RANDOM VARIABLE - fxn outcomes → real #s

ex: randomized experiment - rolling 6 sided die

$$X = \begin{cases} 1 & \text{if die is } 1 \\ \vdots \\ 6 & \text{if die is } 6 \end{cases}$$

$$Y = \begin{cases} 1 & \text{if die is even} \\ 0 & \text{if die is odd} \end{cases}$$

experiments:

1. N students randomly assign each student a birthday
   X_i = i^{th} student's birthday

2. N students, each one turns in a homework, randomly shuffle

$$X_i = \begin{cases} 1 & \text{if student } i \text{ gets his/her homework back} \\ 0 & \text{otherwise} \end{cases}$$

$E[x]$ - expectation of x

$$\sum_{e \in \text{outcome}} P_r(e)\, X(e)$$

↑ probability of getting e.

average value of X w.r.t. P_r

Q: 10 students, run experiement #2 how many students are expected to get their own homework

$E[X+Y] = E[X] + E[Y]$ ← linearity of expectation

A: $E[X_1 + X_2 + X_3 \ldots] = E[X_1] + E[X_2] + \ldots$

$= n\,E[X_1] = n \cdot \frac{1}{n} = 1$ (symmetry)

Prob. student 1 gets his homework back = 1

10 students expected # of pairs of students w/ the same birthday?

$X_{ij} = \begin{cases} 1 & \text{if student } i \text{ and student } j \text{ have the same birthday} \\ 0 & \text{otherwise} \end{cases}$

$$E\left[\sum_{\substack{i,j \\ i \neq j}} X_{ij}\right] = \sum_{\substack{i,j \\ i \neq j}} E[X_{ij}] \quad \text{(linearization)}$$

$$= \binom{n}{2} E[X_{12}] \quad \text{(symmetry)}$$

$$= \frac{n(n-1)}{2} \frac{365}{(365)^2} = \frac{n(n-1)}{365 \cdot 2}$$

for $n = 10$ 0.12 expected pairs

needle of length 1

(lined) paper with lines of 1 inch

Probability [needle intersects a line]

What is the prob that # of pairs w/ the same birthday is 'close' to the distribution?

Markov's inequality

$$P(X > t \cdot E[X]) \leq \frac{1}{t} \quad \text{or} \quad P(X \geq t) \leq \frac{E[X]}{t}$$

Prob (# of pairs of students > 3)

Prob (# of pairs of students > 0.12 · 25) < $\frac{1}{25}$

Variance ;

$$var(X) = E\left((X - E[x])^2 \right)$$

expected deviation of x from it's expected value

$$Var(X) = E[\,X^2 - 2X\,E[x] + (E[x])^2\,]$$
$$= E[x^2] - 2\,E[x]\cdot E[x] + (E[x])^2$$
$$= E[x^2] - \cancel{2}(E[x])^2$$

CHEBYSHEV inequality

$$Pr[\,|X - E[x]| > t\,] < \frac{VAR(x)}{t^2}$$

markov's inequality

$$P(X > t) < \frac{E[x]}{t}$$

take $x' = (X - E[x])^2$

$t' = t^2$

$$P(X' > t') < \frac{E[x']}{t'}$$

$$P\left((X - E[x])^2 > t^2 \right) < \frac{E[(x - E[x])^2]}{t^2}$$

$$P(\,|X - E[x]| > t\,) < \frac{E[(x - E[x])^2]}{t^2}$$

$$P(\,|X - E[x]| > t) < \frac{var(x)}{t^2}$$

MAX CUT NP - COMPLETE

randomized algorithm to "approximate" max cut
output a cut of size $\geq \frac{1}{2}$ max

for each vertex v let
v be in S with prob $\frac{1}{2}$

consider E to be the set of edges ~~in the~~ spanning
the optimal max cut

for every $e \in E$ let $X_e = \begin{cases} 1 & \text{if } S, S^c \text{ cuts } e \\ 0 & \text{otherwise} \end{cases}$

$$E_S[\text{cut}(S, S^c)]$$
$$> E_S\left[\sum_{e \in E} X_e\right] = \sum_{e \in E} E[X_e] = \frac{|E|}{2}$$