

# Go Track

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PowerUp! SG Tech Traineeship -  
Software Engineering



# Learning Objectives – Mod 9

At the end of the course, participants should be able to:

- Define what a data structure is.
- Examine the different data structures available.
- Demonstrate the different usage of each data structures discussed.

# Data Structures

- Structures that come with programming language may not suit a problem.
- Therefore, sometimes it is necessary to create own data structures, that
  - Stores
  - Search
  - Retrieve data
- Sometimes data also needs to be manipulated or processed in specialized ways.
- Creating own data structures allows more control and flexibility to the kind of structures that can be constructed, and its supporting functions that it can have.

# Data Structures

## Types of Data Structures

- Linked lists
- Stacks
- Queues
- Binary Trees

# Linked List

- A *Linked List* is a data structure consisting of a set of nodes, where each node consists of:
  - item (to store the data item)
  - next (to store a pointer that links to next node)
- The first node is usually accessed via a pointer in separate variable commonly called *head* (or *firstnode*).



# Linked List

- Declaring a simple linked list node structure in Go.

```
type Node struct {  
    item itemType // to store the data item  
    next *Node // pointer to point to next node  
}  
  
type linkedList struct {  
    head *Node  
    size int  
}
```



# Linked List

- A new node can be created with its members initialized as follows:

```
newNode := &Node{"Mary", nil}
```

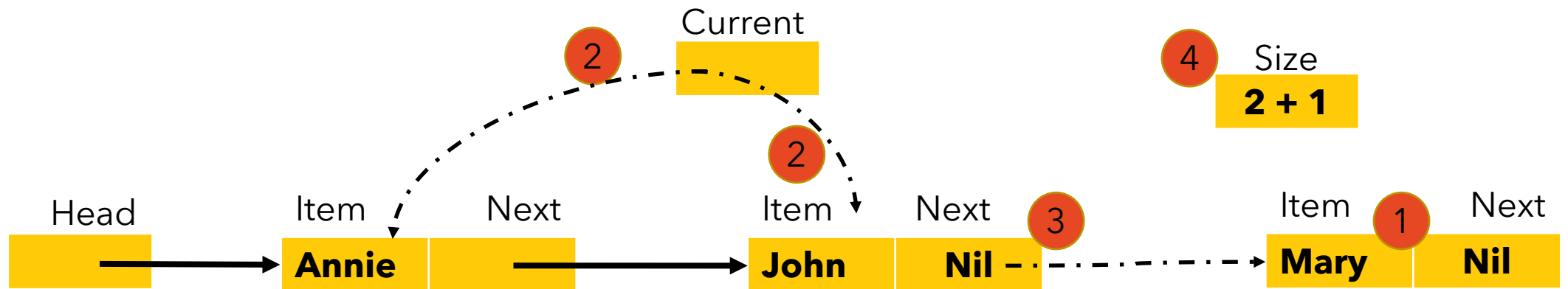
or

```
newNode := &Node{  
    item: "Mary",  
    next: nil,  
}
```



# Linked List

- To add an item to the end of the list:
  - create a new node to store the item
  - traverse to the last node
  - make the last node's pointer to point to the new node
  - increase the size by 1



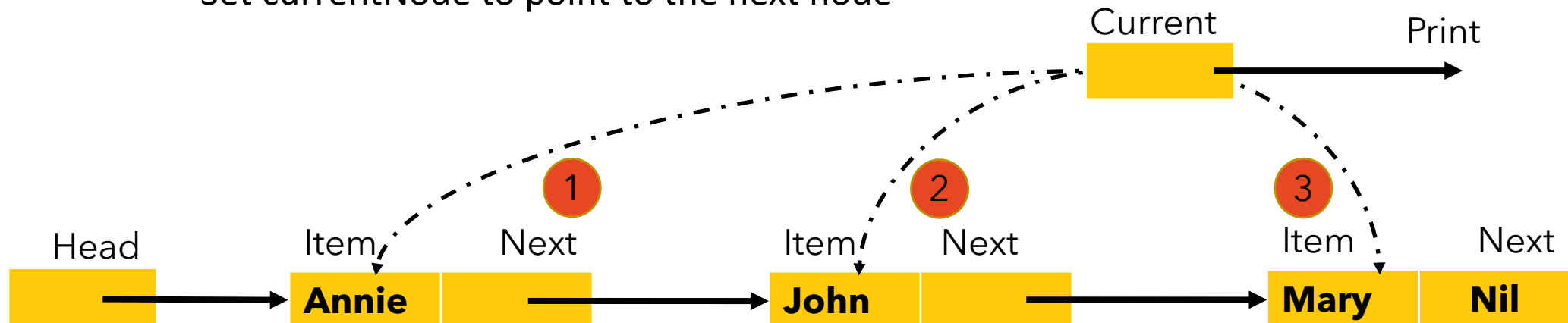


# Linked List

```
func (p *LinkedList) addNode(name string) error {
    newNode := &Node{
        item: name,
        next: nil,
    }
    if p.head == nil {
        p.head = newNode
    } else {
        currentNode := p.head
        for currentNode.next != nil {
            currentNode = currentNode.next
        }
        currentNode.next = newNode
    }
    p.size ++
    return nil
}
```

# Linked List

- To display all the nodes in linked list:
  - Set a currentNode pointer to point to the first node of the list
  - While currentNode is not null
    - Retrieve the item from the node pointed by currentNode
    - Display the item
    - Set currentNode to point to the next node



# Linked List

```
func (p *linkedList) printAllNodes() error {
    currentNode := p.head
    if currentNode == nil {
        fmt.Println("Linked list is empty.")
        return nil
    }
    fmt.Printf("%+v\n", currentNode.item)
    for currentNode.next != nil {
        currentNode = currentNode.next
        fmt.Printf("%+v\n", currentNode.item)
    }

    return nil
}
```

# Linked List

- There can be many possible other functions for the linked list:
  - Adding item at a certain position
  - Removal of item from list
  - Retrieval of item at certain position from list
  - And many more...

# Linked List

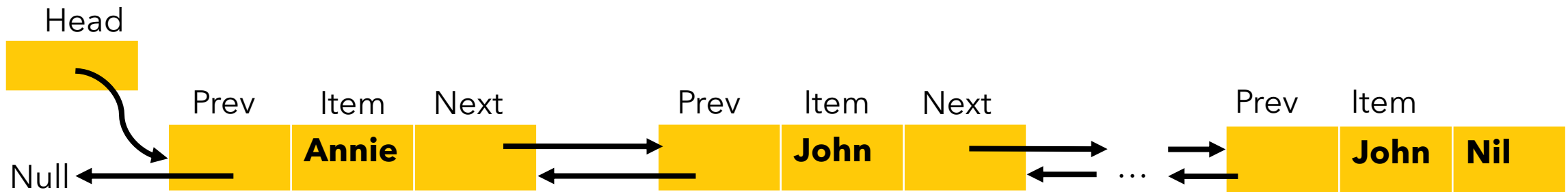
- Advantages of implementing Linked list:
  - Size of the list is not fixed, can grow as large as is necessary.
  - Easy to add/delete items by simply updating pointers, without shifting items but have to traverse to the correct location first.
  - Does not waste storage - use only necessary amount of memory (dynamic allocation)

# Linked List

- Disadvantages of implementing Linked list:
  - access to items is sequential - need to traverse through the list sequentially to access an item
  - Worst case complexity?  $O(n)$  Adding item or removing the last item in the list.
  - Requires additional space to store the pointers/linkages

# Doubly Linked List

- A *Doubly Linked List* is a data structure consisting of a set of nodes, where each node consists of:
  - item (to store the data item)
  - next (to store a pointer that links to next node)
  - prev (to store a pointer that links to the previous node)
- The first node is usually accessed via a pointer in separate variable commonly called *head* (or *firstnode*).
- The last node is commonly called *tail* (or *lastnode*).



# Doubly Linked List

- Declaring a simple doubly linked list node structure in Go.

```
type Node struct {  
    item itemType // to store the data item  
    prev *Node    // pointer to point to prev node  
    next *Node    // pointer to point to next node  
}  
  
type linkedList struct {  
    head *Node  
    tail *Node  
    size int  
}
```





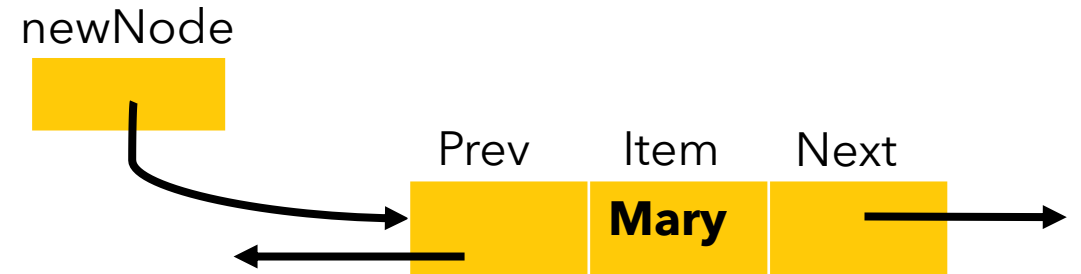
# Doubly Linked List

- A new doubly node can be created with its members initialized as follows:

```
newNode := &Node{"Mary", nil, nil}
```

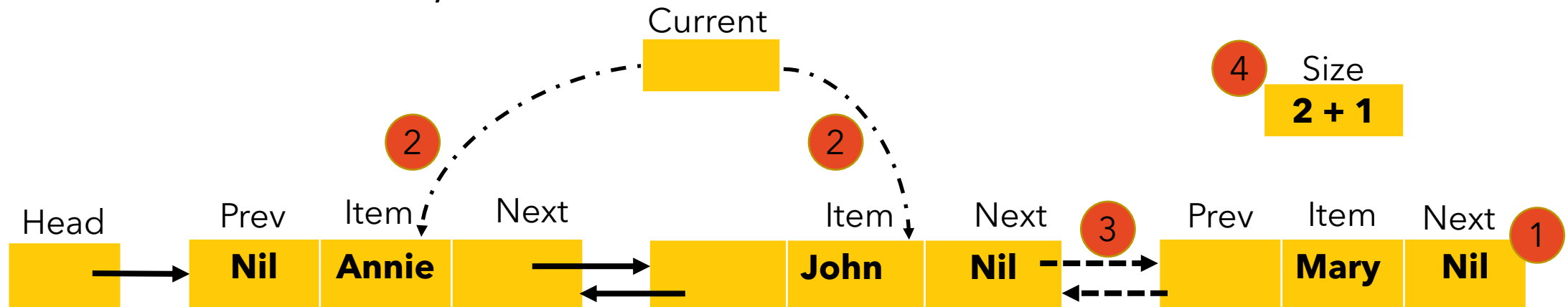
Or

```
newNode := &Node{  
    item: "Mary",  
    next: nil,  
    prev: nil,  
}
```



# Doubly Linked List

- To add an item to the list:
  - create a new node to store the item
  - traverse to the selected node
  - make the last node's pointer to point to the new node
  - Make the new node's prev pointer to point to the last node
  - increase the size by 1

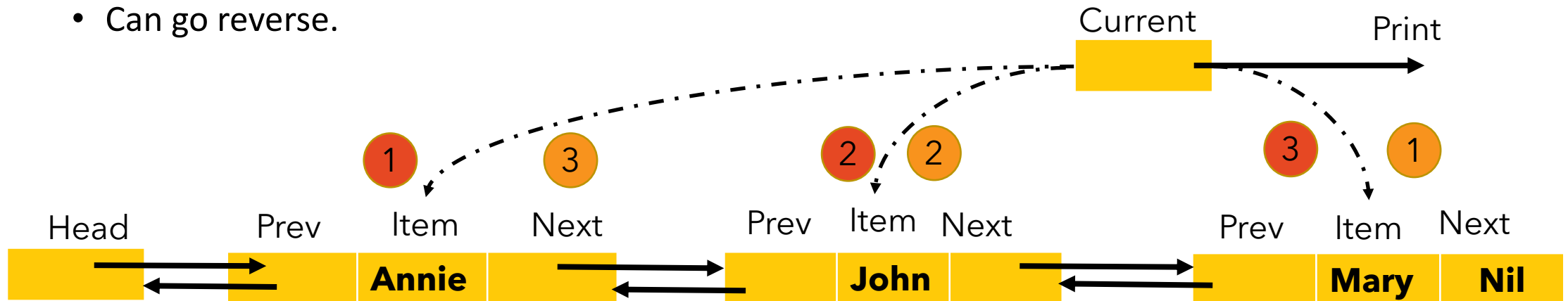


# Doubly Linked List

```
func (p *LinkedList) addNode(name string) error {
    newNode := &Node{
        item: name,
        next: nil,
        prev: nil,
    }
    if p.head == nil {
        p.head = newNode
        p.tail = newNode
    } else {
        currentNode := p.head
        for currentNode.next != nil {
            currentNode = currentNode.next
        }
        newNode.prev = currentNode
        currentNode.next = newNode
        p.tail = newNode
    }
    p.size ++
    return nil
}
```

# Doubly Linked List

- To display all the nodes in doubly linked list:
- Set a currentNode pointer to point to the first node of the list
- While currentNode is not null
  - Retrieve the item from the node pointed by currentNode
  - Display the item
  - Set currentNode to point to the next node
- Can go reverse.



# Doubly Linked List

- To display in forward display

```
func (p *dblinklist) printAllNodes() error {
    currentNode := p.head
    if currentNode == nil {
        fmt.Println("DB Link list is empty")
        return nil
    }
    fmt.Printf("%+v\n", currentNode.item)
    for currentNode.next != nil {
        currentNode = currentNode.next
        fmt.Printf("%+v\n", currentNode.item)
    }
    return nil
}
```

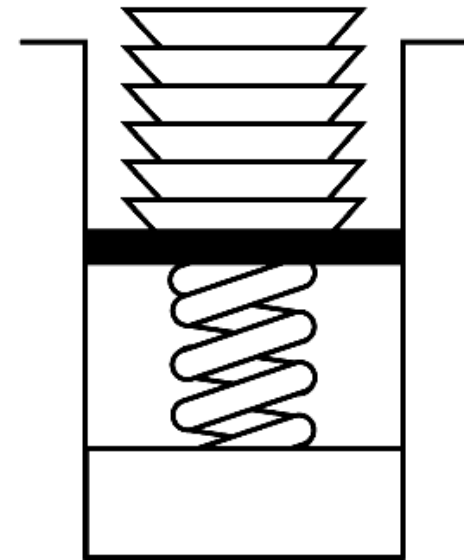
# Doubly Linked List

- To display in reverse display

```
func (p *dblinklist) printAllNodesReverse() error {
    currentNode := p.tail
    if currentNode == nil {
        fmt.Println("DB Link list is empty")
        return nil
    }
    fmt.Printf("%+v\n", currentNode.item)
    for currentNode.prev != nil {
        fmt.Printf("%+v\n", currentNode.prev.item)
        currentNode = currentNode.prev
    }
    return nil
}
```

# Stack

- A *Stack* is data structure for organizing data.
- Important property: *LIFO (Last-in First-Out)*
- Last item placed on the stack will be removed first.
- e.g. stack of plates, books etc
- only item at top of stack is visible and can be retrieved.
- new items can only be added to top of stack
- items can only be removed from top of stack
- Much simpler to implement than a linked list.



