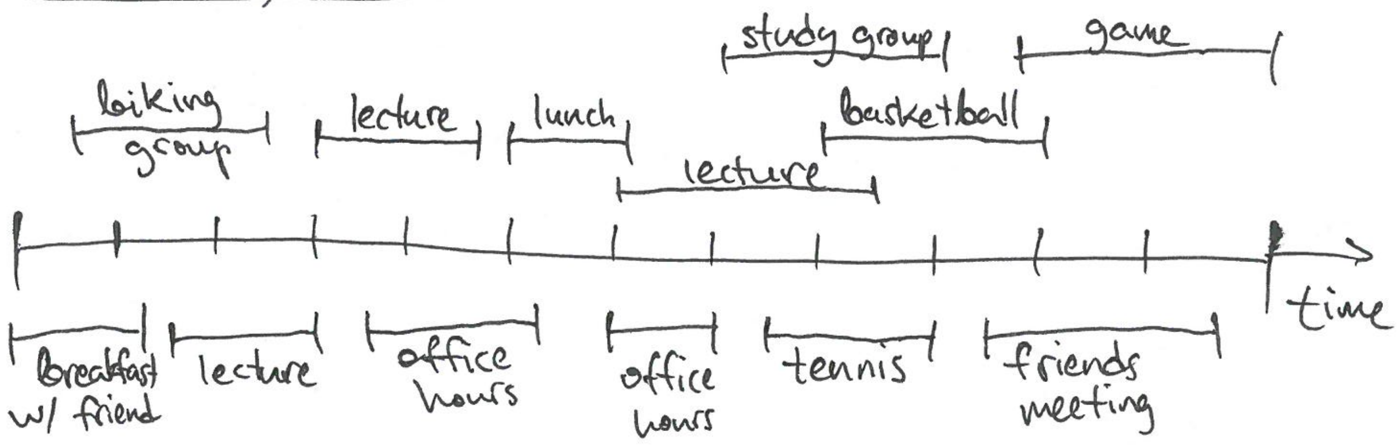
Planning Your Time

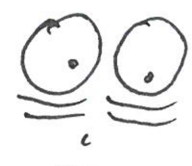


Input Activities $i = 1 \dots n$; each has a start time s_i & a finish time f_i.

A schedule Subset of non-overlapping activities $i_1 \dots i_k$

$$s_{i_1} < f_{i_1} \le s_{i_2} < f_{i_2} \dots < f_{i_k}$$

(A) "I want to do as many activities as possible!" (i.e., max k).



People often use greedy strategies:
pick an activity that looks "best" now, then "deal with the consequences" move on to next. E.g.

* Pick shortest activity (i.e., min $f_i - s_i$)
* Pick most urgent activity (i.e., min s_i)
* Pick earliest ~~deadline~~ finish (i.e., min f_i)

<u>Advantage</u> Easy to implement, efficient to find next activity.

Run-time for the three algorithms we suggested: $O(n \lg n)$

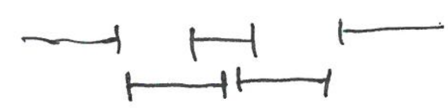
1 sort the activities ((i) according to $f_i - s_i$ or $\frac{s_i}{f_i}$)
(ii) sort $s_1 \ldots s_n, f_1 \ldots f_n$

2 pick min activity, rule out overlapping activities.
(i.e, start time between start & finish of activity)

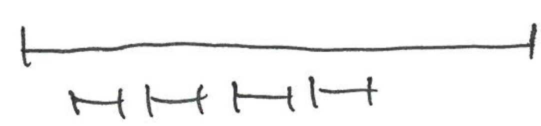
Step 1 takes $O(n \lg n)$ time. Step 2 takes $O(n)$ time overall. *

<u>Disadvantage</u> May not find optimal solution.

* Shortest activity - one short activity may rule out two activities

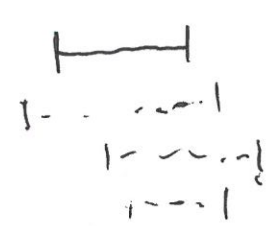


* Most urgent activity - urgent activity may rule out many activities that start later

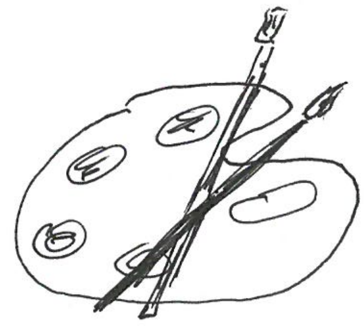


* Earliest finish time - works for previous examples, and in fact works in general as we'll see later in the term.

