SIMPLE HARMONIC MOTION
CHAPTER OVERVIEW

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MODULE 4 : SIMPLE HARMONIC MOTION

1.1 **INTRODUCTION**

Any motion that repeats itself after certain period is known as **periodic motion** and since such motion can be represented in terms of sine or cosine function , it is called **harmonic motion.**

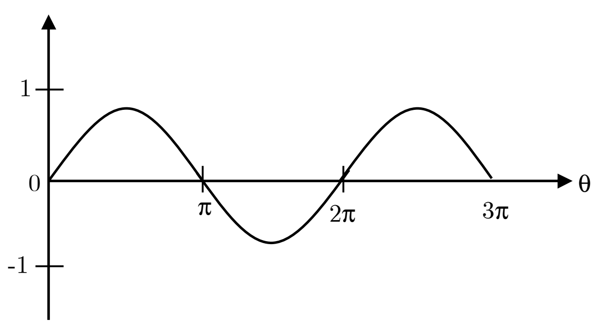
Harmonic motion - Is the motion of the body whose expression for the displacement of a periodic motion can be described in terms of sine or cosine function.

A periodic function - Is the function whose value its repeats after a definite interval of time. sinθ and cosθ both are the periodic function because $\sin(\theta + 2\pi n) = \sin\theta$,

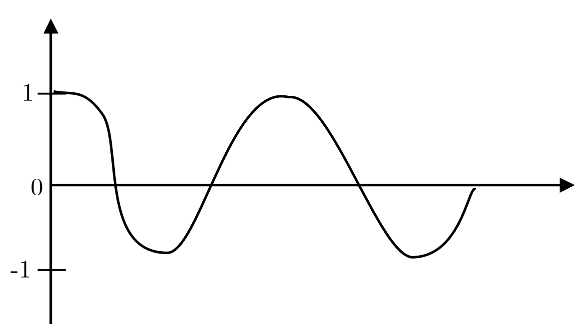
$\cos(\theta + 2\pi n) = \cos\theta$ where $n = 1, 2, 3$

GRAPHS OF SINE AND COSINE FUNCTIONS

Graph of y = sinθ



Graph of y = cosθ



1.2 **Periodic motion and oscillatory motion**

Periodic motion - Is the motion of the body which repeated itself after fixed (regular) interval of time. The fixed interval of time after which the motion is repeated is called periodic time (T) of motion.

Examples of periodic motion

1. The motion of hands of a clock is periodic motion.
2. Motion of the planet (Earth) around the sun
3. Motion of the moon around the Earth
4. Swinging of the bob of a simple pendulum
5. Oscillating mass on a spring
6. Heart beats of person , its period is about 0.83sec for a normal person.

Oscillatory (vibratory) motion - Is the motion in which a body moves to and fro or back and forth repeatedly about a fixed point (called mean position or equilibrium position) in a definite interval of time. In such a motion, the body is confined within well definite limits (called extreme positions) on either side of mean position. A periodic and bounded motion of a body about a fixed point is called an **oscillatory of vibratory or harmonic motion.**

Examples of oscillatory motion

1. Oscillations of simple pendulum.
2. Vibrations of mass attached to a spring
3. Vibration of guitar string.
4. Vibration of air molecules as a sound wave passes.
5. The motion of liquid contained in U – tube.

Characteristics of oscillatory motion.

1. It is a periodic motion.
2. It is to and fro motion along the same path about a fixed point called equilibrium or mean position.
3. The body is confined within well defined limits called extreme positions.

1.3 **Difference between periodic motion and oscillatory motion.**

All oscillatory motions are periodic motions because each oscillatory motion is completed in a definite interval of time. But all periodic motions may not be oscillatory. For example, the revolution of Earth around the sun is a periodic motion but not an oscillatory motion, because the basic concept of to and fro (or back and froth) motion about some mean position for oscillatory motion is not present in this motion.

1.4 **Harmonic and non – harmonic oscillation.**

Harmonic oscillation - Is the oscillation which can be expressed in terms of single harmonic function (i.e. sine or cosine function). Mathematically, a simple harmonic oscillation can be expressed as

$$y = A\sin\omega t = A\sin\left(2\pi t/T\right) \text{ OR}$$

$$y = A\cos\omega t = A\cos\left(2\pi t/T\right)$$

Where

y = displacement of body measured from mean position at any instant of time t,

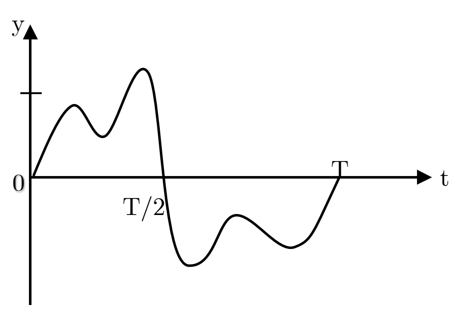
A = Amplitude of displacement of a body

ω = Angular frequency (Velocity)

f , T = frequency and periodic time of oscillation.

Non – harmonic oscillation - Is the oscillation which cannot be expressed in terms of single harmonic function. A non – harmonic oscillation is a combination of two or more than two harmonic oscillations. Mathematically, Non – harmonic oscillation may be expressed as $y = a\sin\omega t + b\cos\omega t$

Graphical representation of the Non – harmonic oscillation.

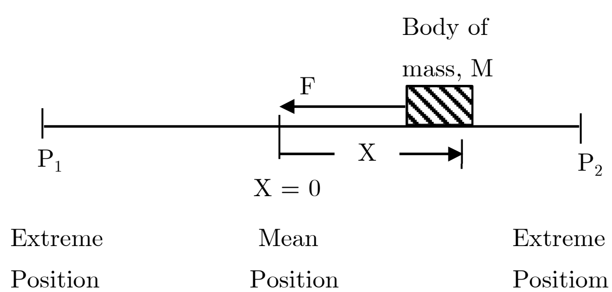


RESTORING FORCE

For a body to undergo oscillatory motion it must be acted upon by an unbalanced force called **Restoring force**.

Restoring force - Is defined as an elastic force which required to return back the body when move towards to the mean position. i.e Restoring force is the force which always acts to accelerated a body in the direction of its equilibrium (mean position)

Expression of the restoring force



According to the Hooke's law

$F \alpha - X$, $F = -KX$, $F = -KX = M\omega^2 X = -Ma$

X = displacement of a particle measured from mean position.

M = Mass of a body, ω = Angular velocity

a = acceleration of a body , K = force constant

Negative sign shows that the direction of the force F is in opposite to the direction of the displacement , X.

The restoring force on the body can be depends on the following factors:-

1. Displacement, X of the body measured from the mean position.
2. The inertia of the body i.e mass of the body.
3. The angular frequency (velocity) ω of the body.

Characteristics of the restoring force in s.h.m

1. The direction of the restoring force should be in a direction opposite to the direction of the displacement of the body.
2. The magnitude of the restoring force should be directly proportional to the displacement of the body.

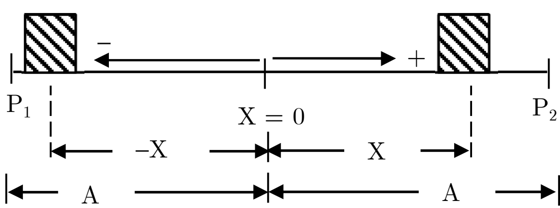
Conditions necessary for the body to oscillates.

For a body to oscillates or vibrates three conditions must be satisfied.

1. The body must have inertia to keep it moving across the mid – point of its path.
2. There must be a restoring force to accelerate the body towards to the mid – point [F = - Ma = - KX]
3. The frictional force acting on the body against its motion must be very small.

Force constant, k - Is defined as restoring force per unit displacement $K = \frac{F}{X}$ S.I Unit of the force constant is Nm^{-1}. The value of k depends upon the length, thickness , material of the wire e.t.c

Consider the figure below which shows the oscillation of the body about their mean position



The point X = 0 is called mean position because it lies the middle of the line of oscillation. It is also called **equilibrium position** because at this point the resultant force acting on the particle is equal to zero.

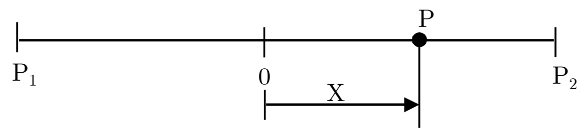
2.1 SIMPLE HARMONIC MOTION (S.H.M)

S.H.M is defined as a to and fro motion of the body in which its acceleration is directly proportional to the displacement and its always directed towards to the mean position or equilibrium i.e Simple harmonic motion is a special type of periodic motion, in which a particle moves to and fro respectedly about a mean (i.e equilibrium) position under a restoring force which is always directed toward the mean position and whose magnitude at any instant directly proportional to the displacement of the particle from the mean position at that instant.

Mathematically

Consider a particle executing S.H.M along the x – axis between points P_1 and P_2 into O as mean position , where $OP_1 = -A$ and $OP_2 = A$

Let X be displacement at any time , t



$a \; \alpha -x \;, \quad a = -\omega^2 x$

$$a = \frac{d^2x}{dt^2} = -\omega^2 x$$

The restoring force acting on the particle at that instant is F = -KX. S.H.M is the type of oscillatory motion . The origin of the name S.H.M is of all the periodic or harmonic motion (e.g circular motion , elliptical motion e.t.c). It is the simplest periodic motion for this reason it is called **simple harmonic motion S.H.M** is named so since is the simplest type of oscillatory motion.

Conditions for the body to execute s.h.m

1. The acceleration of the body is directly proportional to the displacement measured from the mean position.
2. The acceleration of the body must be always directed towards to the mean or equilibrium position i.e acceleration must be oppositely directed to the displacement.

2.2 TYPES OF S.H.M

There are two types of S.H.M

1. Linear simple harmonic motion
2. Angular simple harmonic motion

Linear s.hm - Is the motion of the body in S.H.M which takes place under action of restoring force $F \; \alpha -X$ or $a \; \alpha -x$, $F = -KX$ or $a = -\omega^2 x$ Negative sing on an acceleration shows that the direction of acceleration is in opposite to the displacement , X.

Differential form of equation of linear S.H.M

Since a = $-\omega^2 x$ but $a = \frac{d^2x}{dt^2}$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

This represent differential form of equation of liner S.H.M

Examples of linear S.H.M

1. Oscillation of a loaded spring
2. Oscillation of loaded helical spring
3. Vibrations of atoms and molecules
4. Vibrations of a turning fork
5. Oscillation of simple pendulum
6. Liquid oscillating in a u – tube
7. Floating cylinder on the liquid
8. Oscillation of the ball on neck chamber
9. Oscillation of piston in a gas filled cylinder.
10. Motion of body dropped in a tunnel along the diameter of the earth ect.

Angular s.h.m - Is the motion of the body in S.H.M which takes place under acting of the restoring torque if a rigid body rotates about a fixed axis so that at any instant its angular acceleration is proportional to its angular displacement from a fixed line through that axis , the body is said to execute Angular S.H.M i.e $\tau \ \alpha -\theta$ or $\tau = -c\theta$, C = Constant of proportionality.

Differential form of equation of angular S.H.M

$$\frac{d^2\theta}{dt^2} \alpha -\theta \ , \quad \frac{d^2\theta}{dt^2} = -\omega^2\theta$$

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$$

This represent differential form of equation of an angular S.H.M.

TERMINOLOGIES

Examples of angular S.H.M

1. Oscillation of torsional pendulum
2. Oscillation of compound pendulum
3. Vibration of a balance wheel of a watch etc

Terminologies applied to the s.h.m

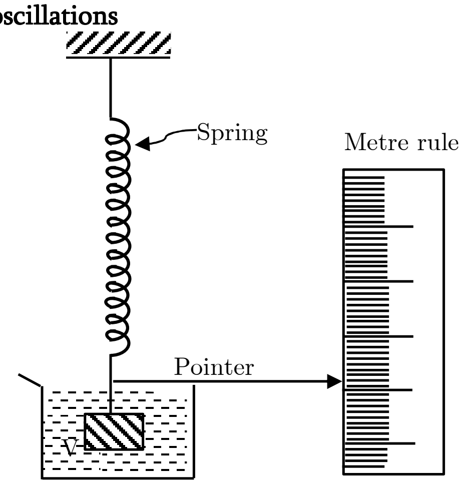
1. **Oscillator** - Is a body executing oscillatory motion.

2. **Restoring force** - Is the force which maintain the body to oscillates about their mean position [F = -KX = -Mω^2X]

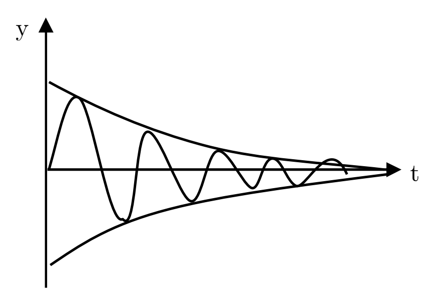
3. **Equilibrium position** - Is the position at which the body would come to rest if it were to loss all of its energy. At this position , no net force act on oscillating body.

4. **Damped simple harmonics oscillation** - Is the simple harmonic system oscillates with a decreasing amplitude with time. Many oscillating bodies do not move back and forth between precisely fixed limits because frictional forces dissipate the energy of motion. The dissipative force or damping force are active in the system which are generally the frictional or viscous forces. Consider the figure below which shows the oscillation of block of mass M in viscous liquid.

Graph of displacement against time for damped oscillations



Graph of displacement against time for damped oscillations



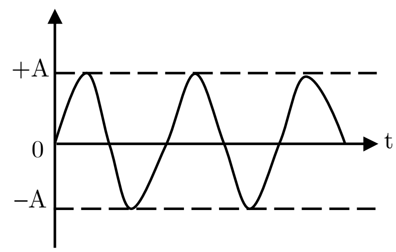
Examples of damped oscillations

1. The oscillations of the bob of a simple pendulum in air or medium are damped oscillations.
2. The oscillations of string in air or any medium

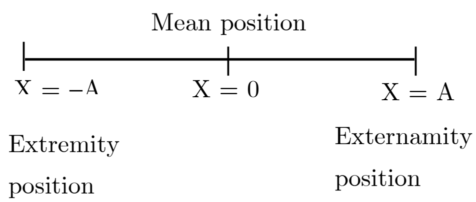
5. Un damped simple harmonic oscillation

Is the simple harmonic system which oscillates with a constant amplitude which does not change with time.

Graph of displacement against time for un damped oscillation.



6. Extreme positions - Are the end points of oscillation on either side of the equilibrium position



7. Amplitude (a) - Is the maximum displacement of particle which executing S.H.M

8. Linear displacement (x or y) - Is the distance covered by the particle in S.H.M which can be measured from mean position at any instant of the time.

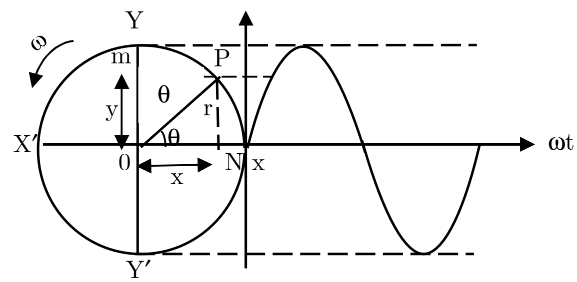
Equations of linear displacement of particle executing S.H.M

(i) $y = A\sin\omega t$
(ii) $y = A\cos\omega t$
(iii) $y = A\sin(\omega t + \Phi)$
(iv) $y = A\cos(\omega t + \Phi)$

Each symbol have usual meaning

Derivation of equation of linear displacement
Method I:

Analysis of S.H.M in terms of uniform circular motion S.H.M can be described as the projection along a diameter of uniform circular motion. Suppose a particle is moving in anticlockwise direction with a uniform angular velocity in along the circumference of radius r and centre O. This circle is known as **circle of reference** while the particle p is known as **reference particle**.



From the figure above

$$\sin\theta = \frac{\overline{PN}}{\overline{\overline{OP}}} = \frac{y}{r}$$

$$y = r\sin\theta = r\sin\omega t \quad [r = A]$$

$$y = r\sin\omega t = A\sin\omega t$$

Also $x = A\cos\theta = A\cos\omega t$

Where

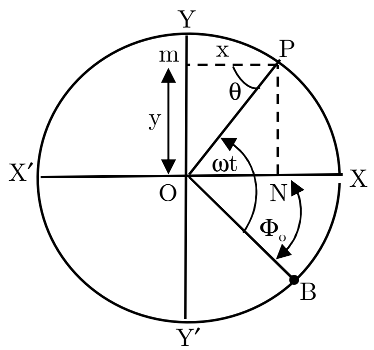
y , x = displacement of a particle
r , A = radius or amplitude
ω = angular velocity , t = time
T = periodic time , f = frequency
Φ = initial phase angle (epoch)
ωt or $\omega t \pm \Phi$ = argument of sine or cosine function
$\sin\omega t$ or $\cos\omega t$ = oscillating term.

Another definition

S.H.M is defined as the projection of a uniform circular motion on any diameter of a circle of reference

Important points

1. If projection of P is taken on diameter XOX' then point N will be executing S.H.M. here ON = X displacement at time, t. $X = A\cos\theta = A\cos\omega t$
2. If B is the starting position of the particle of reference such that <BOX and < BOP = ωt as shown below .



then

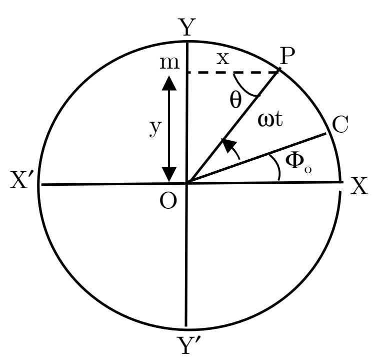
$\theta = \omega t - \Phi_o$

$y = A\sin\theta$

$y = A\sin(\omega t - \Phi_o)$

also $X = A\cos(\omega t - \Phi_o)$

3. If C is the starting point of the particle of reference such that < COX = Φ_o and <COP = ωt



$\theta = \;<XOP = \omega t + \Phi_o$

$y = A\sin(\omega t + \Phi_o)$

$x = A\cos(\omega t + \Phi_o)$

Method II

Derivation of equation of displacement of a particle in S.H.M by using calculus approach method.

From the definition of S.H.M

$$a = -\omega^2 y \text{ but } a = \frac{dv}{dt} = v\frac{dv}{dy}$$

$$v\frac{dv}{dy} = -\omega^2 y\ ,\ vdv = -\omega^2 ydy$$

$$\int vdv = -\omega^2\int ydy$$

$$\frac{v^2}{2} = \frac{-\omega^2 y^2}{2} + C \text{ where } y = A\ ,\ v = 0$$

$$C = \frac{\omega^2 A^2}{2}$$

$$\frac{V^2}{2} = \frac{-\omega^2 y^2}{2} + \frac{\omega^2 A^2}{2} = \frac{\omega^2}{2}\left(A^2 - y^2\right)$$

$$V^2 = \omega^2\left(A^2 - y^2\right)$$

$$V = \pm\omega\sqrt{A^2 - y^2}$$

For the case:

$$V = \omega\sqrt{A^2 - y^2}$$

$$\frac{dy}{dt} = \omega\sqrt{A^2 - y^2}$$

$$\frac{dy}{\sqrt{A^2 - y^2}} = \omega dt$$

$$\int\frac{dy}{\sqrt{A^2 - y^2}} = \int\omega dt$$

$$\sin^{-1}\left(y/A\right) = \omega t + \Phi_o$$

$$y = A\sin\left(\omega t + \Phi_o\right)$$

If $\omega t = 0$, $y = 0$, $\Phi_o = 0$. Then the equation of the displacement of the particle stating from origin is given by $y = A\sin\omega t$

For the case $V = -\omega\sqrt{A^2 - y^2}$

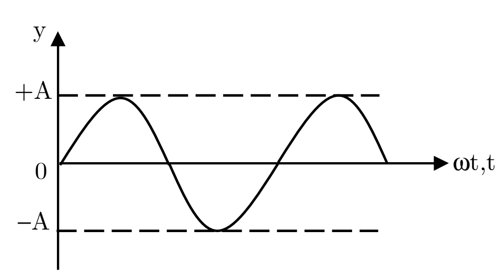
$$\frac{dy}{dt} = -\omega\sqrt{A^2 - y^2}$$

$$\int\frac{-dy}{\sqrt{A^2 - y^2}} = \int\omega dt$$

$$y = A\cos\left(\omega t + \Phi_0\right)$$

Graph of y against time , t (θ)

Let y = Asinωt



Examples of branches of physics applied to the S.H.M

1. In mechanics or mechanical oscillation , $y = r\sin\omega t$.
2. In waves (progressive waves) $y = A\sin(\omega t \pm kx)$
3. In electricity (A.C theory) $I = I_o\sin\omega t$ or $I = I_o\cos\omega t$.

9. **Velocity in s.h.m –** Is defined as the rate of change of the displacement of the particle from the mean position at any instant of time

$$V = \frac{dy}{dt}$$

S.I Unit of velocity is m/s

Expression of the velocity of particle in S.HM

Since

$$y = A\sin\omega t$$

$$V = \frac{dy}{dt} = \frac{d}{dt}\left[A\sin\omega t\right] = A\omega\cos\omega t$$

$$V = A\omega\cos\omega t$$

Also

$$V = A\omega\cos\omega t \;,\quad \cos\omega t = V/A\omega$$

$$y = A\sin\omega t \;,\quad \sin\omega t = y/A$$

Since

$$\cos^2\omega t + \sin^2\omega t = 1$$

$$\left(\frac{V}{A\omega}\right)^2 + \left(\frac{y}{A}\right)^2 = 1$$

$$\frac{V^2}{A^2\omega^2} + \frac{y^2}{A^2} = 1$$

This represent equation of an ellipse

Also

$$\frac{V^2}{A^2\omega^2} + \frac{y^2}{A^2} = 1$$

$$V^2 + \omega^2 y^2 = A^2\omega^2$$

$$V^2 = \omega^2\left(A^2 - y^2\right)$$

$$V = \pm\omega\sqrt{A^2 - y^2}$$

At the equilibrium (mean) position

When $y = 0$, $V = V_{max}$

$$V_{max} = \pm\omega\sqrt{A^2 - O^2} = \pm\omega A$$

$$V_{max} = \pm\omega A = \pm 2\pi f A = \pm\frac{2\pi A}{T}$$

Thus the velocity in S.H.M is not uniform throughout the motion. It is maximum at the mean position at the extreme position

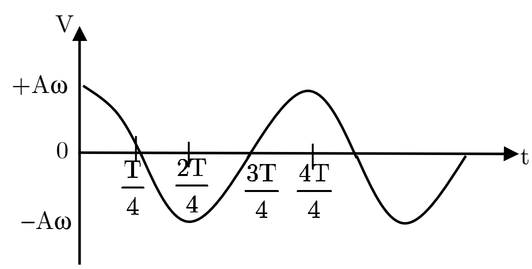
At the extreme positions, the velocity of particle is minimum or zero.

$$y = A \;,\quad V = \omega\sqrt{A^2 - A^2} = 0$$

Note :

The velocity of particle in S.H.M is **maximum at the mean position and minimum or zero** at the extreme positions. The maximum value of the velocity is called **velocity amplitude** in S.H.M in magnitude $V_{max} = \omega A$ the direction of velocity is either forwards to the mean position or away from the mean position.

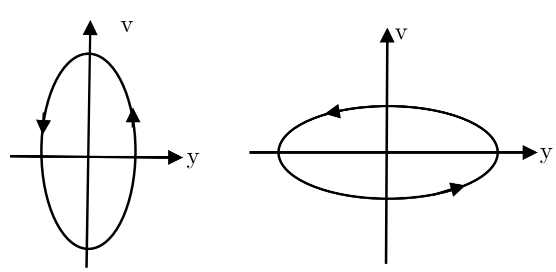
Graph of v against time



Graph of v against y

$$\frac{y^2}{A^2} + \frac{V^2}{A^2\omega^2} = 1$$

For $\omega > 1$ For $\omega < 1$



10. Acceleration - Is defined as the rate of change of velocity of particle execute S.H.M

$$\text{i.e } a = \frac{dv}{dt}$$

SI unit of acceleration is m/s^2
Expression of acceleration is S.HM
Since

$$V = A\omega\cos\omega t$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[A\omega\cos\omega t\right]$$

$$a = -A\omega^2\sin\omega t \text{ or}$$

$$a = -\omega^2 y \text{ or } a = -\omega^2 x$$

Note that

1. Negative sign shows that the direction of acceleration is in opposite to the direction of the displacement.
2. A square of ω emphasize that always acceleration is a positive quantity.
3. Value of acceleration at the different position.

At the mean (equilibrium) point

When $y = 0$, $a = -\omega^2 y = 0$
acceleration of particle in S.H.M is equal to zero at the mean position.

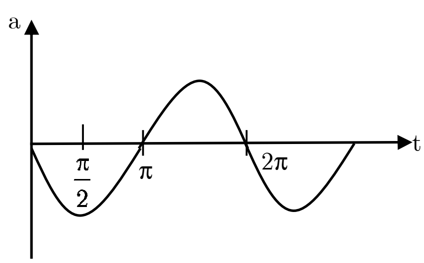
At the extreme position

When $y = \pm A$, $a = a_{max}$

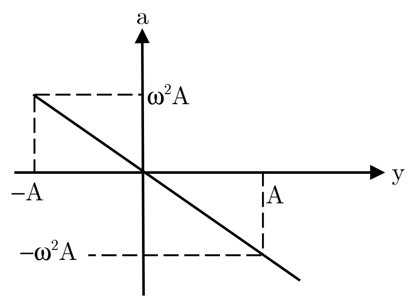
$a_{max} = \pm\omega^2 A = \pm(2\pi f)^2 A$

Acceleration in S.H.M is maximum at the end point of oscillation (extreme position)

Graph of acceleration against time

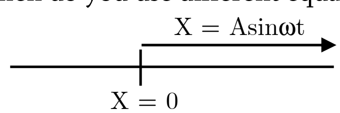


Graph of acceleration against displacement



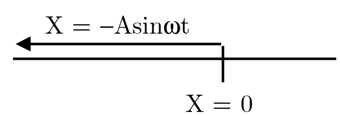
4. The velocity in S.H.M is in phase with acceleration , when particle is moving from extreme position to mean position and is in opposite phase (out phase) with acceleration . When the particle is moving from mean position to extreme position.
5. When do you use different equations

(i)



$$V = \omega\sqrt{A^2 - X^2}$$

(ii)



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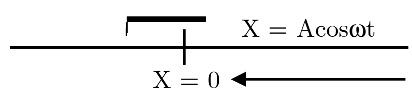

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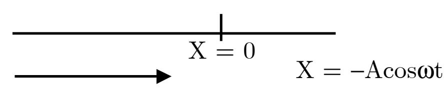


11. Periodic time (T) in s.h.m - Is the time taken by the oscillating particle to complete one oscillation
SI unit of T is second (s)

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

For the S.H.M

$$a = \omega^2 y \text{ (in magnitude)}$$

$$\omega = \sqrt{\frac{a}{y}}, \quad T = \frac{2\pi}{\omega}$$

$$T = 2\pi\sqrt{\frac{y}{a}} = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

12. Frequency, f - Is the number of oscillations or vibrations completed per unit time i.e is the reciprocal of periodic time

$$f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\text{Acceleration}}{\text{Displacement}}}$$

SI unit of f is Hetz or per second (S^{-1})

13. Angular frequency , ω - Angular frequency of a body executing periodic motion is equal to the product of frequency of a body with factor 2π i.e $\omega = 2\pi f$ SI unit of ω is $rads^{-1}$.

14. **Phase** - In oscillatory motion , the phase of a vibrating particle is the argument of sine or cosine function involved to represent the generalized equation of motion of the vibrating particle. If the displacement of a particle at the instant of time, t is represented by the equation

$$y = a\sin(\omega t + \Phi_o) = a\sin\left[\frac{2\pi t}{T} + \Phi_o\right]$$

$\omega t + \Phi_o = \Phi =$ phase of oscillation at any time t.

Phase – is the fraction of the time period that has elapsed since the vibrating particle last left its mean position in the positive direction.

15. Epoch (initial phase) - Is the phase of a vibrating particle corresponding to the time, $t = 0$. It is denoted by Φ_o

SOLVED EXAMPLES

Example 1

The displacement X in metre of the particle from the equilibrium position of the particle moving with S.H.M is given by $X = 0.05\sin 6t$. Where *t* is the time in seconds measured from an instant when $x = 0$

(i) State the amplitude of oscillation

(ii) Calculate the time period of the oscillation and the maximum acceleration of the particle.

Solution

Given that $X = 0.05\sin 6t$…………….(1)

$X = A\sin\omega t$………………..(2)

On comparing equation (1) and (2)

(i) Amplitude , $A = 0.05m$

(ii) $\omega = 6rads^{-1}$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{6} = \frac{\pi}{3}\sec$$

Acceleration of the particle

$$a = \frac{d^2x}{dt^2} = \frac{d}{dt}\left[\frac{a}{dt}(0.05\sin 6t)\right]$$

$$a = \frac{d}{dt}(0.3\cos 6t) = -1.8\sin 6t$$

When

$\sin 6t = \pm 1$, $a = a_{max}$

$a_{max} = \pm 1.8m/s^2$ or

$a_{max} = 1.8m/s^2$ (in magnitude)

Example 2

Which of the following forces would cause on object to move in simple harmonic motion?

(a) $F = -0.5X^2$

(b) $F = -2.5y$

(c) $F = 98X$

(d) $F = -7\theta$

Solution

Both (b) and (d) will give S.H.M because they give force which is proportional to the displacement and minus sign indicates acceleration towards the centre (a) does not produce S.H.M since its motion is not proportional to displacement similarly (c) does not produce S.H.M since its motion is not towards the centre.

Example 3

The displacement y of mass vibrating with S.H.M is given by $y = 20\sin 10\pi t$ when y is in mm , and t is in second. Find

(i) The amplitude

(ii) Period

(iii) The velocity at $t = 0$

Solution

Given that $y = 20\sin 10\pi t$…………(1)

Standard equation of displacement in S.H.M

$y = A\sin 2\pi ft$…………..(2)

on comparing equation (1) and (2)

(i) Amplitude $A = 20mm$

(ii) $10\pi = 2\pi f \Rightarrow f = 5s^{-1}$

$$T = \frac{1}{f} = \frac{1}{5} = 0.2\text{second}$$

(iii) $V = \frac{dy}{dt} = \frac{d}{dt}\left[20\sin 10\pi t\right]$

$V_{(t)} = 200\pi\cos(10\pi t)$

At t =0 , $V_{(0)} = 200\pi\cos(10\pi \times 0)$

$V_{(0)} = 200\pi mm/s$

Example 4

A body oscillates with S.H.M according to the equation

$$x = (5.0m)\cos\left[(2\pi rads^{-1})t + \pi/4\right]$$

at t = 1.5s , calculate the

(a) Displacement
(b) Speed
(c) Acceleration of the body.

Solution

Here

$$\omega = 2\pi rads^{-1} \quad , \quad T = \frac{2\pi}{\omega} = 1\,sec$$

(a) $x = 5\cos\left[2\pi\times 1.5 + \pi/4\right]$

$x = -3.535m$

(b) Speed

$$V = \frac{dx}{dt} = \frac{d}{dt}\left[5\cos\left(2\pi t + \pi/4\right)\right]$$

$$V = -5\times 2\pi\sin\left(2\pi t + \pi/4\right)$$

$$= -5\times 2\pi\sin\left[2\pi\times 1.5 + \pi/4\right]$$

$$V = 22.22m/s$$

(c) $a = \frac{dv}{dt} = \frac{d}{dt}\left[-10\pi\sin\left(2\pi t + \pi/4\right)\right]$

$$a = -20\pi^2\cos\left[2\pi t + \pi/4\right]$$

Now

$$a = -20\pi^2\cos\left[2\pi\times 1.5 + \pi/4\right]$$

$$a = 139.56m/s^2$$

Example 5

A particle moving in a straight line has velocity V given by $V^2 = \alpha - \beta y^2$ where α and β are constant and y is its distance from a fixed point in the line. Show that the motion of particle is S.H.M. Find its time period and amplitude.

