Intuitionistic fuzzy sets applied to color image processing

Agustina Bouchet[1], Susana Montes[1] and Irene Díaz[2]

[1]Department of Statistics and Operations Research and Mathematics Didactics, University of Oviedo, Spain
[2]Department of Computing. University of Oviedo, Spain

Abstract
Segmentation of color images is a challenge which is in constant development as they provide more information than a gray-scale image. In particular, the analysis of biomedical images is specially helpful for many purposes. In this work, a method to segment leukocytes is presented. This method is based on the combination of the use of RGB color space, intuitionistic fuzzy sets and K-means. The results show a good performance of the proposed method achieving the segmentation of the objects of interest.

Keywords
Intuitionistic fuzzy set, color image, segmentation, leukocytes

1. Introduction

Color images provide more and better information than gray-level images, especially in areas such as medicine or biology, among others. As a consequence, its use has increased exponentially during the last decades thanks to the development of the new techniques together with the improvement of many computing infrastructures. Thus, new techniques for color image processing for correcting, segmenting, measuring and/or quantifying the structures contained in them represent a challenge as clinical information about the anatomic structures could provide a more accurate diagnosis.

The analysis of blood cell images is an important task in the pathological laboratories for the detection of diseases such as infections, haemorrhage, leukaemia, anaemia or inflammation, among others. Shape and number of white blood cells, also called leukocytes, in the cell images are a very important characteristics for the detection of leukareaemia. Thus, an accurate segmentation of leukocytes is an important task which has being study in several papers. A selected references about this topic are [1, 2, 3, 4, 5, 6, 7, 8, 9].

In [10] authors developed an automatic method to segment leukocytes using intuitionistic fuzzy sets. This kind of sets give a flexible mathematical framework to cope with the uncertainty and imprecision in the images. These sets consider not only the membership degree but also the uncertainty that the membership degree (called hesitancy) implies. In that case, HSV color space is used to model the image and only the hue component is considered. Also, the similarity

WILF 2021: 13th International Workshop on Fuzzy Logic and Applications, December 20–22, 2021, Vietri sul Mare, Italy
bouchetagustina@uniovi.es (A. Bouchet); montes@uniovi.es (S. Montes); sirene@uniovi.es (I. Díaz)
0000-0003-1516-8976 (A. Bouchet); 0000-0002-4701-2207 (S. Montes); 0000-0002-3024-6605 (I. Díaz)

between each point of the processing image and a fixed point is measured. This point is obtained using fuzzy mathematical morphology [11]. The main drawbacks of this approach are the following. Firstly, information in the image is lose because only one component of the image is considered discarding the information contained in the other two components. Secondly, the similarity is measured between points, thus, the color information of the neighbours of these points is not considering. Finally, the computational cost of the method is very high due to the use of the fuzzy mathematical morphology. To overcome these difficulties, in this work we propose an improvement of the one proposed in [10]. The method is based on using the RGB color space in order to consider the information provided by the three components and on using K-means clustering algorithm for the selection of the fixed pixel. Finally, we evaluate the similarity in a neighbourhood taking into account the characteristic of non-homogeneous color of leukocytes.

The paper is organized as follows. Section 2 provides all the necessary theoretical concepts for the development of the method. In Section 3, the proposed method is described. Section 4 shows the experimental results. Finally, conclusions are discussed in Section 5.

2. Preliminaries

2.1. Color model

There are many different color representations as RGB, HSV, HSL, CIE L*a*b, YCbCr, among others [12, 13]. RGB is the most straightforward representation system to manage color images, widely used in computer system and hardware devices for color image display. However, the choice of a suitable color space representation is still a challenging task in the processing and analysis of color images [14]. Thus, RGB space is chosen in this work because the three components of the image have the same nature, representing a quantity of a certain primary color [15]. RGB space is the only codification such that all the components represent the same concept, the level of saturation of a primary color. One possible approach to process color images is the individual processing of these images. Using this approach, it is possible to extract shapes and significant details of the images. An image in this model is composed by three independent images, each of them corresponding with a primary color: red (R), green (G) and blue (B). The individual processing of these three planes allows extracting meaningful details of the images for a later segmentation. Therefore, a color image can be defined as follow:

A color image in the RGB color space is a function $f : D_f \subset \mathbb{R}^2 \to \mathbb{R}^3$, where D_f is the domain of the image, such that $f(i, j) = (r_{ij}, g_{ij}, b_{ij})$, being $r_{ij}, g_{ij}, b_{ij} \in \{0, \ldots, 255\}$.

