MODULE 4 : ELECTROSTATICS

ELECTROSTATICS
Sub-topics;
1. ELECTRIC FORCE AND COULOMB'S LAW
2. ELECTRIC FIELD
3. ELECTRIC POTENTIAL
4. CAPACITANCE.

INTRODUCTION

Electrostatics – Is the branch of physics which deals with charges at the rest. The word **''ELECTROSTATICS''** means electricity at the rest. Sometimes is known as Static-Electricity.

IMPORTANCE OF ELECTROSTATICS

1. Electrostatics generation can produce high voltages such as 10 volts. Such high voltages are required for X - Ray work and nuclear bombardment.
2. We use principles of electrostatics for spray paints powder.
3. The principle of electrostatics are used to present pollution.
4. The problems of preventing sparks and breakdown of insulators in high voltage engineering are essentially electrostatic.
5. The development of lightning rod and capacitor are the outcomes of electrostatics.

FRICTIONAL ELECTRICITY

Is the electrostatic charge (ie charges at rest) developed on insulating bodies when they are rubbed against each other. When two objects rubbing with each other, there is transferring of electric charges. When two objects rubs each other exerts a force and this force is known as ''Electric force''

ELECTRIC FORCE- Is the force experienced on the charged bodies or electric charges.

ELECTRIC CHARGE - Is the charges acquired by the body due to the deficiency or excess of electrons fron the normal due to the share.

Electric charge- Is the charge(s) acquired by the body when two bodies rubbing each other which may be either positive or negative charges.

TYPES OF ELECTRIC CHARGES

(i) Positive charge.
(ii) Negative charge.

POSITIVE CHARGE - Is the charge which possessed by the body after rubbing having defficiency of the electrons.

NEGATIVE CHARGE - Is the charge which possessed by the body after rubbing having efficiency of the electrons. Fig.

The name of the object, which acquires	
POSITIVE CHARGE	**NEGATIVE CHARGE**
Glass rod	Silk cloth
Fur or cat skin	Ebonite rod
Woolen cloth	Amber
Woolen cloth	Plastic object

PROPERTIES OF ELECTRIC CHARGES

1. Like charges repel each other and unlike charges attract each other.
2. The magnitude of elementary is negative or positive charge is same and is equal to 1.6×10^{-19}C.
3. The force between charges varies as the inverse square of their separation.
4. The charge is quantized ie equal to $\pm ne$ where n is an integer and $e = 1.6 \times 10^{-19}$C.
5. The electric charge of a system is always conserved.

6. Electric charge is a scalar quantity.
7. The magnitude of a charge on a body is independent of the speed of the body.

CONSERVATION OF ELECTRICC CHARGES

This is the property that the total charges on an isolated system remains constant.

LAW OF CONSERVATION OF THE ELECTRIC CHARGES

State that'' The algebraic sum of positive and negative charges in an isolated system remain always remain constant''. In other words, the net charge of isolated system remain unaltered.

The following examples explain the law of conservation of charge;-

1. In all nuclear transformations, the proton number is found to remain unchanged.

   $^{1}_{0}n + ^{235}_{92}u \rightarrow ^{141}_{56}Ba + _{36}kr^{92} + 3^{1}_{0}n + E$

   E = energy

   Proton number before fission = 0 + 92 = 92

   Proton number after fission

   = 56 + 36 + 3(0) = 92

   Thus, net charge (proton number) is same before and after the nuclear fission of ^{235}U
2. When a glass rod is rubbed with silk, negative charges appear on the silk while an equal amount of positive charges appears on the glass rod. Then, the net charges on the glass – rod system is remains equal to zero both before and after rubbing. Hence electric charges are conserved.

QUANTIZATION OF CHARGE

The magnitude of charge on a proton or an electron($e = 1.6 \times 10^{-19}C$) is called ''elementary charge''. Since protons and electrons are the only charged particles constituting the matter, the charge on an object must be integral multiple of $\pm e$.

Mathematically, the charge on any object must always be equal to

$q = \pm ne$, [n = 1, 2, 3,-------]

''Quantization of electric charge''

Refers that the electric charge of an object is an integral multiple of the fundamental charge.ie The charges carried by the body is not fraction.

CONDUCTOR - Is the substance which possessing the electric charge(s) **Examples:** Copper, Iron, Zinc, aluminium, etc

INSULATOR (BAD CONDUCTOR)

Is the substance which does not possessing the electric charges. **Examples:** Rubber, Cotton, dry piece of wood, waxy, etc

CHARGING - Is the process of electrifying body.

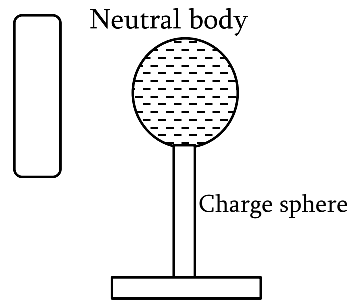
METHOD OF CHARGING

1. Rubbing (Friction) method
2. Shaining method
3. Electrostatic Induction
4. Corona discharge method

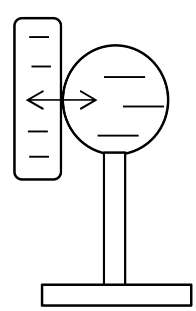
1. RUBBING(FRICTION) METHOD

When two objects are rubbed together electrons moves form one body to another body, so that both objects becomes charged.

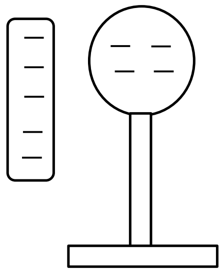
(i) Neutral body



(ii) On contact and rubbing process



(iii) After rubbing



Conclusion: After rubbing a neutral body on charged sphere. The body will possessing the negative charges and they fly apart from each other since they contain similar charges of equal amount. The sign carried by the charged body can be tested by using Gold Leaf Electroscope.

Note that: Rubbing process can be occurred for the two possible cases:-

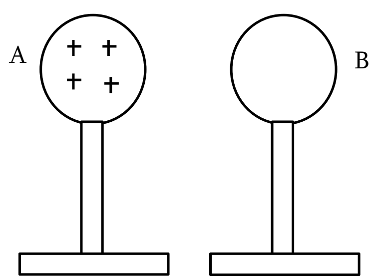
(i) For two charged bodies.

(ii) For a charged body and neutral body

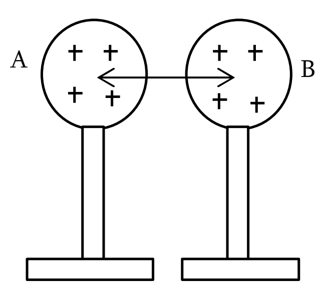
2. CHARGING BY CONTACT (SHAIRING)

This is the method of electrification in which a charged body is brought into contact with a neutral body. The neutral body will then acquire charge.

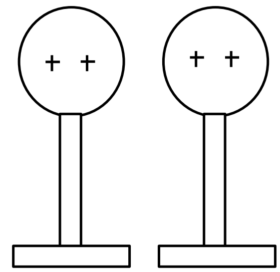
(i) Before charging



(ii) On contact



(iii) After charging



After charging the two bodies, fly apart due to the existence of repulsive force. Since the two bodies carries similar electric charges.

3. ELECTROSTATIC INDUCTION

Is the temporary electrification of a conductor, when a charged body is brought near it. In contrast to conduction, there is no transfer of electrons between the two bodies during charging by induction. It is because no physical contact takes place between the charging body and the conductor.

There are four steps involved on the charging the body by the Electrostatic Induction:-

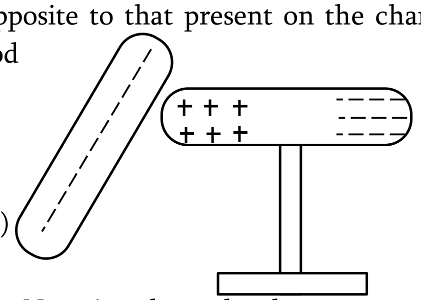
(i) Bring a charged rod near the body

(ii) Keeping the charged rod nearby, touch the body with fore finger or corrected to the Earth.

(iii) Remove the finger first and then the charged rod.

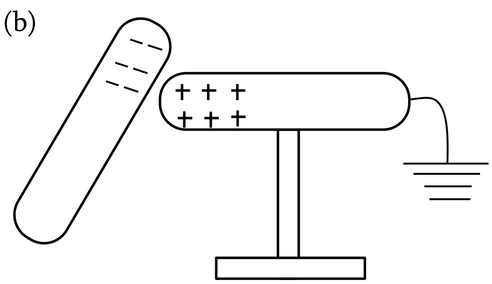
(iv) The body is now charged with a charge opposite to that present on the charging rod

(a)

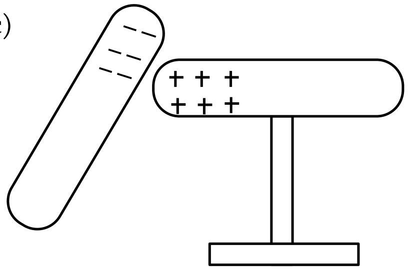


Negative charged rod

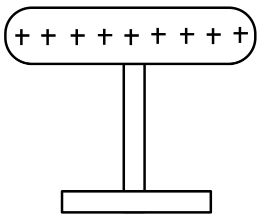
(b)



(c)



(d)



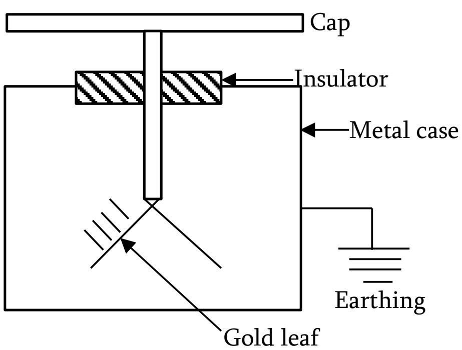
Note that: it is possible to charge the uncharged conductor in this way if the conductor is

(i) Splitting the conductor into two rods.

(ii) Momentary Earth (e.g. touching the sphere with the finger)

GOLD LEAF ELECTROSCOPE

Is the device which is used to test or detecting the electric charges on the given body



TEST OF A CHARGE

Repulsion is the sure test for electrification. Explain.

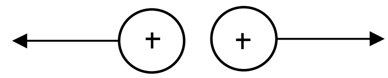
Reason: Attraction can exist between a charged body and uncharged body as well as between bodies having opposite charges. However, repulsion will only take place between bodies having similar charges. Hence repulsion is the sure test for electrification.

I. FUNDAMENTAL LAWS OF ELECTROSTATICS.

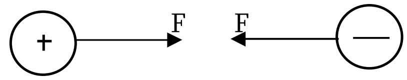
1. FIRST LAW OF ELECTROSTATICS

State that 'like charges repel, unlike charges attract

Like charges



Unlike charges



The quantity of electric charges carried by the body is given by

$Q = Ne = It$

$$N = \frac{It}{e}$$

N = Number of the electrons

e = electronic charges, t = time

I = electric current

Electric current – is defined as the rate of change of electric charges flows

$$I = \frac{dQ}{dt}, \quad Q = \int_{t_1}^{t_2} Idt$$

Unit of electric charge

$Q = It = \text{Ampere} \times \text{Second}$

$1C = 1Asec$

$\therefore$ S.I. Unit of electric charge is Coulomb (C)

One coulomb – is the charge which flows when a current of one ampere passes for one second i.e one coulomb is that charge which repels an equal and similar charge with a force of 9×10^9N when placed in vacuum (or air) at a distance of one metre from it.

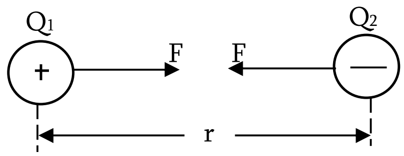
1 micro coulomb ($1\mu C$) = 10^{-6}C

1 nano coulomb (1nC) = 10^{-9}C

1 pico coulomb (1pc) = 10^{-12}C

2. **COULOMB'S LAW OF ELECTROSTATICS**
State that 'The magnitude of the electric force of attraction or repulsion between two point charges is directly proportional to the product of their charges and inversely proportional to the square of their distance apart'.

Mathematically



$$F \propto \frac{Q_1Q_2}{r^2}$$

$$F = \frac{KQ_1Q_2}{r^2}$$

Where K is a constant of proportionality. It's value depends upon the nature of the medium in which the two charges are located.

(i) In S.I. Unit, when two charges are located in vacuum or air $K = \frac{1}{4\pi\varepsilon_o}$

ε_o = Absolute permittivity of free space or air

$$F_a = \frac{1}{4\pi\varepsilon_o}\cdot\frac{Q_1Q_2}{r^2}$$

The absolute permittivity of free space (air) is measured to be

$$\varepsilon_o = 8.854\times10^{-12}N^{-1}C^2m^{-2}$$

$$\frac{1}{4\pi\varepsilon_o} = 9\times10^9 Nm^2C^{-2}$$

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_o r^2} = 9\times10^9\cdot\frac{Q_1Q_2}{r^2}$$

(ii) If two electric charges are situated in the given medium

$$K = \frac{1}{4\pi\varepsilon}$$

ε = electric permittivity of the given medium

$$F_m = \frac{Q_1Q_2}{4\pi\varepsilon r^2}$$

Electric permittivity (ε) – is the property of a medium and effects the magnitude of electric force between two point of charges.

According to the coulomb's law

$$F = \frac{Q_1Q_2}{4\pi\varepsilon r^2}$$

$$\varepsilon = \frac{Q_1Q_2}{4\pi F r^2}$$

Unit of ε (permittivity of given medium)

$$\varepsilon = \frac{C^2}{Nm^2} = C^2N^{-1}m^{-2}$$

∴ S.I. Unit of ε is $N^{-1}C^2m^{-2}$

Dimensional of ε

$$[\varepsilon] = \frac{[Q]^2}{[F][r]^2} = \frac{A^2T^2}{MLT^{-2}\cdot L^2}$$

$$[\varepsilon] = [M^{-1}L^{-3}T^4A^2]$$

RELATIVE PERMITTIVITY OR DIELECTRIC CONSTANT (k or εr) – Is defined as the ratio of permittivity of the given medium to the permittivity of free space or vacuum.

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_o}$$

$$\varepsilon = \varepsilon_r\varepsilon_o$$

Relative permittivity have no unit

$$\text{Now } F = \frac{Q_1Q_2}{4\pi\varepsilon r^2} = \frac{Q_1Q_2}{4\pi\varepsilon_r\varepsilon_o r^2}$$

Another definition of relative permittivity, K or εr

The electric force between the same two electric charges in air.

$$F_{air} = \frac{1}{4\pi\varepsilon_o}\cdot\frac{Q_1Q_2}{r^2}$$

Electric force between the same two electric charges hold at the same distance apart in a medium of absolute permittivity (ε)

$$F_{medium} = \frac{1}{4\pi\varepsilon_r\varepsilon_o}\cdot\frac{Q_1Q_2}{r^2}$$

$$= \frac{1}{\varepsilon_r}\left(\frac{Q_1Q_2}{4\pi\varepsilon_o r^2}\right)$$

$$F_{med} = \frac{F_{air}}{F_{med}}$$

$$\varepsilon_r = \frac{F_{air}}{F_{med}}$$

Relative permittivity or dielectric constant of a medium is the ratio of the force between two charges placed at a certain distance a part in air to the force between the same two charges placed the same distance apart in that medium.

LIMITATION OF COULOMB'S LAW

1. It holds for point charges only
2. It holds for stationary charges only
3. Coulomb's law applied only on point charges is at the distance, r defined un ambiguously.

Note that:

(i) The direction of the force that one charge exerts on another is determine by considering the relative signs of the charges.

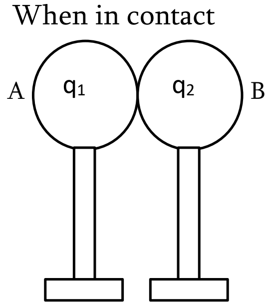
(ii) If $Q_1Q_2 > O$, the nature of the force is the repulsive force. in this case $Q_1Q_2 < O$, the nature of the force is the attractive.

(iii) Coulomb's law obey's Newton's third law of motion.

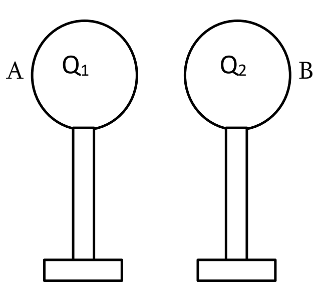
SHAIRING OF CHARGES

When two charges conductors (body) are brought into contact, the charges are redistributed on their surface according to the shapes of the bodies. If the bodies are of the same size the charges will distributed equally among them. Consider the distribution of charges on the identical charged spheres.

(i) When in contact



(ii) After separation



Different cases

Case 1 : If q_1 and q_2 are both positive or negative charges $Q_1 = Q_2 = \frac{q_1 + q_2}{2}$

Also. $F = \frac{1}{4\pi\varepsilon_o} \cdot \frac{Q^2}{r^2} \quad [Q_1 = Q_2 = Q]$

Case 2: If q_1 is positive and q_2 is negative and that $|q_1| > |q_2|$

$$Q_1 = Q_2 = \frac{|q_1| - |q_2|}{2}$$

Case 3: If q_1 is positive and q_2 is negative and that $|q_1| < |q_2|$

$$Q_1 = Q_2 = \frac{|q_2| - |q_1|}{2}$$

COMPARISONS BETWEEN ELECTROSTATIC FORCES AND GRAVITATIONAL FORCES

POINT OF SIMILARITIES.

Newton's universal law of gravitation

$$F = \frac{GM_1M_2}{r^2}$$

Coulomb's law, $F_e = \frac{1}{4\pi\varepsilon} \cdot \frac{Q_1Q_2}{r^2}$

(i) Both laws obey's inverse square law

(ii) Both are contral forces i.e forces act along the lines joining the centre of the bodies

(iii) Both are conservative forces i.e work done by them does not depend upon the path followed.

(iv) Both involve a property of the interacting particles – the mass in one case and the charge in the other.

POINTS OF DIFFERENCES.

Gravitational force	Electrostatic force
Gravitational forces are	Electric forces may be

always attractive	attractive or repulsive depending upon the sign of the charges.
The gravitational constant G is independent of nature of the medium	Electrical constant $K\left(=\frac{1}{4\pi\varepsilon_o}\right)$ depends upon the nature of the medium
Gravitational force operate over very large distances.	Electrostatic forces operate over small distance
Gravitational force is small compared to the coulomb's force	Electrostatic forces are extremely large as compared to the gravitational force.

PRINCIPLE OF SUPERPOSITION

State that 'The electric force experienced by a charge due to the other charges is the vector sum of electric forces acting on it due to these other charges'.

$$\vec{F} = \overline{F_1} + \overline{F_2} + \text{-------} + \overline{F_n}$$

EXAMPLES

Example 1

(a) (i) What does $q_1 + q_2 = 0$ signify in electrostatics?

(ii) Electrostatic experiments cannot be conducted successfully on humid days. Explain.

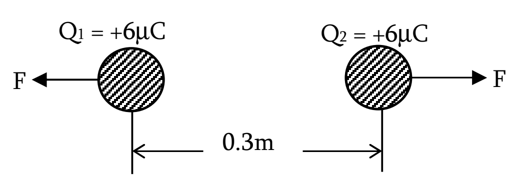
(b) Find the force between two point charges +6μC and +5μC situated 0.3m apart in air. Is the force of attraction or repulsion?

Solution

(a) (i) The charges q_1 and q_2 are equal and opposite.

(ii) The humid air becomes conducting therefore, the static charge on the apparatus leaks off to the air. For this reason, electrostatic experiments do not work well on humid days.

(b)



Apply coulomb's law

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_1Q_2}{r^2}$$

$$F = \frac{9\times10^9\times\left(5\times10^{-6}\right)\left(6\times10^{-6}\right)}{(0.3)^2}$$

$F = 3.0N$

The force is the repulsive force since $Q_1.Q_2 > O$

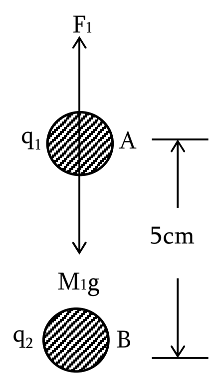
Example 2

A free pith – ball of 8g carries a positive charge of 5×10^{-8}C. What must be the nature and magnitude of charge that should be given to a second pith – ball fixed 5cm vertically below the former pith – ball so that the upper pith – ball is stationary?

Solution

Charge on the pith – ball A

$q_1 = 5\times10^{-8}C$



The weight M_1g of the pith – ball A acts vertically down wards. Let q_2 be charge on the pith – ball B held 5cm below the pith – ball A, so that the pith – ball A remain stationary. It can be possible only, if the charges on two pith – balls are of same signs i.e if charge on the pith – ball A is positive, charge on B should be positive.

$F_1 = M_1g$ ....(i)

Apply coulomb's law

$$F_1 = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q_1q_2}{r^2} \text{.......(ii)}$$

(i) = (ii)

$$\frac{q_1 q_2}{4\pi\varepsilon_o r^2} = M_1 g$$

$$9\times10^9 \cdot \frac{q_1 q_2}{r^2} = M_1 g$$

$$q_2 = \frac{M_1 g r^2}{9\times10^9 \times q_1}$$

$$= \frac{8\times10^{-3}\times9.8(0.05)^2}{9\times10^9\times5\times10^{-8}}$$

$$q_2 = 4.36\times10^{-7}\,C \text{ (Positive)}$$

Example 3 NECTA 1992/P1/13

(a) Calculate the value of two equal charges if they repel one another with a force of 0.1N when situated 0.5m apart in vacuum.

(b) What would be the value of charges when were situated in another medium with a force of 0.1N whose permittivity was ten times of air.

Given that $\frac{1}{4\pi\varepsilon_o} = 9\times10^9$

Solution

(a) Apply the coulomb's law

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{r^2} \quad [Q_1 = Q_2 = Q]$$

$$F = 9\times10^9 \cdot \frac{Q^2}{r^2}$$

$$Q = \sqrt{\frac{Fr^2}{9\times10^9}} = \sqrt{\frac{0.1\times(0.5)^2}{9\times10^9}}$$

$$Q = 1.7\times10^{-6}\,C = 1.7\mu C$$

(b) $F = \frac{1}{4\pi\varepsilon} \cdot \frac{Q^2}{r^2}$ but $\varepsilon = 10\varepsilon_o$

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_2}{10r^2}$$

$$Q = \sqrt{10}\left[\frac{Fr^2}{9\times10^9}\right]^{1/2}$$

$$Q = \sqrt{10}\times1.7\mu c$$

$$Q = 5.30\mu C$$

Example 4

(a) (i) Vehicles carrying inflammable materials usually have long chains that hang down and drag on the ground. Why?

(ii) Repulsion is sure test for electrification. Explain.

(b) A small sphere is given a charge of +20μC and a second sphere of equal diameter is given a charge of -5μC. The two spheres are allowed to touch each other and then separated 10cm apart. Assuming air as the medium; what force exists between them?

Solution

(a) (i) When a vehicle is in motion, its tyres rub against the road and get charged due to friction. Further, due to friction of air, the body of the vehicle also gets charged. If the accumulated charge becomes excessive, sparking may occur and the inflammable material may catch fire. Since the chain ropes are touching the ground, the charge leaks to the Earth. hence the danger of fire is avoided.

(ii) Refer to your notes

(b) Initial total electric charges before touches.

$$Q_o = Q_1 + Q_2$$

After the two sphere, charges then the spheres are separated after acquire the same amount of electric charges.

$$Q_f = Q + Q = 2Q$$

Apply the law of conservation of the electric charges.

$$2Q = Q_1 + Q_2$$

$$Q = \frac{Q_1 + Q_2}{2} = \frac{20 + -5}{2}$$

$$Q = 7.5\mu c$$

Apply the coulomb's law

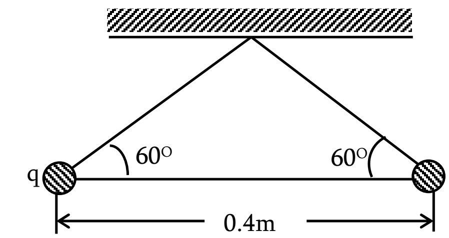
$$F = 9\times10^9 \cdot \frac{Q^2}{r^2}$$

$$= \frac{9\times10^9\times(7.5\times10^{-6})^2}{(0.1)^2}$$

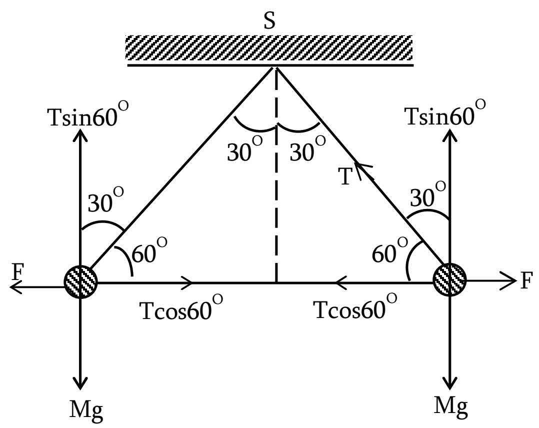
$$F = 50.62N \text{ (Repulsive)}$$

Example 5

Two identical balls, each of mass 0.10gm carry identical charges and are suspended by two threads of equal length. At equilibrium, they positions themselves as shown in the figure below. Calculate the charge on either ball.



Solution



Each of the two balls is in equilibrium under the action of the following forces;

(i) The electrostatic repulsive force, F

(ii) The weight Mg acting vertically downwards.

(iii) The tension T in the string directed towards point, s at the equilibrium.

$T\sin 60^o = Mg$……….(1)

$T\cos 60^o = F$………..(2)

Dividing equation (1) by (2)

$$\frac{T\sin 60^0}{T\cos 60^o} = \frac{Mg}{F}$$

$$\tan 60^0 = Mg/F$$

$$F = \frac{Mg}{\tan 60^o}$$

Apply the coulomb's law

$$F = \frac{Q^2}{4\pi\varepsilon_o r^2} = \frac{Mg}{\tan 60^o}$$

$$9\times10^9\cdot\frac{Q^2}{r^2} = \frac{Mg}{\tan 60^o}$$

$$Q = \left[\frac{Mgr^2}{9\times10^9\cdot\tan 60^o}\right]$$

$$= \left[\frac{(0.4)^2\times0.1\times10^{-3}\times9.8}{9\times10^9\tan 60^o}\right]^{\frac{1}{2}}$$

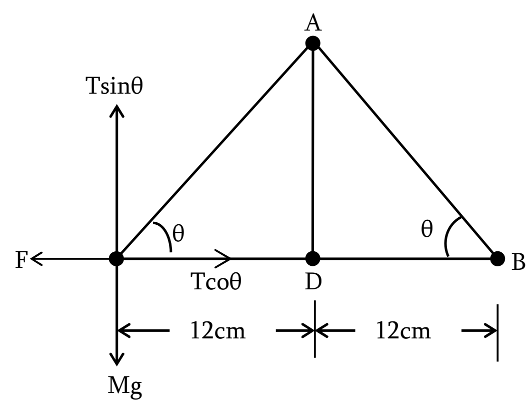
$$Q = 0.1\times10^{-6}C = 0.1\mu c$$

Example 6

Two small sphere, each having a mass of 0.1g are suspended from a point by thread 20cm long. They are equally charged and they repel each other to a distance of 24cm. What is the charge on each sphere? Given that

$$\frac{1}{4\pi\varepsilon_o} = 9\times10^9 C^{-2}Nm^2$$

Solution



By using Pythagoras theorem

$$\overline{AD} = \sqrt{\overline{AB}^2 - \overline{BD}^2} = \sqrt{20^2 - 12^2}$$

$$\overline{AD} = 16cm$$

At the equilibrium

$$T\sin\theta = Mg \;,\; T\cos\theta = F$$

$$\frac{T\sin\theta}{T\cos\theta} = \frac{Mg}{F} \;,\; \tan\theta = \frac{Mg}{F}$$

From the figure above

$$\tan\theta = \frac{16}{12} = \frac{4}{3}$$

$$\frac{4}{3} = \frac{Mg}{F} \;,\; F = \frac{3}{4}Mg$$

Apply coulomb's law

$$F = 9\times10^9\cdot\frac{Q^2}{r^2}$$

$$9\times10^9\frac{Q^2}{r^2} = \frac{3}{4}Mg$$

$$Q = \left[\frac{3Mgr^2}{4\times9\times10^9}\right]^{\frac{1}{2}}$$

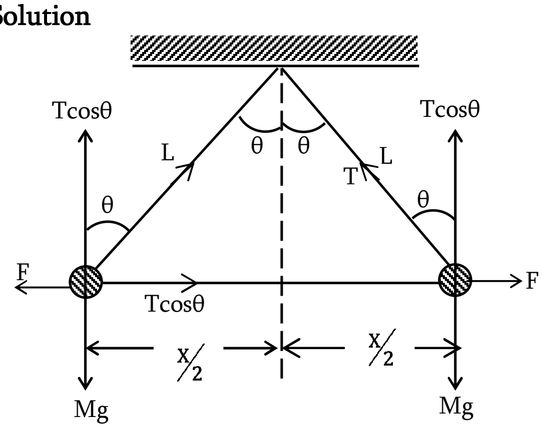
$$= \left[\frac{3\times0.1\times10^{-3}\times9.8\times(0.24)^2}{4\times9\times10^9}\right]^{\frac{1}{2}}$$

$$Q = 6.850\times10^{-8}C$$

Example 7

Two similar balls of mass M are hung from silk thread of length L and carry similar charges q. Prove that the separation between the balls is given by $X = \left[\frac{q^2}{2\pi\varepsilon_o Mg}\right]^{\frac{1}{3}}$. Assuming that the angle which the string makes with the vertical is very small.

Solution



At the equilibrium of either balls

$$F = T\sin\theta\ ,\ Mg = T\cos\theta$$

$$\frac{F}{Mg} = \frac{T\sin\theta}{T\cos\theta} = \tan\theta$$

$$F = Mg\tan\theta$$

Apply the coulomb's law

$$F = \frac{q^2}{4\pi\varepsilon_o X^2}$$

$$\frac{q^2}{4\pi\varepsilon_o X^2} = Mg\tan\theta\ ....(1)$$

From the figure above

$$\sin\theta \simeq \tan\theta = \frac{X}{2L}\ ....(2)$$

Since θ is very small angle

$$\frac{q^2}{4\pi\varepsilon_o X^2} = \frac{Mgx}{2L}$$

$$X^3 = \frac{2L}{4\pi\varepsilon_o Mg}$$

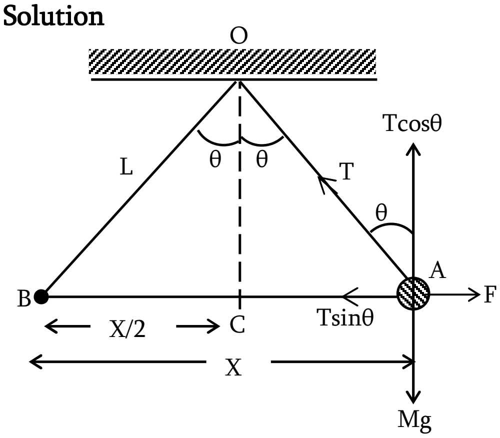
$$X = \left[\frac{q^2L}{2\pi\varepsilon_o Mg}\right]^{\frac{1}{3}}\ \text{hence shown}$$

Example 8

Two small sphere each having mass M kg and charge q coulomb are suspended from a point by insulating thread each L metre long but of negligible mass. If θ is the angle, each threads makes with the vertical when equilibrium has been attained show that

$$q^2 = \left(4MgL^2\sin^2\theta\tan\theta\right)4\pi\varepsilon_o$$

Solution



At the equilibrium

$$T\sin\theta = F\ ,\ T\cos\theta = Mg$$

$$\frac{T\sin\theta}{T\cos\theta} = \frac{F}{Mg}$$

$$F = Mg\tan\theta$$

Apply coulomb's law

$$F = \frac{q^2}{4\pi\varepsilon_o X^2}$$

$$\frac{q^2}{4\pi\varepsilon_o X^2} = Mg\tan\theta$$

$$q^2 = 4\pi\varepsilon_o\left(Mgx^2\tan\theta\right)$$

But $\sin\theta = \dfrac{X}{2L}$, $X = 2L\sin\theta$

$$q^2 = \left[4MgL^2\sin^2\theta\tan\theta\right]4\pi\varepsilon_o$$

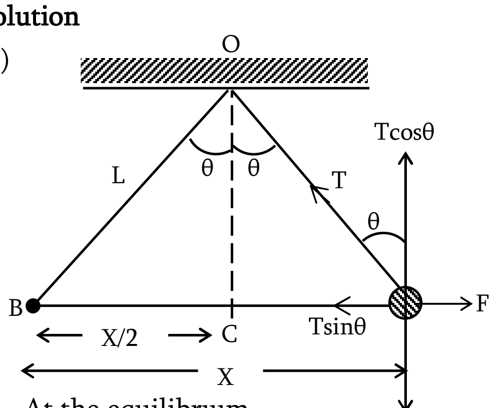
Example 9

(a) Two similar balls of mass, M are hang from silk threads of length L and carry similar charges q. assume that θ is so small that tanθ can be replaced by its approximation show that $X = \left[\dfrac{q^2L}{2\pi\varepsilon_o Mg}\right]^{\frac{1}{3}}$. Where X is the separation between the ball. If L = 1.2m, M = 10gm and X = 5cm; What is q?

(b) Assuming that each ball is losing charge at the rate of 1.0 × 10-9c/s. At what instantaneous relative speed, dx/dt do the balls approach each other initially?

Solution

(a)



At the equilibrium

$T\sin\theta = F$, $T\cos\theta = Mg$

$$\frac{T\sin\theta}{T\sin\theta} = \frac{F}{Mg}, \quad \tan\theta = \frac{kq^2}{x^2}$$

Apply coulomb's law

$$F = \frac{kq^2}{x^2}$$

$$\tan\theta = \frac{kq^2}{mgx^2}$$

From the figure above

$$\sin\theta = \frac{x}{2L}$$

Since $\theta \to 0$, $\sin\theta \approx \tan\theta$

$$\frac{X}{2L} = \frac{Kq^2}{Mgx^2}$$

$$K = \frac{1}{4\pi\varepsilon_o}$$

$$X = \left[\frac{q^2L}{2\pi\varepsilon_o Mg}\right]^{\frac{1}{3}}$$

Now, make the subject q

$$q = \sqrt{\frac{2\pi\varepsilon_o Mgx^3}{L}}$$

$$= \sqrt{\frac{2\pi\varepsilon_o \times 0.01\times 9.8\times(0.05)^3}{1.2}}$$

$$q = 2.38\times10^{-8}C$$

(b) Differentiate X w.r.t time, t

$$\frac{dx}{dt} = \left[\frac{2L}{2\pi\varepsilon_o Mg}\right]^{\frac{1}{3}}\cdot\frac{2}{3}q^{-1/3}\frac{dq}{dt}$$

M = 0.01Kg, L = 1.2M

$$q = 2.38\times10^{-8}C, \quad \frac{dq}{dt} = 10^{-9}C/S$$

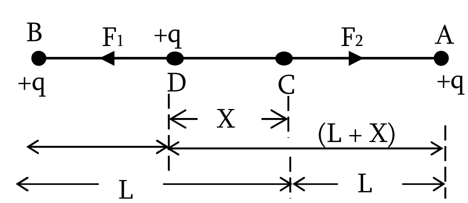
$$\frac{dx}{dt} = 1.3996\times10^{-3}m/s \simeq 1.4mm/s$$

Example 10

A particle of mass M and charge +q is located midway between two fixed charged particles each having a charge +q and at a distance 2L apart. Assuming that the middle charge moves along the line joining the fixed charges. Calculate the frequency and periodic time of oscillation when it is slightly displaced.

Solution

Let +q be the charge placed at the mid – point be slight displaced to the lift.



Apply the coulomb's law of the electrostatics

$$F_1 = \frac{q^2}{4\pi\varepsilon_o(L+X)^2} \quad , \quad F_2 = \frac{q^2}{4\pi\varepsilon_o(L-X)^2}$$

Let F = Net force

$$F = F_1 - F_2$$

$$= \frac{q^2}{4\pi\varepsilon_o(L+X)^2} - \frac{q^2}{4\pi\varepsilon_o(L-X)^2}$$

$$= \frac{q^2}{4\pi\varepsilon_o}\left[\frac{1}{(L+X)^2} - \frac{1}{(L-X)^2}\right]$$

$$= \frac{q^2}{4\pi\varepsilon_o}\left[\frac{(L-X)^2-(L+X)^2}{(L+X)^2(L-X)^2}\right]$$

$$F = \frac{-4q^2LX}{4\pi\varepsilon_o(L^2-X^2)^2}$$

Since $L >>> X$, $L^2 - X^2 \simeq L^2$

$$F = \frac{-q^2LX}{\pi\varepsilon_o L^4} = \frac{-q^2X}{\pi\varepsilon_o L^3}$$

$$Ma = \frac{-q^2}{\pi\varepsilon_o L^3}X$$

$$a = -\left[\frac{q^2}{\pi\varepsilon_o ML^3}\right]X$$

$a \propto -x$. This shows that the charged particle executed S.H.M $a = -\omega^2 x$

$$-\omega^2 x = -\left[\frac{q^2}{\pi\varepsilon_o ML^3}\right]X$$

$$\omega = \sqrt{\frac{q^2}{\pi\varepsilon_o ML^3}} \quad \text{but } \omega = 2\pi f$$

$$2\pi f = \sqrt{\frac{q^2}{\pi\varepsilon_o ML^3}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{q^2}{\pi\varepsilon_o ML^3}}$$

Periodic time, T $\quad T = \dfrac{1}{f} = \dfrac{2\pi\sqrt{\pi\varepsilon_o ML^3}}{q}$

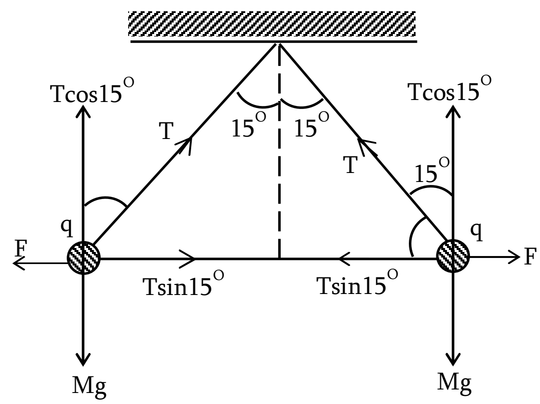
Example 11

Two identical charged sphere are suspended by strings of equal lengths. The string make an angle 30^O with each other when suspended in a liquid of density $800kgm^{-3}$, the angle remains the same. What is the dielectric constant of the liquid? The density of the material of sphere is $1600kgm^{-3}$.

Solution

Let M be the mass of each sphere in air of charge q each and separated at a distance, r.

In air



At the equilibrium

$$T\sin 15^o = F \ , \ T\cos 15^o = Mg$$

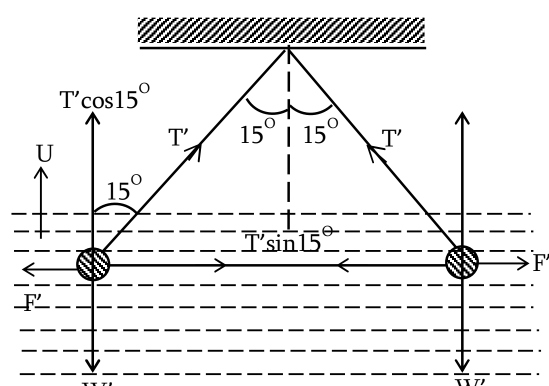
$$\frac{T\sin 15^o}{T\cos 15^o} = \frac{F}{Mg}$$

But $\quad F = \dfrac{1}{4\pi\varepsilon_o}\cdot\dfrac{q^2}{r^2}$ (coulomb's law)

$$\tan 15^o = \frac{q^2}{4\pi\varepsilon_o Mgr^2} \quad \text{.........(1)}$$

When sphere are suspended in a liquid.

Suppose the two sphere are suspended in a liquid of dielectric constant K as shown in the figure below. The weight of the sphere decreases to W'(say) due to upthrust and electrostatic force between the spheres decreases to F' (say) due to liquid as the electric medium.



$$F' = \frac{1}{4\pi\varepsilon_o k}\cdot\frac{q^2}{r^2}$$

At the equilibrium

$$T'\sin 15^o = F'$$
$$T'\cos 15^o = W'$$
$$\tan 15^o = \frac{F'}{W'}$$
$$\tan 15^o = \frac{q^2}{4\pi\varepsilon_o r^2 kw'} \text{..........(2)}$$

From equation (1) and (2)

$$Mg\tan 15^o = \frac{q^2}{4\pi\varepsilon_o r^2}$$
$$\rho vg\tan 15^o = \frac{q^2}{4\pi\varepsilon_o r^2} \text{..........(3)}$$

Also $KW'\tan 15^o = \frac{q^2}{4\pi\varepsilon_o r^2}$

The apparent weight of the sphere

$$W' = W - U$$
$$W' = \rho vg - \delta vg = vg(\rho - \delta)$$
$$kvg(\rho-\delta)\tan 15^o = \frac{q^2}{4\pi\varepsilon_o r^2} \text{.........(4)}$$

(3) = (4)

$$kvg(\rho-\delta)\tan 15^o = \rho vg\tan 15^o$$
$$k = \frac{\rho}{\rho-\delta} = \frac{1600}{1600-800}$$
$$k = 2$$

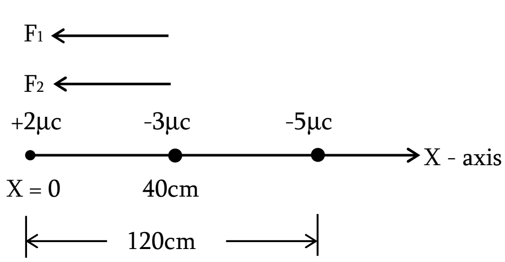
Example 12

Three point charges are placed at the following points on the x – axis; 2μC at x = 0; -3μC at x = 40cm and -5μC at x = 120cm. calculate force on the -3μC charge.

Solution

Let F_1 = force of attraction exerted on -3μC by +2μC

F_2 = force of repulsion exerted on -3μC by -5μC



Apply coulomb's law of electrostatics.

$$F_1 = 9\times10^9\cdot\frac{Q_1Q_2}{r^2}$$
$$F_1 = 9\times10^9\times\frac{3\times10^{-6}\times2\times10^{-6}}{(0.4)^2}$$
$$F_1 = 0.3375\text{N (towards left)}$$

Again $F_2 = \frac{9\times10^9\times3\times10^{-6}\times5\times10^{-6}}{(0.8)^2}$

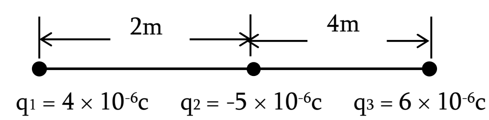
$$F_2 = 0.211\text{N (toward left)}$$

Resultant force on -3μC

$$F = F_1 + F_2$$
$$= 0.3375 + 0.211$$
$$F = 0.5485\text{N (towards left)}$$

Example 13

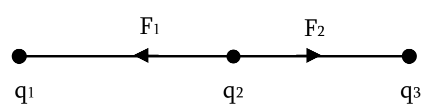
Find the force on the centre charge in the figure below



Solution

Let F_1 = electric force exerted on q_2 due to the q_1

F_2 = electric force exerted on q_2 due to the q_3



Apply the coulomb's law

$$F = \frac{1}{4\pi\varepsilon_o}\cdot\frac{Q_1Q_2}{r^2}$$
$$F_1 = \frac{9\times10^9\times4\times10^{-6}\times5\times10^{-6}}{2^2}$$
$$F_1 = 0.045\text{N (toward to the left)}$$

Also $F_2 = \frac{9\times10^9\times5\times10^{-6}\times6\times10^{-6}}{4^2}$

$F_2 = 0.0169N$ (towards to the right)

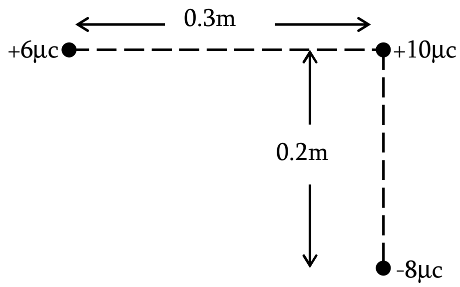
Resultant force on q^2

$F = F_1 - F_2 = 0.045 - 0.016$

$F = 0.0281N$

Example 14

Calculate the resultant force on the 10μC of charge in the figure below



Solution

Let F_1 = Magnitude of the force exerted on 10μC of charge by 6μC of charge.

F_2 = Magnitude of the force exerted on 10μC due to -8μC.

Apply coulomb's law

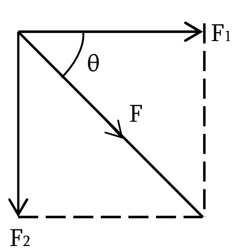
$$F_1 = \frac{9\times10^9\times6\times10^{-6}\times10\times10^{-6}}{(0.3)^2}$$

$F_1 = 6N$ (towards to the right of 10μC)

Also F

$$F_2 = \frac{9\times10^9\times10\times10\times10^{-6}\times8\times10^{-6}}{(0.2)^2}$$

$F_2 = 18N$ (Vertically downward)



F = Resultant force

$F^2 = F_1^2 + F_2^2$

$F = \sqrt{F_1^2 + F_2^2}$

$F = \sqrt{6^2 + 18^2} = 18.97N$

$F = 18.97N$

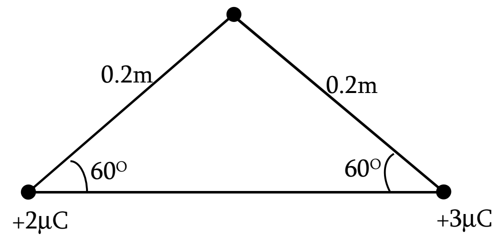
Let θ is the angle which F makes with F_1

$$\tan\theta = \frac{F_2}{F_1} = \frac{18}{6} = 3$$

$\theta = 71.57^\circ$

Example 15

The charges shown in the figure below are stationary. Find the force on 4μC charge due to the other two charges.



Solution

Let F_1 = Magnitude of force on the charge +4μC due to +2μC

F_2 = Magnitude of force on the charge +4μC due to +3μC.

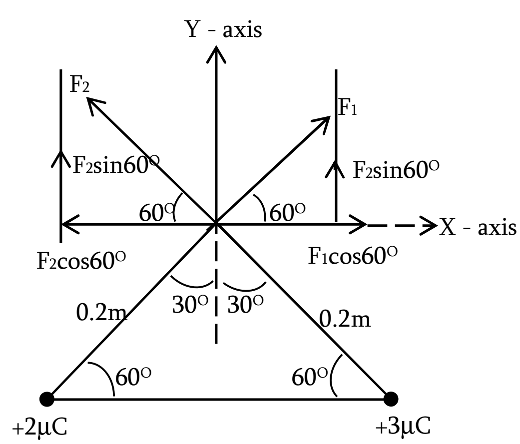
Apply coulomb's law

$$F_1 = \frac{9\times10^9\times4\times10^{-6}\times2\times10^{-6}}{(0.2)^2}$$

$F_1 = 1.8N$

$$\text{Also } F_2 = \frac{9\times10^9\times4\times10^{-6}\times3\times10^{-6}}{(0.2)^2}$$

$F_2 = 2.7N$



Net horizontal component of force

$F_X = (F_2 - F_1)\cos60^\circ$

$= (2.7 - 1.8)\cos60^\circ$

$$F_X = 0.45N \text{ (towards left)}$$

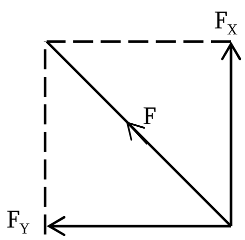
Net vertical component of force

$$F_y = (F_1 + F_2)\sin 60^o$$

$$= (2.7 + 1.8)\sin 60^o$$

$$F_y = 3.9N \text{ (Vertical upwards)}$$

Resultant force



$$F^2 = F_X^2 + F_y^2$$

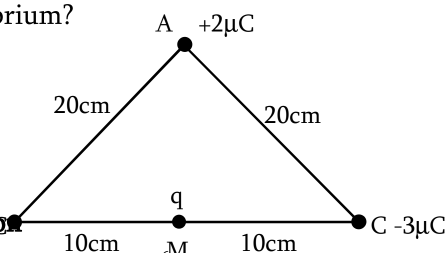
$$F = \sqrt{F_X^2 + F_y^2}$$

$$= \sqrt{(3.9)^2 + (0.45)^2}$$

$$F = 3.9N$$

Example 16

Three point charges of and are kept at the vertices A, B and C respectively of an equilateral triangle of side 20cm a shown in the figure below. What should be the sign and magnitude of the charge to be placed at the mid – point M of side BC, so that the charge at A remains in equilibrium?



Solution

$$q_A = 2\mu c = 2\times10^{-6}C$$

$$q_B = q_C = -3\mu C = -3\times10^{-6}C$$

$$AB = AC = BC = 20cm = 0.2m$$

Let q be charge placed at the point M, so that the charge at A remain in equilibrium force on charge q_A due to q_B

$$F_B = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q_A q_B}{AB^2} = \frac{9\times10^9\times2\times10^{-6}\times3\times10^{-6}}{(0.2)^2}$$

$$F_B = 1.35N \text{ (along AB)}$$

It follows that the force on charge q_A due to q_C are inclined at an < 60^O. Therefore their result is given by

$$F = \sqrt{F_B^2 + F_e^2 + 2F_B F_C \cos 60^o}$$

$$= \sqrt{(1.35)^2 + (1.35)^2 + 2\times1.35\times1.35\times0.5}$$

$$F = 2.34N \text{ (along AM)}$$

Let q be the charge placed at the point M, so that the charge at the point A remains in equilibrium. It will be so if force on charqe due to charqe q is equal and opposite to F. It follows that charge q is POSITIVE in nature.

$$\frac{1}{4\pi\varepsilon_o}\cdot\frac{q_A q}{AM^2} = 2.34$$

$$AM = \sqrt{20^2 - 10^2} = 10\sqrt{3}cm$$

$$AM = 0.1\sqrt{3}m$$

$$\frac{9\times10^9\times2\times10^{-6}\times q}{(0.1\times\sqrt{3})^2} = 2.34$$

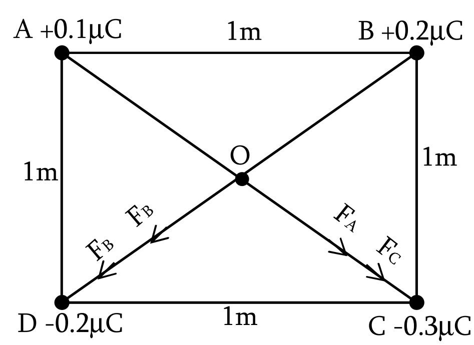
$$6\times10^5 q = 2.34$$

$$q = 0.39\times10^{-6} = 0.39\mu C$$

Example 17

Point charges having values +0.1µC, +0.2µc, -0.3µC and -0.2µC are placed at the corners A , B , C and D respectively of a square of side one metre. Calculate the magnitude of the force on the charge of +1µC placed at the centre of the square.

Solution



$$\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 \text{ (Pythagorous theorem)}$$

$$\overline{AC}^2 = 1^2 + 1^2 = 2$$

$$\overline{AC} = \sqrt{2}m$$

$$\overline{AO} = \frac{1}{2}\overline{AC} = 0.5\sqrt{2}m$$

Apply coulomb's law of electrostatics

$$F = 9\times10^9 \cdot \frac{Q_1Q_2}{r^2}$$

$$F_A = 9\times10^9 \times \frac{(0.1\times10^{-6})(1\times10^{-6})}{2(0.5)^2}$$

$$F_A = 0.0018N$$

$$F_C = \frac{9\times10^9 \times (0.3\times10^{-6})(1\times10^{-6})}{2(0.5)^2}$$

$$F_C = 0.0054N$$

Both F_A and F_C acts in the same direction.

