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Collaborators: none

1 Problem 1-2: A New Order

Let G be an undirected graph on N vertices where each vertex has degree at most 2.

1.1 (a)

Suppose that we perform a BFS of G. Let $v_1, \ldots, v_N$ be the vertices of G in the order they are visited in the search. Prove or disprove: every edge in G is of the form (v_i, v_{i+1}) or (vi, v_{i+2}) for some i.

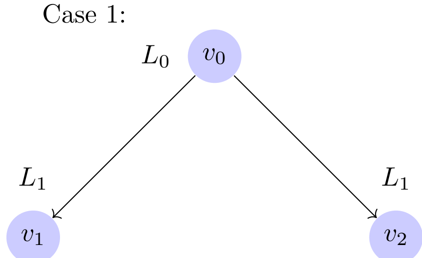
Lemma 1.1. *If G be an undirected graph on N vertices where each vertex has degree at most 2, then every layer L_i in a BFS has at most 2 vertices.*

Proof. **Proof By Induction:** On i , L_i = { number of vertices in layer L_i at distance i}

Base Case: $i = 0$
$i = 0$. $L_0 = \{v_0\}, v_0$ only vertex in layer L_0. This base case holds because there is only one vertex in L_0

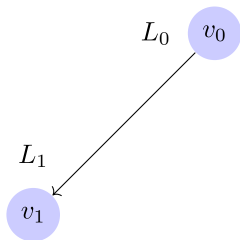
Base Case: $i = 1$

Case 1:



This case holds true because v_0 has a degree of 2 and layer L_1 has 2 vertices.

Case 2:

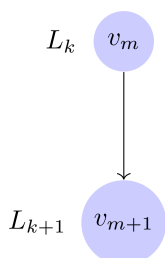


This case holds true because v_0 has a degree of 1 and layer L_1 has 1 vertices.

Induction Hypotheses:
$\forall\ i \leq k \quad L_i = \{\text{Has at most 2 vertices}\}$

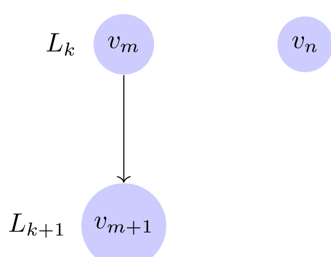
Induction Step:
For $k+1$ we want to show $L_{k+1} = \{$ Has at most 2 vertices$\}$

Case 1:
Layer L_k has 1 vertex and it has 1 child.



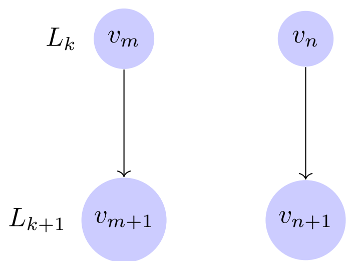
Layer L_{k+1} has one vertex, and thus Case 1 holds true.

Case 2:
Layer L_k has 2 vertices and only one vertex has 1 child.



Layer L_{k+1} has one vertex, and thus Case 2 holds true.

Case 3:
Layer L_k has 2 vertices and each vertex has 1 child.



Layer L_{k+1} has 2 vertices, and thus Case 3 holds true.

Case 4:
Layer L_k has 2 vertices and each vertex has 0 children.
A BFS search will never reach L_{k+1} because there are no additional vertices to search.

$\square$