Solution

Given that $V^2 = \alpha - \beta y^2$

Differentiate it w.r.t time , t

$$\frac{d}{dt}(V^2) = \frac{d}{dt}\left[\alpha - \beta y^2\right]$$

$$2V\frac{dv}{dt} = -2\beta y\frac{dy}{dt}$$

$$2va = -2\beta yv$$

$$a = -\beta y \ , a \propto -y$$

It will execute S.H.M or linear S.H.M , $a = -\omega^2 y$

$$-\omega^2 y = -\beta y \Rightarrow \omega^2 = \beta$$

$$\omega = \sqrt{\beta}$$

The period , $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\beta}}$ and

$V = 0$, $y = A = r =$ Amplitude

$$0^2 = \alpha - \beta A^2 \text{ or } A = \sqrt{\alpha/\beta}$$

Amplitude , $A = \sqrt{\frac{\alpha}{\beta}}$

Example 6

Calculate the period for a particle executing S.H.M with acceleration of $16m/s^2$ at a distance of 4m from equilibrium position.

Solution

Since

$$a = -\omega^2 x$$

$$a = \omega^2 x$$

$$\omega = \sqrt{\frac{a}{x}} \quad , \quad T = 2\pi\sqrt{\frac{a}{x}}$$

$$T = 2\times 3.14\sqrt{\frac{4}{16}}$$

$$T = 3.14\,sec$$

Example 7

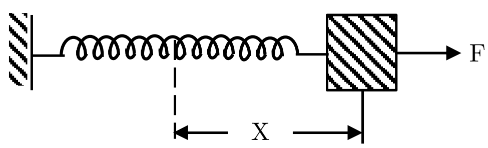
(a) (i) A mass M(kg) is attached to the end of a spring of force constant k(Nm^{-1}) show that $k = M\omega^2$ where ω is the angular velocity.

(ii) From the equilibrium position a particle oscillating in a S.H.M is displaced by a distance X measured in metre from an instant when X = 0 and t is in second is given by X = 0.08sin9t. Determine the period of oscillations and maximum acceleration of the particle

(b) A body oscillates vertically in S.H.M with amplitude of 30mm and frequency of f = 5.0Hz calculate the acceleration of the particles.
 (i) At the extremities of the motion
 (ii) At the centre of the motion
 (iii) At the position mid – way between the centre and extremity.

Solution

(a) (i) consider the figure below



According to the Hooke's law

Restoring force , F = -KX

But F = Ma

$$Ma = -kx \Rightarrow a = \left(\frac{m}{k}\right)x$$

For S.H.M

$$a = -\omega^2 x$$

$$-\omega^2 x = -\left(\frac{k}{m}\right)x$$

$$\omega^2 = k/m$$

$$K = M\omega^2$$

(ii) Given that X = 0.08sin9t…………(1)

Standard equation of displacement of S.H.M

X = Asinωt…………………..(2)

On comparing equation (1) and (2)

$$\omega = 9\,rad/s \quad , \quad T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{9} = 0.6985\,sec$$

The maximum acceleration

$$a_{max} = \omega^2 A = 9^2 \times 0.08$$

$$a_{max} = 6.48m/s^2$$

(b) (i) At extremities x = ± A

$$a = \pm\omega^2 A = \pm(2\pi f)^2 A$$

$$= \pm[2\pi\times 5]^2 \times 30\times 10^{-3}$$

$$a = 29.6m/s^2$$

(ii) At the centre X = 0 , a = 0

(iii) $X = \pm A/2 \quad , \quad a = \pm(2\pi f)^2 \cdot A/2$

$$a = \pm[2\pi\times 5]^2 \times 15\times 10^{-3}$$

$$a = 14.80m/s^2$$

Example 8

(a) Why do we say that velocity and acceleration of a body executing S.H.M are out of phase?

(b) A particle execute S.H.M has period of 4seconds and amplitude of 2cm. find
 (i) The maximum velocity
 (ii) Velocity at half way of its maximum displacement.

Solution

(a) When a body is executing S.H.M, its acceleration is zero at the mean position and maximum at the extreme positions. On the other hand , the velocity is maximum at the mean position and zero at the extreme positions. Further acceleration is always directed towards the mean position for this. For this reason, the velocity and acceleration of a body executing are out of phase.

(b) (i) since $V = \omega\sqrt{A^2 - y^2}$

When y = 0 , V = Vmax

$$V_{max} = \omega A = \frac{2\pi}{T}A = \frac{2\pi}{4}\times 2\times 10^{-2}$$

$$V_{max} = 3.14\times 10^{-2}m/s = 3.14cm/s$$

(ii) $y = A/2 \quad , \quad V = ?$

$$V = \frac{2\pi}{T}\cdot\sqrt{A^2 - \frac{A^2}{4}} = \frac{2\pi A}{T}\sqrt{1-\frac{1}{4}}$$

$$= \frac{2\pi}{4}\times 2\times 10^{-2}\sqrt{\frac{3}{4}}$$

$$V = 2.72\times 10^{-2}m/s$$

Example 9

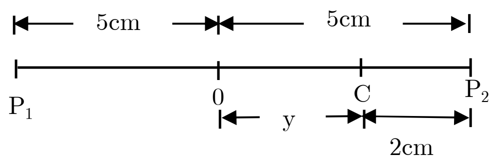
(a) What are the characteristics of S.H.M?

(b) A particle moves with S.H.M between two points which are 10cm apart. At the instant when the particle is 2cm from one end of these points its acceleration is 48cm/s² find.
 (i) Its velocity at the same instant
 (ii) Its period
 (iii) The maximum velocity

Solution

(a)
- The particle moves to and fro about the mean position in a straight line.
- The displacement , velocity and acceleration all vary sinusoidally with time but are not in phase.
- The acceleration is always directed toward the mean position.
- The restoring force is directly proportional to displacement but acts in a direction opposite to displacement
- It is a periodic motion.

(b)



Displacement of the particle from mean position $y = 5 - 2 = 3cm$

(i) In magnitude $a = \omega^2 y$

$$\omega = \sqrt{\frac{a}{y}} = \sqrt{\frac{48}{3}} = 4rad/s$$

Now

$$V = \omega\sqrt{A^2 - y^2} = 4\sqrt{5^2 - 3^2}$$
$$V = 16cm/s$$

(ii) $$T = \frac{2\pi}{\omega} = \frac{2\pi}{4} = 1.57\,sec$$

(iii) $$V_{max} = \omega A = 4\times 5 = 20cm/s$$

Example 10

(i) A particle is moving with S.H.M in a straight line when the distance of the particle from mean position has the value of X_1 and X_2 , their corresponding values of the velocities are U_1 and U_2 respectively. Show that the time period of the motion is given by

$$T = 2\pi\left[\frac{X_2^2 - X_1^2}{U_1^2 - U_2^2}\right]^{1/2}$$

(ii) The velocity of a particle moving in a straight line is given by the equation $V = \sqrt{A^2 - X^2}$ where K and X are constants and X is the distance of the particle from a fixed point in the line. Prove that the motion is the S.H.M and find the period of the motion.

Solution

(i) Since

$$V^2 = \omega^2\left(A^2 - X^2\right)$$

Now

$$U_1^2 = \omega^2\left(A^2 - X_1^2\right)$$
$$U_2^2 = \omega^2\left(A^2 - X^2\right)$$

Takes

$$U_1^2 - U_2^2 = \omega^2\left(A^2 - X_1^2\right) - \omega^2\left(A^2 - X_2^2\right)$$
$$U_1^2 - U_2^2 = \omega^2\left(X_2^2 - X_1^2\right)$$
$$\omega = \left[\frac{U_1^2 - U_2^2}{X_2^2 - X_1^2}\right] \quad \text{but} \quad T = \frac{2\pi}{\omega}$$
$$T = 2\pi\left[\frac{X_2^2 - X_1^2}{U_1^2 - U_2^2}\right]^{1/2}$$

(ii) Given

$$V = K\sqrt{A^2 - X^2}$$
$$V^2 = K^2\left(A^2 - X^2\right) = K^2A^2 - K^2X^2$$

Differentiate V w.r.t time

$$\frac{d}{dt}\left(V^2\right) = \frac{d}{dt}\left(K^2A^2 - K^2X^2\right)$$
$$2V\frac{dv}{dt} = -2K^2X\frac{dx}{dt}$$
$$2va = -2k^2xv$$
$$a = -k^2x \;,\; a \propto -x$$

Hence its execute S.H.M

For S.H.M

$$a = -\omega^2 x = -k^2x$$
$$\omega = k$$
$$T = \frac{2\pi}{\omega} = \frac{2\pi}{k}$$

Example 11

A particle executing S.H.M along a straight line has a velocity of 4m/s when its displacement is 3m and velocity of 3m/s when the displacement is 4m. Find the time taken to travel 2.5m from positive extremely of it oscillation.

Solution

$V_1 = 4ms/s$, $X_1 = 3m$

$V_2 = 3m/s$, $X_2 = 4m$

From the equation $V^2 = \omega^2\left(A^2 - X^2\right)$

$$16 = \omega^2\left(A^2 - 9\right) \ldots\ldots\ldots\ldots\ldots\ldots(i)$$

$$9 = \omega^2\left(A^2 - 16\right) \ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

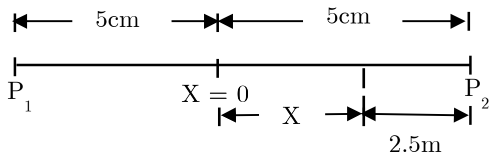
Dividing equation (i) by (ii)

$$\frac{16}{9} = \frac{\omega^2\left(A^2 - 9\right)}{\omega^2\left(A^2 - 16\right)} = \frac{A^2 - 9}{A^2 - 16}$$

$$16\left(A^2 - 16\right) = 9\left(A^2 - 9\right)$$

$72A^2 = 175$, $A = 5m$

Also $16 = \omega^2\left(5^2 - 9\right)$, $\omega = 1rads^{-1}$



Now $X = 5 - 2.5 = 2.5m$

Since the particle travel from positively extreme point to left then

$$X = A\cos\omega t$$

$$2.5 = 5\cos\omega t \quad \left[\omega = 1\right]$$

$$\frac{2.5}{5} = \cos t$$

$$\cos\left(\pi/3\right) = \cos t$$

$$t = \frac{\pi}{3}\sec = 1.047\sec$$

Example 12

The velocity of a particle executing S.H.M is 16cm/s at a distance 8cm and 8cm/s at a distance 12cm from mean position. Determine the amplitude of the motion.

Solution

Since

$$V = \omega\sqrt{A^2 - X^2} \quad , \quad V^2 = \omega^2\left(A^2 - X^2\right)$$

$$16^2 = \omega^2\left(A^2 - 8^2\right) \ldots\ldots\ldots\ldots(i)$$

$$8^2 = \omega^2\left(A^2 - 12^2\right) \ldots\ldots\ldots(ii)$$

(i)/(ii)

$$\frac{16^2}{8^2} = \frac{\omega^2\left(A^2 - 8^2\right)}{\omega^2\left(A^2 - 12^2\right)}$$

On simplifying

$$A = \pm 13.06cm$$

Example 13

A body moving with S.H.M with period 10sec. find its acceleration at a distance of 20cm from the centre of its motion. If the distance between its two extreme positions is 2m , how long does it take to travel 70cm from one of these positions?

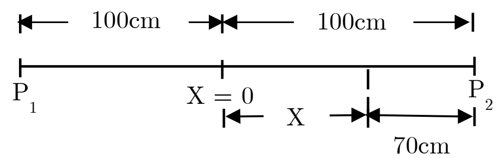
Solution

Case 1 : Let a = acceleration of a body

$$a = \omega^2 y = \frac{4\pi^2 y}{T^2} = \frac{4\pi^2 \times 20}{100}$$

$$a = 7.9cm/s^2$$

Case 2: consider the figure below



$X = 100 - 70 = 30cm$

Since $X = A\cos\omega t$

$$t = \frac{\cos^{-1}\left(X/A\right)}{2\pi/T} = \frac{\cos^{-1}\left(30/100\right)}{2\pi/10}$$

$$t = 2.01\sec$$

Example 14

(a) (i) Give the definition of a simple harmonic motion.

(ii) Using the calculus approach , derive the relations $v = \omega\sqrt{r^2 - y^2}$ and $y = r\sin\omega t$, where v = linear velocity , r = radius of the circle , the amplitude y = displacement at any instant time , t , ω = angular velocity.

(b) A particle moving with S.H.M has velocities of 4cm/s and 3cm/s at distance of 3cm and 4cm respectively from its equilibrium position find the

(i) Amplitude of oscillation

(ii) Period

(iii) Velocity of the particle as its passes through the equilibrium position.

(c) A light spiral spring is loaded with mass of 50kg and its extend by 10cm. calculate the period of small vertical oscillation.

Solution

(a) (i) Refer to your notes

(ii) From definition of S.H.M

$$a = -\omega^2 y \quad \text{but} \quad a = V\frac{dv}{dy}$$

$$V\frac{dv}{dy} = -\omega^2 y \Rightarrow Vdv = -\omega^2 ydy$$

$$\int vdv = -\omega^2\int ydy$$

$$\frac{v^2}{2} = \frac{-\omega^2 y^2}{2} + C$$

Where $V = 0$, $y = r$, $C = \dfrac{\omega^2 r^2}{2}$

$$\frac{V^2}{2} = \frac{-\omega^2 y^2}{2} + \frac{\omega^2 r^2}{2}$$

$$V^2 = \omega^2\left(y^2 - y^2\right) \quad \text{or} \quad V = \omega\sqrt{r^2 - y^2}$$

But $\quad V = \dfrac{dy}{dt} = \omega\sqrt{r^2 - y^2}$

$$\int \frac{dy}{\sqrt{r^2 - y^2}} = \int \omega dt$$

$$\sin^{-1}\left(y/r\right) = \omega t + \Phi$$

$$y/r = \sin\left(\omega t + \Phi\right) \quad \text{or}$$

$$y = n\sin\left(\omega t + \Phi\right)$$

When $t = 0$, $y = 0$, $\Phi = 0$

$y = r\sin\omega t$ hence shown.

(b) Since

$$V^2 = \omega^2\left(r^2 - y^2\right)$$

$$4^2 = \omega^2\left(r^2 - 3^2\right)$$

$$16 = \omega^2\left(r^2 - 9\right) \ldots\ldots\ldots\ldots\ldots(i)$$

Also

$$3^2 = \omega^2\left(r^2 - 4^2\right)$$

$$9 = \omega^2\left(r^2 - 16\right) \ldots\ldots\ldots\ldots(ii)$$

Takes $\dfrac{16}{9} = \dfrac{\omega^2\left(r^2 - 9\right)}{\omega^2\left(r^2 - 16\right)}$

On solving , $r = 5cm$

(ii) T = Periodic time

$$16 = \omega^2\left(25 - 9\right)$$

$$\omega = \sqrt{\frac{16}{25 - 9}} = 1rads^{-1}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{1} = 6.28\,sec$$

$$T = 6.28second$$

(c) $T = 2\pi\sqrt{\dfrac{e}{g}} = 2\times 3.14\times\sqrt{\dfrac{0.1}{9.8}}$

Example 15

A particle executing S.H.M has a maximum displacement of 4cm and its acceleration at distance of 1cm from its mean position is 3cm/s². What will be its velocity when it is at distance of 2cm from its mean position.

Solution

Since $\quad a = \omega^2 y$ where $a = 3cm/s^2$, $y = 1cm$

$$3 = \omega^2 \times 1 \;,\quad \omega = \sqrt{3}rad/s$$

Also $\quad V = \omega\sqrt{A^2 - y^2}$

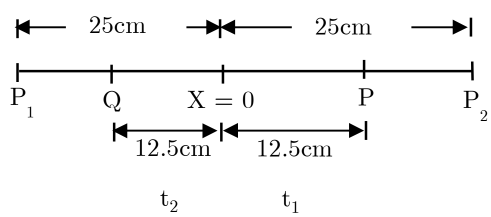
$$= \sqrt{3}\cdot\sqrt{4^2 - 2^2}$$

$$V = 6cm/s$$

Example 16

A particle executes S.H.M of amplitude 25cm and time period of 3seconds. What is the minimum time required for the particle to move between two points 12.5cm on either sides of the mean position

Solution



Time taken by the particle to move from O to P

Since $\quad X_1 = A\sin\omega t_1$

$$12.5 = 25\sin\left(\frac{2\pi t_1}{3}\right)$$

$$\frac{1}{2} = \sin\left(2at_1/3\right)$$

$$t_1 = 0.25\,sec$$

∴ Time taken by particle to move between two points on either side of the mean position is given by $t = t_1 + t_2$ but $t_1 = t_2$

$t = 2t_1 = 2 \times 0.25 = 0.5sec$

Example 17

In what time after it motion begins will a particle oscillate according to the equation $y = 7\sin(0.5\pi t)$ moves from the mean position to maximum displacement.

Solution

Given that $y = 7\sin(0.5\pi t)$

But $y = A = 7$, $7 = 7\sin(0.5\pi t)$

$$1 = \sin(0.5\pi t)$$

$$\sin\left(\pi/2\right) = \sin(0.5\pi t)$$

$$t = 1.0 \text{ second}$$

Example – 18

A particle execute S.H.M of amplitude 30cm and time period 4s. What is the minimum time required for the particle to move from mean position to a point 15cm.

Solution

As $y = A\sin\left(\frac{2\pi t}{T}\right)$

$$15 = 30\sin\left(\frac{2\pi}{4}t\right) \text{ or } \sin\left(\frac{2\pi t}{4}\right) = \frac{1}{2}$$

$$\sin\left(\frac{\pi}{6}\right) = \sin\left(\frac{2\pi t}{4}\right)$$

$$t = \frac{1}{3}\text{second}$$

Example 19

(a) If the displacement of a moving point any time be given by an equation of the form of $X = a\cos\omega t + b\sin\omega t$. Show that the motion is S.H.M.

(b) If a = 3cm , b = 4cm and ω = 2rad/s , determine the period amplitude , maximum velocity and maximum acceleration of the motion.

Solution

(a) Given that $X = a\cos\omega t + b\sin\omega t$

$$\frac{dx}{dt} = \frac{d}{dt}\left[a\cos\omega t + b\sin\omega t\right]$$

$$\frac{dx}{dt} = -a\omega\sin\omega t + b\omega\cos\omega t$$

Again

$$\frac{d^2x}{dt^2} = -a\omega^2\cos\omega t - b\omega^2\sin\omega t$$

$$\frac{d^2x}{dt^2} = -\omega^2\left(a\cos\omega t + b\sin\omega t\right)$$

$$a = \frac{d^2x}{dt^2} = -\omega^2 x \ , \quad a\ \alpha - x$$

Hence its excute S.H.M

(b) Periodic time , $T = \frac{2\pi}{\omega} = \frac{2\pi}{2}$

$$T = 3.14\text{second}$$

Amplitude $A = \sqrt{a^2 + b^2}$

$$A = \sqrt{3^2 + 4^2} = 5cm$$

The maximum velocity $V_{max} = \omega A$

$$V_{max} = 2\times 5 = 10cm/s$$

The maximum acceleration

$$a_{max} = \omega^2 A = 2^2 \times 5$$

$$a_{max} = 20cm/s^2$$

Example 20

The equations of simple harmonic motion is given by $y = 3\sin 5\pi t + 4\cos 5\pi t$ where *y* is in cm and *t* in second. Determine the amplitude, period and initial phase.

Solution

Given that $y = 3\sin 5\pi t + 4\cos 5\pi t$ ....(i)

Let $y = A\sin(5\pi t + \Phi)$ ....(1)

$$y = A\cos\phi\sin 5\pi t + A\sin\phi\cos 5\pi t \text{ ....(ii)}$$

On comparing equation (i) and (ii)

$A\cos\Phi = 3$, $A\sin\Phi = 4$

Then $A^2\left(\cos^2\Phi + \sin^2\Phi\right) = 3^2 + 4^2 = 25$

$A^2 = 25$, $A = 5cm$

Again

$$y = 5\sin(5\pi t + \Phi)$$

$$y = A\sin(2\pi f + \Phi)$$

Then $5\pi = 2\pi f$, $f = 5/2 \ sec^{-1}$

Periodic time

$$T = \frac{1}{f} = \frac{2}{5} = 0.4\sec$$

$T = 0.4 \text{ sec}$

Let Φ = initial phase

Take

$$\frac{A\sin\Phi}{A\cos\Phi} = \frac{4}{3} \quad \text{or} \quad \tan\Phi = \frac{4}{3}$$

$$\Phi = \tan^{-1}\left(\frac{4}{3}\right) = 53^\circ 8'$$

$\therefore$ Initial phase $\Phi = 53^\circ 8'$

Example 21

At the ends of tree consecutive seconds the distances of a point moving with S.H.M form its mean position , measured in the same direction are 1 , 5 and 5. Show that the period of a complete oscillation is $T = \dfrac{2\pi}{\cos^{-1}\left(3/5\right)}$

Solution

The displacement of a particle measured from the mean position $y = A\sin\omega t$

Let t be the initial time

$y_1 = A\sin\omega t = 1$..........(i)

$y_2 = A\sin(\omega t_2)$ but $t_2 = t + 1$

$5 = A\sin\omega(t + 1) = A\sin(\omega t + \omega)$.......(ii)

$y_3 = A\sin\omega(t + 2)$

$5 = A\sin(\omega t + 2\omega)$.........(iii)

Adding equation (i) and (ii)

$$1 + 5 = A\sin\omega t + A\sin\left(\omega t + 2\omega\right)$$

$$6 = A\left[\sin\omega t + \sin\left(\omega t + 2\omega\right)\right]$$

By using factor formula

$$6 = A\left[2\sin\left(\omega t + \omega\right)\cos\omega\right]$$

$$6 = \left[A\sin\left(\omega t + \omega\right)\right]2\cos\omega$$

$$6 = 5 \times 2\cos\omega$$

$$6 = 10\cos\omega \ , \quad \omega = \cos^{-1}\left(3/5\right)$$

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\cos^{-1}\left(\frac{3}{5}\right)}$$

Example 22

A particle is moving with S.H.M while making an execursion from one position of rest to the other its distance from the middle point of its path at three consecutive seconds are observed to be X_1 , X_2 and X_3 prove that the time of a complete oscillation is

$$T = \frac{2\pi}{\cos^{-1}\left[\frac{x_1 + x_3}{2x_2}\right]}$$

Solution

The displacement of the particle from the mean position $X = A\sin\omega t$

Let t be the initial time

$x_1 = A\sin\omega t$ ....(i)

$x_2 = A\sin\omega\left(t + 1\right) = A\sin\left(\omega t + \omega\right)$......(ii)

$x_3 = A\sin\omega\left(t + 2\right) = A\sin\left(\omega t + 2\omega\right)$.....(iii)

Adding equation (i) and (iii)

$$x_1 + x_3 = A\sin\omega t + A\sin\left(\omega t + 2\omega\right)$$

$$x_1 + x_3 = A\left[\sin\omega t + \sin\left(\omega t + 2\omega\right)\right]$$

By using factor formula

$$x_1 + x_3 = 2\cos\omega\left[A\sin\left(\omega t + \omega\right)\right]$$

$$x_1 + x_3 = 2x_2\cos\omega$$

$$\cos\omega = \frac{x_1 + x_3}{2x_2}$$

$$\omega = \cos^{-1}\left[\frac{x_1 + x_3}{2x_2}\right]$$

Periodic time

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\cos^{-1}\left[\frac{x_1 + x_3}{2x_2}\right]}$$

$$T = \frac{2\pi}{\cos^{-1}\left[\frac{x_1 + x_3}{2x_2}\right]}$$

Example 23

A 10kg brass load suspended by brass wire 10m long is observed to vibrate vertically in S.H.M at a frequency of 10Hz. What is the cross – sectional area of the wire? Young's modulus of brass = $9 \times 10^{10}Nm^{-2}$

Solution

Since $T = \frac{1}{f} = 2\pi\sqrt{\frac{e}{g}}$

$$\frac{1}{f^2} = \frac{4\pi^2 e}{g} \Rightarrow e = \frac{g}{(2\pi f)^2}$$

$$e = \frac{9.8}{(2\pi \times 10)^2} = 2.48 \times 10^{-3} m$$

Again $E = \frac{\text{stress}}{\text{strain}} = \frac{mg}{A} \Big/ \frac{e}{L}$

$$A = \frac{MgL}{Ee} = \frac{10 \times 9.8 \times 10}{9 \times 10^{10} \times 2.48 \times 10^{-3}}$$

$$A = 4.39 \times 10^{-6} m^2 = 4.39mm^2$$

Example 24

A copper rod of length 2m and radius 3mm hangs down from a ceiling. A 9kg object is attached to the other lower end of the rod. The rod acts as a spring and the object oscillates vertically with a small amplitude. Ignoring the rod mass find the frequency of the S.H.M. Young's modulus of copper is $1.1 \times 10^{11}Nm^{-2}$

Solution

Since $E = \frac{MgL}{\pi r^2 e} \Rightarrow e = \frac{mgL}{\pi r^2 E}$

$$e = \frac{9 \times 9.8 \times 2}{3.14 \times (3 \times 10^{-3})^2 \times 1.1 \times 10^{11}}$$

$$e = 5.67 \times 10^{-5} m$$

The frequency of oscillation

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{e}} = \frac{1}{2 \times 3.14} \cdot \sqrt{\frac{9.8}{5.67 \times 10^{-5}}}$$

$$f = 66.2Hz$$

Example 25

A man stands on a weighing machine placed on a horizontal platform. The machine reads 50kg. By means of a suitable mechanism, the platform is made to execute harmonic vibrations up and down with a frequency of 2 vibrations per second. What will be the effect on the reading of the weighing machine? The amplitude of vibration of platform is 5cm. Take $g = 10m/s^2$.

Solution

Maximum acceleration

$$a_{max} = (2\pi f)^2 A = (2\pi \times 2)^2 \times 0.05$$

$$a_{max} = 7.9m/s^2$$

The max. force on the man

$$F_{max} = M(g + a_{max})$$

$$= 50(10 + 79)$$

$$F_{max} = 895N = 89.5kgf$$

The min . Force on the man

$$F_{min} = M(g - a_{max})$$

$$= 50(10 - 79)$$

$$= 105n = 10.5kgf$$

$$\therefore F_{min} = 10.5kgf$$

∴ The reading of weighing machine varies between 10.5kgf and 89.5kgf.

Example 26

A particle oscillating with S.H.M has a speed of $V = 8m/s$ and an acceleration of $a = 12m/s^2$ when is 3m from its equilibrium position. Find :-

(a) Amplitude of the motion
(b) Maximum velocity and
(c) Maximum acceleration

Solution

(a) Given that $a = \omega^2 y$, $\omega^2 = a/y$

$$V = \omega\sqrt{A^2 - y^2} = \sqrt{\frac{a}{y}}\sqrt{A^2 - y^2}$$

$$V = \sqrt{\frac{a}{y}(A^2 - y^2)}$$, on solving from A

$$A = \sqrt{\frac{v^2 y + ay^2}{a}} = \sqrt{\frac{8^2 \times 3 + 2 \times 3^2}{12}}$$

$$A = 5m$$

(b) $V_{max} = \omega A = A\sqrt{\frac{a}{y}} = 5\sqrt{\frac{12}{3}}$

$$V_{max} = 10m/s$$

(c) $a_{max} = \omega^2 A = \frac{aA}{y} = \frac{12 \times 5}{3} = 20m/s$

Example 27

The shortest distance travelled by a particle executing S.H.M from position in 2 seconds is equal to $\sqrt{3}/2$ times its amplitude. Determine its time period.

Solution

Given t = 2sec , $y = \frac{\sqrt{3}}{2}A$, T = ?

By using the equation , $y = A\sin\omega t$

$$\frac{\sqrt{3}}{2}A = A\sin\left[\frac{2\pi\times 2}{T}\right]$$

$$\frac{\sqrt{3}}{2} = \sin\left[\frac{4\pi}{T}\right] \quad \text{But} \quad \frac{\sqrt{3}}{2} = \sin\left(\frac{\pi}{3}\right)$$

$$\sin\left(\frac{4\pi}{T}\right) = \sin\left(\frac{\pi}{3}\right)$$

$$4\pi/\pi = \frac{\pi}{3}$$

$$T = 12\text{ Seconds}$$

Example 28

Given that $y = \sin^2\omega t$ show that it does not represent S.H.M what is the periodic of the given function.

Solution

$$y = \sin^2\omega t$$

$$\frac{dy}{dt} = (\sin\omega t\cos\omega t)\,\omega = \sin 2\omega t$$

$$a = \frac{d^2y}{dt^2} = (\omega\cos 2\omega t)\,2\omega = 2\omega^2\cos 2\omega t$$

Since the acceleration is not directly proportional to the negative of displacement, the function does not represent S.H.M

Now

$$y = \sin^2\omega t = \frac{1}{2}(1-\cos 2\omega t)$$

The given function is clearly a periodic function with an angular frequency , 2ω

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

Example 29

A particle moving with S.H.M passes through two point A and B , 56cm apart , with the same velocity having accupied 2scond in passing from A and B after another 2seconds it return to B.

Find

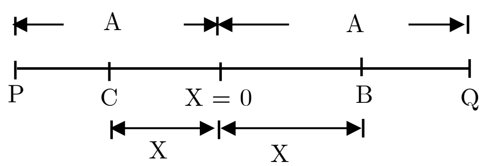
(a) The period

(b) The amplitude of oscillation.

Solution

Let A be the amplitude of oscillation the points are at equal distance from the mean position

$$X = \frac{56}{2} = 28\text{cm}$$



(a) Let t_1 = time taken by the particle to move from C to O since

$$X = A\cos\omega t_1 \quad , \quad t_1 = \frac{1}{\omega}\cos^{-1}\left(\frac{X}{A}\right)$$

t_2 be time taken by the particle to move from O to B

$$X = A\sin\omega t_2 \quad , \quad t_2 = \frac{1}{\omega}\sin^{-1}\left(X/A\right)$$

Now $t = t_1 + t_2$ but $t = 2\sec$

$$2 = \frac{1}{\omega}\cos^{-1}\left(\frac{X}{A}\right) + \frac{1}{\omega}\sin^{-1}\left(\frac{X}{A}\right)$$

$$2\omega = \cos^{-1}\left(\frac{X}{A}\right) + \sin^{-1}\left(\frac{X}{A}\right)$$

$$\text{But } \cos^{-1}\left(\frac{X}{A}\right) + \sin^{-1}\left(\frac{X}{A}\right) = \frac{\pi}{2}$$

$$2\omega = \frac{\pi}{2} \quad , \quad \omega = \frac{\pi}{4}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi/4} = 8.0\sec$$

$$T = 8.0\sec$$

(b) Since OC = OB , then $t_1 = t_2$

$t = 2t_1 = 2t_2$

$2t_2 = 2$, $t_2 = 1\text{sec}$

$t_1 = t_2 = 1\text{sec}$

Now

$$X = A\sin\left[\frac{2\pi t_2}{T}\right]$$

$$28 = A\sin\left[\frac{2\pi \times 1}{8}\right] = A\sin\left[\frac{\pi}{4}\right]$$

$$28 = A \cdot \frac{\sqrt{2}}{2}$$

$\therefore$ Amplitude $A = 28\sqrt{2}$

EXERCISE 5.1

1. A body execute S.H.M of time period 16seconds. Its velocity is found to be 4m/s , two second after it has passed through the mean position. Find the amplitude .
[ans. 14.4m]

2. Obtain the equation of S.H.M of a particle whose amplitudes is 0.04m and whose frequency 50Hz. The initial phase is $\pi/3$ rad.
(Ans. $y = 0.04\sin(100\pi t + \pi/3)$)

3. (a) Explain the meaning of the following terms as applied in S.H.M
   (i) Period
   (ii) Amplitude
   (iii) Restoring force
   (b) How can a uniform motion in a circle be related to a simple harmonic motion?

