Problem Set 3

This problem set is due at **10:00 am** on **Tuesday, February 14th**.

Problem 3- 1: Updating MST after edge addition

Given an undirected, connected, weighted graph $G = (V, E)$, a minimum spanning tree T of G, and $e \in V \times V - E$, we would like to find a minimum spanning tree of the graph $G' = (V, E \cup \{e\})$. Show an algorithm for this task that is as efficient as possible. Analyze the run-time of the algorithm and prove its correctness.

Problem 3- 2: Shortest paths and spanning trees

- Prove or disprove. Given an undirected, connected, weighted graph, if you run an MST algorithm on the graph, then the weight of the shortest path between any two vertices is the weight of the unique path between them in the tree.
- Prove or disprove. Let $G = (V, E)$ be an undirected, connected, weighted graph. Suppose we run Dijkstra's algorithm on G starting a vertex $u \in V$. For every vertex $v \in V$ let p_v denote the edges of the shortest path from u to v that is found by Dijkstra. Let $E' = \cup_{v \in V} p_v$.
  (a) (V, E') is a spanning tree of G;
  (b) (V, E') is a minimum spanning tree of G.

Problem 3- 3: Median finding

You are given two *sorted* lists of numbers $l_1 = [a_1, \ldots, a_n], l_2 = [b_1, \ldots, b_m]$. Describe a divide and conquer algorithm that is as efficient as possible to find the median of the numbers $a_1, \ldots, a_n, b_1, \ldots, b_m$. Write down your algorithm and prove that it is correct. Find the recurrence relation for the runtime of the algorithm and solve it.