MODULE 8 : FLUID

FLUIDS MECHANICS
SUB TOPICS
1. FLUID DYNAMICS
2. VICOSITY

1. **FLUID DYNAMICS**
   A FLUID is the substance that can flow and may be either liquids or gases.

   LIQUID is the substance which don't have rigidity but have surface examples of liquids water, sea water, mercury, alcohol, etc.

   GAS is the substance which has neither rigidity nor surface examples of gases: air, oxygen, hydrogen gas etc.
   Note that:
   There are marked differences between liquids and gases:
   - Gases are easily compressed whereas liquids are nearly incompressible.
   - Liquid has definite size but gas expands to fill any closed vessel containing it.

   FLUID DYNAMICS is the study of fluids in motion.

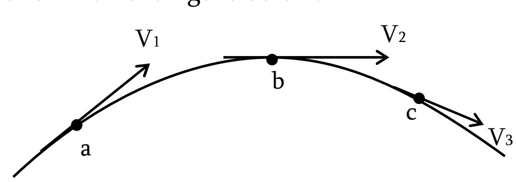
   HYDRODYNAMICS is the branch of physics which deals with the study of properties of fluids in motion.

   AERODYNAMICS is the branch of physics which deals with study of the interaction between the air and the solid bodies moving through it.

TERMINOLOGIES USED IN FLUIDS MECHANICS.

1. **STEADY AND UNSTEADY FLUID FLOW**
   - Steady fluid flow is the fluid flow if velocity at each point of space remain constant with time.
   - Unsteady fluid flow is the fluid flow whose velocity at each point of space varies with time.
2. **COMPRESSIBLE AND INCOMPRESSIBLE FLUID FLOW**
   - Compressible fluid flow is the fluid flow in which density varies with time.
   - Incompressible fluid flow is the fluid flow in which density remain constant with time.
3. **ROTATIONAL AND IRROTATIONAL FLUID FLOW.**
   - Rotational fluid flow is the fluid flow which produces net angular velocity.
   - Irrotational fluid flow is the fluid flow which does not produce net angular velocity.
4. **VISCOUS AND NON – VISCOUS FLUID FLOW**
   - Viscous fluid flow is the fluid flow which possessing internal friction force.
   - Non – viscous fluid flow is the fluid flow which does not possessing internal friction force.
5. **IDEAL FLUID** is the fluid flow which is incompressible and non – viscous.
6. **LINE OF FLOW** is the path taken followed by the particles of fluid flow.

   TYPES OF FLUID FLOW
   The fluid flow is of two types viz:
   (i) Streamline flow or steady flow
   (ii) Turbulent flow
7. **STREAMLINE FLOW OR STEADY FLOW**
   Is the fluid flow in which fluid particles that pass any given point follow the same path at the same speed. i.e is the fluid flow which is steady with respect to time.
   Consider the liquid flowing through the pipe as shown on the figure below.



Let V_1, V_2 and V_3 be the velocities of the liquid at points a, b and c respectively. If the liquid continuous to flow at a time, velocity of whatever liquid particles happen to be at a still be V_1, that at b still V_2 and at C still be V_3. The fluid flow is said to be streamline or steady fluid flow. The fixed path followed by an orderly procession of particles in the steady flow of a liquid is called '**streamline'**. **A streamline** may be defined as a path, straight or curved such that tangent to it any point indicates the direction of flow of the fluid at that point.

PROPERTIES OF STREAMLINE

(i) The tangent at any point of the streamline gives direction of velocity of the liquid at that point/

(ii) Two streamline cannot intersect

(iii) At a particular points of the streamline, the velocity of fluid is constant. However at different points of streamline, the velocity may be different.

(iv) Crowding of streamlines represent a faster flow of the liquid.

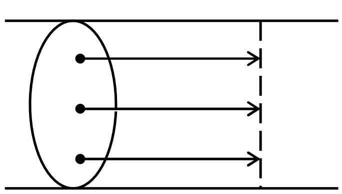
LAMINAR FLOW AND VELOCITY PROFILE

- **LAMINAR FLOW** it is a spherical case of streamline or steady flow in which the liquids flows as a series or parallel layers (laminae) and no one layer crosses another layer. The smooth streamline flow us known as **'LAMINAR FLOW**.

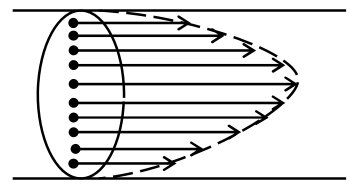
- **VELOCITY PROFILE** is the surface obtained by joining the heads of velocity vectors for the particles in section of a flowing liquid

Example

(i) Flow of non – viscous liquid



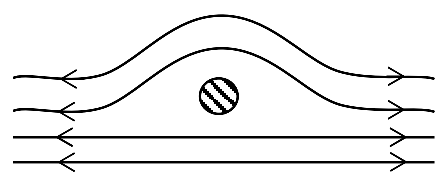
(ii) Flow of viscous liquid



- **TURBULENT FLOW** is the fluid flow which is unsteady with respect to the time. The flow of a liquid is said to be turbulent or disorderly if its velocity is greater than its critical velocity. When velocity of a liquid exceed the critical velocity, the paths and velocities of the liquid particles begin to change continuously and haphazardly. The flow losses all it orderliness and is called turbulent. In such a flow most of the energy needed to driven the liquid is dissipated in setting addies and whirl pools in it.

Example

(a) $V << V_C$



(b) $V >>> V_C$

CAUSES OF TURBULENT FLUID FLOW

(i) This is due to the too great of a speed of fluid flow

(ii) A brupt change in areas or direction

Examples of turbulent fluid flow

1. The smoke from a cigarette after rising a short distance.
2. The flow of water just behind the boat or ship
3. The air flow behind a moving bus or train
4. The sound produced by whistling and by wind instruments result from the turbulent flow of air.

DIFFERENCES BETWEEN STREAMLINE AND TURBULENT FLOW

Streamline flow	Turbulent flow
The velocity of liquid d is less than critical velocity	The velocity of liquid is greater than critical velocity.
Viscosity play a dominant role.	Density play a dominant role
The flow is regular and orderly	The flow is irregular and disorderly.

8. CRITICAL VELOCITY (V_C)

Is the velocity which possessed by the fluids when changes from streamline (laminar) flow into turbulent flow i.e the critical velocity of a liquid is that velocity of liquid up to which its flow is streamlined and above its flow is turbulent. It is the maximum possible velocity possessed by a liquid in streamline.

CRITICAL VELOCITY BY METHOD OF DIMENSIONS.

Let the critical velocity Vc of a liquid depend on.

(i) Coefficient of viscosity, η
(ii) Density ρ of the liquid
(iii) Diameter D of the pipe.

Let $V_C \alpha \eta^X \rho^y D^Z$

$$V_C = Re\,\eta^x \rho^y D^z$$

Re = dimensionless constant known as Reynolds number. x, y and z any real numbers.

Dimensionally

$$[V_C] = [n]^x [\rho]^y [D]^z$$

$$M^0LT^{-1} = \left(ML^{-1}T^{-1}\right)^X \left(ML^{-3}\right)^y L^Z$$

$$M^0LT^{-1} = M^{x+y}L^{-x-3y+z}T^{-x}$$

Equating powers of M, L and T

M: 0 = x + y ……(i)
L: 1 = -x -3y + z ……(ii)
T: -1 = -x…….(iii)

On solving equations (i), (ii) and (iii)

X = 1 , y = -1 , z = -1

$$V_C = \frac{Re\,\eta}{\rho D} = \frac{Re\,\eta}{2\rho r}$$

9. TERMINAL VELOCITY, V_T

Is the constant (maximum) velocity attained by the spherical body in the viscous fluid.

10. STAGNATION is the point where by the velocity of moving fluid is equal to zero.

11. TUBE OF FLOW AND STREAM TUBE

- Tube of flow is the group of streamline i.e is the tubular region of the flowing fluid or boundaries are defined by a jet of streamlines.
- Stream tube is the tube whose surface is made up by streamlines across which there is no transport of the fluid.

Note that

In streamline flow of a liquid, the energy needed to drive the liquid is used up only in overcoming the viscous drag between the layers. But the streamline flow is possible only so long as the liquid velocity does not exceed a certain limiting value for it. This limiting value of velocity is **called critical velocity.**

12. REYNOLD'S NUMBER (N_R or Re)

Is a pure number which determine the type o flow of liquid through a pipe.

According to Reynolds formula

$$V_C = \frac{N_R \eta}{\rho D}$$

$$N_R = \frac{V_C \rho D}{\eta}$$

The value of NR plays a key role in determining the nature of flow. So the nature of flow depends upon the flowing factors:

(i) Velocity (ii) diameter (iii) density
(iii) coefficient of viscosity

Experimentally, it can be shown that for the fluid flow in a cylindrical tube or pipe:-

(i) If Re < 2000, the nature of fluid flow is the streamline (laminar) flow.
(ii) If Re > 3000, the nature of fluid flow is the turbulent flow.
(iii) If 2000 ≤ Re ≤ 3000, the fluid flow is the unstable flow.

REYNOLDS NUMBERS IS DIMENSIONLESS

$$[N_R] = \frac{[V_C][\rho][D]}{[\eta]} = \frac{LT^{-1}ML^{-3}L}{ML^{-1}T^{-1}}$$

$$[N_R] = [M^0L^0T^0] = 1$$

So, Reynolds number is dimensionless

IMPORTANCE OF REYNOLDS NUMBER

- It determines the nature of liquid flow (laminar or turbulent) through a pipe it is pure number (i.e dimensionless)
- It lead to the law of similarity

13. RATE OF FLOW

Is the volume of a liquid that passes the cross – section of a vessel (eg pipe) in one second. It is denoted by the symbol Q

$$\text{Rate of flow } Q = \frac{V_o}{t}$$

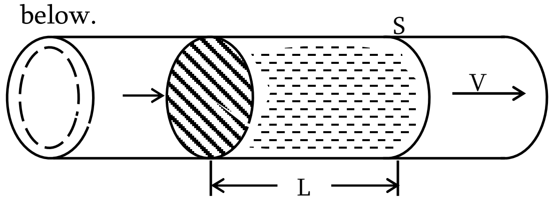
Vo = Volume of liquid that passes the cross – section of a pipe.

t = time taken

The S.I unit of rate of flow of liquid is m3/s.

Expression of rate of flow

Consider a pipe of uniform cross – section area, A in which there is flowing of liquid at an average velocity V as shown on the figure below.



$$\text{Rate of flow } Q = \frac{V_o}{t} = \frac{AVt}{t}$$

$$Q = AV$$

This is called 'discharge equation'.

ASSUMPTIONS

While discussing fluid flow, we generally make the following assumptions:

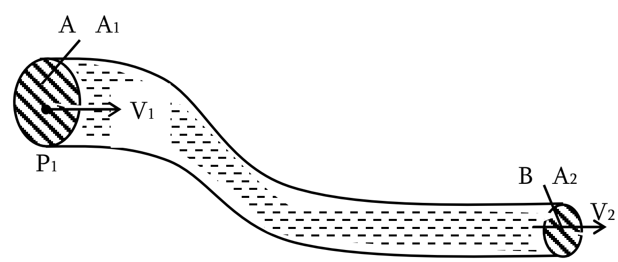
(i) The fluid flow is non - viscous i.e there is no internal friction between the adjacent layers of fluid.
(ii) The fluid is incompressible. This means that density of the fluid is constant.
(iii) The fluid motion is steady. The steady flow of a fluid means that the velocity, density and pressure at each point in the fluid do not change with time.

EQUATION OF CONTINUITY

DEFINITION CONTINUITY EQUATION – is the mathematical representation of the rate of volume of the incompressible fluid flow. the equation of continuity state that the mass rate has the same value at every position along a tube which has only one entry point and single outlet for the fluid flow. This equation is **called ' Equation of continuity'**.

DERIVATION OF CONTINUITY EQUATION

Consider the steady flow of a liquid through a pipe of varying cross – sectional area as shown on the figure below.



Let A_1 and A_2 be cross – sectional area at A and B respectively. V_1 and V_2 be the velocities of liquid at A and B respectively. Rate of mass of fluid (liquid). Flow

$$\frac{dm}{dt} = \rho AV$$

$$\text{At the point A: } \frac{dM_1}{dt} = \rho_1 A_1 V_1$$

$$\text{At the point B: } \frac{dM_2}{dt} = \rho_2 A_2 V_2$$

Applying the law of conservation of mass

$$\frac{dm}{dt} = \text{constant}$$
$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$
$$\rho AV = \text{constant}$$

This represent continuity equation for the compressible fluid flow. For an incompressible fluid flow.

$$\rho_1 = \rho_2 = \rho$$

Now $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

$$\rho A_1 V_1 = \rho A_2 V_2$$

$A_1 V_1 = A_2 V_2$ OR $AV = \text{constant}$

This represent continuity equation for the incompressible fluid flow.

Q = AV

Q = Rate of volume of fluid flow.

Let d_1 and d_2 be diameters of the pipe at point A and B respectively.

$$A_1 V_1 = A_2 V_2$$
$$\frac{\pi d_1^2}{4} V_1 = \frac{\pi d_2^2}{4} V_2$$
$$V_1 d_1^2 = V_2 d_2^2$$
$$V_2 = V_1 \left[\frac{d_1}{d_2}\right]^2$$

Additional concepts

From the equation of continuity for an incompressible fluid flow

$$Q = AV$$
$$V = \frac{Q}{A}$$

The following points may be noted:

1. AV = constant, Q . This equatin is called the equation of continuity for the streamline of an incompressible an non – viscous fluid flow.

2. $V = \frac{Q}{A}$, $V \alpha \frac{1}{A}$. The velocity of flow increases as the cross – sectional area decreases and vice – versa.

3. In the case of a pipe, the cross – sectional area is proportional to the square of the radius R of the pipe $\left[A = \pi R^2 \ , \ A \alpha R^2\right]$, $V \alpha \frac{1}{R^2}$

4. The cross – sectional area available to flowing water goes on increasing as we approaching the bottom of canal or river. So, the velocity flow decreases. This explain why the deep water run slows.

5. Since $A_1 > A_2$, $V_1 < V_2$, $P_1 > P_2$ Thus, the velocity of liquid flow at B is greater than that A($V_2 > V_1$). This means that the liquid experiences an accelerating force between A and B. the accelerating force can be present only if the pressure at A is greater than the pressure at B always fluid flows from the region of higher pressure to the region of the lower pressure.

LAW OF CONSERVATION OF MASS

State that ‘ As a fluid moves and deforms, new fluid is neither created nor destroyed’. The continuity equation implies the law of conservation of mass. In other words, continuity equation expresses conservation of mass in a mathematical form.

CONTINUITY PRINCIPLE

State that ‘ For an incompressible fluid flowing in a pipe, the product AV is a constant , at every cross – section of the pipe where A is the cross – sectional area and V is the velocity of fluid flow’ i.e AV = constant.

NUMERICAL EXAMPLE

Example – 01

Water flows through a horizontal pipe of varying cross – section at the rate of 10m³/minute. Determine the velocity of water at a point where the radius of the pipe is 10cm.

Solution

Rate of discharge $Q = \frac{10m^3}{1\,min} = \frac{10m^3}{60\,sec}$

$$Q = \frac{1}{6} m^3 s^{-1}$$

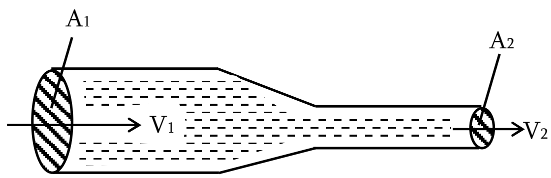
Since $Q = AV = \pi R^2 V$

$$V = \frac{Q}{\pi R^2} = \frac{1/6}{3.14 \times (0.1)^2}$$
$$V = 5.3m/s$$

Example – 02

Water flows through a pipe of internal diameter 20cm at the speed of 1m/s. what should be the diameter of the nozzle be if the water is to emerge at the speed of 4m/s?

Solution



Apply continuity equation

$$A_1V_1 = A_2V_2$$

$$\frac{\pi d_1^2}{4}V_1 = \frac{\pi d_2^2}{4}V_2$$

$$d_1^2V_1 = d_2^2V_2$$

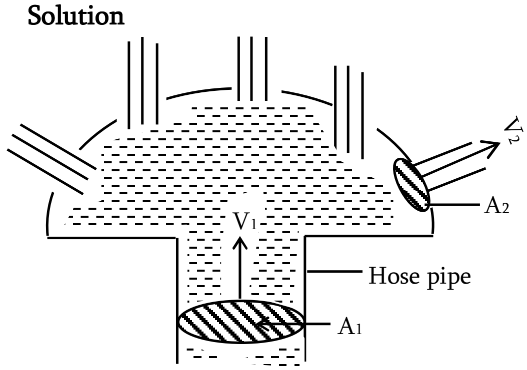
$$d_2 = d_1\sqrt{\frac{V_1}{V_2}} = 20\text{cm}\sqrt{\frac{1}{4}}$$

$$d_2 = 10\text{cm} = 0.1\text{m}$$

Example – 03

A lawn sprinker has 20holes each of cross – sectional area $2.0 \times 10^{-2}\text{cm}^2$ and is connected to the hose pipe of cross – sectional area 2.4cm^2. If the speed of water in a hose pipe is 1.5m/s. estimate the speed of water as it's emerges from the holes.

Solution



Let A_1 = Cross – sectional area of hose pipe

A_2 = Cross – sectional area of each hole

N = number of holes

Apply continuity equation

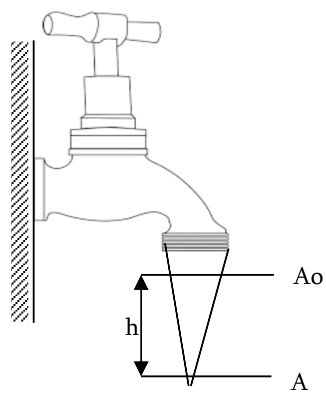
$$NA_2V_2 = A_1V_1$$

$$V_2 = \left(\frac{A_1}{A_2}\right)\frac{V_1}{N} = \left(\frac{2.4}{2\times10^{-2}}\right)\frac{1.5}{20}$$

$$V_2 = 9.0\text{m/s}$$

Example – 04

In the figure below shows how the stream of water emerging from a faucet 'necks down' as it falls figure shows two levels separated by a vertical distance h. The cross – sectional areas A_O and A are marked in the figure. At what rate does water flow from the tap?



Solution

Apply equation of continuity

$A_OV_O = AV$

Let V_O and V are the water velocities at the corresponding levels.

Again $V_2 - V_0^2 = 2gh$

$$V = \frac{A_0V_0}{A}$$

Now $V_0^2 = V^2 - 2gh = \left[\frac{A_0V_0}{A}\right]^2 - 2gh$

$$2gh = V_0^2\left[\frac{A_0^2}{A^2} - 1\right]$$

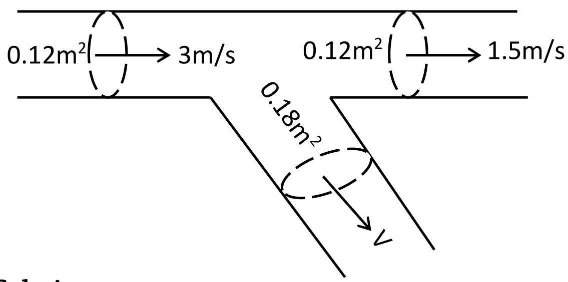
$$V_0 = \sqrt{\frac{2gA^2h}{A_0^2 - A^2}} = A\cdot\sqrt{\frac{2gh}{A_0^2 - A^2}}$$

Rate of volume of fluid flow

$$Q = A_0V_0 = A_0A\sqrt{\frac{2gh}{A_0^2 - A^2}}$$

Example – 05

An incompressible liquid travels as shown in the figure below. Calculate the speed of the fluid in the lower branch.



Solution

Apply the continuity equation

$$0.18V + 0.12 \times 1.5 = 0.12 \times 3$$
$$0.18V = 0.36 - 0.018$$
$$V = 1\text{m/s}$$

Example – 06

Water at 20°C is pumped through a horizontal smooth pipe 15cm in diameter and discharges into the air. If the pump contains, a flow velocity of 30cm/s.

(a) What is the nature of the flows?

(b) What is the discharge rate per second? Given that viscosity of water at 20°C is $1.00 \times 10^{-3}\text{Nsm}^{-2}$ and density of water is 1000kgm^{-3}.

Solution

(a) Reynolds number

$$N_R = \frac{\rho VD}{\eta} = \frac{1000 \times 0.3 \times 0.15}{1 \times 10^{-3}}$$
$$N_R = 45,000$$

Since $N_R > 3000$, Therefore the nature of the water flow is turbulent.

(b) Discharged rate

$$Q = AV = \frac{\pi D^2 V}{4}$$
$$= \frac{3.14 \times (0.15)^2 \times 0.3}{4}$$
$$Q = 0.0212\text{m}^3/\text{s}$$

Example – 07

(a) (i) Deep water runs show why?

(ii) Why two streamlines cannot cross each other?

(b) (i) What is the importance of Reynolds number

(ii) Approximately what volume of water per second can flow through a pipe 2.0cm in diameter before turbulent flow will occur? The critical value of Reynolds number is 2000. Viscosity of water is 0.801×10^{-3}Pas

Solution

(a) (i) According to equation of continuity $AV = \text{Constant}$ i.e $V \alpha \frac{1}{A}$. For the deep water, area of x – section (A) becomes large so that V is small for this reason, deep water runs slow.

(ii) The tangent at any point on a streamline gives the direction of flow of liquid at that point. If two streamlines cross each other at a point, then two tangents can be drawn at that point. It means that the liquid has two velocities along two different directions at this point. This is against the direction of streamline flow. Therefore, two streamlines cannot cross each other.

(b) (i) It determines the nature of liquid flow (laminar or turbulent) through a pipe. It is pure number (i.e dimensionless).

(ii) The Reynolds number $N_R = \frac{\rho VD}{\eta}$

The critical value of NR will give you the maximum flow speed.

$$V_{max} = \frac{N_R \eta}{\rho D} = \frac{2000 \times 0.801 \times 10^{-3}}{1000 \times 2 \times 10^{-2}}$$
$$V_{max} = 0.0801\text{m/s}$$

Maximum volume rate of laminar flow

$$Q_{max} = AV_{max} = \frac{\pi D^2}{4} \cdot V_{max}$$
$$= \frac{3.14\left(2 \times 10^{-2}\right)^2}{4} \times 0.0801$$
$$Q_{max} = 25.2 \times 10^{-6}\text{m}^3 = 25.2\text{cm}^3/\text{s}$$

EXERCISE

1. A 20 litres bucket can be filled with a water using a water whose 3.00cm in diameter in 2 minutes. Calculate the speed with which the water leaves the hose? **Answer** 0.2364m/s.

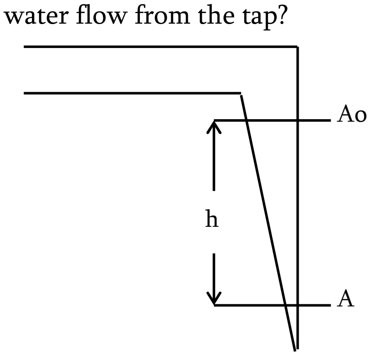
2. What should be the maximum average velocity of water in a tube of diameter 2cm so that the flow is laminar? Viscosity of water is $0.001Nsm^{-2}$, for laminar flow, $N_R = 1000$. **Answer** 0.05m/s.

3. A liquid is flowing through a horizontal pipe line of varying cross – section at a certain cross – section, the diameter of the pipe is 5×10^{-2}m and the velocity of flow of the liquid is 25×10^{-2}m/s. calculate the velocity of flow at another cross – section where the diameter is 1×10^{-2}m. **Answer** 6.25m/s.

4. A pipe of 0.03m internal diameter is connected to a lawn sprinker having 25holes each of diameter 0.5mm and water is flowing at 1m/s. what is the velocity of water in the pipe.

5. The cross – sectional area A_O of the Aorta (the major blood vessel emerging from the heart) of a normal resting person is $3 \times 10^{-4}m^2$ and the speed Vo of the blood is 0.3m/s. A typical capillary (diameter = 6μm) has a cross – sectional area A of $3 \times 10^{3}m^2$ and a flow speed of 5×10^{-5}m/s. How many capillaries does each have? **Answer** 6×10^9.

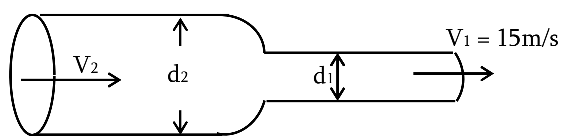
6. Figure below, how the stream of water emerging from the water tap 'necks down' as it falls. The cross – sectional area Ao is $1.2cm^2$ and A is $0.35cm^2$. The two levels are separated by a vertical distance h = 45mm. at what rate does water flow from the tap?



7. A garden hose having an internal diameter of 2cm is connected to a lawn sprinkler that consists of an enclosure with 24holes each of 0.125cm in diameter. If water in the hose has a speed of 90cm/s, at what speed does it leave the sprinkler hole? **Answer** 9.6m/s.

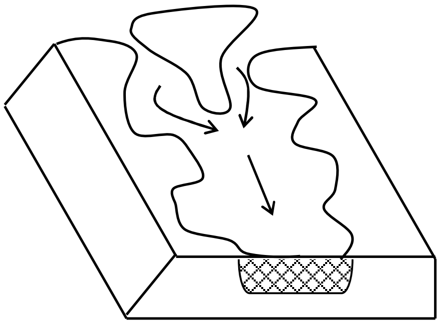
8. Water flows through a horizontal pipe of varying cross – sectional at the rate of $60m^3$ per minute. Find the velocity of water at a point where diameter of the pipe is 0.25m by applying equation of continuity also find the velocity at the point where diameter is 0.45m. **Answer** $V_1 = 20.37$m/s, $V_2 = 6.29$m/s.

9. Water flows through a horizontal pipe and is delivered into the atmosphere at a speed of 15m/s as shown in figure below. The diameter of the left and right section of the pipe are 0.05m and 0.03m respectively.
   (a) What volume of water is delivered into the atmosphere during a 10min period?
   (b) What is the flow speed of water in the left section of the pipe.



Answer (a) 6.6m3 (b) 0.54m/s.

10. (a) What does velocity increase when water flowing in broader pipe enters a narrow pipe?
   (b) Figure below shows the confluence of two streams to form a river. One stream has a width of 8.2m, depth of 3.4m and current speed of 2.3m/s. The other stream is 6.8m wide, 3.2m deep and flows at 2.6m/s. The width of the river is 10.7m and the current speed is 2.9m/s. what is its depth?



Solution

(a) For a steady flow, the equation of continuity tells us that AV = constant. Here A is the area of cross – section of the pipe and V is the velocity of liquid flow. Where enters a narrow pipe, the area of cross – section (A) decreases and hence velocity (V) of water flow increases

(b) Apply continuity equation

$$10.7 \times y \times 2.9 = 8.2 \times 3.4 \times 2.3 + 6.8 \times 3.2 \times 2.6$$

y = required depth of river

$$31.03y = 64.124 + 56.576$$

$$y = 3.89\text{cm}$$

11. (a) What is the principle on which continuity equation is based?

(b) A water pipe is 10cm in diameter and has a constriction of 2cm diameter. If the velocity of flow in the main pipe is 0.84m/s. calculate:

(i) The velocity of flow in the constriction.

(ii) Rate of discharge of water through a pipeline

Answer (b)(i) 21m/s (ii) 0.0066m³/s.

TOTAL ENERGY OF MOVING LIQUID

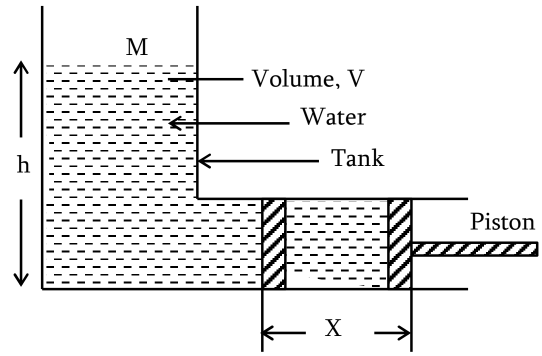
There are three types of the energy possessing by the fluid or liquid:

(i) Kinetic energy (k.e) due to its motion

(ii) Potential energy due to its position w.r.t a choosen reference level.

(iii) Pressure energy due to the pressure of the liquid.

Consider the figure below which shows the wide tank containing a liquid (water).



- **Expression of the kinetic energy**

$$k.e = \frac{1}{2}MV^2$$

M = Mass of flowing liquid

V = velocity

$$\text{k.e per unit mass} = \frac{V^2}{2}$$

$$\text{k.e per unit volume} = \frac{1}{2}\rho V^2$$

- **Expression of the potential energy**

p.e = mgh

g = Acceleration due to gravity

h = height of liquid inside of the tank

p.e per unit mass = kgh

p.e per unit volume = ρgh

ρ = density of the liquid

- **Expression of the pressure energy**

Pressure energy is the same as the amount of work done required by the piston to undergo displacement

X due to creation of the pressure

A = cross – sectional are of piston

P = pressure of the liquid

Work done , W = FX

W = PAX = P(AX)

But AX = Vo = Volume

Ep = W = PVo

$$\text{Pressure energy per unit mass} = \frac{PV_o}{M} = \frac{P}{\rho}$$

Pressure energy per unit volume = P

The total energy of the moving liquid

$E = E_P + p.e + k.e$

$$E = PV_o + Mgh + \frac{1}{2}MV^2$$

Total energy per unit mass

$$= \frac{P}{\rho} + \frac{V^2}{2} + gh$$

Total energy per unit volume

$$= P + \frac{1}{2}\rho V^2 + \rho gh$$

BERNOULLI'S THEOREM

In 1738, Daniel Bernoulli established for the streamline flow of an ideal fluid by making use of principle of conservation of energy.

DEFINITION BERNOULLI'S EQUATION is an equation for the steady flow of a non – viscous, incompressible fluid flows which relates the pressure p, the fluid speed, V and the height h at any two points on the streamline. Bernoulli's equation is given by.

$$P + \frac{1}{2}\rho V^2 + \rho gh = \text{constants}$$

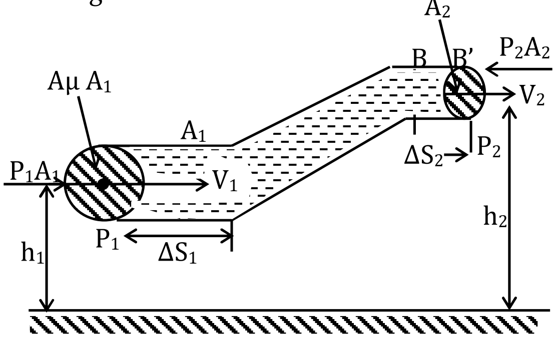
$$P + \frac{1}{2}\rho V^2 + \rho gh = \text{constant}$$

P = pressure of the liquid
ρ = density of the liquid
V = velocity of liquid flow
g = acceleration due to gravity
h = height (elevation) from the reference point (level)

Bernoulli's principle is based on the law of conservation of energy (mechanical energy)

DERIVATION OF BERNOULLI'S EQUATION

Consider the two points at different elevation (height) in the flow pipe i.e h_1 and h_2 as shown in the figure below.



Let A_1, A_2 = cross – sectional area at A and B respectively.
V_1, V_2 = Velocities of liquid at A and B respectively.
P_1, P_2 = Liquid, pressure at A and B respectively.

The corresponding change in work done by the liquid in the pipe due to the small displacement of fluids from the point AB to A'B'.

$\Delta W = F\Delta S$ but $F = PA$
$\Delta W = PA\Delta S = P\Delta V$ $(A\Delta S = \Delta V)$
At point A : $\Delta W_1 = P_1\Delta V$
At point B: $\Delta W_2 = P_2\Delta V$
Net change of work done
$\Delta W = AW_1 - \Delta W_2 = (P_1 - P_2)\Delta V$.....(i)

The net change in work done on the liquid flow in the pipe is due to the changes of both kinetic enrgy and potential energy

$\Delta W = \Delta k.e + \Delta p.e$

$$\Delta W = \frac{1}{2}\Delta M\left(V_2^2 - V_1^2\right) + \Delta Mg\left(h_2 - h_1\right) \text{....(ii)}$$

According to the work – energy Conservation principle (i) = (ii)

$$\left(P_1 - P_2\right)\Delta V = \frac{1}{2}\Delta M\left(V_2^2 - V_1^2\right) + \Delta Mg\left(h_2 - h_1\right)$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right) + \rho\left(h_2 - h_1\right)$$

Since $\Delta M / \Delta V = \rho$

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho gh_2 \text{ or}$$

$$P + \rho gh + \frac{1}{2}\rho V^2 = \text{constant}$$

P = Pressure of the liquid
$\frac{1}{2}\rho V^2$ = Kinetic energy per unit volume
ρgh = Potential energy per unit volume

BERNOULLI'S PRINCIPLE

state that ' for an incompressible , irrotational, non – viscous fluid undergoing steady flow, the pressure plus potential energy per unit volume plus the kinetic energy per unit volume is always constants for all points in the pipe.

CONDITIONS NECESSARY FOR BERNOULLIS EQUATION TO BE APPLICABLE.

1. The fluid is non – viscous
2. The fluid flow is irrotational fluid flow
3. The fluid is steady fluid flow
4. The fluid flow is incompressible

DIFFERENT FORMS OF BERNOULLI'S EQUATION

1. $P + \frac{1}{2}\rho V^2 + \rho gh = \text{constant}$ each term have the same dimension with the pressure.

2. Dividing ρ both side of equation (1) above
$\frac{p}{\rho} + gh + \frac{V^2}{2} = \text{constant}$,
$\frac{P_1}{\rho} + gh_1 + \frac{V_1^2}{2} = \frac{P_2}{\rho} + gh_2 + \frac{V_2^2}{2}$ each term have the same dimension as the dimension of energy per unit mass. For the streamline flow of an ideal fluid (non – viscous and incompressible) the sum of the pressure energy, kinetic energy and potential energy per unit mass is always constant.

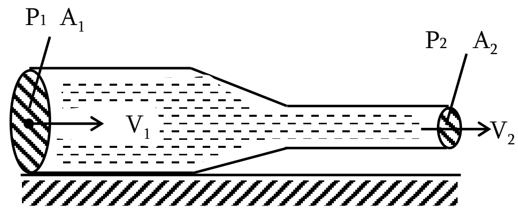
3. From equation (2) dividing by g both side
$\frac{P}{\rho g} + \frac{V^2}{2g} + h = \text{constant}$,
$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_2$ each term in this equation has the dimension of length or height so each term is called 'ahead'
$\frac{P}{\rho g} = \text{pressure head}$
$\frac{V^2}{2g} = \text{velocity head}$
h = gravitational head
'For the streamline flow of an ideal fluid, the sum of pressure head, velocity head and gravitational head is always a constant.

ADDITIONAL CONCEPTS

1. Since $A_1 > A_2$, $V_1 < V_2$, $P_1 > P_2$ also $P + \rho gh + \frac{\rho V^2}{2} = \text{constant}$. If the fluid is at rest, then V = 0. P + ρgh = pressure of a static pressure'. Then term ½ρV² is called the 'dynamic pressure'. It represents the pressure of fluid by virtue of its motion.

2. If the liquid stops its motion the term disappear on Bernoulli's equation is ½ρV² (kinetic energy per unit volume) p + ρgh = constant.

3. Bernoulli's equation for the horizontal pipe



For the horizontal pipe, h1 = h2 = h recall
$P_1 + \frac{1}{2}\rho V_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho gh_2$
$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$ i.e
$P + \frac{1}{2}\rho V^2 = \text{constant}$

4. In driving the equation, we have an effect assumed that the pressure and velocity are uniform over an cross – section of the tube. This is not so for real (viscous) fluid and so it only applied strictly to a single for streamline fluid flow. In additional, actual fluids especially gases are compressible. Thus the equation is not applicable these.

LIMITATIONS OF BERNOULLI'S THEOREM

1. When a fluid is in motion, it experiences viscous drag. This has not taken into account.

2. The derivation of Bernoulli's equation has been based on the assumption that there is no loss of energy. However, some energy of the fluids get converted into heat energy and is lost.

3. When the fluids moves along a curved path, the energy due to the centrifugal force should be taken into consideration.

4. In the derivation of Bernoulli's equation it is assumed that rate of flow of liquid is constant (i.e $A_1V_1 = A_2V_2$) But this is not correct in actual practice. Thus in the case of a liquid flowing through a pipe the velocity of flow is maximum at the centre and goes on decreasing towards the walls of the pipe. Therefore, we should taken average velocity of the liquid.

NUMERICAL EXAMPLES

Example – 19

(a) (i) Name three physical principles which apply to both liquids and gases.

(ii) What are the implications of Bernoulli's theorem when the fluid is at rest?

(b) At what speed will the velocity head of a stream of water be equal to 40cm?

Solution

(a) (i)
- Bernoulli's theorem
- Pascal principle
- Archimedes' principle

(b) Bernoulli's theorem state that

$$P_1 + \rho g h_1 + \frac{1}{2}\rho V_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho V_2^2$$

The equations of hydrostatics are special cases of Bernoulli's theorem the velocity is zero everywhere i.e $V_1 = V_2 = 0$

$$P_1 + \rho g h_1 = P_2 + \rho g h_2$$

$$P_1 - P_2 = \rho g (h_2 - h_1)$$

Velocity head, $\frac{V^2}{2g} = h$

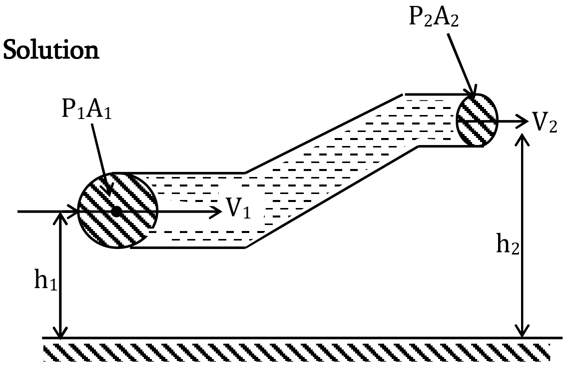
$$V = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.4}$$

$$V = 2.8\text{m/s}$$

Example – 20

The velocity at a certain point in a flow pipe is 1.0m/s at the gauge pressure is $3.0 \times 10^5\text{N/m}^2$. The cross – sectional area at a point 10m above the first is half the first point if the flowing fluid is the pure water, calculate the gauge pressure at the second point. (Density of water = 1000kgm^{-3}).

Solution



$V_1 = 1\text{m/s}$, $P_1 = 3 \times 105\text{Nm}^{-2}$.

