Sample problems - Test 2 [1]

1. A commander is located at one node p in a network. His subordinates constitute a node set S. The enemy needs to cut off the communication between the commander and his subordinates (the commander should not be able to communicate to any of his subordinates). Enemy needs $w(e)$ effort to remove an edge e in the network. Compute the minimum effort required to cut off the communication between the commander and his subordinates.

2. Prove or disprove (give counter examples) for the following:

   (a) For any maximum flow allocation, for all pairs (u, v) either flow in edge (u, v) or flow in (v, u) must be 0.

   (b) There exists one maximum flow for which for all pairs (u, v) either flow in edge (u, v) or flow in (v, u) is 0.

   (c) If all edges have unique capacities, the network has a unique minimum cut.

   (d) If we add a positive number b to the capacity of every edge, the minimum cut remains the same.

3. Consider a network with a source and a destination. An edge is called upward critical if increasing the capacity of an edge increases the maximum flow. Does all networks have an upward critical edge? An edge is called downward critical if decreasing the capacity of the edge decreases the maximum flow. Does all networks have a downward critical edge? Justify your answer in both cases.

4. There are M faculties and N courses. Every faculty ranks 2 courses in order of preference. A faculty can teach one course and a course can be taught by one faculty only. Find a feasible course allocation if one exists (A feasible course allocation allows a faculty to teach one of the two courses he prefers). Find a k-feasible course allocation if one exists. (A faculty is dissatisfied if he is allotted his second choice course. A k-feasible allocation is one which dissatisfies at most k faculties).

5. Consider a row of n coins of values $v_1, \ldots, v_n$, where n is even. We play a game against an opponent by alternating turns. In each turn, a player selects either the first or last coin from the row, removes it from the row permanently, and receives the value of the coin. Use dynamic programming to determine the maximum possible amount of money we can definitely win if we move first.

6. You are given n types of coin denominations of values $1 = v_1 < v_2 < \cdots < v_n$. Give a dynamic programming algorithm which makes change for an amount of money C with as few coins as possible.

7. You have a set of n integers each in $0, 1, 2, \ldots, K$ Partition these integers into two subsets such that you minimize $|S_1 - S_2|$, where S_1 and S_2 denote the sums of the elements in each of the two subsets.

[1] Problems chosen from collections of problems by Prof. Sarkar from UPenn and Prof. Dean from Clemenson University.