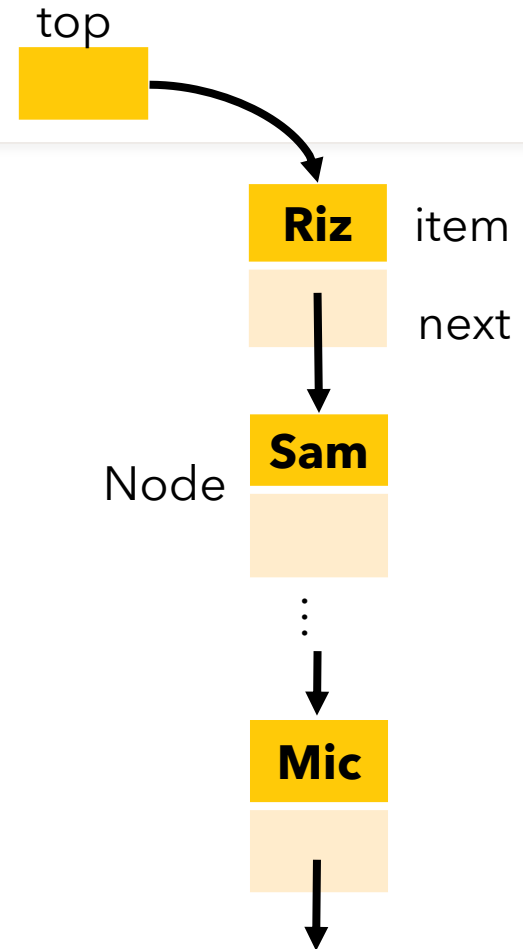
# Stack

- Possible operations of a typical stack:
  - add an item to the top of stack (*push*)
  - remove an item from top of stack (*pop*)
  - retrieve/ look at item at top of stack (*peek* or *getTop*)



# Stack

- A structure similar to linked list can be used to implement a stack.
- Similarly, it can consist of nodes, each of which contains:
  - item (to store the data item)
  - next (to store a pointer that links to next node)
- The first node is usually accessed via a pointer in separate variable called *top* (or *topNode*).



# Stack

- A new node in stack is like a linked list, just rearranged.

```
newNode := &Node{"Mary", nil}
```

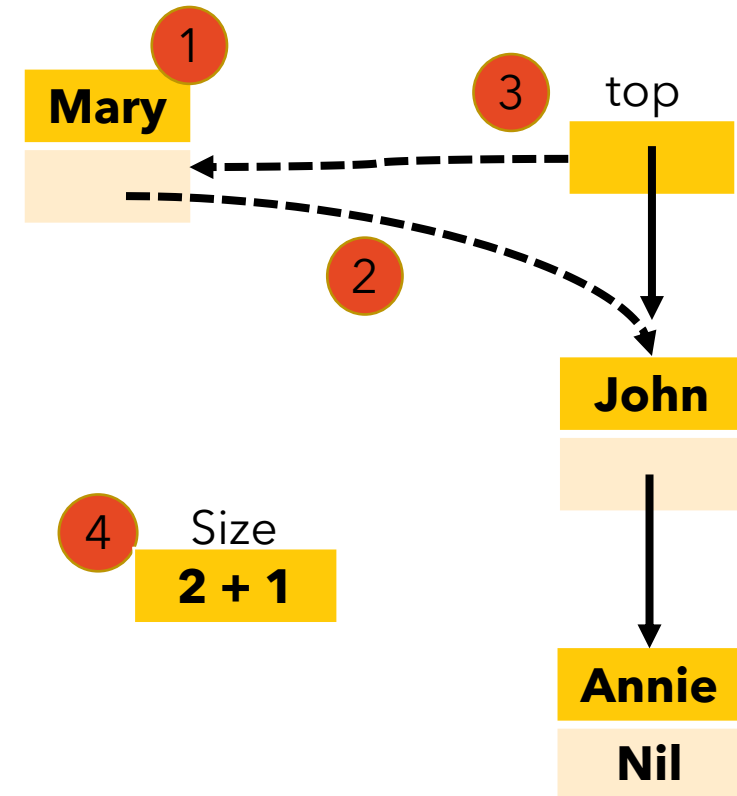
Or

```
newNode := &Node{  
  item: "Mary", // to store the data item  
  next: nil, // to store the data item  
}
```



# Stack

- To *push* a new item on top of a stack:
  - create a new node to store the item
  - make the new node's next point to the top
  - make the top point to new node
  - increase the size by 1

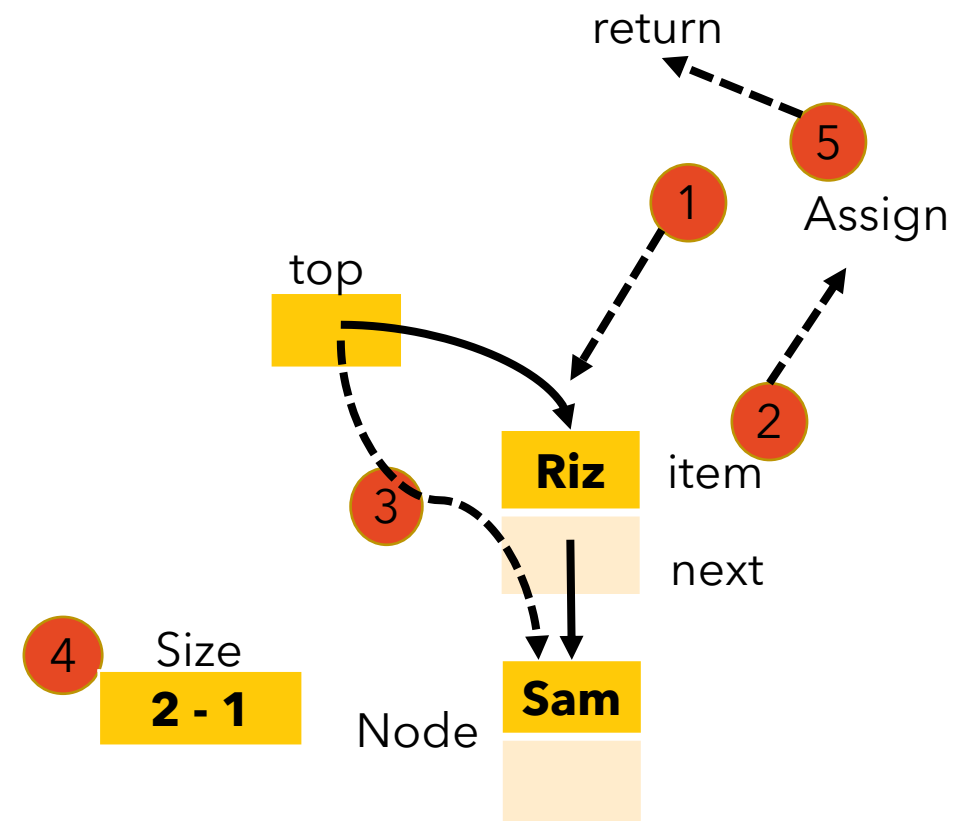


# Stack

```
func (p *stack) push(name string) error {  
    newNode := &Node{  
        item: name,  
        next: nil,  
    }  
    if p.top == nil {  
        p.top = newNode  
    } else {  
        newNode.next = p.top  
        p.top = newNode  
    }  
    p.size ++  
    return nil  
}
```

# Stack

- To pop an item from top of a stack:
  - Need to check if top is pointing to any node or nil
  - Otherwise, capture the item at top node.
  - Need to check if size is 1, make top point to nil
  - Else, make the top point to the next node in stack
  - Decrease the size by 1
  - Return the popped item.



# Stack

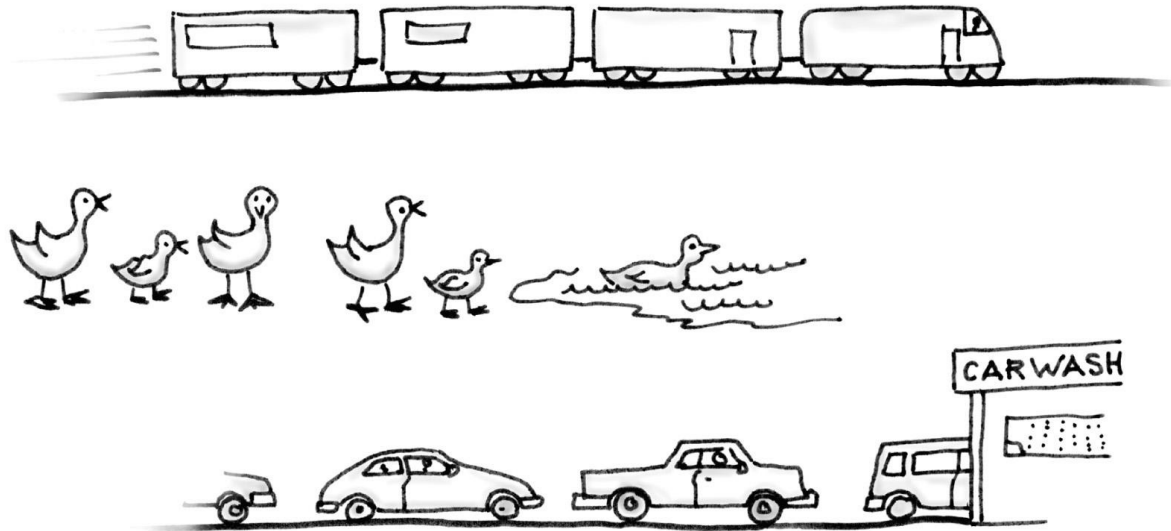
```
func (p *stack) pop() (string, error) {  
    var item string  
    if p.top == nil {  
        return "", errors.New("Empty Stack!")  
    }  
    item = p.top.item  
  
    if p.size == 1 {  
        p.top = nil  
    } else {  
        p.top = p.top.next  
    }  
    p.size--  
    return item, nil  
}
```

# Stack

- Stacks are used in several applications:
  - Checking for balanced parenthesis in compilers
  - Converting infix to postfix expressions
  - Evaluation of postfix expressions
  - Simulation of applications that has LIFO behaviour
  - And so on...

# Queue

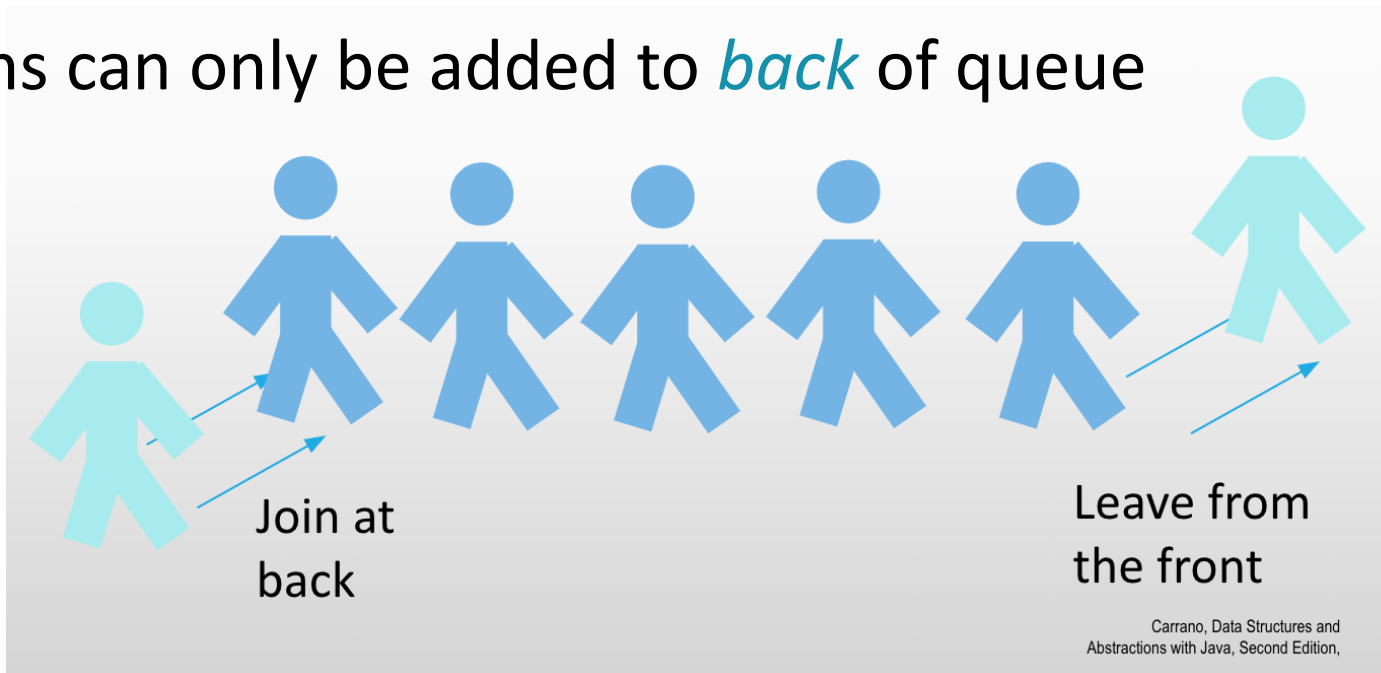
- A *queue* is another form of data structure for organizing data.
- Important property: *FIFO (First-in First-Out)*
- First item placed on the queue will be removed first.





# Queue

- operations can only occur at the queue's two ends
- Items can only be removed from *front* of queue
- New items can only be added to *back* of queue

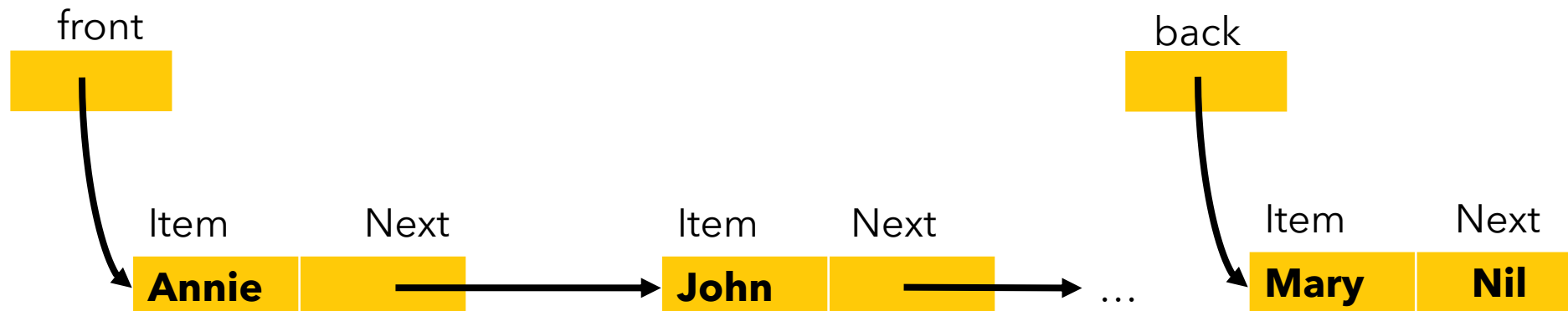


# Queue

- Possible operations of a typical queue:
  - add an item to back of queue(*enqueue*)
  - remove an item from front of queue (*dequeue*)
  - retrieve/ look at item at front of stack (*getFront*)

# Queue

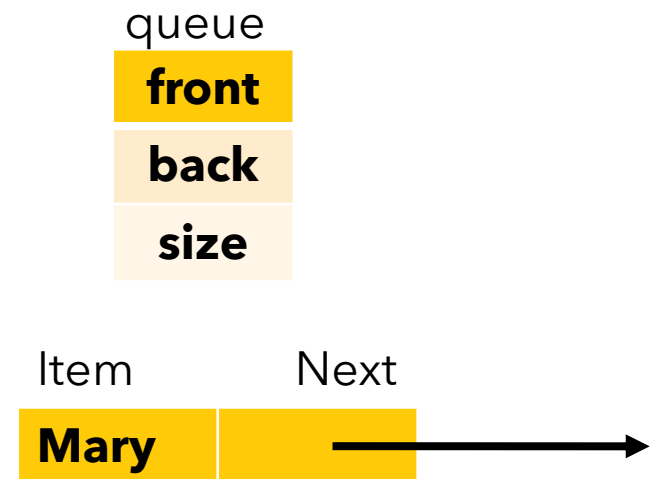
- A structure similar to linked list can be used to implement a queue.
- Similarly, it can consist of nodes, each of which contains:
  - item (to store the data item)
  - next (to store a pointer that links to next node)
- a pointer *front* (or *frontNode*) to point to the node at front position.
- a pointer *back* (or *backNode*) to point to the node at back position.



