ESS385 Systems Analysis Final Project

Case VI: Vegetation Growth under Water-limited Conditions and Desertification

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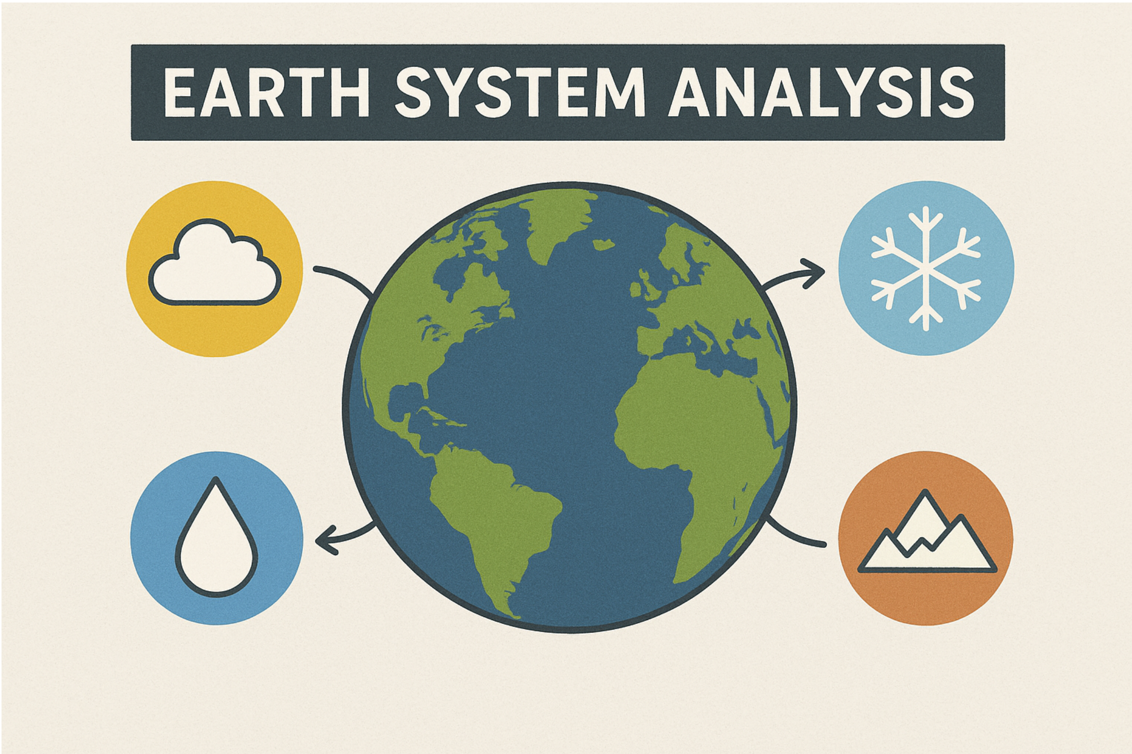
November 2025



Contents

1 Introduction

1.1 General Overview

During the ESS385 Systems Analysis block course we were introduced to the basic theoretical as well as practical methodology of how to grasp, describe and model earth relevant systems, such as ecological models like the Lotka-Voltera interaction model. From descriptions of how certain species interact (e.g. compete, regulate, etc.) we learned how to derive mathematical equations and how to further implement them in the MATLAB programming language to model time series and scatter plots. For the last part of this block course, we are now asked to perform a whole workflow without structured guidance in order to show our skills on a newly designed earth system analysis.

1.2 Analysis Overview

For the final assignment, the Earth subsystem of the terrestrial biosphere is studied, with a specific focus on vegetation growth under water-limited conditions. The phenomenon of interest is the interaction between vegetation dynamics and desertification processes. According to the United Nations Convention to combat Desertification that same is defined as the "... land degradation in arid, semi-arid and dry sub-humid areas resulting from various factors, including climatic variations and human activities" ("Article 1. Use of Terms", n.d.). Understanding vegetation growth under such conditions is essential for analyzing how ecosystems respond to environmental changes such as climate variability. This topic is closely related to the challenges that we as humans face on Earth today. Ongoing and increasing warmth of the Earth's temperature related to human-made climate change foster desertification in already dry areas (Burrell et al., 2020). To link our analysis to potential threats of the biosphere related to dry ecosystems and ongoing desertification due to climate change the following research questions were studied:

- How does varying the infiltration rate of bare soil ($\mathbf{W_0}$) influence the stability and equilibrium levels of soil water and vegetation in the water–vegetation system?
- How does changing the plant-density sensitivity of infiltration ($\mathbf{k_2}$) affect the strength of vegetation–water feedbacks and the potential for stability or vegetation collapse?
- How does a changing non-constant precipitation influence and foster change in vegetation presence as well as the stability of present vegetation?