$h = h_2 - h_1 = 10\text{m}$, $A_1 = 2A_2$ $(A_2 = A_1/2)$

Applying continuity equation

$$A_1V_1 = A_2V_2$$

$$V_2 = \left(\frac{A_1}{A_2}\right)V_1 = \left(\frac{2A_2}{A_2}\right) \times 1\text{m/s}$$

$$V_2 = 2\text{m/s}$$

Apply Bernoulli's equation

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

$$P_2 = P_1 + \rho g (h_1 - h_2) + \frac{1}{2}\rho (V_1^2 - V_2^2)$$

$$= P_1 - \rho g h + \frac{1}{2}\rho (V_1^2 - V_2^2)$$

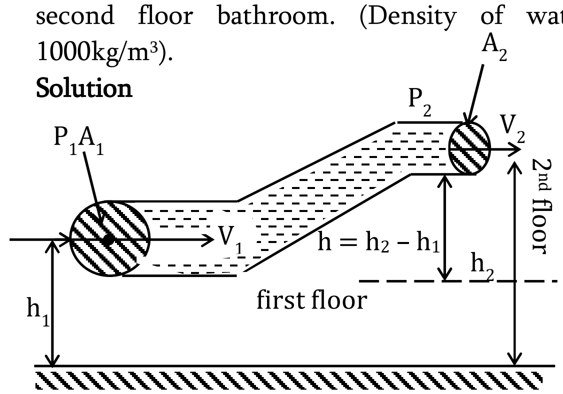
$$= 3 \times 10^5 - 1000 \times 9.8 \times 10 + \frac{1}{2} \times 1000 [1^2 - 2^2]$$

$$P_2 = 2.035 \times 10^5 \text{Nm}^{-2}$$

Example – 21

Water is supplied to a house at ground level through a pipe of inner diameter 2.0cm at an absolute pressure of 4.0×10^5Pa. The pipe leading to the second floor bath room 5metre above has an inner diameter of 1.0cm. If the velocity of flow at the ground level inlet is 4m/s. Find the flow velocity and pressure at the pipe outlet in the second floor bathroom. (Density of water = 1000kg/m^3).

Solution



Apply continuity equation

$$A_1V_1 = A_2V_2$$

$$\frac{\pi d_1^2 V_1}{4} = \frac{\pi d_2^2 V_2}{4}$$

$$V_2 = V_1\left[\frac{d_1}{d_2}\right]^2 = 4\left[\frac{2}{1}\right]^2$$

$$V_2 = 16.0\text{m/s}$$

Apply Bernoulli's equation

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

$$P_2 = P_1 + \frac{1}{2}\rho\left(V_1^2 - V_2^2\right) + \rho g\left(h_1 - h_2\right)$$

$$= P_1 + \frac{1}{2}\rho\left[V_1^2 - V_2^2\right] - \rho g\left(h_2 - h_1\right)$$

$$= 4\times10^5 + \frac{1}{2}\times1000\left[4^2 - 16^2\right] - 1000\times9.8\times5$$

$$P_2 = 2.31\times10^5\,\text{Pa}$$

Example – 22

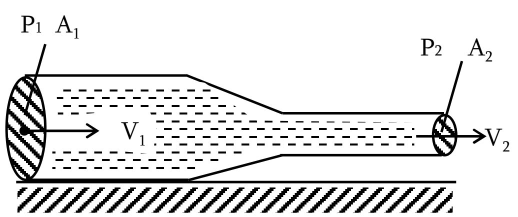
(a) Derive the continuity equation as applied to the fluid motion.

(b) Oil of density 850kg/m³ along a horizontal pipe whose cross – sectional area at one end is 50mm² and the other is 25mm². The difference between the pressure at the ends is $3.3\times10^2\text{Nm}^{-2}$. Calculate the velocity of the oil flowing at the larger cross – sectional area.

Solution

(a) See your notes

(b)



Apply the continuity equation

$$A_1V_1 = A_2V_2$$

$$50V_1 = 25V_2$$

$$V_2 = 2V_1 \text{..........(i)}$$

Apply Bernoulli's equation for the horizontal pipe.

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho\left[V_2^2 - V_1^2\right]$$

$$\Delta P = \frac{1}{2}\rho\left[\left(2V_1\right)^2 - V_1^2\right]$$

$$V_1 = \sqrt{\frac{2\Delta P}{3\rho}} = \sqrt{\frac{2\times3.3\times10^2}{3\times850}}$$

$$V_1 = 0.5087\text{m/s}$$

Example – 23

Water is flowing through a horizontal pipe of varying cross – section. At a certain point where the velocity is 0.24m/s the pressure of water is 0.01metre of mercury. What is the pressure at the point where the velocity is 0.48m/s. (density of mercury and water are 13600 and 1000kgm^{-3} respectively).

Solution

Apply Bernoulli's equation for the horizontal pipe

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{P_2}{\rho} + \frac{V_2^2}{2}$$

$$P_1 = h_1gd \quad , \quad P_2 = h_2gd$$

$$\frac{h_1gd}{\rho} + \frac{V_1^2}{2} = \frac{h_2gd}{\rho} + \frac{V_2^2}{2}$$

$$h_2 = h_1 + \frac{\rho}{2gd}\left[V_1^2 - V_2^2\right]$$

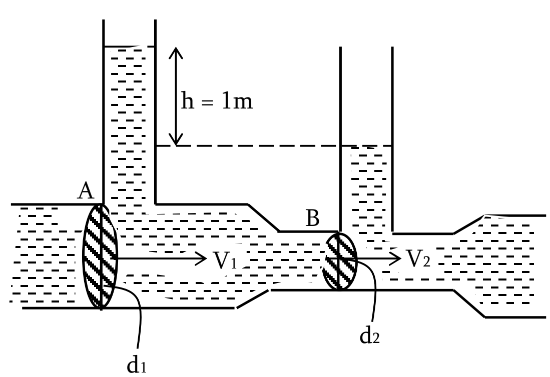
$$= 0.01 + \frac{1000}{2\times9.8\times13600}\left[\left(0.24\right)^2 - \left(0.48\right)^2\right]$$

$$h_2 = 0.0094\text{m}$$

Example – 24

A pipe is running full of water at certain point A it tapers from 60cm in diameter to 20cm at B. The pressure difference between A and B is 1m of water column. Find the rate of flow of water through the pipe.

Solution



Let P_1 and P_2 be the pressures at A and B respectively and V_1 and V_2 be corresponding velocities.

Rate of volume of water flows.

$$Q = A_1V_1 = A_2V_2$$

$$\frac{\pi d_1^2 V_1}{4} = \frac{\pi d_2^2 V_2}{4}$$

$$V_1 d_1^2 = V_2 d_2^2$$

$$V_2 = V_1\left[\frac{d_1}{d_2}\right]^2 = V_1\left[\frac{60}{20}\right]^2$$

$$V_2 = 9V_1 \text{.........(i)}$$

Gravitational potential energy remain constant.

Apply Bernoulli's equation for the horizontal pipe

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho\left[V_2^2 - V_1^2\right]$$

$$\rho gh = \frac{1}{2}\rho\left[(9V_1)^2 - V_1^2\right]$$

$$V_1 = \sqrt{\frac{gh}{40}} = \sqrt{\frac{9.8\times 1}{40}}$$

$$V_1 = 0.495\text{m/s}$$

Now

$$Q = \frac{\pi d_1^2}{4}V_1 = \frac{3.14\times(0.6)^2\times 0.495}{4}$$

$$Q = 0.1399\text{m}^3/\text{s}$$

Example – 25

The flow of blood in a large artery of an Anaesthetised dog is diriverted through a venturimeter. The wider part of the meter has a cross – sectional area equal to that of the artery, A = 8mm². The narrower part has an area, a = 4mm². The pressure drop in the artery is 24Pa. What is the speed of the blood in the artery? Given that the density of blood = 1.06 × 103kgm⁻³).

Solution

A = 8mm², a = 4mm², $P_1 - P_2$ = 24Pa

V_1 = ? Let A = a_1, a = a_2

Apply continuity equation for the horizontal pipe

$$P_1 - P_2 = \frac{1}{2}\rho\left[V_2^2 - V_1^2\right]$$

$$P_1 - P_2 = \frac{1}{2}\rho V_1^2\left[\left(\frac{a_1}{a_2}\right)^2 - 1\right]$$

$$V_1^2 = \frac{2(P_1 - P_2)}{\rho\left[\left(\frac{a_1}{a_2}\right)^2 - 1\right]} = \frac{2\times 24}{1060\times 3}$$

$$V_1 = \sqrt{\frac{48}{1060\times 3}}$$

$$V_1 = 0.123\text{m/s}$$

Example – 26

In a horizontal pipeline of uniform area of cross – section, the pressure falls by 5N/m² between two points separated by a distance of 1km. what is the change in kinetic energy per Kg of the oil flowing at these points? (Density of oil = 800kgm⁻³)

Solution

According to the Bernoulli's principle

$$\frac{P}{\rho} + gh + \frac{V^2}{2} = \text{constant}$$

For the horizontal pipe

$$\frac{P}{\rho} + \frac{V^2}{2} = \text{constant}$$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{P_2}{\rho} + \frac{V_2^2}{2}$$

$$\frac{P_1 - P_2}{\rho} = \frac{1}{2}\left(V_2^2 - V_1^2\right)$$

Change in k.e per unit mass

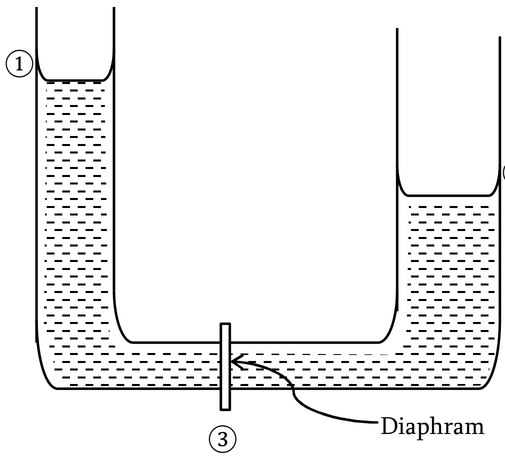
$$= \frac{1}{2}\left(V_2^2 - V_1^2\right) = \frac{P_1 - P_2}{\rho}$$

$$\Delta\text{k.e/m} = \frac{5}{800} = 6.25\times 10^{-3}$$

$$\text{k.e per kg} = 6.25\times 10^{-3}\text{Jkg}^{-1}$$

Example – 27

Consider a uniform U – tube with a diaphragm at the bottom and filled with a liquid to different heights in each limb as shown in the figure below



Now imagine that the diaphragm is punctured so that the liquid flows from left to the right.

(a) Show that the application of Bernoulli's particles to points (1) and (2) lead to a contradiction.

(b) Explain why Bernoulli's equation or principle is not applicable here.

Solution

Let the height of the liquid in the limbs (1) and (2) be y_1 and y_2 and the velocity of the liquid be V_1 and V_2 respectively

Apply Bernoulli's equation at the points (1) and (2)

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g y_2$$

$$P_1 = P_2 = P = \text{Atmospheric pressure}$$

$$V_1 = V_2 = 0$$

$$\rho g y_1 = \rho g y_2$$

$$y_1 = y_2$$

This is a contradiction because y_1 and y_2 is not equal as shown in the figure above.

(b) Bernoulli's principle cannot be applicable here because the flow is not steady.

Example – 28

A horizontal pipe of diameter 20cm has a constriction of diameter 4cm. the velocity of water in the pipe is 2m/s and pressure is $107 N/m^2$. Calculate the velocity and pressure at the constriction.

Solution

According to the continuity equation

$$A_1V_1 = A_2V_2$$

$$V_2 = \frac{A_1V_1}{A_2} = \frac{\pi(0.1)^2}{\pi(0.02)^2} \times 2$$

$$V_2 = 50 m/s$$

According to the Bernoulli's principle for the horizontal pipe.

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_1 = 10^7 N/m^2 \ , \ V_2 = 50 m/s \ , \ V_1 = 2 m/s$$

$$P_2 = P_1 + \frac{1}{2}\rho\left[V_1^2 - V_2^2\right]$$

$$= 10^7 + \frac{1}{2} \times 1000\left[2^2 - 50^2\right]$$

$$P_2 = 8.752 \times 10^6 Nm^{-2}$$

Example – 29

(a) (i) Can Bernoulli's equation be used to describe the flow of water through a rapid in a river?

(ii) Does it matter if one uses gauge instead of absolute pressure in applying Bernoulli's equation.

(b) A liquid is kept in a cylindrical vessel which is rotated along its axis. The liquid rises at the side. If the radius of the vessel is 0.05m and the speed of rotation is 2rev per second. Find the difference in the height of the liquid at the centre of the vessel and at its sides.

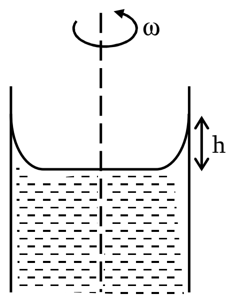
Solution

(a) (i) No Bernoulli's equation can be applied only when the flow of liquid is streamline.

(ii) No, provided the atmospheric pressure at the two points where Bernoulli's equation is applied do not differ significantly.

(b) When the cylindrical vessel rotated the velocity at the sides is higher.
According to the Bernoulli's theorem

$$P + \frac{1}{2}\rho V^2 = \text{constant}$$



Hence the pressure at sides is lower. Since the pressure at a given horizontal level must be equal, the liquid rises at sides to difference in height of the liquid at the centre of the vessel and at its sides.

$$\frac{1}{2}\rho V^2 = \rho g h$$

$$\rho V^2 = 2\rho g h$$

$$\omega^2 r^2 = 2gh \quad \text{but} \quad \omega = 2\pi f$$

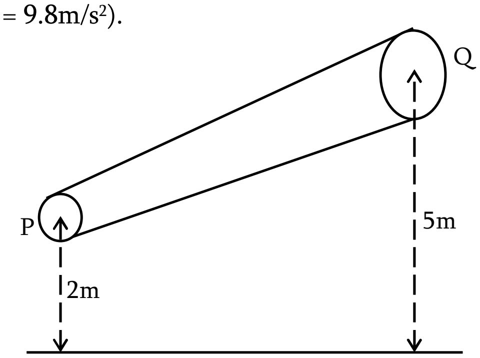
$$h = \frac{(2\pi f)^2 r^2}{2g}$$

$$h = \frac{\left[3.14 \times 2 \times 2 \times 0.05\right]^2}{2 \times 9.8}$$

$$h = 0.02\text{m}$$

Example – 30

A non – viscous liquid of constant density 1000kg/m³ flows in a streamline motion along a tube of variable cross – section. The tube is kept inclined in the vertical plane as shown in the figure below. The area of cross – section of the tube at the points P and Q at heights 2m and 5m are respectively $4 \times 10^{-3}\text{m}^2$ and $8 \times 10^{-3}\text{m}^2$. The velocity of the liquid at point P is 1m/s. Find the work done per unit volume by the gravity and pressure force as the liquid flows from point P to the point Q. ($g = 9.8\text{m/s}^2$).



Solution

Apply the continuity equation

$$A_P V_P = A_Q V_Q$$

$$V_Q = \frac{A_P V_P}{A_Q} = \frac{4 \times 10^{-3} \times 1}{8 \times 10^{-3}}$$

$$V_Q = 0.5\text{m/s}$$

Since the liquid flows from P to Q the work done per unit volume against gravity

Wg = mass per unit volume × g × ($h_2 - h_1$)

$$= 1000 \times 9.8 \times (5 - 2)$$

$$= 2.94 \times 10^4 \text{J/m}^3$$

∴ Work done per unit volume by gravity

$$W_g = -2.94 \times 10^4 \text{J/m}^3$$

Suppose Wp is the work done per unit volume by the pressure force. Ten according to work – energy theorem.

$$W_p + W_g = \frac{1}{2}\rho\left(V_Q^2 - V_P^2\right)$$

$$W_P = \frac{1}{2}\rho\left[V_Q^2 - V_P^2\right] - W_g$$

$$= \frac{1}{2} \times 1000\left[(0.5)^2 - 1^2\right] - \left[-2.94 \times 10^4\right]$$

$$W_P = 2.94 \times 10^4 \text{J/m}^3$$

EXERCISE NO. 2

1. The reading of a pressure meter attached with a closed water pipe is $4.0 \times 10^5\text{Nm}^{-2}$. On opening the valve of the pipe, the reading of the pressure meter is reduced to $3.2 \times 10^5\text{Nm}^{-2}$. Calculate the velocity of water flowing in the pipe. **Answer.** 12.65m/s.

2. Water flows through a constricted pipe at a uniform rate at one point, where the pressure is 2.5×10^4Pa, the diameter is 8.0cm at another point 0.5m higher, the pressure is 1.5×10^4Pa, and the diameter is 4.0cm. find
   (i) The speed of flow in the lower and upper section.
   (ii) The rate of flow through the pipe
   Answer
   (i) 0.83m/s (lower); 3.3m/s (upper)
   (ii) $4.15 \times 10^{-3}\text{m}^3/\text{s}$.

3. (a) State and prove Bernoulli's theorem for fluids and give the assumptions used in deriving it.
   (b) Water flows through a pipe whose internal diameter is 2×10^{-2}m at a speed of 1m/s. what should be the diameter of the nozzle if the water is to emerge at a speed of 4m/s?

4. (a) State Bernoulli's theorem. Prove that the total energy possessed by a flowing ideal liquid is conserved starting assumptions used.
   (b) Water flows through a horizontal pipe of non – uniform cross – section. The pressure is 0.01m of mercury where the velocity of flow is 0.35m/s. Find the pressure at point where velocity is 0.65m/s. **Answer** 8.9×10^{-3}m of Hg.

5. A garden hose has an inside cross – sectional area 0f 3.60cm^2 and the opening in the nozzle is 0.250cm^2. the water velocity is 50cm/s in a segment of the hose that lies on the ground (see figure below).
   (a) With what velocity does the water leave the nozzle when it is held 1.50m above the ground?
   (b) What is the water pressure in the hose on the ground? Given pressure at the nozzle in 1atm and density of water = 1000kg/m^3.

   Answer (a) 7.20m/s (b) 1.41×10^5Pa

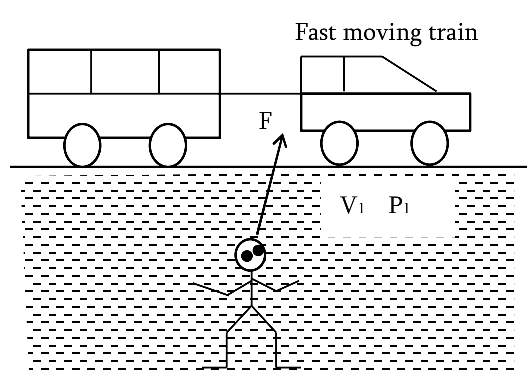
FIG 56

APPLICATIONS OF BERNOULLI'S PRINCIPLE

1. Sunction affect
2. Aerofoil lift
3. Blowing off roof of house
4. Fluid flow in the wide tank (Torricelli's theorem)
5. Venturimeter
6. Pitot – static tube
7. Thrust on the rocket
8. Spinning of the ball (magnus effect)
9. Two moving parallel ships
10. Spray gun, the atomizer
11. Jets and nozzle
12. Bunsen burner
13. Filter pump and carburetor
14. Ping pong ball kept on a stream of water.
15. Law of hydrostatic pressure
16. Attracted disc paradox e.t.c

1. SUCTION EFFECT

This is an effect experienced by person standing closer to the plat form at a station when a fast moving train passes him or her. The fast moving air between the person and the train produces a decreases in pressure and the excess pressure on the other side pushes the person on the other side pushes the person toward the train i.e it is dangerous to stand near the plat form where the fast moving train or bus passes him or hear because near the train the air moving with high velocity produces a low pressure. But the pressure behind the person standing near the fast moving train is the atmospheric pressure which is comparatively high. This pressure difference pushes the person towards the train. This is accordance to the Bernoulli's principle



According to the Bernoulli's equation for the horizontal pipe.

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

ρ = density of moving air

Since $P_2 > P_1$, $V_1 > V_2$

$$P_2 - P_1 = \frac{1}{2}\rho\left[V_1^2 - V_2^2\right]$$

The force required to push person towards to the fast moving train

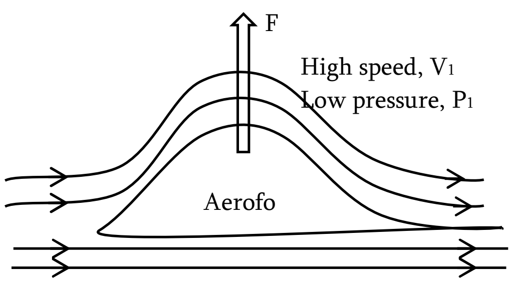
$$F = \left(P_2 - P_1\right)A = \frac{1}{2}\rho A\left[V_1^2 - V_2^2\right]$$

A = effectively area of a person

2. AEROFOIL LIFT (STRUCTURE)

Aerofoil is a solid piece properly shaped so that an upward vertical force acts on it, when it moves horizontally through the air. Aeroplane gets lift using this principle. It based on Bernoulli's principle.

Consider the figure below



The upper surface of the aerofoil is more curved than the lower surface, so fluids (air) flow faster over the top surface than the bottom surface. The streamlines are closer above because of the higher velocity. Therefore the pressure above becomes less than that of below of the foil. This pressure difference gives rise to an upward lift force is **called Dynamic lift force of the Air craft**.

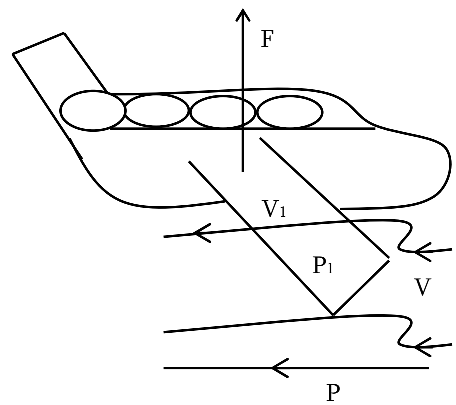
Definition Dynamic lift – is the resultant upward force exerted on a body which is moving relative to the fluid. Aerofoil lift can be applied especially on the following :-

(i) Aircraft wings, turbine blades and propellers.

(ii) Motion of the bird/fish

Expression of a dynamic lift force on the wing(s) of Aeroplane.

Consider the motion of air along the wings of Aeroplane as shown on the figure below.



Let V_1 , V_2 be the velocity of air at the top and bottom respectively. P_1, P_2 be pressure of air at the top and bottom of wings of Aeroplane.

Apply the Bernoulli's principle of horizontal pipe.

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

Since $P_2 > P_1$, $V_2 < V_1$

$$P_2 - P_1 = \frac{1}{2}\rho\left[V_1^2 - V_2^2\right]$$

Dynamic lifting force

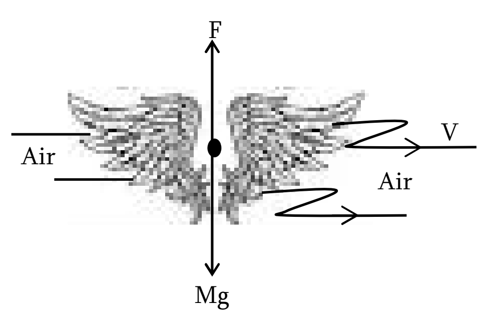
$$F = \left(P_2 - P_1\right)A = \frac{1}{2}\rho A\left[V_1^2 - V_2^2\right]$$

ρ = Density of air

A = Effective or total area of wings of Aeroplane.

Expression of the velocity of imparted air along the wing(s) of a bird.

Consider the motion of air along the wings to a bird as shown on the figure below.



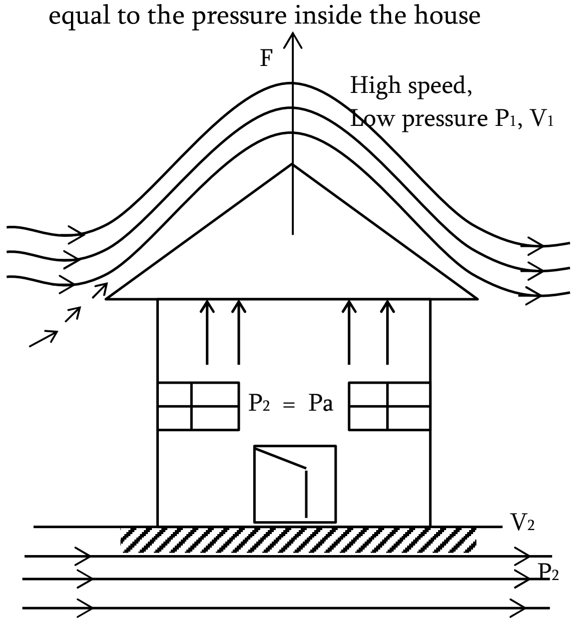
M = mass of wing of a bird
g = Acceleration due to gravity
F = Upthrust
At the equilibrium of the bird
F = Mg
According to the Newton's second law of motion.

$$F = \rho A V^2$$
$$\rho A V^2 = Mg$$
$$V = \sqrt{\frac{Mg}{\rho A}}$$

ρ = Density of air
A = Total area of wing of a bird

3. BLOWING OFF ROOF OF THE HOUSE

When there is no storm the outside pressure is equal to the pressure inside the house



Low speed, high pressure

When winds blows as shown in the figure above, high speed of air over the top of the roof produces a low pressure. The pressure under the roof (inside of the house) is greater equal to the atmospheric pressure due to the low speed of air molecules. The pressure difference at the top and inside of the house causes the lift of the roof of house and this is accordance to the Bernoulli's principle. Expression of net upward force.

$F = A(P_a - P_1)$
A = total area of the roof of house

4. Why does smoke go up a chimney

It is partty because hot air rises (i.e density decreases). But Bernoulli's theorem also plays a role. Because the wind blows across the top of a chimney, the pressure is less there than inside the house. Hence air and smoke are pushed up the chimney. Even on an apparently still night, there is usually enough ambient air flow at the top of chimney to allow upward flow of smoke.

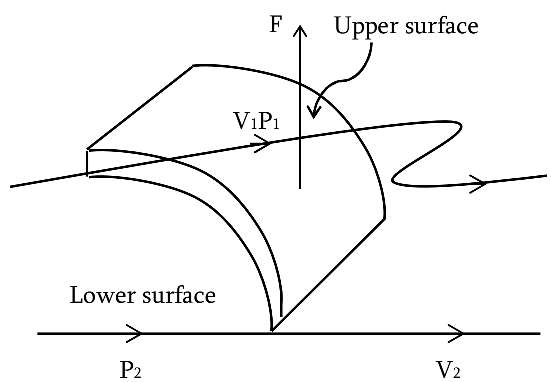
NUMERICAL EXAMPLES

Example – 36

Air flows over the upper surface of the wings of jet plane at a speed of 340m/s and past the lower surface at 280m/s. determine the lift force if its base has a total wings are of 50m². The density of flowing air is 1.29kgm⁻³.

Solution

$V_1 = 340$m/s , $V_2 = 280$m/s
$A = 50$cm² , $\rho = 1.29$kgm^{-3} , F = ?



Apply the Bernoulli's equation for the horizontal pipe.

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$
$$P_2 - P_1 = \frac{1}{2}\rho\left[V_1^2 - V_2^2\right]$$

Lifting force, $F = (P_2 - P_1)A$

$$F = \frac{1}{2}\rho A\left[V_1^2 - V_2^2\right]$$
$$= \frac{1}{2}\times 1.29\times 50\left[(340)^2 - (280)^2\right]$$
$$F = 1199.7\times 10^3\,\text{N} = 1199.7\,\text{KN}$$

Example – 37

(a) Air flows past the upper surface of an Aeroplane at 300m/s and past the lower surface at 250m/s. the surface area of the wings is 16m^2 and the plane is flying at an elevation where density of air is 1.0kgm^{-3}. What is the upward force on a wings?

Solution

$$F = \frac{1}{2}\rho A\left[V_1^2 - V_2^2\right]$$

$$= \frac{1}{2}\times 1\times 16\left[(300)^2 - (250)^2\right]$$

$$F = 220{,}000\text{N} = 220\text{KN}$$

(b) What happens when the roof blows off a house in a violent wind strong? Hints: see your notes.

Example – 38

(i) State the Bernoulli's principle and the equation of continuity.

(ii) Distinguish between Dynamic lift' and 'upthrust'.

(iii) A bat of mass 1100g hovers upwards by beating its wings of effective area 0.4m^2. Estimate the velocity of the imparted to the air by the beatings of the wings. Assuming the air to be at S.T.P weather conditions. The density of air = 1.29kgm^{-3}.

Solution

(i) See your notes.

(ii) Dynamic lift is the resultant upward force exerted on a body which is moving relative to a fluid while upthrust is the upward force experienced by the body when is totally or partially immersed in the fluid.

(iii) Since $V = \sqrt{\dfrac{Mg}{\rho A}} = \sqrt{\dfrac{1.1\times 9.8}{1.29\times 0.4}}$

Example – 39

Air flows over the upper surfaces of the wings of an aeroplane at a speed of 120m/s and past the lower surface of the wings at 110m/s. Calculate the lift force on the aeroplane if tis has a total wing area of 20m^2.(density of air = 1.29kgm^{-3}).
Answer 2.97×10^4N.

Example – 40

A bird of mass 0.50kg hovers by beating its wings of effective area 0.30m^2.

(i) What is the upward force of the air on the bird?

(ii) What is the downward force on the air as it beats its wings?

(iii) Estimate the velocity imparted to the air, which has a density of 1.3kgm^{-3} by the beatings of the wings. Which of Newton's laws is applied in each of (i), (ii) and (iii).
Answer (i) 5.0N (ii) 5.0N (iii) 3.6m/s

Example – 41

(a) Why air is blown in between two balls suspended close to each other they are attracted towards each other why?

(b) Why does flag flutter when strong winds are blowing on a certain day?

Solution

(a) This is because when air is blowing velocity is increased and the pressure is decreases. On the other side of the balls, pressure is high, so they are attracted towards to each other.

(b) If the velocity of wind is high then the pressure is low and vice – versa. Thus a pressure difference is created on the two sides of the flag and hence it flutters.

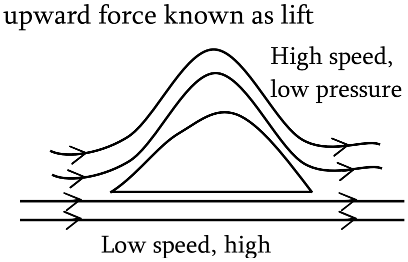
Example – 42

(a) Why is air craft wing gives stream lined shape?

(b) What causes of a turbulent flow in a fluid?

Solution

(a) The shape of cross – section of air craft wing is so designed that the velocity of the air above the wing is greater than that below. According to Bernoulli's theorem the pressure of air below is greater than that above it consequently, the air craft experiences and upward force known as lift



(b) See your notes.

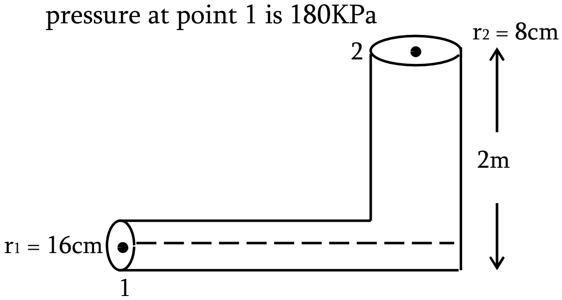
Example – 43

(a) (i) Outline any four assumptions made in fluid mechanics.

(ii) What is the 'Principle of mass continuity and hence state the principle of mass continuity:

(iii) In a steady flow of fluid, the velocity V at a point is constant; can there be accelerated motion of fluid particles? If so, what produces the net force for this acceleration?

(b) Water flows through a pipe as shown in the figure 1 at a rate of 80litre per second. if the pressure at point 1 is 180KPa



Find (i) The velocity at point 1

(ii) The velocity at point 2

(iii) The pressure at point 2

Density of water = 1000kgm^{-3}

g = 9.8m/s^2.

(c) (i) Is viscosity of liquid related to its density?

(ii) A metal plate 0.04m^2 in area is lying on a liquid layer of thickness 10^{-3}m and coefficient of viscosity 14Nsm^{-2}. Calculate the horizontal force needed to move the plate with a speed of 0.040m/s.

Solution

(a) (i)
- Laminar or steady fluid flow.
- Irrotatinal fluid flow
- Incompressible fluid flow
- Non – viscous fluid flow

(ii) This is the statement of the conservation of mass in fluid mechanics. State that ' In any steady state process, the rate at which mass enters a system is equal to the rate at which mass of fluid leaves the system provided there is no leakage of the fluid in the system.

(iii) Yes. The pressure differences in the tube of fluid flow creates a force which tend to accelerate the flowing of a liquid.

(b) (i) $\frac{v}{t} = 80\,\text{L}/\text{S} = 0.80\text{m}^3/\text{s} = Q$

P_1 = 180KPa = 180,000Nm^{-2}

Now $Q = A_1V_1$, $V_1 = \frac{Q}{\pi r_1^2}$

$$V_1 = \frac{0.80}{\pi(0.16)^2}$$

$$V_1 = 0.995\text{m/s}$$

(ii) $V_2 = \frac{Q}{\pi r_2^2} = \frac{0.08}{\pi(0.08)^2}$

$V_2 = 3.981\text{m/s}$

(iii) According to the Bernoulli's equation

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

$$P_2 = P_1 + \rho g(h_1 - h_2) + \frac{1}{2}\rho(V_1^2 - V_2^2)$$

$$= 180,000 + 1000 \times 9.8[0-2] + \frac{1}{2} \times 1000$$

$$\times\left[(0.995)^2 - (4.981)^2\right]$$

$$P_2 = 152971\text{Nm}^{-2} = 152.971\text{kPa}$$

(c) (i) Viscosity and density are not related liquid with similar densities can have very difference in viscosity. Density remains generally the same regardless of the temperature of a liquid but viscosity generally changes quit dramatically with temperature so viscosity is very sensitive to temperature change.

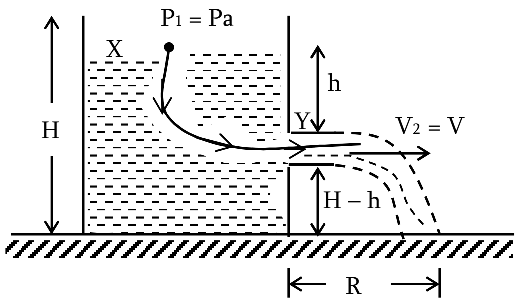
(ii) $F = \eta A \frac{dv}{dx}$

$$= 14 \times 0.04 \times \frac{0.04}{10^{-3}}$$

$$F = 22.4\text{N}$$

5. FLOWING OF THE LIQUID FROM THE WIDE TANK

Consider an ideal liquid of density ρ contained in a vessel having an orifice y near its bottom. the vessel is assumed to be sufficiently wide and the opening y sufficiently narrow. So the velocity of the liquid on the its free surface can be taken as zero.



At X : $P_1 = Pa$, $V_1 = 0$, $h_1 = h$
At Y : $P_2 = Pa$, $V_2 = V$, $h_2 = 0$
Applying the Bernoulli's theorem

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

$$Pa + \frac{1}{2}\rho(0)^2 + \rho gh = Pa + \frac{1}{2}\rho V^2 + \rho g(0)$$

$$\rho gh = \frac{1}{2}\rho V^2$$

$$V^2 = 2gh$$

$$V = \sqrt{2gh}$$

This is the equation of velocity of efflux. Let a body fall freely from the level of the free surface of the liquid. Let V be the velocity acquired from the free surface of the liquid to the orifice level.

Now, $U = 0$, $a = g$

$$V^2 - U^2 = 2gh$$

$$V^2 - 0^2 = 2gh$$

$$V = \sqrt{2gh}$$

This proves Torricelli's theorem

Torricelli's theorem

State that 'The velocity of imaginary liquid (velocity of efflux) is same as that would be attained if it fall under gravity. Sometimes Torricelli's theorem is known as 'Law of efflux.

Definition: Velocity of efflux is the velocity of emerging fluid from the orifice

$$V = \sqrt{2gh}$$

- Expression of horizontal range

  The escaping liquid from orifice is in the form of a parabola.

  Let t = time taken by the liquid to strike on the ground.

$$H - h = Ut + \frac{1}{2}gt^2 \quad , \quad U = 0$$

$$H - h = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2(H-h)}{g}}$$

Horizontal range

$$R = V.t = \sqrt{2gh} \times \sqrt{\frac{2(H-h)}{g}}$$

$$R = \sqrt{4h(H-h)}$$

$$R = 2\sqrt{h(H-h)}$$

Condition for the maximum horizontal range.

$$R^2 = 4h(H-h)4Hh - 4h^2$$

Differentiate R w.r.t h

$$2R\frac{dR}{dh} = \frac{d}{dh}\left[4Hh - 4h^2\right]$$

$$2R\frac{dR}{dh} = 4H - 8h$$

When $R = R_{max}$, $\frac{dR}{dh} = 0$

$$0 = 4H - 8h \quad , \quad h = \frac{H}{2}$$

- Expression of the maximum horizontal range.

  Since

$$R = 2\sqrt{h(H-h)}$$

$$R_{max} = 2\sqrt{H/2\left(H - H/2\right)} = 2\sqrt{H/2 \,.\, H/2}$$

$$R_{max} = H$$

DEFINITION

- **DISCHARGE RATE** – is the volume of liquid passing a given cross – section in a unit time.

  $$\text{Discharge rate } Q = AV = \frac{\text{volume}}{\text{time}}$$

  S.I unit of discharge rate is m^3/s.

- Mean velocity – is the discharge rate per unit cross – section area.

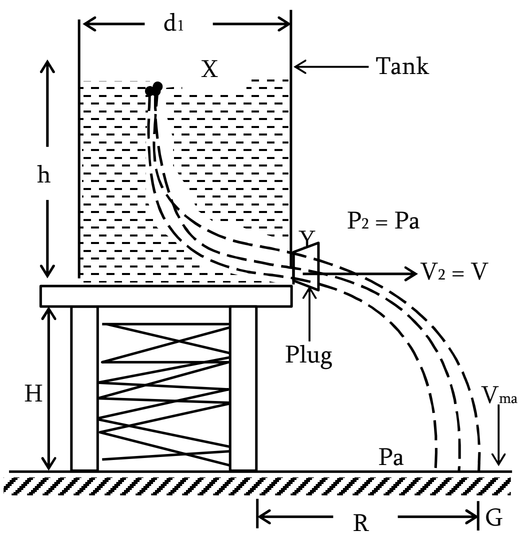
  $$V_m = \frac{Q}{A}\left(m/s\right)$$

- **MASS FLOW RATE** – is the mass of the fluid passing a given section per unit time. S.I unit of mass flow rate is kg/s.

- **UNIFORM FLOW** – is the flow in which cross – sectional area and velocity of the stream of the fluid are the same at each cross – section.

SPECIAL CASE FOR THE FLOWING OF LIQUID FROM THE WIDE TANK.

Consider a cylindrical tank of diameter d1 rest on the top of platform of height H. If initially the tank is full filled with the height



If a small hole is made at the bottom of one side of the tank. If the pheg is removed from orifice, water flows out from the tank as shown in the figure above

(i) Expression of the velocity of efflux

According to the Torricelli's theorem

$$V = V_2 = \sqrt{2gh}$$

(ii) Expression of the maximum velocity of water strike on the ground level.