2.2. Intuitionistic fuzzy sets

Intuitionistic fuzzy sets introduce uncertainty in the membership degree which is known as the hesitance degree. It is well known that traditional fuzzy set theory assigns a membership degree to each element and the non-membership degree is classically computed as one minus the membership degree. However, human thinking often involves uncertainty or imprecision and sometimes standard fuzzy sets are not enough because the presence of hesitance or uncertainty or the partial unknowledge in the membership definition [10, 16].

Following this idea, Atanassov [17] introduced the concept of intuitionistic fuzzy set, which aims to reflect the fact that the degree of non-membership is not always equal to one minus the membership degree due to the presence of some hesitation. In this way, the definition of intuitionistic fuzzy sets (IFS) is the following:

Given a referential set X, an intuitionistic fuzzy set A (IFS) of X is defined as

$$A = \{(x, \mu_A(x), \nu_A(x))/x \in X\} \tag{1}$$

where $\mu_A(x) \to [0, 1]$ and $\nu_A(x) \to [0, 1]$ are the membership degree and the non-membership degree of an element $x \in X$ w.r.t. the set A. They satisfy $0 \le \mu_A(x) + \nu_A(x) \le 1, \forall x \in X$.

An intuitionistic or hesitation degree is also introduced by Atanassov, which arises due to the lack of knowledge about the membership degree. The hesitation degree $\pi_A(x)$ is given by $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x), \forall x \in X$.

This index represents the ignorance or intuition in the construction of the fuzzy sets [18] while, in the context of image segmentation, indicates the knowledge/ignorance of the expert when assigning a pixel either to the background or to the object. In this sense, the hesitation degree is equal to zero when the expert is absolutely sure that the pixel belongs either to the background or to the object and this value increases with respect to the ignorance/intuition of the expert as to whether the pixel belongs to the background or to the object [18].

2.3. Intuitionistic fuzzy generator

Intuitionistic fuzzy generators are used to construct IFS and they are defined as a function $\varphi : [0, 1] \to [0, 1]$ satisfying [19]: $\varphi(x) \le 1 - x, \forall x \in [0, 1]$. Therefore, $\varphi(0) \le 1$ and $\varphi(1) = 0$.

An important intuitionistic fuzzy generator is defined by Sugeno [20] and the IFS generated is given by:

$$A_\lambda = \left\{ \left(x, \mu_A(x), \frac{1 - \mu_A(x)}{1 + \lambda \mu_A(x)} \right) /x \in X \right\} \tag{2}$$

By its good properties, Sugeno type intuitionistic fuzzy generator we will consider in our approach in order to create an intuitionistic fuzzy image for modeling a color image. Really, we are not dealing with IFSs in the strict sense, but we use this framework to formalize our method.

2.4. Intuitionistic fuzzy divergence

A intuitionistic fuzzy divergence (IFD) is a measure of the difference between two intuitionistic fuzzy sets. In the literature, there are several definitions of IFD [6, 8, 21]. However, the IFD introduced in [16] is used in this work due to the relevance of the hesitation degree in its definition. Let A and B be two IFS, the value of this IFD associated to them is:

$$\begin{aligned} IFD(A,B) &= \sum_i 2 - [1 - \mu_{AB}(x_i)]e^{\mu_{AB}(x_i)} - \\ &- [1 + \mu_{AB}(x_i)]e^{-\mu_{AB}(x_i)} + \\ &+ 2 - [1 - \mu_{AB}(x_i) - \pi_{AB}(x_i)]e^{\mu_{AB}(x_i) - \pi_{AB}(x_i)} \\ &- [1 + \pi_{AB}(x_i) + \mu_{AB}(x_i)]e^{-\pi_{AB}(x_i) - \mu_{AB}(x_i)} \end{aligned}$$

