EXAMPLES:

1. (a) (i) What are the basic rules of dimensional analysis.
   (ii) What is the importance of dimensional analysis inspite of its drawn backs?
   (b) The Van der Waals equation for real gases is given by

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

where p is the pressure , R is gas constant , v is the volume and T represent absolute temperature what are the dimensions of the constants 'a' and "b"?

Solution

(a) (i) Dimensional analysis is based on two simple rules:-

- We can add or subtract quantities only if they have the same dimensions. For example we cannot add an area to a force to obtain a meaningful sum.
- An equation is correct if each and every term on the two sides of an equal sign has the same dimension eg. A = B + C i.e [A] = [B] = [C]

(ii) In many physical situations it is very difficult to obtain the formula of a physical quantity it is because the mathematical analysis involved is too difficult in such situations , dimensional analysis can be powerful tool.

(b) Given that

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

Dimensionally

$$[p] = ML^{-1}T^{-2} \ , \ [V] = L^3$$

Since a/v^2 is added on the sides of pressure than have the same dimension with pressure.

$$\frac{[a]}{[v]^2} = [p]$$

$$[a] = [p][v]^2$$

$$= ML^{-1}T^{-2}(L^3)^2$$

$$[a] = ML^5T^{-2}$$

Also b is subtracted form v , thus have the same dimension with volume , v

$$[b] = [v] = L^3$$

2. (a) (i) Define the terms fundamental units and derived units , giving one example for each.
   (ii) Why the units of mass length and time are called fundamental units?
   (b) The wavelength λ of wave associated with momentum of the particle is given by

$$\lambda = h/p$$

Where h is the constant and p represent momentum.

(i) Determine the dimensions of h.
(ii) Suggest the two possible units of the constant , h.

Solution

(a) (i) Refer to your notes.
(ii) The units of these quantities can be defined itself without depends on any other unit(s) of the quantities.

(b) (i) Given that

$$\lambda = h/p$$

$$h = \lambda p$$

Dimensionally

$$[\lambda] = L \qquad [P] = MLT^{-1}$$

$$[h] = [\lambda]\ [p] = L \cdot MLT^{-1}$$

$$[h] = ML^2T^{-1}$$

(ii) Possible units of h

$$h = kgm^2s^{-1} = kgms^{-2} = Nms$$

$$= Js$$

Two possible units of h are kgm^2s^{-1} or Js.

3. (a) Define the terms:-
   (i) Measurement
   (ii) Physical quantity
   (iii) Dimensional analysis.

(b) (i) State the main features or characteristics of units.

(ii) The position X of a particle depend upon time t according to the equation

$$x = at + bt^2.$$

Determine the dimensions and units of 'a' and 'b'. What are the physical quantities denoted by them.

Solution

(a) Refer to your notes

(b) (i) Refer to your notes.

(ii) Given that : $x = at + bt^2$

Dimensionally

$$[x] = L \quad [t] = T$$

Apply principle of dimensional homogeneity.

$$[a][t] = [x]$$

$$[a] = \frac{[x]}{[t]} = \frac{L}{T} = LT^{-1}$$

$$[a] = LT^{-1}$$

Unit of a is m/s and a represent velocity

Again ,

$$[b][t]^2 = [x]$$

$$[b] = \frac{[x]}{[t]^2} = \frac{L}{T^2} = LT^{-2}$$

$$[b] = LT^{-2}$$

Unit of b is m/s^2 and b represent acceleration.

4. (a) The velocity v of the particle depends upon time according to the relation.

$$v = at + \frac{b}{t + c}$$

What are the dimensions of 'a' , 'b' and 'c'?

(b) (i) What is meant by the statement than 'an equation is homogeneous with respect to its units?

(ii) The stress , s required to fracture a solid can be expressed as

$$s = k\sqrt{\frac{\lambda E}{d}}$$

Where k is dimensionless constant , E is the young's modulus and d is the distance between the planes of atoms separated by the fracture. If the equation is dimensionally consistent , find the dimensions of the physical quantity λ and suggest the meaning of this quantity.

Solution

(a) Since $v = at + \frac{b}{t + c}$

Dimensionally

$$[v] = LT^{-1} \; , \; [t] = T$$

Apply principle of dimensions homogeneity.

$$[a][t] = [v]$$

$$[a] = \frac{[v]}{[t]} = \frac{LT^{-1}}{T} = LT^{-2}$$

$$[a] = LT^{-2}$$

Since C is added on the time , ten C have dimension of time.

$$[c] = [t] = T$$

Also

$$\frac{[b]}{[t + c]} = [v]$$

$$[b] = [v][t + c] = LT^{-1}T$$

$$[b] = L$$

(b) (ii) The statement means that each and every term of the expression or equation has been expressed in the same unit.

(ii) Given that

$$s = k\sqrt{\frac{\lambda E}{d}}$$

$$s^2 = k^2 \frac{\lambda E}{d}$$

$$\lambda = \frac{s^2 d}{k^2 E}$$

Dimensionally

$$[s] = ML^{-1}T^{-2} \qquad [d] = L$$

$$[E] = ML^{-1}T^{-2}$$

Now,

$$[\lambda] = \frac{[s]^2 d}{[E]}$$

$$= \frac{[ML^{-1}T^{-2}]^2 \cdot L}{ML^{-1}T^{-2}}$$

$$[\lambda] = ML^{\circ}T^{-2} = MT^{-2}$$

The quantity λ is an elastic constant of the solid.

5. (a) State two advantages of dimensional analysis.
   (b) The force F is given in terms of time 't' and displacement by the equation

$$F = A\cos BX + C\sin Dt$$

What are the dimensions of D/B?

Solution

(a) Refer to your notes

(b) Since BX and Dt are angles and hence are dimensionless quantities.

$$[B][X] = [D][t] = 1$$

$$\left[\frac{D}{B}\right] = \frac{[X]}{[t]} = \frac{L}{T} = LT^{-1}$$

$$\left[\frac{D}{B}\right] = LT^{-1}$$

6. (a) The variables displacement X velocity V and acceleration are related by an equation

$$v^n = 2ax$$

Where n is an integer constant without dimension. What must be the value of n for the formula to dimensionally and consistent?

(b) The number of particles of crossing unit area perpendicular to x – axis in a unit time is given by

$$n = \frac{-D(n_2 - n_1)}{(x_2 - x_1)}$$

Where n_1 and n_2 are the number of particles per unit volume for the value of x_1 and x_2 respectively. What are the dimensions of diffusion constant D?

Solution

(a) Given that

$$v^n = 2ax$$

Dimensionally

$$[v] = LT^{-1} \quad , \quad [a] = LT^{-2}$$

$$[x] = L$$

Now , $[v]^n = [a][x]$

$$[LT^{-1}]^n = LT^{-2} \cdot L$$

$$[LT^{-1}]^n = [LT^{-1}]^2$$

$$\underline{n = 2}$$

(b) Given that

$$n = -D\frac{(n_2 - n_1)}{(x_2 - x_1)}$$

$$D = -n\frac{(x_2 - x_1)}{(n_2 - n_1)}$$

Dimensionally

$$n = \frac{\text{number of particles}}{\text{Area} \times \text{time}}$$

$$[n] = \frac{1}{[A][t]} = \frac{1}{L^2T} = L^{-2}T^{-1}$$

$$n_2 - n_1 = \frac{\text{number of particles}}{\text{volume}}$$

$$[n_2 - n_1] = \frac{1}{L^3} = L^{-3}$$

$$x_2 - x_1 = \text{distance} \quad , \quad [x_2 - x_1] = L$$

Now

$$[D] = \frac{[n][x_2 - x_1]}{[n_2 - n_1]}$$

$$= \frac{L^{-2}T^{-1} \cdot L}{L^3}$$

$$[D] = L^2T^{-1}$$

7. NECTA 2006/P1/1(b)
   (i) Distinguish between fundamental of physical quantities and derived of physical quantities giving one example for each.
   (ii) An equation showing a body that is accelerating vertically upwards is given by $s = at^2 - bt^3$ where s and t are measured in meter and seconds respectively. Determine the dimensions and units of 'a' and 'b'

Solution

(i) Refer to your notes

(ii) Given that $s = at^2 - bt^3$

Dimensionally

$[s] = L$, $[t] = T$

Assume that the given equation is the dimensionally correct.

$$[a][t]^2 = [s]$$

$$[a] = \frac{[s]}{[t]^2} = \frac{L}{T^2} = LT^{-2}$$

$$[a] = LT^{-2}$$

Unit of a is m/s^2

Also

$$[b][t]^3 = [s]$$

$$[b] = \frac{[s]}{[t]^3} = \frac{L}{T^3} = LT^{-3}$$

$$[b] = LT^{-3}$$

Unit of b is m/s^3

8. (a) What is the basis of principle homogeneity of dimensions?
   (b) Find the dimensions of a/b in the equation

$$p = \frac{a - t^2}{bx}$$

Where p is the pressure and x is distance , t is the time.

Solution

(a) Refer to your notes

(b) Given that $p = \frac{a - t^2}{bx}$

$$p = \frac{a}{bx} - \frac{t^2}{bx}$$

Dimensionally

$[p] = ML^{-1}T^{-2}$, $[X] = L$, $[t] = T$

Apply principle of dimensional analysis

$$\frac{[a]}{[b][x]} = [p]$$

$$\left[\frac{a}{b}\right] = [p][x] = ML^{-1}T^{-2}L$$

$$\left[\frac{a}{b}\right] = MT^{-2} = ML^{\circ}T^{-2}$$

9. (a) The velocity of a body moving in viscous medius is given by

$$V = \frac{A}{B}\left[1 - e^{-t/B}\right]$$

Where t is a time , A and B constants . Find dimensions of A.

(b) The equation relating the current I through a semiconductor diode to the applied potential v at temperature T is given by

$$I = I_{\circ}e^{-qv/KT}$$

Where the exponential function, q is the electronic charge, and k is the Boltzmann's constant. Find the units of k.

Solution

(a) Given that

$$V = \frac{A}{B}\left(1 - e^{-t/B}\right)$$

$$V = \frac{A}{B} - \frac{A}{B}e^{-t/B}$$

Dimensionally

$$[V] = LT^{-1} \quad , \quad [t] = T$$

The term $\frac{t}{B}$ is a dimensionless

$$\frac{[t]}{[B]} = 1 \quad , \quad [t] = [B]$$

$$[t] = [B] = T$$

Assume that the equation is the dimensionally correct

$$\frac{[A]}{[B]} = [V]$$

$$[A] = [V][B] = LT^{-1}T$$

$$[A] = M^{\circ}LT^{\circ} = L$$

$$I = Ie^{-qv/KT}$$

Also $\frac{qv}{KT}$ = dimensionless

$$\frac{[q][v]}{[k]} = 1$$

$$[k] = \frac{[q][v]}{[T]} = \frac{CV}{Kelvin}$$

$$[k] = J/Kelv$$

∴ SI unit of K is JK^{-1}

10. The equation of a wave is given by

$$y = r\sin\omega\left[x/v - k\pi\right]$$

Where the symbols have their usual meanings. What are the dimension of x and k?

Solution

Given that :

$$y = r\sin\omega\left[x/v - k\pi\right]$$

$$y = r\sin\left[\frac{\omega x}{v} - k\omega\pi\right]$$

The dimension of argument or angle is $M^oL^oT^o$ or 1

$$\frac{[\omega][x]}{[v]} = 1$$

$$[x] = \frac{[v]}{[\omega]} = \frac{LT^{-1}}{T^{-1}}$$

$$[x] = L$$

Also $[k][\omega] = 1$

$$[k] = \frac{1}{[\omega]} = \frac{1}{T^{-1}} = T$$

$$[k] = T$$

EXERCISE – 1

1. (a) A force is given by $F = at + bt^2$ where t is a time. What are dimensions of 'a' and 'b'?
   (b) What is the dimension of a/b in the expression? $F = a\sqrt{x} + bt^2$ Where F is the force , x is the displacement and t is the time.
   Answer : (a) [a] = MLT^{-3} , [b] = MLT^{-4}
   (b) $\left[\frac{a}{b}\right] = \left[M^{\circ}L^{-1/2}T^2\right]$

2. The displacement of a particle moving along the x – axis is given by $x = at + bt^2 - ct^3$ where t is the time. Find the dimensions of a , b and c
   Answer : [a] = LT^{-1} [b] = LT^{-2} [c] = LT^{-3}

3. In the relation $p = \frac{\alpha}{\beta}e^{-\frac{\alpha z}{k\theta}}$ where p is the pressure , z is distance , k is the Boltzmann constant , θ is the temperature state the dimensional formula of β.
   Answer : [$M^oL^2T^o$]

4. (a) Why do we use square bracket round M , L and T?
   (b) Turpentine oil is flowing through a tube of length L and radius r. The pressure difference between two ends of the tube is p. the viscocity of the oil given by.

$$n = p\frac{(r^2 - x^2)}{4VL}$$

Where v is the velocity of the oil at a distance x from the axis of the tube. What is the dimension of n? Answer : (b) $ML^{-1}T^{-1}$

5. Given the relation

$$v = at + \frac{c}{t+d}$$

Where v is the velocity and t is time

(i) What are the dimensions of a , c and d?

(ii) What does d represents

Answer: (i) [a] = LT^{-2} [c] = L [d] = T

(ii) Time

6. (a) (i) What is a dimensional equations?

(ii) Give two uses of dimensional equations.

(b) The speed v of an object is given by the equation

$$v = \alpha t^3 - \beta t$$

Where t is a time. What are the dimensions of α and β.

7. The number of particles n acrossing a unit area perpendicular to x – axis in a unit time is given as

$$n = \left[\frac{D(n_2^2 - n_1^2)}{(x_2^2 - x_1^2)}\right]^2$$

Where n1 and n_2 are the number of particles per unit volume for the values of x_1 and x_2 respectively. What are the dimensions of diffusion constant , D ?

8. (a) Is it possible for two quantities to have the same dimensions but different units? Support your answer with an example and an explanation.

(b) A student wish to determine integer value of the exponent in the equation y = $c^n at^2$. Dimensions of y , a and t known. It is known that c no dimensions can dimension analysis be used to determine n? account for your answer.

Hints:

(a) Yes , it is possible for example the dimensions of torque and work both are ML^2T^{-2} . However, torque has a unit of Nm and work has a unit joule (J).

(b) No , dimensional analysis cannot used to determine the value of the exponent n. this is because c is a dimensionless constant.

9. The equation relating current I through a semiconductor diode to the applied potential difference v at temperature T is

$$I = I_o e^{-\frac{ev}{kT}}$$

Where e is the electron charge and k is the constant known as Boltzmann constant. What is the dimension of k? Answer: $ML^2T^{-2}K^{-1}$

EXAMPLES

11. (a) Differentiate between the physical equation and dimension equation .

(b) Use the method of dimensional analysis to find the expression for drag force F given that F is a function of radius R , density ρ and velocity , v

Solution

(a) Physical equation is a mathematical expression with physical parameters with some unknown which are supposed to be found. While dimensional equation is the equation obtained by equating the physical quantity with its dimensional formula

(b) $F \alpha R^x \rho^y V^z$

$F = KR^x \rho^y V^z$

Where k is the dimensionless constant x , y and z are any real numbers

Dimensionally

[F] = MLT^{-2} [R] = L

[ρ] = ML^{-3} [V] = LT^{-1}

Now

[F] = $[R]^x [\rho]^y [V]^z$

$MLT^{-2} = L^x (ML^{-3})^y (LT^{-1})^z$

On equating indices / powers

M : 1 = y ……………………..…(i)

L : 1 = x – 3y + z ……………(ii)

T : -2 = -z …………………....(iii)

On solving z = 2 , y = 1

$1 = x - 3 + 2$

$x = 2$

$F = KR^2\rho V^2$

12. (a) The frequency f of a note produced by a wire stretched between two supports depends on the distance L between the supports , mass per unit length of wire μ and the tension (T) in the wire. Use dimensional analysis to find how f is related to l , μ and T.