# Queue

- Format for declaring a queue structure.

```
type Node struct {  
    item itemType // to store the data item  
    next *Node // pointer to point to next node  
}  
  
type queue struct {  
    front *Node  
    back *Node  
    size int  
}
```



# Queue

- Recall the creation of node

```
newNode := &Node{"Mary", nil}
```

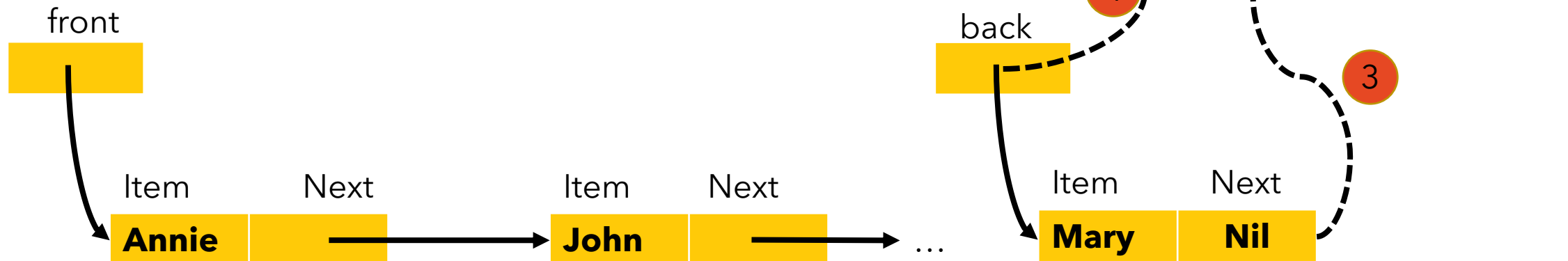
Or

```
newNode := &Node{  
  item: "Mary", // to store the data item  
  next: nil, // to store the data item  
}
```



# Queue

- To *enqueue* a new item at the back of the queue:
  - need to consider case of empty queue.
  - create a new node to store the item
  - make the back node's next pointer to point to the new node
  - make the back pointer point to the new node
  - increase the size by 1

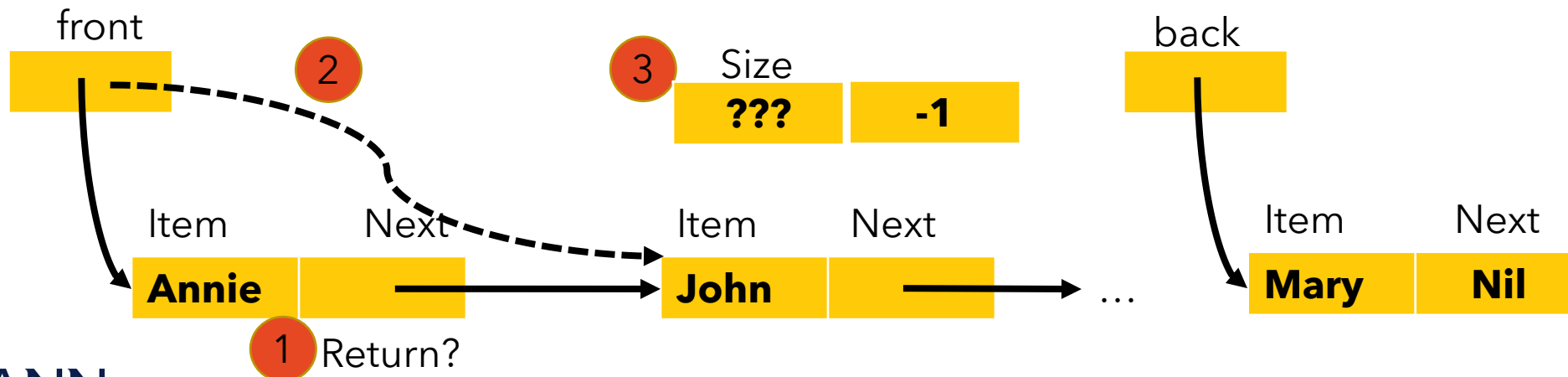


# Queue

```
func (p *queue) enqueue(name string) error {
    newNode := &Node{
        item: name,
        next: nil,
    }
    if p.front == nil {
        p.front = newNode
    } else {
        p.back.next = newNode
    }
    p.back = newNode
    p.size++
    return nil
}
```

# Queue

- To dequeue an item from front of a queue:
  - Capture the item of the node at front
  - Make the front point to the next node in queue
  - Return the item if needed.
  - Reduce size by 1
  - Need to consider case of only 1 node.





# Queue

```
func (p *queue) dequeue() (string, error) {
    var item string

    if p.front == nil {
        return "", errors.New("Empty Queue!")
    }

    item = p.front.item
    if p.size == 1 {
        p.front = nil
        p.back = nil
    } else {
        p.front = p.front.next
    }
    p.size--
    return item, nil
}
```

# Queue

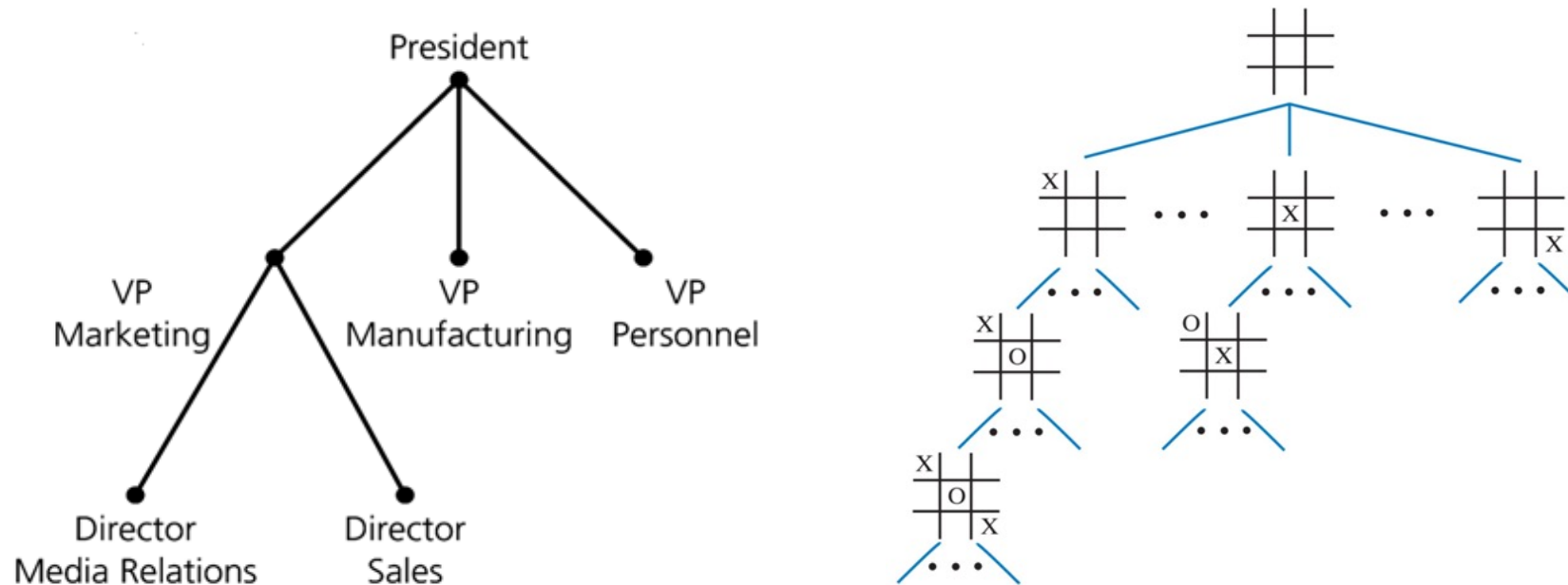
- Advantages of implementing pointer-based Queue:
  - Size of the list is not fixed - can grow as large as or shrink as is necessary
  - Adding & removing data involves manipulation of linkages including front and back ,  $O(1)$  time complexity
  - Does not waste storage - use only necessary amount of memory (dynamic allocation)
- Disadvantages of implementing pointer-based Queue:
  - Additional memory needed to store the linkages.

# Queue

- Queues are used in several applications:
  - Processing jobs that are added in and cleared in FIFO manner e.g. CPU scheduling, disk scheduling, enterprise resource planning, customer service management, order handling, call centres, queue management etc etc
  - Implementing buffers to hold data that are transferred asynchronously (receiving not at same rate as sending) e.g. IO buffers, file IO
- And many many more...

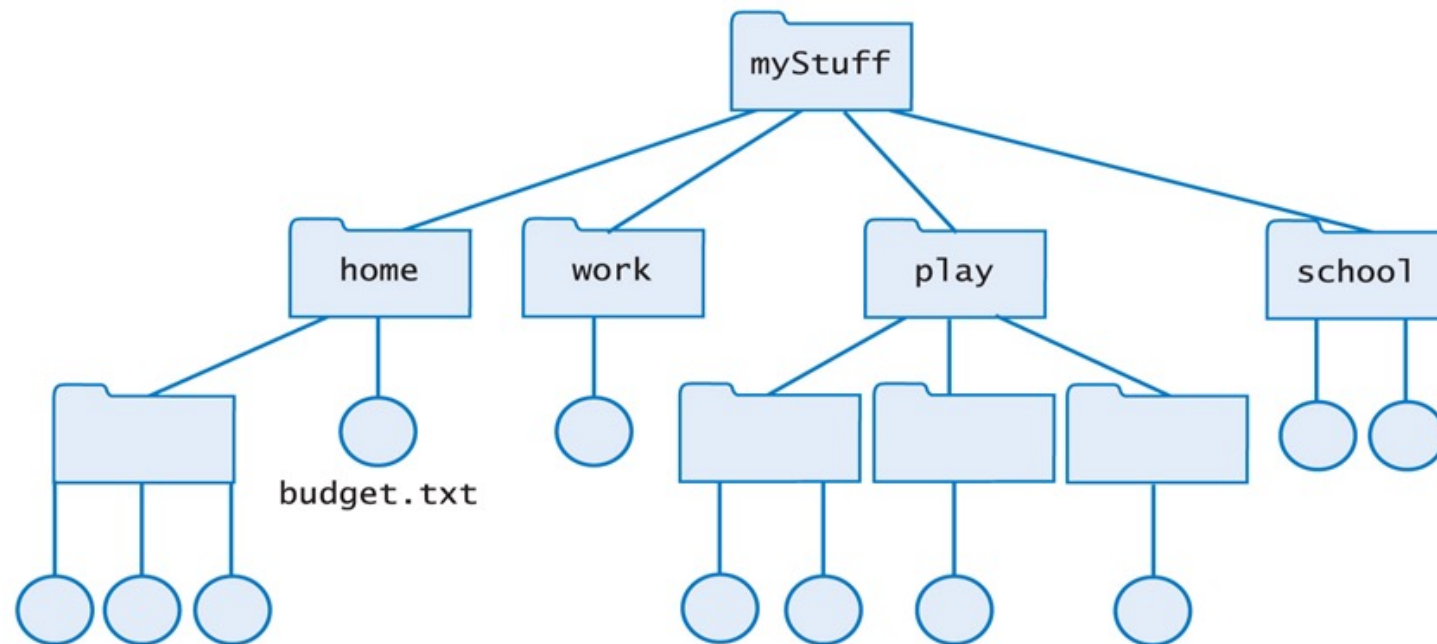
# Tree

- A *tree* is a data structure that organizes data in hierarchical order.
- e.g. organizational chart , game tree, family tree, file directories, expression trees, decision trees



# Tree

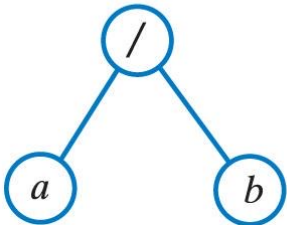
- Another example: directory structure...



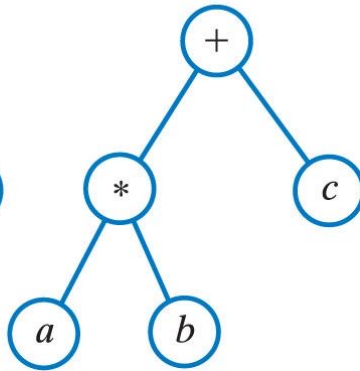
# Tree

- Expression trees:

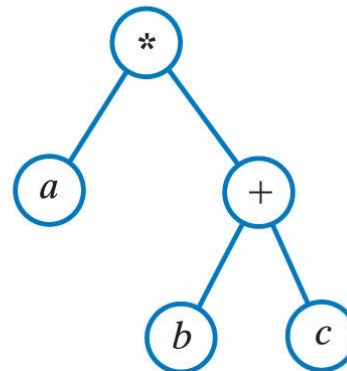
(a)  $a / b$



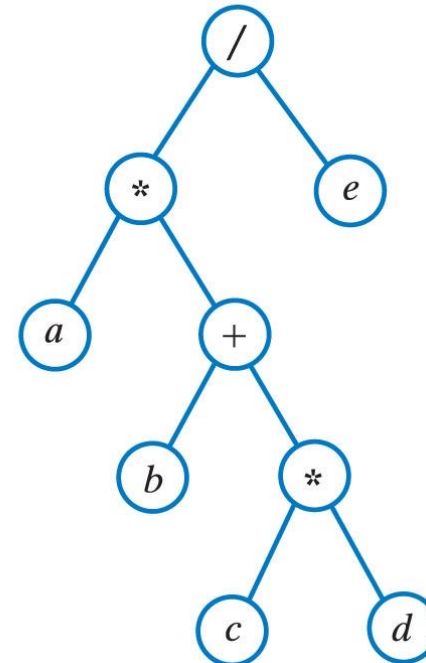
(b)  $a * b + c$



(c)  $a * (b + c)$

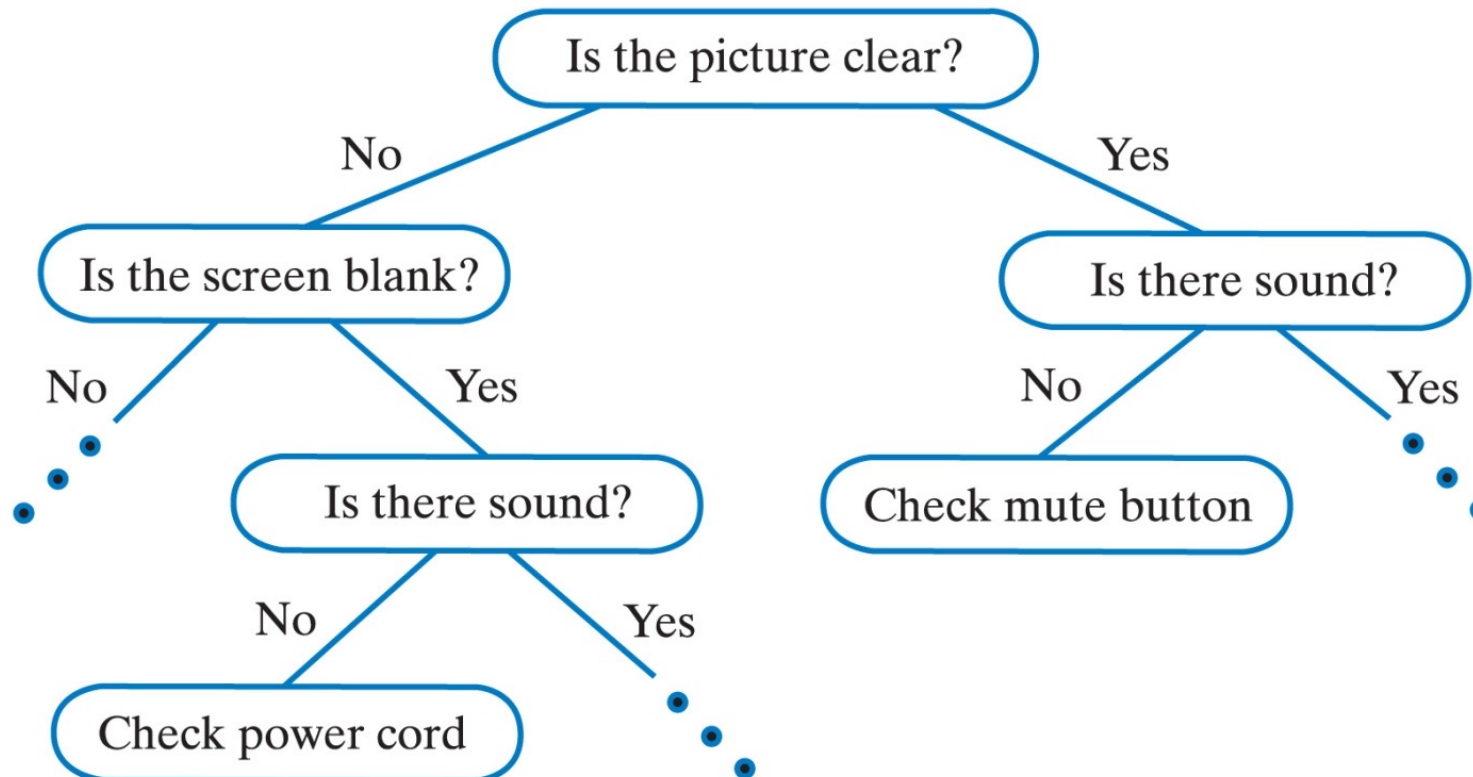


(d)  $a * (b + c * d) / e$



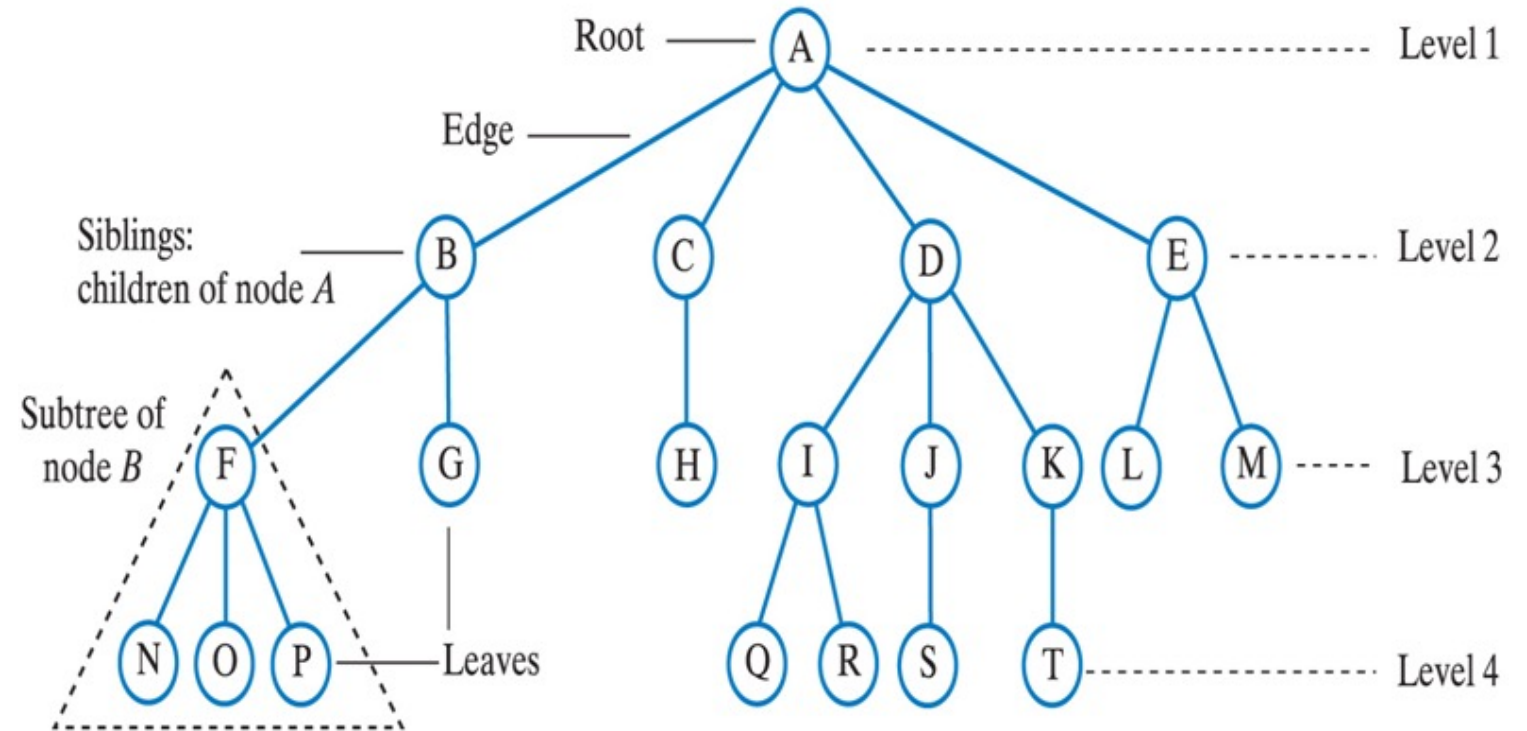
# Tree

- Decision trees:



# Tree

- Tree Terminology:
  - Trees are composed of *nodes* and *edges*





# Tree

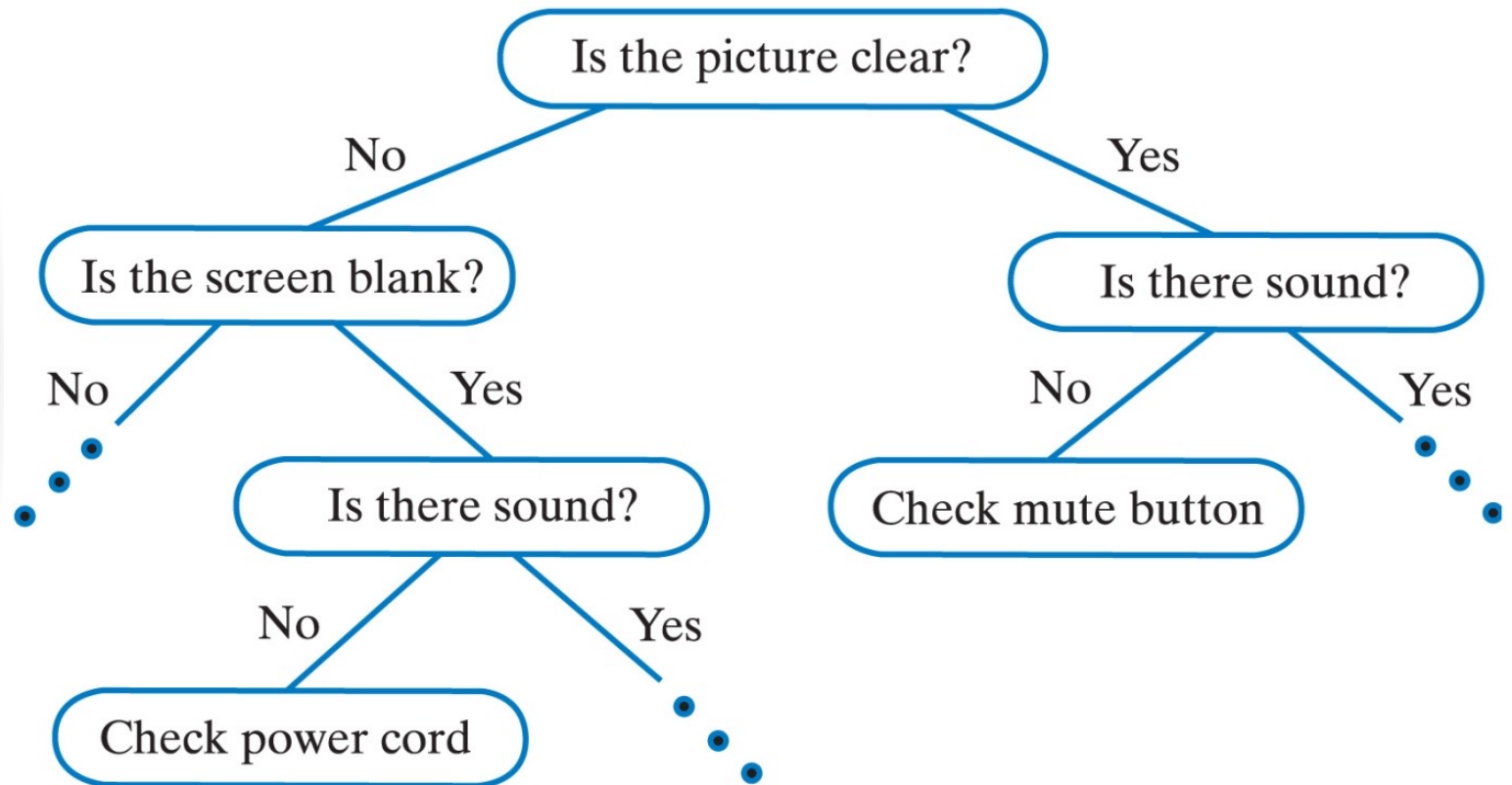
- *root* - the node at the top of the tree
- *leaf* - a node with no children
- *level (or height/ depth)* - the number of generations from the root , longest path from root to leaf
  - not to be confused with depth of a node - path from root to the node
- *height* - the number of levels in the tree
- *parent* - node with nodes (children ) below it
- *children* - nodes below a given node (parent)

# Tree

- Types of Trees
  - *General Tree* - a tree with any number of subtrees
  - *Binary Tree* - a tree with at most 2 subtrees
  - *Binary Search Tree* - a binary tree that is ordered
    - values on the left subtree  $<$  value of parent
    - values on the right subtree  $>$  value of parent
  - *AVL Tree* - a binary search tree that is balanced
    - the heights of any node's two subtrees differ by at most 1

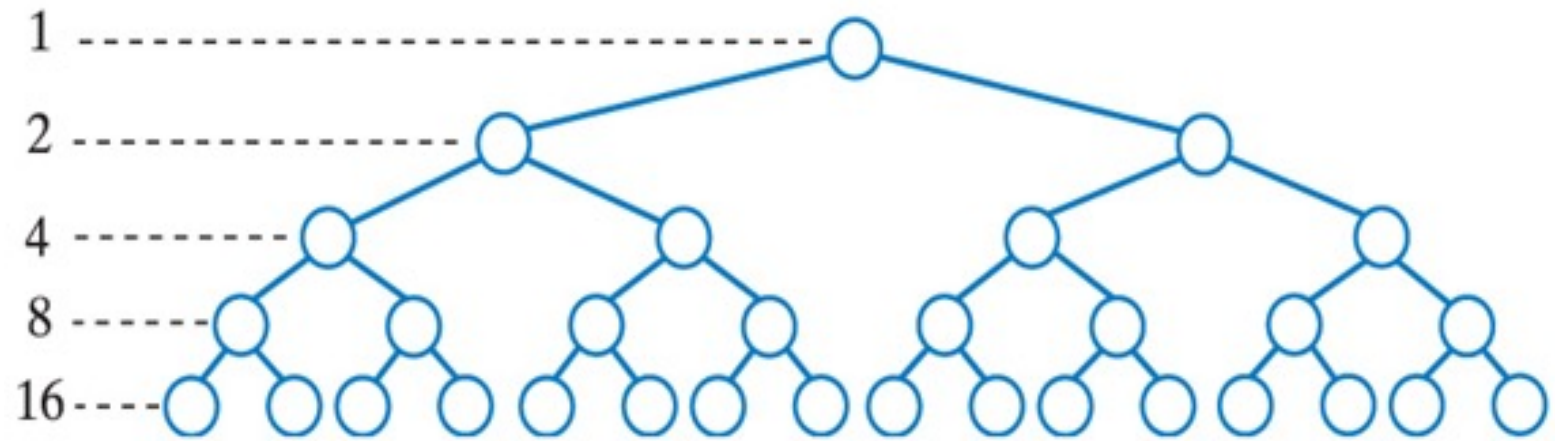
# Binary Tree

- A *binary tree* is a tree that has **at most 2 subtrees**.



# Binary Tree

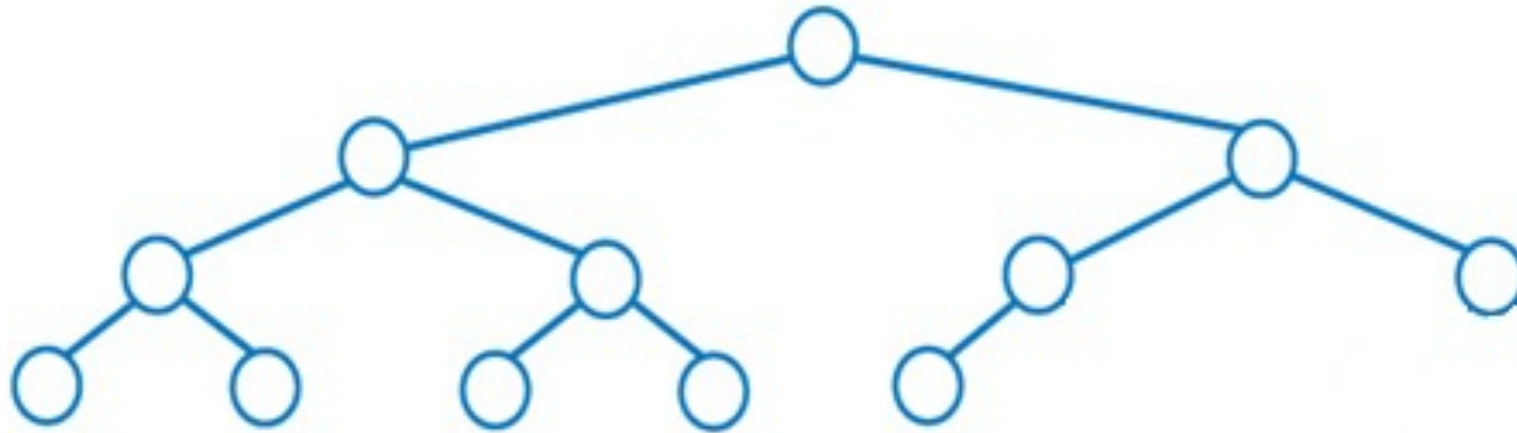
- A binary tree is *full* if every node (except the leaf nodes) has two children



The number of nodes in a full binary tree of height  $h$ ,  $n = 2^h - 1$   
The height of a full binary tree with  $n$  nodes that is full is  $h = \log_2(n + 1)$

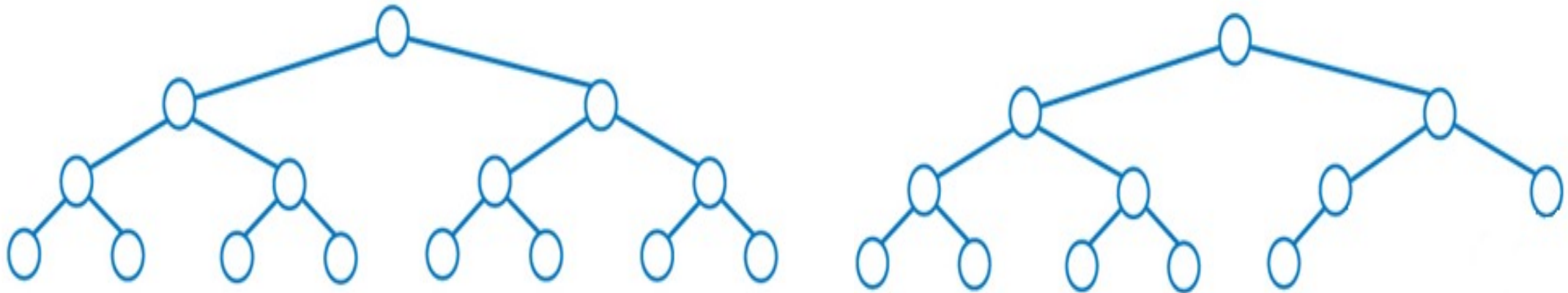
# Binary Tree

- A binary tree is *complete* if
  - it is full to all the levels except the last level and
  - the last level is filled from left to right



# Binary Tree

- A binary tree is *balanced* if the heights of any node's two subtrees differ by at most 1