According to the Bernoulli's equation

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_3 + \frac{1}{2}\rho V_3^2 + \rho g h_3$$

$$Pa + \frac{1}{2}\rho\left(0\right)^2 + \rho g\left(H+h\right) = Pa + \frac{1}{2}\rho V_{max}^2 + \rho g\left(0\right)$$

$$\rho g\left(H+h\right) = \frac{1}{2}\rho V_{max}^2$$

Attentive method

By using principle of conservation of mechanic energy

p.e + k.e = constant

$$Mg\left(H+h\right) + \frac{1}{2}M\left(0\right)^2 = Mg\left(0\right) + \frac{1}{2}MV_{max}^2$$

$$Mg\left(H+h\right) = \frac{1}{2}MV_{max}^2$$

$$V_{max} = \sqrt{2g\left(H+h\right)}$$

(iii) Expression of the time taken in order the tank to become empty

Apply the continuity equation

$$A_1V_1 = A_2V_2$$

But $V_2 = \sqrt{2gh}$, $V_1 = -\frac{dh}{dt}$

Negative sign show that the height of the liquid in the tank decreases as time increases

$$-A_1\frac{dh}{dt} = A_2\sqrt{2gh}$$

$$dt = \frac{-A_1}{A_2}\cdot\frac{dh}{\sqrt{2g}\cdot\sqrt{h}}$$

$$dt = \frac{-A_1}{A_2\sqrt{2g}}\cdot h^{1/2}dh$$

$$\int_0^t dt = \frac{-A_1}{A_2\sqrt{2g}}\int_{ho}^{0} h^{-1/2}dh$$

$$\left[t\right]_0^t = \frac{-A_1}{A_2\cdot\sqrt{2g}}\left[\frac{h}{1/2}\right]_{ho}^{o}$$

$$t - 0 = \frac{-A_1}{A_2\cdot\sqrt{2g}}\cdot\left[2\sqrt{h}\right]_{ho}^{0}$$

$$= \frac{2A_1}{A_2\sqrt{2g}}\sqrt{ho} = \frac{A_1}{A_2}\cdot\sqrt{\frac{4ho}{2g}}$$

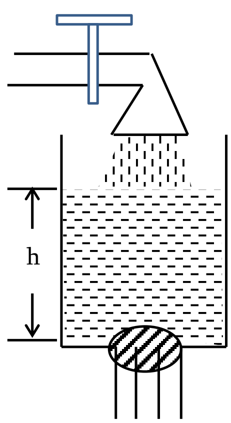
$$t = \frac{A_1}{A_2}\cdot\sqrt{\frac{2ho}{g}}$$

A_1 = cross – sectional area of wide to tank
A_2 = cross – sectional area of plug orifice
ho = Initial height of water inside of the tank.

Example – 44

Water flows into a tank of large cross – sectional area at the rate of $100m^3/s$ and flows out at the same rate through a hole at the base of a tank. If the hole has an area of $10m^2$. Find the height to which the water risen in the tank ($g = 9.8m/s^2$).

Solution



Apply continuity equation
$Q = AV$
According to the Torricelli's theorem

$$V = \sqrt{2gh}$$

$$Q = A\cdot\sqrt{2gh}$$

$$h = \frac{1}{2g}\left[\frac{Q}{A}\right]^2$$

$$h = \frac{1}{2\times 9.8}\left[\frac{100}{10}\right]^2$$

$$h = 5.10m$$

Example – 45

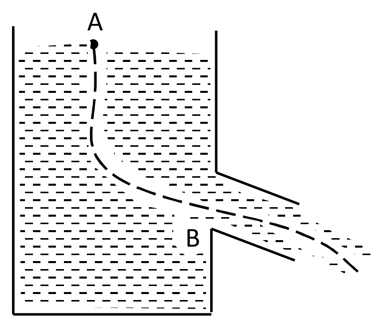
(a) State Bernoulli's principle and give the conditions under which it is applicable.
(b) Use Bernoulli's principle or otherwise explain the following:
 (i) Aerofoil lift
 (ii) Torricelli's theorem

Example – 46

(a) In a steady fluid flow, the velocity vector V at any point is constant. Can there be accelerated motion of the fluid particles? Explain.
(b) Fluid flow can be said to be
 (i) Rotational or irrotational
 (ii) Compressible or incompressible
 (iii) Viscous or non – viscous.
 Briefly explain each of the above fluid flow characteristics.
(c) Bernoulli's equation is given by

$$P + \frac{1}{2}\rho V^2 + \rho gh = \text{constant}$$

 (i) Explain each of the quantities involves.
 (ii) Under what conditions is equation applicable?
(d) The diagram below shows water discharging from an orifice in large tank. Apply Bernoulli's to a streamline connecting point A and B to show that the speed of efflux is $V = \sqrt{2gh}$

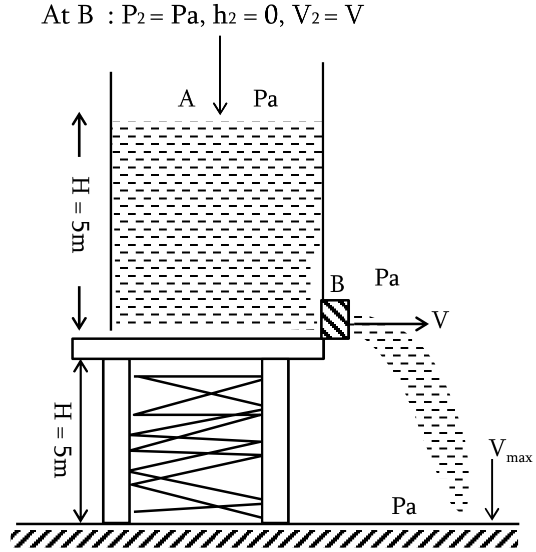


Example – 47

A cylindrical tank 1m is radius rests on a platform 5m high. Initial the tank is filled with water to a height of 5m. A plug whose area is $10^{-4}m^2$ is removed from an orifice on the side of the tank at the bottom calculates:-
(i) Initial speed with which the water flows from the orifice.
(ii) Initial speed with which water strikes the ground ($g = 10m/s^2$).

Solution

(i) At A : $P_1 = Pa$, $h_1 = h$, $V_1 = 0$
 At B : $P_2 = Pa$, $h_2 = 0$, $V_2 = V$



Apply Bernoulli's principle

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

$$Pa + \frac{1}{2}\rho(0)^2 + \rho g h = Pa + \frac{1}{2}\rho V^2 + \rho g(0)$$

$$V = \sqrt{2gh} = \sqrt{2\times10\times5}$$

$$V = 10\text{m/s}$$

(ii) Again, Bernoulli's equation

$$Pa + \frac{1}{2}\rho(0)^2 + \rho g(H+h) = Pa + \frac{1}{2}\rho V_{max}^2 + \rho g(0)$$

$$V_{max} = \sqrt{2g(H+h)} = \sqrt{2\times10[5+5]}$$

$$V_{max} = 14.1\text{m / s}$$

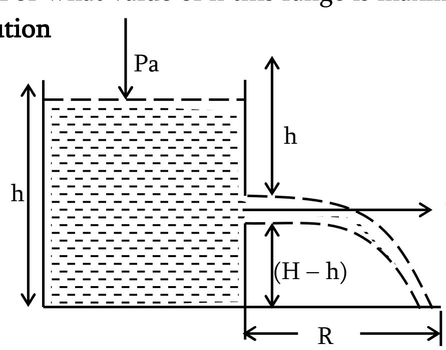
Example – 48

A water stands at a depth H in a tank whose side wall are vertical. A hole is made on one of the walls at a depth, h below the water surface.

(i) At what distance R from the foot of the wall does the emerging stream of water strike the floor?

(ii) For what value of h this range is maximum?

Solution



(i) Apply Bernoulli's equation

$$Pa + \frac{1}{2}\rho(0)^2 + \rho g h = Pa + \frac{1}{2}\rho V^2 + \rho g(0)$$

$$V^2 = 2gh$$

$$V = \sqrt{2gh}$$

Time taken by water to fall through the height (H – h)

$$H - h = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2(H-h)}{g}}$$

Horizontal range, R = V.t

$$R = \sqrt{2gh}\cdot\sqrt{\frac{2(H-h)}{g}}$$

$$R = 2\sqrt{h(H-h)}$$

(ii) Since

$$R = 2\sqrt{h(H-h)} \quad \text{or} \quad R^2 = 4h(H-h)$$

$$R^2 = 4Hh - 4h^2$$

$$\frac{d}{dh}\left[R^2\right] = \frac{d}{dh}\left[4Hh - 4h^2\right]$$

$$2R\frac{dR}{dh} = 4H - 8h$$

When $R = R_{max}$, $\frac{dR}{dh} = 0$

$$0 = 4H - 8h$$

$$h = \frac{H}{2}$$

Example – 49

(b) State Bernoulli's equation and the continuity equation.

(c) A cylindrical tank of diameter 90cm rests on top of platform 6m high. Initial the tank is filled with water to a depth of 3m. A plug whose area is 3cm² is removed from the side of the tank at the bottom.

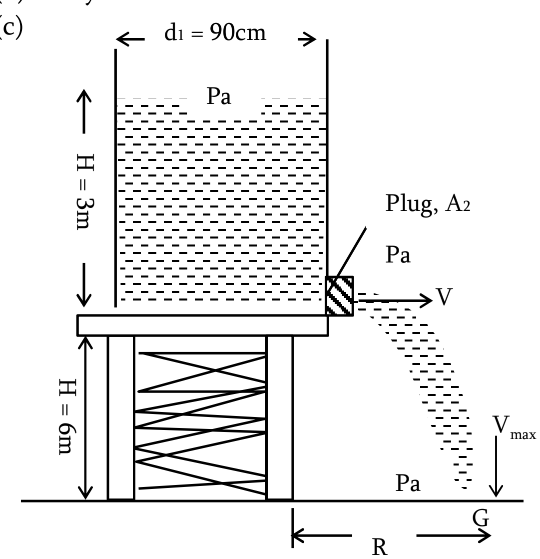
(i) At what speed will water strike the ground?

(ii) How long will it take for the tank to be empty?

Solution

(b) See your notes

(c)



(i) Apply Bernoulli's equation

$$Pa + \frac{1}{2}\rho(0)^2 + \rho g(H+h) = Pa + \frac{1}{2}\rho V_{max}^2 + \rho g(0)$$

$$V_{max}^2 = 2g(H+h)$$

$$V_{max} = \sqrt{2g(H+h)} = \sqrt{2\times 9.8(6+3)}$$

$$V_{max} = 13.28\text{m/s}$$

(ii) Let A_1 = Cross – sectional area of the tank

A_2 = cross – sectional area of the small hole

$$A_1V_1 = A_2V_2$$

$$V_2 = \sqrt{2gh} \quad , \quad V_1 = {dh}/{dt}$$

Minus sign show that the height h inside of tank decreases with increase of the time.

$$-A_1\frac{dh}{dt} = A_2\cdot\sqrt{2gh}$$

$$\frac{-\pi d^2}{4}\cdot\frac{dh}{dt} = A_2\sqrt{2gh}$$

$$dt = \frac{-\pi d^2}{4A_2}\cdot\frac{dh}{\sqrt{2gh}}$$

$$\int_0^t dt = \frac{-\pi d^2}{4A_2}\cdot\int_2^0 \frac{h^{1/2}dh}{\sqrt{2g}}$$

$$t = \frac{3.14(0.9)^2}{4\times 3.10^{-4}\times\sqrt{2\times 9.81}}\left[2\sqrt{h}\right]_0^3$$

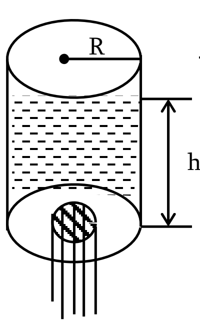
$$t = 1658.80\,\text{sec}$$

Example – 50

A cylinder tank of height 0.4m is open at the taop and has a diameter 0.16m, water is filled it up to a height of 0.16m. Calculate how long will it take to empty the tank through a hole of radius 5 × 10^{-3}m in its bottom

Solution

Apply the continuity equation



$$-\pi R^2\frac{dh}{dt} = \pi r^2 V$$

$$V = \sqrt{2gh}$$

$$-R^2\frac{dh}{dt} = \left(\frac{r}{R}\right)^2\cdot\sqrt{2g}\cdot h^{1/2}$$

$$-h^{-1/2}dh = 0.0173dt$$

$$dt = \frac{-1}{0.0173}h^{-1/2}dh$$

$$\int_0^t dt = \frac{-1}{0.173}\int_{0.16}^0 h^{-1/2}dh$$

$$t = 46.265\text{second}$$

Example – 51

(a) State the continuity principle

(b) Derive Bernoulli's equation. Hence state Bernoulli's principle of an incompressible fluid flowing in a pipe.

(c) Water is supplied to a both tube from an overhead cylindrical simtank of maximum volume 1500litres and cross – sectional diameter 1.5m. The tank is placed on a plat form raised at a height of 4.5m above ground. A supply pipe of diameter 3.5cm runs horizontally for a length of 2.0m from the tank and vertically downwards for a depth of 4m where it is connected to the baht tap. Determine the velocity and pressure of water at the tap's orifice when fully opened.

Solution

(a) And (b) see your notes

(c) At the outlet of the simtank

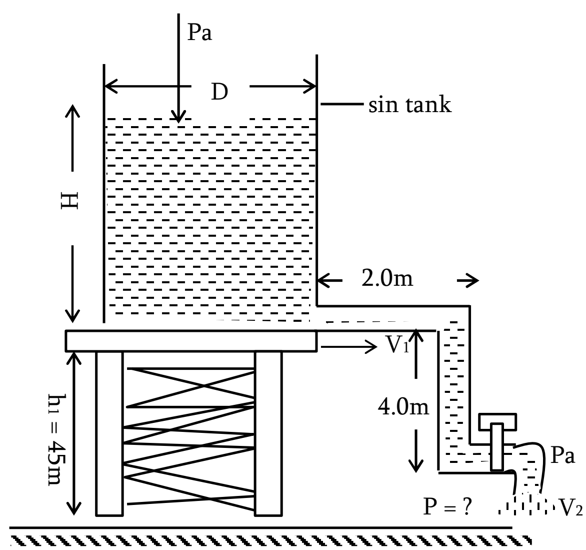
$$P_1 = 1.0\times 10^5\,\text{Pa} \quad , \quad h_1 = 4.5\text{m}$$

$$V_1 = \sqrt{2gH}\ \text{(Torricelli's theorem)}$$

$$V_o = AH = \frac{\pi D^2 H}{4}$$

$$H = \frac{4V_o}{\pi D^2} = \frac{4\times 1.5}{3.14\times(1.5)^2}$$

$$H = 0.85\text{m}$$



$V_1 = \sqrt{2\times 9.8\times 0.85} = 4.08\text{m/s}$

At the taps of orifice

$P_2 = P_1 = 1.01\times 10^5\,\text{Pa}$

$h_2 = 4.5 - 40 = 0.5\text{m} \quad , \quad V_2 = ?$

Apply Bernoulli's equation

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

$$V_2^2 = V_1^2 + 2g\left(h_1 - h_2\right)$$

$$V_2 = \sqrt{V_1^2 + 2g\left(h_1 - h_2\right)}$$

$$= \sqrt{\left(4.08\right)^2 + 2\times 9.8\left(4.5 - 0.5\right)}$$

$$V_2 = 9.75\text{m/s}$$

$\therefore$ The velocity at the tap's orifice

$$V_2 = 9.75\text{m/s}$$

Pressure at the orifice

$$P = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

$$= 1.01\times 10^5 + \frac{1}{2}\times 1000\left(9.75\right)^2 + 1000\times 9.8\times 0.5$$

$$P = 1.53\times 10^5\,\text{Pa}$$

Example – 52

(a) A water barrel stands on a table of height , h . If a small hole is punched in the side of the barrel at its base is found that the resultant stream of water strike the ground at a horizontal distance R from the barrel. What is the depth of water in the barrel.

(b) A hole of area 1mm² opens in the pipe near the lower end of a large water – storage tank and a stream of water shoots from it. If the top of the water in the tank is 20m above the point of the tank, how much water escapes in 1 seconds?

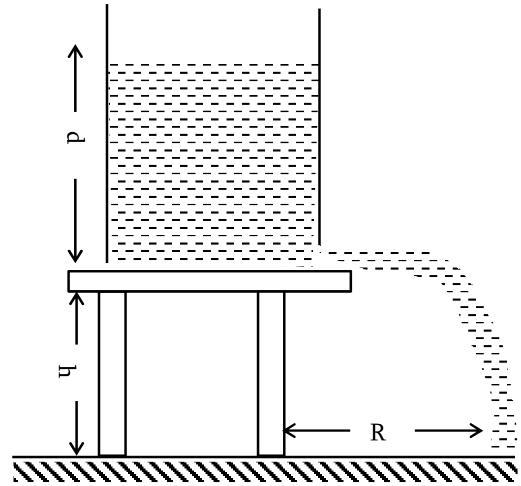
Solution

(a) $V = \sqrt{2gd}$ (Torricelli's theorem)

Time taken by the water to strike on the ground.

$$h = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2h}{g}}$$



Horizontal range, R = V.t

$$R = \sqrt{2gd}\times\sqrt{{}^{2h}\!/_{g}} = \sqrt{4dh}$$

$$R^2 = 4dh$$

$$d = \frac{R^2}{4h}$$

(b) Rate of volume of water flows

$$Q = A\sqrt{2gh}$$

$$= 1\times\sqrt{2\times 9.8\times 10^3\times 10^3}$$

$$Q = 19800\text{mm/s} = 19.8\text{m}^3\text{/sec}$$

Example – 53

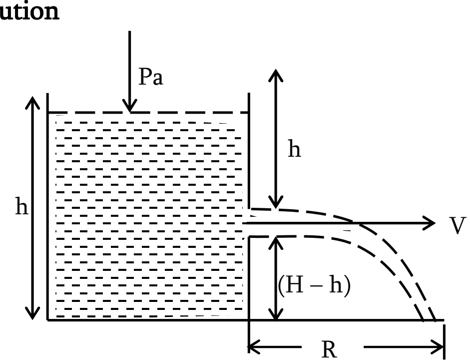
A tank is filled with water up to a height H. a hole is punched in one of the walls at a depth h1 below the water surface.

(a) Find the distance from the foot of the wall at which the stream strikes the floor.

(b) Is it possible to make second hole at another depth so that the stream also has the same range? If so find its depth?

Solution

(a)



According to the Torricelli's theorem velocity of efflux $V = \sqrt{2gh_1}$

Time taken by water to strike on the ground

$$H - h_1 = \frac{1}{2}gt^2 \quad , \quad t = \sqrt{\frac{2(H-h_1)}{g}}$$

Horizontal range, R = V.t

$$R = \sqrt{2gh_1} \cdot \sqrt{\frac{2(H-h_1)}{g}}$$

$$R = 2\sqrt{h_1(H-h_1)}$$

(b) Let the second hole be directed at a depth h_2 below the free surface of water.

$$R_2 = 2\sqrt{h_2(H-h_2)}$$

Since $\quad R = R_2$

$$2\sqrt{h_1(H-h_1)} = 2\sqrt{h_2(H-h_2)}$$

$$h_1H - h_1^2 = h_2H - h_2^2$$

$$h_2H - h_1H - h_2^2 + h_1^2 = 0$$

$$(h_2 - h_1)H + (h_1^2 - h_1^2) = 0$$

$$(h_2 - h_1)H - (h_2^2 - h_1^2) = 0$$

$$(h_2 - h_1)H - (h_2 - h_1)(h_2 + h_1) = 0$$

$$(h_2 - h_1)(H - h_2 - h_1) = 0$$

$$h_2 - h_1 = 0 \text{ or } H - h_2 - h_1 = 0$$

$$h_2 = h_1 \text{ or } h_2 = H - h_1$$

∴ The second hole is at a depth $H - h_1$ from the free surface so that it also give the same range.

Example – 54

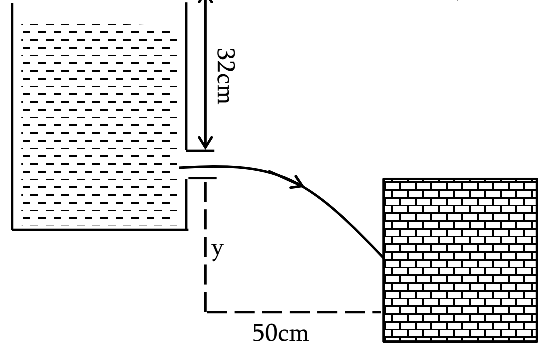
Water from a tank issues into air through a horizontal nozzle and sticks a fixed wall at right angle to the nozzle and 50.0cm from it. If the depth of the nozzle below the free surface of water in the tank is 32cm, calculate:-

(i) Velocity of efflux of water

(ii) The vertical distance below the nozzle where the jet strikes the wall.

Solution

x = 50cm, h = 32cm



(i) According to the Torricelli's theorem velocity of efflux , $V = \sqrt{2gh}$

$$V = \sqrt{2 \times 980 \times 32}$$

$$V = 250.44\text{cm/s}$$

(ii) y = ?

The liquid coming out of the nozzle becomes horizontally projected. Projectile whose equation of trajectory

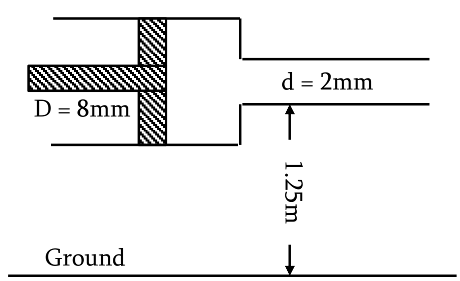
$$X^2 = \frac{2V^2y}{g}$$

$$y = \frac{X^2g}{2V^2} = \frac{(50)^2 \times 980}{2 \times (250.44)^2}$$

$$y = 19.53\text{cm}$$

Example – 55

Consider a horizontally oriented syringe containing water located at a height of 1.25m above the ground. The diameter (D) of the plunger is 8mm and the diameter (d) of the nozzle is 2mm. the plunger is pushed with a constant speed of 0.25m/s. Find the horizontal range of water stream on the ground. Take $g = 10\text{m/s}^2$.



Solution

Apply the continuity equation

$$A_1V_1 = A_2V_2$$

$$\frac{\pi d^2}{4}V_1 = \frac{\pi d^2}{4}V_2$$

$$V_2 = \left(\frac{D}{d}\right)^2 V_1$$

Horizontal range

$$R = V_2 t \quad \text{but} \quad t = \sqrt{\frac{2h}{g}}$$

$$R = \left(\frac{D}{d}\right)^2 \cdot V_1 \cdot \sqrt{\frac{2h}{g}}$$

$$= \left(\frac{8}{2}\right)^2 \times 0.25\sqrt{\frac{2\times 1.2s}{10}}$$

$$R = 2m$$

Example – 56

(a) A tank having cross – sectional area, A is filled with water to a height H_1. If a hole of cross – sectional area 'a' is made at the bottom of the tank, then find the time taken by the water level to decrease from H_1 to H_2.

(b) A cylindrical tank of height 0.4m is open at the top and has a diameter 0.16m. Water is filled in it up to a height of 0.16m. Calculate how long will it take to empty the tank through a hole of radius 5×10^{-3}m in its bottom.

Solution

(a) Let h be the height of the water level at any instant. Then the rate of decrease of water level is $-dh/dt$

$$-A\frac{dh}{dt} = av = a\sqrt{2gh}$$

$$\frac{-dh}{dt} = \frac{a}{A}\cdot\sqrt{2gh}$$

$$\int_{H_1}^{H_2} h^{-1/2}\,dh = \frac{a}{A}\sqrt{2g}\int_0^t dt$$

$$-2\left[h^{1/2}\right]_{H_1}^{H_2} = \frac{a}{A}\sqrt{2gt}$$

$$2\left[\sqrt{H_1}-\sqrt{H_2}\right] = \frac{a}{A}\cdot\sqrt{2g}\ t$$

$$t = \frac{A}{a}\left[\sqrt{\frac{2}{g}}\left(\sqrt{H_1}-\sqrt{H_2}\right)\right]$$

$$t = \frac{A}{a}\cdot\sqrt{\frac{2}{g}}\left[\sqrt{H_1}-\sqrt{H_2}\right]$$

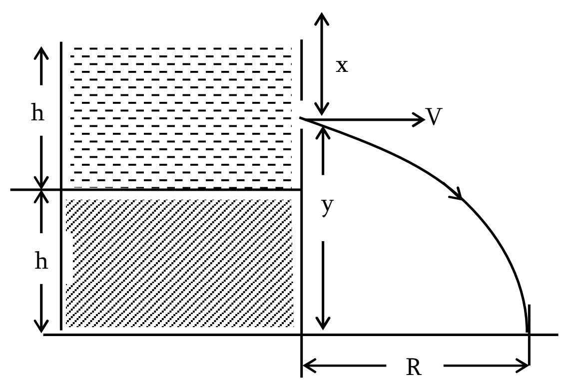
(b) $$t = \left(\frac{0.08}{5\times10^{-3}}\right)^2\sqrt{\frac{2}{9.8}}\left[\sqrt{0.16}-\sqrt{0}\right]$$

$$t = 46.3\,\text{sec}$$

Example – 57

A tank is mounted so that its base is at a height H above the horizontal ground. The tank is filled with water to a depth h. a hole is punched in the side of the tank at a depth X below the water surface. Find the value of X so that the range of the emerging stream is maximum.

Solution



If t is the time taken by the stream to reach on the ground.

$$y = \frac{1}{2}gt^2 \quad \text{or} \quad t = \sqrt{\frac{2y}{g}}$$

$$R = V.t \quad \text{but} \quad V = \sqrt{2gx}$$

$$R = \sqrt{\frac{2y}{g}}\times\sqrt{2gx} = \sqrt{4yx}$$

$$y = H + h - x$$

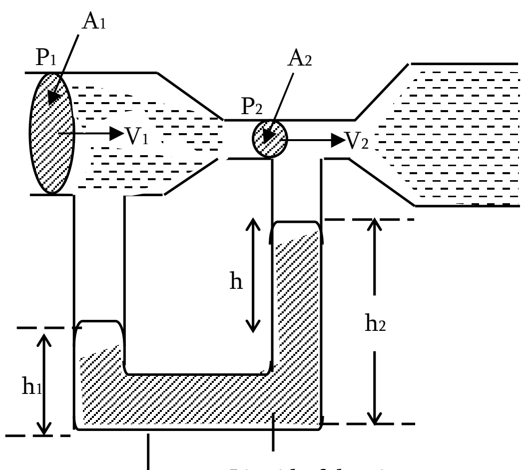
$$R = 2\sqrt{x\left(H+h-x\right)}$$

For R to be maximum $\frac{dR}{dx} = 0$

Solving , we get $X = \frac{H+h}{2}$

6. FLOWMETER – VENTURIMETER

Venturimeter – is the device which is used to measure the rate of volume of liquid flowing through the pipe. It is gauge used for measuring the rate of flow of a fluid when the motion of the fluid is steady the venture tube consists of two horizontal wide tubes joined by a narrow co – axial tube called the throat of the venturimeter. Consider the figure below which show diagram of the venturimeter.



Manometer is used to determine the pressure difference between the pipe at the point A and B. This can be shown by the difference in height of the liquid column in the manometer.

Apply the continuity equation

$$A_1V_1 = A_2V_2$$

$$V_1 = \left(\frac{A_2}{A_1}\right)V_2$$

$$V_1^2 = \frac{A_2^2}{A_1^2}V_2^2 \text{.......(1)}$$

Apply Bernoulli's theorem for the horizontal pipe

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

Since $P_1 > P_2$; $A_1 > A_2$; $V_2 > V_1$

$$P_1 - P_2 = \frac{1}{2}\rho\left[V_2^2 - V_1^2\right] \text{.......(2)}$$

Putting equation (1) into (2)

$$P_1 - P_2 = \frac{1}{2}\rho\left[V_2^2 - \frac{A_2^2V_2^2}{A_1^2}\right]$$

$$\frac{2(P_1 - P_2)}{\rho} = V_2^2\left[\frac{A_1^2 - A_2^2}{A_1^2}\right]$$

$$V_2 = A_1\sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

Rate of volume of the liquid flow

$$Q = A_2V_2 = A_2V_2$$

$$Q = A_1A_2 \cdot \sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

The manometer contains mercury

$$P_1 - P_2 = h\rho mg$$

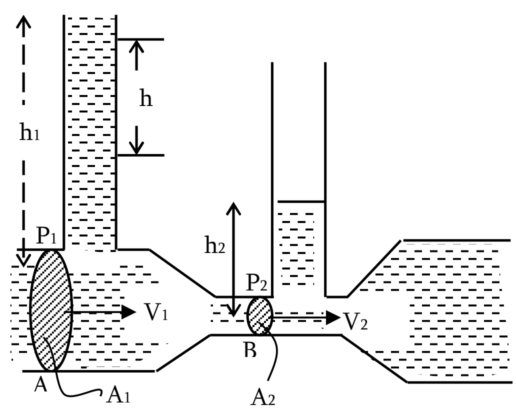
$$Q = A_1A_2 \cdot \sqrt{\frac{2\rho mgh}{\rho(A_1^2 - A_2^2)}}$$

Let: $$K = A_1A_2 \cdot \sqrt{\frac{2\rho mg}{\rho(A_1^2 - A_2^2)}}$$

$$Q = K\sqrt{h} \quad \text{or} \quad Q \alpha \sqrt{h}$$

SPECIAL CASE OF VENTURIMETER

Consider the flowing of liquid (water) on the flow – meter as liquid itself on the tube determine pressure difference as shown on the figure below



Applying the Bernoulli's theorem for horizontal flow at A and B

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho\left[V_2^2 - V_1^2\right] \text{.......(1)}$$

The level of liquid in the manometer at A is higher than the level of liquid in the manometer at B. this shows that P_1 is greater than P_2. Let h be the difference in the levels of the liquid.

$$P_1 - P_2 = h\rho g \text{ ....(2)}$$

(1) = (2)

$$h\rho g = \frac{1}{2}\rho\left[V_2^2 - V_1^2\right]$$

$$2gh = V_2^2 - V_1^2 \text{ ....(3)}$$

Applying the continuity equation

$$A_1V_1 = A_2V_2 = Q$$

$$V_1 = \frac{Q}{A_1}, \quad V_2 = \frac{Q}{A_2}$$

$$2gh = \frac{Q}{A_2^2} - \frac{Q}{A_1^2}$$

$$2gh = Q^2\left[\frac{A_1^2 - A_2^2}{A_1^2A_2^2}\right]$$

$$Q^2 = 2gh\frac{A_1^2A_2}{A_1^2 - A_2^2}$$

$$Q = A_1A_2\sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

$$\text{Let } K = \sqrt{2g}\cdot\frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}$$

K = Constant for given venturimeter

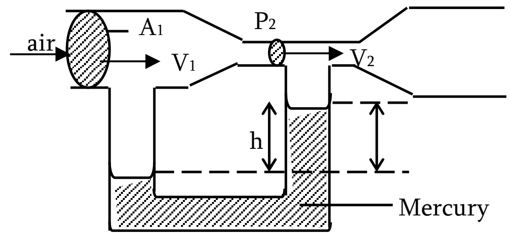
$$Q = K\sqrt{h}, \quad Q \propto \sqrt{h}$$

DEFINITION

- **ABSOLUTE PRESSURE** – Is the actual or total pressure of the gas or liquid.
- **GAUGE PRESSURE** – Is the difference between the absolute pressure and atmospheric pressure.
- **ATMOSPHERIC PRESSURE** – Is the pressure exerted by the atmosphere.

Example – 59

Air flows through the horizontal main tube of the venturimeter as shown on the figure below. If the U – tube of the meter contains mercury, find the mercury – level difference h between the two arms, given that radii of the wide and narrow parts of the main tube be r_1 = 1.0cm and r_2 = 0.5cm respectively and let speed of air entering the meter be V_1 = 15cm/s, the density of air , ρ_{air} = 1.3kgm^{-3} and density of mercury is ρm = 13600kgm^{-3}.



Solution

Apply the continuity equation

$$A_1V_1 = A_2V_2$$

$$\pi r_1^2V_1 = \pi r_2^2V_2$$

$$V_2 = \left(\frac{r_1}{r_2}\right)^2 V_1$$

Apply the Bernoulli's equation for the horizontal pipe.

$$P_1 + \frac{1}{2}\rho_{air}V_1^2 = P_2 + \frac{1}{2}\rho_{air}V_2^2$$

Since $A_1 > A_2$, $P_1 > P_2$

$$\Delta P = P_1 - P_2 = \frac{1}{2}\rho_{air}\left[V_2^2 - V_1^2\right]$$

$$\Delta P = \frac{1}{2}\rho_{air}\left[V_1^2\left(\frac{r_1}{r_2}\right)^4 - V_1^2\right]$$

$$\rho_m gh = \frac{1}{2}\rho_{air}V_1^2\left[\left(\frac{r_1}{r_2}\right)^4 - 1\right]$$

$$h = \frac{\rho_{air}V_1^2}{2g\rho_m}\left[\left(\frac{r_1}{r_2}\right)^4 - 1\right]$$

$$= \frac{1.3(15)^2}{2\times 9.8\times 13600}\left[\left(\frac{1}{0.5}\right)^4 - 1\right]$$

$$h = 0.016\text{m} = 1.6\text{cm}$$

Example – 60

A venturimeter has a throat of diameter 0.06m. The diameter of the horizontal pipeline where the venturimeter is inserted is 0.1m. The pressure difference between the main line and the throat is 0.32m of water. Calculate the rate of volume of water in the pipe line.

Solution

$$A_1 = \pi r_1^2 = (0.05)^2\pi = 0.0025\pi\text{m}^2$$

$$A_2 = \pi r_2^2 = (0.03)^2\pi = 0.0009\pi\text{m}^2$$

Rate of flow of water

$$Q = A_1A_2\sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

$$= (0.0025\pi)(0.0009\pi)\sqrt{\frac{2\times 9.8\times 0.32}{(2.5\times 10^{-3}\pi)^2 - (9\pi\times 10^{-4})^2}}$$

$$Q = 0.00762\text{m}^3/\text{sec}$$

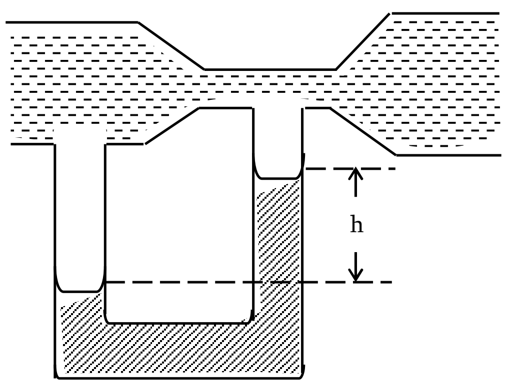
Example – 61

The cross – section area of the pipe shown in the figure below is 50cm² at the wider portions and 20cm² at the constriction. The rate of flow of water through the pipe is 4000cm³/s. Find.

(i) The velocity at the wide and the narrow portions.

(ii) The pressure difference between the points.

(iii) The difference in height between the mercury columns in the U – tube



Solution

(i) Let V_1 = Velocity at wide portions

$$Q = A_1V_1$$

$$V_1 = \frac{Q}{A_1} = \frac{4000}{50} = 80\text{cm/s}$$

$$V_1 = 0.8\text{m/s}$$

Let V_2 = velocity at narrow portion

$$V_2 = \frac{Q}{A_2} = \frac{4000\text{cm}^3/\text{s}}{20\text{cm}/\text{s}} = 200\text{cm}/\text{s}$$

$$V_2 = 2\text{m}/\text{s}$$

(ii) Applying the Bernoulli's equation for the horizontal pipe.

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho\left[V_2^2 - V_1^2\right]$$

$$= \frac{1}{2}\times 13600\left[2^2 - (0.8)^2\right]$$

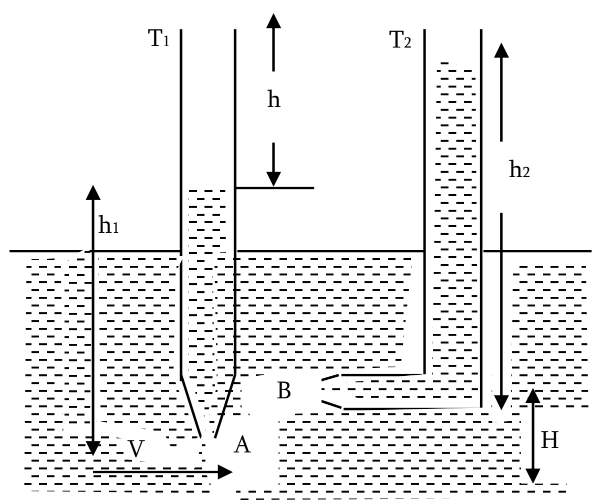
$$P_1 - P_2 = 1680\text{Pa}$$

(iii) $h = \frac{P_1 - P_2}{\rho g} = \frac{1680}{13600\times 9.8}$

$$h = 0.013\text{m}$$

7. PITOT – STATIC TUBE

It is a device used for measuring the velocity of flow and hence the rate of flow at any depth in a flowing liquid. It is based on Bernoulli's principle the arrangement consists of two vertical tubes T_1 and T_2 with small openings at lower ends at the same height H above the ground



The plane of the aperture at A is parallel to the direction of flow of the liquid. The tube T_2 is L – shaped so that the plane of the operture at B is perpendicular to the direction of flow of the liquid.

Let: P_1 = Liquid pressure at A

Total energy per unit volume at A

$$= P_1 + \frac{1}{2}\rho V^2 + \rho gH$$

Since the nose B is perpendicular to the direction of flow, therefore the liquid will be stopped at B. B is the called Stagnation point.

Let P_2 = Pressure of liquid at B. It is called stagnation pressure.

Total energy per unit volume at B

$= P_2 + \rho g H$

Applying Bernoulli's theorem to points A and B

$$P_1 + \frac{1}{2}\rho V^2 + \rho g h = P_2 + \rho g H$$

$$P_2 - P_1 = \frac{1}{2}\rho V^2$$

But $P_2 - P_1 = h\rho g$, ρ is the density of the liquid.

$$\rho g h = \frac{1}{2}\rho V^2$$

$$V = \sqrt{2gh} = \sqrt{\frac{2(P_2 - P_1)}{\rho}}$$

Rate of liquid flow, $Q = AV$

$$Q = A\sqrt{2gh} = A \cdot \sqrt{\frac{2(P_2 - P_1)}{\rho}}$$

Note that:

1. The total pressure within the moving fluid cab be considered to have two components.
   (i) Static pressure which it would have if the fluid is at rest.
   (ii) Dynamic pressure which is the pressure equivalent of its velocity.

DEFINITION

STATIC PRESSURE – is the actual pressure of the liquid at a point due to its rest position of the fluid.

DYNAMIC PRESSURE is the pressure exerted by the fluid due to its own motion.

$$Pd = \frac{1}{2}\rho V^2$$

TOTAL PRESSURE – is the sum of dynamic and static pressure along the tube of flow.

$$P_T = P_S + Pd$$

$$P_T = P_S + \frac{1}{2}\rho V^2$$

$$V = \sqrt{\frac{2(P_T - P_S)}{\rho}}$$

The pressure which can be exerted on the fluid at A and B i.e static and pitot tube respectively is due to the atmospheric pressure and pressure due to the height.

