MODULE 7 : ROTATION

ROTATION DYNAMICS (RIGID BODIES)

A rigid body - Is the body with perfectly definite and unchanging shape.

- The geometrical shape and size of rigid body do not undergo any changes during the motion of rigid body.
- A rigid body may be regarded as an assembly of point masses.
- Rigid body is a body whose particles preserve their relative positions when an external force or torque is applied on it.

Examples of rigid bodies

Metre rule , disc , sphere , wheels , solid cylinder , hoop , e.t.c.

CONCEPT OF MOMENT OF INERTIA OF A RIGID BODY

QUALITATIVELY

Moment of inertia – is the opposition offered by a rigid body to change its angular velocity when the external torque is applied on the rigid body.

QUANTITATIVELY

Moment of inertia of rigid body

Is defined as the sum of the product s of the masses of all the particles constituting the body and the squares of their respectively distance from the axis of rotation.

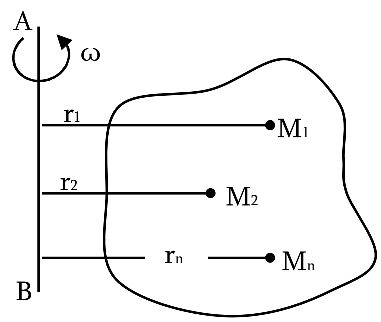
- The line joining the centres of all circular paths is called the axis of rotation. A body can rotate about a fixed axis. This is known as Axis of rotation. M.I is a scalar quantity.

Mathematically

To find the M.I of a body about an axis of rotation AB, we assume that the body is made up of large number of particles of masses M_1 , M_2 , M_3…………situated at distance r_1 , r_2 , r_3…..from the axis of rotation

M.I of the whole body about AB is

$$I = I_1 + I_2 + I_3 + \ldots\ldots + I_n$$
$$= M_1r_1^2 + M_2r_2^2 + M_3r_3^2 + \ldots\ldots + M_nr_n^2$$



$$\boxed{I = \sum_{i=1}^{n} m_i r_i^2} \quad \text{or} \quad I = \sum mr^2$$

S.I unit of M.I is Kgm^2

Dimension of M.I

$$[I] = [m][r]^2 = ML^2$$

FACTOR AFFECTING MOMENT OF INERTIA

The M.I of a body depends on:

1. The mass of the body ($I \alpha M$)
2. The radical distance r from the axis of rotation ($I \alpha r^2$).
3. Position of the axis of rotation
4. The distribution of the mass about the axis of rotation.
5. The shape and size of the rigid body

CALCULATION OF MOMENT OF INERTIA.

Assumptions made on deriving the M.I of the rigid body:

(i) When an object consists of a number of mass points , its M.I about a given axis is equal to the sum of the moment of inertia of individual mass point about that axis.

$$I = \sum_{i=1}^{n} m_i r_i^2$$

For the rigid body consists of a continuous distribution of mass

$$I = \int r^2 dm$$

(ii) It is necessary to express the element of mass in terms of their coordinates and it's is defined as mass density or linear density.

- For the distribution of mass in the linear form.

  Linear density, $\lambda = \frac{\text{mass}}{\text{length}} = \frac{M}{L}$

- For three or two dimensions

  Mass density , $\rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{v}$

  $$\rho = \frac{\text{Mass}}{\text{Area}} = \frac{M}{A}$$

1. MOMENT OF INERTIA OF UNIFORM ROD (SLENDER)

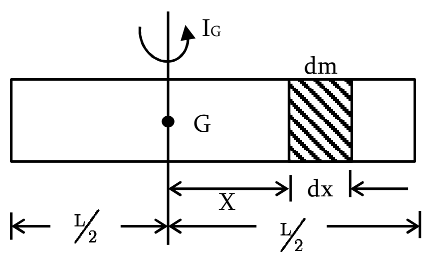
(i) A bout an axis through its centre and perpendicular to its length.

$$I_G = \frac{ML^2}{12}$$

M = mass of the uniform rod

L = length of the uniform rod

Derivation : $I_G = \frac{ML^2}{12}$



Mass per unit length of the whole rod

$$\lambda = M/L$$

Consider a small element of length dx of the rod at a distance x from the centre , G

$$\lambda = \frac{dm}{dx} \;,\; dm = \lambda dx = \frac{M}{L}dx$$

M.I of the small element about the centre, G

$$dI_G = x^2 dm$$

$$dI_G = \frac{M}{L}x^2 dx$$

Total M.I of the rigid body

$$I_G = \frac{M}{L}\int_{-L/2}^{L/2} x^2 dx = \frac{M}{L}\left[\frac{x^3}{3}\right]_{-L/2}^{L/2}$$

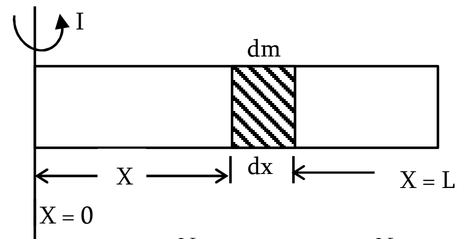
$$I_G = ML^2/12$$

$$I_G = \frac{ML^2}{12}$$

(ii) About an axis passing through the end of the uniform rod.

$$I = \frac{ML^2}{3}$$

Derivation: $I = \frac{ML^2}{3}$



Now : $I = \int_0^L x^2 dm = \frac{M}{L}\int_0^L x^2 dx$

$$= \frac{M}{L}\left[\frac{x^3}{3}\right] = \frac{M}{3L}\left[L^3 - 0^3\right]$$

$$I = \frac{ML^2}{3}$$

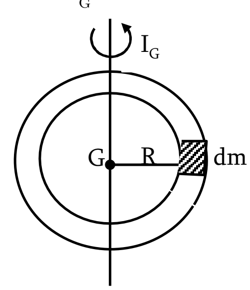
2. M.I OF THIN UNIFORM RING (HOOP) ABOUT AN AXIS PASSING THROUGH THE CENTRE AND PERPENDICULAR TO THE PLANE OF THE RING

$I_G = MR^2$

M = Mass of the hoop or ring

R = radius of the ring (hoop)

Derivation : $I_G = MR^2$



M.I of small element about the centre

$$dI_G = R^2 dM$$

$$\int dI_G = \int R^2 dm$$

$$I_G = MR^2$$

3. M.I OF A CIRCULAR DISC ABOUT AN AXIS THROUGH THE CENTRE OF DISC AND PERPENDICULAR TO THE PLANE OF THE DISC.

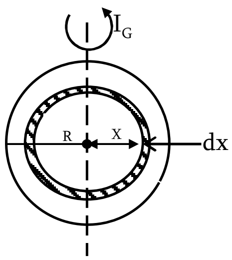
$$I_G = \frac{1}{2}MR^2$$

M = mass of the disc
R = radius of the disc

Derivation : $I_G = \frac{1}{2}MR^2$

Area of the disc A = πR^2

Mass per unit area of the disc , $\rho = \frac{M}{\pi R^2}$



Consider the small element of concentric ring of radius , X mass of small element of the ring

$$dm = \rho dA \quad \text{but} \quad dA = 2\pi x dx$$

$$= \left(\frac{M}{\pi R^2}\right) \cdot 2\pi x dx$$

$$dm = \frac{2mxdx}{R^2}$$

M.I of the small element of disc about the centre

$$dI_G = x^2 dm$$

$$dI_G = \frac{2Mx^3dx}{R^2}$$

$$I_G = \frac{2M}{R^2}\int_0^R x^3 dx$$

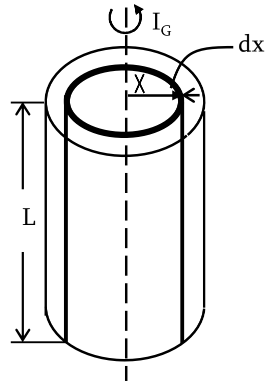
$$I_G = \frac{1}{2}MR^2$$

4. MOMENT OF INERTIA OF UNIFORM SOLID CYLINDER ABOUT AN AXIS OF SYMMETRY OR CENTRE.

$$I_G = \frac{1}{2}MR^2$$

M = Mass of the solid cylinder
R = radius of the cylinder

Derivation $I_G = \frac{1}{2}MR^2$



Let ρ = mass per unit volume
For the whole solid cylinder

$$\rho = \frac{M}{\pi R^2 L}$$

M.I of small element of solid cylinder about the centre.

$$dI_G = x^2 dm$$

But $\rho = \frac{dm}{dv}$, $dm = \rho dv$, $dv = 2\pi \times L dx$

$$dm = 2\pi x L \rho dx$$

$$dm = \frac{M}{\pi R^2 L} \cdot 2\pi x L dx$$

$$dm = \frac{2M}{R^2}xdx$$

Now $dI_G = x\left[\frac{2M}{R^2}xdx\right]$

$$dI_G = \frac{2Mx^3}{R^2}dx$$

$$I_G = \frac{2M}{R^2}\int_0^R X^3 dx = \frac{2M}{R^2}\left[\frac{x^4}{4}\right]_0^R$$

$$I_G = \frac{1}{2}MR^2$$

5. M.I OF THIN WALLED CYLINDER (UNIFORM HOLLOW CYLINDER) ABOUT THE CENTRE OF MASS.

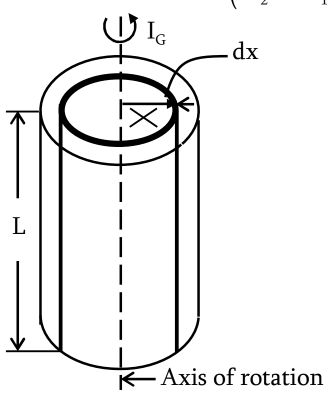
$$I_G = \frac{1}{2}M\left(R_1^2 + R_2^2\right)$$

M = Mass of hollow cylinder
R_1 = inner radius
R_2 = outer radius

Derivation : $I_G = \frac{1}{2}M\left(R_1^2 + R_2^2\right)$

Mass of small element of the cylinder

$$dM = \frac{2Mxdx}{\left(R_2^2 - R_1^2\right)}$$



M.I of small element of hollow cylinder

$$dI_G = x^2dm$$

$$dI_G = \frac{2Mx^3dx}{\left(R_2^2 - R_1^2\right)}$$

Total M.I

$$I_G = \frac{2M}{\left(R_2^2 - R_1^2\right)}\int_{R_1}^{R_2} x^3dx$$

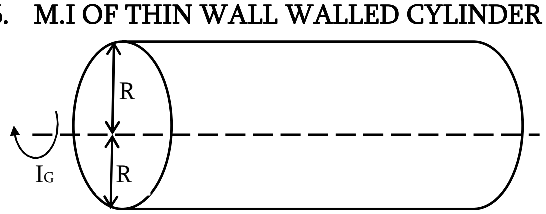
$$= \frac{2M}{\left(R_2^2 - R_1^2\right)}\left[\frac{x^4}{4}\right]_{R_1}^{R_2}$$

$$= \frac{2M}{4\left(R_2^2 - R_1^2\right)}\left[R_2^4 - R_1^4\right]$$

$$= \frac{M}{2\left(R_2^2 - R_1^2\right)}\left[\left(R_2^2 - R_1^2\right)\left(R_2^2 + R_1^2\right)\right]$$

$$I_G = \frac{M}{2}\left(R_1^2 + R_2^2\right)$$

6. M.I OF THIN WALL WALLED CYLINDER



$$R_1 = R_2 = R$$

$$I_G = \frac{M}{2}\left[R^2 + R^2\right]$$

$$I_G = MR^2$$

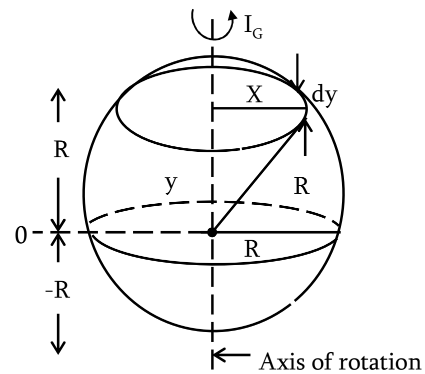
7. M.I OF A UNIFORM SOLID SPHERE ABOUT ITS CENTRE (DIAMETER)

$$I_G = \frac{2}{5}MR^2 = 0.4MR^2$$

M = Mass of solid sphere
R = radius of solid sphere

Derivation : $I_G = \frac{2}{5}MR^2$

Consider a small disc of thickness dy at a distance y from the centre of the sphere



$$R^2 = x^2 + y^2 \text{ (Pythagoras theorem)}$$

$$x = \sqrt{R^2 - y^2} \quad , \quad x^2 = R^2 - y^2$$

Volume of small element of solid sphere. The small element is in the form of disc.

$$dV = \pi x^2 dy$$

Mass of small element of solid sphere

$$dm = \rho dv = \pi x^2 \rho dy$$

M.I of small element (disc) about the centre

$$dI_G = \frac{1}{2}x^2dm$$

$$= \frac{\pi\rho}{2}x^4dy = \frac{\pi\rho}{2}\left[x^2\right]^2 dy$$

$$= \frac{\pi\rho}{2}\left[R^2 - y^2\right]^2 dy$$

$$dI_G = \frac{\pi\rho}{2}\left[R^4 - 2R^2y^2 + y^4\right]dy$$

$$I_G = \frac{\pi\rho}{2}\int_{-R}^{R}\left[R^4 - 2R^2y^2 + y^4\right]dy$$

$$I_G = \frac{8\pi\rho R^5}{15}$$

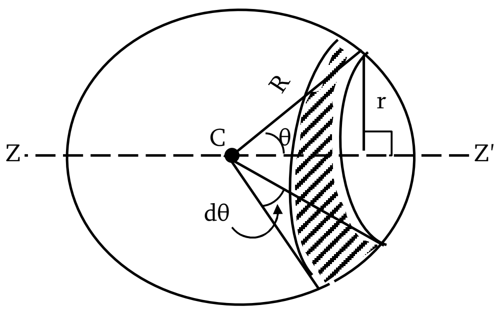
But $\rho = \dfrac{M}{\frac{4}{3}\pi R^3} = \dfrac{3M}{4\pi R^3}$

$$I_G = \frac{2}{5}MR^2$$

8. M.I OF UNIFORM SPHERICAL SHELL (HOLLOW SPHERE) ABOUT THE CENTRE.

$$I_G = \frac{2}{5}MR^2$$

Derivation



Let: δ = surface density

$$\delta = \frac{M}{4\pi R^2}$$

The hoop shown above has a radius Rsinθ

$r = R\sin\theta$

circumference , $C = 2\pi R\sin\theta$

the width of the hoop $= Rd\theta$ so the area ,

$$dA = (2\pi R\sin\theta)Rd\theta$$

$$dM = \delta dA$$

$$= \frac{M}{4\pi R^2}\cdot 2\pi R^2\sin\theta d\theta$$

$$dM = \frac{M}{2}\sin\theta d\theta$$

Now ; $dI = r^2 dm$

$$= (R\sin\theta)^2\cdot\frac{M}{2}\sin\theta d\theta$$

$$dI = \frac{MR^2}{2}\sin^3\theta d\theta$$

The M.I of the entire shell is the M.I of the various hoops.

$$I = \int_0^\pi \frac{MR^2}{2}\sin^3\theta d$$

$$= \frac{MR^2}{2}\int_0^\pi (1-\cos^2\theta)\sin\theta d\theta$$

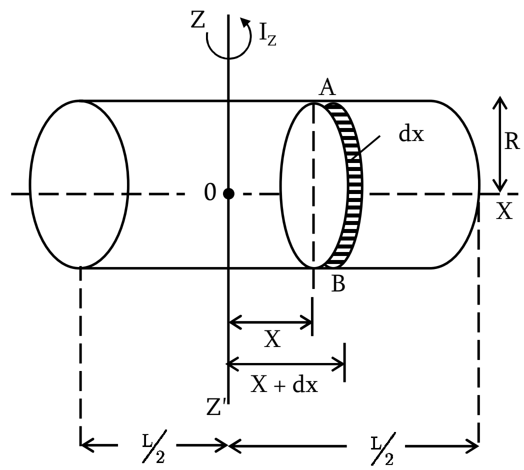
$$= \frac{MR^2}{2}\left[-\cos\theta\right]_0^\pi + \int_0^\pi \cos^2\theta(-\sin\theta d\theta)$$

$$= \frac{MR^2}{2}\left\{\left[-(-1)-(-1)\right]+\left[\frac{\cos^3\theta}{3}\right]_0^\pi\right\}$$

$$= \frac{MR^2}{2}\cdot\frac{4}{3}$$

$$I_G = \frac{2}{3}MR^2$$

9. M.I OF CYLINDER ABOUT AN AXIS THROUGH THE CENTRE AND PERPENDICULAR TO AXIS OF SYMMETRY.



The elementary portion of the cylinder between the two planes will be a circular of radius R and thickness , dx. The volume of the elementary portion (circular disc) of the cylinder.

$$V = \text{Area}\times\text{thickness} = \pi R^2 dx$$

Now

$$dm = \rho v \quad \left[\rho = \frac{M}{\text{Volume}}\right]$$

$$= \frac{M}{\pi R^2 L}\cdot\pi R^2 dx$$

$$dm = \frac{Mdx}{L}$$

Now, M.I of disc about its

Diameter $= \dfrac{1}{4}\text{mass}\times(\text{radius})^2$

M.I of this elementary portion of the cylinder about its diameter AB.

$$= \frac{1}{4}R^2 dm = \frac{MR^2 dx}{4L}$$

Let dI be the M.I of this elementary portion of the cylinder about axis ZZ′.

dI = (M.I of elementary portion about its diameter AB) + X^2dm.

(parallel axes theorem)

$$dI = \frac{MR^2}{4L} \cdot dx + \frac{Mx^2 dx}{L}$$

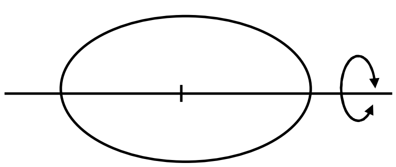
$$dI = \frac{M}{L}\left[\frac{R^2}{4} + X^2\right] dX$$

$$I = \int_{-L/2}^{L/2} \frac{M}{L}\left(\frac{R^2}{4} + X^2\right) dx$$

$$I = M\left[\frac{R^2}{4} + \frac{L^2}{12}\right]$$

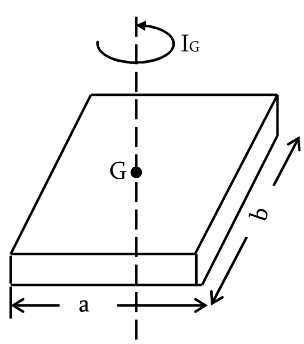
10. M.I OF DISC OF RADIUS R ABOUT ITS DIAMETER

$$I = \frac{1}{4}MR^2$$



11. M.I OF THE SLAB ABOUT PERPENDICULAR ABOUT ITS DIAMETER.

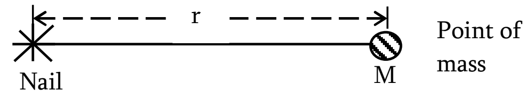
$$I_G = \frac{M}{12}\left(a^2 + b^2\right)$$



Example – 01

A point mass 200gm is attached to one end of string and another end is attached to a nail. The mass is made to rotate in a circle of radius 20cm. What is the moment of inertia of the particle about the axis of the nail?

Solution

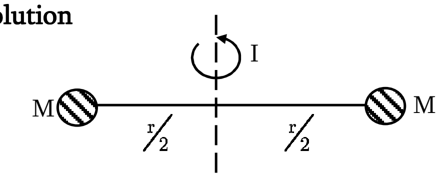


$$M.I = Mr^2$$
$$= 0.20 \times (0.2)^2$$
$$= 0.4 kgm^2$$

Example – 02

The distance between the two atoms in a diatomic molecules is 1.21 × 10^{-10}m. The mass of each oxygen atom is 2.66 × 10^{-26}kg. Treating the atoms as point masses. Find the moment of inertia of the molecule about an axis passing perpendicular to the line joining the centre of the atoms.

Solution



M.I of the system

$$I = M\left(\frac{r}{2}\right)^2 + M\left(\frac{r}{2}\right)^2$$

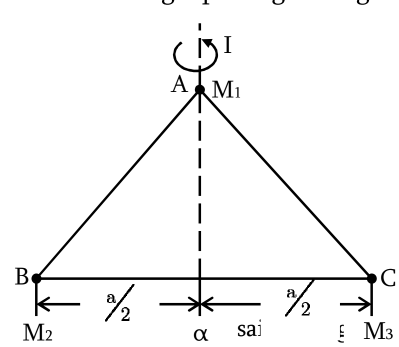
$$I = \frac{1}{2}Mr^2$$

$$I = \frac{1}{2} \times 2.66 \times 10^{-26} \times \left(1.21 \times 10^{-10}\right)^2$$

$$I = 1.95 \times 10^{-46} Kgm^2$$

Example – 03

Three mass points M_1, M_2, M_3 are located at the vertices of an equilateral triangle of length ‘a’ as shown in the figure below. What is the moment of inertia of the system about an axis along the altitude of the triangle passing through M_1?



Solution

M.I of the system about the altitude AQ

$$I = M_1r_1^2 + M_2r_2^2 + M_3r_3^2$$

$$= M_1(0)^2 + M_2\left(\frac{a}{2}\right)^2 + M_3\left(\frac{a}{2}\right)^2$$

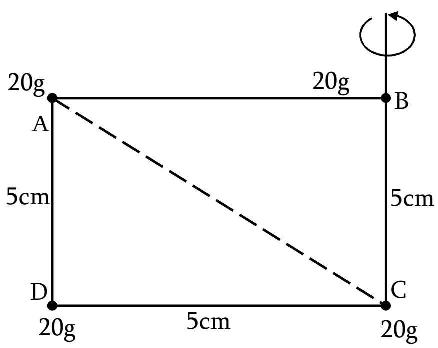
$$I = \frac{a^2}{4}(M_2 + M_3)$$

Example – 04

Four small bodies A , B , C and D which can be considered as particles connected by rods of negligible masses as shown in figure below. Find the M.I of the system:

(i) About an axis coinciding with rod BC and

(ii) About an axis passing through A and perpendicular to the plane of diagram.



Solution

(i) M.I about an axis coinciding with BC

$$I = 20(AB)^2 + 20(BB)^2 + 20(CC)^2 + 20(DC)^2$$

$$BB = CC = 0 \quad , \quad AB = DC = 5cm$$

$$I = 20(5)^2 + 20(0)^2 + 20(0)^2 + 20(5)^2$$

$$I = 1000gcm^2$$

(ii) M.I about an axis through the point A and perpendicular to the plane of the diagram.

$$I = 20(AA)^2 + 20(BA)^2 + 20(CA)^2 + 20(DA)^2$$

Now

$$CA = \sqrt{(AB)^2 + (BC)^2} = \sqrt{5^2 + 5^2}$$

$$\overline{CA} = 5\sqrt{2}cm$$

$$I = 20(0)^2 + 20(5)^2 + 20(5\sqrt{2})^2 + 20(5)^2$$

$$I = 2000gcm^2$$

Example – 05

Calculate the moment of inertia about a transverse axis through the centre of a disc , whose radius is 20cm. its density is $9gcm^{-3}$ and its thickness is 7cm.

Solution

Mass of the disc

$$M = \pi R^2 t\rho$$

$$= \frac{22}{7} \times (20)^2 \times 7 \times 9$$

$$M = 79200g$$

M.I of the disc about the centre , $I = \frac{1}{2}MR^2$

$$= \frac{1}{2} \times 79200 \times (20)^2$$

$$I = 1.584 \times 10^7 gcm^2$$

Example – 06

(a) (i) How is moment of inertia of a rigid body measured?

(ii) Why is the moment of inertia also called rotational inertia?

(b) What are the factors on which moment of inertia of a body depends?

(c) Derive an expression for the moment of inertia of a hollow sphere (thick spherical shell) about a diameter. Given that the M.I of a solid sphere of mass M and radius R about a diameter is $\frac{2}{5}MR^2$

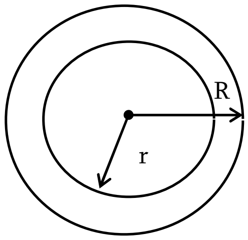
Solution

(a) (i) The moment of inertia of a rigid body about a given axis of rotation is the sum of the products of the masses of its particles and the squares of their respective perpendicular distances from the axis of rotation.

(ii) The moment of inertia is also called rotational inertia because it is a measure of inertia of a rotating body

(b) Refer to your notes

(c) Consider a solid sphere. Suppose a smaller concentric sphere is removed from the solid sphere we get a hallow or a thick spherical shell as shown below.



r = internal radius of the hallow sphere
R = External radius

Volume of the shell $= \frac{4}{3}\pi\left(R^3 - r^3\right)$

Shell $\rho = \frac{M}{\frac{4}{3}\pi\left(R^3 - r^3\right)} = \frac{3M}{4\pi\left(R^3 - r^3\right)}$

M.I of the solid sphere of radius about a diameter

$$= \frac{2}{5}(\text{mass})(\text{radius})^2$$

$$I_S = \frac{2}{5}\times\frac{4}{3}\pi R^3 \rho R^2 = \frac{8}{15}\pi R^5 \rho$$

M.I of the solid sphere of radius r about a diameter.

$$I_r = \frac{8}{15}\pi r^5 \rho$$

M.I of the hollow sphere about the diameter, I

$$I = I_S - I_r$$

$$= \frac{8}{15}\pi\rho R^5 - \frac{8}{15}\pi\rho r^5$$

$$= \frac{8}{15}\pi\rho\left(R^5 - r^5\right)$$

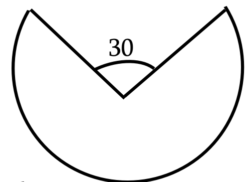
$$= \frac{8\pi}{15}\cdot\frac{3M}{4\pi\left(R^3 - r^3\right)}\cdot\left(R^5 - r^5\right)$$

$$I = \frac{2}{5}M\frac{\left(R^5 - r^5\right)}{\left(R^3 - r^3\right)}$$

Example – 07

From a complete ring of mass , M and radius R a 30° sector is removed. What is the moment of inertia of the complete ring about an axis passing through the centre of the ring and perpendicular to the plane of the ring?

Solution



Mass of incomplete ring

$$= M - \frac{M}{2\pi}\times\frac{\pi}{6} = \frac{11M}{12}$$

M.I of incomplete ring

$$I = \frac{11}{12}MR^2$$

THEOREM OF PERPENDICULAR AND PARALLEL AXES.

We can often simplify the calculation of the M.I for various rigid bodies by using two important theorem of M.I:-

(i) Theorem of perpendicular axes.
(ii) Theorem of parallel axes.

THEOREM OF PERPENDICULAR AXES.

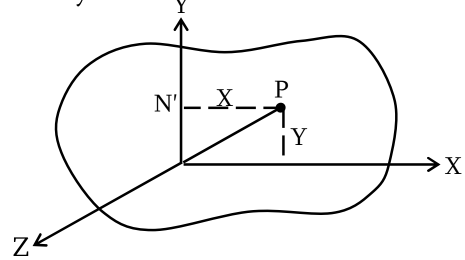
State that 'The moment of inertia of a plane lamina body about an axis perpendicular to its plane is equal to the sum of the moments of inertia of lamina about any two mutually perpendicular axes in its own plane intersecting at the point through which the perpendicular axes passes'.

i.e $I_Z = I_X + I_y$ (in xy – plane)

Proof: $I_Z = I_x + I_y$

Consider a plane lamina lying in the XOY plane as shown below. The lamina body can be made up by a large number of particles.

Consider a particle of mass M at P from P, drop perpendiculars PN and PN' on X – axis and Y – axis respectively.



By Pythagoras theorem

$r^2 = x^2 + y^2$

M.I of whole lamina body about Z – axis

$$I_Z = \sum mr^2 = \sum m\left(x^2 + y^2\right)$$

$$= \sum mx^2 + \sum my^2$$

But $\sum mx^2 = I_y$, $\sum my^2 = I_X$

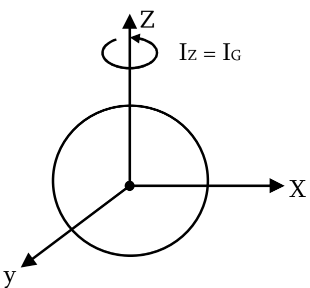
$$I_Z = I_X + I_y$$

I_X = M.I of the whole of lamina about X – axis

I_y = M.I of the whole of lamina about y - axis

APPLICATIONS OF PERPENDICULAR AXES THEOREM.

1. M.I of the disc about its diameter.



Apply the perpendicular axes theorem

$$I_z = I_x + I_y$$

Due to the symmetry , $I_x = I_y$

$$I_z = 2I_x = 2I_y$$

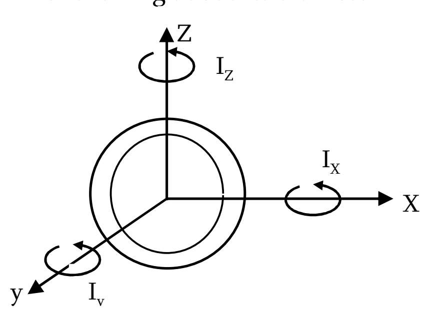
$$I_x = I_y = \frac{1}{2}I_z$$

But $I_z = I_G = \frac{1}{2}MR^2$

$$I_x = I_y = \frac{1}{2}\left[\frac{1}{2}MR^2\right]$$

$$I_x = I_y = \frac{1}{4}MR^2$$

2. M.I of the ring about its diameter



Apply perpendicular axes theorem

$$I_z = I_x + I_y$$

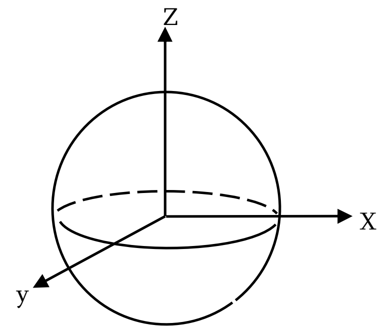
Due to the symmetry , $I_x = I_y$

$$I_z = 2I_x = 2I_y$$

$$I_x = I_y = \frac{1}{2}I_z = \frac{1}{2}\left[MR^2\right]$$

$$I_x = I_y = \frac{1}{2}MR^2$$

3. M.I of solid sphere about its diameter



Apply perpendicular axes theorem

$$I_z = I_x + I_y$$

Due to the symmetry property

$$I_x = I_y$$

$$I_z = 2I_x = 2I_y$$

$$I_x = I_y = \frac{1}{2}I_z$$

But $I_z = I_G = \frac{2}{5}MR^2$

$$I_x = I_y = \frac{1}{2}\left[\frac{2}{5}MR^2\right]$$

$$I_x = I_y = \frac{1}{5}MR^2$$

THEOREM OF PARALLEL AXES

State that 'The moment of inertia of a body about any axis is equal to its moment of inertia about a parallel axis through its centre of gravity plus the product of the mass of the body and the square of the perpendicular distance between two parallel axes.

$$I = I_G + Mh^2$$

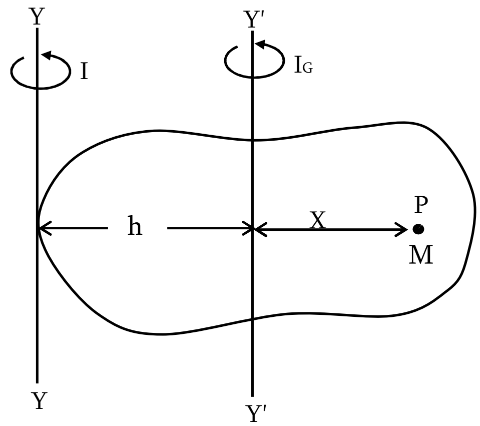
I_G = M.I of the rigid body about the centre

M = Mass of the body

h = perpendicular distance between the parallel axes.

