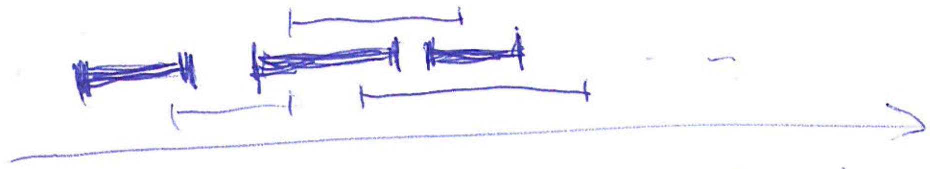
Earliest finish time maximizes # of activities

Recall



Given activities want to find the largest non-overlapping subset.

Eearliest finish time would pick activities as above (note that each time we pick an activity we have to give up on all overlapping activities).

In the first lecture we discussed the problem & the algorithm.

Today we'll prove correctness.

Lemma At every step there is an optimal solution that picks all the activities the algorithm picked

Pf By induction on the number of activities the algorithm picks.

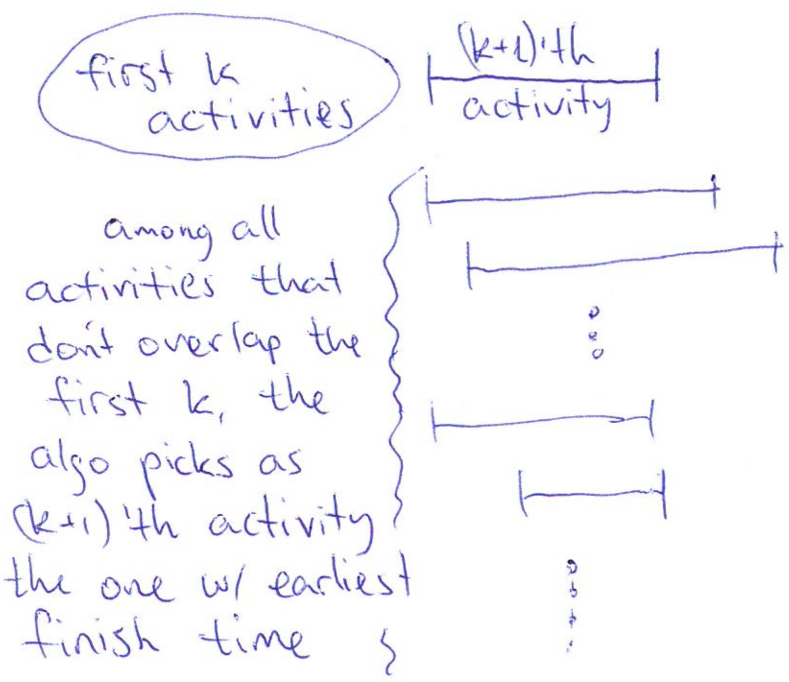
Base 0 activities $\rightarrow$ there exists an optimal solution and it satisfies the claim.

Hypothesis After the algo picked k activities there is an optimal solution Opt that contains the activities

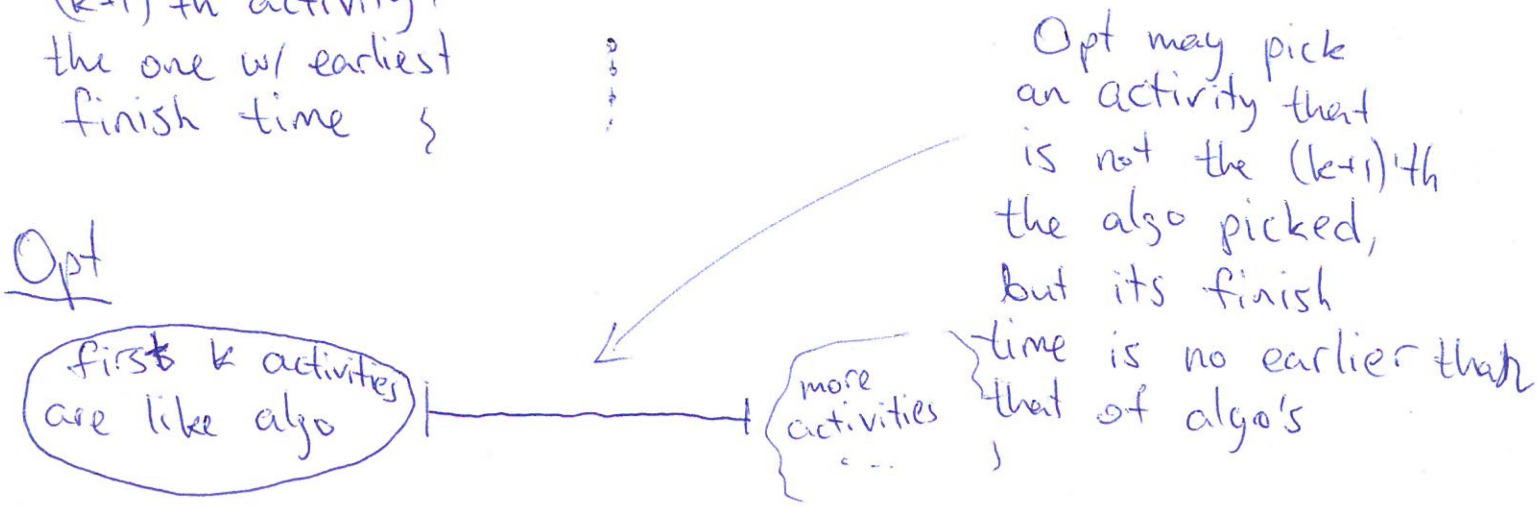
We'll show that:

<u>Step</u> After the algo picked the $(k+1)$'th activity there is still an optimal solution Opt' that picks all $k+1$ activities.

<u>Algorithm</u>



<u>Opt</u>



<u>Opt'</u> exchange the $(k+1)$'th ~~alg~~ activity that Opt picked w/ $(k+1)$'th activity that algo picked. Note:

* Opt' feasible.
* Opt' has as many activities as Opt.

Therefore, lemma holds. ∎