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1   Problem 1-1: Growth

Sort the following functions so f appears before g if $f = O(g)$:
$n^{0.99}$, $\log_{1.1} n$, 10^{1249}, $(\log_2 n)^2$, $2^{(\ln \ln n)^2}$, 10^n, $\ln \ln n$, 2^{n^2}, $(\log_{10} n)^n$,
$1000n + 10^{10}$.
Provide a one line explanation for each pair of consecutive functions in the sorted list.

1.1   (a)

The first function given is $n^{0.99}$. This function is slightly less than linear since linear is just n^1 or just n. Therefore, I will use $n^{0.99}$ as my base value and put in first on my sorted list.

The current sorted list is:
$n^{0.99}$

1.2   (b)

The second function given is $\log_{1.1} n$. We need to change this function to base 2.
$\log_{1.1} n = \frac{\log_2 n}{\log_2 1.1} = 7.27 \, log_2 n \equiv O(\log_2 n)$
Since logarithmic complexity is less than linear complexity in our sorted list, this function will become the first function in the sorted list.

The current sorted list is:
$\log_{1.1} n$, $n^{0.99}$

1.3   (c)

The third function given is 10^{1249}. Although this function may seem large, it is in fact constant since there are no variables.
For values of n, $10^{1249} < \log_{1.1} n$. Thus for $n > 1.1^{10^{1249}}$, the logarithmic function $\log_{1.1} n$ is less than 10^{1249}.
Thus, we can immediately determine that this will be the first value in the sorted list since constant complexity is less than logarithmic and linear complexity.

The current sorted list is:

10^{1249}, $\log_{1.1} n$, $n^{0.99}$

1.4 (d)

The fourth function given is $(\log_2 n)^2$. Taking log of $(\log_2 n)^2$, we get $2 \log_2 (\log_2 n)$. As we determined in subsection b, $\log_{1.1} n = 7.27 \log_2 n$. Taking the log of $7.27 \log_2 n$, we get $\log_2 7.27 + \log_2 (\log_2 n)$.
Thus, $(\log_2 n)^2$ has a lightly larger complexity than $\log_{1.1} n$ in constant factors.

The current sorted list is:
10^{1249}, $\log_{1.1} n$, $(\log_2 n)^2$, $n^{0.99}$

1.5 (e)

The fifth function given is $2^{(\ln \ln n)^2}$. Taking log of $2^{(\ln \ln n)^2}$, we get $(\ln \ln n)^2$. Since we can't determine it's place from this, we take another log and get $2 \ln \ln \ln n$. Taking the log of log of $(\log_2 n)^2$, we get $1 + \log \log \log n$. Thus, $2^{(\ln \ln n)^2}$ has a lightly larger complexity based on constant factors than $(\log_2 n)^2$.

The current sorted list is:
10^{1249}, $\log_{1.1} n$, $(\log_2 n)^2$, $2^{(\ln \ln n)^2}$, $n^{0.99}$

1.6 (f)

The sixth function given is 10^n. This function is exponential, which is greater than the linear function $n^{0.99}$.

The current sorted list is:
10^{1249}, $\log_{1.1} n$, $(\log_2 n)^2$, $2^{(\ln \ln n)^2}$, $n^{0.99}$, 10^n

1.7 (g)

The seventh function given is $\ln \ln n$. Taking $\ln \ln n$ exponentially as a power of 2, we get $\ln n$. Similarly, taking $\log_{1.1} n$ exponentially as a power of 2, we get $2^{7.27} n$. Thus, $\ln \ln n$ has a lower complexity than $\log_{1.1} n$.

The current sorted list is:
10^{1249}, $\ln \ln n$, $\log_{1.1} n$, $(\log_2 n)^2$, $2^{(\ln \ln n)^2}$, $n^{0.99}$, 10^n

1.8 (h)

The eighth function given is 2^{n^2}. Taking the log of 2^{n^2}, we get n^2. Similarly, taking the log of 10^n, we get $n \log_2 10$. Since $n^2 > n \log_2 10$ when $n > \log_2 10$, we can prove that 2^{n^2} has a higher complexity than 10^n.

The current sorted list is:
10^{1249}, $\ln \ln n$, $\log_{1.1} n$, $(\log_2 n)^2$, $2^{(\ln \ln n)^2}$, $n^{0.99}$, 10^n, 2^{n^2}

1.9 (i)

The ninth function given is $(\log_{10} n)^n$. Taking the log of base 10 of $(\log_{10} n)^n$, we get $n \log_{10} \log_{10} n$. Taking the log of base 10 of 10^n, we get n. Since $n \log_{10} \log_{10} n > n$, we can prove that $(\log_{10} n)^n$ has a higher complexity than 10^n. Taking the log of base 10 of 2^{n^2}, we get $n^2 \log_2 n$. A quadratic complexity is greater than a linearithmic complexity. Therefore, $(\log_{10} n)^n$ has a lower complexity than 2^{n^2}.

The current sorted list is:
10^{1249}, $\ln \ln n$, $\log_{1.1} n$, $(\log_2 n)^2$, $2^{(\ln \ln n)^2}$, $n^{0.99}$, 10^n, $(\log_{10} n)^n$, 2^{n^2}

1.10 (j)

The last function given is $1000n + 10^{10}$. This function is linear. This has a higher complexity in terms of constant factor than $n^{0.99}$, which is slightly less than linear.

The current sorted list is:
10^{1249}, $\ln \ln n$, $\log_{1.1} n$, $(\log_2 n)^2$, $2^{(\ln \ln n)^2}$, $n^{0.99}$, $1000n + 10^{10}$, 10^n, $(\log_{10} n)^n$, 2^{n^2}