Proof

Let I be the moment of inertia of a plane lamina about an axis YY. Let G be the centre of gravity of the lamina. Y'Y'is an axis parallel to the given axis and passing through the centre of gravity G of the lamina body.



M.I of the particle about YY = $m(x + h)^2$

M.I of the whole lamina body about YY.

$$I = \sum m(h+x)^2 = \sum m(x^2 + 2xh + h^2)$$

$$= \sum mx^2 + \sum mh^2 + \sum 2mxh$$

But $I_G = \sum mx^2$, $h^2 \sum m = Mh^2$. The lamina will balance itself about its centre of gravity so the algebraic sum of the moments of the weights of the constituent particles about the centre of gravity , G should be zero.

i.e $\sum mxg = 0$ or $g\sum mx = 0$ $[g \neq 0]$

$$I = I_G + Mh^2$$

Note that:

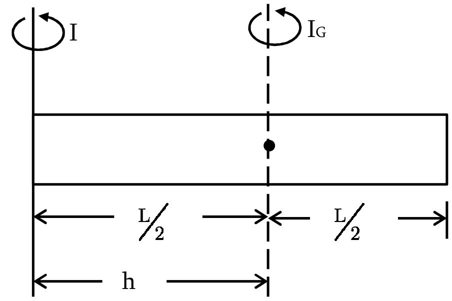
This theorem is applicable not only to a plane lamina but also to a three – dimensional body.

APPLICATIONS OF PARALLEL AXES THEOREM

1. M.I of the rod about an axis of the rotation at the end of the rod.

   Apply the parallel axes theorem

$$I = I_G + Mh^2 = \frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2$$

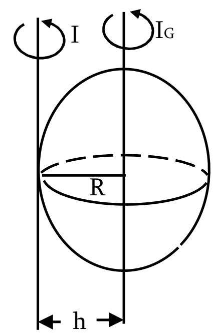


$$I = \frac{ML^2}{12} + \frac{ML^2}{4} = \frac{ML^2}{3}$$

$$I = \frac{ML^2}{3}$$

2. M.I of the solid sphere about an axis tangential to the sphere and perpendicular to the plane of sphere.

   Apply parallel axes theorem

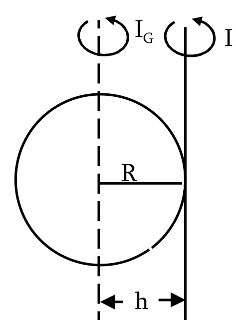


$$I = I_G + Mh^2$$

$$= \frac{2}{5}MR^2 + MR^2$$

$$I = \frac{7}{5}MR^2$$

3. M.I of the disc about an axis tangential and perpendicular to the plane of the disc.



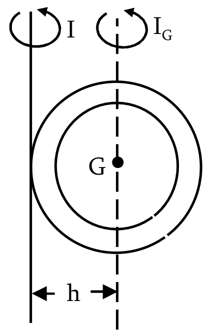
Apply parallel axes theorem

$$I = I_G + Mh^2$$

$$= \frac{1}{2}MR^2 + MR^2$$

$$I = \frac{3}{2}MR^2$$

4. M.I of the ring about an axis parallel to one of the diameters and touching the ring



Apply the parallel axes theorem

$$I = I_G + Mh^2$$

$$I = MR^2 + MR^2 = 2MR^2$$

$$I = 2MR^2$$

RADIUS OF GYRATION , K

Is defined as the distance from the axis of rotation at which the whole mass of the body were supposed to be concentrated without altering its moment of inertia about that axis. It is denoted by K and S.I unit is metre (m).

Mathematically

$$I = Mr^2 \quad , \quad I = I_G \quad , \quad r = k$$

$$I_G = MK^2$$

$$K = \sqrt{\frac{I_G}{M}}$$

Another definition of radius of gyration.

Consider a rigid body which consists n particles each of mass , m.

M.I of a body of mass M and radius of gyration K is given by

$$I = M_1r_1^2 + M_2r_2^2 + \cdots\cdots + M_nr_n^2$$

Let $m_1 = m_2 = \cdots\cdots = m_n = m$

$$I = m\left[r_1^2 + r_2^2 + \cdots\cdots + r_n^2\right]$$

$$I = MK^2 = nmk^2$$

$$nmk^2 = m\left[r_1^2 + r_2^2 + \cdots\cdots + r_n^2\right]$$

$$k = \sqrt{\frac{r_1^2 + r_2^2 + \cdots\cdots + r_n^2}{n}}$$

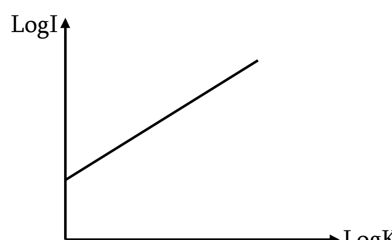
The radius of gyration of a body about an axis is equal to the root mean square distance of the various particles constituting of the body from the axis of rotation. Radius of gyration of a body about an axis of rotation may also be defined as the square root mean square distance of the particles from the axis of rotation and its square when multiplied with the mass of the body gives moment of inertia of the body about that axis.

Additional concepts.

From the concept of moment of inertia and radius of the gyration, the following conclusions may be drawn:-

1. The moment of inertia of a body depends upon mass of the body as well as the manner in which mass is distributed about the axis of rotation. It is because M.I of the body changes with change in position of the axis of rotation.
2. The radius of gyration of a body is not constant quantity. Its value changes with change of location of the axis of rotation.
3. $I = MK^2$

$$\log^I = \log^M + 2\log K$$



EXAMPLES

1. Expression of radius of gyration of uniform rod about an axis passing through the center and perpendicular to its length.

$$I_G = MK^2 = \frac{ML^2}{12}$$

$$K = \frac{L}{\sqrt{12}}$$

2. Expression of radius of gyration of uniform circular disc about an axis passing through the centre and perpendicular to the plane of disc.

$$I_G = \frac{1}{2}MR^2 = MK^2$$

$$K = \frac{R}{\sqrt{2}}$$

3. Expression of radius of gyration of uniform solid sphere about an axis through the centre.

$$I_G = \frac{2}{5}MR^2 = MK^2$$

$$K = \sqrt{\frac{2}{5}}R$$

NUMERICAL EXAMPLES

Example – 08

If the radius of a sphere is 5cm , calculate the radius of gyration.

(i) About its diameter

(ii) About its tangent

Solution

(i) M.I of the sphere about its diameter

$$I_G = \frac{2}{5}MR^2 = MK^2$$

$$K = \sqrt{\frac{2}{5}}\ R = \sqrt{\frac{2}{5}} \times 5cm$$

$$K = 3.162cm$$

(ii) Apply parallel axess theorem

$$I = I_G + Mh^2$$

$$= \frac{2}{5}MR^2 + MR^2$$

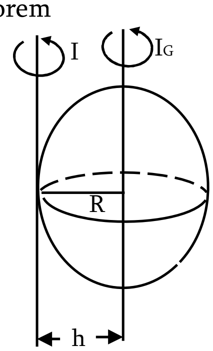
$$I = \frac{7}{5}MR^2$$

But $I = MK^2$

$$MK^2 = \frac{7}{5}MR^2$$

$$K = \sqrt{\frac{7}{5}}\ R = \sqrt{\frac{7}{5}} \times 5cm$$

$$K = 5.92cm$$



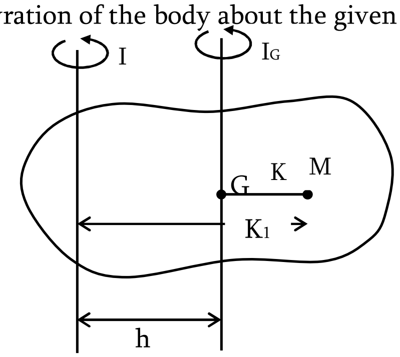
Example – 09

(a) Radius of gyration of a body about an axis at a perpendicular distance of 6cm from its centre of mass is 10cm. Find its radius of gyration about a parallel axis through its centre of mass.

(b) The diameter of flywheel increases by 1%. What will be the percentage increase in moment of inertia about axis of symmetry?

Solution

(a) K_1 = 10cm , h = 6cm , K = ?, K_1 = radius of gyration of the body about the given axis.



Apply parallel axes theorem

$$I = I_G + Mh^2$$

$$MK_1^2 = MK^2 + Mh^2$$

$$K_1^2 = K^2 + h^2$$

$$K = \sqrt{K_1^2 - h^2} = \sqrt{10^2 - 6^2}$$

$$K = 8cm$$

(b) $I = MR^2$

$$\log_e^I = \log_e\left(MR^2\right) = \log_e^M + \log_e^{R^2}$$

$$\log_e^I = \log_e^M + 2\log_e^R$$

On differentiating

$$\frac{dI}{I} = 2\frac{dR}{R}$$

$$\frac{dI}{I} \times 100\% = 2\left[\frac{dR}{R} \times 100\%\right]$$

$$= 2 \times 1\%$$

$$\frac{dI}{I} \times 100\% = 2\%$$

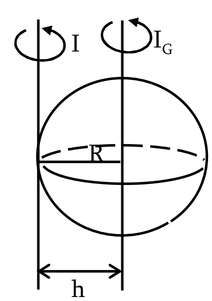
Example – 10

(a) Calculate the moment of inertia of a uniform disc of mass 0.4kg and radius 10cm about an axis through its edge and perpendicular to the plane of the disc.

(b) Find the radius of gyration of sphere of radius 15cm without an axis 5cm from the centre ($I_G = 0.4MR^2$)

Solution

(a) Apply parallel axes theorem



$$I = I_G + Mh^2$$

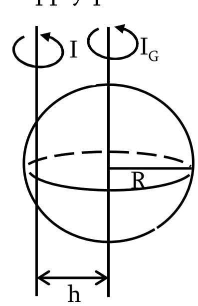
$$= \frac{1}{2}MR^2 + MR^2$$

$$I = \frac{3}{2}MR^2$$

$$= \frac{3}{2} \times 0.4 \times \left(0.1\right)^2$$

$$I = 6.0 \times 10^{-3}Kgm^2$$

(b) Apply parallel axes theorem



$$I = I_G + Mh^2$$

$$MK^2 = 0.4MR^2 + Mh^2$$

$$K = \sqrt{0.4R^2 + h^2}$$

$$= \sqrt{0.4(0.15)^2 + (0.05)^2}$$

$$K = 0.11\text{m}$$

Example – 11

(a) Define angular acceleration and moment of inertia of rigid body.

(b) Some rivers flows towards the equator and hence transport more sediment from higher to lower latitudes. How does the process affect the earth rotation?

Solution

(b) This process tends to show down the earth rotation, due to the deposition of sediment and particles which can be carried by the river tends to increase the moment of inertia.

Example – 12

(a) The mass of a disc is 700gm and its radius of gyration is 20cm. What is its moment of inertia if it rotates about an axis passing through its centre and perpendicular to the face of the disc?

(b) The mass of a flywheel is concentrated on the rim why?

Solution

(a) $I = MK^2$

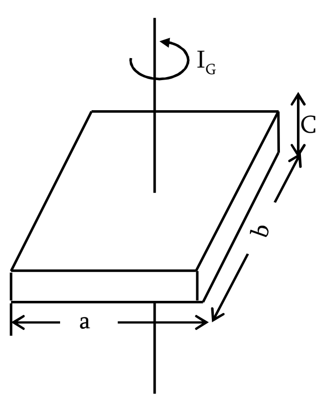
$$= 0.7 \times (0.2)^2$$

$$I = 0.028\text{Kgm}^2$$

(b) This is to increase the moment of inertia. Hence its opposition to any change in uniform rotatory motion is large. So when a fly wheel of large M.I is used the engine runs smoother and steadier.

Example – 13

Derive an expression of moment of inertia of slab about perpendicular axis through the centre as shown on the figure below.



Solution

Consider the mass of small portion

$$\frac{dM}{M} = \frac{adx}{ab} = \frac{dx}{b}$$

$$dM = \frac{Mdx}{b}$$

M.I of the strip of mass ,dM

$$dI_G = \frac{1}{2}a^2dm$$

Apply the parallel axes theorem

$$dI = dI_G + h^2dM \qquad h = x$$

$$dI = \frac{a^2}{12}dM + x^2dM$$

$$I = \frac{a^2}{12}\int_0^M dM + \frac{M}{b}\int_{-b/2}^{b/x} x^2dx$$

$$I = \frac{M}{12}\left(a^2 + b^2\right)$$

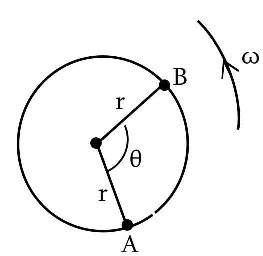
ROTATIONAL MOTION

A rigid body executes translation motion when each particle of the body has the same displacement in same time interval. Example motion of the car in the straight line and velocity and acceleration possessed by the car are known as linear velocity and linear acceleration respectively. A rigid body execute rotation motion when each of the particles (except those on the axis of the rotational) moves in the circle. The velocity and acceleration possessed by the particle is known as Angular velocity (ω) and angular acceleration (α).

ANGULAR QUANTITIES

Are those quantities which describes the rotation motion of the rigid body.

Consider the figure below which shows the particles moving in the circular path



- **ANGULAR VELOCITY (ω)**
  Is defined as angular displacement per unit time.
  $$\omega = \frac{\theta}{t}$$
  Angular velocity is defined as the rate of change angular displacement.
  $$\omega = \frac{d\theta}{dt}$$

- **ANGULAR ACCELERATION (α)**
  Is defined as angular velocity possessed by the body per unit time taken.
  $$\alpha = \frac{\text{change in angular velocity}}{\text{time taken}}$$
  $$\alpha = \frac{\omega - \omega_0}{t}$$
  ω_0 = initial angular velocity
  ω = final angular velocity
  Angular acceleration is defined as the rate of change of angular velocity.
  $$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \omega\frac{d\omega}{d\theta}$$
  Relationship between linear acceleration and angular acceleration.
  $$a = \alpha r$$

Equations of uniform accelerated motion

Angular motion	Linear motion	
$\omega = \omega_0 + \alpha t$	$V = u + at$	a,α-constant
$\theta = \omega_0 + \frac{1}{2}\alpha t^2$	$S = ut + \frac{1}{2}at^2$	”
$\omega^2 = \omega^2_0 + 2\alpha\theta$	$V^2 = U^2 + 2as$	”

DERIVATION OF EQUATION OF UNIFORM ROTATION MOTION.

1. $\omega = \omega_0 + \alpha t$

   From the equation
   $$\alpha = \frac{d\omega}{dt}\ ,\ d\omega = \alpha dt$$
   $$\int_{\omega_0}^{\omega} d\omega = \alpha\int_0^t dt$$
   $$\left[\omega\right]_{\omega_0}^{\omega} = \alpha\left[t\right]_0^t$$
   $$\omega - \omega_0 = \alpha(t - 0)$$
   $$\omega = \omega_0 + \alpha t$$

2. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

   From the equation , $\omega = \frac{d\theta}{dt}$
   $$d\theta = (\omega_0 + \alpha t)dt$$
   $$\int_0^\theta d\theta = \int_0^t (\omega_0 + \alpha t)dt$$
   $$\left[\theta\right]_0^\theta = \left[\omega_0 t + \frac{1}{2}\alpha t^2\right]_0^t$$
   $$\theta - 0 = \omega_0 t + \frac{1}{2}\alpha t^2$$
   $$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

3. $\omega^2 = \omega_0^2 + 2\alpha\theta$

   For the equation
   $$\alpha = \omega\frac{d\omega}{d\theta}$$
   $$\omega d\omega = \alpha d\theta$$
   $$\int_{\omega_0}^{\omega} \omega d\omega = \alpha\int_0^\theta d\theta$$
   $$\left[\frac{\omega^2}{2}\right]_{\omega_0}^{\omega} = \alpha\left[\theta\right]_0^\theta$$
   $$\omega^2 - \omega_0^2 = 2\alpha(\theta - 0)$$
   $$\omega^2 = \omega_0^2 + 2\alpha\theta$$

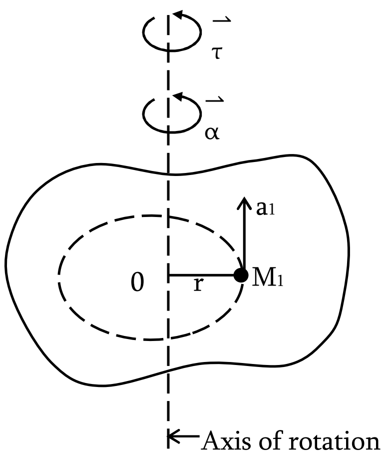
FUNDAMENTAL EQUATIONS OF ROTATION
ANGULAR ACCELERATION (α)
$$\alpha = \frac{dw}{dt} = \omega\frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

1. **Relationship between torque and angular acceleration.**

   TORQUE (τ) – Is the product of the applied force (F) and the perpendicular distance from the axis of the rotation
   $$\tau = Fd_\perp$$
   S.I unit of the torque is **Nm**

Dimensional formula of torque $\left[\tau\right] = \left[F\right]\left[d\right] = \left[ML^2T^{-2}\right]$

- **For the case of rigid body**
External torque acting on a rigid body is the sum of the moments of the forces acting on all the constituent particles of the rigid body about the axis of rotation. Now consider the rotational of the rigid body about the given axis of rotation as shown on the figure below.



A rigid body of moment of inertia I rotating about an axis through its centre of mass O under the effect of an external torque. Torque τ_1 acting on a particle of mass M_1 about the given axis.

$$\tau_1 = F_1r_1 = M_1a_1r_1 = M_1\left(\alpha r_1\right)r_1$$

$$\tau_1 = M_1\alpha r_1^2$$

Similarly

$$\tau_2 = M_2\alpha r_2^2 \quad , \quad \tau_3 = M_3\alpha r_3^2$$

Total external torque

$$\begin{aligned}\tau &= \tau_1 + \tau_2 + \cdots\cdots + \tau_n \\ &= M_1r_1^2 + M_2r_2^2\alpha + \cdots\cdots + N_nr_n^2\alpha \\ &= \alpha\left[M_1r_1^2 + M_2r_2^2 + \cdots\cdots + M_nr_n^2\right]\end{aligned}$$

$$\tau = I\alpha$$

TORQUE – is defined as the product of moment of inertia and angular acceleration of the rigid body about a given axis of rotation.

$$\tau = Fd = I\alpha = I\frac{d^2\theta}{dt^2}$$

Additional concept

$$\tau = I\alpha = I\frac{d\omega}{dt}$$

$$\tau dt = Id\omega$$

$$\int_0^t \tau dt = \int_{\omega_0}^{\omega} Id\omega$$

$$\tau\left[t\right]_0^t = I\left(\omega - \omega_0\right)$$

$$\tau t = I\left(\omega - \omega_0\right)$$

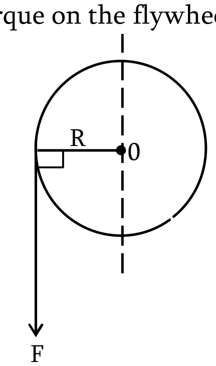
$$\tau t = \text{Angular impulse}$$

ANGULAR IMPULSE – is defined as the product of external applied torque and time or is the change of angular momentum.

$$\text{Angular impulse} = \tau t = I\left(\omega - \omega_0\right)$$

EXPRESSION OF THE TORQUE ON THE DIFFERENT SYSTEMS.

(a) Torque on the flywheel



Torque
$\tau = FR$

Also $\tau = I\dfrac{d^2\theta}{dt^2} = FR$

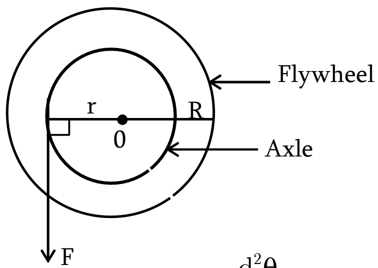
$$\frac{d^2\theta}{dt^2} = \frac{FR}{I}$$

F = Applied force
R = Radius of flywheel
I = M.I of the flywheel .

Special case of the flywheel

Case 1:

If the flywheel is mounted on the axle and applied force is acting on the axle as shown in the figure below



Torque $\tau = Fr = I\dfrac{d^2\theta}{dt^2}$

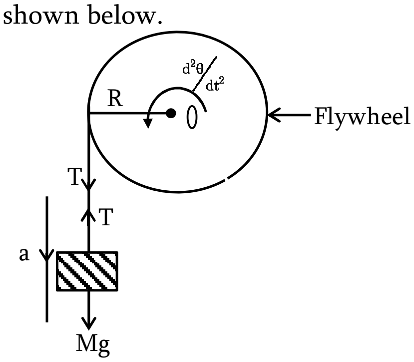
I = M.I of flywheel and axle

r = Radius of the axle

$$\frac{d^2\theta}{dt^2} = \frac{Fr}{I}$$

Case 2:

Consider the flywheel which is wounded with the string and at the end of the string there is attachment of the block of mass , m as shown below.



- **Expression of acceleration of the block**

Resultant force on the block of mass , m.

$$mg - T = ma \text{ ....(i)}$$

Torque on the flywheel

$$\tau = TR = I\alpha$$

$$T = \frac{I\alpha}{R} \quad \text{but } a = \alpha R \text{ , } a = a/R$$

$$T = \frac{Ia}{R^2} \text{ ....(ii)}$$

Putting equation (ii) into (i)

$$mg - \frac{Ia}{R^2} = ma$$

$$mg = \left(m + \frac{I}{R^2}\right)a$$

$$a = \frac{mg}{m + \dfrac{I}{R^2}}$$

- **Expression of the tension on the string.**

$$T = \frac{Ia}{R^2} = \frac{I}{R^2}\left[\frac{mg}{m + I/R^2}\right]$$

$$T = \frac{mg}{1 + \dfrac{mR^2}{I}}$$

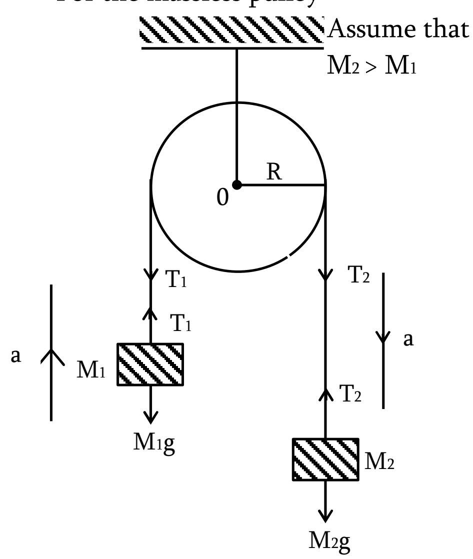
- **Expression of the torque**

$$\tau = TR = \frac{mgR}{1 + \dfrac{mR^2}{I}}$$

(b) Torque on the pulley system resultant force on the pulley system.

$$\tau = FR = \left(M_2 - M_1\right)gR$$

For the massless pulley



As the pulley has a finite mass , the two tension T_1 and T_2 are not.

- **Expression of the linear acceleration of the masses.**

Resultant forces on the masses

M_1: $T_1 - M_1g = M_1a$ ....(i)

M_2: $M_2g - T_2 = M_2a$ ......(ii)

Adding equation (i) and (ii)

$$T_1 - M_1g + M_2g - T_2 = \left(M_1 + M_2\right)a$$

$$T_1 - T_2 + \left(M_2 - M_1\right)g = \left(M_1 + M_2\right)a$$

Torque on the pulley

$$\tau = \left(T_2 - T_1\right)R = I\alpha$$

$$T_2 - T_1 = \frac{I\alpha}{R} \quad \text{but} \quad \alpha = \frac{a}{R}$$

$$T_2 - T_1 = \frac{Ia}{R^2}, \quad T_1 - T_2 = \frac{Ia}{R^2}$$

Now equation (iii) becomes

$$\frac{-Ia}{R^2} + (M_2 - M_1)g = (M_1 + M_2)a$$

$$(M_2 - M_1)g = \left(M_1 + M_2 + \frac{I}{R^2}\right)a$$

$$a = \frac{(M_2 - M_1)g}{M_1 + M_2 + \frac{I}{R^2}}$$

- **Expression of angular acceleration**

$$\alpha = \frac{a}{R}$$

$$a = \frac{(M_2 - M_1)gR}{(M_1 + M_2) + I}$$

$$\frac{a}{R} = \frac{(M_2 - M_1)gR}{(M_1 + M_2)R^2 + I}$$

$$\alpha = \frac{(M_2 - M_1)gR}{I + R^2(M_1 + M_2)}$$

- **Expression of the tension T_1 and T_2**

$$T_1 - M_1 g = M_1 a$$

$$= M_1 g + M_1 g \frac{(M_2 - M_1)}{M_1 + M_2 + \frac{I}{R^2}}$$

$$T_1 = M_1 g \left[1 + \frac{M_2 - M_1}{M_1 + M_2 + \frac{I}{R^2}}\right]$$

$$T_1 = M_1 g \left[\frac{2M_2 + \frac{I}{R^2}}{M_1 + M_2 + \frac{I}{R^2}}\right]$$

Again

$$M_2 g - T_2 = M_2 a$$

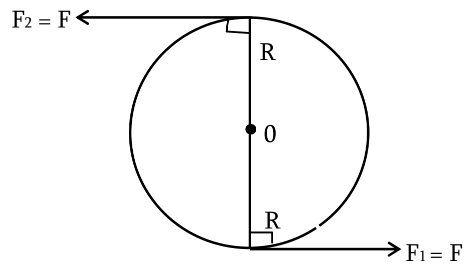
$$T_2 = M_2 g - M_2 a$$

$$T_2 = M_2 g - \frac{M_2 g (M_2 - M_1)}{M_1 + M_2 + \frac{I}{R^2}}$$

$$= M_2 g \left[\frac{M_1 + M_2 + \frac{I}{R^2} - M_2 + M_1}{M_1 + M_2 + \frac{I}{R^2}}\right]$$

$$T_2 = M_2 g \left[\frac{2M_1 + \frac{I}{R^2}}{M_1 + M_2 + \frac{I}{R^2}}\right]$$

(c) Torque on the moving coil meter



Couple force are those forces which have equal in magnitude and are in opposite direction separately.

$\tau = FR + FR$

$\tau = 2FR = Fd$

d = diameter of moving coil meter.

2. Relationship between angular momentum , moment of inertia and angular velocity.

Angular momentum of a rotating

Rigid body about a given axis is the sum of moments of linear momentum of the constituent particles of the body about the given axis.

Angular momentum = linear momentum × perpendicular distance.

$L = mvr$

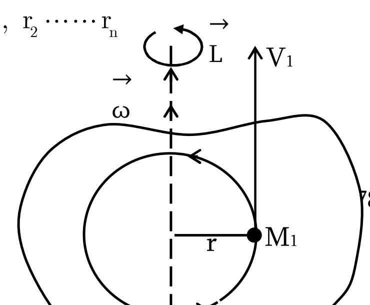
M = mass of the particle

V = linear velocity

r = perpendicular distance.

Consider the rotational of the rigid body which consists of n – particles about the given axis of rotation.

Let $M_1, M_2 \cdots\cdots M_n$ be the masses of the constituent particles of the body whose their respectively distances from the axis of rotation are $r_1, r_2 \cdots\cdots r_n$



Angular momentum of the first particle

$$L_1 = M_1V_1r_1 = M_1\omega r_1r_1 = M_1r_1^2\omega$$

For the second particle

$$L_2 = M_2r_2^2\omega$$

For the third particle

$$L_3 = M_3r_3^2\omega$$

For the nth particle

$$L_n = M_nr_3^2\omega$$

Total angular momentum of n – particles on the given rigid body

$$L = L_1 + L_2 + L_3 + \cdots\cdots + L_n$$

$$= M_1r_1^2\omega + M_2r_2^2\omega + \cdots\cdots + M_nr_n^2\omega$$

$$= \left(M_1r_1^2 + M_2r_2^2 + \cdots\cdots + M_nr_n^2\right)\omega$$

$$L = I\omega$$

Angular momentum is defined as the product of the moment of inertia of rigid body and angular velocity about the given axis of rotation.

$$L = I\omega = MVr$$

S.I unit of angular momentum is kgm^2s^{-1}

Dimensional of angular momentum

$$[L] = [M][V][r] = ML^2T^{-1}$$

3. Relationship between torque (τ) and angular momentum.

From the equation

$$\tau = I\alpha = I\frac{d\omega}{dt}$$

Since $L = I\omega$

(Differentiate L w.r.t time)

$$\frac{dL}{dt} = I\frac{d\omega}{dt}$$

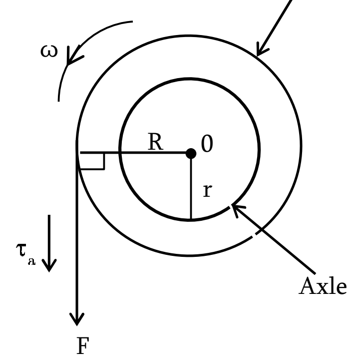
$$\tau = \frac{dL}{dt}$$

Torque – is defined as the rate of change of angular momentum.

- **FRICTION TORQUE (τ_f)**

  Is the torque which opposes the rotational motion of the rigid body about the given axis of the rotation. Always the friction torque is opposes the direction of the applied force (external torque)

  Consider the rotation of the flywheel about the given axis of rotation as shown on the figure below.



The friction force between the axle and flywheel lead to the formation of friction torque. If the applied force is removed on the flywheel then its comes into rest.