The above stated research questions are of different "type". The first two cover the main analysis of the mathematical model that was derived from the connection and feedback between precipitation, soil water availability and vegetation growth. However, the last question aims to connect the two other questions and link it to the ongoing desertification process as well as highly fluctuant precipitation. It is designed on a more "reality-based" modeling, trying to estimate and show potential changes in vegetation growth as well as soil water availability over time using changing precipitation.

2 Studied System

2.1 Components

Table 1: Description of state variables and parameters used in the model.

	State variable/Parameter	units	dimensions
P	State variable (amount of vegetation)	g/m^2	mass per area
W	State variable (soil water availability)	mm	height
b	Parameter (relative respiration rate)	$days^{-1}$	$time^{-1}$
d	Parameter (relative death rate)	$days^{-1}$	$time^{-1}$
$\mathbf{r_w}$	Parameter (relative water loss rate)	$days^{-1}$	$time^{-1}$
$\mathbf{c_{max}}$	Parameter (uptake rate)	$L/(g \cdot day)$	volume per mass per time
PPT	Parameter (precipitation)	mm/day	height per time
e	Parameter (efficiency factor)	$(g/m^2)/mm$	biomass per water height
$\mathbf{W_0}$	Parameter (infiltration rate of bare soil without plants)	"dimensionless"	"dimensionless"
$\mathbf{k_1}$	Parameter (soil water availability for half max. uptake rate)	mm	height
$\mathbf{k_2}$	Parameter (plant density for half max. infiltration rate)	g/m^2	mass per area
t	time	days	time

2.2 System Assumptions

In order for our mathematical analysis and modeling using MATLAB it is essential to make some system assumptions to simplify the model. Only trough these are we able to model ongoing processes related to plant density, water availability and desertification. Among others the most important assumptions are the following:

- Constant parameters: Except for the last modeling process the parameters are assumed to be fix resulting in a non-changing environment. Thus, stochastic events such as climate or weather-related phenomena were not taken into account.
- Single water and plant dependency: Even tough we have a factor e that marks the efficiency of plants of turning water into biomass as well as respiration and mortality rate, the system essentially only depends on the water availability and plant density. Other factors, such as nutrients, were assumed to have no influence on the system.
- Non-consideration of rooting depth: In our model we did not take into account the depth of the plants roots. However this can be misleading as soil water might not be available for shorter rooting depths. This can overestimate the soil water availability for plants.
- No human impact: Despite being highly important for humans as well, our model does not take into account any human activities such as water extraction or any kind of plant use.

2.3 Stock and Flow Diagram

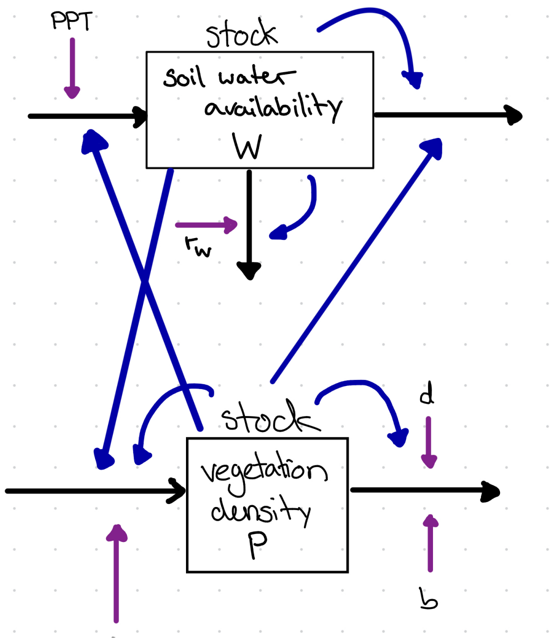


Figure 1: Stock and Flow Diagram of the Studied System

3 Mathematical Model

3.1 Mathematical Equations

As stated in Section 1 Introduction, the idea behind this project was to model the interaction between two different variables using a differential equation, that is later implemented in a MATLAB model. For our later code implementation we used the following equations to model the change of soil water content (1) and plant density (2).

$$\frac{dW}{dt} = PPT \cdot f_2(P) - f_1(W) \cdot P - r_w W \tag{1}$$

$$\frac{dP}{dt} = e \cdot f_1(W) \cdot P - (d + b) \cdot P \tag{2}$$

The water-vegetation relationship is described by the functions f_1(W) and f_2(P), which are described as followed:

$$f_1(W) = c_{max} \cdot \frac{W}{W + k_1}$$

$$f_2(P) = \frac{P + k_2 \cdot W_0}{P + k_2}$$

3.2 Unit consistency check

The units and dimensions were derived from literature and the information given. To verify that they were identified correctly, a unit consistency check was performed. The units of the parameters were substituted into the model equations, and it was confirmed that all terms have the same overall units, ensuring dimensional consistency.

