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# 1 Problem 1-1: Growth

Sort the following functions so $f$ appears before $g$ if $f = O(g)$:
$n^{0.99}$, $\log_{1.1} n$, $10^{1249}$, $(\log_2 n)^2$, $2^{(\ln\ln n)^2}$, $10^n$, $\ln\ln n$, $2^{n^2}$, $(\log_{10} n)^n$, $1000n + 10^{10}$.
Provide a one line explanation for each pair of consecutive functions in the sorted list.

## 1.1 (a)

The first function given is $n^{0.99}$. This function is slightly less than linear since linear is just $n^1$ or just $n$. Therefore, I will use $n^{0.99}$ as my base value and put in first on my sorted list.

The current sorted list is:
$n^{0.99}$

## 1.2 (b)

The second function given is $\log_{1.1} n$. We need to change this function to base 2.
$\log_{1.1} n = \frac{\log_2 n}{\log_2 1.1} = 7.27\, log_2 n \equiv O(\log_2 n)$
Since logarithmic complexity is less than linear complexity in our sorted list, this function will become the first function in the sorted list.

The current sorted list is:
$\log_{1.1} n$, $n^{0.99}$

## 1.3 (c)

The third function given is $10^{1249}$. Although this function may seem large, it is in fact constant since there are no variables.
For values of $n$, $10^{1249} < \log_{1.1} n$. Thus for $n > 1.1^{10^{1249}}$, the logarithmic function $\log_{1.1} n$ is less than $10^{1249}$.
Thus, we can immediately determine that this will be the first value in the sorted list since constant complexity is less than logarithmic and linear complexity.

The current sorted list is:

$10^{1249}$, $\log_{1.1} n$, $n^{0.99}$

## 1.4 (d)

The fourth function given is $(\log_2 n)^2$. Taking log of $(\log_2 n)^2$, we get $2\log_2(\log_2 n)$. As we determined in subsection b, $\log_{1.1} n = 7.27\log_2 n$. Taking the log of $7.27\log_2 n$, we get $\log_2 7.27 + \log_2(\log_2 n)$.
Thus, $(\log_2 n)^2$ has a lightly larger complexity than $\log_{1.1} n$ in constant factors.

The current sorted list is:
$10^{1249}$, $\log_{1.1} n$, $(\log_2 n)^2$, $n^{0.99}$

## 1.5 (e)

The fifth function given is $2^{(\ln \ln n)^2}$. Taking log of $2^{(\ln \ln n)^2}$, we get $(\ln \ln n)^2$. Since we can't determine it's place from this, we take another log and get $2 \ln \ln \ln n$. Taking the log of log of $(\log_2 n)^2$, we get $1 + \log \log \log n$. Thus, $2^{(\ln \ln n)^2}$ has a lightly larger complexity based on constant factors than $(\log_2 n)^2$.

The current sorted list is:
$10^{1249}$, $\log_{1.1} n$, $(\log_2 n)^2$, $2^{(\ln \ln n)^2}$, $n^{0.99}$

## 1.6 (f)

The sixth function given is $10^n$. This function is exponential, which is greater than the linear function $n^{0.99}$.

The current sorted list is:
$10^{1249}$, $\log_{1.1} n$, $(\log_2 n)^2$, $2^{(\ln \ln n)^2}$, $n^{0.99}$, $10^n$

## 1.7 (g)

The seventh function given is $\ln \ln n$. Taking $\ln \ln n$ exponentially as a power of 2, we get $\ln n$. Similarly, taking $\log_{1.1} n$ exponentially as a power of 2, we get $2^{7.27}n$. Thus, $\ln \ln n$ has a lower complexity than $\log_{1.1} n$.

The current sorted list is:
$10^{1249}$, $\ln \ln n$, $\log_{1.1} n$, $(\log_2 n)^2$, $2^{(\ln \ln n)^2}$, $n^{0.99}$, $10^n$

## 1.8 (h)

The eighth function given is $2^{n^2}$. Taking the log of $2^{n^2}$, we get $n^2$. Similarly, taking the log of $10^n$, we get $n \log_2 10$. Since $n^2 > n \log_2 10$ when $n > \log_2 10$, we can prove that $2^{n^2}$ has a higher complexity than $10^n$.

The current sorted list is:
$10^{1249}$, $\ln \ln n$, $\log_{1.1} n$, $(\log_2 n)^2$, $2^{(\ln \ln n)^2}$, $n^{0.99}$, $10^n$, $2^{n^2}$

## 1.9 (i)

The ninth function given is $(\log_{10} n)^n$. Taking the log of base 10 of $(\log_{10} n)^n$, we get $n \log_{10} \log_{10} n$. Taking the log of base 10 of $10^n$, we get $n$. Since $n \log_{10} \log_{10} n > n$, we can prove that $(\log_{10} n)^n$ has a higher complexity than $10^n$. Taking the log of base 10 of $2^{n^2}$, we get $n^2 \log_2 n$. A quadratic complexity is greater than a linearithmic complexity. Therefore, $(\log_{10} n)^n$ has a lower complexity than $2^{n^2}$.

The current sorted list is:
$10^{1249}$, $\ln \ln n$, $\log_{1.1} n$, $(\log_2 n)^2$, $2^{(\ln \ln n)^2}$, $n^{0.99}$, $10^n$, $(\log_{10} n)^n$ , $2^{n^2}$

## 1.10 (j)

The last function given is $1000n + 10^{10}$. This function is linear. This has a higher complexity in terms of constant factor than $n^{0.99}$, which is slightly less than linear.

The current sorted list is:
$10^{1249}$, $\ln \ln n$, $\log_{1.1} n$, $(\log_2 n)^2$, $2^{(\ln \ln n)^2}$, $n^{0.99}$, $1000n + 10^{10}$, $10^n$, $(\log_{10} n)^n$ , $2^{n^2}$