Resultant torque

$$\tau = \tau_a - \tau_f$$

τ_a = Applied torque

τ_f = Friction torque.

$$I\alpha = \tau_a - \tau_f$$

$$\alpha = \frac{\tau_a - \tau_f}{I}$$

When I = M.I of flywheel and axle of system.

Also $\tau = FR$

$$FR = \tau_a - \tau_f$$

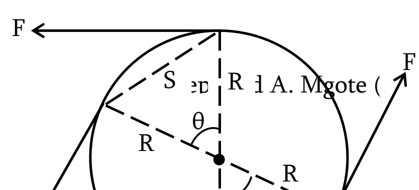
$$F = \frac{\tau_a - \tau_f}{R}$$

F = Applied force

R = Radius of the flywheel

4. WORK DONE BY COUPLE FORCES

Consider the rotation of the moving coil meter about the given axis of the rotational as shown below.



Work done corresponding to each force

$$W = Fs = FR\theta \quad \left[S = R\theta\right]$$

Total work done by the couple forces

$$W = FR\theta + FR\theta$$

$$= 2FR\theta = \left(2FR\right)\theta$$

$$W = \tau\theta$$

Also $dw = \tau d\theta \;,\; w = \int \tau d\theta$

5. POWER OF THE COUPLE FORCES

Power – is the rate of doing work

$$P = \frac{dw}{dt} \quad \text{but} \quad \tau\theta = w$$

$$P = \frac{d}{dt}\left[\tau\theta\right] = \tau\frac{d\theta}{dt} = \tau\omega$$

$$P = \tau\omega$$

6. LAW OF CONSERVATION OF ANGULAR MOMENTUM IN THE CASE OF RIGID BODIES.

The law of conservation of angular momentum in rotational motion is as important as the law of conservation of linear momentum in translational motion. We know that external torque acting on a system about an axis is equal to the time rate of change of angular momentum of the system about that axis i.e

$\tau = {dL}/{dt}$ if no external torque act i.e

$$\tau = 0 \;,\; \frac{dL}{dt} = 0 \text{ or } L = I\omega = \text{constant}$$

law of conservation of angular momentum state that ‘ If no external torque acts on a system, the total angular momentum of the system remains conserved’ i.e ‘ The total angular momentum of the system of rotating rigid body remain constant provided that there is no external torque acts about the given axis of rotation’. If $L_1 \;,\; L_2 \cdots\cdots L_n$ are the angular momentum of the constituent particles of a system , then it follows that $L_1 + L_2 + \cdots\cdots + L_n = \text{constant}$. In this case of a rotating non – rigid body , then during rotation , its moment of inertia may vary due to the change of distribution of mass about the axis of rotation. In such case , if moment of inertia changes from I_1 to I_2 then angular velocity must change from ω_1 to ω_2.

$$I\omega = \text{constant}$$

$$I_1\omega_1 = I_2\omega_2$$

$$I = \frac{\text{constant}}{\omega} \;,\; I \;\alpha\; \frac{1}{\omega}$$

Also $I_1\left(2\pi f_1\right) = I_2\left(2\pi f_2\right)$

$$I_1 f_1 = I_2 f_2$$

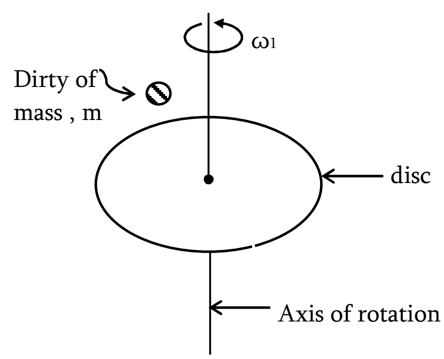
i.e $\quad$ if $=$ constant

$$I = \frac{\text{constant}}{f} \;,\; I \;\alpha\; \frac{1}{f}$$

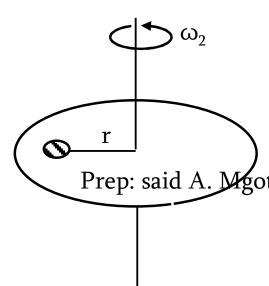
Simple illustrations of law of conservation of angular momentum.

1. **Falling of dirty on the rotating disc. i.e determination of M.I of disc.**

When the dirty of particle of mass m is falling on rotating disc will tend to lower the frequency of rotation of the disc due to the increases of the mass of the disc which tend to increases the M.I of the disc. Before the dirty falling on the disc.



Let $I_1 = I_d$ = M.I of the disc about the given axis of the rotation. Initial angular momentum of the disc $L_1 = I_1\omega_1 = 2\pi I_1 f_1 \ldots\ldots$..(i) finally , when the dirty is falling on the disc reduces the frequency of the rotation to f_2



M.I of the system

$$I_2 = Id + Mr^2$$

Final angular momentum

$$L_2 = \left(Id + Mr^2\right)\omega_2 \ldots\ldots\text{(ii)}$$

Apply the principle of conservation of the angular momentum.

(i) = (ii)

$$Id\omega_1 = \left(Id + Mr^2\right)\omega_2$$

$$I_d\omega_1 = I_d\omega_2 + Mr^2\omega_2$$

$$I_d\left[\omega_1 - \omega_2\right] = Mr^2\omega_2$$

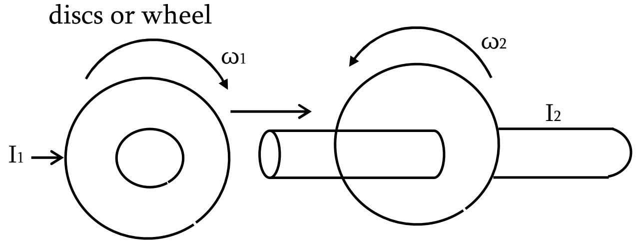
$$I_d = Mr^2\left[\frac{\omega_2}{\omega_1 - \omega_2}\right]$$

$$\text{Also } I_d = Mr^2\left[\frac{2\pi f_2}{2\pi\left(f_1 - f_2\right)}\right]$$

$$I_d = Mr^2\left[\frac{f_2}{f_1 - f_2}\right]$$

2. Collision of two rotating discs or wheels.

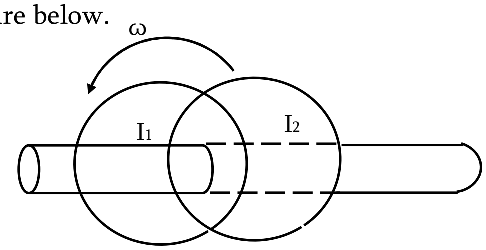
Consider the figure below which shows the rotation of the disc about the given axis of the rotation. Before the collision between two discs or wheel



Initial total angular momentum before collision

$$Li = I_1\omega_1 + I_2\omega_2$$

After the collision of the two discs rotates with common angular velocity as shown on the figure below.



Total final angular momentum

$$L_f = \left(I_1 + I_2\right)\omega$$

Apply the principle of conservation on of angular momentum.

$$L_f = Li$$

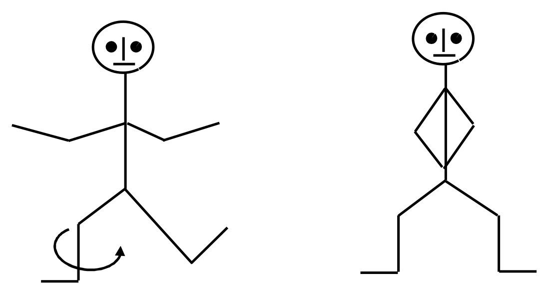
$$\left(I_1 + I_2\right)\omega = I_1\omega_1 + I_2\omega_2$$

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

APPLICATIONS (DAILY EXAMPLES) OF THE PRINCIPLE OF CONSERVATION OF ANGULAR MOMENTUM.

1. A ballet dancer. i.e A ballet dancer can vary her angular speed by out stretching her arms and legs. A ballet dancer makes use of law of conservation of angular momentum to vary her angular speed. When she starts to spin and stretches her arm , the moment of inertia is large and the angular velocity , ω is small. When suddenly folds her arms and brings the stretched leg close to the other leg, her angular velocity increases due to the decrease in moment of inertia i.e $I\omega$ = constant , $I \propto \frac{1}{\omega}$

Show rotation   Fast rotation



Out stretched Their arms and legs, High I and low ω or f

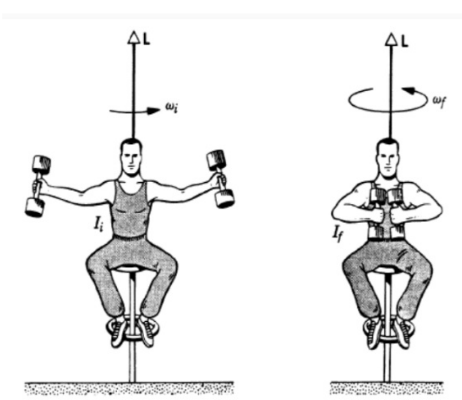
folding their legs and low I and high ω or f.

2. The angular velocity of a planet around the sun increases when it comes near to the sun. when a planet revolving around the sun in a elliptical orbit comes near the sun, the moment of inertia of the planet about the sun decreases in order to conserve angular momentum , the angular velocity shall increases similarly when

the planet is very far away from the sun , there will be decrease in the angular velocity.

3. When a bullet is fired from a rifle , the bullet possesses not only linear velocity but also some spin its axis. Since the angular momentum has to be conserved , therefore the bullet maintains its direction of the motion.
4. A diver jumping from a spring board performs some saults in air. The diver leaves the high diving board with stretched their arms and legs. This decreases moment of inertia and hence increases angular velocity. He then performs some saults as the diver is about to touch the surface of water , he stretches out his limbs by doing this he increases its moment of inertia and decreases its angular velocity.

   $I\omega$ = constant , $I \propto \frac{1}{\omega}$
5. A man standing on rotating turn table, holding his arms extended with a heavy weight in each hand. A turn table is free to rotates. If someone sets the man is slow rotation and then drops his hands to his sides, he will rotate much more rapidly. The total mass of rotating system remain constant. Hence , a decrease in the distance of the heavy weight from the axis of rotation results in a corresponding decrease in the moment of inertia of the system about the axis of rotation because the moment of inertia depends on the distribution of its mass about the axis. Since the angular momentum of the system remains constant the angular velocity of rotating system is increased due to the decrease in the moment of inertia of the system.



6. Determination of egg is hard boiled or half boiled when a raw egg is spun , the yolk which is denser moves away from the axis of rotation. Since moment of inertia ($I = Mr^2$) , when the distance r from the axis of rotation increases , I increases. When I increases , ω decreases. It is hard to spin a raw egg (i.e raw egg will spin at a small rate) on the other hand , when a hard – boiled egg is spun , its yolk does not move away from the axis of rotation. The moment of inertia I remains constant and the egg will rotate faster like a rigid.

NEWTON'S LAWS OF ROTATIONAL MOTION

1. **First law of rotational motion**

   State that 'Everybody continues in its state of rest or uniform rotational motion about its fixes axis its acted by some external torque to change that state'.
2. **Second law of rotational motion.**

   State that ' The rate of change of angular momentum of a body about a fixes axis of rotation is directly proportional to the torque applied and takes place in the direction of the torque'. i.e . $\tau \propto I\frac{(\omega - \omega_0)}{t}$

   $$\tau = KI\frac{(\omega - \omega_0)}{t}$$

   But $\frac{\omega - \omega_0}{t} = \alpha$

   $$\tau = KI\alpha$$

   In S.I unit $K = 1$

   $$\tau = I\alpha$$
3. **Third law of rotational motion**

   State that 'To every external torque applied there is equal and opposite restoring torque i.e To every torque there is an equal and opposite torque.

ROTATIONAL KINETIC ENERGY OF RIDID BODY

Definition

SLIPPING – Is the rotation of rigid body without translation.

SLIDDING – is the translation without rotating of the rigid body.

ROLLING – Is the translation plus the rotation of the rigid body.

Rotational kinetic energy of a body is the energy possessed by the body by virtual of rotational motion. Rotational kinetic energy of the rigid body about the given axis of rotation is given by

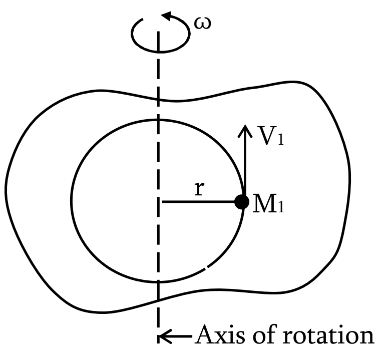
$$K.E = \frac{1}{2}I\omega^2$$

I = M.I of rigid body

ω = Angular velocity about the given axis of rotation.

Derivation : $K.E = \frac{1}{2}I\omega^2$

Consider a rigid body rotating with uniform angular velocity , ω about a given axis.



Let M_1 , $M_2 \cdots\cdots M_n$ be the masses of the different particles constituting on the body where their respectively distances from the axis of rotation are r_1 , r_2 , $r_3 \cdots\cdots r_n$

Let V_1 , $V_2 \cdots\cdots V_n$ be the linear velocities of the particles $V_1 = \omega r_1$, $V_2 = \omega r_2$, $V_n = \omega r_n$

k.e of each particle

$$k.e = \frac{1}{2}MV^2 = \frac{1}{2}M\omega^2 r^2$$

k.e of the first particle

$$k.e_1 = \frac{1}{2}M_1\omega^2 r_1^2$$

k.e of the second particle

$$k.e_2 = M_2\omega^2 r_2^2$$

k.e of the nth particle

$$k.e_n = \frac{1}{2}M_n\omega^2 r_n^2$$

Total k.e of the rotating rigid body about a given axis is equal to the sum of kinetic energies of all the constituent particles about the axis of rotation is given by.

$$K.E = k.e_1 + k.e_2 + \cdots\cdots + k.e_n$$

$$= \frac{1}{2}M_1\omega^2 r_1^2 + M_2\omega^2 r_2^2 + \cdots \frac{1}{2}M\omega^2 r_2^2$$

$$= \frac{1}{2}\omega^2\left[M_1 r_1^2 + M_2 r_2^2 + \cdots\cdots + Mr_n^2\right]$$

$$= \frac{\omega^2}{2}\sum_{i=1}^{n} M_i r_i^2$$

$$K.E = \frac{1}{2}I\omega^2$$

Where : I = Moment of inertia of rigid body

ω = angular velocity of the body about the given axis of rotation.

Relationship between rotational k.e and angular momentum.

$$K.E = E = \frac{1}{2}I\omega^2$$

(Multiply by I both side)

$$I.E = \frac{I^2\omega^2}{2} = \frac{(I\omega)^2}{2}$$

$$I.E = \frac{L^2}{2}$$

$$E = \frac{L^2}{2I}$$

$$\text{Also } L = \sqrt{2IE}$$

Conceptual problem

The moments of inertia of two rotating bodies A and B are IA and IB respectively such that $I_A > I_B$, their momenta are equal . Which of the two has greater rotational kinetic energy.

Answer. $E = \frac{L^2}{2I}$, $E \propto \frac{1}{I}$

Now: $I_A > I_B$, $E_B > E_A$

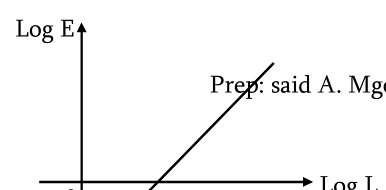
Note that

$$E = \frac{L^2}{2I}$$

$$\log E = \log L^2 - \log(2I)$$

$$\log E = 2\log L - \log(2I)$$

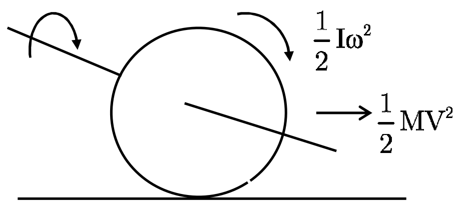
Graph of log E against L



Expression of kinetic energy of the rolling rigid body.

The total kinetic energy of rolling rigid body is equal to the sum of the rotational k.e and translational kinetic energy.

$$K.E = K.E_T + K.E_R$$



$$K.E = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}\left[I\omega^2 + MV^2\right]$$

But : $V = \omega R$

$$K.E = \frac{1}{2}\left[I\omega^2 + M\omega^2R^2\right]$$

$$K.E = \frac{\omega^2}{2}\left[I + MR^2\right]$$

Again: $\omega = \frac{V}{R}$

$$K.E = \frac{1}{2}I\left(\frac{V}{R}\right)^2 + \frac{1}{2}MV^2$$

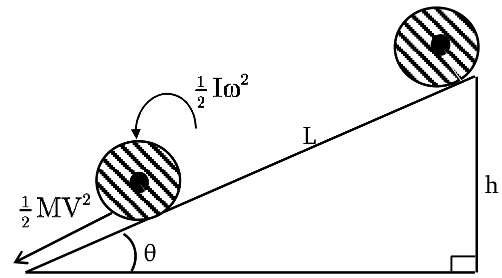
$$K.E = \frac{V^2}{2}\left[\frac{I}{R^2} + M\right]$$

I = M.I of rigid body
M = Mass of rigid body
V = Linear velocity
ω = Angular velocity

ACCELERATION OF A BODY ROLLING DOWN AN INCLINED PLANE.

- From energy consideration

  Consider a body of a mass M and radius R rolling down (without slipping) on an inclined plane of inclination , θ. The gravitational potential energy of the body (resting at the top of the inclined plane) is converted into kinetic energy of translation as well as rotation.



Total energy of rigid body when is at the top of inclined plane.

$$E = Mgh........(i)$$

Total energy of rigid body when reached at the bottom of the inclined plane.

$$E = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$

But : $\omega = \frac{V}{R}$

$$E = \frac{1}{2}MV^2 + \frac{IV^2}{2R^2}$$

$$E = \frac{V^2}{2}\left(M + \frac{I}{R^2}\right).......(ii)$$

Apply the law of conservation of the energy

(i) = (ii)

$$\frac{V^2}{2}\left(M + \frac{I}{R^2}\right) = Mgh$$

$$V^2 = \left(M + \frac{I}{R^2}\right) = 2Mgh$$

$$V^2 = \frac{2Mgh}{M + \frac{I}{R^2}}$$

If the body starts from rest

$$V^2 = 2aL$$

$$2aL = \frac{2MgL\sin\theta}{M + \frac{I}{R^2}}$$

$$a = \frac{Mg\sin\theta}{M + \frac{I}{R^2}}$$

Expression of acceleration of the body in term of radius of gyration.

$$I = MK^2$$

$$a = \frac{Mg\sin\theta}{M + \frac{MK^2}{R^2}}$$

$$a = \frac{g\sin\theta}{1 + \frac{K^2}{R^2}}$$

Expression of the velocity , V of the body when reached at the bottom.

$$V^2 = \frac{2MgL\sin\theta}{M + \frac{I}{R^2}}$$

$$V = \left[\frac{2MgL\sin\theta}{M + \frac{I}{R^2}}\right]^{\frac{1}{2}}$$

Expression of the time taken by the rigid body to reached at the bottom of the inclined plane from the equation.

$$S = ut + \frac{1}{2}at^2$$

But : $S = L$, $U = 0$

$$L = \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2L}{a}} = \sqrt{2L\left[\frac{M + \frac{I}{R^2}}{Mg\sin\theta}\right]}$$

Additional concepts

1. If the two different rigid bodies rolling along the inclined plane, the body which have higher velocity or acceleration is the one which can get or reached to the bottom of the inclined plane first.
2. (i) K.E of rolling body

$$K.E = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$

$$K.E = \frac{M\omega^2}{2}\left(R^2 + K^2\right)$$

(ii) $$\frac{\text{Rotational K.E}}{\text{Translation K.E}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}MV^2} = \frac{MK^2\omega^2}{MR^2\omega^2}$$

$$\frac{\text{Rotatinal K.E}}{\text{Translational K.E}} = \frac{K^2}{R^2}$$

$$\frac{\text{Rotational K.E}}{\text{Total energy of rolling body}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}MV^2 + \frac{1}{2}I\omega^2}$$

$$= \frac{MK^2\omega^2}{MR^2\omega^2 + MK^2\omega^2}$$

$$\frac{K.E_R}{E} = \frac{K^2}{R^2 + \omega^2}$$

(iii) $$\frac{K.E_T}{E} = \frac{\frac{1}{2}MV^2}{\frac{1}{2}MV^2 + \frac{1}{2}I\omega^2}$$

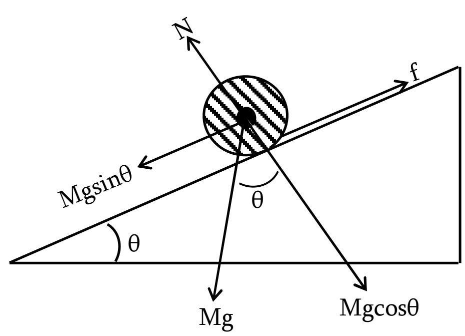
$$= \frac{MV^2}{MV^2 + MK^2V^2/R^2}$$

$$\frac{K.E_T}{E} = \frac{1}{1 + \frac{K^2}{R^2}} = \frac{R^2}{R^2 + K^2}$$

(iv) $$T = \sqrt{2L\left(\frac{I/R^2 + M}{Mg\sin\theta}\right)} = \sqrt{2L\left(\frac{1 + K^2/R^2}{g\sin\theta}\right)}$$

- **Solid cylinder (Rigid body) rolling without slipping on an inclined plane.**

Consider a solid cylinder of mass , M and radius R rolling down a plane inclined at an angle θ with the horizontal. Assume that the cylinder rolls without slipping. The conditions of rolling without slipping is that each instant, the point of contact P (rather than the line of contact) is momentarily at rest and the cylinder is rotating about that axis. The centre of mass of the cylinder moves in a straight line.



Forces available on the cylinder:-

- The weight (Mg) of the cylinder acting vertically downwards through the centre of mass.
- The normal reaction force N of the inclined plane.
- The frictional force , f acting upward and parallel to the inclined plane.

(i) Linear acceleration of the cylinder

Net force acting downward and parallel to the inclined plane

$Ma = Mg\sin\theta - f$ ....(1)

Torque acting on the cylinder due to the friction force.

$$\tau = fR = I\alpha$$

$$f = \frac{I\alpha}{R} = \frac{Ia}{R^2} \text{ ....(2)}$$

Putting equation (2) into (1)

$$Ma = Mg\sin\theta - \frac{Ia}{R^2}$$

$$Ma + \frac{Ia}{R^2} = Mg\sin\theta$$

$$a = \frac{Mg\sin\theta}{M + I/R^2} = \frac{g\sin\theta}{1 + \frac{K^2}{R^2}}$$

Since M.I of solid cylinder about the center

$$I = \frac{MR^2}{2}$$

$$a = \frac{g\sin\theta}{1 + \frac{\frac{1}{2}MR^2}{MR^2}} = \frac{2}{3}g\sin\theta$$

$$a = \frac{2}{3}g\sin\theta$$

This is the expression for the linear acceleration of the solid cylinder clearly a < g.

(ii) Frictional force , f on the cylinder

$$f = Mg\sin\theta - Ma$$

$$= Mg\sin\theta - M\left(\frac{2}{3}g\sin\theta\right)$$

$$f = \frac{1}{3}Mg\sin\theta$$

Clearly that , $f < Mg$

(iii) Conditions for rolling without slipping

The condition for rolling without slipping is $f \le f_{ms}$

$$\frac{1}{3}Mg\sin\theta \le \mu_s N$$

$$\frac{1}{3}Mg\sin\theta \le \mu_s Mg\cos\theta$$

$$\frac{1}{3}\tan\theta \le \mu_s$$

$$\mu_s \ge \frac{1}{3}\tan\theta$$

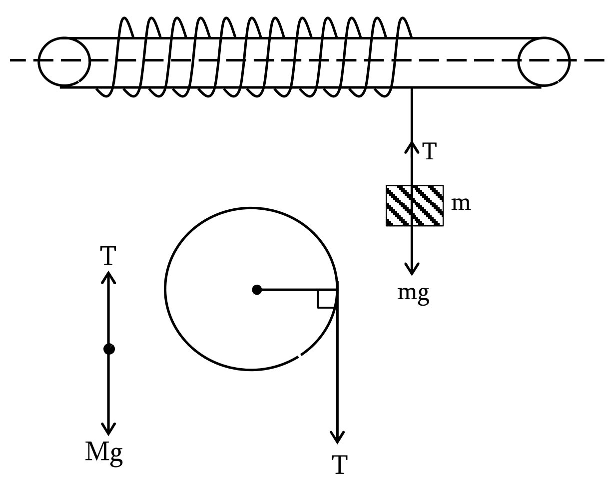
This is the condition for rolling without slipping of a solid cylinder on an inclined plane. Hence , avoid slipping , $\mu_s \ge \frac{1}{3}\tan\theta$ if $\mu_s < \frac{1}{3}\tan\theta$, then the cylinder .

(iv) For maximum value of θ for rolling without slipping

$$\mu_s = \frac{1}{3}\tan\theta \quad \text{or} \quad \theta = \tan^{-1}(3\mu)$$

MOTION OF A POINT MASS TIED TO A STRING WOUND OVER A SOLID CYLINDER.

Consider a solid cylinder of mass M and radius R mounted symmetrically on a horizontal axle so that it is free to rotate about its axis. Let a heavy point mass , M hang from a string would over the cylinder



(i) Acceleration , a of the point mass

$$ma = mg - T \text{ ....(1)}$$

If I be the moment of inertia of the cylinder about its axis and α the angular acceleration.

$$\tau = I\alpha = TR$$

$$TR = \frac{Ia}{R} \text{ or } T = \frac{Ia}{R^2}$$

Equation (1) becomes

$$ma = mg - \frac{Ia}{R^2}$$

$$ma + \frac{Ia}{R^2} = mg$$

$$a = \frac{mg}{m + \frac{I}{R^2}} = \frac{g}{1 + \frac{I}{mR^2}}$$

This is expression for a linear downward acceleration of the point of mass clearly , a < g.

(ii) Angular acceleration α of the cylinder

$$\alpha = \frac{a}{R} = \frac{g/R}{I + \frac{I}{mR^2}}$$

(iii) Tension T in the string

$$T = \frac{Ia}{R^2}$$

$$T = \frac{I}{R^2}\left[\frac{g}{1 + I/mR^2}\right]$$

$$T = \frac{mg}{1 + \frac{mR^2}{I}}$$

If M is the mass of the cylinder

$$I = \frac{1}{2}MR^2 \quad , \quad \frac{R^2}{I} = \frac{2}{M}$$

$$T = \frac{mg}{1 + \frac{m2}{M}} = \frac{g}{1 + \frac{M}{2m}}$$

$$T = \frac{g}{1 + \frac{M}{2m}}$$

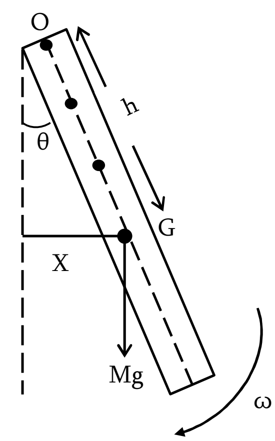
Also : $T = m(g - a)$

$$T = m\left(g - \frac{g}{1 + \frac{M}{2m}}\right)$$

$$T = mg\left[1 - \frac{1}{1 + \frac{M}{2m}}\right]$$

THE COMPOUND PENDULUM

The simple pendulum theoretically has the mass of the bob concentrated at one point , but this is impossible to achieve exactly in practice. Most pendulum are compound with an oscillating mass spread out over a definite volume of space. Consider a rigid object oscillating about a fixed horizontal axis O near one end. Let G be the centre of gravity of a compound pendulum of mass , m that oscillates about a point , O with OG = h



If the pendulum is moved so that the line OG is displaced through an angle θ then restoring couple

$$\tau = -Mgx$$

$$x = h\sin\theta$$

$$\tau = -Mg\sin\theta$$

If θ is very small angle $\sin\theta \simeq \theta$

$$\tau = -Mgh\theta \quad \text{but} \quad \tau = \frac{Id^2\theta}{dt}$$

$$I\frac{d^2\theta}{dt^2} = -Mgh\theta$$

$$\frac{d^2\theta}{dt^2} = -\left(\frac{Mgh}{I}\right)\theta$$

$$\frac{d^2\theta}{dt^2}\alpha - \theta$$

This implies angular S.H.M .

For angular S.H.M

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta$$

$$-\omega^2\theta = -\left(\frac{Mgh}{I}\right)\theta$$

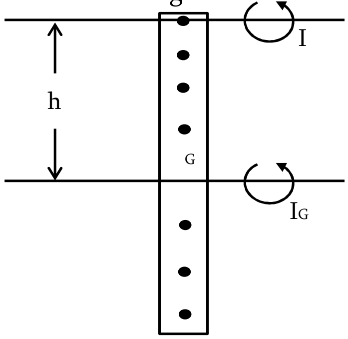
$$\omega = \sqrt{\frac{Mgh}{I}}$$

Periodic time of oscillation

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{Mgh}}$$

Note that:

Consider the figure below



Applying parallel axes theorem

$$I = I_G + Mh^2 \quad \text{but } I_G = MK^2$$

$$= MK^2 + Mh^2$$

$$I = M\left(K^2 + h^2\right)$$

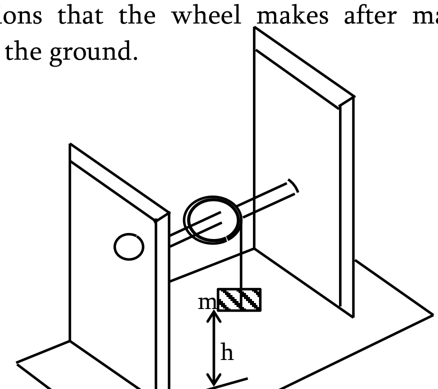
Now $T = 2\pi\sqrt{\frac{I}{Mgh}} = 2\pi\sqrt{\frac{M\left(K^2+h^2\right)}{Mgh}}$

$$T = 2\pi\sqrt{\frac{k^2+h^2}{gh}}$$

DETERMINATION OF MOMENT O INERTIA OF A FLYWHEEL

Consider a block of mass , M hanging on a frictional wheel that is free to rotate about its axle as shown in the figure below.

Let f be frictional energy per revolution experienced by the wheel. n_1 be number of revolutions that the wheel makes before mass , m reaches the ground and n_2 be number of revolutions that the wheel makes after mass m reaches the ground.



Before the mass , m reaches the ground

Apply the conservation of energy

p.e of mass , m = k.e of wheel + k.e mass , m + frictional energy.