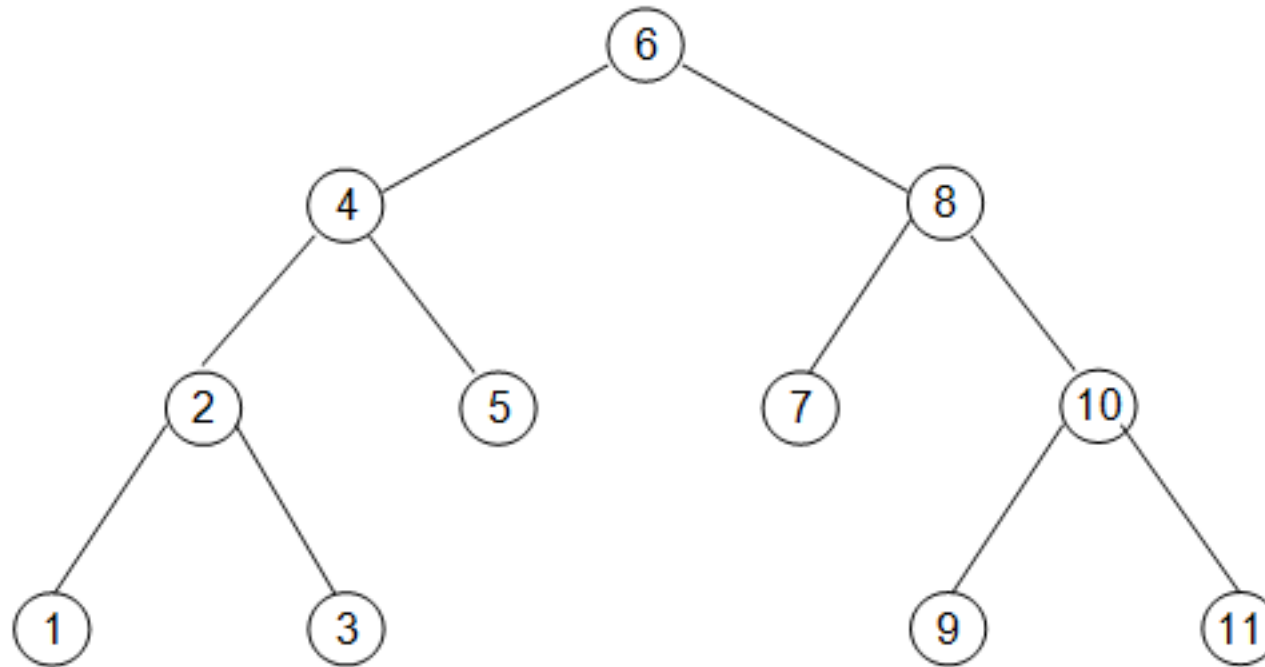


# Binary Tree

- Traversals of a Binary Tree
  - Inorder* : Left-Root-Right
  - Preorder* : Root-Left-Right
  - Postorder* : Left-Right-Root
  - Level order* : Level by Level

# Binary Tree

- *Inorder Traversal* : Left-Root-Right

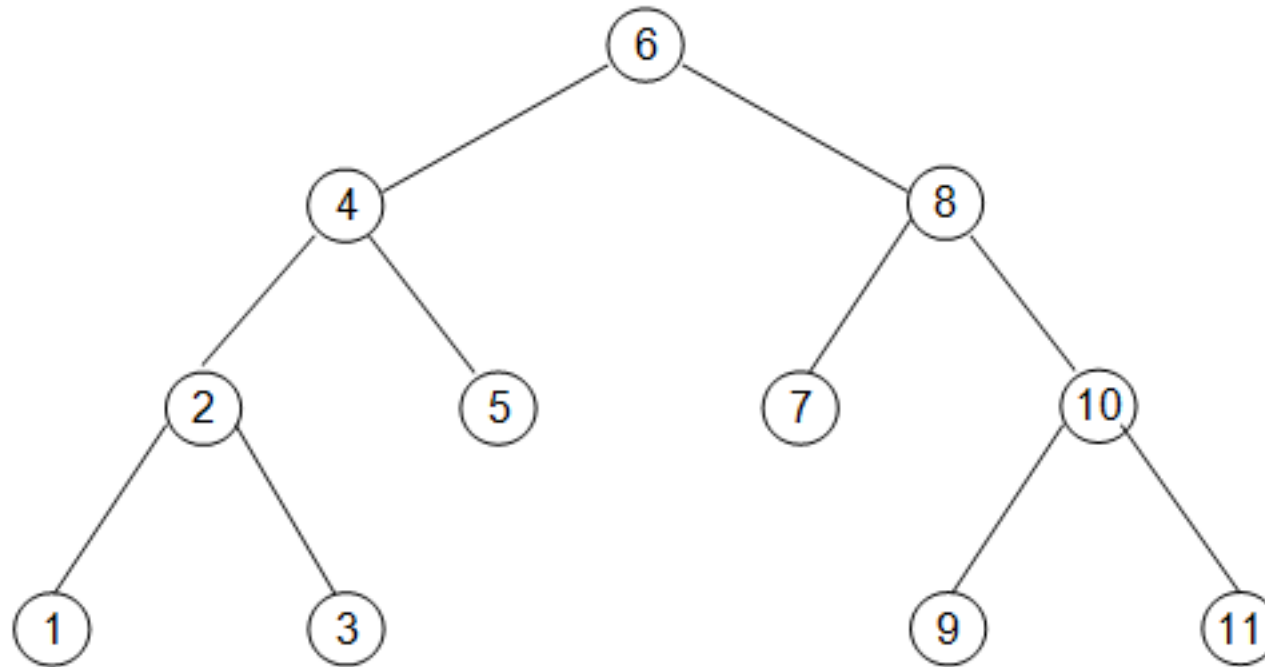


1 2 3 4 5 6 7 8 9 10 11



# Binary Tree

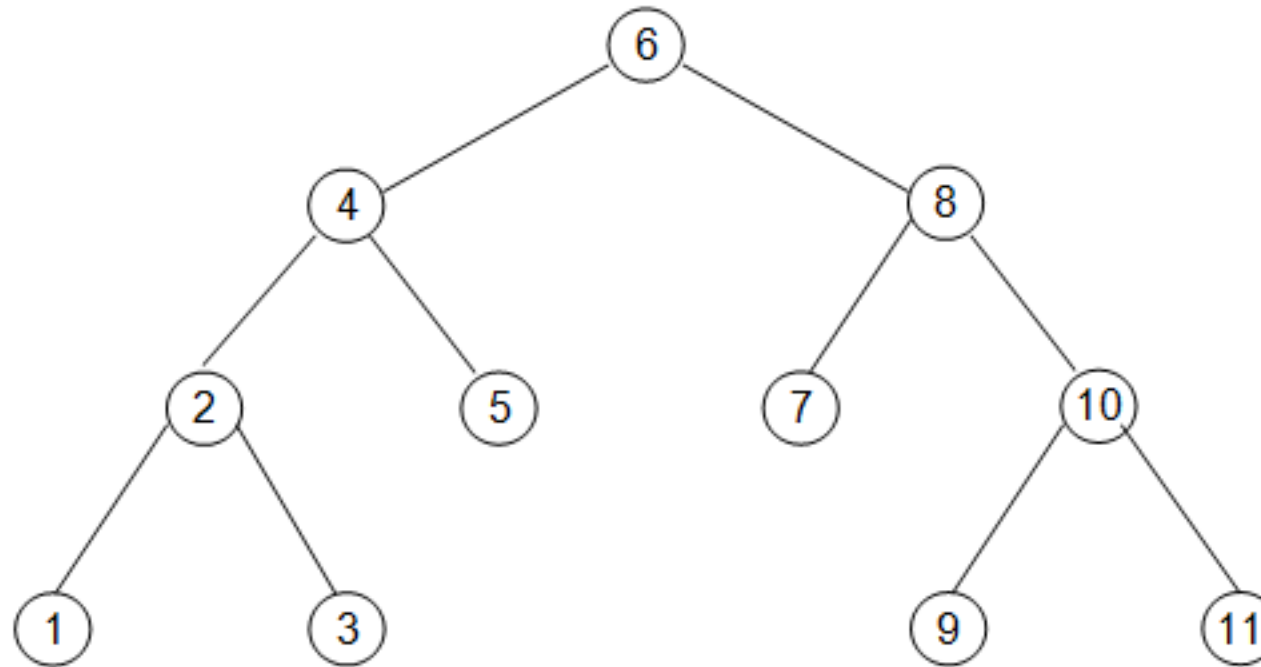
- *Preorder Traversal* : Root-Left-Right



6 4 2 1 3 5 8 7 10 9 11

# Binary Tree

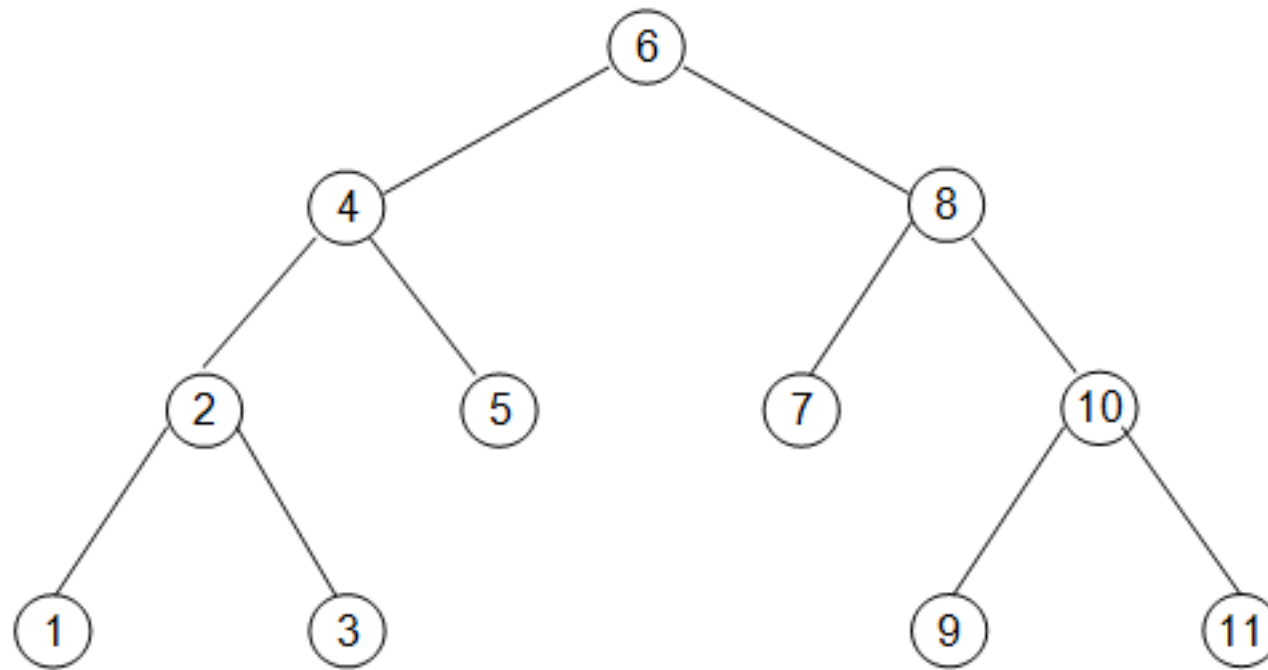
- *Postorder Traversal* : Left-Right-Root



1 3 2 5 4 7 9 11 10 8 6

# Binary Tree

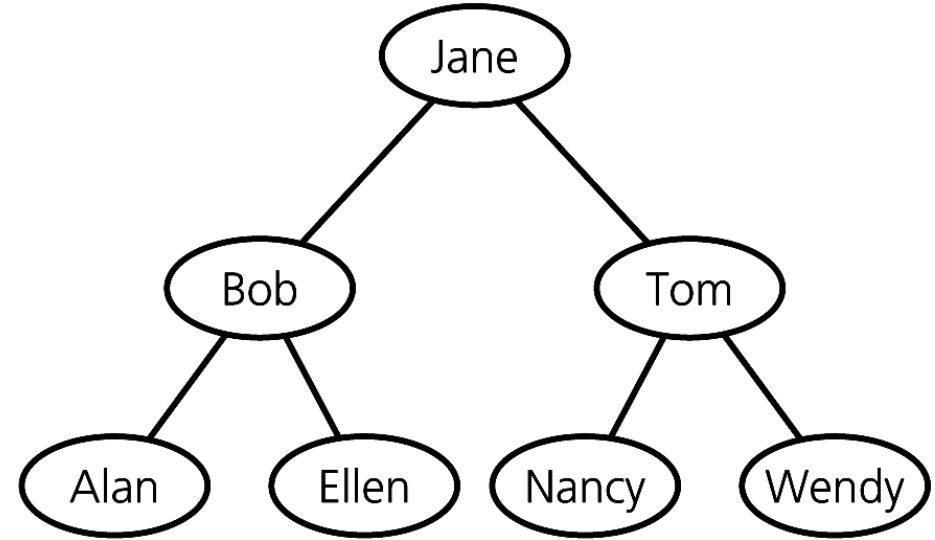
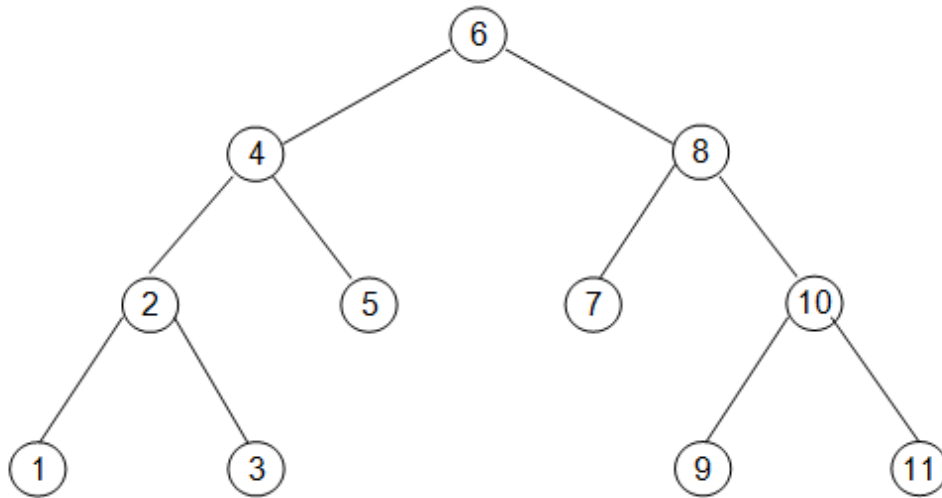
- *Level order Traversal* : Level by level



6 4 8 2 5 7 10 1 3 9 11

# Binary Search Tree

- A binary search tree is a binary tree that is ordered
  - values in the left subtree  $<$  value of parent
  - values in the right subtree  $>$  value of parent

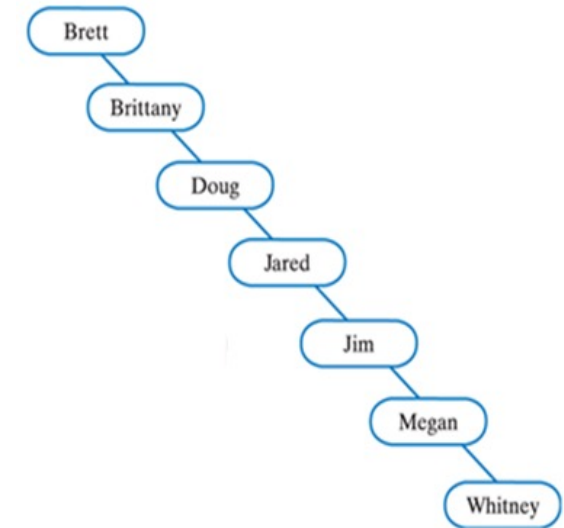
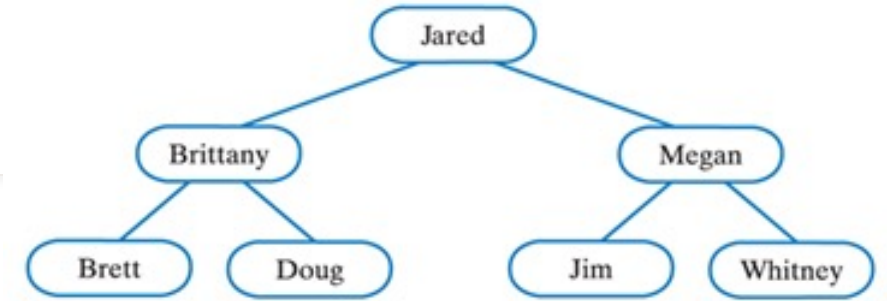


# Binary Search Tree

- All the operations in a BST require a search that begins at the root
- Number of comparisons is directly proportional to the height of the tree.
- Worst case (balanced) =  $O(\log_2 n)$
- Worst case (unbalanced) =  $O(n)$ , where  $n$  = number of nodes in the binary search tree
- Therefore, it is important that a binary search tree is *balanced*
- A binary search tree usually becomes unbalanced during an insert or removal of item.
- So how to make it balanced? Find out more about AVL trees!

