MODULE 10 : ELASTICITY

Definition Elasticity - is the property of a body by virtual of which the body regains its original length , volume and shape after the deforming forces have been removed.

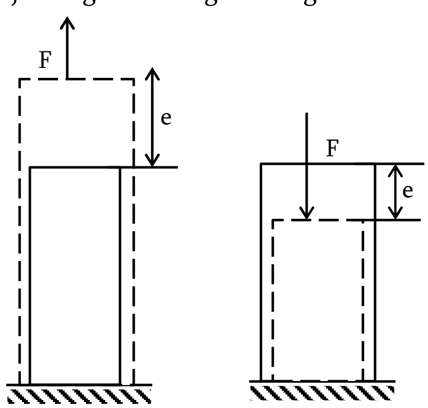
TYPES OF ELASTICITY

There are three types of elasticity:-

1. Elasticity of material due to extension or compression .
2. Bulk or volume elasticity
3. Elasticity of rigidity

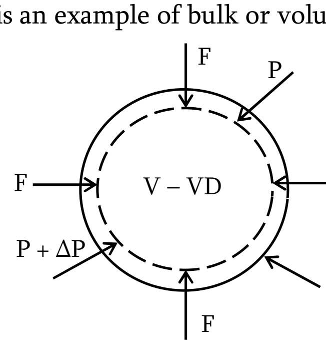
ELASTICITY OF EXTENSION OR COMPRESSION

The deforming forces causes a change in length of an object on removal of the deforming force , then an object regain its original length



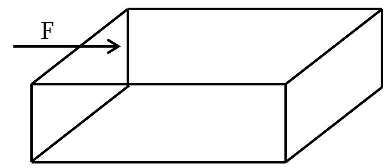
BULK OR VOLUME ELASTICITY

Is the kind of elasticity in which the deforming force produces a change in volume and after removal of deforming force its regain original volume. The deforming force produces a change in volume , since the rubber ball is pressed from all sides , its volume decreases when the pressure is removed , the ball regain its original volume. This is an example of bulk or volume elasticity.



ELASTICITY OF RIGIDITY

Is the tensile forces produces a change in the shape or size of an object is known as **'elasticity of rigidity'**



SOME IMPORTANT TERMS

1. **RIGID BODY** – is the body which is not deformed under the action of various forces.

2. **TENSILE FORCE** – is the load (force) which stretches the material. Sometimes tensile force is known 'deforming force'.

3. **ELASTIC DEFORMATION** – A material is said to undergo elastic deformation if its exactly to its original shape and size on the removal of the load.

4. **ELASTIC LIMIT OF A MATERIAL** – is the maximum amounts by which it can be stretched and still regain original shape after the distorting forces are removed. The maximum stress from which can elastic body will recover its original state after the removal of the deforming force is called **'ELASTIC LIMIT'** – It is the upper limit of deforming force up to which, if deforming force is removed , the body regains its original form completely if deforming force is increased beyond that limit , the body losses its property of elasticity and gets permanently deformed. Elastic limit is the property of a body whereas elasticity is the property of the material.

5. **YIELD POINT** – Is the point beyond elastic limit at which the material changes from elasticity to plasticity.

6. **ELASTIC BODIES** – are those bodies which possess the property of elasticity.

TYPES OF ELASTIC BODY
(i) Perfectly elastic body
(ii) Perfectly inelastic body

PERFECTLY ELASTIC BODY
Is the body which regains its exact original shape and size immediately after the removal of the deforming force. Examples quartz and phosphor bronze.

PERFECTLY INELASTIC BODY
Is the body which does not have any tendency at all to regain its original shape and size on the removal of the slightest deforming force. Example wet clay , paraffin wax.

7. **PLASTIC BODIES**
Are those bodies which do not show any tendency to recover their original form after the deforming forces are removed. Example : Putty , plasticine , clay , kneaded flour.

8. **PLASTICITY** – is the property of remaining deformed even after the removal of deforming forces.

9. **PLASTIC DEFORMATION** – A material is said to be under plastic deformation if its doesn't return exactly to its original shape and size on the removal of the load.

10. **DUCTILITY** - A ductile material is the material which can be permanently stretched i.e ductile material – is the material which lengthen considerably and undergo plastic deformation before they break. Ductile material can be drawn into wires examples of ductile materials copper , lead wrough iron.

11. **BRITTLENESS**
brittle material – is the material which cannot be permanently stretched it breaks soon after elastic limit reached. Brittle materials can only undergo deformation. Brittle material are often very strong in compression and weak in extension. Example glass , high carbon steel , cast iron.

12. **STIFFNESS** – Is the resistance which materials offers to having its size and shape changed. Shiffness of material is measured by using Young's modulus.

13. **BREAKING STRESS** – Is the stress at the breaking point.

14. **STRENGTH OR ULTIMATE TENSILE STRESS (UTS)** of material is the greatest stress it undergoes before breaking i.e is the maximum stress which can be applied to a materials beyond which the material will break.

15. **YIELD STRESS** – Is the stress when the material begins plastic behavior.

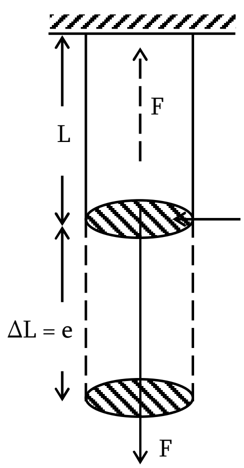
16. **ELASTIC ENERGY (STRAIN ENERGY)** – Is the amount of energy stored in the wire i.e The work done in stretching or deforming a wire gets stored in the wire in the form of its elastic potential energy is called strain energy.

17. **TENSILE STRESS , STRAIN AND YOUNG'S MODULUS**
(i) **STRESS** – is defined as the internal force of restitution per unit area of a deformed body quantitatively.
Stress – Is defined as deforming force acting per unit area of the body.

TYPES OF STRESS
There are three types of stress:
(a) Normal stress
(b) Tangential or shearing stress
(c) Hydrostatic stress or hydraulic stress.

NORMAL STRESS – if the deforming force acts normally over an area of a body , then the internal force of restitution per unit area of the deformed body is known as normal stress. **There are two types of normal stress:** (i) Tensile stress (ii) Compressive stress.

TENSILE STRESS – Is defined as the restoring force developed per unit cross – sectional area of a body when the length of the body increases in the direct of the deforming force. Consider a stretching force F applied to a material of original length L and the cross – sectional area , A



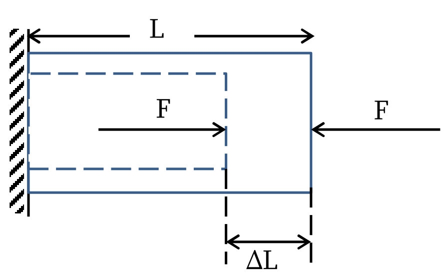
$$\text{Tensile stress} = \frac{\text{Applied force}}{\text{Cross - sectional area}}$$

$$\delta = \frac{F}{A}$$

SI. Unit of tensile stress is Nm^{-2}(Pa)

$$\left[\text{stress}\right] = \left[ML^{-1}T^{-2}\right]$$

COMPRESSIVE STRESS – Is defined as the restoring force developed per unit cross – sectional area of body when the body is compressed i.e when its length decreases under the action of the deforming force.



(b) **TANGENTIAL OR SHEARING STRESS**
When the deforming force acting tangentially to the surface of a body changes the shape of the body then the stress set up in the body is called 'tangential stress'.

(c) **HYDROSTATIC STRESS OR HYDRAULIC STRESS**
The internal restoring force per unit area developed in a body when the body is compressed uniformly from all sides is called hydrostatic stress or hydraulic stress.

(ii) **STRAIN** – is defined as the ratio of change in configuration to the original configuration of the body.

TYPES OF STRAIN

There are three types of strain

(a) Tensile or longitudinal or linear strain.
(b) Bulk or volumetric strain
(c) Shear strain

- Tensile strain – is defined as the ratio of change in length to the original length.

$$\text{Tensile strain}\left(\varepsilon\right) = \frac{\text{Extension}}{\text{Original length}}$$

$$\varepsilon = \frac{\Delta L}{L} = \frac{e}{L}$$

Tensile strain is the dimensionless quantity and has no unit.

- Bulk or volumetric strain - Is defined as the ratio of change in volume to the original volume within the elastic limit

$$\text{Bulk strain} = \frac{\Delta V}{V}$$

- Shear strain – is defined as the ratio of the relative displacement of one plane to its distance from the fixed plane within the elastic limit.

$$\text{Shear strain} = \tan\theta = \frac{\Delta L}{L}$$

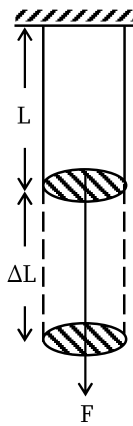
TYPES OF MODULI OF ELASTICITY

There are three types of moduli of elasticity

1. Young's modulus of elasticity
2. Modulus of rigidity
3. Bulk modulus of elasticity

1. YOUNG'S MODULUS OF ELASTICITY

Consider a wire of length L and cross – sectional area , A stretched by a force F through a distance ΔL or e.



ΔL = e = extension of the wire
L = original length of the wire
A = cross – sectional area

$$\text{Tensile stress} = \frac{\text{applied force}}{\text{cross - sectional area}}$$

$$\text{Tensile strain} = \frac{\text{extension}}{\text{original length}}$$

YOUNG'S MODULUS (E or Y)

Is defined as tensile stress per tensile strain

$$E = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

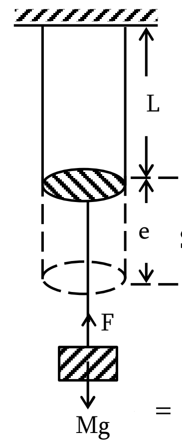
$$E = \frac{\delta}{\varepsilon} = \frac{F/L}{\Delta L/L} = \frac{FL}{A\Delta L}$$

$$E = \frac{FL}{A\Delta L} = \frac{FL}{Ae}$$

S.I Unit of Young's Modulus is Nm^{-2} or Pa.
Its dimensional formula is $[ML^{-1}T^{-2}]$

Additional concepts

(i) Suppose the block of mass , M is suspended on the wire which is fixed at one point as shown in the figure below.



F = Mg

$$A = \pi r^2 = \frac{\pi d^2}{4}$$

r = radius of the wire
d = diameter of the wire

Since

$$E = \frac{FL}{Ae}$$

$$= \frac{FL}{Ae} = \frac{MgL}{\pi r^2 e} = \frac{4MgL}{\pi d^2 e}$$

(ii) HOOKE'S LAW AND YOUNG'S MODULUS

HOOKE'S LAW

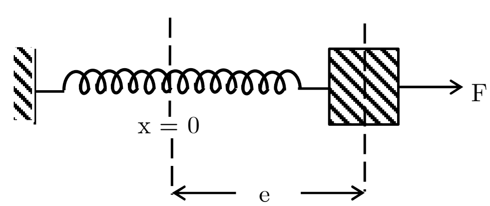
State that 'Provided that elastic limit is not exceeded , the extension (e) provided in the wire is directly proportional to the applied force (load)' i.e $F \propto e$……within the elastic limit

$F \propto e$, $F = Ke$

F = applied force (load)

e = extension of the wire

k = constant of the proportionality known as the force constant.



FORCE CONSTANT (K)

Is defined as the applied force (load) per unit extension

$$K = \frac{F}{e}$$

S.I unit of force constant is Nm^{-1}. Now , stress is a measure of the deforming force and strain is the measure of distortion. Therefore we can restate Hooke's law as follows 'within the elastic limit , the strain produced in a elastic body is directly proportional to the stress'. i.e stress α strain

$$\frac{\text{stress}}{\text{strain}} = \text{Young's Modulus} = \text{constant}$$

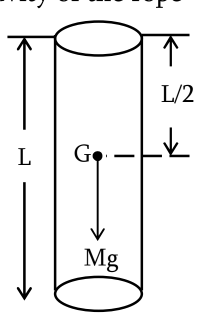
(iii) Problem of thick uniform rope hanging from the ceiling of a room.

Let: M = Mass of rope , L = length of rope

A = cross – sectional area of the rope

ΔL = increase in length of the rope due to its own weight.

The weight of the rope act at the centre of gravity of the rope



$$E = \frac{Mg \times L/2}{A\Delta L}$$

$$E = \frac{MgL}{2A\Delta L}$$

$$\Delta L = \frac{MgL}{2AE}$$

But

$$M = LA\rho$$

$$\Delta L = \frac{(\rho AL)gL}{2AE}$$

$$\Delta L = \frac{\rho g L^2}{2E}$$

APPROXIMATE VALUE OF E OR YOUNG'S MODULUS.

Material	E($\times 10^{11}$Pa)
Aluminum	0.70
Brass	0.91
Copper	1.1
Glass	0.55
Iron	1.9
Lead	0.16
Nickel	2.1
Steel	2.0
Tungsten	3.6
Rubber	1.0 $\times 10^6$

NUMERICAL EXAMPLES

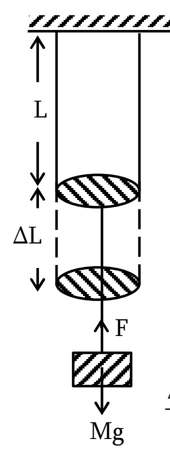
1. (a) What do you mean by the following terms:-
   (i) Elastic bodies (ii) plastic bodies
   (iii) Elastic energy (iv) stress (v) strain

   (b) What is the percentage increase in length of a steel wire of diameter 4.0×10^{-2}cm , when a load of 5kg is suspended from its free end if the other is securely supported (Young's Modulus of the steel $E = 2.0 \times 10^{11}Nm^{-2}$) , $g = 9.81 m/s^2$.

Solution

(a) Refer to your notes

(b)



Since $E = \frac{\text{stress}}{\text{strain}}$

$$E = \frac{FL}{A\Delta L}$$

But

$$F = Mg \quad , \quad A = \frac{\pi d^2}{4}$$

$$\frac{\Delta L}{L} = \frac{4Mg}{\pi d^2 E}$$

$$\frac{\Delta L}{L}\times 100\% = \left[\frac{4Mg}{\pi d^2 E}\right]\times 100\%$$

$$\frac{\Delta L}{L}\times 100\% = \left[\frac{4\times 5\times 9.81}{3.14\times\left(4\times 10^{-4}\right)^2\times 2\times 10^{11}}\right]\times 100\%$$

$$\frac{\Delta L}{L}\times 100\% = 2\%$$

2. (a) Define elasticity. Explain the cause of elasticity
   (b) (i) What are the factors affecting elasticity.
   (ii) A string 60cm long stretched by 2cm by the application of a load of 200gm. What will be the length when a load of 500gm is applied?

Solution

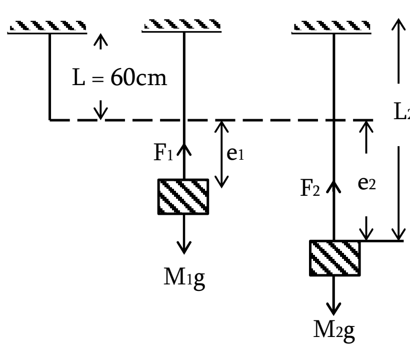
(a) Refer to your notes

Cause of elasticity

When a body is compressed , the atoms constituting the body come close to each other. Due to this , the mean distance between the atoms decreases. A restoring force (repulsive nature) comes into play, which tends to send the atoms back to their normal separation on removal of deforming force. On the other hand when body is elongated , the mean distance between the atoms increases . now , the restoring force is attractive in nature and it tends to bring the atoms back to their normal separation on removal of deforming force.

(b) (i)
- Effect of hammering and rolling
- Effect of annealing
- Effect of the presence of impurities.
- Effect of temperature

(ii)



Since $E = \frac{FL}{Ae} = \frac{MgL}{Ae}$

$$e = \frac{MgL}{EA}$$

$$e_1 = \frac{M_1 gL}{EA}, \quad e_2 = \frac{M_2 gL}{EA}$$

Takes

$$\frac{e_2}{e_1} = \frac{M_1 gL}{EA}\Big/\frac{M_2 gL}{EA}$$

$$\frac{e_2}{e_1} = \frac{M_2}{M_1} \Rightarrow e_2 = e_1\left[\frac{M_2}{M_1}\right]$$

$$e_2 = 5\text{cm}$$

Now length $L_2 = L + e$

$$L_2 = 60 + 5$$

$$L_2 = 65\text{cm}$$

3. (a) What mass should be suspended from the end of steel wire 2m in length and 2mm in diameter to increase the length by 1mm?
   (Young's Modulus = $19\times 10^{10}\text{Nm}^{-2}$)
   (b) A steel wire of length 3.6m and cross – section $2.5\times 10^{-5}\text{m}^2$ under a given load. What is the ratio of the Young's Modulus of steel to that of copper?

Solution

(a) Young's Modulus

$$E = \frac{FL}{Ae} = \frac{MgL}{\pi r^2 e}$$

$$M = \frac{\pi r^2 eE}{gL}$$

But

$$r = \frac{d}{2} = \frac{2\text{mm}}{2} = 1\text{m}$$

$$M = \frac{3.14\times\left(1\times 10^{-3}\right)^2\times 1\times 10^{-3}\times 19\times 10^{10}}{9.81\times 2}$$

$$M = 30.5\text{kg}$$

(b) For steel: $L_1 = 3.6\text{m}$, $A_1 = 2.5\times 10^{-5}\text{m}^2$.
For copper: $L_2 = 2.4\text{m}$, $A_2 = 3.2\times 10^{-5}\text{m}^2$.
Let F be the stretching force and e , the increase in length in each case. If E_1 and E_2 are Young's Modulus of steel and copper respectively.

$$E_1 = \frac{FL_1}{A_1e} \text{ and } E_2 = \frac{FL_2}{A_2e}$$

$$\frac{E_1}{E_2} = \frac{FL_1}{A_1e} \Big/ \frac{FL_2}{A_2e}$$

$$\frac{E_1}{E_2} = \left(\frac{L_1}{L_2}\right)\left(\frac{A_2}{A_1}\right)$$

$$= \frac{3.6 \times 3.2 \times 10^{-5}}{2.5 \times 2.4 \times 10^{-5}}$$

$$\frac{E_1}{E_2} = 1.92$$

4. (a) Two wires of the same length and material are stretched by the same force. If the radii of the wires are in the ratio 1:2. What is the ratio of elongation produced.

   (b) The breaking stress of a material is $106Nm^{-2}$ if the density of the material is $3.0 \times 10^3 kgm^{-3}$, then what should be the length of the wire made if this material so that it breaks under its own weight?

Solution

(a) $\frac{r_1}{r_2} = 1:2 = \frac{1}{2}$

Since $E = \frac{FL}{Ae} = \frac{FL}{\pi r^2 e}$

$$e = \frac{FL}{\pi r^2 E}$$

$$e_1 = \frac{FL}{\pi r_1^2 E} \quad , \quad e_2 = \frac{FL}{\pi r_2^2 E}$$

$$\frac{e_2}{e_1} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{1}{2}\right)^2$$

$$\frac{e_2}{e_1} = 1:4$$

(b) $\text{Breaking stress} = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$

$$\text{Breaking stress} = \frac{Mg}{A} = \frac{\rho ALg}{A}$$

$$L = \frac{\text{Breaking stress}}{\rho g}$$

$$= \frac{10^6}{3 \times 10^3 \times 9.8}$$

$$L = 34.01m$$

5. A stone of 0.5kg is attached to one end of a 0.8m long aluminum wire of 0.7mm diameter and suspended in a horizontal plane at a rate such that the wire makes an angle of 85° with vertical. Find the increase in the length of the wire. (Young's Modulus of Aluminum = $7 \times 10^{10}Nm^{-2}$)

Solution

$g = 9.8m/s^2$

$M = 0.5kg$

$L = 0.8m$

$\theta = 85^\circ$

$E = 7.0 \times 10^{10}Nm^{-2}$

$d = 0.7mm$

$e = ?$

At the equilibrium

$$T\cos\theta = Mg$$

$$T = \frac{Mg}{\cos\theta} = \frac{0.5 \times 9.8}{\cos 85^\circ}$$

$$T = 56.19N$$

$$\text{Now } E = \frac{FL}{Ae} = \frac{4TL}{\pi d^2 e}$$

$$e = \frac{4 \times 56.19 \times 0.8}{3.14 \times \left(0.7 \times 10^{-3}\right)^2 \times 7 \times 10^{10}}$$

$$e = 1.67 \times 10^{-3}m = 1.67mm$$

6. (a) (i) Distinguish between elasticity and plasticity of materials.

   (ii) Elasticity has different meaning in physics and in our daily life. Comment.

   (b) By how much a rubber string length 10cm increases in length under its own weight when suspended vertically $g = 10m/s^2$ (density of rubber $= 1.5 \times 103kgm^{-3}$ Young Modulus of Rubber $= 5.0 \times 108Nm^{-2}$)

Solution

(a) (i) Refer to your notes

   (ii) In daily , life a body is said to be more elastic , if large deformation or strain is produced on subjecting the material to a given stress. However in physics, it is exactly opposite. A body is said to be more elastic, if a small strain is produced on applying the given stress.

(b) Let A = cross – sectional area of rubber string.

$$M = \rho V_o = \rho AL$$

$$\text{stress} = \frac{Mg}{A} = \frac{\rho ALg}{A} = \rho Lg$$

$$= 1.5\times10^3\times10\times10$$

$$\text{stress} = 1.5\times10^5\,Nm^{-2}$$

The weight of the string acts at its centre of gravity therefore , it will produce extension only in 5m length of the wire

$$\text{strain} = \frac{\Delta L}{L} = \frac{\Delta L}{5}$$

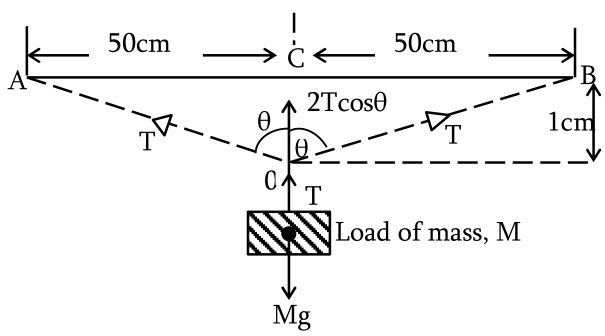
$$E = \frac{\text{stress}}{\text{strain}} = \frac{1.5\times10^5}{\Delta L/5}$$

$$\Delta L = \frac{1.5\times10^5\times5}{E} = \frac{1.5\times5\times10^5}{5\times10^8}$$

$$\Delta L = 1.5\times10^{-3}m = 1.5mm$$

7. A steel wire of diameter 0.8mm and length 1m is damped firmly at two points A and B which are 1m apart in the horizontal plane. A body is hung from the midpoint of the wire such that mid points sags 1cm from original position , calculate the mass of the body given $Y = 2.0\times10^{11}Nm^{-2}$.

