Problem Set 5

This problem set is due at **10:00 am** on **Tuesday, March 21st**.

Problem 5- 1: Job assignment

You are given a matrix M such that $M_{(worker,\ task)}$ = amount of time a worker can dedicate to the given task, and a list T where $T[task]$ = amount of time required to complete the task. Multiple workers can be assigned to the same task and each worker can work for a maximum of 10 hours. Decide by reducing to max-flow, if there exists an assignment of workers to tasks such that every task gets completed. Prove that your reduction is correct.

Problem 5- 2: Broken Keys

You are given an old, worn out keyboard and you would like to calculate the number of keystrokes you have to perform to change one string to another. Because the keyboard is worn out, some keys need to be pressed multiple times for the characters to be printed. Let $N[x]$ denote the number of times the character x needs to be pressed for x to be printed. Write a dynamic program to calculate the minimum number of keystrokes needed to change string1 to string2 given *N*, string1 and string2. You are allowed to delete and enter characters (Note that it might take many keystrokes to enter a character!). Changing the position of the cursor does not cost anything (imagine you can do this with a mouse). Calculate the runtime and prove the correctness of your algorithm.

Problem 5- 3: Different paths

Let $G = (V, E)$ be a graph with nonnegative edge weights. Are the shortest paths found by Bellman-Ford the same as the ones found by Dijkstra's algorithm? If yes, prove your result. If no, give a counterexample and specify the different paths found by the algorithms. Assume that there exists an order on E and among the choices made by both algorithms, the edges are picked according to this order. This means that during the course of a run, if Dijkstra / Bellman-Ford have a choice between edges to be picked, they will pick these edges in the prespecified order.