$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 + n_1 f \text{ ....(i)}$$

After the mass , m reaches the ground

k.e of wheel = frictional energy

$$\frac{1}{2}I\omega^2 = n_2 f$$

$$f = \frac{I\omega^2}{2n_2} \text{.......(ii)}$$

Putting equation (ii) into (i)

$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 + \frac{n_1 I\omega^2}{2n_2}$$

$$V = \omega r$$

$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}m\omega^2 r^2 + \frac{n_1 I\omega^2}{2n_2}$$

$$\omega = \sqrt{\frac{2mgh}{I + mr^2 + \frac{n_1 I}{n_2}}}$$

$$V = r\sqrt{\frac{2mgh}{mr^2 + I\left(1 + {}^{n_1}\!/\!_{n_2}\right)}}$$

Again $\quad 2mgh = I\omega^2 + m\omega^2 r^2 + \frac{n_1 I\omega^2}{n_2}$

$$2mgh - m\omega^2 r^2 = I\left[\omega^2 + \frac{\omega^2 n_1}{n_2}\right]$$

$$I = \frac{2mgh - m\omega^2 r^2}{\omega^2\left(1 + {}^{n_1}\!/\!_{n_2}\right)}$$

$$I = \frac{m\left(2gh - \omega^2 r^2\right)}{\omega^2\left(1 + {}^{n_1}\!/\!_{n_2}\right)}$$

about an axis through the centre of gravity and perpendicular to the plane of the system.

The table below summaries the analogy between linear and angular quantities

S.No	**Linear quantity or equation**	**Angular quantity or equation**	**Relation between linear and angular quantities**
1.	Position x	Angular position θ	
2.	Linear velocity v	Angular velocity ω	$v = \omega r$
3.	Linear acceleration a	Angular acceleration α	$a = r\alpha$
4.	Mass m	Moment of inertia I	$I = \int r^2 dm$
5.	Force $\vec{F}$	Torque $\vec{\tau}$	$\vec{\tau} = \vec{r} \times \vec{F}$
6.	Newton's second law for linear motion F = ma	Newton's second law for rotational motion $\tau = I\alpha$	
7.	Work = $\vec{F} \cdot \vec{S}$	Work = $\vec{\tau} \cdot \overrightarrow{d\theta}$	
8.	Power = $\vec{F} \cdot \vec{V}$	Power = $\vec{\tau} \cdot \vec{\omega}$	
9.	$K.E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$	$K.E = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$	
10.	Linear momentum $\vec{p} = m\vec{v}$	Angular momentum $\vec{L} = I\vec{\omega}$	$\vec{L} = \vec{r} \times \vec{p}$
11.	Newton's second law in terms of linear momentum $\vec{F} = \frac{\overrightarrow{dp}}{dt}$	Newton's second law in terms of angular momentum $\vec{\tau} = \frac{\overrightarrow{dL}}{dt}$	
12.	Conservation of linear momentum on collision if no external force acts.	Conservation of angular momentum on collision if no external torque acts.	
13.	Equations of motion $s = \left(\frac{v_0 + v}{2}\right)t$ $v = v_0 + at$ $s = v_0^t + \frac{1}{2}at^2$ $v^2 = v_0^2 + 2as$	Equation of motion $\theta = \left(\frac{\omega_0 + \omega}{2}\right)t$ $\omega = \omega_0 + at$ $\theta = \omega_0^t + \frac{1}{2}at^2$ $\omega^2 = \omega_0^2 + 2\alpha\theta$	

Example – 14

(a) (i) Why is moment of inertia called rotational inertia?

(ii) Give the physical significance of moment of inertia.

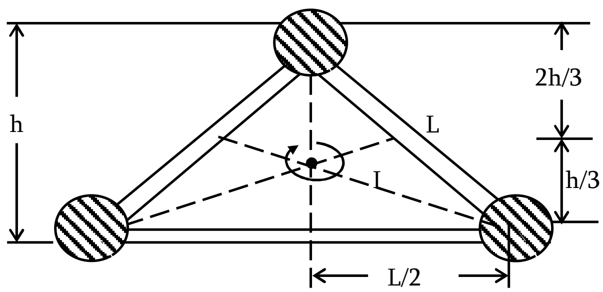
(b) Calculate moment of inertia of system made by connecting three spherical balls of mass, M using rods of length , L if the system rotates

Solution

(a) (i) The moment of inertia is called rotational inertia for the reason that it gives the measure of inertia of a body during its rotational motion.

(ii) The moment of inertia of body plays the same role in rotational motion as its mass does in linear motion.

(b) Let I = Moment of inertia of the system.



By using Pythagoras theorem

$$h = \sqrt{L^2 - \frac{L^2}{4}} = \sqrt{L^2 - \left(\frac{L}{2}\right)^2}$$

$$h = \frac{L\sqrt{3}}{2}$$

Since $r_1 = r_2 = r_3 = r = \frac{2h}{3} = \frac{2}{3} \cdot \frac{L\sqrt{3}}{2}$

$$r = \frac{L\sqrt{3}}{3}$$

Now $I = M_1r_1^2 + M_2r_2^2 + M_3r_3^2$

$$= Mr^2 + Mr^2 + Mr^2$$

$$= 3Mr^2 = 3M\left(\frac{L\sqrt{3}}{3}\right)^2$$

$$I = ML^2$$

Example – 15

If the radius of a sphere is 5cm. calculate the radius of gyration.

(i) About its diameter

(ii) About its tangent.

Solution

(i) M.I of the sphere about its diameter.

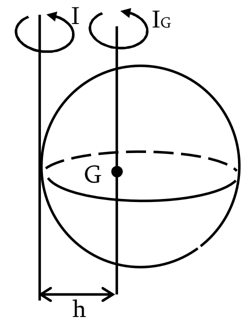
$$I_G = \frac{2}{5}MR^2 \quad \text{but} \quad I_G = MK^2$$

$$MK^2 = \frac{2}{5}MR^2$$

$$K = R\cdot\sqrt{\frac{2}{5}} = 5\times\sqrt{\frac{2}{5}}$$

$$K = 3.162\text{cm}$$

(ii) Apply parallel axes theorem



$$I = I_G + Mh^2$$

$$= \frac{2}{5}MR^2 + MR^2$$

$$I = \frac{7}{5}MR^2$$

But $I = MK^2$

$$MK^2 = \frac{7}{5}MR^2$$

$$K = R\cdot\sqrt{\frac{7}{5}} = 5\times\sqrt{\frac{7}{5}}$$

$$K = 5.92\text{cm}$$

Example – 16

(a) If no external torque acts on a body , will its angular velocity remain conserved?

(b) A wheel is rotating at 90revmin^{-1}, What is the torque required to bring it to rest in 5 revolutions if the moment of inertia of the wheel is 0.80kgm^2.

Solution

(a) If no external torque acts ($\tau = 0$) then angular momentum of the body will remain conserved i.e $I\omega$ = constant as in the rotational motion , the moment of inertia (I) of the body can change due to the change in position of the axis of rotation, the angular speed may not remain conserved. However , if the position of the axis of rotation also remains fixed , the angular speed will remain conserved.

(b) $\omega_0 = 2\pi f_0 = \frac{2\pi\times 90}{60} = 3\pi rads^{-}$

$$1\text{ rev} \longrightarrow 2\pi$$

$$5\text{ rev} \longrightarrow \theta$$

$$\theta = \frac{5\text{rev}\times 2\pi}{1\text{rev}}$$

$$\theta = 10\pi\text{rad}$$

Now $\omega^2 = \omega_0^2 + 2\alpha\theta$

The wheel will comes to rest , $\omega = 0$

$$0^2 = \omega_0^2 + 2\alpha\theta$$

$$\alpha = \frac{-\omega_0^2}{2\theta} = \frac{-(3\pi)^2}{2\times 10\pi}$$

$$\alpha = -0.45\pi rads^{-2}$$

Negative sign show that the flywheel is showing down.

$$\tau = I\alpha$$

$$= 0.80\times(-0.45\pi)$$

$$\tau = -0.36\pi Nm$$

Negative sign show that the torque is applied in the opposite direction to that of the original rotation.

Example – 17

(a) If no external torque acts on a body , will its angular velocity remain constant? Explain.

(b) A flywheel rotates on a bearing which exerts a constant frictional torque of 12Nm an external torque of 36Nm acts on the flywheel for a time of 15sec, after which the time is removed. If the angular velocity of the flywheel increases from zero to 60rads^{-1} in 15second period.

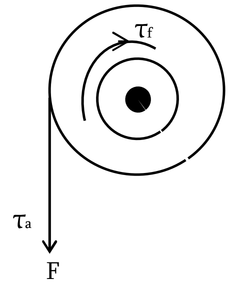
(i) Calculate the moment of inertia of the flywheel.

(ii) Find at what time the flywheel will come to rest.

Solution

(a) If no external torque acts on body, angular momentum (L) of the body will remain constant. Now $L = I\omega$ therefore , angular velocity will remain constant , only if the moment of inertia of the body remains constant.

(b) (i)



Resultant torque

$$\tau = \tau_a - \tau_f$$

τ_a = applied or external torque

τ_f = frictional torque

$$\tau = 36 - 12 = 24Nm$$

$$\tau = 24Nm$$

Since $\omega_0 = 0$

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{60-0}{15}$$

$$\alpha = 4rads^{-2}$$

Again $\tau = I\alpha$, $I = \frac{\tau}{\alpha}$

$$I = \frac{24}{4}$$

$$I = 6.0kgm^2$$

(ii) When the external torque is removed the flywheel shows down and net torque τ_1 of -12Nm now acts on it due to friction.

$$\alpha_1 = \frac{\tau_1}{I} = \frac{-12}{6} = -2rads^{-2}$$

$$\omega_0 = 60rads^{-2} \quad , \quad \omega = 0$$

Finally the flywheel comes to rest($\omega = 0$)

$$t_1 = \frac{\omega - \omega_1}{t} = \frac{0-60}{-2}$$

$$t_1 = 30\,sec$$

$\therefore$ After $t_1 = 30$ second the external torque is removed.

Example – 18

(a) (i) Define radius of gyration of a body.

(ii) Find the radius of gyration of a solid sphere of a diameter 2.0m rotating about a diameter as an axis.

(b) A wheel of radius 0.72m and moment of inertia 4.8kgm^2 has a constant force of 10N applied tangentially at the rim calculate.

(i) Angular acceleration

(ii) The angular speed after 4.0seconds from rest

(iii) The number of revolutions made in 4.0 seconds.

(c) The maximum and minimum distance of a comment from the sun are 1.40×10^{12}m and 7.0×10^{10}m. If its velocity nearest to the sun is 6.0×10^4m/s. Find its velocity in the furthest position from the sun. State assumptions made in your calculations.

Solution

(a) (i) Refer to your notes

(ii) M.I of a sphere

$$I_G = MK^2 = \frac{2}{5}MR^2$$

$$K = \sqrt{2/5}\,R = 1.0m \times \sqrt{2/5}$$

$$K = 0.63m$$

(b) (i) Torque on the wheel

$$\tau = I\alpha = FR$$

$$\alpha = \frac{FR}{I} = \frac{10 \times 0.72}{4.8}$$

$$\alpha = 1.5rads^{-2}$$

(ii) $\omega = \omega_0 + \alpha t = 0 + 1.5 \times 4$

$$\omega = 6.0rads^{-1}$$

(iii) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$$= 0 \times 4 + \frac{1}{2} \times 1.5 \times 4^2$$

$$\theta = 12rad$$

$$N = \frac{\theta}{2\pi} = \frac{12}{2\pi}$$

$$N = 1.91 \simeq 2 \text{ revolution}$$

(c) Apply the principle of conservation of angular momentum.

$$MV_1r_1 = MV_2r_2$$

$$V_1 = \frac{V_2r_2}{r_1} = \frac{6 \times 10^4 \times 7 \times 10^{10}}{1.4 \times 10^{12}}$$

$$V_1 = 3.0 \times 10^3 m/s$$

Example – 19

(a) What do you understand by the term moment of inertia of a rigid body?

(b) (i) State the perpendicular axes theorem of moment of inertia for a body in the form of lamina.

(ii) Calculate the moment of inertia of a thin circular disc of radius 50cm and mass 2kg about an axis along a diameter of the disc.

(c) A wheel mounted on an axle that is not frictionless is initially at rest. A constant external torque of 50Nm is applied to the wheel for 20second. At the end of the 20sec, the wheel has an angular velocity of 600revmim-1. The external torque is the n removed and the wheel comes to rest after 120sec more.

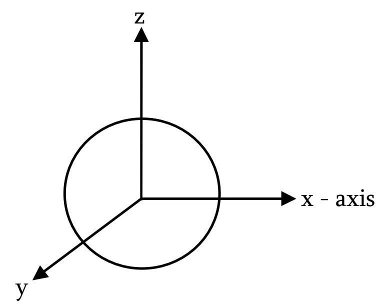
(i) Determine the moment of inertia of the wheel

(ii) Calculate the frictional torque which is assumed to be constant.

Solution

(a) Refer to your notes

(b) (ii)



Apply perpendicular axes theorem

$$I_Z = I_X + I_y$$

Due to the symmetry property.

$$I_X = I_y$$

$$2I_X = 2I_y = I_Z$$

$$I_X = I_y = \frac{1}{2}I_Z = \frac{1}{2}I_G$$

$$I_X = I_y = \frac{1}{2}\left(\frac{1}{2}MR^2\right) = \frac{1}{4}MR^2$$

$$= 0.25 \times 2 \times (0.5)^2$$

$$I_X = I_y = 0.125kgm^2$$

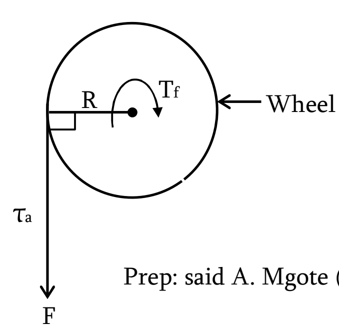
(c) $\tau_a = 50Nm$, $t_1 = 20\,sec$

$$\omega = \frac{6000 \times 2\pi}{60} = 20\pi rads^{-1}$$

$$t_2 = 120\,sec$$

$$\tau_f = \text{frictional torque}$$

(i) Before the external torque is removed.



Net to torque $\tau = \tau_a - \tau_f = I\alpha$

$$\alpha = \frac{\tau_a - \tau_f}{I}$$

$$\frac{\omega}{t_1} = \frac{50 - \tau_f}{I}$$

$$\frac{20\pi}{20} = \frac{50 - \tau_f}{I}$$

$$\pi = \frac{50 - \tau_f}{I} \text{..........(i)}$$

When the external torque is removed

$$\omega = 0,\ \tau_a = 0\ ,\ \omega_0 = 20\pi \text{rads}^{-1}$$

$$\alpha = \frac{\omega - \omega_0}{t_2} = \frac{-\omega_0}{t_2}$$

$$\frac{-I_f}{I} = \frac{-20\pi}{120}$$

$$\frac{\pi}{6} = \frac{\tau_f}{I} \text{...............(ii)}$$

On solving equations (1) and (2)

(ii) $\tau_f = \dfrac{\pi I}{6} = 7.14$

$$\tau_f = 7.14\text{Nm}$$

Example – 20

(a) Why are spokes fitted in the cycle wheel?

(b) If two circular disc of different radii and the same weight and thickness are made from metals of different densities , which disc will have the larger moment of inertia about the central axis?

(c) The moment of inertia of two rotating bodies A and B about the same axis are I_A and I_B ($I_A > I_B$) and their angular momenta are equal which has greater rotational kinetic energy?

Solution

(a) The spokes of cycle wheel increase its moment of inertia , the greater is the opposition to any change in uniform rotational motion. As a result , the cycle runs smoother and steadier. If the cycle wheels had no spokes , the cycle would be driven in jerks and hence unsafe.

(b) $I_1 = \dfrac{1}{2}MR_1^2\ ,\ I_2 = \dfrac{1}{2}MR_2^2$

$$\frac{I_1}{I_2} = \frac{R_1^2}{R_2^2}$$

Now; $M = \pi R_1^2 t\rho_1\ ;\ R_1^2 = \dfrac{M}{\pi t\rho_1}$

$$R_2^2 = \frac{M}{\pi t\rho_2}$$

$$\frac{R_1^2}{R_2^2} = \frac{\rho_2}{\rho_1}$$

Now; $\dfrac{I_1}{I_2} = \dfrac{\rho_2}{\rho_1}\ ,\ I \propto \dfrac{1}{\rho}$

Therefore the disc which have greater density have smaller rotational inertia.

(c) Rotational K.E $= \dfrac{1}{2}I\omega^2$

$$\text{K.E} = \frac{1}{2}I\frac{L^2}{I^2} = \frac{L^2}{2I}$$

Let K_A and K_B are the rotational k.e of A and B respectively. Since the angular momentum of the two bodies are the same.

$$\frac{K.A}{K_B} = \frac{L^2}{2I_A}\bigg/\frac{L^2}{2I_B} = \frac{I_B}{I_A}$$

Since $I_A > I_B$, then $K_B > K_A$

Example – 21

(a) Explain each of the following in terms of rotational dynamics:-

(i) It is easier to loosen a nut on the bolt using a long spanner than using a short one.

(ii) An ice – skater spins more easily while her arms folded than when her arms stretched.

(iii) In hand – driven grinding machine , handle is put near the circumference of the wheel or stone.

(b) The radius of gyration of a body about an axis at a distance 8cm from its centre of mass is 12cm. Find its radius of gyration about a parallel axis through its centre of mass.

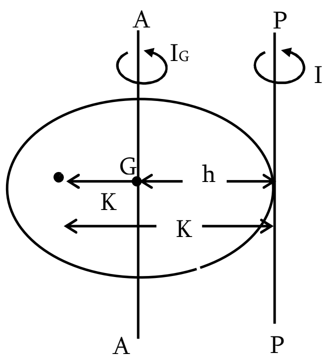
Solution

(a) (i) It is easier to loosen a nut on the bolt using a long spanner than using a short one because with a long spanner more torque is exerted to rotate the nut by applying a small force , while with a short spanner move force is required to produce the needed torque to rotate the nut.

(ii) With airms folded more mass is brought close to the axis of rotation , hence the moment of inertia is small but the angular velocity is large on the other hand , if her arms are outstretched some mass is displaced away from the axis of rotation and the moment of inertia is large but angular velocity is small. The angular momentum is conserved in each case.

(iii) For a given force , torqued can be increased if the perpendicular distance of the point of application of force from the axis of rotation is increased. This explain as to why handle is put near the circumference of the wheel or stone.

(b)



M.I about an axis PP[1] = I = MK²

M.I of the body through a parallel axis through its centre of mass $I_G = MK_1^2$

Apply parallel axes theorem

$$I = I_G + Mh^2$$
$$MK^2 = MK_1^2 + Mh^2$$
$$K_1 = \sqrt{K^2 - h^2} = \sqrt{12^2 - 8^2}$$
$$K_1 = 4\sqrt{5}\text{cm}$$

Example – 22

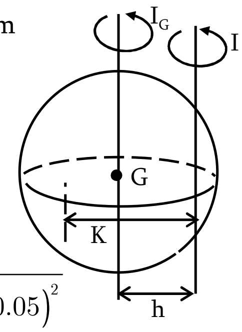
(a) How a swimmer jumping from a height is able to increase the number of loops mad in the air?

(b) Find the radius of gyration for a sphere of radius 15cm without an axis 5cm from the centre. (Given that $I_G = 0.4Mr^2$).

Solution

(a) The swimmer can increase the number of loops in air by pulling his arms and legs in wards i.e by decreasing the moment of inertia by doing so the angular velocity ω increases because angular momentum (L = Iω) remain constant.

(b) Apply parallel axes theorem



$$I = I_G + Mh^2$$
$$I = 0.4Mr^2 + Mh^2$$
$$I = MK^2$$
$$MK^2 = M\left(0.4r^2 + h^2\right)$$
$$K = \sqrt{0.4r^2 + h^2}$$
$$= \sqrt{0.4(0.15)^2 + (0.05)^2}$$
$$K = 0.11\text{m}$$

Example – 23

Two rings of equal mass and thickness but of different materials are acted upon by the same torque about an axis passing through their centre and perpendicular to the plane of the rings. If the radii of the rings are in ratio 1:4, find the ratio of their angular accelerations.

Solution

Let R_1 and R_2 be the radii of the two rings , each having mass , M.

$$I_1 = MR_1^2 \ , \ I_2 = MR_2^2$$
$$\frac{I_1}{I_2} = \frac{MR_1^2}{MR_2^2} = \left(\frac{R_1}{R_2}\right)^2 = \left(\frac{1}{4}\right)^2$$
$$\frac{I_1}{I_2} = \frac{1}{16}$$

If α_1 and α_2 are the angular accelerations produced in them, due to the torque τ acting on them.

$$\tau = I_1\alpha_1 = I_2\alpha_2$$
$$\frac{\alpha_1}{\alpha_2} = \frac{I_2}{I_1} = 16$$
$$\frac{\alpha_1}{\alpha_2} = 16:1$$

Example – 24

A grind stone has a moment of inertia of $1.6 \times 10^{-3}kgm^{-2}$ when a constant torque is applied, the flywheel reaches an angular velocity of $1200revmin^{-1}$ in 15seconds. Assuming its started from rest, find:-

(i) The angular acceleration
(ii) The torque applied
(iii) The angular turned
(iv) The work done on the flywheel by the torque.

Solution

$$\omega_0 = 0 \ , \ \omega = \frac{2\pi\times1200}{60} = 40\pi rads$$

(i) $$\alpha = \frac{\omega-\omega_0}{t} = \frac{40\pi}{15}$$

$$\alpha = 8.38rads^{-2}$$

(ii) $$\tau = I\alpha = 1.6\times10^{-3}\times8.38$$

$$\tau = 0.0134Nm$$

(iii) $$\omega = \left(\frac{\omega_0+\omega}{2}\right)$$

$$\theta = \left(\frac{\omega_0+\omega}{2}\right)t = \left(\frac{0+40\pi}{2}\right)\times15$$

$$\theta = 300\pi rad$$

(iv) $$w = \tau\theta = 0.0134\times300\pi$$

$$w = 12.6J$$

Example – 25

A flywheel on a motor increase its rate of rotation uniformly from $120revmin^{-1}$ to $300revmin^{-1}$ in 10seconds. Calculate:

(a) It angular acceleration
(b) Its angular displacement in this time.

Solution

$$\omega_0 = 2\pi f_0 = \frac{2\pi\times120}{60} = 4\pi rads^{-1}$$

$$\omega = 2\pi f = \frac{2\pi\times300}{60} = 10\pi rads^{-1}$$

(a) $$\alpha = \frac{\omega-\omega_0}{t} = \frac{10\pi-4\pi}{10}$$

$$\alpha = 0.6\pi rads^{-2}$$

(b) $$\theta = \left(\frac{\omega+\omega_0}{2}\right)t = \left(\frac{4\pi+10\pi}{2}\right)\times10$$

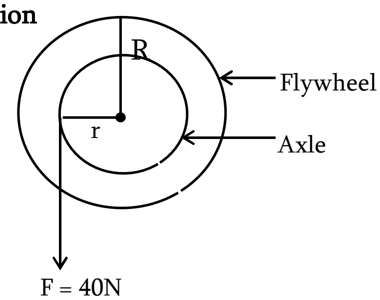
$$\theta = 70rad$$

Example – 26

A heavy flywheel of mass 15kg and radius 0.2m is mounted on a horizontal axle of radius 0.01m and negligible mass compaired with flywheel neglecting friction, find:-

(i) The angular acceleration if a force of 40N is applied tangentially to the axle.
(ii) The angular velocity of the flywheel after 10second

Solution



(i) M.I of flywheel $$I = \frac{1}{2}MR^2$$

$$\tau = I\alpha = \frac{MR^2\alpha}{2} = Fr$$

$$\alpha = \frac{2Fr}{MR^2} = \frac{2\times40\times0.01}{15\times0.2^2}$$

$$\alpha = 1.3rads^{-2}$$

(ii) $$\omega = \omega_0 + \alpha t$$

$$= 0 + 1.3\times10$$

$$\omega = 13rads^{-1}$$

Example – 27

The moment of inertia of a solid flywheel about its axis is 0.1kgm2. It is set in rotation by applying a tangential force of 20N with a rope wound round the circumference of the radius of the wheel being 0.1m

(i) Calculate the angular acceleration of the flywheel.
(ii) What would be the acceleration if a mass of 2kg were hung from the end of the rope.

Solution

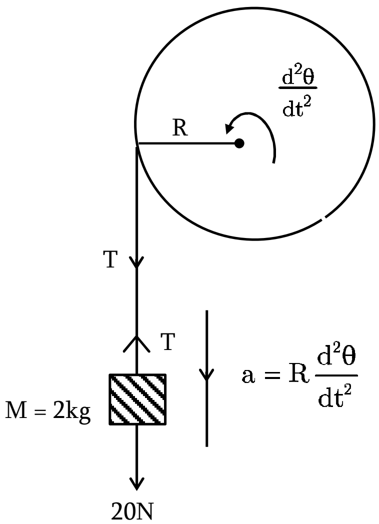
(i) Torque on the flywheel.

$$\tau = FR = I\frac{d^2\theta}{dt^2}$$

$$\alpha = \frac{d^2\theta}{dt^2} = \frac{FR}{I} = \frac{20\times 0.1}{0.1}$$

$$\alpha = 20\text{rads}^{-2}$$

(ii) If a mass M of 2kg hung from the end of the rope ,its move down with acceleration , a.



Resultant force on the mass

$$Mg - T = Ma \text{ ....(1)}$$

Torque on the flywheel

$$\tau = TR = I\frac{d^2\theta}{dt^2} \text{ ....(2)}$$

Linear acceleration

$$a = \alpha R = R\frac{d^2\theta}{dt^2}$$

Now , equation (i) becomes

$$Mg - T = MR\frac{d^2\theta}{dt^2}$$

(Multiply by R both side)

$$MgR - TR = MR^2\frac{d^2\theta}{dt^2} \text{ ....(3)}$$

Putting equation (2) into (3)

$$MgR - I\frac{d^2\theta}{dt^2} = MR^2\frac{d^2\theta}{dt^2}$$

$$MgR = \left(I + MR^2\right)\frac{d^2\theta}{dt^2}$$

$$\alpha = \frac{d^2\theta}{dt^2} = \frac{MgR}{I + MR^2}$$

$$= \frac{2\times 10\times 0.1}{0.1 + 2\left(0.1\right)^2}$$

$$\alpha = 1.6.7\text{rads}^{-2}$$

Since $a = \alpha R$

$$= 16.7\times 0.1$$

$$a = 1.67\text{m/s}^2$$

Example – 28

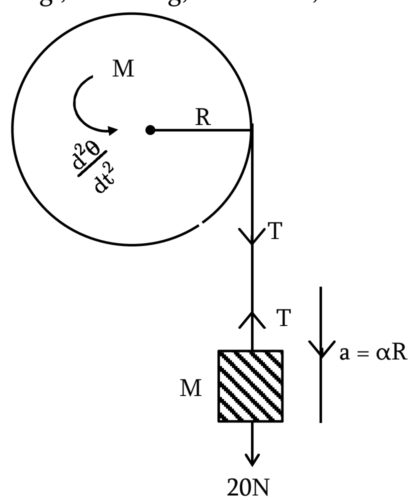
A 3kg block hang from a string wound on 40kg wheel. The wheel has a radius of 0.75m and radius of gyration of 0.6m. Find:-

(i) The angular acceleration of the wheel

(ii) The distance the blocks falls in first 10seconds.

Solution

m = 3kg , M = 40kg, R = 0.75m, K = 0.6m



(i) Let α = angular acceleration

M.I of the flywheel , I = MK^2

Resultant force on the block

$$ma = mg - T$$

Torque $\tau = TR = I\alpha$

$$T = \frac{I\alpha}{R} \quad \text{but} \quad \alpha = {}^{a}\!/\!_{R}$$

$$T = \frac{Ia}{R^2}$$

$$ma = mg - \frac{Ia}{R^2}$$

$$a\left(m + {}^{I}\!/\!_{R^2}\right) = mg$$

$$a = \frac{mg}{m + \frac{I}{R^2}} = \frac{mg}{m + \frac{MK^2}{R^2}}$$

$$a = \frac{3\times 9.8}{3+\frac{40(0.6)^2}{(0.75)^2}} = 1.03\text{m/s}^2$$

$$\alpha = \frac{a}{R} = \frac{1.03}{0.75}$$

$$\alpha = 1.37\text{rads}^{-2}$$

(ii) $S = ut + \frac{1}{2}at^2 \quad u = 0$

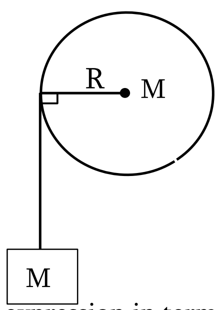
$$= \frac{1}{2}\times 1.03\times 10^2$$

$$S = 51.5\text{m}$$

Example – 29

(a) Define 'Angular momentum' and state a mathematical relation between angular momentum (L) of a body and its moment of inertia (I)

(b) A pulley of mass M and radius R mounted on an axle is free rotate about an axis through its centre and perpendicular to its plane (see figure below). A light cord is wrapped around the rim of the wheel and mass, M is suspended from the end of the cord.

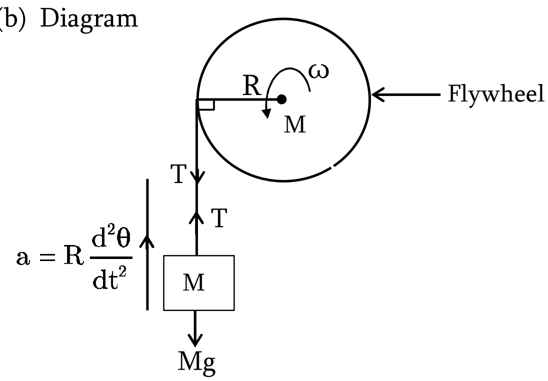


Find an expression in terms of M, m and R for
(i) The angular acceleration of the disc.
(ii) Tension in the cord.

