Review Session

exam: 3 questions

- DP / Greedy / Flow
  - Max flow alg
  - Reductions to flow
  - DP
  - Greedy

P1:

M families,

Family i has $a(i)$ members

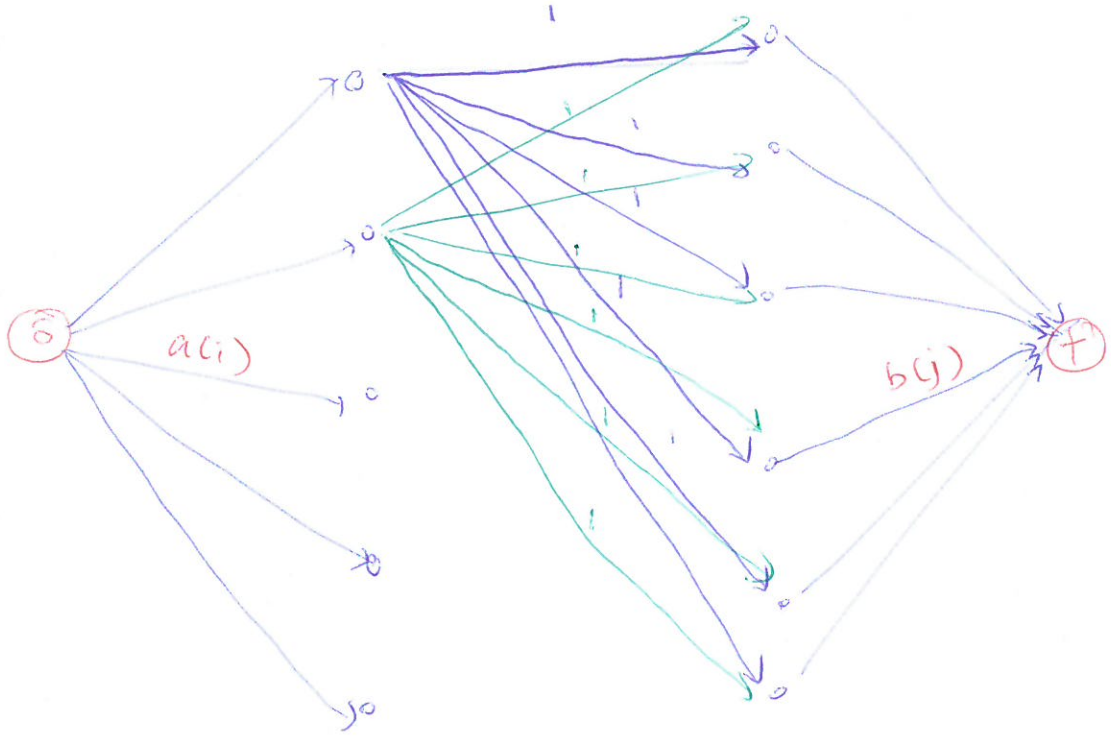
- No two members of the same family sit at the same table.

- There are N tables.

Table j can seat $b(j)$ people

find an assignment of family members to tables if such exists

① define a flow network s.t. there exists a valid assignment, then you can get this assignment from the max flow.



Claim: if max flow $= \sum_{i \in F} a(i)$

Then the assignment of family members to tables is given by the edges with flow 1 across the bipartite graph $G = (F \cup T, E_{FT})$

Pf:

assignment is valid if

① everyone is assigned

② no table is assigned more than its capacity

③ No two members of the same family have the same flow

① True b/c max flow $= \sum_{i \in F} a(i)$

② capacity of edges going into t (sink) corresponds to table capacities

③ due to capacities of E_{FT} and nonexistence of parallel edges

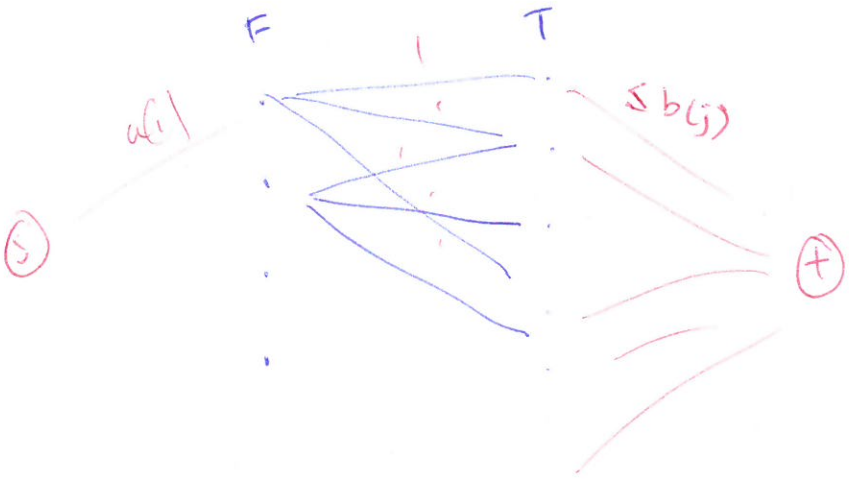
Claim: If a valid seating exists then there is a flow of value $\sum_{i \in F} a(i)$

pf: $S(1): \{1, 2, 3\}$

S: Families $\longrightarrow \{\{\text{tables}\}\}$

no two members sit at the same table.
& table i has less than $b(i)$ people seated at it