At point A: $P_1 = Pa + \rho g h_1$

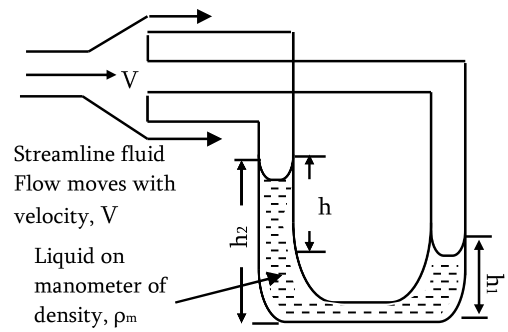
At point B: $P_2 = Pa + \rho g h_2$

$$P_2 - P_1 = \rho g (h_2 - h_1)$$

$$V = \sqrt{\frac{2\rho g (h_2 - h_1)}{\rho}}$$

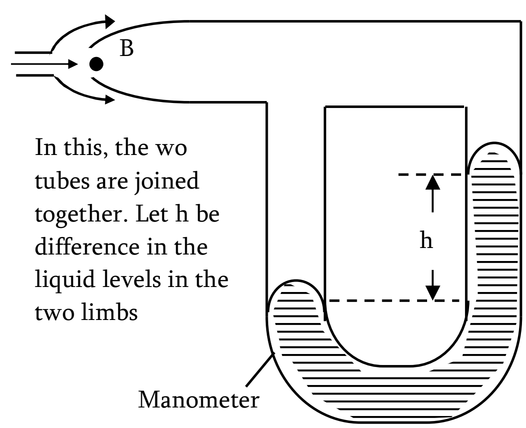
$$V = \sqrt{2g(h_2 - h_1)}$$

2. Gases cannot be used as manometer fluid and therefore the type of PITOT – STATIC tube used to measure the velocities has the form of the following.



$V = \sqrt{\frac{2\Delta P}{\rho}}$ But $\Delta P = \rho m g h$

3. The modified version of the PITOT tube is called the Prandtl tube is shown on the figure below.



$$\frac{1}{2}\rho V^2 + \rho g h = \rho m g h$$

$$\frac{1}{2}\rho V^2 = h(\rho m - \rho) g$$

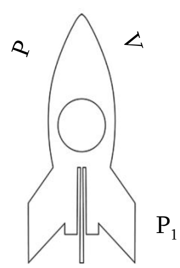
ρ = density of flowing fluid

ρm = density of the liquid inside of the manometer.

The reading of this instrument does not depend on atmospheric pressure. When this instrument is mounted on an aircraft it shows the velocity of the aircraft with respect to the surrounding air so is also **'an air speed indicator'.**

8. THRUST ON THE ROCKET

The rocket is propelled from the change of momentum resulting from the fluid that is ejected from it. A flow out of orifice gives rise to the thrust or reaction which in turns to drives the rocket forward. If A is the cross – sectional area, the reaction force can be calculated as follows:



Apply the Bernoulli's equation

$$P_1 + \frac{1}{2}\rho V_1^2 = P + \frac{1}{2}\rho V^2$$

$$P_1 - P = \frac{1}{2}\rho\left(V^2 - V_1^2\right)$$

$$V \gg V_1 \ , \ V^2 - V_1^2 \simeq V^2$$

$$P_1 - P = \frac{1}{2}\rho V^2$$

$$V^2 = 2\frac{\left(P_1 - P\right)}{\rho} \quad , \quad V = \sqrt{\frac{2\left(P_1 - P\right)}{\rho}}$$

Since $F = \rho A V^2 = 2A\left(P_1 - P\right)$

Lift force $F = 2A\left(P_1 - P\right)$

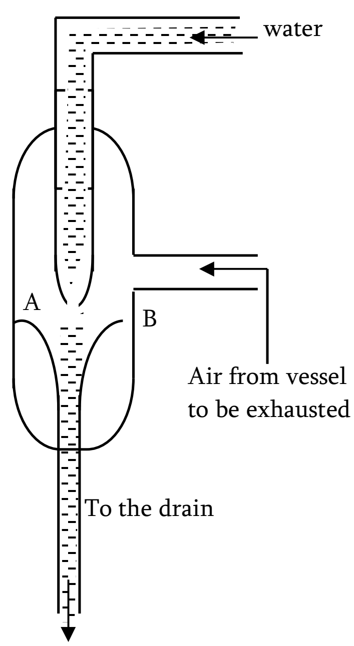
Also $F = \left(P_1 - P\right)A = \frac{1}{2}\rho A\left(V^2 - V_1^2\right)$

9. MOVING TWO PARALLEL SHIP

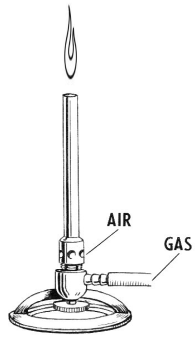
It is dangerous, for the two parallel ships to moves very fast in the parallel direction sea water in between these ships will have high velocity and by Bernoulli's theorem, pressure will be low. But on the other side pressure will be high. This pressure difference can make the two ships get pressed against each other.

10. FILTER PUMP

Filter pump is a device which is used to produce the pressure in a vessel. The working of filter pump is based on Bernoulli's principle. In the figure below, A is the small orifice through which water is issues out in the form of a jet. The velocity of water at the orifice A is high and hence pressure at A becomes less than the pressure of air in the vessel connected to the side tube B. So air from the vessel is sucked in wards, this is carried by the water flowing downwards. Hence the pressure of the air in the vessel is produced.

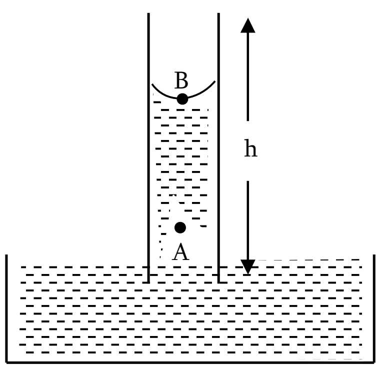


11. BUNSEN BURNER



In a Bunsen burner, the gas comes out of the nozzle with high velocity. According to the Bernoulli's principle, the pressure in the steam of the burner decreases. So , air from the atmosphere rushes into the burner. The mixture of air and gas moves up the burner. It burn at the top.

12. LAW OF HYDROSTATIC PRESSURE



Bernoulli's principle can be employed to calculate the difference of pressure between the points A and B (figure above). Since the liquid is at rest, therefore V is zero. Let h be the height of B above A. Let X be the height of A above the ground. Let P_1 and P_2 be pressures at A and B respectively

Total energy per unit volume at A

$= P_1 + \rho gx$

Total energy per unit volume at B

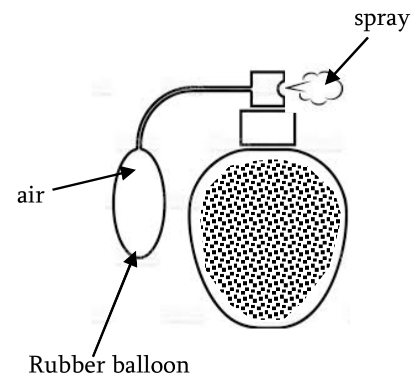
$= P_2 + \rho g\ (h + x)$

Applying bernoulis principle

$P_1 + \rho ghx = P_2 + \rho g\ (h + x)$

$P_1 – P_2 = \rho gh$

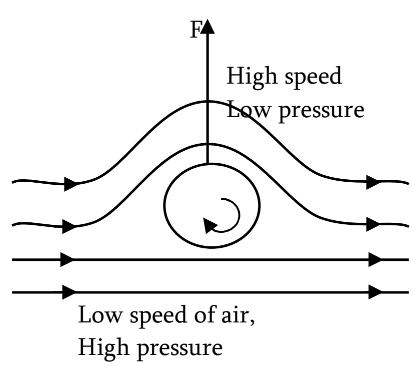
13. ATOMIZER OR SPRAYER



It is based on Bernoulli's principle it is caused to spray liquid. It is generally used in perfumes and deodorant bottles. When the rubber balloon is pressed, the air in the horizontal tube passes with large velocity. According to the Bernoulli's theorem, the pressure in the tube will be reduced. But the pressure in the container is equal to atmospheric pressure. This pressure difference makes the liquid rise in the vertical tube. On the top of the vertical tube, the liquid is blown away through the nozzle in the form of fine spray.

14. SPINNING OF BALL (MAGNUS EFFECT)

If a tennis ball is cut or a golf ball sliced, it spins as it travels through the air and experiences a side way force which cause it to curve in flight. This is due to air being dragged round by the spinning ball, thereby increasing the air flow on one side and decreases on the other side which result the pressure difference



EXAMPLES

Example – 62

(a) (i) If twos ships are moving parallel and close to each other, they experience an attractive force. Why.

(ii) The accumulation of snow on an aeroplane wing may reduce the lift. Explain.

(b) The static pressure in a horizontal pipeline is 4.3×10^4Pa, the total pressure is 4.7×10^4Pa and the area of cross – sectional is $20cm^2$. The fluid may be considered to be incompressible and non – viscous and a density of $1000kgm^{-3}$. Calculate

(i) The flow velocity in the pipe line

(ii) The flow volume in rate wise.

Solution

(a) (i) When two ships come close to each other, the air velocity between the narrow – gap increases and so pressure decreases. The pressure on the outer surfaces of the ship is then greater than the pressure in the gap. Therefore, the ships are pulled towards each other and sometimes they collide.

(ii) Due to accumulation of snow on the wings of the aeroplane, the structure of the wings no longer remains as that of aerofoil. As a result , the net upward force (i.e lift) is decreases.

(b) (i) V = Velocity of flow

$$P_T = P_S + \frac{1}{2}\rho V^2$$

$$V = \sqrt{\frac{2(P_T - P_S)}{\rho}} = \sqrt{\frac{2(4.7 - 4.3)\times 10^4}{1000}}$$

$$V = 2.83m/s$$

(ii) The rate of volume of liquid flow

$$Q = AV$$

$$Q = 0.002 \times 2.83$$

$$Q = 5.66 \times 10^{-3} m^3/s$$

Example – 63

(c) Write the continuity and Bernoulli's equations as applied to fluid dynamics

(d) (i) Under what conditions is the Bernoulli's equation applicable?

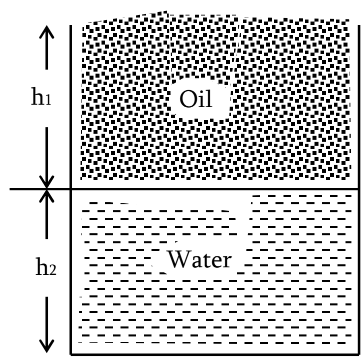
(ii) Discus tow (2) applications of the Bernoulli's equations

(e) (i) Develop an equation to determine the velocity of a fluid in a venturimeter pipe.

(ii) What amount of fluid passes through a section at any given time.

Example – 64

A wide tank is filled with water and kerosene figure shows. The tank has a small hole at the bottom. The height of water layer is 40cm and that of the kerosene layer is 30cm. find the velocity of the water flow neglecting the viscosity. Given that relative density of kerosene is 0.80



Solution

$$h_1\rho_1 g + h_2\rho_2 g = \frac{1}{2}\rho_2 V_2$$

$$V = \sqrt{2\left[h_2 + \frac{h_1\rho_1}{\rho_2}\right]}$$

$$= \sqrt{2\times 9.8\left[0.4 + 0.3\left(\frac{8000}{1000}\right)\right]}$$

$$V = 3.54m/s$$

Example – 65

(a) (i) Write down the Bernoulli's equation for the fluid flow in a pipe and indicate the term which will disappear when the flow of fluid is stopped.

(ii) Water flows into a tank of large cross – section area at a rate of $10^{-4}m^3/sec$ but flows out from a hole of area $1cm^2$ which has been punched through the base. How high does the water rise in the tank?

(iii) At two points on a horizontal tube of varying circular cross – sectional carrying water, the radii are 1cm and 0.4cm and the pressure difference between these points is 4.9cm of water. How much liquid flows through the tube per second?
Density of water = $1000kgm^{-3}$
Acceleration due to gravity, g = $9.8m/s^2$

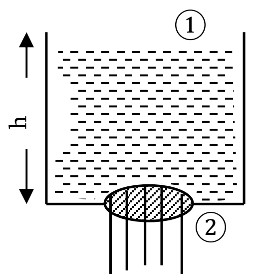
Solution

(i) Bernoulli's equation

$$P + \frac{1}{2}\rho V^2 + \rho gh = \text{constant}$$

The term which disappear when the fluid stopped is $\frac{1}{2}\rho V^2$

(ii) Apply Bernoulli's equation



$$P_1 + \frac{1}{2}\rho V_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho gh_2$$

$$P_2 + \frac{1}{2}\rho(0)^2 + \rho gh = Pa + \frac{1}{2}\rho V^2 + \rho g(0)$$

$$h = \frac{V^2}{2g} \quad \text{bu} \quad \text{t } Q = AV,\ V = Q/V$$

$$h = \frac{1}{2\times 9.8}\left[\frac{10^{-4}}{10^{-4}}\right]^2$$

$$h = 0.051m$$

(iii) Apply the Bernoulli's equation for the horizontal pipe.

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right)$$

$$\rho gh = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right)$$

$$9.8\times 1000\times 4.9\times 10^{-2} = \frac{1}{2}\times 1000\left(V_2^2 - V_1^2\right)$$

$$V_2^2 - V_1^2 = 0.9604$$

Apply continuity equation

$$A_1V_1 = A_2V_2$$

$$\pi r_1^2 V_1 = \pi r_2^2 V_2$$

$$\frac{V_1}{V_2} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{0.4}{10}\right)^2 = 0.16$$

$$V_1 = 0.16V_2$$

Now $V_2^2 - (0.16V_2)^2 = 0.9604$

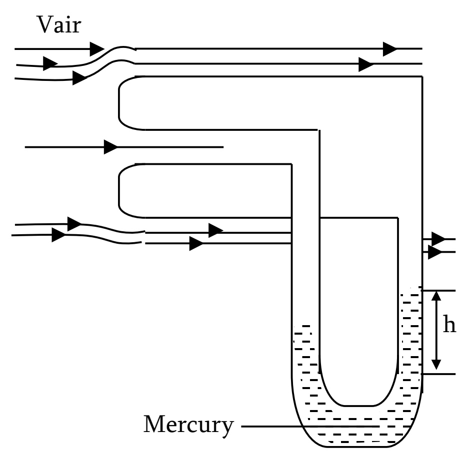
$$V_2 = 0.993m/s$$

Rate of volume of fluid flow

$$Q = \pi r_2^2 V_2 = 4.99\times 10^3 m^3/\text{sec}$$

Example – 66

A pitote tube is shown in the figure below. The fluid in the tube is mercury of density $13{,}600kgm^{-3}$ and the difference in the levels of mercury in the tube is 6cm. what is the speed of air flow? Density of air is $1.25kgm^{-3}$.



Solution

$$\frac{1}{2}\rho V^2 = h(\rho_m - \rho)g$$

$$V = \sqrt{\frac{2h(\rho_m - \rho)g}{\rho}}$$

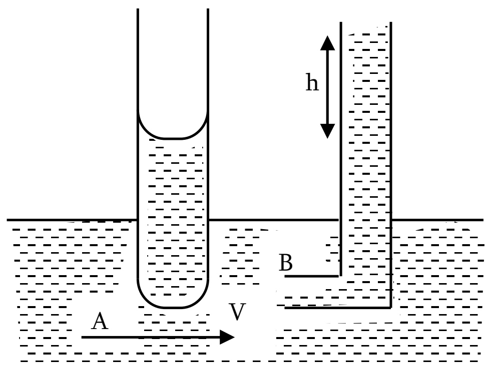
$$V = \sqrt{\frac{2\times 6\times 10^{-2}\times(13600 - 1.25)\times 9.8}{1.25}}$$

$$V = 113.1m/s$$

Example – 67

Pitot tube is fixed in a main of diameters 25cm and the difference of pressure indicated by the gauge is 6cm of water column. Find the volume of water passing through the main in two minutes.

Solution



The aperture A is parallel to the whereas B is facing the flow. At B the kinetic energy per unit volume is reduced to $\frac{1}{2}\rho V^2$ to zero, where V is the velocity of flow. Hence pressure increases and the water rises to a higher level.

$$\frac{1}{2}\rho V^2 = \rho gh$$

$$V = \sqrt{2gh}$$

Rate of flow $= AV = \pi r^2 V$

$$AV = \pi r^2\sqrt{2gh}$$

$$\frac{V_o}{t} = \pi r^2\sqrt{2gh}$$

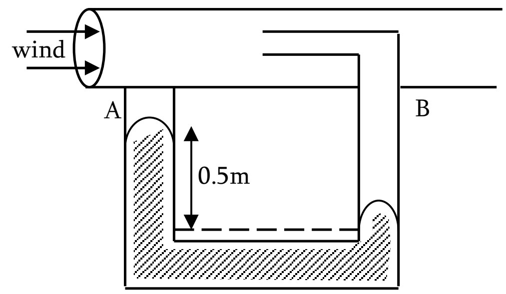
$$V_o = \pi r^2\sqrt{2gh}\ t$$

$$V_o = 3.14(0.125)^2\sqrt{2\times 9.8\times 0.06}\times 120$$

Volume , $V_O = 6.3846m^3$.

Example – 68

A pitot tube mounted on an aeroplane contains alcohol and show a level difference of 50cm. What is the speed of the plane relative to air (Density of alcohol = $800kgm^{-3}$)
density of air = $1kgm^{-3}$ wind.



Solution

Pitot tube is used to measure speed of aeroplane.
Apply Bernoulli's theorem at A and B

$$Pa + \frac{1}{2}\rho V_a^2 = Pa + \frac{1}{2}\rho V_B^2$$

$V_B = 0$ because its opening is perpendicular to the wind and its velocity is brought to zero.

$$\frac{1}{2}\rho V_A^2 = P_B - P_A$$

$$\frac{1}{2}\rho V_A^2 = h\delta g$$

$$V_A = \sqrt{\frac{2h\delta g}{\rho}} = \sqrt{\frac{2\times 0.5\times 800\times 9.8}{1}}$$

$$V_A = 88.54m/s$$

Example – 69

A fully loaded Boeing aircraft has a mass of 3.3×10^5kg. Its total wing area is $500m^2$. It is in level flight with speed of 960km/hr.

(a) Estimate the pressure difference between the lower and upper surfaces of the wings

(b) Estimate the fractional increase in the speed of air on the upper surface of the wing relative to the lower surface. (the density of air ρ = $1.2kgm^{-3}$)

Solution

(a) The height of boeing is balanced by the upward force due to the pressure difference.

$$A\Delta P = 3.3\times 10^5\times 9.81$$

$$\Delta P = \frac{3.3\times 10^5\times 9.81}{A} = \frac{3.3\times 10^5\times 9.81}{500}$$

$$\Delta P = 6.5\times 10^3 Nm^{-2}$$

(b) We ignore the small height difference between the top and bottom sides. The pressure difference between them is

$$\Delta P = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right)$$

V_2 = Speed of air over the upper Surface and V1 speed under the bottom surface

$$\Delta P = \frac{\rho}{2}\left(V_2 - V_1\right)\left(V_2 + V_1\right)$$

$$V_2 - V_1 = \frac{2\Delta P}{\rho\left(V_2 + V_1\right)}$$

Taking the average speed

$$V_{av} = \frac{V_2 + V_1}{2} = 960\text{km/hr} = 267\text{m/s}$$

$$\frac{V_2 - V_1}{V_{av}} = \frac{\Delta P}{\rho V_{av}^2} = 0.08$$

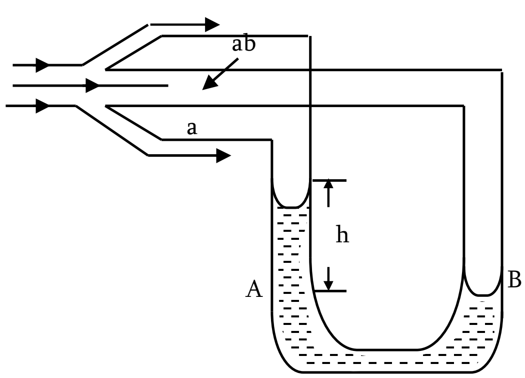
Example – 70

(a) How pitot tube is used to measure the speed of an aeroplane?

(b) A pitot tube is mounted on an aeroplane wing to measure the speed of the plane. The tube contains alcohol and shows a level difference of 40cm. what is the speed of the plane relative to air? Given specific gravity of alcohol = 0.8 and density of air = 1kgm^{-3})

Solution

(a) The arrangement is shown in figure below



The limb B of the U – tube is bend at the top at right angle. The limb A is also bent at right angle and ends into a wide tube. The wide tube has two openings at 'a' and 'a'. Both the tubes are joined together as shown above. The arrangement is mounted on the wing of the aeroplane. The manometer tube contains alcohol or any other liquid of low density. When the aeroplane flies, air flows, past the tube. The openings at 'a' and 'a' are parallel to the direction of flow. The opening of the limb B is perpendicular to the direction of air flow. The velocity of air entering this opening is reduced to zero and the pressure increases. Thus, the level of liquid in the limb A is raised and that in B is lowered. Let this difference, h. Applying Bernoulli's theorem to the point a and b

$$P_a + \frac{1}{2}\rho V_a^2 = P_b + \frac{1}{2}\rho V_b^2$$

Where P_a and P_b are the pressures and V_a and V_b are velocities at a and b respectively. But $V_b = 0$ ($\therefore$ Air comes to rest at B).

$$P_a + \frac{1}{2}\rho V_a^2 = P_b$$

$$P_b - P_a = \frac{1}{2}\rho V_a^2 \quad \left[P_b - P_a = \rho_{mgh}\right]$$

$$\frac{1}{2}\rho V_a^2 = \rho_{mgh}$$

$$V_a = \sqrt{\frac{2\rho_{mgh}}{\rho}}$$

For this equation, the velocity V_a of air flow which is also the velocity of the aeroplane can be calculated.

(b) $h = 40\text{cm} = 0.4\text{m} \quad \rho_m = 0.8\times10^3\text{kgm}^{-3}$

$$V_a = \sqrt{\frac{2\times0.4\times0.8\times10^3\times9.8}{1}}$$

$$V_a = 79.2\text{m/s}$$

Example – 72

(a) (i) Distinguish between static pressure, dynamic pressure and total pressure when applied to streamline or laminar fluid flow and write down expressions at a point in the fluid in terms of fluid velocity, the fluid density, ρ, pressure , P and the height h of the point with respect to a datum.

(ii) The static pressure in a horizontal pipeline is 4.3×10^4Pa, the total pressure is 4.7×10^4Pa and the area of cross – section is $20cm^2$. The fluid may be considered to be incompressible and non – viscous and has a density of $1000kgm^{-3}$. Calculate the flow velocity and the volume flow rate in the pipeline.

(b) (i) State Newton's law of viscosity and hence deduce the dimensions of the coefficient of viscosity.

(ii) In an experiment to determine the coefficient of viscosity of motor oil, the following measurements are made:-

- Mass of glass sphere = 1.2×10^{-4}kg
- Diameter of glass sphere = 1.2×10^{-3}m
- Terminal velocity of sphere = 5.4×10^{-2}m/s
- Density of oil = $860kgm^{-3}$

(c) (i) briefly explain the carburetor of a car as applied to Bernoulli's theorem

(ii) Three capillaries of the same length but with internal radii 3R, 4R and 5R are connected in series and a liquid flows through them under streamline conditions. If the pressure across the third capillary is 8.1mm of liquid, find the pressure across the first capillary.

(d) Give reasons for the following observations as applied in fluid dynamics:

(i) A flag flutter when strong winds are blowing on a certain day

(ii) A parachute is used while jumping an aeroplane.

(iii) Hot liquids flow faster than cold ones.

Solution

(a) (i)

- Static pressure – is the actual pressure of the fluid at a point due to its rest position of the fluid.
- Dynamic pressure is the pressure exerted by the fluid due to its motion.

  Dynamic pressure = $\frac{1}{2}\rho V^2$

- Total pressure is the sum of dynamic and static pressure along the tube of flow.

$$P_T = P_S + P_D$$

Bernoulli's equation

$$P + \frac{1}{2}\rho V^2 + \rho gh = \text{constant}$$

(ii) Since $P_T = P_S + \frac{1}{2}\rho V^2$

$$V = \sqrt{\frac{2(P_T - P_S)}{\rho}}$$

$$V = \sqrt{\frac{2(4.7 - 4.3)\times 10^4}{1000}}$$

$$V = 2.83m/s$$

Volume flow rate, Q = AV

$Q = 0.002 \times 2.83$

$Q = 5.66 \times 10^{-3}m^3/sec$

(b) (i) Newton's law of viscosity state that 'The viscous force F acting tangentially on a layer of the liquid flow is directly proportional to the area A of the layer and proportional to the velocity gradient'.

i.e $F \propto A\frac{dv}{dr}$

$$F = \eta A\frac{dv}{dr}$$

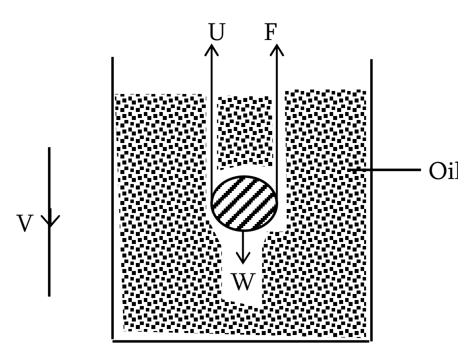
Dimension of coefficient of viscosity

$$\eta = \frac{F}{A \cdot \frac{dv}{dr}}$$

$$[\eta] = \frac{[F]}{[A]\left[\frac{dv}{dr}\right]} = \frac{MLT^{-2}}{L^2T^{-1}}$$

$$[\eta] = ML^{-1}T^{-1}$$

(ii)



U = Upthrust

F = Viscous drag force

W = weight of sphere

At the equilibrium of the sphere

$$F + U = W$$

$$F = W - U = Mg - \frac{4}{3}\pi r^3 \rho g$$

But $F = 6\pi\eta vr$

$$6\pi\eta vr = mg - \frac{4}{3}\pi r^3 \rho g$$

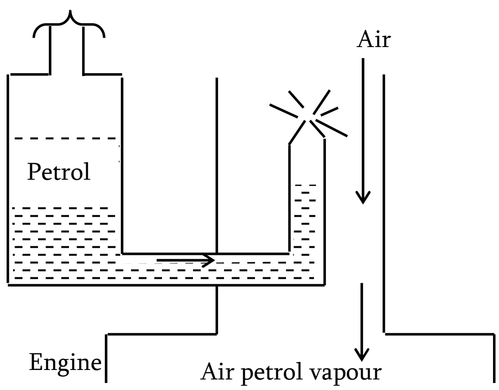
$$\eta = \frac{mg}{6\pi vr} - \frac{4}{3}\frac{\pi r^3 \rho g}{6\pi vr}$$

$$\eta = \frac{mg}{6\pi vr} - \frac{2r^2 \rho g}{9vr}$$

$$= \frac{9.8 \times 1.2 \times 10^{-4}}{6 \times 3.14 \times 5.4 \times 10^{-2} \times 2 \times 10^{3}} - \frac{2\left(2 \times 10^{-3}\right)^2 \times 860 \times 9.8}{9 \times 5.4 \times 10^{-2} \times 2 \times 10^{-3}}$$

$$\eta = 0.45 Nm^{-2}s$$

(c) (i) Principle operation of carburetor



Carburator of engine for vehicle works equation at one stage of its cycle, the engine draws in air. This rushes past the fine nozzle of a pipe connected to the petrol tank and lowers the air pressure at the some petrol is then forced out of the tank by atmospheric pressure through the nozzle in a fine spray. The petrol vapour mixes the air and so provides the air – petrol mixture required for the engine.

(ii) According to the Poiseulli's formula rate of flow through capillary.

$$Q = \frac{\pi P R^4}{8\eta L}$$

Rate of flow through the individual capillaries are:

$$Q_1 = \frac{\pi P_1 (3R)^4}{8\eta L}, \quad Q_2 = \frac{\pi P_2 (4R)^4}{8\eta L}$$

$$Q_3 = \frac{\pi P_3 (5R)^4}{8\eta L}$$

Since the capillaries are series , then

$Q_1 = Q_2 = Q_3 =$ Constant

$$\frac{\pi P_1 (3R)^4}{8\eta L} = \frac{\pi P_2 (4R)^4}{8\eta L} = \frac{\pi P_3 (5R)^4}{8\eta L}$$

$$81P_1 = 256P_2 = 625P_3$$

$$P_1 = \frac{625}{81}P_3, \quad P_3 = 8.1\text{mm of liquid}$$

$$P_1 = \frac{625}{81} \times 8.1$$

$$P_1 = 62.5\text{mm} = 6.25\text{cm of liquid}$$

(d) (i) When strong winds blow on the surface of a flag, the velocity of the wind is different at the different points on the flag. According to Bernoulli's theorem, the pressure will also be different at these points. The pressure difference causes the flag to flutter.

(ii) Viscous drag, $F = 6\pi\eta vr$. The open parachute has a large surface area (i.e. large r) and experiences a large viscous force while descending as a result, the terminal velocity becomes very small and the person does not get hurt.

(iii) The coefficient of viscosity of a liquid decreases with the increases in temperate. Therefore hot liquid will flow faster than the cold one.

Example – 73

(a) (i) Write down the Bernoulli's equation for liquid flow in a pipe and indicate the term which will disappear when the fluid is stopped.

(ii) Name the principle on which the continuity equation is based.

(iii) Basing on the applications of Bernoulli's principle, briefly explain why two ships are moving parallel and close to each other experience an attractive force.

(b) (i) A sphere is dropped under gravity through a fluid of viscosity, η. Taking average acceleration as half of the initial acceleration, show that the time taken to attain terminal velocity is independent of fluid density.

(ii) Water is flowing through a horizontal pipe having different cross – sections at two points A and B. The diameters of the pipe at A and B are 0.6m and 0.2m respectively. The pressure difference between points A and B is 1m column of water. Calculate the volume of water flowing per second.

(c) (i) The flow rate of water from a tap of diameter 1.25cm is 3 litres per minute. The coefficient of viscosity of water is $10^{-3}Nsm^{-2}$. Determine the Reynolds number and then state type of flow of water.

(ii) Air is moving fast horizontally past an air – plane. The speed over the top surface is 60m/s and under the bottom surface is 45m/s. calculate the pressure difference.

Given that $g = 9.8m/s^2$.

Density of air = $1.293kgm^{-3}$

Density of water = $1000kgm^{-3}$.

Solution

(a) (i) $P + \frac{1}{2}\rho V^2 + \rho gh = \text{constant}$

The term which will disappear on Bernoulli's equation when the fluid stopped is $\frac{1}{2}\rho V^2$

(ii) The continuity equation is based on the principle of law of conservation of mass we know that fluid is conserved, as a fluid moves and deforms new fluid is neither created nor destroyed. The continuity equation is a mathematical statement of this fact that fluid is conserved.

(iii) When two ships come close to each other, the air velocity between the narrow gap increased and so pressure decreases. The pressure on the outer surface of the ship is then greater than the pressure in the gap. Therefore, the ships are pulled towards each other and sometimes they may collide.

(b) (i) Average acceleration

$$\bar{a} = \frac{0+a}{2} = \frac{a}{2}$$

If t is the time taken by the sphere to attain terminal velocity, V_T.

$$V_T = 0 + \bar{a}t = \frac{at}{2}$$

Since

$$F = W - U = \frac{4}{3}\pi r^3 g(\rho - \delta)$$

$$F = Ma \ , \ a = \frac{F}{M}$$

$$a = \frac{\frac{4}{3}\pi r^3 g(\rho-\delta)}{\frac{4}{3}\pi r^3 \rho} = \frac{g(\rho-\delta)}{\delta}$$

$$\text{Now}, \ V_T = \frac{at}{2} = \frac{g(\rho-\delta)t}{2\rho}$$

$$t = \frac{2V_T\rho}{(\rho-\delta)g}$$

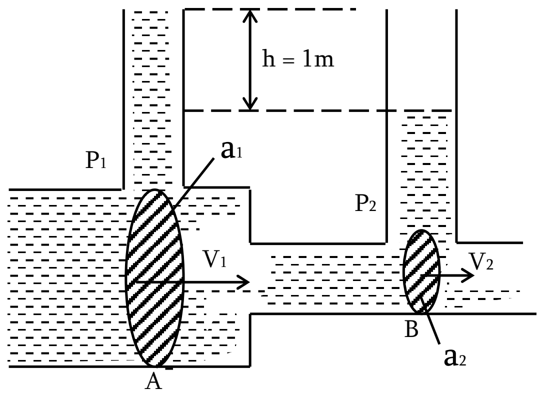
$$\text{But } V_T = 2r^2g\frac{(\rho-\delta)}{9\eta}$$

$$t = \frac{2\rho}{(\rho-\delta)g}\times\frac{2r^2g(\rho-\delta)}{9\eta}$$

$$t = \frac{4r^2\rho}{9\eta}$$

∴ The time taken by the sphere to attain terminal velocity is independent of the density δ of the fluid.

(ii)



Apply the Bernoulli's equation for the horizontal pipe.

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

From the equation of continuity

$$a_1 v_1 = a_2 v_2$$

$$v_1 = \frac{a_2 v_2}{a_1}$$

$$P_1 + \frac{1}{2}\rho\left(\frac{a_2}{a_1}\right)^2 V_2^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho V_2^2\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

$$V_2 = a_1\sqrt{\frac{2(P_1 - P_2)}{\rho(a_1^2 - a_2^2)}}$$

Rate flow , $Q = a_2 v_2$

$$Q = a_1 a_2\sqrt{\frac{2(P_1 - P_2)}{\rho(a_1^2 - a_2^2)}}$$

Since $a_1 = \dfrac{\pi(0.6)^2}{4} = 0.09\pi m^2$

$$a_2 = \frac{\pi(0.2)^2}{4} = 0.01\pi m^2$$

$$P_1 - P_2 = h\rho g = 1\times1000\times9.8 = 9,800 Nm^{-2}$$

$$Q = 0.09\pi\times0.01\pi\sqrt{\frac{2\times9800}{1000\left[(0.09\pi)^2 - (0.01\pi)^2\right]}}$$

(c) (i) $D = 1.25cm, Q = 3litre/min$

$\eta = 10^{-3} Nsm^{-2}$

$$Q = \frac{3\times10^{-3}}{60} = 50\times10^{-6} m^3/s$$

$$V = \frac{Q}{A} = \frac{50\times10^{-6}}{\frac{\pi}{4}\left(1.25\times10^{-2}\right)^2}$$

$$V = 0.4074 m/s$$

$$N_R = \frac{\rho VD}{\eta} = \frac{1000\times0.4074\left(1.25\times10^{-2}\right)}{10^{-3}}$$

$$N_R = 5092.5$$

Since the value of NR is greater than 3000, the flow of water from the tap is turbulent.

(ii) Apply the Bernoulli's equation for horizontal pipe

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right)$$

$$= \frac{1}{2}\times1.293\left(60^2 - 45^2\right)$$

$$P_1 - P_2 = 1018.24 Nm^{-2}$$

Example – 74

(a) (i) What is meant by an Ideal fluid?

(ii) Define the term critical velocity with respect to the fluid flow.

(b) (i) State the Torricelli's theorem

(ii) Water enters into an open tank at a rate of $0.025 m^3/s$ and leaves through a small hole of diameter 0.10m at its base. Calculate the maximum height to which water can rise.

(c) (i) Write one form of Bernoulli's equation and from it identify the static pressure and dynamic pressure.

(ii) Water flows in a pipe of cross – sectional area $40 cm^2$. If at a certain point the static pressure is $1.12 \times 10^5 Pa$ and the total pressure is $2.12 \times 10^5 Pa$, calculate the speed and volume per second of water passing in the pipe.

(d) (i) using the Bernoulli's equation in 1(c) (i)

(i) Derive the lift force of an aircraft. Give one assumption made in arriving at your answer.

(ii) Explain how the takeoff of an aircraft can be achieved $g = 9.8 m/s^2$

Density of water = $1000 kg/m^3$.

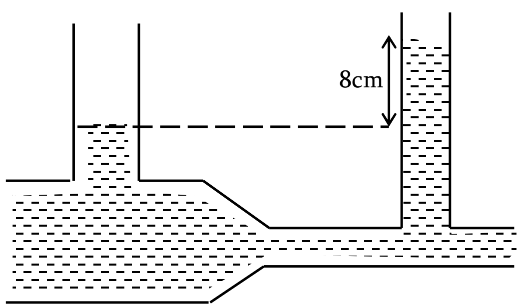
Example – 75

(a) (i) A person standing near a fast – moving train has a danger of falling towards the train. Why?

(ii) Why the wings of an aeroplane are rounded outwards while flatted in wards?

(iii) Mention three important features of an Ideal liquid.

(b) A horizontal tube has different cross – sections at point A and B. the diameter of the tube at points A and B are 4cm and 2cm respectively. The two manometers arms are fixed at points A and B (figure below). When a liquid of density $800kgm^{-3}$ flows through the tube, the difference of pressure between the arms of manometer is 8cm. calculate the rate of liquid, deriving the necessary formula.



Solution

(a) (i) The fast moving train reduces the pressure between person and train. Due to larger pressure on the other side of person, the person gets pulled towards to the train and this is according to the Bernoulli's principle.

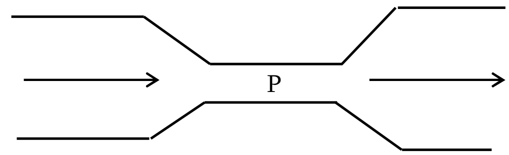
(ii) The special design of the wings increases velocity at the upper surface and decreases velocity at the lower surface. So according to Bernoulli's theorem, the pressure on the upper side is less than the pressure on the lower side. This difference of pressure provides lift.

(iii)
- It is incompressible
- It is non – viscous
- It cannot withstand any shearing stress.

(b) Assignment to the student.

Example – 76

(a) State the equation of continuity for a compressible fluid flowing through a pipe



A horizontal pipe of diameter 36cm tapers to a diameter 18cm at P. An Ideal gas at a pressure of 2×10^5Pa is moving along the pipe at a speed of 30m/s. The pressure of gas at P is 1.80×10^5Pa. assuming that temperature of the gas remain constant. Calculate the speed of the gas at P.

(b) State the Bernoulli's equation for an incompressible fluid, giving the meanings of the symbols in the equation.

(c) For the gas in (a) recalculate the speed at P on the assumption that it can be treated as an incompressible fluid, and use Bernoulli's equation to calculate the corresponding value for the pressure at P. Assume that in the wider part of the pipe the gas speed is still 30.0m/s, the pressure is still 2.00×10^5Pa and at this pressure the density of the gas is $2.60kgm^{-3}$.

Solution

(a) Continuity equation for the compressible fluid flow.

