Zurich University
of Applied Sciences

School of
Engineering

P03 – Logic exercises

Following Russell & Norvig's finding that *"a student of AI must develop a talent for working with logical notation"* [AIMA, p. 290], this lab description collects some exercises to get acquainted with formulating and manipulating known facts in logical notation, and to do inference to arrive at new conclusions. For the sake of keeping the linguistic extravaganza found in one reference[1], some tasks are formulated in German.

1. Working with propositional logic

1.1 Which of the following is correct?[2]

- False ⊨ True.
- True ⊨ False.
- (A ∧ B) ⊨ (A ⇔ B).
- A ⇔ B ⊨ A ∨ B.
- A ⇔ B ⊨ ¬A ∨ B.
- (A ∧ B) ⇒ C ⊨ (A ⇒ C) ∨ (B ⇒ C).
- (C ∨ (¬A ∧ ¬B)) ≡ ((A ⇒ C) ∧ (B ⇒ C)).
- (A ∨ B) ∧ (¬C ∨¬D ∨ E) ⊨ (A ∨ B).
- (A ∨ B) ∧ (¬C ∨¬D ∨ E) ⊨ (A ∨ B) ∧ (¬D ∨ E).
- (A ∨ B)∧ ¬(A ⇒ B) is satisfiable (see V05).
- (A ⇔ B) ∧ (¬A ∨ B) is satisfiable.
- (A ⇔ B) ⇔ C has the same number of models as (A ⇔ B) for any fixed set of proposition symbols that includes A, B, C.

[1] Materials (script, exercises and exams) of Hellwig Geisse's lecture on „Methoden der KI" at Fachhochschule Giessen Friedberg, Germany, spring semester 2003 – the AI lecture this lecturer (stdm) had the pleasure to attend during his own studies.
[2] Exercise 7.4 from [AIMA].

1.2 Decide whether each of the following sentences is satisfiable or unsatisfiable[3]
Verify your decisions using truth tables or the equivalence rules (see V06a).

- $Smoke \Rightarrow Smoke$
- $Smoke \Rightarrow Fire$
- $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$
- $Smoke \lor Fire \lor \neg Fire$
- $((Smoke \land Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \lor (Heat \Rightarrow Fire))$
- $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \land Heat) \Rightarrow Fire)$
- $Big \lor Dumb \lor (Big \Rightarrow Dumb)$

[3] Based on exercise 7.10 from [AIMA].

1.3 Wumpus world navigation[4]

Suppose an agent has progressed to the following point, having perceived nothing in [1,1], a breeze in [2,1], and a stench in [1,2], and is now concerned with the contents of [1,3], [2,2], and [3,1]:

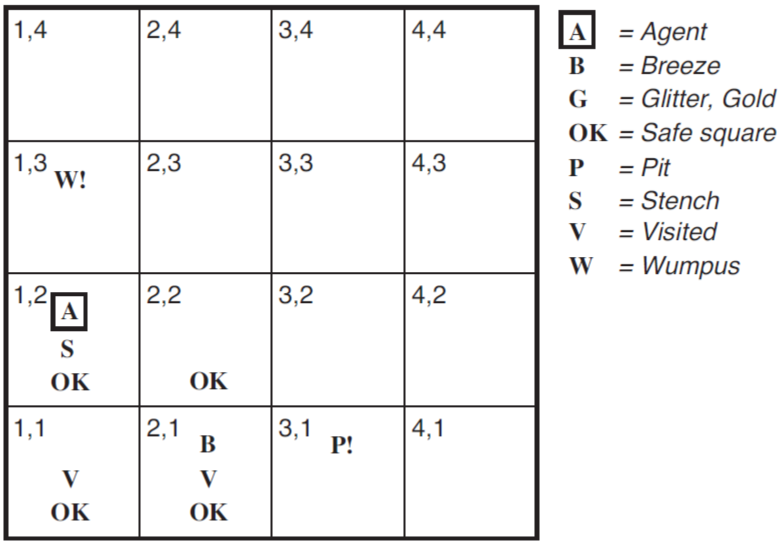

Figure 1: From [AIMA, Fig. 7.4a, p. 239].

Each of these can contain a pit, and at most one can contain a wumpus. Following the example the slide “Entailment in the wumpus world, contd.”, construct the set of possible worlds (you should find 32 of them).

Mark the worlds in which the KB is true and those in which each of the following sentences is true:

- α2 = “There is no pit in [2,2].”
- α3 = “There is a wumpus in [1,3].”

Hence show that KB ⊨ α2 and KB ⊨ α3.

[4] Exercise 7.1 from [AIMA].