Solution



At the equilibrium

$$2T\cos\theta = Mg$$

$$M = \frac{2T\cos\theta}{g}\ \ldots\ldots\ldots\ldots\ldots(i)$$

$$\text{Since } E = \frac{TL}{Ae} \Rightarrow T = \frac{EAe}{L}$$

Extension of the wire

$$e = \overline{AO}+\overline{OB}-\overline{AB}$$

$$\text{But } \overline{AO} = \overline{OB}$$

$$e = 2\overline{AO}-\overline{AB}$$

By using Pythagoras theorem

$$\overline{AO} = \sqrt{\overline{OC}^2+\overline{AC}^2}$$

$$= \sqrt{1^2+50^2} = 50.009999cm$$

$$2\overline{AO} = 100.019998cm$$

$$e = 100.019998-100$$

$$e = 0.019998-100$$

From the figure above

$$\cos\theta = \frac{\overline{OC}}{\sqrt{\overline{BC}^2+\overline{OC}^2}} = \frac{1}{\sqrt{50^2+1^2}}$$

$$\cos\theta = \frac{1}{50.01}$$

From equation (i)

$$M = \frac{2T\cos\theta}{g} = \frac{2EAe}{gL}\cos\theta$$

$$M = 0.082kg$$

8. A 45kg traffic light is suspended with two steel wires of equal lengths and radii of 0.5cm. if the wires make an angle of 15° with horizontal , what is the fractional increase in their length due to the weight of the light. Given that the Young's Modulus of steel = $2.0\times10^{11}Nm^{-2}$.

Solution

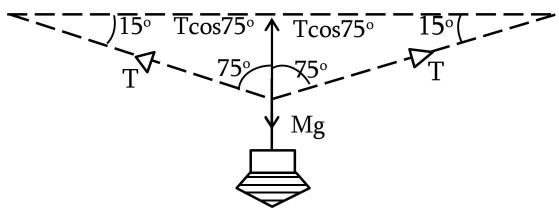
$E = 2.0\times10^{11}Nm^{-2}$, or 0.5cm = 0.005m

The cross – sectional area of the wire

$$A = \pi r^2 = \pi(0.005)^2$$

$$A = 7.85\times10^{-5}m^2$$

The traffic light of weight Mg is suspended with two steel wires each of length L (say) a shown in figure below



Let T be the tension in each of the two wires and ΔL be extension produced in their lengths. At the equilibrium of the system.

$$T\cos 45^\circ + T\cos 75^\circ = Mg$$

$$T = \frac{Mg}{2\cos 75^\circ} = \frac{45\times 9.8}{2\times 0.2588}$$

$$T = 852N$$

The tension in the wire acts as the stretching force for it.

$$E = \frac{TL}{A\Delta L}$$

$$\frac{\Delta L}{L} = \frac{T}{AE} = \frac{852}{7.85\times 10^{-5}\times 2\times 10^{11}}$$

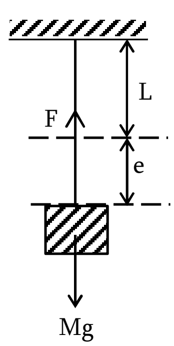
$$\frac{\Delta L}{L} = 5.42\times 10^{-5}$$

9. A wire loaded by a weight of density 7.8gcm-3 is found to be length 95cm on immersing the weight in water the length decreased by 0.19cm. Find the original length of the wire.

Solution

Let L be the length and A, the area of cross – section of the wire. If V is the volume of the weight attached to the wire , then

In air



$$F = Mg = \rho Vg$$

$$= 7800\times 9.8V$$

The extension of the wire

$$e = 95 - L = 0.95 - L$$

Now , Young's Modulus

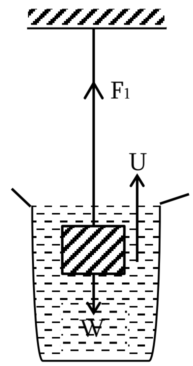
$$E = \frac{FL}{Ae}$$

$$E = \frac{76,440VL}{A(0.95-L)} \quad \ldots\ldots(i)$$

In water

Apparent weight

$$F_1 = F - \text{Weight of water displaced}$$



$$F_1 = V\rho g - V\delta g$$

$$= Vg(\rho - \delta)$$

$$= Vg(7800 - 1000)$$

$$F_1 = 66,640V$$

$$L_1 = 95 - 0.19 = 94.81cm$$

Now extension

$$e_1 = 94.81 - L = 0.9481 - L$$

Now

$$E = \frac{F_1L}{Ae_1}$$

$$E = \frac{66,640VL}{A(0.9481-L)} \quad \ldots\ldots.(2)$$

$$(1) = (2)$$

$$\frac{76,440VL}{A(0.95-L)} = \frac{66,640VL}{A(0.9481-L)}$$

$$L = 0.9352m = 93.52cm$$

10. A wire is loaded with weight of density $9.0 \times 10^3 kgm^{-3}$ and its length is found to be 0.98m. On immersing the weight in water the length is shorted by 2.5mm. Find the original length of the wire.
Answer : 95.75cm

11. A mass of 20kg is attached to one end of a steel of wire 50cm long and is rotated in a horizontal circle. The area of cross – section of the wire is $10^{-6}m^2$ and the breaking stress for it is 4.8×10^7Pa. calculate the maximum velocity with which the mass can be rotated.

Solution

For a body rotating in a circular path the centripetal force acting on it is $M\omega^2 r$.

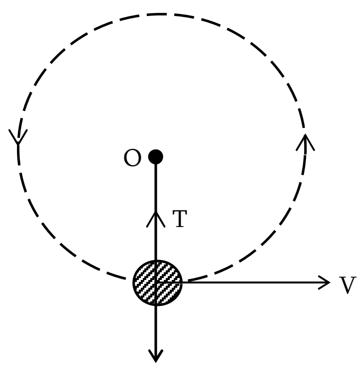
$$\text{Breaking stress} = \frac{M\omega^2 r}{A}$$

$$4.8\times10^7 = \frac{20\times0.5\omega^2}{10^{-6}}$$

$$\omega = 2.2\text{rad/s}$$

12. A sphere of mass 3kg is attached to one end of a steel wire of length 1m and radius 1mm. it is whirled in a vertical circle with an angular velocity of 2rev/s. what is the elongation of the wire when the weight is at the lowest point of its path? E for steel = $20\times10^{10}\text{Nm}^{-2}$.

Solution



Let T be tension in the wire when the sphere is at the lowest position.

$$T = M\omega^2 R + Mg = M\left(\omega^2 R + g\right)$$

Since

$$E = \frac{FL}{Ae} = \frac{TL}{Ae}$$

$$e = \frac{TL}{EA} = \frac{M\left(\omega^2 R + g\right)L}{\pi r^2 E}$$

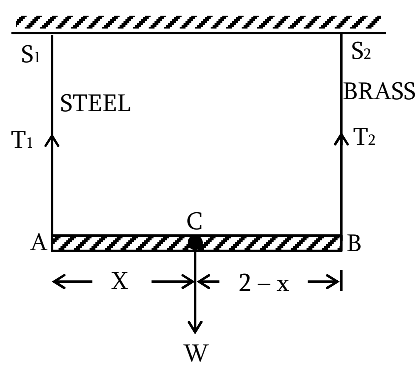
$$e = \frac{3\times\left[\left(2\pi\times2\right)^2\times1+9.8\right]\times1}{20\times10^{10}\times3.14\times\left(1.0\times10^{-3}\right)^2}$$

$$e = 8.004\times10^{-4}\text{m}$$

13. A light rod of length 2m is suspended from the ceiling horizontally by means of two vertical wires of equal length tied to its ends one of the wires is made of steel and is of cross – section 10^{-3}m^2 and the other is of brass of cross – section $2\times10^{-3}\text{m}^2$. Find out the position along the rod at which a weight may be hung to produce;
(a) Equal stresses in both wires
(b) Equal strains in both wires?
Young's Modulus for brass = 10^{11}Nm^{-2}
Young's Modulus for steel = $2\times10^{11}\text{Nm}^{-2}$

Solution

Figure below shows a light rod AB suspended by means of the two wires S, A (steel) and S_2B(Brass)



Let A_1 and A_2 be the areas of cross – section of the two wires. Suppose that a weight W is suspended from a point C at a distance X from the end A of the rod.
Let T_1 and T_2 be the tensions in the wires S_1A and S_2B respectively.

(a) When the stresses in the two wires are equal.

Stress in the steel wire = stress in the brass wire

$$\frac{T_1}{A_1} = \frac{T_2}{A_2}$$

$$\frac{T_1}{T_2} = \frac{A_1}{A_2} = \frac{10^{-3}}{2\times10^{-3}}$$

$$\frac{T_1}{T_2} = 0.5 \text{ ....(i)}$$

As the suspended system of the rod and two wires is in equilibrium. Apply the principle of moment of a force.

$$T_1 \times AC = T_2 \times BC$$
$$T_1 X = T_2(2-X)$$
$$\frac{T_1}{T_2} = \frac{2-X}{X} \text{ ....(ii)}$$
$$\text{(i)} = \text{(ii)}$$
$$\frac{2-X}{X} = 0.5$$
$$1.5X = 2$$
$$X = 1.333\text{m} = 133.3\text{cm (From steel)}$$

(b) When strains in the two wires are equal

Now ,

$$E = \frac{\text{stress}}{\text{strain}} \quad , \quad \text{strain} = \frac{\text{stress}}{E}$$

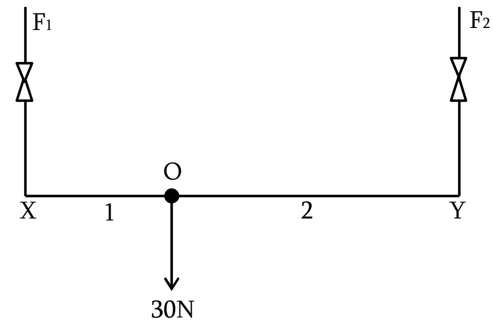
Strain of steel = strain of brass wire

$$\frac{T_1}{A_1 E_1} = \frac{T_2}{A_2 E_2}$$
$$\frac{T_1}{10^{-3} \times 2 \times 10^{11}} = \frac{T_2}{2 \times 10^{-3} \times 10^{11}}$$
$$T_1 = T_2$$

Again for the equilibrium of the system

$$T_1 X = T_2(2-X)$$
$$X = 2 - X$$
$$X = 1\text{m} = 100\text{cm (From steel wire)}$$

14. Two vertical wires X and Y , suspended at the same horizontal level are connected by a light rod XY at their ends as shown in the figure below. The wires have the same length L and cross – sectional area , A. A weight of 30N is replaced XO:OY = 1:2 both wires are stretched at the end of the rod XY then remains horizontal. If the wire X has a Young's Modulus E_1 of $1.0 \times 10^{11}\text{Nm}^{-2}$. Calculate the Young's Modulus E_2 of the wire assuming the elastic limit is exceed for both wires.



Solution

$E_2 = ?$ $E_1 = 1.0 \times 10^{11}\text{Nm}^{-2}$

$$XO : OY = \frac{1}{2} \quad , \quad W = 30\text{N}$$

At the equilibrium

$$F_1 + F_2 = 30 \text{ ....(i)}$$

Takes moment about O

Clockwise moment About O = Anti-clockwise moment

$$F_1 \overline{OX} = F_2 \overline{OY}$$
$$\frac{F_2}{F_1} = \frac{\overline{OX}}{\overline{OY}} = \frac{1}{2}$$
$$2F_2 = F_1 \text{ ....(i)}$$

Putting equation (ii) into (i)

$$2F_2 + F_2 = 30\text{N}$$
$$3F_2 = 30\text{N} \quad , \quad F_2 = 10\text{N}$$
$$F_1 = 30 - F_2 = 20\text{N}$$

Since $L_1 = L_2 = L$, $A_1 = A_2 = A$

$$e_1 = e_2 = e$$

Now

$$E = \frac{FL}{Ae}$$
$$\frac{E}{F} = \frac{L}{Ae} = \text{constant}$$
$$\frac{E_1}{F_1} = \frac{E_2}{F_2}$$
$$E_2 = E_1\left[\frac{F_2}{F_1}\right] = 1.0 \times 10^{11}\left[\frac{10}{20}\right]$$
$$E_2 = 0.5 \times 10^{11}\text{Nm}^{-2}$$

ARRANGEMENTS OF THE BARS

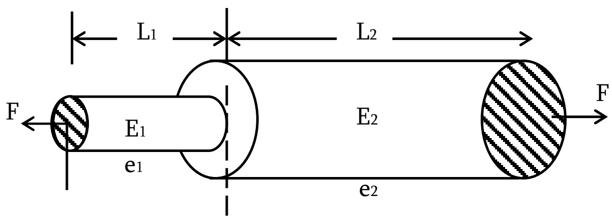
Bars can be connected into two ways:-

(i) Bars in series connection

(ii) Bars in parallel connection

1. BARS IN SERIES CONNECTION

When two or more bars are connected in series force actin on each other is the same but extension (elongation) of each bar is different



Composite bar – is the bar which can be formed when two or more bars are joined end to end. Total extension of the composite bar

$$e_T = e_1 + e_2$$

But

$$e_1 = \frac{FL_1}{A_1E_1} \;,\; e_2 = \frac{FL_2}{A_2E_2}$$

Now

$$e_T = \frac{FL_1}{A_1E_1} + \frac{FL_2}{A_2E_2}$$

$$e_T = F\left[\frac{L_1}{E_1A_1} + \frac{L_2}{E_2A_2}\right]$$

$$F = \frac{e_T}{\frac{L_1}{E_1A_1} + \frac{L_2}{E_2A_2}} = \frac{e_1 + e_2}{\frac{L_1}{E_1A_1} + \frac{L_2}{E_2A_2}}$$

Special case

If the two bars have the same physical condition.

$$A_1 = A_2 = A \;,\; L_1 = L_2 = L$$

$$e_1 = e_2 = e$$

$$F = \frac{e_T}{\frac{L}{A}\left[\frac{E}{E_1} + \frac{1}{E_2}\right]}$$

$$F = \frac{Ae_T}{L\left(\frac{1}{E_1} + \frac{1}{E_2}\right)} \quad \text{OR}$$

$$F = \frac{Ae_TE_1E_2}{L\left(E_2 + E_1\right)}$$

Let

E = total effectively value of Young's modulus of composite bar

$$e_T = e + e = 2e$$

$$L_T = L_1 + L_2 = 2L$$

$$F = \frac{AEe_T}{L_T}$$

$$\frac{AE(2e)}{2L} = \frac{A(2e)}{L\left[\frac{1}{E_1} + \frac{1}{E_2}\right]}$$

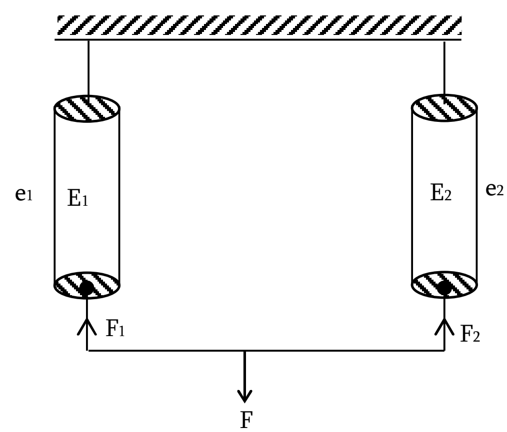
$$\frac{E}{2} = \frac{1}{\frac{1}{E_1} + \frac{1}{E_2}}$$

$$\frac{2}{E} = \frac{1}{E_1} + \frac{1}{E_2}$$

$$\frac{1}{E} = \frac{1}{2}\left[\frac{1}{E_1} + \frac{1}{E_2}\right]$$

(ii) BARS IN PARALLEL CONNECTION

When two or more bars are in parallel connection , extension of each bar is the same and force applied on each bar is different.



Total force of the system

$$F = F_1 + F_2$$

$$F = \frac{E_1A_1e}{L} + \frac{E_2A_2e}{L}$$

$$F = \frac{e}{L}\left[E_1A_1 + E_2A_2\right]$$

If

$$A_1 = A_2 = A$$

$$F = \frac{Ae}{L}\left[E_1 + E_2\right]$$

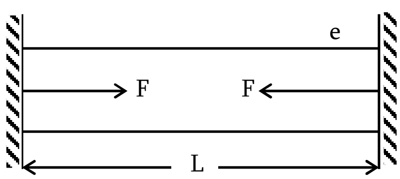
$$\frac{FL}{Ae} = E_1 + E_2$$

$$E = E_1 + E_2$$

E = Total Young's Modulus in the parallel connection.

FORCE DUE TO EXPANSION OR CONTRACTION OF THE ROD.

Consider a rod of length L and cross – sectional area , A subjected to the source of heat in such a way that it expands by e and then prevented from contraction as shown in the figure below.



LINEAR EXPANSIVITY (α)

Is defined as fractional increase in length per unit rise of temperature in degree

$$\alpha = \frac{e}{L\Delta\theta}$$

$$e = \Delta L = \alpha L\Delta\theta$$

L = Original length

$\Delta\theta$ = Temperature rise

$e = \Delta L$ = Extension

Now, Young's modulus of the rod

$$E = \frac{FL}{Ae}$$

$$F = \frac{EAe}{L} = \frac{E}{A}\cdot\alpha L\Delta\theta$$

$$F = EA\alpha\Delta\theta$$

Also

$$F = EA\alpha\left(\theta_2 - \theta_1\right)$$

θ_1 = Initial temperature of the rod

θ_2 = final temperature of the rod

A = cross – sectional area

Tensile stress of the rod

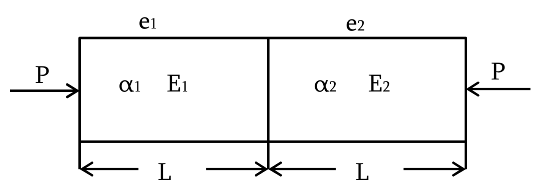
$$\text{Stress} = \frac{F}{A}$$

$$\text{Stress} = E\alpha\left(\theta_2 - \theta_1\right) = E\alpha\Delta\theta$$

$$\text{Thermal stress} = E\alpha\Delta\theta$$

Special case

If the two bars of the same cross – sectional area and length L are subjected on the same source of heat and temperature rise $\Delta\theta$ causes the expansion of e_1 and e_2 corresponding to the first and second bar respectively as shown on the figure below



Total extension or expansion of the composite bar / rod.

$$e = e_1 + e_2$$

$$e_1 = \alpha_1 L\Delta\theta \ , \ e_2 = \alpha_2 L\Delta\theta$$

$$e = \left(\alpha_1 + \alpha_2\right)L\Delta\theta \ \ldots\ldots..(i)$$

Since Young's Modulus of the rod

$$E = \frac{FL}{Ae} = \frac{PL}{e}$$

$$e = \frac{PL}{E}$$

$$e_1 = \frac{PL}{E_1} \ , \ e_2 = \frac{PL}{E_2}$$

Now

$$e = e_1 + e_2$$

$$e = \frac{PL}{E_1} + \frac{PL}{E_2}$$

$$e = PL\left[\frac{1}{E_1} + \frac{1}{E_2}\right]$$

$$e = PL\left[\frac{E_2 + E_1}{E_1E_2}\right] \text{........(ii)}$$

(i) = (ii)

$$PL\left[\frac{E_1 + E_2}{E_1E_2}\right] = (\alpha_1 + \alpha_2)L\Delta\theta$$

$$P = \frac{E_1E_2(\alpha_1 + \alpha_2)\Delta\theta}{E_1 + E_2}$$

P = Pressure due to the force of contraction.

ENERGY STORED IN THE WIRE

The amount of energy stored in the wire sometimes is known as strain energy and can be denoted by using symbol of W. The energy stored in the wire is given by

$$W = \frac{1}{2}F\Delta L = \frac{1}{2}Fe$$

F = Force applied

$e = \Delta L$ = extension

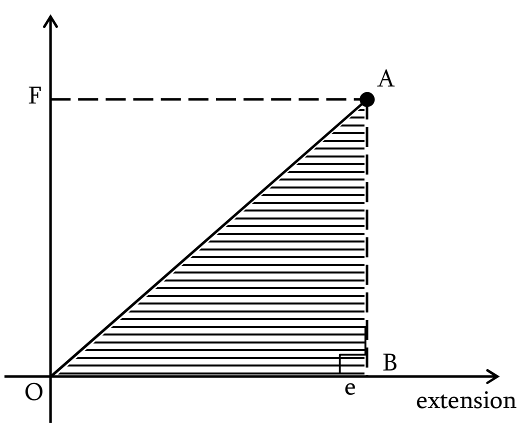
Derivation of W = ½ Fe

Method 1:

By the graphical method. According to the Hooke's law

F = ke

Graph of force against extension , e



Total area under the graph of force , F against extension of the wire represent the total energy in the wire.

Area of the triangle OAB

$$\text{Area of } \Delta OAB = \frac{1}{2}\overline{BA} \times \overline{OB}$$

$$= \frac{1}{2}Fe$$

$$W = \frac{1}{2}Fe$$

Method 2:

By using integration method. Consider a wire of length L and area of cross – section A suspended from a rigid support. Suppose that a normal force F is applied at its free end and its length increase by X. Then , Young's Modulus of the material of the wire is given by

$$E = \frac{F/A}{X/L}$$

$$F = \frac{EAX}{L}$$

Suppose that the length of the wire is increased by an infinitesimally small amount dx under the action of a constant force , F. The small amount of work done

$$dw = Fdx$$

$$dw = \frac{EAX}{L}dx$$

The amount of work done in stretching the wire by a length , e

$$W = \int_0^e \frac{EAX}{L}dx$$

$$= \frac{EA}{L}\left[\frac{X^2}{2}\right]_0^e$$

$$W = \frac{EAe^2}{2L} = \frac{1}{2}\left(\frac{EAe}{L}\right)e$$

$$W = \frac{1}{2}Fe$$

DIFFERENT FORMS OF THE EXPRESSION OF ENERGY STORED IN THE WIRE.

1. $W = \frac{1}{2}Fe$
2. Since F = Ke

$$W = \frac{1}{2}(ke)e \qquad \text{K = Force constant}$$

$$W = \frac{1}{2}Ke^2$$

3. $W = \frac{EAe^2}{2L}$

4. Since $F = EA\alpha\Delta\theta$

$$W = \frac{1}{2}EA\alpha e\Delta\theta$$

ENERGY DENSITY (U)

Is defined as the amount of energy stored per unit volume of the wire.

$$\text{Energy density} = \frac{\text{Energy stored}}{\text{Volume}}$$

$$U = \frac{W}{V_o} = \frac{W}{AL}$$

A = cross – sectional area

L = length of the wire

S.I Unit of energy density is Jm^{-3}.

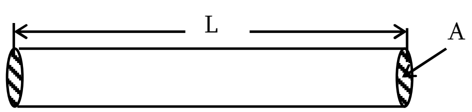
The energy density on the wire is given by

$$U = \frac{1}{2}\text{stress} \times \text{strain}$$

Derivation $U = \frac{1}{2} \times \text{stress} \times \text{strain}$

Method 1: By the analytical method.