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$\rho AV = \text{Constant}$$

ρ = density of fluid

A = cross – sectional area of pipe

V = velocity of fluid

Given that

$P_1 = 2 \times 105Pa$, $P_2 = 1.8 \times 10^5Pa$

$\rho_1 = 2.60kgm^{-3}$, $\rho_2 = ?$

$V_1 = 30m/s$, $V_2 = ?$

From the ideal gas

$$\frac{P}{\rho T} = \text{constant} \ , \ T = \text{constant}$$

$$\frac{P_1}{\rho_1} = \frac{P_2}{\rho_2} \ , \ \rho_2 = \left(\frac{P_2}{P_1}\right)\rho_1$$

$$\rho_2 = 2.60\left[\frac{1.8\times10^5}{2.0\times10^5}\right]$$

$$\rho_2 = 2.34kgm^{-3}$$

Apply continuity equation of compressible fluid flow.

$$\rho_2 A_2 V_2 = \rho_1 A_1 V_1$$

$$V_2 = \left(\frac{\rho_1}{\rho_2}\right)\left(\frac{A_1}{A_2}\right)V_1$$

$$= \left(\frac{\rho_1}{\rho_2}\right)\left(\frac{\pi d_1^2/4}{\pi d_2^2/4}\right)V_1$$

$$V_2 = \left(\frac{\rho_1}{\rho_2}\right)\left(\frac{d_1}{d_2}\right)^2 V_1$$

$$= \left(\frac{2.60}{2.34}\right)\left(\frac{36}{18}\right)^2 \times 30\text{m/s}$$

$$V_2 = 133\text{m/s}$$

(b) $P + \frac{1}{2}\rho V^2 + \rho g h = \text{constant}$

P = Pressure of fluid

ρ = density of fluid

V = velocity of fluid flow

g = Acceleration due to gravity

h = height

(c) Apply the continuity equation for incompressible fluid flow.

$$A_1 V_1 = A_2 V_2$$

$$\frac{\pi d_1^2}{4} V_1 = \frac{\pi d_2^2}{4} V_2$$

$$V_2 = \left(\frac{d_1}{d_2}\right)^2 \cdot V_2$$

$$= \left(\frac{36}{18}\right)^2 \times 30\text{m/s}$$

$$V_2 = 120\text{m/s}$$

Apply Bernoulli's equation for horizontal pipe.

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_2 = P_1 + \frac{1}{2}\rho\left(V_1^2 - V_2^2\right)$$

$$= 2\times10^5 + \frac{1}{2}\times 2.6\left[30^2 - 120^2\right]$$

$$P_2 = 1.82\times10^5\text{Pa}$$

Example – 77

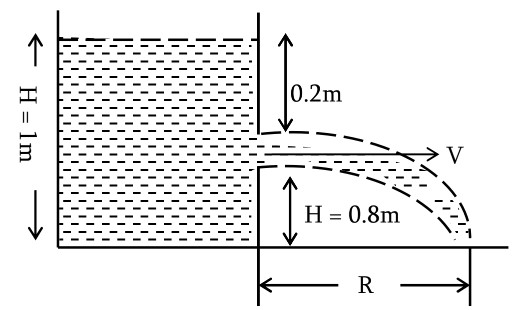
A large tank contain water to a depth of 1.0m. water emerging from a hole in the side of tank 20cm below the level of the surface. Calculate:-

(a) The speed at which the water emerges from the hole.

(b) The distance from the base of the tank at which water strikes the floor on which the tank is standing. If a second hole were to be drilled in the wall of the tank vertically below the first hole, at what height above the base of the tank would this second hole have to be if the water issuing from it were to hit the floor at the same point as that from the first hole?

Solution

(a)

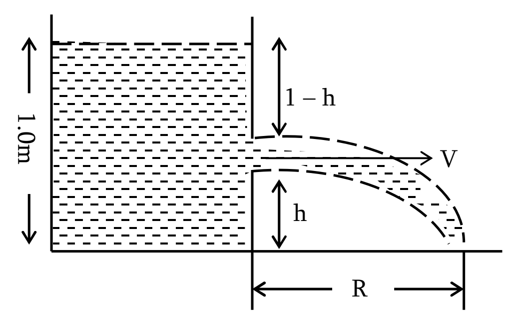


According to the Torricelli's theorem

$$V = \sqrt{2gh} = \sqrt{2\times10\times0.2}$$

$$V = 2\text{m/s}$$

(b) Let the second hole be dilled at the height h above the base of the tank.



Let V = Velocity of efflux

$$V = \sqrt{2g(1-h)}$$

$$h = \frac{1}{2}gt^2$$

$$R = v.t = \sqrt{\frac{2h}{g}}\times\sqrt{2g(1-h)}$$

$$\text{But } R = V_1t_1 = 2.0\sqrt{\frac{2\times 0.8}{10}}$$

$$R = 0.8\text{m}$$

$$\text{Now} \quad 0.8 = \sqrt{2h(1-h)}$$

$$4h(1-h) = 0.64$$

$$h^2 - h + 0.16 = 0$$

On solving h = 0.8m or 0.2m.

∴ The second hole must be drilled at a height of 20cm above the base of the tank if it has to be drilled below the first hole.

Example – 78

(a) Explain why:-

(i) To keep a piece of perpendicular horizontal, you should blow over, not under it.

(ii) When we try to close together tap with our fingers, fast jets of water push through the openings between out fingers.

(iii) The size of the needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection.

(iv) A plane is in level at constant speed and each of its two wings has an area of 25m². If the speed of air is 180km/hr over the upper wing surface; determine the planes mass (take air density to be 1kgm⁻³).

Solution

(a) (i) When we blow over the piece of paper, the velocity of air above the paper increases and pressure of air on it decreases. (Bernoulli's principle). The result is that pressure below the paper becomes greater than the pressure above the paper. Therefore, piece of paper remains horizontal.

(ii) As we try to close a water tap with our fingers, the area of outlet of water jet is reduced. According to equation of continuity (AV = constant) the velocity of water increases enormously.

(iii) According to the Bernoulli's principle

$$P + \frac{1}{2}\rho V^2 + \rho gh = \text{constant}$$

The size of the needle controls the velocity (V) of flow and thumb controls pressure (P). It is clear from the above equation that total energy of medicine to be injected depends upon the second power of velocity and first power of pressure therefore, velocity has more influence in the process. For this reason, the needle has a better control over the flow of medicine.

(iv) Due to small area of cross – section of the hole, the fluid flows out of the hole with a large velocity and hence possesses large momentum since no external force is acting on the system to conserve momentum, the vessel acquires a backward velocity. As a result, the vessel experience back ward thrust.

(b) According to the Bernoulli's equation for horizontal pipe.

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_2 - P_1 = \frac{1}{2}\rho\left(V_1^2 - V_2^2\right)$$

$$V_1 = 234\text{km/hr} = 65\text{m/s}$$

$$V_2 = 180\text{km/hr} = 50\text{m/s}$$

$$P_2 - P_1 = \frac{1}{2}\times 1\left[65^2 - 50^2\right]$$

$$P_2 - P_1 = 862.5\text{Nm}^{-2}$$

Upward lift, F $(P_2 - P_1)$ × Area of two wings

$$F = 862.5\times 2(25)\text{N}$$

Let: M = Mass of plane

$$F = Mg \ , \ M = \frac{F}{g}$$

$$M = \frac{862.5\times 50}{9.8}$$

$$M = 4.4\times 10^3\text{kg}$$

EXERCISE NO. 3

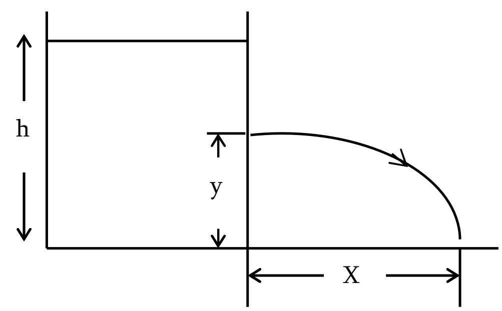
1. A water tank has hole at a distance 7m from free water surface. Find the velocity of water through the hole. If the radius of the hole is 2mm, What is the rate of flow of water? **Answer** 11.7m/s, 1.47 × 10⁻⁴m³/s.

2. The diameters of a pipe at two points where a renturimeter is connected is 5cm and 8cm and the difference of level in it is 4cm. Calculate the mass of water flowing through the pipe of second. **Answer** $0.158kgs^{-1}$.

3. (a) (i) State the Bernoulli's theorem.
   (ii) The reading of pressure meter attached with a closed pipe is 3.5×10^5Pa on opening the valve of the pipe the reading of pressure meter is reduced to 3×10^5Pa. Calculate the speed of water flowing in the pipe.
   (iii) Explain the origin of the lift on an aeroplane at the takeoff.
   (b) (i) Define the term 'coefficient of viscosity.
   (ii) Explain the effect of temperature and pressure on the viscosity of liquids.
   (iii) Two drops of equal size are falling through air with a constant speed of 10m/s. if the drops coalesce, what would be the new constant speed?
   (c) Water is flowing with a mean velocity of 2m/s along horizontal pipe of internal diameter 1cm and length 5m. At the end of the pipe a vertical tube open to air is connected. To what height will water rise up this tube? Viscosity of water is 0.0011kg/ms.

4. (a) (i) Explain the terms Laminar flow and turbulent flow.
   (ii) Mention three (3) applications of Bernoulli's principle.
   (b) (i) Distinguish between static pressure and dynamic pressure.
   (ii) A pilot – static tube fitted with a pressure gauge in used to measure the speed of a boat at sea. Given that the speed of the boat does not exceed 10m/s and the density of sea water is $1050kgm^{-3}$; calculate the maximum pressure on the gauge.
   (c) A large water tank stands on frictionless surface. The sea over a circular hole of radius 0.5cm in the wall of the tank raptures. If the level of water above the hole is 1m, calculate thrust on the tank. Density of water is $1000kgm^{-3}$.
   (d) (i) Define the term surface energy as applied to liquid surface.
   (ii) If a number of little droplets of a liquid of density ρ, surface tension T and specific heat capacity C each of radius r, coalesce to form a single drop of radius R, what will the rise in temperature (θ) be?

5. (a) A cylinder of large cross – sectional area, containing water, stands on horizontal bench. The water surface is at a height h above the bench. Water emerges horizontally from a hole in the side of the cylinder, at a height y above the bench.



   (i) Use Bernoulli's equation to derive expressions for the speed at which the water emerges from the hole and the speed at which it hits the bench.
   (ii) Derive expressions for the time for the water to travel from the hole to the bench and for X, the horizontal distance the water travels from the cylinder.
   (b) Draw a diagram of a pitot – static tube, and with reference to Bernoulli's equation, explain how the tube may be used to measure the speed of a boat in sea water.

6. (a) What do you understand by the equation of continuity as applied to a fluid motion?
   (b) Derive Bernoulli's equation for an incompressible fluid.
   (c) A simple garden syringe used to produce a jet of water consists of a piston of area

4.00cm^2 which moves in a horizontal cylinder which has a small hole of area 4.00mm^2 at its end. If the force on the piston is 50N, calculate a value for the speed at which water is forced out of the small hole, assuming the speed of the piston is negligible. Density of water = 1000kgm^{-3}.

(d) Explain why speed of the piston may be ignored. **Answer** (c) 15.8m/s

Explanations questions MASWALI YANATAKIWA KUINGIZWA.

TUTORIAL SHEETS NO: 3 (B)
BASED ON BERNOULLI'S EQUATION AND CONTINUITY EQUATION.

1. A liquid is flowing through a horizontal pipe line of varying cross – section. At a certain point, the diameter of the pipe is 6cm and the velocity of flow of liquid is 2cm/s. Calculates the velocity of flow at another point where the diameter is 1.5cm. **Answer** 32cm/s.

2. At what speed will the velocity of a stream of water be equal to 20cm of mercury column taking g = 10m/s^2 and density of mercury = 136000kg/m^3. **Answer** 7.3756m/s.

3. Calculate the total energy per unit mass possessed by water at a point, where the pressure is 98 × 10^{3}Nm^{-2}, velocity is 0.1m/s and height of water level from the ground is 0.2m (g = 9.8m/s^2) **Answer** 99.965J/kg.

4. Water at a pressure of 4 × 10^{4}N/m^2 flows at 2m/s through a horizontal pipe of 0.02m^2 cross – sectional area which reduces to 0.01m$^{2.}$ What is the pressure in the smaller cross – section of the pipe? **Answer** 3.4 × 10^{4}Pa

5. The reading of pressure meter attached with a closed pipe is 3.5 × 10^{5}N/m^2. On opening the valve of the pipe, the reading of the pressure meter is reduced to 3.0 × 10^{5}N/m^2. Calculate the speed of water flowing in the pipe (density of water = 1000kgm^{-3}) **Answer** 10m/s.

6. An aeronautic engineer observes that on the upper and the lower surface of the wing of an aeroplane the speeds of the air are 120m/s and 90m/s respectively during flight. What is the lift on the wing of aeroplane if its area is 3.2m^2. Given density of air is 1.29kg/m^3. **Answer** 13003.2N.

7. Calculate the minimum pressure required to force the blood from the heart to the top of the head (vertical distance = 40cm) assume that the density of blood to be 1040kg/m^3. Friction is to be neglected (g = 9.80m/s^2). **Answer** 4,076.8N/m^2.

8. A drum of 40cm radius has a capacity of 440dm^3 of water. It contains 396dm^3 of water and is placed on a solid of exactly the same size as of drum. If a small hole is made at lower end of the drum perpendicular to its length, find the horizontal range of water on the ground in the beginning. Given g = 10m/s^2. **Answer** 166cm.

9. Water from a tap emerges vertically downward with an initial speed of 1.0m/s. the cross – sectional area of the tap is 10^{-4}m^2. Assume that the pressure is constant throughout the stream of water and that the flow is steady. What is the cross – sectional area of the stream 0.15m below the tap? Use g = 10m/s^2. **Answer** 5.0 × 10^{-5}m^2.

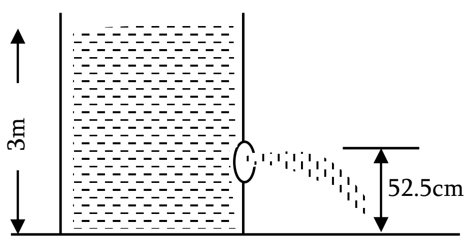
10. The diameter of a pipe at two points, where a venturimeter is connected is 8cm and 5cm and the difference of levels in it is 4cm. Calculate the volume of water flowing through the pipe per second (g = 980cm/s^2) **Answer** 1889ccs^{-1}.

11. The flow of blood in a large artery of an anaesthetized dog is diverted through a vetnurimeter. The wide part of the meter has a cross – sectional area equal to that of the artery i.e 8mm^2. The narrower part has an area 4mm^2. The pressure drop in the artery is 24Pa. What is the speed of the blood in the artery? Given that density of the blood = 060kg/m^3. **Answer** 0.123m/s.

12. Consider a cylinder of radius, R at its bottom, there is a hole of radius, r. The cylinder is filled up to the height h and the hole is open. If t is the time in which the cylinder is emptied, then find the relation between t and h. answer $t = \frac{2R^2\sqrt{h}}{r^2\sqrt{2g}}$, or $t \alpha \sqrt{h}$

13. Each of the two wings of an aeroplane has an area $30m^2$. The speeds of the air on the upper and lower surfaces of the wing of aeroplane are 90m/s and 70m/s respectively. If the plane is in level flight at constant speed, find the uplift and the mass of the aeroplane. **Answer** 1.24×10^5N, 1.26×10^4kg.

14. Water is filled in a cylindrical beaker to a height of 3m as shown in the figure below. The ratio of the cross – sectional area of the orifice and the beaker is 0.1. Find the speed of the liquid coming out from the orifice given $g = 10m/s^2$. **Answer** 7.04m/s



15. Water flows through a horizontal pipe. The area of cross – section at one place $A_1 = 10cm^2$, velocity of water flow is 1m/s and pressure is 2000Pa at another place area $A_2 = 5cm^2$. What is the pressure at area A_2? **Answer** 500Pa.

16. The diameter of a pipe at two points, where a vetnurimeter is connected is 9cm and 4cm and the difference of levels in its is 4cm. calculate the volume of water flowing through the pipe per second. **Answer** $10930cm^3$/sec.

17. A hole of area 1mm2 opens near the bottom of a large – water storage tank, and a stream of water shoots from it. If the top of water in the tank is to be kept at 20m above the point of leak, how much water in litre/sec should be added to the reservoir tank to keep this level? ($g = 10m/s^2$) **Answer** 20mL/sec.

18. Air of density $1.3kg/m^3$ flows horizontally with a speed of 106km/hr. A house has a plane roof of area $40m^2$. Find the magnitude of aerodynamic lift on the roof. **Answer** 2.25×10^4N

19. (a) A liquid flows through a horizontal pipe of varying cross – section at a rate of 10 litres per minute. Find the velocity of liquid at a point where radius of pipe is 5cm.
    (b) Water flowing at a speed of 0.5m/s through a horizontal pipe of internal diameter 3cm. determine the diameter of the nozzle if the water is to emerge at a speed of 3m/s. **Answer** (a) 0.021m/s (b) 1.225cm.

20. Calculate the speed at which the velocity of stream of water will be equal to 30cm of mercury column, given that $\rho_{Hg} = 13600kgm^{-3}$, $g = 10m/s^2$ answer 9.033m/s.

21. A pressure meter when attached with a closed horizontal pipe shows a reading of $5 \times 10^5N/m^2$. When the valve of the pipe is opened, the reading of the pressure meter falls to $3.5 \times 10^5N/m^2$. Determine the speed of the water flowing in the pipe. **Answer** $10\sqrt{3}m/s$

22. A fully loaded Boeing aircraft has a mass of 3.3×10^5kg. If its total wing area is $500m^2$. It is in level flight with a speed of 960km/hr.
    (a) Estimate the pressure difference between the lower and upper surfaces of the wings.
    (b) Estimate the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface. The density of air is $\rho = 1.2kgm^{-3}$, $g = 9.8m/s^2$ **Answer** (a) $6.5 \times 10^3N/m^2$ (b) 0.08.

23. Two horizontal pipes of different diameter are connected together and water is allowed to flow through them. The speed of water in the first pipe is 5m/s and the pressure is $1.5 \times 10^4 N/m^2$. What will be the speed and the pressure of water in the second pipe? The diameters of the first and second pipes are 2cm and 4cm respectively. **Answer** 1.25m/s, $0.328 \times 104 N/m^2$.

24. (a) Calculate the velocity head of water which is flowing with a velocity of 5m/s in a horizontal pipe.
    (b) A pilot tube is mounted in a main pipe of diameter 40cm. If difference in pressure indicated by the gauge is 6cm of water column, calculate the volume of air passing through the main pipe in two minutes. **Answer** (a) 1.28 (b) $12.54m^3$.

25. The diameters of a horizontal tube at points A and B are 6cm and 3cm, respectively. Two manometer limbs are attached at A and B. when water is allowed to flow through the tube, pressure difference of 10cm is noted between the limbs. Calculate the rate of flow of water in the tube. **Answer** $110cm^3$/sec.

26. On the wall of a cylindrical water tank, two holes are made as shown in the figure. Water coming out from these holes hits the ground at the same point. What will be the ratio h_1/h_2?

    Answer $\frac{h_1}{h_2} = 1$

27. A cylindrical tank of cross – sectional area a_1 is filled with water to a height h. A hole of cross – sectional area a_2 is made at the bottom. If $a_1 = 5a_2$, then find the:-
    (i) Initial velocity with which the water falls in tank.
    (ii) Initial velocity with which water emerges out from the hole.
    (iii) Time taken to make the tank empty

    Answer (i) $V_1 = \sqrt{\frac{gh}{12}}$ (ii) $V_2 = 5\sqrt{\frac{gh}{12}}$

    (iii) $t = \frac{4\sqrt{3h}}{\sqrt{g}}$

28. The cylindrical tube of a spray pump has a cross – section of $8cm^2$ one end of which has 40holes each of diameter 1.0mm. If the liquid flow inside the tube is 1.5m per minute, what is the speed of ejection of liquid through the holes? **Answer** 0.637m/s.

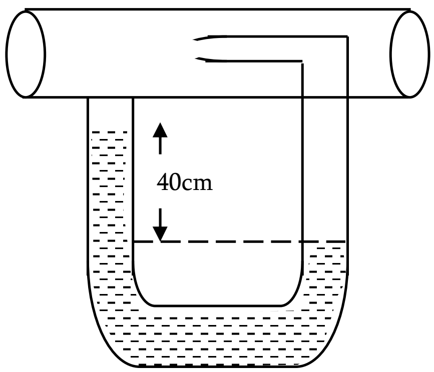
29. (a) What is the largest average velocity of blood flow in an artery of radius 2×10^{-3}m, if the flow must remain laminar?
    (b) What is the corresponding flow rate? (Taking viscosity of blood to be 2.084×10^{-3}Pas) **Answer** (a) 0.983m/s (b) $1.23 \times 10^{-5} m^3/s$.

30. A plane is in level flight at constant speed and each of its two wings has an area of $25m^2$. If the speed of air is 180km/hr over the upper wing surface, determine the mass of the plane. (Take air density to be $1kgm^{-3}$) **Answer** 4400g

31. In the diagram, shown, the difference in the two tubes of the manometer is 5cm the cross – section of the tube at A and B is $6mm^2$ and $10mm^2$ respectively. Find the rate of volume of water flows through the tube. **Answer** $7.5cm^3$/sec

32. The flow of blood in a large artery of an anesthetized dog is diverted through h a venturimeter. The wider part of the meter has a cross – sectional area equal to that of the artery i.e $8mm^2$. The narrower part has an area $4mm^2$. The pressure drop in the artery is 24Pa. What is the speed of the blood in the artery? Given that the density of the blood = $1.06 \times 10^3 kgm^{-3}$. **Answer** 0.123m/s.

33. A pilot tube is mounted on an aeroplane wing to measure the speed of the plane. The tube contains alcohol and shows a level difference of 40cm as shown in figure below. What is the speed of the plane relative to air? Given that relative density of alcohol = 0.8 and density of air = $1kgm^{-3}$. **Answer** 79.2m/s



34. Calculate the rate of flow of gycerime of density $1.25 \times 10^3 kg/m^3$ through the cross – section of a pipe, if the radii of its ends are 0.1m and 0.04m and the pressure drop cross its length is $10Nm^{-2}$. **Answer** $6.434 \times 10^{-4} m^3/s$.

35. What is flowing steadily through a horizontal pipe of non – uniform cross – section. The pressure of water is $4 \times 10^4 N/m^2$ at a point, where velocity of flow is 2m/s. What is the pressure at a point where cross – section reduces to $0.01m^2$? **Answer** $3.4 \times 10^4 N/m^2$.

36. Water is maintained at a height of 10m in a tank. Calculate the diameter of a circular aperture needed at the base of the tank to discharge water at the rate of $26.4m^3min^{-1}$. Given that $g = 9.8m/s^2$. **Answer** $g = 9.8m/s^2$.

37. A small hole is made at a height of $1/\sqrt{2}$ m from the bottom of a cylindrical water tank and at a depth of $\sqrt{2}m$ from the free surface of water tank. Find the distance, where the water emerging from the hole strikes the ground. **Answer** 2m.

38. A venturimeter is 37.5cm is diameter in the mains and 14cm diameter in the throat. The difference between the pressure of water in the mains and the throat is 23cm of mercury. Find the rate of discharge of water from the venturimeter. **Answer** $1.402 \times 10^5 cm^3/s$

39. The radii of the two parts of venturimeter are 20cm and 10cm respectively. When the venturimeter is connected to a water pipe, the levels of water in the manometer tubes differ by 10cm. Find the rate of flow of water through the pipe. **Answer** $4.54 \times 10^4 cm^3/s$.

40. What should be the average velocity of water in a tube of diameter 0.4cm so that the flow is (a) laminar and (b) turbulent. The viscosity of water is $10^{-3}Nm^{-2}s$. **Answer** 0.5m/s (b) 0.75m/s.

FREQUENTLY ASKED SHORT AND LONG ANSWER QUESTIONS.

1. (a) Give the meaning of the following terms as used in fluid dynamics:
   (i) Critical velocity (ii) ideal fluid
   (iii) Incompressible fluid
   (iv) Streamline flow (v) Turbulent flow
   (b) State and prove equation of the continuity.

2. (a) What is Reynolds number? On what factor, does it depend.
   (b) State and prove Bernoulli's theorem for a non – viscous liquid.
   (c) Outline six (6) practical applications of Bernoulli's theorem.

3. (a) Explain the meaning of the following terms as applied to fluid dynamics.
   (i) Compressible fluid
   (ii) Steady fluid flows
   (iii) Viscous fluid flow
   (iv) Line of flow
   (v) Irrotational fluid flow.
   (b) State four (4) properties of the streamline.
   (c) Differentiate between streamline and turbulent fluid flow (give three (3) points)

4. (a) Define the following terms:-
   (i) Rotational fluid flow
   (ii) Non – viscous fluid flow
   (iii) Velocity profile
   (iv) Terminal velocity.
   (b) Under what conditions Bernoulli's equation is applicable?
   (c) State four (4) limitations of the Bernoulli's principle.

5. (a) (i) State the law of conservation of mass
   (ii) State the continuity principle
   (b) Explain the meaning of following terms.
   (i) Tube of flow (ii) stream tube
   (iii) Stagnation
   (c) State the term which disappear on then Bernoulli's equation when the liquid stops its motion.

6. (a) (i) What is the nature of flow of a liquid if Reynold's number for that liquid is 2500?
   (ii) Give the significance of Reynold's number.
   (b) What is venturimeter? Give an expression for the rate of flow of a liquid using this device.

7. Write short notes on (i) Magnus effect
   (ii) Blowing off the roofs
   (iii) Lift on an Aeroplane

8. (a) State and explain Torricelli's theorem
   (b) What are the various forms of energies of a liquid in motion? write their relation in terms of unit mass.
   (c) Write a note on (i) Atomizer
   (ii) Curved motion of spinning ball.

9. (a) What do you understand by critical velocity? Establish a relation for it by using method of dimensions.
   (b) A large tank filled with water to height h is said to be emptied through a small hole at the bottom. Find the ratio of time taken for the level of water to fall down from h to h/2 and h/2 to zero. **Answer** (b) $\sqrt{2}-1$

10. According to Bernoulli's theorem, for the streamline flow of an ideal liquid, is given by

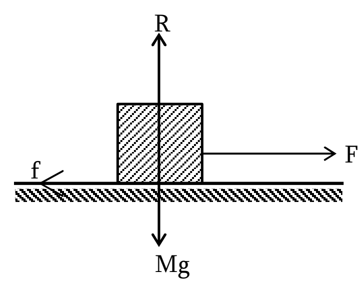
$$\frac{P}{\rho}+\frac{V^2}{2}+gh = \text{constant}$$

Where the symbols have usual meaning.

2. VISCOSITY (FLUID FRICTION)

IN SOLID

Friction force is the force which possess the motion of an object between two surface which are in contact together. Consider the figure below which shows the motion of block on rough surface.



Coefficient of friction force

$$\mu = \frac{F}{R} = \frac{f}{R}$$

$$f = \mu R = \mu Mg$$

Each symbol have usual meaning. Let us pour equal amounts of water and honey in two similar funnels. It will be observed that water flows out of the funnel very quickly on the other hand, honey is extremely show in flowing down. This difference in the behavior of two liquids indicates that the internal frictional force' in the two liquids have different values. When the fluid flowing possessing the internal frictional force within thir molecules (layers of fluids). This indicates that there must be some internal frictional force which opposes relative motion between different layers. This force is called **'Viscous force'**. The phenomena is called **'Viscosity'**.

Qualitatively definition

Viscosity is the property of fluid (liquid or gas) by virtue of which it opposes the relative motion between its different layers i.e viscosity or fluid friction is a kind of internal resistance which offers an opposition to the motion of one layer of fluid past over another layer.

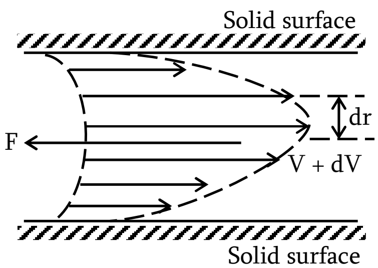
- Both liquids and gases exhibit viscosity although liquids are much more viscous than gases.
- The greater the viscosity, the less easy it is for the fluid to flow and the more sticky it feels.
- The external force required to maintain the relative velocity between the different layers of the fluid is a measure of the viscosity of the fluid.

CAUSE OF VISCOSITY

The main cause of viscosity is due to molecular forces of attraction between the adjacent layers of fluid when one layer is moving relative to the others.

Quantitatively definition

Consider a liquid flowing steadily over a solid horizontal surface. The layer of the liquid in contact with the solid horizontal surface is at rest. So this layer is fixed layer. The velocities of other layers increase uniformly with the increase in distance from fixed layer to the centre as shown below.



Newton's suggested that, if the liquid layer has an area A , viscous force (F) exerted on it by the adjacent layer depends on:

(i) Area A, of the layer
(ii) Distance (dr) of separation of the adjacent layers.
(iii) Relative velocity (dv) between the adjacent layers.

According to Newton's law of viscous flow'. The viscous force (F) acting tangentially on a layer of the liquid is proportional to

(i) The area A of the layer and
(ii) The velocity gradient, dv/dr

$$F \propto A\frac{dv}{dr} \quad \text{or} \quad F = \eta A\frac{dv}{dr}$$

Where η is the coefficient of the viscosity of liquid. It depends upon the nature of the liquid. Negative sign shows that the viscous force acts in a direction opposite to the direction of the motion of the liquid.

In magnitude : $F = \eta A\frac{dv}{dr}$

$$\eta = \frac{F}{A\cdot\frac{dv}{dr}}$$

DEFINITION VISCOSITY OR COEFFICIENT OF VISCOSITY - Is defined as viscous force per unit area per unit velocity gradient i.e coefficient of viscosity of a liquid is the viscous force acting tangentially per unit area of a liquid having a unit velocity gradient is a direction perpendicular to the direction of flow of the liquid.

$$\eta = \frac{F}{A\cdot\frac{dv}{dr}} = \left(\frac{F}{A}\right)\cdot\frac{1}{\frac{dv}{dr}}$$

$$\eta = \frac{\text{tangential stress}}{\text{velocity gradient}}$$

Unit of viscosity (η)

$$\eta = \frac{N}{m^2s^{-1}} = Nm^{-2}s \quad \text{or} \quad Pas$$

∴ S.I unit of viscosity is Nsm^{-2} or Pas or $Kgm^{-1}s^{-1}$ or decapoise.

- The coefficient of viscosity of a liquid is said to be one decapoise if a tangential force of 1newton m^{-2} of the surface is required to maintain a relative velocity of 1ms-1 between two layers of liquid 1m a part.

Relation between decapoise and poise

1 decapoise $= 1Nsm^{-2} = 10^5 \times (10^2)^{-2}$dyne

$Scm^{-2} = 10$poise

1 centipoise $= \frac{1}{100}$ poise

1 micropoise = 10-6 poise

Dimensional formula of coefficient of viscosity

We know that $\eta = \frac{F}{A \cdot dv/dr} = \frac{Fdr}{Adv}$

$$[n] = \frac{[F][dr]}{[A][dv]} = \frac{[ML^2T^{-2}]}{[L^2][LT^{-1}]}$$

$$[n] = [ML^{-1}T^{-1}]$$

DEFINITION

- **VELOCITY GRADIENT** is the change in velocity per unit distance between adjacent layer in a direction perpendicular to the velocity

  Velocity gradient $= \frac{dv}{dr} = \frac{dv}{dx}$

  S.I unit of velocity gradient is per second (s-1)

  $$\left[\frac{dv}{dr}\right] = T^{-1} = [M^0L^0T^{-1}]$$

- **VISCOUS FORCE** is the opposing force within the liquid which opposes the relative motion of one layer over the other layer

- **FLUIDITY** is the measure of the ability of a fluid to flow and is equal to the reciprocal of viscosity

  Fluidity $= \frac{1}{\eta}$

  S.I unit of fluidity is $Pa^{-1}s^{-1}$.

NEWTONIAN AND NON – NEWTONIAN FLUID

Newtonian fluid is the fluid with which tangential stress is directly proportional to the velocity gradient i.e $\frac{F}{A} \alpha \frac{dv}{dr}$

Examples for any pure liquids (eg water) and gases η is independent of the velocity gradient at a particular temperature.

- Viscostatic oils have about the same value of η whether cold or hot
- For some liquids such as paints, glues and liquid cements, η decreases as the tangential stress increases and these are said to be **'THIXOTROPIC'**.
- **Non – Newtonian** fluid is the fluid with which tangential stress is not proportional to the velocity gradient.

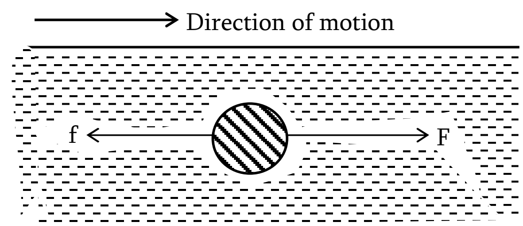
MOTION OF SPHERICAL BODY ON THE VISCOUS FLUID.

There are two types of motion of an object through the viscous fluid.

(i) Horizontal motion (ii) vertical motion.

HORIZONTAL MOTION OF AN OBJECT THROUGH VISCOUS FLUID.

Consider the motion of the spherical body through the viscous fluid as shown on the figure below



F = Applied external force

f = viscous drag force.

Generally , viscous drag force (f) on a body moving through the viscous fluid can be depends on the following:

(i) The shape of the body (spherical or oval shape); k

(ii) Coefficient of viscosity of the liquid, η

(iii) Velocity V of an object mathematically , f α ηv, f = kηv

DEFINITION VISCOUS DRAG force is the opposing force on the body which is moving along the viscous fluid

$$f = k\eta v$$

Now, consider an object of mass M moving along the viscous fluid as shown on the figure above. Resultant force on an object.

$$F - f = Ma$$
$$M\frac{dv}{dt} = F - k\eta v$$
$$\frac{dv}{dt} = \frac{-F}{M}\left(\frac{k\eta v}{F} - 1\right)$$
$$\frac{dv}{dt} = \frac{-k\eta}{M}\left(V - \frac{F}{K\eta}\right)$$
$$\frac{dv}{V - \frac{F}{K\eta}} = \frac{-K\eta}{M}\cdot dt$$
$$\int_0^V \frac{dv}{V - \frac{F}{K\eta}} = \frac{-K\eta}{M}\int_0^t dt$$
$$\left[\log_e\left(V - \frac{F}{K\eta}\right)\right]_0^V = \frac{-K\eta}{M}\left[t\right]_0^t$$
$$\log_e\left(V - \frac{F}{K\eta}\right) - \log_e\left(\frac{-F}{K\eta}\right) = \frac{-K\eta t}{M}$$
$$\log_e\left(\frac{V - \frac{F}{K\eta}}{\frac{-F}{K\eta}}\right) = \frac{K\eta t}{M}$$

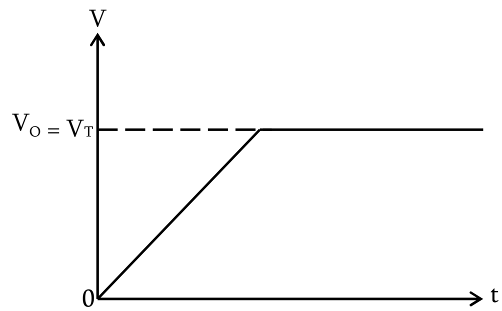
In exponential from

$$\frac{V - \frac{F}{K\eta}}{\frac{-F}{K\eta}} = e^{-k\eta t/M}$$
$$V - \frac{F}{K\eta} = \frac{F}{K\eta}e^{k\eta t/M}$$
$$V = \frac{F}{K\eta} - \frac{F}{K\eta}e^{-k\eta t/M}$$
$$V = \frac{F}{K\eta}\left[1 - e^{-k\eta t/M}\right]$$

Where $t = 0$, $V = 0$
When $t = \alpha$, $V = V_T$
V_T = terminal velocity

$$V_T = \frac{F}{K\eta}\left[1 - e^{-\infty}\right] = \frac{F}{K\eta}$$
$$V_T = \frac{F}{K\eta}$$

GRAPH OF V AGAINST TIME ,T



Definition terminal velocity is the constant maximum velocity attained by the body moving through a viscous fluid.

STOKE'S LAW

Derivation of the stoke's law

This can be derived by using method of dimensional analysis. Stokes performed may experiments on the motion of small spherical bodies in different liquids. He conclude that the viscous force, F acting on the small sphere of radius r depends on:

(i) Coefficient of viscosity η of the liquid
(ii) Velocity of spherical body ,V
(iii) Radius , r.

$F \alpha \eta^a r^b v^c$

$F = k\eta^a v^c r^b$

Where k is a dimensionless constant

Dimensionally

$$[F] = [\eta]^a [r]^b [V]^c$$
$$MLT^{-2} = \left[ML^{-1}T^{-1}\right]^a [L]^b \left[LT^{1-}\right]^c$$
$$M'L'T^{-2} = M^a L^{-a+b+c} T^{-a-c}$$

On equating indices (power)

M: 1 = a…….(i)

L: 1 = -a + b + c …..(ii)

T: -2 = -a – c …….(iii)

On solving , a = b = c = 1

Now f = kηvr

The value of k as found experimentally comes out to be 6π. F = 6πηvr

This relation is known as stoke's law. Stokes law state that 'for steady motion of a small spherical body, smooth and rigid and moves in a liquid of infinite extent, the viscos drag force experienced on the body is given by $F = 6\pi\eta vr$.

Stoke's law is valid under the following assumptions:-

(i) The viscous medium is homogenous and infinite extent.
(ii) The spherical body is perfectly rigid and smooth.
(iii) The medium is continuous i.e the size of the moving body is much larger than the distance between the molecules of the medium.
(iv) The body does not slip in the medium.
(v) The velocity of the spherical body is less than critical velocity. So, the motion of the body through the fluid does not give rise to turbulent motion.

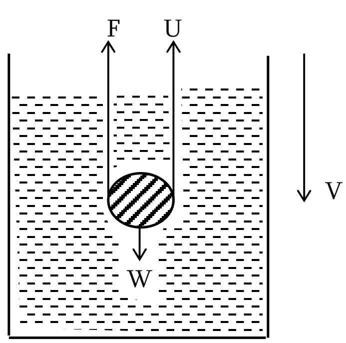
METHODS OF DETERMINATION OF VISCOSITY OF FLUIDS

The following are methods used to determine the viscosity of the fluids.

(i) By using stoke's law i.e vertical motion of spherical body through viscous fluid.
(ii) By using poiseull's formulae
(iii) By using the constant pressure head method.

1. VERTICAL MOTION OF SPHERICAL BODY ON VISCOUS FLUID (STOKE'S LAW)

Consider a sphere of radius r and density ρ falling under gravity in a fluid of density, δ as shown on figure below.



