Problem Set 4

This problem set is due at **10:00 am** on **Tuesday, March 7th**.

Problem 4- 1: Longest path

Given a directed acyclic graph (DAG) G give a dynamic programming algorithm to find the longest path in this graph. Prove that your algorithm is correct and analyze its runtime. (Hint: First topologically sort the DAG)

Problem 4- 2: OCD-2

Recall the problem 'OCD' from PS-2 where you had to store L gallons of oil in containers of capacities $1, 2, \ldots, 2^{1000}$. This time you need to store $L \geq 3$ gallons of oil and you are given access to a factory that can only make containers of capacities $1, 3, 4$. Once again, you would like to store the L gallons of oil in as few different containers as possible while ensuring that every container you store the oil in is full.

(a) Show that you cannot use the greedy algorithm from PS-2 to solve this problem. Give an L such that the greedy algorithm does not find the optimal number of containers.

(b) Give a dynamic programming algorithm to solve this problem. Prove that your algorithm is correct and analyze its runtime.