$$\frac{dW}{dt} = PPT \cdot f_2(P) - f_1(W) \cdot P - r_w W \qquad \text{[Have all terms in mm/day]}$$

$$\frac{[mm]}{[day]} = \frac{[mm]}{[day]} \cdot \frac{\frac{[g]}{[m^2]} + \frac{[g]}{[m^2]}}{\frac{[g]}{[m^2]} + \frac{[g]}{[m^2]}} - \frac{[L]}{[g] \cdot [day]} \cdot \frac{[mm]}{[mm] + [mm]} \cdot \frac{[g]}{[m^2]} - \frac{[mm]}{[day]} \qquad \text{[Plug in units]}$$

$$\frac{[mm]}{[day]} = \frac{[mm]}{[day]} - \frac{[L]}{[m^2] \cdot [day]} - \frac{[mm]}{[day]} \qquad \text{[Simplify all the fractional parts]}$$

This results in the correct units for all three terms.

$$\frac{dP}{dt} = e \cdot f_1(W) \cdot P - (d + b) \cdot P \qquad \text{[Have all terms in g/m}^2\text{/day]}$$

$$\frac{\frac{[g]}{[m^2]}}{[day]} = \frac{\frac{[g]}{[m^2]}}{[mm]} \cdot \frac{[L]}{[g]\,[day]} \cdot \frac{[mm]}{[mm] + [mm]} \cdot \frac{[g]}{[m^2]} - \left(\frac{1}{[day]} + \frac{1}{[day]}\right) \cdot \frac{[g]}{[m^2]} \qquad \text{[Plug in units]}$$

$$\frac{\frac{[g]}{[m^2]}}{[day]} = \frac{\frac{[g]}{[m^2]}}{[mm]} \cdot \frac{[mm]}{[day]} - \frac{\frac{[g]}{[m^2]}}{[day]} \qquad \text{[Simplify]}$$

This results in the correct units for all two terms.

3.3 Parameters

Reasonable starting values for the different parameters were obtained either from the script given or from literature. A list with all parameters is provided here:
$\mathbf{P}$ = 5-10 g/m^2 according to (Rietkerk et al., 2002)
$\mathbf{W}$ = 0.2 according to Model
$\mathbf{b}$ = 0.15-0.35 according to script
$\mathbf{d}$ = 0.1 according to script
$\mathbf{r_w}$ = 0.2 acccording to (Rietkerk et al., 2002)
$\mathbf{c_{max}}$ = 0.5 according to script
$\mathbf{PPT}$ = 0.65 mm/day according to (Moreno-de las Heras et al., 2015)
$\mathbf{e}$ = 10 (g/m^2)/mm according to script
$\mathbf{W_0}$ = 0.2-0.9 according to script
$\mathbf{k_1}$ = 3 mm according to script
$\mathbf{k_2}$ = 5-25 g/m^2 according to script

3.4 MATLAB implementation

Our MATLAB scripts and explanations are hosted on GitHub and can be found following this link

3.5 System Summary

The parameters described in Section 2.1 Components, together with the underlying assumptions, provide a solid foundation for constructing the model that estimates vegetation growth under water-limited conditions and assesses desertification processes. Since these parameters can vary over time and the system itself depends on these factors such as soil texture, it becomes important to examine how the model behaves when these parameters change.
In the following, it is outlined how the research questions introduced in Section 1.2 Analysis Overview are addressed by analyzing and comparing the model's behavior under varying parameter settings:

- How does varying the infiltration rate of bare soil ($\mathbf{W_0}$) influence the stability and equilibrium levels of soil water and vegetation in the water–vegetation system? The parameter $\mathbf{W_0}$ represents the amount of water that would infiltrate bare soil if there weren't any plants. Changing this parameter between different simulations, can be seen as a good way to investigate the infiltration of precipitation into the soil.

- How does changing the plant-density sensitivity of infiltration ($\mathbf{k_2}$) affect the strength of vegetation–water feedbacks and the potential for stability or vegetation collapse? The parameter $\mathbf{k_2}$ is the plant density where the system reaches half of the maximum infiltration rate. By manipulating it from "high" ($\mathbf{k_2}$ = 5) to "low" ($\mathbf{k_2}$ = 25) the influence of vegetation on the water infiltration can be simulated.