4. In what time after its motion will particle oscillating according to the equation $x = 7\sin(0.5\pi t)$ move
   (i) From the mean position to its extreme position
   (ii) From the mean position to the displacement equal to half of the amplitude
   [Ans. (i) 1sec (ii) 1/3 sec]

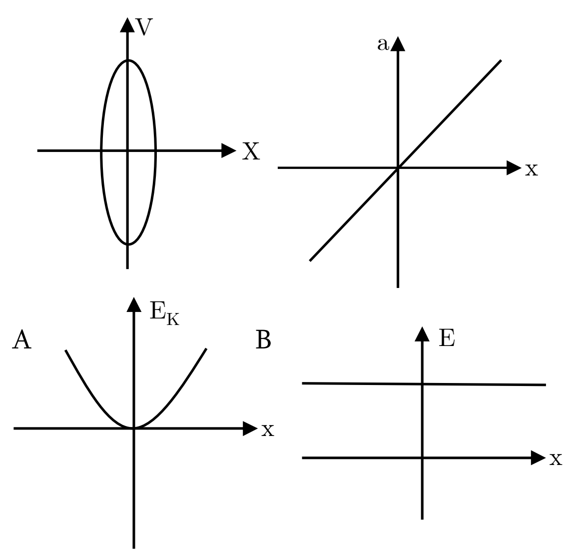
5. The maximum acceleration of a particle vibrating in S.H.M is A and maximum velocity V. Calculate the amplitude and the period of oscillation.
$$\left[\text{Ans. } r = V^2/A \;,\; T = \frac{2\pi V}{A}\right]$$

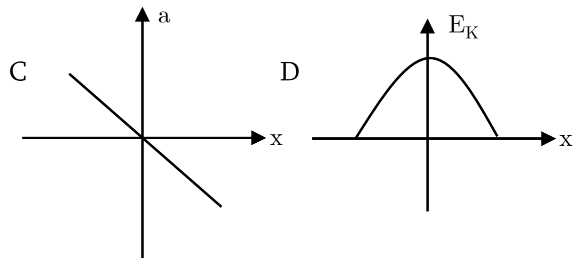
6. A body described S.H.M with an amplitude of 5cm and a period of 0.2seconds. Find the acceleration and the velocity of the body when the displacement.
   (a) 5cm (b) 3cm (c) 0
   Answer (a) $a = -5\pi^2 m/s^2$, $V = 0$
   (b) $a = -3\pi^2 m/s^2$, $V = 0.4\pi m/s$
   (c) $a = 0$, $V = 0.5\pi m/s$

7. (a) A particle is moving with S.H.M of period 16seconds and amplitude 10m. Find the speed of the particle when it is 6.0m from its equilibrium position.
   (b) How far is the particle in (a) above from its equilibrium position 1.5sec after passing through it. What is its speed at this time
   [Ans. (a) 3.1m/s (b) 5.6m , 3.3m/s]

8. A butcher throws a cut of beef on a spring scale which then oscillates about an equilibrium position with the period T = 0.5sec the amplitude of vibrations being A = 2.0cm and having displacement of 4.0cm , determine;
   (a) Frequency
   (b) Maximum acceleration and
   (c) Minimum velocity
   [ans. (a) 2Hz (b) 3.16m/s² (c) 0.25m/s]

9. Some of the following graphs refer to S.H.M , where V is the velocity , a is the acceleration, E_K is the kinetic energy , E is the total energy and X is the displacement from the mean position. Which graphs are correct





[ans. A , D , E, F]

10. (a) Show that in S.H.M, the acceleration of the particle is directly proportional to the displacement at at the given instant.
    (b) The S.H.M of a particle is given by y = 3sinωt + 4cosωt what is the amplitude of motion.

    [ans. (a) let x = Asinωt , a α – x , (b) 5unit]

11. A particle executing S.H.M along a straight line has a velocity of 4m/s , when at a distance of 3m from its mean position and 3m/s , when at a distance of 4m from it. Find the time its takes to travel 2.5m from the position extremity of its oscillation. [ans. 1.048sec]

12. A particle moves with S.H.M in a straight line in the first second after starting from rest , it travels a distance X_1cm and in the next second it travels a distance X_2cm in the same direction. Prove that the amplitude of oscillation is

$$r = \frac{2x_1^2}{3x_1 - x_2}$$

Hint :

As the particle starts from rest it must start from the extreme position. When t = 0 , x = r where r is the required amplitude

Since x = rcosωt

$$r - x_1 = r\cos x_1 = r\cos \ldots\ldots..(i)$$

$$r - (x_1 + x_2) = r\cos x_2 = r\cos 2\omega$$

$$r - x_1 - x_2 = r[\, 2\cos^2\omega - 1 \,]\ldots\ldots..(ii)$$

on solving equation (i) and (ii)

we get $r = \dfrac{2x_1^2}{3x_1 - x_2}$

EXAMPLES OF SYTEM THAT EXCUTE S.H.M

1. Vibrating a loaded horizontal spring
2. Vertical oscillations of a loaded helical spring
3. Simple pendulum
4. Liquid oscillating in a U – tube
5. Floating loaded test tube or cylinder
6. Oscillation of the ball on neck chamber
7. Oscillation of piston in a gas filled cylinder
8. Motion of a body dropped in tunnel along the diameter of the Earth.
9. A compound pendulum
10. A torsional pendulum
11. A vibrating cantilever

SHOWING THAT A GIVEN MOTION OF THE BODY (PARTICLE) IS A S.H.M

There are two methods which can be used to show that the system execute S.H.M

Method I :

Force consideration method

Steps involved to that the system execute S.H.M

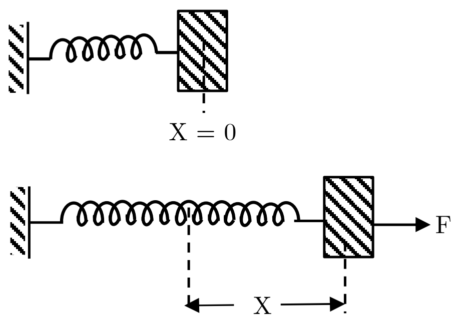
1. Identify all forces acting on the body when is in its equilibrium position.
2. Identify the forces on the body when is slightly displaced from its equilibrium position.
3. Obtain the expression of the resultant force when the body is at the point from equilibrium position.
4. Obtain expression of the acceleration a , using Newton's second law of motion , F = Ma.
5. Can the expression for the acceleration is step(4) be expressed in terms of force i.e a = -(positive quantity) y? if YES , the motion is the S.H.M if NOT , the motion is not S.H.M.
6. After identifying that the motion is S.H.M , then obtain the expression of the periodic time of the oscillation by equating with $a = -\omega^2 y$

$$T = \frac{2\pi}{\omega}$$

Method II:
By displacement consideration method.
Examples of linear simple harmonic motion

1. **Oscillation of loaded horizontal spring**

Consider a block of mass, M on a horizontal surface attached to a spring of a spring constant , k as shown on the figure below

(i)
(ii)


If the force F is applied on the block of mass, M is slightly displaced from the mean position and then released, it is execute S.H.M.

Hooke's law state that 'Provided that elastic limit is not exceeded, the extension of an elastic material is directly proportional to the applied force' i.e F α X , F = KX

K = force constant

Restoring force

F = -KX

Minus sign shows that the direction of the force, F is opposite to the direction of the displacement X.

F = Ma (Newton's second law)

Ma = -KX ,

$$a = -\left[\frac{K}{M}\right]X \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)$$

$a = \alpha - x$

This show that the body execute S.H.M

For the S.H.M, $a = -\omega^2 x \ldots\ldots\ldots(ii)$

(i) = (ii)

$$-\omega^2 x = -\left(\frac{k}{m}\right)x$$

$$\omega^2 = {}^{k}\!/_{m} \qquad k = m\omega^2$$

Angular velocity

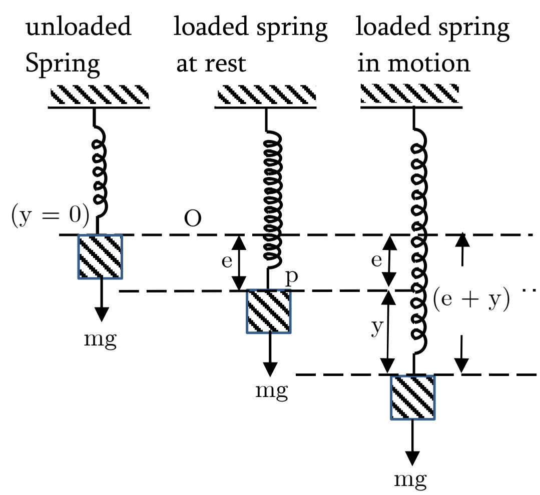
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Periodic time of oscillation

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$$

2. **Vertical oscillations of a loaded helical spring**

Consider a massless helical spring suspended vertically as shown in figure (i) when a mass M is attached on it , the spring stretches to point p (figure (ii) shown). Suppose the system is stretched to a further displacement *y* and then released, then the system execute S.H.M



At the equilibrium for the case when loaded spring at rest

$k.e = mg \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$

for the loaded spring in motion , the restoring force.

$F = -[K(e + y) - mg]$

$F = -[Ke + ky - mg] \ldots\ldots\ldots\ldots(2)$

Putting equation (1) into (2)

$F = -[Ke + Ky - Ke]$

$Ma = -Ky$

$$a = -\left[\frac{k}{m}\right]y \text{ , } a \;\alpha - y$$

Thus , the system execute S.H.M

Now $a = -\omega^2 y$, then $\omega^2 = k/m$

$$T = 2\pi\sqrt{\frac{M}{K}}$$

Also $ke = mg$; $\frac{e}{g} = \frac{m}{k}$

$$T = 2\pi\sqrt{\frac{M}{K}} = 2\pi\sqrt{\frac{e}{g}}$$

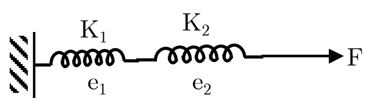
ARRANGEMENT OF THE SPRING

Spring can be arranged in

1. Series connection
2. Parallel connection

1. Spring in series connections

When two springs can be arranged in series connection, then the force acting on each spring is the same (constant) and produces different extension on each spring. Let K_1 and K_2 be force constants on each spring which produces extension e_1 and e_2 respectively.



Total extension, $e_T = e_1 + e_2$

But from Hooke's law

$$\frac{-F}{K_S} = -\frac{F}{K_1} + \frac{-F}{K_2}$$

Dividing both side by –F gives

$$\frac{1}{K_S} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$K_S = \frac{K_1K_2}{K_1 + K_2}$$

Generally for n – spring in series connection

$$\frac{1}{K_S} = \frac{1}{K_1} + \frac{1}{K_2} + \cdots\cdots + \frac{1}{K_n}$$

For identical springs

$$K_1 = K_2 = K_n = K$$

$$\frac{1}{K_S} = \frac{1}{K} + \frac{1}{K} + \cdots\cdots + \frac{1}{K} = \frac{n}{K}$$

$$K_S = K/n$$

Addition concepts for the spring in series connections

1. Periodic time, Ts of oscillation of two spring in series connections.

$$T_S = 2\pi\sqrt{\frac{M}{K_S}} = 2\pi\sqrt{\frac{M(K_1 + K_2)}{K_1K_2}}$$

For the two identical springs

$$K_1 = K_2 = K$$

$$T_S = 2\pi\sqrt{\frac{2M}{K}}$$

2. Let T_1 and T_2 be the periodic time of each spring whose force constants are K_1 and K_2 respectively

Now $\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}$

$$\frac{1}{m\omega^2} = \frac{1}{m\omega_1^2} + \frac{1}{m\omega_2^2}$$

$$\frac{1}{\omega^2} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2}$$

Also $\frac{1}{(2\pi f)^2} = \frac{1}{(2\pi f_1)^2} + \frac{1}{(2\pi f_2)^2}$

$$\frac{1}{f^2} = \frac{1}{f_1^2} + \frac{1}{f_2^2}$$

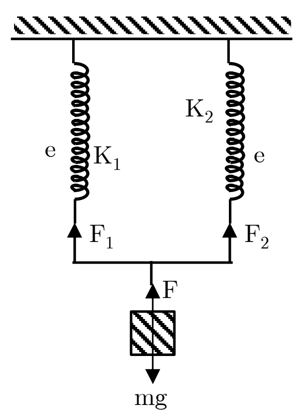
Total periodic time for the two spring in series connection

$$T^2 = T_1^2 + T_2^2$$

$$T = \sqrt{T_1^2 + T_2^2}$$

2. Spring in parallel connection

When two springs can be arranged in parallel connection, the same extension is produced in both springs, but the force in the spring is different.



Total force applied

$$F = F_1 + F_2$$

$$K_pe = K_1e + K_2e$$

$$K_p = K_1 + K_2$$

For n – springs connected in parallel.

$$K_p = K_1 + K_2 + \cdots\cdots + K_n$$

For *n* identical springs

i.e $K_1 = K_2 = \cdots\cdots\cdots = K_n = K$

$K_P = nK$

Additional concepts for the springs in parallel connection

1. Expression of the periodic time for the two springs in parallel connection.

$$T_p = 2\pi\sqrt{\frac{M}{K_p}} = 2\pi\sqrt{\frac{M}{K_1 + K_2}}$$

For the two identical springs

$$K_1 = K_2 = K$$

$$T_p = 2\pi\sqrt{\frac{M}{2K}}$$

2. Let T_1 and T_2 be the periodic times of each spring whose force constants are K_1 and K_2 respectively arranged in parallel connection.

Now $K_p = K_1 + K_2$

$$m\omega^2 = m\omega_1^2 + m\omega_2^2$$

$$\omega^2 = \omega_1^2 + \omega_2^2$$

Also

$$(2\pi f)^2 = (2\pi f_1)^2 + (2\pi f_2)^2$$

$$\{f^2 = f_1^2 + f_2^2\}$$

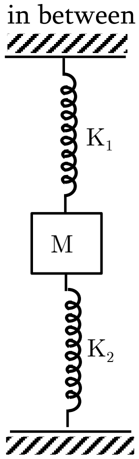
$$\frac{1}{T^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2}$$

$$T = \frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2}}$$

SPECIAL CASES FOR THE SPRING

1. **A mass between two springs**

Here , the body of weight Mg is connected in between the two springs as shown below



When the body is pulled to one side through a small distance *y*, one spring gets compressed by length *y* and the other springs stretched by length *y*. the restoring forces F_1 and F_2 set up in both the spring will act same direction, then

$F_1 = K_1 y$ and $F_2 = -K_2 y$

Total restoring force

$$F = F_1 + F_2 = -K_1 y - K_2 y$$

$$F = -(K_1 + K_2)y$$

$$Ma = -(K_1 + K_2)y$$

$$a = -\frac{(K_1 + K_2)}{M}, \quad a\ \alpha - y$$

Again $a = -\omega^2 y = -\dfrac{(K_1 + K_2)y}{M}$

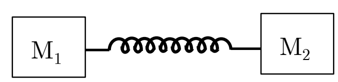
$$\omega = \sqrt{\frac{K_1 + K_2}{M}}$$

Periodic time , $T = 2\pi\sqrt{\dfrac{M}{K_1 + K_2}}$

2. In a two particles system as shown in the figure below if K is the force constant of the spring, the time period of vibration of the system is

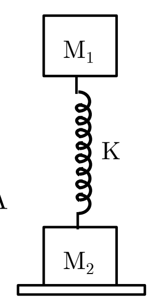
$T = 2\pi\sqrt{\dfrac{\mu}{K}}$ where

$\mu = \dfrac{M_1 M_2}{M_1 + M_2}$ = reduced of mass



3. In a system of two particle if only one mass M_1 is oscillating as shown in the figure below, then time period.

$$T = 2\pi\sqrt{\frac{M_1}{K}}$$



If A is the amplitude of oscillation of M_1, then the maximum reaction of the table on the mass M_1 is $R_{max} = (M_1 + M_2)g + KA$ and the minimum reaction of the table on the mass M_2, $R_{min} = (M_1 + M_2)g - KA$

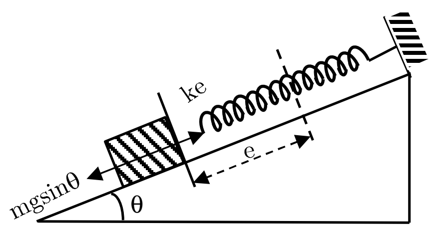
4. If a wire of length L , area of cross – section A , Young's modulus E is stretched by suspending a mass, M then periodic time, T of the oscillation is given by

$$T = 2\pi\sqrt{\frac{e}{g}}$$

Since $E = \frac{FL}{Ae}$, $\frac{e}{g} = \frac{ML}{EA}$

$$T = 2\pi\sqrt{\frac{ML}{EA}}$$

5. S.H.M on the spring lies on smooth inclined plane. Consider the figure below which shows a block of mass M attached at the end of the spring and extends the spring by amount e from the equilibrium position and comes into the rest.



At the equilibrium of the spring

$$Ke = mg\sin\theta$$

If the block is pulled downward by the distance X and then released it is execute S.H.M restoring force

$$F = -\left[K\left(x+e\right) - mg\sin\theta\right]$$

$$ma = -\left[kx + ke - ke\right]$$

$$ma = -kx \ , \ a = -\left(\frac{k}{m}\right)x$$

$a \propto -x$, its execute S.H.M

since

$$T = 2\pi\sqrt{\frac{M}{K}} \text{ But } mg\sin\theta = ke$$

$$T = 2\pi\sqrt{\frac{Me}{Ke}} = 2\pi\sqrt{\frac{Me}{mg\sin\theta}}$$

$$T = 2\pi\sqrt{\frac{e}{g\sin\theta}}$$

Workdone in stretching spring

The force F in a spring whose extension is X and which obeys Hooke's law is given by F = KX. If the extension is increased by a small distance X under the action of this force, then dW = Fdx

Total workdone to increases the extension from, O to X.

$$w = \int_0^x kxdx = \frac{1}{2}kx^2$$

$$w = \frac{1}{2}kx^2 = \frac{Fx}{2} = \frac{F^2}{2k}$$

SOLVE EXAMPLES

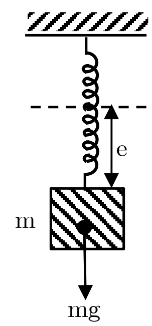
Example 1

A mass of 0.1kg was attached to a free end of a vertical spring and the upper end was fixed. The spring was extended by 0.04cm and the spring was then pulled down by a small distance and released so that it oscillated freely. Find the period of two such springs connected in:-

(a) Series connection

(b) Parallel connection

Solution



First obtain the value of the force constant , K of the spring. Since F = Ke (Hook's law)

Now $ke = mg$

$$k = \frac{mg}{e} = \frac{0.1\times 9.8}{0.04\times 10^{-2}}$$

$$k = 2450\text{Nm}^{-1}$$

(a) For spring in series

$$T_s = 2\pi\sqrt{\frac{m\left(k_1+k_2\right)}{k_1k_2}} \text{ but } k_1 = k_2 = k$$

$$T_s = 2\pi\sqrt{\frac{2m}{k}} = 2\times 3.14\sqrt{\frac{2\times 0.1}{2450}}$$

$$T_s = 0.058\text{ sec}$$

(b) For spring in parallel

$$T_p = 2\pi\sqrt{\frac{m}{k_1+k_2}} = 2\pi\sqrt{\frac{m}{2k}}$$

$$= 2\times 3.14\sqrt{\frac{0.1}{2\times 2450}}$$

$$T_p = 0.028\text{ sec}$$

Example 2

A 3.0kg ball is attached to a spring of negligible mass and with a spring constant $k = 40Nm^{-1}$. The ball is displaced 0.10m from equilibrium and then released. What is the maximum speed of the ball as it undergoes S.H.M?

Solution

Maximum speed occurs at the equilibrium

$$V_{max} = \omega A$$

But $$\omega = \sqrt{\frac{k}{m}}$$

$$V_{max} = A\sqrt{\frac{k}{m}} = 0.1\sqrt{\frac{40}{3.0}}$$

$$V_{max} = 0.37m/s$$

Example 3

A car with a mass of 1300kg is a constructed so that its frame is supported by four springs. Each spring has a force constant of 20000Nm^{-1}. If two people riding in the car have a combined mass of 160kg, find the frequency of vibration of the car after it is driven over a pothole in road.

Solution

First obtain total mass of the system,

M = mass of car + mass of the people

M = 1300 + 160 = 1460kg

For the given extension, e

The total force constant for combined four springs

$$K_T = 4K$$

$$f = \frac{1}{2\pi}\sqrt{\frac{4k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4\times 20000}{1460}}$$

$$f = 1.18Hz$$

Example 4

A spring compressed by 0.1m develops a restoring force 10N. A body of mass 4kg is placed on it. Deduced

(i) The force constant of the spring

(ii) The depression of the spring under the weight of the body ($g = 10m/s^2$)

(iii) The period of oscillation, if the body is left free.

Solution

(i) $$k = \frac{F}{e} = \frac{10}{0.1} = 100Nm^{-1}$$

(ii) $$y = \frac{mg}{k} = \frac{4\times 10}{100} = 0.4m$$

(iii) $$T = 2\pi\sqrt{\frac{m}{k}} = 2\times\frac{22}{7}\cdot\sqrt{\frac{4}{100}}$$

$$T = 1.26\ sec$$

Example 5

A mass of M attached to a spring oscillates with a period 2seconds. If the mass is increased by 2kg, the period increased by 1sec. find the initial mass, M and state assumption made in your calculation.

Solution

Assume that Hooke's law is obeyed

Since $$T = 2\pi\sqrt{\frac{m}{k}}$$

Case 1: $T = 2s$, $M = m$

Then $$2 = 2\pi\sqrt{\frac{m}{k}}$$ or

$$k = \frac{4\pi^2 m}{4} = \pi^2 m$$

Case 2: $T = 2+1 = 3s$, $m = m+2$

Then $$3 = 2\pi\sqrt{\frac{m+2}{k}}$$

$$3 = 2\pi\sqrt{\frac{m+2}{\pi^2 m}} = 2\sqrt{\frac{m+2}{m}}$$

$$9m = 4m + 8$$

$$m = 1.6kg$$

Example 6

A mass of 0.1kg is attached to the free end of a vertical spring whose upper end is fixed and the spring extends by 0.04m now the mass is pulled down a small distance 0.02m and then released find.

(i) It period

(ii) The maximum force acting on it , during the oscillation [$g = 10m/s^2$]

Solution

(i) $$T = 2\pi\sqrt{\frac{e}{g}} = 2\times 3.14\sqrt{\frac{0.04}{10}}$$

$$T = 0.40\ sec$$

(ii) The maximum force

$$F_{max} = Ma_{max} = M\omega^2 A$$

$$= 0.1\left[\frac{2\times 3.14}{0.4}\right]^2 \times 0.02$$

$$F_{max} = 0.49N$$

Example 7

The time period of a body suspended by a spring is T. what will be the new time period if its spring is cut into two equal parts and

(i) The body is suspended by one part

(ii) Suspended by both parts in parallel.

Solution

Periodic time before the spring being cut

$$T = 2\pi\sqrt{\frac{m}{k}}$$

(i) On cutting the spring in two equal parts , the length of a one part is halves and $e_1 = e/2$.

Let K_1 = New force constant

Apply Hooke's law

F = Ke = constant

$$ke = k_1 e_1 = \frac{k_1 e}{2}$$

$$k_1 = 2k$$

New period time

$$T_1 = 2\pi\sqrt{\frac{M}{K_1}} = 2\pi\sqrt{\frac{M}{2K}}$$

$$T_1 = \frac{T}{\sqrt{2}}$$

(ii) If the body is suspended from both parts in parallel then the total force constant for the spring in parallel connection

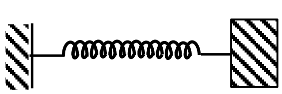
$$K_P = K_1 + K_2 = 2K + 2K = 4K$$

New periodic time

$$T_2 = 2\pi\sqrt{\frac{M}{4K}} = \frac{1}{2}\cdot 2\pi\sqrt{\frac{M}{K}}$$

$$T_2 = \frac{T}{2}$$

Example 8



The horizontal spring in the figure above was found to be stretched 3cm from equilibrium position when a force of 0.75N acting on it, then a mass of 0.12kg was attached to the end of the spring and pulled 4cm long along a horizontal frictionless table from the equilibrium position. When the body was released executed a S.H.M

(a) What was the force constant of the spring?

(b) What was the force extended by the spring on the 0.15kg body just before it was released?

Solution

(a) K = Force constant

$$K = \frac{F}{e} = \frac{0.75}{3\times 10^{-2}} = 25Nm^{-1}$$

(b) Force extended by the spring

F = KX (By Hooke's law)

$$F = 25\times 4\times 10^{-2} = 1.0N$$

Example 9

A spring vibrates with a frequency of 2.4Hz when 0.8kg hangs from it. What will be its frequency be if only 0.5kg hangs from it?

Solution

$$\text{Since } f = \frac{1}{2\pi}\sqrt{\frac{K}{M}}, \quad f \propto \sqrt{\frac{1}{M}}$$

$$f_2 = f_1\cdot\sqrt{\frac{M_1}{M_2}} = 2.4\sqrt{\frac{0.8}{0.5}}$$

$$f_2 = 3.0Hz$$

Example 10

A 4.0kg body is suspended by a spring when in addition to it , another body of mass 0.6kg is suspended , the spring further stretches by 2.0cm . If the second body is removed and the first is made to oscillates then what will be the time period? [g = 9.8m/s²]

Solution

The force constant on the spring

$$K = \frac{M_2 g}{e} = \frac{0.6\times 9.8}{0.02} = 2.94Nm^{-1}$$

The period of oscillation

$$T = 2\pi\sqrt{\frac{M_1}{K}} = 2\times 3.14\sqrt{\frac{4}{294}}$$

$$T = 0.73\,sec$$

Example 11

A spring is such that a load of 5gm produces an extension of 1cm. A mass of 0.1kg hangs at the end of the spring in equilibrium vertically a distance of 6cm and released. Find:-

(a) The period of vibration of 0.1kg mass
(b) The maximum velocity of the mass
(c) The velocity of the mass when it is 4cm above equilibrium position ($g = 9.8m/s^2$)

Solution

The force constant of the spring

$$K = \frac{Mg}{e} = \frac{0.005\times 9.8}{0.01} = 4.9N/m$$

(a) $$T = 2\pi\sqrt{\frac{M}{K}} = 2\pi\sqrt{\frac{0.1}{4.9}}$$

$$T = 0.90\,\text{sec}$$

(b) $$V_{max} = \omega A = \left(\frac{2\pi}{T}\right)A = \left(\frac{2\times 3.4}{0.9}\right)\times 0.06$$

$$V_{max} = 0.42m/s$$

(c) $$V = \omega\sqrt{A^2 - X^2} = \left(\frac{2\pi}{T}\right)\cdot\sqrt{A^2 - X^2}$$

$$= \left(\frac{2\times 3.14}{0.9}\right)\sqrt{(0.06)^2 - (0.04)^2}$$

$$V = 0.31m/s$$

Example 12

A small piece of metal is suspended from a light spiral spring produces an extension of 7.9cm. It is then displaced vertically so that it perform S.H.M of 100 vibrations of which are observed to take 57sec. What value does this experiment give for the acceleration due to gravity?

Solution

Periodic time oscillation

$$T = \frac{t}{n} = \frac{57}{100} = 0.57\,\text{sec}$$

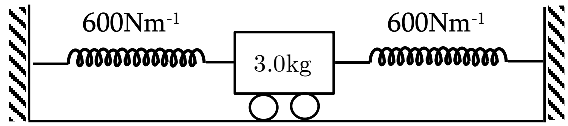
Since $$T = 2\pi\sqrt{\frac{e}{g}}, \quad g = \frac{4\pi^2 e}{T^2}$$

$$g = \frac{4(3.14)\times 0.079}{(0.57)^2}$$

$$g = 9.6m/s^2$$

Example 13

A trolley of mass 3.0kg is connected to the identical springs each of force constant $600Nm^{-1}$ as shown in the figure below. If the trolley is displaced from its equilibrium position by 5.0cm and released



What is

(i) The period of ensuring oscillations
(ii) The maximum speed of the trolley.
(iii) How much is the total energy dissipated as heat by the time the trolley come to rest due to the dumping force.

Solution

The total force constant when the spring are in parallel $K = K_1 + K_2 = 600 + 600 = 1200Nm^{-1}$.

(i) $$T = 2\pi\sqrt{\frac{M}{K}} = 2\times 3.14\sqrt{\frac{3}{1200}} = 0.314\,\text{sec}$$

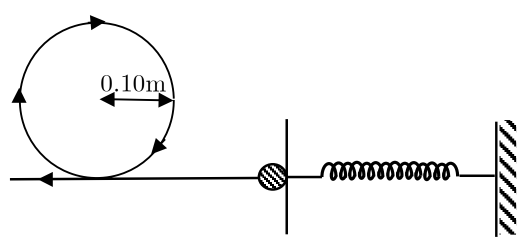
(ii) $$V_{max} = 2\pi A/T = \frac{2\times 3.14\times 0.05}{0.314} = 1.0m/s$$

(iii) Total energy dissipated

$$E = \frac{1}{2}KA^2 = \frac{1}{2}\times 1200\times(0.05)^2 = 1.5J$$

Example 14

A compressed spring is used to propel a ball bearing along a track which contains a circular loop of radius 0.10m in a vertical plane. The springs obeys Hooke's law and requires a force of 0.20N to compress it 1.0mm



(a) The spring is compressed by 30mm calculate the energy stored in the spring.
(b) A ball – bearing of mass 0.025kg is placed against the end of the spring which is then released. Calculate

(i) The speed with which ball bearing leaves the spring.
(ii) The speed of the ball at the top of the loop.
(iii) The force exerted on the ball y the rack at top of the loop ($g = 10m/s^2$)

Assume that the effect of friction can be ignored.

Solution

(a) Force constant on the spring

$$K = \frac{F}{e} = \frac{0.2}{1\times10^{-2}} = 200Nm^{-1}$$

Energy stored

$$E = \frac{1}{2}KX^2$$

$$E = \frac{1}{2}\times200\times\left(30\times10^{-3}\right)^2$$

$$E = 9.0\times10^{-2}J$$

(b) (i) Apply the law of conservation of energy

Loss in elastic p.e in elastic = gain in elastic k.e of the ball

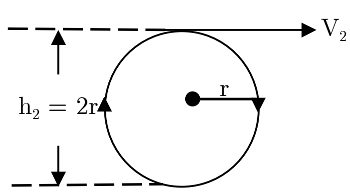
$$\frac{1}{2}KX^2 = \frac{1}{2}MV^2$$

$$9\times10^{-2} = \frac{1}{2}MV^2$$

$$V = \left[\frac{2\times9\times10^{-2}}{0.025}\right]^{1/2}$$

$$V = 2.7m/s$$

(ii) Let V_2 be speed of the reached at the top of the loop.



Apply the law of conservation of mechanical energy

(p.e + k.e) of ball At the top of loop = Elastic p.e on the spring

$$Mgh_2 + \frac{1}{2}MV_2^2 = \frac{1}{2}KX^2$$

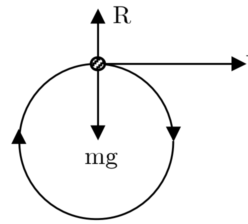
$$Mgh_2 + \frac{1}{2}MV_2^2 = 9\times10^{-2}$$

$$V_2 = \left[\frac{18\times10^{-2}}{M} - 4gr\right]^{1/2}$$

$$= \left[\frac{18\times10^{-2}}{0.02} - 4\times10\times0.1\right]^{1/2}$$

$$V_2 = 1.8m/s$$

(iii) Let R = force exerted on the ball at the top of the loop



$$\frac{MV_2^2}{r} = Mg - R$$

$$R = M\left[g - \frac{V_2^2}{r}\right]$$

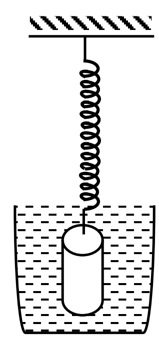
$$= 0.025\left[10 - \frac{(1.8)^2}{0.1}\right]$$

$$R = 0.55N$$

Example 15

A solid cylinder of a mass 5kg and diameter 12cm is suspended by a spring of force constant K = 2N/cm and hangs partially submerged in water. Find the period of small vertical oscillations neglecting viscos drag force. Given that the density of water $\rho = 1000kgm^{-3}$

Solution



If the cylinder is pulled down by a distance y , then the restoring force

$$F = -\left[Ky + \rho Agy\right]$$

$$F = -\left[K + \rho Ag\right]y$$

A = cross sectional area of cylinder

For S.H.M $F = -M\omega^2 y$

$$-M\omega^2 y = -\left[k + \rho Ag\right]y$$

$$\omega = \sqrt{\frac{k+\rho Ag}{m}}$$

Let T = Period time

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k+\rho Ag}}$$

But $A = \dfrac{\pi d^2}{4}$

$$T = 2\times 3.14\sqrt{\dfrac{5}{200+1000\times 9.8\times\dfrac{(0.12)^2}{4}}}$$

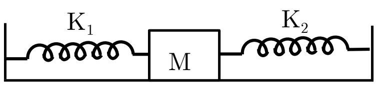
$$T = 0.8\,\text{sec}$$

Example 16

Suppose that two springs in figure below have different springs constants K_1 and K_2 show that the frequency f of oscillation of the block is given by

$$f = \sqrt{f_1^2 + f_2^2}$$

Where f_1 and f_2 are the frequencies at which would oscillates if connected only to spring 1 or only to spring



Solution

Periodic time , $T = 2\pi\sqrt{\dfrac{M}{K}}$

$$K = \frac{4\pi^2 M}{T^2}$$

For spring 1: $K_1 = \dfrac{4\pi^2 M}{T_1^2}$

For spring 2: $K_2 = \dfrac{4\pi^2 M}{T_2^2}$

Total force constant for springs in the parallel connection

$$K = K_1 + K_2$$

$$\frac{4\pi^2 M}{T^2} = \frac{4\pi^2 M}{T_1^2} + \frac{4\pi^2 M}{T_2^2}$$

$$\frac{1}{T^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2}$$

$$f^2 = f_1^2 + f_2^2$$

$$f = \sqrt{f_1^2 + f_2^2}$$

Example 17

A particle has a time period of 1.0sec under the action of a certain force and 2sec under the action of another force. Find the time period when the forces are acting in the same direction simultaneously.

