$W = \frac{20}{9}gm = 0.021778N$$

When the ring just touches liquid both weight of ring and surface tension forces acts downward.



By principle of moment of force

$$\left(W+F\right)\times 45 = 36\times 5$$

$$W+F = 10gm = 0.0392N$$

$$F = 0.0392 - W = 0.0392 - 0.0218$$

$$F = 0.0174N$$

But $F = 4\pi R\gamma$

$$\gamma = \frac{F}{4\pi R} = \frac{0.174N}{4\times 3.14\times 2\times 10^{-2}m}$$

$$\gamma = 0.693Nm^{-1}$$

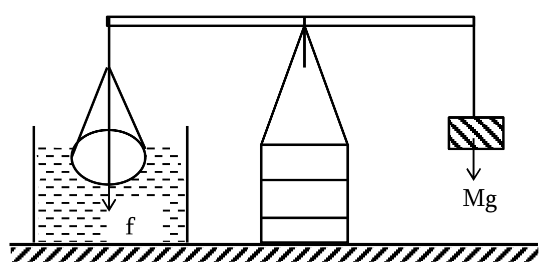
Example – 79

A ring of glass is cut from a tube of 8.4cm internal diameter and 8.8cm external diameter. This ring with its lower edge horizontal is suspended from one arm of balance. So that the lower edge is contact with water in a container and is in the same plane as the free horizontal surface of water remove from the ring. It is found that an additional mass of 3.97gm must be placed on the other scale pan to compensate for the pull of surface tension on the ring.

(i) Calculate the surface tension of water

(ii) If the edge of the ring were 1mm higher than the free horizontal surface of water, how would this affect the extra mass to achieve a balance?

Solution



(i) Force due to the surface tension is given by

$$F = \gamma\times \text{total length}$$

$$F = 2\pi r\left(r+R\right)$$

Where r and R are the internal and external radii of the tube. Here extra weight will produce the force due to the surface tension.

$$Mg = 2\pi\gamma\left(r+R\right)$$

$$\gamma = \frac{Mg}{2\pi(r+R)}$$

$$= \frac{3.97\times10^{-3}\times9.8}{2\times3.14(0.042+0.044)}$$

$$\gamma = 0.072Nm^{-1}$$

(ii) If the edge of the ring is 1.00mm higher, the weight of water drawn up the surface tension must add to the other scale pan. This weight is given by:

$$W = \rho Vg$$

$$W = \pi(R^2 - r^2)\rho gh$$

$$= 3.14\left[0.044^2 - 0.042^2\right]\times0.001\times1000\times9.8$$

$$W = 0.00529kg$$

$$W = 5.29gm$$

Example – 80

(a) A hydrometer has a cylindrical glass stem of diameter 0.5cm. It floats in water of density 1000kgm-3 and surface tension $7.2 \times 10^{-2}Nm^{-1}$. A drop of liquid detergent is added reduces the surface tension to $5.0 \times 10^{-2}Nm^{-1}$. What will be the change in length of exposed portion of the glass.

(b) A glass plate has a length of 15cm and thickness 0.2cm. It weighs 25gf in air. The plate is suspended vertically from a spring balance. With the long side horizontal and half immersed in water. Find the weight of the plate.

Solution

(a) Change in surface Tension force = weight of liquid displaced

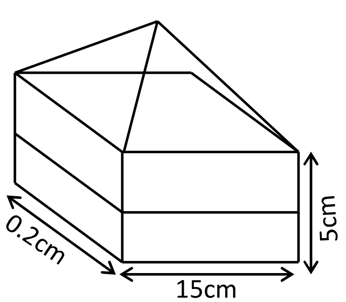
$$2\pi r(\gamma_1 - \gamma_2) = \pi r^2 \rho gh$$

$$h = \frac{2(\gamma_1 - \gamma_2)}{r\rho g}$$

$$h = \frac{2(7.2-5.0)\times10^{-2}}{0.25\times1000\times9.8}$$

$$h = 1.796\times10^{-3}m$$

(b)



Apparent weight plate

$$= (Mg + \gamma L) - \text{upthrust}$$

L = Perimeter , $L = 15\times2 + 0.2\times2$

$$L = 30.4cm = 0.304m$$

Upthrust = weight of liquid displaced.

$$= 15\times10^{-2}\times2.5\times10^{-2}\times0.2\times10^{-2}\times1000\times9.8$$

$$U = 78.5\times10^{-3}N = 7.35\times10^{-2}N$$

Apparent weight

$$= (25\times10^{-3}\times9.8 - 7\times10^{-2}\times0.304) - 7.35\times10^{-2}$$

∴ Apparent weight = 0.153N

Example – 81

(a) (i) consider the statement that surface tension of soap solution is $2.0 \times 10^{-2}Nm^{-1}$. What does the statement mean?