- How does a changing non-constant precipitation influence and foster change in vegetation presence as well as the stability of present vegetation? To investigate the final research question, a modification of the model was required. Instead of using a single continuous precipitation value, the model now draws a random daily precipitation amount between 0 and 1 mm. This results in an average precipitation of 0.5 mm per day, which is less than in the previous two models. This adjustment allows simulation of a more realistic precipitation pattern while simultaneously representing reduced rainfall conditions intended to mimic a mild desertification process. Under the limited model that was used, this adjustments formed an appropriate approach to investigate the last research question.

4 Results

4.1 State Space Portraits

To derive the null isoclines of the system, both equations (1) and (2) were set equal to zero and solved for their respective variables. However, the null-isocline for the vegetation cover equation (2) has got two solutions. One is trivially being when P = 0, which means that there is no vegetation present. The second one can be derived as the following isocline and represents a vertical line:

$$W = \frac{k_1}{\frac{e \cdot c_{\max}}{d+b} - 1}$$

For the null-isocline of the soil water equation (2) the mathematical process of setting the equation to 0 can not be solved manually. Thus, another approach was conducted using MATLAB. First all helper functions where pluged in properly:

$$\frac{dW}{dt} = PPT \cdot \frac{P + k_2 \cdot W_0}{P + k_2} - c_{max} \cdot \frac{W}{W + k_1} \cdot P - r_w W$$

After that a "point field" over an area of interest was generated. This area of interest had the dimensions of 0.01 to 50 on the y-axis and 0.01 to 1 on the x-axis to cover a broad area of potential soil water-plant cover pairs. For every point the differential equation was then evaluated using plausible input values according to Section 3.3 Parameters and all the points, where the evaluation equaled 0 were stored. This procedure allowed for a computational extraction of points, where the null-isocline is present. Plotting these points resulted in the final null-isocline for the soil water content and can be seen in Figure 2 State Space Portrait just below.

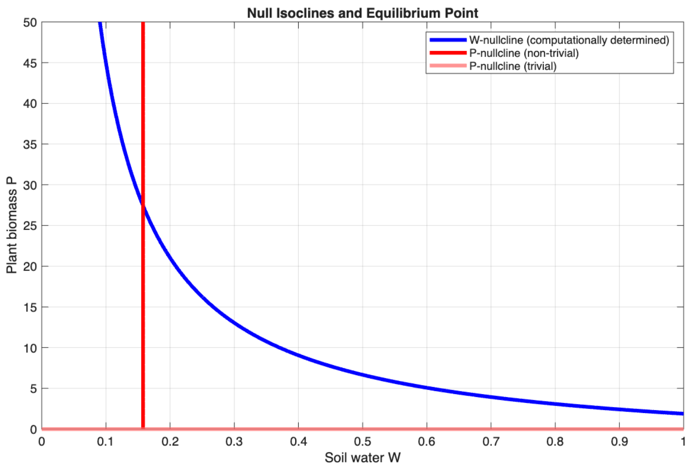


Figure 2: State Space Portrait with constant Precipitation showing a stable equilibrium

The stability of this equilibrium can be assessed by evaluating the derivatives on either side of the null-isoclines. For the non-trivial P null-isocline, we observe that at tiny values of both P and W, plant density decreases slightly. By definition, this changes to an increase on the right side of the vertical null-isocline. Determining the rate of change along the W null-isocline is more challenging, as it can only be described implicitly. This complexity arises because soil water dynamics are influenced not only by external precipitation but also by internal factors, such as plant density, the infiltration rate of bare soil $\mathbf{W_0}$ and the contribution of plant density to half of the maximum infiltration rate $\mathbf{k_2}$. Unlike simpler cases with linear null-isoclines, here both the magnitude and direction of change are not easily determined. However, plugging in values at different regions in the state-space grid suggest growing soil water content to the left and shrinking soil water content to the right. This results in some kind of a (semi) spiral movement towards the equilibrium. Nonetheless, the magnitudes of the changes can vary substantially, making the identification of the equilibrium challenging. Hence, another simulation was used to safely determine the stability of the equilibrium, which can be seen in Figure 3 Sensitivity Analysis on the following page.