(b) The mass (M) of the tangent stone that can be moved by a flowing river depends on the velocity (V) of the river , the density (ρ) of the water and the acceleration due to gravity , g . Find how M is related to V , g and ρ.

Solution

(a) $f \;\alpha\; L^x\mu^yT^z$

$f = kL^x\mu^yT^z$

K is the dimensionless constant , x and z are any real numbers.

Dimensionally

$[F] = T^{-1}$ $\quad [L] = L$

$[\mu] = ML^{-1}$ $\quad [T] = MLT^{-2}$

Now

$[F] = [L]^x[\mu]^y[T]^z$

$M^0L^0T^{-1} = L^x(ML^{-1})^y(MLT^{-2})^z$

$M^0L^0T^{-1} = M^{y+z}L^{x-y+z}T^{-2z}$

On equating indices/ power

M : 0 = y + z………….(i)

L : 0 = x – y + z………(ii)

T : -1 = -2z……………(iii)

On solving simultaneously

x = -1 $\quad$ y = -½ $\quad$ z = ½

f = KL-1μ-½ T½

$$f = \frac{K}{L}\left(\frac{T}{\mu}\right)^{1/2} = \frac{K}{L}\sqrt{\frac{T}{\mu}}$$

(b) $M \;\alpha\; V^x\rho^yg^z$

$M = KV^x\rho^yg^z$

Where k is dimensionless constant x , y and z are any real numbers

Dimensionally

$[M] = [V]^x[\rho]^y[g]^z$

$M^1L^0T^0 = (LT^{-1})^x(ML^{-3})^y(LT^{-2})z$

$ML^0T^0 = MyL^{x-3y+z}T^{-x-2z}$

On equating indices

M : 1 = y………………..(i)

L : 0 = x – 3y + z……...(ii)

T : 0 = -x – 2z…………(iii)

On solving simultaneously

X = 6 , y = 1 , z = -3

$M = KV^6g^{-3}\rho$

$$M = \frac{KV^6\rho}{g^3} \quad \text{or} \quad M \;\alpha\; \frac{V^6\rho}{g^3}$$

13. The velocity of transverse waves along a string can be expressed in terms of the tension in the string (T) and mass per unit length (μ) in the string. Use the method of dimensional analysis to derive the expression of V in terms T and μ.

Solution

$V \;\alpha\; T^x\mu^y$, $V = KT^x\mu^y$

Where x , y and k are dimensionless constant.

Dimensionally

$[V] = LT^{-1}$ $\quad [T] = MLT^{-2}$

$[\mu] = ML^{-1}$

Now

$[V] = [T]^x[\mu]^y$

$M^0LT^{-1} = (MLT^{-2})^x(ML^{-1})^y$

$M^0LT^{-1} = M^{x+y}L^{x-y}T^{-2X}$

On equating indices

M : 0 = x + y………………(i)

L : 1 = x – y ……………...(ii)

T : -1 = -2x………………..(iii)

On solving : x = ½ , y = -½

$V = KT^{½}\mu^{-½}$

$$V = K\sqrt{\frac{T}{\mu}}$$

14. (a) (i) With the help of example distinguish between dimensions and unit.

(ii) What is the basic requirement for a physical relation to be correct?

(b) The rate flow of volume v/t of the fluid in the pipe of length L found to depend on the pressure gradient (p/L) coefficient of viscosity (n) and the radius (r) of the pipe. Using dimensional analysis obtain a relation between v/t and the given quantities.

Solution

(a) (i) Refer to your notes

(ii) Dimensional consistency is the basic requirement for a physical relation to be correct , it of the course not sufficient.

(b) $\frac{v}{t} \alpha \left(\frac{p}{L}\right)^x n^y r^z$

$\frac{v}{t} = k\left(\frac{p}{L}\right)^x n^y r^z$

Where k is the dimensionless constant , x , y and z are any real numbers

Dimensionally

$[v/t] = L^3T^{-1}$ $[A/L] = ML^{-2}T^{-2}$

$[n] = ML^{-1}T^{-1}$ $[r] = L$

Now

$$\left[\frac{v}{t}\right] = \left[\frac{p}{L}\right]^x [n]^y [r]^z$$

$M^0L^3T^{-1} = (ML^{-2}T^{-2})^x (ML^{-1}T^{-1})^y L^z$

$M^0L^3T^{-1} = M^{x+y} L^{-2x-y+Z} T^{-2x-y}$

On equating indices

M : $0 = x + y$……………...…(i)

L : $3 = -2x - y + z$……….(ii)

T : $-1 = -2x - y$…………..(iii)

On solving simultaneous

$X = 1 , y = -1 , z = 4$

$\frac{v}{t} = K\left(P/L\right)^1 n^{-1} r^4$

$\frac{v}{t} = \frac{kpr^4}{nL}$

15. NECTA 2001/P1/1(b)

(i) Mention any two uses of dimension analysis

(ii) The velocity (v) of a liquid beyond which streamline flows ceases and turbulence begins depends on the radius (r) of the tube density (ρ) and viscosity (n) of the liquid. Using dimensions , obtain an expression of v in terms of r , ρ and n.

Solution

(i) Refer to your notes

(ii) $V \alpha r^x \rho^y n^z$

$V = kr^x \rho^y n^z$

Where k , x , y and z are dimensionless constants.

Dimensionally

$[V] = LT^{-1}$, $[r] = L$ $[\rho] = ML^{-3}$

$[n] = ML^{-1}T^{-1}$

Now

$[V] = [r]^x [\rho]^y [n]^z$

$M^0LT^{-1} = L^x(ML^{-3})^y(ML^{-1}T^{-1})^z$

$M^0LT^{-1} = M^{y+z} L^{x-3y-z} T^{-z}$

On equating indices/powers

M : $0 = y + z$……..……(i)

L : $1 = x - 3y - z$…….(ii)

T : $-1 = -z$…………....(iii)

On solving $x = -1 , y = -1 , z = 1$

$V = kr^{-1}\rho^{-1}n = \frac{kn}{\rho r}$

16. NECTA 2002 /P1/1(b)

(i) What are dimensional equation?

(ii) State any two uses of dimensional equation?

(iii) A gas bubble from an explosion under water is found to oscillate with period T , which is proportional to P^a d^b and E^c where p is pressure , d is the density and E is the energy of the explosion. Find the values of the a , b and c . Hence determine the units of the proportionality.

Solution

(i) and (ii) Refer to your notes

(iii) $T \alpha P^a d^b E^C$

$T = KP^a d^b E^C$

Dimensionally

$[T] = [P]^a [d]^b [E]^c$

$M^0LT = (ML^{-1}T^{-2})^a(ML^{-3})^b (ML^2T^{-2})^c$

$M^0L^0T^1 = M^{a+b+c} L^{-a-3b\_2c} T^{-2a-2c}$

On equating indices / powers

M : $0 = a + b + c$…………...….(i)

L : $0 = -a - 3b + 2c$………….(ii)

T : $1 = -2a - 2c$………………(iii)

On solving simultaneously

$$a = \frac{-5}{6}\quad , \quad b = \frac{1}{2}\quad , \quad c = \frac{1}{3}$$

Since k is constant of proportionality i.e dimensionless constant , therefore k has no unit.

17. NECTA 2003 /P1/1(a)
   (i) State the universal law and gravitation and find the dimension of G.
   (ii) The viscosity n of the gas depend on the mass M, the effective diameter d and the mean speed of the molecules v , use dimensional analysis to find and expression for n. hence estimates the diameter of methane (CH_4) molecule given that n = 2 × 10^{-5}kgm-1s^{-1} for helium and n = 1.1 × 10^{-5}kgm^{-1}s^{-1} for methane and that the diameter of the helium is 2.1 × 10^{-10}m.

   Solution

   (i) The law state that "The magnitude of gravitational force of attraction between two heavy bodies in the inverses is directly proportional to the product of masses and inversely proportional to the square of their distance apart".

$$F = \frac{GM_1M_2}{r^2}$$

Dimension of G

$$G = \frac{Fr^2}{M_1M_2}$$

$$[G] = \frac{[F][r]^2}{[M_1][M_2]}$$

$$= \frac{MLT^{-2}\cdot L}{MM}$$

$$[G] = M^{-1}L^3T^{-2}$$

   (ii) n = km^{x}d^{y}v^z

   Where k , x , y and z are dimensionless constants.

   Dimensionally

   [n] = [m]^x [d]^y [z]^z

   MLT^{-1} = M^{x}L^y(LT^{-1})^z

   M^{-1}T^{-1} = M^x L^{y+z} T^{-z}

   On equating indices

   M : 1 = x…………….(i)

   L : -1 = y + z………(ii)

   T : -1 = -z…………(iii)

On solving : x = 1 , y = -1 , z = 1

n = kmd-2v

$$n = \frac{kmv}{d^2}$$

Mass of helium (molar mass) M_1 = 4

Mass of methane (molar mas) M_2 = 16

$$d = \sqrt{\frac{kmv}{n}}$$

Methane : $d_2 = \sqrt{\frac{km_2v}{n_2}}$

Helium : $d_1 = \sqrt{\frac{km_2v}{n_1}}$

$$\frac{d_2}{d_1} = \sqrt{\frac{km_2v}{n_2}}\Big/\sqrt{\frac{km_1v}{n_1}}$$

$$d_2 = d_1\sqrt{\left(\frac{m_2}{m_1}\right)\left(\frac{n_1}{n_2}\right)}$$

$$= 2.1\times10^{-10}\sqrt{\left(\frac{16}{4}\right)\left(\frac{2\times10^{-5}}{1.1\times10^{-5}}\right)}$$

$$d_2 = 5.6633\times10^{-10}\,\text{m}$$

18. NECTA 2004/P1/1(a)
   (i) What is meant by the term dimensions of physical quantity?
   (ii) Give two uses of dimensions analysis.
   (iii) Use the method of dimension to obtain the relationship between the lift force per unit wing span on an aircraft wing of width L moving with velocity v through air of density , ρ on the parameter L , V and ρ

   Solution

   (iii) Let ϕ = lift force per width of wing span

$$\phi \propto l^xv^y\rho^z$$

$$\phi = kL^xv^y\rho^z$$

K, x, y and z are any real numbers.

Dimensionally

$$[\Phi] = \left[\frac{F}{L}\right] = \frac{MLT^{-2}}{L} = ML^0T^{-2}$$

[L] = L [V] = LT-1

$[\rho] = ML^{-3}$

Now,

$$[\Phi] = [L]^x [V]^y [\rho]^z$$

$$ML^0T^{-2} = L^x \left(LT^{-1}\right)^y \left(ML^{-3}\right)^z$$

$$ML^0T^{-2} = M^z L^{x+y-3z} T^{-y}$$

On equating indices/powers

M : $1 = z$……………….(i)

L : $0 = x + y - 3z$……..(ii)

T : $-2 = -y$……………..(iii)

On solving simultaneously

$x = 1$, $y = 2$, $z = 1$

$\Phi = KL\rho V^2$

19. Viscosity force (F) on the sphere moving in a fluid is found to depend on radius of the sphere (r) coefficient of viscosity (n) of the fluid and the speed (v) of the speed. Find an expression of force (f) that relates the given quantities.

Solution

$F \alpha r^x n^y v^z$

$F = k r^x n^y v^z$

Where k , x , y and z are dimensionless quantities.

Dimensionally

$[f] = MLT^{-2}$ $\quad [r] = L$

$[n] = ML^{-1}T^{-1}$ $\quad [V] = LT^{-1}$

Now

$[F] = [r]^x [n]^y [v]^z$

$MLT^{-2} = L^x (ML^{-1}T^{-1})^y (LT^{-1})^z$

$MLT^{-2} = M^y L^{x-y+z} T^{-y-z}$

On equating indices / powers

M : $1 = y$ …………………(i)

L : $1 = x - y + z$…………(ii)

T : $-2 = -y - z$ …………..(iii)

On solving : $x = y = z = 1$

$\therefore F = krnv$

20. (a) A liquid having small depth but large volume is forced by an applied pressure p above it to escape with velocity , v through a small hole below if v , is given by

$$V = cp^x \rho^y$$

Where ρ is the liquid density c , x and y are dimensionless constant.

(i) Determine x and y

(ii) If v = 14m/s when p = 1×10^5Pa and ρ = 1000kgm^{-3}. Deduce C.

(b) After being deformed , a spherical drop of liquid will execute periodic vibrations about its sphere . the frequency f of vibrations of the drop will depend on the surface tension γ of the drop , its density ρ and on the radius r of the drop . Using the method of dimensions obtain an expression for the frequency of these vibrations in terms of the related physical quantities.

Solution

(a) $V = cp^x \rho^y$

Dimensionally

$[v] = LT^{-1}$ $\quad [P] = ML^{-1}T^{-2}$

$[\rho] = ML^{-3}$ $\quad [V] = [P]^x [\rho]$

y

$M^0LT^{-1} = (ML^{-1}T^{-2})^x (ML^{-3})^y$

$M^0LT^{-1} = M^{x+y} L^{-x-3y} T^{-2x}$

On equating indices or powers

M : $0 = x + y$…………….(i)

L : $1 = -x - 3y$…………(ii)

T : $-1 = -2x$……………(iii)

On solving : $x = ½$, $y = -½$

(ii) $V = C\sqrt{p/\rho}$

$$14 = C\sqrt{\frac{10^5}{1000}}$$

$C = 1.4$

(b) $F \alpha r^x \rho^y \gamma^z$

$F = kr^x \rho^y \gamma^z$

K , x , y and z dimensionless quantities

Dimensionally

$[f] = T^{-1}$ $\quad [r] = L$

$[\rho] = ML^{-3}$ $\quad [\gamma] = MT^{-2}$

Now

$[f] = [r]x [\rho] y [\gamma]z$

$M^0L^0T^{-1} = L^x (ML^{-3})^y (MT^{-2})^z$

$M^0L^0T^{-1} = M^{y+z} L^{x-3y} T^{-2z}$

On equating indices / powers

M : $0 = y + z$……………...(i)

L : $0 = x - 3y$…………….(ii)

T : $-1 = -2x$………………(iii)

On solving $x = \frac{3}{2}$, $y = -½$, $z = ½$

$$f = kr^{-3/2}\rho^{-1/2}\gamma^{1/2}$$

$$f = k\sqrt{\frac{\gamma}{\rho r^3}}$$

21. NECTA 2007/P1/1(b)
    (ii) The frequency f of a note given by an organ pipe depends on the length L the air pressure p and the air density D . use the method of dimensions to find a formula for the frequency.
    (iii) What will be new frequency of a pipe whose original frequency was 256Hz if the air density fall by 2% and the pressure increases by 1%.

    Solution

    (ii) $F \propto L^x p^y d^z$

    $F = KL^x p^y d^z$

    K = constant of proportionality.

    x , y and z are any real numbers.