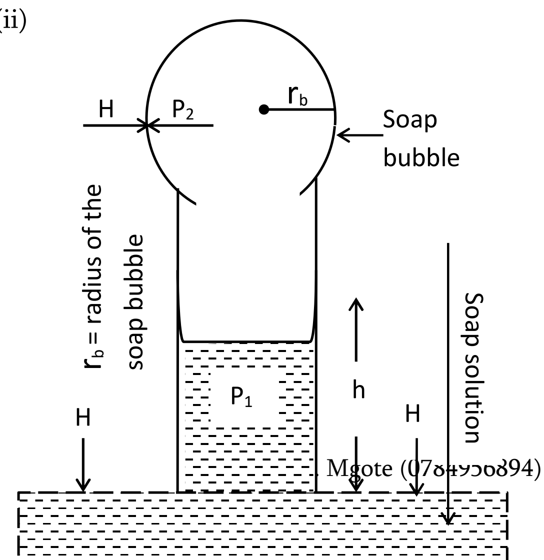
(ii) The lower end of a vertical glass tube 2.0mm in diameter dips into a soap solution and at the upper end there is a bubble of radius 20mm of the same soap solution. If the soap solution, density $1.0gm/cm^3$ rises 0.44cm up the tube, what is the surface tension?

(b) A spherical alcohol drop of radius 2.0mm is to be broken into small spherical drops of equal size each of radius 0.25mm. If the surface tension of alcohol is $2.56 \times 10^{-2}Nm^{-1}$. Find the work done.

Solution

(a) (i) The statement means that the force of $2.0\times10^{-3}N$ acts on a unit length and perpendicular to one side of liquid describe in any direction in a surface of a liquid.

(ii)



Let r = radius of the tube

r = 20mm = 20 × 10^{-3}m, h = 0.44cm

$$r_b = \frac{2mm}{2} = 1\times10^{-3}m$$

Excess pressure on the soap bubble

$$P_2 - H = \frac{4\gamma}{r_b}$$

$$P_2 = \frac{4\gamma}{r_b} + H \text{........(1)}$$

Excess pressure due to the rise of the liquid column in glass tube.

$$P_1 + \rho gh = H$$

$$P_1 = H - \rho gh \text{......(2)}$$

For the curved meniscus of the liquid in the narrow tube

[subtract equation (2) from (1)]

$$P_2 - P_1 = \left(\frac{4\gamma}{r_b} + H\right) - (H - \rho gh)$$

$$\frac{2\gamma}{r} = \frac{4\gamma}{r_b} + \rho gh$$

$$\gamma = \frac{\rho ghrr_b}{2r_b - 4r}$$

$$= \frac{1000\times9.8\times0.44\times10^{-2}\times20\times10^{-3}\times1\times10^{-3}}{2\times1\times10^{-3} - 4\times20\times10^{-3}}$$

$$\gamma = 2.4\times10^{-2}\,Nm^{-1}$$

(b) Number of the drops formed

$$n = \left(\frac{R}{r}\right)^3 = \left(\frac{2}{0.25}\right)^3 = 512\text{drops}$$

$$\text{Work done, } W = 4\pi\gamma\left[nr^2 - R^2\right]$$

$$W = 4\times3.14\times2.56\times10^{-2}$$

$$\left[512\left(0.25\times10^{-3}\right)^2 - \left(2\times10^{-3}\right)^2\right]$$

$$W = 9.0\times10^{-6}J$$

Example – 82

(a) (i) Why is not possible to separate two pieces of paper joined by glue or gum?

(ii) A liquid drop of diameter D breaks up into 27tiny drops. Find the resulting change in energy. Given surface tension of liquid is γ.

(b) (i) Define the angle of contact.

(ii) A clean glass capillary tube is held vertically in water raised to height of 7cm. the tube is down depressed and only 5cm of its length is above water. What will be the angle of contact.