Solution

(a) Refer to your notes

(b) Diagram



(i) Resultant force on the block

$$mg - T = ma \ldots\ldots\ldots\ldots(1)$$

Torque on flywheel

$$\tau = FR = I\alpha \quad \text{but } I = \frac{1}{2}MR^2$$

$$FR = \frac{1}{2}MR^2\alpha \ , \quad F = T$$

$$TR = \frac{1}{2}MR^2\alpha \qquad 2T = MR\alpha \qquad T = \frac{1}{2}MR\alpha$$

$$a = \alpha R$$

From equation (1)

$$mg - T = mR\alpha$$

$$mg - \frac{1}{2}MR\alpha = mR\alpha$$

$$mg = \left(mR + \frac{1}{2}MR\right)\alpha$$

$$\alpha = \frac{2mg}{(2m+M)R}$$

(ii) Tension on the cord

$$T = \frac{1}{2}MR\alpha$$

$$T = \frac{Mmg}{M+2m}$$

Example – 30

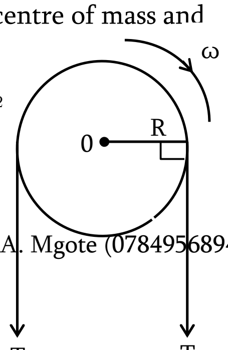
A flywheel of mass 50kg is made in form of a circular disc of radius 10cm and is driven by the belt whose tensions at the points where runs on and off the rim of the wheel are 20N and 49N respectively. If the wheel is rotating at a certain speed of 180revolutions per minute, find how long it will be before the speed has reached 720revolutions per minute while the flywheel is rotating at this later speed , the belt slip off and break applied. Find the constant working couple required to stop the wheel in 12second.

Solution

Case 1:

The flywheel is of the form of the disc , so as its M.I about an axis through its centre of mass and perpendicular to the plane.

$$I = \frac{1}{2}MR^2 = \frac{1}{2}\times 50\times 0.1^2$$



$$I = 0.25kgm^2$$

Resultant tension on the string

$$T = T_2 - T_1 = 49 - 20$$

$$T = 29N$$

The resultant tension in the string produces torque in the wheel.

Torque $\tau = TR = I\alpha$

$$\alpha = \frac{IR}{I} = \frac{29\times0.1}{0.25}$$

$$\alpha = 11.6rads^{-2}$$

$$\omega = 2\pi f = 2\pi\times\frac{720}{60} = 24\pi rads^{-1}$$

Since $\alpha = \dfrac{\omega - \omega_0}{t}$, $t = \dfrac{\omega-\omega_0}{\alpha}$

$$t = \frac{24\pi - 6\pi}{11.6} = 4.87\,sec$$

∴ It will take 4.87sec before the speed reached 720revolutions per minute.

Case 2: during deceleration

$$\omega_0 = 2\pi f = 2\pi\times\frac{720}{60} = 24\pi rads^{-1}$$

$$\omega = 0$$

$$\alpha = \frac{\omega-\omega_0}{t} = \frac{0-24\pi}{12}$$

$$\alpha = -2\pi rads^{-2}$$

Opposing torque $\tau = -I\alpha$

$$\tau = -\left(-2\pi\times0.25\right)$$

$$\tau = 1.57Nm$$

Example – 31

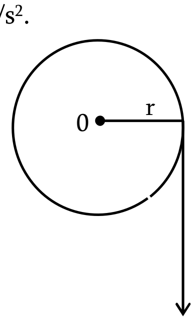
A solid cylinder of mass 50kg and radius 0.5m is free to rotate about its axis which is horizontal. A string is wound round the cylinder with one end attached to its and the hanging freely. Find the tension in the string required to produce an angular acceleration of 2rev/s².

Solution

Torque , $\tau = Tr = I\alpha$

$$T = \frac{I\alpha}{r}$$

But $I = \dfrac{1}{2}Mr^2$



$$T = \frac{1}{2}Mr^2\cdot\frac{\alpha}{r}$$

$$= \frac{1}{2}Mr\alpha = \frac{1}{2}\times50\times0.5\times2\times3.14\times2$$

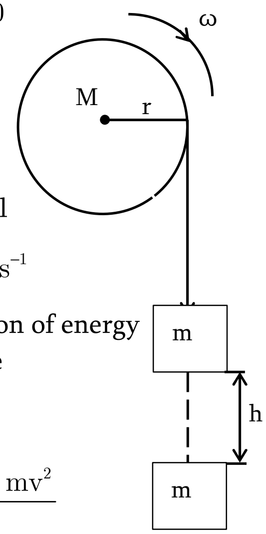
$$T = 157N$$

Example – 32

A mass of 0.5kg hangs from the rim of a wheel of radius 0.2m by a light string which is wound round the wheel. When released from rest the mass falls through a distance of 5m in 10seconds , rotating the wheel. Find the moment of inertia of the wheel. (g = 10m/s²).

Solution

Let V be the velocity after falling through 5m



$$h = \left(\frac{v+u}{2}\right)t \text{ but } u = 0$$

$$5 = \left(\frac{0+v}{2}\right)\times10$$

$$V = 1m/s$$

Angular velocity of flywheel

$$\omega = \frac{V}{r} = \frac{1}{0.2} = 5rads^{-1}$$

Apply the law of conservation of energy

Gain in K.e = loss in p.e

$$\frac{1}{2}I\omega^2 + \frac{1}{2}MV^2 = mgh$$

$$I = \frac{2mgh - mv^2}{\omega^2}$$

$$= \frac{2\times0.5\times10\times5 - 0.5\times1^2}{5^2}$$

$$I = 1.98kgm^2$$

Example – 33

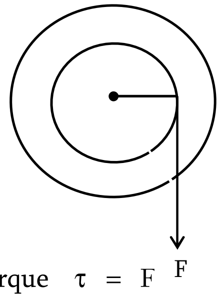
A flywheel is mounted on a horizontal axle which has a radius of 0.06m. A constant force of 50N is applied tangentially to the axle. If the moment of inertia of system (flywheel and axle) is 4kgm². Calculate:

(a) The angular acceleration of the flywheel

(b) The number of revolutions that the flywheel makes in 16seconds assuming that its starts from rest.

Solution

I = 4kgm², F = 50N, r = 0.06m



(a) Torque $\tau = F \quad I\alpha$

$$\alpha = \frac{Fr}{I} = \frac{50 \times 0.06}{4}$$

$$\alpha = 0.75 rads^{-2}$$

(b) If θ is the angle turned through in the time, t = 16sec.

$$\omega_0 = 0$$

$$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 0.75 \times 16^2$$

$$\theta = 96rad$$

Number of revolution

$$N = \frac{\theta}{2\pi} = \frac{96}{2\pi}$$

$$N = 15rev \text{ (approx.)}$$

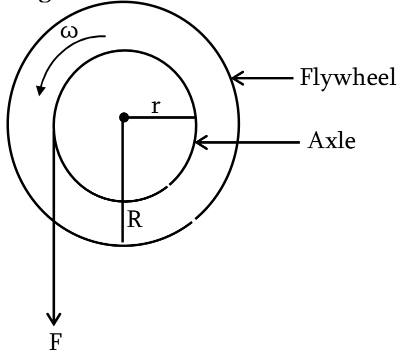
Example – 34

A flywheel of moment of inertia 0.30kgm² mounted on a fixed axle accelerates uniformly from rest to an angular velocity of 6rads⁻¹ in 12sec. find:-

(a) The angular acceleration
(b) The torque causing the wheel to accelerates.
(c) The number of revolution in this 12seconds period.

Solution

I = 0.30kgm², ω = 60rads-1, $\omega_0 = 0$, t = 12sec



(a) $$\alpha = \frac{\omega - \omega_0}{t} = \frac{60 - 0}{12}$$

$$\alpha = 5rads^{-2}$$

(b) $$\tau = I\alpha = 0.3 \times 5$$

$$\tau = 1.5Nm$$

(c) $$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 5 \times 12^2$$

$$\theta = 360rad$$

$$N = \frac{\theta}{2\pi} = \frac{360}{2\pi} \simeq 57rev$$

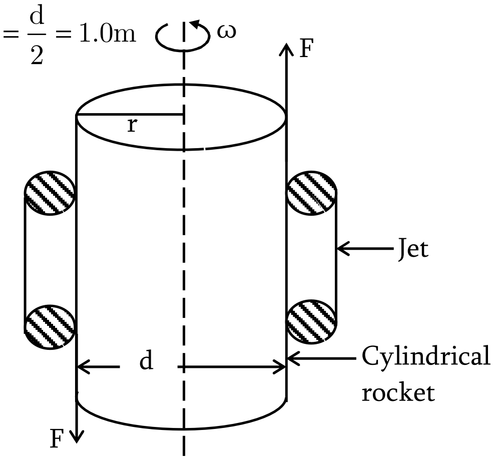
Example – 35

A cylindrical rocket of diameter 2.0m develops a spinning motion in space of period 2.0sec about an axis of these cylinder to eliminate this spin two jets motors which are attached to the rocket at the opposite ends of a diameter are fired until the spinning motion cease each motor turns the rocket in the same direction and provides a constant thrust of 4.0 × 10³N in the direction tangentially to the surface of the rocket and in the plane perpendicular to its axis. If the moment of inertia of the rocket about its cylindrical axis is 6.0 × 10⁵kgm². Calculate the number of revolutions made by the rocket during the firing and the time for which the motors are fired.

Solution

F = 4 × 10³N, I = 6.0 × 10⁵kgm², d = 20cm

$$r = \frac{d}{2} = 1.0m$$



Torque $\tau = -Fd = I\alpha$

F = applied force

d = distance between the coplanar force

$$\alpha = \frac{-Fd}{I} = \frac{-4 \times 10^3 \times 2}{6 \times 10^5}$$

$$\alpha = \frac{-8}{600} rads^{-2}$$

(Negative sign means retardation)

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi rads^{-1}$$

Since $\omega^2 = \omega_0^2 + 2\alpha\theta$

$$0^2 = \pi^2 + 2\left(\frac{-8}{600}\right)\theta$$

$$\theta = 37.5\pi^2 rad$$

Number of revolution

$$N = \frac{\theta}{2\pi} = \frac{37.5\pi^2}{2\pi}$$

$N = 58.9$ revolutions.

Time taken

$$\omega = \omega_0 + \alpha t$$

$$0 = \pi + \frac{-8}{600}t$$

$$t = 235.6 \sec$$

Example – 36

Two masses $M_1 = 15kg$ and $M_2 = 10kg$ are attached to the ends of a cord which passed over the pulley on an Atwood's machine. The mass of the pulley is $M = 10kg$ and its radius is $R = 0.1m$. Calculate the tensions in the cord, the acceleration and the number of revolution made by the pulley at the end of 2seconds from the starts.

Solution

As the pulley has a finite mass, the two tensions T_1 and T_2 are not equal.

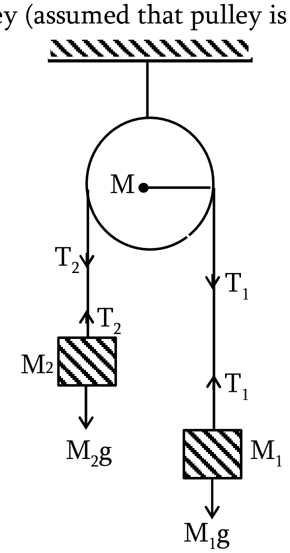
Let: a = linear acceleration

Resultant forces

M_1: $M_1g - T_1 = M_1a$………..(i)

M_2: $T_2 - M_2g = M_2a$……….(ii)

Here the tensions in the cords exert torque on the pulley (assumed that pulley is a solid disc)



Torque on the pulley.

$$\tau = (T_1 - T_2)R$$

But $\tau = I\alpha = \frac{1}{2}MR^2\alpha \quad \left[\alpha = \frac{a}{R}\right]$

$$\tau = \frac{1}{2}MR^2 \cdot \frac{a}{R} = \frac{1}{2}MRa$$

Now; $(T_1 - T_2)R = \frac{1}{2}MRa$

$$T_1 - T_2 = \frac{1}{2}Ma \text{……..(iii)}$$

Adding equation (i) and (ii)

$$(M_1 - M_2)g = (M_1 + M_2)a + (T_1 - T_2) \text{…(iv)}$$

Putting equation (iii) into (iv)

$$(M_1 - M_2)g = (M_1 + M_2)a + \frac{Ma}{2}$$

$$a = \frac{(M_1 - M_2)g}{M_1 + M_2 + M/2}$$

$$= \frac{(15 - 10) \times 9.8}{15 + 10 + 10/2}$$

$$a = 1.63m/s^2$$

Tension , T_1

$$T_1 = M_1(g - a) = 15(9.8 - 1.63)$$

$$T_1 = 114.3N$$

Also $T_2 = M_2(g + a) = 10(9.8 + 1.63)$

$$T_2 = 114.3N$$

Angular acceleration

$$\alpha = \frac{a}{R} = \frac{1.63}{0.1}$$

$$\alpha = 16.3rads^{-2}$$

Since $\theta = \frac{1}{2}\alpha t^2 \quad (\omega_0 = 0)$

$$= \frac{1}{2} \times 16.3 \times 2^2$$

$$\theta = 32.6rad$$

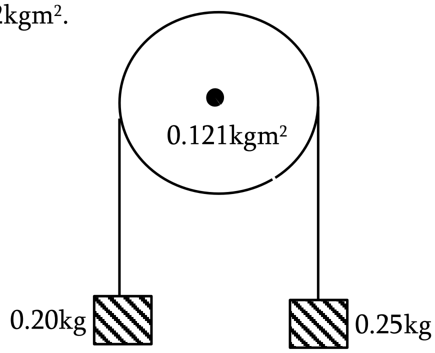
Number of revolution , $N = \frac{\theta}{2\pi}$

$N = 32.6$

$N = 5.2$ Revolutions.

Example – 37

Masses 0.20kg and 0.25kg suspended as in the figure below , from a light cord which passes over a wheel of radius 0.15m and moment of inertia 0.12kgm².



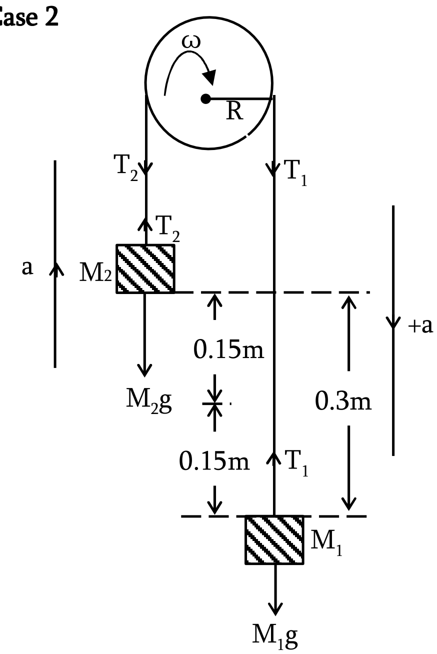
Initially , the two masses are held at the same horizontal level. Explain what happens when they are released from rest if the cord does not slip on the wheel. Assume that the wheel rotates freely about its axis , calculate the angular velocity of the wheel and the speed of the each mass , when the vertical distance between the masses is 0.3m.

Solution

Case 1:

When they are released from the rest the block of mass $M_1 = 0.25$kg moves downward while the block of mass $M_2 = 0.2$kg moves upward with the same linear acceleration.

Case 2



Resultant forces on each block

M_1: $M_1 - T_1 = M_1a$

$2.5 - T_1 = 0.25a$……..(i)

M_2: $T_2 - M_2g = M_2a$

$T_2 - 2 = 0.2a$……..….(ii)

Adding equation (i) and (ii)

$$T_2 - T_1 + 2.5 - 2 = 0.45a$$

$$T_1 - T_2 = 0.5 - 0.45a \text{ ……….(iii)}$$

Torque on the flywheel

$$\tau = (T_1 - T_2)R = I\alpha$$

$$T_1 - T_2 = \frac{I\alpha}{R} = \frac{Ia}{R^2} \text{ …….(iv)}$$

Putting equation (iv) into (iii)

$$\frac{Ia}{R^2} = 0.5 - 0.45a$$

$$\frac{0.12a}{(0.15)^2} = 0.5 - 0.45a$$

$$a = 0.865\text{m/s}^2$$

$$\alpha = \frac{a}{R} = \frac{0.865}{0.15} = 0.577\text{rads}^{-2}$$

Since $V^2 = u^2 + 2as \quad (u = 0)$

$$V = \sqrt{2as} = \sqrt{2\times 0.0865\times 0.15}$$

$$V = 0.161\text{m/s}$$

Angular velocity , ω

$$\omega = \frac{V}{R} = \frac{0.161}{0.15}$$

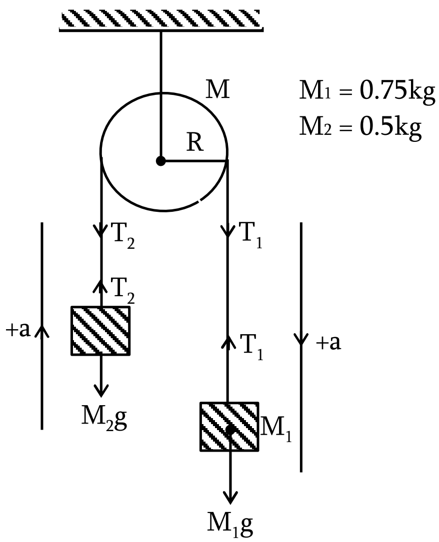
$$\omega = 1.074\text{rads}^{-1}$$

Example – 38

In an at woods , machine the pulley mounted in horizontal frictionless bearing has radius $R = 0.05$m. the cord passing over the pulley carries a block of mass $M_1 = 0.75$kg at one end and block of mass 0.5kg on other end when set free from rest , the heavier block is observed to fall a distance of 1metre in 10seconds. Compute the moment of inertial of the pulley.

Solution

$M_1 = 0.75$kg , $M_2 = 0.5$kg



Since the heavier blocks falls through distance S in t = 10sec.

$$u = 0$$

$$S = \frac{1}{2}at^2 \quad , \quad a = \frac{2s}{t^2}$$

$$a = \frac{2\times 1}{10^2} = 0.02\text{m/s}^2$$

Linear velocity at the end of 10sec

$$V = u + at = 0 + 0.02\times 10$$
$$V = 2\text{m/s}$$

The loss in p.e

$$\Delta p.e_1 = M_1gh = 0.75\times 9.81\times 1$$

$$\Delta p.e_1 = 7.35\text{J}$$

The lighter mass ascends 1m.

$$\Delta p.e_2 = M_2gh = 0.5\times 9.81\times 1$$

$$\Delta p.e_2 = 4.9\text{J}$$

Since no friction on the pulley.

Apply the law of conservation of energy

Loss in p.e = gain of k.e of two blocks + rotational k.e of the pulley.

$$\Delta p.e = \frac{1}{2}\left(M_1 + M_2\right)V^2 + \frac{1}{2}I\omega^2$$

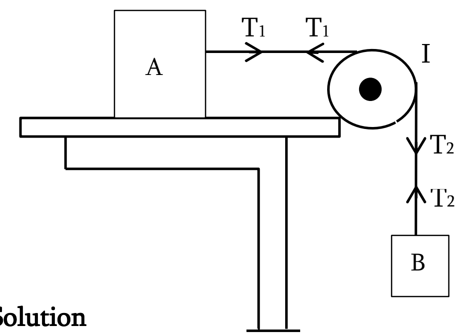
$$= \frac{1}{2}\left(M_1 + M_2\right)V^2 + \frac{1}{2}I\left(\frac{V}{R}\right)^2$$

$$2.45 = \frac{1}{2}\times 1.25\times\left(0.2\right)^2 + \frac{I}{2}\left(\frac{0.2}{0.05}\right)^2$$

$$I = 0.3031\text{kgm}^2$$

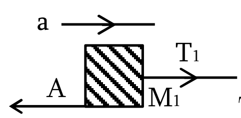
Example – 39

The pulley in the figure below has radius R and a moment of inertia , I. The rope does not ship over the pulley and the pulley spins on a frictionless axle. The coefficient of kinetic friction between block A and the table top is μ_k. The system is released from rest , and block B descends. Block A has a mass M_1 and block B has mass, M_2. Calculate the speed of block B as a function of the distance d that it has descended.



Solution

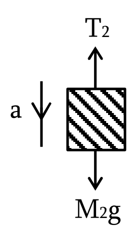
Consider FBD of body A



$$T_1 - f = M_1a$$

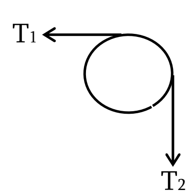
$$T_1 - \mu M_1g = M_1a \text{ ....(i)}$$

Body B



$$M_2g - T_2 = M_2a \text{ ....(ii)}$$

For the pulley



Torque on the pulley

$$\left(T_2 - T_1\right)r = I\alpha$$

$$T_2 - T_1 = \frac{Ia}{r} \quad \text{........(iii)}$$

Adding equation (i) , (ii) and (iii)

$$M_2g - \mu M_1g = \left(M_1 + M_2 + \frac{I}{r^2}\right)a$$

$$a = \frac{\left(M_2 - \mu M_1\right)g}{M_1 + M_2 + \frac{I}{r^2}}$$

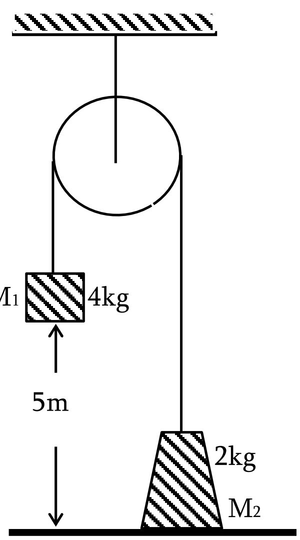
Since $V^2 = U^2 + 2ad \quad (u = 0)$

$$V = \sqrt{2ad}$$

$$V = \sqrt{2gd\left[\frac{M_2 - \mu M_1}{M_1 + M_2 + I/r^2}\right]}$$

Example – 40

The pulley as shown on the figure below has radius 0.16m and moment of inertia 0.38kgm². The rope does not ship on the pulley rim use energy methods to calculate the speed of the 4kg block just before it strikes the floor.



Solution

Apply the law of conservation of energy

$$p.e_1 = \text{k.e of pulley} + p.e_2 + k.e_1 + k.e_2$$

$$M_1gh = \frac{1}{2}I\omega^2 + M_2gh + \frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2$$

$$V_1 = V_2 = V \quad , \quad \omega = \frac{V}{r}$$

$$2\left(M_1 - M_2\right)gh = \frac{IV^2}{r^2} + \left(M_1 + M_2\right)V^2$$

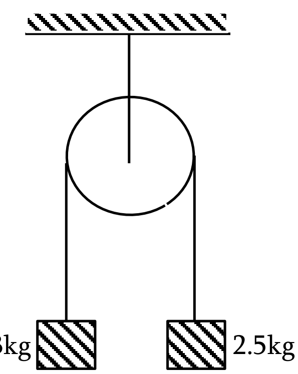
$$V = \sqrt{\frac{2\left(M_1 - M_2\right)gh}{I/r^2 + M_1 + M_2}}$$

$$= \sqrt{\frac{2\left(4-2\right)\times 9.8\times 5}{0.38/0.16^2 + 4 + 2}}$$

$$V = 3m/s$$

Example – 41

Two bodies of mass 2.5kg and 3kg are hanging on the pulley by mean of light inextensible string as shown in the figure below. Radius of the wheel is 0.2m and its mass is 0.5kg. Calculate time taken for distance separation between bodies to be 3m assuming that the bodies were initially at the same height.



Solution

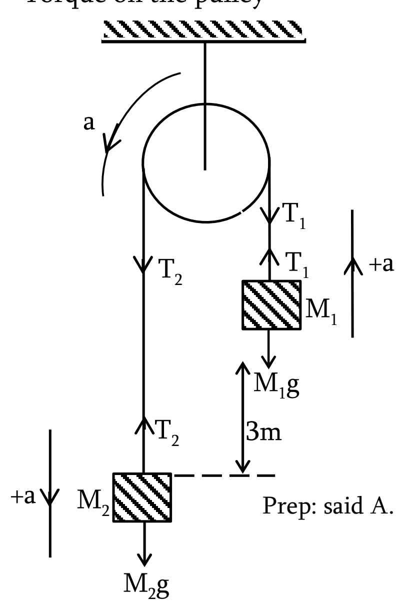
$M_1 = 2.5kg$, $M_2 = 3.0kg$

Resultant force on

M_1: $T_1 - M_1g = M_1a$ ......(i)

M_2: $M_2g - T_2 = M_2a$ .....(ii)

Torque on the pulley



$$(T_2 - T_1)r = I\alpha \quad \left[\alpha = \frac{a}{r}\right]$$

$$T_2 - T_1 = \frac{Ia}{r^2} \;.....(iii)$$

(i) + (ii) + (iii)

$$(M_2 - M_1)g = \left(M_1 + M_2 + \frac{I}{r^2}\right)a$$

$$a = \frac{(M_2 - M_1)}{M_1 + M_2 + \frac{I}{r^2}} \quad \left[I = \frac{1}{2}Mr^2\right]$$

$$= \frac{(3 - 2.5)\times 9.8}{3 + 2.5 + \frac{0.5}{2}}$$

Since $S = Ut + \frac{1}{2}at^2$

$$1.5 = \frac{1}{2}\times 0.852t^2$$

$$t = 1.88\text{ sec}$$

Example – 42

A flywheel with an axle 1cm in diameter is in frictionless bearing and set in motion by applying a steady tension of 2N to a thin thread wound tightly round the axle. The moment of inertia of the system about the axis of rotation is 5 × 10^{-4}kgm^2 calculate.

(a) Angular acceleration of the flywheel when 1m has been pulley off the axle.

(b) The constant retarding couple which must then be applied to stop the flywheel in one turn with tension on the rope completely removed.

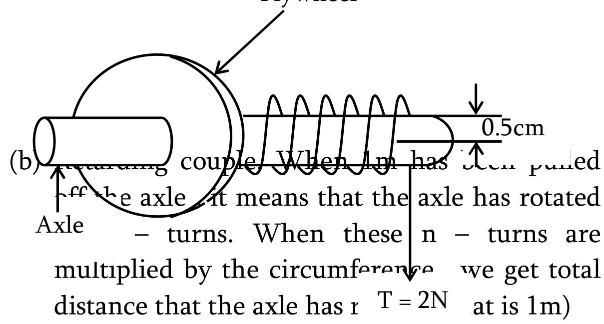
Solution

(a) $\tau = Tr = I\alpha \quad \left(\alpha = \frac{a}{r}\right)$

$$\alpha = \frac{Ir}{I}$$

$$\alpha = \frac{2\times 5\times 10^{-3}}{5\times 10^{-4}}$$

$$\alpha = 20\text{rads}^{-2}$$



(b) Retarding couple. When 1m has been pulled off the axle it means that the axle has rotated n – turns. When these n – turns are multiplied by the circumference we get total distance that the axle has moved (that is 1m)

$$2\pi rn = 1$$

$$n = \frac{1}{2\pi r}$$

Total angular displacement for the axle after n – revolutions.

$$\theta = 2\pi n = 2\pi\times\frac{1}{2\pi r} = \frac{1}{r}$$

$$\theta = \frac{1}{0.5\times 10^{-2}} = 200\text{rad}$$

Since $\omega^2 = \omega_0^2 + 2\alpha \quad (\omega_0 = 0)$

$$\omega^2 = 8000\text{rad}^2\text{s}^{-2}$$

Now to stop the flywheel in one complete turn

$$\theta_2 = 2\pi\text{rad}$$

$$\omega_2^2 = \omega^2 + 2\alpha_2\theta_2$$

$$0 = 8000 + 2\alpha_2\times 2\pi$$

$$\alpha_2 = \frac{2000}{\pi}\text{rads}^{-2}$$

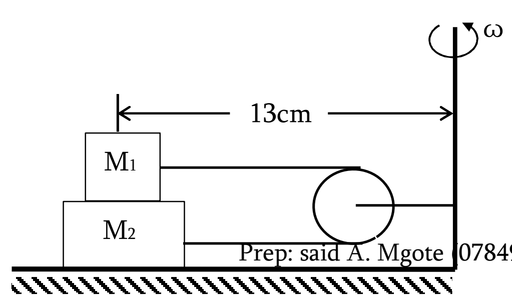
$$\tau = I\alpha_2$$

$$= 5\times 10^{-4}\times\frac{2000}{\pi}$$

$$\tau = 0.318\text{Nm}$$

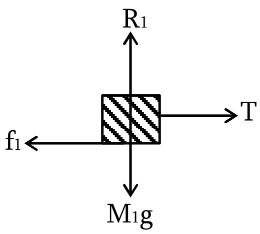
Example – 43

In a turn table arrangement as shown in the figure below. A block of mass M_1 = 0.9kg and M_2 = 1.7kg and they are 13cm from the axis of rotation. The coefficient of static friction between the blocks is 0.1 , the pulley being smooth. Find the angular speed of rotation of the turn table for which the blocks just to slide.



Solution

Consider the FBD for M_1



Resultant force on M_1

$$T - f_1 = M_1\omega^2 r$$

$$T - \mu M_1 g = M_1\omega^2 r$$

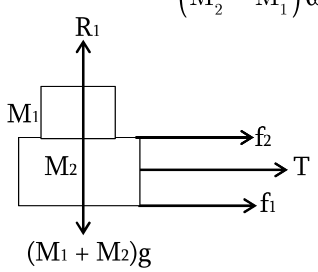
Consider the FBD for M_2

Resultant force on M_2

$$T + \mu M_1 g + \mu\left(M_1 + M_2\right)g = M_2\omega^2 r \ldots\text{(ii)}$$

Takes (ii) – (i)

$$\left(M_2 - M_1\right)\omega^2 r = \left(3M_1 + M_2\right)\mu g$$



$$\omega = \sqrt{\frac{\mu g\left(3M_1 + M\right)}{\left(M_2 - M_1\right)r}}$$

$$= \sqrt{\frac{0.1\times 9.8\left(3\times 0.9 + 1.7\right)}{\left(1.7 - 0.9\right)\times 13\times 10^{-2}}}$$

$$\omega = 6.4\text{rads}^{-1}$$

Example – 44

(a) A flywheel starts rotating from rest because of an external torque 1.5Nm. The torque is removed after the flywheel rotated by 48 revolutions it continues to rotate by 145 revolutions more before coming to rest. If the total time taken is 50 seconds , calculate the moment of inertia of the flywheel and the maximum angular velocity by it.

(b) A wheel of radius 50cm having 30spokes each of mass 100gm is travelling forward at 5m.s. If the mass of the rim of the wheel is 1000g, calculate the moment of inertia , kinetic energy and angular momentum of a wheel.

Solution

(a) Torque acting on the flywheel $\tau = 1.5\text{Nm}$

Number of revolutions completed $N = 48$

Angular displacement covered

$$\theta = 2\pi N = 2\pi\times 48 = 96\pi\text{rad}$$

Since $$\theta = \left(\frac{\omega_0 + \omega}{2}\right)t = \left(\frac{0 + \omega}{2}\right)t_1$$

$$96\pi = \frac{\omega t_1}{2}$$

$$\omega t_1 = 192\pi \ldots\ldots\text{(i)}$$

When the torque is removed , let α_2 be the angular retardation and t_2 be the time taken by the flywheel before coming to rest.