Resultant force of F_A and F_C

$$F_1 = F_A + F_C$$
$$= 0.0018 + 0.0054$$
$$F_1 = 0.0072N$$

Again $F_B = \frac{9\times10^9 \times (0.2\times10^{-6})(10^{-6})}{2(0.5)^2}$

$$F_B = 0.0036N$$

Also $F_D = \frac{9\times10^9 \times (0.2\times10^{-6})(10^{-6})}{2(0.5)^2}$

$$F_D = 0.0036N$$

Both F_D and F_B act in the same direction. Resultant force

$$F_2 = F_B + F_D$$
$$= 0.0036 + 0.0036$$
$$F_2 = 0.0072N$$

The angle between F_1 and F_2 is 90O. So resultant force $F^2 = F_1^2 + F_2^2$, $F = \sqrt{F_1^2 + F_2^2}$

$$F = \sqrt{(0.0072)^2 + (0.0072)^2}$$

$$F = 0.01018N$$

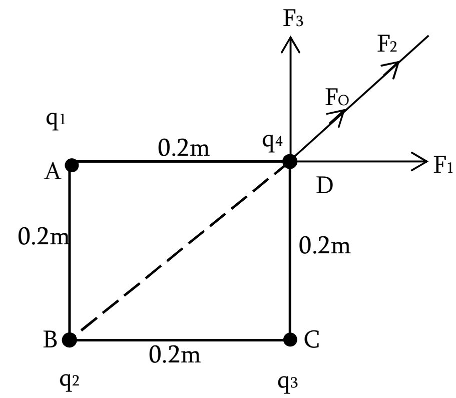
Example 18

Four equal point charges each 16μC are placed on the four corners of a square of side 0.2m. Calculate the force on any one of the charges.

Solution

Let $\overline{AB} = \overline{BC} = \overline{CD} = \overline{AD} = 0.2m$

$$q_1 = q_2 = q_3 = q_4 = 16\mu = 16\times10^{-6}C$$



Force exerted on q4 due to q1

$$F_1 = \frac{9\times10^9 \times 16\times10^{-6} \times 16\times10^{-6}}{(0.2)^2}$$

$F_1 = 57.6N$ along AD produced

Force exerted on q_4 due to q_2

$$F_2 = \frac{9\times10^9 \times 16\times10^{-6} \times 6\times10^{-6}}{2(0.2)^2}$$

$F_2 = 28.8N$, along BD produced.

Force exerted on q4 due to q3

$$F_3 = \frac{9\times10^9 \times 16\times10^{-6} \times 16\times10^{-6}}{(0.2)^2}$$

$F_3 = 57.6N$ along CD produced.

As F_1 and F_2 are perpendicular to each other, so their resultant force

$$F_o = \sqrt{F_1^2 + F_3^2} = \sqrt{(57.6)^2 + (57.6)^2}$$

$$F_o = 57.6\sqrt{2} = 81.5N$$

(In the direction of F_2)

Resultant force on q_4

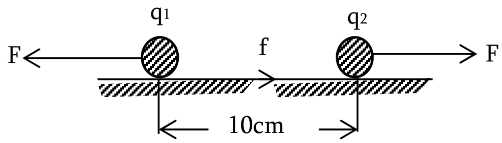
$$F = F_o + F_2 = 28.8 + 81.5$$

$F = 110.3N$, along BD produced.

Example 19

Two particles, each having a mass of 5gm and charge $1.0 \times 10^{-7}C$ stay in limiting equilibrium on a horizontal table with a separation of 10cm between them. The coefficient of friction between each particle and the table is the same. Find coefficient of friction, μ

Solution



At the equilibrium

Electrostatic force = Friction force

$$\frac{q_1 q_2}{4\pi\varepsilon_o r^2} = \mu Mg$$

$$\mu = \frac{q_1 q_2}{4\pi\varepsilon_o r^2 Mg}$$

$$\mu = \frac{9\times10^9\times\left(1\times10^{-7}\right)^2}{(0.1)^2\times5\times10^{-3}\times9.8}$$

$$\mu = 0.18$$

Example 20

Two identical sphere having charges of opposite sign attract each other with a force of 0.108N when separated by 0.5m. The spheres are connected by a conducting wire which then removed and there after they repel each other with a force of 0.036N. What were the initial charges on the sphere.

Solution

Let $+q_1$ and q_2 be the initial charges on the two spheres.

- When to sphere attract each other

$$F = 9\times10^9\cdot\frac{q_1 q_2}{(0.5)^2}$$

$$q_1 q_2 = \frac{0.108\times(0.5)^2}{9\times10^9}$$

$$q_1 q_2 = 3\times10^{-12}\ \ldots\ldots\text{(i)}$$

- When two spheres are connected by the wire, they share the charges equally charge on each sphere.

$$q = \frac{q_1 - q_2}{2}$$

Force of repulsion between them

$$F = \frac{kq^2}{r^2}$$

$$0.036 = \frac{9\times10^9}{(0.5)^2}\left[\frac{q_1 - q_2}{2}\right]^2$$

$$\left(q_1 - q_2\right)^2 = 4\times10^{-12}$$

$$q_1 - q_2 = 2\times10^{-6}\ \ldots\ldots\text{(ii)}$$

Now $\left(q_1 + q_2\right)^2 = \left(q_1 - q_2\right)^2 + 4q_1q_2$

$$\left(q_1 + q_2\right)^2 = \left(2\times10^{-6}\right)^2 + 4\times3\times10^{-12}$$

$$= 16\times10^{-12}$$

$$q_1 + q_2 = 4\times10^{-6}\ \ldots\ldots\text{(iii)}$$

On solving equation (i) and (iii)

$$q_1 = 3\times10^{-6}\text{C}\ ,\ q_2 = 10^{-6}\text{C}$$

Example 21

A charge Q is to be divided on two objects. What should be the value of the charges on the two objects so that the force between the objects can be maximum.

Solution

Let q and Q − q be the charges on the two objects.

Apply Coulomb's law of the electrostatics

$$F = \frac{q(Q-q)}{4\pi\varepsilon_o r^2} = \frac{qQ - q^2}{4\pi\varepsilon_o r^2}$$

$$\frac{df}{dq} = \frac{1}{4\pi\varepsilon_o r^2}\cdot\frac{d}{dq}\left(qQ - q^2\right)$$

$$\frac{df}{dq} = \frac{1}{4\pi\varepsilon_o}\left[Q - 2q\right]$$

When $F = F_{max}$, $\frac{df}{dq} = 0$

$$0 = \frac{1}{4\pi\varepsilon_o}\left[Q - 2q\right]$$

$$0 = Q - 2q$$

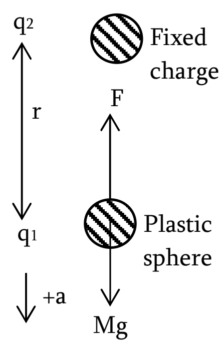
$$q = \frac{Q}{2}$$

Example 22

A small plastic sphere coated with a thin metalized surface has a mass 0.05kg and carries a charge of +8nc. It is suspended by light insulating thread at a point 3cm below the centre of a small fixed conducting sphere carrying -5nc. When the thread is cut; with what acceleration does the plastic sphere starts to fall?

Solution

Resultant force on the plastic sphere



$$Ma = Mg - F$$

$$Ma = Mg - \frac{Kq_1q_2}{r^2}$$

$$a = g - \frac{Kq_1q_2}{Mr^2}$$

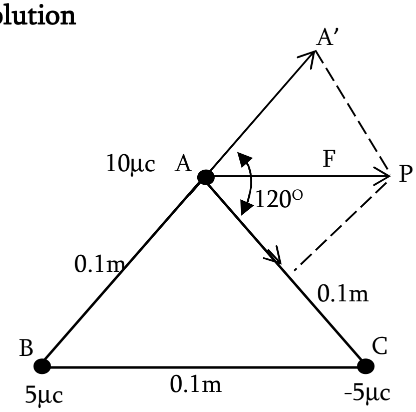
$$= 9.8 - \frac{9\times10^9\times5\times8\times10^{-18}}{0.05\times\left(3\times10^{-2}\right)^2}$$

$$a = 9.792\text{m}/\text{s}^2$$

Example 23

Three charges 10μC, 5μc and -5μC are placed in air at three corners A, B and C of an equilateral triangle of side 0.1m. Find the resultant force experienced by charge placed at corner A.

Solution



Apply coulomb's law of electrostatics

$$F_{AB} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_Aq_B}{AB^2}$$

$$= \frac{9\times10^9\times10\times10^{-6}\times5\times10^{-6}}{(0.1)^2}$$

$$F_{AB} = 45\text{N}$$

Also $F_{AC} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_Aq_C}{AC^2}$

$$= \frac{9\times10^9\times10\times10^{-6}\times5\times10^{-6}}{(0.1)^2}$$

$$F_{AC} = 45\text{N (attractive)}$$

Let F = Resultant force on charge at A

$$F = \sqrt{F_{AB}^2 + F_{AC}^2 + 2F_{AB}F_{AC}\text{Cos}120^\circ}$$

$$= \sqrt{45^2 + 45^2 + 2\times45\times45\times(-0.5)}$$

$$F = 45\text{N}$$

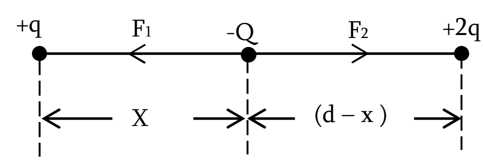
The resultant force act on the charge 10μC is long AP i.e parallel to the side BC of the ΔABC

Example 24

Two point electric charges of values q and 2q are kept at a distance d apart from each other in air. A third charge Q is to be kept along the same line in such a way that the net force acting on q and 2q is zero. Calculate the position of charge Q in terms of q and d.

Solution

For the equilibrium of charges q and 2q, the charge Q must have sign opposite to that q and 2q. Suppose it is placed at distance X from q and if q and 2q are positive and then Q is negative.



At the equilibrium, $F_1 = F_2$

$$\frac{kqQ}{X^2} = \frac{KQ2q}{(d-x)^2}$$

$$2x^2 = (d-x)^2$$

$$\sqrt{2}x = d-x$$

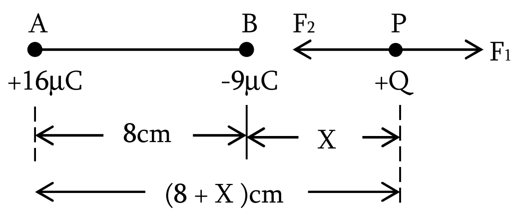
$$x = \frac{d}{1+\sqrt{2}} = \left(\sqrt{2}-1\right)d$$

Example 25

Two point charges of +16μC and -9μC are 8cm apart in air. Where can a third charge be so that no net electrostatic force act on it?

Solution

Let the third charge +Q be located at P a distance X charge -9μC as shown in the figure below.



Force at P due to the charge at A along $\overline{AP}$

$$F_1 = K\cdot\frac{16\times10^{-6}Q}{(X+0.08)^2}$$

Force at P due to the charge at B along $\overline{PB}$

$$F_2 = K\cdot\frac{9\times10^{-6}Q}{X^2}$$

At the equilibrium at P

$$F_1 = F_2$$

$$K\cdot\frac{16\times10^{-6}Q}{(X+0.08)^2} = K\cdot\frac{9\times10^{-6}Q}{X^2}$$

$$\frac{16}{(X+0.08)^2} = \frac{9}{X^2}$$

$$\frac{4}{X+0.08} = \frac{3}{X}$$

$$X = 0.24\text{m} = 24\text{cm}$$

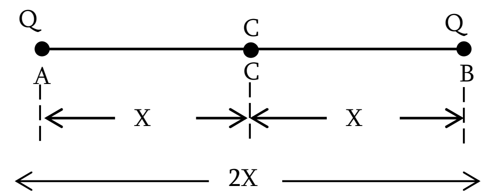
Example 26

(a) A charge q is placed at the centre of the line joining the two charges (each of magnitude Q). Prove that the system of three charges will be in equilibrium if $q = {}^{-Q}\!/_{4}$

(b) Two point charges +Q and +4Q are placed at a distance 'a' apart on a horizontal plane where should the third charge placed for it to be in equilibrium?

Solution

(a)



In the system to be at equilibrium, considering charge at B.

Resultant force when charge is at B.

$$F = \frac{KQq}{X^2} + \frac{KQ^2}{(2X)^2}$$

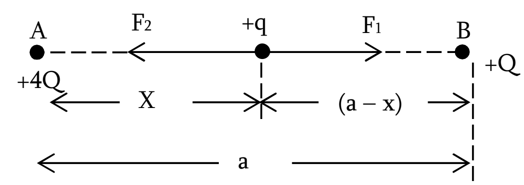
But F = 0

$$0 = \frac{KQq}{X^2} + \frac{KQ^2}{4X^2}$$

$$\frac{KQq}{X^2} = -\frac{KQ^2}{4X^2}$$

$$q = \frac{-Q}{4}$$

(b) Let the point charge +q be placed at a distance X from the charge +4Q



At the equilibrium of the system

$$F_1 = F_2$$

$$\frac{Kq(4Q)}{X^2} = \frac{KqQ}{(a-x)^2}$$

$$\frac{4}{X^2} = \frac{1}{(a-X)^2}$$

$$X = \frac{2a}{3}$$

Example 27 NECTA 1992/P4/2

The distance r between the electron and the proton in the hydrogen atom is about 5.3 × 10-11m. What is the magnitude of ;-

(a) The electrical force and
(b) The gravitational force between the two particles

Given that;
Mass of electron Me = 9.1×10^{-31}kg
Mass of proton Mp = 1.7×10^{-27}kg
Charge of an electron, e = 1.6×10^{-19}C
G = 6.7×10^{-11}Nm^{2}kg^{-2}

$$\frac{1}{4\pi\varepsilon_o} = 9\times10^9\,\text{Fm}^{-1},$$

Solution

(a) Apply the coulomb's law

$$Fe = \frac{1}{4\pi\varepsilon_o}\cdot\frac{e^2}{r^2}$$

$$= \frac{9\times10^9\times\left(1.6\times10^{-19}\right)^2}{\left(5.3\times10^{-11}\right)^2}$$

$$Fe = 8.202\times10^{-8}\,\text{N}$$

(b) Apply the Newton's universal law of gravitation.

$$F_g = \frac{GM_eM_P}{r^2}$$

$$= \frac{6.7\times10^{-11}\times9.1\times10^{-31}\times1.7\times10^{-27}}{\left(5.3\times10^{-11}\right)^2}$$

$$F_g = 3.69\times10^{-47}\,\text{N}$$

Comment

On comparing between Fe and Fg

$$\frac{F_e}{F_g} = \frac{8.202\times10^{-8}}{3.69\times10^{-47}}$$

$$\frac{F_e}{F_g} = 2.223\times10^{39}$$

$$\therefore\ F_e \gg> F_g$$

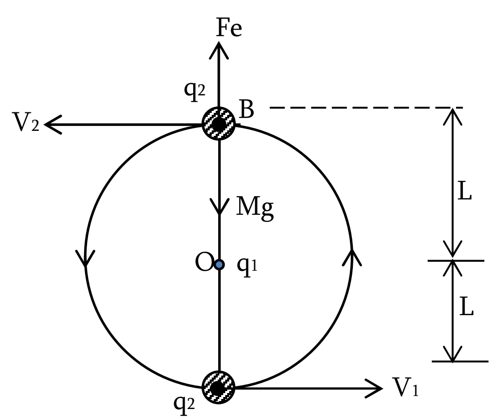
Example 28

A small ball of mass 2 × 10^{-3}kg having a charge of 1μC is suspended by a string of length 0.8m. another identical hall having the same charge is kept at the point suspension. Determine the minimum horizontal velocity, which should be imported to the lower ball so that it can make complete revolution.

Solution

Reasoning:

- The suspended charged ball will be just able to make complete revolution, if the necessary centripetal force required at the highest point of the vertical circular path is provided by the resultant of its weight and electrostatic repulsion between the charged balls.
- When the suspended charged ball goes from lowest to the highest point, the electrostatic potential energy remains the same as it rises up against gravity, the increase in g.p.e of the ball is equal to the decrease in its kinetic energy.



At the highest point B

It is requires that the tension in the string at the highest point is zero.

Resultant force on charged ball at B

$$\frac{MV_2^2}{L} = Mg - Fe \quad [r = L]$$

$$\frac{MV_2^2}{L} = Mg - \frac{1}{4\pi\varepsilon_o}\cdot\frac{q_1q_2}{r^2}$$

$$V_2^2 = gL - \frac{1}{4\pi\varepsilon_o}\cdot\frac{q_1q_2L}{ML^2}$$

$$V_2^2 = gL - \frac{1}{4\pi\varepsilon_o}\cdot\frac{q_1q_2}{ML}$$

$$= 9.8\times0.8 - \frac{9\times10^9\times10^{-6}\times10^{-6}}{2\times10^{-3}\times0.8}$$

$$V_2^2 = 2.215m^2/s^2$$

When the suspended ball goes from A to B

Increase in the g.p.e

$$g.p.e = 2MgL = 2\times10^{-3}\times2\times9.8\times0.8$$

$$g.p.e = 31.36\times10^{-3}J$$

Degrease in kinetic energy

$$\Delta k.e = \frac{1}{2}MV_1^2 - \frac{1}{2}MV_2^2 = \frac{1}{2}M\left(V_1^2 - V_2^2\right)$$

$$\Delta k.e = \frac{1}{2}\times2\times10^{-3}\left(V_1^2 - 2.215\right)$$

From conservation of mechanical energy

$$\frac{1}{2}\times2\times10^{-3}\left(V_1^2 - 2.215\right) = 31.36\times10^{-3}$$

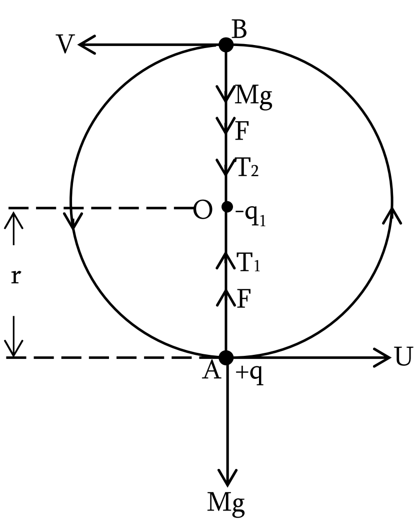
$$V_1 = 5.79m/s$$

Example 29

A ball of mass 10^{-2}kg and having charge $+3\times10^{-6}$C, is tied at one end of a 1m long thread. The other end of the thread is fixed and a charge -3×10^{-6}C is placed at this end. The ball can move in the circular orbit of radius 1m in the vertical plane. Initially, the ball is at the bottom. Find the minimum horizontal velocity of the ball so that it will be able to complete the full circle.

Solution

Let U be the minimum horizontal velocity of the ball at the bottom i.e at A and V be its horizontal velocity at the top point B.



At the point B $\frac{MV^2}{r} = Mg + F + T_2$

Starting with minimum velocity at the point A, the ball will just complete the vertical circle if the tension in the string becomes zero as the ball reaches the top B

$$\frac{MV^2}{r} = mg + \frac{1}{4\pi\varepsilon_o}\cdot\frac{q^2}{r^2}\ldots\ldots..(1)$$

Apply the law of conservation of energy for points A and B

$$\frac{1}{2}MU^2 - \frac{1}{2}MV^2 = 2Mgr$$

$$MV^2 = MU^2 - 4Mgr\ldots\ldots..(2)$$

Multiply equation (1) by r

$$MV^2 = Mgr + \frac{1}{4\pi\varepsilon_o}\cdot\frac{q^2}{r}\ldots\ldots.(3)$$

(2) = (3)

$$MU^2 - 4Mgr = Mgr + \frac{1}{4\pi\varepsilon_o}\cdot\frac{q^2}{r}$$

$$U = \left[5gr + \frac{1}{4\pi\varepsilon_o}\cdot\frac{q^2}{Mr}\right]^{\frac{1}{2}}$$

$$U = 7.62m/s$$

Example 30

Two small charged objects are attached to a horizontal spring one at each end. The magnitudes of the charges are equal and the spring constant is 220N/m. The spring is observed to be stretched by 0.02m relative to its unstrained length fo 0.32m. Determine;-

(a) Possible algebraic signs and
(b) The magnitude of the charges

Solution

(a) The charges are either positive or both negative for the spring to stretch.
(b) When the charges are at their initial position, the coulomb repulsive force between the charges causes the spring to

stretch. Since two forces are used to stretch the spring, the extension each force produce is ½ (0.02)m = 0.01m

$$\frac{kq^2}{x^2} = k_1 e$$

$$q = x\sqrt{\frac{k_1 e}{k}}$$

$$= 0.34\sqrt{\frac{220\times0.01}{9\times10^9}}$$

$$q = 5.3\times10^{-6}C$$

Example 31: 2011/P1/11

(a) (i) State coulomb's law of electrostatic (01 mark)

(ii) An α – particle has a mass of 6.68 × 10-27kg and charge of +2e. Two α – particles are situated at a distance of 1.6 × 10^{-11}m apart in vacuum. How does the electrostatic force compare with the gravitational attraction between them? (03 marks)

(b) Point charges each of +2 × 10^{-9}C are placed at each of three corners of a square of side 0.20m. What would be the magnitude and direction of the resultant force on another point charge of -1 × 10^{-9}C if it is located at the;

(i) Centre of the square? (03 marks)

(ii) Vocant corner of the square (03 marks)

e = 1.6 × 10-19C

Permittivity of free space

$$\varepsilon_o = 8.854\times10^{-12}Fm^{-1}$$

Solution

(a) (i) Refer to your notes

(ii) Electrostatic force

$$F = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q^2}{r^2}$$

But q = +2e

$$F = \frac{1}{4\times3.14\times8.854\times10^{-12}}\left[\frac{2\times1.6\times10^{-19}}{1.5\times10^{-11}}\right]^2$$

$$F = 4.0925\times10^{-6}N$$

Gravitational force.

$$F_g = \frac{GM_1M_2}{r^2}$$

$$= \frac{6.7\times10^{-11}\times\left(6.68\times10^{-27}\right)^2}{\left(1.5\times10^{-11}\right)^2}$$

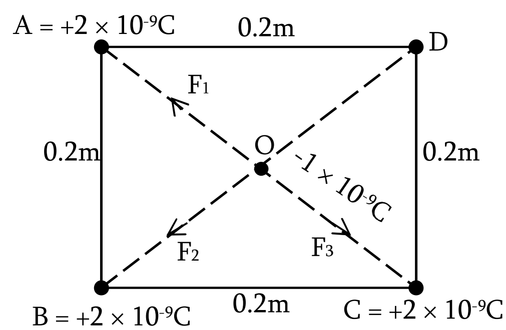
$$F_g = 1.329\times10^{-41}N$$

Now

$$\frac{F}{F_g} = \frac{4.0925\times10^{-6}}{1.329\times10^{-41}} = 3.1\times10^{35}$$

This shows that Electrostatic force is much greater than gravitational force.

(b) (i) At the center of square.



By using pythagorous theorem.

$$\overline{BD}^2 = \overline{BC}^2 + \overline{DC}^2$$

$$\overline{BD}^2 = (0.2)^2 + (0.2)^2$$

$$\overline{BD} = \sqrt{2(0.2)^2} = 0.2\sqrt{2}m$$

From the figure above

$$\overline{AO} = \overline{BO} = \overline{CO} = \overline{DO} = 0.1\sqrt{2}m$$

Let F_1 = Force on change at O due to A

F_2 = Force on charge at O due to charge at B.

F_3 = force on charge due at O due to charge at C.

Apply Coulomb's law

$$F = 9\times10^9 \cdot \frac{Q_1Q_2}{r^2}$$

$$F_1 = 9\times10^9 \times \frac{2\times10^{-9}\times1\times10^{-9}}{\left(0.1\sqrt{2}\right)^2}$$

$$F_1 = 9.0\times10^{-7}\,\text{N (toward A)}$$

$$F_1 = F_2 = F_3$$

$$F_1 = F_2 = F_3 = 9.0\times10^{-7}\,\text{N}$$

F_1 and F_3 acting in opposite direction. Then resultant force between F_1 and F_3 is equal to zero.

∴ Resultant electric force

$$F_1 = F_2 = 9.0\times10^{-7}\,\text{N}$$

(along $\overrightarrow{OB}$)

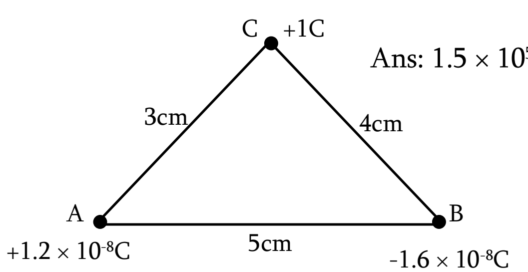
(iii) -1×10^{-9c} is located at the point D.

(Left to the student assignment)

EXERCISE NO 1

1. The electrostatic force of repulsion between two equal positively charged ions $3.7\times10^{-9}\,\text{N}$, when they are separated by a distance of $5\overset{0}{\text{A}}$. How many electrons are missing from each ion? Ans 2.

2. In the triangle ABC shown in the figure below charges of $+1.2\times10^{-8}$C, -1.6×10^{-8}C and +1C are placed at points A, B and C respectively. The lengths of the sides of the triangle are given. Find the total force on +1C charge located at C.



Ans: 1.5×10^5N

3. Charges of +2μC, +3μC and -8μC are placed at the vertices of an equilateral triangle of side 10cm. calculate the magnitude of force acting on -8μC charge due to other two charges.

4. Two small spheres, each of mass 0.20g and charge q coulomb are suspended from a point by insulating threads each of length 50cm but of negligible mass. If each string makes an angle of 37^O with the vertical when equilibrium has been reached, calculate the charge on each sphere. Ans 0.24μC

5. It is required to hold four equal point charges +q in equilibrium at the corners of a square. Find the point charge that will do this if placed at the centre of the centre of the square. Ans $-q\dfrac{2\sqrt{2}+1}{4}$

6. In hydrogen atom, an electron of mass 9.1×10^{-31}kg revolves about a proton in a circular orbit of radius $0.53\overset{0}{\text{A}}$. Calculate the centripetal acceleration and angular velocity of electron ($e = 1.6\times10^{-19}$C).

   Ans. $9.01\times10^{22}\text{m/s}^2$, 4.1×10^{16}rad/sec.

7. Two point charges of mass M, charge Q are suspended at a common point by two threads of negligible mass and length L. Show that at equilibrium the inclination angle α of each thread to the vertical is given by $Q^2 = 16\pi\varepsilon_0 MgL^2\sin^2\alpha\tan\alpha$ if α is very small, show that $\alpha = \sqrt[3]{\dfrac{Q^2}{16\pi\varepsilon o MgL^2}}$

8. A small brass sphere having a positive charge of 1.7×10^{-8}C is made totouch another sphere of the same radius having a negative charge of 3×10^{-9}C. Find the force between them when they are separated by a distance of 20cm. what will be the force between them

when they are immersed in an oil of dielectric constant 3?

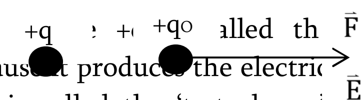
Ans. 1.1×10^{-5}N, 0.367×10^{-5}N

9. Two pith balls of mass 0.5g each are suspended from a common point O by means of silk threads, each of length 20cm. when the balls are given equal and similar charges, they repel each other so that the two threads makes an angle of 60^{O} with each other. Determine charge on each ball.

Ans. 1.12×10^{-7}C

II. ELECTRIC FIELD AND ELECTRIC FIELD INTENSITY (E)

DEFINITION ELECTRIC FIELD – Is defined as a region round electric charge where electric force is experienced or exerted. Electric fields are represented by the electric lines of force. the direction of an electric line of force gives the direction of the electric field



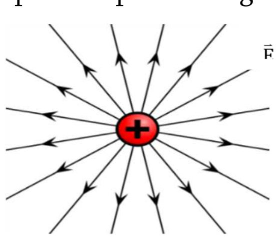
The ... alled th ... ce charge' becaus... produc... the electri... The charge +qo is called the 'test charge' ... test charge should be as small as possible so that its presence does not affect the electric field due to the source charge. Electric field is the vector quantity. The magnitude of electric field at the given point is known as **electric field intensity (E)**

ELECTRIC LINES OF FORCE

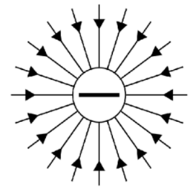
Electric line of force – is the path along which a small positive test charge would move if free to do so i.e electric lines(s) of force is a line such that tangents to it's the direction of the force on a small positive charge at that point.

Example: field lines due to some charge configuration.

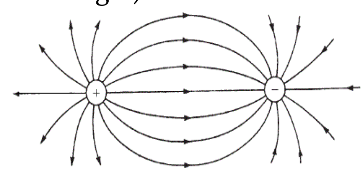
1. Single positive point charge



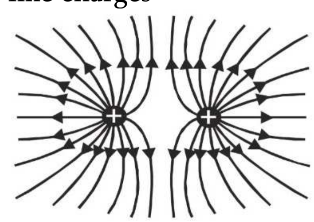
2. Single negative point charge



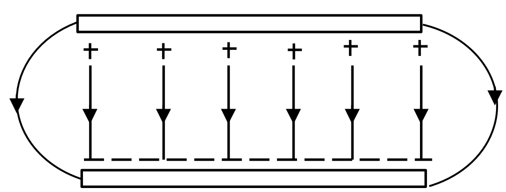
3. For unlike charges (Two equal and opposite point charges)



4. For like charges



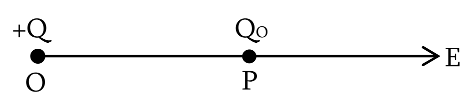
5. A pair of parallel conducting plates when one plate has a positive charges and other equal negative charges



ELECTRIC FIELD INTENSITY AT A POINT IN AN ELECTRIC FIELD.

Is defined as the electric force per unit charge exerted at the point. Is defined as the force experienced by unit positive charge placed at that point. Sometime electric field intensity is known as electric field strength and can be denoted by E

Mathematically



$$\text{Electric field intensity} = \frac{\text{Electric force}}{\text{Electric charge}}$$

$$E = \frac{F}{Q_o}$$

S.I. Unit of electric field intensity is NC^{-1} or Vm^{-1}.

Dimensional of E

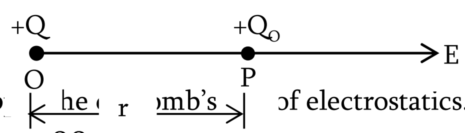
$$E = \frac{F}{Q_o}$$

$$[E] = \frac{[F]}{[Q_o]} = \frac{MLT^{-2}}{AT}$$

$$[E] = [MLT^{-3}A^{-1}] = MLT^{-3}A^{-1}$$

EXPRESSION OF ELECTRIC FIELD INTENSITY AT A GIVEN POINT OF CHARGE.

Consider a point charge +Q located at O and a small positive test charge $+Q_o$ at a distance r from O in space as shown in the figure below



Applying the coulomb's law of electrostatics.

$$F = \frac{QQ_o}{4\pi\varepsilon_o r^2}$$

Electric field strength at the point, P

$$E = \frac{F}{Q_o} = \frac{QQ_o}{4\pi\varepsilon_o r^2} \Big/ Q_o$$

$$E = \frac{Q}{4\pi\varepsilon_o r^2} \text{ in air/ space}$$

For the given medium

$$E = \frac{Q}{4\pi\varepsilon r^2} = \frac{Q}{4\pi\varepsilon_r\varepsilon_o r^2}$$

Where ε_o = permittivity of free space

ε = permittivity of given medium

ε_r = relative permittivity

Expression of electric force in terms of electric field, E

$$E = F/Q$$

$$F = EQ$$

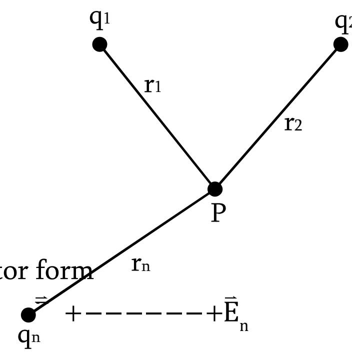
For an electron, expression of electric force is given by F = Ee

E = electric field intensity

e = electronic charge.

ELECTRIC FIELD STRENGTH DUE TO THE GROUP OF POINT CHARGES.

The resultant electric field strength due to the group of point charges at the given point can be obtained by using principle of superposition. Therefore, the electric field intensity at a point P due to n point charges (q_1, q_2……..) is equal to the vector sum of electric field due to the q_1, q_2, q_3 --------- at the point P



In vector form

$$\vec{E} = \vec{E}_1 + \text{-------} + \vec{E}_n$$

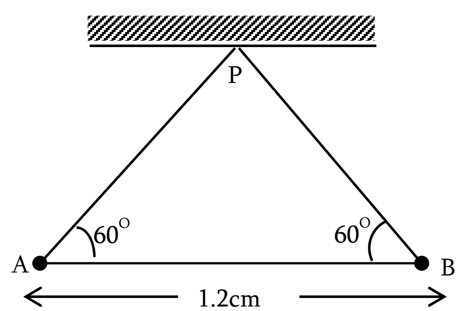
Example 32 NECTA 2016/P2/5

(a) (i) State coulomb's law of electrostatics (01 mark)

(ii) Define electric field strength E at any point (01 mark)

(iii) Mention two common properties of electric field lines (02 marks)

(b) Two identical balls each of mass 0.8kg carry identical charge and they are separated by thread of equal length. At equilibrium they positioned themselves at a distance of 1.2mcm as shown in figure 2. Calculate the charge in neither ball (05 marks)

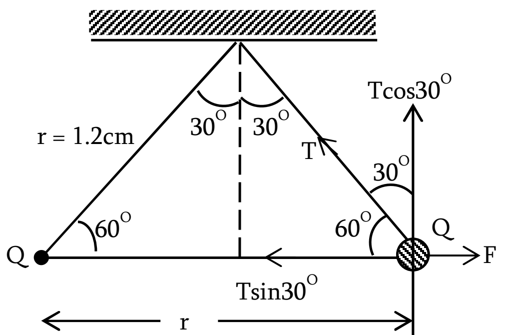


Solution

(a) (i) (ii) Refer to your notes

(iii)
- Electric lines of force start from a positive charge and end on a negative charge.
- Electric lines of force leave or enter the charged surface normally.
- Electric lines of force never intersect each other.

(b) FBD



Apply the coulomb's law

$$F = \frac{KQ^2}{r^2}$$

Apply the equilibrium

$$T\sin 30^{\circ} = FT \quad \cos 30^{\circ} = Mg$$

$$\frac{T\sin 30^{\circ}}{T\cos 30^{\circ}} = \frac{F}{Mg}$$

$$\tan 30^{\circ} = F/Mg$$

$$F = Mg\tan 30^{\circ}$$

$$\frac{KQ_2}{r^2} = Mg\tan 30^{\circ}$$

$$Q = \sqrt{\frac{Mgr^2\tan 30^{\circ}}{K}}$$

$$= \sqrt{\frac{0.8\times 9.8\times\left(1.2\times 10^{-2}\right)\tan 30^{\circ}}{9\times 10^{9}}}$$

$$Q = 2.69\times 10^{7}\,C$$

Example 33

(a) What is the electric field strength at a point 10cm away from a +6μC point charge in space (air).

(b) Assuming that the charge on an atom is distributed uniformly in a sphere of radius 10^{-10}m, what will be the electric field at the surface of gold atom? For gold, z = 79.

Solution

(a) $E = 9\times 10^{9}\cdot\frac{Q}{r^2} = \frac{9\times 10^{9}\times 6\times 10^{-6}}{(0.1)^2}$

$$E = 5.4\times 10^{6}\,N/C$$

(b) $r = 10^{-10}\,m$, $Q = Ze$

$$Q = 79\times 1.6\times 10^{-19}\,C$$

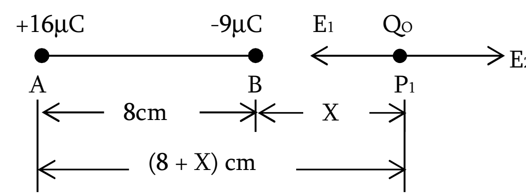
$$E = 9\times 10^{9}\frac{Q}{r^2} = \frac{9\times 10^{9}\times 79\times 1.6\times 10^{-19}}{\left(10^{-10}\right)^2}$$

$$E = 1.1376\times 10^{13}\,N/C$$

Example 34

Two point charges of +16μC and -9μC are placed 8cm apart in air. Determine the point of the point at which the result field is zero.

Solution



At the equilibrium at the point P

$$E_1 = E_2$$

$$\frac{K16}{(8+X)^2} = \frac{K9}{X^2}$$

$$\frac{16}{9} = \left[\frac{8+X}{X}\right]^2$$

$$X = 24cm$$

∴ Electric field is zero at a point 24cm to the right of -9μc charge.

Example 35

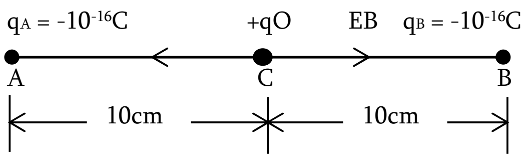
Two equal charges of -10^{-16}C each are 20cm apart in air. Calculate

(a) Electric field at a point mid – way between them.

(b) Force acting on a charge of -10^{-16}C kept at point midway between them.

Solution

(a)



Electric field intensity at C due to charge at A

$$E_A = 9\times10^9 \cdot \frac{q_A}{r^2}$$

$$= 9\times10^9 \times \frac{10^{-6}}{(0.1)^2}$$

$$E_A = 9\times10^5 N/C \text{ (toward left)}$$

$$E_B = 9\times10^5 N/C \text{ (toward right)}$$

Resultant electric field at C

$$E = E_A - E_B$$

$$E = 0$$

(b) Electric force $F = E_{qo}$

$$F = O$$

Example 34

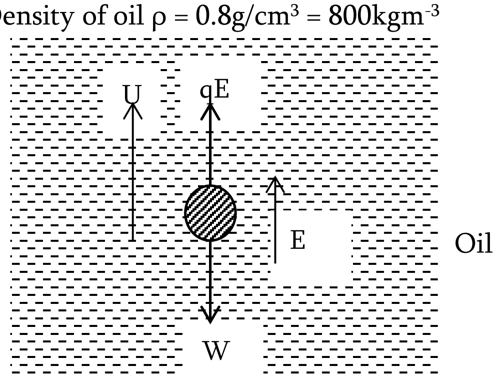
A copper ball of density 8.6gcm^{-3}, 1cm in diameter is immersed in oil o density 0.8gcm^{-3}. What is the charge on the ball, if it remains just suspended in oil in an electric field of intensity 3600Vm^{-1} acting in the inward direction.

Solution

Radius of copper ball $R = \frac{1}{2}\text{cm} = 5\times10^{-3}\text{m}$

Density of copper ball $\rho = 8.6\text{g/cm}^3 = 8600\text{kgm}^{-3}$

Density of oil $\rho = 0.8\text{g/cm}^3 = 800\text{kgm}^{-3}$



At the equilibrium

$$U + qE = W$$

$$qE = W - U$$

$$= \frac{4}{3}\pi R^3 \rho g - \frac{4}{3}\pi R^3 \rho g$$

$$qE = \frac{4}{3}\pi R^3 g(\rho - \delta)$$

$$q = \frac{4\pi R^3 g(\rho - \delta)}{3E}$$

$$= \frac{4\times3.14\times(5\times10^{-3})^3\times9.8(8600-800)}{3\times3600}$$

$$q = 1.11\times10^{-5}C$$

Example 35

A charged dust particle of radius 5 × 10^{-7}m is located in a horizontal electric field having on a intensity of 6.28 × 105Vm^{-1}. The surrounding medium is air with coefficient of viscosity $\eta = 1.6\times10^{-5}Nsm^{-2}$. If the particle moves with a uniform horizontal speed of 0.02m/s. Find the number of electrons on it.

Solution

When the charged dust particle moves with uniform speed,

Electric force = Force of viscosity of air

On dust particle

$$qE = 6\pi\eta vr$$

But $q = Ne$

$$NeE = 6\pi\eta vr$$

$$N = \frac{6\pi\eta vr}{eE}$$

$$= \frac{6\times3.14\times1.6\times10^5\times5\times10^{-7}\times0.02}{1.6\times10^{-19}\times6.28\times10^5}$$

$$N = 30$$

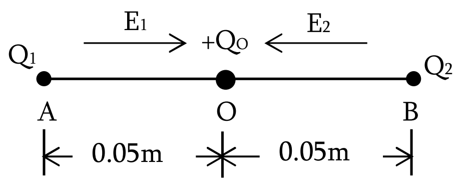
Example 36

Two point charges $Q_1 = +0.2C$ and $Q_2 = +0.4C$ are placed 0.1m apart. Calculate the electric field at

(a) The mid – point between the charges.
(b) A point on the line joining Q_1 and Q_2 such that it is 0.05m away from Q_2 and 0.15m away from Q_1.

Solution

(a) Let O be the mid – point between the two charges.



Electric field at O due to Q_1

$$E_1 = \frac{KQ_1}{r^2} = \frac{9\times10^9\times0.2}{(0.05)^2}$$

$$E_1 = 7.2\times10^{11}\,N/C\left(\text{acting along } \overline{AO}\right)$$

Electric field at O due to Q_2

$$E_2 = \frac{KQ_2}{r^2} = \frac{9\times10^9\times0.4}{(0.05)^2}$$

$$E_2 = 14.4\times10^{11}\,N/C\left(\text{along } \overline{BO}\right)$$

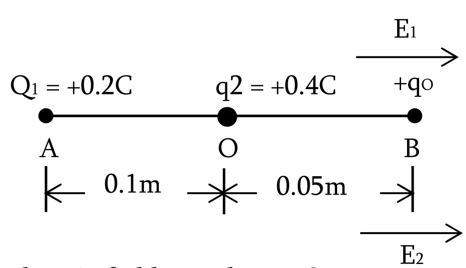
Resultant electric field at O

$$E = E_2 - E_1$$

$$= (14.4-7.2)\times10^{11}$$

$$E = 7.2\times10^{11}\,N/C \text{ acting along BO}$$

(b)



Electric field at P due to Q_1

$$E_1 = \frac{KQ_1}{r_1^2} = \frac{9\times10^9\times0.2}{(0.15)^2}$$

$$E_1 = 8.0\times10^{10}\,N/C(\text{along AP})$$

Electric field at P due to Q_2

$$E_2 = \frac{KQ_2}{r_2^2} = \frac{9\times10^9\times0.4}{(0.05)^2}$$

$$E_2 = 1.44\times10^{12}\,N/C\ (\text{along AP})$$

Resultant electric field at P

$$E = E_1 + E_2$$

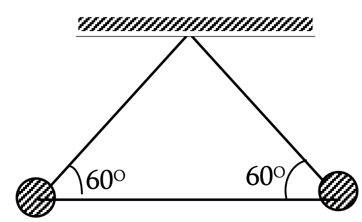
$$= 1.44\times10^{12} + 8\times10^{10}$$

$$E = 1.52\times10^{12}\,N/C\ (\text{along AP})$$

Example 37 NECTA 1996/P1/12

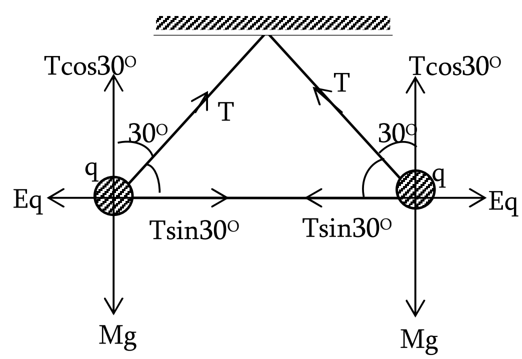
Two polythene ball each of mass 0.1gm has in contact with each other from silk thread figure.

(i) Below shown, they are charged. Whey they are given the same charge of 2μC each of the two ball they fly apart. In equilibrium their threads makes an angle of 60° with each other. Calculate the electric field strength between charged ball.



Solution

Consider the FBD as shown below.



At the equilibrium on either charged ball

$$E_q = T\sin30^\circ \text{ and } Mg = T\cos30^\circ$$

$$\frac{E_q}{Mg} = \frac{T\sin30^\circ}{T\cos30^\circ} = \tan30^\circ$$

$$E_q = Mg\tan30^\circ$$

$$E = \frac{Mg\tan30^\circ}{q}$$

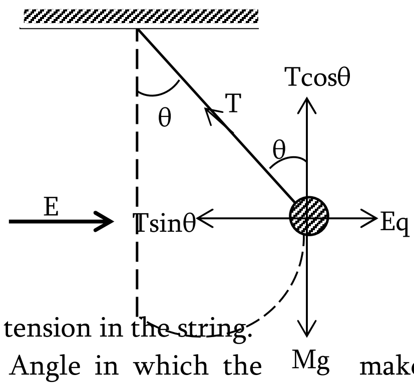
$$= \frac{0.1\times10^{-3}\times9.8\tan30^\circ}{2\times10^{-6}}$$

$E = 282.90 N/C$

Example 38

A pendulum of mass 80 milligram carrying a charge of $2.0\times10^{-8} c$ is at rest in a horizontal uniform electric field of $2.0\times10^{4} Vm^{-1}$. Find the tension in the thread of the pendulum and angle it makes with the vertical

Solution.



Let T = tension in the string.

θ = Angle in which the makes with vertical.

When the bob is in equilibrium

$$T\sin\theta = E_q,\ T\cos\theta = M_g$$

$$\frac{T\sin\theta}{T\cos\theta} = \frac{E_q}{M_g}$$

$$\tan\theta = \frac{E_q}{M_g}$$

$$\tan\theta = \frac{2\times10^{-8}\times2\times10^{4}}{80\times10^{-6}\times9.8} = 0.51$$

$$\theta = 27^\circ$$

$$\text{Also } T = \frac{qE}{\sin\theta} = \frac{2\times10^{-8}\times2\times10^{4}}{\sin27^\circ}$$

$$T = 8.81\times10^{-4} N$$

Example 39

A simple pendulum consists of a small sphere of mass M suspended by a thread of length L. The sphere carries a positive charge q. The pendulum is placed in a uniform electric field of strength E directed vertically upwards. With what period will the pendulum oscillate if the electrostatic force acting on the sphere is less than the gravitational force? Assume the oscillation to be small.

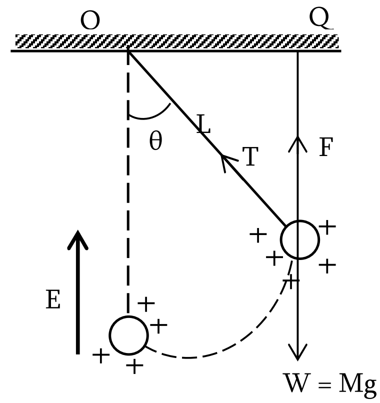
Solution

The available forces are :-

(i) Weight of sphere W = Mg

(ii) Electric force F = Eq in direction of E

(iii) Tension in the thread



Since F<W

Torque due to W about O.

$T_2 = T(\overline{OQ})$ anticlockwise.

Torque due to T about O is zero.

Net torque.

$$I = T_1 - T_2$$

$$I = (W-F)\overline{OQ} \text{ clockwise.}$$

Since torque is in opposite to θ i.e the effect torque is to bring the bob at the equilibrium.

$$I = -(W-F)\overline{OQ}$$

$$\text{But } \sin\theta = \frac{\overline{OQ}}{L},\ \overline{OQ} = L\sin\theta$$

$$I = -(W-F)L\sin\theta.$$

If θ is very small angle measured in radian.

$$\sin\theta \simeq \theta$$

$$I = -(W-F)LQ$$

$$\frac{Id^2\theta}{dt^2} = -(W-F)LQ$$

$$\frac{d^2\theta}{dt^2} = \frac{-(W-F)LQ}{I}$$

$$\frac{d^2\theta}{dt^2} = \frac{-(W-F)LQ}{ML^2}$$

$$\frac{d^2\theta}{dt^2} = \frac{-(Mg-Eq)\theta}{ML}$$

Then $\frac{d^2\theta}{dt^2} \infty -Q$. Hence the system execute angular S.H.M

For angular S.H.M

$$\frac{d^2\theta}{dt^2} = -W^2\theta$$

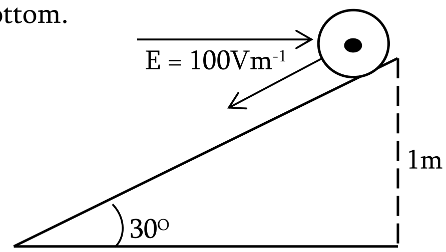
$$-\omega^2\theta = -\left[g - \frac{Eq}{M}\right]\frac{Q}{L}$$

$$\omega = \sqrt{\frac{g - q\left(\frac{E}{M}\right)}{L}}$$

$$T = \frac{2\pi}{W} \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g - Eq/m}}$$

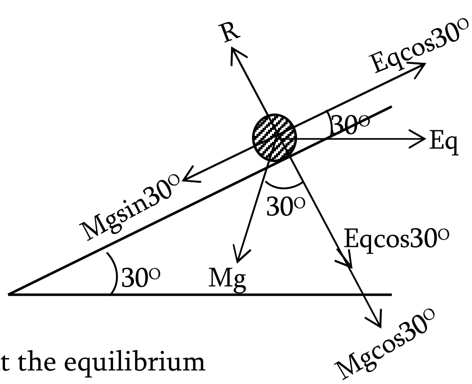
Example 40

An inclined plane making an angle of 30° with the horizontal is placed in a uniform horizontal electric field of $100Vm^{-1}$ see figure below. A particle of mass 1Kg and charge 0.01C is allowed to slide down from rest from a height of 1m. If The coefficient of friction is 0.2. Find the time it will take for the particle to reach the bottom.



Solution

Consider FBD



At the equilibrium

$$R = qE\sin 30^\circ + Mg\cos 30^\circ$$

Frictional force on the particle

$$F = \mu R = \mu\left(Mg\cos 30^\circ + qE\sin 30^\circ\right)$$

$$= 0.2\left[1\times 9.8\cos 30^\circ + 0.01\times 100\sin 30^\circ\right]$$

$$F = 1.799N$$

Resultant force on the particle

$$Ma = Mg\sin 30^\circ - qE\cos 30^\circ - F$$

$$a = \frac{Mg\sin 30^\circ - qE\cos 30^\circ - F}{M}$$

$$= \frac{1\times 9.8\sin 30^\circ - 0.01\times 100\cos 30^\circ - 1.799}{1}$$

$$a = 2.235m/s^2$$

If S is the actual distance travelled by the particle.

$$\sin 30^\circ = \frac{1}{S}, \quad S = 2m$$

$$\text{Now}, \ S = ut + \frac{1}{2}at^2$$

Particle starting from the rest

$$U = 0$$

$$S = \frac{1}{2}at^2$$

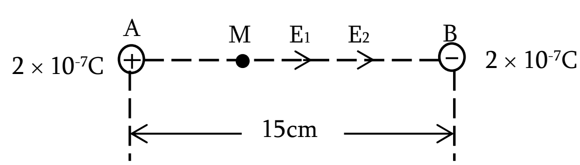
$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\times 2}{2.235}}$$

$$t = 1.34\sec$$

Example 41

Two equal and opposite charges of magnitude $2\times 10^{-7}C$ are placed 15cm apart. Find the magnitude of electric field intensity at a point a mid-way between the charges. What force would acts on a proton (charge = $+1.6 \times 10^{-19}C$) placed there?