Consider a wire of cross – sectional area , A and length L as shown below



Volume of the wire = AL

$$\text{Energy store} = \frac{1}{2}Fe$$

$$\text{Energy density} = \frac{W}{AL} = \frac{\frac{1}{2}Fe}{AL}$$

$$U = \frac{1}{2}\left(\frac{F}{A}\right)\left(\frac{e}{L}\right)$$

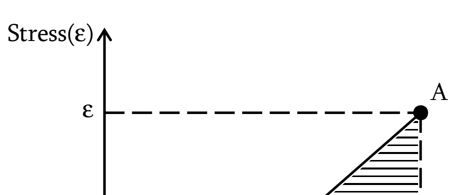
$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

Method 2: By graphical method

Since $E = \frac{\text{Stress}}{\text{Strain}}$

Stress = E × Strain

Stress α Strain



The area under the graph of stress against strain represent energy density of the wire.

$$\text{Area of } \Delta OAB = \frac{1}{2}\overline{AB} \times \overline{OB}$$

$$W = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

NUMERICAL EXAMPLES

15. (a) A steel rod of length 0.60m and cross – sectional area $2.5 \times 10^{-5}m^2$ at a temperature of 100^oC is damped so that when it cools was unable to contract. Find the tension in the rod when it has cooled to 20^OC.

(b) A spring 60cm long is stretched by 2cm for the application of load of 200gm. What will be the length when a load of 500g is applied?

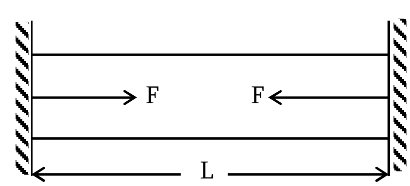
(c) Calculate the percentage increase in length of a wire of diameter 2.2mm stretched by a load of 100kg (Young's Modulus of wire is 12.5 × 1010N $^{-2}$).

Young's Modulus of steel = 2.0 × 1011Pa

Linear expansivity of steel = $1.6 \times 10^{-7}C^{-1}$

Solution

(a)



$$\text{Linear expansivity } \alpha = \frac{e}{L\Delta\theta}$$

$$e = \alpha L\Delta\theta = \alpha\left(\theta_2 - \theta_1\right)$$

$$\text{Since } E = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{e/L}$$

$$E = \frac{FL}{Ae}$$

$$F = \frac{EAe}{L} = \frac{EA\alpha L\left(\theta_2 - \theta_1\right)}{L}$$

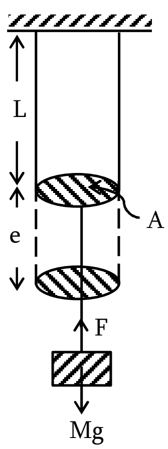
$$F = EA\alpha\left(\theta_2 - \theta_1\right)$$

$$= 2\times10^{11}\times2.5\times10^{-5}\times1.6\times10^{-7}\left(100-20\right)$$

$$F = 64N$$

(b) Refer to a solution of examples (2b)(ii)

(c)



$$\text{Since } E = \frac{FL}{Ae}$$

But

$$F = Mg \ , \ A = \frac{\pi d^2}{4}$$

$$E = \frac{MgL}{\frac{\pi d^2}{4}\cdot e}$$

$$E = \frac{4MgL}{\pi d^2 e}$$

$$\frac{e}{L}\times100\% = \frac{4Mg}{\pi d^2 E}\times100\%$$

$$\frac{\Delta L}{L}\times100\% = \frac{4\times100\times9.8}{3.14\times\left(2.2\times10^{-3}\right)^2\times12.5\times10^{10}}\times100\%$$

$$\frac{\Delta L}{L}\times100\% = 0.2\%$$

16. (a) (i) Define tensile stress and tensile strain

(ii) Calculate the work done in a stretching copper wire of 100cm long and $0.03cm^2$ cross – sectional area when a load of 120N is applied

(b) (i) A 45kg traffic light is suspended with two steel wires of equal lengths and radii of 0.5cm. If the wires make and angle of 15^{O} with the horizontal, what is the fractional increase in their length due to the weight of the light?

Young Modulus of copper = 1.1×10^{11}Pa

Young modulus of steel = $2.0\times{}^{11}$Pa

Solution

(a) (i) see your notes

(ii) L = 100cm , A = 0.03m2 , F = 120N

Energy stored in the wire

$$W = \frac{1}{2}Fe$$

$$\text{Since } E = \frac{FL}{Ae} \ , \ e = \frac{FL}{AE}$$

$$W = \frac{1}{2}F\left[\frac{FL}{AE}\right] = \frac{F^2L}{2AE}$$

$$= \frac{\left(120\right)^2\times1}{2\times0.03\times10^{-4}\times1.1\times10^{11}}$$

$$W = 0.0218J$$

(b) (i)
- effect of presence of impurities
- Effect of temperature
- Effect of annealing (any two)

(ii) See solution example 8

17. (a) A cylindrical copper rod of length 0.5m and diameter 4.0×10^{-2}m is fixed between two rigid supports at a temperature of 20^{O}C. The temperature of the rod is raised to 70^{O}C.

(i) Calculate the force exerted on the rigid support at 70^{O}C.

(ii) What is the energy stored in the rod at 70^{O}C.

Given that Young Modulus of copper = $1.2\times10^{11}Nm^{-2}$, linear expansion α copper = $1.7\times10^{-5}K^{-1}$

(b) Why are the bridges declared unsafe after long use?

Solution

(a) (i) $F = EA\alpha(\theta_2 - \theta_1)$

But $A = \frac{\pi d^2}{4}$

$$F = \frac{\pi d^2 E\alpha}{4}[\theta_2 - \theta_1]$$

$$= \frac{3.14\times(4\times10^{-2})^2\times1.2\times10^{11}\times1.7\times10^{-5}(70-20)}{4}$$

$$F = 1.28\times10^5 N$$

(ii) $W = \frac{1}{2}Fe$ but $e = \alpha L\Delta\theta$

$$= \frac{1}{2}\times1.28\times10^5\times0.5\times1.7\times10^{-5}\times50$$

$$W = 27.2J$$

(b) A bridge is subjected to forces of varying amounts due to the flow of traffic over it in the other words , the bridge is subjected to varying stresses as a result it becomes weaker i.e the strain produced by a given stress increases . If the elastic limit of the bridge is exceeded, it may collapse for this reason, bridges are declared unsafe after long use.

18. Two wires each of 1metre long and 1mm² cross – section area , one of the steel and the other of brass are connected end to end. What tensile force would be required to extend the whole wire by 1mm? (Young modulus of steel is $2.0 \times 10^{11}Nm^{-2}$ and for brass is $1.0 \times 10^{11}Nm^{-2}$).

Solution

Let E_1 = Young modulus of steel

E_2 = Young modulus of brass

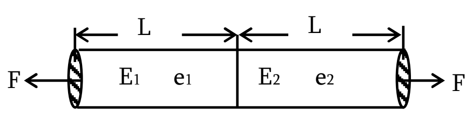
$E_1 = 2.0 \times 10^{11}Nm^{-2}$

$E_2 = 1.0 \times 10^{11}Nm^{-2}$

$L_1 = L_2 = L = 1m$

$A = 1mm^2 = 10^{-6}m^2$

$e = 1mm = 1 \times 10^{-3}m$



Total extension of the wire

$$e = e_1 + e_2$$

But $E = \frac{FL}{Ae}$, $e = \frac{FL}{AE}$

$$e = \frac{FL}{AE_1} + \frac{FL}{AE_2} = \frac{FL}{A}\left[\frac{1}{E_1} + \frac{1}{E_2}\right]$$

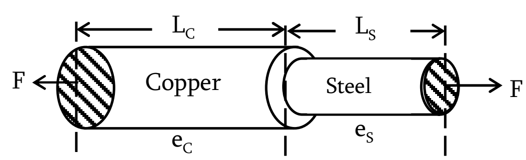
$$F = \frac{Ae}{L}\left[\frac{E_1E_2}{E_1 + E_2}\right]$$

$$F = \frac{10^{-6}\times10^{-3}}{1}\left[\frac{2\times10^{11}\times1\times10^{11}}{(2+1)\times10^{11}}\right]$$

$$F = 66.7N$$

19. Copper rod of a length 3m and cross – section area of 0.5cm² it fastened end to end to a steel rod of cross – section area 0.2cm² and opposite pulls are applied at the ends of the combination. If the rods have equal elongation. Find the length of the steel rod. Given that Young's Modulus of copper is $11 \times 10^{10}Nm^{-2}$ and Young modulus of steel is $21 \times 10^{10}Nm^{-2}$.

Solution



Young's Modulus , $E = \frac{FL}{Ae}$

$$F = \frac{EAe}{L}$$

Now $F_S = F_C$

$$\frac{E_SA_S}{L_S} = \frac{E_CA_C}{L_C}$$

$$L_S = L_C\left(\frac{E_S}{E_C}\right)\left(\frac{A_S}{A_C}\right)$$

$$= 3m\left[\frac{21\times10^{10}}{11\times10^{10}}\right]\left[\frac{0.2}{0.5}\right]$$

$$L_S = 2.29m$$

20. (a) Show that the energy stored per unit volume in a stretched wire is equal to the half the product of the stress and the strain.

(b) A catapult consists of two rubber cords each of unstretched length 10.0cm and area of cross – section $0.40cm^2$. Assuming that all the energy stored in the stretched cord is converted into kinetic energy of the missle. Calculate the maximum height to which a stone of mass 100gm could be projected if each of the cord were stretched by 5.0cm. Young's modulus of rubber = $1.00 \times 107Nm^{-2}$.

Solution

(a) Refer to your notes

(b) Energy stored in the rubber

$$W = \frac{1}{2}Fe \quad \text{but } F = \frac{EAe}{L}$$

$$W = \frac{EAe^2}{2L} \text{ ....(i)}$$

Kinetic energy of the stone

$$k.e = \frac{1}{2}mv^2 \text{ ....(ii)}$$

Apply the law of conservation of energy

(i) = (ii)

$$\frac{EAe^2}{2L} = \frac{1}{2}MV^2$$

$$V^2 = \frac{EAe^2}{ML} \text{ ....(iii)}$$

Again

Loss in p.e = gain in k.e of stone

$$Mgh = \frac{1}{2}MV^2$$

$$gh = \frac{Ee^2A}{2ML}$$

$$h = \frac{EAe^2}{2MgL}$$

$$= \frac{1.00\times10^7 \times 0.40\left(0.05\right)^2}{2\times9.81\times0.1\times10}$$

$$h = 10.2m$$

21. (a) Steel is more elastic than rubber. Explain , why .

(b) The rubber chord of catapult has a cross – sectional area of $1mm^2$ and total unstretched length 10cm. it is stretched to 12cm and then released to project a missle of mass 5gm. If Young's modulus E for the rubber is $5.0 \times 10^8Nm^{-2}$; Calculate the velocity of projection. Assume that total elastic energy of catapult is converted into kinetic energy without any loss of heat.

Solution

(a) Consider two wires , one of steel and the other made of rubber. Both the wires are identical i.e they possess the same length (L) and the same area of cross – section (A). If they are subjected to the same deforming force , F then extension in the steel wire (L_s) will be less than that in the rubber wire (L_r) i.e $L_s < L_r$

$$\text{Now } E_S = \frac{FL}{Ae_S} \text{ and } E_r = \frac{FL}{Ae_r}$$

Since $e_s < e_r$, it follows that $E_s > E_r$

(b) Young's Modulus [e = 12 – 10 = 2cm]

$$E = \frac{FL}{Ae} \quad , \quad F = \frac{EAe}{L}$$

$$F = \frac{5\times10^8 \times 1\times10^{-6} \times 2\times10^{-2}}{0.1}$$

$$F = 100N$$

Apply the law of conservation of energy

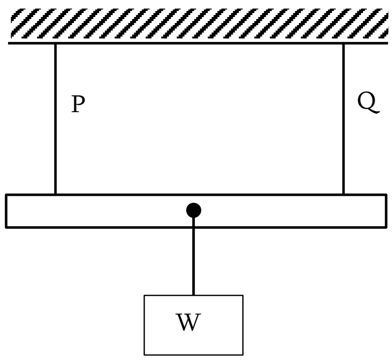
k.e of missle = work done

$$\frac{1}{2}MV^2 = \frac{1}{2}Fe$$

$$V = \sqrt{\frac{Fe}{M}} = \sqrt{\frac{100\times2\times10^{-2}}{5\times10^{-3}}}$$

$$V = 20m/s$$

22. What is strain energy? A piece of rod 1.05m long whose weight is negligible is supported at its end by wires Q and P of equal length as shown in figure 1 below.



The cross – sectional area of P is $1mm^2$ and that of Q is $2mm^2$. At what point along the bar should the weight be suspended in order to produce.
(i) Equal stress of P and Q
(ii) Equal strain of P and Q.
Given Young's Modulus of
$P = 2.4 \times 10^{11} Nm^{-2}$ and that of wire
$Q = 1.6 \times 10^{11} Nm^{-2}$.

23. A wire of area of cross – section $3mm^2$ breaks under a force of 225N . Find the rise in temperature of the wire at the time of breaking. Given that Y = 117GPa. Density of the wire = $8930kgm^{-3}$, specific heat capacity $C = 380JKg^{-1}K^{-1}$.

Solution

Let $\Delta\theta$ = temperature rise
Apply the law of conservation of energy
Elastic p.e stored = heat produced in the wire

$$\frac{1}{2}F\Delta L = MC\Delta\theta$$

$$\text{But} \quad \Delta L = \frac{FL}{AY} \quad , \quad M = \rho LA$$

$$\frac{1}{2}F\frac{FL}{AY} = \rho ALC\Delta\theta$$

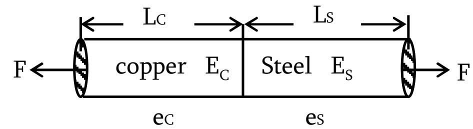
$$\Delta\theta = \frac{F^2}{2YA^2\rho C}$$

$$\Delta\theta = \frac{(225)^2}{2\times117\times10^9\times(3\times10^{-6})^2\times8930\times380}$$

$$\Delta\theta = 0.007^\circ C$$

24. A composite wire of uniform diameter 3mm consisting of copper wire of length 2.2m and a steel wire of length 1.6m stretches under a load by 0.7mm. Calculate the load , given that the Young's Modulus of copper is $1.1 \times 10^{11}Pa$ and for steel is $2.0 \times 10^{11}Pa$.

Solution



$$\text{Young's Modulus , } E = \frac{FL}{Ae}$$

$$e = \frac{FL}{AE}$$

$$\text{For copper:} \quad e_C = \frac{FL_C}{AE_C}$$

$$\text{For steel:} \quad e_S = \frac{FL_S}{AE_S}$$

Total extension

$$e = e_S + e_S$$

$$e_C + e_S = \frac{FL_C}{AE_C} + \frac{FL_S}{AE_S}$$

$$e_C + e_S = \frac{F}{A}\left[\frac{L_C}{E_C} + \frac{L_S}{E_S}\right]$$

$$F = \frac{A(e_C + e_S)}{\frac{L_C}{E_C} + \frac{L_S}{E_S}}$$

$$F = \frac{\pi d^2}{4}\frac{(e_C + e_S)}{\left[\frac{L_C}{E_C} + \frac{L_S}{E_S}\right]}$$

But

$$e_S + e_C = 0.7mm = 0.7 \times 10^{-3}m$$

$$F = \frac{3.14\times(3\times10^{-3})^2\times0.7\times10^{-3}}{4\left[\frac{2.2}{1.1\times10^{11}} + \frac{1.6}{2.2\times10^{11}}\right]}$$

$$F = 176.7N$$

25. (c) (i) Define the following terms as applied to the strength of materials : elasticity , elastic limit and ultimate strength

(ii) A steel wire of length L1 = 4.7m and cross – section $A_1 = 3.0 \times 10^{-5}m^2$ stretches by the same amount of as a copper wire of length $L_2 = 3.5m$ and cross – section $A_2 = 4.0 \times 10^{-5}m^2$ under a given load. What is the ratio of Young's Modulus of steel to that of copper?

(d) A cylindrical copper wire and cylindrical steel wire each of length 1.5m and diameter 2mm are joined at one end to form a composite wire is loaded until its length becomes 3.003m. calculate the force applied and strain in copper wire.

Young's Modulus of copper = $117 \times 10^9 Nm^{-2}$

Young Modulus of steel = $210 \times 10^9 Nm^{-2}$

Solution

(c) (i) Refer to your notes (ii) E_1/E_2 = ?

Extension of steel = Extension of copper

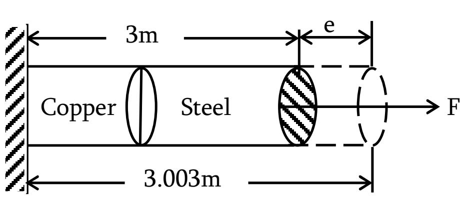
$$\frac{FL_1}{A_1E_1} = \frac{FL_2}{A_2E_2}$$

$$\frac{E_1}{E_2} = \left(\frac{A_2}{A_1}\right)\left(\frac{L_1}{L_2}\right)$$

$$\frac{E_1}{E_2} = \left(\frac{4\times10^{-5}}{3\times10^{-5}}\right)\left(\frac{4.7}{3.5}\right)$$

$$\frac{E_1}{E_2} = 1.79$$

(d) Diagram



Now, Young Modulus

Copper $e_1 = \frac{FL}{Ae}$, $e_2 = \frac{FL}{AE_2}$

Total extension

$$e = e_1 + e_2 = \frac{FL}{AE_1} + \frac{FL}{AE_2}$$

$$e = \frac{FL}{A}\left[\frac{1}{E_1} + \frac{1}{E_2}\right]$$

$$e = \frac{FL}{A}\left[\frac{E_2+E_1}{E_1E_2}\right]$$

$$F = \frac{Ae}{L}\left[\frac{E_1E_2}{E_1+E_2}\right]$$

But

$$A = \frac{\pi d^2}{4}$$

$$F = \frac{\pi d^2 e}{4L}\left[\frac{E_1E_2}{E_1+E_2}\right]$$

But

$$e = 3.003 - 3.000 = 0.003m$$

$$F = \frac{3.14\left(2\times10^{-3}\right)^2\times0.003}{4\times1.5}\left[\frac{117\times210}{117+210}\right]\times10^9$$

$$F = 471.86N$$

Strain in the copper wire

$$strain = \frac{stress}{E_1} = \frac{F/A}{E_1}$$

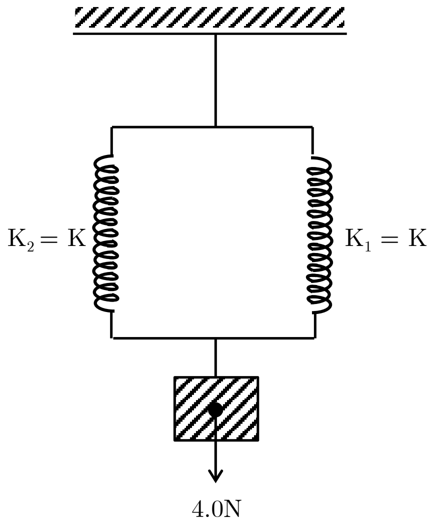
$$= \frac{F}{AE_1} \text{ But } A = \frac{\pi d^2}{4}$$

$$= \frac{4F}{\pi d^2 E_1}$$

$$= \frac{4\times471.86}{3.14\times\left(2\times10^{-3}\right)^2\times117\times10^9}$$

$$strain = 1.284\times10^{-3}$$

26. A load of 40N is suspended from a parallel two spring's system as shown in the diagram below. If the spring constant of each spring is $20Nm^{-1}$. Calculate elastic energy



Solution

$K_1 = K_2 = 20Nm^{-1}$, $F = 40N$

Total force constant for the springs in parallel connection

$K_P = K_1 + K_2 = 20 + 20$

$K_P = 40Nm^{-1}$

Elastic energy $W = \dfrac{F^2}{2K_P}$

$W = \dfrac{4 \times 4}{2 \times 40}$

$W = 0.2J$

27. A wire of length 5.0m of uniform cross – sectional area and of uniform cross – sectional area and of radius 1.0mm is exerted by 1.5mm. when subjected to a uniform tension of 100N. Calculate the strain energy per unit volume stored in the wire. (Assume the Hooke's law is obey in process)

Solution

$$U = \frac{1}{2} \times \text{stress} \times \text{strain}$$

$$= \frac{1}{2}\left(\frac{F}{A}\right)\left(\frac{e}{L}\right) = \frac{1}{2}\left(\frac{F}{\pi r^2}\right)\left(\frac{e}{L}\right)$$

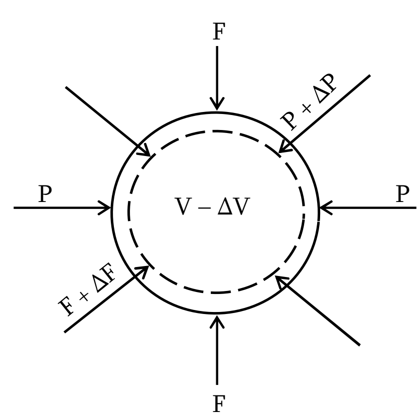
$$= \frac{1}{2}\left[\frac{100}{3.14\times\left(10^{-3}\right)^2}\right]\left[\frac{1.5\times10^{-3}}{5}\right]$$

$$U = 9.0 \times 10^{-8}\,Jm^{-3}$$

28. (a) Define the following terms:-
   (i) Tensile stress
   (ii) Tensile strain
   (iii) Young Modulus.
   (b) (i) Derive the expression for the work done in stretching a wire of length L by a load W through an extension X.
   (ii) A vertical wire made of steel of length 2.0m and 1.00mm diameter has a load of 5.0kg applied to its lower end. What is the energy stored in the wire.
   (c) A copper wire 2.0m long and 1.22×10^{-3}m diameter is fixed horizontally to two rigids supports 2.0m. Find the mass in kg of the load, which when suspended at the midpoint of the wire , produces a sag of 2.0cm at the point. Given that steel wire have Young's Modulus E = $2.0 \times 1011Nm^{-2}$.

2. BULK MODULUS

This refers to the situation in which the volume of a substance is changed due to the application of the forces. Consider a sphere of volume V and surface area A as shown below. Suppose that a force F which acts uniformly over the wholes surface of the sphere , decreases its volume by ΔV.



BULK STRESS

Is defined as the increase in force per unit area. Sometime Bulk stress is known as Normal stress.

$$\text{Normal stress} = \frac{\text{Increase in force}}{\text{Area}}$$

$$= \frac{(F + \Delta F) - F}{A} = \frac{\Delta F}{A}$$

$$\text{Normal stress} = \Delta P$$

SI unit of Bulk stress is Pascal (Pa) or Nm^{-2}.

BULK STRAIN (VOLUMETRIC STRAIN)

Is defined as the change in volume per unit original volume.

Volumetric strain

$$= \frac{(V - \Delta V) - V}{V} = \frac{-\Delta V}{V}$$

The negative sign indicates that on increasing the stress , volume of the sphere decreases.

BULK MODULUS , K

Is defined as the ratio of the normal stress to the volumetric strain.

$$\text{Bulk Modulus} = \frac{\text{Bulk stress}}{\text{Bulk strain}}$$

$$K = \frac{\Delta P}{-\Delta V/V} = \frac{-V\Delta P}{\Delta V}$$

$$K = -\frac{V\Delta P}{\Delta V}$$

S.I unit of Bulk Modulus is **Pa or Nm^{-2}**.

