Problem Set 2

This problem set is due at **10:00 am** on **Tuesday, February 7th**.

Problem 2- 1: Pedantic Sandy

Sandy would like to travel on a graph $G = (V, E, w : E \to \mathbb{R}^{\geq 0})$, i.e. a graph with vertex set V, edge set E and non-negative edge weights, which are determined by w. She starts with her left foot on the first vertex. She can only place one foot on any given vertex and alternates feet (i.e. if she starts with a left foot on the first vertex, the next vertex she's on will have her right foot on it). Sandy wants to know what the shortest path to every vertex is, such that the foot which ends on the final vertex is the left foot. Write an algorithm to help her do this. (HINT: Consider the graph G' with vertex set $V' = V \times \{\textit{left}\} \cup V \times \{\textit{right}\}$ edge set $E' = \{((u, \textit{left}), (v, \textit{right}))|(u, v) \in E\} \cup \{((u, \textit{right}), (v, \textit{left}))|(u, v) \in E\}$ and weight function $w'((u, \cdot), (v, \cdot)) = w((u, v))$. Use G' to solve the problem)
Write a paragraph or a sentence summarizing the problem you are solving and what your results are, then describe your algorithm in English and, if helpful, pseudocode. Prove your algorithm is correct and analyze the running time of the algorithm.

Problem 2- 2: OCD

You have to store L gallons of oil where L is a natural number. You own a factory that can make as many 1-gallon, 2-gallon, 2^2-gallon, ..., 2^{1000}-gallon containers as you would like. However, you would like to store the L gallons of oil in as few different containers as possible while ensuring that every container you store the oil in is full. Consider the following algorithm -

Input: L
Result: Smallest number of containers you can fill L gallons of oil in, while ensuring that no container has any space left and the list the capacities of these containers
$L' = L$;
$count = 0$;
$Arr = []$;
while $L' > 0$ **do**
  $A = \max\{2^i \leq L' | i \in [1000]\}$;
  $count = count + 1$;
  $Arr = Arr.append(A)$;
  $L' = L' - A$;
end
Output: $(count, Arr)$

Does this algorithm terminate and give you a correct answer? If yes, write a proof. If no, give a counterexample.