Solution



M is the mid – point of AM and BM electric field strength at M due to charge at A

$$E_1 = 9\times 10^9 \times \frac{2\times 10^{-7}}{(0.075)^2}$$

$$E_1 = 0.32\times10^6 N/C \text{ along AM}$$

Electric field strength at M due to charge at B

$$E_2 = 9\times10^9\times\frac{2\times10^{-7}}{(0.075)^2}$$

$$E_2 = 0.32\times10^6 N/C \text{ along MB}$$

Resultant electric field strength at M

$$E = E_1 + E_2$$

$$= (0.32+0.32)\times10^6$$

$$E = 0.64\times10^6 N/C \text{ along MB}$$

Force on the proton at M

$$F = E_q$$

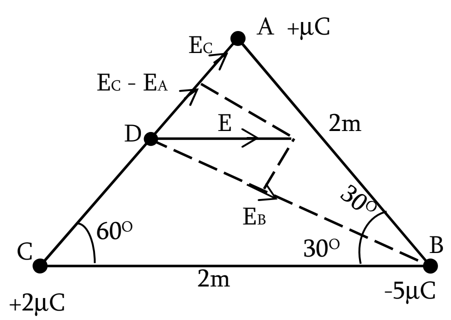
$$= 0.64\times10^6\times1.6\times10^{-19}$$

$$F = 1.024\times10^{-13} N \text{ along MB}$$

Example 42

Point charges having values $+2\mu c$ are placed at corners A, B and C respectively of an equilateral triangle of side 2m in free space. Determine the magnitude of intensity at the point D mid-way between A and C.

Solution



Since $E = 9\times10^9\cdot\frac{Q}{r^2}$

The electric field intensity EA at D due to the charge at A.

$$\overline{AD} = \frac{1}{2}\overline{AC} = \frac{1}{2}\times2m$$

$$\overline{AD} = 1m$$

$$E_A = 9\times10^9\times\frac{10^{-6}}{12}$$

$$E_A = 9\times10^3 N/C$$

The electric field intensity Ec at D due to the charge at C.

$$E_C = 9\times10^9\times\frac{2\times10^{-6}}{12}$$

$$E_C = 18\times10^3 N/C$$

The magnitude of the resultant of Ec and EA is given by

$$E_C - E_A = (18-9)\times10^3$$

$$E_C - E_A = 9\times10^3 N/C$$

Consider ΔCDB

$$\cos30^o = \frac{BD}{2}$$

$$\overline{BD} = 2\cos30^o$$

The electric field intensity EB at D due to the charge at B is given by

$$E_B = 9\times10^9\times\frac{5\times10^{-6}}{(\sqrt{3})^2}$$

$$E_B = 15\times10^3 N/C$$

Let E = Magnitude of resultant electric field intensity.

$$E^2 = E_B^2 + (E_C - E_A)^2$$

$$E = \sqrt{E_B^2 + (E_C - E_A)^2}$$

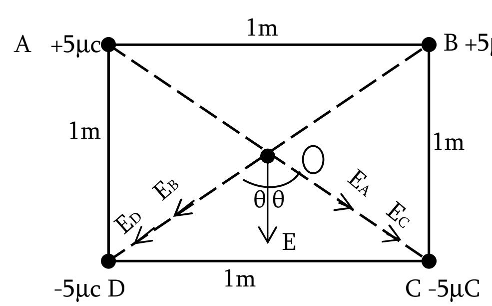
$$= \sqrt{(15\times10^3)^2 + (9\times10^3)^2}$$

$$E = 1.749\times10^4 N/C$$

Example 43

Four charges $+5\mu c$, $+5\mu c$, $-5\mu c$ and $-5\mu c$ are placed at the corners A,B,C and D of a square of side 1m. Calculate the electric field intensity at the centre of the square.

Solution



Now $\overline{AO} = \overline{BO} = \overline{CO} = \overline{DO}$

By using pythagorous theorem

$$\overline{AC} = \sqrt{\overline{DC}^2 + \overline{DA}^2} = \sqrt{1^2 + 1^2}$$

$$\overline{AC} = \sqrt{2}\text{m}$$

But $\overline{OA} = \frac{\overline{AC}}{2} = \frac{\sqrt{2}}{2} = 0.707\text{m}$

Electric field intensity at O due to the charge at A

$$E_A = 9\times10^9 \times \frac{5\times10^{-6}}{(0.707)^2}$$

$E_A = 9\times10^4 \text{N/C along } \overline{AO}$

Similarly

$E_A = E_B = E_C = E_D$

$E_B = 9\times10^4 \text{N/C along } \overline{BO}$

$E_C = 9\times10^4 \text{N/C along } \overline{OC}$

$E_D = 9\times10^4 \text{N/C along } \overline{OD}$

Then $E_1 = E_A + E_C = 18\times10^4 \text{N/C along } \overline{OC}$

$E_2 = E_B + E_D = 18\times10^4 \text{N/C along } \overline{OD}$

Resultant electric field intensity

$$E = 2E_1 \cos 45^o$$

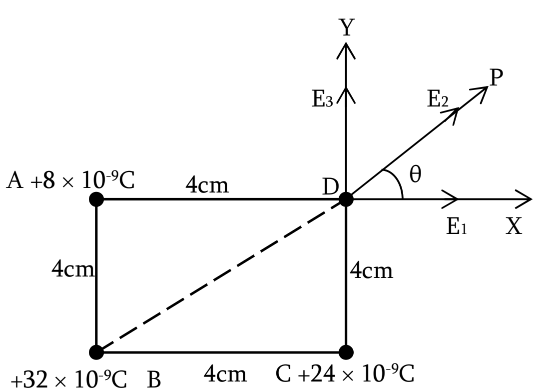
$$= 2\times18\times10^4 \times \cos 45^o$$

$$E = 25.45\times10^4 \text{N/C}$$

Resultant electric field intensity acts vertically downwards parallel to AD and BC.

Example 44

Three point charges of $+8 \times 10^{-9}$C, $+32 \times 10^{-9}$C and $+24 \times 10^{-9}$C are placed on the corners A, B and C of a square ABCD having each side 4cm. Find the electric field intensity at the corner D. Assume that the medium is air.



Solution

By using pythagorous theorem

$$\overline{BD} = \sqrt{BC^2 + CD^2}$$

$$= \sqrt{(0.04)^2 + (0.04)^2}$$

$$\overline{BD} = 0.0566\text{m}$$

Magnitude of the electric field intensity at D due to charge $+8.0 \times 10^{-9}$C is

$$E_1 = 9\times10^9 \times \frac{8\times10^{-9}}{(0.04)^2}$$

$E_1 = 4.5\times10^4 \text{N/C}$ along DX magnitude of electric field intensity at D due to charge $+32 \times 10^{-9}$C is

$$E_2 = 9\times10^9 \times \frac{32\times10^{-9}}{(\sqrt{2}\times0.04)^2}$$

$E_2 = 9\times10^4 \text{N/C}$ along DP magnitude of electric field at D due to charge $+24 \times 10^{-9}$C

$$E_3 = 9\times10^9 \times \frac{24\times10^{-9}}{(0.04)^2}$$

$E_3 = 13.5\times10^5 \text{N/C}$ along DY

Resolve horizontally

$$E_X = E_1 + E_2 \cos\theta$$

$$= 4.5\times10^4 + 9\times10^4 \cos 45^o$$

$$E_X = 19.86\times10^4 \text{N/C}$$

Resolve vertically

$$E_y = E_3 + E_2 \sin 45^o$$

$$= 9\times10^4 \sin 45^o + 13.5\times10^4$$

$$E_y = 19.86\times10^4 \text{N/C}$$

Resultant electric field intensity

$$E = \sqrt{E_X^2 + E_y^2}$$

$$= \sqrt{(10.86\times10^4)^2 + (19.86\times10^4)^2}$$

$$E = 22.63\times10^4 \text{N/C}$$

Direction of E

$$\beta = \tan^{-1}\left[\frac{E_y}{E_X}\right]$$

$$= \tan^{-1}\left[\frac{19.86\times10^4}{10.86\times10^4}\right]$$

$$\beta = 61.32^o$$

Example 45

(a) Define electric field strength and state its S.I unit.

(b) How is the direction of the electric field is specified.

Solution

(c) The direction of the electric field at appoint is specified by the direction of the force that would act on a positive charge placed at that point.

Example 46

(a) If the distance from a point charge is double, how is the field strength is charged?

(b) A point charge of $+2.0 \times 10^{-7}$C is situated 40mm from a point charge of -3×10^{-7}C in a vacuum. What is the field strength mid-way between two charges? $\varepsilon_o = 8.85\times10^{-12}C^2N^{-1}m^2$.

(c) A point charge +q is placed 20mm from another point charge of -2q in a vacuum. At what position is the electric field strength to be zero?

Solution

(a) Since $E = \frac{KQ}{r^2}$

Let E_1 = initial electric field when $r = r_1$

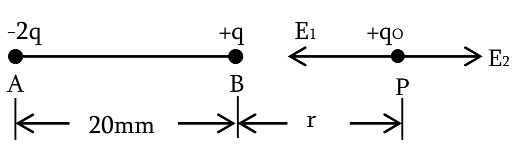
E_2 = Electric field when $r_2 = 2r$

$$E_1 = \frac{KQ}{r^2}, \quad E_2 = \frac{KQ}{(2r)^2}$$

$$E_2 = \frac{KQ}{4r^2} = \frac{E_1}{4}$$

Therefore it reduced to a quarter of its initial value

(b)



Resultant electric field at O

$$E = E_1 + E_2$$

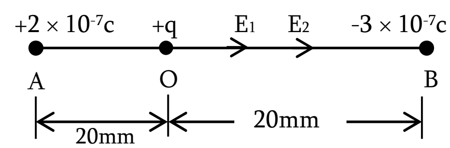
$$= \frac{KQ_1}{r^2} + \frac{KQ_2}{r^2}$$

$$= \frac{k}{r^2}\left[Q_1 + Q_2\right]$$

$$= \frac{9\times10^9}{\left(20\times10^{-3}\right)^2}\left[2+3\right]\times10^{-7}$$

$$E = 11.25\times10^6 N/C$$

(c)



At the equilibrium at point P

$$E_1 = E_2$$

$$\frac{kq}{r^2} = \frac{k(2q)}{(20+r)^2}$$

$$2r^2 = (20+r)^2$$

On solving r = 48

$\therefore$ The point is 48mm from the +q charge and 68mm from -2q

Example 47

(a) Can two like charges attract each other?

(b) Is it possible to produce a high voltage on our body without getting shock?

(c) A bird perches on a bare high power line and nothing happens on the bird. A man standing on the ground touches the same line gets a fatal shock. Why?

Solution.

(a) Yes. If one charge is very large and is placed very close to each other than the larger charge will induce an opposite and equal charge on the other body. The induced charge is greater than the charge present on the body, so there will be attraction.

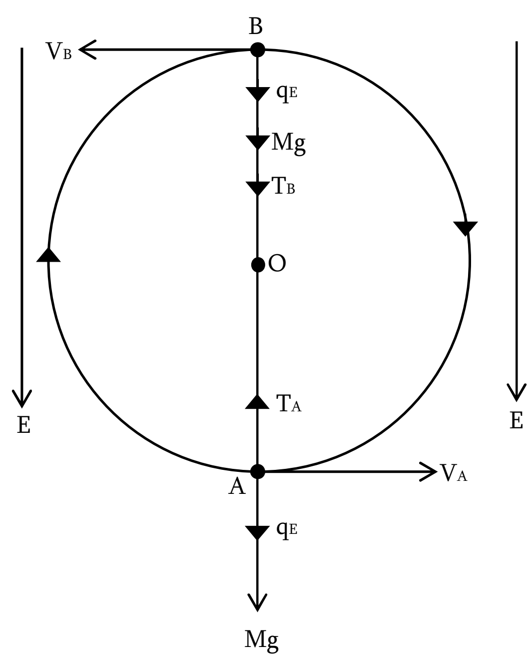
(b) Touch a high voltage line wire by wearing shoes with insulating soles. The body voltage will be high. But the potential difference between the body and the line wire is zero.

(c) No current passes through the bird because the circuit is not completed and it body is at the same voltage as the line wire. When a man touches the same line the circuit is competed. There is a large potential difference between his hand and feet. A large current flows through his body and he gets a fatal shock. But if the man wears shoes with insulating he will get a shock.

Example 48

A uniform electric field of strength $10^6 Vm^{-1}$ is directed vertically downwards. A particle of mass 0.01Kg and charge 10^{-6} Coulomb is suspended by an inextensible thread of length 1m. The particle is displaced slightly from its mean position and released. Calculate the true period of its oscillation. What minimum velocity should be given to the particle at rest so that it competes a full circle in a vertical plane without the thread getting slack? Calculate the maximum and the minimum in this situation.

Solution



Downward acceleration due to the electric field

$$a = \frac{qE}{M} = \frac{10^{-6}\times10^{6}}{0.01}$$

$$a = 100 m/s^2$$

Total acceleration which acting downwards on the particle.

$$g' = g + a = 9.8 + 100$$

$$g' = 109.8 m/s^2$$

Periodic of oscillation of the charged particle will be

$$T = 2\pi\sqrt{\frac{L}{g'}} = \frac{2\times22}{7}\sqrt{\frac{1}{109.8}}$$

$$T = 0.6 sec$$

For motion in a vertical circle minimum velocity required at the lowest point.

$$V_{min} = \sqrt{5g'L} = \sqrt{5\times109.8\times1}$$

$$V_{min} = 23.43 m/s$$

This happens when tension at the highest point

$$T_A = T_{min} = 0$$

Tension at the lowest point in that case would be

$$T_A = T_{max} = 6Mg$$

$$= 6\times0.01\times109.8$$

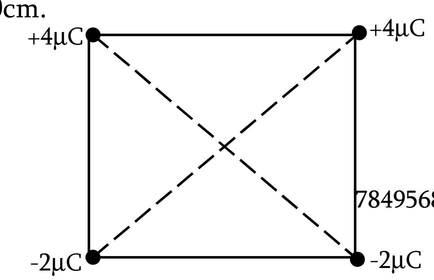
$$T_{max} = 6.588N$$

EXERCISE NO 2

1. A free pith ball of mass 6g carries a positive charge of $\frac{1}{3}\times10^{-7}C$ .What is the nature and magnitude of charge that should be given to a second pith ball fixed 5cm vertically below the farmer pith ball so that the upper pith ball is stationary? Ans. POSITIVE CHARGE $4.9\times10^{-7}C$

2. (a) Three point charges of $+0.33\times10^{-8}C$, $+0.33\times10^{-8}C$ and $0.165\times10^{-8}C$ are at the point A,B and C of a square ABCD. Find the electric field at the corner D.

   (b) Find the electric field intercity at the centre of the square shown in the figure below. Give that diagonal of square is 60cm.



784956894)

Ans. (a) $1.63 \times 10^5 N/C$ (b) $8.5 \times 10^5 N/C$

3. Two point charges of $+5 \times 10^{-19}C$ and $+20 \times 10^{-19}C$ are separated by a distance 2m. Find the point on the line joining them at which electric field intensity is zero. Ans $^2/_3$ m

ELECTRICITY LINES OF FORCE

An electric line of force is the path along which a unit positive charge would move, ii it is free to do so i.e A line of force is a curve drawn in a field such that the tangent to it any point gives the direction of the field at that points

PROPERTIES OF ELECTRIC LINES OF FORCE

1. The lines of force start from the positive charge and end at the negative charge.
2. The lines of force originate (from a positive charge) or terminate (at a negative charge) always at right angles to the surface of the charge.
3. The lines of force never intersect each other.
4. The lines of force do not pass through a conductor. It indicates that the electric field inside a conductor is always zero.
5. The relative closeness of lines of force in different regions of space gives the idea about the relative strengths of the electric field in the different regions.
6. The lines of force contract longitudinally ie lengthwise. This property of lines of force leads to explain attraction between two unlike charges.
7. The lines of force exert a lateral pressure on each other. This property of lines of force leads to explain the repulsion between two like charges.

IMPORTANCE OF ELECTRIC LINES OF FORCE

The electric lines of force can be used to represent the electric field due to appoint charge or a system of point charges. It may be noted that through the lines of force are imaginary, the electric field they represent is real.

UNIFORM AND NON-UNIFORM ELECTRIC FIELD.

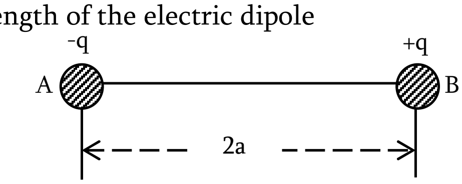
- Uniform electric field means that the magnitude and direction of the electric field intensity (E) are same everywhere.
  Note that:
  electric field between the plates of a charged parallel plate capacitor is nearly uniform.
- Non-uniform electric field means that the magnitude and direction of the electric field strength varies from one point towards to the another point.

ELECTRIC DIPOLE

Is a system of two equal and opposite charges separated by a small or certain distance.

ELECTRIC DIPOLE MOMENT

Is defined as the product of either charge and the length of the electric dipole



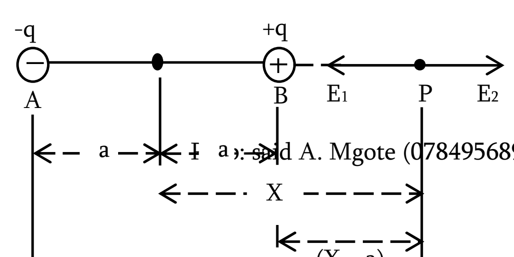
Let P = electric dipole moment

$P = 2qa$

S.I. Unit of electric dipole moment is Coulomb metre (Cm). Electric dipole moment is the vector quantity $\vec{P} = q(\overline{2a})$

1. **ELECTRIC FIELD ON AN AXIAL LINE OF AN ELECTRIC DIPOLE.**

   Consider an electric dipole consisting of charges –q and +q, separated by a distance of 2a and places in free space. Let p be a point on the axis at a distance X from centre of the dipole



E_1 = Electric field at P due to –q
E_2 = Electric field at P due to +q

Now $E_1 = \frac{kq}{(a+x)^2}$, $E_2 = \frac{kq}{(x-a)^2}$

Resultant electric field intensity at P.

$$E = E_2 - E_1$$

$$= \frac{kq}{(x-a)^2} - \frac{kq}{(x+a)^2}$$

$$= kq\left[\frac{1}{(x-a)^2} - \frac{1}{(x-a)^2}\right]$$

$$= kq\left[\frac{(x+a)^2-(x-a)^2}{(x-a)^2(x+a)^2}\right]$$

$$= kq\left[\frac{(x^2+2ax+a^2)-(x^2-2ax+a^2)}{(x^2-a^2)^2}\right]$$

$$E = kq\frac{4ax}{(x^2-a^2)^2} \quad \text{but } K = \frac{1}{4\pi\varepsilon_o}$$

$$E = \frac{1}{4\pi\varepsilon_o}\cdot\frac{4qax}{(x^2-a^2)^2}$$

Let P = 2qa

$$E = \frac{1}{4\pi\varepsilon_o}\cdot\frac{2PX}{(x^2-a^2)^2}$$

Special case: when dipole is of very small length i.e a <<< x

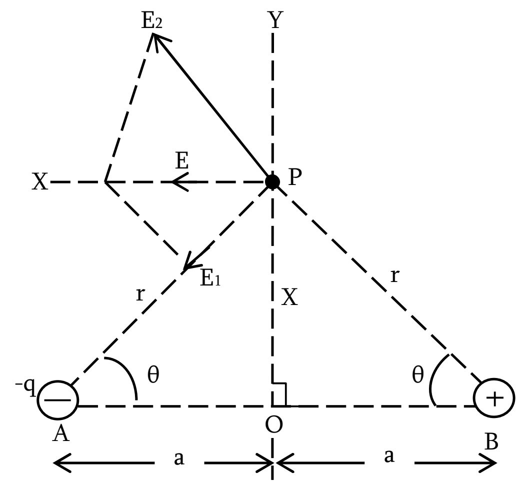
$$x^2 - a^2 \simeq x^2$$

$$E = \frac{1}{4\pi\varepsilon_o}\cdot\frac{2PX}{X^4}$$

$$E = \frac{1}{4\pi\varepsilon_o}\cdot\frac{2P}{X^3}$$

2. **ELECTRIC FIELD ON EQUITORIAL LINE OF AN ELECTRIC DIPOLE**

Let p be a point on equatorial line of the dipole (right bisector of the length of dipole) at a distance X from the centre of the dipole.



By using pythagorous theorem

$$r^2 = a^2 + x^2$$

$$r = (a^2+x^2)^{\frac{1}{2}}$$

$$\cos\theta = \frac{a}{r} = \frac{a}{(x^2+a^2)^{\frac{1}{2}}}$$

Now $E_1 = E_2 = \frac{Kq}{r^2}$

Resolve E_1 and E_2 horizontally

$$E = E_1\cos\theta + E_2\cos\theta$$

$$E = 2E_1\cos\theta$$

$$E = \frac{2kq\cos\theta}{r^2}$$

$$E = 2kq\cdot\frac{a}{r^3}$$

$$E = \frac{1}{4\pi\varepsilon_o}\cdot\frac{2qa}{(a^2+x^2)^{\frac{3}{2}}}$$

Let 2qa = P

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{P}{\left(a^2 + x^2\right)^{\frac{3}{2}}}$$

Since a <<< x

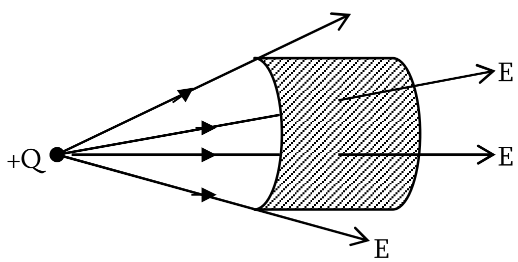
$$E = \frac{P}{4\pi\varepsilon_o X^3}$$

Examples of electric dipole

Water (H_2O), Chloroform ($CHCL_3$), Ammonia (NH_3)

ELECTRIC FLUX AND GAUSS'S LAW

ELECTRIC FLUX - Is the measure of the number of electric lines of forces passing through a given surface.



QUANTITATIVELY DEFINITION

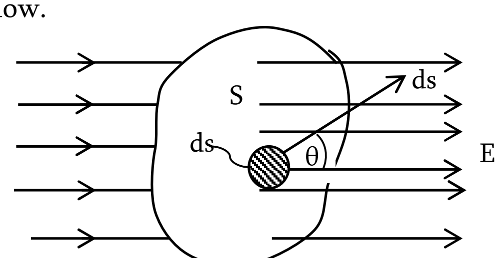
Electric flux - is defined as the product of the electric field strength and the perpendicular area, A.

$$\Phi_E = E \cdot A$$

Generally, electric flux

$$\Phi_E = EA\cos\theta$$

Consider the closed surface where by the electric field passed through the surface as shown figure below.



$$d\Phi_E = E\cos\theta ds$$

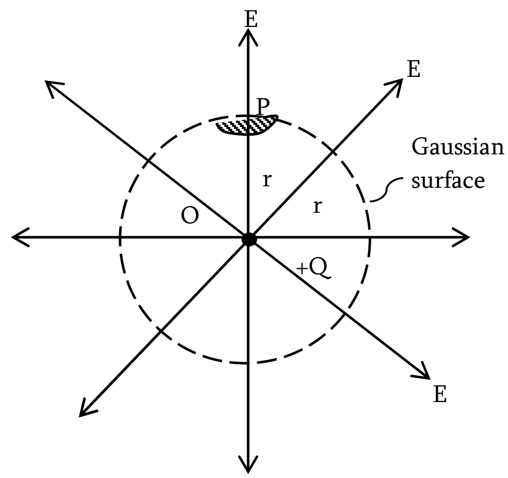
$$\oint d\Phi_E = \oint E\cos\theta ds$$

$$\Phi_E = EA\cos\theta$$

S.I. unit of the electric flux is Nm^2/C

ELECTRIC FLUX FROM POINT OF CHARGE

Consider a sphere of radius r drawn in space enclosed with positive charge Q in a free space.



Electric field intensity at the point P

$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

Assume that the surface is the spherical closed surface.

$$\Phi_E = E \cdot A = \frac{Q}{4\pi\varepsilon_o r^2} \cdot 4\pi r^2$$

$$\Phi_E = \frac{Q}{\varepsilon_0}$$

This is an expression of Gauss's law

GAUSS'S LAW

State that 'The total electric flux through any closed surface is equal to $1/\varepsilon_o$ times the net change enclosed by the surface' i.e

$$\Phi_E = \oint \vec{E} \cdot d\vec{s} = \frac{Q}{\varepsilon_o}$$

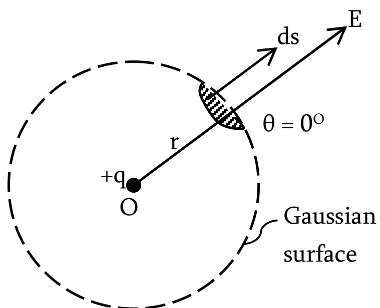
GAUSSIAN SURFACE – around a charge distribution is a closed surface such that the electric field intensity at all points on the surface is the same and electric flux through the surface along the normal to the surface at every point on the surface i.e The closed surface enclosed by the point positive of charge.

PROPERTIES OF GAUSSIAN SURFACE

1. It is an imaginary closed surface
2. It is never touches the charge
3. The surface can have any orbitray shape
4. The Gaussian surface is cylindrical for a line charge and cylindrical charge distribution but spherical for a point charge and a spherical charge distribution.

PROOF OF GAUSS'S LAW

Consider a single point charge +q located at a point O as shown in the figure below



Total electric flux

$$\Phi_E = \oint E ds \cos 0^0$$

$$= E\oint ds \text{ but } \oint ds = 4\pi r^2$$

$$\Phi_E = E \cdot 4\pi r^2$$

But $E = \frac{Q}{4\pi\varepsilon_o r^2}$

$$\Phi_E = \frac{1}{4\pi\varepsilon_o r^2} \cdot 4\pi r^2$$

$$\Phi_E = \frac{Q}{\varepsilon_0}$$

CHARGE DENSITIES

(a) **Linear charge density (λ)** – is the amount of charge distributed over a line per unit length of the line

$$\lambda = \frac{\text{charges distribution}}{\text{length}}$$

$$\lambda = \frac{Q}{L}$$

S.I. unit of λ is C/m

(b) **Surface charge density**, δ is defined as the quantity of charge per unit area of surface of conductor

$$\delta = Q/A$$

S.I unit of δ is C/m^2

(c) **Charged density ρ** – is defined as the amount of the electric charges per unit volume

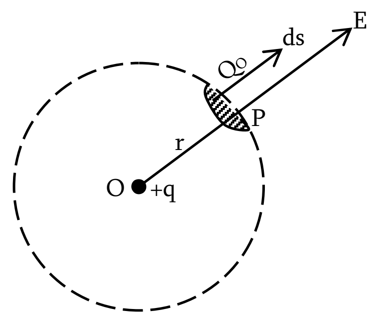
$$\rho = \frac{Q}{V_0}$$

S.I. unit of ρ is C/m^3.

APPLICATIONS OF GAUSS'S LAW

1. **DERIVATION OF COULOMB'S LAW FROM GAUSS'S THEOREM.**

Consider a single point charge +q located at O



Electric flux at the point P

$$\Phi_E = E \times 4\pi r^2$$

Apply the Gauss's law

$$\Phi_E = \frac{Q}{\varepsilon_o}$$

$$\frac{Q}{\varepsilon_0} = 4\pi r^2 E$$

$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

The magnitude of electric force at P.

$$F = EQ_0$$

$$F = \frac{QQ_0}{4\pi\varepsilon_o r^2}$$

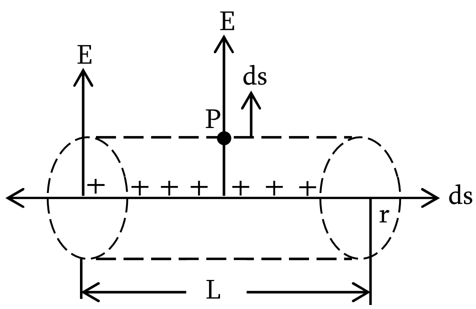
This is the Coulomb's law of electric force between two points charges Q and Q_O separated at a distance, r .

Note:

(i) Gauss's law simply says that the number of field lines crossing a closed surface depends only on the enclosed charge.

(ii) Gauss's law is true only if inverse square between point charges is true.

2. **ELECTRIC FIELD INTENSITY DUE TO LINE OF CHARGES.**



Let λ = linear surface charged density

$$\lambda = \frac{Q}{L}$$

Now, total electric flux on curved point of the cylinder is given by

$$\Phi_E = \oint E ds = E\oint ds$$

But $\oint ds = 2\pi rL$

$$\Phi_E = E \cdot 2\pi rL$$

Apply Gauss's law

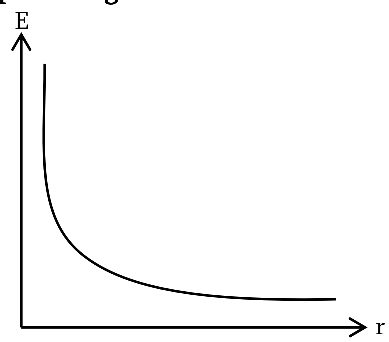
$$\Phi_E = \frac{Q}{\varepsilon_0}$$

$$E \cdot 2\pi rL = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{2\pi\varepsilon_0 rL} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$E \propto \frac{1}{r}$$

Graph of E against r



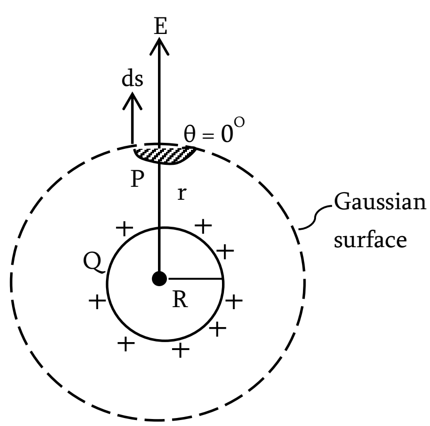
3. **ELECTRIC FIELD INTENSITY DUE TO A UNIFORMLY CHARGED SPHERICAL SHELL.**

Charged spherical shell (Hollow sphere) is the sphere whose electric charges are located on the surface.

Consider a spherical shell of radius R and centre O. Let +Q be the uniform charge on the surface of the sphere we are required to obtain electric field intensity at a point P at a distance r from the centre of the sphere.

Different cases:-

Case 1: electric field outside the shell. i.e (when point P lies outside of the shell)



Apply the Gauss's law.

$$\Phi_E = \oint E ds = E \cdot 4\pi r^2$$

$$\Phi_E = Q/\varepsilon_0$$

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_o}$$

$$E = \frac{Q}{4\pi\varepsilon_o r^2} \text{.........(i)}$$

If δ is the surface charge density of the sphere shell,

$$\delta = \frac{Q}{4\pi R^2}$$

$$Q = 4\pi R^2\delta \text{..........(ii)}$$

Putting equation (ii) into (i)

$$E = \frac{4\pi R^2\delta}{4\pi\varepsilon_0 r^2}$$

$$E = \frac{\delta R^2}{\varepsilon_0 r^2} \quad E \alpha \frac{1}{r^2}$$

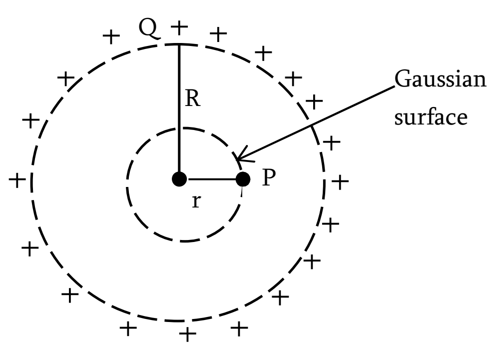
Case 2: electric field on the surface of shell. In this case, point P is on the surface of the shell and r = R

$$E = \frac{Q}{4\pi\varepsilon_o R^2}$$

$$= \frac{4\pi R^2\delta}{4\pi\varepsilon_o R^2}$$

$$E = \frac{\delta}{\varepsilon_0} = \text{Constant}$$

Case 3: Electric field inside the shell (i.e r < R)

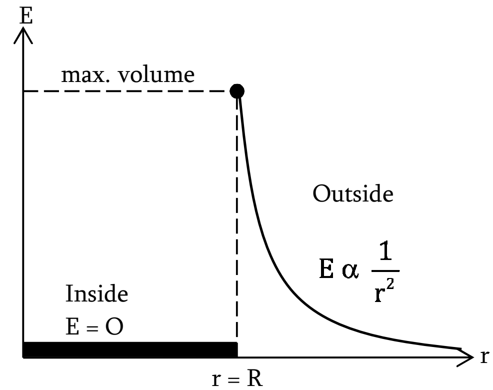


Inside of spherical shell no charges Q = O

$$\Phi_E = \frac{Q}{\varepsilon_0} = \frac{0}{\varepsilon_0} = 0$$

$$E = 0$$

GRAPH OF E AGAINST r FOR SPHERICAL SHELL



4. **ELECTRIC FIELD INTENSITY DUE TO A NON – CONDUCTING UNIFORMLY CHARGED SOLID SPHERE.**

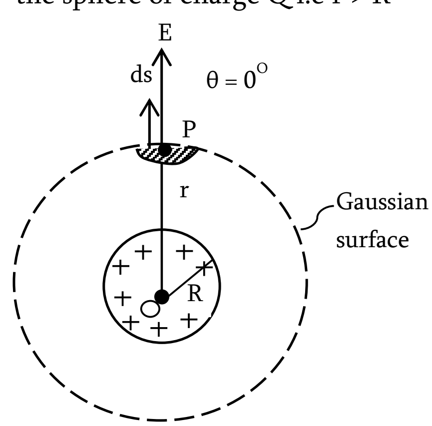
Charged solid sphere is the sphere whose electric charges are distributed within the sphere. If ρ is the surface charged density

$$\rho = \frac{Q}{4/3\,\pi R^3}$$

$$Q = \frac{4}{3}\pi R^3\rho$$

Different case:

Case 1: Electrical field outside the solid sphere i.e when the point P lies outside of the sphere of charge Q i.e r > R



Total electric flux at P

$$\Phi_E = \oint E ds = E\oint ds$$

$$\Phi_E = E \cdot 4\pi r^2 \ldots\ldots\ldots\ldots(i)$$

Apply Gauss's law

$$\Phi_E = \frac{Q}{\varepsilon_0} \ldots\ldots\ldots\ldots\ldots(ii)$$

(i) = (ii)

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_o}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \text{ but } Q = \frac{4}{3}\pi R^3 \rho$$

$$E = \frac{\frac{4}{3}\pi R^3 \rho}{4\pi\varepsilon_0 r^2} = \frac{\rho R^3}{3\varepsilon_0 r^2}$$

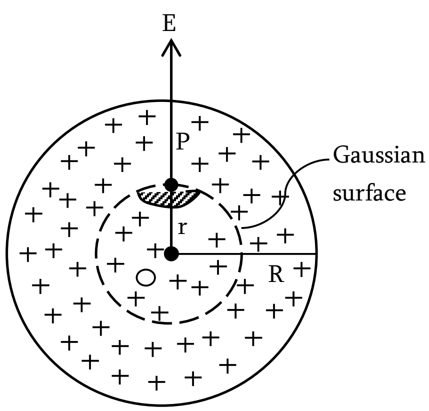
$$E = \frac{\rho R^3}{3\varepsilon_0 r^2} \quad r > R, \; E \,\alpha\, \frac{1}{r^2}$$

Case 2. Electric field on the surface of solid sphere. In this case r = R

$$E = \frac{\rho r^3}{3\varepsilon_o r^2} = \frac{\rho r}{3\varepsilon_o}$$

$$E = \frac{\rho r}{3\varepsilon_o} = \frac{\rho R}{3\varepsilon_0} \quad E \,\alpha\, r$$

Case 3. Electric field inside the solid sphere. In this case the point P is lies inside of solid sphere i.e (r < R).



Let Q_1 be charges on the Gaussian surface

$$Q_1 = \frac{4}{3}\pi r^3 \rho$$

$$Q = \frac{4}{3}\pi R^3 \rho$$

$$\frac{Q_1}{Q} = \frac{\frac{4}{3}\pi r^3 \rho}{\frac{4}{3}\pi R^3 \rho} = \frac{r^3}{R^3}$$

$$Q_1 = \frac{Qr^3}{R^3}$$

According to the Gauss's law

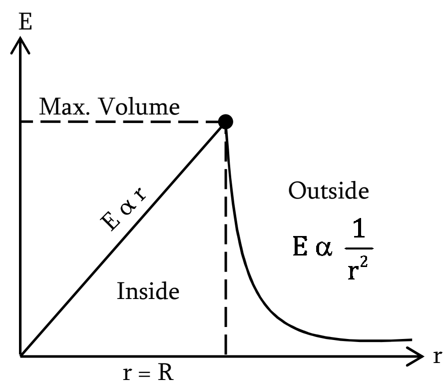
$$\Phi_E = E \cdot 4\pi r^3 = \frac{Q_1}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{Qr^3}{\varepsilon_o R^3}$$

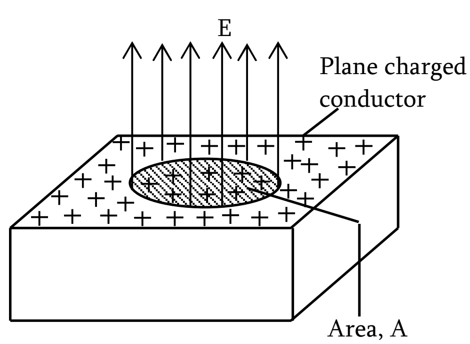
$$E = \frac{Qr}{4\pi\varepsilon_o R^3} = \frac{\frac{4}{3}\pi R^3 \rho r}{4\pi\varepsilon_o R^3}$$

$$E = \frac{\rho r}{3\varepsilon_o} \quad E \,\alpha\, r$$

GRAPH OF E AGAINST r FOR SOLID CHARGED SPHERE



5. **ELECTRIC FIELD INTENSITY FOR PLANE SURFACE CHARGED CONDUCTOR.**

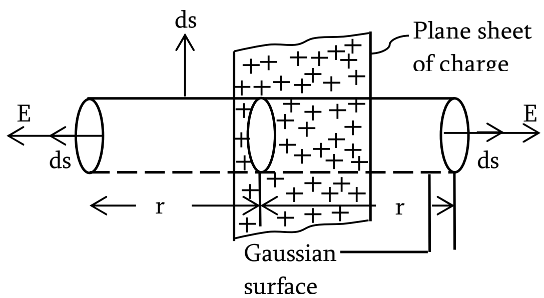


Apply the Gauss's law

$$\Phi_E = \frac{Q}{\varepsilon_0} = E \cdot A$$

$$E = \frac{Q}{A\varepsilon_0} = \frac{\delta}{\varepsilon_0}$$

6. **ELECTRIC FIELD INTENSITY DUE TO INFINITE PLANE SHEET OF CHARGE**



At the end θ = O^O^. The electric flux passing through the ends of caps.

$$\Phi_E = \oint Eds + \oint Eds$$

$$= 2\oint Eds = 2E\oint ds$$

$$\Phi_E = 2EA$$

According to the Gauss's law

$$\Phi_E = \frac{Q}{\varepsilon_0}$$

$$2EA = \frac{Q}{\varepsilon_o}$$

$$E = \frac{Q}{2A\varepsilon_0} = \frac{\delta}{2\varepsilon_0}$$

ELECTRIC POTENTIAL

Electric field stores the electric potential energy.

Electric potential at a point in an electric field – is the amount of work done in moving a unit positive charge from infinity to that point against the electrostatic force. the reason why we start from infinity is that the potential at the infinity is taken to be zero.

Mathematically

$$\text{Electric potential} = \frac{\text{work done}}{\text{change}}$$

- Electric potential is a scalar quantity because work done and electric charge are scalars.
- The potential energy of charge q at a point where potential is V is given by
  P.e of q, W = QV

S.I. unit of electric potential is Volt (V)

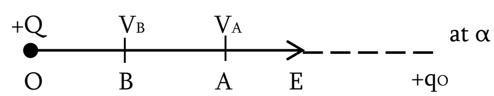
Definition of one volt – the electric potential at a point in the electric field is 1V if 1 joule of work is done in bringing a unit positive charge (i.e +1C) from infinity to that point against the electrostatic force.

Dimensional formula of potential

$$[V] = \frac{[W]}{[Q]} = [ML^2T^{-3}A^{-1}]$$

ELECTRIC POTENTIAL DIFFERENCE

Consider a point charge +Q placed at point O in space as shown in the figure below. The points A and B are in the electric field of charge +Q.



$$\text{Electric potential at A, } V_A = \frac{W_{\alpha A}}{q_o}$$

$$\text{Electric potential at B, } V_A = \frac{W_{\alpha B}}{q_o}$$

$$V_B - V_A = \frac{W_{\alpha B}}{q_o} - \frac{W_{\alpha A}}{q_o} = \frac{W_{AB}}{q_o}$$

Clearly, $V_B > V_A$ i.e point B is at higher electric potential than point A.

Let $V = V_B - V_A = V_{BA}$

V = Electric potential difference

$$V = V_B - V_A = \frac{W_{BA}}{q_o}$$

ELECTRIC POTENTIAL DIFFERENCE OR (POTENTIAL DIFFERENCE) between two points in an electric field may be defined as the amount of work done per unit positive test charge in moving it from one point to the other (without acceleration) against the electrostatic force due to the electric field. The potential difference between two point A and B in an electric field is

$$V_B - V_A = \frac{W_{AB}}{q_o}$$

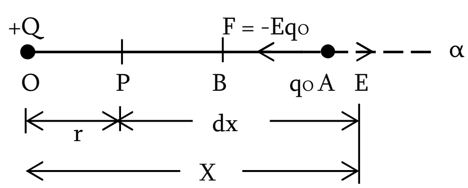
Work done in taking test charge from point A to B, WAB may be:
(i) Positive (ii) negative (iii) zero.
Then the electric potential at B will be (a) higher (b) lower or (c) same as that at A respectively.

Note that:
The following points are important note on the electric potential:

1. Potential is a property of a point in the electric field.
2. It is assumed that the test charge does not affect the electric field .
3. Electric potential at the infinity is equal to zero.
4. Electric charge moves from the region of the higher potential to the region of the lower potential.

EXPRESSION OF ELECTRIC POTENTIAL DUE TO A SINGLE POINT CHARGE.

Consider a point charge +Q placed at O in free space. It is described to obtain an electric potential at P due to +Q



Electric field at a point A

$$E = \frac{Q}{4\pi\varepsilon_0 X^2}$$

The amount of work in moving test charge q_o from A to B

$$dw = fdx$$

$$dw = -Eq_o dx$$

$$= \frac{-Qq_o dx}{4\pi\varepsilon_o X^2}$$

Total amount of work done in bringing a small positive test charge from infinity to point r.

$$W = \frac{-Qq_o}{4\pi\varepsilon_o}\int_{\alpha}^{r} x^2 dx$$

$$= \frac{-Qq_o}{4\pi\varepsilon_o}\left[\frac{-1}{x}\right]_{\alpha}^{r}$$

$$= \frac{-Qq_o}{4\pi\varepsilon_o}\left[\frac{-1}{r} - \frac{-1}{\alpha}\right]$$

$$W = \frac{Qq_o}{4\pi\varepsilon_o}$$

Electric potential at point P

$$V = \frac{W}{q_o} = \frac{Qq_o}{4\pi\varepsilon_o r}/q_o$$

$$V = V_{(r)} = \frac{Q}{4\pi\varepsilon_o r} = \frac{KQ}{r}$$

Electric potential at the infinity

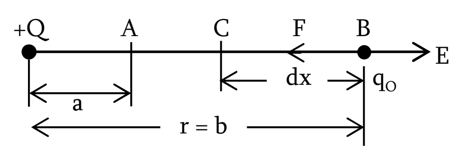
$$V_{\alpha} = \frac{1}{4\pi\varepsilon_o}\left[\frac{1}{\alpha}\right] = 0$$

ELECTRIC POTENTIAL AT INFINITY POINT

Is the work done in moving point positive charge from infinite to the point.

POTENTIAL DIFFERENCE FORMULA

Consider an electric field produced by a single positive charge (+Q) at distance, r as shown in the figure below



Apply the coulomb's law

$$F = \frac{Qq_o}{4\pi\varepsilon_o r^2}$$

Change in work done to move small distance, dr

$$dw = -Fdr$$

$$= \frac{-Qq_o dr}{4\pi\varepsilon_o r^2}$$

$$W = \frac{-Qq_o}{4\pi\varepsilon_o}\int_b^a r^{-2}dr$$

$$= \frac{-Qq_o}{4\pi\varepsilon_0}\left[\frac{-1}{r}\right]_b^a$$

$$= \frac{-Qq_o}{4\pi\varepsilon_o}\left[\frac{-1}{a} - \frac{-1}{b}\right]$$

$$W = \frac{Qq_o}{4\pi\varepsilon_o}\left[\frac{1}{a} - \frac{1}{b}\right]$$

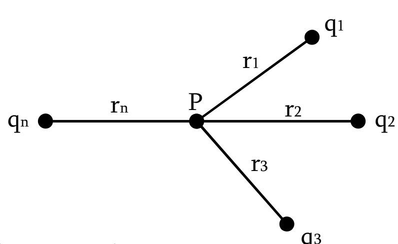
Potential difference between two points

$$V = \frac{W}{q_o} = \frac{Qq_o}{4\pi\varepsilon_o}\left[\frac{1}{a} - \frac{1}{b}\right] / q_o$$

$$V = \frac{Q}{4\pi\varepsilon_o}\left[\frac{1}{a} - \frac{1}{b}\right]$$

POTENTIAL AT A POINT DUE TO GROUP OF POINT CHARGES

Electric potential obeys superposition principle. Therefore, electric potential at any point P due to group of point charges P due to group of point charges $q_1, q_2, q_3 ------ q_n$ is equal to the algebraic sum of potential due to $q_1, q_2, q_3 ------ q_n$ at point P. Let the distance of $q_1, q_2, ------ q_n$ be $r_1, r_2, ------ r_n$ respectively from point P.



Total potential at P

$$V = V_1 + V_2 + -------- + V_n$$

$$= \frac{KQ_1}{r_1} + \frac{KQ_2}{r_2} + -------- + \frac{KQ_n}{r_n}$$

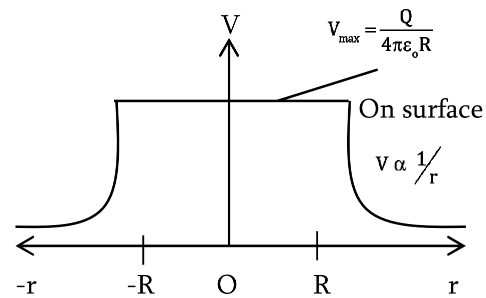
$$= K\left[\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + ------ + \frac{Q_n}{r_n}\right]$$

$$V = \frac{1}{4\pi\varepsilon_o}\left[\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + ----- + \frac{Q_n}{r_n}\right] \text{ or}$$

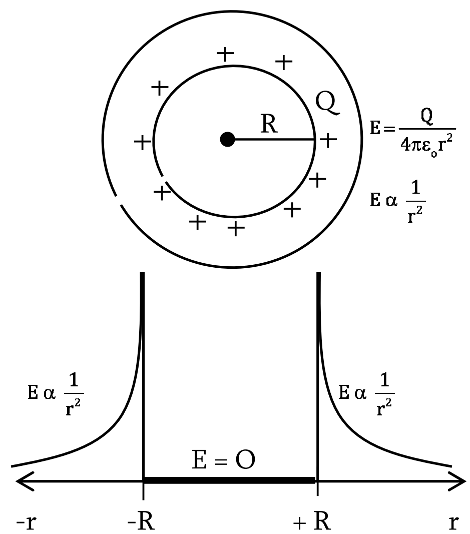
$$V = \frac{1}{4\pi\varepsilon_o}\sum_{i=1}^{n}\frac{Q_i}{r_i}$$

GRAPH OF ELECTRIC POTENTIAL AGAINST DISTANCE, r

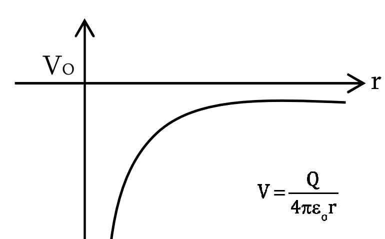
Since $V = \dfrac{Q}{4\pi\varepsilon_o r}$



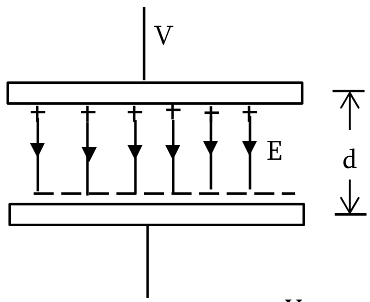
Graph of E against r for the charged spherical conductor.



GRAPH OF A POTENTIAL FOR A NEGATIVE POINT OF CHARGE



Electric potential gradient – is the charge of electric potential per unit distance between two parallel plates

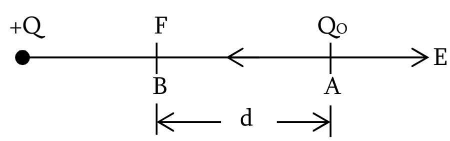


Electric field intensity, $E = \frac{V}{d}$

S.I. Unit of electric potential gradient is Volt per metre (Vm^{-1})

RELATIONSHIP BETWEEN ELECTRIC FIELD INTENSITY AND POTENTIAL DIFFERENCE

Consider two point A and B in the electric field of charge +Q as shown below



The charge in work done by Qo to move from point A to B

$$dw = Fdr$$

$$dw = -EQ_0 dr$$

Minus sign shows that E and F are in opposite direction.

$$dw = Q \cdot dv$$

$$Q \cdot dv = -EQ \cdot dr$$

$$dv = -Edr$$

$$E = -\frac{dv}{dr}$$

Negative sign shows that as dv increases dr decreases

Note that

$$\text{Since } E = \frac{Q}{4\pi\varepsilon_o r^2} = \frac{1}{r}\left[\frac{1}{4\pi\varepsilon_o r}\right]$$

$$\text{But } V = \frac{Q}{4\pi\varepsilon_o r}$$

$$E = \frac{V}{r} \text{ or } E = \frac{-dv}{dr}$$

Also $dv = -Edr$

$$V = -\int Edr$$

Example 49 NECTA 2015/P2/5

(a) (i) Differentiate electric potential from electric potential difference (02 marks)

(ii) Sketch a graph of variation of electrical potential from the centre of a follow charged conducting sphere of radius r, up to infinity. Explain the shape of the graph.