Solution

Let the period of oscillation be T_1 under the action of force F_1 and T_2 under the action of force F_2.

Let F be the resultant force and T be periodic time of oscillation under the action of the resultant force.

$$F = F_1 + F_2$$

$$M\omega^2 y = M\omega_1^2 y + M\omega_2^2 y$$

$$\omega^2 = \omega_1^2 + \omega_2^2$$

$$\left(\frac{2\pi}{T}\right)^2 = \left(\frac{2\pi}{T_1}\right)^2 + \left(\frac{2\pi}{T_2}\right)^2$$

$$\frac{1}{T^2} = \frac{1}{T_2^2} + \frac{1}{T_2^2}$$

$$T = \frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2}} = \frac{1\times 2}{\sqrt{1^2 + 2^2}}$$

$$T = 0.894\,\text{sec}$$

Example 18

A body executes S.H.M under a time period of 0.8sec. It has a time period of 0.6sec under the action of another force. Calculate the time period when both the forces act in the same direction simultaneously.

Solution

In S.H.M , $T = 2\pi\sqrt{\dfrac{\text{displacement}}{\text{acceleration}}}$

But $F = Ma$

$$T = 2\pi\sqrt{\frac{My}{Ma}} = 2\pi\sqrt{\frac{My}{F}}$$

Now $T \propto \dfrac{1}{\sqrt{F}}$, $T = \dfrac{K}{\sqrt{F}}$

$$0.8 = \frac{K}{\sqrt{F_1}} \;\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$0.6 = \frac{K}{\sqrt{F_2}} \;\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Dividing equation (1) by (2)

$$\frac{0.8}{0.6} = \sqrt{\frac{F_2}{F_1}} \quad \text{OR} \quad F_2 = F_1\left(\frac{16}{9}\right)$$

Again

$$T \; \alpha \; \frac{1}{\sqrt{F_1 + F_2}}$$

$$T = \frac{K}{\sqrt{F_1 + F_2}} \;\ldots\ldots\ldots\ldots\ldots(3)$$

Dividing equation (3) by (1)

$$\frac{T}{0.8} = \sqrt{\frac{F_1}{F_1 + F_2}} = \sqrt{\frac{F_1}{F_1 + \frac{16}{9} F_1}}$$

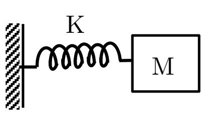
$$T = 0.8 \times \frac{3}{5}$$

$$T = 0.48 \text{ sec}$$

Example 18

A spring of constant K = 0.5N/m and an attached mass M oscillates on a smooth horizontal table when mass is position X_1 = 0.1m , its velocity is V_1 = -1m/s , at X_2 = 0.2m it has velocity V_2 = 0.5m/s. Find the value of attached mass , M and the amplitude A of the motion.

Solution



The K.E of a body attached with spring of spring constant K at any point X is given by

K.E = E – P.E

E = Total energy , p.e = potential energy

$$\frac{1}{2}MV^2 = \frac{1}{2}KA^2 - \frac{1}{2}KX^2$$

As per question

$$\frac{1}{2}MV_1^2 = \frac{1}{2}K\left(A^2 - X_1^2\right) \text{ and}$$

$$\frac{1}{2}KA^2 = \frac{1}{2}MV_1^2 + \frac{1}{2}KX_1^2 \ldots\ldots\ldots\ldots\ldots(1)$$

$$\frac{1}{2}KA^2 = \frac{1}{2}MV_2^2 + \frac{1}{2}KX_2^2 \ldots\ldots(2)$$

(1) = (2)

$$\frac{1}{2}MV_1^2 + \frac{1}{2}KX_1^2 = \frac{1}{2}MV_2^2 + \frac{1}{2}KX_2^2 \text{ or}$$

$$M\left(V_2^2 - V_1^2\right) = K\left(X_1^2 - X_2^2\right)$$

$$M\left[(0.5)^2 - (-1)^2\right] = 0.5\left[(0.1)^2 - (0.2)^2\right]$$

On solving , M = 0.02kg

Let A = amplitude of a motion

From equation (1)

$$A = \left[\frac{MV_1^2 + KX_1^2}{K}\right]^{\frac{1}{2}}$$

$$= \left[\frac{0.02(-1)^2 + 0.5(0.1)^2}{0.5}\right]^{\frac{1}{2}}$$

$$A = 0.22\text{m}$$

Example 20

Three springs are connected to a mass M = 100g as shown in the figure 22 below. Given that K = 2.5N/M

(a) What is the effective spring constant of the combination of springs?

(b) When mass M oscillates find the time period of the vibration.

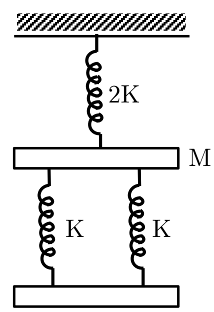


Figure 22

Solution

(a) In the given combination springs , three springs of spring constant , 2K , K and K are in parallel , then the effective spring constant.

$$K_P = 2K + K + K = 4K$$

$$K_P = 4 \times 25 = 10\text{Nm}^{-1}$$

(b) Time period of oscillation

$$T = 2\pi\sqrt{\frac{M}{K_P}} = 2\pi\sqrt{\frac{0.100}{10}}$$

$$T = 0.628 \text{ sec}$$

Example 21

A spring stretches 1cm when loaded with 90gm. With only a light pan hanging on the spring, a mass of 75gm is dropped from a height of 10cm into the pan find

(a) The resulting extension of the spring
(b) The vertical distance between the highest and lowest positions of the oscillating pan.
(c) The periodic time of the oscillation (take g = 9.8m/s²)

Solution

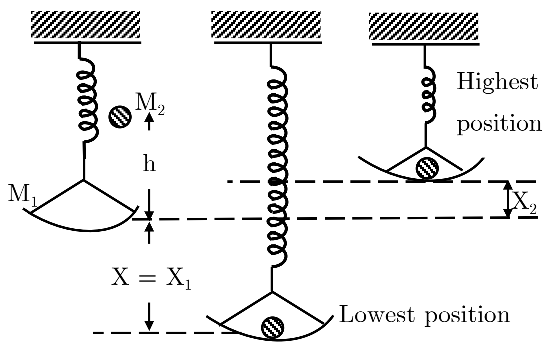
The spring constant of the spring

$$K = \frac{Mg}{e}$$

$$K = \frac{0.09\times9.8}{0.01}$$

$$K = 88.2N/m$$

(a) Since the pan is light , the mass do not lose any of its kinetic energy on impact with the pan. Let vertical height the mass is dropped is h and the resulting extension of the spring is X. The mass descent a total distance of h + x as shown on the figure below



Apply the law of conservation of energy

Loss in p.e Of mass = gain in elastic potential energy of the spring

$$Mg(h+x) = \frac{1}{2}KX^2$$

$$0.075\times9.8(0.1+X) = \frac{1}{2}\times88.2X^2$$

$$44.1X^2 - 0.735X - 0.0735 = 0$$

On solving quadratically

$X_1 = 0.05m = 5cm$, $X_2 = 0.033m$

∴ The extension of spring X = 0.05m = 5cm

Note that:

Minus sign shows that the pan would then rise above its initial position as it oscillates.

(b) Vertical distance between the highest and lowest position i.e

$$X_1 + X_2 = 0.05 + 0.033$$

$$X_1 + X_2 = 0.083m = 8.3cm$$

Alternatively

The required distance = extension produced by the mass when hung.

According to the Hooke's law

$$Ky = Mg$$

$$y = \frac{Mg}{K} = \frac{0.075\times9.8}{88.2}$$

$$y = 0.083m = 8.3cm$$

(c) Periodic time

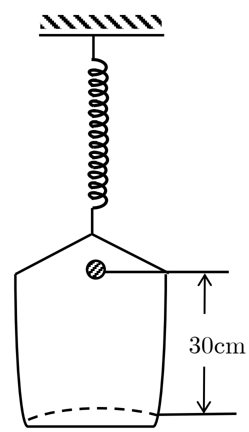
$$T = 2\pi\sqrt{\frac{M}{K}} = 2\times3.14\sqrt{\frac{0.075}{88.2}}$$

$$T = 0.183\,sec$$

Example 22

A frame of mass 200gm when suspended from a coil spring is found to stretch the spring 10cm a lump of putty of mass 200gm is dropped from rest on the frame from a height 30cm find

(a) The maximum distance the frame moves downward
(b) The amplitude of the resulting oscillation of the frame
(c) The period of oscillation



Solution

Reasoning

The frame has a mass , part of kinetic energy of the lump of putty will be loss during the impact with the frame. The frame and the lump of putty both will have k.e is converted into elastic p.e of the spring as the lump of putty is in impact with the frame it causes it to extend due to the impact.

(a) The extension produced due to the frame

$$X_1 = \frac{Mg}{K} = 10cm$$

Let X2 be the extension produced due to the loss of energy of the frame and lump of putty after impact with the frame now initial speed of the putty just before hitting the frame.

Apply the law of conservation of energy

$$\frac{1}{2}M_1U^2 = M_1gh$$

$$U = \sqrt{2gh} = \sqrt{2\times9.8\times0.3}$$

$$U = 2.425m/s$$

Let V = Common velocity .

Apply the principle of conservation of linear momentum.

$$M_1U + M_2X0 = (M_1 + M_2)V$$

$$V = \frac{M_1U}{M_1 + M_2} = \left(\frac{200}{200+200}\right)\times2.425$$

$$V = 1.2125m/s$$

Apply the law of conservation of energy

Loss of k.e of frame and putty at maximum extension of the spring = elastic p.e stored in the spring.

$$\frac{1}{2}(M_1 + M_2)V^2 = \frac{1}{2}KX_2^2$$

$$X_2 = \left[\frac{(M_1+M_2)}{K}\right]^{1/2} \quad \text{but} \quad K = \frac{M_1g}{X_1}$$

$$= \left[\frac{(0.2+0.2)(1.2125)^2}{19.6}\right]^{1/2}$$

$$X_2 = 0.1732m = 17.32cm$$

∴The maximum distance the frame moves down

$$X_1 + X_2 = (10+17.32)cm$$

$$X_1 + X_2 = 27.32cm$$

(b) Let A = Amplitude of the oscillation

Since

$$V = \omega A = A\sqrt{\frac{K}{M_1+M_2}}$$

$$1.2125 = A\sqrt{\frac{19.6}{0.2+0.2}}$$

$$A = 0.173m = 17.3cm$$

(c) Periodic time

$$T = 2\pi\sqrt{\frac{M_1+M_2}{K}} = 2\times3.14\sqrt{\frac{0.2+0.2}{19.6}}$$

$$T = 0.90\,sec$$

Example 23

A particle in S.H.M is describe by the displacement function

$$x_{(t)} = B\sin(\omega t + \alpha) \quad ; \quad \omega = 2\pi/T$$

If the initial position (at t = 0) of the particle is 1cm and its initial velocity is πcm/s. What are its amplitude and initial phase angle? The angular frequency of the particle is πs^{-1}

Solution

Here at t = 0 , x = 1cm , V = πcm/s , $\omega = \pi s^{-1}$

Since $x = B\sin(\omega t + \alpha)$

At t = 0

$$1 = B\sin(\omega x0 + \alpha)$$

$$1 = B\sin\alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)$$

Now

$$V = \frac{dx}{dt} = \frac{d}{dt}\left[B\sin(\omega t+\alpha)\right]$$

$$V = B\omega\cos(\omega t + \alpha)$$

Since at t = 0 , V = πcm/s

$$\pi = B\pi\cos(\omega x0 + \alpha)$$

$$B\cos\alpha = 1 \ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

Squaring and adding the equation (i) and (ii)

$$(B\cos\alpha)^2 + (B\sin\alpha)^2 = 1^2 + 1^2$$

$$B = \sqrt{2}$$

Also

$$\frac{B\sin\alpha}{B\cos\alpha} = \frac{1}{1} = 1$$

$$\tan\alpha = 1 \ , \ \alpha = \frac{\pi}{4} \text{ or } \frac{5\pi}{4}$$

$$\therefore \ \text{Amplitude} \quad B = \sqrt{2}\text{cm}$$

$$\text{Initial phase angle}, \ \alpha = \frac{\pi}{4}\text{rad}$$

Example 24

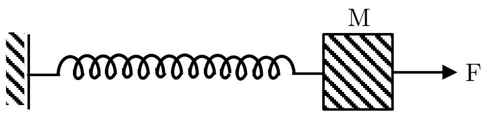
A spring of force constant 1200Nm^{-1} is mounted horizontally on a horizontal table. A mass of 3.0kg attached to the free end of the spring is pulled sideways to a distance 2cm and released.

(i) What is the frequency of the oscillation of the mass?

(ii) What is the maximum acceleration of the mass?

(iii) What is the maximum speed of the mass

Solution



M = 1200Nm^{-1}

M = 3kg

A = 2cm

(i) $$f = \frac{1}{2\pi}\sqrt{\frac{K}{M}} = \frac{1}{2\times 3.14}\sqrt{\frac{1200}{3}}$$

$$f = 3.18\text{Hz}$$

(ii) Since , $$\omega = \sqrt{\frac{K}{M}} \ , \ \omega^2 = \frac{K}{M}$$

Maximum acceleration

$$a_{max} = \omega^2 A = \frac{KA}{M}$$

$$= \frac{1200\times 2\times 10^{-2}}{3}$$

$$a_{max} = 8.0\text{m}/\text{s}^2$$

(iii) Maximum velocity

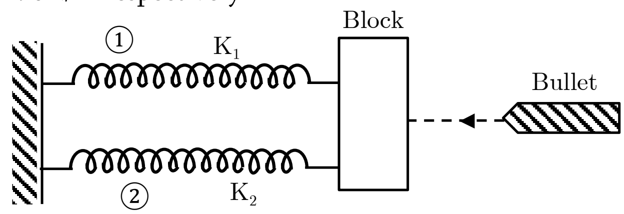
$$V_{max} = \omega A = A\sqrt{\frac{K}{M}}$$

$$= 2\times 10^{-2}\cdot\sqrt{\frac{1200}{3}}$$

$$V_{max} = 0.40\text{m}/\text{s}$$

Example 25

Two springs are attached to a block of 9.9kg as shown in the figure below , the block is at the rest on a frictionless surface. A bullet of mass 100gm is with a velocity of 100m/s at the block. The bullet strikes and embeds itself on the block. The bullet then oscillates with a block in S.H.M if the springs constant for the springs 1 and 2 are 196N/m and 490N/m respectively



(a) Calculate the amplitude of the block

(b) Solve for the frequency of oscillation

(c) Find the total energy of oscillation and initial kinetic energy of the bullet why?

Solution

Given that

$K_1 = 196\text{Nm}^{-1}$

$K_2 = 490\text{Nm}^{-1}$

$M = 9.9\text{Kg}$

$Mb = 100\text{gm} = 0.1\text{kg}$

$Ub = 100\text{m}/\text{s}$

Apply the principle of conservation of linear momentum

$$M_b U_b = (M_b + M)V$$

$$V = \frac{M_b U_b}{M_b + M} = \frac{0.1\times 100}{0.1 + 9.9}$$

$$V = 1.0\text{m}/\text{s}$$

(a) Apply the law of Conservation of energy

Loss in k.e of bullet block spring System = gain in elastic p.e on the spring

$$\frac{1}{2}(M_b + M)V^2 = \frac{1}{2}KA^2$$

$$A = \left[\frac{(M_b + M)V^2}{K}\right]^{1/2}$$

But $K = K_1 + K_2$

$$A = \left[\frac{(9.1+0.1)\times 1^2}{490+196}\right]^{1/2}$$

$$A = 0.121M = 12.1cm$$

(b) Frequency , $f = \frac{1}{2\pi}\sqrt{\frac{K_1+K_2}{M+M_b}}$

$$= \frac{1}{2\times 3.14}\sqrt{\frac{49.+196}{9.9+0.1}}$$

$$f = 1.319Hz$$

(c) Total energy of oscillation

$$E = \frac{1}{2}(K_1+K_2)A^2$$

$$= \frac{1}{2}\left[(490+196)(0.121)^2\right]$$

$$E = 596.738J$$

Initial kinetic energy of the bullet

$$E_0 = \frac{1}{2}M_bU_b^2 = \frac{1}{2}\times 0.1\times(100)^2$$

$$E_0 = 500J$$

Comment

The two values are not the same since there are some of the amount of kinetic energy of the bullet which can be converted into heat energy and sound energy when stricking and embedded on the block of wood during the impact.

Example 26

A block of mass M at rest on a horizontal frictionless table is attached to a rigid support by a support by a spring of force constant K. A bullet of mass, m and velocity u strikes the block as shown in the figure below. The bullet remain embedded in the block determine:

(a) The velocity of the block immediately after collision.

(b) The amplitude of resulting S.H.M

Solution

(a) Apply the principle of conservation of linear momentum.

$$mu + mxo = (M+m)v$$

$$V = \frac{mu}{M+m}$$

(b) Apply the law of conservation of energy

Loss of k.e of Bullet block system = gained in elastic p.e on the spring

$$\frac{1}{2}(M+m)V^2 = \frac{1}{2}KA^2$$

$$KA^2 = (M+m)\left[\frac{mu}{M+m}\right]^2$$

$$KA^2 = \frac{(mu)^2}{M+m}$$

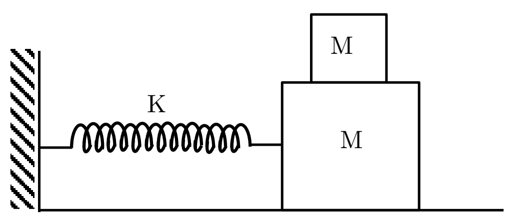
$$A = \frac{mu}{\sqrt{K(M+m)}}$$

Example 27

The friction coefficient between two blocks shown in the figure is μ and the horizontal plane is smooth.

(a) If the system is slightly displaced and released, find the time period.

(b) Find the magnitude of the frictional force between the block when the displacement from the mean position X. What can be amplitude if the upper block does not slip, relative to the lower block?



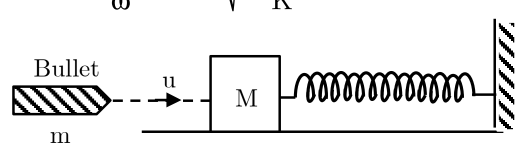
Solution

(a) For small amplitude , the two blocks oscillates together in this case

$$\omega = \sqrt{\frac{K}{M+m}}$$

Periodic time

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{M+m}{K}}$$



(b) The acceleration of two blocks at displacement X from the mean position.

$$a = -\omega^2 x = \frac{-kx}{M+m}$$

Force on upper block

$$F = ma = \frac{-mkx}{M+m}$$

This force is provided by friction of the lower block in magnitude of frictional force

$$F = \frac{mkx}{(M+m)}$$

The maximum frictional force

$$F_{max} = \frac{mKA}{(M+m)} \quad [\text{since } X = A]$$

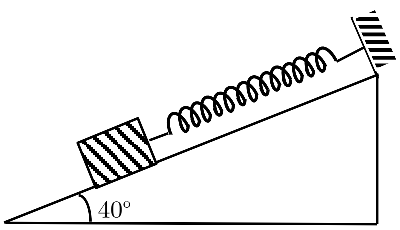
$$\mu mg = \frac{mKA}{M+m}$$

∴ Amplitude of oscillation

$$A = \frac{\mu(M+m)g}{K}$$

Example 28

A block weighing 14.0N, which slides without friction on a inclined plane is connected to the top of the incline by a massless spring of unscratched length 0.45m and the spring constant 120Nm^{-1} as shown on the figure below



(a) How far from the top of the incline does the block stop?

(b) If the block is pulled slightly down the incline and released, what is the period of the resulting oscillations?

Solution

(a) Let L = unstretched length
X = extension of the spring
But the system not oscillating according to the hooke's law

$$F = KX = Mg\sin\theta$$

$$x = \frac{mg\sin\theta}{k}$$

The total length of the spring

$$L = L + X = L + \frac{mg\sin\theta}{K}$$

$$= 0.45 + \frac{14}{120}\sin 40^\circ$$

$$L = 0.52m = 52m$$

(b) Since $w = mg$, $m = \frac{w}{g} = \frac{14}{10}$

$$m = 1.4kg$$

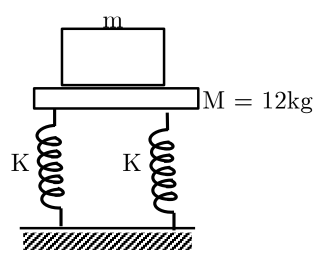
Periodic time

$$T = 2\pi\sqrt{\frac{M}{K}} = 2\pi\sqrt{\frac{1.4}{120}}$$

$$T = 0.68\text{ sec}$$

Example 29

A tray of mass 12kg is supported by a two identical springs shown in figure below. When the tray is pressed down slightly and released, it excuted simple harmonic motion period of 1.5sec. What is force constant? When the block mass m is placed on the tray, period of S.H.M changes to 3.0sec. What is the mass of block?



Solution

Case 1:

Total force constant for the two springs in parallel condition.

$K_P = K + K = 2K$

Periodic time of tray before placed a block of mass m on a top of it .

$$T_1 = 2\pi\sqrt{\frac{M}{2K}} \Rightarrow T_1^2 = \frac{4\pi^2 M}{2K}$$

$$K = \frac{2\pi^2 M}{T_1^2} = \frac{2\times 3.14^2\times 12}{1.5^2} = 5.17Nm^{-1}$$

Case 2

After a block of mass m placed at the top of the tray

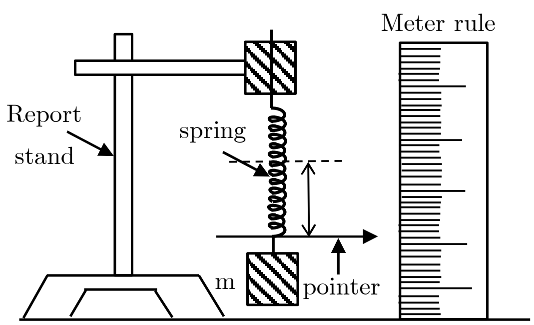
$M_T = M + m = 12 + m$, $T_2 = 3.0\,\text{sec}$

$$T_2 = 2\pi\sqrt{\frac{12+m}{2k}} \Rightarrow 3^2 = \frac{4\times 3.14^2(12+m)}{2\times 105.17}$$

m = 36.05kg

1. **DETERMINATION OF g BY USING A MASS ON AN OSCILLATION SPRING.**

   A mass M suspended on a spring causes an extension e when set into vertical oscillation periodic time, T of the oscillation can be obtained.



Periodic time , T $= 2\pi\sqrt{\frac{M + M_1}{K}}$

Ms = effective mass of the spring

K = force constant

Since ke = mg , $m = \frac{ke}{g}$

Now

$$T^2 = 4\pi^2\left[\frac{M}{K} + \frac{M_S}{K}\right]$$

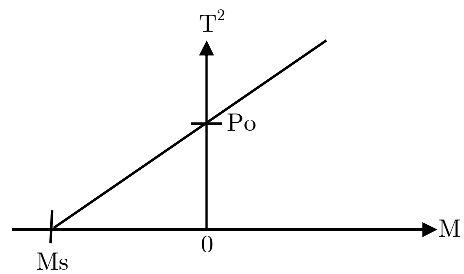
$$= 4\pi^2\left[\frac{Ke/g}{K} + \frac{M_s}{K}\right]$$

$$T^2 = \left(\frac{4\pi^2}{g}\right)e + \frac{4\pi^2 M_s}{K}$$

From the equation

$$T^2 = \left(\frac{4\pi^2}{K}\right)M + \frac{4\pi^2 M_S}{K}$$

GRAPH OF T^2 AGAINST M



Since slope $= \frac{4\pi^2}{K}$

$$K = \frac{4\pi^2}{\text{slope}}$$

The effective mass of the spring is determined from the horizontal axis intercepts

M – axis intercept = Ms

Also can be obtained from the vertical intercept

$$P_O = \frac{4\pi^2 M_s}{K}$$

$$P_O = M_s \times \text{slope}$$

$$M_S = \frac{P_o}{\text{slope}}$$

Periodic time of the helical spring is given by

$$T = 2\pi\sqrt{\frac{M}{K}}$$

On this formula , you may assume that the mass of the spring is very neglible or small if the mass of the spring is taken into account , then the total mass of the system is M + Ms.

Note that :

Ms is less than the actual mass of the spring it is the lowest coil which oscillates with the full amplitude of suspended body.

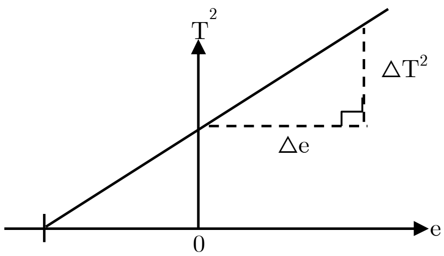
Effective mass $= \frac{1}{3}$Ms

$$T = 2\pi\sqrt{\frac{M + \frac{1}{3}Ms}{K}}$$

(for the practical purpose)

Thus a graph of T^2 against e is straight line and has a gradient of $4\pi^2/g$ and therefore enable g to be determined such a graph can be obtained by adding a number of different masses to the spring and measuring the static extension e which each produces together with the corresponding period of oscillation, T

AGAINST T² AGAINST e



$$\text{Slope} = \frac{\Delta T}{\Delta e} = \frac{4\pi^2}{g}$$

$$g = \frac{4\pi^2}{\text{slope}}$$

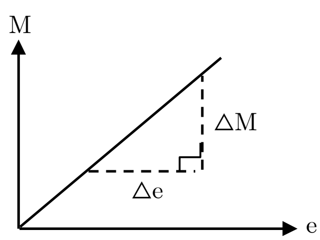
Note that

1. From the graph above when e = 0 , T^2 – intercept = $\frac{4\pi^2 M_s}{K}$ and therefore Ms can be obtained if K is known. The value of K is found by plotting a graph of m against e.

GRAPH OF M AGAINST e

Since $mg = ke$

$$m = \left(\frac{k}{g}\right)e$$



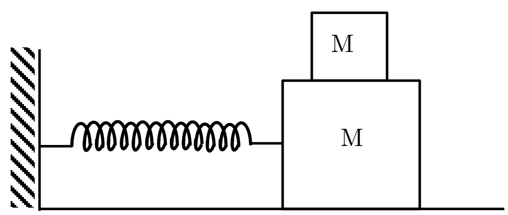
$$\text{Slope} = \frac{k}{g} \Rightarrow k = g \times \text{slope}$$

EXERCISE 5.2

1. It is found that a load of mass 200gm stretches a spring by 10.0cm. The same spring is then stretched by an additional 5.0cm and released find :-
   (a) Spring constant
   (b) Period of vibration and frequency
   (c) Maximum acceleration and
   (d) Velocity through equilibrium position
   [ans. (a) 19.6Nm⁻¹ (b) 0.63s , 1.58Hz
   (c) 4.9m/s² (d) 0.49m/s]

2. A mas of 0.5kg is vibrating in a system in which the constant of the spring used is 100N/m. the amplitude of vibration is 0.2m determine.
   (a) Energy of the system
   (b) The maximum velocity
   (c) The potential energy and kinetic energy when x =0.1m and
   (d) The maximum acceleration
   [ans. (a) 2J (b) 3.83m/s (c) 0.5J, 1.5J (d) 40m/s²]

3. Two blocks (m = 1.0kg and M = 10kg] and a spring of force constant K = 200Nm⁻¹ are arranged on a horizontal frictionless as shown in the figure below. The coefficient of static friction between two block is 0.40. what is the maximum possible amplitude of the S.H.M if no slippage is to occur between the blocks



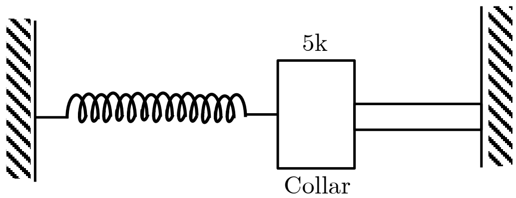
[Ans 0.22m or 22cm]

4. The analysis of S.HM in the chapter is ignored the mass of the spring . How does the spring's mass change the characteristics of the motion.

5. A car with a mass of 1300kg is constructed so that its frame is supported by four springs each spring has a force constant of 20,000N/m. If two people reading in the car have a combined mass of 160kg find the frequency of vibration of the car after it is driven over a pothole in the road [ans 1.18Hz]

6. A 5kg collar is attached to a spring of force constant 500N/m, It slides without friction on a horizontal road as shown in figure below. The collar is displaced from its equilibrium position by 10cm and released calculate

(i) The period of oscillation
(ii) The maximum speed and
(iii) The maximum acceleration of the collar
[ans (i) 0.628sec (ii) 1.0cm/s (iii) 10m/s^2]

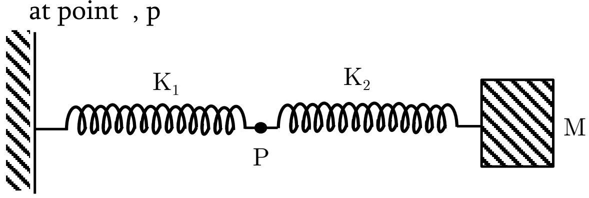


7. A body of mass 10kg is suspended by a massless coil spring of natural length 40cm and force constant $2 \times 10^3 Nm^{-1}$. What is the stretched length of the spring ? If the body is pulled down further stretching the spring to a length of 48cm and the released , what is the frequency of oscillations of the suspended mass? [g = 10m/s^2] [ans 45cm , 2.25Hz]

8. Two particles A and B of equal masses are suspended from massless spring of spring constant K_1 = 4N/m and K_2 = 8N/m respectively. If the maximum velocities during oscillation are equal , find the ratio of amplitudes of A and B [ans. $\sqrt{2}$]

9. Which of the following relationship between the acceleration a and the displacement x of a particle involve S.H.M
(a) $a = 0.7x$
(b) $a = -200x^2$
(c) $a = -10x$
(d) $a = 100x^3$ [ans. (c)]

10. the mass M shown in figure below oscillates in S.H.M with amplitude A what is the amplitude at point , p



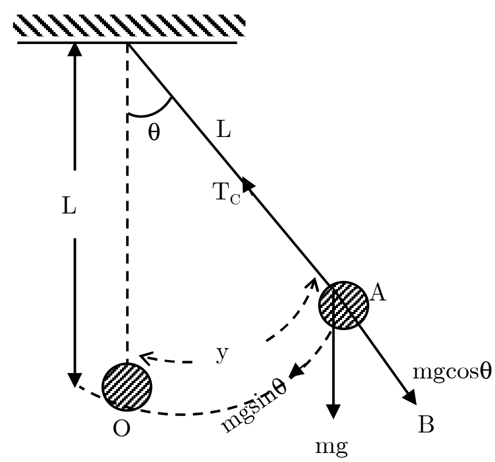
$$\left[\text{Ans. } X_1 = \frac{K_2 A}{K_1 + K_2}\right]$$

SIMPLE PENDULUM

A simple pendulum is the most common examples of bodies execute S.H.M

A simple pendulum is a heavy point mass suspended by a weightless, inextensible and perfectly flexible string from a rigid support about which it is free to oscillate.