COMPRESSIBILITY (β)

The compressibility of a material is the measure of how easily the material is compressed.

Definition compressibility – is defined as the reciprocal of the bulk modulus of the substance

$$\beta = \frac{1}{K} = \frac{-\Delta V}{V\Delta P}$$

S.I unit of compressibility is Pa^{-1} or $N^{-1}m^2$.

APPROXIMATE VALUES OF K

Material	K× 10^{11}Pa	Material	K× 10^{11}Pa
Aluminum	0.7	Iron	1.0
Brass	0.61	Lead	0.077
Copper	1.4	Nickel	2.6
Glass	0.37	Steel	1.6
		Tungsten	2.0

Additional concepts

(i) Solid and liquids are relatively incompressible i.e they have small value of compressibility (β) or large value of bulk modulus (k) and these values are almost independent of temperature and pressure. On the other hand , gases are easily compressed (β is small, k is larger and the value of β and k strongly depends on the temperature and pressure).

(ii) BULK MODULUS OF A GAS

A gas has volume elasticity because the volume of a gas can be changed by applying pressure. Consider a certain mass of perfect gas enclosed in a cylinder. Let its pressure be P and volume be V. suppose the pressure be increased to P + dP such that the volume is reduced to V – dV.

$$\text{Stress} = dP$$

$$\text{Strain} = \frac{V - dV - V}{V} = \frac{-dV}{V}$$

Negative sign shows that the volume decreases as pressure increases.

$$\text{Bulk Modulus of a gas} = \frac{\text{Bulk stress}}{\text{Bulk strain}}$$

$$K = \frac{dP}{-dV/V} = -V\frac{dP}{dV}$$

Gases can be expanded either under Isothermal expansion or adiabatic expansion.

- **ISOTHERMAL EXPANSION**

Is the expansion of the gas which takes place under the constant temperature.

For isothermal process.

$$PV = \text{a constant}$$

$$\frac{d}{dV}[PV] = \frac{d}{dV}[C]$$

$$P\frac{dV}{dV} + V\frac{dP}{dV} = 0$$

$$PdV + VdP = 0$$

$$P = -V\frac{dP}{dV} = \text{Isothermal elasticity}$$

$$\text{Kiso} = P$$

$$P = \text{Pressure of a gas}$$

Compressibility of gas

$$\beta = \frac{1}{\text{Kiso}} = \frac{1}{P}$$

- Adiabatic expansion

  Is the expansion of the gas which takes place when no amount of energy or leave from the system.

  For an adiabatic process.

$$PV^{\gamma} = \text{Constant}$$

$$\frac{d}{dV} = \left[PV^{\gamma}\right] = V^{\gamma}\frac{dP}{dV} + P\frac{d}{dV}\left(V^{\gamma}\right) = \frac{d}{dV}(C)$$

$$V^{\gamma}\frac{dP}{dV} + \gamma PV^{\gamma-1} = 0$$

$$V^{\gamma}\frac{dP}{dV} = -\gamma PV^{\gamma-1}$$

$$V^{\gamma}\frac{dP}{dV} = -\gamma PV^{\gamma}\cdot V^{-1}$$

$$-V\frac{dP}{dV} = \gamma P$$

$$\text{Kadi} = -V\frac{dP}{dV} = \gamma P$$

$$\text{Kadi} = \gamma P \quad , \quad \gamma = {}^{C_P}\!/\!_{C_V}$$

Compressibility of the gas under Adiabatic process.

$$\beta\text{adi} = \frac{1}{\text{Kadi}} = \frac{1}{\gamma P}$$

Also

$$\text{Kadi} = \gamma\text{Kiso}$$

$$\frac{\text{Adiabatic elasticity}}{\text{Isothermal elasticity}} = \frac{\gamma P}{P} = \gamma$$

This means adiabatic elasticity is γ times the isothermal elasticity.

VELOCITY OF SOUND WAVE

1. Velocity of sound wave in solid can be depends on Young's Modulus (E) and density of the solid substance (ρ).

$$V = \sqrt{{}^{E}\!/\!_{\rho}}$$

2. Velocity of sound in gases

$$V = \sqrt{\frac{K}{\rho}}$$

K = Bulk Modulus of gas

ρ = density of the gas

- Under isothermal condition

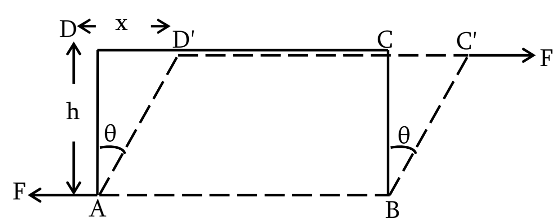
$$V = \sqrt{\frac{P}{\rho}}$$

- Under adiabatic condition

$$V = \sqrt{\frac{\gamma P}{\rho}}$$

3. SHEAR MODULUS OR MODULUS OF RIGIDITY (η)

   Refer as the situation in which the shape of the substance changes due to the application of tensile stress. Consider the tensile force (F) applied to the object as shown in the figure below.



Definition Shear stress – is defined as the tangential force per unit area.

$$\text{Shear stress} = \frac{\text{tangential force}}{\text{Area}}$$

Shear strain – is defined as the tangent of the angle of shear.

Shear strain $= \tan\theta$

If θ is very small angle in radian , $\tan\theta \simeq \theta$

$$\text{Shear strain} = \tan\theta \simeq \frac{X}{h}$$

MODULUS OF RIGIDITY OR SHEAR MODULUS , η or G

Is defined as shear stress per unit shear strain.

$$\text{Modulus of Rigidity} = \frac{\text{Shear stress}}{\text{Shear strain}}$$

$$\eta = \frac{F''}{A\tan\theta} = \frac{F''}{A\left(X/h\right)}$$

$$\eta = \frac{F''}{A\tan\theta} = \left(\frac{F''}{A}\right)\left(\frac{h}{X}\right)$$

S.I unit of Modulus of the Rigidity is Pascal (Pa) or Nm^{-2}.

Approximate values of η

Material	$\eta \times 10^{11}$Pa
Aluminum	0.30
Brass	0.36
Copper	0.42
Glass	0.23
Iron	0.70
Lead	0.056
Nickel	0.77
Steel	0.84
Tungsten	1.50

FACTORS AFFECTING ELASTICITY

Following are found to affect the elasticity of a material:-

1. Effect of hammering and rolling
   Hammering and rolling result in decrease in the elasticity of the material due to break – up of crystal grains into smaller units and hence elasticity of the material increases.
2. Effect of annealing.
   Annealing results in increase in the plasticity of the material due to formation of large crystal grains. Hence, the elasticity of the material decreases.
3. Effect of the presence of impurities.
   The effect of the presence of impurities in a material can be both ways i.e it can increase as well as decrease the elasticity of the material. The type of effect depends upon the nature of the impurity present in the material.
4. Effect of temperature
   In most cases , the increase in temperature of the material causes decrease in the elasticity of the material. The elasticity of invar does not change with change of temperature.

POISSON'S RATIO (δ)

When a wire is suspended from one end and loaded at the other end the length of the wire increases and its diameter decreases i.e when a wire is stretched , it becomes longer but thinner. The linear strain (also called primary strain) is in the direction of the applied force. The lateral strain (also called secondary strain) is at right angles to the direction of the applied force.

Definition

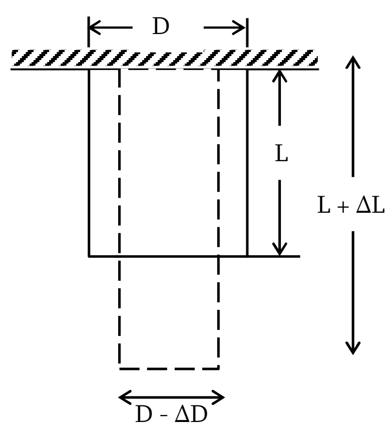
POISSON'S RATIO – is the ratio of the lateral strain to the longitudinal strain.

$$\delta = \frac{\text{Lateral strain}}{\text{Linear strain}}$$

δ has no dimensions and no units

Let L and D be the original length and diameter respectively of a wire.

Let ΔL and ΔD be the respective change in them.



$$\text{Longitudinal strain} = \frac{\Delta L}{L}$$

$$\text{Lateral strain } \beta = \frac{-\Delta D}{D}$$

$$\text{Poisson's ratio } \delta = \frac{\beta}{\alpha} = \frac{\Delta D}{D} \times \frac{L}{\Delta L}$$

$$\delta = \frac{-L}{D} \cdot \frac{\Delta D}{\Delta L}$$

The minus sign indicate that δ is positive i.e the length increases as the diameter decreases. Theoretically , the limiting values of δ are -1 and 0.5 in actual practice it lies between 0.2 and 0.4 for most of the materials.

Material	Poisson's ratio (δ)
Aluminum	0.16
Brass	0.26
Copper	0.32
Glass	0.19
Iron	0.27
Lead	0.43
Nickel	0.36
Steel	0.19
Tungsten	0.20

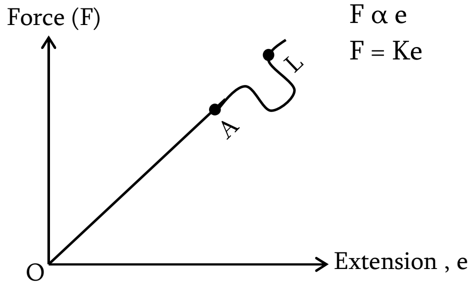
FOUR IMPORTANT RELATIONS BETWEEN Y , K , η and δ

1. $\eta = \frac{Y}{2(1 + \delta)}$
2. $\frac{9}{Y} = \frac{3}{\eta} + \frac{1}{K}$
3. $\delta = \frac{3K - 2\eta}{6K + 2\eta}$
4. $K = \frac{Y}{3(1 - 2\delta)}$

GRAPHS OF ELASTICITY

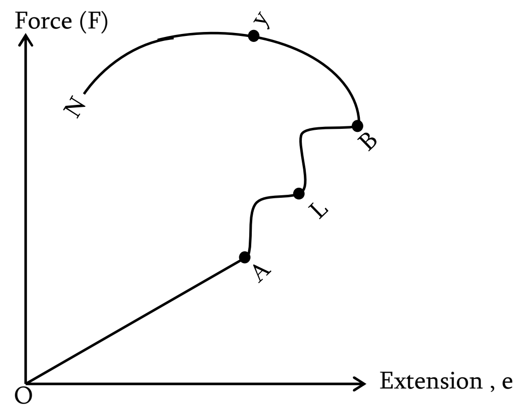
1. **Graph of a force against extension**

(a) For the brittle substance



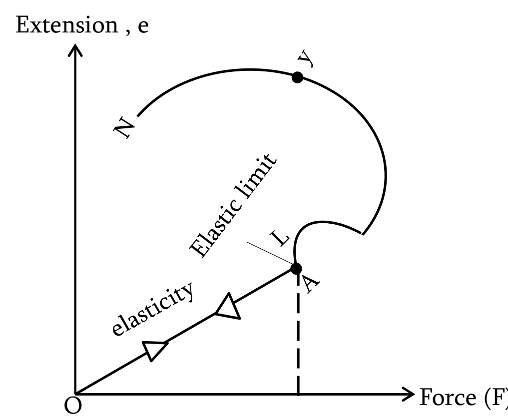
$F \propto e$
$F = Ke$

(b) For the ductile material

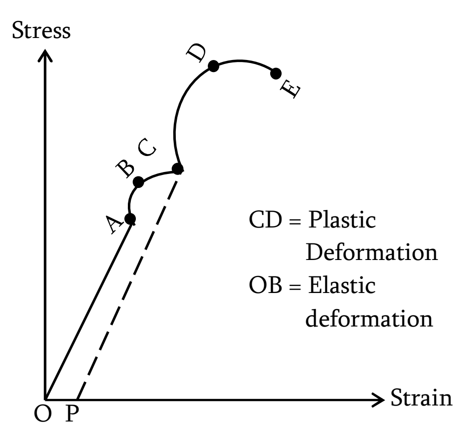


A = Proportionality limit
L = elastic limit
Y = ultimate strength
N = fracture point
OL = Elastic deformation
BY = Plastic deformation

(c) Graph of extension against force



2. **Graph of stress against strain , ductile material.**



CD = Plastic Deformation
OB = Elastic deformation

Young's Modulus of a material.

$$E = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{Stress} = \text{Exstrain} \text{ , Stress } \alpha \text{ Strain}$$

Where

A = Proportionality limit

B = Elastic limit

C = Yield point

D = breaking (maximum) stress

E = Wire breaks (breaking point)

Main features from the graph above:-

(i) **REGION OA** (Hooke's law region)

The portion OA of the graph is straight line showing that up to point A , strain produced in the wire is directly proportional to the stress i.e strain α stress. In this portion , the material of the wire obeys Hooke's law.

$$\text{Slope} = \frac{\Delta\delta}{\Delta\varepsilon} = E \text{ , } \begin{matrix} \delta = \text{stress} \\ \varepsilon = \text{strain} \end{matrix}$$

The main significant of the slope represent the Young's Modulus of the wire of a material. The point A is called proportionality limit.

Definition

LIMIT OF PROPORTIONALITY – Is the greatest stress a material can sustain a material without departure from a linear stress – strain relation. If the applied force is removed at any point between O and A the wire regains its original length.

REGION AB

The portion of AB of the graph is not a straight line showing that this region strain is not proportional to the stress up to B , the wire returns to regional length , when the stress is reduced to zero. The slope of the graph is decreased; this means that strain increases more rapidly with stress.

REGION OB

This is the elastic deformation region. The point B is called Elastic limit

Definition

ELASTIC LIMIT – is the maximum stress which a body can sustain and still regain its original shape and size when the load is removed.

REGION BC

If the stress is increased beyond the elastic limit , a point C is reached at which there is marked increase in extension. This point is called yield point. Between B and C the material becomes plastic i.e if the wire is unloaded at any point between B and C , the wire does not quite come back to its original length. The extension not recoverable after removing the load is known as 'Permanent set'. However, this permanent deformation is not serious enough to be important in practice we must keep the stress below the yield point. Here OP is the permanent set.

REGION CD

If the stress is increased beyond point C , the wire lengthens rapidily until we reach point D at the top of the curve. The point D is called the ultimate strength or breaking stress. Beyond point D , even smaller than at C may continue to stretch the wire until it breaks.

CLASSIFICATION OF MATERIAL ON ELASTIC PROPERTIES

On the basis of elastic properties , the materials can be classified as :-

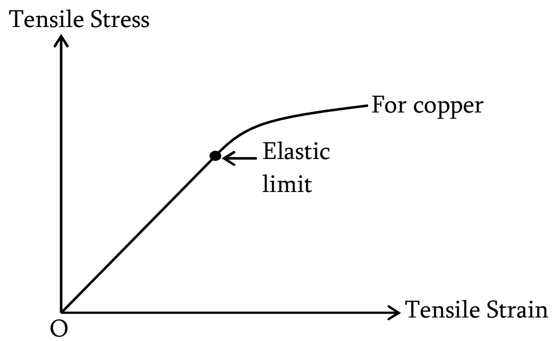
(i) Ductile materials

(ii) Brittle material

(iii) Elastomers.

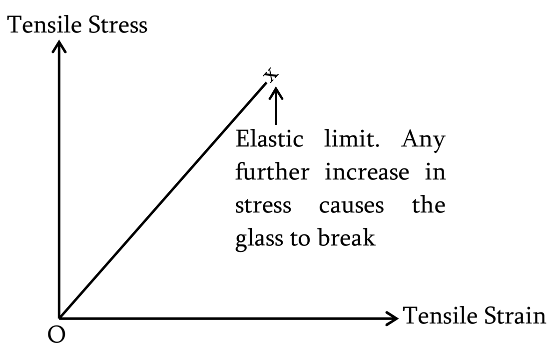
(i) DUCTILE MATERIALS

Are those materials which show large plastic range beyond elastic limit eg. Copper , silver , iron , aluminum e.t.c.



(ii) BRITTLE MATERIALS

Are those materials which show small plastic range beyond elastic limit eg. Glass , cast iron , ceramics e.t.c. A brittle material cannot be permanently stretched, it breaks soon after the elastic limit is reached.



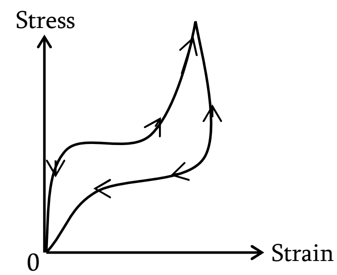
NOTE

- Materials which can be hammered into thin sheets are called 'Malleable' plastic deformation also allows a metal to be malleable i.e it can be hammered to give any shape. Eg. Gold , silver , lead e.t.c

(iii) ELASTOMERS

Are those materials which do not obey Hooke's law within elastic limit (i.e within elastic limit , stress – strain curve is not a straight line. A substance that can be elastically stretched to a large value of strain is called an Elestomer. Example : Rubber. The elastic tissue tissue of aorta is an elastomer in our body.

STRESS – STRAIN CURVE FOR RUBBER (Elastic Hysteresis)



When the stress applied on a body is decreased to zero , the strain will not be reduced to zero immediately for some substances (eg – vulcanized rubber) , the strain lags behind the stress. The lagging of strain behind stress is called **'Elastic hysterisis'**. The lack of coincidence of the curves for increasing and decreasing stress in known as **elastic hysteresis** . The stress – strain graph for increasing and decreasing load encloses a loop as shown in the figure above. The area of the loop gives the energy dissipated during its deformation.

ELASTIC AFTER EFFECT

The delay in regaining the original state by a body after the removal of the deforming force is called 'Elastic after effect'. In galvanometers and electrometers , the suspensions made from quartz and phosphor – bronze are used as the elastic after effect is negligible in wires of these materials.

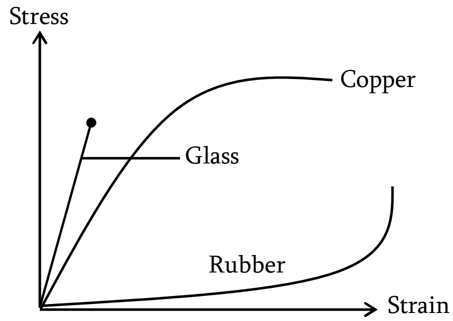
ELASTIC FATIQUE

Is defined as the loss in the strength of a material caused due to repeated alternating strains to which the material is subjected. Elastic fatique occurs , when a metal is subject to repeated loading and directions for this reason , a hard wire is broken by bending it repeatedly in opposite directions.

CREEP

The gradual increase in strain which occurs when a material is subjected to stress for a long period of time is known as Creep.

STRESS – STRAIN CURVES



NUMERICAL EXAMPLES

29. (a) Define the following materials as classified on the basis of elastic properties:-
    (i) Ductile materials
    (ii) Brittle materials
    (iii) Elastomers

(b) (i) Briefly explain why the stretching of a coil spring is determined by its shear modulus.
    (ii) A copper wire of negligible mass , 1m long and cross – sectional area $10^{-5}m^2$ is kept on a smooth horizontal table with one end fixed. A ball of 1kg is attached to the other end. The wire and the ball are rotating with an angular velocity of 35rad/s if the elongation of the wire is 10^{-3}m; find Young's modulus of wire. If on increasing the angular velocity to 100rad/s; the wire breaks down , find the breaking stress.

(c) (i) differentiate bulk modulus from shear modulus.
    (ii) Two wires , one of steel and one of phosphor bronze each 1.5m long and 2mm diameter are joined end to end as a composite wire of length 3cm. what tension in the composite wire will produce total extension of 0.064cm?

Young's modulus for steel = $2.0 \times 10^{11}Nm^{-2}$
Young's modulus for bronze = $1.2 \times 10^{11}Nm^{-2}$

Solution

(a) Refer to your notes

(b) (i) It is because when a coil spring is stretched , there is neither a change in the length of the coil nor a change in its volume. Only change that takes place is the change in shape of the coil spring is determined by its shear modulus.

(iii) Stretching force on the wire is given by.

$$F = M\omega^2 r = 1\times1\times(20)^2$$

$$F = 400N$$

$$\text{Stress in wire} = \frac{F}{A} = \frac{400}{10^{-6}}$$

$$\frac{F}{A} = 4\times10^{8}\,Nm^{-2}$$

Strain in wire

$$= \frac{\Delta L}{L} = \frac{10^{-3}}{1} = 10^{-3}$$

Young' s modulus

$$E = \frac{\text{stress}}{\text{strain}}$$

$$E = \frac{4\times10^{8}}{10^{-3}} = 4\times10^{11}\,Nm^{-2}$$

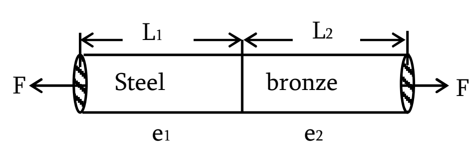
$$E = 4.0\times10^{11}\,N/m^2$$

$$\text{Breaking stress} = \frac{Mr\omega_B^2}{A}$$

$$= \frac{1\times1\times(100)^2}{10^{-6}}$$

Breaking stress = $10^{10}Nm^{-2}$.

(c) (i) Refer to your notes
    (ii) Diagram $L_1 = L_2 = L$



Young's modulus

$$E = \frac{FL}{Ae}\ ,\quad e = \frac{FL}{AE}$$

For steel: $e_1 = \dfrac{FL}{AE_1}$

For bronze: $e_2 = \dfrac{FL}{AE_2}$

Total extension $e = e_1 + e_2$

$$e = \frac{FL}{AE_1} + \frac{FL}{AE_2} = \frac{FL}{A}\left[\frac{1}{E_1} + \frac{1}{E_2}\right]$$

$$e = \frac{FL}{A}\left[\frac{E_2 + E_1}{E_1E_2}\right]$$

$$F = \frac{Ae}{L}\left[\frac{E_1E_2}{E_1 + E_2}\right] \quad \text{but } A = \frac{\pi d^2}{4}$$

$$F = \frac{\pi d^2 e}{4L}\left[\frac{E_1E_2}{E_1 + E_2}\right]$$

$$= \frac{3.14\times(2\times10^{-3})^2\times0.064\times10^{-2}}{4\times1.5}\left[\frac{2\times10^{11}\times1.2\times10^{11}}{(2+1.2)\times10^{11}}\right]$$

$$F = 10.191\text{N}$$

30. (a) Calculate the elastic potential energy per unit volume of water at the depth of 1km. compressibility (β) of water = 5 × 10^{-10}SIunit, density of water = 1000kgm^{-3}.

(b) A boy has a catapult made of rubber cord of length 42cm and diameter 6.0mm.the boy stretches the cord by 20cm to catapult a pebble of mass 20g. the pebble flies off with a speed of 20m/s. find the young's modulus for rubber ignore the change in the cross-section of the cord in stretching.