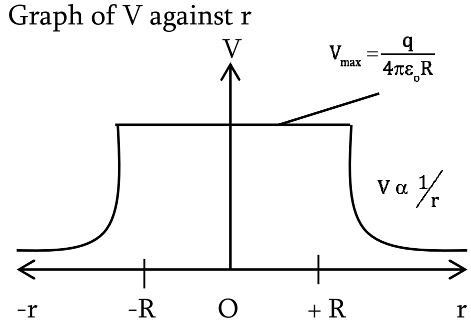
(b) Two bodies A and B are 0.1m apart. A point charge of $3 \times 10^{-3}\mu c$ is placed at A and a point charge of $-1 \times 10^{-9}\mu c$ is placed at B. C is the point on the straight line between A and B, where the electric potential is zero. Calculate the distance between A and C (06 marks)

(c) A square ABCD has each side of 100cm. four point charges of $+0.04\mu c$, $-0.05\mu c$, $+0.06\mu c$ and $+0.05\mu c$ are placed at A, B, C and D respectively. Calculate the electric potential at the centre of the square (07 marks)

Solution

(a) (i) Refer to your notes

(ii) Graph of V against r



Electrical potential is given by

$$V = \frac{q}{4\pi\varepsilon_o r}$$

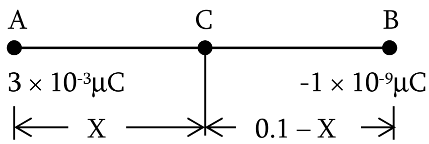
On the surface of hallow charged conducting sphere electric potential is maximum and constant.

$$V_{max} = \frac{q}{4\pi\varepsilon_o R}$$

Outside of hollow charged conducting sphere electrical potential decreases with increase of radius r

$$V = \frac{q}{4\pi\varepsilon_o r} \quad \left(V \alpha \frac{1}{r}\right)$$

(b)



Total electric potential at C

$$V = V_A + V_B$$

$$V = \frac{K.3\times10^{-3}}{X} + \frac{K\left(-1\times10^{-9}\right)}{\left(0.1-X\right)}$$

But $V = 0$

$$0 = \frac{K\left(3\times10^{-3}\right)}{X} + \frac{K\left(-1\times10^{-9}\right)}{\left(0.1-X\right)}$$

$$\frac{K\left(3\times10^{-3}\right)}{X} = \frac{K\left(1\times10^{-9}\right)}{0.1-X}$$

$$\frac{0.1-X}{X} = \frac{1\times10^{-9}}{3\times10^{-3}}$$

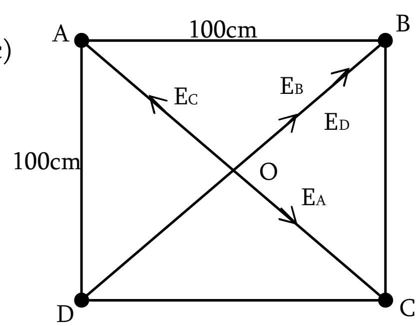
$$\frac{0.1-X}{X} = 3.33\times10^{-7}$$

$$0.1-X = 3.33\times10^{-7}X$$

$$0.1 = 1.000000333X$$

$$X = 0.099999966\text{m}$$

(c)



$$q_A = +0.04\mu c$$

$$q_B = -0.05\mu c$$

$$q_C = +0.06\mu c$$

$$q_D = +0.05\mu c$$

By using pythagorous theorem

$$\overline{DB} = \sqrt{\left(100\right)^2 + \left(100\right)^2} = 100\sqrt{2}\text{cm}$$

$$\overline{BD} = \sqrt{2}\text{m}$$

$$\overline{OA} = \overline{OB} = \overline{OC} = \overline{OD}$$

$$\overline{OD} = \frac{1}{2}\overline{BD} = \frac{\sqrt{2}}{2}$$

Electric field intensity $E = \dfrac{kq}{r^2}$

$$E_A = 9\times10^9\times\frac{0.04\times10^{-6}}{\left(\sqrt{2}/2\right)^2}$$

$$E_A = 720\text{N/C (along OC)}$$

$$E_B = \frac{9\times10^9\times0.05\times10^{-6}}{\left(0.5\sqrt{2}\right)^2}$$

$$E_B = 900\text{N/C (along OB)}$$

$$E_C = 9\times10^9\times\frac{0.06\times10^{-6}}{\left(0.5\sqrt{2}\right)^2}$$

$$E_C = 1{,}080\text{N/C (along OA)}$$

Also

$$E_D = 9\times\frac{10^9\times0.05\times10^{-6}}{\left(0.5\sqrt{2}\right)^2} = 900\text{N/C (along OB)}$$

Resultant electric field between E_B and E_D

$$E_1 = E_B + E_D = 900 + 900$$

$$= 1800\text{N/C (along OB)}$$

Resultant electric field between E_A and E_C

$$E_2 = E_C - E_A$$

$$E_2 = 1080 - 720 = 360\text{N/C (along OA)}$$

Resultant electric field at the centre O

$$E = \sqrt{E_1^2 + E_2^2}$$

$$= \sqrt{\left(360\right)^2 + \left(1800\right)^2}$$

$$E = 1835.65\text{N/C}$$

Example 50 NECTA 2000/P1/8

(a) (i) What is the electric potential at a point in an electrostatic field?

(ii) Derive an expression for electric potential at a point a distance a from a positive point charge, Q.

(b) Positive charge is distributed over a solid spherical volume of radius R and the charge per unit volume, δ.

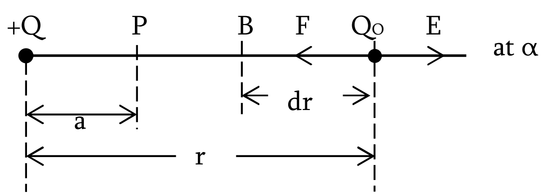
(i) Show that electric field inside the volume at a distance r < R from the centre is given by $E = \frac{\delta r}{3\varepsilon_o}$

(ii) What is the elastic field at appoint r > R (i.e outside the spherical volume)

Solution

(a) (i) Refer to your notes

(ii) Electric potential at a point of distance a from a charge, Q.



Apply coulomb's law

$$F = \frac{KQQ_o}{r^2}$$

Work done due to the small displacement

$$dw = -fdr$$

$$dw = \frac{-KQQ_o dr}{r^2}$$

Total work done

$$w = -KQQ_o \int_{\alpha}^{a} r^{-2} dr$$

$$= -KQQ_o \left[\frac{-1}{r}\right]_{\alpha}^{a}$$

$$= -KQQ_o \left[\frac{-1}{a} - \frac{-1}{\alpha}\right]$$

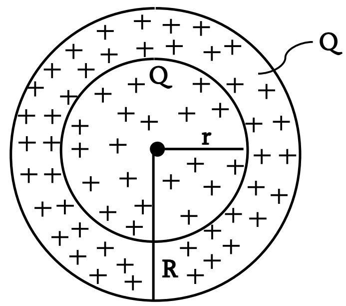
$$w = \frac{KQQ_o}{a}$$

Electric potential at P

$$V = W/_{a_o} = \frac{KQQ_o}{a}/Q_o$$

$$V = \frac{KQ}{a} = \frac{Q}{4\pi\varepsilon_o a}$$

(b) (ii) Electric field inside of spherical volume at a distance r < R



Let Q′ = Electric charges on the Gaussian surface

$$Q' = \frac{4}{3}\pi r^3 \delta$$

Also $Q = \frac{4}{3}\pi R^3 \delta$

$$\frac{Q'}{Q} = \frac{r^3}{R^3}$$

According to the Gaussian law

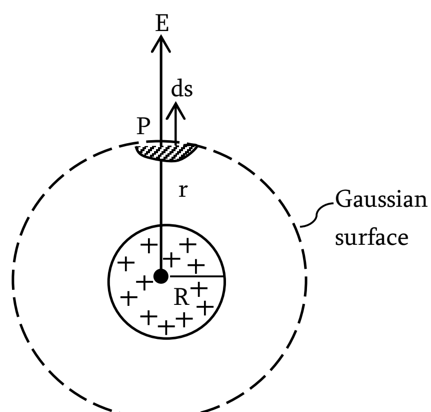
$$\Phi_E = \frac{Q'}{\varepsilon_o} = E \cdot 4\pi r^2$$

$$E \cdot 4\pi r^2 = \frac{Qr^3}{\varepsilon_o R^3}$$

$$E = \frac{Qr}{4\pi\varepsilon_o R^3} = \frac{4/3\,\pi R^3 \delta r}{4\pi\varepsilon_o R^3}$$

$E = \frac{\delta r}{3\varepsilon_o}$ Hence shown

(ii) When r > R



Total electric flux at P

$$\Phi_E = E \cdot 4\pi r^2$$

According to the Gauss's law

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_o r^2} \cdot \frac{\frac{4}{3}\pi R^3\delta}{4\pi\varepsilon_0 r^2}$$

$$E = \frac{\delta R^3}{3\varepsilon_0 r^2}$$

Example 51

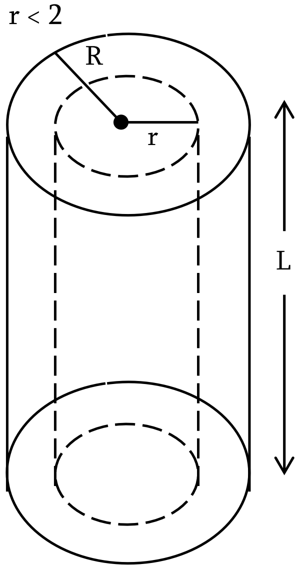
Charge is distributed uniformly throughout an infinitely long cylinder of radius R. Show that E at s distance r from the cylinder axis (r < R) is given by $E = \frac{\rho r}{2\varepsilon_o}$, where ρ is density of charge. What result do you expect for r > R?

Solution

Let r = radius of Gaussian surface

L = length of the cylinder

Case 1: r < 2



Area of curved surface of Gaussian = 2πrL

Electric flux, $\Phi_E = E.2\pi rL$

According to the gauss's law

$$\Phi_E = \frac{Q}{\varepsilon_o}$$

$$E.2\pi rL = \frac{Q}{\varepsilon_o}$$

$E.2\pi rL = \frac{1}{\varepsilon_o}$ [ρ × volume of enclosed by Gaussian surface]

$$E \cdot 2\pi rL = \frac{1}{\varepsilon_o} \cdot \rho \times \pi r^2 L$$

$$E = \frac{\rho r}{2\varepsilon_o} \quad (E \, \alpha \, r)$$

Case 2: If r >R, then the charges enclose

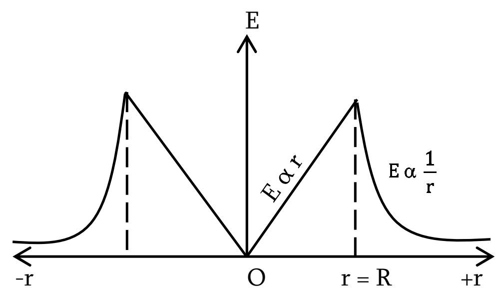
$$Q = \pi R^2 L\rho$$

Electric flux

$$E \cdot 2\pi rL = \frac{\pi R^2 L\rho}{\varepsilon_o}$$

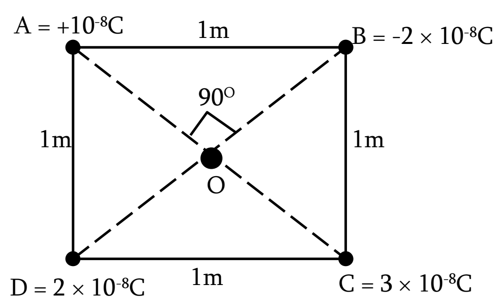
$$E = \frac{R^2\rho}{2\varepsilon_o r} \quad \left(E\alpha\frac{1}{r}\right)$$

Note: Graph of E against r for a charged infinitely long straight cylindrical rod (cylindrical symmetry)



Example 52

Calculate the potential at the centre O of the square shown



Solution

By pythagorous theorem

$$BD = \sqrt{1^2 + 1^2} = \sqrt{2}m$$

Since $OB = OC = OA = OD$

$$\overline{OB} = \frac{1}{2}BD = \frac{\sqrt{2}}{2} = 0.707m$$

Potential at O due to charges at A, B, C and D is given by

$$V = \frac{1}{4\pi\varepsilon_o}\left[\frac{Q_1 + Q_2 + Q_3 + Q_4}{\overline{OB}}\right]$$

$$= \frac{9\times10^9}{0.707}\left[1+-2+3+2\right]\times10^{-8}$$

$$V = 509.2V$$

Example 53 NECTA 2007/P1/12

(a) Two similar balls of mass m are hung from silk thread of length "a" and carry a similar charge q. Assume $\tan\theta \simeq \sin\theta$. To this approximation, show that $X = \left(\frac{q^2a}{2\pi\varepsilon_o mg}\right)^{1/3}$ where X is the distance of separation.

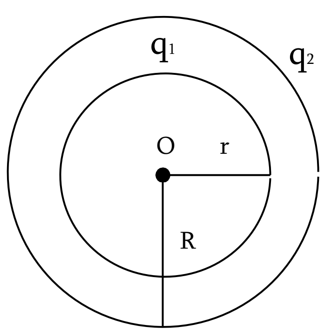
(b) A charge Q is the distributed over two concentric hollow sphere of radii r and R (R > r) such that densities are the same. Calculate the potential at the common centre of the two spheres.

Solution

(a) See solution example 9(a) pg 31

(b) If q_1 and q_2 are the charges on the two sphere of radii r and R respectively, the surface density is given by

$$\delta = \frac{q_1}{4\pi r^2} = \frac{q^2}{4\pi R^2}$$



$$\frac{q_1}{r^2} = \frac{q_2}{R^2}$$

$$\frac{q_1}{q_2} = \frac{r^2}{R^2}$$

$$\frac{q_1}{q_2}+1 = \frac{r^2}{R^2}+1$$

$$\frac{q_1+q_2}{q_2} = \frac{r^2+R^2}{R^2}$$

But $q_1+q_2 = Q$ (total charge)

$$\frac{Q}{q_2} = \frac{r^2+R^2}{R^2}$$

$$\frac{q_2}{R} = \frac{QR}{r^2+R^2}$$

Similarly $\frac{q_1}{r} = \frac{Qr}{r^2+R^2}$

Total electric potential at the centre

$$V = V_1+V_2$$

$$V = \frac{q_1}{4\pi\varepsilon_o r}+\frac{q_2}{4\pi\varepsilon_o R}$$

$$= \frac{1}{4\pi\varepsilon_o}\left[\frac{q_1}{r}+\frac{q_2}{R}\right]$$

$$= \frac{1}{4\pi\varepsilon_o}\left[\frac{Qr}{r^2+R^2}+\frac{QR}{r^2+R^2}\right]$$

$$V = \frac{Q}{4\pi\varepsilon_o}\left[\frac{r+R}{r^2+R^2}\right]$$

Example 54

(a) A charge of 2C is moved from a point 2m away from a charge of 1C to appoint 1m away from that charge. What is the work done by the agent?

(b) ABC is a night angled triangle where AB and BC are 25cm and 60cm respectively. A metal sphere of 2cm radius charged to a potential of 9×10^5 V is placed at B. Find the amount of work done in carrying a positive charge of 1C from c to A.

Solution

(a) Potential at point 1m away from 1C charge is

$$V_1 = 9\times10^9\times\frac{1}{1} = 9\times10^9 \text{ Volts.}$$

Potential at point 2m away from 1C charge is

$$V_2 = 9\times10^9\times\frac{1}{2} = 4.5\times10^9 \text{ Volts}$$

Potential difference

$$V = V_1-V_2 = (9-4.5)\times10^9$$

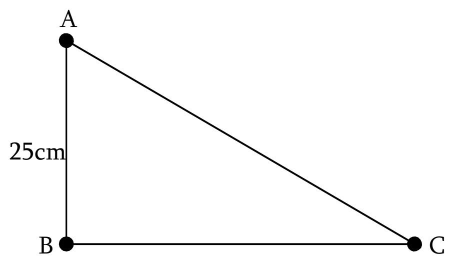
$$V = 4.5\times10^9 \text{ Volt}$$

Work done, W = QV

$$W = 2\times4.5\times10^9$$

$$W = 9\times10^9 \text{ Joule}$$

(b)



Potential of the charged sphere is given by

$$V = 9\times10^9\frac{q}{r}$$

$$9\times10^5 = 9\times10^9\times\frac{q}{0.02}$$

$$q = 2\times10^{-6}C$$

Potential at A due to charge

$q = 2\times10^{-6}C$ is

$$V_A = 9\times10^9\times\frac{q}{AB} = 9\times10^9\times\frac{2\times10^{-6}}{0.25}$$

$$V_A = 7.2\times10^4 V$$

Potential at point C due to charge

$q = 2\times10^{-6}C$

$$V_C = 9\times10^9\times\frac{q}{BC} = 9\times10^9\times\frac{2\times10^{-6}}{0.6}$$

$$V_C = 3\times10^4 C$$

P.d between point C and A

$$V = V_A - V_C$$

Work done, $W = Q(V_A - V_C)$

$$W = 1\times(7.2-3)\times10^4$$

$$W = 4.2\times10^4 J$$

Example 55 NECTA 2013/P1/11

(a) (i) Describe coulomb's law and give the dimension of each quantity (03 marks)

(iii) Briefly explain how you can demonstrate that there are two types of charges in nature (02 marks)

(b) (i) Define electric potential (01 mark)

(ii) A radioactive source in the form of metallic sphere of radius 1.0cm emits β – particles at the rate of 5.0 × 10^{10} particles per second. if the source is electrically insulated, how long will it take for its electric potential to be raised by 2Volts? (assuming that 40% of the emitted β – particles escape the source) (04 marks)

Solution

(a) (i) Refer to your notes

(ii) It can be shown experimentally that like charges repel each other while unlike charges attract each other, in other words, if the two charges are of the same nature (i.e both positive or both negative) the force between them is of repulsion. On other hand, If one charge is positive and the other negative, the force between them is of attraction.

(b) (i) Refer to your notes.

(ii) Suppose the potential of the sphere rises by 2V in time t seconds.

Number of β – particles emitted in time , t N = 5 × 10^{10}t

Number of β – particles escaped in t seconds.

$$N_1 = \frac{40}{100}N = \frac{40}{100}\times5\times10^{10}t$$

$$N_1 = 2\times10^{10}t$$

Charge acquired by metallic sphere in time t second.

$$Q = N_1 e$$

$$Q = 1.6\times10^{-19}\times2\times10^{10}t$$

$$Q = 3.2\times10^{-19}t$$

Electric potential on the surface of a sphere of radius R

$$V = 9\times10^9\cdot\frac{Q}{R}$$

$$2 = 9\times10^9\times\frac{3.2\times10^{-9}t}{10^{-2}}$$

$$t = 6.94\times10^{-4}\text{ sec}$$

Example 56

A radioactive source in the form of a metal sphere of diameter 10^{-3}m emits β – particles at a constant rate of 6.25 × 10^{10} particles per second. If the source is electrically insulated, how long will it take for its potential to rise by 1.0V, assuming that 80% of emitted β – particles escape from the surface?

Ans. 6.944×10^{-6} sec.

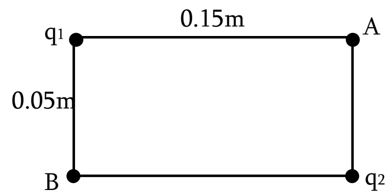
Example 57 NECTA 2008/P1/13

(a) What is meant by (i) an electric field (ii) a magnetic field (02 marks)

(b) (i) Define electric potential (02 marks)

(c) (i) What is an electric line force? (01 mark)

(ii) In figure 3 below $q_1 = -5 \times 10^{-5}$C and $q_2 = +2 \times 10^{-6}$C . Calculate the work done in moving a third charge $q_3 = 3 \times 10^{-6}$C from B to A along the diagonal of the rectangle.



Solution

(a) And (b) (i) Refer to your notes

(b) (ii) Let R and r be the radii of the bigger drop and n – small droplets of water respectively.

Apply the law of conservation of mass

Mass of 8 droplets = mass of the bigger drop

$$8\times\frac{4}{3}\pi r^3\rho = \frac{4}{3}\pi R^3\rho$$

$$R^3 = 8r^3$$

$$R = 2r = 2\times1\times10^{-3}$$

$$R = 2\times10^{-3}\text{m}$$

Charge on the bigger drop

$$Q = N_q = 8\times10^{-8}\text{C}$$

Potential on the bigger drop

$$V = 9\times10^9\cdot\frac{Q}{R} = \frac{9\times10^9\times8\times10^{-8}}{2\times10^{-3}}$$

$$V = 3600 \text{ Volts.}$$

(c) (i) Refer to your notes

(ii) Potential at the corner B

$$V_B = \frac{q_1}{4\pi\varepsilon r_1}+\frac{q_2}{4\pi\varepsilon_o r_2}$$

$$= 9\times10^9\left(\frac{5\times10^{-6}}{0.05}+\frac{2\times10^{-6}}{0.15}\right)$$

$$V_B = -7.8\times10^5 \text{ Volts}$$

Potential at corner A

$$V_A = 9\times10^9\left(\frac{-5\times10^{-6}}{0.15}+\frac{2\times10^{-6}}{0.05}\right)$$

$$V_A = 6\times10^6 \text{ Volts}$$

Potential difference between A and B

$$V_{AB} = V_A - V_B = 6\times10^6-\left(-7.8\times10^5\right)$$

$$V_{AB} = 8.4\times10^5 \text{ Volt}$$

Work done in moving charge 3×10^{-6}C from B to A

$$W = QV_{AB}$$

$$= 3\times10^{-6}\times8.4\times10^5$$

$$W = 2.52\text{J}$$

Example 58

Water drops 2mm in diameter fall from a jet at a potential of 100V into a hallow insulated sphere of radius 0.05m. Find the maximum potential reached by the sphere, assuming that it would be possible for the process to continue until it was full.

Solution

Number of drops required to make the square full

$$N = \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \left(\frac{R}{r}\right)^3$$

Charge carried by a single drop

$$q = 4\pi\varepsilon_o rv$$

$$Q = N_q = 4\pi\varepsilon_o rvV$$

Potential on the sphere

$$V_S = \frac{Q}{4\pi\varepsilon_0 R} = \frac{4\pi\varepsilon_0 rvN}{4\pi\varepsilon_0 R}$$

$$= \left(\frac{r}{R}\right)VN = \left(\frac{r}{R}\right)V\cdot\left(\frac{R}{r}\right)^3$$

$$V_S = V\left[\frac{R}{r}\right]^2 = 100\left[\frac{0.05}{0.001}\right]^2$$

$$V_S = 250000 \text{ Volt.}$$

Example 59

(a) Twenty seven drops of mercury are charged simultaneously to the same potential of 10V. What will be the potential if all the charged drops are made to combine to form one large drop? Assume the drops to be spherical.

(b) 64 small liquid drops, each carrying a charge of 0.5μc and each of diameter 0.04m are combined to form a bigger drop. Calculate the potential at the surface of bigger drop.

Ans. (a) 90volt (b) 3.6×10^6 volt

Example 60

The potential at the surface of a spherical drop of water carrying a charge 1.5×10^{-6}C is 250V

(i) Find the radius of the drop

(ii) Two such drops of the same radius and charge combine to form a single spherical at the surface of the new drop so formed.

Ans. (i) 54m (ii) 396.8V

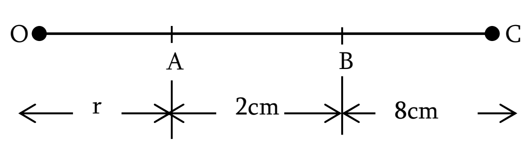
Example 61 NECTA 2009/P1/13

(a) (i) Define the term 'Electric field intensity in terms of force (01 mark)

(ii) Give two (2) possible units in which the electric field intensity can be measured (01 mark)

(iii) If there is an electric field intensity 300 S.I. Units at the surface of the Earth. what is the charge per square metre on the Earth surface? (2.5 marks)

(b) In figure 4.0 below, when we move from A to B, the potential changes by one unit. The potential difference between B and C is given to be two units. Determine the value of charge Q placed at O



Solution

(a) (i) Refer to your notes

(ii) Two possible units of E are N/C and Vm^{-1}

(iii) E = 300N/C $\varepsilon_o = 8.85 \times 10^{-12} Fm^{-1}$

$$\text{Now } E = \frac{\rho}{\varepsilon_o}$$

$$\delta = E\varepsilon_o$$

$$= 300 \times 8.85 \times 10^{-12}$$

$$\delta = 2.655 \times 10^{-9} C/m^2$$

(b) Let VA, VB and VC be the potential ate the points A, B and C respectively.

Let OA = r

Given that $V_A - V_B = 10$ Volt

$$V_A = \frac{KQ}{r}, \quad V_B = \frac{KQ}{r+2}$$

$$V_A - V_B = \frac{KQ}{r} - \frac{KQ}{r+2}$$

$$1 = KQ\left[\frac{1}{r} - \frac{1}{r+2}\right]$$

$$1 = KQ\left[\frac{r+2-r}{r(2+r)}\right]$$

$$1 = KQ\left(\frac{2}{r(r+2)}\right) \text{.....(i)}$$

$$\text{Also } V_C = \frac{KQ}{r+10}$$

$$V_B - V_C = 2$$

$$2 = \frac{KQ}{r+2} - \frac{KQ}{r+10}$$

$$= KQ\left(\frac{r+10-r-2}{(r+2)(r+10)}\right)$$

$$2 = KQ\left(\frac{8}{(r+2)(r+10)}\right) \text{.....(ii)}$$

Dividing equation (ii) by (i)

$$\frac{2}{1} = \frac{8KQ}{(r+2)(r+10)} \Big/ \frac{2KQ}{r(r+2)}$$

$$2 = \frac{4r}{r+10}, \quad 2r + 20 = 4r$$

$$r = 10cm$$

Putting r = 10cm = 0.1m in (i)

$$1 = 9 \times 10^9 \, Q\left(\frac{1}{0.1} - \frac{1}{0.1+0.02}\right)$$

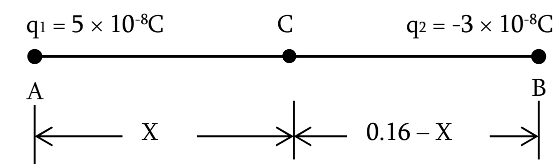
$$Q = 6.67 \times 10^{-11} C$$

Example 62

Two charges 5×10^{-8}C and -3×10^{-8}C are located 16cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at the infinity to be zero.

Solution

Let the potential be zero at point C at a distance X from the charge q_1.



$$V_C = \frac{1}{4\pi\varepsilon_o}\left[\frac{q_1}{X}+\frac{q_2}{0.16-X}\right]$$

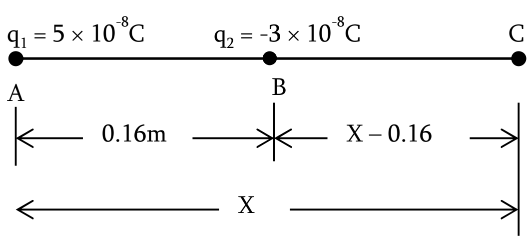
$$0 = \frac{1}{4\pi\varepsilon_o}\left[\frac{5\times10^{-8}}{X}+\frac{-3\times10^{-8}}{0.16-X}\right]$$

$$\frac{5}{X} = \frac{3}{0.16-X}$$

$$X = 0.1\text{m} = 10\text{cm}$$

$\therefore$ The point of zero potential lies 10cm from charge $q_1\left(=5\times10^{-8}\text{C}\right)$

The other possibility is that point of zero potential C may lie on AB produced at a distance X from q_1.



$$\text{Now} \quad V_C = \frac{1}{4\pi\varepsilon_o}\left[\frac{5\times10^{-8}}{X}-\frac{3\times10^{-8}}{X-0.16}\right]$$

$$0 = \frac{5\times10^{-8}}{X}-\frac{3\times10^{-8}}{X-0.16}$$

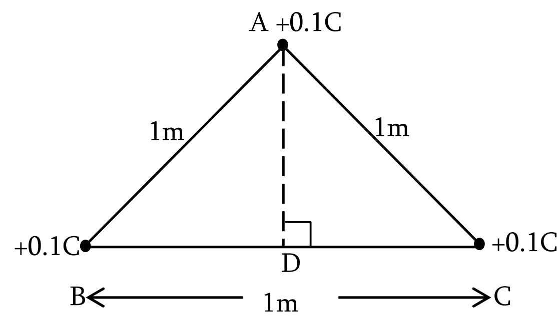
$$\frac{5}{X} = \frac{3}{X-0.16}$$

$$X = 0.4\text{m} = 40\text{cm}$$

$\therefore$ The point of zero potential C also lies on AB produced at a distance 40cm from $q_1 = 5\times10^{-8}\text{C}$

Example 63

Three charges of +0.1C each are placed at the corners of an equilateral triangle as shown on the figure below



If the energy is supplied at the rate of 1KW, how many days would be required to move the charge at A to a point D which is the mid – point of the line BC.

Solution

Potential at A due to charges at B and C is given by $V_A = \frac{1}{4\pi\varepsilon_o}\left[\frac{0.1}{1}+\frac{0.1}{1}\right]$

$$V_A = 18\times10^8 \text{ Volts}$$

Potential at D due to charges at B and C is given by $V_D = \frac{1}{4\pi\varepsilon_o}\left[\frac{0.1}{0.5}+\frac{0.1}{0.5}\right]$

$$V_D = 36\times10^8 \text{ Volt}$$

$$\text{Now } V_D - V_A = (36-18)\times10^8$$

$$V_{DA} = 18\times10^8 \text{ Volt}$$

Work done in moving charge 0.1C from A to D is given by

$$W = QV_{DA}$$

$$= 0.1\times18\times10^8$$

$$W = 1.8\times10^8\text{J}$$

$$\text{Power} = \frac{w}{t}, \quad t = \frac{w}{p}$$

$$t = \frac{1.8\times10^8}{1000} = 1.8\times10^5\text{sec}$$

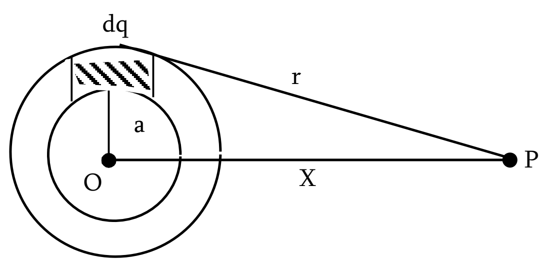
$$t = \frac{1.8\times10^5}{24\times50}$$

$$t = 2..08\text{days (approx.)}$$

Example 64

Calculate the potential at a point on the axis of a ring of charge of radius a. given that charge on the ring = q and distance of the observation point from the centre of the ring is X.

Solution



The potential at the point P due to an elementary charge dq is given by

$$dV = \frac{1}{4\pi\varepsilon_o}\cdot\frac{dq}{r}.$$

Total potential at P due to ring of charge

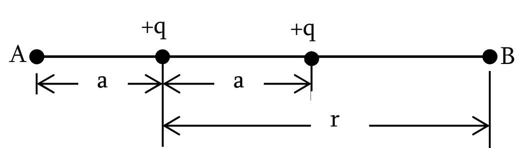
$$V = \frac{1}{4\pi\varepsilon_o r}\int dq = \frac{q}{4\pi\varepsilon_o r}$$

$r = \sqrt{a^2+x^2}$ (Pythagoras theorem)

$$V = \frac{q}{4\pi\varepsilon_o\sqrt{a^2+x^2}}$$

Example 65

Calculate the potential at P due to the charge configuration shown in figure below. If r>>>a, then how will you modify the result.



Solution

The potential at P due to the given charge configuration is the sum of the potentials due to the charge –q, + q and +q. these charges are at distances r + a, r and r – a respectively from the point P.

$$V = \frac{1}{4\pi\varepsilon_o}\left[\frac{-q}{r+a}+\frac{q}{r}+\frac{q}{r-a}\right]$$

$$= \frac{q}{4\pi\varepsilon_o}\left[\frac{-1}{r+a}+\frac{1}{r}+\frac{1}{r-a}\right]$$

$$V = \frac{1}{4\pi\varepsilon_o}\left[\frac{q}{r}+\frac{2qa}{r^2-a^2}\right]$$

If r >>> a, then

$$r^2-a^2 \simeq r^2$$

$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{r}+\frac{2}{r^2}\right)$$

Note that: when r >> a, the potential at P is simply the potential of a dipole and an isolated charge at distance r.

Example 66

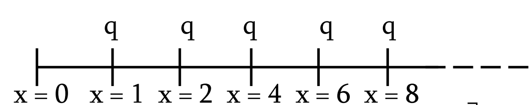
An infinite number of charges, each equal to q are placed along the x – axis at x = 1, x = 2, x = 4, x = 8m,---------and so on.

(i) Calculate the potential and the electric field at the point X = 0 due to this set of charges.

(ii) What will be the potential and electric field if in the above set of charges, the conservative charges have opposite signs.

Solution

(i) Let us first consider the case when charges are of the same sign.



$$v = \frac{}{4\pi\varepsilon_o}\left[\frac{}{1}+\frac{}{2}+\frac{}{4}+\frac{}{8}+------\right]$$

$$V = \frac{1}{4\pi\varepsilon_o}\left[\frac{1}{1}+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+------\alpha\right]$$

Sum of the infinite terms

$$S_\alpha = \frac{G_1}{1-r} = \frac{1}{1-\frac{1}{2}}$$

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+----\alpha = \frac{1}{1-\frac{1}{2}}$$

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{2^2}+-----+\alpha = 2$$

$$\text{Now } V = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{2^2}+----+\alpha\right)$$

$$V = \frac{2}{4\pi\varepsilon_o} = \frac{q}{2\pi\varepsilon_o}\text{Volt}$$

Since all the charges are of the same sign therefore at X = 0, due to all the charges are in the same direction. Electric field intensity at X = 0 due to all charges is given by

$$E = \frac{1}{4\pi\varepsilon_o}\left[\frac{q}{1^2}+\frac{q}{2^2}+------+\alpha\right]$$

$$= \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{2^4}+-----\right]$$

$$E = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{1-\frac{1}{4}}\right]$$

$$E = \frac{q}{3\pi\varepsilon_o}N/C$$

(ii) Let us now consider a case when the charges are of opposite sign in case, the potential at X = 0.

$$V = \frac{q}{4\pi\varepsilon_o}\left[\frac{q}{1}-\frac{q}{2}+\frac{q}{2^2}-\frac{q}{2^3}+----\right]$$

$$= \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{1}-\frac{1}{2}+\frac{1}{2^2}-\frac{1}{2^3}+----\right]$$

$$= \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{1-\left(\frac{1}{2}\right)}\right] = \frac{q}{4\pi\varepsilon_o}\left(\frac{2}{3}\right)$$

$$V = \frac{q}{6\pi\varepsilon_o}\text{ Volt}$$

The electric field intensity at X = 0

$$E = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{2^4}-\frac{1}{2^6}+----\right]$$

$$= \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{1-\left(\frac{1}{4}\right)}\right]$$

$$E = \frac{q}{5\pi\varepsilon_o}N/C$$

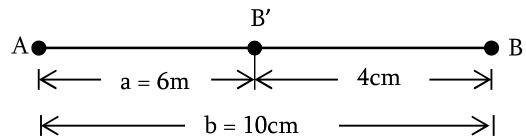
Example 67

Two positive charges of 12μc and 8μc respectively are 10cm apart. Find the work done in bringing them 4cm closer

(Assume $\frac{1}{4\pi\varepsilon_o} = 9\times10^9 F^{-1}m$)

Solution

Suppose the 12μc is fixed in position



$$V = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$= 9\times10^9\times12\times10^{-6}\left[\frac{1}{0.06}-\frac{1}{0.1}\right]$$

$V = 720000$ Volt

The work done in moving the 8μc from 10cm to 6cm.

$W = QV$

$= 8\times10^{-6}\times720000$

$W = 5.8J$

Note that: The very high potential is due to small quite charge.

Example 68

The electric potential due to a point charge at a distance of 10cm is 100V. Calculate the strength of the electric field at that point.

Solution

$$\text{Since } E = \frac{Q}{4\pi\varepsilon_o r^2}, V = \frac{Q}{4\pi\varepsilon_o r}$$

$$E = \frac{1}{r}\left(\frac{Q}{4\pi\varepsilon_o r}\right) = \frac{V}{r}$$

$$E = \frac{100}{0.1}$$

$E = 1000V/m$

Example 69

The electric field outside a charged long straight wire is given by $E = \frac{1000}{r}Vm^{-1}$ and is directed outwards. What is the sign of the charge on the wire? If two points A and B are situated such that $r_A = 0.2m$ and $r_B = 0.4m$. Find the value of $V_B - V_A$

Solution

$$V_B - V_A - \int_A^B Edr = -\int_{0.2}^{0.4} \frac{1000dr}{r}$$

$$= -1000\left[\log_e r\right]_{0.2}^{0.4}$$

$$V_B - V_A = -693.1$$

Example 70

A solid metallic sphere of radius 0.20m is given a charge 10nc. Calculate the electric field strength at these distances from the centre.

(a) 0.10m (b) 0.20m (c) 0.40m

Solution

Charge density

$$\rho = \frac{3q}{4\pi R^2}$$

$$= \frac{3\times10\times10^{-9}}{4\pi(0.2)^1}$$

$$\rho = 2.984\times10^{-7}Cm^{-3}$$

(a) r = 0.1

$$E = \frac{\rho r}{3\varepsilon_o} = \frac{2.984\times10^{-7}\times0.1}{3\times8.85\times10^{-12}}$$

E = 1124N/C

(b) r = R = 0.2

$$E = \frac{\rho R}{3\varepsilon_o} = \frac{2.984\times10^{-7}\times0.2}{3\times10^{-12}\times8.85}$$

E = 2248N/C

(c) $E = \frac{\rho R^3}{3\varepsilon_o r^2}$

$$= \frac{2.984\times10^{-7}\times(0.2)^3}{3\times8.85\times10^{-12}\times(0.4)^2}$$

$$E = 561.95857N/C$$

EXERCISE NO 3

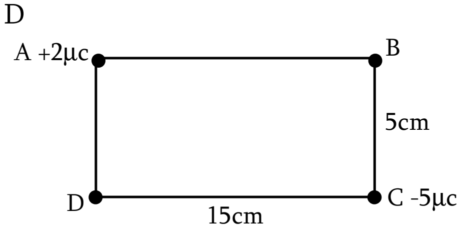
1. Two charges 3×10^{-8}C and -2×10^{-8}C are located 15cm apart. At what point on the line joining the two charges is the electrical potential zero? Take potential at infinity to be zero.

   Ans. 9cm from charge of 3×10^{-8}C

2. ABCD is a square of side 0.2m charges of 2×10^{-9}, 4×10^{-9}, 8×10^{-9}C are placed at the corners A,B and C respectively. Calculate the work required to transfer a charge of 2×10^{-9} coulomb from corner D to the centre of the square

   Ans.6.27×10^{-7}J

3. The side of a rectangle ABCD are 15cm and 5cm. Two point charges of +2μc and -5μc are placed at the corners A and C respectively as shown in figure below. Calculate the work done in carrying charge of 3μc from point B to D



   Ans. 2.52J

4. The electric potential V(x) in a region along the X – axis varies with the distance X (in metre) according to the relation $V(x) = 4X^2$. Calculate the force experienced by a 1μc charge placed at point X = 1m

   Ans. -8×10^{-6}N

5. Two point charges are placed on x – axis, a 2μc charge at X = 100cm, and -1μc at X = 40cm. calculate the potential at point X = 100cm.

   Ans: 5000V

6. A point charge A of 5×10^{-9}C is placed at distance of 6cm from another point charge B of 3×10^{-9}C. The charge B is brought to words charge A from 6cm to 5cm. calculate the work done

   Ans. 4.5×10^{-7}J

7. A point charge of $6 \times 10^{-8}C$ is situated at $X = 0$. How much work will be done in taking an electron from point $X = 3m$ to $X = 6m$?

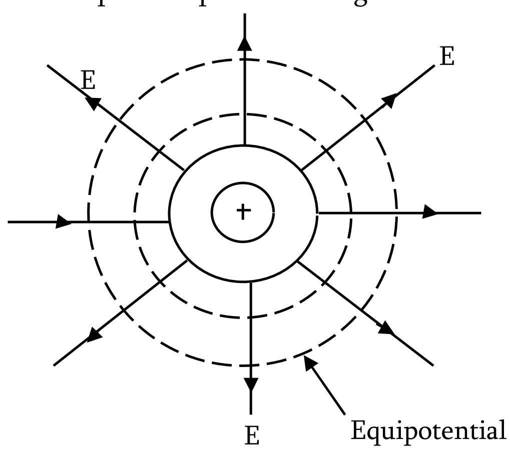
Ans. $1.44 \times 10^{-17}J$

EQUIPOTNTIAL SURFACE

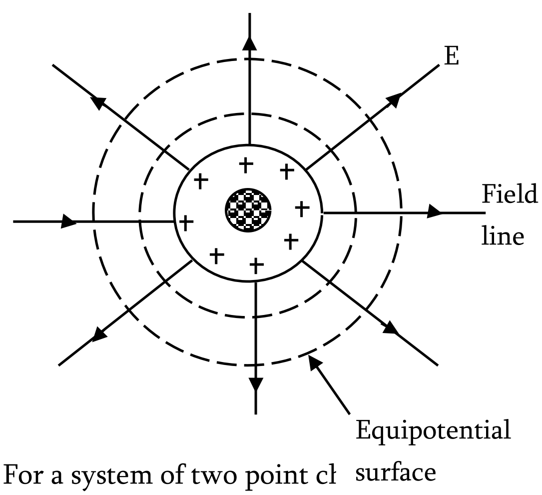
Equipotential surface – is the any surface which has the same electrostatic potential at every point. (every where)

- It is the locus having the same electrostatic potential at every point. (every where)
- For an equipotential surface, the change in the potential difference must be equal to zero ($\Delta V = 0$)

(i) Isolated positive point of charge

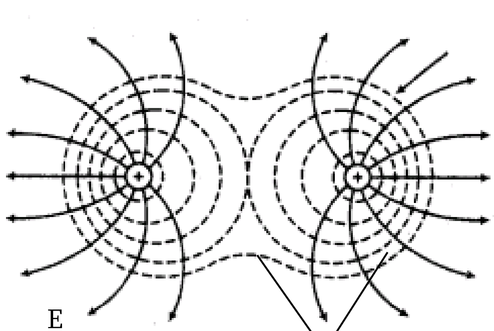


(ii) for a charged conducting s

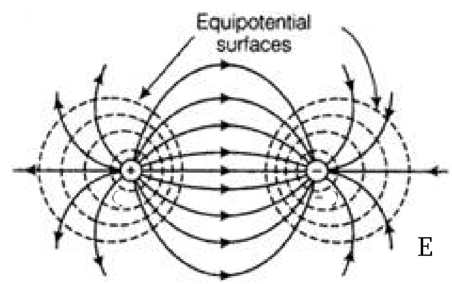


(iii) For a system of two point cl

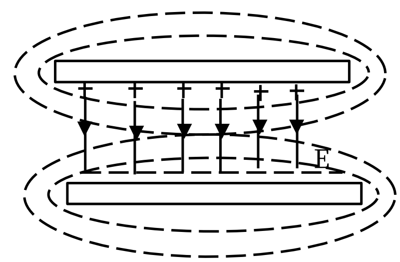
(a) For like charges



(b) For unlike charges



(iv) For a pair of parallele conducting plates when one plate has a positive charge and theother an equal negative charge.



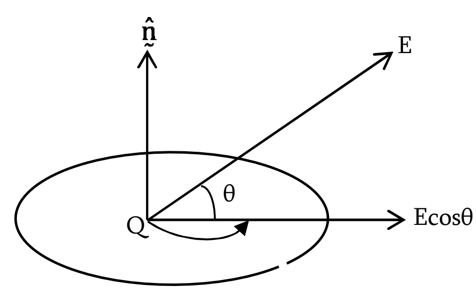
PROPERTIE OF EQUIPOTENTIAL SURFACE

1. No work done in moving a test charge over on equipotential surface. Since for an equipotential surface $\Delta V = 0$

   $W = Q\Delta V = Q \times O$

   $W = O$

2. The electric field is always at right angle to the equipotential surface. Suppose the electric field E makes an angle θ with surface as illustrated in the figure below



The horizontal force parallel to the surface causes a test charge to move along it.

$$F = qE\cos\theta$$
$$dw = Fdr$$
$$dw = qE\cos\theta dr$$
But $dw = 0$
$$0 = \cos\theta$$
$$\theta = \cos^{-1}(0) = 90^0$$

3. The equipotential surfaces tell the direction of the electric field.
4. The equipotential surface help to distinguish regions of strong field from those of weak field since.

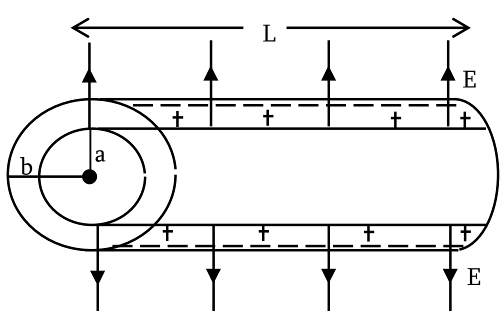
$$E = \frac{-dv}{dr}, \quad dr = \frac{-dv}{E}$$

dv = constant, then $dr \propto \frac{1}{E}$

i.e. the spacing between the equipotential surface will be lesser win the regions, where the electric is stronger and vice – vers.
Therefore equipotential surfaces are closer together, where the electric field is stronger and father apart where the field is weaker.

5. No two equipotential surfaces can intersect each other.

THE POTENTIAL DIFFERENCE BETWEEN THE CONCENTRIC CHARGED CYLINDERS.



Since $Q = \lambda L$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Now $E = \frac{-dv}{dr} = \frac{\lambda}{2\pi\varepsilon_o r}$

$$dv = \frac{-\lambda}{2\pi\varepsilon_o r}dr$$

$$\int_{V_b}^{V_a} dv = \frac{-\lambda}{2\pi\varepsilon_o}\int_b^a \frac{dr}{r}$$

$$V_a - V_b = \frac{-\lambda}{2\pi\varepsilon_o}\left[\log_e^r\right]_b^a$$

Let $V_a - V_b = V$

$$V = \frac{\lambda}{2\pi\varepsilon_o}\log_e\left(b/a\right)$$

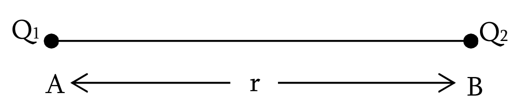
$$V = \frac{1}{2\pi\varepsilon_o L}\log_e\left(b/a\right)$$

ELECTRICAL POTENTIAL ENERGY OF A SYSTEM OF CHARGES.

Is the work done required to assemble the system of charge by bringing them from the infinity i.e is defined as the work required to be done to bring the charges constituting the system to their respective locations from infinity

(i) **EXPRESSION OF THE ELECTROSTATIC POTENTIAL ENERGY DUE TO THE TWO POINT OF CHARGES.**

Consider two point charges Q_1 and Q_2 initially are at infinity. Suppose Q_1 is the first charge to be brought from infinity to its original position. For this, no work is required. It is because, when charge Q_1 is moved, no electrostatic force due to any other charge opposes it. Now, if charge Q_2 is brought from infinity to a point B which is at a distance r from point A, then work done on moving charge Q_2 from infinity to a point B, this numerically equal to the electric potential at point B

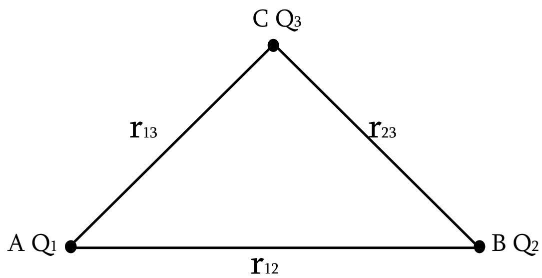


$$V_B = \frac{KQ_1}{r}$$

$$U = Q_2 V_B$$

$$U = \frac{KQ_1Q_2}{r} = \frac{Q_1Q_2}{4\pi\varepsilon_o r}$$

(ii) **EXPRESSION OF ELECTROSTATIC POTENTIAL ENERGY OF A SYSTEM OF THREE CHARGES IN THE FORM OF TRIANGULAR FORM**



Let us consider three charges Q_1, Q_2 and Q_3 separated by distance r_{12}, r_{13} and r_{23} as shown in the figure above. The electrostatic potential energy of the system of the three charges is equal to the work done to assemble them in this, configuration starting from infinite separation. Work done to bring first charge (say Q_1) from the infinity to the point A is equal to zero, because there is no other electric charge to attract or repel it. Work done to bring the charge, Q_2

$$U_2 = Q_2 V_1 \text{ but } V_1 = \frac{KQ_1}{r_{12}}$$

$$U_2 = \frac{KQ_1Q_2}{r_{12}}$$

Work done to bring the charge Q3

$$U_3 = (V_1 + V_2)Q_3 \text{ , } V_2 = \frac{KQ_2}{r_{23}}$$

$$U_3 = \left(\frac{KQ_1}{r_{13}} + \frac{KQ_2}{r_{23}}\right)Q_3$$

$$U_3 = K\left[\frac{Q_1Q_3}{r_{13}} + \frac{Q_2Q_3}{r_{23}}\right]$$

Total electrostatic potential energy

$$U = U_1 + U_2 + U_3$$

$$= 0 + \frac{KQ_1Q_2}{r_{12}} + \frac{KQ_1Q_3}{r_{13}} + \frac{KQ_2Q_3}{r_{23}}$$

$$U = \frac{1}{4\pi\varepsilon_o}\left[\frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_2Q_3}{r_{23}}\right]$$

Generally

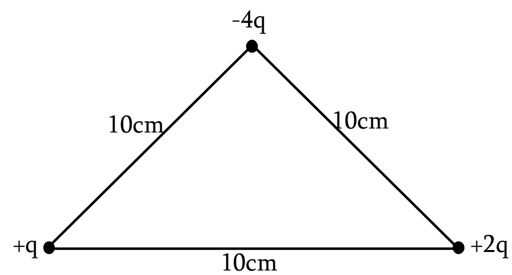
$$U = \frac{KQ_1Q_2}{r_{12}} + \frac{KQ_1Q_3}{r_{13}} + \frac{KQ_1Q_4}{r_{14}}$$

$$U = K\sum_{\text{all pairs}}^{n} \frac{Q_iQ_j}{r_{ij}}$$

EXAMPLES

Example 71

Three point charges are arranged at three vertices of a triangle as shown in the figure below. Given that q = 10^{-7}C. Calculate the electrostatic potential energy.'



Solution

$$Q_1 = +q \text{ , } Q_2 = +2 \text{ , } Q_3 = -4q$$

Total electrostatic potential energy

$$U = \frac{k}{r}\left[Q_1Q_2 + Q_1Q_3 + Q_2Q_3\right]$$

$$= \frac{9\times10^9}{0.1}\left[q(29) + q(-4q) + (2q)(-4q)\right]$$

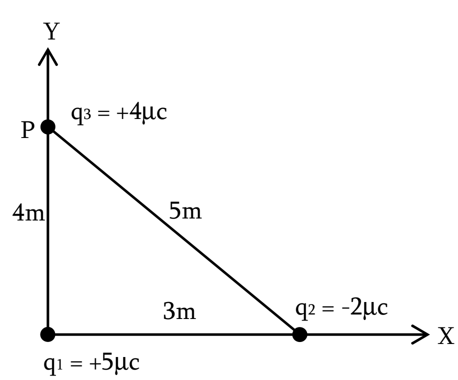
$$= \frac{9\times10^9}{0.1}\left[2q^2 - 4q^2 - 8q^2\right]$$

But $q = 10^{-7}$

$$U = 9.0\times10^{-3}\,J$$

Example 72

Find the total potential energy of the system of point charges shown in the figure below.