There are three forces available which are acting on the following sphere:-

(i) Weight , W = Mg of the sphere acting vertically downwards through the centre of gravity of the sphere.
(ii) Upthrust U acting vertically upward due to buoyancy
(iii) Viscous drag force F acting vertically upward $F = 6\pi\eta vr$ (stoke's law)

Net downward force = W – (U + F). This force is responsible for the downward acceleration of the sphere i.e when the spherical body is in a liquid starts to move from rest, the body has initial acceleration since the sphere is denser than the liquid so there is net force downward acting on the body and medium. As the velocity of the spherical body increases, the viscous drag F also increases. A stage is reached when the weight of the sphere is just balanced by the sum of the upthrust and the viscous drag. At this state, there is no resultant force on the sphere. Consequently , the sphere begins to move with constant velocity, V. This constant maximum velocity of the sphere is known as the terminal velocity. When the spere moves with terminal velocity i.e at the equilibrium.

$$F + U = W$$

$$F = W - U \quad \ldots\ldots(1)$$

Since $F = 6\pi\eta vr$ (stoke's law)

$$W = Mg = \frac{4}{3}\pi r^3 \rho g$$

U = weight of the liquid displaced (Archimedes principle)

$$U = \frac{4}{3}\pi r^3 \delta g$$

$$6\pi\eta vr = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \delta g$$

$$6\pi\eta vr = \frac{4}{3}\pi r^3 g(\rho - \delta)$$

$$V = \frac{2r^2(\rho - \delta)g}{9\eta}$$

Viscosity of the liquid is given by

$$\eta = \frac{2r^2(\rho - \delta)g}{9\eta}$$

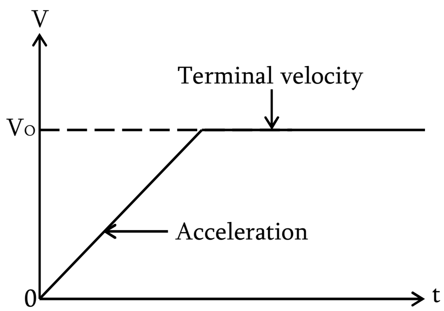
DISCUSSION

From the express of the terminal velocity ,

$$\eta = \frac{2r^2(\rho - \delta)g}{9\eta}$$

We can conclude the following:-

1. The variation of the velocity of the sphere with time is shown graphically below. As is clear from the graph, the sphere accelerates for some time and then attains the constant velocity (Vo) (terminal velocity).



2. The terminal velocity of a sphere is directly proportional to the sphere of the radius of the sphere ($V \propto R^2$). This means that for a given medium, the terminal velocity of a large sphere is greater than that of a small sphere of the same material. **For this reasons:** this explains as to why larger rain drops fall with greater velocity as compared to the smaller rain drops.
   Additional concept: $V \propto r^2$.
   $$\frac{V_1}{V_2} = \frac{r_1^2}{r_2^2}$$

3. $V \propto (\rho - \delta)$
   (i) If $\rho > \delta$, then the sphere shall attain terminal velocity is in downward direction.
   (ii) If $\rho < \delta$, V is negative. This means that the body instead of falling moves upwards and the sphere shall attain terminal velocity in upward direction.

   For this reason
   - This explain bubbles of air rise up in water.
   - This explain the rise of gas bubbles in soda water.

4. The terminal velocity is inversely proportional to the coefficient of viscosity of fluid ($v \propto 1/n$). Thus, a given sphere shall attain less terminal velocity in a more viscous fluid.

EFFECT OF COALESCENCE OF TERMINAL VELOCITY

Consider n droplets each moving with a terminal velocity Vo. Suppose they coalesce to constitute a single large drop of radius , R we are required to obtain expression of the new terminal velocity of large drop.

Apply the law of conservation of mass.

Volume of large drop = total volume of n small drops

$$\frac{4}{3}\pi R^3 = n\left[\frac{4}{3}\pi r^3\right]$$

$$R^3 = nr^3 \quad , \quad R = n^{1/3}r$$

$$n = \left(\frac{R}{r}\right)^3$$

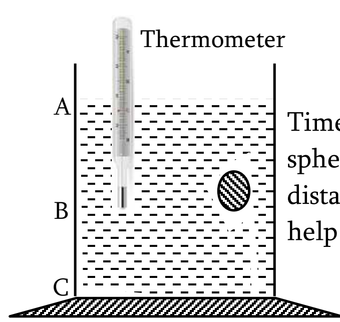
Since $V \propto (\text{radius})2$

$$\frac{V}{Vo} = \frac{R^2}{r^2} = \frac{\left(n^{1/3}r\right)}{r^2} = n^{2/3}$$

$$V = n^{2/3}Vo$$

PRACTICAL APPLICATIONS OF STOKE'S LAW

1. Determination of coefficient of viscosity of very viscous liquids like gases or pitch et.c. The experimental liquid is taken in a tall and wide glass jar. A small steel sphere is taken and is gently dropped in the jar. Two points B and C separated by a distance S are marked near the bottom as shown in figure below. Let us assume that when the sphere reaches B, it has acquired the terminal velocity, V.



Time taken by the sphere to fall through a distance *s* in noted with help of stopwatch

$$V = \frac{s}{t}$$

Now, $$V = \frac{2r^2g(\rho-\delta)}{9\eta}$$

$$\frac{s}{t} = \frac{2r^2g(\rho-\delta)}{9\eta}$$

$$\eta = \frac{2}{9}r^2g(\rho-\delta)\times\frac{t}{s}$$

Knowing r, g , ρ and δ, the coefficient of viscosity η of the liquid is calculated. A thermometer is dipped in the liquid to known the temperature at which η is being calculated.

2. In Millikan's experiment for the determination of electronic charge, we make use of stoke's law.

3. It is used to find the size of small particles.

4. We also make use of stoke's law of explaining the following.
   (i) A man coming down with the help of a parachute acquires only a limited velocity.
   (ii) The falling rain drops can acquire only a limited velocity.
   (iii) Formation of a clouds.

2. DETERMINATION OF VISCOSITY OF THE FLUID BY USING POISEULLI'S FORMULAE.

Poiseulli's formula (equation) is the equation which is used to determine the rate of volume of the fluid flow through a pipe and viscosity of the fluid flow.

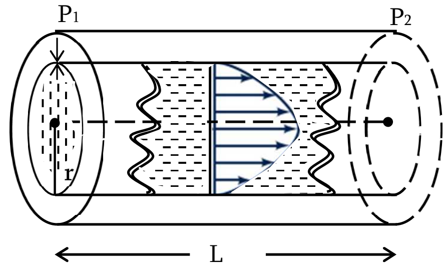
Poiseulli's equation is given by

$$\frac{v}{t} = \frac{\pi p r^4}{8\eta L} = \frac{\pi(P_1-P_2)r^4}{8\eta L}$$

DERIVATION OF POISEULLI'S EQUATION

Method 1: derivation by dimensional analysis.

Consider a liquid flowing steadily through a horizontal capillary tube of length *l* and radius r. in order to maintain the flow, there must exist a pressure difference P across the ends of the tube



It has been observed that the volume V of the liquid flowing per second through the tube depends upon on :-

(i) Coefficient of viscosity, η of the liquid
(ii) Radius of the tube, r
(iii) Pressure gradient, P/L

Let $$\frac{V}{t} \propto \eta^a r^b \left(P/L\right)^C$$

$$\frac{V}{t} = k\eta^a r^b\left(\frac{P}{L}\right)^c$$

Where k is dimensionless constant of proportionality

Dimensionally

$$\left[\frac{V}{t}\right] = [n]^a[r]^b\left[\frac{P}{L}\right]^c$$

$$M^0L^3T^{-1} = (ML^{-1}T^{-1})^a L^b (ML^{-2}T^{-2})^C$$

$$M^0L^3T^{-1} = M^{a+c}L^{-a+b-2c}T^{-a-2c}$$

On equating indices (powers)

M: 0 = a + c …..(i)

L : 3 = -a + b – 2c…..(ii)

T : -1 = -a – 2c……(iii)

On solving these equations , we get

a = -1 , b =4 and c = 1

$$\frac{v}{t} = k\eta^{-1}r^4\left(\frac{P}{L}\right)^1$$

$$\frac{v}{t} = \frac{kpr^4}{\eta L}$$

Mathematical investigation show that

$$K = \frac{\pi}{8}$$

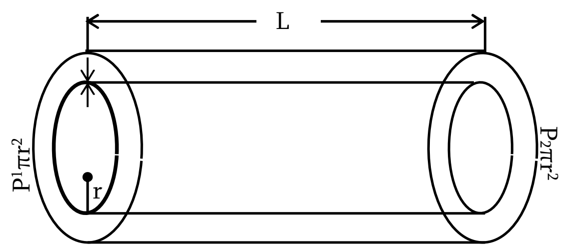
$$\frac{V}{t} = \frac{\pi p r^4}{8\eta L}$$

Assumptions used to develop the poiseulli's formulae.

1. The fluid flow is steady and parallel to the axis of the tube.
2. There must be existence of pressure gradient
3. The capillary tube is horizontal so, the gravity has no effect on the flow of liquid through the tube.
4. The pressure at the inlet of the pipe should be greater than the pressure at the end of the out let end.
5. The liquid is viscous fluid.
6. The liquid can withstand small shearing stress.
7. The pressure over any cross – section of the tube is constant. So there is no radial flow of the liquid.

METHOD 2. DERIVATION OF POISEULLI'S FORMULA MATHEMATICALLY.

Consider the flowing of viscous fluid along the pipe whose length is L with radius r as shown on the figure below.



If P is the excess pressure and L is the length of the pipe.

$$F = -\eta A\frac{dv}{dr} \qquad A = 2\pi rL$$

$$F = -2\pi\eta rL\frac{dv}{dr}$$

But $F = PA_1 = P\pi r^2$

$$\frac{dv}{dr} = \frac{-pr}{2\eta L}$$

$$dv = \frac{-pr}{2\eta L}dr$$

$$\int dv = \frac{-p}{2\eta L}\int rdr$$

$$V = \frac{-pr^2}{4\eta L} + C$$

When V = 0 , r = R

$$0 = \frac{-PR^2}{4\eta L} + C$$

$$C = \frac{PR^2}{4\eta L}$$

Now. $V = \dfrac{-Pr^2}{4\eta L} + \dfrac{PR^2}{4\eta L}$

$$V = \frac{P}{4\eta L}\left(R^2 - r^2\right)$$

Volume per second of liquid coming out of the pipe. Consider a cylindrical shell between r and r + dr, since the shell of liquid has the same velocity, v at every part.

Volume per sec from pipe = 2πrvdr

Total volume per second

$$\frac{v}{t} = \int_0^R 2\pi rvdr$$

$$= \int_0^R \frac{2\pi p}{4\eta L}\left(R^2 r - r^3\right)dr$$

$$= \frac{\pi p}{2\eta L}\left[\frac{R^2r^2}{2} - \frac{r^4}{4}\right]_0^R$$

$$\frac{V}{t} = \frac{\pi PR^4}{8\eta L} = \frac{\pi\left(P_1 - P_2\right)R^4}{8\eta L}$$

POISEULLI'S LAW state that 'The volume rate of flow of the liquid is inversely proportional to the viscosity as might be expected and proportional to the pressure gradient along the pipe and varies as fourth power of the pipe radius'.

Limitation of poiseulli's law

The law is true only for laminar flow but not for turbulent. Flow

Note that

(i) Stoke's law is strictly the law applies only to a fluid of infinity extent.

(ii) Stoke's law does not hold if the sphere is moving so fast that conditions are not streamline.

SPEED OF BULK FLOW

Is defined as rate of volume of flow per unit cross – sectional area of the pipe. Steady flow occurs only when the speed of bulk flow is less than a certain value Vc. Since Poiseulli's formulae applies only to steady flow, it does not hold when the speed of bulk flow exceeded Vc. Experiment shows that for cylindrical pipes.

$$V_c \simeq \frac{1100\eta}{\rho r}$$

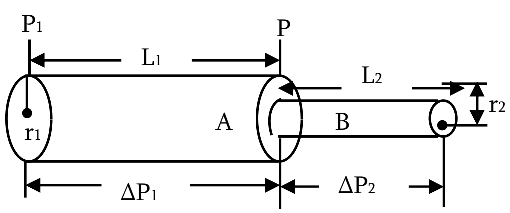
Vc = critical value of speed of fluid flow
η = coefficient of viscosity of fluid
r = radius of the pipe
ρ = density of fluid flow.

FLUID FLOW IN CONNECTED TUBE (PIPES)

Tube or pipes can be connected into two ways:
(i) Series connection of the tubes
(ii) Parallel connection of the tubes.

SERIES CONNECTION OF THE PIPES

When the two tubes of different diameters and lengths are connected in series, the rate of volume of fluid flow is same in all the tubes



- Expression of the rate of volume of the fluid flow. According to the poiseull's formula

$$\frac{v}{t} = \frac{\pi \Delta p r^4}{8\eta L}$$

For the tube A

$$Q = \frac{v}{t} = \frac{\pi(p_1 - p)r_1^4}{8\eta L_1}$$

$$p_1 - p = \frac{8\eta L_1 Q}{\pi r_1^4} \quad \ldots\ldots\ldots(i)$$

For the tube B

$$Q = \frac{v}{t} = \frac{\pi(p - p_2)r_2^4}{8\eta L_2}$$

$$p - p_2 = \frac{8\eta L_2 \frac{1}{2}}{\pi r_2^4} \quad \ldots\ldots\ldots(2)$$

(1) + (2)

$$p_1 - p + p - p_2 = \frac{8\eta L_1 Q}{\pi r_1^4} + \frac{8\eta L_2 Q}{\pi r_2^4}$$

$$\pi(p_1 - p_2) = 8\eta Q\left[\frac{L_1}{r_1^4} + \frac{L_2}{r_2^4}\right]$$

$$\frac{v}{t} = Q = \frac{\pi(p_1 - p_2)}{8\eta\left[\frac{L_1}{r_1^4} + \frac{L_2}{r_2^4}\right]}$$

- Expression of the total pressure difference across the ends of pipes.

$$\Delta P = \Delta P_1 + \Delta P_2$$

- Expression of the pressure at the function point of the two pipes

Since $\frac{v}{t}$ = constant

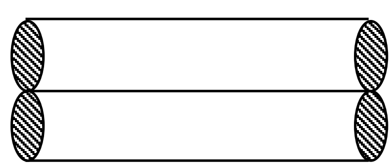
$$\left(\frac{v}{t}\right)_1 = \left(\frac{v}{t}\right)_2$$

$$\frac{\pi(P_1 - P)r_1^4}{8\eta 4} = \frac{\pi(P - P_2)r_2^4}{8\eta L_2}$$

$$P = \frac{P_1 r_1^4 L_2 + P_2 r_2^4 L_1}{r_1^4 L_2 + r_2^4 L_1}$$

2. PARALLEL CONNECTIN OF THE PIPES

For the pipes in parallel connection, the total rate of volume of water flows is equal to the sum of the rate of liquid flows on each pipe.



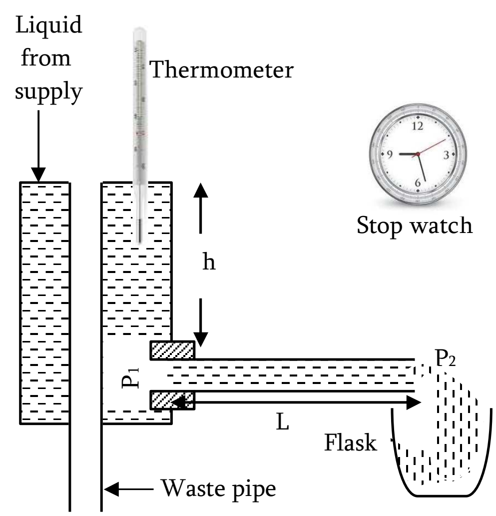
$$\frac{v}{t} = \left(\frac{v}{t}\right)_1 + \left(\frac{v}{t}\right)_2 + \cdots\cdots + \left(\frac{v}{t}\right)_n$$

$$\frac{v}{t} = \left(\frac{v}{t}\right)_1 + \left(\frac{v}{t}\right)_2$$

$$\frac{v}{t} = \frac{\pi(P_1 - P_2)}{8\eta L}\left[r_1^4 + r_2^4\right]$$

3. MEASUREMENT OF VISCOSITY , H OF THE LIQUID BY USING CONSTANT PRESSURE HEAD APPARATUS (POISEULLI'S FORMULA).

The method is based on poiseulli's formula for laminar flow and is suitable for a liquid which flows readily and is available in large quantity eg. Water. This can be illustrated on the figure below.



The liquid is supplied from a constant head apparatus and made to flow along a capillary tube of radius , r small enough for a laminar flow to be attained. The liquid flow down the wide waste pipe to ensure that the head of liquid is kept at a height, h for the whole experiment. The pressure head h can be changed by either lowering or raising the waste pipe.

Measurement

- The length L of the tube can be measured with a metre rule.
- The pressure difference $\Delta P = \rho gh$, h is measured by metre rule.
- The volume rate can be measured by conical flask.

$$\frac{v}{t} = \frac{\text{volume of liquid collected}}{\text{time taken}}$$

- Time taken is recorded by the stop watch
- Temperature of the liquid must be recorded since coefficient of viscosity varies with temperature.

  Mathematically

The pressure at the inlet end.

$$P_1 = Pa + \rho gh$$

Pressure at out let end

$$P_2 = Pa$$

$$\Delta P = P_1 - P_2 = \rho gh$$

$$\frac{\Delta P}{L} = \frac{P_1 - P_2}{L} = \frac{\rho gh}{L}$$

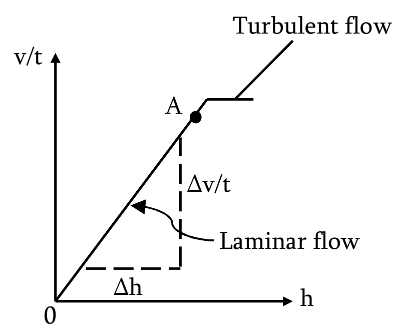
According to the Poiseulli's equation

$$\frac{v}{t} = \frac{\pi\left(P_1 - P_2\right)r^4}{8\eta L}$$

$$\frac{v}{t} = \frac{\pi r^4 \rho gh}{8\eta L}$$

$$\eta = \frac{\pi r^4 \rho ght}{8VL}$$

GRAPH OF v/t AGAINST H



The viscosity of the liquid can only be obtained from the linear part of the graph

$$\text{slope} = \frac{\Delta \, v/t}{\Delta h}$$

$$\frac{v}{t} = \left(\frac{\pi r^4 \rho g}{8\eta L}\right)h$$

$$\text{slope} = \frac{\pi r^4 \rho g}{8\eta L}$$

$$\eta = \frac{\pi r^4 \rho g}{8L \times \text{slope}}$$

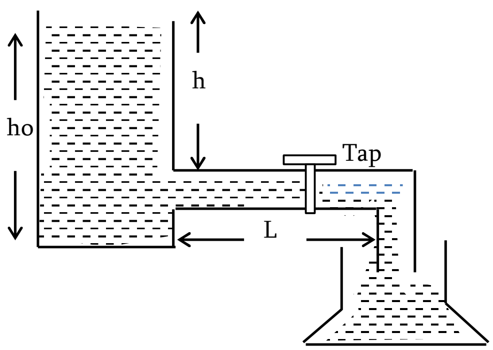
SOURCES OF ERRORS AND PRECAUTIONS

1. The capillary tube must be kept horizontally so that there is no flow of liquid under the gravity.

2. The flow of liquid must be slow and streamlined.
3. The diameter of the tube must be determined on both ends in perpendicular direction and the average must be taken.
4. A narrow bore capillary tube may be used so that the flow is steady corresponding to the observable pressure difference.

THEORY OF FLOWING OF THE LIQUID FROM WIDE TANK

Consider the steady flow of liquid through a capillary tube of radius r and length L as shown the figure below



The pressure difference between the ends of the capillary tube is ρgh. According to the Poiseulli's equation. Rate of volume of liquid flows

$$\frac{v}{t} = \frac{\pi p r^4}{8\eta L} = \frac{\pi\rho g h r^4}{8\eta L} \ldots\ldots.(1)$$

According to the continuity equation

$$\frac{dv}{dt} = -A\frac{dh}{dt} \ldots\ldots\ldots..(2)$$

[Negative sign shows dh decreases as time, dt increases]

(1) = (2)

$$-A\frac{dh}{dt} = \left(\frac{\pi\rho g r^4}{8\eta L}\right)h$$

$$\frac{dh}{dt} = -\left[\frac{\pi\rho g r^4}{8\eta LA}\right]h$$

Let $C = \frac{\pi\rho g r^4}{8\eta LA}$

$$\frac{dh}{dt} = -Ch$$

$$\frac{dh}{h} = -Cdt$$

$$\int_{ho}^{h}\frac{dh}{h} = -C\int_{0}^{t}dt$$

$$\left[\log_e h\right]_{ho}^{h} = -Ct$$

$$\log_e h - \log_e ho = -Ct$$

$$\log_e\left(h/ho\right) = -Ct$$

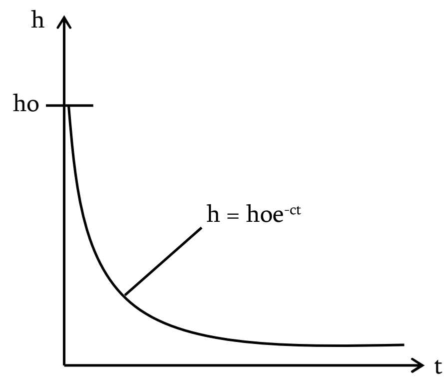
$$h/ho = e^{-ct}$$

$$h = hoe^{-ct}$$

$$C = \frac{\pi\rho g r^4}{8\eta LA}$$

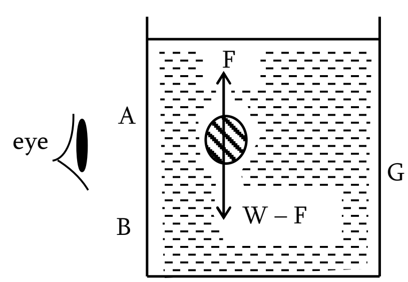
ρ = density of fluid flow
g = Acceleration due to gravity
r = radius of capillary tube
η = viscosity of the liquid
A = cross – sectional area of the tank
L = Length of capillary tube.

GRAPH OF h AGAINST TIME , T



COMPARING OF VISCOSITIES OF THE LIQUIDS

Stoke's formula can be used to compare the coefficients of viscosity of very viscous liquids such as a glycerine or treacle. Consider the motion of the same sphere moving through the liquids of the different viscosities.



Since the time taken to fall through a certain distance in different liquid is recorded and terminal velocities now are noted.

From the stoke's formula

$$\eta = \frac{2r^2g(\rho-\delta)}{9V}$$

For the first liquid

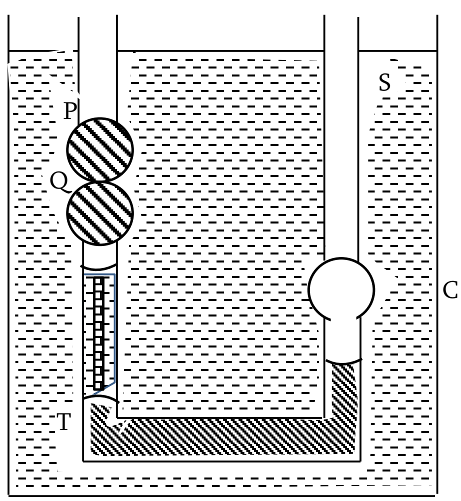
$$\eta_1 = 2r^2g\frac{(\rho-\delta_1)}{9V_1}$$

$$\frac{\eta_1}{\eta_2} = \frac{V_2}{V_1}\left(\frac{\rho-\delta_1}{\rho-\delta_2}\right)$$

$$\frac{\eta_1}{\eta_2} = \frac{V_2(\rho-\delta_1)}{V_1(\rho-\delta_2)}$$

COMPARISON OF VISCOSITIES OSTWALD VISCOMETER.

An Ostward viscometer, which contain a vertical capillary tube T, is widely used for comparing the viscosities of two liquids. The liquid is introduced at S, drawn by sunction above P and the time t1 taken for the liquid level to fall between the fixe marks P, Q is observed. The experiment is then repeated with the same volume of a second liquid and the time t_2 for the liquid level to fall from P to Q is noted.



Suppose the liquids have respective densities ρ_1, ρ_2. Then since the average head h of liquid forcing it through T is the same in each case, the pressure excess between the ends of T = $h\rho_1 g$, $h\rho_2 g$ respectively. If the volume between the marks P, Q is V. according to the Poiseulli's equation

$$\frac{v_1}{t_1} = \frac{\pi(h\rho_1 g)r^4}{8\eta_1 L} \text{..........(1)}$$

For the second liquid

$$\frac{V}{t_2} = \frac{\pi(g\rho_2 g)r^4}{8\eta_2 L} \text{............(2)}$$

(1) / (2)

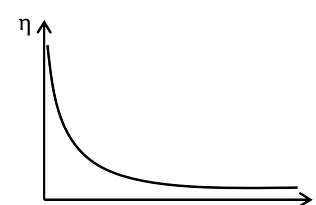
$$\frac{t_1}{t_2} = \frac{\eta_1\rho_2}{\eta_2\rho_1}$$

$$\frac{\eta_1}{\eta_2} = \frac{t_1}{t_2}\cdot\frac{\rho_1}{\rho_2}$$

By knowing t_1, t_2 and desnities ρ_1, ρ_2 , the coefficients of viscosity can be compared.

FACTORS AFFECTING VISCOSITY OF THE FLUIDS

1. Effect of temperature
   - For the liquid. The viscosity of the liquid is due to the intermolecular force of attraction between liquid layers moving at the different speed. Intermolecular force reduces with rise in temperature. Therefore the viscosity of the liquid decreases with rise in temperature.



   - For gases viscosity of the gas is due to the transfer of momentum between neighbouring layers of the gases. It is directly proportional to the average speed of the gas molecules. Since the average speed increases with rise of temperature, thus the viscosity of the gases increases with rise of temperature.

2. Effect of pressure on viscosity
   - For the liquid in general pressure produces a much smaller effect than temperature on the viscosity of the liquid. In this case of liquids having low value of viscosity like ether, water , the change is small. But for liquids having large value of viscosity like minerals oil change in much greater. Viscosity of water decreases with pressure but for the other liquid viscosity increases with pressure.
   - For the gases. The viscosity of a gas is independent of the pressure at ordinary pressure. But at low pressure viscosity continually fall with decrease in pressure. At high pressure, the viscosity increases with pressure.

Fluid	n(Nsm^{-2})
Glycerine (20^{o}C)	1.5
Motor oil (0^{o}C)	0.11
Motor oil (20^{o}C)	0.03
Blood (37^{o}C)	4×10^{-3}
Water (20^{o}C)	1.0×10^{-3}
Water (90^{o}C)	0.32×10^{-3}
Gasoline (20^{o}C)	2.9×10^{-4}
Air (20^{o}C)	1.8×10^{-5}
CO2 (20^{o}C)	1.5×10^{-5}
Castor oil (20^{o}C)	2.42
Castor oil (40^{o}C)	2.13

COMPARISONS BETWEEN VISCOSITY AND SOLID FRICTION

POINTS OF SIMILARITY

(i) Both opposes motion
(ii) Both arise due to the intermolecular forces
(iii) Both comes into play due to the relative motion.

POINT OF DIFFERENCES

VISCOSITY	SOLID FRICTION
The viscous force is directly proportional to the surface area of a contact of liquid layers	It is independent of the area of solid surface in contact.
The viscous force is directly proportional to the relative two layers of a liquid.	It is independent of the relative velocity of one body with respect to another body in contact.
Force of viscosity is independent on the normal reaction force.	Friction force on solid depend on the normal reaction force.
Viscous force depends upon the shape of the body moving through the fluid.	Friction force is independent of the shape of the bodies in contact.

APPLICATIONS OF THE VISCOSITY

1. The viscosity of blood depends upon the concentration of red blood corpuscle. Therefore, viscosity of blood can be used to detect blood corpuscle deficiency.
2. Viscosity of air liquid is used in providing damping torque in measuring instruments.
3. Oil used as lubricant should have proper value of viscosity.
4. The viscosity of oil help in applying brakes.
5. Blood circulation through arteries depend upon the viscosity of the blood.

Note that

Most of liquids however become less viscous as they become warmer. In water cars are more difficult to start from cold because the engine oil is much more viscous than when warm. Viscosity of oil is especially made with the same viscosity when cold or warmer.

NUMERICAL EXAMPLE

1. There is 1mm thick layer of glycerine between a flat plate of area 100cm2 and a big plate. If the coefficient of viscosity of glycerine is 1.0kgm-1s-1, then how much force is required to move the plate with a velocity of 7cm/s.

Solution

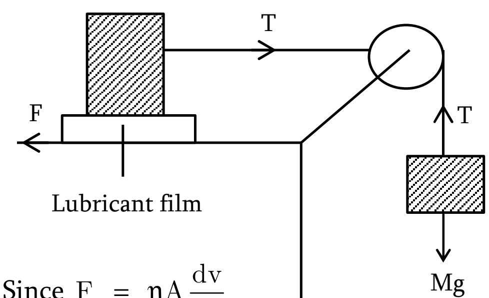
$$F = \eta A \frac{dv}{dr} = \frac{1 \times 100 \times 10^{-4} \times 7 \times 10^{-2}}{1 \times 10^{-3}}$$

$$F = 0.7N$$

2. A metal plate of surface area $0.3m^2$ is connected to a body of mass 16gm through a string passing over a massless frictionless pulley. A lubricant having a film thickness 0.6mm is placed between the plate and the surface. When the system is left free the plate moves to the right with a constant speed of 0.170m/s. calculate the coefficient of viscosity of the lubricant.

Solution

$T = F = Mg$



Since $F = \eta A \dfrac{dv}{dx}$

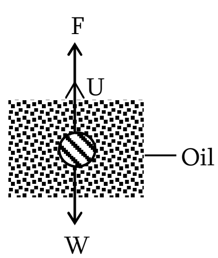
$$\eta = \frac{Fdx}{Adv} = \frac{Mgdx}{Adv}$$

$$\eta = \frac{0.016\times 9.8\times 0.6\times 10^{-3}}{0.30\times 0.17}$$

$$\eta = 1.8\times 10^{-5}\,Nsm^{-2}$$

3. An Iron sphere of diameter 10mm falls through a column of out density $940kgm^{-3}$. The density of iron is $7.8 \times 10^3 kgm^{-3}$. The coefficient of viscosity of the oil is $4.48Nsm^{-2}$. Calculate the terminal velocity attained by the ball.

Solution



$$V = \frac{2r^2g(\rho-\delta)}{9\eta}$$

$$= \frac{2\times(5\times 10^{-3})^2\times 9.8(7800-940)}{9\times 4.48}$$

$$V = 0.0834m/s$$

4. (a) Two vertical drops of water are falling through air with a steady velocity of 10cm/s. if the drops combine to form a single drop what would be the terminal velocity of the single drop.
   (b) With what terminal velocity will an air bubble 0.8mm in diameter rise in a liquid of viscosity of $0.15Nsm^{-2}$ and specific gravity is 0.9? Density of air is $1.293kgm^{-3}$.

Solution

(a) $V \propto (\text{Radius})^2$

$$\frac{V}{V_o} = \frac{R^2}{r^2} = \left(\frac{R}{r}\right)^2$$

Apply the law of conservation of mass

$$2\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$$

$$R = 2^{1/3}r$$

Now $V = V_o\left[\dfrac{2^{1/3}r}{r}\right]^2 = V_o\cdot 2^{2/3}$

$$V = 10cm\times 2^{2/3}$$
$$V = 15.9cm/s$$

(b) $V = 2r^2g\dfrac{(\rho-\delta)}{9\eta}$

$$= \frac{2\times(4\times 10^{-4})^2\times 9.8(1.293-0.9\times 10^3)}{9\times 0.15}$$

$$V = -0.0021m/s = 0.21cm/s$$

Negative sign shows that the bubble will rise up.

5. An oil drop, falls through air with a terminal velocity of $5 \times 10^{-4}m/s$.
   (i) Calculate the radius of the drop
   (ii) The terminal velocity of a drop of half of this radius.
   Velocity of oil = $1.8 \times 10^{-5}Nsm^{-2}$.
   Density of oil = $900kgm^{-3}$
   Neglecting the density of air as compaired to that of oil.

Solution

(i) By the stoke's law

$$V = \frac{2r^2g(\rho-\delta)}{9\eta} \quad , \quad \delta = 0$$

$$V = \frac{2r^2g\rho}{9\eta}$$

$$r = \left[\frac{9\eta V}{2\rho g}\right]^{\frac{1}{2}}$$

$$= \left[\frac{9\times5\times10^{-4}\times1.8\times10^{-5}}{2\times900\times9.8}\right]^{\frac{1}{2}}$$

$$r = 2.14\times10^{-6}\,\text{m}$$

(ii) Velocity α (radius)²

Let V = terminal velocity of the drop when its radius is halved $R = R_o/2$

$$\frac{V}{V_o} = \left[\frac{R}{R_o}\right]^2 = \left[\frac{R_o/2}{R_o}\right]^2$$

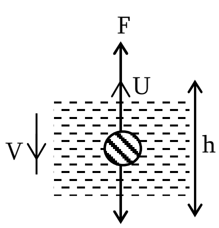
$$V = \frac{1}{4}V_o = \frac{1}{4}\times5\times10^{-4}$$

$$V = 1.25\times10^{-4}\,\text{m/s}$$

6. A metallic sphere of radius 1.0×10^{-3}m and density 1.0×10^{4}kgm^{-3} enters a tank of water after a free fall through a height h in earth's gravitational field. If its velocity remain unchanged after entering through water, determine the value of h. given that viscosity of water = 1.0×10^{-3}Nsm^{-2}, g = 10m/s^2 and density of water = 1000kgm^{-3}.

Solution

Velocity attained by sphere fall under gravity $V = \sqrt{2gh}$. The terminal velocity in the fluid



$$V = \frac{2r^2g(\rho-\delta)}{9\eta}$$

$$\sqrt{2gh} = \frac{2r^2g(\rho-\delta)}{9\eta}$$

$$h = \frac{1}{2g}\left[\frac{2r^2g(\rho-\delta)}{9\eta}\right]^2$$

$$h = 20\text{m}$$

7. A sphere is dropped under gravity through a fluid of viscosity, η. Taking average acceleration as half of the initial acceleration. Show that the time taken to attain terminal velocity is independent of fluid density.

Solution

When the sphere enters the fluid its first moves with maximum acceleration a as the velocity increases, the viscous force also increases after sometimes the acceleration becomes zero and the sphere now moves with (constant) terminal velocity, V.

$$\bar{a} = \frac{0+a}{2} = \frac{a}{2}$$

If t is the time taken by the sphere to attain terminal velocity , V

$$V = 0+\bar{a}t = \frac{at}{2}$$

Also $\quad F = W - U = \frac{4}{3}\pi r^3 g(\rho-\delta)$

(Stoke's law)

But $\quad F = Ma \quad , \quad a = \frac{F}{M}$

$$a = \frac{\frac{4}{3}\pi r^3 g(\rho-\delta)}{\frac{4}{3}\pi r^3\rho}$$

$$a = \frac{g(\rho-\delta)}{\rho}$$

Now $\quad V = \frac{at}{2} = \frac{g(\rho-\delta)}{2\rho}t \;.......(i)$

Also , terminal velocity

$$V = \frac{2r^2g(\rho-\delta)}{9\eta}\;......(ii)$$

(i) = (ii)

$$\frac{g(\rho-\delta)t}{2\rho} = \frac{2r^2g(\rho-\delta)}{9\eta}$$

$$t = \frac{4r^2\rho}{9\eta}$$

∴ The time taken by the sphere to attain terminal velocity is independent of the density ρ of the fluid.

8. (a) (i) Why do the clouds seem floating in the sky?

(ii) Why a parachute descends slowly where as a stone dropped from the same height falls rapidly?

(b) Eight rain drops of radius 10^{-3}m each falling down with a terminal velocity of 0.05m/s coalesce to form a bigger drop. Calculate the terminal velocity of bigger drop.

Solution

(a) (i) The terminal velocity of a rain drop is directly proportional to the square of radius of the drop. When falling, large drop have high terminal velocities while small drops have small terminal velocities. The small drops fall so slowly that could seems to be floating.

(ii) The open parachute has a large surface area. Therefore the viscous force of air on the parachute is larger than that on a falling stone. For this reason, parachute descends slowly i.e it has a small terminal velocity.

(b) Radius of each small drop r = 10^{-3}m.

Terminal velocity of each drop Vo = 0.05m/s.

Apply the law of conservation of volume

Volume of bigger drop = volume of 8 drops.

$$\frac{4}{3}\pi R^3 = 8\times\frac{4}{3}\pi r^3$$

$$R = 2\times10^{-3}\text{m}$$

Since Vα (radius)2

$$\frac{V}{V_o} = \frac{R^2}{r^2} = \left(\frac{R}{r}\right)^2$$

$$V = V_o\left[\frac{R}{r}\right]^2 = 0.05\left[\frac{2\times10^{-3}}{10^{-3}}\right]^2$$

$$V = 0.2\text{m/s}$$

9. (a) Draw diagrams to show the forces acting on an object falling through a viscous liquid.

(i) At the instant of release

(ii) When it has reached its terminal velocity. Write down an equation force the forces acting on the object in (ii)

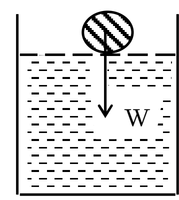
(b) In an experiment to determine the coefficient of viscosity of motor oil the following measurement were made:-

- Mass of glass sphere = 1.2×10^{-4}kg
- Diameter of sphere = 4.0×10^{-3}m
- Terminal velocity of sphere = 5.4×10^{-2}m/s

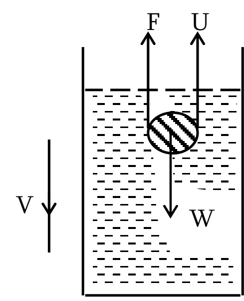
Calculate the viscosity of the oil.

Solution

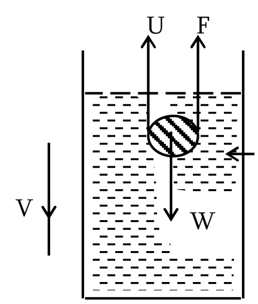
(a) (i) Diagram of the force instant of releasing an object through viscous fluid.



(b) When attain terminal velocity , V



F = Viscous drag force
U = upthrust
W = weight of an object



U = upthurst
F = viscous drag force
W = weight of sphere

At the equilibrium of the sphere

$$F = W - U$$

$$6\pi\eta Vr = Mg - \frac{4}{3}\pi r^3\rho g$$

$$\eta = \frac{Mg}{6\pi Vr} - \frac{2r^2\rho g}{9Vr}$$

$$\eta = 0.45\text{Nm}^{-2}\text{s}$$

10. (a) (i) Water flows faster than honey?
(ii) If water in one flask and glycerine in the other are violently shaken and kept on the table, then which one will come to rest earlier?
(iii) What is the effect of temperature on the viscosity of liquids and gases?
(b) Glycerine flow steadily through a horizontal tube of length 1.5m and radius 1.0cm. if the amount of glycerine collected per second at one end is $4.0 \times 10^{-3}kgs^{-1}$, what is pressure difference between two ends of the tube. Given density of glycerine is $1.3 \times 10^3 kgm^{-3}$ and viscosity of glycerine is $0.83Nsm^{-2}$ also check if the assumption of laminar flow in the tube is correct?

Solution

(a) (i) According to Poiseulli's formula,

$$\frac{v}{t} = Q = \frac{\pi PR^4}{8\eta L}$$

It is clear that for given P, R and L , $Q \alpha \frac{1}{\eta}$ where η is the coefficient of viscosity. Since η for water is less compared to honey , Q for water is greater than for honey. For this reason, water flows faster than honey

(ii) The glycerine comes to rest earlier because the viscosity of glycerine is greater than that of the water.