Solution

(a) Energy per unit volume

$$U = \frac{1}{2}\times \text{stree}\times\text{strains}$$

But

$$K = \frac{\text{stress}}{\text{strain}} \quad ; \quad \text{strain} = \frac{\text{stress}}{K}$$

$$= \frac{1}{2}\beta(\text{stress})^2$$

$$= \frac{1}{2}\times5\times10^{-10}\times[1000\times1000\times9.8]^2$$

$$U = 2.4\times10^4\,\text{J/m}^3$$

(b) Apply the law of conservation of energy

Potential energy Stored in the cord = kinetic energy of the stone

$$\frac{1}{2}F\Delta L = \frac{1}{2}MV^2$$

$$F = \frac{MV^2}{\Delta L}$$

$$\text{Stress in the cord} = \frac{F}{A} = \frac{MV^2}{\pi r^2\Delta L}$$

$$\text{strain} = \frac{\Delta L}{L}$$

$$\text{Young's modulus},\ E = \frac{\text{stress}}{\text{strain}}$$

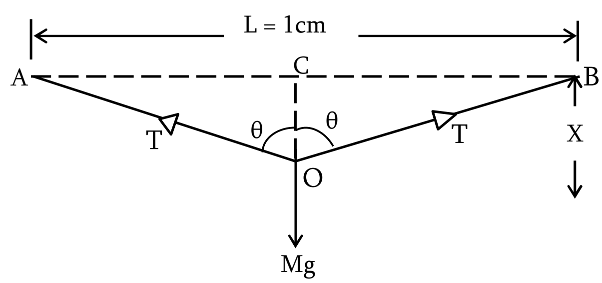
$$E = \frac{MV^2L}{\pi r^2(\Delta L)^2}$$

$$= \frac{(20\times10^{-3})\times(20)^2\times(42\times10^{-2})}{\pi(3\times10^{-3})^2\times(20\times10^{-2})^2}$$

$$E = 2.97\times10^6\,\text{N/m}^2$$

31. A mild steel wire of length 1m and cross – sectional area 0.5 × 10^{-2}cm^2 is stretched , well within its elastic limit , horizontally between two pillars. A mass of 100g is suspended from the mid – point of the wire. Calculate the depression at the mid – point. Given that young's modulus for steel = 2.0 × 10^{11}Nm^{-2}.

Solution



Let C be the midpoint of the wire AB

The increase in length of the wire

$$\Delta L = \overline{AO} + \overline{OB} - \overline{AB} \quad \text{but } \overline{AO} = \overline{OB}$$

$$\Delta L = 2\overline{AO} - \overline{AB}$$

If X is the depression at the mid – point

$$\overline{AO}^2 = \overline{AC}^2 + \overline{OC}^2$$

$$\overline{AO} = \left[\overline{AC}^2 + \overline{OC}^2\right]^{\frac{1}{2}}$$

$$\overline{AO} = \overline{AC}\left[1+\frac{\overline{OC}^2}{\overline{AC}^2}\right]^{\frac{1}{2}}$$

Here

$$\overline{AC} = \overline{BC} = \frac{L}{2} = \frac{1}{2} = 0.5m$$

$$\overline{AO} = 0.5\left[1+\frac{X^2}{\left(\frac{1}{2}\right)^2}\right]^{\frac{1}{2}}$$

Now

$$\Delta L = 2\times\frac{1}{2}\left[1+\frac{X^2}{\left(\frac{1}{2}\right)^2}\right]^{\frac{1}{2}} - 1$$

$$\Delta L = \left[1+4X^2\right]^{\frac{1}{2}} - 1$$

$$= \left(1+\frac{1}{2}4X^2\right) - 1$$

$$\Delta L = 2X^2$$

IF t is the tension in each segment of the wire.

$$T\cos\theta + T\cos\theta = mg$$

$$2T\cos\theta = Mg$$

$$T = \frac{Mg}{2\cos\theta}$$

$$\cos\theta = \frac{\overline{OC}}{\overline{\overline{AO}}} = \frac{X}{\left[AC^2+X^2\right]^{\frac{1}{2}}}$$

$$= \frac{X}{AC}\times\left[1+\frac{X}{AC^2}\right]^{-\frac{1}{2}}$$

$$= \frac{X}{AC} - \frac{1}{2}\frac{X^3}{AC^3} \simeq \frac{X}{AC} = \frac{X}{0.5}$$

$$\cos\theta = 2X$$

Now

$$T = \frac{Mg}{2\times 2x} = \frac{Mg}{4x}$$

$$E = \frac{FL}{A\Delta L} = \frac{TL}{A\Delta L}$$

$$E = \frac{MgL}{4A\times\left(2x^2\right)} = \frac{MgL}{8x^3A}$$

$$X = \left[\frac{MgL}{8EA}\right]^{\frac{1}{3}}$$

$$A = 0.5\times10^{-6}m^2$$

$$X = \left[\frac{0.1\times9.8\times1}{8\times2\times10^{11}\times0.5\times10^{-6}}\right]^{\frac{1}{3}}$$

$$X = 1.07\times10^{-2}m = 1.07cm$$

32. (a) Differentiate between tensile and shear stress.

(b) A lift is designed to hold a maximum 12 people. The lift cage has a mass of 500kg and distance from the top of the building to the ground is 50m.

(i) What is the minimum cross – sectional area should the cable have in order to support the lift and the people in it?

(ii) Why should the cable have to be thicker than the minimum cross – sectional area in 2(b)(i) in practice?

(iii) How much will the lift cable in 2(b)(i) above stretched if 10 people get into the lift at the ground floor assuming that the lift has a cross – sectional of 1.36cm? the ass of an average person = 70kg , young's modulus of steel = $2 \times 10^{11}Nm^{-2}$, and tensile strength of steel = $4.0 \times 10^8Nm^{-2}$

Solution

(a) Refer to your notes

(b) (i) Total weight in the lift

W = Weight of 12 people + weight of cage

W = (12 × 70 + 500) × 9.8

W = 13,132N

$$\text{Tensile strength} = \frac{\text{Force}}{\text{Area}}$$

$$\varepsilon = \frac{W}{A_{min}} \quad , \quad A_{min} = \frac{W}{\varepsilon}$$

$$A_{min} = \frac{13,132}{40\times10^8}$$

$$A_{min} = 3.283\times10^{-5}m^2$$

(iii) In practice , the cable should be thicker than the minimum cross – sectional area in order to avoiding accident and reduce the normal tensile stress to the cage since area of $3.283 \times 10^{-5}m^2$ is minimum hence breaking stress must be small compared to the actual stress.

(iv) Total weight = weight of 10 people + weight of the cage

$= (70 \times 10 + 500) \times 9.8$

$W = 11760N$

Since $E = \frac{FL}{Ae} = \frac{WL}{Ae}$

$$e = \frac{WL}{AE} = \frac{11760 \times 50}{1.36 \times 10^{-4} \times 2 \times 10^{11}}$$

$e = 0.02m$

33. (a) Why do spring balances show wrong readings after they have been used for a long time?

(b) What will be the density of lead under a pressure of $2 \times 10^8 N/m^2$? Density of lead $= 11.4 \times 10^3 kg/m^3$ and bulk modulus of lead $K = 8 \times 10^9 N/m^2$.

Solution

(a) If a material is repeatedly stressed and unstressed , it becomes weaker i.e the strain produced by a given amount of stress increases. For this reason , the spring balances which have been used for a long time given wrong reading.

(b) Bulk modulus $k = -\frac{vdp}{dv}$

$$dv = -\frac{vdp}{k} = \frac{-2 \times 10^8 v}{8 \times 10^9}$$

New volume of lead $v_1 = v + dv$

$$v_1 = v + \frac{-v}{40} = \frac{39v}{40v}$$

ρ_1 = new density

Apply the conservation of mass

$$\rho_1 v_1 = \rho v$$

$$\frac{39v}{40}\rho_1 = v \times 11.4 \times 10^3$$

$$\rho_1 = 11.4 \times 10^3 \times \frac{40}{39}$$

$$\rho_1 = 11.69 \times 10^3 kgm^{-3}$$

34. If the normal density of sea water is $1000kgm^{-3}$. What will be its density at a depth of 3km? Given that the compressibility of water = 0.000048 per atmosphere? 1 atmosphere $= 1.01 \times 10^5 N/m^2$.

Solution

Consider a sample of mass , M gram having a volume , V

$\rho = \frac{M}{V}$ ......(i)

Bulk modulus of the water

$$B = \frac{1}{\text{Compressibility}} = \frac{1\text{atm}}{0.000048}$$

$$= \frac{1.01 \times 10^5}{0.000048}$$

$B = 2.1041 \times 10^9 Nm^{-2}$

Depth $h = 3km = 3000m$

Pressure $dp = \rho gh$

$$B = -V\frac{dp}{dv} \Rightarrow \frac{dv}{v} = \frac{-dp}{B}$$

$$\frac{dv}{v} = -\frac{\rho gh}{B} = \frac{-1000 \times 9.8 \times 3000}{2.104 \times 10^9}$$

$$\frac{dv}{v} = -0.01397$$

New volume , $v_1 = v + dv$

$$v_1 = v - 0.01397v$$

$$= v[1 - 0.01397]$$

$$v_1 = 0.98603v$$

New or final density

$$\rho_1 = \frac{m}{v_1} = \frac{m}{0.98603v}$$

$$\rho_1 = \frac{\rho}{0.98603} = \frac{1000}{0.98603}$$

$$\rho_1 = 1014.1679 kgm^{-3}$$

35. What is the density of ocean water at a depth where the pressure is 80 atmosphere. Given that its density at the surface is $1.03 \times 10^3 kgm^{-3}$. Compressibility of water $= 45.3 \times 10^{-11} Pa^{-1}$.

Solution

Density at the surface

$$\rho = 1.03 \times 10^3 kgm^{-3}$$

Compressibility $= \dfrac{1}{K} = 45.3\times10^{-11}\,Pa^{-1}$

Pressure $P = 80atm = 80\times1.01\times10^{5}\,Pa$

Let ΔV = change in volume

$$K = \frac{PV}{\Delta V}\ ,\quad \Delta V = \frac{PV}{K}$$

$$\Delta V = 80\times1.01\times10^{5}\times45.8\times10^{-11}\,V$$

$$\Delta V = 0.0037V$$

New volume , $V_1 = V-\Delta V$

$$= V - 0.0037V$$

$$V_1 = 0.9963V$$

Apply the law of conservation of mass

$$\rho_1 V_1 = \rho V$$

$$0.9963V\rho_1 = \rho V$$

$$\rho_1 = \frac{\rho}{0.9963} = \frac{1.03\times10^{3}}{0.9963}$$

$$\rho_1 = 1.0338\times10^{3}\,kgm^{-3}$$

36. Find the change in volume which $1m^3$ of water will undergo when taken from the surface to bottom of sea one mile deep. Given that elasticity of water = 20,000 atm.

Solution

$V = 1m^3$, h = 1mile = $1760\times3\times30.48$

h = 160,934.4 = 160.9344m

ΔV = change in volume

$$K = V\frac{dP}{dV}\ \text{(In magnitude)}$$

$$dp = \rho gh$$

$$K = \frac{\rho ghv}{\Delta v}\ ,\quad \Delta v = \frac{\rho ghv}{K}$$

$$\Delta V = \frac{1000\times9.8\times160.9344\times1}{20,000\times1.0\times10^{5}}$$

$$\Delta V = 7.785\times10^{-3}\,m^3$$

37. A sphere contracts in volume by 0.01%. When taken to the bottom of sea 1km deep. Find the bulk modulus of the material of sphere. Given the density of sea water = $1000kgm^{-3}$ and $g = 9.8m/s^2$

Solution

Bulk modulus $K = \dfrac{VdP}{dV}$

But $dP = \rho gh$

$$dv = \frac{\rho ghv}{k}$$

$$\frac{dv}{v} = \frac{1000\times9.8\times1000}{k}$$

$$k = 9.8\times10^{10}\,Nm^{-2}$$

38. One cm3 of water is taken from the surface to the bottom of a lake 200m deep. The bulk modulus of water is 2.2×10^4 atmosphere and density of water is $1000kgm^{-3}$ and atmospheric pressure is 10^5Nm^{-2} and $g = 9.8m/s^2$. Calculate the change in volume.

Solution

$$K = \frac{VdP}{dV}\qquad \text{But}\quad dp = \rho gh$$

$$dV = \frac{V\rho gh}{K}$$

$$= \frac{10^{-6}\times1000\times9.8\times200}{2.2\times10^{4}\times1.01\times10^{5}}$$

$$dV = 8.9\times10^{-4}\,cm^3$$

39. The average depth of Indian Ocean is about 3000m. Calculate the fractional compression, $\Delta V/V$ of water at the bottom of the ocean, given that the bulk modulus of water is $2.2\times10^9Nm^{-2}$.

Solution

$$\text{Since}\quad \frac{dV}{V} = \frac{\rho gh}{K}$$

$$= \frac{1000\times9.8\times3000}{2.2\times10^{9}}$$

$$\frac{dV}{V} = 1.36\times10^{-2}\ \text{or}$$

$$\frac{dV}{V}\times100\% = 1.36\%$$

40. A material has normal density ρ and bulk modulus K. find the increase in the density of the material when it is subjected to an external pressure P from all sides.

Solution

$$\frac{\rho_1}{\rho} = \frac{M}{V-\Delta V}\Big/\frac{M}{V}$$

$$\frac{\rho_1}{\rho} = \frac{V}{V-\Delta V}$$

$$\frac{\rho_1}{\rho} = V\left(V-\Delta V\right)^{-1}$$

(using binomial theorem)

$$\frac{\rho_1}{\rho} = 1+\frac{\Delta V}{V}$$

$$\frac{\Delta V}{V} = \frac{\rho_1}{\rho}-1 = \frac{\rho_1-\rho}{\rho}$$

Again $\quad K = \dfrac{\Delta P}{\Delta V/V} = \dfrac{\rho\Delta P}{\rho_1-\rho}$

$$\rho_1-\rho = \frac{\rho\Delta P}{K} = \frac{P\rho}{K} \quad \left[\begin{array}{l}\Delta P = P\\ \text{Given}\end{array}\right]$$

41. Compute the bulk modulus of water from the following data:-
Initial volume = 100 litre.
Final volume = 100.5 litre.
Change in pressure = 100 atm
1atm = 1.013×10^5Pa
Compare the bulk modulus of water with that of air at constant temperature. Explain in simple terms why is the ratio so large.

Solution

Original volume , $\quad V = 100$litres
Change in volume $\Delta V = 100.5 - 100$
$\Delta V = 0.5$ litre.
dp = 100atm = 100 × 1.013 × 105Pa.

Bulk modulus $K = \dfrac{VdP}{dV}$

$$K = \frac{100\times1.013\times10^5\times100}{0.5}$$

$$K = 2.026\times10^9\,\text{Pa}$$

Reason

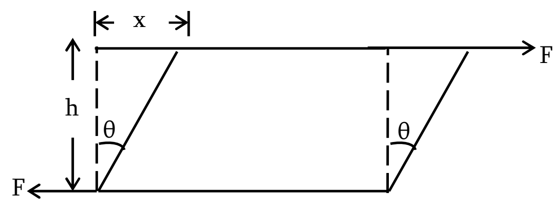
For water k is very large , because change in volume of water is very small when compressed . But for air for the same change in pressure , the change in volume is very large.

42. A certain runners foot touches the ground the shearing force acting on an 8mm thick sole of shoes is 25N distributed over an area 1.5cm². Calculate the value of angle of shear given that the shear modulus of the sole is 1.9×10^5Nm^{-2}.

Solution

h = 8mm , F = 25N , A = 15cm³

η = 1.9 × 105Nm^{-2}, $\quad \theta = ?$



Shear stress $= \dfrac{F}{A}$, Shear strain = tanθ

$$\eta = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F}{A\tan\theta}$$

$$\tan\theta = \frac{F}{\eta A} = \frac{25}{1.9\times10^5\times15\times10^{-4}}$$

$$\tan\theta = 0.08772$$

$$\theta = \tan^{-1}\left(0.08772\right) = 5.013^\circ$$

$$\theta = 5.013^\circ \simeq 0.08772\text{rad}$$

43. (a) Which is more elastic iron or rubber?
(b) A spherical ball contracts in volume by 0.01% when subjected to a normal uniform pressure of 10^8Nm^{-2}. Find the bulk modulus of the material.

Solution

(a) Iron is more elastic than rubber , it is because for a given stress , the strain produced in iron is much smaller than that produced in the rubber.
(b) dp = 10^8Nm^{-2}

$$\frac{dV}{V}\times100\% = 0.01\%$$

$$\frac{dV}{V} = \frac{0.01}{100} = 10^{-4}$$

Bulk modulus $K = \dfrac{dP}{dV/V}$

$$K = \frac{10^8}{10^{-4}} = 10^{12}\,\text{Nm}^{-2}$$

$$K = 1.0\times10^{12}\,\text{Nm}^{-2}$$

44. A square lead slab of side 50cm and thickness 5.0cm is subjected to a shearing force (on its narrow face) of magnitude 9.0×10^4N. The lower edge is riveted to the floor. How much is the upper edge displaced , if the shear modulus of lead is 5.6×10^9Pa?.

Solution

H = 5.6 × 109Pa , F = 9.0 × 104n

Area of the narrow face on which the force is applied.

$A = 50 \times 5 = 250cm^2 = 250 \times 10^{-4}m^2$

Distance of the narrow face from the floor.

$L = 50cm = 50 \times 10^{-2}m$

If the upper edge of the slab gets displaced through a distance , X

$$\eta = \frac{F/A}{X/L} = \frac{FL}{XA}$$

$$X = \frac{FL}{A\eta} = \frac{9.0\times10^4\times50\times10^{-2}}{250\times10^{-4}\times5.6\times10^9}$$

$$X = 3.214\times10^{-4}\,m$$

45. (a) (i) State Hooke's law

(ii) What is the limitation of Hooke's law

(b) A load of 31.4kg is suspended from a wire of radius 10^{-3}m and density $9.0 \times 10^3kgm^{-3}$. Calculate the change in temperature of the wire , if 75% of the work done is converted into heat. Given that young's modulus and heat capacity of the material of the wire are $9.8 \times 10^{10}Nm^{-2}$ and $490Jkg^{-1}K^{-1}$ respectively.

Solution

(a) (i) State that 'within elastic limit , stress is directly proportional to strain

(b) Work done in stretching the wire

$$W = \frac{1}{2}Fe$$

Since $E = \frac{FL}{Ae} = \frac{FL}{\pi r^2 e}$

$$e = \frac{FL}{\pi r^2 E}$$

$$W = \frac{1}{2}F\cdot\frac{FL}{\pi r^2E} = \frac{F^2L}{2\pi r^2E} \quad \ldots(i)$$

The work done in stretching the wire is converted into heat energy if $\Delta\theta$ is rise in temperature , then

$$W = MC\Delta\theta = \left(\pi r^2 L\rho\right)C\Delta\theta \quad \ldots..(ii)$$

(i) = (ii)

$$\pi r^2 L\upsilon C\Delta\theta = \frac{F^2L}{2\pi r^2E}$$

$$\Delta\theta = \frac{F^2}{2\pi^2r^4Y\rho C}$$

$$= \frac{(31.4\times9.8)^2}{2\pi r^2\times\left(10^{-3}\right)^4\times9.8\times10^{10}\times9\times10^3\times490}$$

$$\Delta\theta = 0.011^{\circ}C$$

46. A sphere of 50gm is attached to one end of a steel wire 0.315m diameter end one metre long in order to form a conical pendulum , the other end is attached to a vertical shaft which is set rotating about its axis. Calculate the number of revolutions necessary to extend the wire of elasticity of steel = $2 \times 10^{11}Nm^{-2}$.

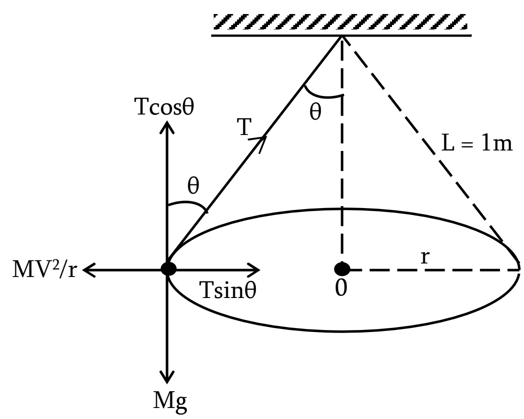
Solution

Let T = tension in the wire

Young's modulus

$$E = \frac{TL}{Ae} = \frac{TL}{\pi r^2 e}$$

$$T = \frac{\pi r^2 eE}{L}$$



$$T = \frac{3.14\times(.1575)^2\times1\times10^{-3}\times2\times10^{11}}{1}$$

$$T = 15.578\times10^6\,N$$

From the figure above

$$T\cos\theta = Mg$$

$$\cos\theta = \frac{Mg}{T} = \frac{50\times10^{-3}\times9.8}{15.578\times10^6}$$

$$\cos\theta = 3.145\times10^{-8}$$

$$\sin\theta = \sqrt{1-\cos^2\theta}$$

$\sin \simeq 1$

Since $r = L\sin\theta = 1m$

Again $T\sin\theta = \frac{MV^2}{r}$

$$V^2 = \frac{rT\sin\theta}{M}$$

$$V^2 = \frac{1\times 15.578\times 10^6\times 1}{50\times 10^{-3}}$$

$$V = 17,651.1\text{m/s}$$

Periodic time

$$T = \frac{2\pi r}{V} = \frac{2\times 3.14\times 1}{17,651.1}$$

$$T = 3.55786\times 10^{-4}\text{ sec}$$

Frequency of revolution

$$f = \frac{1}{T} = 2,810.68\text{Hz}$$

$$f = 2811\text{Hz}$$

47. A simple pendulum is made by attaching a 1.0kg bob to 5m long copper wire of diameter 0.08cm and it has certain period of oscillation next a 10kg bob is substituted for the 1kg bob. Calculate the change in time period if any (young's modulus of copper = $12.4 \times 10^{10}\text{Nm}^{-2}$)

Solution

Young's modulus of copper wire

$$E = \frac{FL}{Ae} = \frac{FL}{A\Delta L} \quad [\Delta L = e]$$

$$\Delta L = \frac{FL}{AE} = \frac{Mg}{\pi r^2 E}$$

The change in length when 1kg bob is attached.

$$\Delta L = \frac{1\times 9.8\times 5}{3.14\left(0.04\times 10^{-2}\right)^2\times 12.4\times 10^{10}}$$

$$\Delta L_1 = 7.86\times 10^{-4}$$

The change in length when 10kg is attached at the end of the wire.

$$\Delta L_2 = \frac{10\times 9.8\times 5}{3.14\times\left(0.04\times 10^{-2}\right)^2\times 12.4\times 10^{10}}$$

$$\Delta L_2 = 7.86\times 10^{-3}\text{m}$$

Periodic time of the simple pendulum

$$T = 2\pi\sqrt{\frac{L}{g}}$$

New periodic time due to the increase in length

$$T' = 2\pi\sqrt{\frac{L+\Delta L}{g}}$$

For the first case

$$T_1 = 2\pi\sqrt{\frac{L+\Delta L_1}{g}}$$

$$= 2\times 3.14\sqrt{\frac{5+7.86\times 10^{-4}}{9.8}}$$

$$T_1 = 4.488\text{ sec}$$

For the second case

$$T_2 = 2\pi\sqrt{\frac{L+\Delta L_2}{g}}$$

$$T_2 = 2\times 3.14\sqrt{\frac{5+7.86\times 10^{-3}}{9.8}}$$

$$T_2 = 4.4915\text{ sec}$$

Change in the periodic time

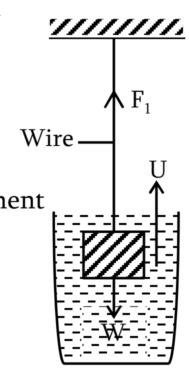
$$\Delta T = T_2 - T_1$$

$$= 4.4915 - 4.488$$

$$\Delta T = 0.0035\text{ sec}$$

48. A metal block of weight 20N and volume $8 \times 10^{-4}\text{m}^3$, completely immersed in oil of density 700kgm^{-3}, is attached to one end of a vertical wire of length 4.00m whose other end is fixed. The length of the wire then increases by 1mm. if the diameter of the wire is 0.6mm. Calculate.
(i) It young's modulus
(ii) The energy stored in the wire
(Assume $g = 10\text{Nkg}^{-1}$)

Solution



W = weight of block in air

W = 20N

ρ = density of oil

$\rho = 700\text{kgm}^{-3}$

Upthrust on metal

= weight of oil displacement

$U = \rho Vg$

$= 8 \times 10\text{-}4 \times 700 \times 10$

U = 5.6N

Force on the wire

$F = W - U = 20 - 5.6$

F = 14.4N

(i) Young's modulus of wire

$$E = \frac{FL}{Ae} = \frac{FL}{\pi r^2 e}$$

$$E = \frac{14.4 \times 4}{3.14 \times \left(0.3 \times 10^{-3}\right)^2 \times 1 \times 10^{-3}}$$

$$E = 2.0 \times 10^{11}\,\text{Nm}^{-2}$$

(ii) Energy store in the wire

$$W = \frac{1}{2}Fe$$

$$= \frac{1}{2} \times 14.4 \times 1 \times 10^{-3}$$

$$W = 7.2 \times 10^{-3}\,\text{J}$$

49. (a) (i) Differentiate between plastic materials and elastic materials in terms of the yield point.