Point of suspension – is the point from which pendulum is suspended. Since the string is very fine , the center of mass of the pendulum concides with the centre of mass of the bob the centre of mass of the bob is known as **point of oscillation.** The distance between the point of suspension and the point of oscillation is called length of the pendulum. A simple pendulum is used to determine the value of the acceleration due to gravity g. Consider the figure below which shows the oscillations of simple pendulum.



There are two forces which act on the bob , the weight (mg) and the tension (T) in the string resolving mg into the rectangular components

(i) Mgcosθ acts along AB opposite to the tension ,T

ii) Mgsinθ acts along AO , tangent to the arc AO and directed towards O. When the bob is slightly displaced to one side from its mean position and released , then oscillates about the mean position in an arc of a circle restoring force , F = -Mgsinθ.
Assume that θ is very small (i.e θ < 5^o) measured in radian.

$$\sin\theta \simeq \theta = \frac{y}{L}$$

$$F = -Mg\sin\theta = -Mg\theta = -\frac{mgy}{L}$$

But F = Ma

$$Ma = \frac{-Mgy}{L} \quad , \quad a = \frac{-gy}{L}$$

$$a = -\left[\frac{g}{L}\right]y \quad , \quad a \; \alpha - y$$

It execute S.H.M

Expression of frequency and periodic time of oscillation of a simple pendulum.

For the S.H.M , a = -ω2y

$$-\omega^2 y = -\frac{gy}{L}$$

$$\omega = \sqrt{\frac{g}{L}} \qquad \text{But} \qquad \omega = 2\pi f$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

$$\text{Periodic time , } T = \frac{1}{f}$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Let n be the number of oscillation of pendulum after time , t

$$T = \frac{\text{time taken}}{\text{number of oscillation}} = \frac{t}{n}$$

$$t = nT = 2\pi n\sqrt{\frac{L}{g}}$$

Periodic time T of simple pendulum depends only on value of g and length of the thread and its independent on the mass of the bob.

Expression of tension on the string

$$T_C = M\left[g\cos\theta + \frac{V^2}{L}\right]$$

Important points

Periodic time of simple pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Following points may be noted

1. **Effect of amplitude**

   T is independent on the amplitude of oscillation. Here is assumed that the amplitude of oscillation is very small (i.e $\theta < 5^o$).

2. **Effect of mass of a bob**

   T is independent on the mass of the bob

   Note:

   Since T is independent on the mass of a bob and amplitude of its oscillation such vibrations where time period T is independent of amplitude is called **ISOCHRONOUS VIBRATIONS**

3. **Effect of length**

   As $T \alpha \sqrt{L}$, therefore T must increase with increase in the value of length (L) of pendulum.

4. **Effect of acceleration due to gravity g.**

   As $T \alpha \frac{1}{\sqrt{g}}$, therefore T will increase with decrease in the value of g , As the value of acceleration due to gravity is less at hills or in mines than that on surface of Earth , hence the time period of simple pendulum increases at hills or inside the mines due to it , the pendulum clock will be slowed down it means that , the pendulum clock will be losing the time at hill or inside the mines.

5. **Effect of weightlessness**

   If simple pendulum is made to oscillate in a satellite or freely falling lift , then the effective value of acceleration due to gravity g' will be zero.

   $$g' = g - g = g - a = 0$$

   $$\text{So } T = 2\pi\sqrt{\frac{L}{O}} = \infty \text{ (Infinite)}$$

   This shows that the pendulum will not oscillate and will remain where left free.

6. **Effect of density of medium**

   If the bob of a simple pendulum is made to oscillate in a fluid whose density is less than the density of the bob , then time period of simple pendulum gets increased.

   Let ρ = density of material of bob

δ = density of liquid

V = volume of the bob

Now $V\rho g' = V\rho g - V\delta g$

$$g' = g\left[1 - \frac{\delta}{\rho}\right]$$

g' = effect value of acceleration due to gravity act on the bob

$$T' = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{L}{g\left(1-\frac{\delta}{\rho}\right)}}$$

$$T = 2\pi\sqrt{\frac{L\rho}{g(\delta-\rho)}} = 2\pi\sqrt{\frac{L}{g}}\cdot\sqrt{\frac{\rho}{\rho-\delta}}$$

T = periodic time of pendulum oscillating in air.

$$T' = T\cdot\sqrt{\frac{\rho}{\rho-\delta}}$$

Since $\rho > \sigma$, $T' > T$

7. **Effect of temperature** - If the bob of the simple pendulum is suspended by a wire then effective length of pendulum will increase with the rise of temperature due to it , the time period of simple pendulum will increase. If $d\theta$ is the change in temperature and α is the coefficient of linear expansion of wire , then the new length of the wire.

$$L' = L(1+\alpha d\theta)$$

$$T' = 2\pi\sqrt{\frac{L'}{g}} = 2\pi\sqrt{\frac{L(1+\alpha d\theta)}{g}}$$

$$T' = 2\pi\sqrt{\frac{L}{g}}\cdot\sqrt{1+\alpha d\theta}$$

$$T' = T[1+\alpha d\theta]^{1/2} \quad \{T' > T\}$$

Hence

$$T' = 1 + \frac{\alpha d\theta}{2}$$

% increase in time period =

$$\left(\frac{T'-T}{T}\right)\times 100\% = 50\alpha d\theta$$

8. **If the bob of the simple pendulum of mass** M is suspended from a wire of natural length , L , radius r and the wire stretched by ΔL due to elasticity , then.

$$T = 2\pi\sqrt{\frac{L+\Delta L}{g}} = 2\pi\sqrt{\frac{L}{g}\left[1+\frac{\Delta L}{L}\right]}$$

If E is the Young's modulus of wire ,then

$$E = \frac{MgL}{\pi r^2 \Delta L} \quad \text{or} \quad \frac{\Delta L}{L} = \frac{Mg}{\pi r^2 E}$$

$$T = 2\pi\sqrt{\frac{L}{g}\left[1+\frac{Mg}{\pi r^2 E}\right]}$$

9. **If a simple pendulum** - is in a carriage which is accelerating with acceleration.

(i) upwards , then $g' = g + a$

$$T = 2\pi\sqrt{\frac{L}{g+a}}$$

(ii) downward , $g' = g + a$

$$T = 2\pi\sqrt{\frac{L}{g-a}}$$

(iii) horizontally , $g' = \sqrt{g^2 + a^2}$

$$T = 2\pi\sqrt{\frac{L}{\sqrt{g^2+a^2}}}$$

10. **Second pendulum** - Is the pendulum whose time period of vibrations is two seconds. The bob of such pendulum while oscillating passes through the mean position after every one. Second i.e beats second.

Length of second pendulum

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$L = \frac{T^2 g}{4\pi^2} = \frac{2^2\times 9.8}{4\pi^2}$$

$$L = 0.993\text{m} = 99.3\text{cm}$$

SPECIAL CASE FOR THE SIMPLE PENDULUM

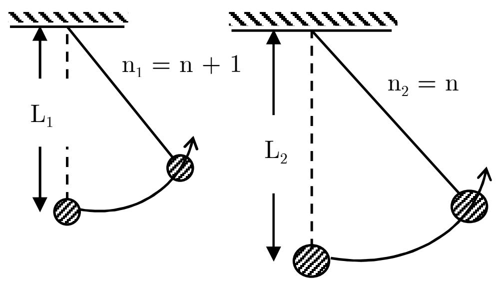
Simultaneous oscillating pendulums

Consider two pendulum of length L_1 and L_2 with periodic time T_1 and T_2 respectively and starting oscillating in the same direction.

Different cases:-

Case : 1

Two pendulums oscillating in the same direction after time t (in phase). Experimentally it can been shown that if two pendulums will be in phase again after time t , then the number of oscillations of shorter pendulum will be one more oscillation compared with longer pendulum



For the shorter pendulum

$$T_1 = 2\pi\sqrt{\frac{L_1}{g}} \quad , \quad t_1 = n_1T_1 = (n+1)T_1$$

For the longer pendulum

$$T_2 = 2\pi\sqrt{\frac{L_2}{g}} \quad , \quad t_2 = n_2T_2 = nT_2$$

For the two pendulum to be inphase after first time , then the time taken

$$t = t_1 = t_2$$

$$(n+1)T_1 = nT_2$$

$$n_2 = n = \frac{T_1}{T_2 - T_1} \quad , \quad n_1 = n+1 = \frac{T_2}{T_2 - T_1}$$

∴ The time taken by the two pendulums to be inphase.

$$t = (n+1)T_1 = nT_2$$

$$t = nT_2 = \frac{T_1T_2}{T_2 - T_1}$$

Note that;

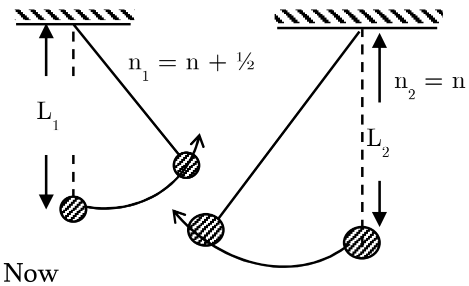
If the pendulums will continue swinging , they will be inphase after every time t seconds. The time to be inphase in kth time is given by

$$t_k = \frac{KT_1T_2}{T_2 - T_1}$$

K = 1 , 2, 3, ---- ----- ----- -----

Case : 2

If the two pendulums are out – phase (antiphase) in time , t . the shorter pendulum makes extra half (½) more oscillation compared with longer pendulum.



Now

$$t = t_1 = t_2$$

$$\left(n + \frac{1}{2}\right)T_1 = nT_2$$

$$n = n_2 = \frac{T_1}{2(T_2 - T_1)} \quad , \quad n_1 = \frac{T_2}{2(T_2 - T_1)}$$

The time taken by the two pendulum to be out – phase.

$$t = n_2T_2 = \frac{T_1T_2}{2(T_2 - T_1)}$$

Generally

$$t_k = \frac{T_1T_2K}{2(T_2 - T_1)} \quad , \quad K = 1\ ,2-----$$

SOLVED EXAMPLES

Example 1

A simple pendulum has a period of 2.0sec and an amplitude of swing 5.0cm calculate the maximum magnitudes of.

(a) The velocity of the bob

(b) The acceleration of the bob

Solution

(a) $V_{max} = \omega A = \left(\frac{2\pi}{T}\right)A = \left(\frac{2\pi}{2}\right)\times 5$

$V_{max} = 15.7cm / s$

(b) $a_{max} = \omega^2 A = \left(\frac{2\pi}{T}\right)^2 \quad , \quad A = \left(\frac{2\pi}{2}\right)^2 \times 5$

$a_{max} = 49.3m / s^2$

Example 2

(a) What will be the time period of seconds pendulum if its length is doubled?

(b) The acceleration due to gravity on the surface of the moon is 1.7m/s². What is the time period of a simple pendulum on the surface of the moon, if its time period on the surface of the earth is 3.5sec? Take g = 9.8m/s² on the surface of the earth.

Solution

(a) $T = 2\pi\sqrt{\frac{L}{g}} = 2$

$$T' = 2\pi\sqrt{\frac{2L}{g}} = \sqrt{2}\cdot 2\pi\sqrt{\frac{L}{g}}$$

$$T' = \sqrt{2}\times 2 = 2.828\,\text{sec}$$

(b) Let g and g' be the acceleration due to gravity on the surface of Earth and moon respectively

On Earth, $T = 2\pi\sqrt{\frac{L}{g}}$

On Moon, $T' = 2\pi\sqrt{\frac{L}{g}}$

$$\frac{T'}{T} = \sqrt{\frac{g}{g'}} \text{ or } T' = T\cdot\sqrt{\frac{g}{g'}}$$

$$T' = 3.5\sqrt{\frac{9.8}{1.7}} = 8.4\,\text{sec}$$

$$T' = 8.4\,\text{sec}$$

Example 3

(a) (i) Define S.H.M

(ii) Prove that, the velocity V of a particle in S.H.M is given by $V = \omega\sqrt{A^2 - y^2}$ where A is the amplitude of oscillation, ω is angular frequency and y is the displacement from the mean position

(b) A simple pendulum has a period of 2.8sec when the length is shortened by 1.0m the period becomes 2.0sec. From this information, determine the acceleration of gravity and the origin length of the pendulum.

Solution

(a) Refer to your notes

(b) Let $T_1 = 2.8$sec, $L_1 = L$, $T_2 = 2.0$sec, $L_2 = L - 1$

Let L be original length of the simple pendulum.

Since $T = 2\pi\sqrt{\frac{L}{g}}$

$$T_1 = 2\pi\sqrt{\frac{L}{g}}$$

$$T_2 = 2\pi\sqrt{\frac{L-1}{g}}$$

Takes $\frac{T_1}{T_2} = \sqrt{\frac{L}{L-1}}$ Make subject L

$$L = \frac{T_1^2}{T_1^2 - T_2^2} = \frac{2.8^2}{2.8^2 - 2^2}$$

$$L = 2.042\text{m}$$

The value of g, $T_1 = 2\pi\sqrt{\frac{L}{g}}$

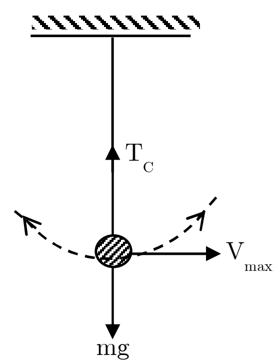
$$g = \frac{4\pi^2 L}{T_1^2} = \frac{4\pi^2 \times 2.042}{(2.8)^2}$$

$$g = 10.2\text{m/s}^2$$

Example 4

A small bob of mass 20gm oscillate as a simple pendulum with amplitude 5cm and period 2sec. Find the velocity of the bob and the tension in the supporting thread when the velocity of the bob is maximum take g = 9.8m/s².

Solution



Velocity of the bob is maximum when is at the mean position

$$V_{max} = \omega A = \left(\frac{2\pi}{T}\right)A$$

$$V_{max} = \left(\frac{2\pi}{2}\right)\times 0.05$$

$$V_{max} = 0.16\text{m/s}$$

Since periodic time , $T = 2\pi\sqrt{\frac{L}{g}}$

$$L = \frac{gT^2}{4\pi^2} \text{............(i)}$$

Tension on the spring at the position

$$T_C = M\left[g + \frac{V_{max}^2}{L}\right] = 0.02\left[9.8 + \frac{(0.16)^2}{L}\right] \text{(ii)}$$

Insert equation (i) into (ii)

$$T_2 = 0.02\left[9.8 + \frac{(0.16)^2\,4\pi^2}{gT^2}\right]$$

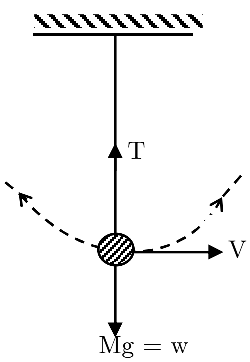
$$= 0.02\left[9.8 + \frac{(0.16)^2\,4\pi^2}{9.8\times 2^2}\right]$$

$$T_C = 19.65\times 10^{-2}\,N$$

Example 5

The bob of a simple pendulum 1.20m long weighs 0.50N . Find the tension T in the suspension when the bob passes through the centre of the oscillation , if the amplitude is 0.06m.

Solution



At the equilibrium position

$$T - mg = \frac{m\omega^2A^2}{L} = \frac{mv^2}{L}$$

$$T = mg + \frac{m\omega^2A^2}{L} \quad \text{But } \omega^2 = \frac{g}{L}$$

$$T = mg + \frac{mgA^2}{L^2} = w\left[1 + \frac{A^2}{L^2}\right]$$

$$T = 0.5\left[1 + \frac{(0.06)^2}{(1.2)^2}\right]$$

$$T = 0.50125N$$

Example 6

The time period of a simple pendulum at a place where acceleration due to gravity g is T . What will be its time period at a place where the acceleration due to gravity is 0.98g?

Solution

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\frac{\Delta T}{T} = \frac{-1}{2}\frac{\Delta g}{g} = \frac{-1}{2}\left[\frac{-0.02g}{g}\right]$$

$$\Delta T = 0.01T$$

New time period , $T' = T + \Delta T$

$$T' = 1.01T$$

Example 7

(a) Define simple harmonic motion

(b) Two simple pendulum of length 0.4m and 0.6m respectively are set oscillating in step

 (i) After what further time will the two pendulum be in step agins?

 (ii) Find the number of oscillations made by each pendulum during the time in s(b)(i) above.

(c) State two examples of S.H.M which are importance to the day life experience

(g = 9.8m/s²)

Solution

(a) Refer to your notes

(b) (i) $L_1 = 0.4m$, $L2 = 0.6m$

Periodic time of simple pendulum

$$T = 2\pi\sqrt{\frac{L}{g}}$$

For the shorter pendulum

$$T_1 = 2\pi\sqrt{\frac{L_1}{g}} = 2\pi\sqrt{\frac{0.4}{9.8}}$$

$$T_1 = 1.2687\text{ sec}$$

For the longer pendulum

$$T_2 = 2\pi\sqrt{\frac{L_2}{g}} = 2\pi\sqrt{\frac{0.6}{9.8}}$$

$$T_2 = 1.5539\text{ sec}$$

Now

$$t = (n+1)T_1 = nT_2$$

$$1.2687(n+1) = 1.5539n$$

On solving n = 4.5

Therefore ,the time by two pendulum to be in step again

$$t = nT_2 = (n+1)T_1$$

$$= 4.5 \times 1.5539$$

$$t = 6.99 \text{ sec}$$

(ii) Number of oscillation for the large pendulum n_2 = 4.5. number of oscillation for shorter pendulum n_1 = n + 1 = 5.5

(c) • Motion of piston in a gas filled cylinder help us for transportation.

• Motion of air molecules are in S.H.M which help for transmission of sound wave.

Example 8

Two simple pendulums of length 1.44m and 1.00m starts swinging at the same time after how much time will be

(i) Out of phase and

(ii) In phase again? (take g = 10m/s^2)

Solution

(i) Let T_1 and T_2 be the time periods of longer length and shorter pendulum respectively and L_1 and L_2 be their corresponding lengths.

$$T_1 = 2\pi\sqrt{\frac{L_1}{g}}$$

$$T_2 = 2\pi\sqrt{\frac{L_2}{g}}$$

Takes

$$\frac{T_1}{T_2} = \sqrt{\frac{L_1}{L_2}} \quad \ldots\ldots\ldots\ldots(i)$$

The time taken by the two pendulum to be out – phase is t.

$$t = nT_1 = \left(n+\frac{1}{2}\right)T_2$$

$$\frac{T_1}{T_2} = \frac{n+\frac{1}{2}}{n} \quad \ldots\ldots\ldots\ldots(ii)$$

(i)=(ii)

$$\frac{n+\frac{1}{2}}{n} = \sqrt{\frac{1.44}{1.00}}$$

On solving n = 2.5 vibrations

Now

$$t = 2\pi n\sqrt{\frac{L_1}{g}} = 2\times 3.14\times 2.5\sqrt{\frac{1.44}{1.00}}$$

$$t = 5.960 \text{ sec}$$

(ii) The time in which two pendulums to be in phase

$$t = (n+1)T_2 = nT_1$$

$$\frac{T_1}{T_2} = \frac{n+1}{n} = \sqrt{\frac{L_1}{L_2}} = 1.2$$

On solving , n = 5.0 vibrations

$$t = nT_1 = 2\times 3.14\times 5\sqrt{\frac{1.44}{1.00}}$$

$$t = 11.92 \text{ sec}$$

Example –9

If the length of simple pendulum is increased by 50% , what is the percentage increase in its time period?

Solution

Time period of a simple pendulum

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Thus as L increases , T increases

$$\Delta T = \frac{2\pi}{\sqrt{g}}\cdot\frac{1}{2}L^{\frac{1}{2}}\Delta L$$

$$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta L}{L}$$

% increase in time period

$$\frac{\Delta T}{T}\times 100\% = \frac{1}{2}\times\frac{50}{100}\times 100$$

$$\frac{\Delta T}{T}\times 100\% = 25\%$$

Example 10

A second pendulum is taken in a carriage. Find the period of oscillation when the carriage moves with an acceleration of 4.2m/s^2.

(i) Vertically upwards

(ii) Vertically downwards

(iii) In horizontal direction

Take g = 9.8m/s^2

Solution

For a second pendulum , T = 2sec

As $T = 2\pi\sqrt{\frac{L}{g}}$ so $2 = 2\pi\sqrt{\frac{L}{g}}$

Or $L = \frac{g}{\pi^2} = \frac{9.8}{\pi^2}$

(i) When the carriage moves up with a = 4.2m/s², time period is

$$T_1 = 2\pi\sqrt{\frac{L}{g+a}}$$

$$T_1 = 2\pi\sqrt{\frac{9.8/\pi^2}{9.8+4.2}}$$

$$T_1 = 1.73\,\text{sec}$$

(ii) When the carriage moves down with a = 4.2m/s².

$$T_2 = 2\pi\sqrt{\frac{L}{g-a}} = 2\pi\sqrt{\frac{9.8/\pi^2}{9.8-4.2}}$$

$$T_2 = 2.64\,\text{sec}$$

(iii) When the carriage moves horizontally with acceleration a = 4.2m/s² , then g and a perpendicular to each other the effective acceleration.

$$g' = \sqrt{g^2+a^2} = \sqrt{9.8^2+4.2^2}$$

$$g' = 10.66\text{m}/\text{s}^2$$

The time period ,

$$T_3 = 2\pi\sqrt{\frac{L}{g}} = 2\times3.14\sqrt{\frac{9.8/\pi^2}{10.66}}$$

$$T_3 = 1.92\,\text{sec}$$

Example 11

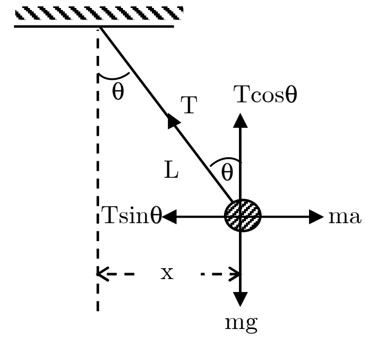
A pendulum bob weighing 0.4kg is hung from roof of a railway carriage by a 91cm string. The carriage is moving at 20m/s round a curve of radius 0.8km.

(a) Find the distance of the bob from the vertical through the point of support and the tension in the string.

(b) Find also the approximate time of a small oscillation whilst the train is moving round the curve.

Solution

(a) Assuming a train is taking a turn to the right



Acceleration of the bob or train toward the centre of the circular arc.

$$a = \frac{v^2}{r} = \frac{20\times20}{800} = 0.5\text{m}/\text{s}^2$$

At the equilibrium of the bob

$$T\sin\theta = Ma\ldots\ldots\ldots\ldots\ldots\ldots(i)$$

$$T\cos\theta = Mg\ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

Dividing equation (1) by (2)

$$\frac{T\sin\theta}{T\cos\theta} = \frac{Ma}{Mg} \Rightarrow \tan\theta = \frac{a}{g}$$

$$\theta = \tan^{-1}\left[\frac{a}{g}\right] = \tan^{-1}\left[\frac{0.5}{9.8}\right]$$

$$\theta = 2.92^\circ$$

Now $X = L\sin\theta = 91\text{cm}\times\sin2.92$

$$X = 4.64\text{cm}$$

Tension in the string

$$T = \frac{Mg}{\cos\theta} = \frac{0.45\times9.8}{\cos(2.92^\circ)}$$

$$T = 4.42\text{N}$$

(b) The bob will sweep a conical pendulum of period of oscillation

$$T = 2\pi\sqrt{\frac{L\cos\theta}{g}} = 2\times3.4\sqrt{\frac{0.91\cos2.92}{9.8}}$$

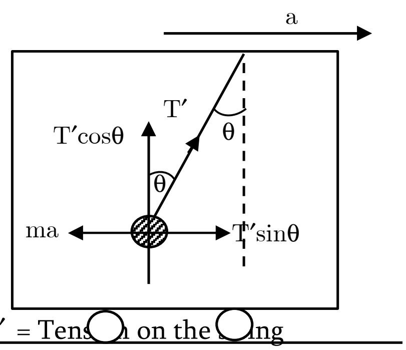
$$T = 1.91\,\text{sec}$$

Example 12

A simple pendulum is suspended from a roof of a trolley which moves horizontally with an acceleration , a . find the time period of oscillation of the pendulum and the angle which the string makes with the vertical in equilibrium position.

Solution

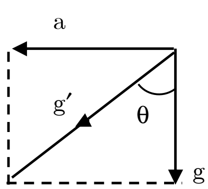
In the reference frame of the trolley , the pendulum is subjected to two accelerations, acceleration due to gravity , g downward and the inertial acceleration , a , horizontally



Let T' = Tension on the string

g' = Effectively value of acceleration due to the gravity.

By using Pythagoras theory



$$(g')^2 = a^2 + g^2$$

$$g' = \sqrt{a^2 + g^2}$$

Periodic time of oscillation

$$T = 2\pi\sqrt{\frac{L}{g'}} = 2\pi\sqrt{\frac{L}{\sqrt{a^2+g^2}}}$$

Let θ be angle between the vertical with the string. From the figure above

$$\tan\theta = \frac{a}{g}$$

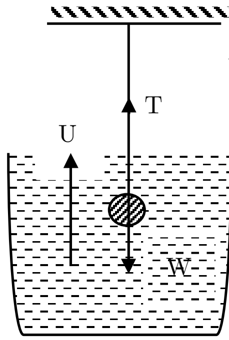
$$\theta = \tan^{-1}\left[\frac{a}{g}\right]$$

Example 13

The bob of a simple pendulum execute S.H.M in water with a period , t. The period of the bob in air is to. What is the relation between t and to? Given that density of the bob is $\frac{4000}{3}\,kgm^{-3}$ neglecting frictional force of water and density of water is $1000kgm^{-3}$.

Solution

Leg g' = effectively value of acceleration due to gravity when bob is in water. At the equilibrium of a-bob



$$\omega' = \delta vg - \rho vg$$

$$vg' = \delta vg - \delta vg$$

$$g' = g\left[1 - \frac{\rho}{\delta}\right]$$

$$g' = g\left[1 - \frac{1000\times 3}{4000}\right] = \frac{g}{4}$$

In air : $t_o = 2\pi\sqrt{\frac{L}{g}}$

In water: $t = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{L}{g/4}}$

$$t = 2t_o$$

Example 14

Determine the time period of small oscillation of a mathematical pendulum , that is a ball suspended by a thread L = 20cm in length of it is located in a liquid and density is n = 3.0times less than that of the ball. the resistance of the liquid is to be neglected take $g = 9.8m/s^2$.

Solution

Let V = Volume of the material of a bob

g' = effective value of g

Now : Apparent = weight of – up thrust of liquid bob

$$\rho vg' = v\rho g = \frac{v\rho g}{n}$$

$$g' = \left(1 - \frac{1}{n}\right)g = \left(\frac{n-1}{n}\right)g$$

Periodic time

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{nL}{g(n-1)}}$$

$$= 2\times 3.14\sqrt{\frac{3\times 20\times 10^{-2}}{9.8(3-1)}}$$

$$T = 1.10\,sec$$

Example 15

A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = kt^2$ ($k = 1m/s^2$) y is the vertical displacement. The time period now become, T_2 what is the ratio T_1^2/T_2^2 given that $g = 10m/s^2$.

Solution

In the first case:

$$T_1 = 2\pi\sqrt{\frac{L}{g}}$$

$$T_1^2 = \frac{4\pi^2 L}{g} \ldots\ldots\ldots\ldots(1)$$

In second case:

Displacement $y = kt^2$

Upward velocity , $v = \frac{dy}{dt} = 2kt$

Upward acceleration , $a = \frac{dv}{dt} = 2k$

$$a = 2\times 1 = 2m/s^2$$

Now periodic time

$$T_2 = 2\pi\sqrt{\frac{L}{g+a}}$$

$$T_2^2 = \frac{4\pi^2 L}{g+a} \ldots\ldots\ldots\ldots(2)$$

Take (1) / (2)

$$\frac{T_1^2}{T_2^2} = \frac{g+a}{g} = \frac{10+2}{10} = \frac{6}{5}$$

$$\frac{T_1^2}{T_2^2} = \frac{6}{5}$$

Example 16

A pendulum clock normally shows correct time on an extremely cold day , its length decreases by 0.2% compute the error in time per day.

Solution

The correct time period of pendulum clock is 2seconds.

Let be its correct length

$$2 = 2\pi\sqrt{\frac{L}{g}} \ldots\ldots\ldots\ldots\ldots\ldots(i)$$

Decrease in length $= 0.2\% = \frac{0.2L}{100}$

Length after contraction

$$L_1 = L - \frac{0.2L}{100} = L\left(1-\frac{0.2}{100}\right)$$

New time period

$$T = 2\pi\sqrt{\frac{L_1}{g}} = 2\pi\sqrt{\frac{L}{g}\left(1-\frac{0.2}{100}\right)} \ldots\ldots\ldots(ii)$$

Dividing equation (ii) by (i)

$$\frac{T}{2} = \left[1-\frac{0.2}{100}\right]^{\frac{1}{2}}$$

$$T = 2\left[1-\frac{0.2}{100}\right]^{\frac{1}{2}} = 2\left[1-\frac{1}{2}\times\frac{0.2}{100}+\right]$$

$$T = \left(2-\frac{0.2}{100}\right)\text{sec}$$

Which is less than 2seconds

Therefore , the clock gain time

Time gained in 2 sec $= \frac{0.2}{100}$ s

Total time gained in 1day $= 24 \times 3600$

$$2\text{ sec} \longrightarrow \frac{0.2}{100}\text{ sec}$$

$$24\times 3600\text{ sec} \longrightarrow t$$

$$t = \frac{0.2}{100}\times\frac{24\times 3600}{2}$$

$$t = 86.4\text{ sec}$$

Example 17

(a) What is the relation between uniform circular motion and S.H.M?

(b) Is motion of a simple pendulum strickly S.H.M?

(c) The motion of a simple pendulum is approximately S.H.M for small angles of oscillation , for large angles of oscillation , time period is greater than $2\pi\sqrt{\frac{L}{g}}$ why?

Solution

(a) uniform circular motion can be thought of as two S.H.M operating at night angles.

(b) It is not strickly simple harmonic because we can make assumption that $\sin\theta \simeq \theta$ which is nearly valid only if θ is very small.

(c) The restoring force tending to bring the pendulum to its mean position is mgsinθ in this arriving at the formula

$$T = 2\pi\sqrt{\frac{L}{g}}$$

take sin ≃ θ i.e restoring force F = mgθ for the large value of θ , sinθ < θ therefore , the restoring force decreases from mgθ to mgsinθ as a result the pendulum takes a longer time to complete one vibrations.

Example 18

For an oscillating simple pendulum, is the tension in the string constant throughout the oscillation? If not , when it is (a) the least (b) the greatest?

Solution

In simple pendulum , when bob is in deflection position , the tension in the string is T = mgcosθ. Since the value of θ is different at different positions , hence tension in the string is not constant throughout oscillations.

(a) At the end points , θ is maximum , the value of cosθ is least , hence the value of tension in the string is least.

(b) At the mean position , the value of θ = 0° and cos0° = 1 , so the value of tension is greatest.

Example 19

At what point in the motion of a simple pendulum is

(a) The tension in the string greatest?

(b) The tension in string least

(c) The radial acceleration greatest?

(d) The angular acceleration least?

(e) The speed greatest?

Solution

(a) At the mean position

(b) At the end point of the oscillations (extreme position)

(c) At the end point of the oscillation

(d) At the mean position

(e) At the mean position

Example 20

A girl sitting on a swing stands up what will be the effect on the periodic time of the swing?

Solution

The periodic time , T will degreasing because in the standing position the location of centre of mass of the girl shift upward due to it the effective length of the pendulum L. Decreases as $T \propto \sqrt{L}$, then T decreases.

Example 21

(a) Can pendulum clocks be used in artificial satellite explain.

(b) How will you determine the period of oscillation of mass without use of a clock? You are given a weightless spring , a metre rule and a known mass.

Solution

(a) No , in a artificial satellite , the body is in weightless state where g = 0 . the time period of pendulum watch.

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{L}{O}} = \alpha$$

It means , inside the satellite pendulum does not oscillate. Hence pendulum watch cannot work in satellite.