(iii) • The viscosity of liquids decreases with the increases in temperature.
• The viscosity of gases increases with increase of temperature

(b) Rate of volume of glycerine

$$\frac{v}{t} = \frac{m}{\rho} = \frac{4\times10^{-3}}{1.3\times10^{3}}$$

$$\frac{v}{t} = 3.077\times10^{-6}m^3/s$$

According to the Poiseulli's formula

$$\frac{v}{t} = \frac{\pi pr^4}{8\eta L}$$

$$P = \left(\frac{v}{t}\right)\cdot\frac{8\eta L}{\pi r^4}$$

$$= \frac{8\times3.077\times10^{-6}\times0.83\times1.5}{3.142\times10^{-8}}$$

$$P = 9.75\times10^{2}Nm^{-2}$$

Let V_1 = Velocity of the liquid

Flowing through the tube

$$\frac{v}{t} = V_1 A = V_1 \pi r^2$$

$$V_1 = \frac{v/t}{\pi r^2} = \frac{3.077\times10^{-6}}{3.142\times10^{-4}}$$

$$V_1 = 9.8\times10^{-3}m/s$$

Let VC be the required critical velocity

$$V_C = \frac{N\eta}{\rho D} = \frac{2000\times0.83}{1.3\times10^{3}\times2\times10^{-2}}$$

$$V_C = 63.85m/s$$

Since $V < V_C$, so the assumption regarding the laminar flow is correct.

11. (a) Distinguish between terminal velocity and critical velocity.
(b) The rate of flow of a liquid of density ρ is via a horizontal uniform capillary tube at the bottom of a vertical tank of uniform cross – section area , A is given by

$$A\frac{dh}{dt} = \frac{\pi\rho g r^4 h}{8\eta L}$$

Where at any time h is the height of the liquid in the tank above the axis of capillary tube of radius r and length L, the coefficient of viscosity is η. Show that $h = hoe^{-ct}$, where ho is the initial height . identify the constant C.

(c) A horizontal tube of diameter $2 \times 10^{-3}m$ and length 0.5m is connected at the bottom of a cubical tank is full of water, after what time will it be one – quarter full. State any assumptions you have made in your calculation.

Solution

(a) And (b) see your notes

(c) Area of cubical tank

$$A = 1m\times1m = 1m^2$$

Since $h = h_o e^{-ct}$

$$C = \frac{\pi\rho r^4 g}{8\eta LA}$$

$$= \frac{3.14\times10^{3}\times\left(1\times10^{-3}\right)^{4}\times9.8}{8\times10^{-3}\times0.5\times1}$$

$$C = 7.7\times10^{-6}\text{s}^{-1}$$

Let h_o = initial height of water

After t sec, $h = \frac{h_o}{4}$

$$\frac{h_o}{4} = h_o e^{-ct}$$

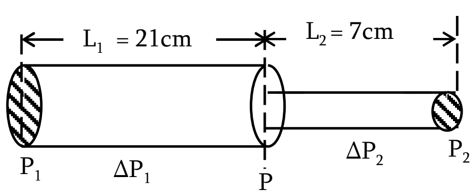
$$\frac{1}{4} = e^{-ct}, \quad 4 = e^{ct}$$

$$t = \frac{1}{C}\log_e 4 = \frac{1}{7.7\times10^{-6}}\log_e 4$$

$$t = 180{,}018.36\,\text{sec}$$

12. Water flows steady through tube which consists of two parts joined together end to end one part is 21cm long and the other is 7cm long and has a diameter of 0.075cm. If the pressure difference between the ends of the tube is 14cm of water. Find the pressure difference between ends of each part and if the other has a diameter of 0.225cm.

Solution



$$d_2 = 0.07\text{cm}, \quad \frac{d_2}{2} = r_2 = 0.035\text{cm}$$

$$d_1 = 0.225\text{cm}, \quad \frac{d_1}{2} = r_1 = 0.1125\text{cm}$$

$\Delta P_1 + \Delta P_2 = 14\text{cm}$ of water.

According to the Poiseulli's formula

$$Q = \frac{\pi\Delta p r^4}{8\eta L}$$

For the large pipe

$$Q = \frac{\pi\Delta p_1 r_1^4}{8\eta L_1}$$

For the smaller pipe

$$Q = \frac{\pi\Delta p_2 r_2^4}{8\eta L_2}$$

Since Q = Constant

$$\frac{\pi\Delta p_1 r_1^4}{8\eta L_1} = \frac{\pi\Delta p_2 r_2^4}{8\eta L_2}$$

$$\frac{\pi\Delta p_1\left(0.1125\times10^{-2}\right)^4}{8\eta\times21\times10^{-2}} = \frac{\pi\Delta p_2\left(0.035\times10^{-2}\right)^4}{8\eta\times7\times10^{-2}}$$

$$3\Delta P_2 = 81\Delta P_1$$

$$\Delta P_2 = 27\Delta P_1$$

Since $\Delta P_1 + \Delta P_2 = 14$

$\Delta P_1 = 0.5\text{cm}$ of water

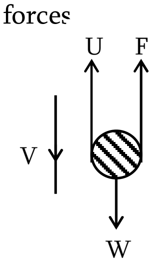
$\Delta P_2 = 13.5\text{cm}$ of water

13. (a) Write down the formula for the viscous drag force on a sphere falling in a liquid as state by stokes law. Explain the symbol used.
(b) (i) When a sphere in a liquid starts to move from rest, what are the magnitude and directions of forces acting on it.
(ii) Why does a sphere in (i)j above has an initial acceleration
(iii) How does the forces change its velocity of sphere increases?
(c) A horizontal tube consists of two parts joining together end to end, one part is 100mm long and has a radius of 1.00mm the other part is 140cm long with a radius of 0.5mm. When water equivalent to that due to 120mm of water flows between the tube ends, what does the pressure difference across each of component of tube(given your answer in terms of mm of water).

Solution

(a) Stoke's formula F = $6\pi\eta vr$
r = radius of spherical body
η = viscosity of fluid
v = terminal velocity

(b) Diagram showing the component of the forces



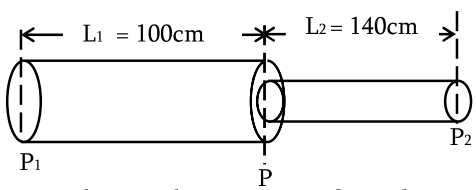
U = upthrust
F = Viscous force
W = Weight of sphere

(i) $W = Mg$ is weight of body acts vertically downward
$U = \delta Vg$ = upthrust acting
$F = 6\pi\eta vr$ viscous drag force acts vertically upward.

(ii) Because the spherical body is denser than that of the fluid, so there is resultant force downward.

(iii) Mg is independent on the velocity upthurst (U) is independent on the velocity $F = 6\pi\eta vr$ i.e viscous drag force is directly proportional to the velocity and is zero when the sphere is just to starts to fall.

(c)



According to the Po formula

$$\frac{v}{t} = \frac{\pi\Delta pr^4}{8\eta L}$$

$$\frac{v}{t} = \text{constant}$$

Now

$$\frac{\pi\Delta p_1 r_1^4}{8\eta L_1} = \frac{\pi\Delta p_2 r_2^4}{8\eta L_1}$$

$$\frac{\Delta P_1 (1.0)^4}{100} = \frac{\Delta P_2 (0.5)^4}{140}$$

$$\Delta P_1 = 0.446\Delta P_2$$

$$\Delta P_1 + \Delta P_2 = 120\text{mm}$$

$$\Delta P_2 = 114.88 \text{ of } H_2O$$

$$\Delta P_1 = 5.12\text{mm} \text{ of water}$$

14. (a) (i) Write down the Poiseulli's equation for a viscous fluid flowing through a tube defining all the symbols.

(ii) What are assumptions are used to develop the equation in (a)(i) above.

(iii) What is meant by Newtonian fluid?

(b) A submarine model is situated in a part of a tube with diameter 5.1cm where water moves at 2.4m/s determine:-

(i) Velocity in the water supply pipe of diameter 25.4cm

(ii) Pressure difference between the narrow and wide tube.

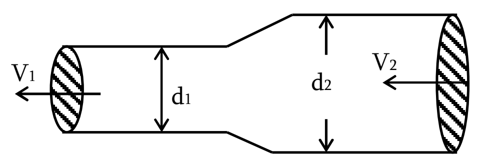
(c) (i) Define the compressibility of a gas in terms of elasticity of gases.

(ii) The bulk modulus of elasticity for the head is $8 \times 10^9 N/m^2$. Find the density of lead if the pressure applied is $2 \times 10^8 Nm^{-2}$.

Solution

(a) Refer to your notes

(b) $d_1 = 5.1cm$ $\quad V_1 = 2.4m/s$
$d_2 = 25.4cm$ $\quad V_2 = ?$



(i) apply the continuity equation

$$A_1V_1 = A_2V_2$$

$$\frac{\pi d_1^2}{4}V_1 = \frac{\pi d_2^2}{4}V_2$$

$$V_2 = V_1\left(\frac{d_1}{d_2}\right)^2 = 2.4\left[\frac{5.1}{25.4}\right]^2$$

$$V_2 = 0.09676\text{m/s}$$

(ii) Apply the continuity for the horizontal pipe

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

Since $A_2 > V_1$, $P_1 < P_2$

$$P_2 - P_1 = \frac{1}{2}\rho\left(V_1^2 - V_2^2\right)$$

$$= \frac{1}{2}\times 1000\left[(2.4)^2 - (0.09676)^2\right]$$

$$P_2 - P_1 = 2,875.32\text{Nm}^{-2}$$

(c) (i) Compressibility of a metal is a measure of how easily three material is compressed i.e compressibility is the reciprocal of bulk modulus

$$K_1 = \frac{1}{K}$$

(ii) Bulk modulus

$$K = -\frac{V\Delta P}{\Delta V}$$

$$\Delta V = \frac{-V\Delta P}{K} = \frac{-2\times10^8 V}{8\times10^9}$$

$$\Delta V = \frac{-V}{40}$$

New volume of lead

$$V_1 = V + \Delta V = V + \frac{-V}{40} = \frac{39}{40}V$$

Apply the law of conservation of mass

$$\rho_1 V_1 = \rho V$$

$$\rho_1 \frac{39V}{90} = 11.4\times10^3 V$$

$$\rho_1 = 11.4\times10^3 \times \frac{40}{39}$$

$$\rho_1 = 11.69\times10^3 \text{kg/m}^3$$

15. (a) A according to the Bernoulli's theorem the water pressure in a horizontal pipe of uniform bore must remain constant. But it goes on decreasing in practice. Why?

(b) In an experiment with Poiseulli's apparatus, the following figures were obtained.

- Volume of water issuing per minute = $7.08 \times 10^{-6}\text{m}^3$.
- Head of water = 0.314m
- Length of tube = 0.564m
- Radius of tube = 0.00514m

Find the coefficient of viscosity.

Solution

(a) The reason for this viscosity of the liquid. The flowing water requires energy to work against viscous force and this energy is imparted by pressure energy. Therefore, the liquid pressure goes on decreasing.

(b) According to the Poiseulli's law

$$\frac{V}{t} = \frac{\pi P r^4}{8\eta L} \quad \text{but} \quad P = \rho g h$$

$$\frac{V}{t} = \frac{\pi\rho g h r^4}{8\eta L}$$

$$\eta = \frac{\pi\rho g h r^4 t}{8VL}$$

$$\eta = \frac{3.14\times1000\times9.8\times0.314\times\left(5.14\times10^{-3}\right)^2\times60}{8\times7.08\times10^{-6}\times0.564}$$

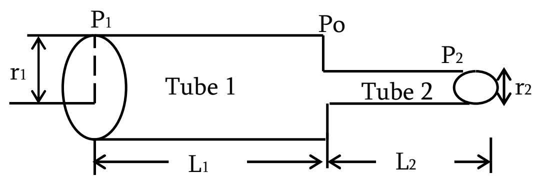
$$\eta = 0.00138\text{kgm}^{-1}\text{s}^{-1}$$

16. Show that if two capillary tubes of radii r_1 and r_2 having lengths L_1 and L_2 respectively are set in series, the rate of flow V is given by

$$V = \frac{\pi P}{8\eta}\left[\frac{L_1}{r_1^4} + \frac{L_2}{r_2^4}\right]^{-1}$$

Where P is the pressure difference across the arrangement and η is the coefficient of viscosity of the liquid.

Solution



Apply Poiseull's equation

$$V = \frac{\pi\Delta P r^4}{8\eta L}$$

For tube 1:

$$V = \frac{\pi\left(P_1 - P_o\right)r_1^4}{8\eta L_1}$$

$$P_1 - P_o = \frac{8\eta L_1 V}{\pi r_1^4} \quad \ldots\ldots(1)$$

For tube 2:

$$V = \frac{\pi\left(P_o - P_2\right)r_2^4}{8\eta L_2}$$

$$P_o - P_2 = \frac{8\eta L_2 V}{\pi r_1^4} \quad \ldots\ldots.(2)$$

Adding equation (1) and (2)

$$P_1 - P_2 = \frac{8\eta V}{\pi}\left[\frac{L_1}{r_1^4} + \frac{L_2}{r_2^4}\right]$$

Let $P_1 - P_2 = P$

$$P = \frac{8\eta V}{\pi}\left[\frac{L_1}{r_1^4} + \frac{L_2}{r_2^4}\right]$$

$$V = \frac{\pi P}{8\eta}\left[\frac{L_1}{r_1^4} + \frac{L_2}{r_2^4}\right]^{-1}$$

Hence shown.

17. (a) (i) Why are machine parts gammed in winter?
(ii) Why does a bigger rain drop fall faster than a smaller rain drop?
(b) A horizontal tube A of length 50cm and radius 0.1mm is joined to another horizontal tube B of length 40cm and radius 0.2mm. if a liquid passing through the tube enters A at a pressure of 85cm of mercury and leaves B at a pressure of 76cm of mercury, what is the pressure at the junctions of the tube?

Solution

(a) (i) Due to low temperature in winter the viscosity of the lubricants used in machine parts increases as a result, the machine parts get jammed.
(ii) The terminal velocity of a rain drop (sphere) is directly proportional to the square of radius of the drop. For this reason, a bigger drop falls faster than a smaller rain drop.

(b) A according to Poiseulli's formula the rate of flow of liquid in tubes A and B is given by

$$Q_1 = \frac{\pi P_1 R_1^4}{8\eta L_1}, \quad Q_2 = \frac{\pi P_2 R_2^4}{8\eta L_1}$$

For the tubes in series connection

$$Q = \text{constant}$$

$$Q_1 = Q_2$$

$$\frac{\pi P_1 R_1^4}{8\eta L_1} = \frac{\pi P_2 R_2^4}{8\eta L_2}$$

$$\frac{P_1}{P_2} = \left(\frac{L_1}{L_2}\right)\left(\frac{R_2}{R_1}\right)^4$$

Let X cm of mercury be the pressure at the junction of the tubes

$$P_1 = (85 - X)\,\text{cm}, \quad P_2 = X - 76$$

$$\frac{85 - X}{X - 76} = \frac{50}{40}\left(\frac{0.02}{0.01}\right)^4$$

$$X = 76.43\text{cm of Hg}$$

18. (a) (i) What are the similarities and differences between solid friction and fluid frictions?
(ii) What is terminal velocity? Up which factors it depends?
(b) Two capillaries of same length and radii in the ratio of 1:2 are connected in series and a liquid flows through this system under streamline conditions. If the pressure across the two extreme end of the combination is 1m of water, what is the pressure difference across the first capillary?

Solution

(a) Refer to our notes
(b) Let L and R be the length and radius of first capillary respectively. The corresponding values for the second capillary are L and 2R according the Poiseulli's formulae

$$Q = \frac{\pi P R^4}{8\eta L} = \text{constant}$$

$$\frac{\pi P_1 R^4}{8\eta L} = \frac{\pi P_2 (2R)^2}{8\eta L}$$

$$P_1 = 16P_2$$

Since $P_1 + P_2 = 1\text{m}$

$$16P_2 + P_2 = 1\text{m}$$

$$P_1 + \frac{P_1}{16} = 1\text{m}$$

$$P_1 = \frac{16}{17}\text{m} = 0.94\text{m}$$

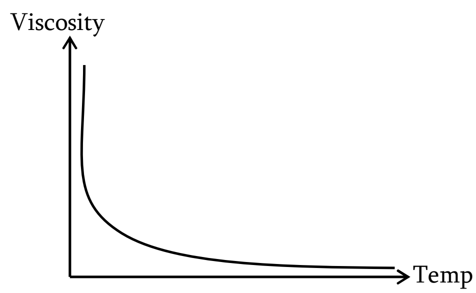
19. NECTA 1990/P2/9
(a) Explain the meaning of coefficient of viscosity.
(b) Describe how you would investigate experimentally the variation of the coefficient of viscosity of water with temperature from about 15°C to 80°C. Explain how the results are deduced from observations made and state any assumptions you involved.

(c) How would a temperature rise affect terminal velocity of a small solid sphere falling through a large volume of viscous liquid? Illustrate your answer with a graph of terminal velocity against temperature.

Solution

(a) Refer to your notes.

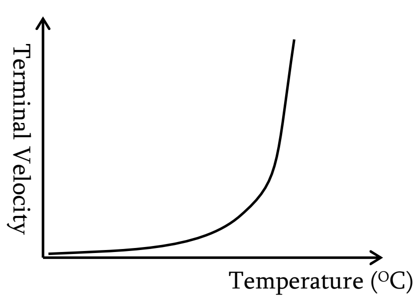
(b) The variation of viscosity of water with temperature from 15°C to 80°C could be investigated by the applying the stoke's law for the terminal velocity of a ball of known radius and density falling through the fluids. Beginning with temperature of 15°C are heat up the water and record the measurement f or the terminal velocity reached at the different temperature. A graph of viscosity against temperature is plotted. It will been seen that as temperatures increases viscosity decreases and the terminal velocity of the ball increases.



Assumption made:

- Wall affects are negligible
- The ball terminal velocity is reached before time of ball is measured.

(c) As temperature rises liquid viscosity decreases and terminal velocity of spherical ball increases .



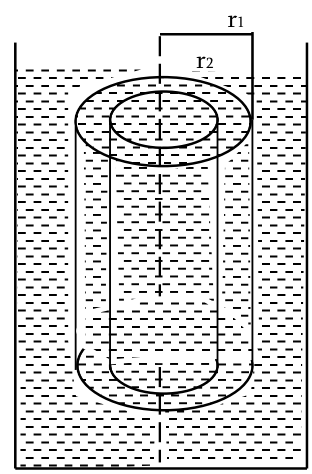
20. (a) Define the coefficient of viscosity and explain how it may be measured for a viscous liquid such as treacle.

(b) A vertical cylinder internal diameter 1.02cm is filled with glycerine. Calculate the velocity attained by a thin walled brass tube 1.00cm in diameter, weighing 2.5gm per length , which is falling coaxial down the cylinder. Neglect the end effect. Viscosity glycerine = $0.83Nsm^{-2}$, $g = 9.8m/s^2$.

Solution

(a) See your notes

(b)



Suppose r is the radius neglecting buoyancy.

Downward force = weight = constant

On cylinder viscous force on liquid between the cylinder.

$$F = -\eta A\frac{dv}{dr}, \quad A = 2\pi rL$$

$$F = -2\pi r\eta L\frac{dv}{dr} \cdot F = mg$$

$$Mg = -2\pi\eta rL\frac{dv}{dr}$$

$$dv = \frac{-mg}{2\pi\eta L}\cdot\frac{dr}{r}$$

$$\int_u^0 dv = \frac{-mg}{2\pi\eta L}\int_{r_2}^{r_1}\frac{dr}{r}$$

$$\left[V\right]_u^0 = \frac{-mg}{2\pi\eta L}\left[\log_e^r\right]_{r_2}^{r_1}$$

$$U = \frac{mg}{2\pi\eta L}\cdot\log_e\left(r_1/r_2\right)$$

$$= \frac{2.5\times10^{-3}\times9.8}{2\times3.14\times0.83\times10^{-3}}\log_e\left(\frac{51}{50}\right)$$

$$U = 9.3\times10^{-3}\,m/s$$

21. (a) Define the coefficient of viscosity and find its dimensions.
(b) State the stoke's law for the viscous force on a sphere of radius, r moving with uniform velocity V through a medium of viscosity, η explain why this law is applicable only if V is less than some critical value Vc. If V is in the form of $V_C = k\eta^x\rho^yV^z$ where K is the dimensionless constant and ρ is the density of the medium. Use the method of dimensions to find the values of x, y and z.
(c) A steel sphere of radius 0.50cm and mass 400g is released from rest inside a large volume of oil coefficient of viscosity $1.2Nm^{-2}s$. assume that the stokes law can be applied even if the velocity is varying, write down the equation of the motion of the sphere and derive an equation relating the time after release to the velocity acquired. Hence determine how soon after release the sphere 1% of the terminal velocity $\left(\log_e^{10} = 2.303\right)$

Solution

(a) And (b) Refer to your notes

(b) Now $V_C = k\eta^x\rho^yr^z$

Dimensionally

$$[V_C] = [\eta]^x[\rho]^y[r]^z$$

$$M^0LT^{-1} = M^{x+y}\cdot L^{-x-3y+z}T^{-x}$$

On equating indices or powers

M: 0 = x + y ………(i)

L: 1 = -x – 3y + z…….(ii)

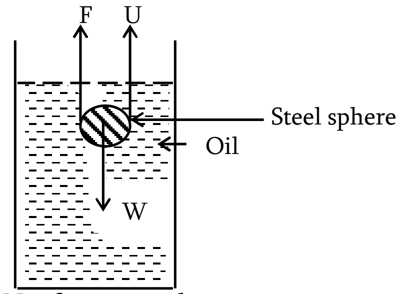
T: -1 = -x………..(iii)

On solving x = 1, y = z = -1

$$V_C = k\eta^1\rho^{-1}r^{-1}$$

$$V_C = \frac{k\eta}{\rho r}$$

(c)



Net force on sphere

$$F = W - U - 6\pi\eta av$$

$$F = M'g - 6\pi\eta av$$

(upthrust can be ignored)

$$M\frac{dv}{dt} = M'g - 6\pi\eta av$$

$$dt = \frac{MdV}{M'g - 6\pi\eta av}$$

$$\int_0^t dt = \int_0^V \frac{MdV}{M'g - 6\pi\eta av}$$

$$t = \frac{M}{6\pi\eta a}\log_e\left(\frac{M'g}{M'g - 6\pi\eta av}\right)$$

At the terminal velocity, Vo

$$M'g = 6\pi\eta aV_o$$

When V is within 1% of Vo

$$t = \frac{M}{6\pi\eta a}\log_e\left[\frac{M'g}{0.01M'g}\right]$$

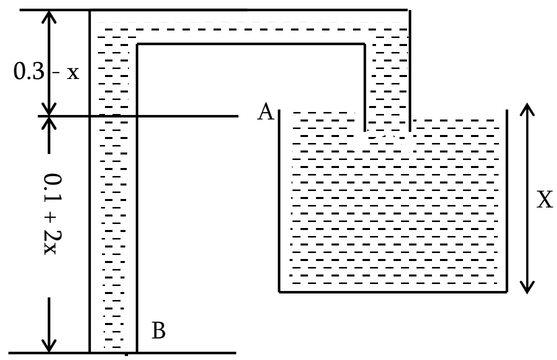
$$= \frac{M}{6\pi\eta a}\log_e(100)$$

$$= \frac{4\times10^{-3}\times2\log_e^{10}}{6\pi\times1.2\times0.5\times10^{-2}}$$

$$t = 0.16\,sec$$

22. A cylindrical bucket of radius 10cm and height 30cm is filled with water, calculate the minimum time that is required to empty the bucket if the process is carried out by means of a flexible siphon tube, (suitably manipulated) of length 70cm and internal diameter 2.00mm. Viscosity of water = $10^{-3}kgm^{-1}s^{-1}$, $g = 10m/s^2$.

Solution



Let X be the height of water in the bucket a time , t then, from the figure above

Head of liquid , AB = 0.7 – 2(0.3 – X)

= 0.1 + 2X

Excess pressure = (0.1 + 2X)ρg……(1)

The volume of liquid in the bucket V = $\pi r^2 x$, where r is the bucket radius.

Since X decreases as t increases.

$$\frac{dv}{dt} = -\pi r^2 \frac{dx}{dt} \quad \ldots\ldots\ldots(2)$$

Using Poiseulli's formula

$$\frac{dv}{dt} = \frac{\pi p a^4}{8\eta L}$$

$$\frac{dv}{dt} = \frac{\pi(0.1+2x)\rho g a^4}{8\eta L}$$

(2) = (3)

$$-\pi r^2 \frac{dx}{dt} = \frac{\pi(0.1+2x)\rho g a^4}{8\eta L}$$

$$-\pi r^2 \frac{dx}{dt} = \frac{\pi a^4 (0.05+x)\rho g}{4\eta L}$$

$$\int_{30}^{0} \frac{dx}{0.05+x} = \frac{a^4 \rho g}{4\eta L r^2}\int_0^T dt$$

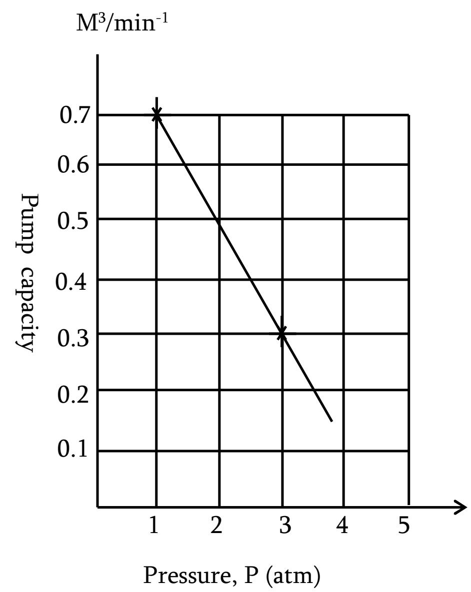
$$= \frac{4\times10^{-3}\times0.7\times(0.1)^2 \log_e\left(35/5\right)}{10^{-12}\times1000\times10}$$

$$T = 5,450\,\text{sec}$$

23. NECTA 1994/P2/3(C)

Your school need a water supply of peak capacity 5 litres per second from a stream which is 2.0km away from the school. The stream is at the same altitude as the school. Enough material to build a water tower 20m high near the stream is available. There is a 2KW pump whose working instruction are given on the graph (see figure below)

GRAPH OF PUMP CAPACITY AGAINST PRESSURE DIFFERENCE BETWEEN INLET AND OUTLET OF THE PUMP.



On the basis of available information:

(i) Decide whether the pump is adequate.

(ii) Calculate a suitable size of the pipe between the water tower and the school.

(iii) State with reason, whether the size you have calculated is maximum or minimum.

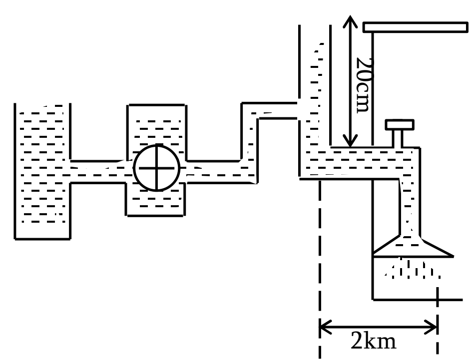
Solution

(i) School need $\frac{v}{t} = 5\text{litre/sec}$

The pressure difference of inlet and outlet

$$P = \rho g h = 1000\times9.8\times20$$

$$P = 1.96\times10^5\,\text{Nm}^{-2} = 1.96\text{atm}$$



Now $\frac{v}{t} = 5\text{litre/sec} = 5\times10^{-3}\text{m}^3/\text{sec}$

$= 5\times10^{-6}\text{m}^3\times60/\text{sec}$

$\frac{v}{t} = 0.3\text{m}^3/\text{min}$

At 1.96at the pump capacity is 0.5m³/min. the school demand is only 0.3m³/min; thus the pump is adequate.

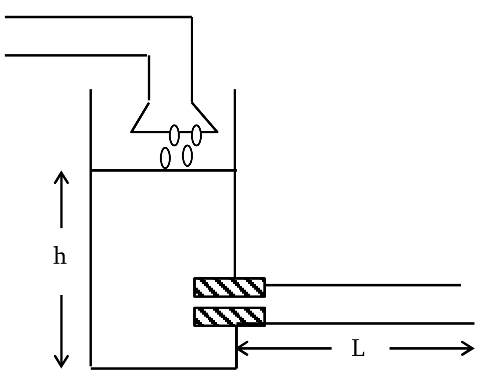
(ii) By using Poiseulli's formula

$$\frac{v}{t} = \frac{\pi\Delta Pr^4}{8\eta L}$$

The pressure difference along the pipe is due to 20m of water.

24. An empty vessel is open at the top has a horizontal capillary tube of length 20cm and internal radius 1.0mm protruding from one of its side walls immediately above the base water flows into the vessel at a constant rate of 1.5cm³/s. at water depth does the water level stop rising? Assume the flow is steady (coefficient of viscosity of water = 10^{-3}Nsm^{-2}, density of water = 1000kgm^{-3}, g =10m/s².

Solution



According to the Poiseulli's formula

$$\frac{v}{t} = \frac{\pi\Delta Pr^4}{8\eta L} = \frac{\pi\rho ghr^4}{8\eta L}$$

$$h = \frac{v}{t}\cdot\frac{8\eta L}{\pi\rho gr^4}$$

$$h = \frac{1.5\times10^{-6}\times8\times10^{-3}\times0.2}{3.14\times1000\times10\times\left(10^{-3}\right)^4}$$

$$h = 7.6\times10^{-2}\text{m}$$

25. NECTA 2010/P2/1
   (b) (i) Distinguish between Dynamic lift and static lift.
   (ii) How much work is done by pressure inforcing 1.4m³ of water through internal diameter if the difference in pressure between the ends of the pipe is 1.2 atmospheres?
   (c) (i) state the Poiseulli's law
   (ii) Castor oil of density 9.6×10^2kgm^{-3} is forced through a pipe of circular cross – section by a pump which maintains a gauge pressure of 950Pa. the pipe has a diameter of 2.6cm and length of 65cm. the castor oil emerging from free end of the pipe at atmospheric pressure is collected after 90seconds a total mass of 1.23kg has been collected. What is the coefficient of viscosity of oil.
   (d) A bird of mass 500gm hovers upwards by beating it wings of effective area 0.3m². Estimate the velocity of imparted to the air by the wings beating. Assuming that air to be at S.T.P in weather condition.

26. Three capillaries of length L_1, L_2 and L_3 and radii r_1, r_2 and r_3 respectively are joined in series. Show that the volume of the liquid flowing per second is given by

$$V = \frac{\pi P}{8\eta\left[\frac{L_1}{r_1^4}+\frac{L_2}{r_2^4}+\frac{L_3}{r_3^4}\right]}$$

Where P is the pressure difference across the combination of capillaries.

27. A cylindrical vessel of radius 5cm has at the bottom horizontal capillary tube of length 20cm and internal radius 0.4mm. If the vessel is filled with water. Find the time taken by it to empty one – half of the contents. Given that the viscosity of water is 0.01poise.

Solution

Let h be the height of water level in the capillary tube at any instant , t and dh be the fall in level in time at the rate of flow of volume is given by

$$\frac{-Adh}{dt} = \frac{\pi pr^4}{8\eta L} \quad \text{but } p = \rho gh$$

A = Area of the vessel

$$\frac{-Adh}{dt} = \frac{\pi \rho g r^4 h}{8\eta L}$$

$$\frac{-dh}{dt} = \left(\frac{\pi \rho g r^4}{8\eta LA}\right)h$$

$$dt = \frac{-8\eta LA}{\pi \rho g r^4}\cdot\frac{dh}{h}$$

$$\int_0^t dt = \frac{-8\eta LA}{\pi \rho g r^4}\int_H^{H/2}\frac{dh}{h}$$

$$t = \frac{-8\eta LA}{\pi \rho g r^4}\left[\log_e^h\right]_H^{H/2}$$

$$= \frac{-8\eta LA}{\pi \rho g r^4}\left[\log_e^{(H/2)} - \log_e^H\right]$$

$$t = \frac{8\eta LA}{\pi \rho g r^4}\log_e^2$$

$$= \frac{8\times 20\times 0.01\times 3.14\times 5^2\times 23026\times 0.3010}{3.14\times 1\times 981\times (0.04)^4}$$

$$t = 11040\,\text{sec}$$

$$t = 3\text{hours } 4 \text{ minutes}$$

28. A soap bubble of radius 4cm and surface tension 30dyne/cm blown a the end of the tube of length 10cm and internal radius 0.20cm. If the coefficient of air is 1.85×10^{-4} C.G.S units, find the time taken by the bubble to be reduced to a radius of 2cm.

Solution

Let R be the radius of the bubble, then the rate of flow of air is given by $V = \frac{4}{3}\pi R^2 \cdot \frac{dR}{dt}$

According to the Poiseulli's formula

$$\frac{dv}{dt} = \frac{\pi pr^4}{8\eta L}$$

$$P = \frac{4\gamma}{R} \text{ (soap bubble)}$$

$$\frac{dv}{dt} = \frac{\pi r^4}{8\eta L}\cdot\frac{4\gamma}{R} = 4\pi R^2\cdot\frac{dR}{dt}$$

$$dt = \frac{8\eta L}{\gamma r^4}\cdot R^3 dR$$

$$\int dt = \frac{8\eta L}{\gamma r^4}\int_{R_2}^{R_1} R^3 dR$$

$$t = \frac{8\eta L}{\gamma r^4}\left[\frac{R_1^4 - R_2^4}{4}\right]$$

$$= \frac{8\times 10\times 1.85\times 10^{-4}}{30(0.2)^4}\left[\frac{4^4 - 2^4}{4}\right]$$

$$t = 296\,\text{sec} = 4 \text{ minutes } 5.6\text{seconds}$$

29. Calculate the velocity of water along the axis of the capillary tube through which it is flowing under a pressure difference of 10cm of water of the length of the capillary tube is 50cm and its diameter is 1mm. the coefficient of viscosity for water is 0.01 poise.

Solution

Since $V = \frac{P}{4\eta L}\left(R^2 - r^2\right)$

Along the axis of tube, r = 0

∴ The velocity of water along the axis of the tube.

$$V = \frac{PR^2}{4\eta L}$$

$$V = \frac{10\times 1\times 980\times (0.05)^2}{4\times 0.01\times 50}$$

$$V = 12.25\,\text{cm/s}$$

30. A cylindrical vessel of radius R has, its bottom , a horizontal capillary tube of length L and internal radius r. vessel is filled with a liquid of density ρ up to the height , it find the time taken by it to empty of its contents.

Solution

Area of the vessel, $A = \pi R^2$.

When the liquid is flowing through the capillary tube, let h be the height of liquid in the vessel at any instant.

Pressure difference between the ends of the tube, $P = \rho gh$

Volume of liquid flowing out per second

$$\frac{dv}{dt} = \frac{-Adh}{dt} \text{........(1)}$$

According to the Poiseulli's law

$$\frac{dv}{dt} = \frac{\pi\rho g r^4 h}{8\eta L} \text{..........(2)}$$

(1) = (2)

$$\frac{-Adh}{dt} = \frac{\pi\rho g r^4 h}{8\eta L}$$

$$dt = \frac{-8\eta AL}{\pi\rho g r^4} \cdot \frac{dh}{h}$$

$$\int_0^t dt = -\frac{8\eta AL}{\pi\rho g r^4}\int_H^0 \frac{dh}{h}$$

$$t = \frac{-8\eta AL}{\pi p r^4}\left[\log_e^h\right]_H^0$$

$$t = \frac{8\eta AL}{\pi\rho g r^4}\log_e H$$

In general, the relation can be written as

$$t = \frac{8\eta AL}{\pi\rho g r^4}\left[\log_e^h - \log_e^h\right]$$

h is the height up to which the vessel is emptied.

31. If a tennis ball is cut or golf ball is sliced it spins as it travels through the air and experiences a side way force which causes it to curve in flight. Explain this phenomena.

32. EZEB 2009/P2/2
   (a) (i) What is lamina and turbulent fluid flow?
   (ii) Two spherical ran drops of equal size are falling vertically through air with terminal velocity 0.15m/s. If the two drops coalesce to form a single large drop, find terminal velocity of large drop.
   (b) A small air bubble of density 1.29kgm-3 and diameter 0.2mm rises vertically in a wide tank of water. If the water past it in streamline, what is the maximum speed does it reach?

33. EZEB 2005/P2/1
   (a) Distinguish between Dynamic lift and upthrust.
   (b) A bird of mass 0.8kg hoves upward by beating the mass of effective wings of area $0.5cm^2$.
   (i) What is the upthrust?
   (ii) Estimate the velocity of imparted to the air by beating of the wings assuming the density of air at S.T.P to be $1.3kgm^{-3}$.
   (c) $200cm^3$ of water flows through a horizontal capillary tube of length 50cm and the bore 1.0mm in 30minutes under constant head of water 30cm high. Calculate the viscosity of water and the rate of flow through a tube of 20cm long and of bore of 0.8mm.

34. A metal sphere of radius 2×10^{-3}m and a mass 3.0×10^{-4} falls under gravity centrally down a wide tube, filled with a liquid at 35°C. The density of the liquid is $700kgm^{-3}$. The sphere attained a terminal velocity of magnitude 40×10^{-2}m/s. the tube is emptied and filled with another liquid at the same temperature and density $900kgm^{-3}$. When the metal surface fall centrally down the tube is found to attain a terminal velocity of magnitude 25×10^{-2}m/s. determine at 35°C, the ratio of coefficient of viscosity of second liquid to that of the first liquid.

Solution

Volume of metal sphere, $V = \frac{4}{3}\pi R^3$

$$V = \frac{4}{3}\times 3.14\times\left(2\times10^{-3}\right)^3$$

$$V = 3.3493\times10^{-8}\,m^3$$

Density of metal sphere

$$\rho = \frac{M}{V} = \frac{3\times10^{-4}}{3.3493\times10^{-8}}$$

$$\rho = 8,957.1kgm^{-3}$$

According to the stoke's law

$$\eta = \frac{2r^2\left(\rho-\delta\right)g}{9V}$$

First liquid

$$\eta_1 = \frac{2r^2(\rho - \delta_1)g}{9V_1}$$

$$\frac{\eta_1}{\eta_2} = \frac{2r^2g(\rho-\delta_2)}{9V_2} \Big/ \frac{2r^2g(\rho-\delta_1)}{9V_1}$$

$$\frac{\eta_2}{\eta_1} = \left(\frac{V_1}{V_2}\right)\left(\frac{\rho-\delta_2}{\rho-\delta_1}\right)$$

$$= \left(\frac{40\times10^{-2}}{25\times10^{-2}}\right)\left[\frac{8,957.1-900}{8,957.1-700}\right]$$

$$\frac{\eta_1}{\eta_2} = 1.56$$

35. When the gas expand isothermally show that, the variation of the viscosity of the gas with pressure is given by the relation

$$P_1V_1 = \frac{\pi a^4}{16\eta L}\left(P_1^2 - P_2^2\right) = P_2V_2$$

Where P_1, P_2 and the respectively pressure at the inlet and outlet of tubes and V_1, V_2 are the corresponding volume per second, L is the length of tube, a = radius of tube and η is the viscosity of the gas.