Solution

(a) (i) The adhesive force between the glue and paper is greater than the force of cohesive between, piece of paper stick together with a large force and it is not possible to separate them.

(ii) Apply the law of conservation of volume

$$27\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3$$

$$r = \frac{D}{6}$$

Increase in surface area

$$\Delta A = A_2 - A_1$$

$$= 27\times4\pi r^2 - 4\pi\left(\frac{D}{2}\right)^2$$

$$\Delta A = 2\pi D^2$$

$$\text{Now: } W = \gamma\Delta A = \gamma\times2\pi D^2$$

$$W = 6.28D^2\gamma$$

(b) (ii) since $h = \frac{2\gamma\cos\theta}{\rho gr} \quad (h\alpha\cos\theta)$

$$\frac{h_2}{h_1} = \frac{\cos\theta_2}{\cos\theta_1}, \quad \cos\theta_2 = \frac{h_2\cos\theta_1}{h_1}$$

$$\cos\theta_2 = \frac{5}{7}\cos0^o = \frac{5}{7}$$

$$\theta_2 = \cos^{-1}\left(\frac{5}{7}\right) = 44.4^o$$

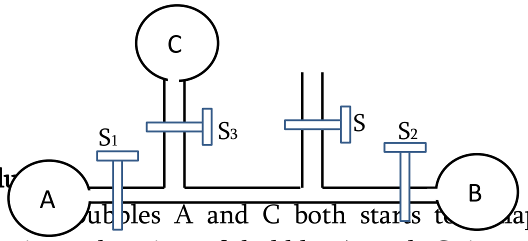
$$\theta_2 = 44.4^o$$

Example – 83 EZEB 2009/P1/4

(a) What do you understand by the term surface tension.

(b) Describe what happens to surface tension of water when some of detergent is added to water?

(c) The given figure shows three soap bubbles A, B and C prepared by blowing the capillary tube having top cocks, S, S_1, S_2 and S_3. If S closed and S_1, S_2 and S_3 remain open. How is the size of the bubbles A, B and C changes.



Solution

(c) Bubbles A and C both starts to collapse since the size of bubble A and C is small compared to the size of bubble B. Smaller the radius, greater the excess pressure. So the pressure inside the bubble A and C will be more than the pressure inside the bubble B. since air flows from higher pressure to the lower pressure, therefore the smaller bubbles A and C will go on reducing (i.e collapse) and bigger bubble B will go on expanding

Example – 84

A thin film of water of thickness 80μm is sand witched between glass plates and forms a circular patch of radius 0.12m. Take the surface tension of water to be $0.072Nm^{-1}$, and the angle of contact to be zero, calculate the normal force needed to separate the plates.

Solution

$$F = \left(\frac{2\gamma}{X}\right)A = \frac{2\times3.14\times(0.12)^2\times0.072}{80\times10^{-6}}$$

$$F = 81.4N$$

Example – 85

Two circular glass plates of radii 7cm are separated by a film of water 0.10mm thick. If the surface tension of water is $7.5 \times 10^{-2}Nm^{-1}$. What force acting at right angles to the plates is needed to pull plates apart?

Solution

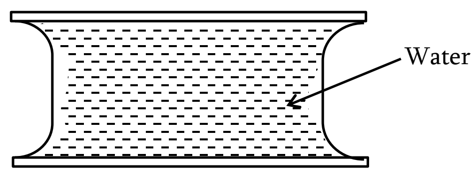
$$F = \left(\frac{2\gamma}{X}\right)A = \frac{2\times3.14\times(0.07)^2\times0.075}{1\times10^{-4}}$$

$$F = 23.09N$$

Example – 86

(a) Explain in terms of pressure differences why water rises up a capillary tube and why the level of a mercury is depressed in a capillary tube.

(b) A drop of water squeezed between two glass plates to form a thin film as shown in figure below. If the water film has a thickness of 0.002mm and on area of $400mm^2$. What is force needed to pull the plates apart? Take the surface tension of the water to be $7.0 \times 10^{-2}Nm^{-1}$.



Solution

(a) The pressure under the curved water surface in the tube is less than the pressure above it, which is approximately the same as the pressure at the surface of the water outside the tube (i.e atmosphere). To equalize the pressure in the water inside and outside the tube at the same level; the water has rise up the inside of the tube. With mercury the pressure under the curved surface in the tube is greater than the pressure above it, and so the level has to be depressed.