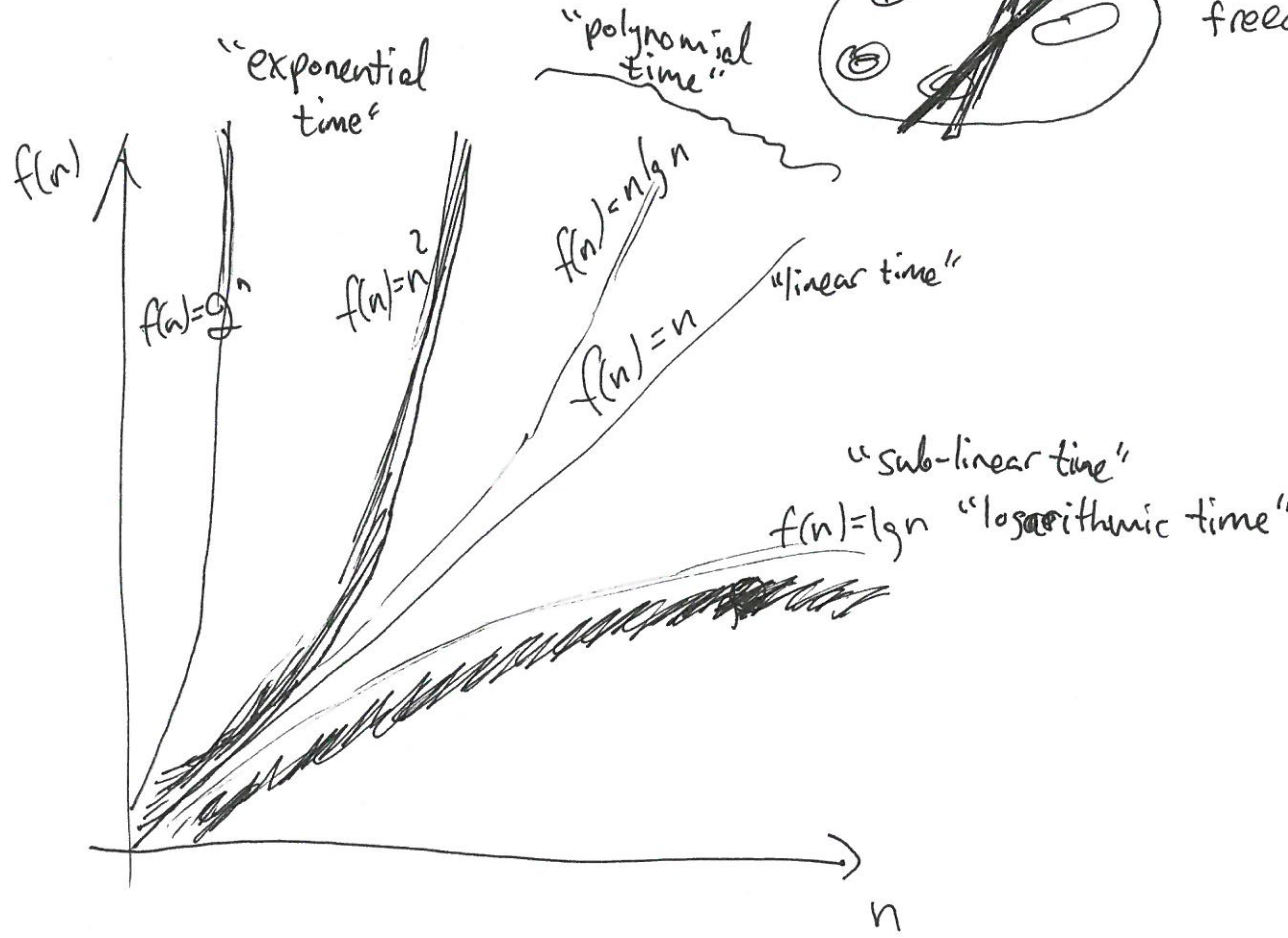
Intuition: put yourself ahead of the game for the next activities.