    Dimensionally

    $[f] = T^{-1}$ $\qquad [L] = L$

    $[p] = ML^{-1}T^{-2}$ $\qquad [D] = ML^{-3}$

    Now

    $[f] = [L]^x [p]^y [D]^z$

    $M^0L^0T^{-1} = L^x (ML^{-1}T^{-2})^y (ML^{-3})^z$

    $M^0L^0T^{-1} = M^{y+z} L^{x-y-3z} T^{-2y}$

    On equating indices / powers

    M : $0 = y + z$…………………(i)

    L : $0 = x - y - 3z$……..……(ii)

    T : $-1 = -2y$…………………(iii)

    On solving $x = -1$, $y = ½$, $z = -½$

    $F = KL^{-1}P^{½} D^{-½}$

    $$F = \frac{K}{L}\sqrt{\frac{P}{D}}$$

(iii) $F = \frac{K}{L}\sqrt{\frac{P}{D}}$

Since K and L re constant

The maximum fractional error on f

$$\frac{Df}{f} = \frac{1}{2}\left[\frac{\Delta p}{p} + \frac{\Delta D}{D}\right]$$

Percentage error

$$\frac{Df}{f}\times 100\% = \frac{1}{2}\left[\frac{\Delta p}{p}\times 100\%\right] + \frac{1}{2}\left[\frac{\Delta D}{D}\times 100\%\right]$$

$$= \frac{1}{2}\times 1\% + \times 2\%$$

$$\frac{\Delta f}{f}\times 100\% = \frac{3}{2}\%$$

$$\frac{Df}{f} = 0.015$$

$\Delta f = 0.015f = 0.015 \times 256$

$\Delta f = 3.84Hz$

New frequency $f' = f + \Delta f$

$f' = 256 + 3.84$

$f' = 259.84Hz$

22. NECTA 2010/P1/1
    (a) Mention two uses of dimensional analysis
    (b) The critical velocity of a liquid in a certain pipe is 3m/s. Assuming that the critical velocity v depends on the density (ρ) of the liquid, its viscosity , n and the diameter of the pipe , d.
        (i) Use the method of dimensional analysis to derive the equation of the critical velocity of the liquid in a pipe of half the diameter.
    (ii) A freely body acquire a velocity $g^x h^y$ after falling through height , h . using dimensions to find the value of x and y

Solution

(a) Refer to your notes

(b) (i) $v \propto \rho^x n^y d^z$

$V = k\rho^x n^y d^z$

K , x , y and z are any real numbers

Dimensionally

$[v] = [\rho]^x [n]^y [d]^z$

$= (ML^{-3})^x (ML^{-1}T^{-1})^y (L)^z$

$M^0LT^{-1} = M^{x+y} L^{-3x-y+z} T^{-y}$

On equating indices or powers

M : $0 = x + y$ …………(i)

L : $1 = -3x - y + z$……..(ii)

T : $-1 = -Y$………………(iii)

$y = 1$, $x = -y = 1$

$1 = -3(-1) - 1 + z$

$1 = 3 - 1 + z = 2 + z$

$z = -1$

$v = k\rho^{-1}n^1d^{-1}$

$$v = \frac{kn}{\rho d}$$

(ii) $v_1 = 3m/s$, $d_1 = d$

$v_2 = ?$ $d_2 = d/2$

$$v_1 = \frac{kn}{\rho d_1} \quad , \quad v_2 = \frac{kn}{\rho d_2}$$

$$\frac{v_2}{v_1} = \frac{kn}{\rho d_2} \bigg| \frac{kn}{\rho d_1} = \frac{d_1}{d_2}$$

$$v_2 = v_1\left[\frac{d_1}{d_2}\right] = 3m/s\left[\frac{d}{d/2}\right]$$

$v_2 = 6m/s$

(c) Student assignment

$x = y = ½$

23. (a) If force (F) , area (A) and density (D) are taken as fundamental units , find the dimensional formula for M , L and T in terms of F , A and D.

(b) A steel ball of radius r is allowed to fall under gravity through a column of a viscous liquid of coefficient of viscosity n after some time the velocity of the ball attains a constant volume V_T. The terminal velocity depends upon

(i) The weight of the ball mg

(ii) Coefficient of viscosity n

(iii) Radius of the ball by the method of dimensions derives the relation for terminal velocity.

Solution

(a) Dimensionally

$[F] = [M^1L^1T^{-2}]$ $[A] = [L^2]$

$[D] = [M^1L^{-3}]$

Now

$M = [DL^3]$ $L = [A^{½}]$

$[M] = [DA^{3/2}]$

Now

$F = [M^1L^1T^{-2}]$

$F = [DA^{3/2}A^{½}T^{-2}]$

$T^2 = DA^2F^{-1}$

$T = [D^{½}A^1F^{-½}]$

(b) $V_T \propto (mg)^a n^b r^c$

$$V_T = k(mg)^a n^b r^c$$

Where k is a dimensionless constant

Dimensionally

$[V_T] = [mg]^a [n]^b [r]^c$

$M^0L^1T^{-1} = (MLT^{-2})^a (ML^{-1}T^{-1})^b L^c$

$M^0L^1T^{-1} = M^{a+b}L^{a-b+c}T^{-2a-b}$

On equating indices

M : $0 = a + b$……………………(i)

L : $1 = a - b + c$……………….(ii)

T : $-1 = -2a - b$………………(iii)

On solving : $a = 1$, $b = c = -1$

$$V_T = k\frac{mg}{nr}$$

EXAMPLES

24. (a) Can dimensional analysis tell you that a physical relation is completely right?

(b) The frequency of vibration of stretched string is a function of tension (T) , length (L) and mass per unit length (μ) . using dimensional analysis to prove that

$$f = \frac{K}{L}\sqrt{\frac{T}{\mu}}$$

Where k is the dimensionless constant

Solution

(a) Even if a physical relation is dimensionally correct , it does not prove that the relation is completely correct it is because the numerical factor in the relation can be wrong. Thus a dimensional check can tell you when a relation is wrong , it cannot tell you that it is completely right.

(b) $f = \frac{K}{L}\sqrt{\frac{T}{\mu}}$

Dimensionally

$[F] = T^{-1}$ $[T] = MLT^{-2}$

$[\mu] = ML^{-1}$

$[L.H.S] = [F] = T^{-1}$

$$[R.H.S] = \frac{1}{[L]}\sqrt{\frac{[T]}{[\mu]}} = \frac{1}{L}\sqrt{\frac{MLT^{-2}}{ML^{-1}}}$$

$$= \frac{1}{L}\cdot LT^{-1} = T^{-1}$$

$[R.H.S] = T^{-1}$

Since [L.H.S] = [R.H.S] = T^{-1} , Therefore the given equation is dimensionally correct.

25. (a) Check the correctness of the following equation.

$$h = \frac{2T\cos\theta}{rdg}$$

Where θ is the angle of contact , d is the density of the liquid , r is the radius of the tube , g is the acceleration due to gravity , h is the height of the liquid and T is the surface tension.

(b) Using dimensional analysis , check the correctness of the following relations:-

(i) $T = 2\pi\sqrt{\frac{L}{g}}$

(ii) $F = 6\pi nvr$

Each symbol have usual meaning

Solution

(a) Given that $h = \frac{2T\cos\theta}{rdg}$

Dimensionally

$[h] = L \quad [T] = MT^{-2}$

$[r] = L \quad [d] = ML^{-3}$

$[g] = LT^{-2}$

Now

$[L.H.S] = [h] = L$

$$[R.H.S] = \frac{[T]}{[r][d][g]} = \frac{MT^{-2}}{LML^{-3}\cdot LT^{-2}}$$

$[R.H.S] = L$

Since $[L.H.S] = [R.H.S] = L$

Therefore the equation is the dimensionally correct.

(b) (i) $T = 2\pi\sqrt{\frac{L}{g}}$

$[L.H.S] = [T] = T$

$$[R.H.S] = \sqrt{\frac{[L]}{[g]}} = \sqrt{\frac{1}{LT^{-2}}}$$

$[R.H.S] = T$

Since $[L.H.S] = [R.H.S] = T$

Therefore the given equation is the dimensionally correct.

(ii) $F = 6\pi nvr$

$[L.H.S] = [F] = MLT^{-2}$

$[R.H.S] = [n][v][r]$

$= ML^{-1}T^{-1}LT^{-1}L$

$[R.H.S] = MLT^{-2}$

Since $[R.H.S] = [L.H.S] = MLT^{-2}$

Thus, the equation is dimensionally correct.

26. NECTA 2005 /P1/(b)

(i) Distinguish between derive and fundamental quantities

(ii) A small liquid drop is distributed its spherical shape and thus set oscillation , the frequency of oscillation is given by $f^2\rho r^3 = k\gamma$ where ρ is the density of the liquid drop r is its radius , γ is the surface tension of the liquid. Show by the dimensional analysis that k is dimensionless constant.

Solution

(i) Refer to your notes

(ii) Given that $f^2\rho r^3 = k\gamma$

$$k = \frac{f^2\rho r^3}{\gamma}$$

Dimensionally

$$[k] = \frac{[f]^2[\rho][r]^3}{[\gamma]}$$

$$= \frac{(T^{-1})^2 ML^{-3}L^3}{MT^{-2}}$$

$[k] = M^0L^0T^0 = 1$

K is the dimensionless constant .

27. Using the method of dimensions indicate which the equation are dimensionally correct and which are not given that f = frequency , γ = surface tension , ρ = density , r = radius , k = dimensionless constant.

(i) $\rho^2 = k\sqrt{\frac{r^3f}{\gamma}}$

(ii) $f = \frac{kr^3\sqrt{\gamma}}{\rho^{1/2}}$

(iii) $f = \dfrac{k\gamma^{1/2}}{\sqrt{\rho r^{3/2}}}$

Solution

Dimensionally

$[f] = T^{-1}$ $[\gamma] = MT^{-2}$

$[\rho] = ML^{-3}$ $[r] = L$

(i) $\rho^2 = k\sqrt{\dfrac{r^3 f}{\gamma}}$

$[L.H.S] = [\rho]^2 = (ML^{-3})^2$

$[L.H.S] = M^2L^{-6}$

Also

$$[R.H.S] = \sqrt{\frac{[r]^3[f]}{[\gamma]}} = \sqrt{\frac{L^3T^{-1}}{MT^{-2}}}$$

$[R.H.S] = M^{½}\ L^{3/2}T^{½}$

Since $[L.H.S] \neq [R.H.S]$, thus the given equation is the dimensionally incorrect

(ii) $f = \dfrac{kr^3\sqrt{\gamma}}{\rho^{1/2}}$

$[L.H.S] = [F] = T^{-1}$

$$[R.H.S] = \frac{[r]^3 \cdot [\gamma]^{1/2}}{[\rho]^{1/2}}$$

$$= \frac{L^3 \cdot (MT^{-2})^{1/2}}{(ML^{-3})^{1/2}} = \frac{L^3 \cdot M^{1/2}T^{-1}}{M^{1/2}L^{-3/2}}$$

$$[R.H.S] = M^0L^{9/2}T^{-1}$$

Since $[R.H.S] \neq [L.H.S]$, then the equation is the dimensionally incorrect.

(iii) $f = \dfrac{k\gamma^{1/2}}{\sqrt{\rho r^{3/2}}}$

$[L.H.S] = T^{-1}$

$$[R.H.S] = \frac{[\gamma]^{1/2}}{[\rho]^{1/2}[r]^{3/2}} = \frac{(MT^{-2})^{1/2}}{(M^{-3})^{1/2} \cdot L^{3/2}}$$

$$= \frac{M^{1/2}T^{-1}}{M^{1/2}L^{-3/2} \cdot L^{3/2}} = T^{-1}$$

$[R.H.S] = T^{-1}$

Since $[L.H.S] = [R.H.S] = T^{-1}$, thus the given equation is the dimensionally correct.

28. (a) Show that the equation relating the current density (J) in the wire to the drift velocity v of the electron is $J = nv_e$ where e is the charge of an electron and n is the electron density.

(b) It is suggested that the pressure p at depth h in a liquid of the density ρ is p = chρg , where g is the acceleration due to gravity. Show that this equation is dimensionally correct.

Solution

(a) $J = nve$

$J = I/A$

Unit on L.H.S. $= Am^{-2}$

Unit on R.H.S $= m^{-3}ms^{-1}c = Am^{-2}$

Since unit of L.H.S and R.H.S of equation is the same. Thus the equation is the dimensionally correct.

(b) Given that

$P = ch^x\rho^y g^z$

Dimensionally

$[P] = ML^{-1}T^{-2}$ $[h] = L$

$[g] = LT^{-2}$ $[\rho] = ML^{-3}$

Now

$[P] = [h]^x[\rho]^y[g]^z$

$ML^{-1}T^{-2} = M^yL^{x-3y+z}T^{-2z}$

On equating indices

M : 1 = y……………..……….(i)

L : -1 = x – 3y + z ………..(ii)

T : -1 = - 2z……………….(iii)

On solving : x = y = z = 1

P = chρg.

Therefore the equation is the dimensionally correct.

29. Check the correctness of the following results

(i) Time period of the satellite is given by

$$T = \sqrt{\frac{3\pi}{\rho G}}$$

G = gravitational constant
ρ = density

(ii) The density of the earth is given by

$$\rho = \frac{3g}{4\pi GR}$$

G = gravitational constant.
R = radius of Earth

(iii) Time period of the tensional oscillation is

$$T = 2\pi\sqrt{\frac{I}{C}}$$

Where I = moment of inertia , c = couple per unit twist [ans. All are correct]

30. (a) Distinguish between fundamental and derived quantities. Give two examples of each

(b) The velocity of propagation on v ripples on surface of a liquid is given by one of the following equations.

(i) $v^2 = \frac{k\rho\lambda}{T}$ (ii) $v^2 = \frac{kT}{\lambda\rho}$

(iii) $v = k\rho\lambda T^2$ (iv) $v = \frac{k\rho T}{\lambda}$

Where k is the a constant , T is the surface tension of the liquid , ρ its density and λ is the wavelength of the ripples. Using the dimensional analysis to determine which is equation is correct.

(c) By graphical method or otherwise use the following data for water to confirm your choice and determine the value of k

Density of water = $1000 kgm^{-1}$
Surface tension of water = $72 \times 10^{-2} Nm^{-1}$
Determine the value of k

V(m/s)	0.70	0.60	0.50	0.40	0.30
λ $\times 10^{-2}$m	0.092	0.125	0.178	0.280	0.50

Solution

(a) Refer to your notes

(b) Dimensionally

$[V] = LT^{-1}$ $[\rho] = ML^{-3}$
$[\lambda] = L$ $[T] = MT^{-2}$

(i) $$v^2 = \frac{k\rho\lambda}{T}$$

$[L.H.S] = [V]^2 = (LT^{-1})^2 = L2T^{-2}$

$$[R.H.S] = \frac{[\rho][\lambda]}{[T]} = \frac{ML^{-3}\cdot L}{MT^{-2}}$$

$[R.H.S] = L^{-2}T^2$

Since [L.H.S] ≠ [R.H.S] , therefore the equation is dimensionally incorrect.

(ii) $$v^2 = \frac{kT}{\lambda\rho}$$

$[L.H.S] = [V]^2 = (LT^{-1})^2$
$[L.H.S] = L^2T^{-2}$

Again

$$[R.H.S] = \frac{[T]}{[\lambda][\rho]} = \frac{MT^{-2}}{L\cdot MT^{-3}}$$

$[R.H.S] = L^2T^{-2}$

Since $[L.H.S] = [R.H.S] = L^2T^{-2}$

Therefore the given equation is the dimensionally correct.

