Problem Set 7

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Problem 7-1: Reductions

Assuming that a variable x_i occurs in k clauses, then for each occurence of x_i, we can substitute with a variable $x_{i,j}$ where $1 \leq j \leq k$ and adding the following clause to the formula: $(x_{i,1} \vee x'_{i,2}) \wedge (x_{i,2} \vee x'_{i,3}) \ldots \wedge (x_{i,k} \vee x'_{i,1})$. Note that $x_i \Rightarrow x_{i,1} \Rightarrow x_{i,2} \Rightarrow \ldots x_{i,k}$. Thus when x_i is true, the value of this additional clause is true. When x_i is false, the value of this clause is still true. Thus, the value of the original formula is not impacted.

The Reduction Algorithm is as follows:

Algorithm

Algorithm 1 3SAT to SAT(10)

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1:  procedure 3SAT TO SAT(10)(φ)   ▷ φ is the boolean formula in which each variable appears in at most 10 clauses
2:      for each variable x_i in φ do
3:          if the count is ≤ 10 then
4:              do nothing
5:          else
6:              for each instance of variable x_i > 10 do
7:                  Let n be number of instances of x_i
8:                  k ← n − 10
9:                  We replace each instance of x_i after 10 by x_{i,j} where 1 ≤ j ≤ k
10:                 Add the clause (x_{i,1} ∨ x'_{i,2}) ∧ (x_{i,2} ∨ x'_{i,3}) ... ∧ (x_{i,k} ∨ x'_{i,1})
11:                 this reduces to true
12:             end for
13:         end if
14:     end for
15: end procedure
```