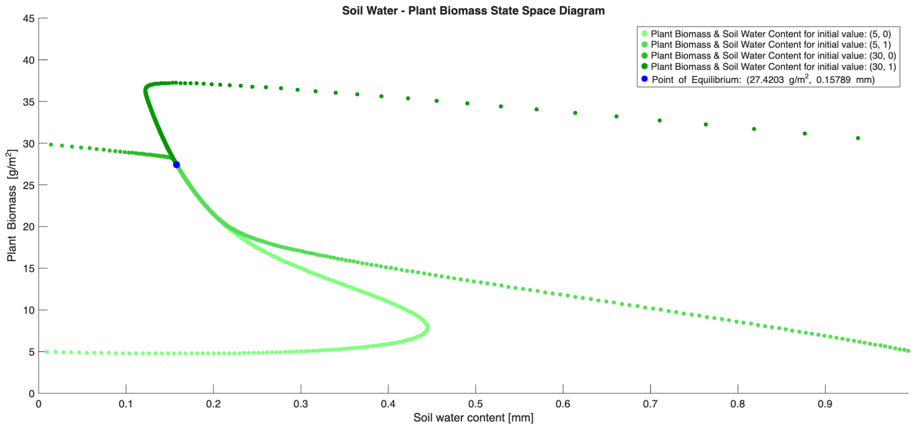


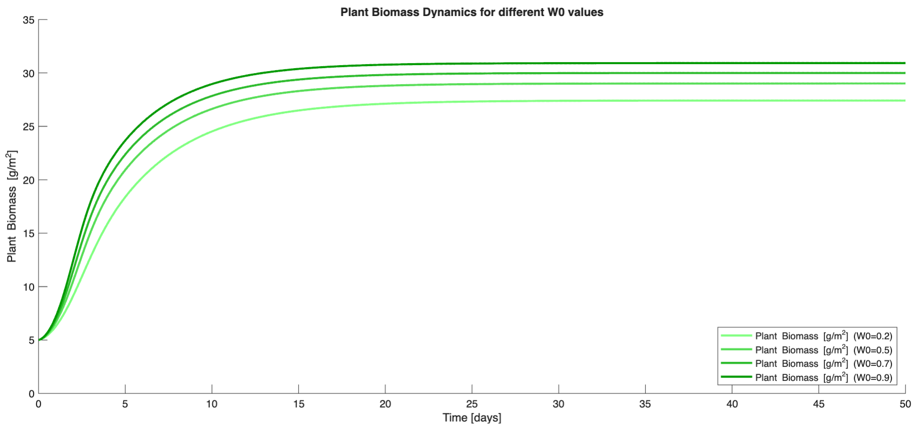
Figure 3: Modeled sensitivity analysis showing stable equilibrium for constant precipitation.

The sensitivity analysis confirms that the equilibrium identified above is indeed stable. The trajectories toward the equilibrium, however, do not follow a straight path but instead exhibit a semi-spiral pattern, particularly for low initial values of both plant biomass and soil water (e.g., P=5, W=0). This pattern reflects an initial rapid increase in soil water, followed by a subsequent rise in plant biomass. On the other hand, initial conditions with low plant biomass and high soil water show the opposite behavior, with a rapid decline in soil water accompanied by an increase in biomass. Regardless of the initial conditions, it is evident that changes in soil water, and thus the magnitude of dW/dt, dominate the early dynamics of the system. As a consequence, trajectories initially evolve mainly in the horizontal direction. As plant biomass responds more slowly to increasing soil water, the system gradually shifts toward a more vertical movement, eventually converging to the equilibrium. The reasons behind this characteristics will form the basis for further analysis of the model and will be explained in more detail in Section 5 Discussion.
Furthermore it has to be said, that this equilibrium only counts for constant precipitation. As stated in Section 3.5 one analysis will cover the influence of a changing precipitation rate on the system. The stability outcome for a non-constant precipitation rate might be different to the one with a constant precipitation rate.

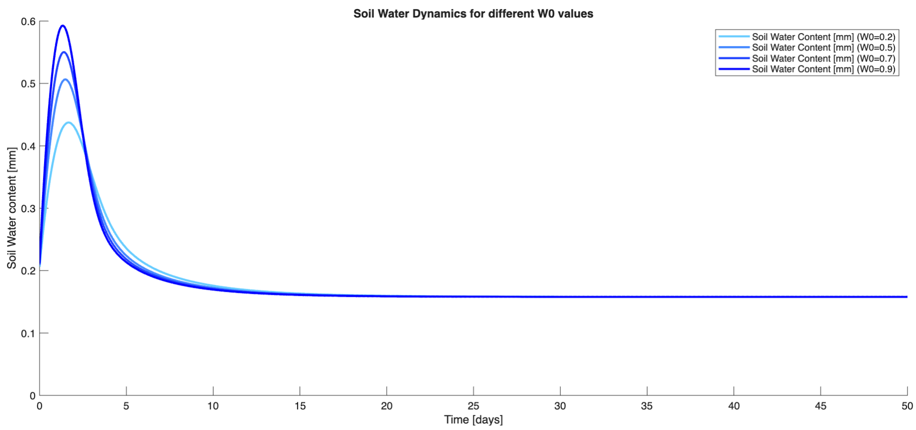
4.2 Numerical Simulations

To run the model a time span of 50 days was applied as this seemed to be a reasonable time span to analyze the development. Three different analysis runs were performed, each of them analyzing the behavior of changing parameters according to the explanations stated in Section 3.5 System Summary. The results are first presented as bare plots and later explained and analyzed in further detail in Section 5 Discussion.