(iii) $$v = k\rho\lambda T^2$$

$[L.H.S] = [V] = LT^{-1}$
$[R.H.S] = [\rho][\lambda][T]^2$
$= ML^{-3}.L(MT^{-2})^2$
$[R.H.S] = M^3L^{-2}T^{-4}$

[L.H.S] ≠ [R.H.S] , thus the equation is dimensionally incorrect

(iv) $$v = \frac{k\rho T}{\lambda}$$

$[L.H.S] = [V] = LT^{-1}$

$$[R.H.S] = \frac{[\rho][T]}{[V]} = \frac{ML^{-3}\cdot MT^{-2}}{LT^{-1}}$$

$[R.H.S] = M^2L^{-4}T^{-1}$

Since [L.H.S] ≠ [R.H.S] , then the equation is incorrect.

(c) Correct equation

$$v^2 = \frac{kT}{\lambda\rho} = \left(\frac{kT}{\rho}\right)\frac{1}{\lambda}$$

$$v^2 = \left(\frac{kT}{\rho}\right)\cdot\frac{1}{\lambda} + 0$$

y = m x + c

EXAMPLES

31. If the density of mercury is $13.6g/cm^3$ converts it value into kgm^{-3} by using dimensional equation

Solution

Dimensional formula of density

$[\rho] = [M^1L^{-3}T^0]$

a = 1 , b = -3 , c = 0

Cgs unit	S.I Unit
$n_1 = 13.6$	$n_2 = ?$
$M_1 = 1gm$	$M_2 = 1kg$
$L_1 = 1cm$	$L_2 = 1m$
$T_1 = 1sec$	$T_2 = 1sec$

$$n_2 = n_1\left[\frac{m_1}{m_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$= 13.6\left[\frac{1}{1000}\right]^1\left[\frac{1}{100}\right]^{-3}\left[\frac{1}{1}\right]^o$$

$$= 13.6\times10^3 kgm^{-3}$$

$$13.6g/cm^3 = 13600kgm^{-3}$$

32. Convert an acceleration of $9.8m/s^2$into km/h^2

Solution

The physical quantity is acceleration having dimensional formula as $M^0L^1T^{-2}$

a = 0 , b = 1 , c = -2

System 1	System 2
$n_1 = 98$	$n_2 = ?$
$M_1 = 1kg$	$M_2 = 1kg$
$L_1 = 1m$	$L_2 = 1km$
$T_1 = 1s$	$T_2 = 1hr$

Applying

$$n_2 = n_1\left[\frac{m_1}{m_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$= 9.8\left[\frac{1}{1}\right]^o\left[\frac{1}{1000}\right]^1\left[\frac{1}{3600}\right]^{-2}$$

$$= 127008kmh^{-2}$$

$$9.8m/s^2 = 1270008kmh^{-2}$$

33. Convert kinetic energy of 5J into erg.

Solution

Dimensional formula of k.e is $M^1L^2T^{-2}$

a = 1 , b =2 , T = -2

System 1 (MKS)	System 2 (Cgs)
$n_1 = 5$	$n_2 = ?$
$M_1 = 1Kg$	$M_2 = 1g$
$L_1 = 1m$	$L_2 = 1cm$
$T_1 = 1sec$	$T_2 = 1sec$

$$n_2 = n_1\left[\frac{m_1}{m_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$= 5\left[\frac{1000}{1}\right]^L\left[\frac{100}{1}\right]^2\left[\frac{1}{1}\right]^{-2}$$

$$n_2 = 5\times10^7$$

$$\therefore 5J = 5\times10^7 erg$$

34. The density of a material in c.g.s system is $8g/cm^3$. In a system of units in which unit of length is 5cm, unit of mass is 20g and unit of time 1sec , what is density?

Solution

The dimensional formula for density is $[M^1 L^{-3}T^0]$

a = 1 , b = -3 , c = 0

System 1	System 2
$M_1 = 1g$	$M_2 = 20g$
$L_1 = 1cm$	$L_2 = 5cm$
$T_1 = 1cm$	$T_2 = 1sec$
$n_1 = 8$	$n_2 = ?$

$$n_2 = n_1\left[\frac{m_1}{m_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$= 8\left[\frac{1}{20}\right]^{\prime}\left[\frac{1}{5}\right]^{-3}\left[\frac{1}{1}\right]^o$$

= 50 units

$8g/cm^3$ = 50 units

35. Find the value of 20J on system which has 10cm , 1kg and ½ minute as the fundamental units of length , mass and time respectively.

Solution

Dimensional formula of energy is $[M^1L^2T^{-2}]$

a = 1 , b = 2 c = -2

System 1	System 2
$n_1 = 20$	$n_2 = ?$
$M_1= 1kg$	$M_2 = 1kg$
$L_1 = 1m$	$L_2 = 10cm$
$T_1 = 1sec$	$T_2 = 30sec$

Applying

$$n_2 = n_1\left[\frac{m_1}{m_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$= 20\left[\frac{1kg}{1kg}\right]^1\left[\frac{10cm}{10cm}\right]^2\left[\frac{1s}{60s}\right]^{-2}$$

$20\ J = 18 \times 10^5$ Units

REVISION QUESTIONS.

36. (a) Why in mechanics the dimensional analysis method cannot be used to determine the relationship of more than three equations?

(b) The acceleration due to gravity g_r at a point outside of the earth's surface at a distance r from the centre of the earth is given by

$$g_r = g\left[\frac{R}{r}\right]^2$$

Where g is the acceleration due to gravity at the earth's surface R is the earth radius. A satellite of mass M is in circular orbit of radius r , it is thought that the orbital time

$$T = KM^a r^b g_r^c$$

Where a , b and c are dimensionless constant use dimensional analysis to find the values of a , b and c hence show that

$$T \alpha r^{3/2}$$

Answer (b) a = o , b = ½ , c = - ½

37. (a) State what is meant by an equation is homogenous with respect to its unit.

(b) Show that the equation

$$x = ut + \frac{1}{2}at^2$$

is homogeneous with respect to its units.

(c) Explain why an equation may be homogeneous with respect to its unit but still be incorrect.

38. (a) Derive the following terms:-

(i) Dimensional constant

(ii) Dimensional variable

(b) (i) Mention six (6) limitations of dimension analysis.

(ii) According to Svedberg, the maximum safe angular velocity, ω at which a solid disc can spin depends only on the radius r of the disc breaking stress s and to density ρ of the material. Find the relation between these quantities.

Answer (b) (ii) $\omega = \frac{k}{r}\sqrt{\frac{s}{\rho}}$

39. (a) While moving through a liquid to speed v , a spherical body experience a retarding force F given by $F = kR^x n^y v^z$ where k is the dimensionless constant , n is the viscosity of the liquid and R is the radius of the body. Determine the numerical values of x , y and z by means of the method of dimensions.

(b) After being deformed, a spherical drop of liquid will execute periodic vibrations about its sphere. The frequency f of vibrations of the drop will depend on the surface tension of the drop its density (ρ) and on the radius (r) of the drop. Using the method of dimensions obtain an expression for the frequency of these vibrations in terms of the related physical quantities.

Answer (a) x = y = z = 1

(b) $f = k\sqrt{\frac{\gamma}{\rho r^3}}$

40. NECTA 1996/P2/1(C)

The period of vibration T of a turning fork may be expected to depend on the density D and Young's modulus Y of the material of which it is made and the length 'a' of its prongs. Which of the following equation represent the relation between T and the other quantities?

(i) $T = \frac{BD^2}{Y(ga^3)^{1/2}}$

(ii) $T = Ba\left(\frac{D}{Y}\right)^{1/2}$

(iii) $T = BY\left(\frac{a}{g}\right)^{1/2}$

B is dimensionless constant and g is the acceleration due to gravity

(iv) The following value (table below) were obtained for a set of geometrically similar turning fork.

Frequency(Hz)	256	288	320	384	480
Length of prong (cm)	12.0	10.6	9.6	8.0	6.4

Use these value (or otherwise) to confirm the choice of equations

41. Check the correctness of the following equations:-

(i) $F = 6\pi nvr$ (ii) $v = \left[\frac{2GR}{M}\right]^{1/2}$

(iii) $v = k\sqrt{\frac{E}{\rho}}$ (iv) $v = \frac{kn}{\rho r}$

(v) $T^2 = 4\pi^2 a^3$, a = radius

(vi) $T^2 = \frac{4\pi^2 a^3}{GM}$

Each symbol have usual meaning.

42. Check the correctness of the following equations by the method of dimensions:-

(i) $t = \sqrt{\frac{\rho r^3}{s}}$ where t = time of oscillations

ρ = density , r = radius ,s = surface tension.

(ii) $\frac{1}{2}mv^2 = mgh$, m = mass

v = velocity, h = height , g = acceleration due to gravity.

(iii) $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ where I is the moment of inertial of the flywheel which acquires an angular velocity ω when the mass m tied to a string descends through a height h and acquires a linear velocity , v.

43. Test the correctness of the following relations:-

(i) $t = kL\sqrt{\frac{\rho}{Y}}$ Where t is the time period of a turning fork , L is the length of prongs ρ is the density of the material whose Young's modulus of elasticity is Y and K is the constant of proportionality.

(ii) $t = 2\pi\sqrt{\frac{k^2 + t^2}{Lg}}$ where t is the time period of a compound pendulum , k the radius of gyration , L the length and g the acceleration due to gravity.

44. (a) The velocity of sound waves v, in medium may be assumed to depend upon density (d) of the medium and its modulus of elasticity (E). Deduce by method of dimensions an expression for V.

(b) wavelength λ of matter wave associated with particle depends upon its mass M , velocity V and plank's constant , h.

obtain dimensionally an expression for λ.

Answer (a) $v = k\sqrt{\frac{E}{d}}$, (b) $\lambda = k\frac{h}{mv}$

45. Reynold number (N_R) a dimensionless quantity determines the conditions of flow of a viscous liquid through a pipe. N_R is a function of the density of the liquid , ρ its average speed v and coefficient of viscosity of liquid n given that N_R is also directly proportional to diameter d of the pipe. Show from dimensional consideration

$$N_R = \frac{d\rho v}{n}$$

46. (a) The tension T in a rotating hoops depends on liner mass density (μ) , radius (r) and angular velocity (ω) of the hoop rotating about an axis through its centre use the method of dimensions to drive the relations between T , μ , r and ω.

(b) The energy per second p conveyed by a travelling wave in string depends on the frequency f , amplitude a , and the product of linear density , μ and speed v of the wave. Use dimensional analysis to derive the formula of p.

Answer (a) $T = \mu r^2\omega^2$ (b) $p = kf^2a^2\mu$

47. (a) The force acting on a body , moving along a circular path depends upon
(i) Mass
(ii) Velocity
(iii) Radius of the circle. Derive an expression for the force.

(b) Check the dimensional homogeneity of equation.

$$a_n = \frac{n^2h^2}{\pi\varepsilon_o Me^2}$$

Where an is the radius of the nth orbit of an electron in the hydrogen atom
ε_o , the absolute permittivity
M_e , the mass of electron
e , the charge on an electron

answer: (a) $F = K\frac{MV^2}{r}$

48. (a) A body moving through air at a speed V experiences a retarding force F given by

$$F = KA\rho V^x$$

Where A is the surface area of the body , ρ is the density of air and k is dimensionless constant. Deduce the value of x.

(b) It has been suggested that for liquids

$$s^3\beta^4 = k$$

a constant , s being the surface tension and β the compressibility show that k is not a dimensionless constant.

answer : (a) x = 2

49. (a) Explain the principle of homogeneity of dimensions.

(b) The power output P of a wind – mill depends on the area A, swept by the windmill blades, the density ρ of air and the speed v of wind. Use the method of dimension to derive the formula of p in term of A , ρ and V

[$P = KA\rho V^2$].

50. (a) If p represent radiation pressure , c represents the speed of light and E represent radiation energy striking a unit area per second. Find the non – zero integer x , y and z such the $P^xE^yC^z$ is dimensionless.

(b) In the equation $v^n = ka^bx$. What must n and b to make the equation dimensionally correct? Where v is the velocity a is the acceleration and x is the displacement.

51. (a) (i) what is a physical quantity ?

(ii) In physics why do we use seven fundamental quantity?
(iii) In defining units of physical we define standard physical quantity. What a characteristics used in define a standard of a physical quantity.
(iv) If dimensions are given physical quantity may not be unique. Explain.

(b) The velocity v of the wave of wavelength λ on the surface of a pool of liquid surface tension and density are δ and ρ respectively is given.

$$v^2 = \frac{g\lambda}{2\pi} + \frac{2\pi\delta}{\rho\lambda}$$

Where g is the acceleration due to gravity , state whether or not the given equation is dimensionally correct?

52. (a) The maximum safe angular velocity ω of which a solid disc can be spin depends on the radius of the disc(R) , breaking force per unit area(s) acting on the disc and density ρ of the material of the disc. By dimensional argument find an expression for ω in term of R , S and ρ.

(b) Explain briefly two types of dimensions.

(c) The depth to which a bullet penetrates a human body depends upon kinetic energy , E and modulus of elasticity , n prove by the method of dimensional analysis that for double penetration , kinetic energy of the bullet must be increased to 8 times.

Example – 01

Consider two thermometers , A and B needed to measure the temperature of hot water of about 99.9°C (true value). The reading on each thermometer are taken in five times.

(i) Which the thermometer is more accurately for the temperature measurement of hot water.

Thermometer A	Thermometer B
99.85°C	101.10 °C
99.80°C	101.15 °C
99.85°C	101.05 °C
100.00°C	101.15 °C
100.15°C	101.05°C

(ii) Which thermometer is more precisely for the measurement?.

Solution

(i) Accuracy can be obtained by finding the average of reading corresponding to each the thermometer.

$$\bar{\theta} = \frac{\theta_1 + \theta_2 + \theta_3 + \cdots\cdots + \theta_n}{n}$$

For thermometer A

$$\overline{\theta_A} = \frac{99.85 + 99.80 + 99.85 + 100 + 100.15}{5}$$

$$\overline{\theta_A} = 99.93^{\circ}C$$

For thermometer B

$$\overline{\theta_B} = \frac{101.10 + 101.15 + 101.5 + 101.15 + 101.05}{5}$$

$$\overline{\theta_B} = 101.10^{\circ}C$$

Therefore , thermometer A is more accurate as it is close to the actual (true) value of temperature of hot water.

(ii) Precision of measurement can be obtained by finding the difference between the maximum and lowest reading.

$$\Delta\theta_A = 100.15 - 98.80 = 0.35^{\circ}C$$

$$\Delta\theta_B = 101.15 - 101.05 = 0.10^{\circ}C$$

Therefore thermometer B has greater precision since have small difference in temperature.

Example – 02

The mass of the body as measured by the students given as 9.2kg and 9.23kg which measurement more accurate? Why?

Solution

9.23kg is more accurate because it has more significant figures (3sgf) meaning that more accurate.

Quiz 1

Three students A , B and C conducted an experiment measuring the diameter of small marble , each student performed two experiments and their result are shown below.

	Experiment 1	Experiment 2
Student A	2.4	2.3
Student B	3.0	5.5
Student C	2.7	3.2

If all the above values are in mm and that the best answer for the diameter of the marble is 3.0mm.

(i) Whom among the above students is more precise? Why?

(ii) Whom among the above students is not precise and not accurate ? why?

