Problem 1

a) $a = \begin{bmatrix} 7 \\ -4 \\ 2 \end{bmatrix}$ $b = \begin{bmatrix} 3 \\ -1 \\ -1 \end{bmatrix}$

$a^T = [7, -4, 2]$ $a^T \cdot b = 7 \cdot 3 + 4 \cdot -1 + 2 \cdot -1 = \underline{23}$

b) $a \cdot b = 3$ $a \cdot c = -5$

$$\begin{array}{l|l} a \cdot (3b + c) & \\ 3 \cdot a \cdot b + a \cdot b & \text{insert} \\ 3 \cdot (3) + (-5) & \\ 9 - 5 & \\ \underline{4} & \end{array}$$

Problem 2

$A = \begin{bmatrix} -2 & 1 \\ -1 & 3 \\ 6 & -4 \end{bmatrix}$ $b = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

a)

$$Ab = \begin{bmatrix} -2 & 1 \\ -1 & 3 \\ 6 & -4 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \underline{\begin{bmatrix} 5 \\ 5 \\ -16 \end{bmatrix}}$$

b) $A^T b = \begin{bmatrix} -2 & -1 & 6 \\ 1 & 3 & -4 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 5 & -16 \end{bmatrix}$

c) $\begin{bmatrix} 1 & -3 & 1 \\ -1 & 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

d) b is in the null space of A since b is not 0, A has not the full rank

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Problem 3

a) $\begin{bmatrix} 2 & -4 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} -1 & 5 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 6 \\ -10 & 8 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 8 & -3 \\ 5 & -4 & 10 \end{bmatrix} \cdot \begin{bmatrix} 2 & -2 \\ 4 & 6 \\ 7 & -1 \end{bmatrix} = \begin{bmatrix} 13 & 45 \\ 84 & -94 \end{bmatrix}$

Problem 4

a) $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$f(x) = x_1^2 + 2x_2^3 - 3x_3 - x_1 x_2 x_3$

$f'(x_1) = 2x_1 - x_2 x_3$

$f'(x_2) = 6x_2^2 - x_1 x_3$

$f'(x_3) = -3 - x_1 x_2$

$\nabla_x f(x) = \left[2x_1 - x_2 x_3 \;,\; 6x_2^2 - x_1 x_3 \;,\; -3 - x_1 x_2 \right]$

b) $f(x) = a^T x + x^T x$

1. derivative of $a^T x$ is a

2. $x^T x$ is $\begin{bmatrix} x_1^2 \\ x_2^2 \\ x_3^2 \\ \dots \end{bmatrix}$, derivative of this is $\begin{bmatrix} 2x_1 \\ 2x_2 \\ 2x_3 \\ \dots \end{bmatrix}$ or $2x$

3. $f'(x) = \underline{a + 2x}$

Problem 5

a) $F(x,y) = \begin{bmatrix} x^2 + y^2 \\ y^3 \end{bmatrix}$

$\frac{d}{dx}(F(x,y)) = \begin{bmatrix} 2x \\ 0 \end{bmatrix}$

$\frac{d}{dy}(F(x,y)) = \begin{bmatrix} 2y \\ 3y^2 \end{bmatrix}$, Jacobi: $\underline{\begin{bmatrix} 2x & 2y \\ 0 & 3y^2 \end{bmatrix}}$

b) $F(x,y) = \begin{bmatrix} x^2 + y^2 \\ xy + 2\cos(x) \end{bmatrix}$

$\frac{d}{dx}(F(x,y)) = \begin{bmatrix} 2x \\ y - 2\sin(x) \end{bmatrix}$

$\frac{d}{dy}(F(x,y)) = \begin{bmatrix} 2y \\ x \end{bmatrix}$, Jacobi: $\underline{\begin{bmatrix} 2x & 2y \\ y - 2\sin(x) & x \end{bmatrix}}$

Problem 6

a) $\frac{df}{dy}(f(y)) = \begin{bmatrix} 2y_1 \\ 2 \end{bmatrix}$

b) $H(x) = \begin{bmatrix} x_1 \cdot x_2 \\ x_1 \end{bmatrix}$, $\frac{dH}{dx}(H(x)) = \begin{bmatrix} x_2 & x_1 \\ 1 & 0 \end{bmatrix}$

c) $\frac{df}{dH(x)} = \begin{bmatrix} 2 & x_1 x_2 \\ 2 \end{bmatrix}$

d) $\left(\frac{df}{dH(x)}\right)^T \cdot \frac{dH}{dx} = \begin{bmatrix} 2 x_1 x_2 & 2 \end{bmatrix} \cdot \begin{bmatrix} x_2 & x_1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 2 x_1 x_2^2 + 2 & 2 x_1^2 x_2 \end{bmatrix}$