(ii) With the help of diagrams given the meaning of shear strain and shear stress.

(b) A force of 6.2×10^4Nacts axially to a structural steel rod of radius 9.5mm and length 81.0cm find:-

(i) The strength in the rod.

(ii) The elongation of the rod under such a load is it safe for such a rod to build bridges which could hold 7 tones? Given reasons for your answer.

(c) (i) Explain the fact that the legs of a mouse are thinner than those of an elephant , yet the weight of each animal is well supported by the legs of the animals.

(ii) A load of 500kg is hanging from a steel wire. If its length of 3cm and cross – sectional area of 0.2cm^2 was found to stretch the wire by 0.4cm above its non – load length ; what are the stress and young's modulus of the wire?

(d) (i) Explain the difference between ductile and brittle materials.

(ii) Discuss the effects of temperature changes on the strength and stiffness of a solid material

($E = 2 \times 10^{11}\text{N/m}^2$)

Solution

(a) (i) Plastic materials have a short plastic range beyond yield point while elastic material have a wide plastic range beyond yield point before deformation.

(ii) Refer to your notes.

(b) (i) $\text{Stress} = \frac{\text{Force}}{\text{Area}}$

$$= \frac{6.2 \times 10^4}{\pi\left(9.5 \times 10^{-3}\right)^2}$$

Stress in the rod = $2.2 \times 10^8\text{N/m}^2$ (approx.)

$$\text{strain} = 1.09 \times 10^{-3}$$

$$e = L \times \text{strain}$$

$$= 1.09 \times 10^{-3} \times 0.81$$

$$= 8.856 \times 10^{-4}\,\text{m} = 0.8856\text{mm}$$

∴ Elongation of the rod ≃ 0.9mm. When subjected to a 7 tonnes load.

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

$$= \frac{7 \times 10^3 \times 9.8}{\pi\left(9.5 \times 10^{-3}\right)^2}$$

Stress = $2.4195 \times 10^8\text{N/m}^2$

Yield strength of steel

= $2.5 \times 10^8\text{N/m}^2$

Stress < yield strength

∴ It is safe for such a rod to be subjected to 7tonnes load.

(c) (i) The mouse is very light compared to an elephant. The weight is well supported by the thin legs since the pressure is small as well. An elephant is heavy for the pressure on the ground and so the reaction on the elephant to be minimum, the surface area of the legs has to be large as well since P = F/A as a result on elephant is well supported by the thick legs.

(ii) $\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{500 \times 9.8}{2 \times 10^{-1} \times 10^{-4}}$

$$= 2.45 \times 10^8\,\text{N/m}^2$$

$$\text{strain} = \frac{\Delta L}{L} = \frac{4 \times 10^{-1}}{3}$$

$$\text{Strain} = 0.1333$$

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{2.45\times10^{8}}{0.1333}$$

$$E = 1.8375\times10^{9}\,\text{N/m}^2$$

(d) (i) Refer to your notes

(ii) Strength of a material is the maximum force which can be applied to the material without breaking it. Stiffness of a solid material is the resistance of a material against deformation. When the temperature is rises , the strength and stiffness of materials decreases since at high temperatures thy creep due to vibratory motion of atoms. This means thy keep deforming even under constant stress.

50. (a) Water is more elastic than air , why?

(b) A square lead slab of side 50cm and thickness 5.0cm is subjected to a shearing force (on its narrow face) of magnitude 9.0 × 104N. The lower edge is riveted to the floor. How much is the upper edge displaced if the shear modulus of the lead is 5.6 × 10^9N/m^2.

Solution

(a) We know that volume elasticity is the reciprocal of compressibility. Now air is more compressible than water. Therefore water is more elastic than air.

(b) The area of cross – section of the face where the force is applied.

A = 50 × 5 = 250cm2 = 250 × $10^{-4}$$m^2$

$$\text{Shear modulus } \eta = \frac{F}{A\theta}$$

Shear strain,

$$\theta = \frac{F}{\eta A} = \frac{9\times10^{4}}{5.6\times10^{9}\times250\times10^{-4}}$$

$$\theta = 6.4\times10^{-4}\,\text{rad}$$

Now

$$\theta = \frac{x}{h}$$

x = displacement of the upper edge

$$x = \theta h$$

$$= 6.4\times10^{-4}\times0.5$$

$$x = 3.2\times10^{-4}\,\text{m}$$

51. A 20kg weight is suspended from a length of copper wire 1mm in radius. If the wire breaks suddenly does its temperature increase or decrease. Calculate the change in temperature specific heat capacity of copper = 4200JK$g^{-1}$$K^{-1}$.

Density of copper , ρ = 9000Kgm^{-3}.

Young's modulus of copper = 12 × 10^{10}Nm^{-2}.

Solution

When the wire is stretched its gained P.E equal to the work done on it. When the wire is suddenly break this potential energy is released as the molecules return to their original position. The potential energy is converted into heat energy that is why temperature is increase in the system.

Apply the law of conservation of energy

Energy stored in the copper wire = heat energy produced

$$MC\Delta\theta = \frac{EAe^2}{2L}$$

$$\rho ALC\Delta\theta = \frac{EAe^2}{2L}$$

$$\rho LC\Delta\theta = \frac{Ee^2}{2L}$$

$$\Delta\theta = \frac{Ee^2}{\rho CL^2} \quad \text{but } e = \frac{FL}{EA}$$

$$= \frac{E}{\rho CL^2}\cdot\frac{F^2L^2}{E^2A^2}$$

$$= \frac{1}{\rho CE}\left(\frac{F}{A}\right)^2 \quad F = Mg,\; A = \pi r^2$$

$$\Delta\theta = \frac{1}{\rho CE}\left[\frac{Mg}{\pi r^2}\right]^2$$

$$= \frac{1}{9000\times4200\times12\times10^{10}}\left[\frac{20\times9.8}{3.14\left(10^{-3}\right)^2}\right]^2$$

$$\Delta\theta = 8.59\times10^{-4}\,\text{K}$$

52. On taking a solid ball from the surface to the bottom of a take of 200m depth the reduction in the volume of the ball is 0.1%. The density of water of the take is $1000kgm^{-3}$. Determine the bulk modulus of elasticity of rubber (take $g = 10m/s^2$)

Solution

$dp = \rho gh = 200 \times 10 \times 1000$

$dp = 2 \times 10^6 Nm^{-2}$

$$\text{Volumetric strain } \frac{dv}{v} = \frac{0.1}{100} = 10^{-3}$$

$$k = 2\times10^9 N/m^2$$

53. A rubber cube of side 20cm has one side fixed while a tangential force is equal to the weight of 400kg is applied to the opposite. Find:-
(i) Shearing strain
(ii) The distance through which the strained side moves.
Given that modulus of rigidity for the rubber is $8.0 \times 10^6 N/m^2$.

Solution

$$\text{(i) Shear stress} = \frac{F}{A} = \frac{400\times9.8}{\left(2\times10^{-2}\right)^2}$$

$$= 9.8\times10^4 N/m^2$$

Modulus of rigidity

$$\eta = \frac{\text{shearing stress}}{\text{shear strain}}$$

$$\text{Shear strain}, \theta = \frac{\text{shear stress}}{\eta}$$

$$\theta = 0.0123$$

$$\text{(ii) } \theta = \frac{x}{h}$$

$$x = \theta h = 0.0123\times0.2$$

$$x = 0.0025m = 0.25cm$$

54. An aluminium cube of each side 4cm is subjected to a tangential force. The top force of the cube is sheared 0.012cm with respect to the bottom. Find:-
(i) Shearing strain
(ii) Shear stress and
(iii) Shear force
Given that modulus of rigidity is $2.08 \times 10^{10} N/m^2$.

Solution

$$\text{(i) Shearing strain } \theta = \frac{x}{h}$$

$$x = \frac{0.012}{4} = 0.003$$

$$\text{(ii) Shear stress} = \eta\times\text{shear strain}$$

$$\text{Shear stress} = 6.24\times10^7 Nm^{-2}$$

(iii) Shearing force = η × area of the cube face

$$= 6.24\times10^7\times16\times10^{-4}$$

$$F = 9.98\times10^4 N$$

55. A cube is subjected to a pressure of 5.0 × 105Nm-2. Each side of the cube is shortened by 1%. Find
(i) The volumetric strain and
(ii) Bulk modulus of elastic of cube material.

Solution

Let L be the initial length of each side of the cube.

Initial volume $V = L^3$

Final length of each side

$$L_1 = L-\frac{L}{100} = \frac{99L}{100}$$

$$\text{Final volume } V_1 = L_1^3$$

$$V_1 = \left(\frac{99L}{100}\right)^3 = \left(\frac{99}{100}\right)^3\cdot L^3$$

Change in volume

$$\Delta V = V_1 - V$$

$$= \left(\frac{99L}{100}\right)^3 - L^3$$

$$\Delta V = L^3\left[\left(\frac{99}{100}\right)^3 - 1\right]$$

(i) Volumetric strain

$$\frac{\Delta V}{V} = \frac{L^3\left[\left(\frac{99}{100}\right)^3-1\right]}{L^3}$$

$$\frac{\Delta V}{V} = -0.03$$

(ii) Bulk modulus

$$K = \frac{\text{Normal stress}}{\text{Volumetric strain}}$$

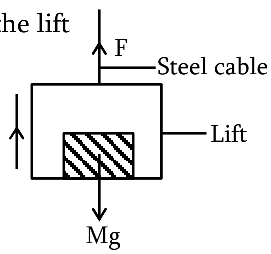
$$K = \frac{5\times10^5}{0.03}$$

$$K = 1.67\times10^7 Nm^{-2}$$

56. (a) Define tensile stress , tensile strain , young' modulus
(b) The maximum upward acceleration of a lift of total mass 2500kg is $0.5m/s^2$. The lift is supported by a steel cable , which has a maximum safe working stress of 1.0×10^8Pa. what minimum area of cross – sectional of cable should be used?
(c) A nylon guitar string62.8cm long and 1mm diameter is tuned by stretching it 2.0cm. Calculate (i) the tension (ii) the elastic energy stored in the string.
Young's modulus of nylon
$= 2 \times 10^9$Pa , $g = 10m/s^2$.

Solution

(a) Refer to your notes
(b) Resultant force on the lift



$$F_{net} = F - Mg$$
$$Ma = F - Mg$$
$$F = Ma + M$$
$$F = M(a+g)$$

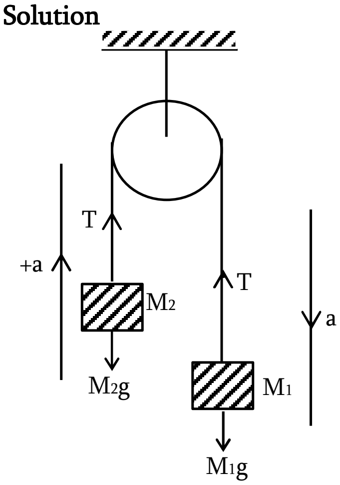
$$\text{Working stress} = \frac{\text{Force}}{\text{Area}}$$
$$A_{min} = \frac{M(a+g)}{\text{Workng stress}}$$
$$= \frac{2500(10+0.5)}{1.0\times10^8}$$
$$A_{min} = 2.625\times10^{-4}m^2$$

(c) (i) Young's Modulus
$$E = \frac{FL}{Ae} = \frac{4FL}{\pi d^2 e}$$
$$F = \frac{\pi d^2 E}{4L}$$
$$= \frac{3.14\times(1\times10^{-3})^2\times2\times10^9}{4\times0.628}$$
$$F = 2,500N$$

(ii) Elastic energy
$$W = \frac{1}{2}Fe$$
$$= \frac{1}{2}\times2,500\times2\times10^{-2}$$
$$W = 25J$$

57. A steel wire of length L and diameter d is placed over a massless , frictionless pulley with one end of the wire connected to a mass M_1 and other end to a mass M_2. When the masses move by how much does the wire stretch?

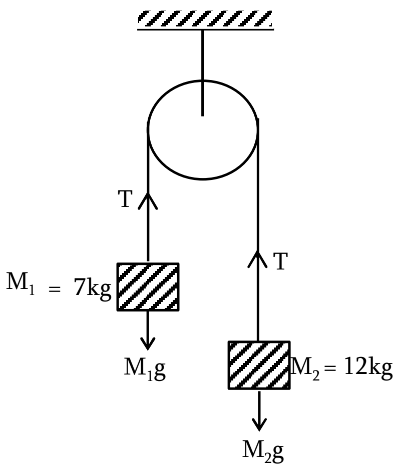
Solution



Let E = Young's modulus of steel
Resultant forces on the mass
M_1: $M_1g - T = M_1a$…………(i)
M_2: $T - M_2g = M_2a$…………(ii)
Adding equation (i) and (ii)
$$(M_1 - M_2)g = (M_1 + M_2)a$$
$$a = \left(\frac{M_1 - M_2}{M_1 + M_2}\right)g$$
From equation (i)
$$T = M_2g + M_2g\left[\frac{M_1 - M_2}{M_1 + M_2}\right]$$
$$T = \frac{2M_1M_2g}{M_1 + M_2}$$
Since $E = \frac{TL}{Ae}$, $A = \frac{\pi d^2}{4}$
$$E = \frac{E\pi L}{\pi d^2 e}$$
$$e = \frac{4L}{\pi d^2 E}\cdot T$$
$$e = \frac{8M_1M_2gL}{\pi d^2 E(M_1 + M_2)}$$

58. Two masses 7kg and 12kg are connected at the two ends of a metal wire that goes over a frictionless pulley. What should be the minimum radius of the wire in order that the wire does not break; if the breaking stress of the metal is $1.3 \times 10^8 N/m^2$?

Solution



The wire will not break, if the

$$\text{Breaking stress} = \frac{\text{Tension in the wire}}{A}$$

$$\varepsilon = \frac{2M_1M_2g}{\pi r^2 \left(M_1 + M_2\right)}$$

$$r^2 = \frac{2M_1M_2g}{\pi\left(M_1 + M_2\right) \times \text{Breaking stress}}$$

$$r = \left[\frac{2\times7\times12\times9.8}{3.14\times19\times1.3\times10^8}\right]^{1/2}$$

$$r = 4.6\times10^{-4}\text{m}$$

59. (a) A steel wire 30m long has a cross – section area of $0.5mm^2$, young's modulus of steel is 2.0×10^{11}Pa. Calculate the force constant of the wire.

(b) A uniform wire of unstretched length 2.49m is attached to two points A and B which are 2.0m a part and in the same horizontal line when a 5kg mass is attached to the mid – point C of the wire, the equilibrium position of C is 0.75m below line AB neglecting the weight of the wire and taking young's modulus for its material to be $2 \times 10^{11}Nm^{-2}$ find:-

(i) The strain in the wire
(ii) The stress in the wire
(iii) The energy stored in the wire

Solution

(a) Young's modulus of steel wire

$$E = \frac{FL}{Ae}$$

According to the Hooke's law

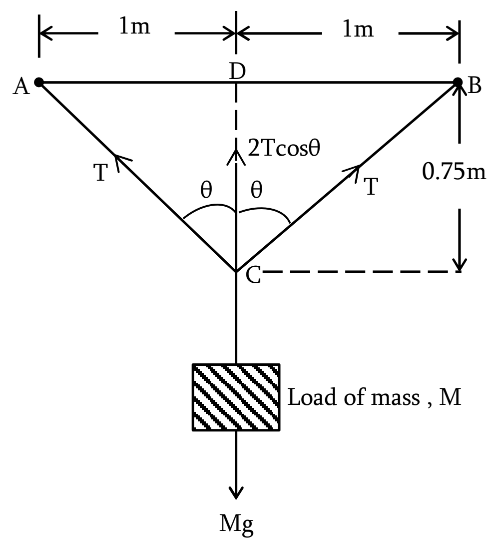
$$F = K.e$$

$$E = \frac{KeL}{Ae} = \frac{KL}{A}$$

$$K = \frac{EA}{L} = \frac{2\times10^{11}\times0.5\times10^{-6}}{30}$$

$$K = 3333.33\text{Nm}^{-1}$$

(b) Diagram



(i) Strain = ?

By Pythagorous theorem

$$\overline{BC}^2 = \overline{DC}^2 + \overline{DB}^2$$

$$\overline{BC} = \sqrt{1^2 + \left(0.75\right)^2}$$

$$\overline{BC} = 1.25\text{cm}$$

New length

$$L_1 = \overline{AC} + \overline{CB} \quad \text{but} \quad \overline{CB} = \overline{AC}$$

$$L_1 = 2\times1.25 = 2.5\text{m}$$

$$\text{Extension , } \Delta L = L_1 - L$$

$$\Delta L = 2.5\text{m} - 2.49 = 0.01\text{m}$$

$$\text{Stain} = \frac{\Delta L}{L} = \frac{0.01}{2.49}$$

$$\text{Strain} = 4.02\times10^{-3}$$

(ii) Stress = ?

$$E = \frac{\text{stress}}{\text{strain}}$$

$$\text{stress} = E \times \text{strain}$$

$$= 2.0 \times 10^{11} \times 4.02 \times 10^{-3}$$

$$\text{Stress} = 8.04 \times 10^{8}\,\text{Nm}^{-2}$$

(iii) At the equilibrium

$$2T\cos\theta = Mg$$

$$T = \frac{Mg}{2\cos\theta}$$

$$\text{But } \cos\theta = \frac{\overline{DC}}{\overline{CB}} = \frac{0.75}{1.25}$$

$$\frac{1}{\cos\theta} = \frac{1.25}{0.75}$$

$$T = \frac{5 \times 9.8}{2} \times \frac{1.25}{0.75}$$

$$T = 40.833\text{N}$$

$$T = 40.833\text{N}$$

A = Cross – sectional area

$$A = \frac{\text{Tension}}{\text{stress}} = \frac{40.833}{8.04 \times 10^{8}}$$

$$A = 5.07873 \times 10^{-8}\,\text{m}^2$$

$$\text{Energy stored } W = \frac{EAe^2}{2L}$$

$$= \frac{2 \times 10^{11} \times 5.07873 \times 10^{-8} \times (0.01)^2}{2 \times 2.49}$$

$$W = 2.04 \times 10^{-1}\,\text{J}$$

60. (a) Obtain an expression for the energy per unit volume of a strained wire in terms of its young's modulus of elasticity and the strain produced.

(b) A steel wire of length 3metres and diameter 1mm is subject to a progressively increase tensile stress. Calculate the increase in the strain energy stored in the wire as the extension of the wire is increased from 3mm to 4mm. (young's modulus for steel = 2×10^{11}N/m²)

Solution

(a) Refer to your notes

(b) Energy stored in the wire

$$W = \frac{1}{2}Fe \quad \text{but} \quad F = \frac{EAe}{L}$$

$$= \frac{1}{2}\left(\frac{EAe}{L}\right)e$$

$$W = \frac{EAe^2}{2L} \quad \text{but} \quad A = \frac{\pi d^2}{4}$$

$$W = \frac{\pi d^2 E e^2}{8L}$$

Let

ΔW = Increase in strain energy stored

$$\Delta W = W_2 - W_1$$

$$= \frac{\pi d^2 E e_2^2}{8L} - \frac{\pi d^2 E e_1^2}{8L}$$

$$\Delta W = \frac{\pi d^2 E}{8L}\left[e_2^2 - e_1^2\right]$$

$$= \frac{3.14 \times \left(3 \times 10^{-3}\right)^2 \times 2 \times 10^{11}}{8 \times 3}\left[\left(4 \times 10^{-3}\right)^2 - \left(3 \times 10^{-3}\right)^2\right]$$

$$\Delta W = 0.183\text{J}$$

61. A rectangular metallic bar one metre long, one cm deep and one cm broad is placed on a smooth table. The young's modulus and modulus of rigidity of metal of the bar are 2×10^{11}N/m² and 8×10^{10}N/m² respectively.

(i) If the bar is rigidly clamped at one end is pulled at the other end with a force 5000N normally to its end cross – section, calculate the elongation of the bar and the work done in elongating the bar.

(ii) If now the base of the bar is rigidly clamped to the bar , how you apply a force of 5000N to produce shearing strain in the bar? Calculate the angle of deformation and the horizontal displacement produced in the top layer of the bar.

Solution

(i) Young's modulus

$$E = \frac{FL}{Ae}, \quad e = \frac{FL}{EA}$$

Work done

$$W = \frac{1}{2}Fe \quad \left[A = 1 \times 1 = 1\text{cm}^2\right]$$

$$= \frac{1}{2}\frac{F^2 L}{EA}$$

$$= \frac{1}{2}\frac{\times(5000)^2 \times 1}{2\times 10^{11}\times 10^{-4}}$$

$$W = 0.625J$$

(ii) $\eta = 8 \times 1010 N/m^2$

to produce shearing strain force must be applied tangentially on the top surface of the bar.