2. [Optional] Formulating sentences in first-order logic

2.1 Hier das neueste aus dem Land der Bloffs und Würgel[5]
[EN: latest news from the fictional country of the Bloffs and Wurgels]

- *Zu jedem Würgel gibt es einen Bloff, der von diesem Würgel gepfennert wird.*
  [EN: To each Wurgel belongs a Bloff that gets pfennert by this Wurgel.]
- *Wenn irgendein Bloff nausert, dann nausern alle Bloffs. (Ausgesprochen merkwürdig!)*
  [EN: If any Bloff nauserts, then all Bloffs nauser. (Really strange!)]
- *Wenn es für jeden Bloff einen Würgel gibt, der diesen Bloff pfennert, dann nausern alle Würgel.*
  [EN: If for each Bloff there exists a Wurgel that pfennerts this Bloff, then all Wurgel nauser.]

Formalisieren Sie diese Aussagen mit Hilfe der folgenden Prädikate:
[EN: Formalise these statements using the following predicates:]

- IstBloff(X), IstWuergel(x), Pfennert(x, y), Nausert(x)

[5] From exam «Methoden der KI», 01.10.1997.

2.2 Hier sind Neuigkeiten aus Bloffonien[6]
[EN: News from Bloffonia]

- *Jeder Bloff, der nörgt, klüpft einen Würgel.*
  [EN: Each Bloff that norgts, klupfts a Wurgel.]
- *Prumm ist ein Bloff, der keinen Würgel klüpft.*
  [EN: Prumm is a Bloff that does not klupf a Wurgel.]

Formalisieren Sie diese beiden Aussagen als Sätze des Prädikatenkalküls erster Stufe. Verwenden Sie die Prädikate Bloff(x), Noergt(x), Wuergel(y) und Kluepft(x, y).
[EN: Formalise these two statements as sentences of FOL. Use the predicates Bloff(x), Norgt(x), Wurgel(y), and Klupft(x, y).]

[6] From exam «Methoden der KI», 30.01.1997.

2.3 Heute dürfen wir die Bloffs zuhause besuchen[7]
[EN:Today we are going to visit the Bloffs at home]

- *Wenn ein Bloff im Horg pummert, dann ist der Horg noch nicht suggi.*
  [EN: If a Bloff pummers in the Horg, then the Horg isn't yet suggi.]
- *Es gibt Bloffs, deren Horg schon suggi ist, die aber in einem anderen Horg noch pummern.*
  [EN: There are Bloffs whos Horg is already suggi but who still pummer in another Horg.]
- *Niemals pummert ein Würgel in einem Horg, in dem bereits ein Bloff pummert.*
  [EN: Never does a Wurgel pummer in a Horg in which already a Bloff is pummering.]

Formalisieren Sie diese Aussagen mit Hilfe der folgenden Prädikate:
[EN: Formalise these statements using the following predicates:]

- IstBloff(x), IstHorg(x), IstWuergel(x), PummertIn(x, y), IstSuggi(x), IstGleich(x, y)
  [EN: IsBloff(x), IsHorg(x), IsWurgel(x), PummersIn(x, y), IsSuggi(x), IsEqual(x, y)]

[7] From exam «Methoden der KI», 26.01.2001.

3. Inference in logical sentences

3.1 [Optional] Die possierlichen Bloffs benehmen sich ausgesprochen merkwürdig[8] (FOL)
(EN: The funny Bloffs act distincively strange (FOL)]

- *Wenn ein Bloff rüpft, dann nörgt er auch.*
  [EN: If a Bloff rupfts, then it also norgts.]
- *Wenn ein Bloff nörgt und zinnt, dann gängert er.*
  [EN: If a Bloff norgts and zinnts, then it gangerts.]

Obwohl die Bloffs sehr scheu sind, kennen wir einen bestimmten Bloff ein bisschen genauer:
[EN: Although the Bloffs are very shy, we do know a specific Bloff a little better:]

- *Prumm ist ein Bloff, der zinnt, aber nicht gängert.*
  [Prumm is a Bloff that zinns but not gangerts.]

Wandeln Sie die drei Aussagen in Klauseln um.
[EN: Translate the three statements into clauses.]

Beweisen Sie durch Wiederspruch in der Klauselmenge: *«Prumm rüpft nicht»*.
[EN: Prove by contradiction in the set of clauses: «Prumm does not rupf.»]

[8] From exam «Methoden der KI», 26.09.1996 (?).

3.2 On unicorns[9] (propositional logic)

Given the following, can you prove that the unicorn is mythical? How about magical? Horned?

- *If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal.*
- *If the unicorn is either immortal or a mammal, then it is horned.*
- *The unicorn is magical if it is horned.*

[9] Exercise 7.2. from [AIMA], adapted from Barwise and Etchemendy (1993).