(b) $$F = \left(\frac{2\gamma}{X}\right)A = \frac{2\times0.07\times400\times10^{-6}}{1\times10^{-6}}$$

$$F = 56N$$

Example – 87

(a) Define bond energy

(b) Estimate the bond energy of water molecules, assuming each water molecules has an average twelve neighbours and that the equilibrium spacing for water molecules is about 10-10m. the surface tension coefficient for water is $0.07Nm^{-1}$ at 20^oC.

Solution

(a) **Bond energy** is the energy needed to separate completely a pair of molecules from the equilibrium spacing.

(b) n = 12 , $\gamma = 0.07Nm^{-1}$.
Each molecule at the surface is about 10^{-10}m from its nearest neighbor. So each surface molecule take up an area of about

$$10^{-10} \times 10 = 10^{-20} m^2$$

N = A = Number of molecules per

$$m^2 = 10^{20}$$

$$\gamma = \frac{AnE}{4}$$

$$E = \frac{4\gamma}{An} = \frac{4 \times 0.07}{10^{20} \times 12}$$

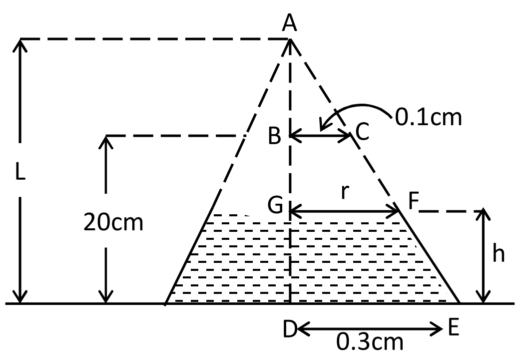
$$E = 2.0 \times 10^{-22} J$$

Example – 88

A tube of conical bore is dipped into water with apex upwards. The length of tube is 20cm and radii at the upper and lower ends are 0.1cm and 0.3cm. Find the weight to which liquid rises in the tube (surface tension of water = 0.08N/m)

Solution

Diagram meniscus has been



By using similarity theorem

$\Delta ABC \cong \Delta ADE$

$$\frac{\overline{\overline{AB}}}{AD} = \frac{\overline{\overline{BC}}}{DE}$$

$$\frac{L-20}{L} = \frac{0.1}{0.3}$$

$$L = 30cm = 0.3m$$

Also $$\frac{\overline{\overline{AD}}}{AG} = \frac{\overline{\overline{DE}}}{GF}$$

$$\frac{0.3 \times 10}{0.3 \times (10-h)} = \frac{0.3}{r}$$

$$r = \frac{0.3-h}{100}$$

Since $$\frac{2\gamma\cos\theta}{r} = \rho gh$$

$$\frac{2\gamma}{\frac{0.3-h}{100}} = 1000 \times 9.8h$$

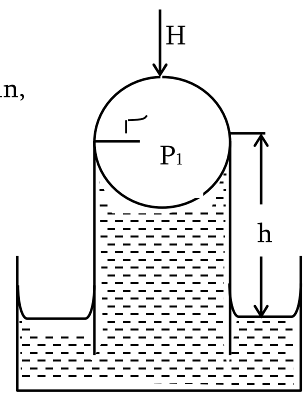
$$h^2 - 30h + \frac{800}{49} = 0$$

On solving, h = 29.45cm or 0.55cm since the length of the tube is only 20cm, h = 0.55cm.

Example – 89

A minute spherical air bubble is raising slowly through a column of mercury contained in a deep air. If the radius of the bubble at the depth of 100cm is 0.1mm. calculate its depth where the radius of the bubble is 0.126mm given that the surface tension of mercury is 4.72×10^{-1}N/m, the atmospheric pressure is 760mmHg and density of mercury is $13600kgm^{-3}$.

