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1 Problem 2-2: OCD

Lemma 1.1. *Given the previous algorithm, the result will always be the smallest number of containers you can fill L gallons of oil in.*

Proof. By Induction: A natural number L, where $L \geq 1$.
Base Case: $L = 1$
Given the algorithm, a 1 gallon container will be used. The base case olds true that the smallest number of containers you can fill L gallons of oil in since only one container is used.
Induction Hypotheses: Given k gallons of oil, there exists a method to store the k gallons of oil in as few different containers as possible while ensuring that every container you store the oil in is full.
Induction Step: For $k + 1$ gallons of oil, we want to show that there also exists a method to store the $k + 1$ gallons of oil in as few different containers as possible while ensuring that every container you store the oil in is full.
It is possible to find such an i where i is between 0 to 1000 where $2^i \leq k + 1$.
$Z = k + 1 - 2^i$
Z is greater than or equal to 0 and it is less than or equal to k.
By Induction Hypotheses , there exists a method to store the Z gallons of oil in as few different containers as possible while ensuring that every container you store the oil in is full. Thus, we conclude the lemma is true. □

Lemma 1.2. *Given the previous algorithm, this algorithm finds the optimal solution.*

Proof. Assume there is an optimal solution θ.
By Induction: A natural number L, where $L \leq 1$, the solution returned by the previous algorithm is the same as θ.
Base Case: $L = 1$
The previous algorithm will therefore return 1 container of size 1-gallon.
θ will also return 1 container of size 1-gallon since that is the optimal solution.
Induction Hypotheses: Given k gallons of oil, the previous algorithm and θ will return the same count and sizes of containers to store the k gallons of oil.
Induction Step: For $k + 1$ gallons of oil, we want to show that the previous algorithm and θ will return the same count and sizes of containers to store the $k + 1$ gallons of oil.
It is possible to find such an i where i is between 0 to 1000 where $2^i \leq k + 1$.
$Z = k + 1 - 2^i$
Z is greater than or equal to 0 and it is less than or equal to k.
Both θ and the previous algorithm will choose the 2^i container because that is the maximum container that can be chosen given the rule that holds the

most oil and contains no empty space. Since $Z \geq 0$ and $Z \leq k$, both θ and the previous algorithm will choose the same sequence of containers by the Induction Hypotheses. Thus, we conclude that the lemma is true. $\square$