(iii) By using results of (iii) Above which student is more accurate?

NUMERICAL EXAMPLES

1. (a) Give the meaning of the following terms as used in error analysis:-
   (i) Absolute error
   (ii) Relative error

   (b) The force F acting on an object of mass 'M' travelling at velocity V in a circle of radius 'r' is given by

$$F = \frac{MV^2}{r}$$

If the measurement are recorded as
$M = (3.5 \pm 0.1)$kg;
$V = (20 \pm 1)$ m/s
$r = (12.5 \pm 0.5)$m ; find the maximum possible.

   (c) Show how you will record the reading of force 'F' in part (6)

Solution

(a) Refer to your notes

(b) (i) Given that $F = \frac{MV^2}{r}$

The maximum fractional error on F

$$\frac{\Delta f}{f} = \frac{\Delta M}{M} + \frac{2\Delta V}{V} + \frac{\Delta r}{r}$$

$$= \frac{0.1}{3.5} + \frac{2\times 1}{20} + \frac{0.5}{12.5}$$

$$\frac{\Delta f}{f} = 0.17$$

(ii) Percentage error

$$\frac{\Delta f}{f}\times 100\% = 0.17\times 100\%$$

$$\frac{\Delta f}{f}\times 100\% = 17\%$$

(c) The value F without error.

$$F = \frac{MV^2}{r} = \frac{3.5(20)^2}{12.5}$$

$$F = 112\text{N}$$

Since $\frac{\Delta f}{f} = 0.17$

$$\Delta f = 0.17F$$

$$\Delta f = 0.17\times 112\text{N} = 19\text{N}$$

The numerical value of F can be recorded as F = (112 ± 19N).

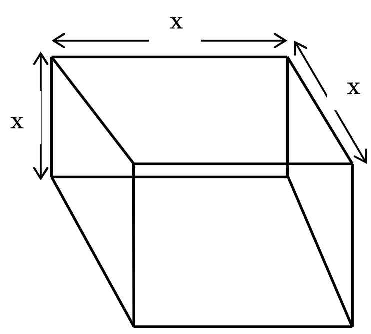
2. (a) Give the meaning of the following terms as used in error analysis.
   (i) Accuracy of measurement
   (ii) Precision of measurement
   (iii) Discrepancy
   (iv) Permissible error

(b) The side of cube is measured as (7.5 ± 0.1)cm. Find the volume of the cube.

Solution

(a) See your notes

(b) Let x be the side of the cube



Volume of the cube $v = x^3$

The maximum fractional error on v

$$\frac{\Delta v}{v} = \frac{3\Delta x}{x} = \frac{3\times 0.1}{7.5}$$

$$\frac{\Delta v}{v} = 0.04$$

Volume of the cube without error

$$v = x^3 = (7.5)^3 = 422cm^3$$

Error on v

$$\Delta v = 0.04v = 0.04\times 422$$

$$\Delta v = 17cm^3$$

Volume of the cube $v = (422 \pm 17)cm^3$

3. (a) (i) Define the term dimension of a physical quantity.

(ii) The number of particles n crossing a unit area perpendicular to x – axis in a unit time is given as

$$n = \frac{-D(n_2 - n_1)}{(x_2 - x_1)}$$

Where n_1 and n_2 are the number of particles per unit volume for the values of x_1 and x_2 respectively. What are the dimensions of diffusion constant , D

(b) (i) Give two basic rules of dimensional analysis

(ii) The frequency , f of a vibrating string depends upon the force applied , F the length L of the string and the mass per unit length , μ using dimension show how f is related to F , L and μ

(c) (i) What is meant by least count of measurement?

(ii) The period of oscillation of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where by 100 vibrations were taken to measure 200 second. If the least count for the time and length of a pendulum of 1m are 0.1sec and 1mm respectively. Calculate the maximum percentage error in the measurement of g.

4. The specific resistance ρ of a thin circular wire of radius r cm , resistance , R ohm and length Lcm is given by

$$\rho = \frac{\pi r^2 R}{L}$$

If r = (0.26 ± 0.02) cm

R = (32 ± 1) Ω

L = (78 ± 0.01) cm. find the percentage error in ρ

Solution

r = (0.26 ± 0.02) cm

L = (78 ± 0.01) cm

Since $\rho = \frac{\pi r^2 R}{L}$, π = constant

The maximum fractional error on ρ

$$\frac{\Delta\rho}{\rho} = \frac{2\Delta r}{r} + \frac{\Delta R}{R} + \frac{\Delta L}{L}$$

$$\frac{\Delta\rho}{\rho}\times 100\% = \left[\frac{2\Delta r}{r} + \frac{\Delta R}{R} + \frac{\Delta L}{L}\right]\times 100\%$$

$$= \left[\frac{2\times 0.02}{0.26} + \frac{1}{32} + \frac{0.01}{78}\right]\times 100\%$$

$$\frac{\Delta\rho}{\rho}\times 100\% = 18\%$$

5. Your given two resistance $R_1 = (4.0 \pm 0.1)\Omega$ and $R_2 = (9.1 \pm 0.2)\Omega$. Calculate their effectively resistance when they are connected in
   (i) Series connection
   (ii) Parallel connection

   Also the percentage error in each case.

   Solution

   (i) In series connection

   $Rs = R_1 + R_2 = (4.0 + 9.1)\Omega$

   $Rs = 13.1\Omega$

   Error on Rs , $\Delta Rs = \Delta R_1 + \Delta R_2$

   $\Delta Rs = 0.1 + 0.2 = 0.3\Omega$

   Effectively value of resistance

   $\underline{Rs = (13.1 \pm 0.3)\Omega}$

   Percentage error on Rs

   $$\frac{\Delta Rs}{Rs}\times 100\% = \frac{0.3}{13.1}\times 100\%$$

   $$\frac{\Delta Rs}{Rs}\times 100\% = 2.3\%$$

   (ii) The maximum fractional error on Rp

   $$\frac{\Delta Rp}{Rp} = \frac{\Delta R_1}{R_1} + \frac{\Delta R}{R} + \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}$$

   $$= \frac{0.1}{4} + \frac{0.2}{9} + \frac{0.1+0.2}{13.1}$$

   $$\Delta Rp = Rp\left[\frac{0.1}{4} + \frac{0.2}{9.1} + \frac{0.3}{13.1}\right]$$

   $$= 2.779\left[\frac{0.1}{4} + \frac{0.2}{9.1} + \frac{0.3}{13.1}\right]$$

   $$\Delta Rp = 0.1942\Omega$$

   Percentage error on Rp

   $$\frac{\Delta Rp}{Rp}\times 100\% = \frac{0.1942}{2.779}\times 100\%$$

   $$\frac{\Delta Rp}{Rp}\times 100\%$$

6. (a) (i) Define the term dimensions of a physical quantity
   (ii) Identify two uses of dimensional equations

   (b) (i) What is the basic requirement for a physical relation to be correct?
   (ii) List two quantities whose dimensions is $[ML^2T^{-1}]$

   (c) (i) The frequency 'f' of vibration of a stretched string depends on the tension 'F' , the length 'L' and the mass per unit length μ of the string. Derive the formula relating the physical quantities by the method of dimensions.
   (ii) Use dimensional analysis to prove the correctness of the relation ,

   $$\rho = \frac{3g}{4\pi RG}$$

   Where ρ = density of the earth,
   g = acceleration due to gravity
   R = radius of the earth and
   G = gravitational constant

7. The viscosity n of a liquid , flowing through a capillary tube of length L and radius r is given by the

   $$\frac{v}{t} = \frac{\pi(p_1 - p_2)r^4}{8nL}$$

   Where p_1 and p_2 are pressure existing at the end of the tube , t is the time taken by liquid of the volume , v to pass through the tube.

   (i) Find an expression for the fractional error in n.

   (ii) Calculate the percentage error in n using the following experimental result:-

   Length $L = (26.0 \pm 0.10)$cm
   Radius $r = (0.65 \pm 0.01) \times 10^{-3}$m
   Pressure $P_1 = (8.10 \pm 0.05) \times 10^3 Nm^{-2}$
   Pressure $P_2 = (5.40 \pm 0.05) \times 10^3 Nm^{-2}$
   Volume $v = (3.23 \pm 0.02)cm^3$
   Time $t = (60.00 \pm 0.20)$ sec

   (iii) Write the experimental value of n (including the order of accuracy).

   Solution

   Let : $p_1 - p_2 = p$

   (i) $\frac{v}{t} = \frac{\pi pr^4}{8\eta L}$, $\eta = \frac{\pi pr^4 t}{8vL}$

   Apply natural logarithm both side

   $$\log_e^n = \log_e\left[\frac{\pi}{8}\cdot pr^4tv^{-1}L^{-1}\right]$$

$$\log_e^{\eta} = \log\left[\frac{\pi}{8}\right] + \log_e^{p} + 4\log_e^{r} + \log_e^{t} + \log_e^{v^{-1}} + \log_e^{L^{-1}}$$

$$\log_e^{\eta} = \log_e\left(\pi/8\right) + \log_e^{p} + 4\log_e^{r} + \log_e^{t} \mp \log_e^{v} \mp \log_e^{L}$$

On differentiating

$$\frac{\Delta\eta}{\eta} = \frac{\Delta p}{p} + 4\frac{\Delta r}{r} + \frac{\Delta t}{t} + \frac{\Delta v}{v} + \frac{\Delta L}{L}$$

But $\frac{\Delta p}{p} = \frac{\Delta p_1 + \Delta p_2}{p_1 - p_2}$

The maximum fractional error on n

$$\frac{\Delta\eta}{\eta} = \frac{\Delta p_1 + \Delta p_2}{p_1 - p_2} + \frac{4\Delta r}{r} + \frac{\Delta t}{t} + \frac{\Delta v}{v} + \frac{\Delta L}{L}$$

(ii) Percentage error on n

$$\frac{\Delta\eta}{\eta} \times 100\% = \left[\frac{\Delta p_1 + \Delta p_2}{p_1 - p_2} + \frac{4\Delta r}{r} + \frac{\Delta t}{t} + \frac{\Delta v}{v}\right] \times 100\%$$

$$= \left[\frac{0.05 + 0.05}{8.10 - 5.40} + \frac{0.02}{3.23} + \frac{4 \times 0.1}{0.65} + \frac{0.1}{26} + \frac{0.2}{60}\right] \times 100\%$$

$$\frac{\Delta\eta}{\eta} \times 100\% = 11.19\%$$

(iii) Experimental value of n

The value of n without error

$$\eta = \frac{\pi(p_1 - p_2)r^4 t}{8VL}$$

$$= \frac{3.14(8.10 - 5.4) \times 10^3 (0.65 \times 10^{-3})^4 \times 60}{8 \times 3.23 \times 10^{-6} \times 26 \times 10^{-2}}$$

$$\eta = 13.52 \times 10^{-3} \text{kgm}^{-1}\text{s}^{-1}$$

Error in viscosity

$$\frac{\Delta\eta}{\eta} = \frac{11.19}{100}$$

$$\Delta\eta = \frac{11.19}{100}\eta = \frac{11.19}{100} \times 13.52 \times 10^{-3}$$

$$= (1.512) \times 10^{-3} \text{kgm}^{-1}\text{s}^{-1}$$

Numerical value of

$\eta = (13.52 \pm 1.512) \times 10^{-3}\text{kgm}^{-1}\text{s}^{-1}$

8. (a) (i) It necessary for a precise experiment to be accurate?

(ii) The diameter of a steel rod is given as (56.47 ± 0.02)mm. What does it mean?

(b) Calculate fractional error of the quantity

$$p = ab - c^2$$

where a = (4.00 ± 0.15) cm

b = (5.00 ± 0.17) cm

c = (3.00 ± 0.13) cm

Solution

(a) (i) Yes , a precise experiment must also be accurately because precision only measures effectiveness in doing the experiment while accuracy measures effectiveness in getting the value.

(i) It means that true value of diameter is unlikely to be less than 56.45mm or greater than 56.49mm

(b) Given that

$$P = ab - c^2$$

Let : $z = ab$

Fractional error on z

$$\frac{\Delta z}{z} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$$

$$\Delta z = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)z = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)ab$$

$$\Delta z = b\Delta a + a\Delta b$$

Let : $w = c^2$

$$\frac{\Delta w}{w} = \frac{2\Delta c}{c}, \quad \Delta w = \frac{2\Delta c}{c} \cdot w$$

$$\Delta w = 2c\Delta c$$

Now : $p = z - w$

$$\frac{\Delta p}{p} = \frac{\Delta z + \Delta w}{z - w} = \frac{b\Delta a + a\Delta b + 2c\Delta c}{ab - c^2}$$

$$\frac{\Delta p}{p} = \frac{5 \times 0.15 + 4 \times 0.17 + 2 \times 3 \times 0.13}{4 \times 5 - 3^2}$$

$$\frac{\Delta p}{p} = 0.2$$

9. (i) What is the meaning of the term 'precision' and 'accuracy' as used in experimental physics.
   (ii) In experiment t determine the volume of glass in a length of glass tubing the following reading were recorded:-
   Length L = (40 ± 1)mm
   External diameter 0 = (12.0 ± 0.2)mm
   Internal diameter d = (10.0 ± 0.2)mm
   If the volume of the glass is calculated by using the relation

$$v = \frac{\pi L}{4}\left(D^2 - d^2\right)$$

   Determine the numerical value of volume , v.

Solution

(i) Refer to your notes

(ii) Given that

$$v = \frac{\pi L}{4}\left[D^2 - d^2\right]$$

$$v = \frac{\pi L}{4}\left[(D-d)(D+d)\right]$$

Since $\pi/4$ is a constant

The maximum fractional error on v.

$$\frac{\Delta v}{v} = \frac{\Delta L}{L} + \frac{\Delta D + \Delta d}{D-d} + \frac{\Delta D + \Delta d}{D+d}$$

$$\frac{\Delta V}{V} = \frac{1}{4} + \frac{0.2+0.2}{12-10} + \frac{0.2+0.2}{12+10}$$

The value of V without error

$$V = \frac{3.14\times 40}{4}\left[12^2 - 10^2\right]$$

$$V = 1382.3\text{mm}^3$$

Error on v

$$\Delta V = \left[\frac{1}{4} + \frac{0.4}{2} + \frac{0.4}{22}\right]\times 1382.3$$

$$\Delta V = 336.15\text{mm}^3$$

Numerical value of v = (1382.3 ± 336.15)mm^3

10. The velocity V of the wave of wavelength λ on the surface of pool of liquid whose surface tension and density are δ and ρ respectively is given by

$$V^2 = \frac{\lambda g}{2\pi} + \frac{2\pi\delta}{\lambda\rho}$$

Where g is the acceleration due to gravity. Show that the equation is dimensionally correct. A vibration of frequency (480 ± 1)Hz produces on the surface of water wave whose wavelength is (0.125 ± 0.00)cm. Assuming that for this wave length the first term on the right hand side of the equation is negligible. Calculate the value which these result give for the surface tension of water. Given that ρ = 1000kgm^{-3} and δ = $7.16 \times 10^{-2}Nm^{-1}$.