Solution



Depth of mercury column,
h_1 = 100cm
Radius of bubble
r_1 = 0.1mm
Atmospheric pressure
H = $1.01 \times 10^5 Nm^{-2}$,
Volume of bubble

$$V_1 = \frac{4}{3}\pi r_1^3$$

Excess pressure inside the spherical air bubble

$$P_1 - (H + \rho g h_1) = \frac{2\gamma}{r_1}$$

$$P_1 = H + \rho g h_1 + \frac{2\gamma}{r_1}$$

Let h_2 = h be depth when the radius of the bubble be r_2 = 0.126mm

Pressure inside the spherical air bubble.

$$P_2 = H + \rho g h_2 + \frac{2\gamma}{r_2}$$

Now, volume of the bubble, $V_2 = \frac{4}{3}\pi r_2^3$

Apply Boyle's law

$$P_1V_1 = P_2V_2$$

$$\left(H+\rho gh_1+\frac{2\gamma}{r_1}\right)\frac{4}{3}\pi r_1^3 = \left(H+\rho gh_2+\frac{2\gamma}{r_2}\right)\frac{4}{3}\pi r_2^3$$

$$h_2 = \frac{1}{\rho g}\left[\left(H+\rho gh_1+\frac{2\gamma}{r_1}\right)\left(\frac{r_1}{r_2}\right)^3-\left(H+\frac{2\gamma}{r_2}\right)\right]$$

$$= \frac{1}{13600\times 9.8}\left[\left(1.01\times10^5+13600\times9.8\times1\right)\right]$$

$$+\left(\frac{2\times0.472}{0.1\times10^{-3}}\right)$$

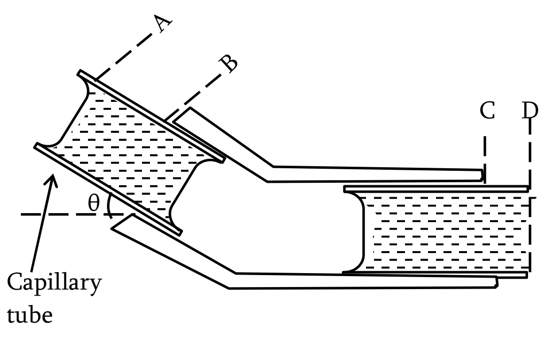
$$\left(\frac{0.1}{0.126}\right)^3-\left(1.01\times10^5+\frac{2\times0.472}{0.126\times10^{-3}}\right)$$

$$h_2 = 0.10\text{m} = 10\text{cm}$$

Example – 90 (Roger B33)

(a) Draw and label a diagram of apparatus suitable for measuring the surface tension of water by Jaeger's method.

(b) The diagram below which is not scale, shows two capillary tubes of uniform bore fitting tightly into a short length of rubber tubing, AB and CD are two threads of water. The capillary tube containing CD is kept horizontal while that containing AB is raised through an angle θ until the water surface at D is both flat and vertical.



(i) Calculate the surface tension of water given that θ is 10.5O, AB is 11.4cm, the radius of the capillary tube at C is 0.72mm and the density of water is 1000kgm-3. The angle of contact between water and glass is zero.

(ii) Suggest an experimental procedure to determine the water surface at D is flat.

Solution

(b) (i) The vertical height of the liquid column AB is

$$h = AB\sin\theta = 11\sin 10.5^{O}$$

$$h = 2.077\text{cm}$$

The excess pressure for the column AB

$$P = \rho gh$$

$$= 1000\times9.8\times2.077\times10^{-2}$$

$$P = 203.59\text{Pa}$$

This is also the excess pressure across the column, CD

$$P = \frac{2\gamma\cos\theta}{r}$$

$$\gamma = \frac{pr}{2\cos\theta} = \frac{0.72\times10^{-2}\times203.5}{2\cos0^{O}}$$

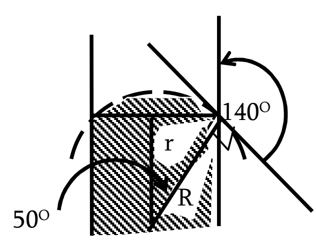
$$\gamma = 0.0733\text{Nm}^{-1}$$

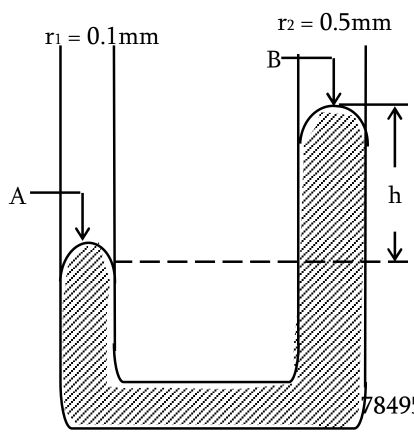
(ii) This is by observing the liquid at D where there are no drops of liquid falling then the surface is flat.