4.2.1 Results of Water Infiltration



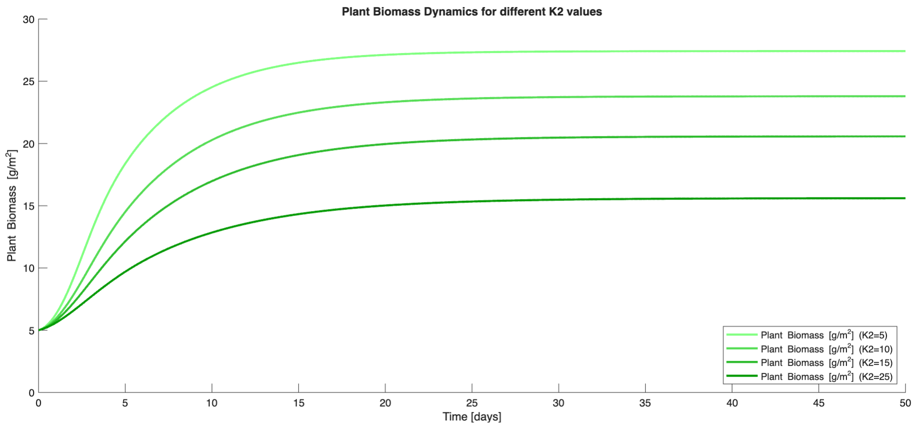
(a) Plant Biomass over Time using different W0 rates.



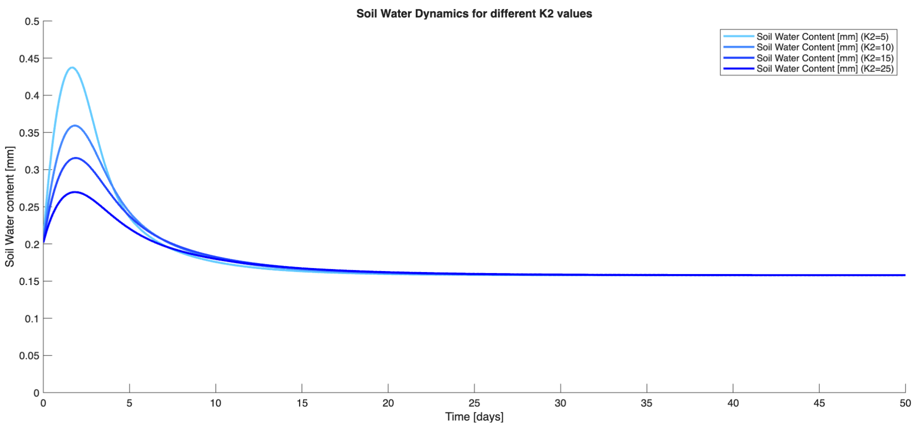
(b) Soil Water content over Time using different W0 rates.

Figure 4: Plant biomass (top) and soil water content (bottom) as a function of different bare soil infiltration rates in the coupled vegetation–soil water model.

4.2.2 Results of the Vegetation Influence on Water Infiltration



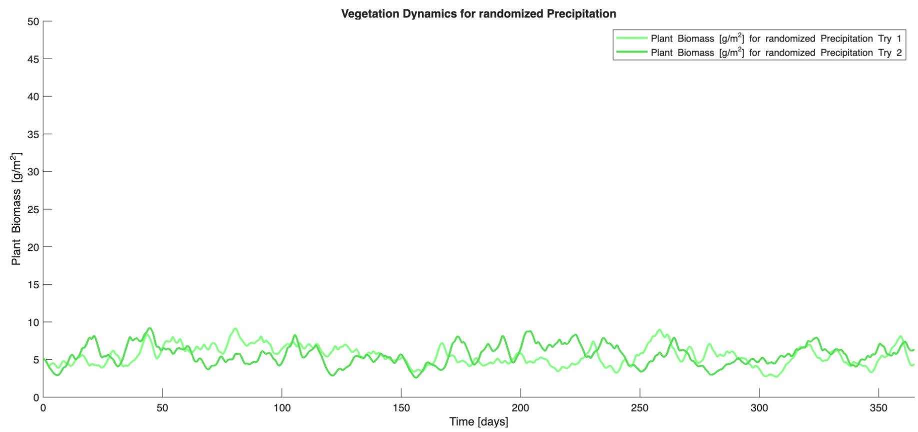
(a) Plant Biomass over Time using different k2 values.