Angular displacement covered.

$$\theta = 2\pi N = 2\pi\times 145 = 290\pi\text{rad}$$

Now $$\theta = \left(\frac{\omega + \omega'}{2}\right)t$$

$$\theta = \left(\frac{\omega + 0}{2}\right)t_2 = \frac{\omega t_2}{2}$$

$$290\pi = \frac{\omega t_2}{2}$$

$$\omega t_2 = 580\pi$$

(i) = (ii)

$$\left(\omega t_2 + \omega t_2\right) = 580\pi + 192\pi$$

$$\omega\left(t_2 + t_1\right) = 772\pi \text{ but } t_1 + t_2 = 50\,\text{sec}$$

$$50\omega = 772\pi$$

$$\omega = 15.44\text{rads}^{-1}$$

Let α be the angular acceleration during the time when the torque was acting

Since $$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\left(15.44\pi\right)^2 = 0^2 + 2\times 96\pi\alpha$$

$$\alpha = 3.899\,\text{rad}/\text{s}^2$$

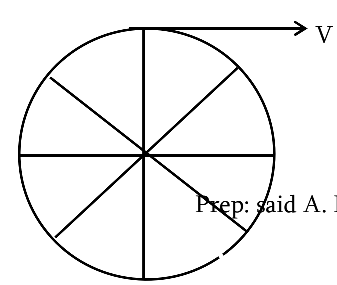
Torque $$\tau = I\alpha$$

$$I = \text{M.I of the flywheel}$$

$$I = \frac{I}{\alpha} = \frac{1.5}{3.899}$$

$$I = 0.3847\text{kgm}^2$$

(b)



Total M.I of the system

I = Ispokes + Irim

Spokes are similar to rods

Rotating about one end.

$$\text{Ispokes} = \frac{1}{3}ML^2 \times \text{number of spokes}$$

$$= \frac{1}{3}\times 0.1\times(0.5)^2\times 30$$

$$\text{Ispoke} = 0.25\text{kgm}^2$$

M.I of the wheel

$$\text{Irim} = MR^2 = 1\times(0.5)^2$$

$$\text{Irim} = 0.25\text{kgm}^2$$

$$I = 0.25+0.25 = 0.5\text{kgm}^2$$

Total energy

$$E = \frac{1}{2}I\omega^2 + \frac{1}{2}MV^2$$

$$= \frac{1}{2}I\frac{V^2}{R^2} + \frac{1}{2}MV^2$$

$$= \frac{V^2}{2}\left[\frac{I}{R^2}+M\right] = \frac{5^2}{2}\left[\frac{0.5+4}{(0.5)^2}\right]$$

$$E = 75J$$

Angular momentum

$$L = I\omega = \frac{IV}{R} = \frac{0.5\times 5}{0.5}$$

$$L = 5\text{kgm}^2\text{s}^{-1}$$

Example – 45

(a) (i) State the principle of conservation of angular momentum.

(ii) How does an ice – skater , a ballet dancer or an acrobat take advantage of principle of conservation of angular momentum?

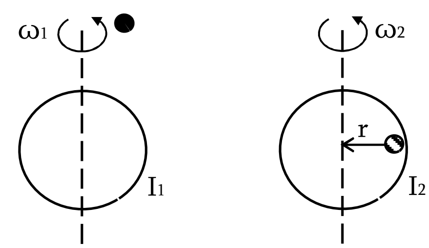
(b) A horizontal wooden disc is rotated about a vertical axis through its centre at 70 r.p.m. A dirt of 300gm falls vertically on the disc and embeds its sharp point in the wood at a distance of 10cm from the axis. If this reduced the rate of rotation of the disc to 50 r.p.m. Determine the moment of inertia of the wooden disc about the vertical axis.

Solution

(a) (i) Refer to your notes

(ii) An ice – skater , a ballet dancer or an acrobat is able to change his angular speed during the course of his performance when the performer stretches out his hand and legs , his moment of inertia increase and the angular speed decrease on the other hand , when he folds his hands and legs near his body , the moment of inertia decreases and he is able to increase his angular speed.

(b) Before the dirty falls After the dirty falls



Apply the principle of conservation of angular momentum.

$$I_1\omega_1 = I_2\omega_2$$

$$I_1(2\pi f_1) = I_2(2\pi f_2)$$

Let I = M.I of the disc

I_0 = M.I of dirty

$$If_1 = (I+I_0)f_2$$

$$I = I_0\left[\frac{f_2}{f_1+f_2}\right] \quad \text{but } I_0 = Mr^2$$

$$= Mr^2\left[\frac{f_2}{f_1+f_2}\right]$$

$$= 0.3\times(0.1)^2\left[\frac{50}{70-50}\right]$$

$$I = 7.5\times 10^{-3}\text{kgm}^2$$

Example – 46

(a) (i) Define the angular velocity of a rotating body and give its S.I unit. A car wheel has its angular velocity changing from 2rads^{-1}

to 30rads^{-1} in 20seconds. If the radius of the wheel is 400mm. Calculate:-

(ii) The angular acceleration

(iii) The tangential linear acceleration of a point on the rim of the wheel.

(b) A large wheel of radius 40cm having 10spokes on its is made to spin about an axle at 3rev per second. A 25cm long arrow is shot parallel to the axle but perpendicular to the surface of the rotating wheel without hitting any of the spokes and enters at a point where one of the spokes has just passed.

(i) What minimum speed should the arrow have?

(ii) Does it matter where (between the axle and the rim you aim?)

(c) (i) A recording disc rotates steadily at 45rev per minute on a turntable. When a small mass of 0.02kg is dropped gently onto the disc at a distance of 0.04m from its axis of rotation and sticks the rate of revolution falls to 36revmin^{-1}. Calculate the moment of inertia of the disc about its centre.

(ii) State and write down the principle used in your calculation in (i) above.

Solution.

(a) (i) Refer to your notes.

(ii) $\alpha = \dfrac{\omega - \omega_0}{t} = \left(\dfrac{30-2}{20}\right)$

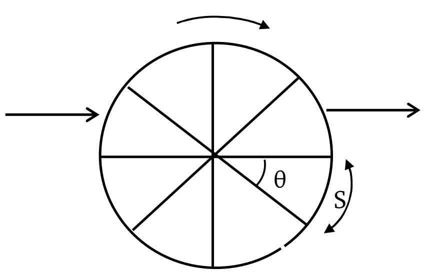
$\alpha = 1.4\text{rads}^{-2}$

(iii) $V = \omega R$

$a = \alpha R = 1.4 \times 0.4$

$a = 0.56\text{m/s}^2$

(b)



Circumference of the wheel

$C = 2\pi r = 2\pi \times 0.4 = 0.8\pi$

The distance between the spokes

$S = \dfrac{C}{N} = \dfrac{0.8\pi}{10}$

$S = 0.08\pi\text{m}$

Speed of the wheel , $V = \omega r = 2\pi fr$

$V = 2\pi \times 2 \times 0.4$

$V = 2.4\pi\text{m/s}$

The time taken by an arrow to travel that distance is the same as time taken by the wheel to turn through an angle , θ.

$$v = \frac{s}{t} \quad , \quad t = \frac{s}{v}$$

$$t = \frac{0.08\pi}{2.4\pi}$$

$$t = \frac{1}{30}\text{ second}$$

Then arrow should travel a distance equal to its length through the wheel without hits the wheel.

$$V_{min} = \frac{L}{t} = \frac{0.25}{1/30}$$

$$V_{min} = 7.5\text{m/s}$$

(ii) Yes, it matter if the arrow is shot near the axle it may collide with the spokes since the relative angular speed at the centre is very low compaired to the point of min

(c) (i) Apply principle of conservation of angular momentum.

$$I_d\omega_0 = I_1\omega_1 \quad , \quad I_1 = I_d + Mr^2$$

$$I_d(2\pi f_0) = (I_d + Mr^2)2\pi f_1$$

$$I_d = Mr^2\left[\frac{f_0}{f_1 - f_0}\right]$$

$$= 0.02 \times (0.04)^2\left[\frac{36}{45-36}\right]$$

$$I_d = 1.28 \times 10^{-4}\text{kgm}^2$$

(ii) Principle of conservation of angular momentum.

Example – 47

Two boys each of mass 25kg on the opposite ends of a horizontal beam of mass 10kg and length 2.6m. The beam is rotating about a vertical axis through its centre at 5revolutions per minute. Find the initial angular momentum. What would be the angular velocity if each body moves 0.6m toward the centre of the beam without to uching the floor. Calculate also the change in kinetic energy of the system.

Solution

Mass of each boy , M = 25kg

Mass of the beam , m = 10kg

Length of the beam L = 2.6m

Distance of each boy from the axis of rotation

$r_1 = 1.3$.

Initial angular velocity

$$\omega_1 = 5\text{r.p.m} = \frac{2\pi \times 5}{60} = \frac{\pi}{6}\text{rads}^{-1}$$

M.I of the system

I_1 = M.I of the rod + M.I of the boy

$$= \frac{ML^2}{12} + 2Mr_1^2$$

$$= \frac{10 \times (2.6)^2}{12} + 2 \times 25 \times (1.3)^2$$

$$I_1 = 90.13\text{kgm}^2$$

Initial angular momentum

$$L_1 = I_1\omega_1 = 90.13 \times \pi/6 \ \text{.........(i)}$$

$$L_1 = 47.168\text{kgm}^2\text{s}^{-1}$$

When each boy moves 0.6m towards the centre , the distance of each boy from the centre.

$$r_2 = 1.3 - 0.6 = 0.7\text{m}$$

New M.I of the system

$$I_2 = \frac{ML^2}{12} + 2mr_2^2$$

$$= \frac{10 \times (2.6)^2}{12} + 2 \times 25 \times (0.7)^2$$

$$I_2 = 30.13\text{kgm}^2$$

Let ω_2 = final angular velocity

Final angular momentum

$$L_2 = I_2\omega_2$$

Apply the principle of conservation of angular momentum

(i) = (ii)

$$I_1\omega_1 = I_2\omega_2$$

$$90.13 \times \frac{\pi}{6} = 30.13 \times 2\pi f_2$$

$$f_2 = 0.25\text{r.p.s} = 15\text{r.p.m}$$

Initial kinetic energy

$$E_1 = \frac{L_1^2}{2I_1} = \frac{1}{2}\frac{(I_1\omega_1)}{I_1} = \frac{1}{2}I_1\omega_1^2$$

$$= \frac{1}{2} \times 90.13 \times \left(\frac{\pi}{6}\right)^2$$

$$E_1 = 12.35\text{J}$$

Final kinetic energy

$$E_2 = \frac{1}{2I_2}(I_2\omega_2)^2$$

$$= \frac{[30.13 \times 0.5\pi]^2}{2 \times 30.13}$$

$$E_2 = 37.13\text{J}$$

Change in K.E , $\Delta E = E_2 - E_1$

$\Delta E = 37.13 - 12.35$

$\Delta E = 24.78\text{J}$

Example – 48

Two discs of moment of inertia I1 and I2 about their respective axes (normal to the disc and passing through the centre) and rotating with angular speed and brought into contact face to face with their axis of rotation coincident.

(i) What is the angular speed of the two discs system?

(ii) Show that kinetic energy of the combined system is less than the sum of initial kinetic energies of two discs. How do you account of this.

Solution

(i) Initial angular momentum of the system

$$Li = I_1\omega_1 + I_2\omega_2$$

Let ω = angular velocity as two discs combine together.

$$L_f = (I_1 + I_2)\omega$$

Before the conservation of angular momentum

$$(I_1 + I_2)\omega = I_1\omega_1 + I_2\omega_2$$

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

(ii) Kinetic energy of the system before the discs combined together.

$$E_1 = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$$

Final kinetic energy of the disc combined together.

$$E_2 = \frac{1}{2}(I_1 + I_2)\omega^2$$

$$= \frac{1}{2}(I_1 + I_2)\left(\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}\right)^2$$

$$E_2 = \frac{1}{2}\frac{(I_1\omega_1 + I_2\omega_2)^2}{(I_1 + I_2)}$$

Change in kinetic energy

$$\Delta E = E_1 - E_2$$

$$\Delta E = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 - \frac{(I_1\omega_1 + I_2\omega_2)^2}{2(I_1 + I_2)}$$

$$\Delta E = \frac{I_1 I_2(\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$$

Example – 49

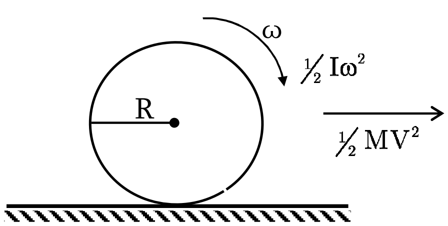
(i) State the principle of conservation of angular momentum and show how the principle can be demonstrated in the life

(ii) A uniform disc of radius R is rotating in its own plane with angular velocity , ω_0 when it is placed flat on a rough table. If μ is the coefficient of sliding friction is independent of velocity show that the time for the disc to come to rest is $\frac{3}{4}\frac{R\omega_0}{\mu g}$ how does the kinetic energy of rotation of the disc vary with time?

Solution

(i) Refer to your notes

(ii) Consider the disc rotating with angular velocity , ω as shown on the figure below.



Total kinetic energy

$$E = \frac{1}{2}MV^2 + \frac{1}{2}I\omega_0^2$$

But $I = \frac{1}{2}MR^2$, $V = \omega_0 R$

Now; $E = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_0^2 + \frac{1}{2}M(\omega_0 R)^2$

$$E = \frac{3}{4}MR^2\omega_0^2 \ldots\ldots(i)$$

Work done by the disc against friction ,

$W = fs = \mu mgs$

$S = R\theta$

$W = \mu MgR\theta \ldots\ldots(ii)$

Apply work – energy theorem

(i) = (ii)

$$\mu mgR\theta = \frac{3}{4}MR^2\omega_0^2$$

$$\theta = \frac{3}{4}\frac{R\omega_0^2}{\mu g}$$

Since $\theta = \omega_0 t$

$$\omega_0 t = \frac{3}{4}\frac{R\omega_0^2}{\mu g}$$

$$t = \frac{3}{4}\frac{R\omega_0}{\mu g} \text{ hence shown.}$$

Kinetic energy

After a time , t the velocity angular velocity

$$\omega = \omega_0 - \alpha t$$

Angular velocity is decreases and becomes equal to zero. Therefore angular acceleration is negative ($\omega = 0$)

$$0 = \omega_0 - \alpha t$$

$$\alpha = \frac{\omega_0}{t} = \frac{\omega_0}{\frac{3}{4}\frac{R\omega_0}{\mu g}}$$

$$\alpha = \frac{4\mu g}{3R}$$

$$k.e = \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left[\omega_0 - \alpha t\right]^2$$

$$= \frac{1}{4}MR^2\left[\omega_0 - \frac{4\mu gt}{3R}\right]^2$$

$$k.e = \frac{4\mu^2 g^2 m}{9}\left(t - \frac{3R\omega_0}{4\mu g}\right)^{\frac{1}{2}}$$

Hence the kinetic energy decreases to zero according to this square law (parabolic) variation with time , t

Example – 50

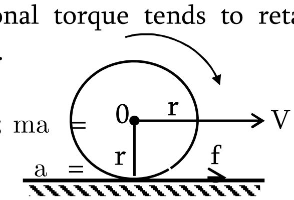
(a) A solid disc and a ring both of the radius 10.0cm are placed on a horizontal table simultaneously, with initial angular speed equal to $10\pi s^{-1}$. Which of the two start to roll earlier? The coefficient of kinetic friction $\mu_k = 0.2$.

(b) How will you distinguish between a hardboiled egg and a raw egg by spinning each on a table top?

Solution

(a) $r = 0.1m$, $\omega_0 = 10\pi rads^{-1}$, $\mu_k = 0.2$.
from the figure , it is clear that frictional force makes the centre of mass to accelerate and the frictional torque tends to retard the angular speed.

Now ; ma =



Since $v = u_0 + at = 0 + \mu_k gt$

$$v = \mu_k gt \ldots\ldots(i)$$

Also $\tau = \mu_k mgr = I\alpha$

$$\alpha = \frac{\mu_k mgr}{I}$$

Since $\omega = \omega_0 - \alpha t$

$$\omega = \omega_0 - \frac{\mu_k mgrt}{I} \ldots\ldots(2)$$

The rolling begins ,when $V = \omega r$
From equation (1) and (2)

$$\mu_k gt = r\left[\omega_0 - \frac{\mu_k mgrt}{I}\right]$$

For the ring , $I = Mr^2$

$$r\omega_0 - \frac{\mu_k mgr^2 t}{mr^2} = \mu_k gt$$

$$r\omega_0 = 2\mu_k gt$$

$$t = \frac{r\omega_0}{2\mu_k g} = \frac{0.1\times 10\pi}{2\times 0.2\times 9.8}$$

$$t = 0.255\text{second}$$

For the disc , $I = \frac{1}{2}Mr^2$

$$r\omega_0 - \mu_k \frac{mgr^2 t}{\frac{1}{2}Mr^2} = \mu_k gt$$

$$r\omega_0 = 3\mu_k gt$$

$$t = \frac{r\omega_0}{3\mu_k g} = \frac{0.1\times 10\pi}{3\times 0.2\times 9.8}$$

$$t = 0.170\text{ sec}$$

Example – 51

(a) If the radius of the earth assumed to be a perfect sphere, suddenly shrinks to half of its present value, the mass of the Earth remaining unchanged, what will be the duration decrease of the day.

(b) A uniform cylinder of radius 20cm is given an initial angular speed 35rad/s about an axis parallel to its length which passes through its centre. The cylinder is gently lowered onto a horizontal frictional surface and released. The coefficient of friction of the surface is $\mu = 0.5$. How long does it take before the cylinder starts to roll without slipping? What distance does the cylinder travel between its release point and the point at which it commences to roll without slipping?

Solution

(a) Apply the law of conservation of angular momentum.

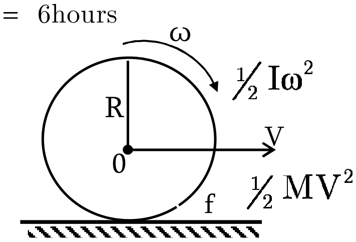
$$I_1\omega_1 = I_2\omega_2$$

$$\frac{2}{5}MR_1^2\left(\frac{2\pi}{T_1}\right) = \frac{2}{5}MR_2^2\left(\frac{2\pi}{T_2}\right)$$

$$T_2 = \left(\frac{R_2}{R_1}\right)^2 T_1 = \left(\frac{R_1}{2R_1}\right)^2 \times 24$$

$$T_2 = 6\text{hours}$$

(b)



The initial velocity of centre of mass is zero i.e $u = 0$.
The frictional force , f causes the centre of mass of the cylinder to accelerate

$$f = \mu mg = ma$$

$$a = \mu g$$

Since $v = u + at$

$$v = \mu gt$$

Frictional torque causes retardation in angular speed. The torque τ about centre of mass of the cylinder if placed on table is given by

$$\tau = -\alpha I = fR$$

$$\mu mgR = -\alpha I$$

$$\alpha = \frac{-\mu MgR}{I}$$

Applying , $\omega = \omega_0 + \alpha t$

$$\omega = \omega_0 - \frac{\mu mgRt}{I}$$

Now, the cylinder stops slipping as soon as the noslip conditions

$V = \omega R$ is satisfied.

$$V = R\left(\omega_0 - \frac{\mu MgRt}{I}\right)$$

$$\mu gt = R\left(\omega_0 - \frac{\mu MgRt}{I}\right)$$

$$\mu gt = \omega_0 R - \frac{\mu MgR^2 t}{I}$$

But $I = \frac{1}{2}MR^2$

$$\mu gt = \omega_0 R - \frac{\omega MgR^2 t}{\frac{1}{2}MR^2}$$

$$t = \frac{R\omega_0}{3\mu g}$$

$$= \frac{0.2 \times 35}{3 \times 0.15 \times 9.8}$$

$$t = 1.59\,\text{sec}$$

Whist it is slipping the cylinder travels a distance.

$$S = \frac{1}{2}at^2 = \frac{1}{2}\mu gt^2$$

$$= \frac{1}{2} \times 0.15 \times 9.8 (1.59)^2$$

$$S = 1.86\text{m}$$

Example – 52

(a) (i) Define the moment of inertia

(ii) State the parallel axes theorem .

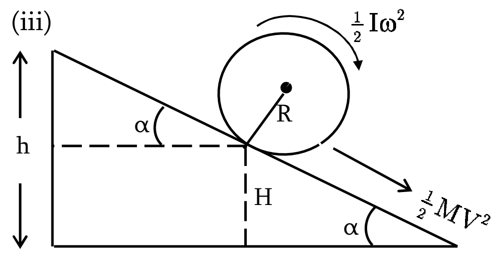
(iii) A solid cylinder of mass 'M' and radius 'R' rolls without slipping down a plane inclined at an angle (α) to the horizontal. Write down the total energy of the cylinder in terms of the mass of cylinder M and velocity V of the centre of mass of cylinder and any other quantities which you defined.

(b) Calculate the maximum speed of a solid cylinder rolling without slopping at the bottom of inclined plane of length 1.6m inclined at 60° to the horizontal. Assume that the cylinder started from the rest at the top of the inclined plane.

Solution

(a) (i) , (ii) Refer to your notes

(iii)



Total k.e of rolling body

$$K.E = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$

Total energy of rolling body

$$E = p.e + k.e$$

$$E = MgH + \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$

(b) Apply the principle of conservation of energy

$$Mgh = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$

$$\omega = V/R$$

$$Mgh = \frac{1}{2}\frac{IV^2}{R^2} + \frac{1}{2}MV^2$$

$$Mgh = \frac{V^2}{2}\left[M + \frac{I}{R^2}\right]$$

But $h = L\sin\alpha$

$$MgL\sin\alpha = \frac{V^2}{2}\left[\frac{I}{R^2} + M\right]$$

$$V = \left[\frac{2MgL\sin\alpha}{M + I/R^2}\right]^{\frac{1}{2}}$$

But $$I = \frac{1}{2}MR^2$$

$$\frac{I}{R^2} = \frac{M}{2}$$

$$V = \left[\frac{2MgL\sin\alpha}{\frac{M}{2}+M}\right]^{\frac{1}{2}} = \left[\frac{4}{3}gL\sin\alpha\right]^{\frac{1}{2}}$$

$$= \left[\frac{4\times 9.8\times 1.6\sin 60^O}{3}\right]^{\frac{1}{2}}$$

$$V = 3.2\text{m/s}$$

Example – 53

(a) (i) State the parallel axes theorem

(ii) What is the radius of gyration of a body?

(iii) Find the moment of inertia of a solid sphere of mass M and radius 'a' about any axis which passes through any point on its surface given that its moment about its centre of gravity is $2/5Ma^2$.

(b) A sphere and a cylinder of the same mass and radius starts from rest at the same point rolls down at the same plane inclined to an angle 30^O to the horizontal:

(i) Which body gets to the bottom first and with what acceleration?

(ii) If the later body has reach the bottom with the acceleration of the former body , what would be the angle of inclination to the horizontal for the body?

(c) A body like hoop , solid cylinder or sphere of mass M and radius 'a' whose moment of inertia about its centre of gravity is MK^2 (Where K is the radius of gyration is made to roll down a slope which is inclined at an angle θ to the horizontal having a total length , h . find the final velocity ,V possessed by the body when it reaches to the bottom of the plane.

(i) In terms of h , k and g

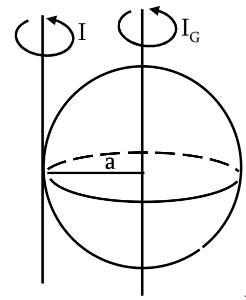
(ii) In terms of θ , k , g and a.

(d) Find also the uniform acceleration of the body down the plane in terms of g, θ , k and a.

Solution

(a) (i) , (ii) Refer to your notes

(ii) apply the parallel axes theorem

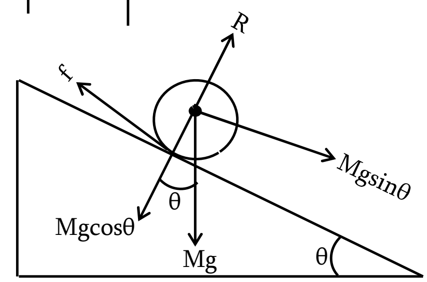


$$I = I_G + Mh^2$$

$$= \frac{2}{5}Ma^2 + Ma^2$$

$$I = \frac{7}{5}Ma^2$$

(b)



From the newton's second law of motion

$Mg\sin\theta - f = Ma$………..(1)

Torque produced by the frictional force

$$\tau = FR = I\alpha$$

$$f = \frac{I\alpha}{R} = \frac{Ia}{R^2} \quad \text{.........(2)}$$

Putting equation (2) into (1)

$$Mg\sin\theta - \frac{Ia}{R^2} = Ma$$

$$Mg\sin\theta = a\left[\frac{I}{R^2}+M\right]$$

$$a = \frac{Mg\sin\theta}{M+\frac{I}{R^2}}$$

(i) Let a_1 = acceleration of sphere

$$a_1 = \frac{Mg\sin\theta}{M+\frac{2M}{5}} = \frac{5Mg\sin\theta}{7M}$$

$$= \frac{5}{7}g\sin\theta = \frac{5}{7}\times 9.8\sin 30^O$$

$$a_1 = 3.5\text{m}/\text{s}^2$$

a_2 = acceleration of the cylinder

$$I = \frac{1}{2}MR^2 \quad , \quad I = \frac{M}{2}$$

$$a_2 = \frac{Mg\sin\theta}{M+\frac{1}{2}} = \frac{2}{3}g\sin\theta$$

$$= \frac{2}{3}\times 9.8\sin 30^O$$

$$a_2 = 3.27m/s^2$$

Since $a_1 > a_2$, therefore the solid sphere will reach at the bottom first than the solid cylinder.

(ii) $\theta = 30^O$

$$a_1 = \frac{5}{7}g\sin 30^O = 3.5m/s^2$$

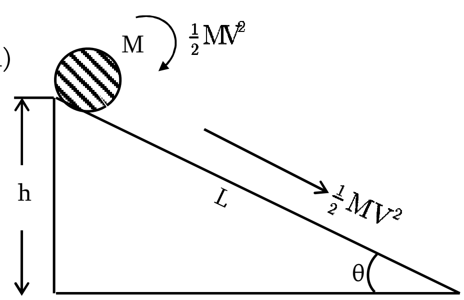
$$a_1 = a_2$$

Let θ_1 = New angle

$$3.5 = \frac{2}{3}g\sin\theta_1$$

$$\theta_1 = 32.39^O$$

(c) (i)



Apply the principle of conservation of energy

$$Mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}MV^2 \qquad \left[\omega = \frac{V}{a}\right]$$

$$Mgh = \frac{1}{2}I\frac{V^2}{a^2} + \frac{1}{2}MV^2 \quad \text{But } I = MK^2$$

$$Mgh = \frac{MV^2}{2}\left[\frac{K^2}{a^2}+1\right]$$

$$V = \left[\frac{2gh}{1+\left(k/a\right)^2}\right]^{\frac{1}{2}}$$

(ii) Since $h = L\sin\theta$

$$V = \left[\frac{2gL\sin\theta}{1+\left(k/a\right)^2}\right]^{\frac{1}{2}}$$

(d) Let a_O = Uniform acceleration down the slope, $U = 0$

$$V^2 = U^2 + 2aL = 2a_O L$$

$$V^2 = 2a_O L \,\ldots\ldots\ldots\text{(i)}$$

Apply the law of conservation of energy

$$MgL\sin\theta = \frac{V^2}{2}\left[M+\frac{I}{a^2}\right]$$

$$V^2 = \frac{2gML\sin\theta}{M+\frac{I}{a^2}}$$

$$2a_O L = \frac{2MgL\sin\theta}{M+\frac{I}{a^2}}$$

$$a_O = \frac{Mg\sin\theta}{M+\frac{I}{a^2}}$$

For the solid sphere, $I = \frac{2}{5}Ma^2$

$$\frac{I}{a^2} = \frac{2}{5}M$$

$$a_O = \frac{Mg\sin\theta}{M+\frac{2}{5}M}$$

$$a_O = \frac{5}{7}g\sin\theta$$

In terms of g, θ k and a

$$I = MK^2$$

$$a_O = \frac{Mg\sin\theta}{M+\frac{MK^2}{a^2}}$$

$$a_O = \frac{g\sin\theta}{1+\left(k/a\right)^2}$$

Example – 54

(a) Define the moment of inertia angular velocity, and angular momentum.

(b) A ballet dancer can spin faster by folding her arms than with arms outstretched. Explain why?

(c) A solid sphere and a cylinder both of the same radius are released simultaneously from the top of a 10m plane inclined at an angle of 30^O with horizontal. Determine which of the body will be first to reach the bottom of the plane? Which of the two objects which have greater angular momentum at the bottom of the plane?

Solution

(a) Refer to your notes

(b) When the arms of the ballet dancer are folded her moment of inertia I decreases but since angular momentum is conserved , her angular velocity increases. $\left(I \propto \frac{1}{\omega}\right)$

(c) For sphere , $I_G = \frac{2}{5}MR^2$

For the cylinder , $I_G = \frac{1}{2}MR^2$

Total energy at the top of plane.

$$E_1 = Mgh = MgL\sin\theta$$

$$= M \times 9.8 \times 10 \sin 30^O$$

$$E_1 = 49M \text{ joule}$$

After bottom of the plane.

$$E_2 = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$

- For the cylinder

$$E_2 = \frac{1}{2}MV_C^2 + \frac{1}{2}\frac{1}{2}MR^2\left(\frac{V_C}{R}\right)^2$$

$$E_2 = \frac{3}{4}MV_C^2$$

By the law of conservation of energy

$$E_1 = E_2$$

$$49M = \frac{3}{4}MV_C^2$$

$$V_C = 8.083 m/s$$

- For the solid sphere

$$E_2 = \frac{1}{2}MV_S^2 + \frac{1}{2}\frac{2}{5}MR^2\left(\frac{V_S}{R}\right)^2$$

$$E_2 = \frac{7}{10}MV_S^2$$

Then $E_1 = E_2$

$$49M = \frac{7}{10}MV_S^2$$

$$V_S = 8.37 m/s$$

Hence the sphere will reach at at the bottom first sine $V_S > V_C$. Therefore sphere will have a greater angular momentum that the cylinder at the bottom of the plane.

Example – 55

(a) How will you distinguish between a hardboiled egg and a row egg by spinning each on the top?

(b) Show that the rotational kinetic energy of a ball rolling over a horizontal plane is 2/7 of its total kinetic energy.