Example – 91

Two lengths of capillary tubing of diameters 0.2mm and 1mm respectively are joined to make a U – tube in which mercury is placed. What is the difference between the levels of the mercury in the two tubes if the surface tension of mercury is 0.46Nm^{-1} and its angle of contact with glass 140^{O}? (Density of mercury = $1.36\times10^4\text{kgm}^{-3}$)

Solution





r = radius of capillary tube
R = radius of curvature of mercury surface

$$\cos 40^{O} = \frac{r}{R} \quad , \quad R = \frac{r}{\cos 40^{O}}$$

Excess pressure at A above atmospheric

$$P_1 = \frac{2\gamma}{R_1} = \frac{2\gamma \cos 40^{O}}{r_1}$$

Excess pressure at B above atmospheric

$$P_2 = \frac{2\gamma \cos 40^{O}}{r_2}$$

Pressure difference between at A and B

$$\rho gh = \frac{2\gamma \cos 40}{r_1} - \frac{2\gamma \cos 40}{r_2}$$

$$9.8 \times 13600 \times h = 2 \times 0.46 \times 10^{4} \cos 40 \left(1 - \frac{1}{5}\right)$$

$$h = 0.0423\text{m}$$

CONCEPTUAL PROBLEMS.

1. (a) What is wetting agent?
   (b) Give reasons for the following statements.
   (i) A mercury barometer always reads less than actual pressure. Why?
   (ii) Why a small drop of mercury is spherical but bigger drops are oval in shape?
   (iii) Why is not possible to separate two pieces of paper joined by glue or gum? Explain.

Solution

(a) A wetting agent is a chemical used in dyeing industry. It decreases the angle of contact between cloth and the dye. As a result, the dye easily penetrates into the cloth. It helps in uniform colouring of the cloth.

(b) (i) Due to capillary action, mercury is depressed in the barometer tube and hence it will always read less than the actual pressure.

(ii) In a small drop, the force due to surface tension is very large as compared to its weight and hence it is spherical in shape. A big drop becomes oval in shape due to its large weight.

(iii) The force of adhesion between the molecules of glue and the paper is very large as compared to the force of cohesion between the glue molecules. For this reason, two pieces of paper joined by glue cannot be separated.

2. Explain the reasons behind:-
   (a) An air bubble in water rises from bottom totop and becomes bigger in size.
   (b) A liquid inside a dropper comes out only when its rubber bulb is pressed hard.
   (c) The end of a thread is often wet by a person before trying to put it through the eye of a needle.

Solution

(a) The fluid pressure increases with depth. So ,pressure at the top is less than the pressure at the bottom. So air bubble will rise from bottom top as fluids have a tendency to move from higher pressure to lower pressure. Now, according to Boyle's law , volume will increase since the pressure decreases at the top (PV = constant) so the bubble will grow in size.

(b) Liquid is held inside the dropper against the atmospheric pressure. So when the rubber bulb is pressed hard, pressure on the liquid becomes more than the atmospheric pressure and it comes out.

(c) On wetting the end of a thread a thin film of water is formed over its fibres. Due to surface tension of the water, the fibres of the thread cling together as a result, the area of cross section of the thread decreases and it becomes easier to put it through the eye of a needle.

3. (a) Why does we ink get absorbed by a blotting paper?
   (b) In summer, cotton dress is preferable give reason.

Solution

(a) The fine pores in the blotting paper act as capillaries, when it is placed on the wet ink, the ink rises due to capillary action.
(b) The cotton dresses have fine pores, in summer, when our body sweats, the sweat is sucked by the cloth due to capillary action.

4. (a) Oil is poured to calm sea waves. Explain, why.
(b) A piece of chalk immersed into water emits bubbles in all direction. Why?

Solution

(a) When oil is poured on water, its surface tension decreases. Due to this, water tends to acquire large surface area. Therefore, when oil is poured, the sea wave get calm.
(b) A piece of chalk has extremely narrow capillaries as it immersed in water, water rises due to capillary action. The air present in the capillaries in chalk is forced out by the rising water as a result, bubbles are emitted from the chalk in all directions.

5. (a) End of a glass tube becomes round on heating. Explain.
(b) It is easier to wash clothes in hot water soap solution. Why?
(c) Water rises in capillary tube, whereas mercury falls in the same tube. Explain.