(b) The mass is made to oscillate , then the periodic time of oscillation

$$T = 2\pi\sqrt{\frac{M}{K}} \quad \text{But } K = \frac{Mg}{X}$$

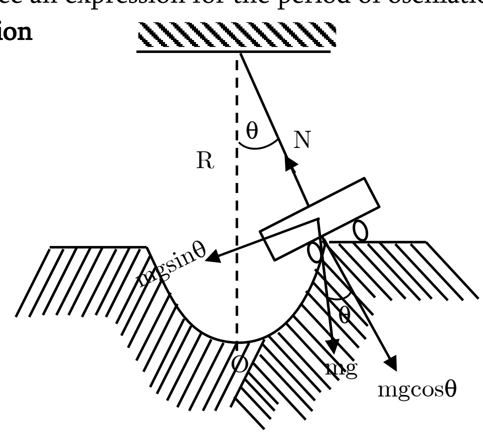
$$\frac{X}{g} = \frac{M}{K} \; , \quad T = 2\pi\sqrt{\frac{X}{g}}$$

Hence , period can be found by measuring extension X produced by mass using a metre rule.

Example 22

The bottom of a dip on a road has a radius of curvature R . A ricks show of mass , M left little away from the bottom oscillates about the dip. Deduce an expression for the period of oscillation.

Solution



When the rickshaw is away from O , the following force act on it.

(i) Weight Mg of the rickshaw acting vertically downwards through its centre of gravity.

(ii) Normal reaction offered by the road to the rickshaw magnitude of restoring force.

$F = mg\sin\theta$

If θ is very small angle measured in radian

$\sin\theta \simeq \theta$

$$K = \frac{Mg\theta}{R\theta} = \frac{Mg}{R}$$

Periodic time

$$T = 2\pi\sqrt{\frac{M}{K}} = 2\pi\sqrt{\frac{M}{Mg/R}}$$

$$T = 2\pi\sqrt{\frac{R}{g}}$$

Example 23

(a) (i) Will a pendulum clock gain or lose when taken to the top of a mountain?

(ii) A girl is swinging a swing in the sitting position what shall the effect on the frequency of oscillation if she stands up.

(b) A pendulum clock gives correct time. What is the error in time per day if the length increased released by 0.05%.

Solution

(a) (i) The time period of a simple pendulum is given by $T = 2\pi\sqrt{\frac{L}{g}}$

On top of mountain , the value of g will decreases therefore , the time period will increase hence the pendulum clock will lose the time

(ii) The system may be roughly regardes as a simple pendulum when the girl stands up the length of the simple pendulum is reduced since $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

So frequency of oscillation increased.

(b) Let T be the correct period and T_1 the time period when the length increased by 0.05%

$$L = L + \frac{0.005L}{100} = \frac{100.05L}{100}$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T_1 = 2\pi\sqrt{\frac{L_1}{g}} = 2\pi\sqrt{\frac{100.05L}{100g}}$$

Takes

$$\frac{T_1}{T} = \sqrt{\frac{100.05}{100}} = \left(1 + \frac{0.05}{100}\right)^{1/2}$$

Expand by using binomial approximation

$$\frac{T_1}{T} = 1 + \frac{1}{2}\times 0.0005$$

$$\frac{T_1 - T}{T} = 0.00025$$

Loss of time per second = 0.00025sec

Loss of time per day

$$= 0.00025\times 24\times 3600 = 21.6\,\text{sec}$$

Example 24

(a) What is the time period of oscillation of a simple pendulum if its bob is made of ice?

(b) Pendulum clock shows correct time if the length increases by 0.1% find the error in time per day.

Solution

(a) In the case of an ice pendulum the period depends on the radius of ice bob since

$$T = 2\pi\sqrt{\frac{L + r}{g}}$$

As the ice melts , radius decreases so period decreases but if the centre of mass of ice remain constant , then there is no change in the period.

(b) Correct number of seconds per day ,

$$f = 24\times 3600 = 86400$$

Let error introduced per day be X second. Incorrect number of seconds per day

$$f_1 = 86400 + X$$

L be original length

L_1 = new length

$L_1 = L + 0.001L = [1 + 0.001]\,L$

Since

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{g}{L_1}}\ ,\quad f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

Takes

$$\frac{f_1}{f} = \sqrt{\frac{L}{L_1}}$$

$$\frac{86400+X}{86400} = \sqrt{\frac{L}{L(1+0.001)}}$$

$$1+\frac{X}{86400} = [1+0.001]^{1/2}$$

$$X = \left[(1+0.001)^{1/2} - 1\right]86400$$

$$X = -43.2\,\text{sec}$$

Example 25

A pendulum clock gain 2minutes a day if the pendulum is 0.812m long by how much its length be changed to make the clock run on time.

Solution

Since

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad \left(T \propto \sqrt{L}\right)$$

At present , the clocks gain 2 minutes per day , its run fast by a proportion

$$\frac{2\,\text{min}}{24\times 60\,\text{min}} = 1.3889\times 10^{-3}$$

Let the present period of the clock be To and the new (correct) period to be T

$$T = T_o + 1.3889\times 10^{-3}T_o$$

$$\frac{T}{T_o} = 1+1.3889\times 10^{-3}$$

Since

$$T = 2\pi\sqrt{\frac{L}{g}}\ ,\quad T_o = 2\pi\sqrt{\frac{L_o}{g}}$$

$$\frac{T}{T_o} = \sqrt{\frac{L}{L_o}} \Rightarrow \frac{L}{L_o} = \left(\frac{T}{T_o}\right)$$

$$\frac{L}{L_o} = \left(1+1.3889\times 10^{-3}\right)^2$$

But

$$Lo = 0.812\text{m}$$

$$L = 0.81426\text{m}$$

$$\Delta L = L - Lo = 2.26\times 10^{-3}$$

∴ The pendulum must strengthened by 2.26mm

Example 26

A simple pendulum set up to swing in front of the second pendulum , period 2sec of clock is seen to gain the two swing in phase at interval of 21sec. what is time of swing of the simple pendulum?

Solution

At the pendulum gain oscillations to be in phase with the second pendulum , it has a period slightly less than 2sec. beat period.

$$T = \frac{1}{f_b} = \frac{1}{f_2 - f_1} = \frac{T_1T_2}{T_1 - T_2}$$

$$21 = \frac{2T_2}{2-T_2}$$

$$T_2 = 1.83\,\text{sec}$$

Example 27

A clock pendulum has a period of 2.00sec. A simple pendulum set up in front of it gains on the clock so that the two vibrate in phase at interval of 22seconds calculate :

(a) The period of the simple pendulum

(b) The fractional change in length of the simple pendulum necessary for the two pendulum to be equal?

Solution

(a) Let T_1 = period of clock pendulum

T_2 = period of simple pendulum

Since 1 vibration $\longrightarrow$ T

2 vibrations $\longrightarrow$ 22sec

$$T = \frac{22\,\text{sec} \times 1\text{vibration}}{2\text{vibrations}}$$

$$T = 11\,\text{sec}$$

T_b = period of a beat

$$T_b = \frac{1}{f_b} = \frac{1}{f_2 - f_1} = \frac{T_1T_2}{T_1 - T_2}$$

$$22 = \frac{2T_2}{2-T_2}$$

On solving T_2 = 1.833sec

(b) $$\frac{\Delta L}{L}\times 100\% = 19\%$$

(Left to the student assignment)

EXERCISE 5.3

1. (a) A simple pendulum has a period of 4.2sec when the pendulum length is shortness by 1m , the period is 3.7sec from these measurements , calculate the acceleration due to gravity and original length of the pendulum.
   (b) If the pendulum is taken from the Earth to the moon where the acceleration of free fall is g/6 , what relative change , if any occurs in the period?
   [Ans. (a) 4.47m , 9.99m/s^2 (b) 2.45.1]

2. The bob of a simple pendulum is suspended by a long string from an inaccessible point and time period of the pendulum is found to be 4.5sec on shortening the pendulum length by 1m , the time period becomes 4.03sec. Calculate the value of the acceleration due to gravity at at the place concerned.
   [Ans. G = 9.845m/s^2]

3. (a) What is the length of a simple pendulum whose time period of oscillation for small amplitude is equal to 2.0second.
   (b) If this pendulum is in lift which accelerates upward at 2.8m/s^2 , by what factor does it period of oscillation change from the original value? (g = 9.8m/s^2)
   [ans. (a) 0.993m (b) 0.88]

4. The length of a simple pendulum execute S.H.M is increased 21%. What is the percentage increase in the time period of the pendulum? [ans 10%]

5. (a) If a second pendulum be lengthened by $\left(\frac{1}{100}\right)^{th}$ of it length , how many seconds will it lose in a day?
   (b) A second pendulum gains 10sec in a day in one place compare the values of g in the two places [ans. (a) 432sec (b) 1.005]

6. Two pendulums of length 90cm and 100cm start oscillating in phase how many oscillation will they be again in the same phase?
   Ans 18.5 (longer length pendulum)
   19.5 (shorter length pendulum)

7. Two simple pendulum of length 0.4m and 0.6m are set off oscillation in step calculate:-
   (a) After what further time the two pendulums will one gain be in step.
   (b) The number of oscillations made by each pendulums at this time.
   Ans 5.46 (shorter pendulum)
   4.46 (longer pendulum)

8. Simple pendulum has time period of 2sec in air the whole arrangement place in a non – viscous liquid whose density is ½ times the density of bob , find the time period of oscillation in liquid. [ans $2\sqrt{2}$ sec]

9. A seconds pendulum is taken from a place where g = 9.8ms/s^2 to a place g = 9.7m/s^2. How would its length be changed in order that its time period remain un affected?
   [ans. Shortened by 0.0101m]

10. The period of a simple pendulum of length L is T_1 and time period of a uniform rod of the same length L pivoted about one end and oscillating in a vertical plane is T_2. Amplitude of oscillations in both cases is small. Find T_1/T_2
    (Ans. $\sqrt{\frac{3}{2}}$)

11. Gravity at poles exceeds gravity at the equator in the ratio of 301:300. A pendulum regulated for poles is taken to the equator. Calculate how many seconds a day it will gain or lose?
    [ans 14sec (loss)]

12. A faulty second's pendulum loses 5sec in a day by how much its length be decreased to keep correct time? [ans.0.011cm]

13. If a pendulum clock is taken to a mountain top , does it gain or loss time , assuming it is correct at a lower elevation? Explain your answer.

4. OSCILLATION OF LIQUID IN A U – TUBE

Consider a liquid of density ρ in a u – tube at equilibrium as shown in figure (i) below

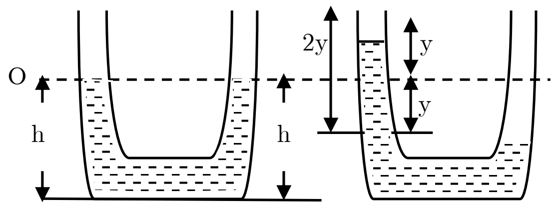


Figure (i) Figure (ii)

If the liquid on one side of the u – tube is depressed by blowing gently down as shown in figure (ii) and released , the liquid will oscillate for a short time about initial position , O before finally comes into rest. Thus oscillation of liquid is a u – tube is in S.H.M. When liquid is depressed by a distance y in one limb it rises by the same amount in the other limb.

Let A = uniform cross- sectional area of the tube. The weight of the liquid column of height 2y provide the restoring force.

Restoring force

F = - 2ρAgy but F = Ma

Ma = - 2ρAgy

$$a = -\left(\frac{2\rho Ag}{m}\right)y$$

Mass of the liquid , M = 2ρAg

$$a = -\left(\frac{2\rho Ag}{2\rho Ah}\right)y = -\left(\frac{g}{h}\right)y$$

a α - y

its execute S.H.M

For the linear S.H.M

$$a = -\omega^2 y$$

$$-\omega^2 y = -\left(\frac{g}{h}\right)y$$

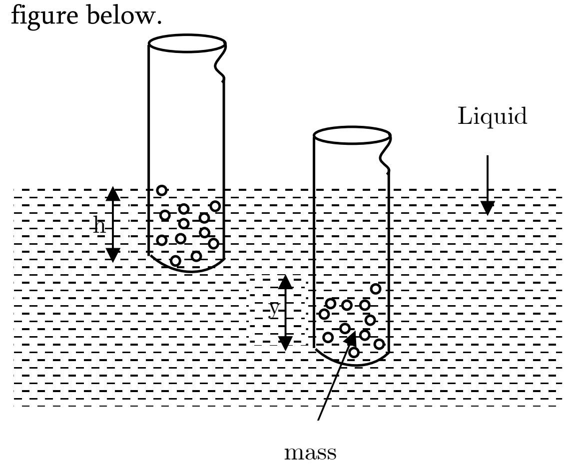
$$\omega = \sqrt{\frac{g}{h}}$$

Periodic time of the oscillation

$$T = 2\pi\sqrt{\frac{h}{g}}$$

5. FLOATING LOADED TEST TUBE (CYLINDER)

Consider a loaded test tube of total mass M floating in a liquid of density ρ as shown in the figure below.



When a loaded test tube is placed in the liquid , it is submerged a distance h below water level when the tube is slightly pushed down and released will oscillate up and down. The restoring force is equal to the excess up thrust due to height y restoring force.

$$F = -(\rho Ag)y \text{ but}$$

$$F = Ma$$

$$Ma = -(\rho Ag)y$$

$$a = -\left(\frac{\rho Ag}{M}\right)y$$

$$a \propto -y$$

For S.H.M $a = -\omega^2 y = -\dfrac{\rho Agy}{M}$

$$\omega = \sqrt{\frac{\rho Ag}{M}}$$

Periodic time

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{M}{\rho Ag}}$$

Mas of cylinder M = Ahδ

δ = density of cylinder

$$T = 2\pi\sqrt{\frac{h\delta}{\rho g}}$$

Note that

Another relation of periodic time at equilibrium of a cylinder (test tube)

Weight of test tube = weight of fluid displaced

$$Mg = \rho Ahg$$
$$M = \rho Ah$$

Since

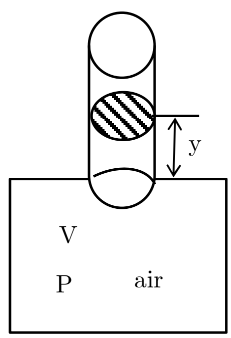
$$T = 2\pi\sqrt{\frac{M}{\rho Ag}} = 2\pi\sqrt{\frac{\rho Ah}{\rho Ag}}$$

$$T = 2\pi\sqrt{\frac{h}{g}}$$

6. OSCILLATIONS OF A BALL IN THE NECK OF AN AIR CHAMBER.

An air chamber of volume V has a neck of an area of cross – section , A into which a ball of mass M can move without friction we can show that when the ball is pressed down some distance and released the ball execute S.H.M let us to find the time period assuming that the pressure – volume variations of the air to be

(a) Isothermal expansion
(b) Adiabatic expansion.



Suppose the ball is depressed by a distance y then change in pressure dp produces restoring force , F.

PV = Constant

Pdv + Vdp = 0

$$dp = -p\frac{dv}{v} = -\frac{pAy}{V}$$

Force

$$F = Adp = -\frac{PA^2y}{V}$$

$$Ma = -\left[\frac{PA^2}{V}\right]y$$

$$a = -\left[\frac{PA^2}{MV}\right]y \quad , \quad a \propto -y$$

Its execute S.H.M

$$\omega^2 = \frac{PA^2}{MV} \quad , \quad \omega = \sqrt{\frac{PA^2}{MV}}$$

Periodic time of oscillation

$$T = \frac{2\pi}{A}\cdot\sqrt{\frac{MV}{P}}$$

(b) For adiabatic expansion

$$PV^{\gamma} = \text{constant}$$

On differentiating

$$\gamma PV^{\gamma-1} + V^{\gamma}\frac{dp}{dv} = 0$$

$$dp = -\gamma\frac{pdv}{v} = -\gamma\frac{PAy}{v}$$

Now

$$F = Adp = -\frac{\gamma PA^2y}{V}$$

$$Ma = -\frac{\gamma PA^2y}{V}$$

$$a = -\left[\frac{\gamma PA^2y}{MV}\right]y \quad , \quad a \propto -y$$

Periodic time

$$T = \frac{2\pi}{A}\sqrt{\frac{MV}{\gamma P}}$$

7. PISTON IN A GAS – FILLED CYLINDER

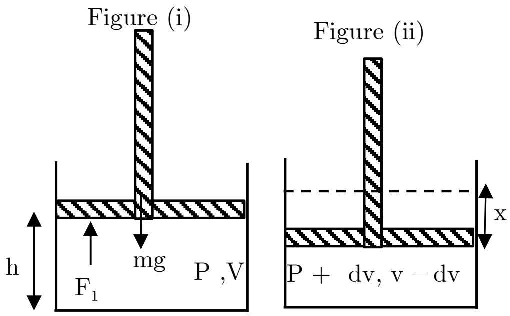
A volume V of air pressure P is contained in a cylinder of cross – sectional area , A by a frictionless light piston of mass M to show that on slightly forcing the piston and the releasing it the piston oscillates with S.H.M of period T is given by

$$T = \frac{2\pi}{A}\sqrt{\frac{MV}{P}}$$

Assumptions made

1. The atmospheric pressure is ignored
2. All dissipative effect are negligible

   Consider the oscillation of the piston as shown in the figure below



At the equilibrium piston (mean position)

$F_1 = PA = Mg$

When piston is displaced downward , the value of air is decreases to V – dV as shown in the figure (ii)

Apply Boyle's law ,

PV = Constant

PV = (P + dP) (V – dV)

PV = PV – PdV + VdP – dPdV

0 = -PdV + VdP [dPdV → 0]

$$PdV = VdP \ , \ dP = \frac{PdV}{V} = \frac{PAX}{V}$$

Restoring force

$$F = -AdP = -\frac{PA^2X}{V}$$

$$Ma = -\frac{PA^2X}{V} \ , \ a = -\left[\frac{PA^2}{MV}\right]X$$

$a \propto -x$

For S.H.M , $a = -\omega^2 x = -\frac{PA^2X}{MV}$

Periodic time of oscillation

$$T = \frac{2\pi}{A}\sqrt{\frac{MV}{P}} \quad \text{or}$$

$$T = 2\pi\sqrt{\frac{MAh}{PA^2}} = 2\pi\sqrt{\frac{Mh}{PA}}$$

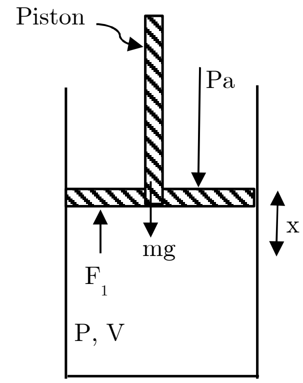
$$= 2\pi\sqrt{\frac{Mh}{Mg}} = 2\pi\sqrt{\frac{h}{g}}$$

$$T = \frac{2\pi}{A}\cdot\sqrt{\frac{MV}{P}} = 2\pi\sqrt{\frac{h}{g}}$$

SPECIAL CASE

If the effect of atmospheric is taken into account.

Let H = Pa = Atmospheric pressure



At the equilibrium of the piston

Mg + PaA = PA

When the piton is lightly displaced by distance X.

Restoring force F = - Adp

Apply Boyle's law PV = constant

On differentiating

$$P + V\frac{dp}{dv} = 0 \ , \ dp = \frac{-PAX}{V}$$

Since

$$F = -M\omega^2 x$$

$$-M\omega^2 y = \frac{-PA^2x}{V}$$

$$M\omega^2 = \frac{(PA)A}{V} = \frac{(Mg + P_aA)A}{Ah}$$

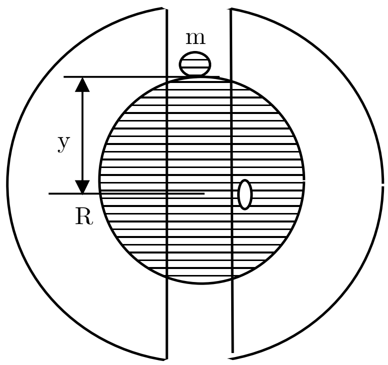
$$\omega = \sqrt{\frac{(Mg + P_aA)}{Mh}}$$

Periodic time , $T = 2\pi\sqrt{\frac{Mh}{P_aA + Mg}}$

8. MOTION OF A BODY DROPPED IN A TUNNEL ALONG THE DIAMETER OF THE EARTH

Suppose the Earth to be a sphere of radius R with centre O.

Let a tunnel be dug along the diameter of the Earth as shown on the figure below. If a body of mass M is dropped at one end of the tunnel the body execute S.H.M about the centre O of the Earth.



Suppose at any instant , the body in the tunnel is at a distance ,y from the centre , O of the Earth , only the inner sphere of radius y will exert gravitational force , F on the body.

Restoring force = gravitational force

$$F = \frac{-GMm}{y^2} \quad \text{but} \quad F = Ma$$

$$ma = -\frac{G}{y^2}\left(\frac{4}{3}\pi y^3 \rho\right)m$$

$$a = -\left(\frac{4}{3}\pi G\rho\right)y \quad , \quad a \propto -y$$

This shows that the body execute S.H.M for S.H.M

$$a = -\omega^2 y = -\left(\frac{4}{3}\pi G\rho\right)y$$

$$\omega = \sqrt{\frac{4}{3}\pi G\rho} \quad \text{but} \quad T = \frac{2\pi}{\omega}$$

$$T = 2\pi\sqrt{\frac{4}{3}\pi G\rho} = \sqrt{\frac{3\pi}{G\rho}}$$

Also

$$\rho = \frac{3g}{4\pi RG} \quad , \quad \rho G = \frac{3g}{4\pi R}$$

$$T = \sqrt{3\pi \times \frac{4\pi R}{3g}} = 2\pi\sqrt{\frac{R}{g}}$$

$$T = \sqrt{\frac{3\pi}{G\rho}} = 2\pi\sqrt{\frac{R}{g}}$$

R = Earth radius
ρ = density of the earth
G = universal gravitational constant

Since

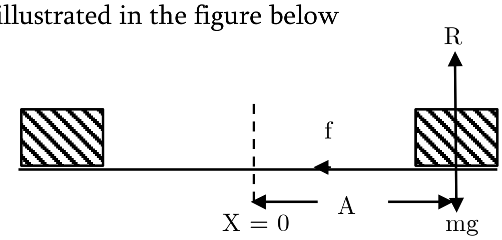
$$R = 6.4\times10^6\,\text{m} \quad , \quad g = 9.8\,\text{m}/\text{s}^2$$

$$T = 2\pi\sqrt{\frac{6.4\times10^6}{9.8}} = 5078\,\text{sec}$$

$$T = 5078\text{seconds}$$

9. S.H.M IN A HORIZONTAL MOTION

Consider a body of mass M in a rough horizontal surface and its execute S.H.M as illustrated in the figure below



Restoring force = frictional force

$$Ma = -\mu mg \Rightarrow a = -\mu g$$

For S.H.M

$$a = -\omega^2 A$$

$$-\omega^2 A = -\mu g \quad , \quad \omega = \sqrt{\frac{\mu g}{A}}$$

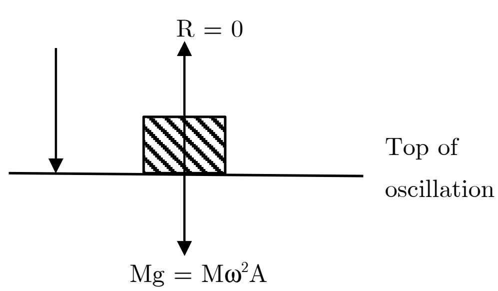
Periodic time of oscillation

$$T = 2\pi\sqrt{\frac{A}{\mu g}}$$

A = amplitude of oscillation
μ = coefficient of frictional force
g = acceleration due to gravity.

10. S.H.M IN A VERTICAL MOTION (S.H.M and g)

If a small coin is placed on a horizontal plat form connected to a vibrator and the amplitude , A is kept constant as the frequency is increased from zero , the coin is increased from zero ,the coin will be heard 'chattering' at a particular frequency of this stage the reaction of the table with the coin becomes zero at some part of every cycle , so that it losses contact periodically with surface.



The maximum acceleration in S.H.M occurs at the end of the oscillation because a α - y

The frequency of oscillation , fo

$$M\omega^2 A = Mg \Rightarrow \omega = \sqrt{g/A}$$

But

$$\omega = 2\pi fo$$

$$fo = \frac{1}{2\pi}\sqrt{\frac{g}{A}}$$

The coin will loses contact with the table when it is moving down with acceleration , g i.e this occurs at the top of oscillation.

SOLVED EXAMPLE S

Example 1

A vertical U – tube of uniform cross – sectional area contain water up to a height to 0.8m. Show that if the water in the tube is S.H.M calculate periodic time , T , take g = 9.8m/s².

Solution

Case 1: see your notes

Case 2: periodic time , T

$$T = 2\pi\sqrt{\frac{h}{g}} = 2\times 3.14\sqrt{\frac{0.8}{9.8}}$$

$$T = 0.51\,\text{sec}$$

Example 2

A mercury column of mass , M oscillates in a U – tube one centimeter of the mercury column weighs 15g . Calculate

(a) Spring constant of motion and

(b) Period of oscillation

Solution

Let the liquid in the right arm be depressed through a distance y metre. Then the liquid in the left arm will rise through a distance , y metre.

Restoring force

(a) T = - weight of unbalanced column of length 2y metre

$$F = \frac{-15\times 10^{-3}\times 9.8}{1\times 10^{-2}}\times 2y$$

$$F = -29.4y \quad \text{but} \quad F = -ky$$

$$-ky = -2.9.4y$$

$$k = 29.4\text{Nm}^{-1}$$

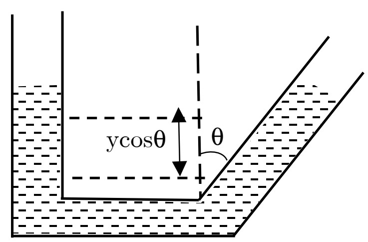
(b) Periodic time

$$T = 2\pi\sqrt{\frac{M}{K}} = 2\pi\sqrt{\frac{M}{29.4}}$$

$$T = 1.159\sqrt{M}\,\text{sec}$$

Example 3

Determine the period of oscillation of mercury of mass M = 200g poured into a bent tube see the figure below whose right arm forms an angle θ = 30° with the vertical. The cross – sectional area of the tube is s = 0.50cm². The viscosity of mercury is to be neglected.



Solution

Let the level of mercury be depressed in the vertical arm of the tube through a distance y. The vertical rise of mercury in the slant portion of the tube = ycosθ

Vertical difference in the length liquid level in two arms h = y + ycosθ = y (1 + cosθ)

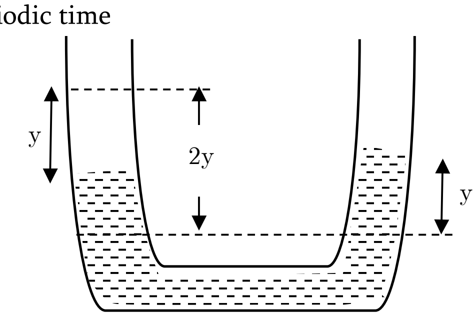
restoring force = -weight of mercury column of length h

$$F = -(1+\cos\theta)y\rho gs$$

$$-ky = -(1+\cos\theta)\rho gsy$$

$$K = (1+\cos\theta)\rho gs$$

Periodic time



$$T = 2\pi\sqrt{\frac{M}{K}} = 2\pi\sqrt{\frac{M}{(1+\cos\theta)\rho gs}}$$

$$= 2\times 3.14\sqrt{\frac{0.2}{\left(1+\cos 30^\circ\right)\times 13600\times 9.8\times 0.5\times 10^{-4}}}$$

$$T = 0.8\,\text{sec}$$

Example 4

A cubical body (side 0.1m and mass 0.002kg) floats in water it is pressed and then released so that it executes S.H.M. Find the time period (g = 10m/s²)

Solution

Let the cubical body floating in water of density ρ with length L inside of the water.

Weight of body = upward thrust due to water

$Mg = AL\rho g$ or $M = AL\rho$

If the cubical body is depressed through distance y , then effective restoring force on the body.

$$F = -\left[A\left(L+y\right)\rho g - AL\rho g\right] = -\left(A\rho g\right)y$$

Since $F = -M\omega^2 y$

$$-M\omega^2 y = -\left(\rho A g\right)y$$

$$\omega = \sqrt{\frac{\rho A g}{M}}\ ,\quad T = 2\pi\sqrt{\frac{M}{\rho A g}}$$

$$T = 2\times\frac{22}{7}\sqrt{\frac{0.002}{\left(0.1\right)^2\times 1000\times 10}}$$

Example 5

Image a tunnel is dug along diameter of the Earth. Show that a body dropped from end of tunnel execute S.H.M. What is the time period of this motion. Assume the Earth to be a sphere of uniform mass density = 5520kgm⁻³ and G = 6.67 × 10⁻¹¹Nm2kg⁻².

Solution

$$T = \sqrt{\frac{3.14\times 3}{6.67\times 10^{-11}\times 5520}}$$

$$T = 5059\,\text{sec}$$

Example 6

A test tube of mass 12g and external diameter 3cm is loaded with 6gm of lead shot so it will float vertically in water. If the test tube is pushed down slightly and then released oscillates vertically with S.H.M

(a) Find the period , T

(b) What would the period be if the test tube were floating in alcohol of density 0.8g/cm³.

Solution

(a) Period time

$$T = 2\pi\sqrt{\frac{M}{\rho A g}}$$

$$= 2\pi\sqrt{\frac{\left(12+6\right)\times 10^{-3}}{A\times 1000\times 9.8}}$$

But $A = \dfrac{\pi d^2}{4} = \dfrac{\pi\left(0.33\right)^2}{4}$

$$T = 0.32\,\text{sec}$$

(b) Since $T \alpha \dfrac{1}{\sqrt{\rho}}$

$$T_2 = T_1\cdot\sqrt{\frac{\rho_1}{\rho_2}} = 0.32\sqrt{\frac{100}{800}}$$

$$T_2 = 0.36\,\text{sec}$$

Example 7

A hydrometer floats upright in a liquid of specific gravity 0.82. The hydrometer sinks in the liquid to a depth of 20cm. what is the period of vertical vibrations of hydrometer when it is depressed slightly and released?

Solution

Periodic time

$$T = 2\pi\sqrt{\frac{h}{g}} = 2\times 3.14\sqrt{\frac{0.2}{9.8}}$$

$$T = 0.90\,\text{sec}$$

Example 8

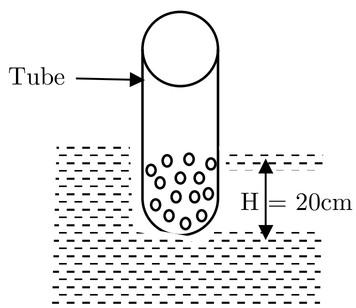
(a) A restoring force is a must for a body to execute S.H.M , explain why?

(b) A weighted glass tube is floated in a liquid with 20cm of its length immersed. It is pushed down through a certain distance and then released, compute the time period of its vibration

Solution

(a) For a body to execute S.H.M it should oscillate about its mean position. When the body is at mean position , it possesses kinetic energy and by virtue of it, the body moves from the mean position to extreme position. The body can return to the mean position only , if it is acted upon by a restoring force.

(b) Figure



$$T = 2\pi\sqrt{\frac{h}{g}}$$

$$= 2\times 3.14\sqrt{\frac{0.2}{9.8}}$$

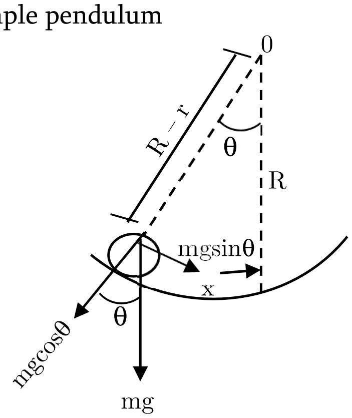
$$T = 0.897\,\text{sec}$$

Example 9

A ball of radius r is made to oscillate in a bowl of radius R , find its time period of oscillation

Solution

The source of restoring force is the gravity as in simple pendulum



Restoring force F = -mgsinθ = -mgθ

Assume that θ → 0°

$$\text{Sin}\theta \simeq \theta = \frac{x}{R-r}$$

$$F = -\frac{Mg.x}{R-r} = -\left[\frac{Mg}{R-r}\right]x$$

$$F\ \alpha -x$$

For S.H.M , $F = -M\omega^2 x$

$$-M\omega^2 x = \frac{-Mgx}{R-r}$$

$$\omega = \sqrt{\frac{g}{R-r}}$$

Periodic time

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R-r}{g}}$$

Example 10

A piston in a car engine performs S.H.M the piston has mass of 0.50kg its amplitude of vibration is 45mm the revolution counter in the car reads 750revolution per minutes. Calculate the maximum force on the piton.

Solution

Assume that gas expand isothermally in the gas filled cylinder.