Solution

Case I

Given that : $V^2 = \frac{\lambda g}{2\pi} + \frac{2\pi\delta}{\lambda\rho}$

Dimensionally

[V] = LT^{-1} [λ] = L [g] = LT^{-2}

[δ] = MT^{-2} [ρ] = ML^{-3}

Now

[L.H.S] = [V $]^2$ = $(LT^{-1})^2$ = L^2T^{-2}

[L.H.S] = L^2T^2

Since 2π is a constant.

$$[\text{ R.H.S }] = [\lambda][g] + \frac{[\delta]}{[\lambda][\rho]}$$

$$= LT^{-2}\cdot L + \frac{MT^{-2}}{L\cdot L^{-3}M}$$

$$= L^2T^{-2} + L^2T^{-2}$$

$$[\text{R.H.S}] = L^2T^2$$

Since [L.H.S] = [R.H.S] = L^2T^{-2}

Therefore the equation is dimensional correct.

Case II

According to the consumption above neglect the first term on R.H.S of the equation.

$$V^2 = \frac{2\pi\delta}{\lambda\rho}$$

$$\delta = \frac{v^2\lambda\rho}{2\pi} \text{ but } v = f\lambda$$

$$\delta = \frac{f^2\lambda^3\rho}{2\pi}$$

Since ρ and 2π are constants.

The maximum fractional errors on δ.

$$\frac{\Delta\delta}{\varepsilon} = \frac{2\Delta f}{f} + \frac{3\Delta\lambda}{\lambda}$$

Error on δ.

$$\Delta\delta = \delta\left[\frac{2\Delta f}{f} + \frac{3\Delta\lambda}{\lambda}\right]$$

$$= 7.16\times10^{-2}\left[\frac{2\times1}{f} + \frac{3\times0.001}{0.125}\right]$$

$$\Delta\delta = 0.21\times10^{-2}$$

Numerical value of surface tension
$\delta = (7.16 \pm 0.21) \times 10^{-2}Nm^{-1}$

11. (a) (i) What is the difference between degree of accuracy and precision.
(ii) In an experiment to determine Young's modulus of a wooden material the following measurements were recorded:-
Length $L = (80.0 \pm 0.05)$cm
Breath $b = (28.65 \pm 0.03)$mm
Thickness $t = (6.40 \pm 0.03)$mm and
Slope $G = (0.035 \pm 0.001)cmgm^{-1}$
Given that the Young's modulus Y is given by.

$$Y = \frac{4}{Gb}\left[\frac{L}{t}\right]^3$$

Calculate the maximum percentage error in the value of Y.

(b) Using the method of dimensions indicate which of the following equations are dimensionally correct and which are not given that f = frequency , γ = surface tension , ρ = density, r = radius and k = dimensionless constant.

(i) $\rho^2 = k\sqrt{\frac{r^3 f}{\gamma}}$

(ii) $f = \frac{kr^3\sqrt{\gamma}}{\rho^{1/2}}$

(iii) $f = \frac{k\gamma^{1/2}}{\sqrt{\rho r^{3/2}}}$

Solution

(a) (i) see your notes

Given that $Y = \frac{4}{Gb}\left[\frac{L}{t}\right]^3$

Since 4 is a constant.
The maximum fractional error on Y

$$\frac{\Delta Y}{Y} = \frac{\Delta G}{G} + \frac{\Delta b}{b} + \frac{3\Delta L}{L} + \frac{3\Delta t}{t}$$

The maximum percentage error.

$$\frac{\Delta Y}{Y}\times100\% = \left[\frac{\Delta G}{G} + \frac{\Delta b}{b} + \frac{3\Delta L}{L} + \frac{3\Delta t}{t}\right]\times100\%$$

$$= \left[\frac{0.001}{0.035} + \frac{0.03}{28.65} + \frac{3\times0.05}{80} + \frac{3\times0.03}{6.40}\right]\times100\%$$

$$\frac{\Delta Y}{Y}\times100\% = 4.56\%$$

(b) Refer to the example 27

12. (a) The time period of oscillation of a simple pendulum in an experiment is recorded as 2.63, 2.56 , 2.71 and 2.80sec respectively, find the
(i) Time period
(ii) Absolute and percentage error.
(b) The time period of oscillation of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

The length L of the pendulum is about 10cm and is known to 1mm accuracy. The period of oscillation is about 0.5sec. The time of 100 oscillations is measured with a watch of 1sec resolution. What is the accuracy in the determination of g?

Solution

(i) Time period

$$\overline{T} = \frac{T_1 + T_2 + T_3 + \cdots + T_n}{n}$$

$$\overline{T} = \frac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5}$$

$$\overline{T} = 2.62\,\text{sec}$$

(ii) Absolute error in each observation
2.62 – 2.63 = -0.01 sec
2.62 – 2.42 = 0.20 sec
2.62 – 2.80 = -0.18 sec
2.62 – 2.56 = 0.06 sec
2.62 – 2.71 = 0.09 sec
Mean absolute error.

$$\overline{\Delta T} = \frac{|-0.01| + |0.2| + |-0.18| + |0.06| + |-0.09|}{5}$$

$\overline{\Delta T} = 0.11\,\text{sec}$

Percentage error

$$\frac{\overline{\Delta T}}{\overline{T}} \times 100\% = \frac{0.11}{2.62} \times 100\%$$

$$\frac{\overline{\Delta T}}{\overline{T}} \times 100\% = 4.2\%$$

(b) Given that : $T = 2\pi\sqrt{\frac{L}{g}}$

$$T^2 = \frac{4\pi^2 L}{g}$$

$$g = \frac{4\pi^2 L}{T^2}$$

Since $4\pi^2$ is a constant.

The maximum fractional error on g

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T}$$

Percentage error

$$\frac{\Delta g}{g} \times 100\% = \left[\frac{\Delta L}{L} + \frac{2\Delta T}{T}\right] \times 100\%$$

$\Delta L = 1\text{mm} = 0.1\text{cm}$, $L = 10\text{cm}$

$T = ?$ $\Delta T = 1\text{sec}$

1 oscillation $\to$ 0.5sec

100 oscillation $\to$ T

$T = 0.5 \times 100$

$T = 50\text{sec}$

$$\frac{\Delta g}{g} \times 100\% = \left[\frac{0.1}{10} + \frac{2 \times 1}{50}\right] \times 100\%$$

$$\frac{\Delta g}{g} \times 100\% = 5\%$$

13. (a) While moving through a liquid at speed V a sphere experiences a retarding force F is given by $F = KR^x\rho^y v^z$ where k is a constant , ρ is the density of liquid and R is the radius of the body. Determine the numerical values of x , y and z by means of the method of dimensions.

(b) In an attempt to determine the acceleration due to gravity a student measures the length L of the simple pendulum using normal laboratory metre rule and time T for one complete oscillation of the pendulum using a stop watch with an accuracy of 0.1 second and $\Delta L = 0.05\text{cm}$. for $L = 0.5\text{m}$, the student obtain $T = 42.6\text{second}$ and goes on to calculate g using the equation

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Find the maximum percentage error introduced into the value g worked out by the student in accuracy of the stop watch as well as the metre rule.

Solution

(a) Given that

$F = KR^x\rho^y v^z$

K = dimensionally constant

x , y and z are any real number

Dimensionally

$[F] = MLT^{-2}$ $[R] = L$

$[\rho] = ML^{-3}$ $[V] = LT^{-1}$

Now

$[F] = [R]^x[\rho]^y[V]^z$

$MLT^{-2} = L^x(ML^{-3})^y(LT^{-1})^z$

$M^1L^1T^{-2} = M^yL^{x-3y+z}T^{-z}$

On equating indices or powers

M : $1 = y$……………………(i)

L : $1 = x - 3y + z$………..(ii)

T : $-2 = -z$………………..(iii)

On solving $x = 6$, $y = 1$, $z = 2$

(b) $T = 2\pi\sqrt{\frac{L}{g}}$

$$T^2 = \frac{4\pi^2 L}{g} \quad , \quad g = \frac{4\pi^2 L}{T^2}$$

Since $4\pi^2$ is a constant

The maximum fractional error on g

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T}$$

$$\frac{\Delta g}{g} \times 100\% = \left[\frac{\Delta L}{L} + \frac{2\Delta T}{T}\right] \times 100\%$$

$$= \left[\frac{0.05}{0.5\times100}+\frac{2\times0.1}{42.6}\right]\times$$

$$\frac{\Delta g}{g}\times100\% = 0.57\%$$

14. An experiment was done to find the acceleration due to gravity , g by using the formula

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Where T = 2.22 second and L = 121.6cm. Given that error due to the stop watch is 0.1sec and if the clock loses 3sec in 3 minutes. Calculate error in measuring value of g.

Solution

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T^2 = \frac{4\pi^2L}{g} \quad , \quad g = \frac{4\pi^2L}{T^2}$$

Since $4\pi^2$ is a constant

The maximum fraction error on g

$$\frac{\Delta g}{g} = \frac{\Delta L}{L}+\frac{2\Delta T}{T}$$

$\Delta t_1 = 0.1\text{sec}$, $\Delta t_2 = ?$

3 sec $\rightarrow$ 3 × 60sec

$\Delta t_2 \rightarrow$ 2.22 sec

$$\Delta t_2 = \frac{3\times2.22}{180}$$

$\Delta t_2 = 0.37\text{sec}$

Total error on the stop watch

$\Delta t = \Delta t_1 + \Delta t_2 = 0.1 + 0.037$

Now , the value of g without error

$$g = \frac{4\pi^2L}{T^2} = \frac{4\times(3.14)^2\times1.216}{(2.22)^2}$$

$g = 9.73\text{m/s}2$

error on the value of g

$$\Delta g = g\left[\frac{\Delta L}{L}+\frac{2\Delta T}{T}\right]$$

$\Delta g = \pm\ 1.21\text{m/s}^2$

15. The surface tension (n) of a liquid of density (D) can be found by introducing the liquid into a u – tube glass , the limbs of which have radii r_1 and r_2. The difference in height of the liquid in the two limbs can be measured and the surface tension (n) can be calculate from the formula

$$\eta\left(\frac{1}{r_1}-\frac{1}{r_2}\right) = \frac{gDh}{2}$$

Where g is the acceleration due to gravity. Estimate the fractional error in n. If h = (1.06 ± 0.005)cm , r_1 = (0.07 ± 0.05) cm and

r_2 = (0.14 ± 0.005)cm.

Solution

Given that :

$$\eta\left(\frac{1}{r_1}-\frac{1}{r_2}\right) = \frac{gDh}{2}$$

$$\eta = \frac{gDh}{2}\left(\frac{r_1r_2}{r_2-r_1}\right)$$

Since $gD/2$ is a constant

The maximum fractional error on n .

$$\frac{\Delta n}{n} = \frac{\Delta h}{h}+\frac{\Delta r_1}{r_1}+\frac{\Delta r_2}{r_2}+\frac{\Delta r_1+\Delta r_2}{r_2-r_1}$$

$$= \frac{0.005}{1.06}+\frac{0.005}{0.07}+\frac{0.005}{0.14}+\frac{0.005+0.005}{0.14+0.07}$$

$$\frac{\Delta\eta}{\eta} = 0.255$$

16. (a) Explain the term limit of precision of a measuring device.

(b) The heat generated in a circuit depends upon the current resistance and time for which current flows. If the errors in measuring the above are 2% , 1% and 1% respectively. Find the maximum error in measuring heat.

(c) The smallest division for the voltmeter and ammeter are 0.1v and 0.01A respectively. If v = IR , find the relative error in the resistance , R when v = 2Volt and I = 0.1A

Solution

(a) Refer to your notes

(b) Electrical power produced $P = I^2R$

Heat energy produced in time ,t

$H = pt = I^2Rt$

The maximum fractional error on H

$$\frac{\Delta H}{H} = \frac{2\Delta I}{I} + \frac{\Delta R}{R} + \frac{\Delta t}{t}$$

Percentage error.

$$\frac{\Delta H}{H}\times 100\% = 2\left(\frac{\Delta I}{I}\times 100\%\right) + \frac{\Delta R}{R}\times 100\%$$

$$+\frac{\Delta t}{t}\times 100\%$$

$= 2 \times 2\% + 1\% + 1\%$

$$\frac{\Delta H}{H}\times 100\% = 6\%$$

(c) $V = IR$

$$R = \frac{V}{I}$$

Maximum fractional error on R.

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$$

Error on voltmeter ,

$$\Delta V = \frac{0.1}{2} = \pm 0.05V$$

Error on Ammeter,

$$\Delta I = \frac{0.01}{2} = \pm 0.005A$$

$$\frac{\Delta R}{R} = \pm\left(\frac{0.05}{2} + \frac{0.005}{0.1}\right)$$

$$\frac{\Delta R}{R} = \pm 0.0275$$

17. In an experiment to determine density of a block of mass 14g it was found that length of the block was (6.0 ± 0.12) cm, width was (4.0 ± 0.1) cm and height was (2.0 ± 0.06)cm.
(a) Which quantity need to be more decorate?
(b) Calculate the experimental value of density of the block.

Solution

Hint:

The most accurate quantity is the one with least fractional error while quantity that needs to be more accurate is the one with the greatest fractional error.

(a) Quantity which need to be more accurate.

$$\frac{\Delta L}{L} = \frac{0.12}{6} = 0.02$$

$$\frac{\Delta W}{W} = \frac{0.1}{4} = 0.025$$

$$\frac{\Delta h}{h} = \frac{0.06}{2} = 0.03$$

Therefore the height needs to be more accurately.

(b) Experimental value of density

$$\rho = \frac{m}{v_o} = \frac{m}{L\omega h} = \frac{14}{6\times 4\times 2}$$

$\rho = 0.292gcm^{-3}$

Error on ρ

The maximum fractional error on ρ.

$$\frac{\Delta\rho}{\rho} = \frac{\Delta L}{L} + \frac{\Delta W}{W} + \frac{\Delta h}{h}$$

$$\frac{\Delta\rho}{\rho} = 0.2 + 0.025 + 0.03$$

$= [\ 0.02 + 0.025 + 0.03\ ]\ \rho$

$= [\ 0.02 + 0.025 + 0.03] \times 0.292$

$\Delta\rho = 0.022gm^{-3}$

Numerical value of the density

$\rho = [0.292 \pm 0.022]\ gcm^{-3}$

18. (a) Differentiate between:-
(i) Error and mistake
(ii) Precision and accuracy
(b) The coefficient of viscosity of liquid is found by using strokes law is given by

$$\eta = \frac{2gr^2(\delta_1 - \delta_2)}{qv}$$

In the experiment the following results were obtained.

Density of steel ball δ_1 = (7800 ± 1.00)kgm^{-3}

Density of oil δ_2 = (126 ± 1.00) kgm^{-3}

Terminal velocity of steel ball

V = (1.00 ± 0.01) m/s

Radius of steel ball r = (6.35 ± 0.05)mm

Determine the numerical value of the viscosity n and maximum percentage error.

(c) (i) What is the advantage of expressing physical quantities in terms of dimensional equations?

(ii) Write the dimensions of a/b in the relation $F = a\sqrt{x} + bt^2$ where F is force , x is distance and t is the time.

19. (a) (i) Distinguish between Random error from systematic error.

(ii) Give a practical example of each term in 1(a)(i) and briefly explain how they can be reduced or eliminated.

(b) (i) Define the term error and mistake

(ii) An experiment was done to find acceleration due to gravity by using the formula

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Where all symbols carry the usual meaning if the clock loses 3 second in 5 minutes , determine the error in measuring 'g' given that T = 2.22sec , L = 121.6cm , ΔT_1 = 0.1sec , ΔL = ±0.05cm.

(c) (i) What is the important of dimensional analysis in spite of its drawn backs.