# Binary Search Tree

- The speed of finding the same element say Megan would be different in a balanced and an unbalanced tree.



# Binary Search Tree

- Possible operations of a binary search tree:
  - Insert
  - Search
  - Traversal
  - Remove

# Binary Search Tree

- The format for declaring the tree structure:

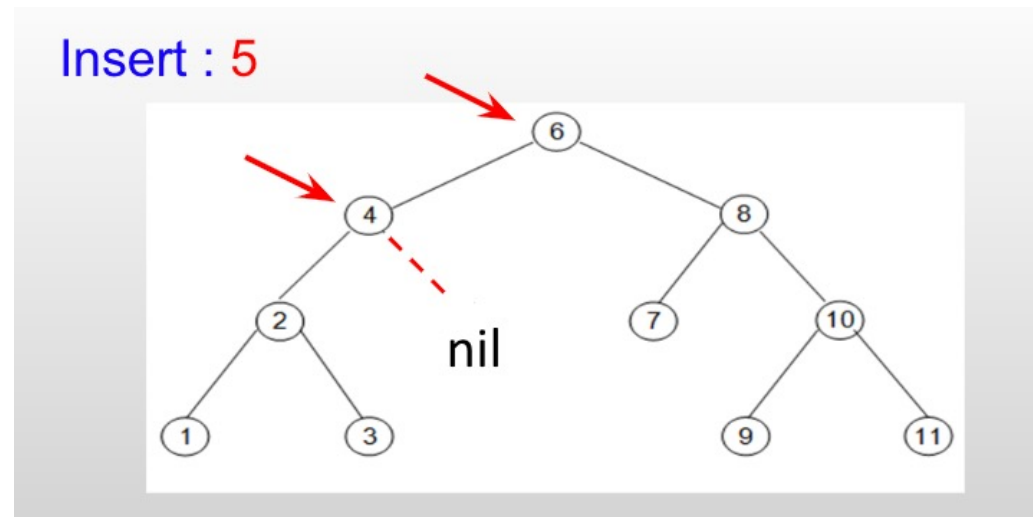
```
type BinaryNode struct {  
    item itemType // to store the data item  
    left *BinaryNode // pointer to point to left node  
    right *BinaryNode // pointer to point to right node  
}  
  
type BST struct {  
    root *BinaryNode  
}
```





# Binary Search Tree

- To *insert a new item into the BST*:
  - Search for the item (pointer will point to nil)
  - Create a new node to store the item
  - Set the pointer pointing to null to point to new node



# Binary Search Tree

```
func (bst *BST) insertNode(t **BinaryNode, item string) error {  
  
    if *t == nil {  
        newNode := &BinaryNode{  
            item:  item,  
            left:  nil,  
            right: nil,  
        }  
        *t = newNode  
        return nil  
    }  
  
    if item < (*t).item {  
        bst.insertNode(&((*t).left), item)  
    } else {  
        bst.insertNode(&((*t).right), item)  
    }  
    return nil  
}
```

# Binary Search Tree

```
func (bst *BST) insert(item string) {  
    bst.insertNode(&bst.root, item)  
}
```

# Binary Search Tree

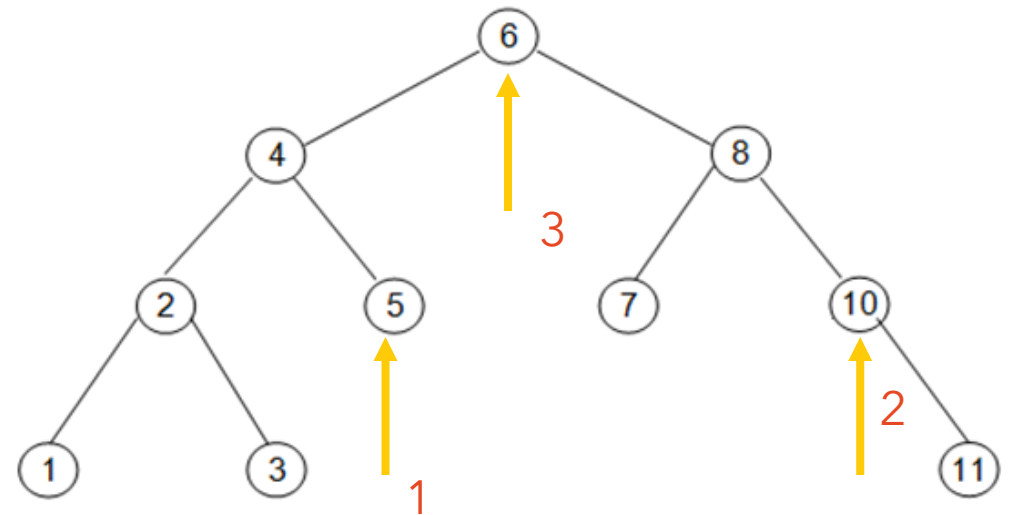
- To *traverse a binary tree in InOrder manner*:
  - Check if the current node is nil
  - If not, recursively call inOrder traversal on left subtree
  - Upon returning, process the current node
  - Then recursively call inOrder traversal on right subtree

# Binary Search Tree

```
func (bst *BST) inOrderTraverse(t *BinaryNode) {  
  
    if t != nil {  
        bst.inOrderTraverse(t.left)  
        fmt.Println(t.item)  
        bst.inOrderTraverse(t.right)  
    }  
}  
  
func (bst *BST) inOrder() {  
    bst.inOrderTraverse(bst.root)  
}
```

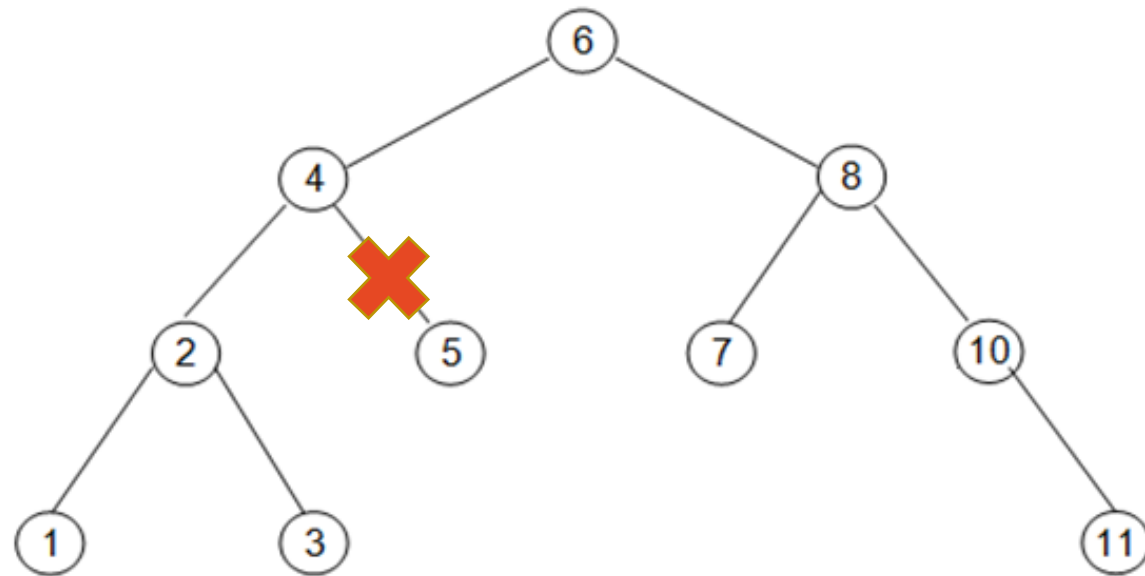
# Binary Search Tree

- To remove a new item into the BST:
  - 3 Distinct Possible Cases
    - Case 1 : node to be deleted has 0 child (is a leaf)
    - Case 2 : node to be deleted has 1 child
    - Case 3 : node to be deleted has 2 children



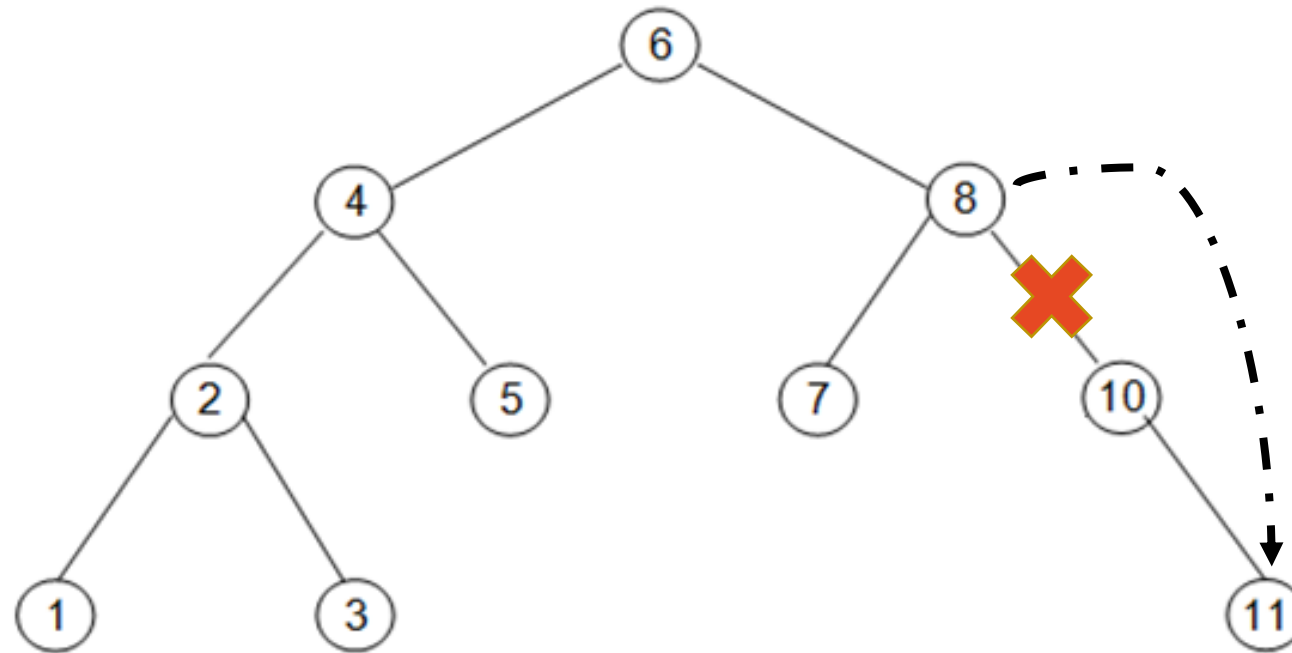
# Binary Search Tree

- Case 1: Deleting a leaf node
  - Simply delete the node by setting pointer pointing to it to point to nil.



# Binary Search Tree

- Case 2: Deleting a node with 1 child
  - Simply delete the node by setting the pointer pointing to it, to point to the node's child (only 1 child)

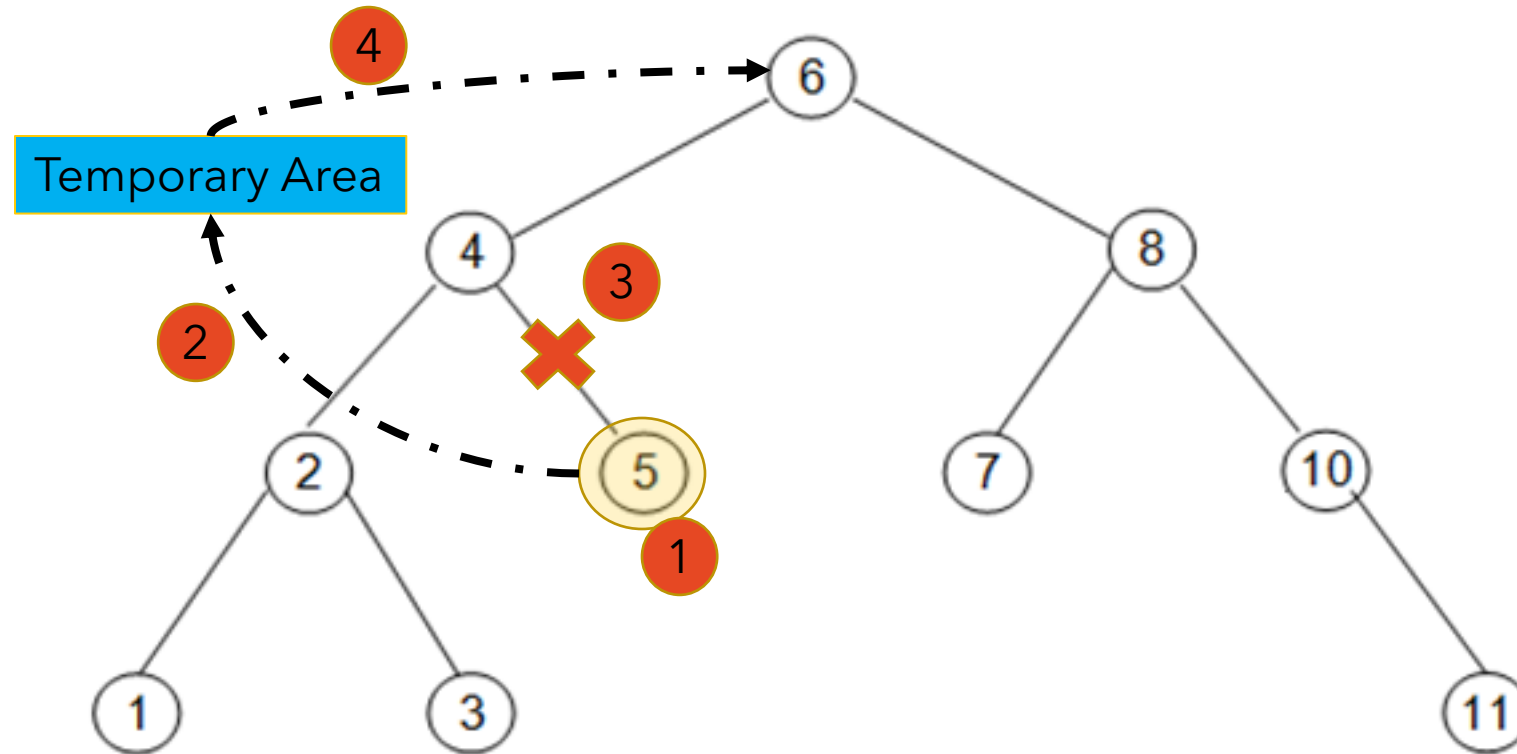




# Binary Search Tree

- Case 3: Deleting a node with 2 children
  1. find the successor (next smaller value) i.e. the rightmost child in the node's left subtree
  2. Store the successor item in a temp variable
  3. delete the successor recursively (either case 1 or case 2)
  4. replace the node's entry with that of the successor (in temp)

# Binary Search Tree



# Binary Search Tree

- Now let's try to figure out how to code
  - PreOrder traversal
  - PostOrder traversal
  - Search
  - Remove