Solution

(a) When an end of a glass tube is heated, the glass at end melts, in molten (liquid) state, in an attempt to acquire minimum surface area due to the property of surface tension, the end of the glass tube becomes spherical i.e round.
(b) A hot water soap solution has considerably lower value of surface tension than that of normal water. Due to low value of surface tension, the hot soap solution wets the dirty cloth in a better way and thus achieves greater cleansing action.
(c) The height up to which a liquid rises in capillary tube

$$h = \frac{2\gamma\cos\theta}{r\rho g}$$

For water, θ is acute and therefore $\cos\theta$ is positive and hence h is positive. Due to this water rises in a capillary tube for mercury θ is obtuse angle and therefore $\cos\theta$ is negative and hence h is negative for this reason, mercury gets depressed in the capillary tube.

6. (a) Sand is a drier soil than clay why?
(b) A large force is required to draw apart normally two glass plates enclosing a thin water film.

Solution

(a) In clay, capillaries are formed due to pores, while in sand, no such capillaries are formed. Due to capillary action water rises in clay and it appears damp. In the absence of capillaries, water cannot rise in sand and hence it is a drier soil.
(b) The thin water film formed between the two glass plates will have concave surface all around. Since on the concave side of a liquid surface, pressure is more, work will have to be done in drawing the plates.

7. Why is a soap solution is a better cleaning agent than ordinary water?

Solution

A cloth has narrow spaces in the form of fine capillaries. The capillary rise is given by

$$h = \frac{2\gamma\cos\theta}{\rho g r}$$

Addition of soap to water reduces the angle of contact, θ. This will increase the value of $\cos\theta$ and hence h. This means that soap water will rise more in the narrow spaces in the cloth and clean fabrics better than water can do alone.

8. A drop of oil placed on the surface of water spreads out, but a drop of water placed on oil contracts. Why?

Solution

The cohesive force between the oil molecules are less than the adhesive force between water molecules and oil molecules. Therefore, the drop of oil placed on water surface spreads out. On the other hand, the cohesive forces between water molecules are greater than the adhesive forces between water molecules and oil molecules. So the drop of water placed on oil surface contracts.

9. (a) Why are soap bubbles almost perfect spheres?
   (b) Why are falling rain drop spherical?

Solution
(a) The soap bubbles are almost perfect spheres because they have large areas and negligible mass. The effect of gravitational forces is almost nil and the shape of the bubbles is maily due to surface tension forces.
(b) The rain drops falling freely under gravity are spherical. It is because every part of the drop is being accelerated to the same and the acceleration cannot. Therefore affect the shape of the drop.

10. What determines the shape of the meniscus?
Solution
The surface of the liquid is generally curved where it is in contact with a solid. Whether the meniscus is concave or convex depends on the liquid concerned and on the solid with which it is contact. If the adhesive force is large compared with the cohesive force, the liquid tends to stick to the wall and so has a concave meniscus. In this case, the angle of contact in less than 90°. On the other hand, if the adhesive force is small compared with the cohesive force, the liquid surface is pulled away from the wall and the meniscus is convex. In this case, the angle of contact is greater than 90°.

11. (a) Why the hot soap is tastier than the cold soap?
    (b) If you float matchsticks on the surface of water between the matchsticks by a hot needle, the matchsticks will fly apart. Why?

Solution
(a) The surface tension of hot soap is less than that of cold soup. Consequently, the hot soap will spread over large area of the torque for this reason hot soup is tastier than the cold soup.
(b) It is because hot needle lowers the surface tension of water between the matchstick and the pull of the water molecules on the outside matchsticks becomes greater than the pull on the inside. As a result the matchsticks fly apart.

12. Give reasons for the following statements:-
    (a) What is the utility of ploughing of a field?
    (b) Air is blowing into a soap bubble. What will be the effect on the pressure inside a soap bubble?
    (c) Greased cotton soaks less than ordinary cotton.

Solution
(a) This is done to break the tiny capillaries through which water can rise and finally evaporate. The ploughing of field helps the soil to retain the moisture.
(b) When air blown into a soap bubble, its radius increases. Since excess pressure is inversely proportional to the radius, therefore the excess pressure will decrease.
(c) This is because the presence of grease reduces the surface tension.