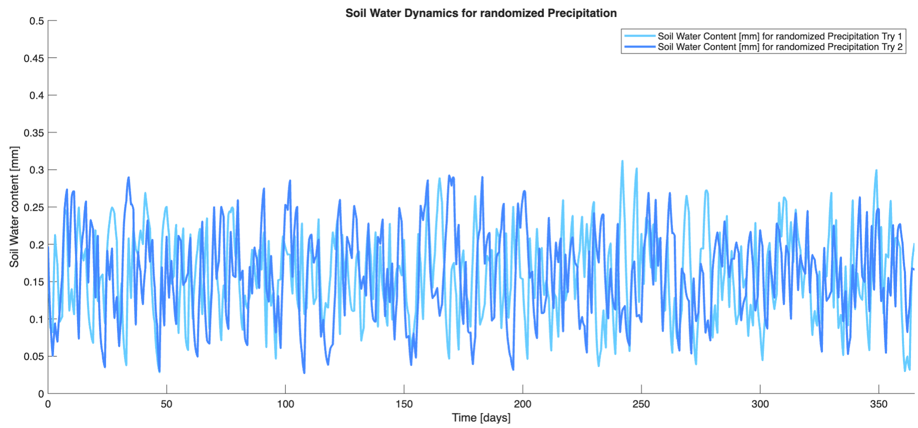
(b) Soil Water content over Time using different k2 values.

Figure 5: Plant biomass (top) and soil water content (bottom) as a function of different vegetation biomass influence values on the precipitation infiltration in the coupled vegetation–soil water model.

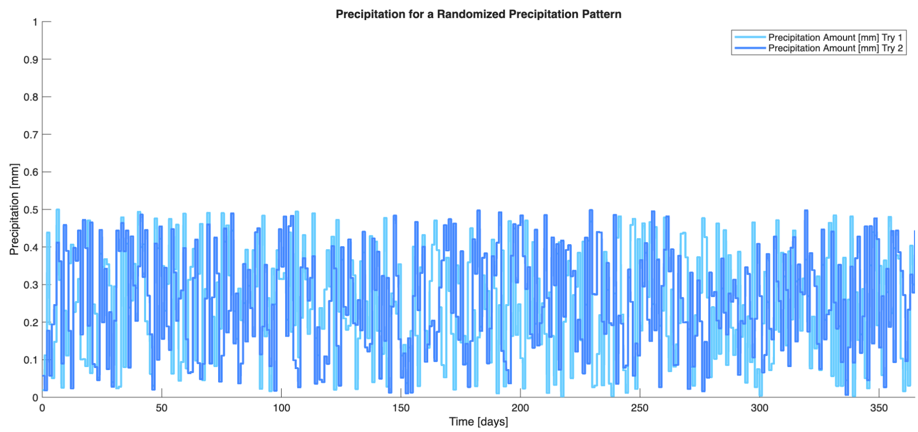
4.2.3 Results of Changing Precipitation Process



(a) Plant Biomass over Time using randomized changing Precipitation.



(b) Soil Water content over Time using randomized changing Precipitation.



(c) Soil Water content over Time using different k2 values.

Figure 6: Plant biomass (top), soil water content (middle) and precipitation (bottom) as a function of randomized changing precipitation in the coupled vegetation–soil water model. The precipitation is ranked between 0 and 1 mm per day over one year resulting in 0 - 365 mm of precipitation per year.

5 Discussion

5.1 Plot Analysis

5.1.1 Analysis of Water Infiltration (W_0)

The results indicate that higher water infiltration rates into bare soil (higher W_0) lead to an immediate increase in soil water availability and consequently to a similar increase in plant biomass. In the first few days the soil water curve is extremely steep, showing that the system responds very quickly to changes in infiltration. These results align with the results that were obtained in Section 4.1 State Space Portraits.

Further, it can be seen that the higher the W_0 the higher and earlier the peak in soil water availability is reached, followed by a faster decline. This can be explained by the interaction between soil water and the plant biomass. As the soil water increases with a higher infiltration rate, the plant biomass also increases rapidly, leading to a higher water consumption through the plants.

Both variables, the plant density as well as the soil water go toward an equilibrium. The soil water availability stabilizes for all different W_0 values at around 0.2 mm. Whereas with a higher W_0 plant densities reach a higher equilibrium. The equilibrium is only possible because the system assumes a constant precipitation.

5.1.2 Analysis of the Vegetation Influence on Water Infiltration (k_2)

The results show that for lower k_2 values (the plant density at which the infiltration rate is at half of the maximum value) the soil water availability reaches a higher peak than for higher values. For low k_2, soil water rises rapidly in the beginning, reaches a higher maximum, and then declines more quickly than in scenarios for higher k_2. In the end, all scenarios converge to a similar equilibrium soil-water level of approximately 0.2 mm.