Solution

The total potential energy U of the system of charges is

$$U = K\left[\frac{Q_1Q_2}{r_{12}} + \frac{Q_2Q_3}{r_{23}} + \frac{Q_1Q_3}{r_{13}}\right]$$

$$= 9\times10^9\left[\frac{(5\times10^{-6})(-2\times10^{-6})}{3} + \frac{(-2\times10^{-6})(4\times10^{-6})}{5} + \frac{(5\times10^{-6})(4\times10^{-6})}{4}\right]$$

$$U = 6.0\times10^{-4}\,J$$

Example 73 NECTA 2017/P2/5

(a) (i) Define the terms capacitance and electric potential (02 marks)

(ii) The capacitance C of a capacitor is full charged by a 200V battery. It is then discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity $2.5 \times 10^2 JKg^{-1}K^{-1}$ and of mass 0.1kg. If the temperature of the block rises by 0.4K, What is the value of C? (06 marks)

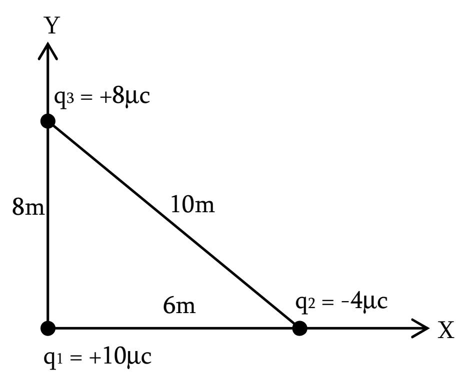
(b) A parallel plate capacitor has plates each of area $0.24m^2$ separated by a small distance 0.50mm. If the capacitor is full charged by a battery of electromotive force of 24V, calculate:-

(i) The capacitance of the capacitor (03 marks)

(ii) The energy stored in the capacitor (03 marks)

(c) (i) Comment on the assertion that, the safest way of protecting yourself from lightening is to inside of a car (02 marks)

(ii) Find the total potential energy of the system of point charges shown in figure 1



Solution

(a) (i) Refer to your notes

(ii) Energy stored in capacitor

$$W = \frac{1}{2}CV^2$$

$$= \frac{1}{2}\times C\times(200)^2$$

$$W = 2C\times10^4\,J$$

Energy that flows through the resistance of coil = heat gained by the block

$$W = MC\Delta\theta$$

$$= 0.1\times2.5\times10^2\times0.4$$

$$W = 10J$$

Apply the law of conservation of energy

$$2C\times10^4 = 10$$

$$C = 500\mu F$$

(b) (i) $A = 0.24m^2$ $d = 0.50mm$ $V = 24$ volt

$$C = \frac{A\varepsilon_o}{d}$$

$$= \frac{0.24\times8.85\times10^{-12}}{0.5\times10^{-3}}$$

$$C = 4.248\times10^{-9}F$$

(ii) Energy stored

$$W = \frac{1}{2}CV^2$$

$$= \frac{1}{2}(4.248\times10^{-9})(24)^2$$

$$W = 1.2234\times10^{-6}J$$

(c) (i) We know that electric field inside a conductor is zero. Since the body of the car is a metal, the electric field inside it is zero. The discharging current due to lighting passes to the Earth through the metallic body of the car. OR the hallow structure of the car provides electrostatic shielding from the lightning

(ii) The total energy U of the system of charge is

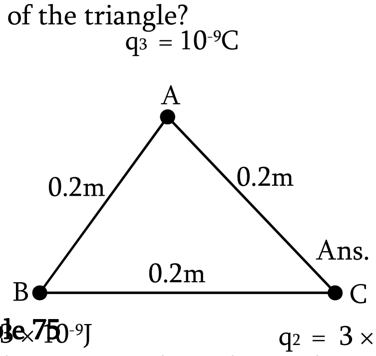
$$U = K\left[\frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_2Q_3}{r_{23}}\right]$$

$$= 9\times10^{9}\left[\frac{(10\times10^{-6})(-4\times10^{-6})}{6}+\frac{(10\times10^{-6})(8\times10^{-6})}{8}+\frac{(8\times10^{-6})(-4\times10^{-6})}{10}\right]$$

$$U = -1.33\times10^{-13}\,J$$

Example 74

Point charges of $3\times10^{-9}C$ are located at two vertices of an equilateral triangle of side 20cm. how much work will be done to bring a test charge of $10^{-19}C$ from infinity up to the third corner of the triangle?

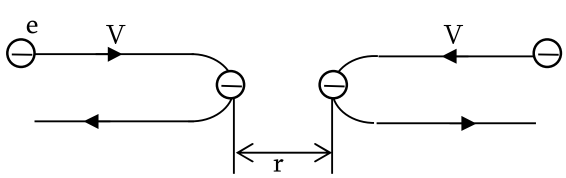


Ans. $2.7\times10^{-7}\,J$

Example 75

Two electrons, each with a velocity of 10^6m/s are released towards each other. What will be the distance of closest approach? Mass electron = $9\times10^{-31}\,kg$.

Solution



Initial total kinetic energy of the two electrons.

$$E = \frac{1}{2}mv^2+\frac{1}{2}mv^2 = mv^2 \ldots\ldots.(i)$$

As the electrons approach each other they experience repulsive electrostatic force. thus, their kinetic energy is converted into potential energy of the system.

Potential energy of electron – electron system.

$$U = \frac{Kq_1q_2}{r} \ldots\ldots\ldots\ldots(ii)$$

Apply the principle of conservation of energy.

$$\frac{Kq_1q_2}{r} = mv^2$$

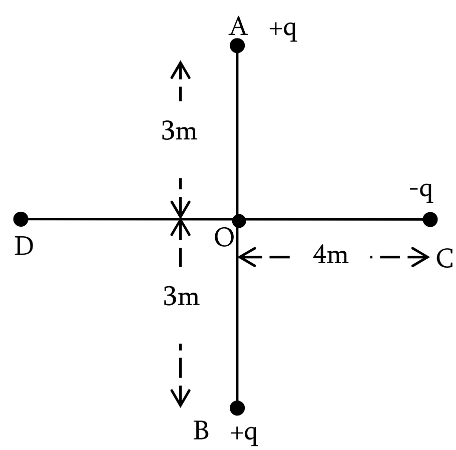
$$r = \frac{Kq_1q_2}{mv^2}$$

$$= \frac{9\times10^{9}\left(1.6\times10^{-19}\right)^2}{9\times10^{-31}\times\left(10^{6}\right)^2}$$

$$r = 2.56\times10^{-10}\,m$$

Example 76

Two fixed, equal positives charges each of magnitude $5\times10^{-5}C$ are located at A and B separated by a distance of 6m. An equal and opposite charges moves towards them along, the line COD, the perpendicular bisector of line AB



The moving charge, when it reaches the point C, at a distance of 4m from O, has kinetic energy of 4J. Determine the distance of the farthest point D which the negative charge will reach before returning towards C.

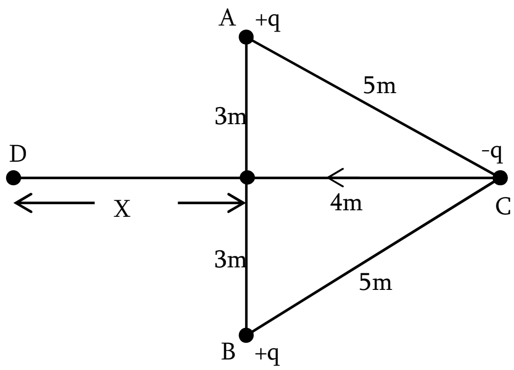
Solution

Here $OA = OB = 3m\overline{OC} = 4m$

By using pythagorous theorem

$$\overline{AC}^2 = \overline{OC}^2+\overline{OA}^2 = 3^2+4^2$$

$\overline{AC} = 5m \qquad q = 5\times10^{-5}C$



The potential energy due to the negative charge at C due to the charges at A and C is.

$$U = K\left[\frac{q_A q_C}{\overline{AC}} + \frac{q_B q_C}{\overline{BC}}\right]$$

$$= 9\times10^9\left[\frac{(5\times10^{-5})(-5\times10^{-5})}{5} + \frac{(5\times10^{-5})(-5\times10^{-5})}{5}\right]$$

$$U = -9J$$

Total energy at C

$$Ec = p.e + k.e$$

$$Ec = -9 + 4 = -5J$$

Let X = distance of point D from O

$$AD = BD = \left[9 + X^2\right]^{1/2}$$

At the point D, the kinetic energy of the negative charge will be zero, the entire energy of -5J will be potential in nature.

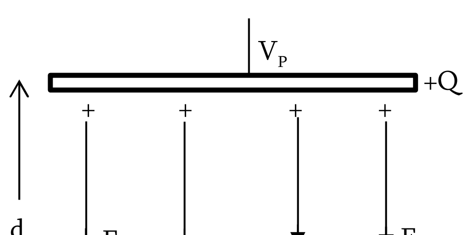
$$9\times10^9\left[\frac{(5\times10^{-5})(-5\times10^{-5})}{(9+X^2)^{1/2}} + \frac{(5\times10^{-5})(-5\times10^{-5})}{(9+X^2)^{1/2}}\right] = -5$$

$$\frac{-45}{(9+X^2)^{1/2}} = -5$$

$$X = \sqrt{27}m = 8.49m$$

MOTION OF CHARGED PARTICLES IN UNIFORM ELECTRIC FIELD.

1. Motion of charge particle (electron) parallel to the electric field E (vertical motion).



Consider the motion of charged particle q(e) along the direction of electric field as shown on the diagram (figure) above.

$$E = \frac{V_P}{d} \ldots\ldots\ldots\ldots\ldots(i)$$

V_P = applied voltage on the plate

d = distance between the plates

Electric force on the charged particle

$$F = E_q = Ma$$

$$a = \frac{E_q}{M} = \frac{E_q}{M} \ldots\ldots\ldots(ii)$$

Important parameters

(i) Expression of the velocity of charged particle in the time, t

$$V = u + at \ , \ u = 0$$

$$V = at$$

$$V = \frac{E_q t}{M} \ldots\ldots\ldots(iii)$$

(ii) Expression of vertical displacement of the charged particle after time, t

$$y = ut + \frac{1}{2}at^2 \quad u = 0$$

$$y = \frac{1}{2}at^2 = \frac{1}{2}\frac{Eqt^2}{m} \ldots\ldots\ldots(iv)$$

(iii) Expression of kinetic energy of charged particle

$$k.e = \frac{1}{2}mv^2$$

$$v^2 = u^2 + 2ay \ , \ u = 0$$

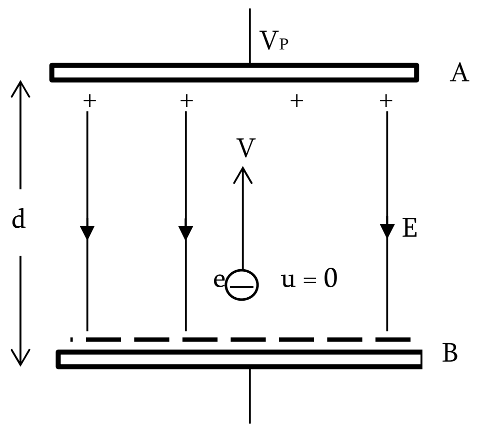
$$v^2 = \frac{2E_{qy}}{m}$$

$$k.e = \frac{1}{2}m\cdot\frac{2E_{qy}}{m}$$

$$k.e = E_{qy} \text{ ....(iv)}$$

Special case

Consider the motion of electron as shown on the figure below



Assume that electron start from rest at the lower plate and reaches at the upper plate with velocity, V. The electric field accelerates the electron straight upward.

Apply the law of conservation of energy.

Energy gained by an electron = kinetic energy of an electron.

$$eV_P = \frac{1}{2}MV^2$$

$$V = \sqrt{\frac{2eV_P}{M}}$$

Expression of time taken by an electron to strike at upper plate

Electric force on an electron

$$Ea = ma$$

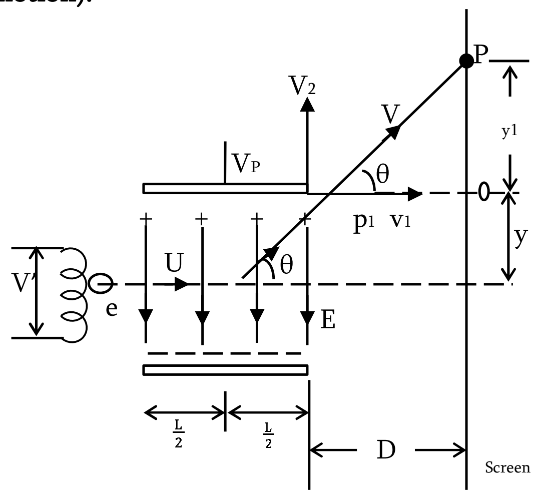
$$a = \frac{Ee}{m}$$

$$t = \frac{v}{a} = \frac{1}{a}\sqrt{\frac{2eV_p}{m}}$$

$$t = \sqrt{\frac{2eV_P}{m\left(eE/m\right)^2}} = \sqrt{\frac{2mV_P}{eE^2}}$$

$$t = \sqrt{\frac{2mV_P}{eE^2}}$$

2. **Motion of charged particle (electron perpendicular to the electric field (horizontal motion).**



Suppose on electron is first accelerated through a potential difference VP and subjected to a uniform electric field produced by two parallel charged plates as shown on the figure above.

Let L = Length each plate

d = distance of the screen from the edge of the plates.

VP = applied voltage on the plates

V = Voltage applied on hot filament.

Apply the law of conservation of energy

Energy gained By an electron = kinetic energy of an electron

$$eV' = \frac{1}{2}mu^2 \text{ ....(1)}$$

Parameters of the motion

(i) Velocity of an electron on emerging out of the electric field. Since the electric field act vertically downward, thus no horizontal force acts on an electron entering the region of the electric field. Therefore horizontal velocity u of an electron is unaffected. The electric field accelerates the electron in the vertically upward direction. Resulting in the vertical deflection. Electric field intensity between two parallel plates.

$$E = \frac{V_P}{d} \ldots\ldots\ldots(2)$$

Electric force on an electron

$$F = Ee = Ma$$

$$a = \frac{Ee}{m}$$

Time taken by the electron to across the electric field.

$$t = \frac{L}{u}$$

The initial vertical component of the velocity of an electron is zero at the point P_1.

$$V_2 = V_y = U_y + at$$

$$V_2 = \frac{Eet}{m} = \frac{EeL}{mu}$$

Velocity of an electron emerging out from the electric field is given by

$$V^2 = V_1^2 + V_2^2 \text{ (Pythagorous theorem)}$$

$$V = \sqrt{U^2 + \left(\frac{EeL}{mu}\right)^2} \ldots\ldots\ldots(3)$$

Direction of V

$$\tan\theta = \frac{V_2}{V_1} = \frac{EeL}{mu}/u$$

$$\tan\theta = \frac{EeL}{mu^2}$$

$$\theta = \tan^{-1}\left[\frac{EeL}{mu^2}\right] \ldots\ldots\ldots(4)$$

(ii) Electron path in an electric field is the parabolic.

Horizontal displacement in time, t

$$x = ut \;,\; t = \frac{x}{u}$$

Vertical displacement

$$y = U_y t + \frac{1}{2}a_y t^2 \;,\; U_y = 0$$

$$y = \left(\frac{Ee}{2m}\right)\frac{x^2}{u^2} = \frac{Eex^2}{2mu^2}$$

Let $\frac{Ee}{2mu^2} = k$

Therefore the path followed by an electron is the parabolic

Note that

$$y = \frac{Eex^2}{2mu^2} = \frac{Eex^2}{4\left(\frac{1}{2}mu^2\right)}$$

$$\text{But } eV' = \frac{1}{2}mu^2$$

$$y = \frac{Eex^2}{4eV'}$$

$$y = \frac{Ex^2}{4V'}$$

(iii) Total vertical deflection

$$y = y_1 + y$$

y = Deflection suffered by the beam in an electric field.

$$y = \frac{EeL^2}{2mu^2}$$

y_1 = deflection suffered by the beam outside of the electric field.

From the figure above

$$\tan\theta = \frac{V_2}{V_1} = \frac{EeL}{mu^2}$$

$$\tan\theta = \frac{y_1}{D} \;,\; y_1 = D\tan\theta$$

Now

$$y = y + y_1$$

$$= \frac{EeL^2}{2mu^2} + D\tan\theta$$

$$= \frac{EeL^2}{2mu^2} + \frac{DEeL}{mu^2}$$

$$y = \left(D + \frac{L}{2}\right)\frac{EeL}{mu^2}$$

$$\text{Also } \frac{1}{2}mu^2 = eV'$$

$$y = \left(D + \frac{1}{2}\right)\frac{EeL}{2\left(\frac{1}{2}mu^2\right)}$$

$$= \left(D+\frac{L}{2}\right)\frac{EeL}{2eV'}$$

$$Y = \left(D+\frac{L}{2}\right)\frac{EL}{2V'}$$

Specific charge

$$\frac{e}{m} = \frac{Yu^2}{EL\left(D+\frac{1}{2}\right)}$$

Note

If an electron accelerated by potential V to acquire a speed, U

$$eV = \frac{1}{2}mu^2$$

$$u = \sqrt{2\left(\frac{e}{m}\right)V}$$

THE ELECTRON VOLT (1ev)

Definition electron volt – is the energy gained by an electron which has been accelerated through a potential difference of one volt applied to the tube. Electron volt is a unit of energy from the expression of work done on moving electric charge, e.

W = eV

$$\text{If } e = 1.6\times10^{-9}C \text{ , } V = 1Volt$$

$$1eV = 1.6\times10^{-19}C\times1V$$

$$1eV = 1.6\times10^{-19}J$$

Note that

1Kev (kilo electron Volt) = 10^3eV = 1.6 × 10^{-6}J

1Mev (Mega electro volt) = 10^6eV = 1.6 × 10^{-13}J

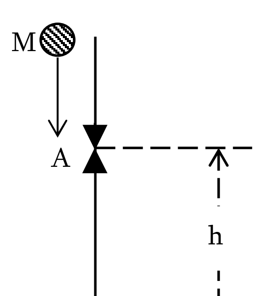
1GeV (Giga electron volt) = 10^9eV = 1.6 × 10^{-10}J

EXPRESSION FOR THE KINETIC ENERGY BY CHARGED PARTICLE IN A ELECTRIC FIELD – CONVERSION OF P.E INTO K.E

First let us consider a body (say a small sphere), falling freely in the Earth gravitational field

Apply the law of conservation of mechanical energy

$$(p.e+k.e)_A = (p.e+k.e)_B$$



As the body falls from A to B, p.e decreases while k.e increases.

Since the body is initially at rest at height, h. when it reaches B it acquires a velocity, V

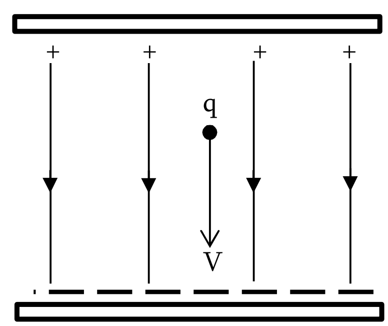
Assume that h = 0 at B

$$M_g h_A + \frac{1}{2}MV_A^2 = M_g h_B + \frac{1}{2}MV_B^2$$

$$M_{gh} = \frac{1}{2}MV^2$$

Decrease in p.e = gained in k.e

Instead of a stone, consider a positive charge moves from plate A to B in the direction of the electric field.



Apply the law of conservation of energy

$$\Delta k.e = \Delta p.e$$

$$k.e_B - k.e_A = p.e_A - p.e_B$$

$$\frac{1}{2}MV_0^2 = qV$$

Example 77

(a) A particle of mass M and charge q is placed at rest in a uniform electric field of strength E and released. Show that the kinetic energy attained affect motion through a distance Y is given by K.E = Eqy.

(b) An electron is travelled horizontally after having been accelerated from rest by a p.d of

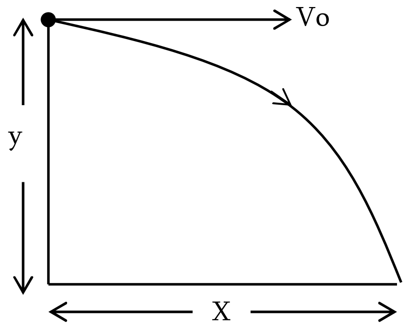
50V. How far would it fall under gravity in travelling 200m in a vacuum.

$$\left(\frac{e}{m} = 1.8\times10^{11}\,Ckg^{1}\right)$$

Solution

(a) Refer to your notes

(b)



Apply the law of conservation of energy

Work done = k.e of an electron

$$eV = \frac{1}{2}MV_0^2$$

$$V_0 = \sqrt{2\left(e/m\right)V} = \sqrt{2\times1.8\times10^{11}\times50}$$

$$V_0 = 4.2\times10^6\,m/s$$

Horizontal displacement

$$X = Vot$$

$$t = \frac{X}{Vo} = \frac{200m}{4.2\times10^6\,m/s}$$

$$t = 4.76\times10^{-6}\,sec$$

Vertical displacement

$$y = \frac{1}{2}gt^2 = \frac{1}{2}\times9.8\times\left(4.76\times10^{-6}\right)^2$$

$$y = 1.10\times10^{-8}\,m$$

Example 78 NECTA 2016/P2/5

(a) Define the following terms;
   (i) Capacitance (01 mark)
   (ii) Charge density (01 mark)
   (iii) Equipotential surface (01 mark)

(b) By using the coulomb's law of electrostatics, derive an expression for the electric field strength E, due to a point charge if the material is surrounded by a material of permittivity, ε and hence show how it relates with charge density, σ (04 marks)

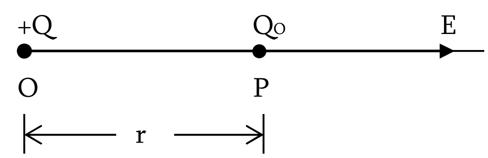
(c) Describe the structure and the mode of action of a simplified version of the van de graaff generator (05 marks)

(d) (i) identify any three factors on which the capacitance of parallel plate capacitor depends (1.5 marks)
   (ii) a proton of mass 16.7×10^{-28}kg falls through a distance of 2.5cm in a uniform electric field of magnitude 2.65×104V/m. determine the time of fall if the air resistance and the acceleration due to gravity, g are neglected (3.5 marks)
   (iii) A parallel plate capacitor is made of a paper 40mm wider and $3.0\times10\text{-}2$mm thick. Determine the length of the paper sheet required to construct a capacitance of 5μF, if it relative permittivity is 2.5 (03 marks)

Solution

(a) Refer to your notes

(b)



Apply the coulomb's law of electrostatics

$$F = \frac{KQQ_0}{r^2}$$

Electric field intensity at P

$$E = \frac{F}{Q_0} = \frac{\frac{KQ_0}{r^2}}{Q_0}$$

$$E = \frac{KQ}{r^2} \quad \text{but } K = \frac{1}{4\pi\varepsilon_r}$$

$$E = \frac{1}{4\pi\varepsilon r^2}$$

Let σ = surface charged density

$$\sigma = \frac{1}{4\pi r^2}, \quad Q = 4\pi r^2\sigma$$

$$E = \frac{4\pi r^2\sigma}{4\pi\varepsilon r^2}$$

$$E = \frac{\sigma}{\varepsilon}$$

(c) Refer to your notes

(d) (i) Capacitance of parallel plate capacitor is given by

$$C = \frac{A\varepsilon_r\varepsilon_o}{d}$$

- Area of one of plate, A ($C \propto A$)
- Distance between two parallel plates, d $\left(C \propto \frac{1}{d}\right)$
- Dielectric constant of the material, $\varepsilon_r \left(C \propto \varepsilon_r\right)$

(ii) Electric force on the proton

$$F = Ee = Ma$$

$$a = \frac{Ee}{M} = \frac{2.65\times10^{4}\times1.6\times10^{-19}}{16.7\times10^{-28}}$$

Since $S = ut + \frac{1}{2}at^2$

Assume that the proton start from rest, u = 0

$$S = \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2s}{a}}$$

$$t = \sqrt{\frac{2\times2.5\times10^{-2}}{2.539\times10^{12}}}$$

$$t = 1.4033\times10^{-7}\text{ sec}$$

(iii) $C = \frac{\varepsilon_0 KA}{d}$, $A = \frac{Cd}{\varepsilon_0 K}$

$$A = \frac{5\times10^{-6}\times3\times10^{-5}}{8.85\times10^{-12}\times2.5}$$

$$A = 6.78\text{m}^2$$

Required length,

$$L = \frac{\text{Area}}{\text{Width}}$$

$$L = \frac{6.78}{40\times10^{-3}}$$

$$L = 169.5\text{m}$$

Example 79

(a) Calculate the electric field strength required to just support a water drop of mass 10^{-7} kg and having a charge 1.6×10^{-19} C .

(b) A particular of mass 10^{-3} kg and charge 5μc is thrown at a speed 20m/s against a uniform electric field of strength 2×10^5 N/C. How much distance will it travel before coming to rest momentarily?

Solution

(a) Let E be the strength of the electric field required to just support the water drop

(b) Force on the charged particle F = Eq

Since the particle is thrown against the electric field.

$$a = \frac{-F}{m} = \frac{E_q}{m}$$

$$= \frac{-2\times10^5\times5\times10^{-6}}{10^{-3}}$$

$$a = -1000\text{m/s}^2$$

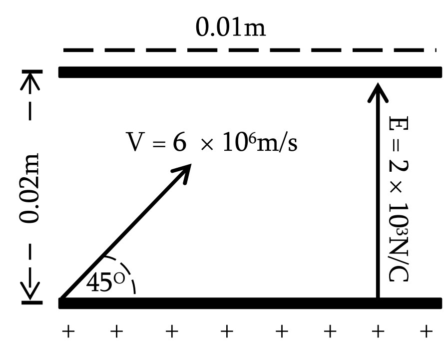
Since $V^2 = U^2 + 2as$

$$S = \frac{V^2 - U^2}{2a} = \frac{0^2 - 20^2}{2\times(-1000)}$$

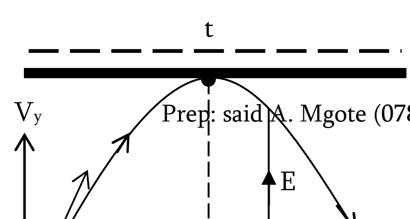
$$S = 0.2\text{m}$$

Example 80

A uniform electric field of strength 2×10^3 N/C is established between two parallel plates of length 0.1m held horizontally at a distance 0.02m apart. An electron is projected at a speed of 6×10^6 m/s making an angle 45° as shown in the figure below. The field is directed vertically upwards will the electron strike the either plate? If it strikes the plate, where does it do so?



Solution



Speed of an electron can be resolved into the following two rectangular components:-

$V_x = V\cos 45^o = 6\times10^6\times0.707$

$V_x = 4.24\times10^6 m/s$ (along horizontal)

As there is no force on an electron in horizontal direction, V_X remain constant.

$V_y = V\sin 45^o = 6\times10^6\times0.707$

$V_y = 4.24\times10^6 m/s$ (along vertical)

Due to electric field, the electron experiences force in downward direction (opposite to its motion), the V_y goes on decreasing.

Force on an electron

$F = Ee$ (downwards)

$Ma = Ee$

$$a = \frac{Ee}{M} = \frac{1.6\times10^{-19}\times2\times10^3}{9.1\times10^{-31}}$$

$a = 3.516\times10^{14} m/s^2$ (downwards)

For motion along vertical

$V_y = 4.24\times10^{-6} m/s$, $s = 0.02m$

$a = -3.516\times10^{14} m/s^2$

Let t = time taken to reach the upper plate

$$S = V_y t + \frac{1}{2}at^2$$

$$0.02 = 4.24\times10^{-6}t + \frac{1}{2}\left(-3.516\times10^{14}\right)t^2$$

$1.758\times10^{14}t^2 - 4.24\times10^6 t + 0.02 = 0$

On solving $t = 6.4\times10^{-9}$ sec

Since t has a finite real value, the electron will strike the upper plate. Suppose that it strikes the upper plate at a distance X from the left end

$X = V_x t$

$= 4.24\times10^6\times6.4\times10^{-9}$

$X = 2.71\times10^{-2} m = 2.71cm$

Example 81

When a point electric charge of $1.5\times10^{-9}C$ is moved between points A and B 5.0mm apart in air in a uniform electric field is $1.5\times10^{-6}J$ of work is done.

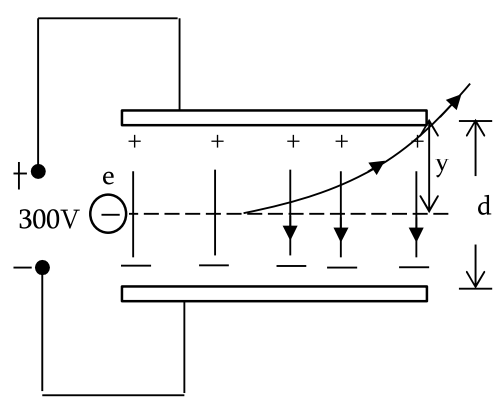
(a) What is the potential difference between A and B?
(b) Calculate the intensity of the electric field.
(c) What is the electrostatic force exerted on the charge by the electric field.

Ans. (a) 1000V (b) $2.0\times10^5 Vm^{-1}$ (c) $3.0\times10^{-4}N$

Example 82

Two plane metal plates 4cm long are held horizontally 3cm apart in a vacuum. The plate is kept at an electric potential 300V and the other plate is Earthed. Electron having a velocity of $1.0\times10^7 m/s$ are ejected horizontally mid – way between the plates and in a direction parallel to the 4cm edge. Calculate the vertical deflection of electron as they emerge from the plate. $\left(e = 1.6\times10^{-19}C,\ Me = 9.1\times10^{-31}kg,\ e/m = 1.8\times10^{11}Ckg^{-1}\right)$

Solution



Electric force on an electron

$Ee = Ma$, $a = E\left(e/m\right)$

Vertical deflection

$$y = \frac{1}{2}at^2 = \frac{1}{2}E\left(e/_m\right)t^2$$

Since $x = ut$, $t = \frac{x}{u}$

$$y = \frac{1}{2}\left(\frac{v}{d}\right)\left(\frac{e}{m}\right)\left(\frac{x}{u}\right)^2$$

$$y = \frac{1}{2}\left(\frac{v}{d}\right)\left(\frac{e}{m}\right)\left(\frac{x}{u}\right)^2$$

$$= \frac{1}{2}\left(\frac{300}{3\times10^{-2}}\right)\times1.8\times10^{11}\times\left(\frac{4\times10^{-2}}{10^7}\right)^2$$

$$y = 0.0144\text{m} = 1.44\text{cm}$$

Example 83

An electron is liberated from the lower of the two large parallel metal plates separated by distance of 20mm. The upper plate has a potential of +2400V relative to the lower plate. How long does the electron take to reach the upper plate? Take $\frac{e}{m} = 1.8\times10^{11}\text{Ckg}^{-1}$ for an electron.

Solution

Force on an electron

Ee = Ma

$$a = \frac{Ee}{m} = \left(\frac{V}{d}\right)\left(\frac{e}{m}\right)$$

$$= 1.8\times10^{11}\times\frac{2400}{0.02}$$

$$a = 2.16\times10^{16}\text{m/s}^2$$

Since $S = ut + \frac{1}{2}at^2$

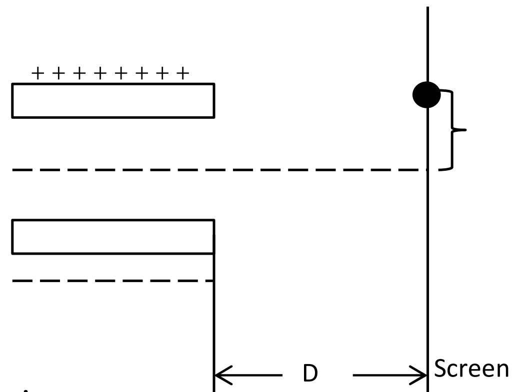
$$u = 0 \ , \ S = \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\times0.02}{2.16\times10^{16}}}$$

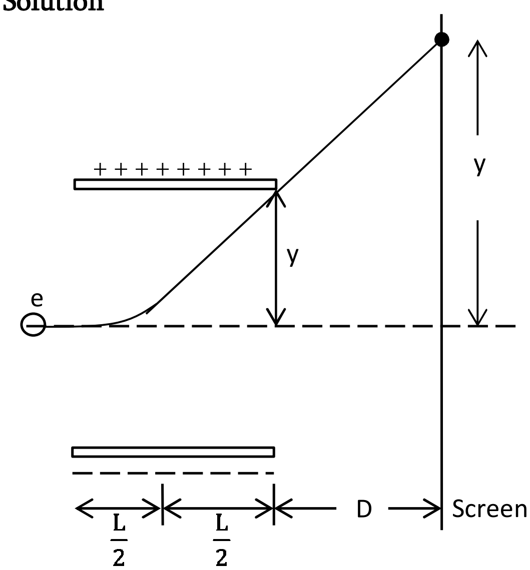
$$t = 1.4\times10^{-19}\text{sec}$$

Example 84

In a cathode ray oscilloscope (CRO) an electron accelerated through 25KV enters a deflecting plate system as shown in the figure below. The magnitude of deflecting field is 20KVm^{-1}. The length of the plates is 5.0cm and the screen is 18cm (D) away from the nearer end of the plates. At what point does the electron hit the fluorescent screen?



Solution



Let M be the mass of the electron V the velocity gained by it when it is accelerated through a potential difference of Vo volt.

Apply the law of conservation of energy

k.e of an electron = energy gained

$$\frac{1}{2}MV^2 = eVo$$

Time taken by the electron to cover length of the plates.

$$t = \frac{L}{V}$$

As the electron enters the deflecting plate system, it experience and acceleration in the positive y – direction given by

$$Ee = Ma \ , \ a = \frac{Ee}{M}$$

Vertical displacement suffered by the electron as it just come out of the deflecting plates.

$$y = \frac{1}{2}at^2 = \frac{Ee}{2m}\cdot\frac{L^2}{V^2}$$

For similar triangles , we have

$$\frac{Y}{y} = \frac{D + L/2}{L/2}$$

$$Y = \frac{2y}{L}\left[D + \frac{L}{2}\right]$$

$$= \frac{2}{L}\cdot\frac{Ee}{2m}\cdot\frac{L^2}{V^2}\left[D + \frac{L}{2}\right]$$

$$Y = \frac{EeL}{MV^2}\left[D + \frac{L}{2}\right] = \frac{EL}{2V_o}\left[D + \frac{L}{2}\right]$$

$$= \frac{1}{2}\times\frac{2\times10^4\times0.05}{25\times10^3}\left[0.18 + \frac{0.05}{2}\right]$$

$$Y = 41\times10^{-3}m = 4.1mm$$

Example 85

An electron beam after being accelerated from rest through a potential difference of 10,000V in vacuum is allowed to impinge normally on a screen, which is fixed. If the incidence current is $100\mu A$, determine the force exerted on the screen, assuming that it brings the electrons to rest.

Solution

Speed of electrons on reading on the screen

$$V = \sqrt{\frac{2eV_P}{M}}$$

$$= \sqrt{\frac{2\times1.6\times10^{-19}\times10{,}000}{9.1\times10^{-31}}}$$

$$V = 5.93\times10^7 m/s$$

Time taken by electrons to strike the target

$$t = \frac{e}{I} = \frac{1.6\times10^{-19}}{100\times10^{-6}}$$

Force exerted on the screen is given by

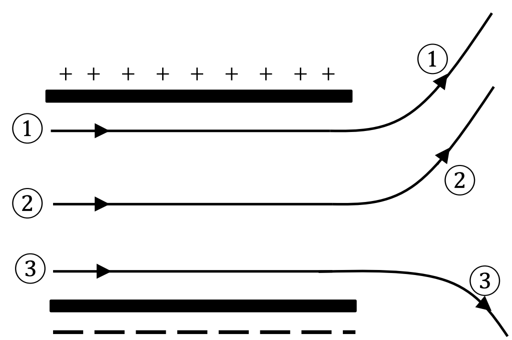
$$F = Ma = M\left[\frac{V-U}{t}\right]$$

$$= \frac{9.1\times10^{-31}\left[0 - 5.93\times10^7\right]}{1.6\times10^{-15}}$$

$$F = -3.373\times10^{-6}N$$

Example 86

The following figure shows the tracks of three charged particles in a uniform electric field. What can we say about the sign of the charges on the three particles? Which particle would have the largest value of e/m i.e charge to mass ratio? Assume that the particles are all initially moving horizontally with the same speed, V.



Solution

Since the particles 1 and 2 are deflected towards the positive plate (in the electric field), they must be negatively charged. For the same reason, the particle 3 must be positively charged. To find which particle has the largest e/m ratio, we notice that the force F on a particle of charge e in an electric field E is given by $F = Ee$. Hence in this case, the vertical acceleration on the particle is a $a = \frac{Ee}{m}$.

Let L be the length over which the electric fields acts. The particle stays in the field for a time L/V

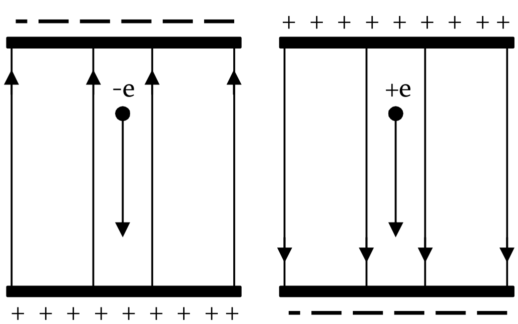
Vertical deflection at the end of the field region.

$$S = \frac{1}{2}at^2 = \frac{eE}{2m}\cdot\frac{L^2}{V^2}$$

Thus s α $e/_m$ for the same value of E, L and V. Since S is seen to be largest fro particle 3, we can say that particle 3 has the largest $e/_m$ ratio of the three particles.

Example 87

An electron falls through a distance of 1.5cm in a uniform electric field of magnitude $2.4\times10^4\,N/C$ (See figure(a)below). The direction of the field is reversed keeping its magnitude unchanged and a proton falls through the same distance (See figure(b)). Compute the time of fall in each case. Contrast the situation (a) with that free fall under gravity.



(a) (b)

Solution

(a) The upward field exerts a downward force eE on the electron

$$a_1 = \frac{eE}{M}$$

Since $u = 0$, $S = ut + \frac{1}{2}at^2$

$$t_e = \sqrt{\frac{2s}{a_1}} = \sqrt{\frac{2SMe}{eE}}$$

$$= \sqrt{\frac{2\times1.5\times10^{-2}\times9.1\times10^{-3}}{1.6\times10^{-19}\times2.4\times10^4}}$$

$$t_e = 2.9\times10^{-19}\,\text{sec}$$

(b) The downward field exerts on downward force eE on the proton.

$$t_p = \sqrt{\frac{2S}{a_p}} = \sqrt{\frac{2SM_P}{eM}}$$

$$\sqrt{\frac{2\times1.5\times10^{-2}\times1.67\times10^{-27}}{1.6\times10^{-19}\times2.4\times10^4}}$$

$$t_p = 1.3\times10^{-7}\,\text{sec}$$

Example 88

The potential difference between the plates of a capacitor is 175V. Midway between the plates, a proton and an electron are released. The electron is released from rest, the proton is projected perpendicularly towards the negative plate with an initial speed. The proton strikes the negative plate at an instant that the electron strikes the positive plate. Ignore the attraction between the two particles and find the initial speed of the proton.

Solution

Electric field intensity between the plates

$$E = \frac{V}{d}$$

$$F_e = M_e a_e = eE = \frac{eV}{d}$$

$$a_e = \frac{eV}{M_{ed}} \;\ldots\ldots\ldots\ldots(i)$$

Similarly acceleration of the proton

$$a_p = \frac{eV}{M_p d} \;\ldots\ldots\ldots..(ii)$$

For an electron , $u = 0$, $s = \frac{d}{2}$

(Electron enters mid – way)

$$\frac{d}{2} = \frac{1}{2}a_c t^2$$

$$t = \sqrt{\frac{d}{a_e}} \;\ldots\ldots\ldots\ldots(iii)$$

For proton

$$\frac{d}{2} = u\cdot\sqrt{\frac{d}{a_e}} + \frac{1}{2}a_p\left(\sqrt{\frac{d}{a_e}}\right)^2$$

Make subject, u.

$$u = \frac{1}{2}d\left(1-\frac{a_p}{a_e}\right)\cdot\sqrt{\frac{a_e}{d}}$$

$$= \frac{d}{2}\left(1-\frac{M_e}{M_p}\right)\frac{1}{d}\sqrt{\frac{eV}{M_e}}$$

$$U = \frac{1}{2}\left(1-\frac{M_e}{M_p}\right)\sqrt{\frac{eV}{M_e}}$$

$$U = \frac{1}{2}\left[1-\frac{9.1\times10^{-31}}{1.67\times10^{-27}}\right]\sqrt{\frac{1.6\times10^{-19}\times175}{9.1\times10^{-37}}}$$

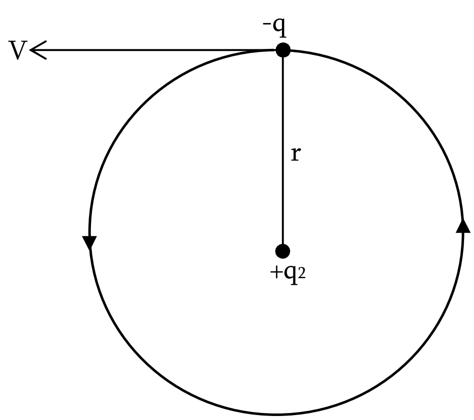
$$U = 2.77\times10^{6}\,m/s$$

Example 89

A particle of mass m and carrying charge $-q_1$ is moving around a charge $+q_2$ along a circular path of radius r. Prove that period of revolution of the charge $-q_1$ about $+q_2$ is given by

$$T = \sqrt{\frac{16\pi^3\varepsilon_o mr^3}{q_1q_2}}$$

Solution



The necessary centripetal force is provided by the electrostatic force of attraction between the two charges i.e

$$\frac{q_1q_2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}$$

$$V = \left[\frac{1}{4\pi\varepsilon_o}\cdot\frac{q_1q_2}{mr}\right]^{\frac{1}{2}}$$

If T is period of revolution of the charge $-q_1$ about q_2 then

$$T = \frac{2\pi r}{V}$$

$$T = \sqrt{\frac{16\pi^3\varepsilon_0 mr^3}{q_1q_2}}$$

Example 90

Two identical particles of mass, m carry a charge Q each. Initially one is at rest on a smooth horizontal plane and the other is projected along the plane directly towards the first particle from a large distance with speed, V. Find the closest distance of approach.

Solution

Let M be the mass of the each of the two particles. When one particle is projected with speed V towards the other particle, suppose that they approach up to a closest distance, d.

Let V_1 and V_2 be their speeds, when the two particles are at a distance, d.

Apply the principle of conservation of linear momentum.

$$mv = mv_1 + mv_2$$

$$v = v_1 + v_2 \,\ldots\ldots\ldots(i)$$

Since the two particles are identical, their speeds at the identical, their speeds at the closest distance of approach must be equal i.e

$V_1 = V_2$

From the equation (i) and (ii)

$$V_1 = V_2 = \frac{V}{2}$$

Initially, only the first particles is moving and the two particles are at large distance from each other.

$\therefore$ Initial energy of the system

$$E_i = \frac{1}{2}mv^2 + 0 = \frac{1}{2}mv^2$$

Final energy of the system

$$E_F = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{Q}{4\pi\varepsilon_o d}$$

$$E_F = \frac{1}{4}mv^2 + \frac{Q^2}{4\pi\varepsilon_o d}$$

Apply the principle of conservation of energy

$$\frac{1}{4}mv^2 + \frac{Q^2}{4\pi\varepsilon_o d} = \frac{1}{2}mv^2$$

$$\frac{Q}{4\pi\varepsilon_o d} = \frac{1}{4}mv^2$$

$$d = \frac{1}{4\pi\varepsilon_o}\cdot\frac{4Q^2}{mv^2}$$

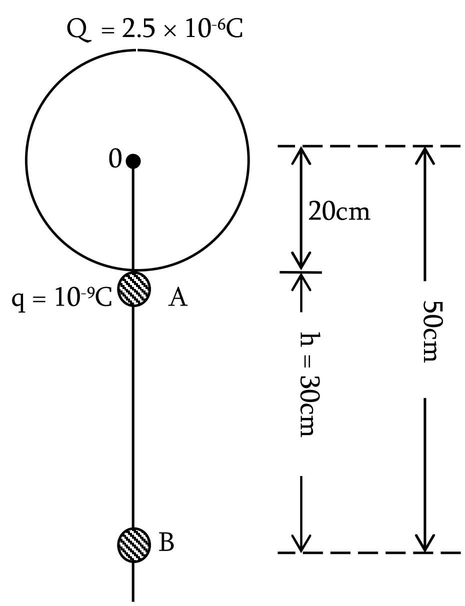
Example 91

A drop of water of mass 18×10^3g falls away from the bottom of charge conducting sphere of radius 20cm, carrying with it a charge of 10^{-9}C and leaning on the sphere a uniformly distributed charge of 2.5×10^{-6}C. What is the speed of the drop after it has fallen 30cm?

$$\frac{1}{4\pi\varepsilon_o} = 9\times10^9\,Jmc^{-2}$$

Solution

The drop of water of mas m and charge q leaves the conducting sphere of charge Q at point A and falls through a distance of 30cm up point B as shown



The drop acquires kinetic energy of the point B at the expense of gravitational potential energy and electrostatic potential energy.

Let V be the speed of the drop on reaching the point B.

Apply principle of conservation of energy.

Increase in k.e of drop = decrease in electrostatic p.e and g.p.e

$$\frac{1}{2}MV^2 = \frac{1}{4\pi\varepsilon_o}\left[\frac{Q_q}{OA} - \frac{Qq}{OB}\right] + mgh$$

h = 30cm = 0.3m, m = 18×10^{-16} kg

Q = 2.5×10^{-6} C, q = 10^{-9} C

$\overline{OA} = r = 20cm = 0.2$

$\overline{OB} = 20 + 30 = 50cm = 0.5m$

$$\frac{1}{2}\times18\times10^{-6}V^2 = 9\times10^9\times25\times10^{-6}\times10^{-9}$$

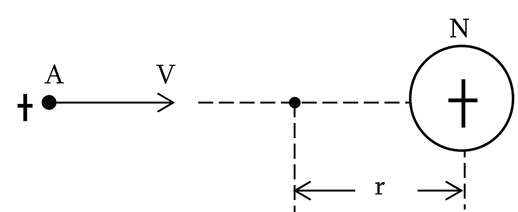
$$\left[\frac{1}{0.2} - \frac{1}{0.5}\right] + 18\times10^{-6}\times9.8\times0.3$$

V = 3.66m/s

Example 92

(a) An electro of charge ($e = 1.6\times10^{-19}$C) is situated in a uniform electric field of intensity $120{,}000Vm^{-1}$. Find force on it, its acceleration, and the time it takes to travel 20mm from rest. ($Me = 9.1\times10^{-31}$kg)

(b) In figure below an α - particle A of charge $+3.2\times10^{-19}$C and mass 6.8×10^{-27}kg is travelling with velocity V of 1.0×10^7m/s directly towards N which has a charge of $+11.2\times10^{-19}$C. Calculate the closest distance of approach of A to N, assuming that A is initially a very long from N compared with the closest distance of approach.



Solution

(a) Electric force on an electron

$$F = Ee = ma$$

$$a = \frac{Ee}{m} = \frac{120{,}000\times1.6\times10^{-19}}{9.1\times10^{-31}}$$

An electron start from rest, U = 0

$$S = \frac{1}{2}at^2 \,,\; t = \sqrt{\frac{2S}{a}}$$

$$t = \sqrt{\frac{2\times20\times10^{-3}}{2.12\times10^{-16}}}$$

$t = 1.37\times10^{-9}\,sec$

Note that:
The extreme shortness of this time is due to the fact that the ratio of charge to mas for an electron is very great $\frac{e}{m} = 1.8\times10^{11}\,Ckg^{-1}$.

(b) Apply the law of conservation of energy

Loss of k.e of α Particle = gained in electrical p.e

$$\frac{1}{2}MV^2 = \frac{Q_1Q_2}{4\pi\varepsilon_0 r}$$

$$r = \frac{Q_1Q_2}{4\pi\varepsilon_0\left(\frac{1}{2}MV^2\right)}$$

$$r = \frac{9\times10^9\times3.2\times10^{-19}\times11.2\times10^{-19}}{\frac{1}{2}\times6.8\times10^{-27}\times\left(1\times10^7\right)^2}$$

$$r = 9.4\times10^{-15}\,m$$

ASSAGNMENT NO 4

1. An electron beam after being accelerated from rest through a potential difference of 5000V in vacuum is allowed to impinge normally on a fixed surface. If the incident current is 50μA, determine the force exerted on the surface assuming that it brings the electrons to rest $\left(e = 1.6\times10^{-19}C\right)$

   Ans. $1.2\times10^{-18}\,N$

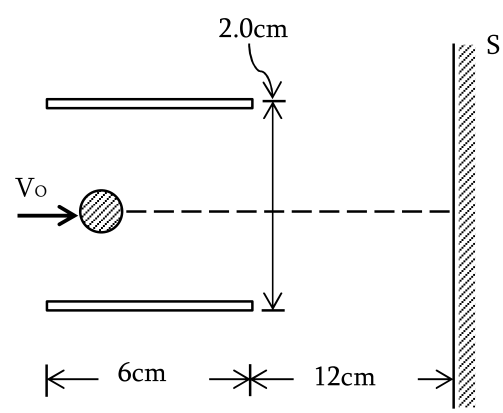
2. Two plane metal plates 4.0cm long are held horizontally 3.0cm apart in a vacuum, one being vertically above the other. The upper plate is at a potential of 300V and the lower is Earthed. Electrons having a velocity of $1.0\times10^7\,m/s$ are injected horizontally mid-way between the plates and in a direction parallel to the 4.0cm edge. Calculate the vertical deflection of the electron beam as it emerges from the plates.

   $\frac{e}{m} = 1.8\times10^{11}\,Ckg^{-1}$.   ans. $1.44\times10^{-2}\,m$

3. (a) What is an electric field? With reference to such a field, define electric potential
   (b) Two plane parallel conducting plates are held horizontal, one above the other, in a vacuum. Electrons having a speed of $6\times10^6\,m/s$ and moving normally to the plates enter the region between them through a hole in the lower plate which is Earthed. What potential must be applied to the other plate so that the electrons fail to reach it? What is the subsequent motion of these electrons? Assume that the electrons do not interact with one other $\left(e/m = 1.8\times10^{11}\,Ckg^{-1}\right)$

   Ans. (b) 100N

4. Cathode ray tubes are often found in oscilloscopes and computer monitors.



As shown in the figure above, an electron with an initial speed of $6.5\times10^6\,m/s$ is projected along the axis midway between the deflection plates of a C.R.O. The uniform electric field between the plates has a magnitude of $1.10\times10^3\,V/m$ and is upward.

(a) What is the force (magnitude and direction) on the electron when it is between the plates?
(b) What is the acceleration of the electron (magnitude and direction) when acted on by the force in part (a)?

(c) How far below the axis has the electron moved when it reaches the end of the plates?

(d) At what angle with the axis is it moving as it leaves the plates?

(e) How far below the axis will it strike the fluorescent screen, s?

Ans. (a) $1.76\times10^{-16}\,N$, downward

(b) $1.93\times10^{14}\,m/s^2$ downward

(c) 8.24 mm (d) 15.4 (e) 4.12cm

5. A proton has an initial velocity of $4.5\times10^5\,m/s$ in the horizontal direction. It enters a uniform electric field of $9.60\times10^3\,N/C$ directed vertically. Ignore any gravitational effects and

(a) Find the time it takes the proton to travel 5.0cm horizontally

(b) Find the vertical displacement of the proton after it has travelled 5.0cm horizontally.

(c) Find the horizontal and vertical components of the proton's velocity after it has travelled 5.0cm horizontally.

Ans. (a) 111ns (b) 5.6mm

(c) V_X = 450km/s V_Y = 102km/s

6. An electron with a speed of $5\times10^8\,cm/s$ enters an electric field of magnitude $1.0\times10^3\,N/C$, travelling along the field in the directions that retards it motion.

(a) How far will the electron travel in the field before stopping momentarily and

(b) How much time will have elapsed?