The maximum force

$$F_{max} = M\omega^2 A$$

$$1\text{rev/s} \longrightarrow 2\pi$$

$$\frac{750\text{rev/s}}{60} \longrightarrow ?$$

$$\omega = 78.53\text{rad / s}$$

$$F_{max} = 0.5\times 45\times 10^{-3}\times(78.53)^2$$

$$F_{max} = 139\text{N}$$

Example 11

A certain engine a piston undergoes a vertical S.H.M with amplitude 7cm a washer rests on top of the piston as the motor is slowly speed up , at what frequency will the washer no longer , stay in contact with the platform?

Solution

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{A}} = \frac{1}{2\times 3.14}\sqrt{\frac{9.8}{0.07}}$$

$$f = 1.88\text{Hz}$$

Example 12

A small body rests on a horizontal diaphram which is making vertical S.H.M vibrations through a distance of 0.4cm. What is the maximum number of vibrations that can be made per seconds. If the body is not be thrown of the diaphram.

Solution

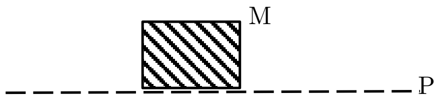
$$\text{Amplitude , } A = \frac{1}{2}\times 0.4\text{cm} = 0.2\text{cm}$$

Frequency of vibration

$$f = \frac{1}{2\pi}\cdot\sqrt{\frac{g}{A}} = \frac{1}{2\times 3.14}\sqrt{\frac{9.8}{0.002}}$$

$f = 11.4Hz$

A small mass , M on a platform P vibrating vertically has an amplitude of 0.05m at what frequency does the reaction of the platform becomes zero at any instant of the cycle and at what point in the cycle and at what point in the cycle does it happen?



Solution

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{A}} = \frac{1}{2\times 3.14}\sqrt{\frac{9.8}{0.05}}$$

$f = 2.23Hz$

The reaction of the platform becomes zero at the top of oscillation.

Example 13

A mass of 1.8kg is placed on a rough horizontal platform which is set into oscillation so it performs S.H.M of amplitude 4mm in the horizontal direction. If the coefficient of static friction is 0.4. Calculate

(a) The frequency which can be obtained just before it begins to slide.

(b) Its corresponding energy of oscillations.

Solution

(a) $$f = \frac{1}{2\pi}\sqrt{\frac{\mu g}{A}} = \frac{1}{2\times 3.14}\sqrt{\frac{0.4\times 9.8}{4\times 10^{-3}}}$$

$f = 4.98Hz$

(b) $$E = \frac{1}{2}M\omega^2A^2$$

$$= \frac{1}{2}\times 1.8\left(2\pi\times 4.98\right)^2\times\left(0.004\right)^2$$

$E = 0.014J$

Example 14

A tray is moving horizontally back and forth in S.H.M at a frequency of 2Hz on this tray is an empty cup obtain the coefficient of static friction between the tray and the cup, given that the cup begins slipping when the amplitude of motion is 5cm.

Solution

Since $f = \frac{1}{2\pi}\sqrt{\frac{\mu g}{A}}$

$$\mu = \frac{\left(2\pi f\right)^2 A}{g} = \frac{\left(2\pi\times 2\right)^2\times 0.005}{9.8}$$

$\mu = 0.8$

Example 15

(a) Define simple harmonic motion and state at what points in such motion a particle has its

(i) Maximum acceleration

(ii) Maximum velocity

Write down expression for the quantities in terms of the amplitude A and the frequency f , of the motion.

(b) Give three examples of S.H.M each from a different branch of physics and in each case state the physical quantities on which the periodic of oscillation depends.

(c) A mass of 100kg is placed in a rough horizontal platform which is then set into vibration of that it performs a S.H.M of amplitude 0.5cm in a horizontal direction. The coefficient of static friction between the surface in contact 0.40 find

(i) The maximum frequency with the amplitude which can be attained before the mass begins to slide.

(ii) The energy of the vibrating mass at this frequency.

Solution

(a) See your notes

(i) At the extremete points of oscillation

$$a_{max} = \pm\left(2\pi f_o\right)^2 A$$

(ii) At the equilibrium positions

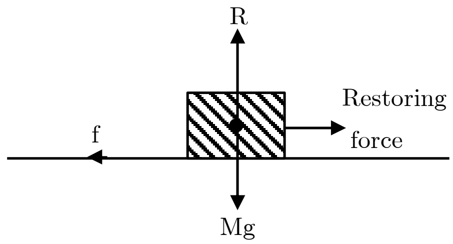
$$V_{max} = \left(2\pi f_o\right)A$$

(b) In mechanics

Simple pendulum its period depends on the length and value of acceleration due to gravity

- In the wages , the period depends on the frequency of the source.
- In an alternating current the period depends on the frequency due to the generator.

(c) (i) Diagram



In magnitude

$$\mu mg = m\omega^2 A$$

$$\mu g = \omega^2 A \quad , \quad f_o = \frac{1}{2\pi}\sqrt{\frac{\mu g}{A}}$$

$$f_o = \frac{1}{2\times 3.14}\sqrt{\frac{0.4\times 9.8}{0.5\times 10^{-2}}}$$

$$f_o = 4.46\text{Hz}$$

(ii) Energy , $E = \frac{1}{2}M\left(2\pi f_o\right)^2 A^2$

$$E = 9.81\times 10^{-3}\text{J}$$

Example 16

A tube T of uniform cross – section area $2 \times 10^{-4}m^2$ is loaded at the bottom so that its total mass 0.04kg. The tube is then placed vertical in a liquid L of density $800kgm^{-3}$ so that it floats up right .

(a) To what depth h will the tube floats?

(b) If the tube is pushed down a small distance and then released , calculate the period of oscillation.

Solution

(a) By the law of floatation

Weight of tube T = weight of the liquid displaced.

$$Mg = \rho Agh$$

$$h = \frac{M}{\rho A} = \frac{0.04}{800\times 2\times 10^{-4}}$$

$$h = 0.25\text{m}$$

(b) Periodic time

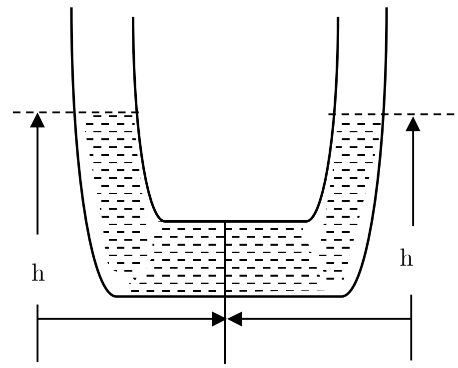
$$T = 2\pi\sqrt{\frac{h}{g}} = 2\times 3.14\sqrt{\frac{0.25}{9.8}}$$

$$T = 1.00\,\text{sec}$$

Example 17

$20cm^3$ of water poured into a glass u – tube of internal radius 7mm when the column of water in the u – tube is displaced it oscillates with the S.H.M . find the period of oscillation

Solution



The length of the tube filled with water

$$L = \frac{V}{A} = \frac{20}{\pi\left(0.7\right)^2}$$

$$L = 12.99\text{cm}$$

$L \simeq 13cm$ (approx.)

Since $2h = L$

$$h = \frac{L}{2} = 6.5\text{cm}$$

Periodic time of oscillations

$$T = 2\pi\sqrt{\frac{h}{g}} = 2\times 3.14\sqrt{\frac{65\times 10^{-2}}{9.8}}$$

$$T = 0.51\,\text{sec}$$

EXERCISE 5.4

1. Thin of several examples in everyday life of motion that at least approximately, simultaneous harmonic motion. In what respect does each derivate from S.H.M.

2. A test tube is loaded with lead short that it floats vertically in a liquid. show that after being further immersed in the liquid , the vertical oscillations of the tube that result after moving the immersing force are simple harmonic motion. The periodic time for vertical oscillations of such a tube when immersed in water is found to be 1.20se and then immersed in paraffin is 1.34sec. What is the relative density of paraffin? [Ans. 0.8]

3. An ideal gas is enclosed in a vertical cylindrical container and supported a freely moving piston of mass , M the piston and cylinder have equal cross – sectional area , A . atmosphere pressure is Po and when the piston is in equilibrium , the volume of the gas is Vo the piston is now displaced slightly from the equilibrium position. Assuming that the system is completely isolated from the surrounding , show that the piston execute S.H.M and find the frequency of oscillation. In an adiabatic process , poisons equation.
PV^{γ} = Constant his obeyed
$$\left[\text{Ans.f} \;=\; \frac{1}{2\pi}\sqrt{\gamma\left[Po + \frac{mg}{A}\right]\frac{A^2}{VoM}}\right]$$

4. Cylindrical vessel with an area of cross – sectional area , A contains a volume V of gas at a pressure P which is just sufficient to support a piston of mass , M which slides freely in the cylinder. The piston is a given a slight displacement and subsequently released. Show that its motion will take be simple harmonic and obtain an expression for the periodic time of this motion if the pressure volume relation of the gas during the accompanying changes is given by the expression PV^{γ} = Constant . Find the periodic time of the motion if M = 0.1kg , A = $10^{-3}m^2$, V = $10^{-3}m^3$, γ = 1.4 , and the external pressure is equivalent to 0.76m of mercury take g = $9.8m/s^2$, density of the mercury = $13600kgm^{-3}$
$$\left[\text{Ans. } T \;=\; 2\pi\sqrt{\frac{MV}{\gamma PA^2}}\right]$$

5. A mass attached by a light spring to the ceiling of an elevator oscillates vertically while the elevator ascends with constant acceleration is the period greater than less than or the same as when the elevator is at rest? Why?

6. (a) (i) What is the criterior for an object to execute S.H.M?

(ii) A body execute S.H.M is associated with acceleration , force acting on it , its velocity and its acceleration which of these three physical quantities and in phase.

(b) (i) Suppose a tunnel is dung through the earth from one side to the other along a diameter. Show that the motion of a particle dropped into the tunnel is S.H.M you may assume that the density of the Earth is uniform.

(ii) A simple pendulum has a period of 1second in a city A where the acceleration due to gravity is $9.66m/s^2$. It is taken to city B where it is found loose 20seconds per day calculate the value of acceleration due to gravity in city B.

7. Imagine a tunnel is dug along a diameter of Earth show that a body dropped from one end of the tunnel execute S.H.M with period of the motion is given by
$$T \;=\; 2\pi\sqrt{\frac{R}{g}}$$
Hence , determine the period given that radius of the Earth R = 6400km and g = $9.8m/s^2$] [ans.5078sec]

8. A vertical U – tube of uniform cross – section contain water up to a height of 0.3m. show that if water on one is depressed and then released its motion up and down the two sides of the tube is S.H.M. Calculate its period [ans. 1.1sec]

9. A test tube weighing 10gm and external diameter 2.5cm is floated vertically in water by placing 20gm of mercury at its bottom. The tube is depressed in water a little and then released. Find the period of oscillation. Take g = $10m/s^2$ [Ans. 0.49sec]

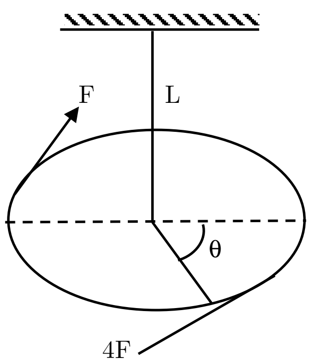
10. A cylindrical woodest block of cross – section 15cm2 and 250gm is floated over water with an extra weight 30gm attached to its bottom. the

cylinder floats vertically from the state of equilibrium , it is slightly depressed and released. If the specific gravity of wood is 0.3 and g = 9.8m/s², deduce the frequency of oscillation of the block. [Ans 1.15Hz]

EXAMPLES OF ANGULAR S.H.M

1. TORSIONAL PENDULUM

Is an object of moment of inertial I which is hung from the end of a torsional wire of length L and radius r and modulus of rigidity (shear modulus) G or n as shown below. When the disc is twisted through a small angle θ , the restoring torque is proportional to the angular displacement



$t\alpha - \theta$

$t = -c\theta$

C = torsional constant

$$I = I\frac{d^2\theta}{dt^2}$$

$$I\frac{d^2\theta}{dt^2} = -c\theta$$

$$\frac{d^2\theta}{dt^2} = -\left(\frac{c}{I}\right)\theta \text{ ....(i)}$$

This shows that object execute angular S.H.M

For angular S.H.M

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta \text{ ....(ii)}$$

(i) = (ii)

$$-\omega^2\theta = -\left(c/I\right)\theta$$

$$\omega^2 = c/I$$

Periodic time of torsional pendulum

$$T = 2\pi\sqrt{\frac{I}{C}}$$

I = Moment of inertial of a disc

C = Torsional constant

DEFINITION

TORSIONAL CONSTANT

Is defined as restoring torque per unit angular displacement

$$C = \frac{\tau}{\theta}$$

SI unit of torsional constant is $Nmdeg^{-1}$ or $Nmrad^{-1}$

Note that

It can be shown that the couple K required to twist a cylindrical wire through an angle θ is given by

$$K = \tau = \frac{\pi na^3\theta}{2L}$$

The torsional constant for solid cylindrical wire of radius a and length , L (The couple required to cause a twist of one radian) is given by

$$C = \frac{\pi na^3}{2L}$$

Periodic time of torsional pendulum now becomes in depressed and then released its motion up and down the two sides of the tube is S.H.M (calculate its period) [Ans.1.1sec]

ANGULAR SIMPLES HARMONIC MOTION

The angular simple harmonic motion of a particle may be defined as the vibratory rotational motion in which its angular acceleration is directly proportional to the angular displacement and is always displacement and is always directed towards the mean position.

i.e. $\frac{d^2\theta}{dt^2}\alpha - \theta$ or $\frac{d^2\theta}{dt^2} = -\omega^2\theta$

periodic time of the angular S.H.M

$$T = 2\pi\sqrt{\frac{\theta}{\alpha}} = 2\pi\sqrt{\frac{\text{Angular displacement}}{\text{Angular acceleration}}}$$

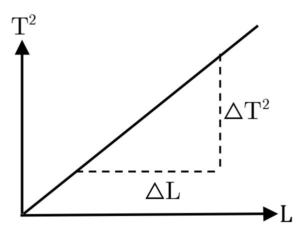
1. EXPERIMENT FOR THE DETERMINATION OF MODULUS OF RIGIDITY OF THE WIRE

A heavy body is known moment of initial I (such as disc or rod) about the axes of rotation is attached to the lower end of a wire and set oscillation about a vertical axis as a shown on the torsional pendulum periodic time

$$T = 2\pi\sqrt{\frac{2IL}{\pi na^3}}$$

$$T^2 = \left(\frac{8\pi I}{na^3}\right)L + 0$$

$$y = mx + c$$

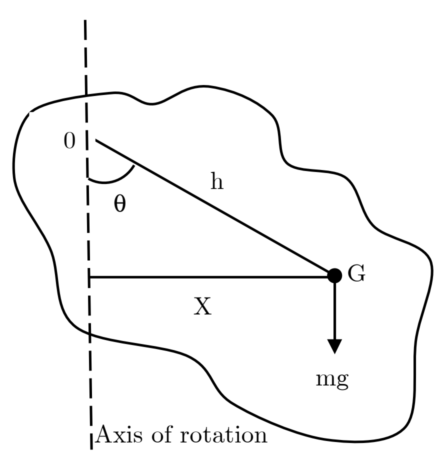


$$\text{Slope} = \frac{8\pi I}{na^3}$$

$$n = \frac{8\pi I}{a^3 \times \text{slope}}$$

2. THE COMPOUND (PHYISCAL) PENDULUM

Any rigid body mounted so that it can swing in a vertical plane about a horizontal axis passing through it is called physical pendulum. Consider a rigid body of mass , M is suspended from an axis through some point O at a distance h from its centre of gravity , G. the moment of inertial about O is I



From the figure above

$$\sin\theta = \frac{x}{n} \quad , \quad x = h\sin\theta$$

If θ is very small angle measured in radian $\sin\theta \approx \theta$

$x = h\sin\theta = h\theta$

restoring torque $\tau = -mgx$

$$\tau = -mgh\theta \text{ but } \tau = I\frac{d^2\theta}{dt^2}$$

$$I\frac{d^2\theta}{dt^2} = -mgh\theta$$

$$\frac{d^2\theta}{dt^2} = -\left(\frac{mgh}{I}\right)\theta \text{ ....(i)}$$

Hence its execute angular S.H.M

For an angular S.H.M

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta \text{ ....(ii)}$$

$$\text{(i)} = \text{(ii)}$$

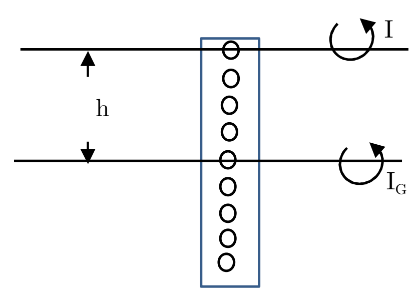
$$\omega^2 = mgh/I$$

Periodic time of compound pendulum

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

Additional concepts

(i) Consider the compound pendulum as shown on the figure below.



Apply parallel axes theorem

$I = I_G + Mh^2$ but $I_G = MK^2$

$$I = M\left(k^2 + h^2\right)$$

Now

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{M\left(k^2+h^2\right)}{mgh}}$$

$$T = 2\pi\sqrt{\frac{k^2+h^2}{gh}}$$

k = radius of gravitation

(ii) **LENGTH OF EQUIVALENT SIMPLE PENDULUM**

A simple pendulum whose periodic time is the same as that of a given compound pendulum is called the equivalent length of a simple pendulum.

For a simple pendulum

$$T = 2\pi\sqrt{\frac{L}{g}}$$

For compound pendulum

$$T = 2\pi\sqrt{\frac{k^2 + h^2}{gh}}$$

The length of equivalent simple pendulum is

$$L = \frac{k^2 + h^2}{h}$$

(iii) PROPERTIES OF COMPOUND PENDULUM

1. Minimum periodic time of oscillation. The period of oscillation is very large , there must therefore be a minimum period for some intermediate value of h. the periodic time of a compound pendulum in minimum when the length of equivalent simple pendulum is minimum.

   For minimum value of L

   $$\frac{dL}{dh} = \frac{d}{dh}\left[\frac{k^2 + h^2}{h}\right] = 0$$

   $$\frac{2h^2 - k^2 - h^2}{h^2} = 0 \text{ or } h^2 - h^2 = 0$$

   $$h_{min} = h = k$$

   Minimum value of L

   $$L_{min} = \frac{k^2 + h^2}{h} = \frac{k^2 + k^2}{k}$$

   Minimum period of oscillation

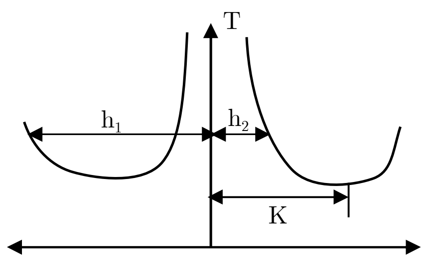
   $$T_{min} = 2\pi\sqrt{\frac{2k}{g}} = 2\pi\sqrt{\frac{2h}{g}}$$

2. Two points can be found one on either side of the centre of gravity such that the period of oscillation for axes through each is the same. They are called centre of suspension and centre of oscillation respectively and are interchangeable. Their distance a part is equal to the length of the equivalent simple pendulum. If h1 and h2 are the length from centre of gravity at which the oscillation through these points gives the same period , then the length of equivalent simple pendulum

   $L = h_1 + h_2 = 2k$

   The square of the radius of gravitation is $k^2 = h_1h$

GRAPH OF T AGAINST h



Determination of g and k by using the compound pendulum

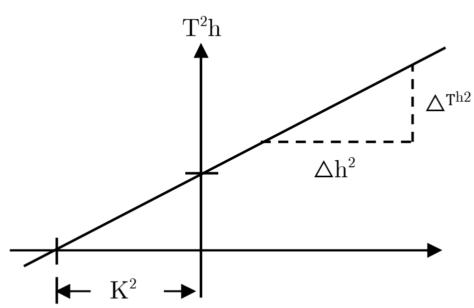
From the equation

$$T = 2\pi\sqrt{\frac{k^2 + h^2}{gh}}$$

$$T^2h = \left(\frac{4\pi^2}{g}\right)h^2 + \frac{4\pi^2k^2}{g}$$

$$y = mx + c$$

GRAPH OF T²h AGAINST



(i) Slope $= \frac{4\pi^2}{g}$, $g = \frac{4\pi^2}{slope}$

(ii) Radius of gravitation , k

T^2h – axis intercept = c

$$c = \frac{4\pi^2k^2}{g} = k^2.\text{slope}$$

$k = \sqrt{\frac{c}{\text{slope}}}$ or x – axis intercept $= -k^2$, then k can be obtained from the equation.

ENERGY CHANGE IN A S.H.M

When a body oscillates in S.H.M it is acted upon by a restoring force which tends to bring it to the equilibrium position due to this force , there is a potential energy (p.e) in the body is in motion , it has a kinetic energy (k.e) during its energy continuously interchanges between k.e and p.e but their sum remain constant (taking the friction negligible) i.e the total energy is always remain constant and this is according to the law of conservation of mechanical energy.

Now

Total energy = p.e + k.e

$$E = Ep + Ek$$

Expression of potential energy

Consider a particle of mass , M execute S.H.M with amplitude A and angular velocity , ω at any instant of time , t displacement of the particle

$X = A\sin\omega t$

Restoring force , $F = M\omega^2X$

If the particle undergoes further small displacement dx , the corresponding change in work done is given by

$$dw = Fdx = M\omega^2 x dx$$

$$w = \int_o^x m\omega^2 x dx$$

$$w = p.e = \frac{1}{2}m\omega^2x^2$$

But $x = A\sin\omega t$

$$p.e = \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2A^2\sin^2\omega g$$

At the mean position

$$x = 0 \quad p.e = \frac{1}{2}m\omega^2(0)^2$$

$$p.e = 0$$

∴ P.E is equal to zero at the mean position

At the extreme positions

$X = A$

$$p.e = E_{P\max} = \frac{1}{2}m\omega^2A^2$$

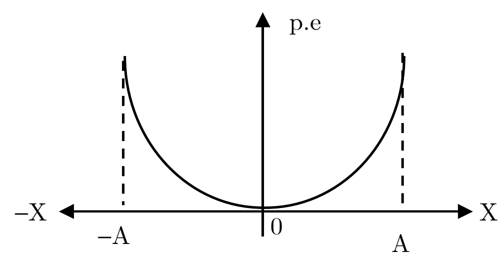
∴ P.E is maximum at the extreme positions.

GRAPH OF P.E AGAINST X

$$p.e = \frac{1}{2}m\omega^2x^2$$

$$y = Bx^2$$

This represent equation of the parabola



Expression of the kinetic energy

$$k.e = E_k = \frac{1}{2}mv^2$$

But $v^2 = \omega^2\left(A^2 - X^2\right)$

$$k.e = \frac{1}{2}m\omega^2\left(A^2 - X^2\right)$$

Also $v = \frac{dx}{dt} = A\omega\cos\omega t$

$$k.e = \frac{1}{2}m\omega^2A^2\cos^2\omega t$$

$$Ek = \frac{1}{2}m\omega^2\left(A^2 - X^2\right) = \frac{1}{2}m\omega^2A^2\cos^2\omega t$$

At the mean position , $X = 0$

$$k.e_{max} = \frac{1}{2}m\omega^2\left(A^2 - 0^2\right) = \frac{1}{2}m\omega^2A^2$$

∴ k.e is maximum at the mean position

At the extreme position , X = A

$$k.e = \frac{1}{2}m\omega^2\left(A^2 - A^2\right) = 0$$

Expression of total energy

$$E = p.e + k.e$$

$$E = \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m\omega^2\left(A^2 - X^2\right)$$

$$E = \frac{1}{2}m\omega^2A^2$$

This shows that total energy of particle in S.H.M is independent on the displacement of particle at any time ,t

Also , $E = p.e + k.e$

$$p.e = \frac{1}{2} m\omega^2 A^2 \sin^2 \omega t$$

$$k.e = \frac{1}{2} m\omega^2 A^2 \cos^2 \omega t$$

Now

$$E = \frac{1}{2} m\omega^2 A^2 \sin^2 \omega t + \frac{1}{2} m\omega^2 A^2 \cos^2 \omega t$$

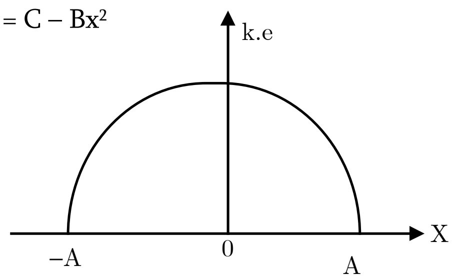
$$= \frac{1}{2} m\omega^2 A^2 \left[\cos^2 \omega t + \sin^2 \omega t\right]$$

$$E = \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} KA^2$$

This shows that total energy of particle in S.H.M is in independent on the time , t

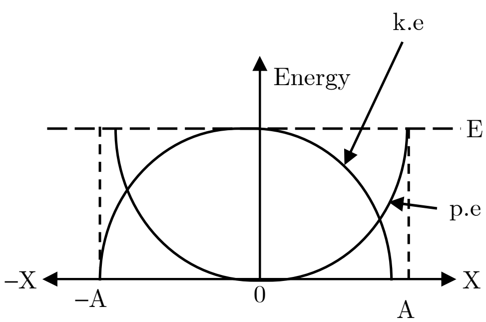
GRAPH OF K.E VS X

$k.e = C - Bx^2$

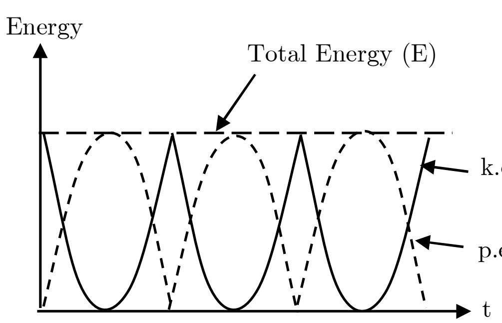


$k.e = c - Bx^2$. This represent equation of the parabola.

GRAPH OF ENERGY AGAINST DISPLACEMENT FOR S.H.M



GRAPH F ENERGY AGAINST TIME FOR S.H.M



APPLICATIONS OF SIMPLE HARMONICS

S.H.M plays a role in functioning of different appliances. These include ; clocks , shock absorbers , musical instrument , hearing gravimeters , seismometer e.t.c

1. HEARING
   The ear functions is due to the S.H. phenomena. The sound wave travel through the air and when they arrive at the eardrum they cause it to vibrate. This signal from eardrum is sent to the brain for interpretation.

2. MUSICAL INSTRUMENTS
   The vibration produced in the string of a guitar causes the air column to excuse S.H.M will result into producing a regular sound.

3. CAR SHORK ABSORBERS
   The springs attached to the car wheels execute S.H.M when the car passing through the pump road. Since the car is in S.H.M , shock absorbers will push the car back to normal place leaving the passengers in pleasant ride.

4. OSCILLATION OF THE NEEDLE OF SEWING MACHINE
   The oscillation of the needle of sewing machine are in S.H.M.

5. The motion of the molecules of solid is nearly simple harmonic. This facts help to explain certain characteristics of the solids.

6. CLOCK
   A large pendulum clock or vibrating quartz crystal are in periodic motion in order to insure that indicates time is accurate. This is due to the fact that oscillation has a constant period because its is in S.H.M thus keep time accurately.

7. GRAVIMETER
   Pendulum execute S.H.M which will enable measurement of local gravity at a given location

SUMMARY FOR CHARACTERISTICS

B	0	C
Max P.E Zero K.E	Zero P.E Max K.E	Max P.E Zero K.E
$t = T/2$ $X = A$ $V = 0$ $a = \omega^2 A$	$t = T/4$ $X = A$ $V = -\omega A$ $a = 0$	$t = 0$ $X = A$ $V = 0$ $a = -\omega^2 A$
$t = T/2$ $X = -A$ $V = 0$ $a = \omega^2 A$	$t = \frac{3}{4}T$ $X = -A$ $V = \omega a$ $a = 0$	$t = T$ $X = A$ $V = 0$ $a = -\omega^2 A$

X

SOLVE EXAMPLE TYPE E

Example 1

A solid sphere of mass 3kg and diameter 0.2m is suspended from a wire. The torque required to twist the wire is 5×10^{-2}Nmrad^{-1}. Calculate the period of oscillation

Solution

Data M = 3kg , R = 0.1m ,
 $c = 5 \times 10^{-2}$Nmrad^{-1} , T = ?

I of sphere about any diameter

$$I = \frac{2}{5}$$

Periodic time , $T = 2\pi\sqrt{\frac{I}{C}}$

$$T = 2\pi\sqrt{\frac{2MR^2}{5C}} = 2\pi\sqrt{\frac{2\times 3\times(0.1)^2}{5\times 5\times 10^{-2}}}$$

$$T = 3.075\,\text{sec}$$

Example 2

A disc of mass 1kg and radius 10cm is suspended by vertical wire at its centre. If the time period is 3.2sec. find the modulus of torsion.

Solution

Since $T = 2\pi\sqrt{\frac{I}{C}}$

$$C = \frac{4\pi^2 I}{T^2}$$

But $I = \frac{1}{2}MR^2$

$$C = \frac{4\pi^2}{T^2}\frac{1}{2}MR^2 = \frac{2\pi^2 MR^2}{T^2}$$

$$C = \frac{2\pi^2\times 1\times(0.1)^2}{(3.2)^2}$$

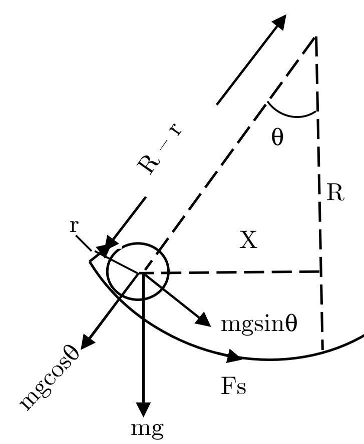
$$C = 1.9\times 10^{-2}\,\text{Nmrad}^{-1}$$

Example 3

A sphere of radius r rolls without slipping on a concave surface of large radius of curvature R show that the motion of the centre of gravity of the sphere is approximately S.H.M with period.

$$T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$$

Solution



From the figure

$$\sin\theta = \frac{x}{R-r}$$

Restoring force

$$F = -\left[Fs + Mg\sin\theta\right]$$

$$Ma = -\left[Fs + Mg\sin\theta\right]\ldots\ldots\ldots\ldots\ldots(i)$$

Torque on the sphere

$$\tau = Fsr = I\alpha$$

$$Fs = \frac{I\alpha}{r} \quad \text{but} \quad I = \frac{2}{5}Mr^2$$

But $a = \alpha r$

$$Fs = \frac{2}{5}Ma\ldots\ldots\ldots\ldots\ldots(ii)$$

$$Ma = -\left[\frac{2}{5}Ma + Mg\sin\theta\right]$$

On simplifying

$$a = \frac{-5}{7}g\sin\theta$$

$$a = -\left[\frac{5g}{7(R-r)}\right]x \quad , \quad a \propto -x$$

It is execute S.H.M

For S.H.M , $a = -\omega^2 x$

$$-\omega^2 x = -\left[\frac{5g}{7(R-r)}\right]x$$

$$\omega = \sqrt{\frac{5g}{7(R-r)}}$$

Periodic time , $T = \frac{2\pi}{\omega}$

$$T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$$

hence shown

Exampl 4

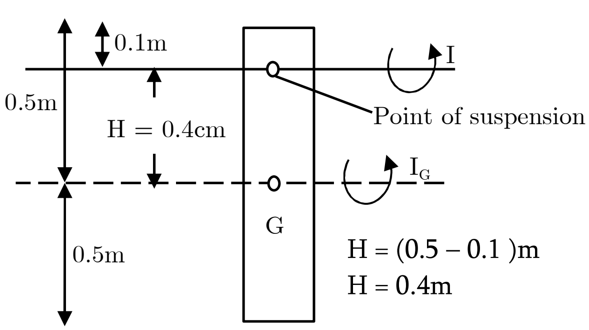
A uniform bar of length 100cm is suspended at 10cm distance from one end and is made to oscillate as a physical pendulum. Calculate

(a) The length of equivalent simple pendulum

(b) The periodic time.

Solution

(a) Let L = Length of equivalent simple pendulum.