Let θ = angle of deformation

$$\eta = \frac{F}{A\theta}, \quad \theta = \frac{F}{\eta A}$$

$$A = 1\times1\times10^{-2} = 10^{-2}m^2$$

$$\theta = \frac{5000}{8\times10^{10}\times10^{-2}}$$

$$\theta = 0.625\times10^{-5}\,rad$$

Displacement in the top layer of the bar

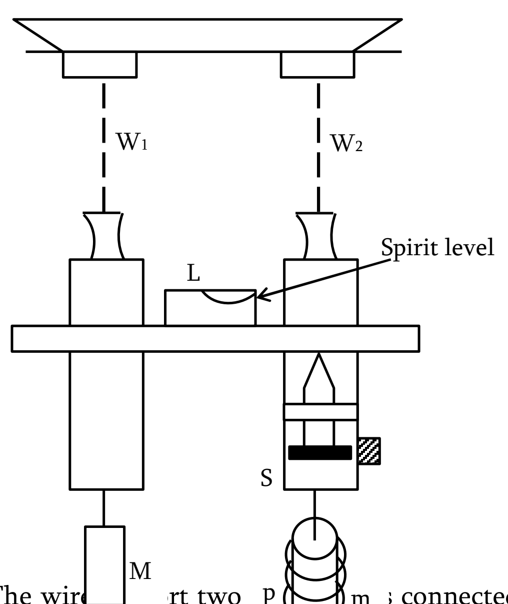
$$X = \text{Depth}\times\theta$$

$$= 1\times10^{-2}\times0.625\times10^{-5}$$

$$X = 0.625\times10^{-7}\,m$$

EXPERIMENT OF DETERMINATION OF YOUNG'S MODULUS FOR A METAL WIRE eg. steel wire.

A standard piece of apparatus called **searles extensometer** can be used to measure Young's Modulus E of steel. Searle's apparatus uses to identical wire , W_1 and W_2 suspended from the same point.



The wires support two frames connected by a spirit level. A spirit level indicates where or not , it is horizontal. A constant mass M loads one wire W_1 and the other wire W_2 is loaded with variable masses m placed on a mass hanger P. weights are placed on P remove any links and the spirit level L is made horizontally by use of the micrometer screw , S when further weights are added to P , wire W_2 stretched but the spirit level L can be returned to the horizontal by turning the micrometer screw , S . This means that the extra extension of the wire W_2 indicated by the micrometer. The extension X is measured for different values of masses, M.

Treatment of results

Young's modulus of steel

$$E = \frac{FL}{AX}$$

$$X = \frac{FL}{AE} \qquad \text{But } F = Mg$$

$$= \frac{MgL}{EA}$$

$$X = \left(\frac{gL}{EA}\right)M$$

L = Original length of the wire

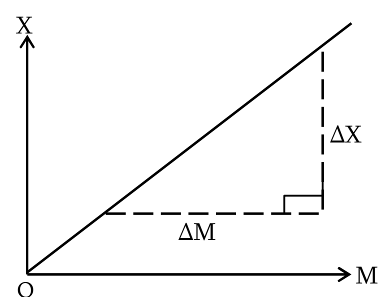
g = Acceleration due to gravity

E = Young's Modulus of a wire

r = radius of the wire

$A = \pi r^2$.

GRAPH OF X AGAINST M



A graph of extensions X against load M is plotted and Young's Modulus , E obtained from the slope of the graph.

$$\text{Slope} = \frac{\Delta X}{\Delta M} = \frac{gL}{AE}$$

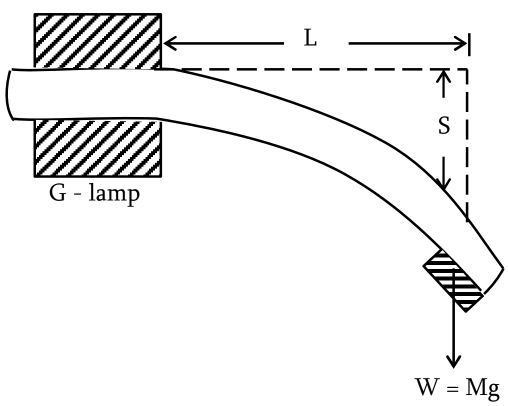
$$E = \frac{gL}{A\text{slope}}$$

NOTE THAT:

1. The use of two wire of identical material enables a number of systematic errors to be eliminated , any temperature fluctuation and yielding of the support affects both wires.
2. It is best to work with long thin wires because the extension is limited to about 0.1% of the original length. A long wire is required for a measurable extension to be obtained. This because thicker specific wires need bigger force which are difficult to handle.

EXPERIMENT OF DETERMINATION OF YOUNG'S MODULUS OF METER RULE FROM THE PERIODIC VIBRATION OF A LOADED CANTILEVER.

Consider the diagram of the loaded cantilever is clamped firmly to the edge of the bench by the G – damp with a definite length L projecting from it.



The depression S due to a load W at the end of the centi lever of length L is given by

$$S = \frac{WL^3}{3IE}$$

- Depress the loaded end slightly and release it so that it vibrates the periodic time of the cantilever is given by

$$T = 2\pi\sqrt{\frac{ML^3}{3IE}}$$

M = Mass of the load

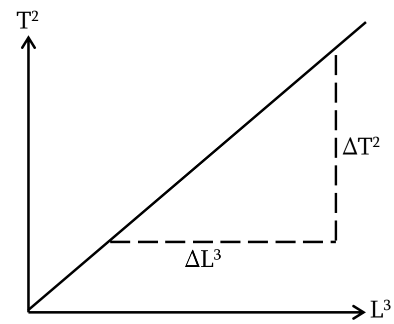
I = moment of inertia of beam of rectangular section

$$I = \frac{bd^3}{12}$$

b = width , d = thickness

$$T^2 = \left(\frac{4\pi^2 M}{3IE}\right)L^3$$

GRAPH OF T^2 AGAINST L^3



$$\text{Slope} = \frac{\Delta T^2}{\Delta L^3} = \frac{4\pi^2 M}{3IE}$$

$$E = \frac{4\pi^2 M}{3I \times \text{slope}}$$

NUMERICAL EXAMPLES

62. With the aid of a diagram describe a simple laboratory experiment to measure Young's Modulus of a wooden bar acting as a loaded cantilever from its period of vibration given that the depression S is given by

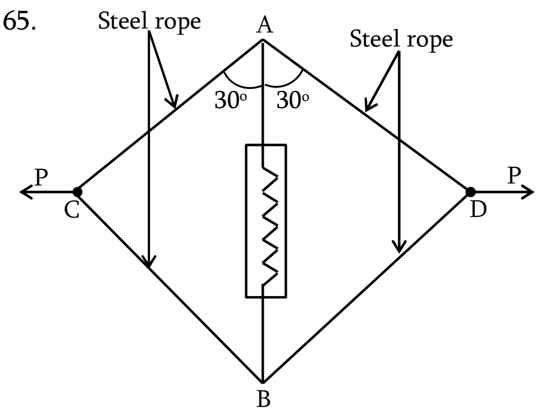
$$S = \frac{WL^3}{3IE}$$

63. (a) Define the terms : stress , strain and Young's Modulus of material.
(b) The young's modulus for steel is greater than that for brass if the two substances are subjected to equal forces , which one could be stretched more? Justify your answer.
(c) A brass wire 2.5m long of cross – sectional area 1.0×10^{-3} cm² is stretched 1.0mm by a load of 0.40kg.
(i) Calculate the Young's Modulus for brass
(ii) Find the percentage increase caused by the load.
(iii) Use the value of Young's modulus obtained in (i) above to obtain the force required to produce a 4.0% strain in the same wire.

(iv) Describe in detail how you would experimentally determine the Young's Modulus for a steel wire.

64. Obtain an expression for the sag of a loaded cantilever in terms of the physical quantities involved to the free end of a light cantilever in the form of a cylindrical rod of dimensions 40×0.5cm is attached a mass of 200gm. The time for small vertical oscillations of the loaded cantilever is found to be 0.65 seconds. What is the Young's Modulus of the cantilever:

answer. $1.3 \times 10^{10} Nm^{-2}$.

65.


A muscle exerciser consists of two steel ropes attached to the ends of a strong spring contained in a telescopic tube , figure above shown when the ropes are pulled sideway in opposite direction as shown in the simplified diagram the spring is compressed. The spring has uncompressed length of 0.80m. The force F is required to compress the spring to a length X (in metre) is calculated from the equation

$$F = 500\ (0.80 - X)$$

The ropes are pulled with equal and opposite forces , P so that the spring is compressed to a length of 0.16m and the ropes makes an angle of 30° with the length of the spring.

(a) Calculate the
   (i) Force , F
   (ii) Work done in compressing the spring.

(b) By considering the force at A or B , calculate the tension in each rope by considering the forces at C or D. Calculate the force P.

Solution

(a) (i) Given that $F = 500\ (0.80 - X)$

The spring is compressed to 0.6m

$$F = 500(0.8 - 0.6)$$

$$F = 100N$$

(ii) According to the Hooke's law

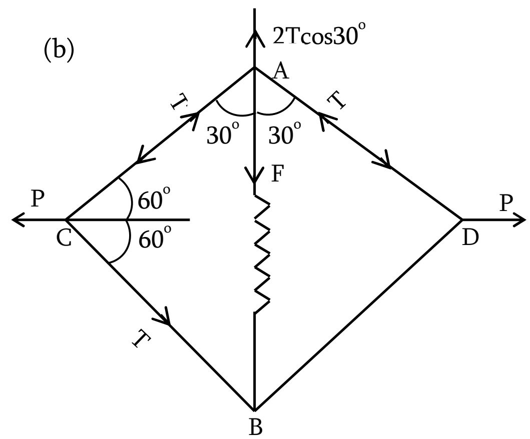
$$F = KX_1 \quad , \quad K = \frac{F}{X_1}$$

$$K = \frac{100}{0.2} = 500N/m$$

Work done by the compressed spring

$$W = \frac{1}{2}KX_1$$

$$= \frac{1}{2} \times 500(0.6)^2$$

(b)


Total vertical tension acts downward

$$T_y = 2T\cos 30^{\circ}$$

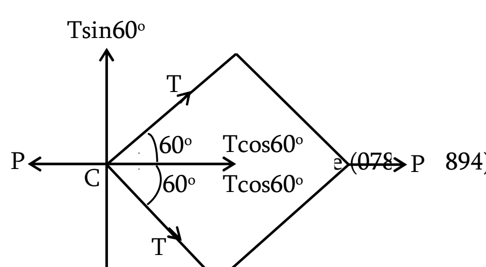
Restoring force

$$F = T_y$$

$$100 = 2T\cos 30^{\circ}$$

$$T = \frac{100N}{2\cos 30^{\circ}}$$

$$T = 57.7N$$

(c)


At the point C

Force

$$P = T\cos\theta + T\cos\theta = 2T\cos\theta$$

$$P = 2 \times 57.7 \times \cos 60^\circ$$

$$P = 57.7N$$

66. (a) A wire 2mm in diameter is just stretched in between two fixed points at a temperature of 50°C. Calculate the tension in the wire , when the temperature falls to 30°C. Coefficient of linear expansion is 11×10^{-6}/°C and Young's Modulus is $2.1 \times 10^{11}N/m^2$.

(b) A square lead slab of side 50cm and thickness 5.0cm is subjected to a shearing force (on its narrow face) of magnitude 9.0×10^4N . The lower edge is riveted to the floor. How much is the upper edge displacement if the shear modulus of lead is 5.6×10^9Pa?

Solution

(a) Tension in the wire = force due to contraction / expansion.

$$T = F = E\alpha A\Delta\theta$$

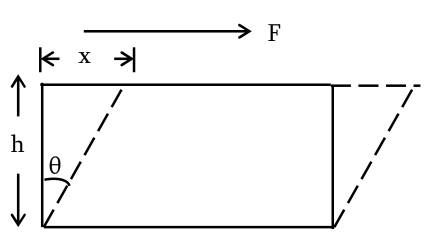
$$A = \pi r^2$$

$$T = \pi r^2 E\alpha(\theta_2 - \theta_1)$$

$$T = 3.14(0.001)^2 \times 2.1\times10^{11} \times 11.1\times10^{-6} \times (50-30)$$

$$T = 145.07N$$

(b)



Since

$$\eta = \frac{F}{A\theta}, \quad \theta = \frac{F}{\eta A}$$

$$\theta = \frac{9\times10^4}{0.025 \times 5.6 \times 10^9}$$

$$\theta = 6.428 \times 10^{-4}\text{rad}$$

Now

$$X = L\theta, \quad L = 0.5m$$

$$X = 0.5 \times 6.428 \times 10^{-4}$$

$$X = 3.2 \times 10^{-4}m$$

67. (a) If S is stress and Y is Young's Modulus of material of a wire. Find the energy stored in the wire for per unit volume in terms of S and Y.

(b) The length of a metallic wire is L1 when the tension in the wire is T1 and is L_2, when the tension is T_2. Find the original length of the wire.

Solution

(a) Energy stored in the wire

Energy stored in the wire per unit volume

$$U = \frac{1}{2} \times \text{stress} \times \text{strain}$$

Now

$$Y = \frac{\text{stress}}{\text{strain}}, \quad \text{strain} = \frac{\text{stress}}{Y}$$

$$U = \frac{1}{2}\text{stress} \times \frac{\text{stress}}{Y}$$

$$U = \frac{S^2}{2Y}$$

(b) Let L and A be the original length and area of cross – section of the wire. If on applying a force F extension produced is e, then

$$Y = \frac{FL}{Ae}$$

In the first case , $F = T_1$ and $e = L_1 - L$

$$Y = \frac{T_1 L}{A(L_1 - L)}$$

In the second case: $F = T_2$, $e = L_2 - L$

$$Y = \frac{T_2 L}{A(L_2 - L)} \quad \text{.......(i)}$$

(i) = (ii)

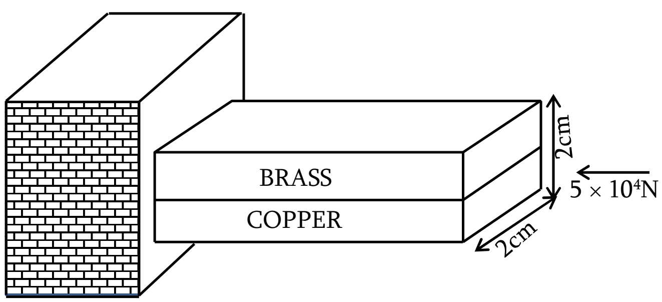
$$\frac{T_1L}{A(L_1 - L)} = \frac{T_3L}{A(L_2 - L)}$$

$$T_1(L_2 - L) = T_2(L_1 - L)$$

$$L(T_2 - T_1) = T_2L_1 - T_1L_2$$

$$L = \frac{T_2L_1 - T_1L_2}{T_2 - T_1}$$

68. A bimetallic rod made of brass and copper has a length of 0.5m and is of rectangular cross – section as shown in figure below. If the rod is subjected to a compressive force of 5×10^4N normal to its cross – section , then find the compressions in each of its two metallic parts. Given that Young's Modulus of brass = 9×10^{10}N/m^2. Young's Modulus of copper = 11×10^{10}N/m^2. Which way the rod will bend towards brass or copper?



Solution

Length of the brass /copper rod L = 0.5m area of cross – section of the brass or copper rod

$A = 2 \times 1 = 2cm^2 = 2 \times 10^{-4}m^2$.

Let E_1 = Young's Modulus of brass

E_2 = Young's Modulus of copper

$E_1 = 9 \times 1010Nm^{-2}$,

$E_2 = 11 \times 1010N/m^2$.

If e_1 and e_2 are the respective compressions of the brass and copper.

$$e_1 = \frac{FL}{AE_1} = \frac{5\times10^4\times0.5}{2\times10^{-4}\times9\times10^{10}}$$

$$e_1 = 1.389\times10^{-3}m$$

Also

$$e_2 = \frac{FL}{AE_2} = \frac{5\times10^4\times0.5}{2\times10^{-4}\times11\times10^{10}}$$

$$e_2 = 1.136\times10^{-3}m$$

Since the length of the brass rod becomes smaller than that of the copper , it will bend toward brass.

69. (i) Why do we prefer steel to copper in the manufacturing of spring?
(v) In stretching a wire , work has to be performed. Why?
(vi) When a wire is stretched , work has to be done. What happens to the work done during the stretching of the wire?

Solution

(i) A better spring will be the one , in which a large restoring force is developed on being deformed. This is turn depends upon the elasticity of the material of the spring. As Young's Modulus of steel is greater than that of copper , steel is preferred to manufacture a spring.

(ii) When a wire is stretched , interatomic forces come into play and these forces oppose the increase in length of the wire. Therefore , in order to stretch the wire , work has to be done against the interatomic forces.

(iii) The work done in stretching the wire is stored in it in the form of the elastic potential energy.

APPLICATIONS OF ELASTICITY

1. Metal ropes are used in cranes to pull up heavy loads. The thickness required for this rope is corresponding to the elastic limit of steel can be found out.
2. Knowledge of elasticity is applied in designing bridges , using less material without reducing the strength.
3. Maximum height of a mountain at any place along its length can be estimated from the elastic behavior of the earth.
4. Elastic materials can be used as vibration absorber. Example tyre rubber has large hysteretic loop so as used as vibration absorber.
5. The metallic parts of machines should not subjected to stress beyond the elastic limit otherwise they will be formed.

6. In a car , it is desirable that as little heat is generated as possible for this reason , in the manufacture of a car tyres that rubber is used which has small hysterisis loop.

ASSIGNMENT TO THE STUDENT EXAMPLES

70. A 40kg boy whose leg bones are $4cm^2$ in area and 50cm long falls through a height of 2m without breaking his leg bones if the bones can stand a stress of $0.9 \times 108N/m^2$, calculate the Young's modulus for the material of the bone (take $g = 10m/s^2$).

Solution

$M = 40kg$, $h = 2m$, $L = 0.5M$

$A = 4 \times 10^{-4}m^2$

Volume of leg = $AL = 4 \times 10^{-4} \times 0.5$

$AL = 2 \times 10^{-4}m^3$

Apply the law of conservation of energy

Loss in gravitation p.e = gain in elastic p.e by both legs

$$mgh = 2\left[\frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume of one leg}\right]$$

$$40 \times 10 \times 2 = 2\left[\frac{1}{2} \times 0.9 \times 10^8 \times \text{strain} \times 2 \times 10^{-4}\right]$$

$$\text{strain} = \frac{40 \times 10 \times 2}{0.9 \times 2 \times 10^4} = \frac{2}{45}$$

Young's modulus

$$E = \frac{\text{stress}}{\text{strain}} = \frac{0.9 \times 10^8}{2/45}$$

$$E = 2.025 \times 10^9 N/m^2$$

71. (i) Define tensile stress , tensile strain and Young's modulus.

(ii) A mass of 11kg is suspended from the ceiling by an aluminum wire of length 2m and diameter 2mm. what is:-

(a) The extension produced

(b) The elastic energy stored in the wire?

The Young's modulus of aluminum is $7 \times 10^{10}Pa$

Answer: (ii) (a) 1mm (b) $5.5 \times 10^{-2}J$

72. (a) An elastic string of cross – sectional area $4mm^2$ required a force of 2.8N to increase its length by one tenth. Find Young's Modulus for the string. If the original length of the string was 1mm; find the energy stored in the string when it is so extended.

(b) A massive store pillar 20m high and of uniform cross – section rests on a rigid base and supports a vertical load of 5.0×10^5N at its upper end. State, with reasons , where in the pillar the maximum compressive stress occurs if the compressive stress in the pillar is not exceed $1.6 \times 10^6Nm^{-2}$, what is the minimum cross – sectional area of the pillar?

Density of the stone = $2.5 \times 10^3kgm^{-3}$

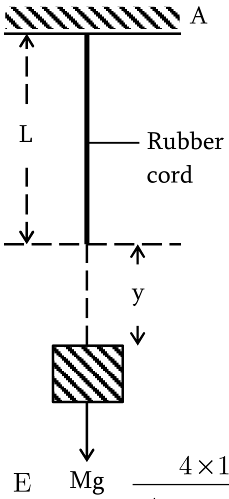
Answer: (a) $7 \times 10^6Nm^{-2}$, 0.14J (b) $0.45m^2$

73. The ends of a uniform wire of cross – sectional area $10^{-6}m^2$ and negligible mass are attached to fixed points A and B which are 1m a part in the same horizontal plane. The wire is initially straight and unstretched to the mid – point of the wire and hangs in equilibrium with the mid – point at a distance 10mm below AB. Calculate the value of Young's modulus of the wire

Answer: $6.25 \times 10^{11}Nm^{-2}$.

74. A rubber cord has a diameter of 5.0mm, and an unstretched length of 1.0m one end of the cord is attached to a fixed support A. when a mass of 1.0kg is attached to the other end of the cord , so as to hang vertically below A , the cord is observed to elongate by 100mm. calculate the Young's modulus of rubber if the 1kg mass is now pulled down a further short distance and then released , what is the period of the resulting oscillation? ($g = 10m/s^2$)

Solution



$L = 1.0m$, $d = 5m$

$M = 1.0kg$

$y = 10mm$

Young's modulus

$$E = \frac{MgL}{AY}$$

y = extension

$$A = \frac{\pi d^2}{4}$$

$$E = \frac{4MgL}{\pi d^2 y}$$

$$E = \frac{4\times1\times10\times1}{3.14\left(5\times10^{-3}\right)^2\times100\times10^{-3}}$$

$$E = 5.1\times10^6\,Nm^{-2}$$

Case 2:

When the point mass m is suspended at the end of the rubber , the rubber elongates under action of the weight Mg , say through y , then

$$E = \frac{Mg/A}{y/L}$$

$$Mg = \frac{EAy}{L}$$

If F is the restoring force set up in the wire , then

$$F = -\frac{EAy}{L}$$

Then negative sing indicate that the restoring force F acts in direction opposite to that in which extension is produced.

$$ma = -\frac{EAy}{L}$$

$$a = -\frac{EAy}{mL}$$

For S.H.M $\quad a = -\omega^2 y$

$$-\omega^2 y = -\frac{EAy}{mL}$$

$$\omega = \sqrt{\frac{EA}{mL}} \quad \text{But} \quad \omega = \frac{2\pi}{T}$$

$$\frac{2\pi}{T} = \sqrt{\frac{EA}{mL}}$$

$$T = 2\pi\sqrt{\frac{mL}{EA}} \quad \text{But} \quad A = \frac{\pi d^2}{4}$$

$$= 2\pi\sqrt{\frac{4mL}{\pi d^2 E}} = \frac{4\pi}{d}\cdot\sqrt{\frac{mL}{\pi E}}$$

$$T = \frac{4\times3.14}{5\times10^{-3}}\cdot\sqrt{\frac{1\times1}{3.14\times5.1\times10^6}}$$

$$T = 0.63\,\text{sec}\,(\text{approx})$$

75. A point mass M is suspended at the end of a massless wire of length L and cross – section A . If Y is Young's modulus of elasticity for wire obtain the frequency of oscillation for simple harmonic motion along the vertical line. Answer: $f = \frac{1}{2\pi}\cdot\sqrt{\frac{YA}{ML}}$

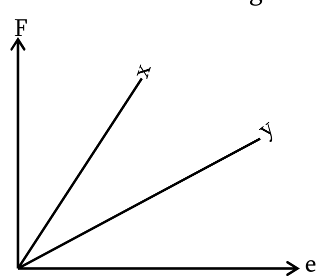
76. A device to project a toy rocket vertically make use of the energy stored in a stretched rubber cord. Assuming that the cord obey's Hooke's law , find the length by which the cord must be extended if the rocket is to be projected to a height of 20.0m.
Mass of rocket = 0.30kg
Young's modulus for rubber = $80 \times 10^8 Nm^{-2}$
Cross – section area of cord = $2.5 \times 10^{-5} m^2$
Answer : 3.5×10^{-2}m.