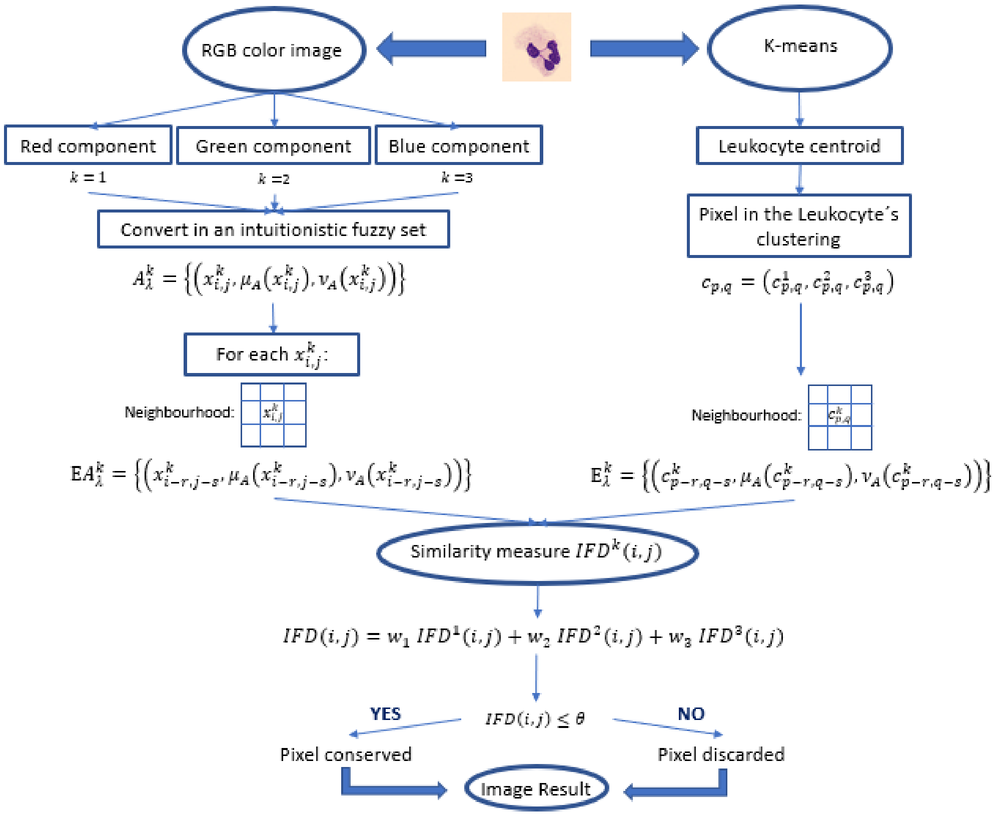

Figure 1: Block diagram of the proposed method.

being $\mu_{AB}(x_i) = \mu_A(x_i) - \mu_B(x_i)$ and $\pi_{AB}(x_i) = \pi_A(x_i) - \pi_B(x_i)$

3. RGB intuitionistic-based method

In this section, the proposed method is developed. This method first considers each component of a RGB image and model each of them as an IFS. Then a pixel from the image is selected as reference to compute the similarity and computes the IFD between a neighbourhoods of each element in the image and a neighbourhood of the pixel selected as reference. If the IFD is less or equal than a fixed value, the point is conserved, otherwise discarded. Figure 3 shows a block diagram of the proposed algorithm. The method to segment the images can be summarised in the following six steps:

1: The first step consists on deciding the color space that it will be used. In this case, RGB model is used for representing the color images. Therefore, let f be a color image, three components are extracted according to the red (R), green (G) and blue (B) components which they be denoted by $k = 1$, $k = 2$ and $k = 3$, respectively. It is important to notice that the proposed method could be used in others color spaces and this choice will depend on the image characteristics and the objective set in the segmentation.