Solution

According to the Poiseulli's law

$$\frac{v}{t} = \frac{\pi P a^4}{8\eta L} \ldots\ldots\ldots\ldots(1)$$

When the gas flows along a tube, the volume increases as the pressure decreases. For a short length dL, the rate of volume of gas flows

$$\frac{v}{t} = \frac{-\pi a^4}{8\eta}\cdot\frac{dP}{dL} \ldots\ldots\ldots(2)$$

$$(1) = (2)$$

$$\frac{-\pi a^4}{8\eta}\cdot\frac{dP}{dL} = \frac{\pi P a^4}{8\eta L}$$

$$PV = P_1V_1 = P_2V_2$$

$$V = \frac{P_1V_1}{P} = \frac{v}{t}$$

P_1, P_2 = Pressure at the inlet and outlet end of the tube and V_1 and V_2 are corresponding volumes per second.

$$\frac{P_1V_1}{P} = \frac{-\pi a^4}{8\eta}\frac{dP}{dL}$$

$$P_1V_1\int_0^L dL = \frac{-\pi a^4}{8\eta}\int_{P_1}^{P_2} P\,dP$$

$$P_1V_1 = \frac{\pi a^4}{16\eta L}\left(P_1^2 - P_2^2\right) = P_2V_2$$

Hence shown.

EXERCISE

36. (a) (i) Explain briefly, what meant by saying that a fluid is viscous.

(ii) Imagine you are kept in a large spherical capsule and is allowed to fall in a vast expanse of viscous medium. State what you expect to feel during the falling.

(iii) Two spherical rain drops are falling vertically through air with a terminal velocity of 0.24m/s. what would be the terminal velocity if these two drops were to coalesce to a form a large spherical drop?

(b) (i) Find the time needed for a sphere of mass, m density falling through a viscous medium of density d to reach the terminal velocity , U

(ii) Compare the time to attain the terminal velocity of two spheres of radius r and 3r to fall through the same viscous liquid.

(iii) Sketch the graph of U against time, t

37. (a) An oil has a coefficient of viscosity of $7.4Nsm^{-2}$ and density of $940kgm^{-3}$. What will be the terminal velocity of a steel sphere of radius 2.0mm falling freely in the oil. Density of the steel = $7800kgm^{-3}$. $g = 9.8m/s^2$.

(b) Estimate the maximum time that would be taken for particle of sand sprinkled. Onto the surface of water of a beaker, filled to a depth of 120mm, to reach the bottom. Assume that the sand particles are spherical and have a diameter of 0.10mm, the viscosity of water is $1.1 \times 10^{-3}Nsm^{-2}$, density of water = $1000kgm^{-3}$, density of sand = $2000kgm^{-3}$, g = $9.8m/s^2$.

Answer (a) 0.025m/s (b) 2.40m/s.

38. The rate of flow of fluid in a tube is given by Poiseulli's equation for laminar flow as

$$\frac{v}{t} = \frac{\pi pr^4}{8\eta L}$$

Each symbol have usually meaning.

(a) If the pressure difference of the pipe is doubled for the same pressure gradient how does the rate of fluid flow through the pipe change?

(b) If the radius of the pipe is double for the same pressure gradient how does the rate of the fluid flow through the pipe change?

Answer (a) double (b) Increase by a factor of 16

39. (a) When the flow of water through a pipe is laminar, what can said about the layer of water in contact with the walls of the pipe?

(b) In an experiment to measure the coefficient of viscosity of water was found to flow in one minute through a capillary tube of internal diameter 1.07mm and length 252mm when the pressure difference of the ends was due to column of water of height 1.75mm. What is the coefficient of viscosity of water?

Answer (a) it is stationary (b) $9.47 \times 10^{-3}Nsm^{-2}$.

40. NECTA 1981/P2/4

(a) Distinguish between streamline and turbulent fluid flow and hence define critical velocity.

(b) A steady wind flows over a large plain on which a mountain is situated. An aeroplane pilot is using the pressure operated altimeter to help him keep the plane at a constant altitude. Explain how the wind might lead to a crash when the plane is in the process of flying over the mountain.

(c) A liquid of density $8.5 \times 10^2 kgm^{-3}$ flows in a tube of diameter 3.0cm at a pressure of $1.6 \times 10^5 Nm^{-2}$. In a venturimeter constriction, the tube has a diameter of 2cm and the pressure drops to $1.0 \times 10^5 Nm^{-2}$. Calculate the rate of flow of the liquid in the tube.

41. (a) Compare the phenomena of friction between solid and viscosity in fluid.

(b) Outline an experiment to investigate the dependence of the rate of fluid through a horizontal capillary tube on the diameter of the tube.

(c) The water supply system to a school is to be increased by replacing the 5cm diameter pipe by a big – bore of 10cm in diameter and its total length is to remain 50m and the pipes remaining horizontal. By what ratio must the pressure be increased if the volume per second is to be double. State any assumptions made.

42. A steel ball bearing of diameter 1.5mm is allowed to fall through a wide vertical tube containing motor oil. After reaching its terminal velocity it is observed to fall through a distance of 0.20m in 28.5 second. Determine the viscosity of the oil given that steel and moter oil densities are $7700kgm^{-3}$ and $820kgm^{-3}$ respectively.

43. An oil drop fall vertically in air and attain a terminal velocity of $3.57 \times 10^{-4}m/s$.

(i) Draw a diagram that indicate the three principal forces acting on the drop and name the forces.

(ii) Draw a sketch graph that shows the variation of velocity with time until the terminal velocity has been reached.

(iii) Calculate the radius of the drop given that the coefficient of viscosity of oil $1.7 \times 10^{-5}Nsm^{-2}$, density of oil is $9.2 \times 10^2 kgm^{-3}$ and the density of air is negligible compared to that of oil.

44. (a) How does the coefficient of viscosity of the gas changes.

(b) Draw and label the apparatus you would use in determining the viscosity of a liquid by

(i) The falling sphere method and

(ii) The constant pressure head method

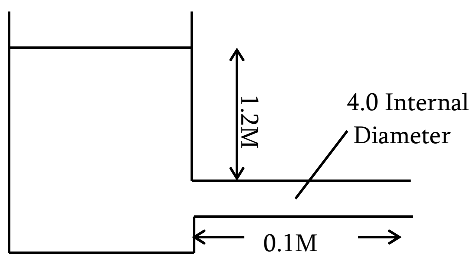
45. (a) (i) State the Newton's law of viscosity and hence deduce the dimensional of coefficient of viscosity.

(ii) The rate of volume flow, dv/dt of liquid of viscosity η, through a pipe of internal radius r and length L is given by the equation

$$\frac{dv}{dt} = \frac{\pi pr^4}{8\eta L}$$

Where P is the pressure difference between the ends of pipe. Show that this equation is dimensionally correct.

(b) The figure below shows a tank containing a light lubricanting oil. The oil flows out of the tank through a horizontal pipe of length 0.10m and internal diameter 4.0mm.



(i) Calculate the volume of oil which flows through the pipe in one minute when the level of oil in the tank is 1.2m above the pipe and does not significantly alter during this time.

- Density of oil = $9.2 \times 10^2 kgm^{-3}$.
- Viscosity of oil = $8.4 \times 10^{-2} Nsm^{-2}$.

(ii) It found that the volume flow is greater at higher temperatures. Assuming that the density changes can be ignored, suggest an explanation for this effect in terms of the nature of viscosity.

(c) Discus how the lubricanting properties of an oil are affected by

(i) The coefficient of viscosity of oil

(ii) Its variation with temperature

46. (a) (i) State the Bernoulli's principle and explain the principle on which it based.

(ii) What are the implication of Bernoulli's principle in the case where the pipe is horizontal.

(b) The table below shows the times of fall of steel balls of different diameters falling through a distance of 50cm in a various liquid whose density is $1.26 \times 10^3 kgm^{-3}$.

Time t(s) × 10-2	8.36	6.89	5.80	4.93
Diameter d, (mm)	2.0	2.2	2.4	2.6

(i) Calculate the uniform velocity V of each ball.

(ii) By potting the graph of V against d^2, explain how the coefficient of viscosity of the liquid may be found ($g = 9.8m/s^2$)

47. (a) (i) State the Bernoulli's theorem for the horizontal flow.

(ii) On which principle does the Bernoulli's theorem based

(iii) A pipe is running fall of water at a certain point A, it tapers from 30cm diameter to 10cm diameter at B, the pressure difference between point A and B is 100cm of water column. Find the rate of flow of water through the pipe.

(c) Two capillaries of the same length and radii in the ratio of 1:2 are connected in series and the liquid flow through the system under streamline conditions. If the pressure across the two extreme ends of the combination is 1m of water; what is the pressure difference across the first capillary? Density of water = $1000kgm^{-3}$ acceleration due to gravity, $g = 9.8m/s^2$.

48. (a) Given the Bernoulli's equation

$$P + \frac{1}{2}\rho V^2 + \rho gh = \text{constant}$$

Where all the symbols carry their usual meaning.

(i) What quantity does each expression on the left hand side of the equation represent?

(ii) Mention any three condition which make the equation to be valid.

(b) Water is supplied to a house level through a pipe of an absolute pressure of 6.5×10^5Paa and velocity of 5m/s. the pipe line leading to the second floor path room 8m above has an inner diameter of 0.75cm. Find the flow velocity and pressure at the pipe outlet in the second floor bathroom.

(c) Define the flowing terms when applied to fluid flow;
(i) Non – viscous fluid
(ii) Steady flow (iii) Line of flow
(iv) Turbulent flow

(d) A horizontal pipeline increases uniformly from 0.080m diameter to 0.160m diameter in the direction of flow of water. When 96litres of water is flowing per second a pressure gauge at the 0.080m diameter section reads 3.5×10^5Pa. what should be the reading of the gauge at the 0.160m diameter section neglecting any loss?
Acceleration due to gravity, g = $9.8m/s^2$
Density of water = $1000kgm^{-3}$.

49. (a) (i) Distinguish between static and Dynamics pressure. Write down expression for the Bernoulli's theorem.
(b) (i) What is meant by an ideal fluid?
(ii) Determine the terminal velocity of rain drop of radius 0.2cm moving in air.
(c) (i) Explain what happen if two bubbles of in equal radii are joined by a tube without bursting.
(ii) Determine the amount of energy required to break a spherical drop of radius 0.5mm into 1000droplets.
(d) (i) Give two applications of Bernoulli's theorem.
(ii) Wind is blowing at 15m/s across the roof of house. Determine the reduction in pressure below atmospheric pressure of the stationary air that accompanies this wind.
(iii) Distinguish between Laminar and Turbulent flow and write down the formular to deduce the Reynolds number defines all physical quantities used g = $9.8m/s^2$.
Density of air = $1.23kgm^{-3}$
Density of water = $1000kgm^{-3}$
Viscosity of air = 1.0×10^{-3}Pas

50. (a) (i) Define the terms streamline and lamina flow.
(ii) State the law of mass continuity and Bernoulli's principle
(iii) Define the pressure terms in Bernoulli's principle.
(b) (i) Explain the working principle of a pilot – static tube (include a diagram for the device)
(ii) A pilot tube is mounted on an airplane wing to determine the speed of the plane relative to the air, which has a density of $1.03kg/m^3$. The tube contains alcohol and indicates a level difference of 2.6cm. What is the plane speed relative to the air? (The density of alcohol is $810kg/m^3$)
(c) (i) A sphere falling vertically through air attains a steady velocity. Explain briefly why a constant velocity is reached.
(ii) The open top of large tank has an area of cross – section of $0.5m^2$. The tank has an opening at the bottom with an area of cross – section $1cm^2$. A load of 20kg is applied on the water surface at the top. Find the velocity of the water coming out of the opening at the time when water level is 50cm above the bottom. g = 9.8N/kg , density of water = $1000kgm^{-3}$.

TUTORIAL SHEET NO 4. (D)
BASED ON VISCOSITY
AND POISEULLE'S FORMULA.

1. Nectar 2021/P2/1
(a) Give the meaning of the following terms as used in fluid dynamics:
(i) Critical velocity
(ii) Incompressible fluid
(iii) Streamline flow
(iv) Turbulent flow

(b) (i) What flows through a pipe of internal diameter 20cm at the speed of 1m/s. what would be the radius of the nozzle if water is expected to emerge at the speed of 4m.s?

(ii) Determine the coefficient of viscosity of the liquid of density $1.47 \times 10^3 kg/m^3$. If an air bubble of radius 1cm is moving through it at the steady rate of 0.2cm/s

(c) (i) Write stoke's equation as applied to motion of a body in a viscous medium and define all symbols used.

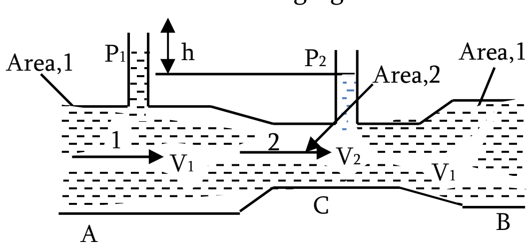
(ii) State two conditions under which stoke's equation is valid.

2. NECTA 2020/P1/1

(a) State two factors which determine the magnitude of viscous force.

(b) Identify two limitations and three importance of applying stoke's law in fluid motion.

(c) A venurimeter consists of two identical wide tubes A and B connected by a narrow tube C. The liquid enters through the wide tube A and after passing through the narrow tube C leaves through the other wide tube B. The entire arrangement is as shown in the following figure



Use the Bernoulli's theorem at points 1 and 2 , to show that an expression for the rate of flow of the liquid is given $Q = A_1A_2 \cdot \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$,where all symbols carry their usual meaning.

(d) A cylindrical tank 1m in radius rests on a platform 5m high. Initially the tank was filled with water to a height of 5m. if a plug of area $10^{-4}m^2$ is removed by an orifice on the side of the tank at the bottom; calculate the initial speed with which the water

(i) Flows from the orifice

(ii) Strikes the ground

3. NECTA 2019/P1/1

(a) (i) Give the meaning of the terms velocity gradient, tangential stress, and coefficient of viscosity as used in fluid dynamics.

(ii) Write down stoke's equation defining clearly the meaning of all symbol used.

(iii) State two assumptions used to develop the equation in 1(a)(ii)

(b) Calculate the terminal velocity of rain drops falling in air assuming that the flow is laminar, the rain drops are sphere of diameter 1mm and the coefficient of viscosity, $\eta = 1.8 \times 10^{-5}Ns/m^2$.

(c) Water flows past a horizontal plate of area $1.2m^2$. If its velocity gradient and coefficient of viscosity adjacent to the plate area $10s^{-1}$ and $1.3 \times 10^{-5}Nsm^{-2}$ respectively; calculate the force acting on the plate.

(d) A horizontal pipe of cross – sectional area $10cm^2$ has one section of cross sectional area $5cm^2$. If water flows through the pipe, and the pressure difference between the two sections is 300Pa, how many cubic meters of water will flow out of the pipe in 1minute?

4. (a) A rain drop of radius 0.4mm falls through air with a terminal velocity of 50cm/s. the viscosity of air is 0.019 Pa – s. find the viscous force on the rain drop.

(b) A drop of water of radius 0.001mm is falling in air. If the coefficient of viscosity of air is $1.8 \times 10^{-5}kgm^{-1}s^{-1}$, what will be the terminal velocity of the drop? Neglect the density of air.

Answer (a) $7.17 \times 10^{-5}N$ (b) $1.2 \times 10^{-4}m/s$

5. In Millikan's oil drop experiment, what is the terminal velocity of a liquid droplet of radius 0.02mm and density 1.2g/cc. take the viscosity of air at the temperature of the experiment to be $1.8 \times 10^{5}Nm^{-2}s$. Find the viscous force on the droplet at the given velocity. Neglect the buoyancy of the droplet due to air. Answer $5.81 \times 10^{-2}m/s$, $3.94 \times 10^{-10}N$.

6. Find the terminal velocity with which an air bubble of density $1kgm^{-3}$ and 0.6mm in diameter will rise in a liquid of viscosity $0.15Nm^{-2}$ and a specific gravity 0.9? What is the terminal velocity of the same bubble in water of coefficient of viscosity $10^{-3}Nm^{-2}s$? **Answer** -0.012m/s , -0.196m/s negative sign shows that air bubble will rise up.

7. Eight spherical rain drops of equal size are falling vertically through air with a terminal velocity of 0.10m/s. What should be the velocity if these drops were to combine to form one large spherical drop? **Answer** 0.4m/s.

8. (a) What is the viscous force on the drop of liquid of radius 02mm moving with constant velocity 4cm/s through a medium of viscosity $1.8 \times 10^{-5}Nm^{-2}s$.
   (b) The terminal velocity of a copper ball of radius 2mm falling through a tank of oil at $20^{O}C$ is 6.5cm/s. compute the coefficient of viscosity of the liquid ($g = 9.8m/s^2$)
   Density of oil = $1.5 \times 10^{3}kgm^{-3}$
   Density of copper = 8.9 × 103kgm-3
   Answer (a) $2.7 \times 10^{-9}m/s$ (b) 0.992decapoise.

9. (a) Two equal drops of water are falling through air with a steady velocity 5cm/s. If the drops combine to form a single drop, what will be new terminal velocity?
   (b) Eight rain drops of radius 1mm each falling down wards with a terminal velocity of 5cm/s coalesce to form a bigger drop. Find the terminal velocity of the bigger drop.
   (c) If n equal rain droplets falling through air with equal steady velocity of 10cm/s coalesce, find the terminal velocity of big drop formed. **Answer** (a) 7.937cm/s (b) 20cm/s (c) $10n^{2/3}cm/s$.

10. There is a conical pipe of radii of its two ends as 0.2m and 0.02m with $20N/m^2$ as the pressure difference along its length. A liquid of density $1.00 \times 10^{3}kg/m^3$ is following through the pipe. Calculate the rate of flow of liquid through the pipe. Answer $2.5 \times 10^{-4}m^3s^{-1}$.

11. The following observations were made in an experiment with Poiseulle's experiment:-
    - Diameter of tube = 0.1cm
    - Density of liquid = $2.4g/cm^3$
    - Volume of liquid collected per minute = $10cm^3$.
    - Pressure head of liquid = 25cm
    - Length of tube = 20cm

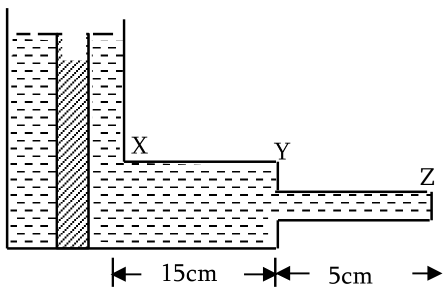
    Using the above data, calculate the viscosity of the liquid. **Answer** $4.32 \times 10^{-2}Nm^{-2}s$.

12. A capillary tube of length 25cm and diameter 0.1cm is fitted horizontally to a vessel full of a liquid of coefficient of viscosity 0.012cgs unit. The depth of the capillary tube below the surface of liquid is 15cm. If the density is $0.8g/cm^3$, calculate the amount of the liquid that will flow out in 7minutes. **Answer** 32.25g

13. While giving a patient a blood transfusion, a bottle is set up to pass the blood through a needle of length 2cm and internal diameter 0.35mm. If the level of blood is 1.4m above needle, calculate the rate of flow and speed of the blood coming out of the needle. Take, coefficient of viscosity of blood = 0.238 poise. **Answer** 10.63cm/s.

14. Two tubes P and Q of lengths 100cm and 30cm have radii 0.1mm and 0.2mm, respectively. A liquid passing through the two tubes is entering P at a pressure of 60cm of mercury and leaving Q at pressure of 55cm of mercury. What is the pressure at the junction of P and Q? **Answer** 55.09cm of Hg.

15. Two capillary tube XY and YZ are joined end to end as shown in the figure. The composite tube is held horizontally with X being connected to a water vessel, giving a constant pressure head of 2cm, while Z is open to the air. If the diameters of tube XY and YZ are in the ratio of 2:1, calculate the pressure difference between Y and Z. Answer 32/19cm of water column.



16. A cylindrical vessel has three identical horizontal tubes of length 40cm, each coming out at heights 0,5 and 9cm respectively. What will be the length of a single over flow tube of the same radius as that of identical tubes which can replace the three tubes if placed at the bottom of the vessel if the level of liquid in the cylindrical vessel is kept constant at 20cm. **answer** 17.39cm.

17. A liquid solution of coefficient of viscosity $2 \times 10^{-3}Nm^{-2}$ made to drive with velocity of $10^{-4}m/s$ along the Xylem vessels of radius ¾ m and length 7μm. Calculate the pressure difference across the length of Xylem vessel. **Answer** 19.9Pascal.

18. (a) Water is flowing through two capillary tubes under constant pressure head. The ratio of their lengths is 3:1 and of diameter is 2:1. Find the ratio of rate of flow of water through two tubes.
    (b) A rubber ball takes 5s to fall through a height of 0.5m inside a large container containing water of specific gravity 1. Calculate the viscosity of water if the ball is 1.24×10^{-3}kg and diameter is 6.6×10^{-3}m **Answer:** (a) 16/3 or 16:3 (b) $1.731Nm^{-2}s$.

19. A cylindrical container of radius 5cm height 5cm , if filled with water. If a horizontal capillary tube of length 10cm and radius 0.3mm is placed at the bottom of the vessel then calculate the time taken to make one third of the water to flow out of capillary. The viscosity of water is 0.01 poise. **Answer** 9826.1sec.

20. A $16cm^3$ volume of water flows per second through a capillary tube of radius r cm and length L cm when connected to a pressure head of h cm of water. If a tube of the same length and radius r/2 is connected to the same pressure head, find the mass of water flowing per minute through the tube. **Answer** 60g

21. In giving a patient a blood transfusion, the bottle is set up so that the level of blood is 1.3m above the needle, which has an internal diameter of 0.36mm and is 3cm in length. If $4.5cm^3$ of blood passes through the needle in one minute, calculate the viscocity of blood. The density of blood is $1020kgm^{-3}$. **Answer** 0.00238Pa

22. A capillary tube of 2mm diameter and 20cm long is fitted horizontally to a vessel kept full of alcohol of density $0.8g/cm^3$. The depth of the centre of capillary tube below the free surface of alcohol is 30cm. if viscocity of alcohol is 0.12 poise, find the amount that will flow in 5 minutes. **Answer** 92.4g

23. If two capillary tubes of radii r_1 and r_2 and having length L_1 and L_2 respectively are connected in series across a head of pressure. Find the rate of flow of the liquid through the tubes, if η is the coefficient of viscosity of the liquid. **Answer** $\frac{v}{t} = \frac{\pi p}{8\eta\left[\frac{L_1}{r_1^4} + \frac{L_2}{r_2^4}\right]}$

24. Two tubes A and B of length 80cm and 40cm have radii 0.1mm and 0.2mm respectively are connected in series end to end. If a liquid passing through two tubes is entering A at a pressure of 82cm of mercury and leaving B at a pressure of 76cm of mercury. Find the pressure at the junction of A and B. **Answer** 76.18cmof Hg.

25. In a plant, a sucrose solution of coefficient of viscosity $1.5 \times 10^{-3}Nsm^{-2}$ in driven at a viscocity of $10^{-3}m/s$ through xylem vessel of radius 2μm and length 5μm. Find the hydrostatic pressure difference across the length of xylem vessel. **Answer** $15Nm^{-2}$.

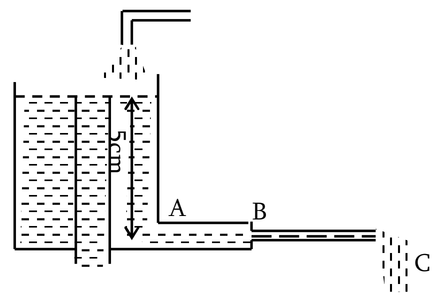
26. Two capillary tubes of length 15cm and 5cm and radii 0.06cm and 0.02cm respectively are connected in series. If the pressure difference across the end faces is equal to pressure of 15cm high water column, then find pressure difference across the (i) first tube and (ii) second tube. Answer (i) 0.54cm of water column, (ii) 14.46cm of water column.

27. The level of liquid in a cylindrical vessel is kept constant at 30cm. it has three identical horizontal tubes A ,B and C of lengths 0 , 5 and 10cm respectively. Calculate the length of single over flow tube of the same radii as that of identical tubes which can replace the three when placed horizontally at the bottom of the cylinder. **Answer** 16cm.

28. A flat plate of 20cm square moves over another similar plate with a thin layer of 0.4cm of a liquid between them. If a force of one kg.wt. moves one of the plate uniformly with a velocity of 1m/s. calculate the strain rate, shearing stress and coefficient of viscosity. **Answer** $250s^{-1}$, $245Nm^{-2}$, 0.98Pa – s.

29. Water flows at the rate of 4litres per second through an orifice at the bottom of tank which contains water 720cm deep. Find the rate of escape of water if additional pressure of $16kg/m^2$ is applied at the surface. **answer** 19.28L/sec

30. Two capillary tubes AB and BC are joined end to end at B. AB = 12cm long and of diameter 4mm, whereas BC is 3cm long and diameter 2mm. the composite tube is held horizontally with A connected to a vessel of water giving a constant head of 5cm and C is open to the air. Find the pressure difference between B and C.



Answer h = 4cm of water column.

31. A large bottle is fitted d with a siphon made of capillary glass tubing. Compare the coefficients of viscosity of petrol and water, if the time taken to empty the bottle in two cases is in the ratio 2:5 given the density of petrol is $0.8gcm^{-3}$. **Answer** 1:2

32. Define terminal velocity. Show that the terminal velocity V of a sphere of radius r, density ρ falling vertically through a viscous fluid of density ρ and coefficient of viscosity η is given by

$$V = \frac{2gr^2(\rho - \delta)}{9\eta}$$

Uses this formula to explain the observed rise of air bubbles in a liquid.

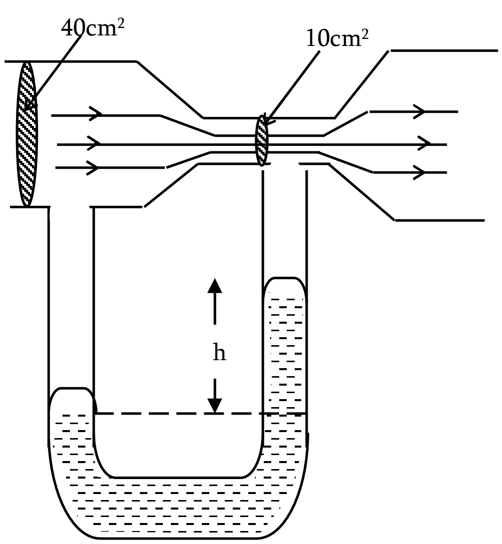
33. Three capillary tubes of lengths L, 2L and L/2 are connected in series. Their radii are r, r/2 and r/3 respectively. If streamline flow is maintained and the pressure difference across the first capillary tube is P find the pressure difference across the other tubes . **Answer** 32P, 40.5p

34. The radius of a pipe carrying a liquid gets decreased by 5% because of deposits on the inner surface. By how much would the pressure difference between the ends of the constricted pipe have to be increased to maintain a constant flow rate? **Answer** p' = 1.23p

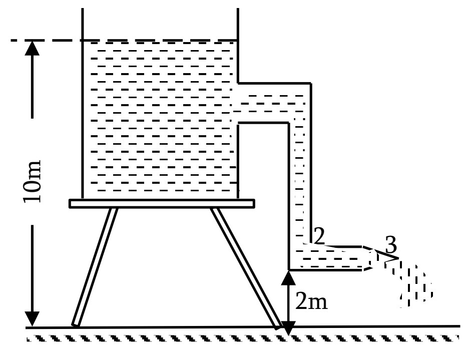
35. An engineer wants to have the same flow rate of water and light machine oil from the pipes of the same length and with the same pressure head. What should be ratio of the radii of the two pipes? Given that viscosity of water = 0.01 poise and that of light machine oil = 11 poise **Answer** 5.76

36. In a hospital, a patient receives a $500cm^3$ blood transfusion though a needle with a length of 5cm and inner radius of 0.03cm. If the blood bag is kept 85cm above the needle, how long the transfusion takes place? Given that the viscosity of blood is 0.017 poise and the density of blood is $1.02g/cm^3$. **Answer** 1,529sec

37. The horizontal pipe as shown in the figure below has a cross – section area of $40cm^2$ at the wider portions and $10cm^2$ at the constriction. Water is flowing in the pipe and the discharge from the pipe is $6.0 \times 10^{-3}m^3/s$. find
    (a) The flow speed at the wide and narrow portions.
    (b) The pressure difference between these portions.
    (c) The difference in height between the mercury columns in the U – shaped tube



38. Water flows steadily from an open tank as shown in the figure below. The elevation of point 1 is 10.0cm, and the elevation of point 2 and 3 is 2.00m. The cross – sectional area at point 2 is $0.0480m^2$, at point 3 it is $0.0160m^2$. The area of the tank is very large compared with cross – sectional area of the pipe. Assuming that the Bernoulli's equation applies, compute:
    (a) The discharge rate in cubic metre per second and
    (b) The gauge pressure at point 2.



Answer (a) $0.2m^3/s$ (b) 6.97×10^4Pa

39. A cylindrical bucket, open at the top, is 25.0cm high and 10.0cm in diameter. A circular hole with a cross – sectional area $1.5cm^2$ is cut in the centre of the bottom of the bucket. Water flows into the bucket from a tube above it at the rate of $2.40 \times 10^{-4}m^3/sec$. how high will the water in the bucket rise?

CONCEPT AND MISCELLANEOUS PROBLEM

1. (a) (i) What is the terminal velocity? Discuss the factors on which it depends.
   (ii) What are the quantities of an ideal liquid?
   (b) (i) Two row boats moving parallel to each other and nearby, pulled towards each other. Explain.
   (ii) The hot liquid flows faster than the colder ones. Explain why.

(c) (i) State the stoke's law. Show that the terminal velocity V of a sphere of radius r, density ρ falling through a viscos fluid of density δ and coefficient of viscosity η is given by

$$V = \frac{2g(\rho-\delta)r^2}{9\eta}$$

Use this formula to explain the observed rise of air bubble.

(ii) Two capillary tube of equal length and inner radii 2r and 4r respectively are added in series and a liquid flows through it. If the pressure difference between the ends of the whole system is 8.5cm of mercury, find the pressure difference between the ends of the first capillary tube.

Solution

(a) (i) Terminal velocity of the body is the maximum constant velocity acquired by the body, while falling freely in a viscous medium. The terminal velocity of a body is given by

$$V = \frac{2r^2g(\rho-\delta)}{9\eta}$$

Terminal velocity depends on the following factors.

- Radius (r) of the body
- Density (ρ) of the body
- Density (δ) of the viscous fluid
- Viscosity (η) of the fluid

(ii)
- The liquid is perfectly incompressible. It means the density of the liquid remains constant irrespective of pressure.
- The liquid is non-viscous i.e there are no tangential forces between layers of liquid in relative motion.
- An ideal liquid cannot withstand any shearing stress, however small the stress may be.

(b) (i) According to Bernoulli's theorem, when velocity of liquid flow increases, the pressure decreases and vice – versa. When two boats move in parallel directions close to each other the stream of water between the boats is set into vigorous motion. As a result, the pressure exerted by water in between the boats becomes less than the pressure of water outside the boats. Due to this difference of pressures, the boats are pulled towards each other.

(ii) The coefficient of viscosity of liquid decreases with rise in temperature. As a result of it, hotter liquid flows faster than the colder one , under the given tangential force.

(c) (i) Refer to your notes

(ii) Let P, P_1 , P_2 be the pressure at the beginning of the a tube, at the joint of two tubes and at the end of the second tube. Since the two tubes are in series so

$$\frac{\pi(P-P_1)(2r)^4}{8\eta L} = \frac{\pi(P_1-P_2)(4r)^4}{8\eta L}$$

$$P-P_1 = (P_1-P_2)16$$

$$P = 17P_1-16P_2 \text{ ....(1)}$$

Given that

$$P-P_2 = 8.5$$

$$P = P_2+8.5 \text{ ......(2)}$$

From equation (1) and (2)

$$P_2+8.5 = 17P_1-16P_2$$

$$P_1-P_2 = \frac{8.5}{17} = 0.5\text{cm of Hg}$$

Pressure difference between the ends of the first tube.

$$\Delta P_1 = P = P_1 = (P-P_2)-(P_1-P_2)$$

$$= 8.5-0.5$$

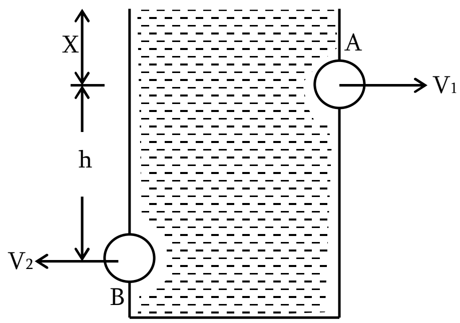
$$\Delta P_1 = 8.0\text{cm of Hg}$$

2. (a) (i) Explain the limitations of Bernoulli's theorem.

(ii) It is advised not to stand near a running train. Why?

(b) There are two holes, each of cross – sectional area, a on the opposite sides of a wide rectangular tank containing a liquid of density ρ. When the liquid flows out the holes, find the net force on the tank.

Given, h is the vertical distance between two holes.



Solution

(a) (i) Refer to your notes

(ii) When fast moving train passes on a rail, then the velocity of air streams in between the rail and the person standing near rail will be very large as compare to the velocity of air streams on the other side of person away from the rail. According to Bernoulli's theorem, the pressure of air will becomes low in between person and rail and is high on the other side of person as a result of this pressure difference a thrust acts on the person which may push the person towards rail side and the person may met with an accident.

(b) When liquid flows out of hole from tank of area cross – section, a with velocity V then force on the tank is equal to rate of change of linear momentum.

$$F = (av\rho)v = \rho a v^2$$

Net force

$$F = a\rho\left(V_2^2 - V_1^2\right)$$

According to the Torricelli's theorem

$$V_1 = \sqrt{2gx}\ , \quad V_2 = \sqrt{2g(h+x)}$$

$$V_1^2 = 2gx \text{ and } V_2^2 = 2g(h+x)$$

$$\Rightarrow F = a\rho\left[2g(h+x) - 2gx\right]$$

$$F = 2a\rho gh$$

3. (a) A bigger rain drop falls faster than a smaller one. Why?

(b) Explain why a parachute is invariably used while jumping from an aeroplane.

(c) Dust generally settles down in a closed room. Explain.

(d) Small air bubbles rise slower than the bigger ones through a liquid. Explain , why?

Solution

(a) When a rain drop attains terminal velocity the viscous force on the drop due to air becomes equal to its weight i.e

$$6\pi\eta vr = \frac{4}{3}\pi r^3 \rho g$$

$$v \propto r^2$$

For this reason, a bigger rain drops falls faster than a smaller one.

(b) We know viscous force $F = 6\pi\eta vr$. Due to its large structure (r large) a parachute experiences a large viscous force and hence while descending through air , it acquires a very small terminal velocity. Due to the low velocity of descent, the person using the parachute will not get hurt.

(c) Dust particles are spheres of very small radii. After acquiring the terminal velocity, they start falling through the air with a constant velocity as the terminal velocity for the dust particles will be very small ($v \propto r^2$), they will settle down in a closed room after some time.

(d) Since $V = \dfrac{2r^2 g(\rho - \delta)}{9\eta}$ where ρ is density of air and δ density of the liquid. Obviously, the factor $(\rho - \delta)$ is negative and hence V will be negative. Since $V \propto r^2$, it follows that the small air bubbles rise (V is negative) slower than the bigger ones through a liquid.

4. (a) According to Bernoulli's theorem , the pressure of water should remain uniform in a pipe of uniform cross – section. But actually it goes on decreasing. Why?

(b) A horizontal tube of diameter 2mm and length 50cm is connected at the bottom of a cubical tank of the side 100cm containing water of coefficient of viscosity $10^{-3}Nm^{-2}s$. If the tank is initially full after what time will the level of water in the tank drop by three quarter full? State any assumption made in your calculations.

(c) Emery powder particles are stirred up in a beaker of water 0.1m deep. Find the radius of the largest particle remaining in suspension after 24hours. Given density of emery = $4000kgm^{-3}$ and the viscosity of water = 10^{-3} decapoise.

Solution

(a) The pressure of water goes on decreasing in pipe of uniform cross – section because of viscosity of water. When water flows, work is done against the viscous force. This work done is taken from the pressure energy. Hence pressure of water falls as it flows through the pipe.

(b) Assuming the flow is steady fluid flow.
Apply Poiseulli's equation

$$\frac{dv}{dt} = \frac{-Adh}{dt} = \frac{\pi pr^4}{8\eta L} = \frac{\pi h\rho gr}{8\eta L}$$

$$\frac{-Adh}{dt} = \frac{\pi h\rho gr^4}{8\eta L}$$

$$\frac{dh}{dt} = -\left(\frac{\pi\rho gr^4}{8\eta AL}\right)h$$

Let: $k = \frac{\pi\rho gr^4}{8\eta AL}$

$$\frac{dh}{dt} = -kh$$

$$\frac{dh}{h} = -kdt$$

$$\int_{ho}^{h}\frac{dh}{h} = -k\int_{0}^{t}dt$$

$$\left[\log_e h\right]_{ho}^{h} = -kt$$

$$\log_e\left(h/ho\right) = -kt$$

In exponential form

$$\frac{h}{ho} = e^{-kt}$$

$$h = hoe^{-kt}$$

$$A = 1m\times 1m = 1m^2,\ \rho = 1000kgm-3$$

$$h = ho - \frac{3}{4}ho = \frac{ho}{4}$$

$$k = \frac{\pi\rho gr^4}{8\eta LA},\ g = 9.8m/s2$$

$$t = 1.80\times 10^5\ sec$$

(c) Assume that the largest particle remains suspended very close to the bottom starting from the surface of water

$$V = \frac{0.1}{24\times 60\times 60}$$

$$V = 1.15741\times 10^{-6}m/s$$

Since $V = \frac{2r^2\left(\rho-\rho'\right)g}{9\eta}$

$$r = \sqrt{\frac{9nv}{2\left(\rho-\rho'\right)g}}$$

$$r = 4.2\times 10^{-7}m$$

5. (a) (i) give the meaning of the term velocity gradient, tangential stress and coefficient of viscosity as used in fluid mechanics.

(ii) Write stoke's equation defining clearly the meaning of all symbol used.

(b) Calculate the terminal velocity of the rain drops falling in air assuming that the flow is laminar, the rain drops are sphere of diameter 1mm and the coefficient of viscosity $n = 1.8\times 10^{-5}Ns/m^2$.

(c) Write flows past a horizontal plate of area $1.2m^2$. If its velocity gradient and coefficient of viscosity adjacent to the plate area are $10s^{-1}$ and $1.3\times 10^{-5}Nsm^{-2}$ respectively, calculate the force acting on the plate.

(d) A horizontal pipe of cross – sectional area $10cm^2$ has one section of cross – sectional area $5cm^2$. If the water flows through the pipe, and the pressure difference between the two sections is 300Pa, how many cubic meters of water will flow out of the pipe in 1minute.
Density of air = $1.29kgm^{-3}$
Density of water = $1000kgm^{-3}$
Acceleration due to gravity $g = 9.8m/s^2$.