Solution

(a) The egg which spins at a lower rate will be raw egg. In a raw egg , the liquid matter tires to get away from the axis of rotation , thereby increasing the moment of inertia. Since $\tau = I\alpha$, α decreases i.e the row egg will spin at a smaller rate. On the other hand, the hardboiled egg will rotate faster like a rigid body.

(b) For the ball

$$I = \frac{2}{5}MR^2$$

$$K.E_R = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{V}{R}\right)^2$$

$$K.E_R = \frac{1}{5}MV^2$$

$$\text{Total K.E} = \frac{1}{2}I\omega^2 + \frac{1}{2}MV^2$$

$$= \frac{1}{5}MV^2 + \frac{1}{2}MV^2$$

$$K.E = \frac{7}{10}MV^2$$

$$\frac{K.E_R}{K.E} = \frac{MV^2}{5} \bigg/ \frac{7MV^2}{10} = \frac{2}{7}$$

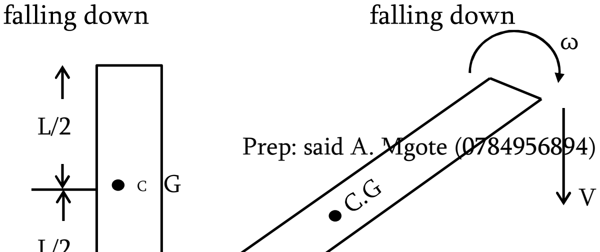
$$K.E_R = \frac{2}{7}K.E$$

Example – 56

A rod of length 30cm , held vertically with one end on the ground is allowed to fall on the ground. Find the angular velocity of the rod when its upper end to uches the ground. Also find the liner velocity of the middle point of the rod at that instant (g = 10m/s²).

Solution

Before the rod falling down | when the rod falling down



Apply the law of conservation of energy

Here the p.e of the rod is converted into rotational k.e

$$\frac{MgL}{2} = \frac{1}{2}I\omega^2$$

$$\omega^2 = \frac{MgL}{I} \quad \text{but } I = \frac{1}{3}ML^2$$

$$\omega^2 = \frac{MgL}{\frac{1}{3}ML^2} = \frac{3g}{L}$$

$$\omega = \sqrt{\frac{3g}{L}} = \sqrt{\frac{3\times10}{0.3}}$$

$$\omega = 10\text{rads}^{-1}$$

Linear velocity of the middle point of the rod.

$$V = \frac{L\omega}{2} = 0.15\times10$$

$$V = 1.5\text{m/s}$$

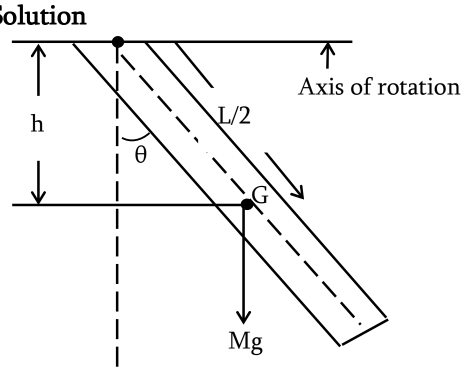
Example – 57

A uniform rod of length 3.0m is suspended at one end so that it can move about an axis perpendicular to its length and it held inclined at 60O to the vertical and then released. Calculate the angular velocity of the rod when

(i) It is inclined at 30O to the vertical

(ii) Reaches the vertical (moment of inertia of rod about end $= \text{mass}\times\frac{(\text{lenght})^2}{3}$)

Solution



From the figure above

$$\cos\theta = \frac{h}{L/2} \quad , \quad h = \frac{1}{2}\cos\theta$$

$$\text{Rotational K.E} = \frac{1}{2}I\omega^2 \ldots\ldots\ldots(1)$$

$$\text{p.e} = Mgh = \frac{MgL\cos\theta}{2}$$

$$\text{Change in p.e} = \frac{MgL}{2}\left[\cos\theta_2 - \cos\theta_1\right]$$

Apply the law of conservation of energy

(1) = (2)

$$\frac{1}{2}I\omega^2 = \frac{MgL}{2}\left(\cos\theta_2 - \cos\theta_1\right)$$

$$\omega = \left[\frac{MgL}{I}\left(\cos\theta_2 - \cos\theta_1\right)\right]^{\frac{1}{2}}$$

$$\text{But } I = \frac{ML^2}{3}$$

$$\omega = \left[\frac{MgL}{ML^2/3}\left(\cos\theta_2 - \cos\theta_1\right)\right]^{\frac{1}{2}}$$

$$\omega = \left[\frac{3g}{L}\left(\cos\theta_2 - \cos\theta_1\right)\right]^{\frac{1}{2}}$$

(i) $\theta_1 = 60^O \ , \ \theta_2 = 30^O$

$$\omega = \left[\frac{3\times9.8}{3}\left(\cos30^O - \cos60^O\right)\right]^{\frac{1}{2}}$$

$$\omega = 1.89\text{rads}^{-1}$$

$$\omega \simeq 1.9\text{rads}^{-1}\ (\text{approx})$$

(ii) $\theta_2 = 0^O \ , \ \theta_1 = 60^O$

$$\omega = \left[\frac{3\times9.8}{3}\left(\cos0^O - \cos60^O\right)\right]^{\frac{1}{3}}$$

$$\omega = 2.2\text{rads}^{-1}$$

Example – 58

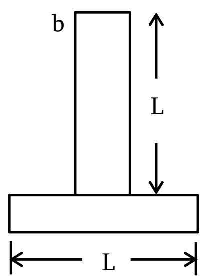
A pendulum is constructed from two identical uniform rods a and b each of length L and mass M connected at right angles to form a 'T' by joining the centre of rod a to one end of rod b. The 'T' is then suspended from the free end of the rod b and the pendulum swings in the plane of the 'T'.

(a) Calculate the moment of inertia of the 'T' about the axis of rotation.

(b) Give an expression for the kinetic and potential energies in terms of the angle θ of the inclination to the vertical of the pendulum.

(c) Derive the equation of the motion of the pendulum.

(d) Show that the period of small oscillation is given by $T = 2\pi\sqrt{\frac{17L}{18g}}$



Solution

(a) The M.I of a thin rod of length L and mass M about its centre is $\frac{ML^2}{12}$

Apply parallel axis theorem givens M.I of rod b

$$I_b = \frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{3}$$

Apply the principle of parallel axes theorem to a rod 'a' gives its M.I about the point of suspension.

$$I_a = \frac{ML^2}{12} + ML^2 = \frac{13}{12}ML^2$$

Total M.I of the system

$$I = I_a + I_b$$

$$= \frac{ML^2}{3} + \frac{13}{12}ML^2$$

$$I = \frac{17}{12}ML^2$$

(b)

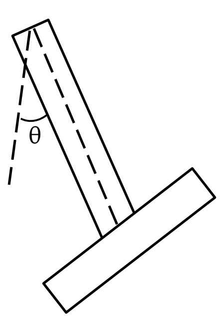


Figure on left hand shows the 'T' at an angle θ to the vertical. The k.e of the system k.e $= \frac{1}{2}I\left(\frac{d\theta}{dt}\right)^2$

At an angle θ the centre of mass of rod b has been raised through a distance $L\frac{(1-\cos\theta)}{2}$ and the centre of mass of rod a has been raised by L $(1 - \cos\theta)$

$$P.E = \frac{MgL(1-\cos\theta)}{2} + MgL(1-\cos\theta)$$

$$P.E = \frac{3MgL}{2}(1-\cos\theta)$$

With respect to the value of $\theta = 0^{o}$

(c) The equation of motion of the pendulum can be derived by using the fact that the total energy of system is constant.

P.E + K.E = Constant

$$\frac{17}{24}ML^2\left(\frac{d\theta}{dt}\right)^2 + \frac{3}{2}MgL(1-\cos\theta) = \text{constant}$$

differentiating this expression w.r.t time gives

$$\frac{17}{24}2.ML^2\left(\frac{d\theta}{dt}\right)\cdot\frac{d^2\theta}{dt^2} + \frac{3}{2}MgL\sin\frac{d\theta}{dt} = 0$$

Which can be rearranged to gives

$$\frac{d^2\theta}{dt^2} = \frac{-18}{17}g\frac{\sin\theta}{L}$$

(d) When the oscillation is small $\sin\theta \simeq \theta$

$$\frac{d^2\theta}{dt^2} = \frac{-18}{17}\left(\frac{g}{L}\right)\theta$$

For an angular S.H.M

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta$$

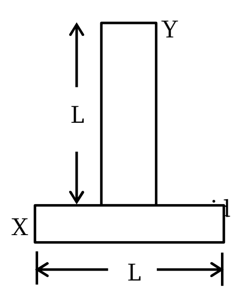
$$-\omega^2\theta = \frac{-18}{17}\left(\frac{g}{L}\right)\theta$$

$$\omega = \sqrt{\frac{18g}{17L}}, \quad \omega = \frac{2\pi}{T}$$

$$T = 2\pi\sqrt{\frac{17L}{18g}}$$ hence shown

Example – 59

A pendulum is constructed from two identical uniform rods X and Y , each of length L and mass M connected at right angles to form a T by joining the centre of rod X to one end of rod Y.



The T is then suspended from the free end of the rod Y and the perpendicular swings in the plane of T about the axis of rotation.

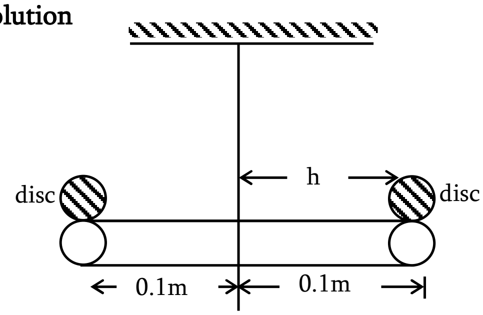
(i) Calculate the moment of inertia I of the T about the axis of rotation.
(ii) Obtain the expression of the k.e and p.e in terms of the angle θ of inclination to the vertical oscillation of the pendulum.
(iii) Show that the period of oscillation is given by

$$T = 2\pi\sqrt{\frac{17L}{18g}}$$

Example – 60

A uniform cylinder 20cm long suspended by a steel wire attached to its mid – point , so that its long axis is horizontally . it found to oscillate with a period of 2sec. when the wire is twisted and released when a small thin disc of mass 10gm is attached to each end , the period is found to be 2.3sec. Calculate the moment of inertia of the cylinder about axis of rotation.

Solution



$$T = 2\pi\sqrt{\frac{I}{Mgh}}$$

For the case of cylinder only

$$T_1 = 2\,\text{sec} \quad , \quad I = I_G$$

$$2 = 2\pi\sqrt{\frac{I_G}{Mgh}} \quad \text{........(1)}$$

For the case of the small disc attached at the end of the cylinder.

$$T_2 = 2.3\,\text{sec}$$

Apply parallel axes theorem

$$I = I_G + 2Mh^2$$

$$T_2 = 2\pi\sqrt{\frac{I_G + 2Mh^2}{Mgh}}$$

$$2.3 = 2\pi\sqrt{\frac{I_G + 2Mh^2}{Mgh}} \quad \text{........(2)}$$

Dividing equation (1) by (2)

$$\frac{2}{2.3} = \sqrt{\frac{I_G}{I_G + 2Mh^2}}$$

$$\left(\frac{2.0}{2.3}\right)^2 = \frac{I_G}{I_G + 2\times 10\times 10^{-3}\times (0.1)^2}$$

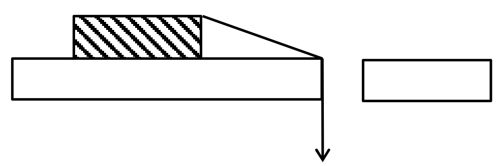
On solving

$$I_G = 6.2\times 10^{-4}\,\text{kgm}^2$$

Example – 61

A block of mass 0.05kg is attached to a cord passing through a hole in a horizontal frictionless surface as shown in the figure below. The block is originally revolving at the distance of 0.2m from hole with an angular velocity of 3rads^{-1}. The rod is then pulled from below shorten the radius of the circle in which the block revolving to 0.1m, the block may be considered point mass.

(a) What is the angular velocity.
(b) Find the change in kinetic energy of the blocks.



Solution

(a) Since $I_1\omega_1 = I_2\omega_2$

$$M_1 r_1^2 \omega_1 = M_1 r_2^2 \omega_2 \qquad [M_1 = M_2]$$

$$\frac{\omega_2}{\omega_1} = \left(\frac{r_1}{r_2}\right)^2$$

$$\omega_2 = \omega_1\left[\frac{r_1}{r_2}\right]^2$$

$$= 3.0\left[\frac{0.2}{0.1}\right]^2$$

$$\omega_2 = 12\text{rads}^{-1}$$

(b) Initial kinetic energy

$$E_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}M_1r_1^2\omega_1^2$$

$$= \frac{1}{2}\times 0.05\times(0.2)^2(3)^2$$

$$E_1 = 0.0095J$$

Final kinetic energy

$$E_2 = \frac{1}{2}I_2\omega_2^2 = \frac{1}{2}M_1r_2^2\omega_2^2$$

$$= \frac{1}{2}\times 0.05\times(0.1)^2\times 12^2$$

$$E_2 = 0.036J$$

Now; $\Delta E = E_2 - E_1 \quad (E_2 > E_1)$

$$= 0.036 - 0.0095$$

$$\Delta E = 0.027J$$

Example – 62

(a) If angular momentum is conserved in a system, whose moment of inertia is decreased , will its rotational kinetic energy be also conserved? Explain.

(b) The initial angular velocity of a circular disc of mass M is ω_1. The two small spheres of mass m are attached gently to two diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc?

Solution

(a) Suppose that the moment of inertia of the system decreases from I to I'. Then , angular speed will increase from ω to ω', such that

$$I\omega = I'\omega'$$

$$\omega' = \frac{I\omega}{I'}$$

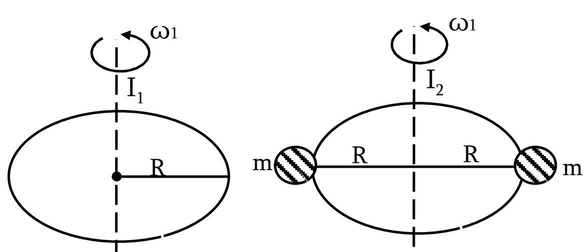
The kinetic energy of rotation of the system.

$$= \frac{1}{2}I'(\omega')^2 = \frac{1}{2}I'\left[\frac{I\omega}{I'}\right]^2$$

$$= \frac{1}{2}\frac{I^2}{I'}\omega^2$$

As I' < I , it follows that the kinetic energy of rotation of the system will increase , when its moment of inertia decreases. Hence , the kinetic energy of rotation will not be conserved.

(b) Before the sphere Attached on the disc   After the two sphere attached to the disc



Initial angular momentum

$$L_1 = I_1\omega_1 = \frac{1}{2}MR^2\omega_1$$

Final angular momentum

$$L_2 = \left[\frac{1}{2}MR^2 + MR^2 + MR^2\right]\omega^2$$

$$L_2 = \left[\frac{1}{2}MR^2 + 2MR^2\right]\omega_2$$

Apply the principle of conservation of angular momentum.

$$L_2 = L_1$$

$$\left[\frac{M}{2} + 2m\right]R^2\omega_2 = \frac{1}{2}MR^2\omega_1$$

$$\omega_2 = \left(\frac{M}{M+4m}\right)\omega_1$$

Example – 63

What will be the duration of the day, if the earth suddenly shrinks to 1/64 of its original volume , mass remaining unchanged? M.I of solid sphere = 2/5 MR^2.

Solution

Let T_1 and ω_1 be the period of revolution and angular velocity respectively of the earth before contraction. Let T_2 and ω_2 be the corresponding quantities after contraction. Let I_1 and I_2 be the moments of inertia of the earth before and after contraction respectively.

Apply the law of conservation of angular momentum.

$$I_1\omega_1 = I_2\omega_2$$

$$\left(\frac{2}{5}MR_1^2\right)\left(\frac{2\pi}{T_1}\right) = \left(\frac{2}{5}MR_2^2\right)\left(\frac{2\pi}{T_2}\right)$$

$$T_2 = \left(\frac{R_2}{R_1}\right)^2 T_1 \;.......(1)$$

Volume after contraction

$$= \frac{1}{64}\text{ volume before contraction}$$

$$\frac{4}{3}\pi R_2^3 = \frac{1}{64}\left(\frac{4}{3}\pi R_1^3\right)$$

$$\frac{R_2}{R_1} = \frac{1}{4}..............(2)$$

Putting equation (2) into (1)

$$T_2 = \left(\frac{1}{4}\right)^2 \times T_1 = \frac{1}{16} \times 24hrs$$

$$T_2 = 1.5hrs$$

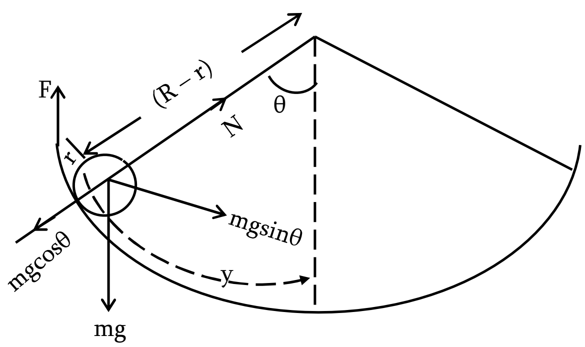
Example – 64

A sphere of radius r rolls without splipping on a concave surface of large radius of curvature R. show that the motion of the centre of gravity of the sphere is approximately simple harmonic motion with a period

$$T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$$

Solution

Let F be the force of static friction between the sphere and the concave mirror and N be normal reaction force



There is no acceleration in a direction normal to the plane.

$$N = Mg\cos\theta.........(i)$$

Resultant force along the plane mirror

$$Mg\sin\theta - F = Ma...............(ii)$$

The torque produced by the force of friction about the centre of mass.

$$\tau = I\alpha = Fr$$

$$F = \frac{I\alpha}{r} = \frac{I}{r}\cdot\frac{a}{r} = \frac{Ia}{r^2}.......(iii)$$

Putting equation (iii) into (i)

$$Mg\sin\theta - \frac{Ia}{r^2} = Ma$$

$$Mg\sin\theta = a\left[M + \frac{I}{r^2}\right]$$

$$a = \frac{Mg\sin\theta}{M + \frac{I}{r^2}}$$

But $I = \frac{2}{5}Mr^2$, $\frac{I}{r^2} = \frac{2}{5}M$

$$a = \frac{Mg\sin\theta}{M + \frac{2}{5}M} = \frac{5}{7}g\sin\theta$$

Restoring force $F = -Ma$

$$F = \frac{-5}{7}Mg\sin\theta$$

From the figure above

$$\sin\theta \simeq \theta = \frac{y}{R-r}$$

If θ is very small angle measured in radian.

$$F = -\frac{5}{7}Mg\left(\frac{y}{R-r}\right)$$

$$Ma = \frac{-5}{7}Mg\left(\frac{y}{R-r}\right)$$

$$a = \frac{-5}{7}g\left(\frac{y}{R-r}\right)$$

$$a = -\left(\frac{5g}{7(R-r)}\right)y.......(iv)$$

For S.H.M

$$a = -\omega^2 y$$

$$(iv) = (iv)$$

$$-\omega^2 y = -\left[\frac{5g}{7(R-r)}\right]y$$

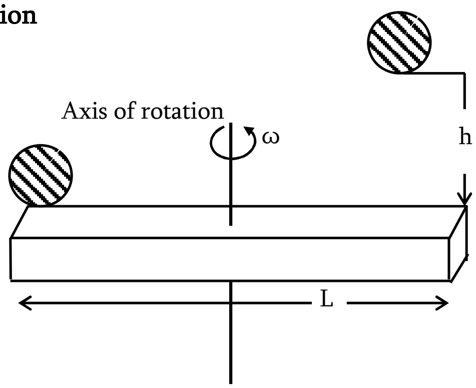
$$\omega = \sqrt{\frac{5g}{7(R-r)}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{7(R-r)}{5g}}$$ Hence shown

Example – 65

A 5.0kg ball is dropped from a height of 12.0m above one end of a uniform bar that pivots as its centre. The bar has mass 8.0kg and is 4.0m in length at the other end of the bar sits another 5.0kg ball , unattached to the bar? The dropped ball sticks to the bar after the collision. How high will the other ball go after the collision?

Solution



The speed of the first ball before it hits the bar is given by

$$V = \sqrt{2gh} = \sqrt{2\times9.8\times12}$$

$$V = 15.34m/s$$

Let ω be the angular velocity of the bar just after collision. Take the axis at the centre of the bar. Initial angular momentum of the ball

$$L_1 = MVr = 5.0\times15.34\times2$$

$$L_1 = 1533.4Kgm^2s^{-1}$$

Immediately after the collision the bar and both balls are rotating together.

Final angular momentum

$$L_2 = \left(\frac{ML^2}{12}+2mr^2\right)\omega$$

Apply the law of conservation of angular momentum

$$L_2 = L_1$$

$$\left(\frac{1}{12}\times8\times4^2\times2\times5\times2^2\right)\omega = 153.4$$

$$\omega = 3.03rads^{-1}$$

Just after the collision the second ball has linear velocity.

$$V = \omega r = 2\times3.03$$

$$V = 6.06m/s$$

Apply the principle of conservation energy

As the second ball moving upward its k.e is converted into potential energy.

$$\frac{1}{2}MV^2 = mgH$$

$$H = \frac{V^2}{2g} = \frac{(6.06)^2}{2\times9.8}$$

$$H = 1.87m$$

Example – 66

(a) State and explain the following.
 (i) Theorem of parallel axes
 (ii) Theorem of perpendicular axes.

(b) A metre stick is held vertically with one end on the floor and is then allowed to fall. Find the velocity of the other end , when it hits the floor , assuming that the end on the floor does not ship.

Solution

(a) Refer to your notes

(b) When the sticks hits the ground p.e of the stick in the vertical Position = k.e of rotation , when it hits the ground.

$$\frac{MgL}{2} = \frac{1}{2}I\omega^2$$

$$MgL = I\omega^2$$

$$\text{But } I = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3}ML^2$$

$$MgL = \frac{1}{3}ML^2\omega^2$$

$$\omega = \sqrt{\frac{3g}{L}}$$

Now , the linear velocity with which stick hits the floor.

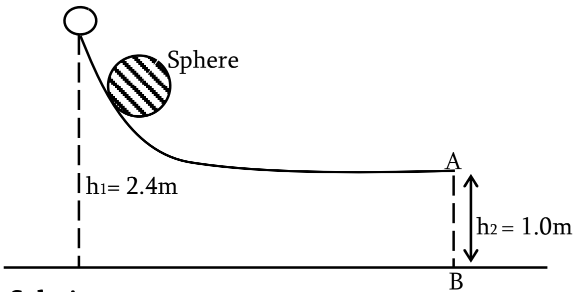
$$V = \omega L = L\cdot\sqrt{\frac{3g}{L}} = \sqrt{3gL}$$

$$= \sqrt{3\times9.8\times1}$$

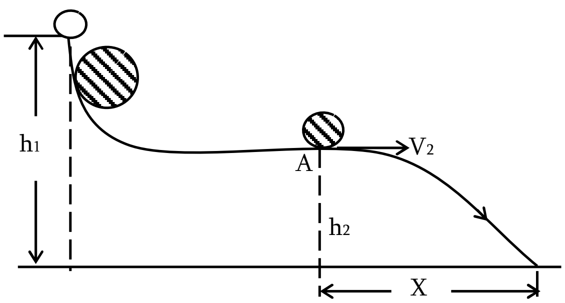
$$V = 5.4m/s$$

Example – 67

A small sphere rolls down without shipping from the top of a track in a vertical plane. The track has an elevated section and a horizontal part. The horizontal part is 1.0m above the ground and the top of the track is 2.4m above the ground. Find the distance on the ground w.r.t the point B (which is vertically below the end A of the track as shown below , where the sphere lands) During the flight as a projectile , does the sphere continues to rotate about its centre of mass?



Solution



Let M be the mass and R ,the radius of the sphere.
When the sphere is at point O
Total energy of sphere at O

$$E_1 = Mgh_1 ....(i)$$

When the sphere is at point A
Total energy of sphere at A

$$E_2 = p.e + k.e$$

$$= Mgh_2 + \frac{1}{2}MV_2^2 + \frac{1}{2}I\omega^2$$

$$= Mgh_2 + \frac{1}{2}MV_2^2 + \left(\frac{2}{5}MR^2\right)\left(\frac{V_2}{R}\right)^2$$

$$E_2 = Mgh_2 + \frac{7}{10}MV_2^2 ....(ii)$$

Apply the principle of conservation of energy

$$E_2 = E_1$$

$$Mgh_2 + \frac{7}{10}MV_2^2 = Mgh_1$$

$$\frac{7}{10}MV_2^2 = g(h_1 - h_2)$$

$$V_2 = \sqrt{\frac{10g}{7}(h_1 - h_2)}$$

$$V_2 = \sqrt{\frac{10 \times 9.8}{7}(2.4 - 1)}$$

$$V_2 = 4.427m / s$$

If t is the time taken by the sphere to reach on the ground

$$h_2 = \frac{1}{2}gt^2 \quad (u = 0)$$

$$t = \sqrt{\frac{2h_2}{g}} = \sqrt{\frac{2 \times 1}{9.8}}$$

$$t = 0.452\,\text{sec}$$

Horizontal range

$$X = V_2 t = 4.427 \times 0.452$$

$$X = 1.9999m \simeq 2m$$

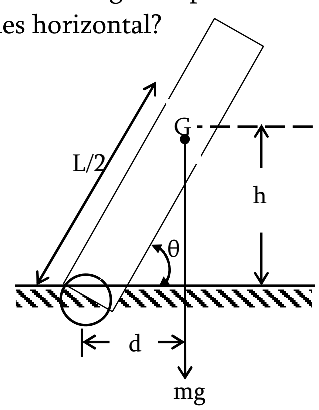
$$X = 2m$$

Example – 68

A uniform draw bridge 8m long is attached to the road way by a frictionless hinge at one end , and it can be raised by a cable attached to the other end. The bridge is at rest , suspended at 60^{O} above the horizontal , when the cable suddenly break;

(a) Find the angular acceleration of the bridge just after the cable breaks.
(b) Could you use the equation $\omega = \omega_0 + \alpha t$, to calculate angular speed of the drawbridge a later time? Explain why?
(c) What is the angular speed of the bridge as it becomes horizontal?

Solution



(a) Torque $= I\alpha = Mgd$

$$\left(\frac{1}{3}ML^2\right)\alpha = Mgd$$

$$\alpha = \frac{3gd}{L^2}$$

$$\cos\theta = \frac{d}{L/2}, \quad d = \frac{L}{2}\cos\theta$$

$$\alpha = \frac{3gL\cos\theta}{2L^2} = \frac{3g\cos\theta}{2L}$$

$$\alpha = \frac{3\times 9.8\cos 60^O}{2\times 8}$$

$$\alpha = 0.92\text{rad/s}^2$$

(b) The angular acceleration , α depends on the angle the bridge makes with the horizontal, therefore α is constant during the motion and $\omega = \omega_0 + \alpha t$ cannot be used.

(c) Apply the principle of conservation of energy

$$Mgh = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2$$

$$Mgh = \frac{1}{6}ML^2\omega^2$$

$$\omega^2 = \frac{6g}{L^2}\cdot\frac{L}{2}\sin 60^O = \frac{3g\sin 60^O}{L}$$

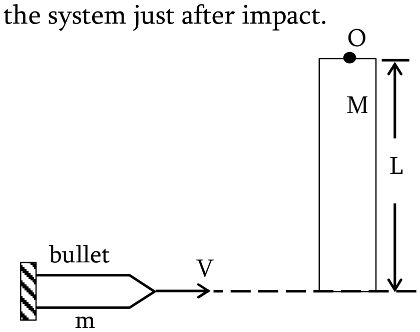
$$\omega = \sqrt{\frac{3g\sin 60^O}{L}} = \sqrt{\frac{3\times 9.8\sin 60^O}{8}}$$

$$\omega = 1.78\text{rads}^{-1}$$

Example – 69

(a) A small ball at the end of a string that passes through a tube is swang in a horizontal circle of radius 0.3m at a speed of 2.8m/s. What will be tangential speed down so as to reduce the radius of the circle to 0.2m?

(b) A rod of length L and mass M is hinged at point O. A small bullet of mass , m hits the rod as shown in the figure below. The bullet gets embedded in the rod. Find the angular velocity of the system just after impact.



Solution

(a) $r_1 = 0.3$m, $r_2 = 0.2$m , $V_1 = 2.8$m/s , $V_2 = ?$

Since there is no external torque acting on the ball , angular momentum can be conserve.

$$MV_1r_1 = MV_2r_2$$

$$V_2 = \frac{V_1r_1}{r_2} = \frac{2.8\times 0.3}{0.2}$$

$$V_2 = 4.2\text{m/s}$$

(b) Initial angular momentum of the system.

Li = Mvl

After the bullet gets embedded , then the system acquires angular velocity , ω.

Final angular momentum of the system.

$$L_f = I\omega$$

I = M.I of the bullet about an axis through O + M.I of rod about the axis through O.

$$I = mL^2 + \frac{1}{3}ML^2$$

$$I = \left(\frac{M+3m}{3}\right)L^2$$

$$L_f = \left(\frac{M+3m}{3}\right)L^2\omega$$

Apply the law of conservation of angular momentum

$$\left(\frac{M+3m}{3}\right)L^2\omega = MvL$$

$$\omega = \frac{3mV}{(M+3m)L}$$

Example – 70

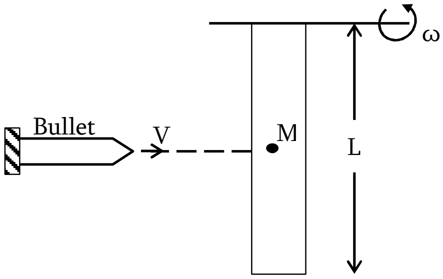
A uniform rod of length 2.5m rests on a frictionless surface. The rod pivots about fixed frictionless axis at one end. The rod is initially at rest a bullet travelling parallel to the horizontal surface and perpendicular to the rod with speed 460m/s strikes the rod at its centre and becomes embedded in it. The mass of the bullet is one – fourth the mass of the rod.

(a) What is the final angular speed of the rod just after the bullets impact? During the collision , why is the angular momentum conserved , but not the linear momentum?

(b) What is the ratio of the kinetic energy of the system after the collision to the kinetic before the collision.