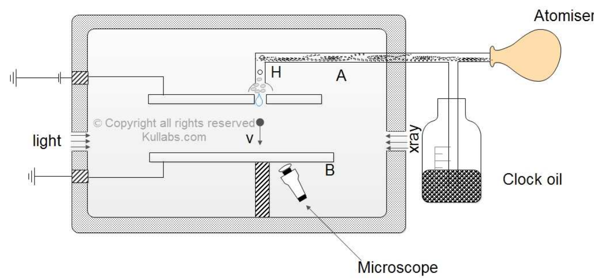
(c) If instead, the region of the electric field is only 8.00mm wide (too small for the electron to stop), what fraction of the electron's initial kinetic energy will be lost in that region?.

(a) 7.12cm (b) 28.5ns (c) 11.2%

MILLIKAN'S OIL DROP EXPERIMENT

This is an experiment used to determine the charge of an electron.

Arrangement of apparatus



A = travelling microscope.

The name oil used in the Millikan's experiment is known as Non – volatile clock oil.

Millikan's apparatus consists of the following parts;-

(i) Double walled chamber (constant temperature bath) this chamber is used to maintain a constant temperature by circulating water between the walls. It's avoid convection current of air which may more the drop and cause error. Also it shields the apparatus from draughts.

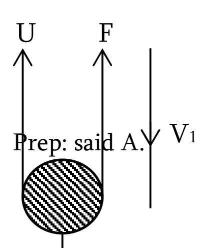
(ii) Atomizer (pump blower) – used to spray oil drop (clock oil) on the capillary tube.

(iii) X – ray tube – used to produce x – ray radiation mainly for ionizing air, Millikan's observed the motion of oil drop and measure the terminal velocity of charged oil droplets.

1. Under action of gravity alone
2. Under the combined action of gravity and electric field opposed to gravity.

1. MOTION OF OIL DROP UNDER GRAVITY ALONE

Suppose the electric field is switched off. As the drop falls under gravity its velocity goes on increasing it's reach the point where by oil drop moves downward with constant terminal velocity V_1, As shown below



The available forces acting on an oil drop are:-

(i) Weight of oil drop acting vertically downward i.e $W = Mg = \frac{4}{3}\pi r^3 \rho g$

(ii) Upthrust (u) due to buoyancy of air acts vertically upward $U = \frac{4}{3}\pi r^3 \sigma g$

(iii) Viscous drag force (F) acting vertically upward.

$F = 6\pi\eta V_1 r$ (By stroke's law)

At the equilibrium

$$W = F + U$$

$$F = E - U$$

$$6\pi\eta V_1 r = W - U \text{ ....(1)}$$

Also

$$6\pi\eta V_1 r = \frac{4}{3}\pi r^3 (\rho - \delta)$$

$$r = \left[\frac{q\eta V_1}{2(\rho - \sigma)g}\right] \text{ ....(2)}$$

Since V_1 is measure and values n, ρ, σ and g are known, the value of radius (r) of spherical droplet can be obtained.

When M = mass of the oil droplet

ρ = density of oil drop

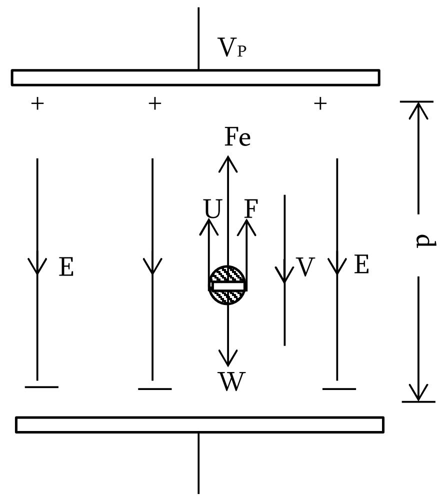
σ = ρa = density of air

η = coefficient of viscosity of air

g = acceleration due to gravity.

2. MOTION OF OIL DROPLET UNDER ACTION OF ELECTRIC FIELD OPPOSED TO GRAVITY.

When the electric field is switched on (applied) between the plates in such a direction that force on the negatively charged oil droplet due to the electric field acts in the vertically upward direction. (i.e. opposite to gravity). Now the oil drop moves downward with new constant terminal velocity, V



Electric field intensity between the two parallel plates.

$$E = \frac{V_P}{d} \text{ ....(3)}$$

VP = applied voltage

d = distance between the two plats

At the equilibrium

$$F_e + F + U = W$$

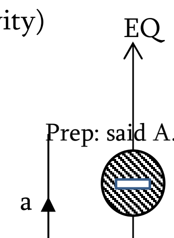
$$F_e = (W - U) - F$$

$$E_q = 6\pi\eta V_1 r - 6\pi\eta V r$$

$$E_q = 6\pi\eta\left[\frac{9\eta V_1}{2g(\rho - \sigma)}\right]^{1/2} (V_1 - V)$$

$$q = \frac{6\pi\eta}{E}\left[\frac{q\eta V_1}{2g(\rho - \sigma)}\right]^{1/2} (V_1 - V)$$

When oil drop pick up excess electric charges then the droplet reaches point and becomes stationary between two parallel plates. (Since the electric force acts on oil drop is directed against the direction of the force of gravity)



Electric force = Weight of drop

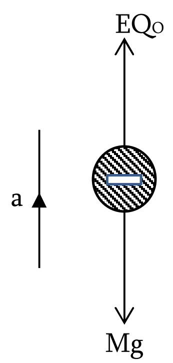
$$EQ = Mg$$

$$Q = \frac{Mg}{E} = \frac{Mgd}{V_P}$$

Now the oil droplet starts moving upward when it pick up more excess electron(s) from ionized air and soon attain a new terminal velocity V_2. When move vertically upward.

- **Expression of initial acceleration of oil drop**

  Assume that charged oil drop pick up extra electron from ionized air



Resultant force on the drop

$$EQ_0 - Mg = Ma$$

But $Q_0 = Q + e$

$$E(Q+e) - Mg = Ma$$

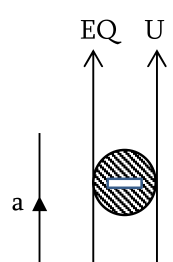
$$EQ + Ee - Mg = Ma$$

But $EQ = Mg$

$$Ee = Ma$$

$$a = \frac{Ee}{M}$$

Now, when charged oil drops moves vertically upward, it's attain new terminal velocity, V_2.



At the equilibrium

$$EQ_0 + U = W + F_2$$

$$EQ_0 = (W - U) + F_2$$

$$= 6\pi\eta V_1 r + 6\pi\eta V_2 r$$

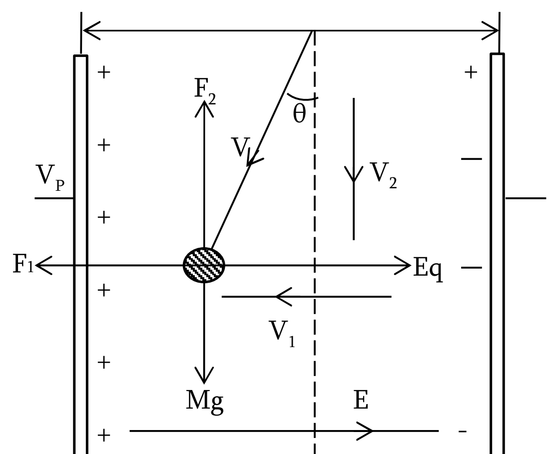
$$EQ_0 = 6\pi\eta r(V_1 + V_2)$$

$$Q_0 = \frac{6\pi\eta}{E}\left[\frac{9\eta V_1}{2g(\rho-\sigma)}\right]^{\frac{1}{2}}(V_1 + V_2)$$

SPECIAL CASE

MOTION OF CHARGED OIL DROP ON THE VERTICAL PARALLEL PLATES.

Consider the figure below which shows the motion of charged oil drop when the plates are in the vertical position.



Let V_1 = horizontal component of velocity

V_2 = Vertical component of velocity

Assume that, the motion of charged oil drop is uniform i.e moderate motion.

$F \propto V$, $F = KV$

K = constant of proportionality

$F_1 = KV_1$, $F_2 = KV_2$

At the equilibrium of charged oil drop

$KV_2 = Mg$................(i)
$KV_1 = Eq$.................(ii)
Dividing equation (i) by (ii)

$$\frac{KV_2}{KV_1} = \frac{Mg}{E_q}$$

$$\frac{V_2}{V_1} = \frac{Mg}{E_q}$$

$$E_q = \left(\frac{V_1}{V_2}\right)Mg$$

From the figure above

$$\tan\theta = \frac{V_1}{V_2}$$

$$E_q = Mg\tan\theta$$

$$q = \frac{Mg\tan\theta}{E} = \frac{Mgd\tan\theta}{V_p}$$

$$q = \frac{Mgd}{V_P}\tan\theta \quad , \quad q \propto \tan\theta$$

CONCLUSIONS

1. Electric charge(s) on an oil drop is the multiple integers of numbers i.e ±1e, ±2e, ±3e, ±4e,---- ----- ---- ----- -----
2. The highest common factor gives the charges on the electron $e = 1.602\times10^{-19}C$
3. Millikan's also obtain the mass of an electron when e/m was known i.e

$$\frac{e}{M} = 1.76\times10^{11}CKg^{-1}$$

Mass of an electron

$$Me = \frac{e}{e/m} = \frac{1.602\times10^{-19}}{1.76\times10^{11}}$$

$$Me = 9.1\times10^{-31}kg$$

PRECAUTIONS

The following precautions may be observed while performing Millikan's oil drop experiments;-

1. The enclosure surroundings Millikan's apparatus must be maintain at constant temperature in order to eliminate error due to convections current.
2. The oritically, any non – volatile oil may be used but in actual practice clock oil is considered best because it gives quite small droplets.
3. The air between the plate should be homogeneous.
4. Stoke's law is not strictly valid for a small spherical drop such as in Millikan's experiment. To find exact value of charge on oil drop, correction has to be applied.
5. The variation has air with temperature and convection currents between the plates must be accounted for.

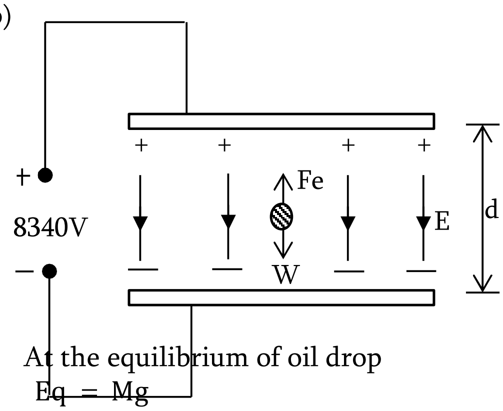
NUMERICAL EXAMPLES

Example 93

(a) Define
   (i) Electric field intensity
   (ii) Difference of potential
(b) A charged oil – drop of radius 0.00013cm is prevented from falling under gravity by the vertical field between two horizontal plates charged to a difference of potential of 8340V. The distance between the plates is 1.6cm and density of oil drop is $920kgm^{-3}$. Calculate the magnitude of charge on the drop.

Solution

(a) Refer to your notes
(b)



At the equilibrium of oil drop

$$Eq = Mg$$

$$\frac{V_q}{d} = \frac{4}{3}\pi r^3\rho g$$

$$q = \frac{4\pi r^3\rho gd}{3V}$$

$$q = \frac{3\times\pi\times\left(0.00013\times10^{-2}\right)^3\times920\times9.8\times1.6\times10^{-2}}{3\times8340}$$

$$q = 1.59\times10^{-19}C$$

Example 94

An oil drop of mass $5\times10^{-15}kg$ carries a charge Q. The drop is stationary between two parallel metal plates 25mm apart with a potential difference of 1000V between them. Determine Q, take g = 10m/s^2.

Solution

Since the oil drop is stationary

Electric force = Weight of drop

$$Eq = Mg$$

$$q = \frac{Mg}{E} = \frac{Mgd}{V}$$

$$q = \frac{5\times10^{-15}\times10\times25\times10^{-3}}{1000}$$

$$q = 1.25\times10^{-18}C$$

Example 95

An oil drop 12excess electron is held stationary under a constant electric field of $2.55\times10^4 Vm^{-1}$ in Millikan's drop experiment. The density of the oil is 1.26gcm-3. Estimate the radius of the oil drop. $\left(g = 9.81m/s^2\ ,\ e = 1.6\times10^{-19}C\right)$

Solution

When the oil drop is held stationary

Weight of oil drop = force on the drop due to electric field

$$\frac{4}{3}\pi r^3\rho g = neE$$

$$r = \left[\frac{3neE}{4\pi\rho g}\right]^{1/3}$$

$$r = \left[\frac{3\times12\times1.6\times10^{-19}\times2.55\times10^4}{4\times3.14\times1.26\times10^3\times9.81}\right]^{1/3}$$

$$r = 0.0981\times10^{-5}m = 9.81\times10^{-4}mm$$

Example 96

In a Millikan's oil drop experiment, six oil drops were observed to have charges of $16\times10^{-19}C$, $3.2\times10^{-19}C$, $8.0\times10^{-19}C$ $6.4\times10^{-19}C$, $2.4\times10^{-18}C$ and $3.2\times10^{-18}C$ respectively. What conclusion can be drawn from these observations.

Solution

$$16\times10^{-19}C = 10\times1.6\times10^{-19}C$$
$$3.2\times10^{-19}C = 2\times1.6\times10^{-19}C$$
$$8.0\times10^{-19}C = 5\times1.6\times10^{-19}C$$
$$6.4\times10^{-19}C = 4\times1.6\times10^{-19}C$$
$$2.4\times10^{-18}C = 15\times1.6\times10^{-19}C$$
$$3.2\times10^{-18}C = 20\times1.6\times10^{-19}C$$

Clearly, the charge on each oil drop is integral multiple of $1.6\times10^{-19}C$ which is the charge on an electron. This indicates that electric charges are quantized.

Example 97 NECTA 2003/P1/11

(a) Define the following
- (i) Electric field intensity
- (ii) Difference of potential. How are these quantities in 11(a) related?

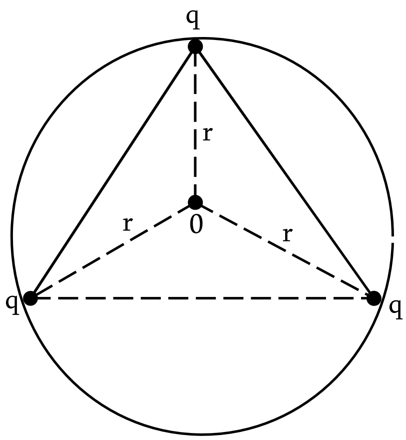
(b) A charged oil drop of radius $1.3\times10^{-6}m$ is prevented from falling under gravity by the vertical field between two horizontal plate charged to a difference of potential of 8340V. The distance between the parallel plate is 16mm and density of oil is 920kgm^{-3}. Calculate the magnitude of the charge on the drop.

(c) Three point charges each carrying a charge q are placed on the circumference of the circle of radius r to form an equilateral triangle. What is the
- (i) Electrical potential at the centre of the circle.
- (ii) Electric field intensity at the centre of the circle.

Solution

(a) And (b) see solution example 93

(c) (i) V = electric potential at the centre of circle.



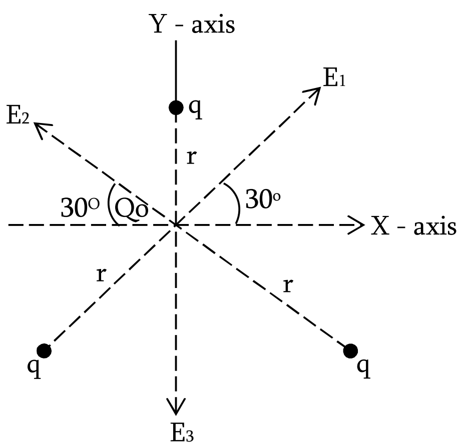
$$V = V_1 + V_2 + V_3$$

$$= \frac{q}{4\pi\varepsilon_0 r} + \frac{q}{4\pi\varepsilon_0 r} + \frac{q}{4\pi\varepsilon_0 r}$$

$$V = \frac{3q}{4\pi\varepsilon_o r}$$

(ii) Let E = Resultant electric field intensity at the centre of the circle.

Since $E = \frac{q}{4\pi\varepsilon_o r^2}$



$$E_1 = E_2 = E_3 = E = \frac{q}{4\pi\varepsilon_o r^2}$$

Resolve electric field intensity

ln x – direction

$$E_X = E\cos 30^o - E\cos 30^o$$

$$E_X = 0$$

ln y – direction

$$E_y = E\sin 30^o + E\sin 30^o - E = 0$$

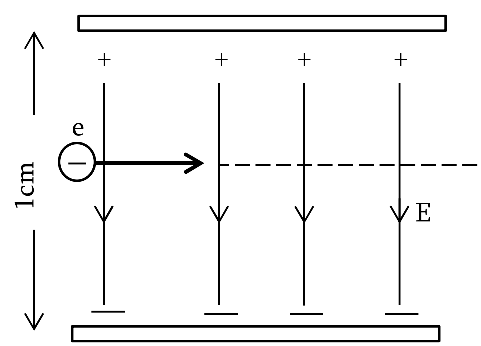
$$E = \sqrt{E_X^2 + E_y^2} = 0$$

Example 98 NECTA 2004/P1/11

(a) (i) State the coulomb's law

(ii) Two point charges A and B are situated 90mm apart. If A has a charge of +2q and B a charge of – 4q were should a point charge of -2q be placed so that it experience no resultant electrostatic force?

(b) (iii) An electron is projected with an initial velocity $V_O = 10^7$m/s into a uniform field in vertically downwards if the electron just misses the upper plates as it emerges from the field. Find the magnitude of the field.



(i) Define electric field strength and state its units.

(ii) How is the direction of the field strength specified?

(c) Two small balls are suspended by insulating threads from a common point. Each balls has a mass of 0.20g and suspension threads are 1.0m long. When the balls are given equal positive charge each suspension thread is found to make an angle of 7° with the vertical. What are the charges carried by the two balls?

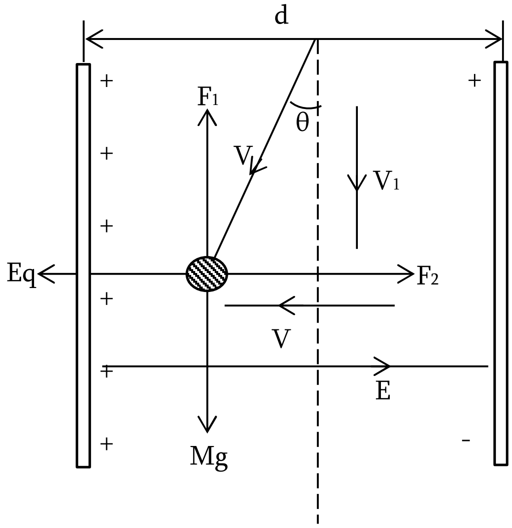
Ans. (a) (i) 217.28mm from +2q
(b) (iii) $1.422 \times 10^4 Vm^{-1}$ (c) 3.99×10^{-8}C.

Example 99

A charged oil drop of mass 6.0×10^{-15}kg falls vertically in air with a steady velocity between two long parallel vertical plates 5mm apart. When a p.d of 3000V is applied between the plates the drop now falls with a steady velocity at an angle of 58° to the vertical. Calculate the charge Q on the drop. Assume that g = 10m/s²)

Solution

The drop falls with steady velocity due to the viscosity of the air and neglecting upthrust.



For the steady motion

F α V, F = KV

$Mg = KV_1$…………..(i)

$EQ = KV_2$…………..(ii)

Dividing equation (ii)/(i)

$$\frac{EQ}{Mg} = \frac{KV_2}{KV_1} = \frac{V_2}{V_1} \quad \left[\tan\theta = \frac{V_2}{V_1}\right]$$

$$\frac{EQ}{Mg} = \tan\theta$$

$$EQ = Mg\tan\theta$$

$$Q = \frac{Mg\tan\theta}{E} = \frac{Mgd}{V}\tan\theta$$

$$Q = \frac{6.0\times10^{-15}\times10\times5\times10^{-3}\tan58^{o}}{3000}$$

$$Q = 1.6\times10^{-19}C$$

Example 100

(i) An oil drop carrying a charge of 2e is kept stationary between two parallel horizontal plates 20mm apart. When a potential difference of 1.2×10^{4}V is applied between them. Calculate the mass of the drop and hence its radius.

(ii) The plates are tilted 60° from the horizontal. Calculate:

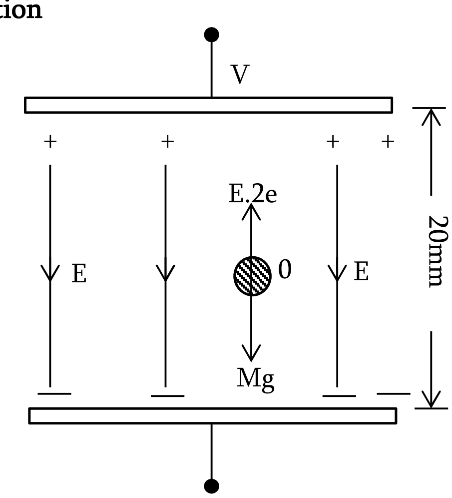
(a) The initial direction of the drop

(b) The new potential difference between the plates needed to make the drop move initially parallel to the plates.

$\left(e = 1.6\times10^{-19}C, oil density = 900kgm^{-3}, g = 10Nkg^{-1}\right)$

Solution

(i)



Electric field strength between two parallel plates.

$$E = \frac{V}{d} = \frac{1.2\times10^{4}}{20\times10^{-3}}$$

$$E = 6.0\times10^{5}Vm^{-1}$$

At the equilibrium

$$Mg = E.2e$$

$$M = \frac{E.2e}{g} = \frac{6\times10^{5}\times2\times1.6\times10^{-19}}{10}$$

$$M = 1.9\times10^{-14}Kg$$

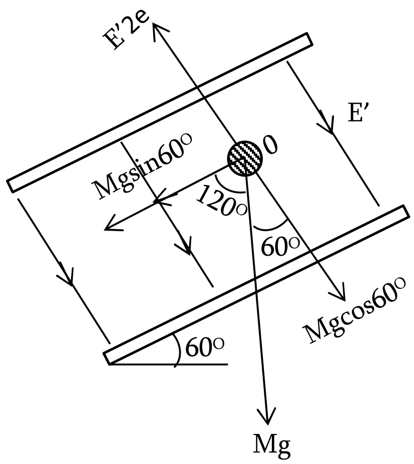
Let a = radius of the oil drop

$$M = \frac{4}{3}\pi a^{3}\rho$$

$$a = \left[\frac{3M}{4\pi\rho}\right]^{1/3} = \left[\frac{3\times1.9\times10^{-14}}{4\times3.14\times900}\right]^{1/3}$$

$$a = 1.7\times10^{-6}m$$

(ii) (a) With plates now titled at 60° from the horizontal, figure below, the electric force on the drop to the plates.



E.2e = Mg, a vertical downward force. The resultant of these two equal force is along the bisector OA of the angle 120° between them. So the drop moves initially along OA.

(b) Suppose a new potential difference V' makes the drop to move initially parallel to the plates. Then the resultant force normal to the plates is zero so.

$$E'\cdot 2e = Mg\cos 60^{\circ}$$

$$\frac{V'}{d}2e = Mg\cos 60^{\circ}$$

$$V' = \frac{Mgd\cos 60^{\circ}}{2e}$$

$$V' = 6.0\times10^{3}\,\text{Volt}$$

Example 101 NECTA 1999/P4/6

An insulated conducting spherical shell of radius 10cm in vacous series a positive charge of $1.0\times10^{-7}\,C$. Calculate the electric field at a point on the surface of the conductor $\left(\frac{1}{4\pi\varepsilon_o} = 9\times10^{9}\,Fm^{-1}\right)$

Ans. $9\times10^{4}\,Nc^{-1}$

Example 102

A and B are two points 60m and 70m respectively from +2.0μC point of electric charge. Calculate:

(a) Potential at A
(b) Potential at B
(c) The energy released when an electric charge of +0.35μC move freely from A to B. $\frac{1}{4\pi\varepsilon_o} = 9\times10^{9}\,Fm^{-1}$

Solution

(a) $V_A = \frac{1}{4\pi\varepsilon_o}\cdot\frac{Q}{r_A} = \frac{9\times10^{9}\times2\times10^{-6}}{60}$

$V_A = 300\text{volt}$

(b) $V_B = \frac{1}{4\pi\varepsilon_o}\cdot\frac{Q}{r_B} = \frac{9\times10^{9}\times2\times10^{-6}}{70}$

$V_B = 257\text{volt}$

(c) $V_{AB} = V_A - V_B = 43\text{volt}$

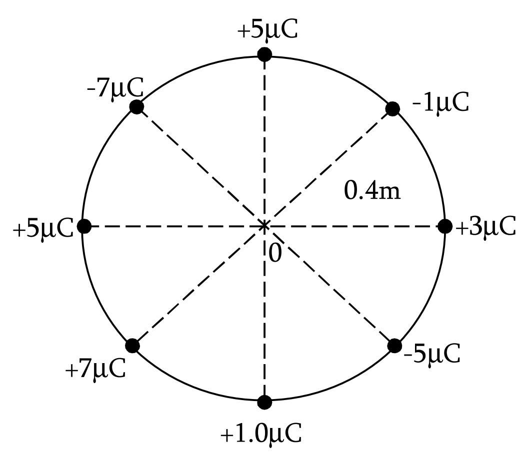
Energy released

$W = QV_{AB} = 43\times0.35\times10^{-6}$

$W = 1.5\times10^{-5}\,J$

Example 103

Eight charges having the values as shown in the figure below are arranged symmetrically on a circle of radius 0.4m in air. Calculate the potential at the centre O.



Solution

The total electric potential at the centre O of the circle.

$$V = \frac{1}{4\pi\varepsilon_o r}\left[Q_1+Q_2+Q_3+Q_4+Q_5+Q_6+Q_7\right]$$

$$= \frac{9\times10^9}{0.4}\left[5+{}^{-}7+15+7+1+-5+3+{}^{-}7\right]\times10^{-6}$$

$$V = 60.75\times10^4\,\text{volt}$$

Example 104

(a) Briefly describe experiment for determination of charge of an electron

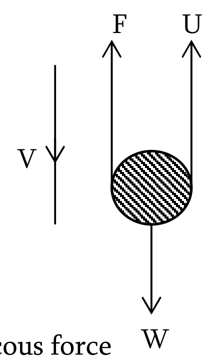
(b) An oil droplet (density = $960kgm^{-3}$) falls through air of viscosity $1.8\times10^{-5}\,Nsm^{-1}$. Neglecting upthrust of air. Calculate the terminal velocity of oil drop if its diameter is $2.4\times10^{-6}\,m$.

(c) What can be done in (b) above to hold the oil drop to be stationary?

Solution

(a) Describe the Millikan's experiment

(b)



F = Viscous force

M = weight of drop

U = upthrust

At the equilibrium F + U = W

$U \simeq 0$

$$F = W$$

$$6\pi\eta Vr = \frac{4}{3}\pi r^3\rho g$$

$$V = \frac{2r^2\rho g}{9\eta} \quad \text{but } r = \frac{d}{2}$$

$$V = \frac{d^2\rho g}{18\eta}$$

$$= \frac{\left(2.4\times10^{-6}\right)^2\times960\times9.8}{18\times1.8\times10^{-5}}$$

$$V = 1.67\times10^{-4}\,m/s$$

(c) Apply an electric field against gravity

Example 105

A charged oil drop of radius $5.0\times10^{-5}m$ and density of $700kgm^{-3}$ remains stationary between horizontal plate $10^{-2}m$ apart when a p.d of 1.1KV is applied to the plates.

(a) Draw a diagram to show the forces acting in oil drop.

(b) Find the charge on the oil drop.

Solution

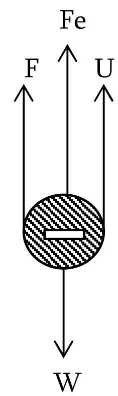
(i) Diagram of forces acting on oil drop

Fe = Electric force

W = Weight of oil drop

F = Viscous drag force

U = Upthrust



(ii) When the oil drop remain stationary

Electric force = Weight of drop

$$EQ = \frac{4}{3}\pi r^3\rho g \quad \left[E=\frac{V}{d}\right]$$

$$Q = \frac{4\pi r^3\rho gd}{3V}$$

$$= \frac{4\times3.14\times\left(5\times10^{-5}\right)^3\times700\times9.8\times10^{-2}}{3\times1100}$$

$$Q = 3.26\times10^{-14}C$$

Example 106

An oil drop of diameter $10^{-5}cm$ carrying two electric charges remains suspended between two charged parallel plates 10mm apart. If the density of oil is $1.8gcm^{-3}$. Calculate the potential difference between the two plates.

Solution

At the equilibrium of oil drop.

Electric force = Weight of drop

$$(ne)E = \frac{4}{3}\pi r^3 \rho g \quad \left[E = \frac{V}{d}\right]$$

$$V = \frac{4\pi r^3 \rho g d}{3ne}$$

$$= \frac{4\times3.14\times\left(10^{-7}\right)^3\times1.8\times10^3\times9.8\times10^{-2}}{3\times2\times1.6\times10^{-19}\times2^3}$$

$$V = 0.288\text{Volt}$$

Example 107

In a Millikan's oil drop experiment a charged oil drop of mass density 880kgm^{-3} is held stationary between two parallel plate 6.0mm apart held at a potential difference of 103volt. When the electric field is switched off the drop is observed to fall a distance of 2.0mm in 35.7 second.

(a) What is the radius of the drop?

(b) Estimate the charge of the drop. How many excess electrons does it carry? Give that the upper plate in the experiment is at a higher potential, viscosity of air = $1.8\times10^{-5}\text{Nsm}^{-2}$. Density of air = 1.29kgm^{-3}

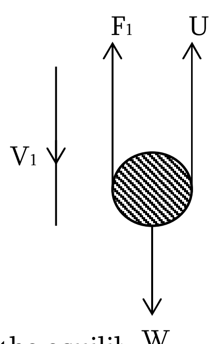
$g = 9.81\text{m/s}^2$

Solution

(a) When the drop falls freely under gravity

$$V_1 = \frac{2\times10^{-3}}{35.7}$$

$$V_1 = 5.6\times10^{-5}\,\text{m/s}$$



At the equilit

$$F_1 + U = W$$

$$6\pi\eta v_1 r = \frac{4}{3}\pi r^3 g\left(\rho_o - \rho_a\right)$$

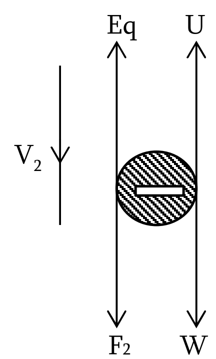
$$r = \left[\frac{9\eta v_1}{2g\left(\rho_o-\rho_a\right)}\right]^{1/2}$$

$$r = \left[\frac{9\times1.8\times10^{-5}\times5.6\times10^{-5}}{2\times9.81\left(880-1.29\right)}\right]$$

$$r = 7.255\times10^{-7}\,\text{m}$$

(b) When the electric field is applied

$$E = \frac{V}{d} = \frac{103}{6\times10^{-3}} = 17{,}166.67\text{Vm}^{-1}$$



When the drop with terminal velocity V_2 until becomes stationary.

$$6\pi\eta r V_2 + W = U + E_q$$

$$E_q = 6\pi\eta V_2 r + W - U$$

$$E_q = 6\pi\eta\left(V_2 + V_1\right)r$$

Since the drop is stationary $V_2 = 0$

$$E_q = 6\pi\eta v_1 r \text{ but } q = Ne$$

$$Ne = \frac{6\pi\eta v_1 r}{E}$$

$$N = \frac{6\times3.14\times1.8\times10^{-5}\times5.6\times10^{-5}\times7.255\times10^{-7}}{1.6\times10^{-19}\times17{,}166.67}$$

$N = 5$

$\therefore$ Number of electrons carries by oil drop

$N = 5$.

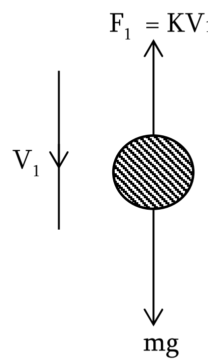
Example 108

In a measurement of the electron charge by Millikan's method, a potential difference of 1.5KV can be applied between horizontally parallel plates 12mm apart. What the electric field switched off, a drop of oil of mass

1.0×10^{-14}kg is observed to fall with constant velocity 400µms^{-1}. When the field is switched on, the drop rises with constant velocity 80µms^{-1} . How many electron charges are there on the drop? (You may assume that the air resistance is proportional to the velocity of the drop and that of air buoyancy may be neglected). Takes g =10m/s^2, e = -1.6 × 10^{-19}C.

Solution

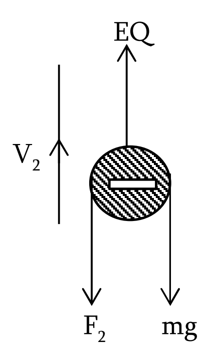
Upthrust can be neglected $(u \simeq 0)$ when the electric field is switched off.



At the equilibrium

$Mg = KV_1$……………..…..(1)

When the electric field is switched on



$F_2 = KV_2$

$EQ = KV_2 + Mg$

$EQ - Mg = KV_2$………..(2)

Dividing equation (2) by (1)

$$\frac{EQ-Mg}{Mg} = \frac{KV_2}{KV_1} = \frac{V_2}{V_1}$$

$$\frac{EQ}{Mg} = \frac{V_2}{V_1}+1$$

$$Q = \frac{Mg}{E}\left[\frac{V_1+V_2}{V_1}\right]$$

$$Ne = \frac{Mgd}{V}\left[\frac{V_1+V_2}{V_1}\right]$$

$$N = \frac{Mgd}{eV}\left[\frac{V_1+V_2}{V_1}\right]$$

$$= \frac{1.0\times10^{-14}\times10\times12\times10^{-3}}{1.6\times10^{-19}\times1.5\times10^{3}}\left[\frac{400+80}{400}\right]$$

$$N = 6$$

Example 109

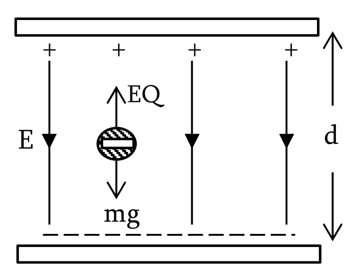
In a version of Millikan's experiment it is found that a charge droplet of mass 1.8×10^{-15}kg just remains stationary when the potential difference between the plates which are 12mm apart is 150V. If the droplet suddenly gains an extra electron.

(a) Calculate the initial acceleration of the droplet and

(b) Find the voltage needed to bring the droplet to rest again.

Solution

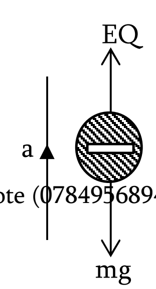
(a) When the oil drop is stationary



$EQ = Mg$…………(1)

When the oil drop pick extra one electron moves vertically upward.

Resultant force on the oil drop



$$EQ_1 - Mg = Ma$$

But $Q_1 = Q + e$

$$E(Q+e) - Mg = Ma$$

$$EQ + Ee - Mg = Ma \text{ ....(2)}$$

Putting equation (1) into (2)

$$Mg + Ee - Mg = Ma$$

$$Ee = Ma$$

$$a = \frac{Ee}{M} = \left(\frac{V}{d}\right)\left(\frac{e}{M}\right)$$

$$= \frac{150 \times 1.6 \times 10^{-19}}{12 \times 10^{-3} \times 1.8 \times 10^{-15}}$$

$$a = 1.11 m/s^2$$

(b) Let N_1 = number of electrons carried by oil drop when is stationary.

Weight of oil drop = Electric force

$$Mg = EQ \quad [Q = N_1 e]$$

$$N_1 = \frac{Mgd}{V}$$

$$N_1 = \frac{1.8 \times 10^{-15} \times 10 \times 12 \times 10^{-3}}{1.6 \times 10^{-19} \times 150}$$

$$N_1 = 9$$

When the charged oil drop gain and extra electron, new total number of electric charges.

$$N = N_1 + e = 9 + 1 = 10$$

$$Q' = Ne$$

Since $\frac{V'Q'}{d} = Mg$

$$V' = \frac{Mgd}{Q'} = \frac{Mgd}{Ne}$$

$$V' = \frac{1.8 \times 10^{-15} \times 10 \times 0.012}{10 \times 1.6 \times 10^{-19}}$$

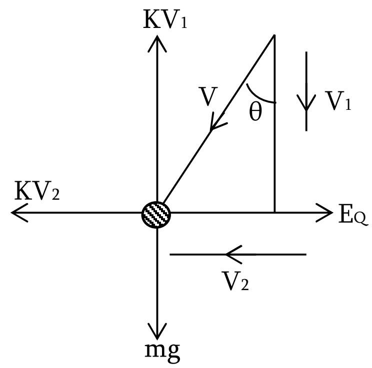
$$V' = 135 \text{Volt}$$

Example 110

An oil drop of mass 3.25×10^{-15}kg falls vertically with uniform velocity through the air between vertical parallel plates which are 2cm apart. When a p.d of 1000V is applied to the plates the drop moving towards the negatively charged plate its path being inclined at 45° to the vertical. If the path of the drop suddenly changes to one at 26°30′ to the vertical and subsequent to one at 37° to the vertical. What conclusion can be drawn?

Solution

Consider the FBD of an oil drop.



Since F α V, F = KV

At the equilibrium

$$KV_1 = Mg$$

$$KV_2 = EQ$$

$$\frac{EQ}{Mg} = \frac{KV_2}{KV_1} = \frac{V_2}{V_1}$$

From the figure above

$$\tan\theta = \frac{V_2}{V_1}$$

$$\frac{EQ}{Mg} = \tan\theta \;,\; Q = \frac{Mgd}{V}\tan\theta$$

Given that:

$$\theta_1 = 45^o \;,\; M = 3.25 \times 10^{-15} kg \;,\; g = 10 m/s^2$$

$$Q_1 = \frac{3.25 \times 10^{-15} \times 10 \times 2 \times 10^{-2} \times \tan 45^o}{1000}$$

$$Q_1 = 6.5 \times 10^{-19} C$$

Since $Q_1 = N_1 e \;,\; N_1 = \frac{Q_1}{e}$

$$N_1 = \frac{6.5 \times 10^{-19}}{1.6 \times 10^{-19}} = 4 \;,\; Q_1 = 4e$$

Now $Q_1 = \frac{Mgd}{V}\tan\theta_1$

Where $\theta = \theta_2 = 26^o 30'$

$$\theta_2 = \frac{Mgd}{V}\tan\theta_2$$

$$\frac{Q_2}{Q_1} = \frac{Mgd}{V}\tan\theta_2 \Big/ \frac{Mgd}{V}\tan\theta_1$$

$$Q_2 = \frac{Q_1\tan\theta_2}{\tan\theta_1} = \frac{4e\tan(26^\circ 30')}{\tan 45^\circ}$$

$$Q_2 = 2e$$

4e changes to 2e

Also

$$Q_3 = \frac{Mgd}{V}\tan\theta_3$$

$$Q_3 = \frac{Q_1\tan\theta_3}{\tan\theta_1} = \frac{4e\tan 37^\circ}{\tan 45^\circ}$$

$$Q_3 = 3e$$

4e changes to 3e

Example 111 (R.M. G – 64)

(a) (i) Which oil drop should be used in Millikan's method.

(ii) Is it possible that charge on an oil drop in Millikan's method to be $\frac{3}{4}e$?

(b) An oil drop of mass 3.2×10^{-15}kg falls vertically with uniform velocity through the air between vertical parallel plates 3cm apart. When a p.d of 2000V is applied between the plates, the drops move with uniform velocity at an angle of 45^{O} to the vertical. Calculate the charge on the drop. The path of the drop suddenly changes becoming inclined at $18^\circ 26'$ to the vertical later the path changes again and becomes inclined at $33^\circ 42'$ to the vertical. Estimate from these data the elementary unit of charge (electron charge).

Solution

(a) (i) Non – volatile clock oil

(ii) No. it is impossible for the charge on an oil drop to be $\frac{3}{4}e$ since electric changes are multiple integer of the number.

(b) Given that $M = 3.2\times10^{-15}$kg, d = 30cm

V_P = 2000V

Case 1: since $Q = \frac{Mgd}{V_P}\tan\theta_1$

$$Q = \frac{3.2\times10^{-15}\times10\times3\times10^{-2}\tan 45^\circ}{2000}$$

$$Q = 4.8\times10^{-19}C$$

Case 2: Elementary unit of charge e = ?

$$Q_1 = \frac{Mgd}{V_P}\tan\theta_1 \;,\; Q_2 = \frac{Mgd\tan\theta_2}{V_P}$$

$$e = Q_2 - Q_1 = \frac{Mgd}{V_P}\left[\tan\theta_2 - \tan\theta_1\right]$$

$$= 4.8\times10^{-19}\left[\tan(33^\circ 42') - \tan 18^\circ 26'\right]$$

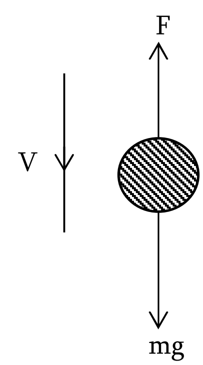
$$e = 1.6\times10^{-19}C$$

Example 112

A charged oil drops falls under gravity with a terminal speed, V the drop is held stationary by applying a suitable electric field in Millikan's set up and is found to carry to 2 excess electrons. Suddenly the drop is observed to move up wards with speed V. Guess what is happened?

Solution

Initially, the charge on the oil drop is q = 2e (since it carries two electrons). When the oil drop falls under the gravity with terminal velocity, V.



Assuming that $u \simeq 0$. At the equilibrium

$Mg = 6\pi\eta vr$ ....(1)

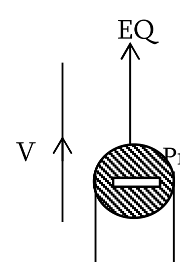
Since oil drop is held by applying electric field.

Weight of drop = Electric force

Mg = 2eE................(2)

When oil drop moves upward with speed V, its means charge(s) on its has changed.

Let Q = New charge on drop



At the equilibrium

$$EQ = Mg + 6\pi\eta vr$$

But $Mg = 6\pi\eta vr$

$$EQ = Mg + Mg = 2Mg$$

From equation (2)

$$EQ = 2(2eE)$$

$$Q = 4e$$

∴ The drop now has 4 excess electrons i.e drop has pick up 2 more electrons from surrounding air.

Example 113

(a) (i) Can we use water drop in Millikan's experiment?

(ii) Why is Millikan's apparatus surrounded by a constant – temperature enclosure?

(b) A charged oil drop of mass $M = 6.4\times10^{-16}$kg is in a horizontally arranged parallel plate capacity with separation d = 10mm. in the absence of a voltage between the plates the drop falls at constant speed $V_1 = 0.078\text{mms}^{-1}$. After a voltage V = 95volt is applied across the capacity, the drops moves uniformly upward with a speed $V_2 = 0.016\text{mms}^{-1}$. Determine the charge q of the drop.

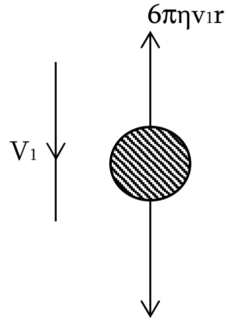
Solution

(a) (i) No, because of danger of evaporation during the experiment.

(ii) Reasons;-

- It avoids convection current of air which may move the drop and cause error.
- It shields the apparatus from the draughts.

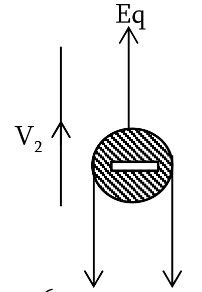
(b) In the absence of a voltage across the plates, the drop falls with constant speed V_1.



Assumption made upthrust can be ignored

$$Mg = 6\pi\eta v_1 r \text{ ....(1)}$$

When voltage V is applied across the plates drop moves upwards with constant speed V_2.



At the equilibrium

$$Eq = 6\pi\eta v_2 r + Mg$$

$$Eq - Mg = 6\pi\eta v_2 r \text{ ....(2)}$$

Dividing equation (2) by (1)

$$\frac{Eq - Mg}{Mg} = \frac{6\pi\eta v_2 r}{6\pi\eta v_1 r}$$

$$\frac{Eq}{Mg} - 1 = \frac{V_2}{V_1}$$

$$q = \frac{Mg}{E}\left[1 + \frac{V_2}{V_1}\right]$$

$$q = \frac{Mg}{V}\left[1 - \frac{V_2}{V_1}\right]$$

$$= \frac{6.4\times10^{-16}\times9.8\times10\times10^{-3}}{95}\left[1 + \frac{0.016}{0.078}\right]$$

$$q = 7.9\times10^{-19}\text{C}$$

Example 114

(a) Why is clock oil used in Millikan's experiment?

(b) Millikan's experiment cannot be performed with bigger drops. Why?

Solution

(a) Reasons

- It is a low – vapour pressure oil and reduces the problem of evaporation.
- It gives quite small droplets.

(b) If bigger drops are used droplets. Millikan's experiment, the electric field required could be of very high magnitude and it may become impossible to produce it.

Example 115

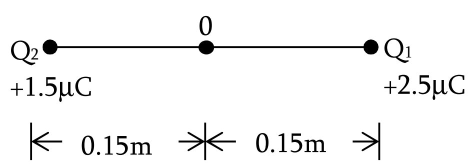
Two tiny sphere carrying charge 1.5μC and 2.5μC are located 30cm apart. Find the potential and electric field.

(a) At the mid – point of the line joining the two charges and.

(b) At a point 10cm from this mid – point in a plane normal to the line and passing through the mid – point.

Solution

(a) (i) the electric potential at the mid – point



Let V1 = potential at O due to Q1
V2 = potential at O due to Q2

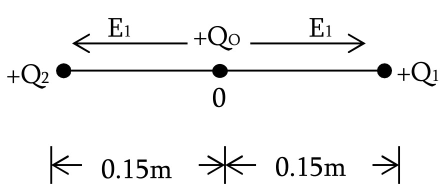
$$V = V_1 + V_2 = \frac{KQ_1}{r} + \frac{KQ_2}{r}$$

$$= \frac{K}{r}\left[Q_1 + Q_2\right]$$

$$= \frac{9\times10^9}{0.15}\left[1.5+2.5\right]\times10^{-6}$$

$$V = 2.4\times10^5\,\text{volt}$$

(ii) Electric field at the mid-point.



Now

$$E = E_1 - E_2 = \frac{KQ_1}{r^2} - \frac{KQ_2}{r^2}$$

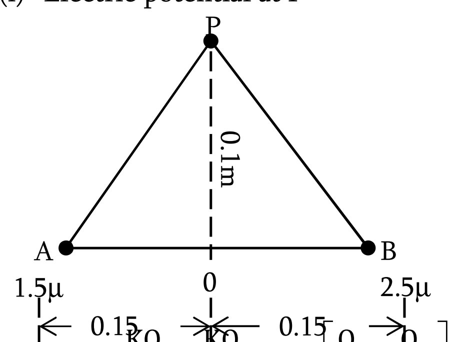
$$= \frac{K}{r^2}\left[Q_1 - Q_2\right]$$

$$= \frac{9\times10^9}{(0.15)^2}\left[2.5-1.5\right]\times10^{-6}$$

$$E = 4.0\times10^5\,\text{Vm}^{-1}$$

The electric field is directed towards the charge of 2.5μC.

(b) (i) Electric potential at P



$$V_P = \frac{KQ_1}{PB} + \frac{KQ_2}{AP} = K\left[\frac{Q_1}{PB} + \frac{Q_2}{AP}\right]$$

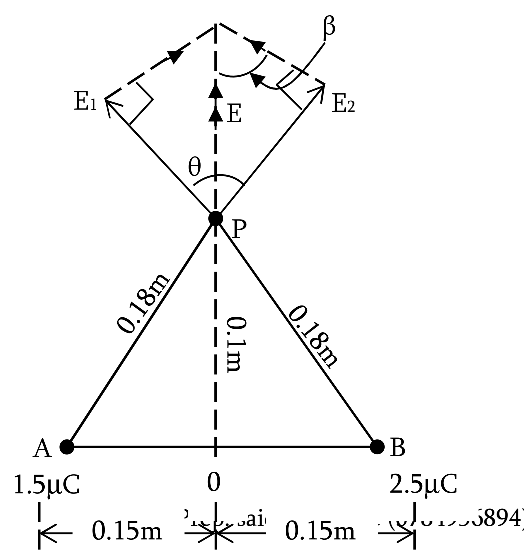
By using pythagorous theorem

$$\overline{PB} = \overline{AP}\sqrt{0.15^2 + 0.1^2} = 0.18\text{m}$$

$$V_P = \frac{9\times10^9}{0.18}\left[1.5+2.5\right]\times10^{-6}$$

$$V_P = 2.0\times10^5\,\text{Volt}$$

(ii) Electric field intensity at point, P.



$$E_1 = 9\times10^9\times\frac{2.5\times10^{-6}}{(0.18)^2}$$

$$E_1 = 0.69\times10^6\,Nc^{-1}$$

Also

$$E_2 = 9\times10^9\times\frac{1.5\times10^{-6}}{(0.18)^2}$$

$$E_2 = 0.42\times10^6\,Nc^{-1}$$

$$\ln\Delta\ AOP\ ,\ \cos\left(\frac{\theta}{2}\right) = \frac{0.10}{0.18} = 0.55$$

$$\frac{\theta}{2} = \cos^{-1}(0.55)$$

$$\theta = 2\cos^{-1}(0.55) = 113.26$$

Let E = Resultant electric field intensity

$$E = \sqrt{E_1^2+E_2^2+2E_1E_2\cos\theta}$$

$$= \sqrt{\begin{array}{l}(0.42\times10^6)^2+(0.69\times10^6)^2\\+0.42\times0.69\times10^{12}\cos113.26\end{array}}$$

$$E = 6.53\times10^5\,Vm^{-1}$$

If β is eh angle which E makes with E1

$$\tan\beta = \frac{E_2\sin\theta}{E_1+E_2\cos\theta}$$

$$= \frac{0.69\times10^6\times0.9191}{0.42\times10^6+0.69\times10^6\cos113.2^\circ}$$

$\beta = 76.60^\circ$ which gives the direction of E.

Example 116

The potential at the surface of a spherical drop of water carrying a charge of $1.5\times10^{-6}C$ is250volt.

(i) Find the radius of the drop

(ii) Two such drops of the same radius and charge combine to form a single spherical drop. Calculate the potential at the surface of the new drop so formed.

