Problem Set 9

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Problem 9-1: Vertex Cover

Algorithm

Algorithm 1 Vertex Cover

procedure VERTEX COVER($G = (V, E)$)
  $C \leftarrow \{\}$
  $M \leftarrow \{\}$
  $E' \leftarrow E$
  while $E' \neq \phi$ **do**
    Let $(u_1, \ldots, u_k)$ be an arbitrary edge e of E'
    $C \leftarrow C \bigcup \{u_1, \ldots, u_k\}$
    $M \leftarrow M \bigcup \{e\}$
    Remove from E' any edge incident on any of the vertices $u_1, \ldots, u_k$
  end while
  return C
end procedure

Complexity

The complexity of this algorithm is $O(n + m)$ where $n = |V|$ and $m = |E|$. In the worst case scenario, you iterate through all the edges and all the vertices connected to a specific edge. Thus the complexity is $O(n + m)$.

Proof

Lemma 0.1. *If $C*$ is the optimal cover of G, then the minimal approximate cover C exists such that $|C| \leq k \times C*$.*

Proof. $C*$ has an endpoint of each edge in M. In addition, by the constuction of M, we know that the endpoint of the edges of M are precisely the set C. Thus, $|C*| \geq |M| \Rightarrow |C| \leq k \times |M| < k \times |C*|$. $\square$