77. (a) Draw a graph of stress against strain for the tensile deformation of metal wire up to the breaking point. Make on your graph the region in which Hooke's law is obeyed. Describe the behavior of the wire when stretched beyond this region. What is the significance of the area between the graph and the strain axis within the Hooke's law region?
(b) A uniform bar of mass 20kg and length 1.0m is supported at the ends by vertical wires, one of steel and the other of brass. The wires are initially 2.0m long and 1.0mm in diameter and upper ends are fixed in the same horizontal plane. Determine the angle between the bar and the horizontal.
Young's modulus for steel = $2 \times 10^{11} Nm^{-2}$

Young's modulus for bras = $9 \times 10^{10} Nm^{-2}$
Answer (b) 1.6×10^{-3}rad.

78. A submerged wreck is lifted from a dock basin by means of a crane to which is attached a steel cable 10m long of cross – sectional area $5cm^2$ and Young's modulus $5 \times 10^{10} Nm^{-2}$. The material being lifted has a mass 10^4kg and mean density $8000kgm^{-3}$. Find the change in extension of the cable as the load is lifted clear of the water. Assume that all times the tension in the cable is the same throughout its length
(Density of water = $1000kgm^{-3}$)
Answer : 5mm.

79. (a) Figure below shows the variation of force F , the load applied to two wires X and Y and their extension ,e. The wires are both iron and have the same length



(i) Which wire has the smaller cross – section?
(ii) Explain how you would use the graph for X to obtain a value for the Young's modulus of iron ,list the additional measurement needed.

(b) A cylindrical copper wire and a cylindrical steel wire , each of length 1.5m and diameter 2mm , are joined at one end to form a composite wire 3m long. The wire is loaded unit its length becomes 3.003m. Calculate the strains in the copper and steel wires and the force applied to the wire.
Young's modulus for copper $= 1.2 \times 10^{11} Nm^{-2}$, for steel $= 2 \times 10^{11} Nm^{-2}$.
Answer : (a) (i) Y
(b) C: 1.25×10^{-3} S: 0.75×10^{-3} , F = 471N

80. (a) Define the terms tensile stress and tensile strain and explain why these quantities are more useful than force and extension for a description of the elastic properties of matters.
(b) A uniform steel wire of density $7800kgm^{-3}$ weighs 16g and is 250cm long. It lengthens by 1.2mm when stretched by a force of 80N. Calculate.
(i) The value of Young's modulus of steel.
(ii) The energy stored in the wire.
Answer (b) (i) $2 \times 10^{11} Nm^{-2}$
(iii) 4.8×10^{-2}J

81. A wire of length 3.0m and cross – sectional area $1.0 \times 10^{-6} m^2$ has a mass of 15kg hung on it. What is the stress produced in the wire? ($g = 9.8m/s^2$). If the young's modulus for the material is $2.0 \times 10^{11} Nm^{-2}$, what is the extension X produced? When extended how much energy is stored in the wire? If the mass of 15kg were allowed to fall through a distance X , what would be the change in its gravitational potential energy? Why this change not equal to the final energy stored in the wire when extended by the mass?
Answer : $1.47 \times 10^8 Nm^{-2}$, 2.2×10^{-3}m ,0.16J

82. A cylindrical copper wire and cylindrical steel wire , each of length 1.000m and having equal diameters are joined at one end to form a composite wire 2.000m long. This composite wire is subjected to a tensile stress until its length becomes 2.002m. Calculate the tensile stress applied to the wire.
The young's modulus for
Copper = 1.2×10^{11}Pa and fro
Steel = 2.0×10^{11}Pa
Answer: 1.5×10^8Pa.

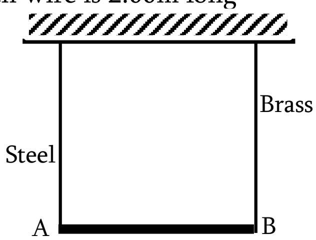
83. (a) A heavy rigid bar is supported horizontally from a fixed support by two vertical wires , A and B , of the same initial length and which experience the same extension. If the ratio of the diameter of A to that of B is 2 and the ratio of Young's modulus of A to that of

B is 2, calculate the ratio of the tension in A to that in B.

(b) If the distance between the wires is D , calculate the distance of wire A from the centre of gravity of the bar.

Answer: (a) 8:1 (b) D/9

84. A light rigid bar is suspended horizontally from two identical wires , one of steel and one of brass as shown in the figure below. Each wire is 2.00m long



The diameter of the steel wire is 0.60mm and the length of the bar AB is 0.20m. When mass of 10.0kg is suspended from the centre of AB the bar remains horizontal.

(i) What is the tension in each wire?

(ii) Calculate the extension of the steel wire and the energy stored in it.

(iii) Calculate the diameter of the brass wire.

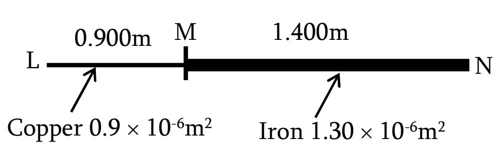
(iv) If the brass wire were replaced by another brass wire of diameter 1.00mm, where should the mass be suspended so that AB would remain horizontal?

Young's modulus for steel = 2.0×10^{11}Pa

Young's modulus for brass = 1.0×10^{11}Pa

Answer: (i) 50N (ii) 1.8×10^{-3}m, 4.4×10^{-2}J (iii) 0.85mm (iv) 0.084m from B

85. A copper wire LM is fused at one end , M to an iron wire MN. The copper wire has length 0.900m and cross – section $0.90 \times 10^{-6}m^2$. The iron wire has length 1.400m and cross – section $1.30 \times 10^{-6}m^2$. The compound wire is stretched; its total length increases by 0.0100m



Calculate

(a) The ratio of the extension of the two wires.

(b) The extension of each wire

(c) The extension applied to the compound wire.

Young's modulus for copper = $1.3 \times 10^{11}Nm^{-2}$

Young's modulus for iron = $2.1 \times 10^{11}Nm^{-2}$

Answer: (a) 3:2 (cu:fe) (b) 6.0mm , 4.0mm (c) 780N

86. A 20m length of continuous steel railway line of cross – sectional area $8.0 \times 10^{-3}m^2$ is welded into place after heating to a uniform temperature of 40°C. (Take young's modulus for steel to be 2.0×10^{11}Pa , its linear expansivity to be $12 \times 10^{-6}K^{-1}$, its density to be $7800kgm^{-3}$, and its specific heat capacity to be $500JKg^{-1}K^{-1}$). Calculate, for normal operating conditions at 15°C.

(a) The tensile strain

(b) The tensile stress

(c) The elastic energy in the rail. How much heat would be required to return the rail to 40°C? Explain briefly why your answer is not the same as that of (c).

Answer: (a) 3.0×10^{-4} (b) 6.0×10^{7}Pa (c) 1.44×10^{3}J; 1.56×10^{7}J.

87. The rubber cord of catapult is pulled back is pulled back until its original length has been doubled. Assuming that the cross – section of the cord is 2mm square, and that Young's modulus for rubber is $10^{7}Nm^{-2}$. Calculate the tension in the cord. If the two arms of the catapult are 6cm apart, and the unstretched length of the cord is 8cm what is the stretching force?

Answer: 40N, 74N.

88. (i) Define compressibility of a gas in terms of the elasticity of gases

(ii) The bulk modulus of elasticity for lead is $8 \times 10^{9}N/m^2$. Find the density of lead if the pressure applied is $2 \times 10^{8}N/m^2$

Density of lead $\rho_{pb} = 11.4 \times 10^{3}kgm^{-3}$.

89. (a) (i) Define the terms : proportional limit , elastic limit, yield point and elasticity.
(ii) Use a sketch graph to show how the extension of the wire varies with applied force and mark the elastic limit and yield point on it. Explain how the magnitude of the Young's modulus is obtained from the graph.
(b) A block of metal weighing 20N with a volume of $8 \times 10^{-4}m^3$ is completely immersed in oil of density $700kgm^{-3}$ then attached to one end of a vertical wire of length 4.0m and diameter of 0.6mm whose other end is fixed. If the length of the wire is increased by 1.0mm. find the
(i) Young's modulus of the wire
(ii) Energy stored in the wire

(c) A rubber cord of a Y – shaped has a cross – sectional area of $4.0 \times 10^{-6}m^2$ and relaxation length of 100mm. If the arms of the catapult are 70mm apart , calculate the
(i) Tension in the rubber
(ii) Force required to stretch it when the rubber cord is pulled back until its length doubles
Young's modulus for rubber = 5.0×10^8Pa

90. (i) What is meant by saying that 'Young's modulus of steel is $2.0 \times 10^{11}Nm^{-2}$?
(ii) A rod of original length 1.2m and area of cross – section $1.5 \times 10^{-4}m^2$ is extended by 3.0mm when the stretching tension is 6.0N. Calculate the Young's modulus for the material of the rod and the energy density of the stretched material.

91. (a) (i) Distinguish between elasticity and plasticity of a material.
(ii) Derive an expression for the energy stored in an elastic string of force constant K if the elastic limit is not exceeded.
(iii) A 5.0kg load is applied to the lower end of a vertical steel wire of length 2.0m and diameter of 1.0mm. Find the energy stored in the wire.
(b) A heavy uniform beam weighing 6.0×10^4N is to be suspended at one end by a steel cable of length 5.0m long and a diameter 0.5m and the other end by nylon rope 5.0m long and diameter of 5.0cm. The rope and the cable are then fastened together above the beam to make it hang.
(i) If the beam will not be horizontal , which end of it will be lower and by how much?
(ii) Find the tension in the rope and in the cable.
(iii) What assumption will you make?
Young's modulus of steel
$E = 2 \times 10^{11}N/m^2$
Young's modulus of nylon
$En = 3.6 \times 10^9N/m^2$

92. (a) (i) Sketch a graph of variation of stress applied to the material against its strain showing proportional limit (A), Elastic limit (L); Yield point (B), Tensile strength (C) and breaking point (F)
(ii) Differentiate between a ductile material and brittle material.
(b) A crane is required to lift a load of up to 1.0×10^5N.
(i) What is the minimum diameter of the steel cable that must be used?
(ii) If a cable of twice the minimum diameter is used and it is 8.0m long when no load is present , how much longer is it when supporting a load of 1.0×10^5N.
(c) (i) What is meant by bulk modulus?
(ii) A marble statue of volume $1.5m^3$ was being transported by a ship at sea. The statue toppled into the ocean when an earth quake caused tidal waves which sank the ship. The statue ended up on the ocean floor 1.0km below the surface. Find the change in volume of the statue in cm^3 due to the presence of water

Density of sea water = $1025 kgm^{-3}$
Bulk modulus for marble = $70 \times 10^9 Pa$
Elastic limit of steel = $3.0 \times 10^8 Pa$
Proportional limit of steel = $2 \times 10^8 Pa$
Tensile strength of steel = $5 \times 10^8 Pa$
Young's modulus of steel = $2 \times 10^{11} Pa$

93. State Hooke's law and describe in detail how it may be verified experimentally for copper wire. A copper wire 200cm long and 1.22mm diameter is fixed horizontally to two rigid supports 200cm long. Find the mass in grams of the load which when suspended at the mid – point of the wire , produces a sag of 2cm at that point. Young's modulus for copper = $12.3 \times 10^{10} Nm^{-2}$.
Answer : 115g

94. A composite wire of uniform diameter 3mm consists of a copper wire of length 2.2m and a steel wire of length 1.6m stretches under a load by 0.7mm. Calculate the load , given that the Young's modulus for copper is $1.1 \times 10^{11} Pa$ and that for steel is $2.0 \times 10^{11} Pa$
Answer: 176.7N

95. A copper wire and steel wire of the same diameter and of length 100cm and 200cm respectively are connected end to end and a force is applied which stretches their combined length by 1cm. Find by how much each wire is elongated if young's modulus for copper and steel are $1.2 \times 10^{11} Pa$ and $2.0 \times 10^{11} Pa$ respectively.
Answer: 0.455cm , 0.545cm

96. Two parallel steel wires A and B are fixed to rigid support at the upper ends and subjected to the same load at the lower ends. The lengths of the wires are in the ratio 4:3. The increase in the length of the wire A is 1mm. calculate the increase in the length of the wire B.
Answer: 2.22m

97. Two different types of rubber are found to have the stress – strain curves shown below in figure. The area of the stress – strain curve of a material is numerically equal to the work done in loading the material and then unloading it.
(a) Which of the two rubber material would you choose for a car tyre?
(b) A heavy machine is to be installed in a factory to absorb vibrations of the machine , block of rubber is placed between the machine and the floor. Which is of the two rubbers would you prefer to use for this purpose?

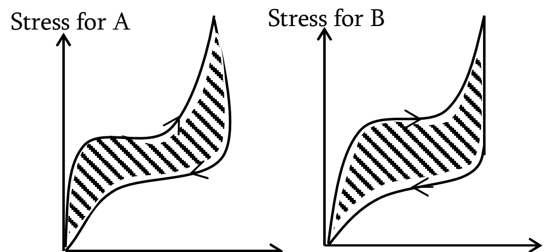


Fig. 98(a) strain   Fig. 98(b) strain

Solution
(a) Rubber A should be used for making the car tyres it is because of the reason that area of the curve i.e work done in case of rubber A is lesser and hence the car tyre of this rubber will not get excessively heated.
(b) Rubber B should be used to absorb vibrations of the machinery. Because of the large area of the curve , a large amount of vibrational energy can be dissipated.

98. (a) Define stress and strain and explain why these quantities are useful in the studying the elastic behavior of a material.
(b) Calculate the minimum tension with which platinum wire of diameter 0.1mm must be mounted between two points in a shout invar frame if the wire is to remain taut when the temperature rises 100K. Platinum has linear expansivity $9 \times 10^{-6} K^{-1}$ and young's modulus $17 \times 10^{10} Nm^{-2}$. The thermal expansion of invar may be neglected.
Answer : (b) 1.2N.

99. (a) Distinguish between Young's modulus the bulk modulus and the shear modulus of a material.

(b) A piece of copper wire has twice the radius of a piece of steel is twice that for the copper . One end of the copper is joined to one end of the steel wire so that both can be subjected to the same longitudinal force by what fraction of its length will steel have stretched when the length of copper has increased by 1%.

Answer: 2%

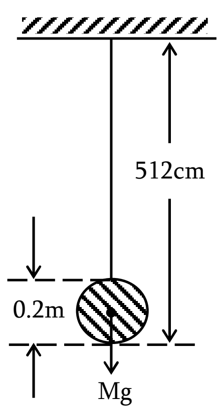
100. A sphere of radius 10cm and mass 25kg attached to the lower end of steel wire which is suspended from the ceiling of a room. The point of support is 521cm above the floor. When the sphere is set swinging as a simple pendulum its lowest point just grazes the floor. Calculate the velocity of the ball at the lower position.

Young's modulus of steel = $20 \times 10^{10} Nm^{-2}$

Unstretched length of wire = 500cm

Radius of the steel wire = 0.05cm

Solution



Stretched length of the wire

$$= 5.21 - 0.2 = 5.01m$$

$$e = 5.01 - 5.0 = 0.01m$$

$$\text{strain} = \frac{e}{L} = \frac{0.01}{5}$$

Let T be the tension in the wire

$$\text{stress} = \frac{T}{A} = \frac{T}{\pi(0.05)^2 \times 10^{-4}}$$

Young's modulus

$$E = \frac{\text{stress}}{\text{strain}}$$

$$20 \times 10^{10} = \frac{T}{\pi(0.05)^2 \times 10^{-4}} \times \frac{5}{0.01}$$

$$T = 314.1N$$

Now

$$T - Mg = \frac{MV^2}{r}$$

V = Velocity of the sphere as it crosses the equilibrium position.

r = The distance of the centre of the sphere from the ceiling.

$$r = 5.01 + 0.1 = 5.11$$

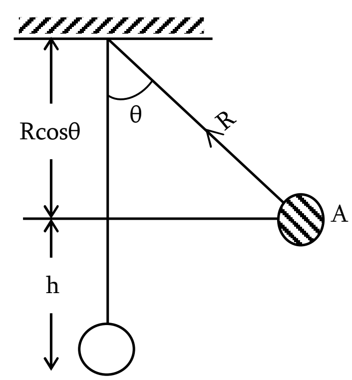
$$314.1 - 25 \times 9.8 = \frac{25V^2}{11}$$

$$V = 3.76m/s$$

101. A load of 981N is suspended from a steel wire of radius 1mm. what is the maximum angle through which the wire with the load can be deflected so that it does not break when the load passes through the position of equilibrium

[Breaking stress is $7.85 \times 10^8 Nm^{-2}$]

Solution



Maximum tension in the string

T_{max} = breaking stress × Area

$= 7.85 \times 10^8 \times \pi (10^{-3})^2$

$T_{max} = 2466.15N$

At the equilibrium for the lowest position

$$T_{max} = \frac{MV^2}{r} + Mg$$

$$T_{max} - Mg = \frac{MV^2}{R}$$

Apply the law of conservation of energy

$$Mgh = \frac{1}{2}MV^2 \left[h = R - R\cos\theta\right]$$

$$MgR\left(1-\cos\theta\right) = \frac{1}{2}MV^2$$

$$\frac{V^2}{R} = 2g\left(1-\cos\theta\right)$$

Now

$$T_{max} - Mg = 2Mg\left(1-\cos\theta\right)$$

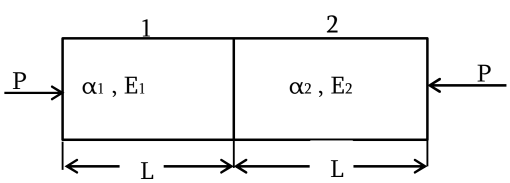
$$\cos\theta = \frac{3Mg - T_{max}}{2Mg}$$

$$\cos\theta = 0.243$$

$$\theta = 75^{\circ}56'$$

102. Two rods of different materials but of equal cross – sections and lengths (1.0m each) are joined to make a rod of length 2.0m. The metal of one rod has coefficient of linear thermal expansion $10^{-5}/^{O}C$ and Young's modulus $3 \times 10^{10} N/m^2$. The other metal has the values $2 \times 10^{-5}/^{O}C$ and $10^{10} N/m^2$ respectively. How much pressure must be applied to the ends of the composite rod to prevent its expansion when the temperature is raised by $100^{o}C$? What will be the separate lengths of two rods at the new temperature?

Solution



Let ΔL_1 and ΔL_2 be the increase in the lengths of rod 1 and 2 due to temperature rise.

$$\Delta L_1 = \alpha_1 L\Delta\theta$$

$$\Delta L_2 = \alpha_2 L\Delta\theta$$

Total increase in the length of the composite bar

$$\Delta L = \Delta L_1 + \Delta L_2$$

$$\Delta L = S\left(\alpha_1 + \alpha_2\right)L\Delta\theta$$

Suppose P is the pressure applied at the ends of the rod to prevent this increase in length.

Let $\Delta L_1'$ and $\Delta L_2'$ be the contractions in lengths of rod 1 and 2 due to pressure P.

$$Y = \frac{\text{stress}\left(\text{pressure}\right)}{\text{strain}} = \frac{PL}{\Delta L}$$

$$\Delta L_1' = \frac{PL}{Y_1}, \quad \Delta L_2' = \frac{PL}{Y_2}$$

$$\text{Total contraction} = \Delta L_1' + \Delta L_2'$$

$$= \frac{PL}{Y_1} + \frac{PL}{Y_2} = PL\left[\frac{Y_1 + Y_2}{Y_1 Y_2}\right] \text{.....(2)}$$

(1) = (2)

$$\left(\alpha_1 + \alpha_2\right)L\Delta\theta = PL\left[\frac{Y_1 + Y_2}{Y_1 Y_2}\right]$$

$$P = \frac{Y_1 Y_2\left(\alpha_1 + \alpha_2\right)\Delta\theta}{Y_1 + Y_2}$$

$$= \frac{3\times10^{10}\times10^{10}\times\left(1+2\right)\times10^{-5}\times100}{3\times10^{10}+10^{10}}$$

$$P = 2.25\times10^{-7}\,Nm^{-2}$$

Let the separate lengths of rods 1 and 2 be L_1 and L_2 at the new temperature respectively.

$$L_1 = L + \Delta L_1 - \Delta L_1'$$

$$= L + \alpha_1 L\Delta\theta - \frac{PL}{Y_1}$$

$$= 1.0\left[1 + 10^{-5}\times100 - \frac{2.25\times10^{7}}{3\times10^{10}}\right]$$

$$L_1 = 1.00025m$$

Similarly $L_2 = L + \Delta L_2 - \Delta L_2'$

$$L_2 = 0.99975m$$

103. (a)

Additional load (kg)	Scale reading/mm
0	2.8
2.0	3.8
4.0	4.5
6.0	5.1
8.0	5.7
10.0	6.3
12.0	6.9
14.0	7.5
16.0	8.1

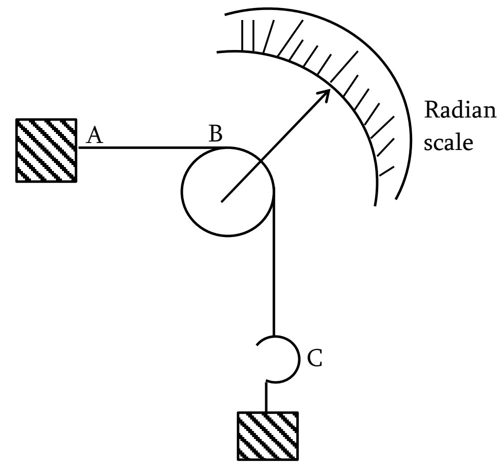
The above table shows readings obtained when stretching a wire supported at its upper end by suspending masses from its lower end. The unstretched length of the wire was 2.23m and its diameter 0.71mm. Using graphical method , determine a value for Young's modulus for the material of the wire.

(b) Describe suitable apparatus for obtaining the above readings and explain the important features of the design.

(c) A student noticed that when a mass of 10.0kg suspended from a wire identical to that described above was pulled downwards and released , it executed vertical oscillations of small amplitude. Use the graph to explain briefly why you would expect the oscillations to be simple harmonic.

104. The diagram represents a simple apparatus for investigating the extension of a stressed wire and for measuring Young's modulus of its material. A copper wire of cross – sectional area $1mm^2$ is firmly clamped at A and passes over a pulley of radius r = 6mm which rotates about a fixed point and carries a pointer that moves over scale graduated in radians. The distance AB is 2m and loads are attached to the free end C.

The following readings were obtained as wire was loaded:



Load in kg	0	1	2	3	4	5	6	7	8
Scale reading θ in rad	0	0.0027	0.053	0.080	0.107	0.138	0.176	0.242	0.434

(i) For each load work out the tension in the wire in N and the extension of the wire in mm (which is given by rθ). Plot a graph of tension (y – axis) against extension (x – axis)

(ii) Discuss the form of this graph , and state what you would expect to observer if the wire were unloaded is steps 1kg at a time.

(iii) Calculate the value of Young's modulus for copper.