2: In this step, the image is modeled as a IFS. Each component of the image has $M \times N$ dimension. However, components are defined as an intuitionistic fuzzy set as follows:

   - The first coordinate $x_{i,j}^k$ represents the value of the pixel (i, j) in the k component with $0 \leq i \leq M$, $0 \leq j \leq N$ and $k = 1, 2, 3$. For example, for the red component $x_{i,j}^1 = r_{i,j}$, for the green one $x_{i,j}^2 = g_{i,j}$ and, finally, for the blue one $x_{i,j}^3 = b_{i,j}$.
   - The second coordinate is the normalised value of the first one. It is obtained applying the membership function:
$$\mu_A(x_{i,j}^k) = \frac{x_{i,j}^k}{255} \tag{3}$$
   - The non-membership function is applied over the value obtained in the above step. The non-membership function is computed by using the Sugeno-type intuitionistic fuzzy generator as follows, where the value of λ chosen is 0.5:
$$\nu_A(x_{i,j}^k) = \frac{1 - \mu_A(x_{i,j}^k)}{1 + \lambda\mu_A(x_{i,j}^k)} \tag{4}$$

   Therefore, for each component k, the intuitionistic fuzzy set is defined as:
$$A_\lambda^k = \left\{ \left(x_{i,j}^k, \mu_A(x_{i,j}^k), \frac{1 - \mu_A(x_{i,j}^k)}{1 + \lambda\mu_A(x_{i,j}^k)} \right) /0 \leq i \leq M \wedge 0 \leq j \leq N \right\} \tag{5}$$

3: The hesitation degree is calculated for each pixel of each component of the image:
$$\pi_A(x_{i,j}^k) = 1 - \mu_A(x_{i,j}^k) - \frac{1 - \mu_A(x_{i,j}^k)}{1 + \lambda\mu_A(x_{i,j}^k)} \tag{6}$$

4: For leukocytes segmentation, a neighbourhood of a point of the object to be segmented is necessary to evaluate the similarity with the image to be analyzed. This neighbourhood is chosen instead of just one point because the leukocytes are, in general, a non-homogeneous color object [10]. An example of this kind of images is later shown in Fig. 4. Therefore, a fixed pixel corresponding to the object to be segmented is selected and a neighbourhood of it is chosen. This selection is made using K-means. For this, a K-means algorithm is applied over the original image and three clustering are obtained, each one for leukocyte, cytoplasm and background, respectively. As the centroid corresponding to the leukocyte is not necessarily a pixel of the original image, an element of the clustering is chosen in order to guarantee that this point belongs to the image. Thus, this point is denoted by $c_{p,q} = (c_{p,q}^1, c_{p,q}^2, c_{p,q}^3)$ where p and q are the position of the pixel in the image. Using this point c as a central point, a neighbourhood of dimension $D \times D$ is selected. This neighbourhood is considered as an intuitionistic fuzzy set in the same way as we considered the image in step 1 and it is denoted by Ec_λ^k, $k = 1, 2, 3$. Let us remark that, from now on, the value for D is 3. This value has been selected in an heuristic way due to the nature of the kind of the images we are processing.

5: In this step the similarity is calculated. Let A^k_λ be the intuitionistic fuzzy set of the color image for each component k given by Eq. 5 and Ec^k_λ the intuitionistic fuzzy set of the neighbourhood for each component k. For each pixel $x_{i,j}$ of the original image, with $0 \le i \le M, 0 \le j \le N$, the following subset is considered with the same size of Ec^k_λ, that is, 3×3:

$$EA^k_\lambda(i,j) = \{(x^k_{i-r,j-s}, \mu_A(x^k_{i-r,j-s}), \nu_A(x^k_{i-r,j-s})/r, s \in \{-1,0,1\}\} \tag{7}$$