Plant biomass shows a similar pattern. For lower k_2 values, a higher maximum in plant biomass is reached. Plant biomass increases rapidly in response to higher soil water availability. Over time, plant biomass stabilizes at different equilibrium levels depending on k_2, for lower k_2 values supporting higher long-term biomass.

5.1.3 Analysis of Changing Precipitation Over Time

The results show that with a changing precipitation over time, the soil water content as well as the plant biomass fluctuate strongly from day to day. This is a more realistic approach for a water limiting system like deserts, as constant daily precipitation is not very realistic.

Plant biomass fluctuates at a less high level than in the simplified models with constant precipitation. However, the plant system reacts less fluctuant than soil water, and its variations occur with a smoother, delayed response. In contrast, soil water content reacts almost immediately to daily rainfall amounts. It shows pronounced peaks and drops closely aligned with the daily precipitation. In Section 4.1 State Space Portraits it was already pointed out that using non-constant precipitation might not converge towards an equilibrium and this is indeed what can be observed in this model. Instead, both parameters, plant biomass and soil water availability stay fluctuant on a low level.

5.2 Answers to Research Questions

1. How does varying the infiltration rate of bare soil (W_0) influence the stability and equilibrium levels of soil water and vegetation in the water–vegetation system?

The model shows that infiltration strongly depends on W_0. When W_0 is high, precipitation falling on dry soil can infiltrate the soil very rapidly, leading to a fast increase in soil water content. In water limited systems, changing the infiltration rate has an instant, short-term effect on the soil water availability and with that on the plant density, even though long-term soil water levels converge. Thus, infiltration capacity is a key determinant of how much rainfall becomes available to plants.

2. How does changing the plant-density sensitivity of infiltration (k_2) affect the strength of vegetation–water feedbacks and the potential for stability or vegetation collapse?
Vegetation affects infiltration through the parameter k_2, which determines the plant density at which infiltration is at half of its maximum. Low k_2 values mean that lower plant biomass already leads to a higher infiltration rate, while high k_2 values delay this effect until vegetation is much denser. Although vegetation reduces the amount of soil water present as it becomes established, a low k_2 also strengthens the early coupling between vegetation and soil water, allowing plants to take advantage of initial water pulses and ultimately reach higher long-term biomass in the model.
3. How does a changing non-constant precipitation influence and foster change in vegetation presence as well as the stability of present vegetation?
With variable precipitation, soil water and vegetation do not reach a steady state. Soil water content fluctuates strongly with rainfall events, and vegetation responds more slowly and smoother, remaining at a variable level that is lower than with continuous rainfall events. This indicates that without a continuous water input, plant growth is more limited. Because vegetation influences infiltration and consumes water, these fluctuations reinforce each other, leading to a persistently dynamic system.

5.3 Interpretation and Conclusion

The comparison of the simulations varying W_0, k_2, and precipitation patterns shows how different components of the system control soil water availability and vegetation dynamics in this simplified model. Overall, the model shows that both parameters W_0 and k_2 have a strong positive effect on vegetation in water-limited systems. Higher bare soil infiltration rates (W_0) and a stronger vegetation influence on infiltration (lower k_2) support higher biomass by making water more rapidly available for plant growth.
For varying precipitation, the system shows a different response. Under these conditions an equilibrium can't be reached. Soil water responds almost immediately to rainfall variability, while biomass shows smoother, delayed reactions. Irregular rainfall leads to instability, lower plant density, and strong dependence on water supplies. This indicates that in water-limiting system a constant water input has a very huge effect on soil water content and plant density.
Overall it can be said that in water-limiting system the most important trait of plants is to react quickly. Fast responses allow them to use available soil moisture before it is lost, enabling higher growth and better long-term biomass compared to slow-responding vegetation.
However, these findings must be interpreted with caution, as the model is highly simplified and additional important factors are not included. Nonetheless with this simplified model general mechanisms can be demonstrated and important factors and their influence on the system can be identified. This suggests that, in order to help a system work against desertification, it may be important to identify ways to increase the water infiltration rate.

5.4 Limitations and Outlook

As we have already mentioned in Section 2.2 System Assumptions, our implemented system has certain limitations, such as single water dependency or non-consideration of the rooting depth. Thus, it might be interesting to couple this analysis with a third dimension which might be the most limiting nutrient or the additional consideration of the rooting depth. Another potentially interesting further development of our implementation might be to model the impact of climate change on the soil water plant biomass model. This could, for example, be done by gradually by decreasing precipitation rates over a long period to try to stimulate a desertification process. Initial values might not be correct, weakness of our model implementation

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