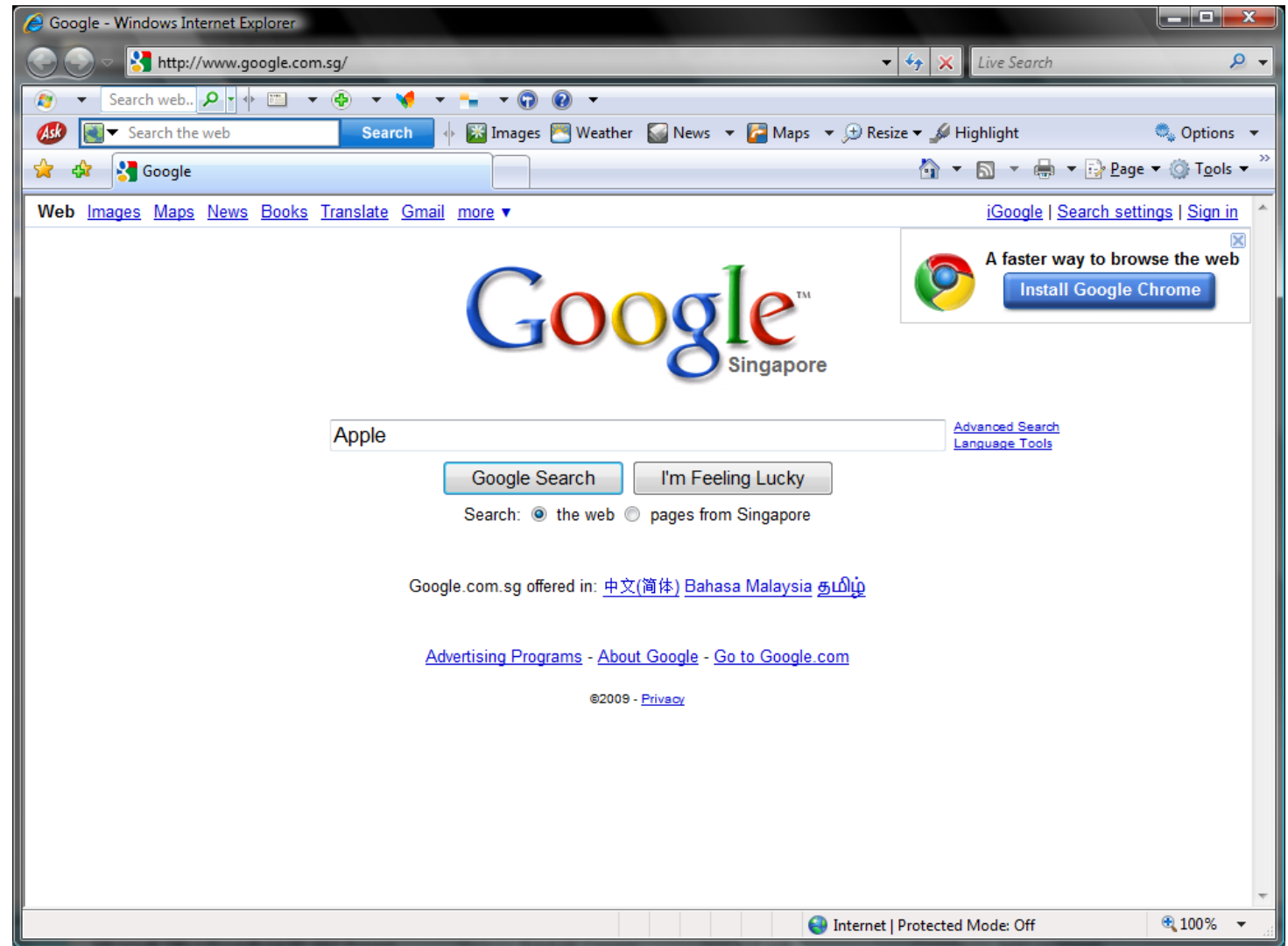
# Learning Objectives – Mod 10

At the end of the course, participants should be able to:

- Define what a search algorithm is.
- Examine the different types of search algorithm.
- Demonstrate the different usage of the search algorithm.

# Searching

- Searching is one of the most common activities in our daily life.
  - e.g. searching for a book, person, tel no, file, movie, song, . . .



# Search

- Types of search
  - Sequential Search – Sorted / Unsorted
  - Binary Search
  - Jump Search
  - Interpolation Search
  - Sublist Search
  - And the list goes on....

# Sequential Search - Unsorted

- Given an unsorted array of data, what are the ways to search for "target" data?

0	1	2	3	4	...	N-1
48	25	95	76	57	...	88

Dave	Eve	Bean	Roy	Ann	...	Gary
------	-----	------	-----	-----	-----	------

Search for value 50 or search for John ?

# Sequential Search - Unsorted

- Search for 76

Data 1 = 76?

<b>48</b>	25	95	76	57	12	33	88
-----------	----	----	----	----	----	----	----

Data 2 = 76?

48	<b>25</b>	95	76	57	12	33	88
----	-----------	----	----	----	----	----	----

Data 3 = 76?

Data 4 = 76?

Yes, item found, stop

48	25	95	<b>76</b>	57	12	33	88
----	----	----	-----------	----	----	----	----



# Sequential Search - Unsorted

- Search for 50

Data 1 = 50?

<b>48</b>	25	95	76	57	12	33	88
-----------	----	----	----	----	----	----	----

Data 2 = 50?

Data 3 = 50?

48	<b>25</b>	95	76	57	12	33	88
----	-----------	----	----	----	----	----	----

...

...

Data N = 50?

No and no more data. Stop. Item not found.

# Sequential Search - Unsorted

Consider the following method

```
int search(itemType[] array, int n, ItemType target)
```

Algorithm:

- Set index to start of array.

- While(not found and not end of array)

  - If item at the index of array equal to target

    - Item found (return index)

  - Else

    - Increment the index by 1

- End of array reached, item not found, return -1

# Sequential Search - Unsorted

- Implementation of said algorithm

```
func search(arrSlice []int, n int, target int) int {  
    for i := 0; i < n; i++ {  
        if arrSlice[i] == target { // found  
            return i  
        }  
    }  
    return -1 // not found  
}
```

# Sequential Search - Unsorted

- In terms of efficiency for unsorted array
  - Worst number of searches =  $N$
  - Average number of comparisons =  $N/2$

0	1	2	3	4	. . .	$N-1$
48	25	95	76	57	...	88

# Sequential Search - Sorted

- Given a sorted array, what are the ways to search for "target" data?

0	1	2	3	4	5		N-1
12	25	33	48	57	76	...	95

# Sequential Search - sorted

- Search for 76

Data 1 = 76?

<b>12</b>	25	33	48	57	76	88	95
-----------	----	----	----	----	----	----	----

Data 2 = 76?

12	<b>25</b>	33	48	57	76	88	95
----	-----------	----	----	----	----	----	----

Data 3 = 76?

Data 4 = 76?

Yes, item found, stop

12	25	33	48	57	<b>76</b>	88	95
----	----	----	----	----	-----------	----	----

# Sequential Search - Sorted

- Search for 50

12	25	33	48	57	76	88	95
----	----	----	----	----	----	----	----

Data 1 = 50? No. Data 2 > 50 ? No, continue to next data.

Data 2 = 50? No. Data 2 > 50 ? No, continue to next data.

Data 3 = 50? No. Data 3 > 50 ? No, continue to next data.

Data 4 = 50? No. Data 4 > 50 ? No, continue to next data.

Data 5 = 50? No. Data 5 > 50 ? Yes. Number exceeded. Stop  
Item not found.

# Sequential Search - Sorted

Consider the following method

```
int search(itemType[] array, int n, ItemType target)
```

Algorithm:

- Set index to start of array.

- While(not found **and not done** and not end of array)

  - If item at the index of array equal to target

    - Item found (return index)

  - Else if item at index greater than target

    - Item not found, done. Return -1

  - Else

    - Increment the index by 1

- End of array reached, item not found, return -1



# Sequential Search - Sorted

- In terms of efficiency for unsorted array
  - Worst number of searches =  $N$
  - Average number of comparisons =  $N/2$  (need not search non-relevant items)

0	1	2	3	4	. . .	$N-1$
12	25	33	48	57	...	95

# Binary Search

- Requires a sorted array.
- Uses divide and conquer

0	1	2	3	4	...	N-1
12	25	33	48	57	...	95

# Binary Search

- Search for 76

$\text{mid} = (0 + 7) / 2 = 3$  (go to somewhere in the middle)

$\text{array}[3] \neq 76$  (compare the values)

$\text{array}[3] < 76$  (in second half)

0	1	2	3	4	5	6	7
12	25	33	48	57	76	88	95

$\text{mid} = (4 + 7) / 2 = 5$  (go to middle)

$\text{array}[5] == 76$  (compare the values)

found, return 5

4	5	6	7
57	76	88	95

# Binary Search

- Search for 80

$\text{mid} = (0 + 7) / 2 = 3$

$\text{array}[3] \neq 80$

$\text{array}[3] < 80$

0	1	2	3	4	5	6	7
12	25	33	48	57	76	88	95

$\text{mid} = (4 + 7) / 2 = 5$

$\text{array}[5] \neq 80$

$\text{array}[5] < 80$

4	5	6	7
57	76	88	95

$\text{mid} = (6 + 7) / 2 = 6$

$\text{array}[6] \neq 80$

$\text{array}[6] > 80$

6	7
88	95

Not found (first > last) , return -1

# Binary Search

Consider the following method

```
int search(itemType[] array, int n, ItemType target)
```

Algorithm:

- Set first to start of array.

- Set last to end of array

- While (first <= last)

  - Set mid = (first + last)/ 2

  - If item at mid is equal to target

    - Item found, return mid

  - Else if target is smaller than item at mid

    - Set last = mid -1

  - Else

    - Set first = mid +1

End of search, item not found. Return -1

# Binary Search

```
func binarySearch(arr []int, n int, target int) int {  
    first := 0  
    last := n  
  
    for first <= last {  
  
        mid := (first + last) / 2  
        if arr[mid] == target { // found  
            return mid  
        } else {  
            if target < arr[mid] {  
                last = mid - 1 // search first half  
            } else {  
                first = mid + 1 // search second half  
            }  
        }  
    }  
    //end of for  
    return -1 // not found  
}
```

# Binary Search - Sorted

- In terms of efficiency for unsorted array
  - Worst number of searches =  $\log_2 N$
  - Average number of comparisons =  $(\log_2 N)/2$

0	1	2	3	4	. . .	N-1
12	25	33	48	57	...	95

# Sequential vs Binary

N	Sequential Search $O(n)$	Binary Search $O(\log^2 n)$
1	1	$n=2 \rightarrow 1$
4	4	2
8	8	3
16	16	4
32	32	5
64	64	6
128	128	7
256	256	8
512	512	9
1024	1024	10
1000000	1000000	20



# Search

Search	Array	Linked-List
Sequential	✓	✓
Binary	✓	✗

- Sequential Search can be used to search for data in arrays and linked-lists  
Binary Search can be used to search for data in arrays but not linked-lists
- Using Binary search on arrays instead of Sequential search
  - Improves the time performance from  $O(n)$  to  $O(\log n)$ .
- Binary search on linked list
  - Locating the mid in every repetition is non-trivial
  - Traversal from the first to the middle node is necessary, taking  $O(n)$  time
  - Eventually gives  $O(n \log n)$ , with  $\log n$  recursive calls/repetitions.

# Learning Objectives – Mod 11

At the end of the course, participants should be able to:

- Define what a sort algorithm is.
- Examine the different types of sort algorithm.
- Demonstrate the different usage of the sort algorithm.

# Sorting

- **Sorting**

- a process that organizes a collection of data
- Either ascending or descending order.

- **Sort Key**

- part of the data item that we would consider during sorting.
  - sorting VIP customers based on amount spent
  - sorting hotel room bookings based on prices
  - sorting popular videos based on number of views

# Sorting

- Types of Sorting
  - Selection
  - Insertion
  - Merge
  - Quick
  - Radix

# Selection Sort

- *Select the largest item, put in its correct place by swapping*
  - Find the *next largest item*, put in its correct place, and so on until array is sorted
  - To put an element in its correct place, it *swaps* position with that element in that location.
  - The array will have a sorted section that grows from end of array and rest of array remain unsorted.
- \* can also sort by smallest item*

# Selection Sort

Example of a selection sort of an array of five integers.

Shaded elements are selected;  
boldface elements are in order.

Initial array:

29	10	14	<b>37</b>	13
----	----	----	-----------	----

After 1<sup>st</sup> swap:

<b>29</b>	10	14	13	<b>37</b>
-----------	----	----	----	-----------

After 2<sup>nd</sup> swap:

13	10	<b>14</b>	<b>29</b>	<b>37</b>
----	----	-----------	-----------	-----------

After 3<sup>rd</sup> swap:

<b>13</b>	10	<b>14</b>	<b>29</b>	<b>37</b>
-----------	----	-----------	-----------	-----------

After 4<sup>th</sup> swap:

<b>10</b>	<b>13</b>	<b>14</b>	<b>29</b>	<b>37</b>
-----------	-----------	-----------	-----------	-----------

Source:

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# Selection Sort

Consider method

```
void selectionSort(ItemType array[], int n)
```

Algorithm:

```
    let last be index of the last item in the subarray of items yet to be sorted
```

```
    for (last = n-1; last >= 1; last--)  
    {  
        select largest item in array[0..last]  
        swap largest item with array[last]  
    }
```

# Selection Sort

```
func selectionSort(arr []int, n int) {  
    for last := n - 1; last >= 1; last-- {  
        // select largest item in array[0..last]  
        largest := indexOfLargest(arr, last+1)  
  
        // swap largest item array[largest] with array[last]  
        swap(&arr[largest], &arr[last])  
    }  
}
```



# Selection Sort

```
func indexOfLargest(arr []int, n int) int {  
    largestIndex := 0 // index of largest item  
    for i := 1; i < n; i++ {  
        if arr[i] > arr[largestIndex] {  
            largestIndex = i  
        }  
    }  
    return largestIndex  
}
```

```
func swap(x *int, y *int) {  
    temp := *x  
    *x = *y  
    *y = temp  
}
```

# Selection Sort

- Worst case:  $O(n^2)$ 
  - *a loop for finding largest within a loop for selection sort, with number of iterations depending on  $n$*
- Average case:  $O(n^2)$
- Does not depend on the initial arrangement of the data
- Only appropriate for *small  $n$*

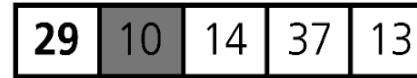
# Insertion Sort

- Partition the array into two regions: **sorted** and **unsorted**
- Take each item from the **unsorted** region and **insert** it into its correct order in the **sorted** region (need to shift elements)

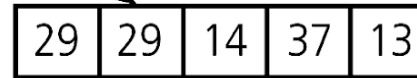
# Insertion Sort

- Example of an insertion sort of an array of five integers.

Initial array:



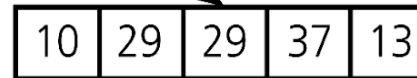
Copy 10



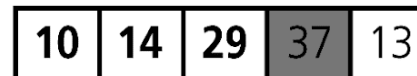
Shift 29



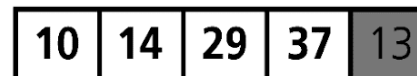
Insert 10; copy 14



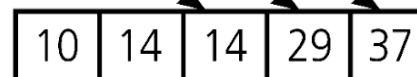
Shift 29



Insert 14; copy 37, insert 37 on top of itself

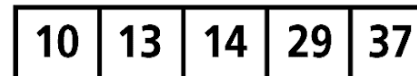


Copy 13



Shift 37, 29, 14

Sorted array:



Insert 13

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# Insertion Sort

Consider method

```
void insertionSort(ItemType array[], int n)
```

Algorithm:

```
    let unsorted be first index of the unsorted region
```

```
    in general, sorted region is array[0..unsorted-1],  
    unsorted region is array[unsorted..n-1]
```

```
    for (int unsorted = 1; unsorted < n; unsorted++)  
    {
```

```
        let nextItem be first item in unsorted region  
        let loc be index of insertion in the sorted region,  
        find the loc in sorted portion for nextItem;  
        shift, if necessary, to make room
```

```
    }
```

# Insertion Sort

```
func insertionSort(arr []int, n int) {  
    for i := 1; i < n; i++ { // sorted: 0..i-1  unsorted: i..n-1  
  
        // 1. copy data  
        data := arr[i] // 1st elt in unsorted region  
  
        // 2. shift larger data to the right  
        last := i  
        for (last > 0) && (arr[last-1] > data) {  
            arr[last] = arr[last-1]  
            last--  
        }  
  
        // 3. insert data  
        arr[last] = data  
    }  
}
```

# Insertion Sort

- Worst case:  $O(n^2)$ 
  - *(a loop within a loop with no. of iterations depending on  $n$ )*
- Average case:  $O(n^2)$
- Best case, when items are already sorted:  $O(n)$ 
  - *( $array[loc-1] > nextItem$  condition is always false)*
- Appropriate for small arrays due to its simplicity
- Prohibitively inefficient for large arrays

# Insertion Sort

- Strengths

- Good when unordered list is mostly sorted
- Need minimum time to verify if list is sorted
- Better with pointer-based implementation (no movement of data)

- Weaknesses

- Every new insertion requires movements/shifting for some inserted items in ordered portion
- When each slot contains large record => movement is expensive
- Array-based implementation is less suitable.