Figure 2 shows an example of a neighbourhood for a generic pixel $x_{i,j}$.
The IFD between each element of EA^k_λ and Ec^k_λ, using Eq. 3, is given by:

$$\begin{aligned} IFD^k(i,j) &= \sum_{r=-1}^{1}\sum_{s=-1}^{1} 2 - [1 - \mu_{Ac}(x_{i-r,j-s})]e^{\mu_{Ac}(x_{i-r,j-s})} - \\ &- [1 + \mu_{Ac}(x_{i-r,j-s})]e^{-\mu_{Ac}(x_{i-r,j-s})} + \\ &+ 2 - [1 - \mu_{Ac}(x_{i-r,j-s}) - \pi_{Ac}(x_{i-r,j-s})]e^{\mu_{Ac}(x_{i-r,j-s}) - \pi_{Ac}(x_{i-r,j-s})} \\ &- [1 + \pi_{Ac}(x_{i-r,j-s}) + \mu_{Ac}(x_{i-r,j-s})]e^{-\pi_{Ac}(x_{i-r,j-s}) - \mu_{Ac}(x_{i-r,j-s})} \end{aligned}$$

being $\mu_{Ac}(x_{i-r,j-s}) = \mu_A(x_{i-r,j-s}) - \mu_A(c_{p-r,q-s})$ and $\pi_{Ac}(x_{i-r,j-s}) = \pi_A(x_{i-r,j-s}) - \pi_A(c_{p-r,q-s})$. Finally, a weighted average is calculated:

$$IFD(i,j) = w_R \times IFD^1(i,j) + w_G \times IFD^2(i,j) + w_B \times IFD^3(i,j) \tag{8}$$

where $w_R + w_G + w_B = 1$.

6: In this step, a decision is taken. If the similarity measure IFD is less or equal than a fixed value, denoted by θ, then the point x_{ij} is conserved; else discarded. The value of θ is heuristically chosen. As a consequence, the segmented leukocytes image is obtained, i.e., for all i and j, $0 \le i \le M, 0 \le j \le N$:

$$\text{If } IFD(i,j) \le \theta \Rightarrow Result^k(i,j) = x^k_{ij} \tag{9}$$

$$\text{If } IFD(i,j) > \theta \Rightarrow Result^k(i,j) = x^k_{ij} \tag{10}$$

where $Result^k(i,j)$ represents the component k of the result image to pixel (i,j), with $k = 1,2,3$. Combining the component, the final image is obtained as follows:

$$Result(i,j) = (Result^1(i,j), Result^2(i,j), Result^3(i,j)). \tag{11}$$

4. Results

In this section, some experiments are conducted to show the results of the application of the proposed method. The images used have been taken from the CellaVision blog [22]. This dataset consists of one hundred 300 × 300 color images. The cell images are generally purple

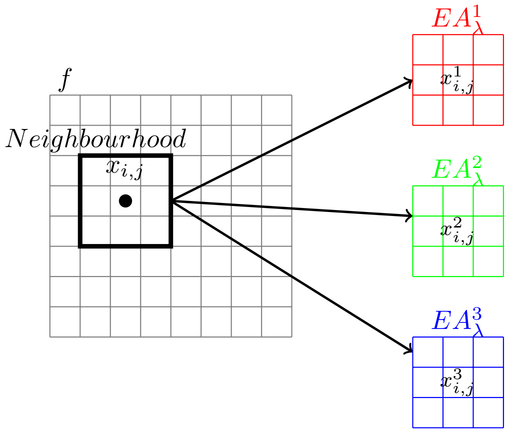

Figure 2: Image decomposition in a neighbourhood.