Solution

(a) Apply the law of conservation of angular momentum for the collision.



$$mvr = (Irod + Ibullet)\omega$$

$$mvr = \left(\frac{1}{3}ML^2 + mr^2\right)\omega$$

$$m = \frac{M}{4}, \quad r = \frac{L}{2}$$

$$\frac{V}{8} = \frac{1gL\omega}{48}$$

$$\omega = \frac{6V}{19L} = \frac{6\times 460}{19\times 2.5}$$

$$\omega = 58.1rad/s$$

Linear momentum is not conserved during collision because of the force applied to the rod at the axis. But since external force acts at the axis it produces no torque and angular momentum is conserved.

(b) Initial kinetic energy of the bullet before collision.

$$K_1 = \frac{1}{2}mV^2 = \frac{1}{2}\cdot\frac{1}{4}MV^2$$

$$K_1 = \frac{1}{8}MV^2$$

Kinetic energy of the system after collision

$$K_2 = \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}\left[Irod + Ibullet\right]\omega^2$$

$$= \frac{1}{2}\left[\frac{1}{3}ML^2 + mr^2\right]\omega^2$$

$$= \frac{1}{2}\left[\frac{1}{3}ML^2 + \frac{M}{4}\cdot\left(\frac{L}{2}\right)^2\right]\omega^2$$

$$= \frac{1}{2}\left[\frac{19}{48}ML^2\right]\omega^2$$

$$= \frac{1}{2}\left[\frac{19}{48}ML^2\right]\left[\frac{6V}{19L}\right]^2$$

$$K_2 = \frac{3}{152}MV^2$$

$$\therefore \frac{K_2}{K_1} = \frac{3}{19}$$

Example – 71

A uniform circular disc of mass 20kg and radius 0.15m is mounted on a horizontal cylindrical axle of radius 0.015m and negligible mass. Neglecting frictional losses in the bearing. Calculate:

(a) The angular velocity acquired from rest by the application for 12seconds of a force of 20N tangentially to the axle.

(b) The kinetic energy of the disc at the end of this period.

(c) The time required to bring to rest if a breaking force of 1N were applied tangentially to its rim.

Solution

(a) M.I of the disc , $I = \frac{1}{2}Ma^2$

Torque due to 20N tangential to the axle

$$\tau = Fr = I\alpha$$

$$\alpha = \frac{Fr}{I} = \frac{Fr}{\frac{1}{2}Ma^2}$$

$$= \frac{20\times 0.015}{\frac{1}{2}\times 20\times(0.15)^2}$$

$$\alpha = 1.333rads^{-2}$$

Since $\omega = \omega_0 + \alpha t$

$$\omega = \omega_0 + \alpha t \quad \text{but} \quad \omega_0 = 0$$

$$\omega = 1.333\times 12$$

$$\omega = 16rads^{-1}$$

(b) $k.e = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}Ma^2\right)\omega^2$

$$= \frac{1}{4}Ma^2\omega^2$$

$$= \frac{1}{4}\times 20\times(0.15)^2\times 16^2$$

$$k.e = 28.8J$$

(c) Decelerating torque , $\tau = Fa$

$$\alpha = \frac{\tau}{I} = \frac{Fa}{\frac{1}{2}Ma^2} = \frac{2F}{Ma}$$

$$\alpha = \frac{2\times 1}{20\times 0.15}$$

$$\alpha = \frac{2}{3}rads^{-2}$$

Time to bring disc to rest

$$t = \frac{\text{initial angular velocity}}{\text{angular deceleration}}$$

$$= \frac{16}{2/3}$$

$$t = 24\,sec$$

Example – 72

(a) A grind stone has a moment of inertia of 600kgm2. A constant couple is applied and the grindstone is found to have a speed of 150revolution per minutes , 10seconds after starting from rest.j calculate the couple applied.

(b) By applying torqued of 980Nm to a flywheel, its angular velocity is increased from 10 to 20 revolutions per second in two minutes? Determine the moment of inertia.

Solution

(a) $\omega = \omega_0 + \alpha t \quad [\omega_0 = 0]$

$$\omega = \alpha t \quad \text{but} \quad \omega = 2\pi f$$

$$\alpha = \frac{2\pi f}{t} = \frac{2\pi}{10}\left[\frac{150}{60}\right]$$

$$\alpha = 1.571rads^{-2}$$

Torque $\tau = I\alpha = 600\times 1.571$

(b) $\omega_0 = 2\pi f_0 = 2\pi\times 10 = 20\pi rads^{-1}$

$$\omega = 2\pi f = 2\pi\times 20 = 40\pi rads^{-1}$$

$$\alpha = \frac{\omega-\omega_0}{t} = \frac{40\pi - 20\pi}{120}$$

$$\alpha = \frac{\pi}{6}rads^{-2}$$

$$I = \frac{\tau}{\alpha} = \frac{980\times 6}{\pi}$$

$$I = 1871.42kgm^2$$

Example – 73

A flywheel of mass 65.4kg is made in the form a circular disc of radius 18.0cm and is driven by the belt whose tension at the point where it runs on and off the rim of the wheel are 2kgwt and 5kgwt respectively if the wheel is rotating at a certain at 60r.p.m , find how long will it be before the speed has reached 210 r.p.m. While the flywheel is rotating at this later speed, the belts ships off and a break applied. Find the constant working couple required to stop the wheel is 7 revolutions.

Solution

The flywheel is in the form of disc

$$I = \frac{1}{2}MR^2 = \frac{1}{2}\times 65.4\times(0.18)^2$$

$$I = 1.05948kgm^2$$

Tension on the belt on one side

$$T_1 = 2kgwt$$

Net tension for the rotation of the wheel

$$T = T_2 - T_1$$

$$T = 5-2 = 3kgwt$$

$$T = 3\times 9.8 = 29.4N$$

Torque on the flywheel

$$\tau = Tr = I\alpha$$

Angular acceleration

$$\alpha = \frac{Tr}{I} = \frac{29.4\times 0.18}{1.05948}$$

$$\alpha = 4.99rads^{-2}$$

$$\omega_0 = 2\pi f_0 = 2\pi\left(\frac{60}{60}\right) = 2\pi rads^{-1}$$

$$\omega = 2\pi f = 2\pi\left(\frac{210}{60}\right) = 7rads^{-1}$$

(i) Time required

$$t = \frac{\omega-\omega_0}{\alpha} = \frac{7\pi - 2\pi}{4.99}$$

$$t = 3.14\,sec$$

(ii) Now , in this case $\omega_0 = 2\pi f$

$$\omega_0 = 2\pi\left(\frac{210}{60}\right) = 7\pi rads^{-1}$$

$$\omega = 0$$

$$N = 7$$

Angle traced, $\theta = 2\pi N$

$$\theta = 2\pi \times 7 = 14\pi \text{rad}$$

Let α = angular retardation

$$\omega^2 - \omega_0^2 = -2\alpha\theta$$

$$\alpha = \frac{49\pi^2}{2\times 14\pi} = \frac{7\pi}{4}\text{rads}^{-2}$$

$$\tau = I\alpha = 1.05948\times\frac{7}{4}\times\frac{22}{7}$$

$$\tau = 5.82\text{Nm}$$

Example – 74

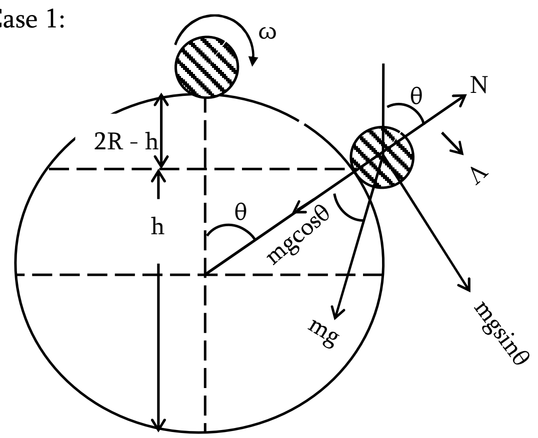
A small sphere is rolling down a large sphere of radius , R. Show that the small sphere will lose contact with the large sphere at a height

$$h = \frac{27}{17}R$$

Show that if a block of ice is used to slide down the large sphere , show also that the block of ice will lose contact with large sphere when $h = \frac{5}{3}R$

Solution

Case 1:



By the law of conservation of energy

Loss in p.e = gain in rolling k.e

$$Mg(2R-h) = \frac{1}{2}I\omega^2 + MV^2$$

But $I = \frac{2}{5}M^2r^2$, $\omega = \frac{V}{r}$

$$Mg(2R-h) = \frac{1}{2}\times\frac{2}{5}Mr^2\left(\frac{V}{r}\right)^2 + \frac{1}{2}MV^2$$

$$V^2 = \frac{10g}{7}(2R-h)\ldots\ldots\ldots.(1)$$

Also $Mg\cos\theta - N = \frac{MV^2}{R}$

When the body loss contact , N = 0

$$Mg\cos\theta = \frac{MV^2}{R}$$

$$V^2 = gR\cos\theta\ldots\ldots\ldots\ldots.(2)$$

$$(1) = (2)$$

$$gR\cos\theta = \frac{10g}{7}(2R-h)$$

$$R\cos\theta = \frac{10}{7}(2R-h)$$

But $\cos\theta = \frac{h-R}{R}$

$$\left(\frac{h-R}{R}\right)R = \frac{10}{7}(2R-h)$$

$$h-R = \frac{10}{7}(2R-h)$$

$$h = \frac{27}{17}R \text{ Hence shown}$$

Case 2:

If block of ice is used instead of a sphere , the potential energy loss is converted into sliding kinetic energy.

Now : $Mg(2R-h) = \frac{1}{2}MV^2$

$$V^2 = 2g(2R-h)\ldots\ldots.(1)$$

Again : $Mg\cos\theta - N = \frac{MV^2}{R}$

For the body to lose contact

$$N = 0$$

$$Mg\cos\theta = \frac{MV^2}{R}$$

$$V^2 = gR\cos\theta = gR\left(\frac{h-R}{R}\right)$$

$$V^2 = g(h-R)\ldots\ldots..(2)$$

$$(1) = (2)$$

$$g(h-R) = 2g(2R-h)$$

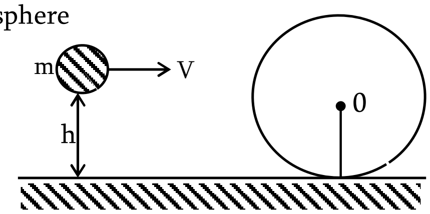
$$h = \frac{5}{3}R \text{ Hence shown.}$$

Example – 75

The sphere of mass M and radius R shown in the figure below , lies on a rough plane when a particle of mass m travelling at a speed V collide and stick with it. The line of motion of the particle is at a height h above the plane. Find

(a) The angular speed of the system about the centre of mass of the sphere just after the collision.

(b) The value of h in terms of R for which the sphere



Starts pure rolling on the plane. Assume that the mass M of the sphere is large compared to the mass of the particle so that the centre of mass of the combined system is not appreciably shifted form the centre of the sphere. Moment of inertia of the sphere about the centre of mass $= \frac{2}{5}MR^2$

Solution

(a) If V_1 is the linear speed of the combined system, conservation of linear momentum gives

$$mV = (M+m)V_1$$

$$V_1 = \frac{mV}{M+m} \quad \text{........(1)}$$

Angular momentum of the particle before collision is Mv (h – R). If the system rotates with angular speed ω after collision, the angular momentum of the system becomes.

$$I\omega = \left(I_{Sphere} + I_{Particle}\right)\omega$$

$$I\omega = \left(\frac{2}{5}MR^2 + mR^2\right)\omega$$

Applying the law of conservation of angular momentum about the centre of mass of the sphere as M>>m,

$$mv(h-R) = \left(\frac{2}{5}MR^2 + mR^2\right)\omega$$

$$\omega = \frac{mv(h-R)}{\frac{2}{5}(M+m)R^2} = \frac{mv(h-R)}{\frac{2}{5}MR^2}$$

$$(M+m \simeq M)$$

$$\omega = \frac{5mv(h-R)}{2MR^2}$$

(b) The sphere will start rolling just after the collision

$$V_1 = \omega R$$

$$\frac{mv}{M+m} = \frac{5mv(h-R)}{2MR^2} \cdot R$$

$$\frac{1}{M+m} = \frac{5(h-R)}{2MR}$$

$$\frac{1}{M} = \frac{5(h-R)}{2MR}$$

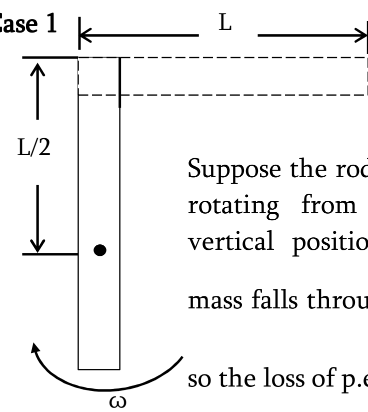
$$h = \frac{7}{5}R$$

Example – 76

uniform rod of length L is freely pivoted at one end. It is initially held horizontally and then released from rest. What is the angular velocity at the instant when the rod is vertical? When the rod is vertical it breaks as its midpoint. What is the largest angle from the vertical reached by the upper part of the rod in its subsequent motion? (assume that no impulsive forces are generated when the rod breaks).

Solution

Case 1



Suppose the rod has mass , m . In rotating from a horizontal to vertical position, the centre of mass falls through a distance $\frac{L}{2}$, so the loss of p.e is $\frac{mgL}{2}$

The M.I of a rod of length L and mass m about the end is $I = \frac{1}{3}mL^2$

K.e of the rod $= \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mL^2\right)\omega^2 = \frac{1}{6}mL^2\omega^2$

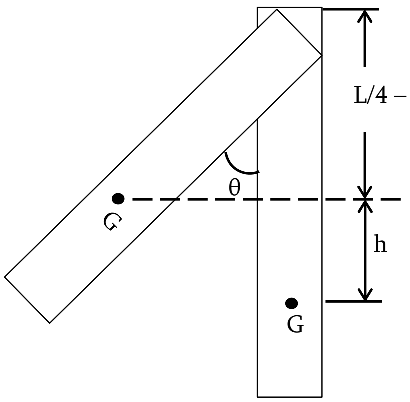
Apply the law of conservation of energy

$$\frac{mgL}{2} = \frac{1}{6}mL^2\omega^2$$

$$\omega = \sqrt{\frac{3g}{L}}$$

Case 2:

When the rod breaks , the mass and the length of the part of the rod are both half of the corresponding values for the unbroken rod.



Now, M.I of the rod

$$I = \frac{1}{3}\left(\frac{m}{2}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{24}mL^2$$

K.e of this part of the rod

$$k.e = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{24}mL^2\right)\times\frac{3g}{L}$$

$$k.e = \frac{1}{16}mgL$$

If the rod now rotates through an angle θ, then

$$\cos\theta = \frac{\frac{L}{4}-h}{\frac{L}{4}} = \frac{L-4h}{L}$$

$$h = \frac{L}{4}\left(1-\cos\theta\right)$$

∴ The centre of mass of the rod will rise through a distance $h = \frac{L}{4}\left(1-\cos\theta\right)$

Grain in p.e $= \left(\frac{m}{2}\right)gh = \frac{mgL}{8}\left(1-\cos\theta\right)$

Apply the law of conservation of energy

$$\frac{1}{16}mgL = \frac{mgL}{8}\left(1-\cos\theta\right)$$

$$1-\cos\theta = \frac{1}{2}$$

$$\theta = 60^{o}$$

Example – 77

(a) (i) Justify the statement that ‘if no external torque acts on a body, its angular velocity will not conserved.

(iv) A car is moving with a speed of 30m/s on a circular track of radius 500m. If its speed is increasing at the rate of $2m/s^2$; find its resultant linear acceleration.

(b) An object of mass 1kg is attached to the lower end of a string 1m long whose upper end is fixed and made to rotate in a horizontal circle of radius 0.6m. If the circular speed of the mass is constant , find the ;-

(i) The tension in the string

(ii) Period of motion

Example – 78

(a) (i) What is meant by moment of inertia of a body.

(ii) List two factors on which the moment of inertia of a body depends.

(b) A thin sheet of aluminium of mass 0.032kg has the length of 0.25m and width of 0.1m. find its moment of inertia on the plane about an axis parallel to the:-

(i) Length and passing through its centre of mass , m.

(ii) Width and passing through the centre of mass , m in its own plane

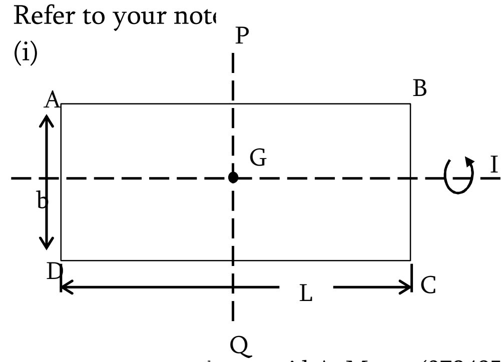
(c) (i) Define the term angular momentum.

(ii) A thin circular ring of mass M and radius r is rotating about its axis with constant angular velocity ω1. If two objects each of mass m are attached gently at the ring , what will be the angular velocity of the rotating wheel?

Solution

(a) Refer to your not

(b) (i)



M.I of lamina about an axis PQ parallel to AD

Similarly M.I of the lamina about an axis RS parallel to AB or DC passing through the centre G

$$I_X = \frac{1}{12}mb^2$$

$$I_X = \frac{1}{12} \times 0.032 \times (0.1)^2$$

$$I_X = 2.67 \times 10^{-5}\,kgm^2$$

(ii) $I = I_X + I_y$ (perpendicular axes theorem)

$$I = \frac{M}{12}(l^2 + b^2)$$

$$= \frac{0.032}{12}(0.25^2 + 0.1)^2$$

$$I = 1.933 \times 10^{-4}\,kgm^2$$

(c) Refer to your notes

Example – 79

(a) (i) Define torque and gives its S.I unit.

(ii) A disc of moment of inertia $2.5 \times 10^{-4}kgm^2$ is rotating freely about an axis through its center at 20rev/min. if some wax of mass 0.04kg is dropped gently on the disc 0.05m from its axis , what will be the new revolutions per minute of the disc?

(b) Explain briefly why a

(i) High diver can turn more somersaults , before striking the water?

(ii) Dancer on skates can spin faster by folding her arms?

(c) A flywheel of moment of inertia 0.4kgm2 is mounted on horizontal axle of radius 0.01m. if a force of 60N is applied tangentially to the axle:

(i) Calculate the angular velocity of the flywheel after 5seconds form rest.

(ii) List down two assumptions taken to arrive at your answer in 6(c) (i)

Solution

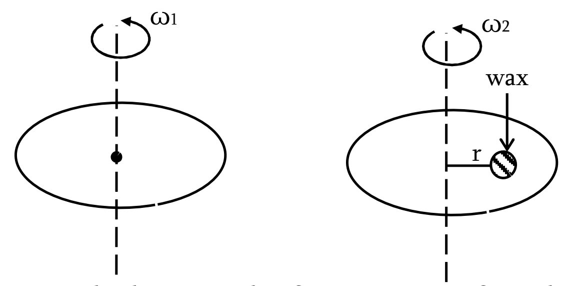
(a) (i) refer to your notes

(ii) $I_d = 2.5 \times 10^{-4}\,kgm^2$

$$f_1 = 20rev/min = \frac{20rev}{60\,sec} = 0.33rev/s$$

$$f_1 = 0.33rev/s \;,\; f_2 = ?$$

Before wax dropped On the disc | After wax dropped on the disc



Apply the principle of conservation of angular momentum.

$$I_d f_1 = (I_d + Mr^2) f_2$$

$$f_2 = \left(\frac{I_d}{Mr^2 + I_d}\right) f_1$$

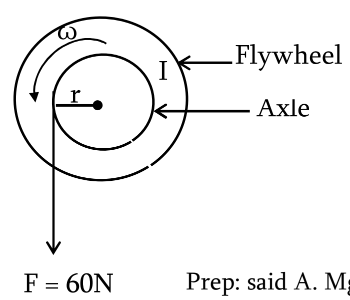
$$= \frac{2.5 \times 10^{-4} \times 20}{2.5 \times 10^{-4} + 0.04 \times (0.05)^2}$$

$$f_2 = 14.29rev\,min^{-1}$$

(b) (i) When a high diver jumps from spring board, he curls his body rolling in his arms and legs. This decreases M.I in which case his angular velocity increases here then performs more somersaults as the diver is about to touch the surface of water , he stretches out his limbs by so doing ,he increases his moment of inertia , thereby reducing his angular velocity.

(ii) By folding her arms , dancer skater reduces her moment of inertia , leading to an increase in her angular speed. This is because when she brings her arms and legs closer to the axis of rotation her M.I about its axis is reduced so angular speed ω increases to ensure that L = Iω is conserved.

(c)



$I = 0.4kgm^2$
$r = 0.01m$

(i) ω = Angular velocity of the flywheel , $t = 5sec$, $\omega_0 = 0$?

Torque on the flywheel

$\tau = Fr = I\alpha$

$\alpha = \frac{Fr}{I} = \frac{60 \times 0.01}{0.4}$

$\alpha = 1.5rads^{-2}$

Since $\omega = \omega_0 + \alpha t$

$\omega = \alpha t = 1.5 \times 5$

$\omega = 7.5rads^{-1}$

(ii) Assumptions made:-

- Moment of inertia of axles is very negligible compaired to the M.I fo the flywheel.
- No friction force between the flywheel and axle.

Example – 80

(a) (i) Define the term moment of inertia.

(v) Briefly explain the meaning of the term radius of gyration.

(b) A uniform solid cylinder of mass M and radius r rolls without shipping about its centre from rest through a distance(s) along a plane inclined at an angle θ. Derive an expression for the linear acceleration *a* of the cylinder down the plane given that the moment of inertia of the cylinder about its centre $I = \frac{1}{2}Mr^2$ (its angular velocity ω and the linear velocity down the plane is V)

(c) A constant torque of 200Nm twists a wheel about its centre. The moment of inertia about its axis is $100kgm^2$ find:-

(i) The angular velocity gained in 4second.

(ii) The kinetic energy gained after 20 revolutions.

Example – 81

(a) Define the following terms

(i) Torque (ii) angular acceleration (iii) radius of gyration.

(b) A cylindrical rocket of diameter 2.0m develops a spinning motion in space of period 2.0seconds about the axis of the cylinder. To stop the spin two jet motors which are attached to the rocket at opposite ends of the diameter are fired until the spinning motion ceases each motor turn the rocket in the same direction and provides a constant thrust of 4×10^3N in the direction of tangential to the surface of the rocket and in plane perpendicular to its axis. If the moment of inertia of the rocket about its cylindrical axis is $6.0 \times 10^5kgm^2$, calculate :-

(i) Angular acceleration of the spinning of the rocket and its angular speed.

(ii) Number of revolutions made by the rocket during firing

(iii) Time for which the motor are fired.

(c) A wheel of radius 50cm and mass 1000gm having 30spokes each of mass 1000gm is travelling forward at 5m/s. If the mass of the rim is 1000gm, determine the;-

(i) Moment of inertia.

(ii) Kinetic energy and angular momentum

(iii) Linear momentum

Example – 82

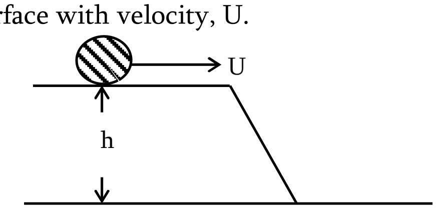
(a) State the principle of angular momentum

(b) An ice skater spins at 4πrad/s with her arms extended.

(i) If the moment of inertia her arms folded is 80% to that with arms extended , what is her angular velocity when she folds her arms.

(ii) Find the fractional change in rotational K.E

(c) A disc rolls without slipping along a horizontal surface with velocity, U.



The disc is then encounters a smooth drop of height 'h' after which it continues with new velocity V at all times the disc remains in vertical plane as shown in the figure above show that $V = \sqrt{U^2 + \frac{4gh}{3}}$

Example – 83

Explain briefly the following phenomena;-

(a) Why is the handle in the flour grinding machine put near the circumference?

(b) A broad handle is use to resolve a screw why?

(c) Why is it more difficult to revolve a stone by tying it to longer string than by trying it to a shorter string?

(d) How is a swimmer jumping into water from a height able to make loop in air.

Solution

(a) A torque is required to revolve the machine. The torque is equal to the multiplication of applied force and its perpendicular distance from the axis of rotation. Clearly, more the distance the more torque can be applied by the same force on account of this , the handle is put near the circumference.

(b) By using broad handle the perpendicular distance of point of action of force from the axis of rotation increases and hence more torque can be applied with the same force. This facilitates to revolution of the screw.

(c) Let the moment of inertia of stone tied to shorter string be F_1 and that of longer string I_2. If τ_1 and τ_2 respectively be torque in the first and second case to produce the same angular acceleration , α then $\tau_1 = I_1\alpha$ and $\tau_2 = I_2\alpha$. Since particle is more distant from the axis of rotation in second case then in first case, we shall have to apply more torque in second case. Evidently it will be easier to revolve stone tied to a shorter string.

(d) This decreases the moment of inertia I of the swimmer since his angular momentum $I\omega$ remains constant , if I decreases his angular velocity increases and we easily forms a loop in air.

Example – 84

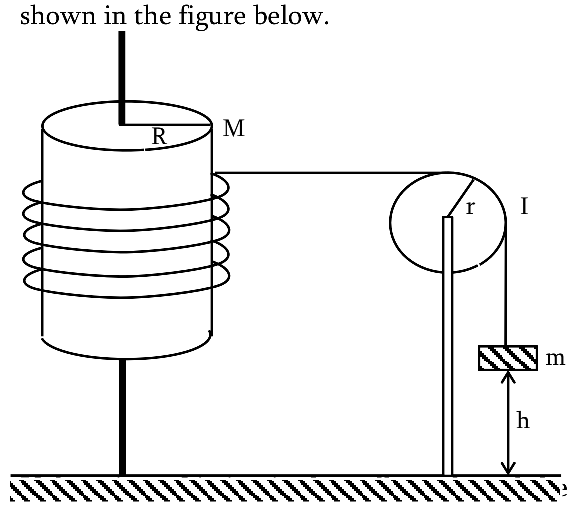
(a) Define angular momentum and give its dimensions.

(b) A grinding wheel in a form of solid cylinder of 0.2m diameter and 3kg mass is rotated at 3600rev/minute.

(i) What is its kinetic energy?

(ii) Find how far it would have to fall to acquire the same kinetic energy as in 2(b)(i) above.

(c) A uniform solid cylinder of mass M and radius R rotates about a vertical axis on a uniform bearing. A massless cord rapped with many turns round the cylinder passes over a pulley of rotational inertia I and radius r and then attached to a small mass , M that is otherwise free fall under the influence of gravity as shown in the figure below.



cord does not slip, what is the speed of the small mass after it has fall a distance h from rest.

Example – 85

(a) (i) discuss how Newton's law of motion are related to angular motion.

(ii) The moment of inertia of a circular disc of mass M and diameter d about an axis through its centre and perpendicular to its plane is given by Md2/8. Deduce an expression for the radius of gyration of the same disc about an axis through its rim and perpendicular to its plane.

(iii) Apply the knowledge you learned in rotation of rigid bodies to explain why spokes are fitted in the cycle wheels.

(b) A grindstone has a moment of inertia of 1.6×10^{-3}kgm^2. When a constant torque is applied , the flywheel reaches an angular

velocity of 1200 rev/min in 15seconds. Assuming it started from the rest , find the:-
(i) Angular acceleration
(ii) Unbalanced torque applied
(iii) Angle turned through in 15sec
(iv) Work done (w) on the flywheel by the torque.

Example – 86
(a) Show that the K.E of rotation of a rigid body about an axis with an angular velocity , ω is given by K.E $= \frac{1}{2}I\omega^2$ where I is the moment of inertia of body about the given axis.
(b) A solid sphere of mass , M and radius 'a' rolls without slipping down a plane inclined at an angle θ to the horizontal obtain an expression for the acceleration of the centre of the solid sphere in terms of g and θ.

Example – 87
(a) (i) Define the moment of inertia of a body.
(ii) State the parallel axes theorem for moment of inertia.
(b) (i) What is the torque (τ)?
(ii) A uniform disc of radius R and mass M is mounted on an axle supported in fixed frictionless bearings. A light cord is wrapped around the rim of the wheel and mass m is attached at the end of the cord. Find the angular acceleration of the disc using the relation $\tau = \frac{dL}{dt}$ and hence the tension in the string or cord.
(c) (i) State the principle of conservation of angular momentum.
(ii) A body stands on the plat form that can only rotate about a vertical axis holding an axle of a rim – loading bicycle , wheel with its axis vertical. The wheel is spinning about this vertical axis with angular velocity ω_0 of the wheel. What will happen?

Example – 88
Calculate the torque τ and angular momentum gained by a steel ball of mass 4.5kg and radius 50mm at the length 12m if the ball is released from the top of the inclined plane 9m above the ground
Ans.0.42Nm ; 1.01Nm.

Example – 89
(a) (i) State the principle of conservation of angular momentum.
(ii) If the earth suddenly contracted to half of its present radius without any external torque on it, by how much would be the day reduced?
(b) (i) Define the term torque?
(ii) A solid cylinder of mass m is placed on a rough inclined plane of inclination θ to the horizontal. Show that the minimum friction force applied (required) for rolling without slipping is ⅓mgsinθ and the minimum coefficient of friction is ⅓tanθ
Ans.(a) (ii) 18hours

Example – 90
(a) What is mean by
(i) Radius of gyration
(ii) Moment of inertia
(b) (i) What can't angular momentum and linear momentum be added.
(ii) A sphere rolls down an inclined plane of 40^{o} from the horizontal. Find its acceleration (M.I is $0.4MR^2$)

Example – 91
(a) (i) Define the term moment of inertia.
(ii) Why is a large torque required to bring about a large change in rotation , if the moment of inertia of the rigid body about that axis is large (02).
(iii) State the principle of conservation of angular momentum. Give two applications of this principle.
(b) (i) An ice – skater is spinning about a vertical axis through his body at a speed of 0.5revolutions per second. He extends his arms horizontally with a weight of mass 2kg in each hand. Assume that the moment of inertia of the skater himself remain constant is $0.8kgm^2$ and the distance of the weights from the axis is 0.9m. Find.

(ii) The total angular momentum of the skater and the weights about the vertical axis.

(c) If the skater in 1(b) pulls his hand to the sides so that the two weights are at distance of 0.2m from the axis of rotation.

(i) Calculate the final rotation speed friction of ice can be neglected.

(ii) What is the change in the total kinetic energy of the skater and the weights? Has this kinetic energy been increased or decreased? How can your account for this change?

Example – 92

(a) In problems involving linear motion the following equation are often used:-