can construct

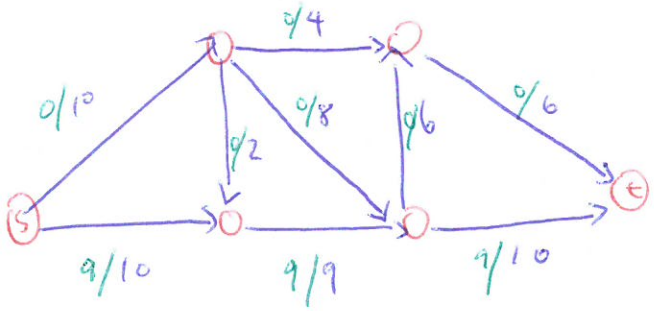


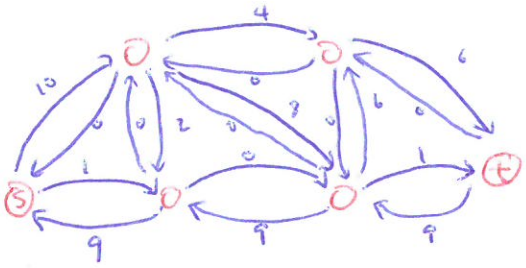
P 2

You are given a flow network s.t. some of the capacities are ∞ but the value of flow is finite, replace all the ∞ capacity edges by finite capacity edges st the max flow value is the same

- replace w/ max of sum of edges going out of s, sum of edges going into T.
  ignores ∞ edges

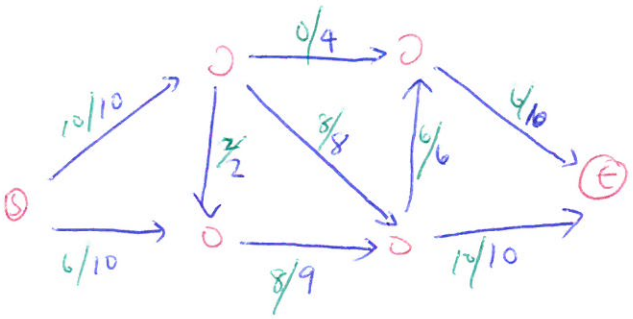
Ford Fulkerson on residual

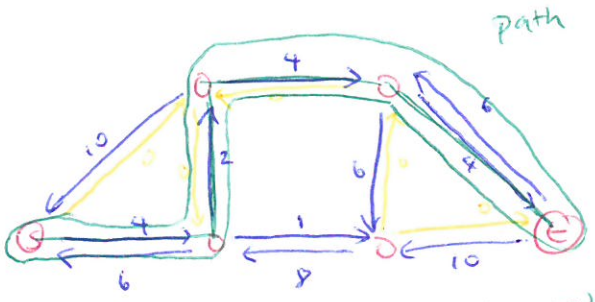




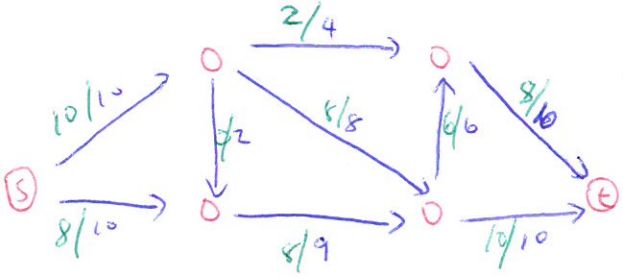
① if there exists a path in the residual graph send flow along the path to increase the flow

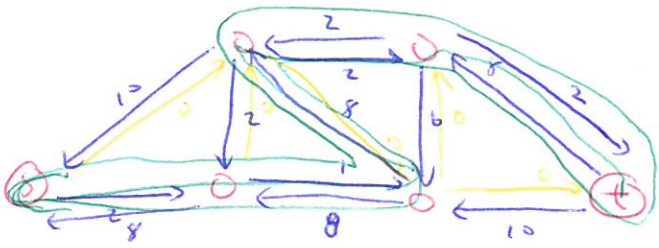
↑ send the bottleneck capacity



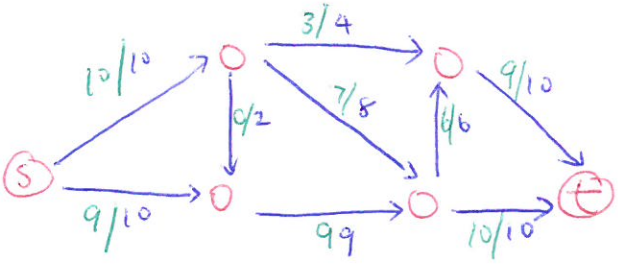


max flow through path -> 2

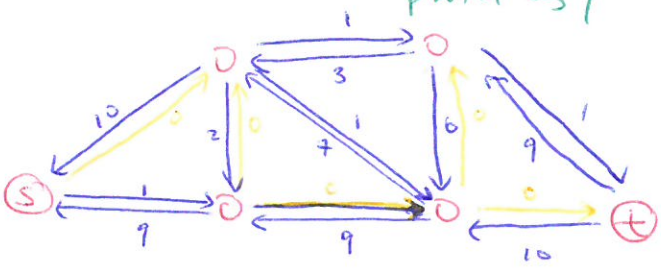




max flow through path -> 1



↑
max flow



no paths!

$$O(|f^*| \cdot |V| + |E|))$$