* Run-times



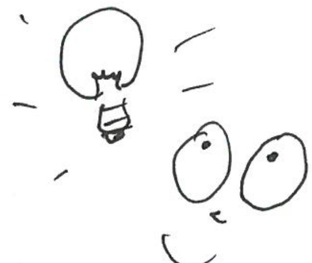
with some artistic freedom...



* We use "O" notation : $f(n) = O(g(n))$ if $\exists m, n_0\ \forall n \geq n_0\ f(n) \leq m \cdot g(n)$

Rationale : one operation can count as two (or more) in a different computational model. We want to count # operations while ignoring such differences.

B "I have priorities"

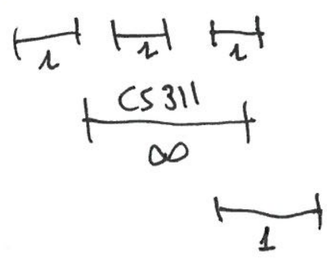


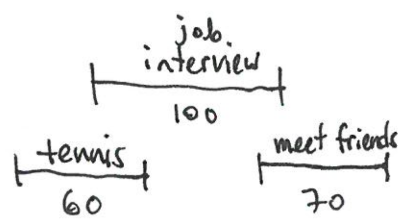
Input Activities as before but now every activity has a weight w_i

Goal $\max \sum_{j=1}^{k} w_{i_j}$ for a schedule $i_1 - i_k$.

Note The no-weights case is a special case with $w_i = 1 \ \forall i$.

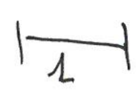
Examples





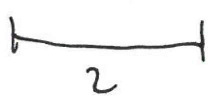
Observation The greedy approach fails!

What's better ??



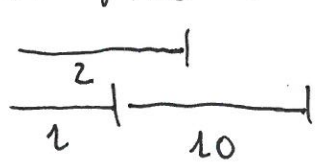
get less, free fast

OR



get more, free later

Depends... Are there exciting oppurtunities ahead?



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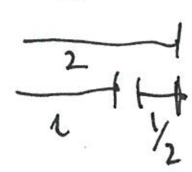
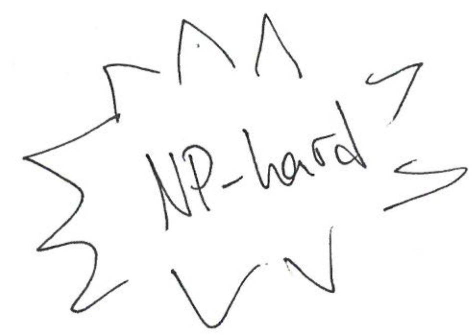


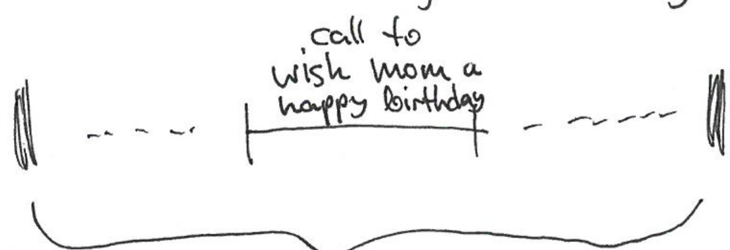
Later in the semester we'll discuss "dynamic programming", a powerful algorithmic paradigm that records information about different alternatives.

<u>Note</u> There can be up to 2^n different schedules. Going through them one by one — using "brute force" — would be extraordinarily inefficient. In contrast — Dynamic programming gives an $O(n \lg n)$ time algo for the problem!!



The most efficient algo we know takes exponential time!!

Ⓒ "I'm flexible in my scheduling"



need half an hour any time during this period.

<u>Input</u> n activities; each has a start time s_i, a finish time f_i and a duration ℓ_i.

<u>Goal</u> Maximize k with $i_1 \sim i_k$ & s.t. $s_{i_j} \le s'_{i_j} < f'_{i_j} \le f_{i_j}$

$s'_{i_1} \le f'_{i_1} \le \cdots \le s'_{i_k} \le f'_{i_k}$