(ii) The following measurement were taken by a student for the length of a piece of rod : 20.92 , 21.11 , 21.02 , 20.99 and 20.69cm. basing on error analysis. find the value of the length of piece of rod and its associated error.

20. (a) (i) What is meant by random error?

(ii) Briefly explain for causes of random errors in measurements.

(b) The period T of oscillation of body is said to be 1.5 ± 0.002s while its amplitude A is 0.3 ± 0.005m and the radius of gyration k is 0.28 ± 0.005m. If the acceleration due to gravity g was found to be related to T, A and K by the equation

$$\frac{gA}{4\pi^2} = \frac{A^2 + K^2}{T^2}$$

Find the :-

(i) Numerical value of g in four decimal places

(ii) Percentage error in g

(c) (i) state the law of dimension analysis

(ii) The largest mass , m of a stone that can be moved by the flowing river depends on the velocity v , the density , ρ of water , and the acceleration due to gravity , g. show that the mass m varies to the sixth power of the velocity of flow.

21. (a) Define the following :-

(i) Dimensional constants

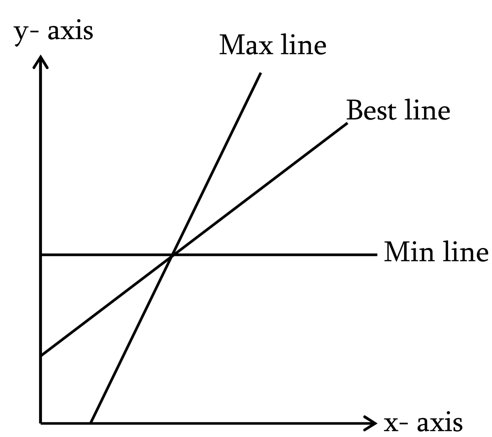
(ii) Dimensionless quantities

(b) (i) define the principle of 'dimensions uniformity' on what principle is based?

(ii) A progressive wave equation is written as $y = a\sin(\omega t - kx)$ where t and x stand for time and distance respectively. Determine the dimensional formula for ω and k

(c) (i) How do random errors differ from systematic errors. (any three differences).

(ii) The experimental data for x and y were plotted and the following linear graph and its accuracy.



If the values of slopes and y – intercepts in these lines are as follows:-

Line	Slope	Difference	y-intercept	Difference
Best fit	2.00	-	3.0	-
Max.line	2.16	0.16	-0.8	-3.8
Min. line	1.81	-0.19	3.1	0.1

Determine the values of the constants β and α with their corresponding maximum errors. If the data were expected to fit the equation y = β + xα

Solution

(a) (b) (c) (i) refer to your notes

(c) (ii) The value of α is the slope of the best line α = 2.00

Error on slope , α

$$D\alpha = \frac{\text{Difference of max line and min line}}{2}$$

$$= \frac{|0.16| + |-0.19|}{2} = 0.175$$

The value of α = 2 ± 0.175

The value of β is the y – intercept of the best fit line β = 3.0.

Error on β

$$\Delta\beta = \frac{\text{Difference of y -intercept max and min}}{2}$$

$$= \frac{|-3.8| + |0.1|}{2} = 1.95$$

The value of β = (3.0 ± 1.95)

22. In determination of uncertainties of ‘a’ and ‘b’ in the equation y = a + bx , three straight lines were plotted one of which is the best line and other two are maximum and minimum line. The values of gradients and y – intercepts for the lines are tabulated below. Find the values of ‘a’ and ‘b’ including their uncertainties.

	Gradient	Y - intercept
Best fit	1.0	2.00
Max line	1.16	-1.50
Min line	0.81	5.20

Solution

Given that y = a + bx

Let : b = slope of best fit

b_1 = slope of max. line

b_2 = slope of min line

Now

$$\Delta b_1 = |b_1 - b| = |1.16 - 1.0|$$

$$\Delta b_1 = 0.16$$

$$\Delta b_2 = |b_2 - b| = |0.81 - 1.0|$$

$$\Delta b_2 = 0.19$$

Error on b

$$\Delta b = \frac{\Delta b_1 + \Delta b_2}{2} = \frac{0.16 + 0.19}{2}$$

$$\Delta b = 0.175$$

Numerical value of b = 1 ± 0.175

a = y – intercept of best fit

a_1 = y – intercept of max line

a_2 = y – intercept of min line

Δa_1 = | a1 – a | = | -1.5 – 2 |

Δa_1 = 3.5

Also

Δa_2 = | a_2 – a | = | 5.20 – 2.00 |

= 3.20

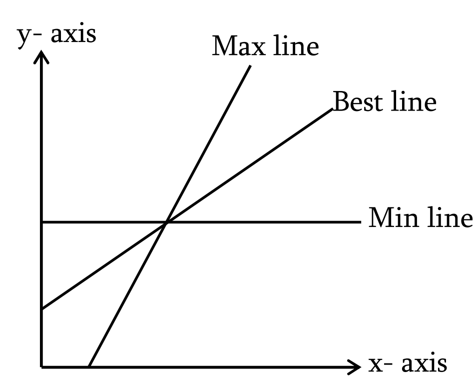
Error on a

$$\Delta a = \frac{\Delta a_1 + \Delta a_2}{2}$$

$$\Delta a = \frac{3.5 + 320}{2} = 3.35$$

∴ Numerical value of a = 2 ± 3.35

23. (a) The experimental data of x and y were plotted and the following linear graph and its accuracy was obtained.



If the values of slopes and y – intercepts in these three lines are as follows.

Line	Slope	Difference	y-intercept	Difference
Best fit	1.00	-	2.0	-
Max.line	1.16	0.16	-1.5	-35
Min. line	0.81	-0.19	5.2	3.2

And if data were expected to fit the equation

$$y = a + bx.$$

Determine the values of the constants a and b with the corresponding maximum error in each.

(b) A rectangular board is measured with a scale having accuracy of 0.2cm. the length and breadth are measured as 33.4cm and 18.4cm respectively find
  (i) The relative error of the area
  (ii) The percentage error of the area
  (iii) The area and its accuracy.

24. Consider the following measurements made in simple pendulum experiment to determine the value of acceleration due to gravity , g

Length L(mm)	Periodic time (sec)
200	0.9
400	1.28
600	1.56
800	1.76
1000	2.02

Determine the numerical value of acceleration due to gravity , g.

Solution

L(m)	T(sec)	$T^2(sec^2)$
0.2	0.90	0.81
0.4	1.28	1.64
0.6	1.56	2.43
0.8	1.76	-3.10
1.0	2.02	4.08

Slope of best line

$$s = \frac{3.76-2.4}{1.0-0.6} = \frac{1.36}{0.4}$$

$$s = 3.4 s^2m^{-1}$$

Slope of maximum line

$$s_1 = \frac{3.2-2}{0.7-0.45} = \frac{1.2}{0.25}$$

$$s_1 = 4.8 s^2m^{-1}$$

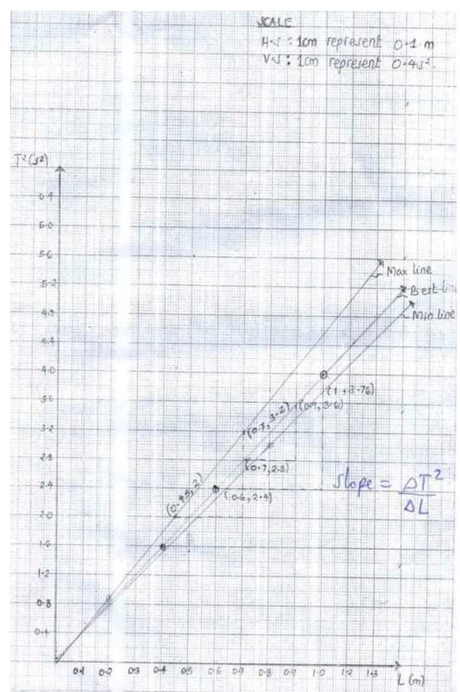
Slope of minimum line

$$s_2 = \frac{3.6-2.8}{0.9-0.7} = 4$$

$$s_2 = 4.0 s^2m^{-1}$$

SPACE

THE GRADE OF T²(S²) AGAINST L(cm)



25. The data below describe the stretching of spring. Plot a graph of the applied force against extension.

Force (N)	External (mm)
2.0	6
3.0	9
4.0	12
5.0	16
6.0	19
7.0	20
8.0	24

9.0	28
10.0	31
11.00	33

Obtain the number value of the force constant.

26. (a) (i) Explain what ±a units , following the value of a parameter , signify in experimental physics.
(ii) The specific resistance ρ of a thin circular wire of radius r cm on resistance R ohms and length L cm is given by

$$\rho = \frac{\pi r^2 R}{L}$$

$$r = \left(0.026 \pm 0.02\right)\text{cm}$$

$$L = \left(78 \pm 0.01\right)\text{cm}$$

$$\rho = \left(0.087 \pm 0.016\right)\Omega\text{cm}$$

Calculate the percentage error in R.

(c) If the error in x is denoted δx determine the formula for $\delta(x^2y^3)$ and hence find the error in (x^2y^3) when x = (5 ± 0.05)cm and y = (10 ± 0.1)cm.

(d) A liquid having small depth but large volume is forced it to escape with velocity V through a small . if v is given $v = cp^x\rho^y$ where ρ is liquid density and c, x and y are dimensionally constants
(i) Determine x and y
(ii) If v = 14m/s when p = 1.0 × 10^5Pa and
ρ = 1000kgm^{-3} deduce c.

27. NECTA 2010/P1/1(c)
(i) Define error
(ii) In an experiment to determine the acceleration due to gravity g, a small ball bearing is timed while falling from rest trough a measured vertical height. The following data were obtained vertical height h = (600 ± 1)mm. Time taken t = (350 ±1)ms. Calculate the numerical value of g from the experimental data, clearly specify the errors.

28. (i) Define error.
(ii) In an experiment to determine the acceleration due to gravity g, a small ball bearing is timed while falling from rest through a measured vertical height. The following data were obtained
Time taken t = (350 ± 1)ms. Calculate the numerical value of g from the experimental data, clearly specify the error.

29. (a) Differentiate between error and mistake.
(b) In determining the resistivity ρ of a certain wire , the following measurement were taken.
Resistance R of the wire = (2.06 ± 0.01) Ω
Diameter d of the wire = (0.57 ± 0.01) mm
Length of the wire = (105.6 ± 0.1) mm
Use the formula

$$\rho = \frac{\pi d^2 R}{4L}$$

Find the relative error in resistivity.
(c) (i) Given three (3) limitation of dimensional analysis.
(ii) After being deformed , a spherical drop of liquid will execute periodic vibration about its sphere. The frequency about (f) of vibration of the drop will depend on the surface tension (γ) of the drop , its density ρ and the radius r of the drop. Using the method of dimensions , obtain an expression for the frequency of these vibrations in terms of the related physical quantities.

30. Compute the numerical value of

$$J = \left(\frac{I^2R}{W+M}\right)\frac{T}{\theta}$$

Given that:-
I = 2.5 ± 0.05 , R = 11.36 ± 0.01
W = 21 ± 1 , M = 155 ± 1
θ = 28 ± 0.5 , T = 298 ± 0.5

31. EZEB 2011/P1/1
(a) (i) State the basic rule of dimensional analysis.
(ii) Find the dimensions of a/b in the equation $p = \frac{a - bt^2}{bx}$
x is distance and t is the time.
(iii) The depth x to which a bullet penetrates in a human body depends upon the coefficient of the viscosity η and kinetic energy (E). Establish the relation among these quantities by method of dimensions.
(b) (i) Define an error
(ii) The focal length of a lens is related to the object distance u and image distance v by the formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

A students records the value of
U = (15.0 ± 0.5) cm and
f = (10.0 ± 0.05) cm calculate the value of v.

32. (a) The pressure p is calculated from the relation.

$$p = \frac{F}{\pi R^2}$$

Where F is the force and R is the radius if the percentage errors is ±2% for F and ±1% for R, calculate the percentage error on p.
(b) In experiment to determine the Young's modulus for the student recorded the following measurements.
Length , L of the wire = 3.25 ± 0.005m
Diameter d of the wire = 0.63 ± 0.02mm
Force F on the wire = 26.5 ± 0.1N
Extension, produced = 1.40 ± 0.05mm
Calculate the Young's modulus of the wire from these measurement and its corresponding error.

33. (a) (i) why is it important to do error analysis whenever taking measurement?
(ii) Is it possible to avoid error why?
(b) The rate of heat flow p in a cable of resistivity ρ , length L and with a diameter d , carrying an electric current I is given by the expression

$$p = \frac{4\rho I^2 L}{\pi d^2}$$

If $\rho = 3 \times 10^{-7}\Omega m$
L = (100 ± 0.1)cm
d = (1.0 ± 0.1)mm and
I = (5 ± 0.1)A
Find an error in measurement of p?

34. (a) The mass of the body as measured by two students is given as 9.2kg and 9.23kg which measured is more accurate? Why?
(b) In the formula $y = a^2x + b$, which quantity should be measured most accurately ? why?

Solution

(a) 9.23kg because it has more significant figures (3sgf) meaning that instrument used is more accurate.

(b) 'a' because its error will be multiplied by the power 2 which is the highest.

35. (a) Which quantity in a given formula should be measured most accurately?
(b) Three lengths are given 3.7cm , 48.78cm and 6.71cm. What do you inter from these reading?
(c) A physical quantity p is given by

$$p = \frac{a^2b^3}{c\sqrt{d}}$$

If the percentage errors of measurement in a , b, c and d are 4% , 2% , 3% and 1% respectively , find the percentage error in p.

Solution

(a) It clear from these readings that measurement have been made by using instrument of different least count.
(b) The quantity in the formula which has maximum power (n) should be

measured most accurately . It is because any error in the measurement of this quantity is multiplied n time in the final result.

(c) Given that $p = \frac{a^2b^3}{c\sqrt{d}}$

The maximum fractional error on p

$$\frac{\Delta p}{p} = \frac{2\Delta a}{a} + \frac{3\Delta b}{b} + \frac{\Delta c}{c} + \frac{1}{2}\frac{\Delta d}{d}$$

Percentage error on p

$$\frac{\Delta p}{p}\times 100\% = 2\left[\frac{\Delta a}{a}\times 100\%\right] + 3\left[\frac{\Delta b}{b}\times 100\%\right]$$

$$+\frac{\Delta c}{c}\times 100\% + \frac{1}{2}\left[\frac{\Delta d}{d}\times 100\%\right]$$

$$= 2\times 4\% + 3\times 2\% + 3\% + \frac{1}{2}\times 1\%$$

$$\frac{\Delta p}{p}\times 100\% = 17.5\%$$

36. (a) What do you understand by absolute error?
(b) If all measurement in an experiment are taken up to same number of significant figures then which measurement is responsible for maximum error?
(c) Discus how error propagate in sum , difference , product and division of quantities.

Solution

(a) The difference in the magnitude of true value and he measured value of a physical quantity is called Absolute (actual) error.
(b) The quantity in the formula which has maximum power responsible for maximum error if all quantities in the formula have the same powers , then the quantity which is least in magnitude is responsible for maximum error.
(c) Refer to your notes.

37. In an experiment with simple pendulum , a time period measured was 40s for 20 vibrations when the length of the pendulum was taken as 100cm.
(i) If the least count of the stop watch is 0.1s and that of the metre scale is 0.1cm , calculate the maximum permissible error in the measurement of g.
(ii) If the actual value of g at DSM is $9.79m/s^2$, calculate the percentage error.