# Merge Sort

- A recursive sorting algorithm

## Strategy

- *Divide* an array into 2 halves
- Sort each half
- *Merge* the sorted halves into one sorted array
- *Divide-and-conquer*

# Merge Sort

- Example of a merge sort with an array of intergers

theArray: 

8	1	4	3	2
---	---	---	---	---

Divide the array in half

1	4	8
---	---	---

2	3
---	---

Sort the halves

Merge the halves:

- $1 < 2$ , so move 1 from left half to tempArray
- $4 > 2$ , so move 2 from right half to tempArray
- $4 > 3$ , so move 3 from right half to tempArray
- Right half is finished, so move rest of left half to tempArray

Temporary array  
tempArray:

1	2	3	4	8
---	---	---	---	---

Copy temporary array back into original array

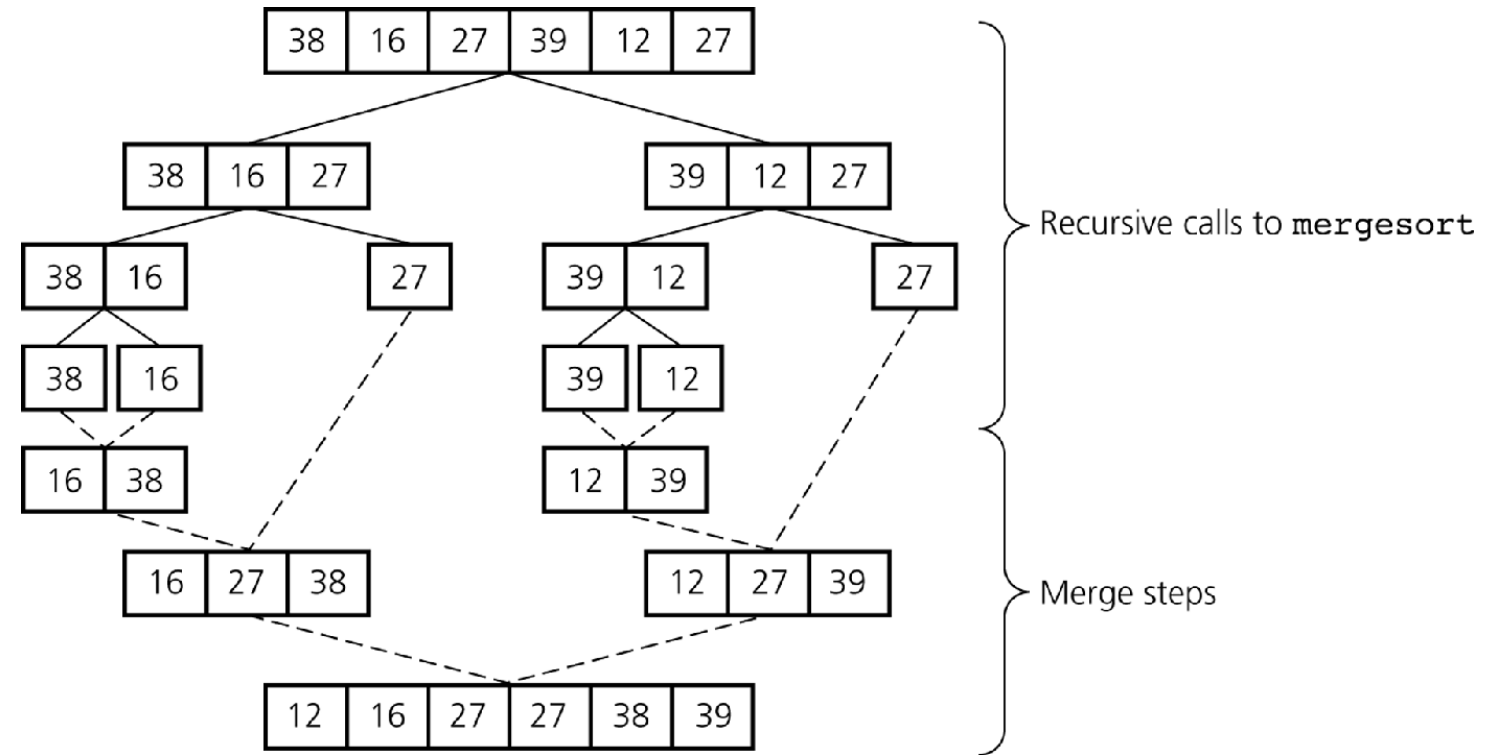
theArray:

1	2	3	4	8
---	---	---	---	---

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# Merge Sort

- Further examples



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# Merge Sort

Consider method

```
void mergeSort(ItemType array[], int first, int last)
```

Algorithm:

```
if (first < last)  // more than 1 items
{
    find index of midpoint
    sort left half array[first..mid]      // recursively
    sort right half array[mid+1..last]    // recursively
    merge the two halves
}
```

# Merge Sort

```
func mergeSort(arr []int, first int, last int) {  
    if first < last { // more than 1 items  
  
        mid := (first + last) / 2    // index of midpoint  
        mergeSort(arr, first, mid)  // sort left half  
        mergeSort(arr, mid+1, last) // sort right half  
        merge(arr, first, mid, last) // merge the two halves  
    }  
}
```

# Merge Sort

```
func merge(arr []int, first int, mid int, last int) {  
    const maxSize int = 8  
    var tempArr [maxSize]int // temporary array  
    // initialize the local indexes to indicate the subarrays  
    first1 := first // beginning of first subarray  
    last1 := mid // end of first subarray  
    first2 := mid + 1 // beginning of second subarray  
    last2 := last // end of second subarray
```

# Merge Sort

```
// while both subarrays are not empty, copy the
// smaller item into the temporary array
index := first1 // next available location in tempArray
for (first1 <= last1) && (first2 <= last2) {

    if arr[first1] < arr[first2] {
        tempArr[index] = arr[first1]
        first1++
    } else {
        tempArr[index] = arr[first2]
        first2++
    }
    index++
}
```

# Merge Sort

```
// finish off the nonempty subarray
// finish off the first subarray, if necessary
for first1 <= last1 {
    tempArr[index] = arr[first1]
    first1++
    index++
}
// finish off the second subarray, if necessary
for first2 <= last2 {
    tempArr[index] = arr[first2]
    first2++
    index++
}
```



# Merge Sort

```
// copy the result back into the original array
for index = first; index <= last; index++ {
    arr[index] = tempArr[index]
}
}
```

# Merge Sort

- Performance is independent of initial order of the array items.
- Worst case:  $O(n \log_2 n)$
- Average case:  $O(n \log_2 n)$ 
  - $\log_2 n$  levels of recursive calls
  - $n$  key comparisons made at each level

# Merge Sort

## Strengths

- an extremely **fast algorithm**, good runtime behaviour
- implements easily when using pointer-based implementation

## Weaknesses

- for array implementation, need **auxillary storage**
- requires data movements again during merging
- costly to implement for array implementation

# Quick Sort

- Fastest general purpose in-memory sorting algorithm in the average case

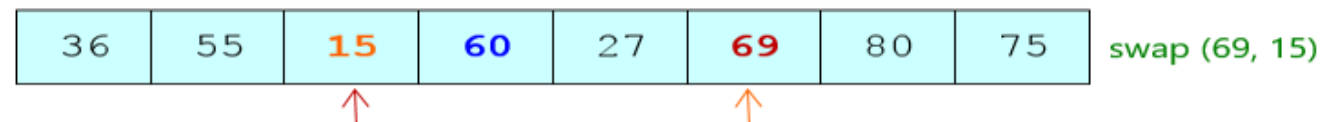
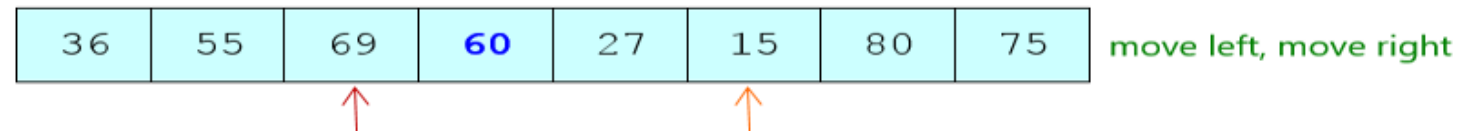
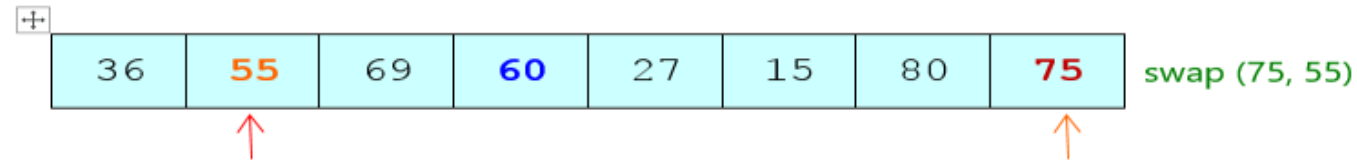
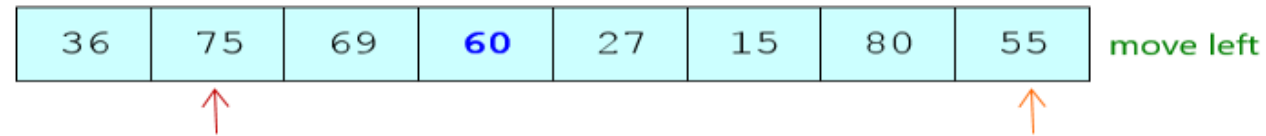
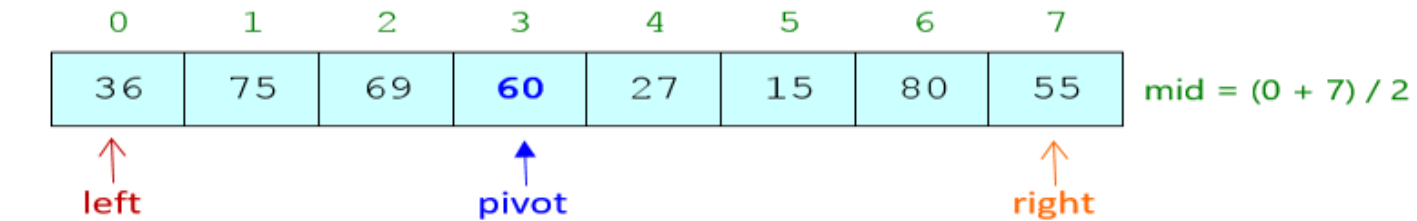
## Strategy

- A *divide-and-conquer* algorithm
  - **Partition** the array into 2 partitions about the pivot
    - Choose an element as **pivot** (*can be any element!*)
    - Elements in each part are rearranged such that
      - items  $<$  pivot on left of pivot (left partition)
      - items  $\geq$  pivot on right of pivot (right partition)
- **Recursively** partition the left and right partitions
  - (until the partition has less than 2 elements)

# Quick Sort

## Partition

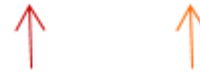
Pivot is middle element



# Quick Sort

36	55	15	60	27	69	80	75
----	----	----	----	----	----	----	----

move left, move right



36	55	15	27	60	69	80	75
----	----	----	----	----	----	----	----

swap (60, 27)



36	55	15	27	60	69	80	75
----	----	----	----	----	----	----	----

move left, move right  
stop, return idx (4)



36	55	15	27	60	69	80	75
----	----	----	----	----	----	----	----

left partition

right partition

# Quick Sort

- Partition

36	55	15	27	60	69	80	75
----	----	----	----	----	----	----	----

left partition

right partition

- Call quicksort to sort:
  - Left partition recursively
  - Right partition recursively

# Quick Sort

Consider method

```
void quickSort(ItemType array[], int left, int right)
```

Algorithm:

```
if (first < last)  // more than 1 items
{
    create the partition: S1, pivot, S2
    sort partition S1 //recursively
    sort partition S2 //recursively
}
```



# Quick Sort

```
func quickSort(array []int, left int, right int) {  
    if left < right { // stop when there are less than 2 elements  
  
        pivotIdx := partition(array, left, right)  
        quickSort(array, left, pivotIdx-1) // sort left partition  
        quickSort(array, pivotIdx+1, right) // sort right partition  
    }  
}
```

# Quick Sort

```
func partition(arr []int, left int, right int) int {  
    pivot := arr[(left+right)/2] // can be any element!  
    for left < right { // stop when there are less than 2 elements  
        for arr[left] < pivot { // move left index  
            left++  
        }  
        for arr[right] > pivot { // move right index  
            right--  
        }  
        if left < right { // swap the elements  
            temp := arr[left]  
            arr[left] = arr[right]  
            arr[right] = temp  
        }  
    }  
    return left // return the pivot index  
}
```

# Quick Sort

- Average case:  $O(n \log_2 n)$ 
  - $\log_2 n$  levels of recursive calls
  - $n$  key comparisons made at each level)
- Best case:  $O(n \log_2 n)$ 
  - When *pivot* always splits array into *equal halves*
- Worst case:  $O(n^2)$ 
  - When at each recursive call, the *smallest or biggest item* is chosen as the *pivot*

# Quick Sort

## Strengths

- *Fast on average*
- No merging required
- *Best case* if pivot always splits data into *equal halves*

## Weaknesses

- Pivot need to be chosen carefully
- Performs badly when
  - *array is already sorted* and *pivot is largest or smallest* element
  - *n is small*
- *due to overhead in recursive calls*

# Radix Sort

- Treats each data element as a character **string**
- Repeatedly organizes the data into groups according to the  $i^{\text{th}}$  **character/digit** in each element

# Radix Sort

0123	2154	0222	0004	0283	1560	1061	2150	Original data
(156 <b>0</b> , 215 <b>0</b> )	(106 <b>1</b> )	(022 <b>2</b> )	(012 <b>3</b> , 028 <b>3</b> )	(215 <b>4</b> , 000 <b>4</b> )	Group by 4 <sup>th</sup> digit			
1560	2150	1061	0222	0123	0283	2154	0004	Combined
(00 <b>0</b> 4)	(02 <b>2</b> 2, 01 <b>2</b> 3)	(21 <b>5</b> 0, 21 <b>5</b> 4)	(15 <b>6</b> 0, 10 <b>6</b> 1)	(02 <b>8</b> 3)	Group by 3 <sup>rd</sup> digit			
0004	0222	0123	2150	2154	1560	1061	0283	Combined
(0 <b>0</b> 04, 1 <b>0</b> 61)	(0 <b>1</b> 23, 2 <b>1</b> 50, 2 <b>1</b> 54)	(0 <b>2</b> 22, 0 <b>2</b> 83)	(1 <b>5</b> 60)	Group by 2 <sup>nd</sup> digit				
0004	1061	0123	2150	2154	0222	0283	1560	Combined
( <b>0</b> 004, <b>0</b> 123, <b>0</b> 222, <b>0</b> 283)	( <b>1</b> 061, <b>1</b> 560)	( <b>2</b> 150, <b>2</b> 154)	Group by 1 <sup>st</sup> digit					
0004	0123	0222	0283	1061	1560	2150	2154	Combined

# Radix Sort

Consider method

```
void radixSort(ItemType array[], int n, int digit)
```

Algorithm:

```
for(j = digit down to 1)
{
    initialize 10 groups to empty
    initialize a counter for each group to 0

    for (i = 0 through n-1)
    {
        k = jth digit of array[i]
        place array[i] at the end of group k
        increase kth counter by 1
    }
    replace the items in theArray with all the items in group 0,
    followed by group 1 and so on.
}
```

# Radix Sort

- Distributing the elements according to value of digit :  $O(n)$ 
  - *n is the number of iterations*
- Combining the elements at the last step :  $O(n)$
- Involves distributing and combining for no of times equal to no of digits
  - *which is a constant, not dependent on n*
- Overall complexity is  $O(n)$ 
  - key is treated as string and must have the same length
  - restrictive, not universal



## Comparing Sorting Algorithm

- Approximate complexity of the different types of sorting algorithm.

Time Complexity	Worse Case	Average Case
Selection Sort	$O(n^2)$	$O(n^2)$
Insertion Sort	$O(n^2)$	$O(n^2)$
Merge Sort	$O(n \log^2 n)$	$O(n \log^2 n)$
Quick Sort	$O(n^2)$	$O(n \log^2 n)$
Radix Sort	$O(n)$	$O(n)$
Tree Sort	$O(n^2)$	$O(n \log^2 n)$