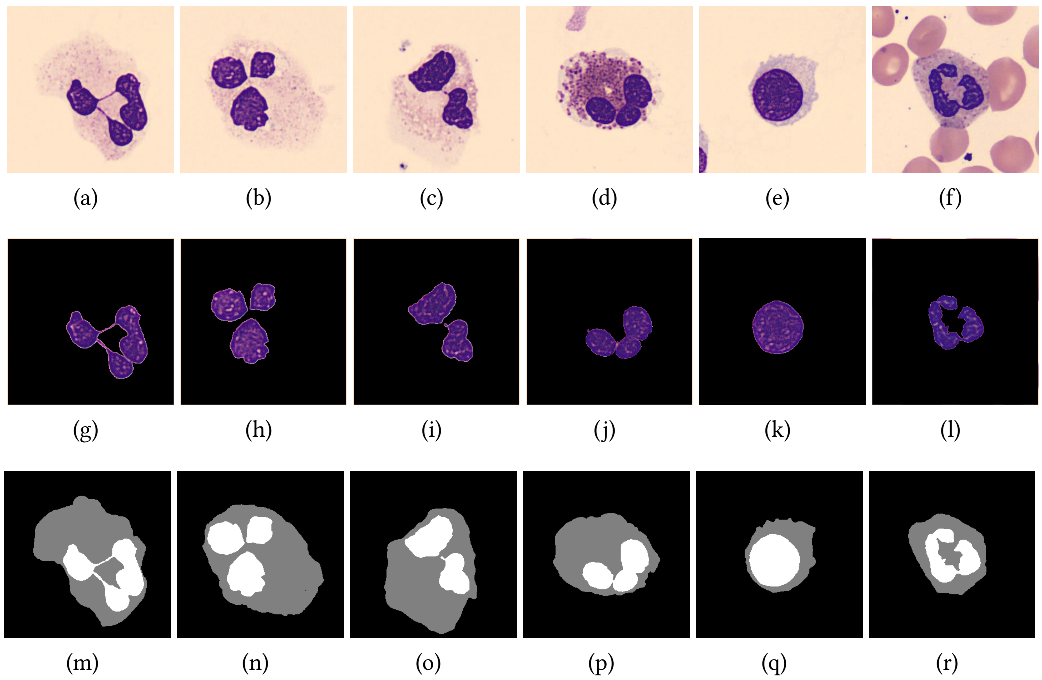

Figure 3: Results of the proposed method. a–f Original Images. g–l Segmentation obtaining by the proposed method. m–r Segmentation given by the experts.

and may contain many red blood cells around the white blood cells. Also, the area related to the nuclei and cytoplasm have been manually segmented by an expert.

As an example, Fig. 4 shows the results of the proposed method applied to six images of the dataset. Original images are shown from (a) to (f), their segmentations are shown from (g) to (l) and finally the expert's segmentation are shown from (m) to (r).

The preliminary results show a good behaviour relative to the resulting color segmentation. This is a first approach to this methodology and some more studies and experiments should be done in order to quantify the segmentation. In particular, for future works we plan to evaluate quantitatively the differences against other methods, in order to validate the proposed algorithm.

5. Conclusions

In this work, it is proposed a new method for color image segmentation of leukocytes based on intuitionistic fuzzy sets. The K-means clustering algorithm is used to select a neighbourhood and then measure the similarity between the image to be analyzed and this neighbourhood. The similarity measure has been used as a tool to segment the objects of interest in an image. As we can see in the results, the performance of the proposed algorithm is high.

As future work, we will validate the proposed method in a more precise way and we will compare our results with other methods developed in the literature. Also, we will study in depth the influence and the values of the parameters λ and θ used in the method.

Acknowledgments

Authors acknowledge financial support by the Spanish Ministry of Science and Technology under Projects TIN-2017-87600-P (Agustina Bouchet and Irene Díaz) and PGC2018-098623-B-I00 (Susana Montes).

References

[1] V. Ananthi, P. Balasubramaniam, A new thresholding technique based on fuzzy set as an application to leukocyte nucleus segmentation, Computer Methods and Programs in Biomedicine 134 (2016) 165–177. doi:https://doi.org/10.1016/j.cmpb.2016.07.002.

[2] S. Arslan, E. Ozyurek, C. Gunduz-Demir, A color and shape based algorithm for segmentation of white blood cells in peripheral blood and bone marrow images 85 (2014) 480–490. doi:https://doi.org/10.1002/cyto.a.22457.

[3] T. Chaira, Intuitionistic fuzzy color clustering of human cell images on different color models, Journal of Intelligent and Fuzzy Systems 23 (2012) 43–51. doi:https://doi.org/10.3233/IFS-2012-0494.

[4] T. Chaira, Accurate segmentation of leukocyte in blood cell images using atanassov's intuitionistic fuzzy and interval type ii fuzzy set theory, Micron 61 (2014) 1–8. doi:https://doi.org/10.1016/j.micron.2014.01.004.