Since $IG = \frac{ML^2}{12} = MK^2$

$$K^2 = \frac{L^2}{12} = \frac{1}{12}$$

Now $L = \frac{K^2 + h^2}{h} = \frac{\frac{1}{2} + (0.4)^2}{0.4}$

(b) Periodic time , $T = 2\pi\sqrt{\frac{L}{g}}$

Example 5

(a) Define simple harmonic motion

(b) Suppose a particle is moving with S.H.M
  (i) Write a relation between the force and position of the particle.
  (ii) Write a relation between the time and the velocity of the particle.

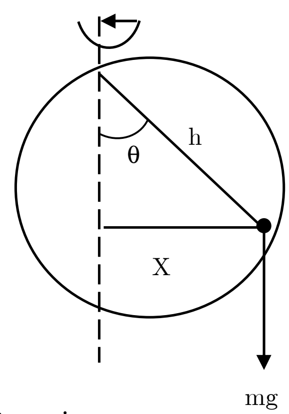
(c) An object in form of a thin disc of radius b is made to oscillate about an axis normal to the disc and through a point on its perimeter. Determine the period , T for small oscillation of the disc in terms of 'b' an any other constants (M.I of thin disc , I = ¾ Mb^2)

Solution

(a) See your notes

(b) (i) $F = -M\omega^2 x$ (ii) $V = A\omega\cos\omega t$

(c) Diagram



Restoring torque

$\tau = -mgx$

$\tau = -mgh\sin\theta$

If θ is very small angle measured in radian , $\sin\theta \simeq \theta$

$$\tau = -mgh\theta$$

$$I\frac{d^2\theta}{dt^2} = -mgh\theta$$

$$\frac{d^2\theta}{dt^2} = -\left(\frac{mgh}{I}\right)\theta$$

It is execute angular S.H.M for an angular S.H.M

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta$$

$$-\omega^2\theta = -\left(\frac{mgh}{I}\right)\theta$$

$$\omega = \sqrt{\frac{Mgh}{I}} \quad , \quad T = \frac{2\pi}{\omega}$$

Periodic time

$$T = 2\pi\sqrt{\frac{I}{Mgh}}$$

$$\text{But } I = \frac{3}{4}Mb^2$$

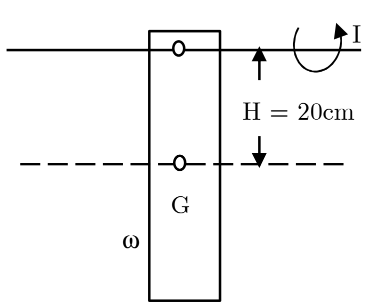
$$T = 2\pi\sqrt{\frac{\frac{3}{4}Mb^2}{Mgb}}$$

$$T = 2\pi\sqrt{\frac{3b}{4g}} = \pi\sqrt{\frac{3b}{g}}$$

Example 6

A thin uniform rod is pivoted about a horizontal axis which passes through a point on the rod 20cm from its centre of gravity. If the time of small oscillations performed by the rod in the vertical plane through the suspension is 1.37sec. calculate the length of the rod [g = 9.81m/s²]

Solution



$$\text{Since} \quad T = 2\pi\sqrt{\frac{h^2+k^2}{gh}}$$

$$T^2 = \frac{4\pi^2\left(h^2+k^2\right)}{gh}$$

$$k^2 = \frac{T^2gh}{4\pi^2} - h^2$$

$$= \frac{(1.37)^2 \times 9.8 \times 0.2 - 0.2^2}{4\pi^2}$$

$$k = 0.231\text{m}$$

For the uniform rod of length , L

$$MK^2 = \frac{ML^2}{12} \quad , \quad L = \sqrt{12}K$$

$$L = \sqrt{12} \times 0.231\text{m}$$

$$L = 0.80\text{m}$$

Example 7

(a) Explain what do you understand by 'Equivalent length of simple pendulum o fa rigid or compound pendulum?

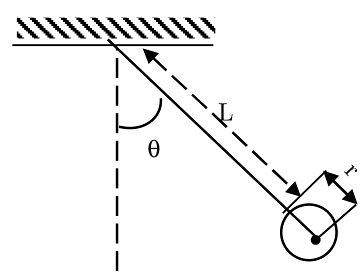
(b) Derive the period of oscillation of a simple pendulum as a special case of a compound pendulum.

(c) Calculate the length of the equivalent simple pendulum for a pendulum consisting of a solid sphere of radius 10cm suspended by a light wire of the length 1m.

Solution

(a) See your notes

(b) Consider a pendulum of mass M and radius r as shown on the figure below.



Assumptions made

(i) The bob is so small that its linear dimensions can be neglected

(ii) The string is in extensible and massless

(iii) Air resistance can be neglected i.e viscosity of air can be neglected.

M.I about the point of support

Apply the parallel Axes theorem

$$I = I_G + Mh^2$$

$$h = L + r \ , \ I_G = \frac{2}{5}Mr^2$$

Periodic time of compound pendulum

$$T = 2\pi\sqrt{\frac{I}{Mgh}}$$

$$= 2\pi\sqrt{\frac{\frac{2}{5}Mr^2 + M(L+r)^2}{M(L+r)g}}$$

$$= 2\pi\sqrt{\frac{\frac{2}{5}r^2(L+r)^2}{g(L+r)}}$$

Since $L \ggg r$, $L + r \simeq L$

$$T = 2\pi\sqrt{\frac{L^2}{gL}} = 2\pi\sqrt{\frac{L}{g}}$$

(c) $K^2 = \frac{2}{5}r^2 = \frac{2}{5}(0.01)^2$

$$K^2 = 4\times10^{-5}$$

Length of equivalent simple pendulum

$$L = \frac{K^2 + h^2}{h} = \frac{4\times10^{-5} + (0.1)^2}{1.1}$$

$L = 1.1\text{m}$ (approx.)

Example 8

A body weighting 500gm makes small oscillations under gravity about a horizontal axis of distance 20cm from its centre of gravity. The length of equivalent simple pendulum is 50cm. find the moment of inertia of the body about its axis of suspension and about a parallel axis through its centre of gravity.

Solution

Length of equivalent simple pendulum

$$L = \frac{k^2 + h^2}{h} = \frac{k^2 + (0.2)^2}{0.2}$$

$$0.5 = \frac{k^2 + (0.2)^2}{0.2}$$

$$k^2 = 0.06\text{m}^2$$

M.I about its centre of gravity

$$I_G = MK^2$$

$$= 0.5 \times 0.06$$

$$I_G = 0.03\text{kgm}^2$$

M.I about an axis through the point of suspension.

$$I = I_G + Mh^2$$

$$= 0.03 + 0.5(0.2)^2$$

$$I = 0.05\text{kgm}^2$$

Example 9

A compound pendulum is formed by suspending a heavy ring of radius 490cm from a string. What is the length of the string when the periodic time is minimize. Find this period

Solution

Radius of ring , r = 4.9m

K = r = 4.9m

For time period to be minimum

$$h_{min} = K = r$$

∴ The ring is suspended on rim and the length of the string has to be zero.

Minimum periodic time

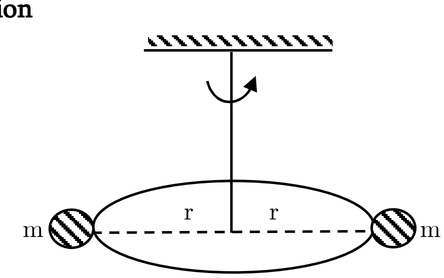
$$T_{min} = 2\pi\sqrt{\frac{2k}{g}} = 2\pi\sqrt{\frac{2\times4.9}{9.8}}$$

$$T_{min} = 6.28\,\text{sec}$$

Example 10

A horizontal disc of mass 200gm is suspended from its middle point by a torsion wire. When slightly twisted it makes small torsional oscillations of period 0.5sec two masses each 22gm are dropped into disc one at each end of the same diameter. What is its period now?

Solution



Initial M.I of disc before attaching two masses

$$I_1 = I_G = \frac{1}{2}Mr^2 = \frac{1}{2}\times0.2r^2$$

$$I_1 = 0.1r^2$$

$$I_1 = 0.5\,\text{sec}$$

M.I of the system after attaching two masses.

$$I_2 = I_G + 2Mr^2$$
$$= 0.1r^2 + 2\times 0.022r^2$$
$$I_2 = 0.144r^2 \quad , \qquad T_2 = ?$$

Since $T = 2\pi\sqrt{\dfrac{I}{C}}$

$$T_1 = 2\pi\sqrt{\frac{I_1}{C}}$$
$$T_2 = 2\pi\sqrt{\frac{I_2}{C}}$$

Takes $\dfrac{T_2}{T_1} = \sqrt{\dfrac{I_2}{I_2}} \Rightarrow T_2 = T_1\sqrt{\dfrac{I_2}{I_1}}$

$$T_2 = 0.5\sqrt{\frac{0.144r^2}{0.1r^2}}$$
$$T_2 = 0.6\,\text{sec}$$

Example 11

A uniform cylinder 20cm long suspended by a steel wire attached to its midpoint so that its long axis is horizontal is found to oscillate with period of 2seconds. When the wire is twisted and released when a small thin disc of mass 10gm is attached to each end the period is found to be 2.3seonds. calculate the moment of inertia of the cylinder about the axis of oscillation.

Solution

Since $T_1 = 2\pi\sqrt{\dfrac{I_1}{C}}$

$$T_2 = 2\pi\sqrt{\frac{I_2}{C}}$$
$$\left(\frac{T_1}{T_2}\right)^2 = \frac{I_1}{I_2}$$

But $I_2 = I_1 + 2Mh^2 = I_1 + 2\times 0.01\times 0.1^2$

$$I_2 = I_1 + 2\times 10^{-4}$$

Now $\left(\dfrac{2.0}{2.3}\right)^2 = \dfrac{I_1}{I_1 + 2\times 10^{-4}}$

On solving , $I_1 = 6.2\times 10^{-4}\,\text{kgm}^2$

Example 12

A particle of mass 0.25kg vibrates with a period of 2.0seconds. if its greatest displacement is 0.4m. what is its maximum kinetic energy?

Solution

Since $K.E = K = \dfrac{1}{2}M\omega^2\left(A^2 - X^2\right)$

$$K_{max} = \frac{1}{2}M\omega^2A^2 \qquad \text{(at x = 0)}$$
$$= \frac{1}{2}\times 0.25\times\left(\frac{2\pi}{2}\right)^2\times(0.4)^2$$
$$K_{max} = 0.197\text{J}$$

Example 13

A body weighing 10kg is executing S.H.M has a velocity of 6m/s after one second of starting from the mean position. If its time period is 6seconds find its K.E , P.E and total energy.

Solution

Data : M = 10kg , V =60m/s , t = 1sec , T = 6sec

Kinetic energy of the body

$$K.E = \frac{1}{2}MV^2 = \frac{1}{2}\times 10\times 6^2$$
$$K.E = 180\text{J}$$

Since $V = A\omega\cos\omega t = A\omega\cos\left(\dfrac{2\pi t}{T}\right)$

$$6 = A\omega\cos\left(\frac{2\pi}{6}\right) = A\omega\times\frac{1}{2}$$
$$A\omega = 12$$

Total energy of the body

$$E = \frac{1}{2}M\omega^2A^2 = \frac{1}{2}\times 10\times(12)^2$$
$$E = 720\text{J}$$

Since $E = P.E + K.E$

$$P.E = E - K.E = 720 - 180$$
$$P.E = 540\text{J}$$

Example 14

A mass of 1kg is executing S.H.M which is given by $x = 60\cos\left(100t + \pi/4\right)$ cm . What is the maximum K.E?

Solution

Given that $x = 60\cos\left(100t + \pi/4\right)$ ....(i)

Standard equation of S.H.M

$$X = A\cos(\omega t + \Phi) \text{ ....(ii)}$$

On comparing equation (i) and (ii)

Maximum kinetic energy

$$K_{max} = \frac{1}{2}MV^2_{max} = \frac{1}{2}M\omega^2A^2$$

$$= \frac{1}{2}\times 1\times 100^2\times\left(6\times10^{-2}\right)^2$$

$$K_{max} = 18J$$

Example 15

An 8kg body performs S.H.M of amplitude 30cm the restoring force is 60N, when the displacement is 30cm. find

(a) Time period

(b) The acceleration P.E and K.E when displacement is 12cm.

Solution

Given : M = 8kg , A = 30cm = 0.3m , F = 60N , y = 0.3m.

(a) By Hooke's law

$$F = ky$$

$$K = \frac{F}{y} = \frac{60}{0.3} = 200Nm^{-1}$$

Periodic time

$$T = 2\pi\sqrt{\frac{M}{K}} = 2\pi\sqrt{\frac{8}{200}}$$

$$T = 1.256 \text{ sec}$$

(b) Here y = 12cm = 0.12m

Acceleration $a = \omega^2 y$

$$a = \left(\frac{2\pi}{T}\right)^{2y} = \left(\frac{2\pi}{1.256}\right)^2\times 0.12$$

$$a = 3m/s$$

$$P.E = \frac{1}{2}ky^2 = \frac{1}{2}\times200\times(0.12)^2$$

$$P.E = 1.44J$$

$$K.E = \frac{1}{2}K\left(A^2 - y^2\right)$$

$$= \frac{1}{2}\times200\left[(0.3)^2 - (0.12)^2\right]$$

$$K.E = 7.56J$$

Example 16

A mass suspended by spring oscillates in 100cm^3 of water the force constant of spring is 9,800Nm^{-1}. The spring is stretched by 10cm and left to oscillate. Calculate the rise in temperature of water when the oscillation stops neglect other energy losses and assume that heat developed is totally taken up by the water. Given that g = 9.8m/s^2 and 1cal = 4.2J.

Solution

Given that : K = 9800Nm-1 , L = 10cm = 0.1m

Now $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

$$f = \frac{1}{2\pi}\sqrt{\frac{9.8}{0.1}} = 1.576s^{-1}$$

Also $Mg = KL$ OR $M = \frac{KL}{g}$

$$M = \frac{9800\times0.1}{9.8}$$

$$M = 100kg$$

Therefore , energy possessed by the oscillating mass.

$$E = 2\pi^2mf^2A^2$$

$$E = 2\pi^2\times100\times(1.576\times0.1)^2$$

$$E = 49.03J = \frac{49.03}{4.2}$$

$$E = 11.674cal$$

Let θ be the rise in temperature of the water

$$MC\theta = 11.67$$

$$100\times1\times\theta = 11.67$$

$$\theta = 0.1167^\circ C$$

Example 17

A restoring force is a must for a body to execute S.H.M explain why?

Solution

For a body to execute S.H.M it should oscillate about its mean position when the body is at the mean position , it possesses kinetic energy and by virtue of its , the body moves from the mean position to extreme position. The body can return to the mean position only , if it is acted upon by restoring force.

Example 18

Does a simple pendulum in a lift moving downwards with an acceleration g execute S.H.M? explain?

Example 19

(a) Derive an equation for the periodic time of oscillation of a mass M attached at the low end of two identical spring fixed in parallel , assuming that damping is minimum and the force constant of each spring is K.

(b) An object of mass 100gm is attached to the lower end of a vertical suspended system of two identical springs fixed in parallel , causing them to extend by 10mm. the mass is then pulled down a further 10mm and released. Determine the

(i) Periodic of oscillation

(ii) The maximum kinetic energy of the mass?

Solution

(a) Refer to your notes

(b) Since ke = mg

$$k = \frac{mg}{e} = \frac{100\times10^{-3}\times9.8}{0.01}$$

$$k = 98Nm^{-1}$$

(i) Periodic time

$$T = 2\pi\sqrt{\frac{M}{2K}} = 2\pi\sqrt{\frac{100\times10^{-3}}{2\times9.8}}$$

$$T = 0.142\text{ sec}$$

(ii) $$K_{max} = \frac{1}{2}M\omega^2A^2 = \frac{1}{2}Ke^2$$

$$= \frac{1}{2}\times98(0.01)^2$$

$$K_{max} = 0.005J$$

Example 20

(a) (i) Define S.H.M and describe the terms Amplitude , period and frequency as applied to S.H.M.

(ii) Explain what is responsible for the continual interchange of potential energy and kinetic energy in a mechanical oscillation at what points in S.H.M is the acceleration greatest? Where is it least?

(b) A small mass of 200gm is attached to one end of helical spring and produces an extension of 15mm. the mass is now set into oscillation of amplitude into oscillation of amplitude 10mm. calculate the

(i) Period of oscillation

(ii) Velocity of the system as it passes the equilibrium point.

(iii) Maximum kinetic energy of the system

(iv) Potential energy of the spring when mass is 5mm below the centre of oscillation.

Solution

(a) (i) refer to your notes

(ii) Both the p.e and k.e depends on the displacement is controlled by the restoring force so the restoring force is the one responsible for the continual interchange of p.e and k.e. the magnitude of acceleration a = ω^2y. acceleration is greatest at the extreme positions and least at the mean position.

(b) (i) $$T = 2\pi\sqrt{\frac{e}{g}} = 2\times3.14\sqrt{\frac{15\times10^{3}}{9.8}}$$

$$T = 0.246\text{ sec}$$

(ii) $$V_{max} = \omega A = \frac{2\pi A}{T}$$

$$= \frac{2\times3.14\times10\times10^{-3}}{0.246}$$

$$V_{max} = 0.26m/s$$

(iii) $$K_{max} = \frac{1}{2}MV_{max}^2 = \frac{1}{2}\times0.2\times(0.26)^2$$

$$K_{max} = 6.76\times10^{-3}J$$

(iv) $$P.E = \frac{1}{2}KX^2 = \frac{1}{2}\left(\frac{mg}{e}\right)x^2$$

$$= \frac{0.2\times9.8\times\left(5\times10^{-3}\right)^2}{2\times15\times10^{-3}}$$

$$P.E = 163\times10^{-3}J$$

Example 21

A mass of 0.5kg connected to a light spring of force constant 20Nm^{-1} oscillates an a horizontal frictionless surface.

(i) Calculate the total energy of the system and the maximum speed of the mass if the amplitude of the motion is 3cm.

(ii) What is the velocity of the mass when the displacement is equal to 2cm?

(iii) Find the kinetic and potential energies of the system when the displacement equal to zero.

Solution

(i) E = Total energy of the system

$$E = \frac{1}{2}KA^2 = \frac{1}{2}\times20\times\left(3\times10^{-2}\right)^2$$

$$E = 9.0\times10^{-3}J$$

At the equilibrium , X = 0

$$E = K.E_{max} = \frac{1}{2}MV^2_{max}$$

$$9.0\times10^{-3} = \frac{1}{2}MV^2_{max}$$

$$V_{max} = \left[\frac{2\times9.0\times10^{-3}}{m}\right]^{\frac{1}{2}}$$

$$= \left[\frac{2\times9\times10^{-3}}{0.5}\right]^{\frac{1}{2}}$$

$$V_{max} = 0.19m/s$$

(ii) Total energy

$$E = k.e + p.e$$

$$E = \frac{1}{2}MV^2 + \frac{1}{2}KX^2$$

$$\frac{1}{2}KA^2 = \frac{1}{2}MV^2 + \frac{1}{2}KX^2$$

$$V = \pm\sqrt{\frac{K}{M}\left(A^2 - X^2\right)}$$

$$= \pm\sqrt{\frac{20}{0.5}\left[3^2 - 2^2\right]\times10^{-4}}$$

$$V = \pm0.14m/s$$

Example 22

(a) A girl is swinging in a swing the sitting position. How is the period of swing affect if she stands up?

(b) A pendulum clock is taken to the moon will it gain or lose time?

Solution

(a) Periodic time of simple pendulum

$$T = 2\pi\sqrt{\frac{L}{g}}$$

When the girl stands up her centre of gravity is raised up this decreases L and hence the period of motion decrease

(b) $$T = 2\pi\sqrt{\frac{L}{g}}$$

The value of g is less in moon than that on earth surface T increases the pendulum takes more time to complete one oscillation.

Example –23

(a) What is the relation between uniform circular motion and S.H.M?

(b) Is the motion of a simple pendulum strickly S.H.M?

(c) The motion of a simple pendulum is approximately S.H.M for small angles of oscillation for large angles of oscillation , time period is greater then $2\pi\sqrt{\frac{L}{g}}$ why?

Solution

(a) Uniform circular motion can be through of as two S.H.M operating at right angles.

(b) It is not strickly simple harmonic because we make the assumption that $\sin\theta \simeq \theta$ which is nearly valid only if θ is very small.

(c) The restoring force tending to bring the pendulum to its means position is mgsinθ in this arriving at the formula

$$T = 2\pi\sqrt{\frac{L}{g}}$$

we take $\sin\theta \simeq \theta$ i.e restoring force $F = Mg\theta$

For the large value of θ $\sin\theta < \theta$, therefore , the restoring force decreases from $mg\theta$ to $mg\sin\theta$ as a result the pendulum takes a larger time to complete one vibration.

Example 24

(a) Define simple harmonic motion (S.H.M) for a particle moving in a straight line.

(b) Use your definition to explain how S.H.M can be represented by the equation

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

(c) A mechanical system is known to perform S.H.M what quantity must be measured in order to determine ω for the system?

Solution

(a) S.H.M is a to and fro motion of a body whose acceleration is directly proportional to its displacement from the mean position and it is always directed towards the mean position.

(b) From the definition above acceleration is directly proportional to x , and opposite directed to x so , $a \alpha -x$, $a = -kx$ the constant $K = \omega 2$ must be positive as 'a' is opposite to x so it appeared squared to emphasize that it is always positive hence the equation of linear S.H.M is given by $\frac{d^2x}{dt^2} = -\omega^2 x$

(c) The quantity to be measured is the period of a oscillation of the system.

Example 25

A body of mass 200gm is execute S.H.M with amplitude of 20mm. the maximum force which acts upon it is 0.064N calculate

(a) Its maximum velocity

(b) Its period of oscillation

Solution

The maximum acceleration of the body

$$a_{max} = \omega^2 A = \frac{f}{m}$$

$$a_{max} = \frac{0.064}{0.2} = 0.32m/s$$

$$\Rightarrow \omega = \sqrt{\frac{a_{max}}{A}} = \sqrt{\frac{0.32}{0.02}}$$

$$\omega = 4rads^{-1}$$

(a) $V_{max} = \omega A = 4 \times 0.02$

$V_{max} = 0.08m/s$

(b) $T = \frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2}$

$T = \frac{\pi}{2}$ second

Example – 26

Can pendulum clocks be used in artificial satellite? Explain

(a) How will you determine the period of oscillation of mass without use of a clock? You are given a weightless spring a metre rule and a known mass.

Solution

(a) No $T = 2\pi\sqrt{\frac{L}{g}}$

Inside of the satellite the effect of g is zero. So will not oscillate $g \rightarrow 0$, $T \rightarrow \alpha$

(b) The mass is made to oscillate the period of oscillate.

$T = 2\pi\sqrt{\frac{M}{K}}$ But $K = \frac{mg}{X}$

$$\frac{X}{g} = \frac{M}{K}$$

$$T = 2\pi\sqrt{\frac{x}{g}}$$

Hence , period can be found by measuring extension x produced by mass using a metre rule.

Example – 27

What is the frequency of oscillation of a simple pendulum mounted in a cabin that is freely falling under gravity?

Solution

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

The value of g inside the freely falling cabin is zero so frequency is zero.

Example 28

(a) What is the origin of name simple harmonic motion?

(b) Give the characteristics of S.H.M

Solution

(a) Of all the periodic or harmonic motion (eg. Circular motion , elliptical motion) it is the simplest periodic motion for this reasons it is called S.H.M.

(b)
- The motion of to and fro and periodic
- The particle is under a restoring force proportional to the displacement from equilibrium positions and always directed towards to the equilibrium position.
- $a = -\omega^2 x$ acceleration is maximum at the extreme position and zero at the equilibrium position.

Example 29

(a) State the conditions for an oscillatory motion to be considered simple harmonic.

(b) A body of mass 0.30kg execute S.H.M with a period of 2.5sec and amplitude 4.0×10^{-2}m. determine

(i) The maximum velocity of the body

(ii) The energy associated with the motion

Solution

(a) State the conditions for S.H.M

(b) (i) $V_{max} = \left(\frac{2\pi}{T}\right)A = \frac{2\pi}{2.5} \times 4 \times 10^{-2}$

$V_{max} = 0.1m/s$

(ii) $E = \frac{1}{2}MV^2_{max} = \frac{1}{2} \times 0.3 \times (0.1)^2 = 1.5 \times 10^{-3}J$

Example 30

(a) What is the period of oscillation of a simple pendulum if its bob made of ice?

(b) A man with a wrist – watch on his hand falls from the top of a tower does the watch give correct time during the free fall?

Solution

(a) In this case of an ice pendulum , the periods depends on the radius of the ice bob as the ice melts radius decreases so period decreases but if the centre of mass of ice remain constant then there will be no change in the period.

(b) Yes the working of the wrist – watch is based on the energy stored in the spring which is independent of acceleration due to gravity.

Example 31

Suppose the bob in a simple pendulum is a hallow sphere with a small hole at the bottom the sphere is filled with water and is made to oscillate. What happens to the period?

Solution

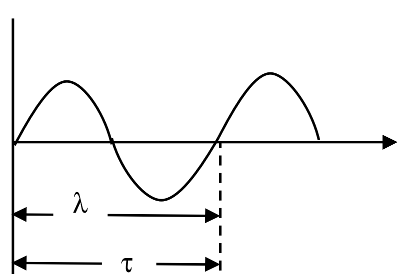
Periodic time T depends on length of simple pendulum which is the distance between centre of gravity of the bob and the point of suspension as the water flows out the C.G shifts downwards so the effective length increases and period increases when the whole water has flows out the C.G gain coincides with centre of the bob and the time period becomes the same as in the beginning thus the period first increases and then decreases to the original value.

Example 32

A sinusoidal voltage is applied to the y – plate of a C.R.O which has a calibrated time base a stationary trace with amplitude of 4.0cm and wavelength of 1.5cm is obtained when the time base is set at 1.0cm ms-1 the time base is then switched off and the trace becomes a vertical line. Calculate the maximum speed of the speed of spot of light on the end of the tube when producing the vertical line.

Solution

The wave form on the screen can been shown her under



Periodic time

$$T = \frac{\lambda \times 1ms}{1cm} = \frac{15cm \times 1ms}{1cm}$$

$$T = 1.5ms = 1.5\times10^{-3s}$$

Maximum speed

$$V_{max} = \omega A = \frac{2\pi A}{T}$$

$$V_{max} = \frac{2\pi \times 0.04}{1.5\times10^{-3}}$$

$$V_{max} = 167.55m/s$$

Example 33

A small piece of cork in a ripple tank oscillates at 0.20m/s , have a wavelength of 15mm and an amplitude of 5mm , what is the maximum velocity of the cork?

Solution

Water wave is a transverse wave as the ripple passes by , the cork will be vibrating up and down simple harmonically with the period of oscillation equal to that of the ripples

Periodic time of the wave

$$T = \frac{\lambda}{V} = \frac{0.015}{0.20}$$

$$T = 0.07\,\text{sec}$$

$$V_{max} = \omega A = \frac{2\pi A}{T}$$

$$= \frac{2\pi \times 0.005}{0.075}$$

$$V_{max} = 0.42m/s$$

Example 34

The displacement – time equation for a particle moving with simple harmonic motion is

$$x = a\sin(\omega t + \varepsilon)$$

(a) Explain what each of the symbols represents, illustrating your answer with a rough graph showing how x varies with time , t.

(b) Write down the velocity – time equation on and draw a corresponding graph showing how the velocity V varies with time , t .

(c) If m is mass of the particle the kinetic energy at displacement x is ½ $M\omega^2(a^2 - x^2)$. Write down the expression for the potential energy at displacement x and the total energy.

(d) The total energy of an atom oscillating in a crystal lattice at temperature T is on average 3KT , where K is the Boltzmann constant , $1.38 \times 10^{-23}JK^{-1}$. Assuming that copper atoms each of mass 1.06×10^{-25}kg , execute S.HM of amplitude 8×10^{-11}m at 300K.calcualte the corresponding frequency.

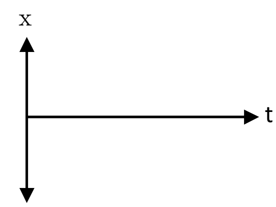
Solution

(a) X – displacement of particle from mean position

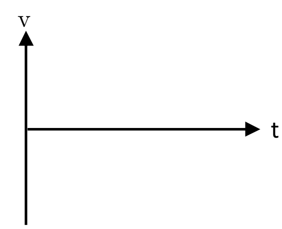
a – amplitude , t = time

ω - angular frequency

ε - Epoch (initial phase angle)



(b)



(c) $P.E = \frac{1}{2}m\omega^2x^2$

Total energy

$$E = P.E + K.E = \frac{1}{2}m\omega^2a^2$$

(d) $E = 3KT$

$$= 3\times1.38\times10^{-23}\times300$$

$$E = 1.242\times10^{-20}J$$

But

$$E = \frac{1}{2}m\omega^2a^2$$

$$\omega = \sqrt{\frac{2E}{ma^2}} \quad \text{but} \quad \omega = 2\pi f$$

$$2\pi f = \sqrt{\frac{2E}{ma^2}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{2E}{ma^2}} = \frac{1}{2\times 3.14}\sqrt{\frac{2\times 1.242\times 10}{1.06\times 10^{-25}}}$$

$$a = 8\times 10^{-11}\text{m}$$

$$f = 9.63\times 10^{11}\text{Hz}$$

Example 35

(a) What is meant by the following terms as used in simple harmonic motion (S.H.M)?
   (i) Periodic motion (01 mark)
   (ii) Oscillatory motion (01 mark)

(b) (i) List four important properties of a particle executing simple harmonic motion (SHM) (04 marks)
   (ii) Sketch a labeled graph that represent the total energy of a particle executing S.H.M (02 marks)

(c) The periodic time of a body execute S.H.M is 4seconds. How much time interval from t = 0 will its displacement be half its amplitude? (02 marks)

Example 36

(a) The equation of simple harmonic motion is given as $x = 6\sin 10\pi t + 8\cos 10\pi t$, where x is in centimeter and t in second. Determine the
   (i) Amplitude (03 marks)
   (ii) Initial phase of motion (02 marks)

(b) (i) Show that the total energy of a body executing simple harmonic motion is independent of time (2.5marks)
   (ii) Find the periodic time of a cubical body of side 0.2m and mass 0.004kg floating in water then pressed and released such that it oscillates vertically (2.5marks)
   $\rho = 1000\text{kgm}^{-3}$
   $g = 9.8\text{m/s}^2$

Example 37

(a) What do you understand by the following terms
   (i) Damped oscillations (01 mark)
   (ii) Un damped oscillations (01 mark)

(b) (i) Sketch the waveform diagram to represent the terms in 4(a)(i) (02 marks)
   (ii) Show that the total energy of a body executing S.H.M is independent of time (0 2marks)

(c) A mass of 0.5kg connected to alight spring of force constant 20Nm oscillates on horizontal frictionless surface. If the amplitude of the motion is 3.0cm calculate
   (i) Maximum speed of the mass (02 marks)
   (ii) Kinetic energy of the system when the displacement is 2.0cm (02 marks)

Example 38

(a) (i) Briefly explain why the motion of a simple pendulum is not strictly simple harmonic (1.5marks)
   (ii) Why the velocity and acceleration of a body executing simple harmonic motion (S.H.M) are out of phase? (1.5mark)

(b) A body of mass 0.30kg execute simple harmonic motion with a period of 2.5sec and amplitude of 4.0×10^{-2}m; determine
   (i) Maximum velocity of the body (1.5 mark)
   (ii) Maximum acceleration of the body (01mark)
   (iii) Energy associated with motion (2.5 marks)

(c) A particle of mass 0.25kg vibrates with a period of 2.0sec if its greatest displacement is 0.4m what is its maximum kinetic energy (02 marks)

Example 39

(a) (i) Outline the mean similarity between simple harmonic motion and uniform circular motion (01 mark)
   (ii) Give the condition for a body to execute each of the motion in (a) above (02 marks)

(b) A bullet of mass 1g strike and get embedded into the bob of a ballistic pendulum of mass 299g. the pendulum in oscillates with amplitude of 0.2m and period of 0.4sec compute the velocity of the bullet before striking the pendulum. (04 marks)

(c) A small smooth spherical bowl of radius of curvature 4m. if it slightly displaced from the bottom point and released it executes simple harmonic motion. Find periodic time (03 marks)