Solution

(i) Potential at the surface of the drop

$$v = \frac{kq}{r}\ ,\ r = \frac{kq}{v}$$

$$r = \frac{9\times10^9\times1.5\times10^{-6}}{250}$$

$$r = 54m$$

(ii) Let R = radius of eh combined drop

Apply the law of conservation of the volume

Volume of the Large drop	=	volume of the two small drops

$$\frac{4}{3}\pi R^3 = 2\left[\frac{4}{3}\pi r^3\right]$$

$$R = 2^{1/3}r$$

The charge of the bigger drop Q = 2q

Potential of the bigger drop at the surface

$$V = \frac{1}{4\pi\varepsilon_o}\cdot\frac{Q}{R} = \frac{1}{4\pi\varepsilon_o}\cdot\frac{2q}{2^{1/3}r}$$

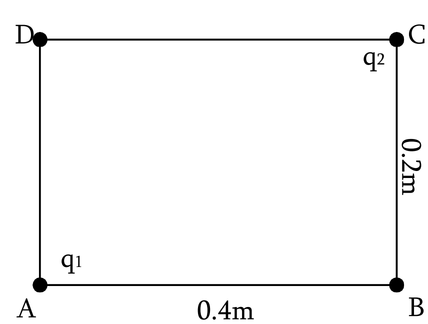
$$= 2^{2/3}\cdot\left[\frac{q}{4\pi\varepsilon_o r}\right]$$

$$= 2^{2/3}\times250$$

$$V = 396.8Volt$$

Example 117

A point charges $q_1 = 0.06\mu C$ is placed at A, $q_2 = -0.04\mu C$ placed at C of rectangle of sides AB = 0.4m and BC = 0.2m. Calculate the potentials at the point B and D. Also find the work done in carrying $1\mu C$ from B to D (see figure below)



Solution

The electric potential at point D

$$V_D = K\left[\frac{q_1}{AD}+\frac{q_2}{CD}\right]$$

$$= 9\times10^9\left[\frac{0.06\times10^{-6}}{0.2}+\frac{-0.04\times10^{-6}}{0.4}\right]$$

$$V_D = 1800\text{volt}$$

The electric potential at B

$$V_D = K\left[\frac{q_1}{AB}+\frac{q_2}{CB}\right]$$

$$= 9\times10^9\left[\frac{0.06\times10^{-6}}{0.4}+\frac{-0.04\times10^{-6}}{0.2}\right]$$

$$V_B = -4500\text{Volt}$$

Work done to carry 1μC from B to D

$$W = q\left(V_D - V_B\right)$$

$$= 1\times10^{-6}\left[1800 - {}^{-}4500\right]$$

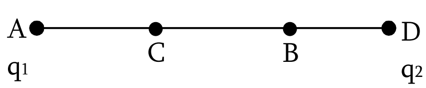
$$W = 6.3\times10^{-3}\text{J}$$

Example 118

Two point charges $q_1 = +2\mu C$ and $q_2 = +3\mu C$ are separated by a distance of 1m in air. Calculate the work done to bring the charges (i) 50cm (ii) 50cm further away than the initial separation.

Solution

(i) Initially the charge q_1 is at A and q_2 is at B. Let us assume that q_1 is fixed at A. when they are brought 50cm closer, q_2 at C.



AB = 1m, AC = 0.5m, AD = 1.5m

Work done to bring q_2 to C

W = change in p.e

$$= \frac{Kq_1q_2}{AC} - \frac{Kq_1q_2}{AB}$$

$$= Kq_1q_2\left[\frac{1}{AC}-\frac{1}{AB}\right]$$

$$= 9\times10^9\times2\times10^{-6}\times3\times10^{-6}\left[\frac{1}{0.5}-\frac{1}{1}\right]$$

$$W = 0.054\text{J}$$

(ii) Work done to take q_2 to D

$$W = Kq_1q_2\left[\frac{1}{AB}-\frac{1}{AD}\right]$$

$$= 9\times10^9\times2\times10^{-6}\times3\times10^{-6}\left[\frac{1}{1}-\frac{1}{1.5}\right]$$

$$W = 0.018\text{J}$$

Example 119

(a) Calculate the velocity acquired by a proton moving under a potential difference of 100volt. Charge proton is $1.6\times10^{-19}C$, its mass 1.67×10^{-27}kg

(b) Calculate the kinetic energy acquired by an alpha particle accelerated through a potential difference of 1,000,000V?

Solution

(a) $$eV = \frac{1}{2}mV_0^2$$

$$V_0 = \sqrt{2\left(e/m\right)V}$$

$$= \sqrt{\frac{2\times1.6\times10^{-19}\times100}{1.67\times10^{-27}}}$$

$$V_0 = 1.38\times10^5\,m/s$$

(b) $$k.e = qv = 2ev$$

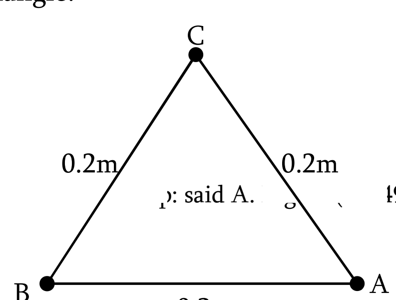
$$= 2\times1.6\times10^{-19}\times10^6$$

$$k.e = 3.2\times10^{-32}\text{J}$$

ASSIGNMENT NO. 5

1. (a) What is mean by
   (i) Electric potential
   (ii) Electric field

   (b) If the point charges at A and B are $3\times10^{-19}C$. How much work is required to bring infinity up to the vertex C of the triangle.



(c) A parallel plate capacitor has a space between the plates filled by two stabs of dielectric constants K_1 and K_2 each of thickness $d/2$ where d is the plate separation of the capacitor. Show that the capacitance. $C = \frac{2\varepsilon_0 A}{d}\left[\frac{K_1K_2}{K_1+K_2}\right]$

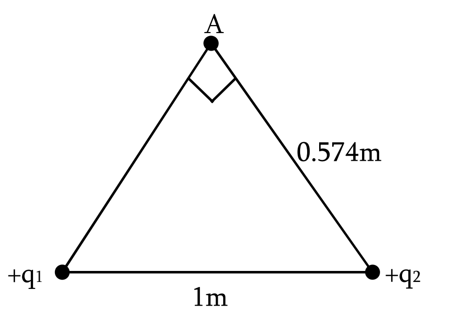
Ans. (b) 2.7×10^{-7} J

2. EZEB 2008/P1/11

(a) (i) Define the term capacitance as applied to a capacitor

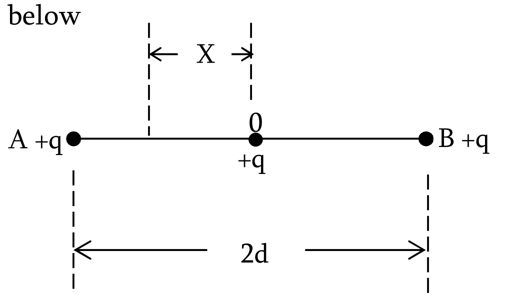
(ii) A parallel plate capacitor with air between its plate is then filled with an insulating material of dielectric constant 2. Calculate the change in the energy stored in the capacitor if the potential difference is 100Volt

(b) Two equal positive charges $q_1 = q_2 = +2.0\mu C$ are situated as shown on the figure below.



Determine the resultant electric field intensity at point A.

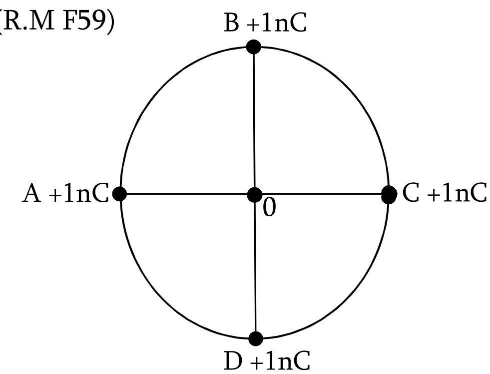
(c) Two equal positive charges are as shown below



If another charge with positive charge +q of mass M is placed at the centre O along the line AB joining them and displaced a small distance X. show that it perform S.H.M with the periodic time.

$$T = \frac{2\pi}{q}\sqrt{\pi\varepsilon_o M d^3}$$

3. (R.M F59)

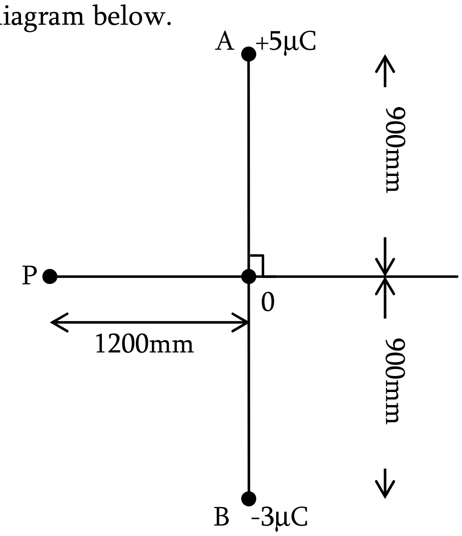


Charges of +1nC at each of point A , B , C and D as shown on the figure above. Given that AOC = 6m, calculate the work done in bringing a charge of +5nC from a distance point to the centre. Assume that $\frac{1}{4\pi\varepsilon_o} = 9\times10^9 F^{-1}m$

An. s 6.0×10^{-8}J

4. (R.M F .60)

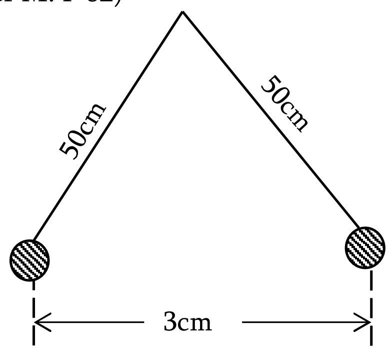
Two point charges of +5μC and -3μC are placed at the points A and B as shown in the diagram below.



Calculate the work done in moving a charge of -3μC from P to Q.

Ans. -24mJ

5. (Roger M. F 62)

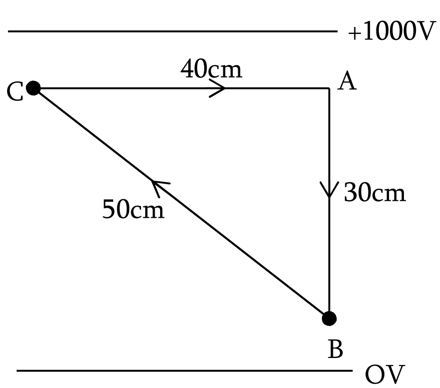


Two light conducting spheres, each 6mm diameter and having a mass of 10mg, are suspended from the same point by the fine insulating fibres 50cm long. Due to the electrostatic repulsion the spheres are in equilibrium when 3cm apart. What is

(a) The force of repulsion between the spheres.
(b) The charge on each sphere
(c) The potential of each sphere (Acceleration due to gravity, g = 10m/s²)

Ans. (a) 3.0×10^{-6} N (b) 5.5×10^{-10} C (c) 1.8×10^{3} V

6. (a) Define the terms:-
   (i) Electric field strength
   (ii) Electric potential, both at point in an electric field .
   (b) An electric field is established between two parallel plates as shown below.



The plates are 50cm a part and a p.d of 1000V is applied between them. A point charge of value +1.0μC is held at point A. It is moved first to B then to C and finally back to A. The distances are shown in the diagram. Calculate.

(i) The force experienced by the charge at A.
(ii) The force experienced by the charge at B.
(iii) The energy required to move the charge from C to A
(iv) The energy involved in moving the charge from A to B
(v) The net energy needed to move the charge along the route ABCA

Ans. (i) 2.0×10^{-3} N
(ii) 2.0×10^{-3} N, **(iii) 0**
(iv) 6.0×10^{-4} N **(v) 0**

7. ABCD is square of side 0.2m charges of 2×10^{-9}, 4×10^{-9}, 8×10^{-9} coulomb are placed at the corners A, B and C respectively. Calculate the work required to transfer a charge of 2×10^{-9} coulomb from corner D to the centre of the square.

Ans. 6.27 × 10^{-7}J

8. Two charges 3×10^{-8}C and -2×10^{-8}C are located 15cm apart. At what point on the line joining the two charges is electrical potential zero? Take the potential at infinity to be zero.
   Ans. X = 0.09m = 9cm from charge of 3×10^{-8}C.

9. Two electrons are moving towards each other each with a velocity of 10^6m/s. what will be the closet distance of approach between them?

Ans. 2.53×10^{-10} m

10. Calculate the voltage needs to balance an oil drop carry 10 electrons, when located between plates of a capacitor, which 5mm apart. Given mass of the drop $=3\times10^{-16}kg$ charge an electron $=1.6\times10^{-19}C$ and $g=9.8m/s^2$

Ans. 9.19 Volt.

11. Two plane parallel conducting plate 15.0mm apart are held horizontal one above the other in air. The upper plate is maintained at a positive potential of 1500V while the lower plate is Earthed. Calculate the number of electrons which must be attached to a small oil drop of mass $4.90\times10^{-15}kg$, if it remains stationary in the air between the plate. (Assume that the density of air is negligible in comparison with that of oil). If the potential of the upper plate is suddenly changed to – 1500V. What is the initial acceleration of the charged drop? Indicate, giving reasons, how the acceleration will change.

Ans. 3 electrons, 19.6m/s^2.

12. In variant of the Millikan's oil drop set up, an oil drop whose radius is measured by separated observation to be 10-6m, fall downward in absence of any electric field with a terminal velocity. When a horizontal electric field is set up by means of two parallel vertical plates held 10mm a part at a p.d of 1500 volt, the drop is seen to fall steadily at an angle of 63^O with the vertical. The density of the oil used in 900kgm-3. Estimate the charge on the drop.

Ans. $4.84\times10^{-19}C$

13. NECTA 2000/P1/13(C)
A small charged oil drop is allowed to fall under gravity in the Millikan's experiment, it is then made to remain stationary under the application of an electric field. Show that the charge Q of the oil drop is given by

$$Q = \frac{6\pi\eta}{E}\left[\frac{9\eta v}{2(\rho_o-\rho_a)g}\right]^{1/2}(V-V')$$

Where η is the coefficient of viscosity of air, V the terminal velocity, ρ_a , ρ_o are densities of air and oil respectively and V the new terminal velocity.

14. An oil drop has a charge of 24e, where $e=1.6\times10^{-19}C$ and is between two plates 4mm apart. The drops fall under gravity with a velocity of $6\times10^{-4}m/s$ and a p.d of 1600V between the plates makes the drop rise with steady velocity V. If the viscosity of air is $1.8\times10^{-5}Nsm^{-2}$ and the density of oil is 900kgm^{-3}. Calculate radius and value of V.

Ans. $r=2.3\times10^{-6}m$, $V=13.7\times10^{-4}m/s$

15. NECTA 2010/P1/13
(a) (i) Without giving any experimental or theoretical detail explain how the results of Millikan's experiment led to the idea that charges comes in 'packets' the size of the smallest packet being carried by an electron (01 mark)
(ii) In the form of Millikan's experiment an oil drop was observed to fall with a constant velocity of $2.5\times10^{-4}m/s$ in the absence of an electric field. When a p.d of 1000V was applied between the plates 10mm apart; the drop remained stationary between them. If the density of oil is $9.0\times10^{2}kgm^{-3}$, density of air is $1.8\times10^{-5}Nsm^{-2}$. Calculate the radius of the oil drop and the number of electric charges carries. (04 marks)
(b) Show that the path of and electron moving in a an electric field is a parabola (05 marks)

16. A charged oil drop of mass 8.22×10^{-16}kg is held stationary by the electric field between two oppositely charged horizontal parallel plates one above, the other at a spacing of 50mm. the top plate is at potential of +840V with respect to the lower plate.
    (a) Is the droplet charged positively or negatively?
    (b) Calculate the droplets charge.
    (c) If the plate voltage is suddenly reversed, what will be the initial acceleration of the droplet?

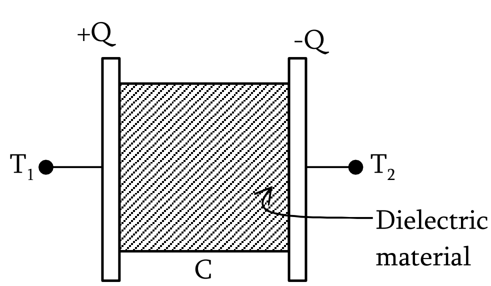
Ans. (a) Negative (b) 4.70×10^{-19}C (c) 19.6m/s².

17. Show that the electric field E necessary to raise an oil drop of mass M and charge q with a speed that is twice the speed fall of the drop when there is no field is $E=\frac{3Mg}{q}$ neglecting the buoyant force.

4. CAPACITANCE

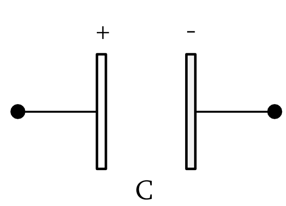
CAPACITOR OR CONDENSER

Definition A capacitor – is a device used for storing electric charges i.e A capacitor is a device which electric charges may be stored, so that it possesses electrical energy. All a capacitors consists two metal plates (conductor) separated by an insulator or dielectric material.

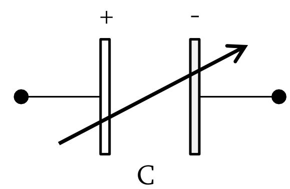


SYMBOL OF A CAPACITOR

(i) Fixed capacitor



(ii) Variable capacitor



TYPES OF CAPACITORS

1. Paper or plastic capacitor
2. Mica capacitor
3. Ceramic capacitor
4. Electrolytic capacitor
5. Air capacitor

Note that

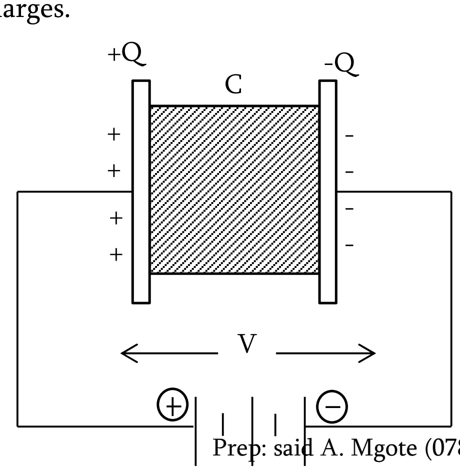
Capacitors are widely used in radii and television tuners, in the starting circuit of electric motors, in automobile ignition system e.t.c

CAPACITANCE OR CAPACITY

Electrical capacitance of a conductor.

Is related to its ability to store the electric charge. When a charge +Q is given to a conductor it potential rises. The greater the charges, the greater the rise in potential.

Consider the circuit below which shows the capacitor being connected on source of electric charges.



Since Q α V

$Q = CV$……….(1)

$$C = \frac{Q}{V} \text{……….(2)}$$

$$V = \frac{Q}{C} \text{……….(3)}$$

C = Constant of proportionality known as capacitance of capacitor.

Q = quantity of electric charges

V = potential difference.

Capacitance (C) – Is defined as the amount of electric charges stored per unit potential difference applied.

$$\text{Capacitance} = \frac{\text{amount of charges stored}}{\text{potential difference}}$$

$$C = \frac{Q}{V}$$

Unit of capacitance

$$C = \frac{Q}{V} = \frac{1\text{coulomb}}{1\text{volt}} = 1F$$

S.I unit of capacitance is the **Farad (F)**

Definition The capacitance of conductor is said to be one farad if its potential is raised by one volt, when one coulomb of charge is given to it farad is the S.I unit of capacitance named so in honour of 'Michael Faraday'. Farad is a large unit of capacitance.

The other small sub – units of capacitance are:-

1milli – farad (1mF) = 10^{-3}F

1micro farad (1μF) = 10^{-6}F

1mono farad (1ηF) = 10^{-9}F

1 pico farad (1pF) = 10^{-12}F

Dimensional of capacitance

$$[C] = \frac{[Q]}{[V]}$$

$$[Q] = [I][t] = AT$$

$$[V] = \frac{[W]}{[Q]} = \frac{ML^2T^{-2}}{AT}$$

$$[V] = ML^2T^{-3}A^{-1}$$

Now

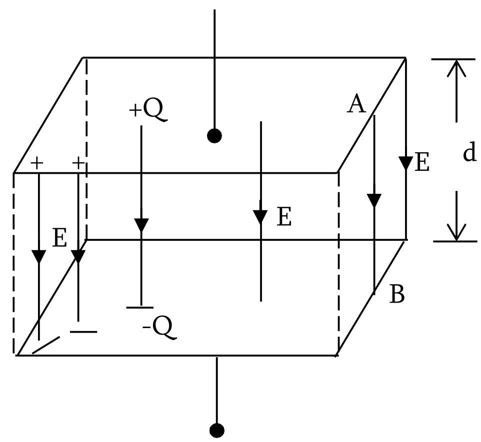
$$[C] = \frac{AT}{ML^2T^{-3}A^{-1}}$$

$$[C] = M^{-1}L^{-2}T^4A^2$$

EXPRESSIONS OF CAPACITANCE AND CAPACITOR

(i) Parallel plate capacitor

A parallel plate capacitor consider of two plane parallel conducting plates separated by a distanced compared with the plates dimensions as shown on the figure below.



Due to the electrostatic induction a charge –Q will appear near side of B and +Q on its further side. The induced positive will get neutralized due to the flow of electrons from the Earth. Thus plate B – Q charges, uniformly distributed on the inner surface of the plates.

1. **Derivation of an expression of capacitance of parallel plates capacitor.**

$$C = \frac{A\varepsilon_0}{d} \quad [\text{Air}]$$

Where

A = area of the one of the plate

d = distance between the two plates

c = capacitance of a capacitor

εo = permittivity of free space.

The surface charge density of the plate

$$\sigma = \frac{Q}{A} \ldots\ldots\ldots(1)$$

By Gauss's theorem, the electric field intensity E between the plate.

$$E = \frac{\sigma}{\varepsilon_0} \ldots\ldots\ldots(ii)$$

Putting equation (i) into (ii)

$$E = \frac{Q}{A\varepsilon_0} \text{ but } E = \frac{V}{d}, \; Q = CV$$

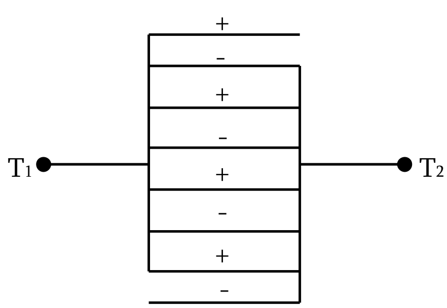
$$\frac{V}{d} = \frac{CV}{A\varepsilon_0}$$

$$C_0 = \frac{A\varepsilon_0}{d} \ldots\ldots(4)$$

If the space between the plates filled with a material whose dielectric constant is εr or K i.e for the given medium instead of air.

$$C = \frac{A\varepsilon}{d} = \frac{A\varepsilon_r\varepsilon_o}{d}$$

2. Expression of capacitance of η – parallel plates capacitor.



For a parallel plates capacitor

$$C = \frac{A\varepsilon_0}{d}$$

Total equivalent capacitance in parallel connection

$$C = C_1 + C_2 + \text{-------} + C_{n-1}$$

$$= \underbrace{\frac{A\varepsilon_0}{d} + \frac{A\varepsilon_0}{d} + \text{--------} + \frac{A\varepsilon_0}{d}}_{(n-1)\text{times}}$$

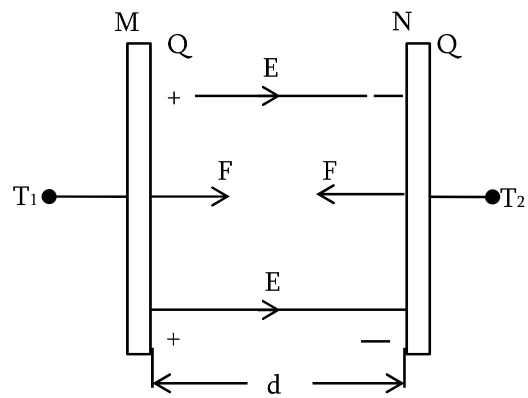
$$\text{For air } C = \frac{(n-1)A\varepsilon_0}{d} \ldots\ldots(6)$$

For the given medium between the plates instead of air/vacuum

$$C = (n-1)\frac{A\varepsilon}{d} = \frac{(n-1)A\varepsilon_r\varepsilon_o}{d}$$

3. Expression of force of attraction between the parallel plates of a charged capacitor.

Consider a parallel plate capacitor consisting of two plates M and N of area A, each separated by a distance d as shown in the figure below.



Electric field intensity E produce by plate M

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

For the air filled between the plates

$$F = \frac{Q^2}{2A\varepsilon_0} \text{----------}(8)$$

If there is a dielectric material between the plates i.e for the given medium

$$F = \frac{Q^2}{2A\varepsilon} = \frac{Q^2}{2A\varepsilon_0\varepsilon_r} \text{---------}(9)$$

Note that:-

$$\text{Let } \sigma = \frac{Q}{A}$$

$$\text{(a)} \quad F = \frac{Q}{2\varepsilon_0\varepsilon_r}\cdot\left(\frac{Q}{A}\right) = \frac{Q\sigma}{2\varepsilon_0\varepsilon_r} \ldots\ldots\ldots(10)$$

Expression of pressure on each plates

$$P = \frac{F}{A} = \frac{Q^2}{2\varepsilon_0 A}/A = \frac{Q^2}{2\varepsilon_0 A^2}$$

For air

$$P = \frac{Q}{2\varepsilon_0 A^2} = \frac{\sigma^2}{2\varepsilon_0} \text{............(11)}$$

For the given medium instead of free space (air)

$$P = \frac{Q}{2\varepsilon_0\varepsilon_0 A^2} = \frac{\sigma}{2\varepsilon_r\varepsilon_0} \text{........(12)}$$

(b) Since

$$F = \frac{Q}{2\varepsilon_0\varepsilon_r A} \text{ but } Q = CV$$

$$= \frac{C^2V^2}{2\varepsilon_0\varepsilon_r A}, \ C = \frac{A\varepsilon_0\varepsilon_r}{d}$$

$$= \frac{V^2}{2\varepsilon_0\varepsilon_r A}\left(\frac{A\varepsilon_0\varepsilon_r}{d}\right)^2$$

$$F = \frac{1}{2}\varepsilon_0\varepsilon_r AE^2 \text{.........(13)}$$

4. **Expression of work done in separating the plates with the charge Q remaining the same before and after separation.**

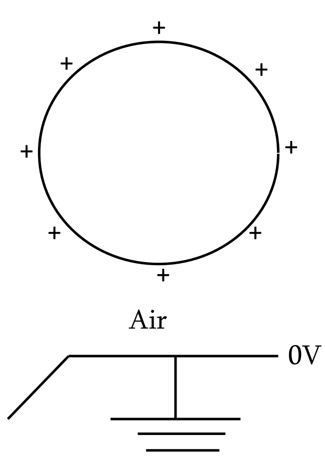
$W = F.d$

$$W = \frac{Q^2 d}{2\varepsilon_o\varepsilon_r A} \text{........(14) Or } W = \frac{(\sigma A)^2 \cdot d}{2\varepsilon_o\varepsilon_r A}$$

$$W = \frac{\sigma^2 Ad}{2\varepsilon_o\varepsilon_r}$$

(ii) Capacitance of an isolates spherical conductor

Consider an isolated spherical conductor of radius R and having a charge Q. Placed in air/ vacuum. The lines force due to the charge on spherical conductor can be assumed to be concentrated at the centre of the sphere. Electric potential at any point on the surface of the conductor.



$$V = \frac{Q}{4\pi\varepsilon_0 R} \text{ but } Q = CV$$

$$V = \frac{CV}{4\pi\varepsilon_0 R}$$

$$C = 4\pi\varepsilon_0 R$$

For the given medium instead of air

$$C = 4\pi\varepsilon_r\varepsilon_0 R$$

FACTORS AFFECTING THE CAPACITANCE OF A CAPACITOR FOR THE PARALLEL PLATES.

- For parallel plate capacitor

$$C = \frac{A\varepsilon_r\varepsilon_0}{d}$$

- For the spherical capacitor

$$C = 4\pi\varepsilon_r\varepsilon_0 R$$

The capacitance of a parallel plates capacitor depends on the following factors:

1. **The area of the plate (conductor)**

If other factors remain constants, the area of the plate A is increases, then capacitance of capacitor C increases C α A

$$\frac{C_1}{C_2} = \frac{A_1}{A_2}$$

2. **The distance between the plates, d**

If d increases, also C decreased

$$C \ \alpha \ \frac{1}{d}$$

$$\frac{C_1}{C_2} = \frac{d_2}{d_1}$$

3. **The dielectric constant of the medium of the material between the plates.**

The increase of the medium increases the capacitance by ε_r times the capacitance of the same conductor witoud any medium in between the plate.

$C \ \alpha \ \varepsilon_r$ or $C \ \alpha \ K$

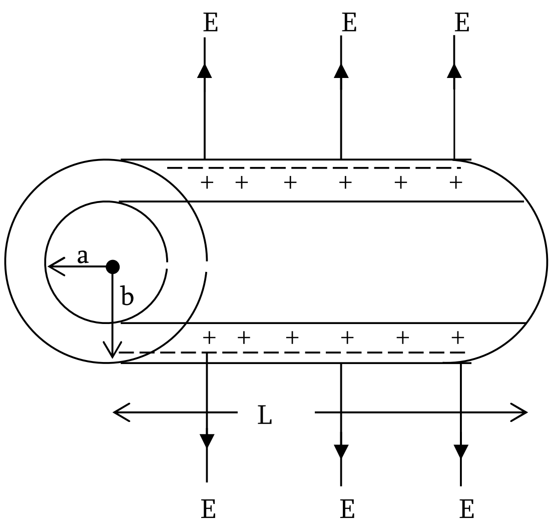
$$\frac{C_1}{C_2} = \frac{K_1}{K_2}$$

4. Capacitance of a conductor can be increased by increasing it size (c α R).

5. The purpose of keeping the Earth metal plate near a charged metal plate is to increase its capacitance. This means that the presence of a conductor near a charged conductor increases its capacitance

III. CYLINDRICAL CAPACITOR

A cylindrical capacitor consists of two coaxial cylindrical shells A and B of inner and outer radii a and b respectively. Let be length of coaxiable cylinder.



Let λ be charges per unit length Q = λl
Apply Gauss's law

$$\Phi_E = \frac{Q}{\varepsilon_0} = EA$$

But $A = 2\pi rL$

$$E\cdot 2\pi rL = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{2\pi\varepsilon_0 rL} = \frac{\lambda}{2\pi\varepsilon_0 rL}$$

Potential difference between the inner and outer surface of cylinder.

$$V = -\int_b^a Edr = \int_a^b Edr$$

$$= \int_a^b \frac{\lambda dr}{2\pi\varepsilon_o r}$$

$$= \frac{\lambda}{2\pi\varepsilon_o}\left[\log_e^r\right]_a^b$$

$$= \frac{\lambda}{2\pi\varepsilon_o}\left[\log_e^b - \log_e^a\right]$$

$$V = \frac{\lambda}{2\pi\varepsilon_0}\log_e\left(b/a\right) \quad \text{but } \lambda = \frac{Q}{L}$$

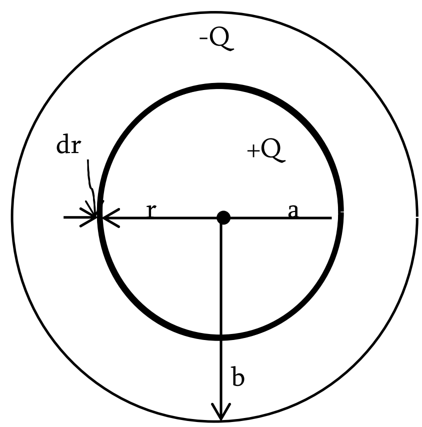
$$V = \frac{Q}{2\pi\varepsilon_0 L}\log_e\left(b/a\right)$$

$$V = \frac{CV}{2\pi\varepsilon_0 L}\log_e\left(b/a\right)$$

$$C = \frac{2\pi\varepsilon_0 L}{\log_e\left(b/a\right)} \quad \text{........(16)}$$

IV. SPHERICAL CAPACITOR

A spherical capacitor consists of two concentric spherical conducting shells A and B of radii a and b for the inner and outer shells respectively.



The electric field intensity at any point P at distance r from the centre of the sphere.

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

The potential difference between A and B is given.

$$V = -\int_b^a E dr = \int_a^b E dr$$

$$= \frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2}$$

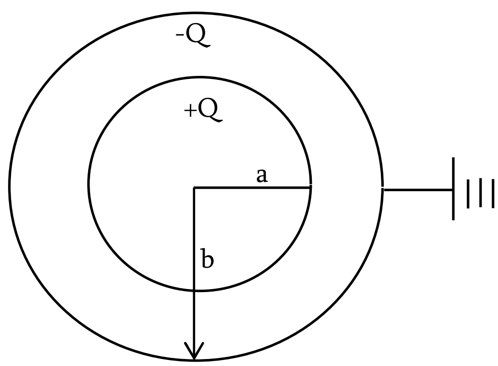
$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{-1}{r}\right]_a^b$$

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right] = \frac{Q}{4\pi\varepsilon_0}\left[\frac{b-a}{ab}\right]$$

$$C = \frac{Q}{V} = \frac{4\pi\varepsilon_0 ab}{b-a}$$

$$C = \frac{4\pi\varepsilon_0 ab}{b-a} \quad ....(17)$$

Alternatively



The potential of the inner sphere

$$V_a = \frac{Q}{4\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0 b} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right]$$

Potential of the outer sphere

$V_b = 0$ since the outer sphere is Earthed.

Potential difference

$$V = V_a - V_b$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right] - 0$$

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{b-a}{ab}\right]; \; Q = CV$$

$$C = \frac{4\pi\varepsilon_0 ab}{b-a} \; \text{(Air)}$$

Special cases:

Case 1: If b >>> a, the capacitance of concentric spheres reduce to that of an isolated sphere.

$$C = \frac{4\pi\varepsilon_0 ab}{b-a}, \; b-a \simeq b$$

$$C = \frac{4\pi\varepsilon_0 ab}{b} = 4\pi\varepsilon_o a$$

This is the capacitance of an isolated sphere.

Case 2: if $b-a \simeq d$ and that $b \simeq a$, the capacitance of concentric spheres reduces to that of a parallel plates capacitor.

$$C = \frac{4\pi\varepsilon_0 ab}{b-a} = \frac{4\pi\varepsilon_0 a^2}{d}$$

Where $a^2 = ab$

$$C = \frac{\left(4\pi a^2\right)}{d} \quad \text{but} \quad 4\pi a^2 = A$$

$$C = \frac{A\varepsilon_o}{d}$$

A = Area of the sphere

This is the capacitance of a parallel plates capacitor.

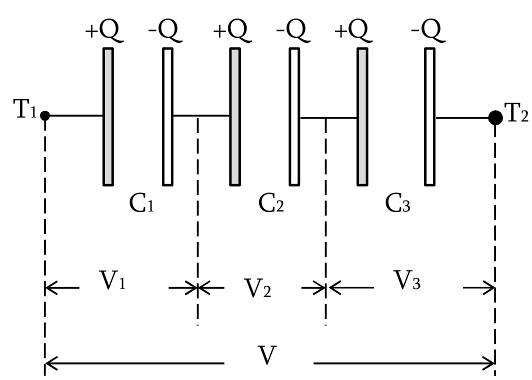
ARRANGEMENT OF CAPACITORS

Capacitors can be arranged into two ways:

(i) Series connection

(ii) Parallel connection

I. CAPACITORS IN SERIES CONNECTION



For the capacitors in series connection:

(i) The charges on the plates of each capacitor are the same.

(ii) Potential difference across each plates of capacitors are different total potential difference across the circuit.

$$V = V_1 + V_2 + V_3$$

$$V_1 = \frac{Q}{C_1},\ V_2 = \frac{Q}{C_2},\ V_3 = \frac{Q}{C_3}$$

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$V = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ \ldots\ldots(18)$$

OR

$$\frac{1}{C_S} = \frac{C_2C_3 + C_1C_3 + C_1C_2}{C_1C_2C_3}$$

$$C_S = \frac{C_1C_2C_3}{C_2C_3 + C_1C_3 + C_1C_2}$$

Note that:

1. For a number of capacitor $C_1, C_2, C_3, \ldots\ldots, C_n$ then the affective capacitance in series is

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \text{--------} + \frac{1}{C_n}$$

2. The reciprocal of the equivalent capacitance C of a number of capacitors connected in series is equal to the sum of the reciprocals of the individual capacitances. If all the capacitors are equal then.

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \text{--------} + \frac{1}{C_n}$$

But $C_1 = C_2 = \text{--------} C_n = C$

$$\frac{1}{C_S} = \frac{1}{C} + \frac{1}{C} + \text{--------} + \frac{1}{C}$$

$$\frac{1}{C_S} = \frac{n}{C}$$

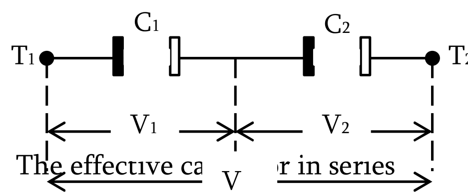
$$C_S = \frac{C}{n}$$

3. For the two capacitors arranged in series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_S = \frac{C_1C_2}{C_1 + C_2}$$

Consider the circuit as shown on the figure below



The effective ca r in series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$Q = C_1V_1 = CV$$

$$\frac{1}{V_1} = \frac{C_1}{V}\cdot\frac{1}{C}$$

$$\frac{1}{V_1} = \left[\frac{1}{C_1} + \frac{1}{C_2}\right]\frac{C_1}{V} \quad \text{or}$$

$$\frac{1}{V_1} = \left[\frac{C_1 + C_2}{C_1C_2}\right]\frac{C_1}{V}$$

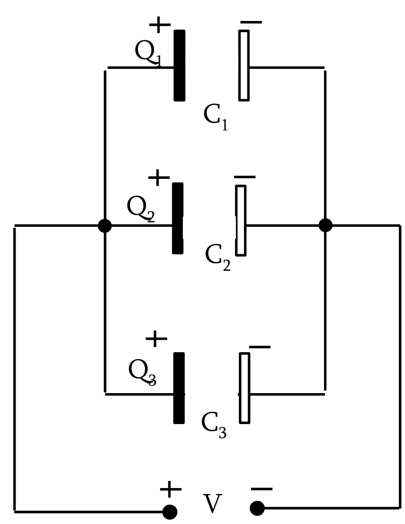
$$V_1 = \left(\frac{C_2}{C_1 + C_2}\right)V \ \text{Also}\ V_2 = \left(\frac{C_1}{C_1 + C_2}\right)V$$

II. CAPACITORS IN PARALLEL CONNECTION

For the parallel connection of the capacitors.

(i) The potential difference across each of the capacitors are the same.

(ii) The charges stored by each capacitor in different



The total electric charges stored.

$$Q = Q_1 + Q_2 + Q_3$$
$$= C_1V + C_2V + C_3V$$

$$C_P V = (C_1 + C_2 + C_3)V$$

$$C_P = C_1 + C_2 + C_3$$

C_P = total effectively capacitance in parallel connection.

Note that

1. For a number of capacitors C_1, C_2….C_n in parallel connection, then the total effectively capacitance.

   $$C_P = C_1 + C_2 + \text{--------} + C_n \text{ or}$$

   $$C_P = \sum_{i=1}^{n} C_i$$

2. For the two capacitors in parallel connection.

   $$C_P = C_1 + C_2$$

3. Suppose a number of capacitors are connected in parallel, then the capacitance is equal to the sum of their individual capacitance.

   $$C_P = C_1 + C_2 + \text{-------} + C_n$$

   If $C_1 = C_2 = \text{----} = C_n = C$

   $$C_P = C + C + \text{------} + C$$

   $$C_P = nC$$

COMPARISONS BETWEEN CAPACITORS IN SERIES AND CAPACITORS IN PARALLEL CONNECTION.

CAPACITORS IN SERIES	CAPACITORS IN PARALLEL
Positive plate of one capacitor is connected to the negative of the other and so on.	Positive plates of all the capacitors are connected to the common terminal and all negatively to the other terminal.
Each capacitors has the same charge Q but p.d, V is different.	Each capacitor has the same p.d, V but charges Q is different.
$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$	$C = C_1 + C_2 + C_3$
The effective capacitance is less than the least among the capacitors.	The effective capacitance is increased
To decrease the capacitance series connection is used.	The effective capacitance is increased. Then to increase capacitance in parallel connection is used.

ENERGY STORED IN A CHARGED CAPACITORS.

When a cell is connected to a capacitor charges flows from the cell to the capacitor beginning with the charge on the capacitor plates is zero, then the charge on the capacitor increases slowly. At any instant there is exists a potential difference between the plates. So to transfer additional charge(s) work must be done. This work done is stored in as the electrical potential energy. The energy store in the capacitor is given by.

$$W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

Q = quantity of electric charges
V = potential difference
C = capacitance of the capacitor.

Derivation of $W = \frac{1}{2}QV = \frac{1}{2}CV^2$

Method 1: By using integration method.

Consider a capacitor of capacitance C charged with charge q such that its potential is V.

$$V = \frac{q}{C}$$

The work done to increases the charges by an amount dq is $dw = vdq = \frac{q}{c}dq$

Total work done to charge the capacitor from O to R

$$W = \int_0^Q \frac{q}{c}dq = \frac{1}{2c}\left[q^2\right]_0^Q$$

$$= \frac{1}{2C}\left[Q^2 - O^2\right] = \frac{Q^2}{2C}$$

But $C = \frac{Q}{V}$

$$W = \frac{Q^2}{2} \times \frac{V}{Q} = \frac{1}{2}QV$$

$$W = \frac{1}{2}QV = \frac{Q^2}{2C}$$

Also $Q = CV$

$$W = \frac{1}{2}(CV) \cdot V = \frac{1}{2}CV^2$$

$$W = \frac{1}{2}CV^2$$

Derivation of $W = \frac{1}{2}CV^2$ by integration method.

$$dw = qdv \text{ but } q = CV$$

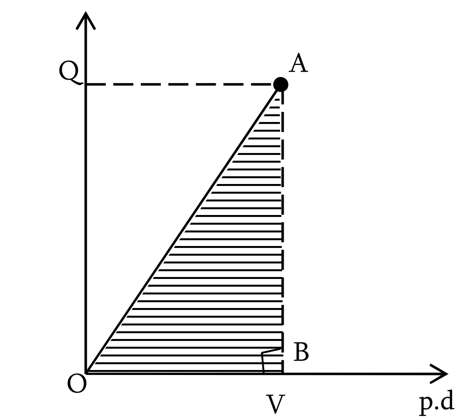
$$dw = cvdv$$

$$\int dw = \int_0^V cvdv = \left[\frac{v^2}{2}\right]_0^V$$

$$W = \frac{1}{2}CV^2$$

Method 2: graphical method.

Since Q = CV [Q α V]. The graph of Q against V is the straight line since Q varies linearly with V.



The area under the graph is equal to the energy stored in the capacitor.

$$\text{Area of } \Delta OBA = \frac{1}{2}(\overline{OA})(\overline{OB})$$

$$= \frac{1}{2}QV$$

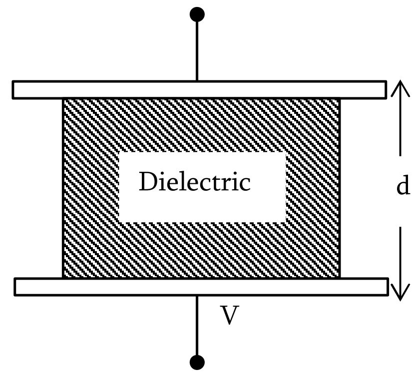
$$W = \frac{1}{2}QV$$

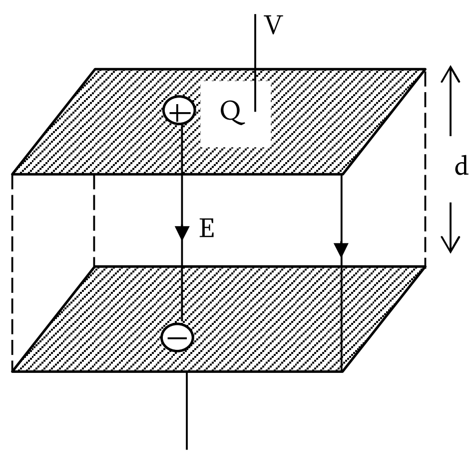
ENERGY DENSITY (U)

Energy density – is the amount of energy stored per unit volume of the electric field.

Derivation of expression energy density (U)

Consider the parallel plates capacitor of capacitance, C. Let d be the distance between the plates, each of area, A.





$$\text{Energy density} = \frac{\text{Energy stored}}{\text{Volume}}$$

$$U = \frac{W}{V_0}$$

S.I. unit of energy density is Jm^{-3}.

Energy stored in the capacitor

$$W = \frac{1}{2}CV^2 \text{ but } C = \frac{A\varepsilon_0}{d}$$

$$W = \frac{1}{2}\frac{A\varepsilon_0 V^2}{d}$$

The volume of space between the plates, $V_0 = Ad$

$$U = \frac{W}{V_0} = \frac{A\varepsilon_0 V^2}{2d} / Ad$$

$$= \frac{1}{2}\varepsilon_o\left(\frac{V}{d}\right)^2$$

$$U = \frac{1}{2}\varepsilon_0 E^2 \text{ (Air)}$$

For the given medium between the parallel plates.

$$U = \frac{1}{2}\varepsilon E^2 = \frac{1}{2}\varepsilon_r\varepsilon_o E^2$$

EXPRESSION FOR THE TOTAL ENERGY WHEN CAPACITORS ARE CONNECTED IN SERIES.

Consider three capacitors C_1, C_2 and C_3 connected in series. The charges flowing through each capacitor will be the same. Let the effective capacitance be C.

$$\frac{1}{C} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

The energy of a charged capacitor

$$W = \frac{Q^2}{2C}$$

Multiply by $Q^2/2$ both side

$$\frac{Q^2}{2C} = \frac{Q^2}{2}\left[\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right]$$

$$\frac{Q^2}{2C} = \frac{Q^2}{2C_1}+\frac{Q^2}{2C_2}+\frac{Q^2}{2C_3}$$

$$W = W_1+W_2+W_3$$

This expression show that the total energy stored in series combination is equal to the sum of the energies stored in the individual capacitor.

EXPRESSION FOR THE TOTAL ENERGY WHEN CAPACITORS ARE CONNECTED IN PARALLEL

When capacitors are connected in parallel, the potential difference across each capacitor C_1, C_2 and C_3 will be the same. The total capacitor in parallel connection.

$$C = C_1+C_2+C_3$$

Multiply by $V^2/2$ both sides

$$\frac{1}{2}CV^2 = \frac{1}{2}C_1V^2+\frac{1}{2}C_2V^2+\frac{1}{2}C_3V^2$$

$$W = W_1+W_2+W_3$$

CHANGE OF ENERGY OF A CAPACITOR

1. By changing the capacitance, Q being constant. If a capacitor is charged and then insulated, Q remains constant. The capacitance C may be varied in three ways:

(i) By changing d only

If d is increased from d1 to d2.

$$C_1 = \frac{\varepsilon_0 A}{d_1},\ C_2 = \frac{\varepsilon_0 A}{d_2}$$

Let E_1 = Initial energy stored

$$E_1 = \frac{Q^2}{2C_1} = \frac{Q^2 d_1}{2A\varepsilon_0}$$

E_2 = Final energy stored

$$E_2 = \frac{Q^2 d_2}{2A\varepsilon_0}$$

$$\Delta E = E_2 - E_1 = \frac{Q^2 d_2}{2A\varepsilon_0} - \frac{Q^2 d_1}{2A\varepsilon_0}$$

$$\Delta E = \frac{Q^2}{2A\varepsilon_0}(d_2 - d_1) \text{ (air)}$$

For the space between the plates is filled with the given medium.

$$\Delta E = \frac{Q^2}{2A\varepsilon_r\varepsilon_0}(d_2 - d_1)$$

(ii) By changing area only

$$C_1 = \frac{\varepsilon_o A_1}{d},\ C_2 = \frac{\varepsilon_0 A_2}{d}$$

$$\text{Then } E_1 = \frac{Q^2 d}{2\varepsilon_o A_1},\ E_2 = \frac{Q^2 d}{2\varepsilon_o A_2}$$

Change of energy

$$\Delta E = E_1 - E_2 \quad (E_1 > E_2)$$

$$\Delta E = \frac{Q^2 d}{2\varepsilon_o}\left[\frac{1}{A_1} - \frac{1}{A_2}\right]$$

(iii) By changing the dielectric constant ε_r only.

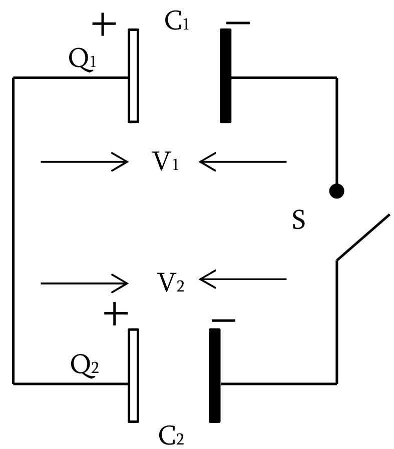
$$E_1 = \frac{Q^2 d}{2\varepsilon_o \varepsilon_{r1} A}, \quad E_2 = \frac{Q^2 d}{2A\varepsilon_0 \varepsilon_{r2}}$$

$$\Delta E = \frac{Q^2 d}{2A\varepsilon_0}\left[\frac{1}{\varepsilon_{r1}} - \frac{1}{\varepsilon_{r2}}\right]$$

2. By changing the capacitance, V being constant.
   (i) If the capacitance is increased from C_1 to C_2.
   Energy gained by capacitor
   $$\Delta E = \frac{1}{2}(C_2 - C_1)V^2$$
   Similarly, if the capacitance decrease from C_1 to C_2.
   Energy lost by capacitor
   $$\Delta E = \frac{1}{2}(C_1 - C_2)V^2$$
   In this case the energy gained must be supplied by work done from the system from outside.
   (ii) Changing d only, by increase from d_1 to d_2
   Energy lost by capacitor
   $$\Delta E = \frac{1}{2}\varepsilon A V^2\left(\frac{1}{d_1} - \frac{1}{d_2}\right) \text{ or}$$
   $$\Delta E = \frac{1}{2}\varepsilon_r \varepsilon_o A V^2\left[\frac{1}{d_1} - \frac{1}{d_2}\right]$$
   (iii) By changing area A only, increasing from A_1 to A_2.
   Energy gained by capacitor.
   $$\Delta E = \frac{\varepsilon_r \varepsilon_o V^2}{2d}(A_2 - A_1)$$
   (iv) By changing ε_r only, increasing from ε_{r1} to ε_{r2}
   Energy gained by capacitor
   $$\Delta E = \frac{\varepsilon_o V^2}{2d}(\varepsilon_{r2} - \varepsilon_{r1})$$
   $$\Delta E = \frac{A\varepsilon_o V^2}{2d}(\varepsilon_{r2} - \varepsilon_{r1})$$

JOINING OF TWO CHARGED CAPACITORS. (LOSS OF ENERGY DURING SHARING OF CHARGES).

Consider two capacitors (or conductors) carrying charges Q_1 and Q_2. Let their capacitance be C_1 and C_2 and potentials be V_1 and V_2 respectively. The two capacitors are connected by a connecting wire.



Before the switch S closed.

$Q_1 = C_1V_1$, $Q_2 = C_2V_2$

Initial total electric charges.

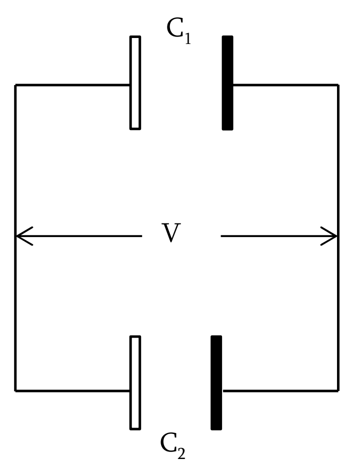
$Q = Q_1 + Q_2$

$Q = C_1V_1 + C_2V_2$...........(i)

Initial total energy

$$W_1 = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

When switch S is closed, charges flows from one capacitor of higher potential to the another capacitor until the potential are the same. This is known as the '**common potential, V'**



Let the charge on them be Q_1' and Q_2' when their common potential is V.