[5] H. Fan, F. Zhang, L. Xi, Z. Li, G. Liu, Y. Xu, Leukocytemask: An automated localization and segmentation method for leukocyte in blood smear images using deep neural networks, J. Biophotonics 12 (2019) e201800488. doi:https://doi.org/10.1002/jbio.201800488.

[6] A. Jati, G. Singh, R. Mukherjee, M. Ghosh, A. Konar, C. Chakraborty, A. K. Nagar, Automatic leukocyte nucleus segmentation by intuitionistic fuzzy divergence based thresholding, Micron 58 (2014) 55–65. doi:https://doi.org/10.1016/j.micron.2013.12.001.

[7] B. Ko, J. Gim, J. Nam, Automatic white blood cell segmentation using stepwise merging rules and gradient vector flow snake, Micron 42 (2011) 695–705. doi:https://doi.org/10.1016/j.micron.2011.03.009.

[8] L. Hasbon Reyes, L. X. Bautista Rozo, F. Rojas Morales, Automatic leukocyte image segmentation: A review, in: 2015 20th Symposium on Signal Processing, Images and Computer Vision (STSIVA), 2015, pp. 1–9. doi:10.1109/STSIVA.2015.7330393.

[9] C. Zhang, X. Xiao, X. Li, Y.-J. Chen, W. Zhen, J. Chang, C. Zheng, Z. Liu, White blood cell segmentation by color-space-based k-means clustering, Sensors (Basel, Switzerland) 14 (2014) 16128 – 16147. doi:https://doi.org/10.3390/s140916128.

[10] A. Bouchet, S. Montes, V. Ballarin, I. Díaz, Intuitionistic fuzzy set and fuzzy mathematical morphology applied to color leukocytes segmentation, Signal, Image and Video Processing 14 (2020) 557–564. doi:https://doi.org/10.1007/s11760-019-01586-2.

[11] I. Bloch, H. Maître, Fuzzy mathematical morphologies: A comparative study, Pattern Recognit. 28 (1995) 1341–1387.

[12] L. Busin, N. Vandenbroucke, L. Macaire, Color spaces and image segmentation, Advances in imaging and electron physics 151 (2008).

[13] G. H. Joblove, D. Greenberg, Color spaces for computer graphics, SIGGRAPH Comput. Graph. 12 (1978) 20–25. doi:10.1145/965139.807362.

[14] J. Angulo, J. Serra, Mathematical morphology in color spaces applied to the analysis of cartographic images, volume 3, 2003, pp. 59–66.

[15] A. Caliman, M. Ivanovici, N. Richard, Probabilistic pseudo-morphology for grayscale and color images, Pattern Recognit. 47 (2014) 721–735.

[16] T. Chaira, A. Ray, A new measure using intuitionistic fuzzy set theory and its application to edge detection, Applied Soft Computing 8 (2008) 919–927. doi:https://doi.org/10.1016/j.asoc.2007.07.004.

[17] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87–96. URL: https://www.sciencedirect.com/science/article/pii/S0165011486800343. doi:https://doi.org/10.1016/S0165-0114(86)80034-3.

[18] P. Couto, P. Melo-pinto, H. Bustince, E. Barrenechea, M. Pagola, Colour image segmentation using a-ifss, in: IFSA-EUSFLAT 2009, 2009.

[19] H. Bustince, J. Kacprzyk, M. V. M. Salillas, Intuitionistic fuzzy generators application to intuitionistic fuzzy complementation, Fuzzy Sets Syst. 114 (2000) 485–504.

[20] M. Sugeno, Fuzzy measures and fuzzy integral: a survey, North Holland, Amsterdam, 1977, p. 89–102.

[21] I. Montes, N. R. Pal, V. Janiš, S. Montes, Divergence measures for intuitionistic fuzzy sets, IEEE Transactions on Fuzzy Systems 23 (2015) 444–456.

[22] X. Zheng, Y. Wang, G. Wang, J. Liu, Fast and robust segmentation of white blood cell images by self-supervised learning, Micron 107 (2018) 55–71. doi:https://doi.org/10.1016/j.micron.2018.01.010.