Solution

(i) $T = 2\pi\sqrt{\frac{L}{g}}$

$$T^2 = \frac{4\pi^2 L}{g}$$

$$g = \frac{4\pi^2 L}{T^2}$$

On differentiating

$$\frac{\Delta g}{g} \quad \frac{\Delta L}{L} + 2\frac{\Delta T}{T}$$

Maximum missible error.

$$\frac{\Delta g}{g}\times 100\% = \left[\frac{\Delta L}{L} + 2\frac{\Delta T}{T}\right]\times 100\%$$

$$= \left[\frac{0.1}{100} + 2\times\frac{0.1}{40}\right]\times 100\%$$

$$= 0.60\%$$

∴ Maximum permissible error = 0.60%

(ii) $g = \frac{4\pi^2 L}{T^2}$, L = 100cm = 1m

$$g = \frac{4\pi^2\times 1}{2^2} = 9.856m/s^2$$

$g = 9.859m/s^2$

$$\%error = \frac{9.8596 - 9.7915}{9.7915}\times 100\%$$

$$\%error = 0.6955\%$$

38. In experiment to determine the value of Young's modulus of elasticity of steel , a wire of length 325cm (measured by a metre scale of least count 0.1cm) is leaded by a mass of 2kg and it is found that it stretches

by 0.227cm (measured by a micrometer having least count 0.001cm) the diameter of the wire as measured by a screw gauge (least count = 0.001cm) is found to be 0.043cm. calculate the maximum per missible error.

Solution

Young's modulus , E

$$E = \frac{\text{stress}}{\text{strain}} = \frac{FL}{Ae} = \frac{4MgL}{\pi d^2 e}$$

The maximum fractionaly error on E

$$\frac{\Delta E}{E} = \frac{\Delta L}{L} + 2\frac{\Delta d}{d} + \frac{\Delta e}{e}$$

% error

$$\frac{\Delta E}{E} \times 100\% = \left[\frac{\Delta L}{L} + 2\frac{\Delta d}{d} + \frac{\Delta e}{e}\right] \times 100\%$$

$$= \left[\frac{0.1}{325} + \frac{0.001}{0.043} \times 2 + \frac{0.001}{0.227}\right] \times 100\%$$

$$\frac{\Delta E}{E} \times 100\% = 5.123\%$$

39. NECTA 1976

A strip of silver of mass (10.01 ± 0.1) gm is (50±0.5)mm long ?(30±0.2)mm wide and (2.0±0.1)mm thick.

(a) Determine the percentage error in the value of the density of silver from the data.

(b) Which of the above measurement need to made most accurately why?

(c) Obtain the density of the silver?

40. In physics the discovery of a new law or principle is acceptable only when experiment approve it in order to get as close to the truth as possible , physicists have not only trying to design more and more perfect instruments but also developed a theory of errors which help in eliminating possible errors in the observations. State two assumptions in which theory of errors originates.

Solution

(i) Instrument used in experiment have some defects or imperfection hence errors are inevitable.

(ii) Experimenter also can subject himself or herself into error or blunders (mistake) due to carelessness or other factors.

(iii) Fluctuation of weather condition such as temperature , wind blow and humidity can subject errors in experiments.

41. The critical magnetic field supplied by passing a current I through a solenoid of diameter D and length L is given by

$$\beta = \frac{\mu_0 nIL}{\sqrt{L^2 + D^2}}$$

Where n is the number of turns per unit length and μ is absolute permeability of free space. Determine the magnitude of the field β and error in the quantity from the following values;

$n = 3920m^{-1}$

$I = 1.92 \pm 0.02A$

$D = 3.5 \pm 0.1cm$

$L = 12 \pm 0.1cm$

$\mu_o = 4\pi \times 10^{-2}Hm^{-1}$

42. (a) Can a dimensional analysis show that a physical quantity is completely right. If NO or YES explain.

(b) Calculate the value of Y in the following relation.

$$Y = \frac{4MgL\sin\theta}{4\pi d^2 e}$$

Where $M = (1000 \pm 0.1)gm$

$L = (200 \pm 0.02)cm$

$d = (0.75 \pm 0.05)$ mm

$e = (0.325 \pm 0.001)$ cm

$g = (9.81 \pm 0.005)$ m/s²

43. (i) State the two common type of error encountered in the experimental physics.

(ii) What is the causes of the error stated in (i) above the how can they be minimized?

(iii) The density of a uniform cylinder was determined by measuring its mass M, length L and diameter , d. calculate the density in (kgm^{-3}) and its error from the following values.

$m = (47.36 \pm 0.01)$ gm
$L = (15.28 \pm 0.05)$ mm
$d = (21.37 \pm 0.04)$ mm

44. Given that $\frac{e}{m} = \frac{8v}{B^2r^2}$ and that

$$B = \frac{\mu_o nI}{\left[1+\left(\frac{D}{L}\right)^2\right]^{1/2}}$$

Where $n = 3920m^{-1}$
$D = (0.35 \pm 0.001)$m
$L = (0.120 \pm 0.001)$m
$I = (1.92 \pm 0.02)$A
$V = (20 \pm 1)$Volt
$\mu_o = 4\pi \times 10\text{-}7Hm^{-1}$

Estimate (i) The value of B and its error
(ii) The value of c/m and it error.

45. In an experiment to determine the apparent cubical expansivity of a liquid by Achimede's principle the following results were obtained:-
Mass of sinker in air $M_1 = (230.2 \pm 0.1)$g
Mass of sinker in cold water $M_2 = (59.1 \pm 0.1)$g
Mass of sinker in warm water $M_3 = (59.6 \pm 0.1)$g
Temperature of cold water $t_1 = (15 \pm 0.5)^oC$
Temperature of warm water $t_2 = (25 \pm 0.5)^oC$
If the cubical expansivity of liquid is given by

$$\gamma = \frac{(M_3 - M_2)}{(M_1 - M_3)(t_2 - t_1)}$$

(i) Determine an expression for the percentage error in v.
(ii) Determine the numerical value of γ and its error.

46. (a) (i) 'Dimension can be treated as algebraic quantities' explain this statement.
(ii) What does this statement mean the density of water is $(1000 \pm 0.5)kgm^{-3}$?
(b) The time for simple pendulum oscillation are recorded as follows;-
0.3, 0.4, 0.5, 0.6, 0.7, 0.8 sec
(i) Determine the mean value of the measure quantities.
(ii) Estimate arithmetic mean of the absolute value.
(iii) The frequency 'f' of a note produce by a taut wire stretched between two support depends on the distance L between the supports the mass per unit length of the wire M and the tension T. Using dimensional analysis to derive the equation of F in terms of L, M, T and K where K is dimensionless constant.

47. A capacitance $c = (2.0 \pm 0.1)\mu F$ is charged to a voltage , $v = (20 \pm 0.2)$ volt. What will be the charge Q on the capacitor?

48. Find out the maximum percentage error while the following observations were taken in the determination of the value of acceleration due to gravity.
Length of thread = 100.2cm
Radius of bob = 2.43cm
Time of one oscillation = 2.2sec. Which quantity will be measured more accurately?

49. (a) Differentiate between error and mistake.
(b) The value of v is to be calculated from the formula $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$ and $f = (20 \pm 0.001)$cm and $u = (32 \pm 0.5)$ cm. calculate:-
(i) The possible % error in f and u
(ii) The possible % error in 1/f , 1/u, 1/v and v
(iii) The actual possible error in the calculated the value of v.

50. In experiment to measure the acceleration of free fall , a steel ball took 807ms to fall a

distance 3.20m from rest. Calculate the value of acceleration of free fall. The uncertainty in the time of fall was ±5ms. What is the percentage uncertainty in the value of the acceleration you have just calculated?

51. The following observation were actually made during an experiment to find the radius of curvature of a concave mirror R using spherometer L = 4.4cm, h = 0.085cm the distance L between the legs of the spherometer was measured with a meter rod and the least count of the spherometer was 0.001cm. calculate the maximum possible error in the radius of curvature given the

$$R = \frac{L^2}{6h} + \frac{h}{2}$$

52. Period of a body execute S.H.M given by

$$T = 2\pi\sqrt{\frac{a^2 + b^2}{12gh}}$$

a = (4 ± 0.05) cm
b = (6 ± 0.05) cm
h = (2 ± 0.05) cm

Calculate the actual value of g including its order of accuracy.

53. In an experiment to measure angles, spectrometer reads up to 6 of an arc. Estimate percentage error in the refractive index of material of glass prism which is given by

$$\mu = \frac{\sin\left(A + \frac{B}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Where A is angle of the prism = 0° and B – angle of minimum derivation = 48°6'

54. In an experiment to determine the coefficient of surface tension of water by the rise in capillary tube the following results were obtained:-
Height of water risen, h = 8.99±0.02cm
Mass of mercury m = 0.088±0.002g
Length of mercury thread in capillary, L = 7.52 ± 0.02cm.
Given that the surface tension of water in given by.

$$\gamma = \frac{h\rho_{\omega}g}{2}\sqrt{\frac{m}{\pi L\rho_{Hg}}}$$

Determine:-
(a) Relative error in measured value of surface tension , γ
(b) The numerical value of γ

55. In a current balanced method of realizing the Ampere , the force between two wires are arranged to give couple between the coil of radii r_1 and r_2 which is balanced by couple produced by force F at a distance x apart. The formula of calculating the current I from the observation is

$$I = \frac{kr_1Fx}{r_2}$$

Where k is numerical factor constant which is exactly known. If r_1 = 0.5m and measured to the nearest 1mm F and X are each measured to the accuracy of 2%, estimate the accuracy to which the value of I can be relied on.

56. A micrometer measures length L_1 as 0.80mm and length L_2 as 0.5mm. If its has a zero error that causes to read low by 0.1mm. What is % error result in the calculation of the ratio $\frac{L_2}{L_1}$ if the zero is over looked?

57. Given that $T = 2\pi\sqrt{\frac{H-h}{g}}$, estimate the percentage error in the g if the percentage error in it is 2% and T is 1.5%.

58. A travelling microscopic can read to 0.1mm. What is the precision of measurement of a distance of 1cm?

59. EZEB 2012/P1/1
(a) Distinguish between systematic and random error (give three difference)

(b) (i) What is the relative importance of errors in the physical world?

(ii) A form six student conducted on experiment in order to determine the surface tension of water $\gamma\omega$ by a rise in capillary tube . She recorded the following data:

Reading of meniscus = (9.92±0.01)cm

Reading of water surface(0.92±0.01)cm

Length of mercury thread in Capillary = (7.51 ± 0.01)cm

Mass of the watch glass = (15.32 ± 0.001)g

Mass of watch glass and mercury = (15.408 ± 0.001)g

Using this information determine the surface tension and its accuracy.

60. (a) Given the data 3.70 , 3.67 , 3.68 , 3.66, and 3.69. If the accurance and precision limits are ±0.03 and 0.02 respectively state (quantitatively) whether the data is accurate or precise.

(b) In determination of final speed v for a toy car , the following data were recorded.

u = 10.20 ± 0.002 m/s

a = 2.0(± 0.01) m/s^2

t = 3.00 ± 0.01s

Given that v = u + at. Find v and its uncertainty.

61. The period of oscillation of a rod depends on its radius r and velocity V. Determine the fractional error in calculating the acceleration due to gravity g if r = (2I0.1)mm and V = 4 (± 0.1)cm/s. The period of oscillation is measured to be 10sec using a stop watch of scale 0.1sec. given that

$$T = \sqrt{\frac{3rv^2}{k+gv}}$$

Where k is a dimensionless constant.

62. (a) give any two advantages

(b) The resistance R of a hallow cylindrical wire of resistivity ρ length , L the outer and inner diameter D and d respectively is given by

$$R = \frac{4\rho L}{\pi\left(D^2 - d^2\right)}$$

Order the determine the resistivity ρ are

R = (25.0 ± 0.2)Ω

L = (1235 ±0.5)cm

d = (0.46 ± 0.01) cm

D = (0.68 ± 0.01) cm find

(i) The maximum possible percentage error in ρ.

(ii) The maximum possible absolute error.

63. The theory of gas flow through small diameter tubes at low pressure is an important consideration of high vacuum technique. One equation which occurs in the theory is given by

$$Q = kr^3\frac{\left(p_1 - p_2\right)}{L}\cdot\sqrt{\frac{M}{RT}}$$

Where k is a number without unit , r is the radius of the tube , p_1 and p_2 are the pressure at each end of the tube of length L , M is the molar mass of the gas (unit $kgmol^{-1}$) and T is the temperature.

(i) Use the equation to find the base unit of Q.

(ii) In using the equation given above the value of r is (1.67 ± 0.03) × 10^{-4}m. What is the percentage uncertainty does this introduce into the value of Q.

64. In an experiment to determine the coefficient of surface tension γ use a U – tube having stems of radius a and b , γ is calculated from.

$$h\rho g = 2\gamma\left[\frac{1}{a} - \frac{1}{b}\right]$$

If h = (0.86 ± 0.01)cm

a = 0.07 ± 0.01cm

b = (0.21 ± 0.02) cm

The uncertainty in g and ρ is not more than $0.05m/s^2$ and $0.5kg/m^3$ respectively. Estimate the order of a accuracy in the calculated volume of γ. Take the density of the liquid to be $960 kg/m^3$. Also calculate the final volume of γ.

65. (a) (i) how can random and systematic errors be minimized during an experiment?
(ii) Estimate the precision to which the Young's modulus, γ of the wire can be determined from the formula

$$\gamma = \frac{4FL}{\pi d^2 e}$$

Given that the applied tension, F = 500N, the length of the loaded wire L = 3cm, the diameter of the wire, d = 1mm, the extension of the wire e = 5mm and the error associated with these quantities are 0.5N, 2mm, 0.01mm and 0.1mm respectively.
(b) (i) State the law of dimensional analysis
(ii) If the speed of the transerve wave along a wire of the tension, T and mass M is given by

$$V = \sqrt{\frac{T}{m}}$$

Apply the dimensional analysis to check whether the given expression is correct or not.

66. (a) (i) Identify two basic rules of dimensional analysis
(ii) The frequency n of vibration of a stretched string is a function of its tension, F length L and mass per unit length, m. use the method of dimensions to derive the formula relating the stated physical quantities.
(b) (i) What causes of systematic error in an experiment? Give four points.
(ii) Estimate the numerical value of

$$\text{drag force } D = \frac{1}{2} C\rho A V^2$$

With its associated error given that the measurement of the quantities C, A, ρ and V were recorded as (10 ± 0.00) units less, $(5 \pm 0.2)cm^2$, $(15 \pm 0.15)g/cm^3$ and (3 ± 0.5) cm/sec respectively.

67. (a) (i) explain briefly the meaning of the term error and mistake
(ii) The resistivity 'ρ' of the material of a wire of resistance 'R' the length 'L' and diameter 'd' is given by

$$\rho = \frac{R\pi d^2}{4L}$$

Show that the percentage error in resistivity is given by

$$\rho = \left(\frac{\Delta R}{R} + \frac{2\Delta d}{d} + \frac{\Delta L}{L}\right) \times 100\%$$

(b) (i) What are the dimensional equations, state any two uses of dimensional equation.
(ii) A gas bubble form an explosion under water is found to oscillate with a period T which is proportional to p^a, d^b, and E^c where p is the pressure, d is the density and E is the energy of explosion. Find the value of a, b and c and hence determine the units of the constants of proportionality