$Q_1' = C_1V$, $Q_2' = C_2V$

Final total electric charges

$$Q_f = Q_1' + Q_2'$$
$$= C_1V + C_2V$$
$$Q_f = (C_1 + C_2)V \text{ ....(ii)}$$

Apply the law of conservation of electric charges

$$Q_f = Q$$

$$(C_1 + C_2) = C_1V_1 + C_2V_2$$
$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

Also $V = \frac{Q_1'}{C_1} = \frac{Q_2'}{C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

Expression of charges acquired by each capacitor after attained the same potential.

$$Q_1' = C_1\left[\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right]$$

Also $Q_2' = C_2\left[\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right]$

Expression of energy loss by the capacitors

Total capacitance after S closed

$C = C_1 + C_2$

Let ΔW = energy loss

Final total energy

$$W_2 = \frac{1}{2}(C_1 + C_2)V^2$$
$$\Delta W = W_1 - W_2 \, (W_1 > W_2)$$
$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}(C_1 + C_2)V^2$$
$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}(C_1 + C_2)\left[\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right]^2$$
$$= \frac{1}{2}\left\{C_1V_1^2 + C_2V_2^2 - \frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2}\right\}$$
$$= \frac{1}{2}\left[\begin{array}{l} C_1^2 + C_2V_2^2 + C_1C_2V_1^2 + C_1C_2V_1^2 \\ - C_1^2V_1^2 - C_2^2V_2^2 - 2C_1C_2V_1V_2 \end{array}\right]$$

$$\Delta W = \frac{1}{2}\frac{C_1C_2}{(C_1 + C_2)}\left[V_1^2 - 2V_1V_2 + V_2^2\right]$$

$$\Delta W = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2 \text{ if } V_1 > V_2 \text{ or}$$

$$\Delta W = \frac{C_1C_2}{2(C_1 + C_2)}(V_2 - V_1)^2 \text{ if } V_2 > V_1$$

Again $Q_1 = C_1V_1$, $Q_2 = C_2V_2$

$$V_1 = \frac{Q_1}{C_1} , V_2 = \frac{Q_2}{C_2}$$

$$\Delta W = \frac{C_1C_2}{2(C_1 + C_2)}\left[\frac{Q_1}{C_1} - \frac{Q_2}{C_2}\right]^2 \text{ or}$$

$$\Delta W = \frac{(Q_1C_2 - Q_2C_1)^2}{2(C_1 + C_2)C_1C_2}$$

Always ΔW is the positive quantity.

Comment

So we conclude that when two charged capacitors are at different potentials are connected charges flows from one capacitor to another capacitor until the potentials are the same. Then there is energy loss. Usually final energy is less than the initial energy i.e energy loss is appeared in form of heat energy being dissipated due to the flowing of charges through the connecting wires.

NUMERICAL EXAMPLES

Example 1

Two conductors have net charges of +20μC and -20μC. If the potential difference between the conductors is 20V. Calculate the capacitance of the system.

Solution

Capacitance of a capacitor

$$C = \frac{Q}{V} = \frac{20\times10^{-6}}{20}$$
$$C = 1.0\times10^{-6}F = 1\mu F$$

Example 2

A liquid drop in air has a capacitance of 20pF

(i) Calculate its radius

(ii) If its potential is 100V, what is the charge on the drop?

Solution

(i) For the spherical capacitor

$$C = 4\pi\varepsilon_0 R \ , \ R = \frac{C}{4\pi\varepsilon_o}$$

$$R = 9\times10^{9}\times2\times10^{-12}$$

$$R = 18\times10^{-3}\,m$$

(ii) Let Q = charge on the drop

$$Q = CV$$

$$= 12\times10^{-12}\times100$$

$$Q = 2\times10^{-10}C$$

Example 3: NECTA 1999/P2/18(a)

(i) What is the capacitance of a capacitor?

(ii) List three factors that govern the capacitance.

Example 4

A parallel plate capacitor has an area of $4.0\times10^{-4}\,m^2$ and a plate separation of 2.00mm.

(i) Calculate its capacitance

(ii) If the plate separation is increased to 3.0mm. Find the capacitance

(iii) If the space between the plate is filled with a medium of dielectric constant of 6.7. calculate its capacitance in the second case $\left[\varepsilon_o = 8.85\times10^{-12}C^2m^2N^{-1}\right]$

Solution

(i) $C = \dfrac{A\varepsilon_0}{d} = \dfrac{8.85\times10^{-12}\times4\times10^{-4}}{2\times10^{-3}}$

$$C = 1.77\times10^{-12}F$$

(ii) $d_1 = 3.0mm \quad C_1 = ?$

$$C_1 = \frac{A\varepsilon_0}{d_1} = \frac{8.85\times10^{-12}\times4\times10^{-4}}{3\times10^{-3}}$$

$$C_1 = 1.18\times10^{-12}F$$

(iii) $C = \dfrac{A\varepsilon_0\varepsilon_r}{d}$

$$= \frac{8.85\times10^{-12}\times6.7\times4\times10^{-4}}{3\times10^{-3}}$$

$$C = 7.90\times10^{-12}F$$

Example 5 NECTA 2005/P1/8

(a) (i) State the coulomb's law (01 mark)

(ii) A spherical metal of radius r carries a charge, Q. Sketch a graph showing the variation of the electric potential V with distance X from the centre of the sphere for points (inside and outside) along a line through the centre of the sphere. Account for the shape of the graph (03 marks)

(b) (i) name the physical properties of a capacitance, hence write the relation (02 marks)

(ii) Show that the energy stored per unit volume of a parallel plate capacitor is given by $U = \frac{1}{2}\varepsilon_0\varepsilon_r E^2$ where the symbols carry their usual meaning (04 marks)

Solution

(a) (i) Refer to your notes.

(ii) See solution example 49 page 142

(b) (i) Area of the one of plate ($C \propto A$)

(ii) Energy constant of material

- The distance between the plates $\left(C \propto 1/d\right)$
- Dielectric constant of material $\left(C \propto \varepsilon_r\right)$

$$C = \frac{A\varepsilon_o\varepsilon_r}{d}$$

$$W = \frac{1}{2}CV^2 \ , \ C = \frac{A\varepsilon_0\varepsilon_r}{d}$$

$$W = \frac{1}{2}\varepsilon_0\varepsilon_r AV^2$$

Since $U = \dfrac{W}{Ad}$

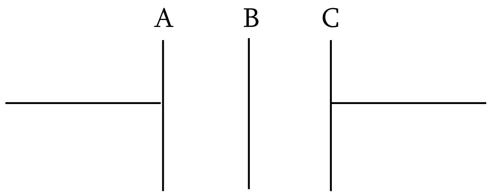
$$U = \frac{1}{2}\cdot\frac{\varepsilon_0\varepsilon_r AV^2}{Ad^2}$$

$$U = \frac{1}{2}\varepsilon_0\varepsilon_r E^2$$

Example 6

A parallel plates capacitor is made of 201 plates separated by paraffinied paper 0.001cm thick of relative permittivity 2.5. The effective size of each plate is 15 × 30cm. what is the capacitance of this capacitor? $\left(\varepsilon_o = 8.85\times10^{-12}C^2N^{-1}m^{-2}\right)$

Solution



Three plates in a parallel plate capacitor, give rise to two capacitor see the above figure. If there are n – plates, then there will be (n – 1) capacitors.

$$C = (n-1)\frac{\varepsilon_o\varepsilon_r A}{d}$$

$$= \frac{(201-1)\times8.85\times10^{-12}\times2.5\times4.5\times10^{-2}}{10^{-5}}$$

$$C = 19.91\times10^{-6}F$$

Example 7

The excess charge of each plate of a parallel plate capacitor is +30μC. The plates are 400cm^2 in area and the separated by 2mm in air.

(i) Calculate the surface density of charge.
(ii) What is electric field between the plates?
(iii) Calculate the potential difference between the plates.
(iv) What is the capacitance of the capacitor?

Solution

(i) Surface charge density

$$\sigma = \frac{Q}{A} = \frac{30\times10^{-6}C}{400\times10^{-4}m^2}$$

$$\sigma = 7.5\times10^{-4}cm^{-2}$$

(ii) $$E = \frac{\sigma}{\varepsilon_0} = \frac{7.5\times10^{-4}}{8.85\times10^{-12}}$$

$$E = 8.47\times10^7 Vm^{-1}$$

(iii) $$V = Ed = 8.47\times10^7\times2\times10^{-3}$$

$$V = 1.694\times10^5 Volt$$

(iv) $$C = \frac{Q}{V} = \frac{30\times10^{-6}}{1.694\times10^5}$$

$$C = 177\times10^{-12}F$$

Example 8

Twenty seven spherical drop of radius 3mm and carrying 10^{-12}C of charge are combined to form a single drop. Find the capacitance of the bigger drop and potential of bigger drop.

Solution

Let r and R be the radii of the small and bigger drops respectively.

Apply the law of conservation of mass or volume

Volume of the Bigger drop = total volume of N – small drops.

$$\frac{4}{3}\pi R^3 = 27\times\frac{4}{3}\pi r^3$$

$$R^3 = 27r^3$$

$$R = 3r = 3\times3$$

$$R = 9mm = 9\times10^{-3}m$$

Capacitance of a bigger drop

$$C = 4\pi\varepsilon_0 R$$

$$= 4\times3.14\times8.85\times10^{-12}\times9\times10^{-3}$$

$$C = 1\times10^{-12}F = 1pF$$

Charge on the large drop

Q = 27 × charge on small drop

Q = 27 × 10^{-12}C

Potential of the bigger drop

$$V = \frac{Q}{C} = \frac{27\times10^{-12}}{10^{-12}}$$

V = 27 Volt

Example 9 (Nelkon 6th Ed. Q.8)

(a) Explain what is meant by Dielectric constant (relative permittivity). State two physical properties desirable in a material to be used as the dielectric in a capacitor.

(b) A sheet of paper 40mm wide and 1.5 × 10^{-2}mm thick between metal foil of the same width used to make a 2.0μF capacitor. If the dielectric constant (relative permittivity) of the paper is 2.5, what length of the paper required? $\left(\varepsilon_o = 8.85\times10^{-12}Fm^{-1}\right)$

Solution

(a) The dielectric constant of a dielectric material is defined as the ratio of capacitance of capacitor with that dielectric between the plates to the capacitance of the same capacitor with air/vacuum between the plates.

The following are physical properties describe in a material to be used as the dielectric in capacitor.

- Dielectric have practically no free electrons.
- When a dielectric is placed in an electric field its molecules are polarized. The effect of this polarization is to weaken the applied electric field within the dielectric.
- The dielectrics of material have the finite value.
- The dielectric strength of the dielectric is finite.
- There is a limit of the electric flux that dielectrics will carry without breaking down.

(b) $C = \dfrac{\varepsilon_0 KA}{d}$, $A = \dfrac{Cd}{\varepsilon_o k}$

$$A = \frac{2\times10^{-6}\times1.5\times10^{-5}}{8.85\times10^{-12}\times2.5}$$

$$A = 1.355932\text{m}^2$$

$$\text{Required length } L = \frac{\text{Area}}{\text{width}} = \frac{1.355932}{40\times10^{-3}}$$

$$L = 33.898\text{m} \simeq 33.9\text{m (approx.)}$$

Example 10

(a) Two capacitors of capacitances 1μF and 0.01μF are charged to the same potential. Which will give more intense electric shock if touched?

(b) Two parallel plate air capacitors have their plate area 100 and 500cm² respectively. If they potential and the distance between the plates of the first capacitor is 0.5mm, what is the distance between the plates of the second capacitor?

Solution

(a) q = CV. Since V is a constant q α C. It means that capacitor having large capacitance will store more charge. Hence, when 1μF capacitor is touched, the discharging current will be high and you will get more intense electric shock than in case of 0.01μF capacitor.

(b) Let A_1 and d_1 be the area of plates and distance between the plates of the first capacitor, and A_2 and d_2 be corresponding value in the case of the second capacitor. If C_1 and C_2 are the capacitances of the two capacitors, then.

$$C_1 = \frac{A_1\varepsilon_0}{d_1} \;,\; C_2 = \frac{A_2\varepsilon_0}{d_2}$$

$$\text{Since } C = \frac{q}{V} = \text{constant}$$

$$C_1 = C_2$$

$$\frac{A_1\varepsilon_0}{d_1} = \frac{A_2\varepsilon_0}{d_2}$$

$$d_2 = \frac{d_1A_2}{A_1} = \frac{500\times0.05}{100}$$

$$d_2 = 0.25\text{cm}$$

Example 11

A parallel pate capacitor has plate area of 25cm² and a separation of 2.0mm between its plates. The capacitor is connected to 12V battery.

(i) Find the charge on the capacitor

(ii) If the plate separation is decreased by 1.0mm, what extra charge is given by the battery to the positive plate?

Solution

$$A = 25\text{cm}^2 = 25\times10^{-4}\text{m}^2, d = 2\text{mm} = 2\times10^{-3}\text{m}$$

$$V = 12\text{volt}$$

$$C = \frac{A\varepsilon_0}{d} = \frac{8.85\times10^{-12}\times25\times10^{-4}}{2\times10^{-3}}$$

$$C = 1.1\times10^{-11}\text{F}$$

(i) The charge on the capacitor

$$Q = CV$$

$$= 1.1\times10^{-11}\times12$$

$$Q = 1.32\times10^{-10}\text{C}$$

(ii) $d' = (2-1)mm = 1mm = 1\times10^{-3}m$

$$C' = \frac{A\varepsilon_0}{d'} = \frac{8.85\times10^{-12}\times25\times10^{-4}}{1\times10^{-3}}$$

$$C' = 2.2\times10^{-11}F$$

$$Q' = C'V = 2.2\times10^{-11}\times12$$

$$Q' = 2.64\times10^{-10}C$$

Extra charge given by the battery to the positive plates.

$$\Delta q = Q'-Q$$

$$= (2.64-1.32)\times10^{-10}C$$

$$\Delta q = 1.32\times10^{-10}C$$

Example 12

A parallel plate capacitor has plate area of $25cm^2$ and separation of 2.0mm between its plates. The capacitor is connected 12V battery.

(i) Find the charge on the capacitor.

(ii) If the plate separation is decreased by 1.0mm, what extra charge is given by the battery to the opposite plate?

Example 12

A spherical capacitor has an inner sphere of radius 9cm and an outer sphere of radius 10cm. the outer sphere is Earthed and the inner sphere is charged. What is the capacitance of the capacitor?

Solution

$$C = \frac{4\pi\varepsilon_0 ab}{b-a} = \frac{ab}{4\pi\varepsilon_o(b-a)}$$

$$= \frac{0.09\times0.1}{9\times10^9(0.1-0.09)}$$

$$C = 0.1\times10^{-9}F = 0.1nF$$

Example 13

The thickness of air layer between the two coatings of a spherical capacitor is 2cm. the capacitor has the same capacitance as the sphere of 1.2m diameter. Find the radii of its surface

Solution

$$C = \frac{4\pi\varepsilon_0 ab}{b-a} = 4\pi\varepsilon_0 R$$

$$b-a = 2cm \text{ and } R = \frac{1.2m}{2}$$

$$R = 60cm$$

$$60 = \frac{ab}{2}$$

$$ab = 120$$

Then

$$(a+b)^2 = (b-a)^2 + 4ab$$

$$(b+a)^2 = 2^2 + 4\times120 = 484$$

$$b+a = 22 \qquad b-a = 2$$

$$2+a+a = 22 \qquad b = a+2$$

$$2a = 20$$

$$a = 10cm \ , \ b = 12cm$$

Example 14

A cylindrical capacitor has two co – axial cylinder of length 15cm and radii 1.5m and 1.4cm. The outer cylinder is Earthed and the inner cylinder is given a charge of 3.5μC. Determine the capacitance of the system and the potential of the inner cylinder. (neglect end effects i.e bending of field lies at the ends).

Solution

$$l = 15cm = 0.15m \quad Q = 3.5\mu C = 3.5\times10^{-6}C$$

$$A = 1.4cm \quad b = 1.5cm$$

$$C = \frac{2\pi\varepsilon_0 L}{\log_e\left(b/a\right)}$$

$$= \frac{2\times3.14\times8.85\times10^{-12}\times0.15}{\log_e\left(1.5/1.4\right)}$$

$$C = 1.2\times10^{-12}F$$

Let V = Electric potential

$$V = \frac{Q}{C} = \frac{3.5\times10^{-6}}{1.2\times10^{-12}}$$

$$V = 2.9\times10^4 \text{ volt}$$

Example 15

(a) Prove that if two spherical shell of a spherical capacitor have their radii approximately equal, the device approximates a parallel plate capacitor.

(b) A capacitor of capacitance C is fully charged by a 200V battery. It is then discharged through a small coil of resistance embedded in a thermally insulated block of specific heat capacity $2.5\times10^{-2}\,JKg^{-1}K^{-1}$ and mass 0.1kg. If the temperature of the block rises by 0.4K. What is the value of capacitance, C?

Solution

(a) For a spherical capacitor.

$$C = \frac{4\pi\varepsilon_\circ ab}{b-a}$$

If $b \simeq a$ then $ab \simeq a^2$

$b-a=d$.

$$C = \frac{4\pi\varepsilon_\circ a^2}{d}$$

But $4\pi a^2 =$ Surface area of sphere.

$$C = \frac{A\,\varepsilon_\circ}{d}$$

This is the capacitance of a parallel plate capacitor whose plate separation is b – a = d.

(b) Energy stored in a capacitor

$$W = \frac{1}{2}CV^2$$

$$W = \frac{1}{2}C\times(200)^2$$

$$W = 2C\times10^4\,J\ldots\ldots\ldots(i)$$

Energy that flows through the resistance of coil=Heat gained by the block.

$$W = MC\Delta\theta$$

$$= 0.1\times2.5\times10^2\times0.4$$

$$W = 10J\ldots\ldots\ldots\ldots\ldots(ii)$$

Applying the law of conservation of energy

(i)=(ii)

$$2C\times10^4 = 10$$

$$C = \frac{1}{2000}F = 500\mu F$$

$$C = 500\mu F.$$

Example 16

Assuming an expression for the potential of an isolated conductor, show that the capacitance of such a sphere will be increased by a factor n if it is enclosed within an Earthed concentric sphere, the ratio of radii of the sphere being $\frac{n-1}{n}$

Solution

The capacitance of an isolated conducting sphere of radius, a is $C = 4\pi\varepsilon_0 a$ when surrounded by an Earthed sphere of radius b, its capacitance becomes.

$$C_1 = \frac{4\pi\varepsilon_0 ab}{b-a}$$

$$\frac{C_1}{C} = \frac{\frac{4\pi\varepsilon_0 ab}{b-a}}{4\pi\varepsilon_0 a}$$

$$= \frac{b}{b-a} = \frac{1}{1-a/b}$$

$$= \frac{1}{1-\frac{(n-1)}{n}}$$

$$\frac{C_1}{C} = n$$

Example 17

Three capacitors of capacitance 3μF, 5μF and 7μF are connected in (i) series (ii) parallel connection. Calculate the effective capacitance in each case.

Solution

$C_1 = 3\mu F$, $C_2 = 5\mu F$, $C_3 = 7\mu F$

(i) The effective capacitance when connected in series.

$$\frac{1}{C_S} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

$$= \frac{1}{3}+\frac{1}{5}+\frac{1}{7}$$

$$\frac{1}{C_S} = 0.676$$

(ii) The effective capacitance when connected in parallel.

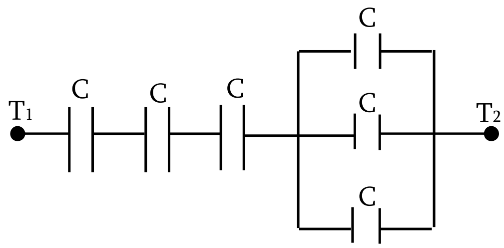
$$C_P = C_1+C_2+C_3$$

$$= 3+5+7$$

$$C_P = 15\mu F$$

Example 18

Three capacitors each of 0.003μF capacitance are connected together in series and also connected in series with three other similar capacitors which are grouped together in parallel. (see the figure below). Calculate the total capacitance.



Solution

The effective capacitance of three capacitors in series.

$$\frac{1}{C_S} = \frac{1}{C}+\frac{1}{C}+\frac{1}{C} = \frac{3}{C}$$

$$C_2 = \frac{C}{3} = \frac{0.003\mu F}{3}$$

$$C_S = 0.001\mu F$$

The effective capacitance of three capacitors in parallel.

$$C_P = C+C+C = 3C$$

$$C_P = 3\times 0.003 = 0.009\mu F$$

The total effective capacitance of CS and CP in series connection.

$$C_T = \frac{C_S C_P}{C_S+C_P} = \frac{0.001\times 0.009}{0.001+0.009}$$

$$C_T = 9.0\times 10^{-4}\mu F$$

Example 19

(a) The effective capacitance of two capacitors C_1 and C_2 when connected in series is 4.55μF and 20μF when connected in parallel. Calculate their individual capacitance.

(b) Three capacitors of capacitance 1μF, 2μF and 3μF are connected in series and a p.d of 110V is applied to the combination. Calculate the potential difference across each capacitors.

Solution

(a) When the capacitors are connected in parallel.

$$C_P = C_1+C_2 = 20\mu F$$

$$C_1+C_2 = 20 \ldots\ldots\ldots(i)$$

When the capacitor are connected in series connected.

$$C_S = \frac{C_1C_2}{C_1+C_2} = 4.55\mu F$$

$$\frac{C_1C_2}{C_1+C_2} = 4.55$$

$$C_1C_2 = 4.55(C_1+C_2)$$

$$= 4.55\times 20$$

$$C_1C_2 = 91 \ldots\ldots\ldots(ii)$$

Takes

$$(C_1-C_2)^2 = (C_1+C_2)^2-4C_1C_2$$

$$= 20^2-4\times 91$$

$$(C_1-C_2)^2 = 36$$

$$C_1-C_2 = 6 \ldots\ldots\ldots(iii)$$

Solving simultaneously equation (i) and (iii)

(b) Total effectively capacitance in series.

$$\frac{1}{C} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3} = \frac{1}{1}+\frac{1}{2}+\frac{1}{3}$$

$$C = \frac{6}{11}\mu F$$

Since V = 110 Volt

Let Q = Charges stored on each capacitor

$$Q = CV = 110\times\frac{6}{11}$$

$$Q = 60\times 10^{-6}C$$

Let the potential difference across C_1, C_2 and C_2 be V_1, V_2 and V_3 respectively.

$$V_1 = \frac{Q}{C_1} = \frac{60\times 10^{-6}}{1.0\times 10^{-6}}$$

$$V_1 = 60\text{Volt}$$

$$V_2 = \frac{Q}{2} = \frac{60\times 10^{-6}}{2\times 10^{-6}}$$

$$V_2 = 30\text{volt}$$

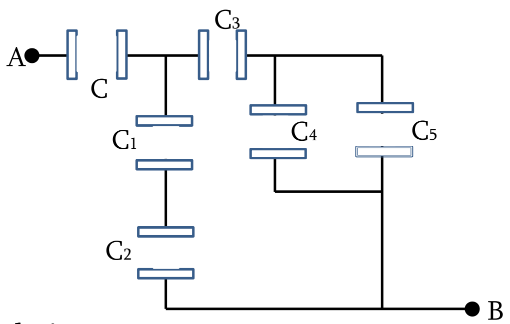
$$V_3 = \frac{Q}{C_3} = \frac{60\times 10^{-6}}{3\times 10^{-6}}$$

$$V_3 = 20\text{volt}$$

Example 20

From the given figure below, find the value of the capacitance, C if the equivalent capacitance between A and B is to be 1μF. All other capacitances are in microfarads.

$C_1 = 8\mu F$, $C_2 = 4\mu F$, $C_3 = 1\mu F$, $C_4 = 4\mu F$, $C_5 = 4\mu F$



Solution

The effective capacitance of C_4 and C_5 are in parallel

$$C_{P1} = C_4 + C_5 = 4+4 = 8\mu F$$

The effective capacitance of C_3 and C_{P1} are in series.

$$C_{S1} = \frac{C_{P1}+C_3}{C_{P1}+C_3} = \frac{1\times 8}{1+8} = \frac{8}{9}\mu F$$

The effective capacitance of C_1 and C_2 in series.

$$C_{S2} = \frac{C_1C_2}{C_1+C_2} = \frac{8\times 4}{8+4} = \frac{8}{3}\mu F$$

Now C_{S1} and C_{S2} are in parallel

$$C_{P2} = C_{S1} + C_{S2} = \frac{8}{9}+\frac{8}{3}$$

$$C_{P2} = \frac{32}{9}\mu F$$

C_{P2} and C are in series which gives 1μF

$$1 = \frac{C_{P2}C}{C_{P2}+C} = \frac{{}^{32}\!/_{9}\,C}{{}^{32}\!/_{9}+C}$$

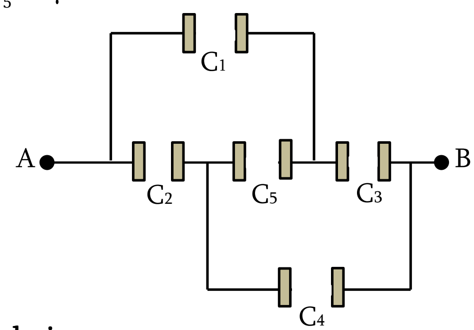
$$\frac{32}{9}+C = \frac{32}{9}C$$

$$C = \frac{32}{23}\mu F = 1.39\mu F$$

$$C = 1.39\mu F$$

Example 21

Calculate the effective capacitance between the terminals A and B show in figure below, if $C_1 = 2\mu F, C_2 = 2\mu F, C_3 = 4\mu F, C_4 = 4\mu F$ and $C_5 = 5\mu F$

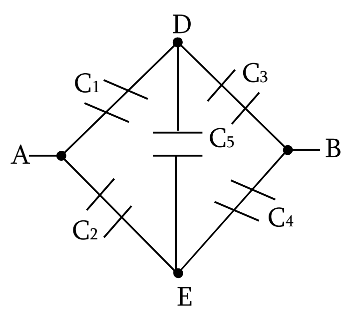


Solution

The network shown in the figure

(i) Above can redrawn as shown in the figure (ii) this similar to a Wheatstone bridge. Hence the bridge is balanced. Under this condition no charge flow through C_5, become the points D and E are at the same potential.

(i)



C_1 and C_3 are in series

$$C_{S1} = \frac{C_1C_3}{C_1+C_3} = \frac{2\times 4}{2+4} = \frac{8}{6}\mu F$$

Again C_2 and C_4 are in series

Then C_{S1} and C_{S2} in parallel

$$C_{AB} = C_{S1} + C_{S2} = \frac{8}{6} + {}^{8}\!/_{6}$$

$$C_{AB} = 2.67\mu F$$

$$C_{AB} = 2.67\mu F$$

Example 22 NECTA 1991/P2/7(a)

What work is done in taking an electron round a closed circular path located at a distance $3.0\times10^{-10}m$ from an electric charge of $3.2\times10^{-19}C$

Solution

The work done is zero, since the closed circular path is an equipotential surface

Example 23

A capacitor of a capacitance 25μF is charged to 500V. Determine the energy stored in the capacitor.

Solution

Energy stored

$$W = \frac{1}{2}CV^2$$

$$W = \frac{1}{2}\times25\times10^{-6}\times(500)^2$$

$$W = 3.125J$$

Example 24

For flash pictures, a photographer uses a capacitor of 30μF and a charger that supplies 3×10^3V. Find charge and energy expanded in Joule for each flash

Ans. $Q = 9\times10^{-2}C$, $W = 135J$

Example 25

The capacitance of a variable radio capacitor can be changed from 50Pf by turning the dial from 0° to 180°, the capacitor is connected to 400V battery.

After charging the capacitor is disconnected from the battery and the dial is turned at 0°

(i) What is the potential difference across the capacitor when the dial read 0°?

(ii) How much work is required to turn the dial if friction is neglected?

Solution

(i) With dial at 0°, the capacitance of the capacitor, $C_1 = 50pF = 50\times10^{-12}F$

With dial at 180° the capacitance of the capacitor, $C_2 = 950pF = 950\times10^{-12}F$.

P.d across $C_2, V_2 = 400V$

Charge on C_2, $Q = C_2V_2 = 950\times10^{-12}\times400$

$$Q = 380\times10^{-9}C.$$

When the battery is disconnected, the charge Q remain the same. Suppose V_1 is the p.d across the capacitor when the dial reads 0°

$$Q = C_1V_1$$

$$V_1 = \frac{Q}{C_1} = \frac{380\times10^{-9}}{50\times10^{-12}}$$

$$V_1 = 7600V.$$

(ii) Work required = Gain in energy of the capacitor.

$$W = \frac{1}{2}C_1V_1^2 - \frac{1}{2}C_2V_2^2$$

$$= \frac{1}{2}\times50\times10^{-12}\times(7600)^2 - \frac{1}{2}\times950\times10^{-12}\times400^2$$

$$W = 1.37\times10^{-3}J$$

Example 26

(a) The two plates of a parallel plate air capacitor are maintained at a p.d of 150V by being connected to a battery. If each plate has an area of 880cm², what energy is needed to increase the distance between them from 5.0cm to 7.0cm?

(b) An air capacitor has plate of area 132cm² while which are 0.50cm apart. What is the force of attraction between if the p.d between them is 600V?

Solution

(a) Energy stored on capacitor

$$W = \frac{1}{2}CV^2 \text{ but } C = \frac{A\varepsilon_o}{d}$$

$$W = \frac{1}{2}\frac{A\varepsilon_oV^2}{d}$$

$$W = \frac{A\varepsilon_oV^2}{2d_1}, \quad W = \frac{A\varepsilon_oV^2}{2d_2}$$

Now

$$\Delta W = \frac{\varepsilon_oAV^2}{2}\left[\frac{1}{d} - \frac{1}{d_2}\right]$$

$$\Delta W = \frac{\varepsilon_o AV^2}{2}\left[\frac{d_2-d_1}{d_1d_2}\right]$$

$$= \frac{8.85\times10^{-12}\times880\times10^{-4}\times(150)^2}{2}$$

$$\left[\frac{(7-5)\times10^{-2}}{7\times10^{-2}\times5\times10^{-2}}\right]$$

$$\Delta W = 5.0\times10^{-8}\,J$$

(b) Force between two parallel plates

$$F = \frac{Q^2}{2A\varepsilon_0} \quad \text{but } Q = CV$$

$$= \frac{C^2V^2}{2A\varepsilon_0}, \quad C = \frac{A\varepsilon_0}{d}$$

$$= \frac{V^2}{2A\varepsilon_o}\left(\frac{A\varepsilon_o}{d}\right)^2 = \frac{1}{2}A\varepsilon_o\left(\frac{V}{d}\right)^2$$

$$F = \frac{1}{2}\times132\times10^{-4}\times8.85\times10^{-12}\left(\frac{600}{0.5\times10^{-2}}\right)^2$$

$$F = 8.4\times10^{-4}\,N$$

Example 27

(a) A 5μF capacitor is fully charged across a 12V battery and connected to uncharged capacitor. The voltage across it is found to be 3volt. What is the capacity of the uncharged capacitor?

(b) Two spherical conductors of capacitance 3.0μF and 5.0μF are charged to potential of 300V and 500V respectively. They are made to touch each other. Calculate the common potential, the charge on each conductors and decreases in electrical energy.

Solution

(a) The common potential V after connection is

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

C_1 = 5μF, V = 300volt, V_1 = 12V, V_2 = 0, C_2 = ?

$$3 = \frac{5\times12 + C_2\times0}{5+2}$$

$$3 = \frac{60}{5+C_2}$$

$$C_2 = 15\mu F$$

(b) $C_1 = 3\times10^{-6}F$, $C_2 = 5\times10^{-6}F$

$V_1 = 300V$, $V_2 = 500V$

Total charges before shairing

$$Q_0 = C_1V_1 + C_2V_2$$

Total charges after shairing

$$Q = (C_1 + C_2)V$$

$$Q = Q_0$$

$$(C_1 + C_2)V = C_1V_1 + C_2V_2$$

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$$= \frac{3\times10^{-6}\times300 + 5\times10^{-6}\times500}{3\times10^{-6} + 5\times10^{-6}}$$

$$V = 425\text{Volt}$$

∴ Common potential V = 425volt

- Let the charges on the sphere be Q_1 and Q_2 after shairing.

  $Q_1 = C_1V = 3\times10^{-6}\times425 = 1275\times10^{-6}C$

  $Q_2 = C_2V = 5\times10^{-6}\times425 = 2175\times10^{-6}C$

  $Q_1 = 1275\mu c$, $Q_2 = 2175\mu C$

- Decrease in energy

$$\Delta W = \frac{C_1C_2}{2(C_1+C_2)}(V_1 - V_2)^2$$

$$= \frac{3\times10^{-6}\times5\times10^{-6}}{2(3+5)\times10^{-6}}[300-500]^2$$

$$\Delta W = 37.5\times10^{-6}\,J$$

Example 28

Two capacitors C_1 and C_2 each of area 36cm² separated by 4cm have capacitances of 6μC and 8μC respectively. The capacitor C1 is charged to a potential difference of 110V whereas the capacitor C_2 is charged to a potential difference of 140V. The capacitors are now joined with plates of like charges connected together.

(i) What will be the loss of energy transferred to heat in the connecting wires? (08 marks)

(ii) What will be the loss of energy per unit volume transferred to heat in the connecting wires? (03 marks)

Solution

$A = 36cm^2$, $d = 4cm$, $C_1 = 6\mu C$, $V_1 = 110V$, $C_2 = 8\mu C$, $V_2 = 140V$

(i) Δw = loss of energy transferred to heat

V = Common potential

Apply the law of conservation of electric charges.

$$(C_1 + C_2)V = C_1V_1 + C_2V_2$$

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$$= \frac{6\times10^{-6}\times110 + 8\times10^{-6}\times140}{(6+8)\times10^{-6}}$$

$$V = 127.143\text{Volt}$$

Initial total energy on the capacitors

$$W_1 = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

$$= \frac{1}{2}[6+8]\times10^{-6}\times(127.143)^2$$

$$W_2 = 0.1132J$$

Final total energy on the capacitors

$$W_2 = \frac{1}{2}(C_1 + C_2)V^2$$

$$= \frac{1}{2}[6+8]\times10^{-6}\times(127.143)^2$$

$$W_2 = 0.1132J$$

Now, $\Delta w = w_1 - w_2$

$$= 0.1147 - 0.1132$$

$$\Delta w = 1.5\times10^{-3}J$$

(ii) Volume, $V = Ad$

$$V = 36\times10^4\times4\times10^{-2}$$

$$V = 1.44\times10^{-4}m^3$$

Let ΔU = loss of energy per unit volume

$$\Delta u = \frac{\Delta w}{v} = \frac{1.5\times10^{-3}}{1.44\times10^{-4}}$$

$$\Delta u = 10.42Jm^{-3}$$

Example 29

The radii of two charged metallic sphere are 5cm and 10cm. both have a charge of 75μC separated. Both the sphere are connected with a conducting wire. Calculate:

(i) the quantity of charge transferred through the wire

(ii) The common potential of the sphere after connecting wire.

Solution

(i) Given that

$r_1 = 5cm = 0.05m$, $r_2 = 10cm = 0.1m$

Since $C = 4\pi\varepsilon_0 r$

$$C_1 = \frac{0.05}{9\times10^9} = 5.56\times10^{-12}F$$

$$C_2 = 4\pi\varepsilon_0 r_2 = \frac{0.10}{9\times10^9}$$

$$C_2 = 11.11\times10^{-12}F$$

$$Q_1 = Q_2 = 75\times10^{-6}C$$

Common potential

$$V = \frac{Q_1 + Q_2}{C_1 + C_2}$$

$$= \frac{2\times75\times10^{-6}}{(5.56\times11.11)\times10^{-12}}$$

$$V = 9\times10^6\text{Volt}$$

Charge on the sphere of capacitor C1 after sharing.

$$Q_1 = C_1V = 5.56\times10^{-12}\times9\times10^6$$

$$Q_1' = 50\times10^{-6}C$$

Charge transferred

$$\Delta Q = Q_1 - Q_1' = Q_2 - Q_1'$$

$$= (75-50)\times10^{-6}$$

$$\Delta Q = 25\times10^{-6}C = 25\mu C$$

(ii) Common potential, $V = 9.0\times10^6\text{Volt}$

Example 30

Eight million small drops each of radius 1mm, coalesce to form a bigger drop. If the charge on each drop is 10^{-10} C, find the potential of the bigger drop?

Solution

Radius of small drop r = 1mm = 10^{-3}m

Radius of the bigger drop, R = ?

Apply the law of conservation of volume

$$\frac{4}{3}\pi R^3 = 8\times10^6\times\frac{4}{3}\pi r^3$$

$$R^3 = 8\times10^6 r^3$$

$$R = \left[8\times10^6 r^3\right]^{1/3}$$

$$R = \left[8\times10^6\times\left(1\times10^{-3}\right)\right]^{1/3}$$

$$R = 0.2\text{m}$$

Total charge on the bigger drop

$$Q = 8\times10^6\times10^{-10}\text{c}$$

$$Q = 8\times10^{-4}\text{C}$$

Potential of the bigger drop

$$V = \frac{Q}{4\pi\varepsilon_o R} = \frac{9\times10^9\times8\times10^{-4}}{0.2}$$

$$V = 3.6\times10^7\,\text{Volt}$$

$$W_2 = \frac{1}{2}CV^2 = \frac{1}{2}\left(C_1+C_2\right)V^2$$

$$= \frac{1}{2}\left(4+2\right)\times10^{-6}\times\left(\frac{400}{3}\right)^2$$

$$W_2 = 5.333\times10^{-2}\,\text{J}$$

Let ΔW = loss of energy due to the shairing

$$\Delta W = W_1 - W_2$$

$$= 8\times10^{-2} - 5.333\times10^{-2}$$

$$\Delta W = 2.667\times10^{-2}\,\text{J}$$

This energy appears in the form of heat and e.m.w radiation.

Example 31

A 4μF capacitor is charged by 200V supply. It is then disconnected from the supply and is connected to another 2μF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation.

Solution

$$C_1 = 4\mu F = 4\times10^{-6}\,F \;,\; V_1 = 200V$$

$$C_2 = 2\mu F = 2\times10^{-6}\,F$$

Energy of the capacitor before shairing

$$W_1 = \frac{1}{2}C_1V_1^2$$

$$= \frac{1}{2}\times4\times10^{-6}\times200^2$$

$$W_1 = 8.0\times10^{-2}\,\text{J}$$

When C_1 is connected to C_2 = 2μF.

Let V be the common potential attained.

$V_2 = 0$ as it is uncharged.

$$V = \frac{C_1V_1+C_2V_2}{C_1+C_2}$$

$$V = \frac{4\times10^{-6}\times200}{\left(4+2\right)\times10^{-6}}$$

$$V = \frac{400}{3}\text{Volt}$$

After energy shairing

Example 32

The plates of a parallel plate capacitor have an area of 90cm² each and are separated by 2.5mm. The capacitor is charged by connecting it to 400V supply.

(a) How much electrostatic energy is stored by the capacitor?

(b) View this energy as stored in the electrostatic field between the plate, obtain the energy per unit volume, U.

Solution

(a) $W = \frac{1}{2}CV^2$ but $C = \frac{A\varepsilon_o}{d}$

$$= \frac{1}{2}\frac{A\varepsilon_0 V^2}{d}$$

$$= \frac{8.85\times10^{-12}\times90\times10^{-4}\times\left(400\right)^2}{2\times2.5\times10^{-3}}$$

$$W = 2.548\times10^{-6}\,\text{J}$$

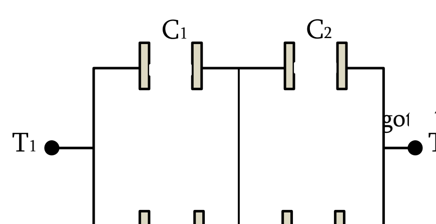
(b) Energy density

$$U = \frac{w}{Ad} = \frac{2.548\times10^{-6}}{90\times10^{-4}\times2.5\times10^{-3}}$$

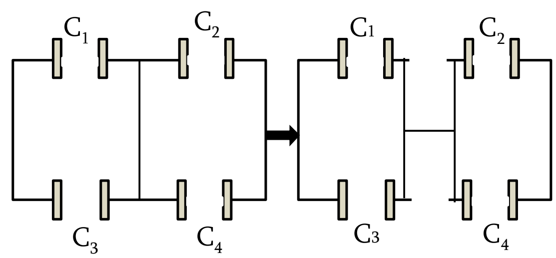
$$U = 0.113\,\text{Jm}^{-3}$$

Example 33

Calculate the equivalent capacitance in below arrangement, assume $C_1 = 1\mu F, C_2 = 3\mu F, C_3 = 2\mu F$



Solution



C_1 and C_3 are in parallel

$C_A = C_1 + C_3 = 1 + 2 = 3\mu F$

$C_B = C_2 + C_4 = 3 + 4 = 7\mu F$

Let C = total capacitance of the circuit above.

$$\frac{1}{C} = \frac{1}{C_A} + \frac{1}{C_B} = \frac{1}{7} + \frac{1}{3}$$

$$C = \frac{7\times3}{7+3}$$

$C = 2.7\mu F$

Example 34NECTA 1989/P1/3

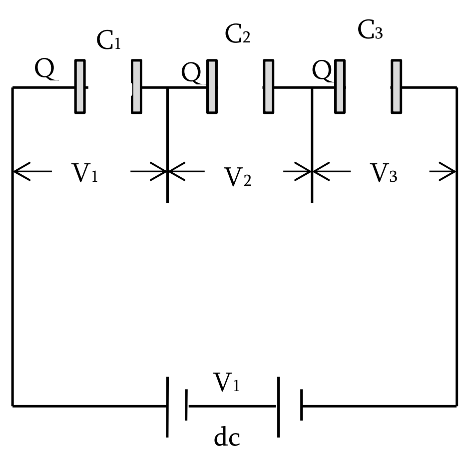
(a) Derive an expression for the equivalent capacitance of their capacitors connected in series.

(b) Three capacitors $2\mu F, 3\mu F$ and $6\mu F$ respectively are connected in series to a 500Vd.c supply. Calculate:-

(i) The charge of each capacitor.

(ii) The potential difference across each capacitor.

Solution



For capacitor in series, charges stored are the same, them

(a) $V = V_1 + V_2 + V_3$

$$= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}.$$

$$V = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Or

$$\frac{1}{C_S} = \frac{C_2C_3 + C_1C_3 + C_1C_2}{C_1C_2C_3}$$

$$C_S = \frac{C_1C_2C_3}{C_2C_3 + C_1C_3 + C_1C_2}$$

(b) $C_1 = 2\mu F,\ C_2 = 3\mu F,\ C_3 = 6\mu F$

$V = 500V$

$$C_S = \frac{2\times3\times6}{3\times6 + 2\times6 + 2\times3}$$

$C_S = 1\mu F$

(i) Charges stored on each capacitor

$Q = CV = 1\times10^{-6}\times500$

$Q = 500\times10^{-6}C = 500\mu C.$

(ii) $V_1 = \frac{Q}{C_1} = \frac{500\times10^{-6}}{2\times10^{-6}}$

$V_1 = 250\text{Volts}.$

$V_2 = \frac{Q}{C_2} = \frac{500\times10^{-6}}{3\times10^{-6}}$

$V_2 = 166.67\text{Volt}$

$V_3 = \frac{Q}{C_3} = \frac{500\times10^{-6}}{6\times10^{-6}}$

$V_3 = 83.33\text{Volt}.$

Example 35

A cylindrical capacitor consists of two coaxial cylinder of radii 4mm and 9mm. the space between the cylinder is filled with a medium of dielectric constant 2.46. Calculate the capacitance per metre length of the capacitor?

Solution

$$\text{Since } C = \frac{2\pi\varepsilon_0\varepsilon_r L}{\log_e\left(b/a\right)}$$

$$= \frac{2\times3.14\times8.85\times10^{-12}\times2.46\times1}{\log_e\left(9/4\right)}$$

$$C = 168.65\times10^{-12}F = 168.65pF$$

Example 36

A spherical capacitor consists of an inner sphere of diameter 6cm and outer sphere of diameter 10cm. the space between the two concentric sphere is filled with a medium of dielectric constant 80. Find the capacitance of the capacitor?

Solution

$$a = 3\times10^{-2}m\ ,\ b = 5\times10^{-2}m.\varepsilon_r = 80$$

$$C = \frac{4\pi\varepsilon_0\varepsilon_r ab}{b-a}$$

$$C = \frac{4\times3.14\times8.85\times10^{-12}\times80\times3\times10^{-2}\times5\times10^{-2}}{(5-2)\times10^{-2}}$$

$$C = 6.67\times10^{-10}F = 667pF$$

Example 37

Show that the force on each plate of a parallel plate capacitor has a magnitude equal to $\frac{1}{2}QE$, where Q is the charge on the capacitor and E is the magnitude of the electric field between the plates.

Solution

Let F be the magnitude of the force between the plates. Work done to increase the plate separation by Δd is given by

$$\Delta W = F\Delta d$$

$$\text{But } \Delta W = \frac{Q^2\Delta d}{2\varepsilon_o A}$$

$$F\Delta d = \frac{Q^2\Delta d}{2\varepsilon_o A}$$

$$F = \frac{Q}{2\varepsilon_0 A} \text{ but } E = \frac{Q}{\varepsilon_0 A}$$

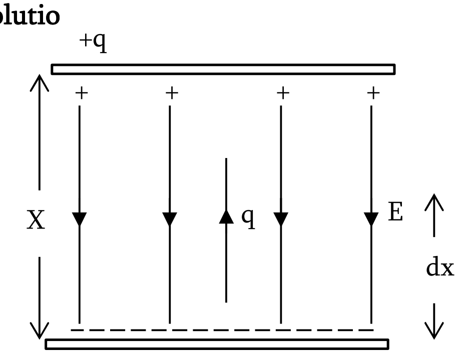
$$= \frac{Q}{2\varepsilon_0 A}[Q]$$

$$F = \frac{1}{2}QE$$

Example 38: NECTA 1998/P1/10(b)

Two opposite charged plates of a capacitor are separated by a distance X. Find an expression for the force of attraction between the plates in terms of A, εo and q i.e the area of the plates, the permittivity of free space, and charge q on the plates respectively.

Solutio



Then, the corresponding change in work done to the small displacement lower plate.

$$dw = Fdx$$

$$dw = -Fdx$$

Negative sing shows that the force, F is the force of attraction.

$$\text{Since } C = \frac{A\varepsilon_0}{X}$$

$$U = \frac{q^2}{2C} = \frac{q^2 X}{2\varepsilon_0 A}$$

$$dU = \frac{q^2 dx}{2\varepsilon_o A}$$

$$\text{But } dw = du$$

$$-Fdx = \frac{q^2 dx}{2\varepsilon_o A}$$

$$F = \frac{-q^2}{2\varepsilon_o A}$$

Example 39: NECTA 1985/P1/11

(a) Write down the formula for:-
   (i) The capacitance of parallel plate capacitor.
   (ii) The effective capacitance for three capacitors in series.

(b) A large parallel plate capacitors of capacitance 5.5μF consists of several plates each of side 30cm by 15cm. if the space between the plates are filled with a material of relative permittivity of 6 and thickness 10^{-2}mm. calculate the number of plates involved.

Solution

(a) (i) $C = \frac{A\varepsilon_0}{d}$

A = area of one of plate
d = distance between the plates
ε_o = permittivity of free space

(ii) $\frac{1}{C} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$ or

$$C = \frac{C_1C_2C_3}{C_2C_3+C_1C_3+C_1C_2}$$

(iii) $C = (n-1)\frac{\varepsilon_o\varepsilon_r A}{d}$

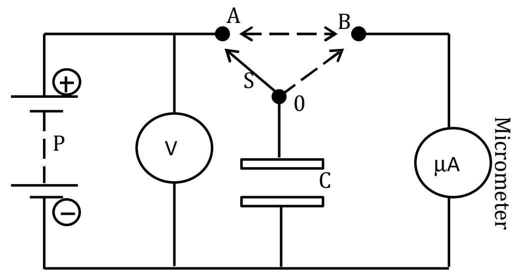
$$n = \frac{Cd}{\varepsilon_o\varepsilon_r A}+1$$

$$= \frac{5.5\times10^{-6}\times10^{-2}\times10^{-3}+1}{3\times10^{-2}\times15\times10^{-2}\times6\times8.85\times10^{-12}}$$

n = 24 plates.

MEASUREMENT OF A CAPACITANCE

The vibrating read switch which consists of a electron magnet can be used. This can be illustrated as shown in the figure below.



The circuit above is used for
(i) Measurement of a capacitance of capacitor.
(ii) Measurement of a permittivity of free space (air) (εo)
(iii) Measurement of relative permittivity (εr).

(i) **Measurement of a capacitance of a capacitor and permittivity of free space (ε_o).**

The circuit above shows the arrangement of apparatus used for measurement of C and ε_o. Let f be the frequency of vibrating read switch. If the switch S is connected at point A, the capacitor gets full charged and discharged when the switch S is the point B. The average current due to the discharged of the capacitor can be registered on the micrometer (μA).

Mathematically

Average current

$$I = \frac{Q}{t} = FQ$$

But, $Q = CV$

$$I = FCV = \eta CV$$

$$C = \frac{I}{FV} = \frac{I}{\eta V}$$

Where ;
I = average current
V = applied voltage from the source
η = f = frequency

For the parallel plates capacitor filled with air (vacuum) between the plates.

$$C_0 = \frac{\varepsilon_0 A}{d}$$

$$\frac{\varepsilon_o A}{d} = \frac{I}{FV}$$

$$\varepsilon_0 = \frac{Id}{fVA} = \frac{I}{\eta VA}$$

(ii) **Measurement of the relative permittivity of given medium, εr.**

Since, $C = \frac{A\varepsilon}{d} = \frac{A\varepsilon_r\varepsilon_0}{d}$

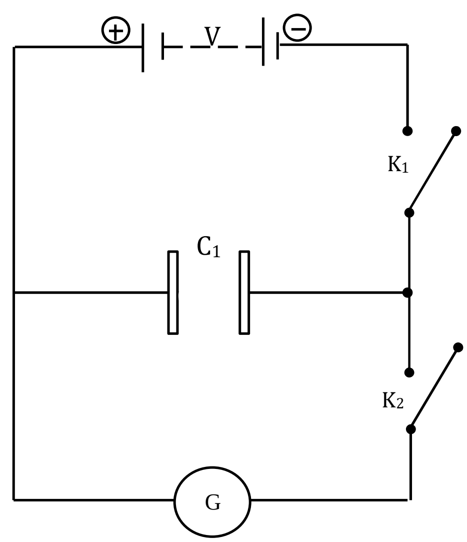
$$\frac{A\varepsilon_r\varepsilon_0}{d} = \frac{I}{FV}$$

$$\varepsilon_r = \frac{Id}{A\varepsilon_0 FV}$$

- Relative permittivity of glass and oil.
  The same method can be used to find the relative permittivity of various solid materials such as glass. If the glass completely fills the space between the two plates and the current in μA is I with the glass and Io with the air between the plates, then for the glass.

$$\varepsilon_r = \frac{C_{glass}}{C_{air}} = \frac{I}{I_0}$$

COMPARISON OF THE CAPACITANCES



Large capacitance of the order of microfarad can be compared with the aid of ballistic galvanometer. In this instrument, the first thrown or deflection is proportional to the quantity of electricity discharged through it. The capacitance C_1 is charged by battery of emf, V and then discharged through the ballistic galvanometer. The corresponding first angle of deflection θ1 is observed. The capacitor is now replaced by another capacitor to the capacitance C_2, charged again by the battery and new deflection is observed when the capacitor is discharged.

Now,

$Q \alpha \theta$

$Q = K\theta$

K = constant of proportionality for the first angle of deflection

$Q_1 = k\theta_1$..............(i)

For the second angle of deflection

$Q_2 = K\theta_2$..............(ii)

Dividing equation (i) by (ii)

$$\frac{Q_1}{Q_2} = \frac{K\theta_1}{K\theta_2} = \frac{\theta_1}{\theta_2}$$

$$\frac{C_1V}{C_2V} = \frac{\theta_1}{\theta_2}$$

$$\frac{C_1}{C_2} = \frac{\theta_1}{\theta_2}$$

CHARGING AND DISCHARGING OF A CAPACITOR THROUGH A RESISTOR, R

A: CHARGING A CAPACITOR THROUGH A RESISTOR